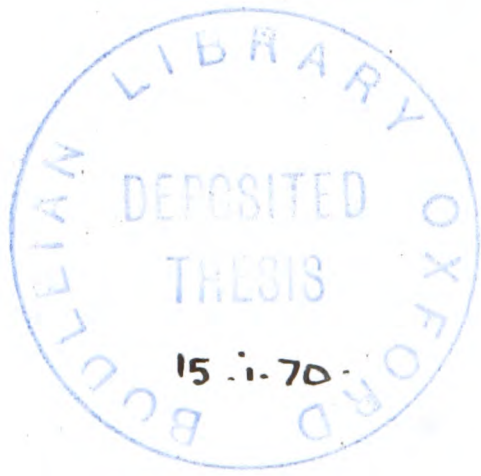
MODALITIES AND FORMAL SYSTEMS

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PART 3

CHAPTER VI

SAYING THAT SOMETHING IS NECESSARY

Introduction to Part 3

Chapters IV and V have dealt with general characteristics of "logics" and the sort of abstraction which is involved in devising a logic. We now turn to one specific case in which such considerations are relevant. What will be done in Part 3 is to examine the notion of necessity - taking it as being typical of "modal" notions; we shall outline a theory of what it is to say that something is necessary, and we shall examine the prospects for symbolizing those utterances which say that something is necessary, given that theory is correct. Thereafter we shall consider some special topics within the field of "necessity", showing in what measure, if any, these special cases lend themselves to symbolization in the form of a logic. Finally some broad conclusions will be stated; these being more to round off our discussion of some topics about necessity, then to close the subject of "modal logic" - for that is a philosophical problem which cannot adequately be covered in one such work as this.

Some attention is due here to the question of what a modality is, and whether we are right to take necessity as a paradigm case. The answer to the question of what modality is, will not be given; we can however say that it is something of the same sort as a claim that something is necessary.

Prior, in his book Formal Logic (680Q;215) cites the examples of Peter of Spain and Isaac Watts when discussing the lengths to which the notion of modality can be extended:

> "... in Peter of Spain 'mode' is used in a broad sense for any sort of qualification of anything, adjectival or adverbial: but more strictly for adverbial qualifications, and most strictly of all for qualifications of the manner of connexion between the subject and the predicate, which is what he takes the ordinary modes of necessity, etc., to be". (680Q;215) referring to Summulae I.28.
>
> "... "There are several other Modes of speaking whereby a Predicate is connected with a Subject: Such as, it is certain, it is doubtful, it is probable, it is improbable, it is agreed, it is granted, it is said by the ancients, it is written, etc., all which will form other Kinds of modal Propositions" ..." Op.Cit. quoting Watts : Logick II,ii,4.

Prior comments on these views that "We can hardly count all the 'modal' operators mentioned by these old and new writers as strictly belonging to the logical form of the propositions into which they enter, and so as constituting the subject-matter of a special branch of logic".

It will be clear that we, unlike Prior, cannot make the division between logical form and logic, which would allow us to echo what he seems to be saying in this passage, viz. that not all the expressions considered are such that sentences which typically use them have to be regarded as having a special logical form. Instead we may say that the sentences standardly employing these various phrasings are only in varying degrees of special interest, and that for many of them it is simply not worth while to seek to create a logic by

reference to which we could discern in them a logical form other than that which they would normally have relative to KC. (Thus as simple true-or-false assertions).

Historically the term "cum modo" - from which we derive the word "modal" - was introduced to characterise assertions in which certain things were said to be necessary, possible or probable. Thus we can always, as pedants, restrict ourselves to the apodeictic and problematic modalities, saying that these are the only "real" modalities. Yet on the other hand the term itself is clearly applicable to a far wider class of cases, and it is difficult to find any etymological reasons for stopping short of Watts' position. Our sympathy with Prior must be based, then on philosophic grounds of the type just sketched or on admitted pedantry.

We approach the matter from two sides. First we say that it seems that the subjects originally discussed as modalities, are in fact the most interesting of the subjects which could be discussed under that heading; secondly, we say that the choice of just one of those, necessity, as an example here is in order since we shall not base any of our arguments on matters which could be influenced by the selection of this or that type of discourse as being modal or not, nor shall we argue that the multiplicity or paucity of different types of talk which amount to saying that something is necessary gives reason for us to reject any particular approach to symbolizing necessity. The aim with which we start our investigations is that where we find a possibility of creating a "logic" for

a particular type of talk about necessity then we shall sketch this. Such a logic will perhaps admit of extension to cover other modalities and if so this will be a positive point. Where no logic seems appropriate, however, we shall argue that point on the basis of the characteristics of the type of talk itself and its usefulness to us as human beings trying to communicate.

As justification of our claim that necessity is one of the more interesting modalities to consider, we need only refer to the various writers who have tried to derive other modalities such as the deontic from it, and to the clear relationship between the notion of natural necessity and that of being in accordance with a law of nature. In discussing necessity we are perhaps not as fresh as if we were to discuss the modality "being said by the ancients that" or to essay to produce an imperative logic analogous to existing modal ones, but we are sure of covering ground whose importance is attested by the multiplicity of writers who have already covered it, and whose interest is shown by the discussions in Part 1 where we concluded that at least one school of thought had gone astray.

Arising from the discussions of Part 2 it is worth restating just what it is that we are seeking when we examine talk about things being necessary. It is stated there that the basis of a logic is a systematically related spectrum of valuations which can be applied to items of a given sort which we wish to symbolize plus

a feature of complexes of those valuations such that we can develop, by abstraction from the valuations applied in particular cases, an algebra for the generation of and conservation of that feature in complexes of given structure.

It may be helpful here to restate the same objective by means of an analogy which gives us more guidance on the nature of our "search". When we measure objects in the physical world, the results which we obtain are expressed in numbers of given units of measurement: the table is six feet by three feet, for example, or in a less familiar phrase the apple-tree on the left is three-hundred-and-twenty-four-appled while that on the right is now only fifteen-appled since we have shaken it. The scientific approach to the world, it is sometimes said, is that approach which attempts to reduce all phenomena to complexes of numerically quantifiable parameters. We may understand the parameters in a very wide sense in this case, as is shown by the apple-tree example, but it is clear that many such parameters can be defined and that they do have in common the feature of being numerically quantifiable.

To say that a parameter is numerically quantifiable, is however nothing but to say that objects (in the widest sense) may be ascribed numerical valuations with respect to those parameters. The numerical valuations can then be compared with the valuations discussed in Part 2, and we can postulate as an interesting feature of them the property of being "real" numerical valuations. The feature cited

may not seem so very interesting, since numbers generated from real numbers by means of the allowed basic arithmetical operations are always real; the definability of complex numbers subject to the same operations adds interest however, and one may find it a matter of interest in principle to discover the algebraic rules governing the arithmetic operations - thus seeing how '+' can be reduced to a successor operation applied repeatedly.

The use of this "scientific" approach to the world as our analogy, allows us to set ourselves the objective of approaching talk about necessity with the "logical" approach; this consists in the attempt to discern parameters of 'things said' ("propositions" or "statements" in the terminology of other writers) such that the values of these parameters (or the valuations which we apply to the things said) can be organised into a system, and such that properties of complexes of these values can be found which interest us in the way "tautologous" interested us for complexes of the values "true" and "false".

As well as the obvious example of "true", "false" and "tautologous", we should keep in mind the less obvious one drawn from our analogy, namely that of the real numbers and the property of "symbolizing a real number". C. L. Hamblin's article "One-valued logic" may also be cited as a reminder that the <u>structure</u> of valuation system can be of interest, without there being any interesting <u>theorems</u> to state - in his case there are no theorems at all. (385A).

Invocations of necessity

The most everyday occurrence of a reference to necessity in our speech is when, in trying to prove some conclusion, we use a claim which we introduce as being necessarily true. This is a situation in which necessity seems to play a role in our argument, and one which prima facie a "logic of necessity" should cover. It is thus a natural place to begin our enquiry whether a special type of logic is possible or necessary.

What we are here concerned with can be described as "invocations of necessity" and contrasted with "assertions of necessity". The latter, although often phrased with the word "necessarily" or the phrase "It is necessary that" are the sort of utterances with which one can agree or disagree, and which admit of being argued for or against on the basis of purported empirical fact; invocations of necessity, by contrast, are either accepted or not accepted when they occur in an argument, and are used to provide premisses for some conclusion never as conclusions themselves.

The second sentence of the following passage is an example of an invocation of necessity:

- This is a white bird. It is necessary for a raven to be black. Hence this is no raven.

The same form of words in other contexts may however be used as an assertion of necessity, e.g.

- It is necessary for a parrot to be black. The raven is species of parrot. Hence, it is necessary for a raven to be black.

In the first example the sentence simply has to be accepted or not; in the second it is the subject of a supposed proof. By saying that in the first example the sentence simply has to be accepted or not, we do not mean that it would be impossible to start an argument about whether ravens are necessarily black; what we mean is that the first argument is about what "this" is, and that the argument about whether ravens are necessarily black or not, is a sidetrack and should be kept distinct.

A further example of the invocation of a necessity in order to establish a conclusion would be the following conversation:

Speaker A : John has bought a luxury villa in Dahomey.

Speaker B : Nonsense, it doesn't have a bath.

Speaker A : So what?

Speaker B : In Dahomey it is necessary for a luxury villa to have a bath.

At the present point in the argument the choice which confronts the first speaker is between accepting what Speaker B has just said and thus letting his own first assertion fall, and starting a sidetrack argument designed to show that there is no reason to accept that luxury villas in Dahomey necessarily have baths. The case now to be considered is that in which the invoked necessity is not challenged and provides one of the claims necessary for the establishment of some conclusion. The interest of this lies in seeking to discover whether the fact that a premiss in an argument

is an invocation of necessity gives us any reason for symbolizing it other than by the same sort of symbolism we would use for a factual claim in the same argument.

A preliminary point which should be noted is that there is no need for any of the words normally associated with necessity, to occur explicitly in an invocation of necessity, and that for this reason it is often hard to tell whether an invocation of necessity is in fact being made. In the second of the above examples the last sentence may be replaced without changing the meaning, as follows:

Speaker B: In Dahomey all luxury villas have baths.

Similarly the following two examples contain invocations of necessity:

(1) Edinburgh is north of London. Dunbar is east of Edinburgh. Whatever is east of something north of a given thing, is north-east of that given thing.† Dunbar is north-east of London.

(2) Speaker A: Aphorism won the 3.30 race.

Speaker B: Oh, it looked to me as though he was second; was the other horse disqualified?

Speaker A: Oh no! You are right that he was second past the post; I had the impression that he was very slow for a winner.

Speaker B: But the winner is the horse which passes the post first without being disqualified.†

Speaker A: Oh, then Aphorism can't have won after all.

That the sentences marked " † " are invocations of necessity can be brought out in two ways. First, it is clear that prefixing "It is necessary that" or infixing "necessarily" in these sentences need not alter their meaning. Secondly, the upshot of any challenge to what is said in these sentences would be a discussion about how to describe things or define terms, and not a discussion about geography or horse-racing i.e. the subjects which are being discussed in the paragraphs concerned.

It is clear that a close relationship exists between the acceptability of an invocation of necessity and the truth of what we have called an "assertion of necessity" which deals with the necessity in question. The relationship between the two types of utterance will be explored in considerable detail presently; for the moment we shall attempt to clarify the conditions under which an invocation of necessity should be taken as acceptable.

The Acceptability of an Invocation of Necessity

Part 2 has presented logical symbolization as being relative to certain valuations and interests. In the propositional calculus ('KC') the truth or falsity of a sequence or combination of assertions can be represented as a function of the truth or falsity of each of the various assertions involved. In a logic based on invocations of necessity (references to such a putative logic are abbreviated to "LIN" - Logic of Invocations of Necessity) it is to be expected that we shall be able to represent the possession by a

sequence or combination of assertions of some interesting valuation as a function of the valuations assigned to each of the various assertions involved, and that some or all of these assertions will be invocations of necessity.

The original and still a normal purpose for the development of logics is to display the validity of arguments. Having disowned any specifically mathematical purposes, our hope in the present approach to a logic of necessity will be to find a symbolism which will better display the validity of arguments. It is to be supposed that "validity" will in this case be similar to - though not necessarily identical with - the concept of validity for the representation of which KC was developed and is used. From this supposition it follows that any part of an argument which is valid and whose validity can be shown in a KC symbolization, will have to be shown valid by any LIN symbolization of the whole argument. If this is to be the case, one may expect that the valuations applicable to the assertions making up the part of the argument will - even when we are considering the basis for LIN- have to be, or be very similar to, those valuations whose applicability forms the starting point for developing KC, viz. "true" and "false". The question thus arises whether these valuations alone would be sufficient for a <u>full</u> symbolization of contexts which involve invocations of necessity.

We have suggested that the basis for the development of KC is our ability to interrelate the truth or falsity of conjunctions or other combinations of assertions and the truth or falsity of the individual component assertions; the fact that truth or falsity of a combination of assertions was for us an extremely important type of valuation made it worthwhile to develop, in KC, a symbolism which represented combinations of assertions as being functions of the values which we could apply to individual assertions contained in the complex.

In the case of combinations of utterances which include one or more invocations of necessity, it is perhaps tendentious to specify the interesting valuations as "true" and "false"; to begin with we may choose the valuations "acceptable" and "unacceptable", on the understanding that what is true is acceptable and what is false is unacceptable. If we consider a sequence of assertions which as a whole is acceptable, it is clear that any invocations of necessity which occur in that sequence must also be acceptable, (the sequence is thus taken as being conjunctive; we must simply intuit the structure in practice, cf. pp. 213 & 255). We thus have a valuation whose applicability to an assertion can be inferred from the valuation of the whole context.

In practice most contexts which contain invocations of necessity also contain assertions about the facts - assertions which can be true or false. These utterances may also be assigned the

valuations "acceptable" and "unnacceptable", with the result that the whole context can be seen as made up of factors each of which is either acceptable or unacceptable, and the valuation of the whole can be represented as a function of the values of the factors, just as was the case for KC.

The net effect of such a change from a logic of utterances which are true or false, to one of utterances which are acceptable or unacceptable, can however readily be seen to be negligible. In any sequence in which only factual assertions occur and which can thus be symbolized in KC, the effect of substituting an invocation of necessity for a factual assertion of the same polarity (i.e. an acceptable for a true, etc.) is merely to create a need to alter the valuation which we have in mind (from true or false to acceptable or unacceptable) but not to alter the symbolization in the least. There is a logic which we may call 'KCa' which is symbolically identical with KC, but which to be interpreted as being based on acceptability-values instead of truth-values and may be applied to contexts where all assertions are either acceptable or unacceptable but not all are either true or false. This is of course merely to say that we may apply KC itself to these contexts, remembering as we do that we are working with a designated valuation which is more inclusive than that of "true".

If LIN is to provide a better symbolization for types of valid argument which are not shown as valid in the KC symbolization, then

it will have to depend on a range of valuations for assertions or parts thereof which though consistent with "acceptable" and "unnacceptable", are related to the valuations of utterance sequences in further <u>independent</u> ways. A pattern for this may be seen in the predicate calculus ('KCP'): in KCP we have a formalism by which we can symbolise the factors which contribute to the truth or falsity of single assertions; the truth of an assertion is for this purpose seen as a function of 'valuations' of the individuals and relations implicitly or explicitly mentioned in it - these valuations being in fact the concepts* of these individuals and relations, which concepts we "assign" to the constants of the symbolism in order to see the consequences for the whole assertions and complexes thereof. If we take KCP as a pattern for LIN we shall have to seek a means of representing the acceptability of an

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*"Concepts" seems the most suitable explanation of what things are assigned to the individual symbols of KCP when they are interpreted; Copi, in <u>Symbolic Logic</u>, by contrast, takes it that the things assigned are the typographical entities used for the symbols (since the symbols regarded as typographical entities are elements of a universe of individual things). It is however hard to feel happy with the use of the term "assign" in such a case. It is none the less important that we are doing something more than merely establishing an arbitrary mapping of the symbols in the formalism onto objects, for it is clear that any formalism no matter how defective can at least have its symbols mapped onto themselves (regarded as 'things') and it is quite probable that many formalisms which do possess interpretations in the sense that their symbols can all be mapped onto objects according to economical "semantic" rules, neither qualify as nor are intended as <u>logics</u> (the mapping concerned being, for instance, from one type of mathematical theory to another).

assertion (whether of fact or an invocation of necessity) as a function of some farther factors.

In particular cases we do not have difficulty in deciding whether an invocation of necessity is acceptable or not; nor is a fair measure of unanimity unusual - in many contexts it is as much an "objective" matter which invocations of necessity are acceptable as it is "objective" which factual claims are true, (opposing "objective" to "based on purely personal reactions"). For example the necessity that one thing is not simultaneously (stationary) in two different places, is one which can in most contexts be invoked acceptably (leaving relevance aside); the acceptability of this invocation is so general that those who dispute it run more risk of being regarded as deviant in intelligence or understanding than in viewpoint. The objectivity of the acceptability of this invocation most probably arises because the general understanding of the difficulties which would arise for our normal communication if this invocation were to be acceptable in one context and unacceptable in others, leads to the assumption that a convention establishing the matter is generally applicable.

This is about as far as the parallel can be drawn. There are, we usually take it, objective truths about the world; there are also, many feel, some objectively acceptable invocations of necessity. These latter may be less clumsily described by talking about necessities which can (objectively) be said to exist; those necessities

are acceptable which really exist. Claims about the facts are stated in terms which lead us to verify them by investigating things and relationships; the difficulty about invocations of necessity is that even though we may talk of them "existing" it is not clear where we should look in order to assure ourselves of their acceptability. What is so pleasant about objective truths is that whenever anyone does question them we can settle the matter by drawing his attention to things and relations; but when someone questions some invocation of necessity which we feel must be acceptable to anyone, we tend to be left saying rather weakly "Well that's just necessarily so. It couldn't be otherwise. You can only deny that by defining your terms abnormally". It seems somehow more obvious that there are norms and standards for the verifications of factual claims than that there are standards for the determination of whether a necessity which has been invoked "really exists".

The case of disagreement is not however the only one; there is also the case where two people agree that the invocation of a given necessity is acceptable. We may ask what basis they have for this agreement. Only if we can for ourselves discern what would be good reasons for the two people so to suppose themselves to agree, will we be in a position to produce the required further analysis of the acceptability in question.* We must thus examine

*It is, of course, possible that due to some misunderstanding the persons concerned only think that they agree. What we must judge is when they in fact do agree and have good reasons for their agreement.

the reasons why people find invocations of necessity acceptable, and we must take up our own normative standpoint in these reasons.

One straightforward obvious reason for accepting an invocation of necessity, is that it seeks to enforce an explicit agreement about the definition of terms. For example, in a discussion between inhabitants of the southern and northern hemispheres about summer-holiday activities it may have been explicitly agreed that "summer" was to be defined in meteorological terms as being the warmest season; anyone who later says "But summer is the warmest season" in an attempt to remind the others of the agreement, or to base some conclusion on that agreement, should be seen as invoking a necessity justifiably or "acceptably". (If on the other hand he uses the same words to make what he believes to be an informative statement he is of course wasting everyone's time since the existence of the necessity in question makes any such factual claim redundant).

An analogous situation occurs when the definition in question, although not explicitly agreed upon, is obviously the normal one to use in such a discussion. If someone in a philosophical discussion states baldly that empiricists believe in innate ideas and we, shocked, recall a normal enough definition of empiricism and on the basis thereof invoke the necessity of an empiricist's not believing in innate ideas as such, then our necessity invocation should be acceptable; if using a term like "empiricist" one should stick to the normal definition or else explicitly announce one's deviant

definition as a basis for the farther discussion.

In practice however, it may well turn out that an invocation of necessity which is aimed at enforcing retention of an assumed definition is in fact not accepted, a different definition is explicitly adopted, and the discussion proceeds with that definition understood. As we are laying down norms for acceptability we need not uncritically approve of such occurrences; if the definition which had been assumed at first, was in fact the reasonable one to assume then of course we shall not agree with the "unacceptable" valuation which the group has ascribed to the invocation of necessity. On the other hand there may have been no one definition which could reasonably have been assumed at the outset. In such a case the person making an invocation of necessity to enforce his personal definition may well feel every bit as justified as in the previous cases, but we shall be less liable to call his invocation "acceptable".* Yet on the other hand if no other definition had had a better title to being assumed as a basis of the discussion, then it seems harsh to dismiss this invocation as unacceptable.

However it is here necessary to reflect on what it is that we are supposed to be attempting to symbolize. The situation in which a disagreement arises and is settled is not one which we normally try to symbolize as such; what we normally symbolize are positions, deductions, and assertions - such as one of the parties to an argument might bring forward. Were we to throw all positions

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*the term is here used to mean "should be accepted" not "could be accepted".

expressed in the course of an argument into the same context, then by the very nature of a disagreement we should obtain a contradiction which would give us no particular basis for finding any one conclusion more validly deduced than any other. It is only when we look at individual viewpoints that we can reasonably expect to find valid deductions or a sensible subject for logical symbolization. When presented with a disagreement, then, our procedure could be either the symbolization separately of the different positions put forward, or the symbolization of the agreement finally reached (if any) plus the deduction which the parties to that agreement believe justifies it.

When in the course of a discussion someone fails to gain acceptance for an invocation of necessity and a disagreement arises, we have two choices. We may set about symbolizing the point of view of the proponent of this invocation - ignoring the disagreement on the part of his audience and following his argumentation through to its conclusion. Or we may wait - perhaps in vain - until the disagreement is resolved, and take the resulting agreed necessity as being subject to our judgement whether its invocation is acceptable in an argumentation which contains only the agreed points from the foregoing argument.

Returning to the person who invokes a necessity trying to enforce a definition which is not obviously the most appropriate to assume in that context, we may say that his invocation is not,

simply on its own, acceptable; the harshness we felt about calling it "unacceptable" springs from the fact that no-one else would have been any more justified in making an invocation based on a different definition, but since in considering him alone we are not considering what would have been reasonable in the social context, but rather what would have been reasonable in the argumentational context, we may say that his invocation is unacceptable because it refers back to no explicitly proposed definition while in this context only an explicitly proposed definition could form an adequate basis for the given invocation.

The position of an invocation of necessity which is incorporated into the agreed standpoint of a group after some disagreement has issued in a definition which that invocation enforces, is simple enough - the invocation is acceptable since the standpoint includes explicit adoption of the definition in question. Should the group incorporate - for example by accepting the argument of one of its members - in its standpoint an invocation which in that context would have to be based on an explicitly agreed definition, then of course the invocation is acceptable only if that definition is explicit in the standpoint.

The Grounds of Invocations of Necessity

Definitions are however not the only bases for invocations of necessity. Other types of explicit or contextually implied prior agreements may also give rise to the acceptability of such invocations

Two examples will enable us to give an idea of the range involved.

It would be stretching things a bit too far to describe the law of non-contradiction as a definition. It is a general principle for the conduct of discussion and deals with the whole class of negating expressions. It gives rise however to copy-book examples of necessities which can be invoked - "It is necessary that a given thing is not both the case and not the case" etc. Still, in practice we treat such necessities just as we do those which arise from definitions.

Suppose that in the course of a discussion it becomes clear that one participant is insensitive to the impropriety of contradicting himself, and that in protest the other participants invoke some necessities based on the law of non-contradiction. If the culprit remains recalcitrant, refusing to accept the invocations of necessity but wishing to contine the discussion, then his position amounts to a refusal to use the whole gamut of negating expressions in the normal way; our only reaction if we are prepared to try to carry on with the discussion is to seek to discover what system he is using for these negating expressions. Discovery of, and agreement upon such a system is little different from the agreement upon definitions which was discussed above. Indeed one of the most natural ways to express our wishes in this case would be to ask the participant to explain or <u>define</u> the terms we regard as negating expressions.

The second example is even farther from talk of definitions; it is the case where a detail or consequence of some theory is cited as a basis for some conclusion, and is cited in such a way as to assume prior acceptance of that theory. "Particles must have mass." (with or without the "must") one might say, or "Above 100°C at normal atmospheric pressure water must evaporate.". Of these two examples the first might plausibly be represented as an enforcement of some definition of "particle" or "mass", and indeed it is most probable that the theory involved would incorporate such definitions, but the second example contains so many terms that it is difficult to see which one of them should have the definition which would make this claim acceptable; and one is more inclined to say that the claim is taken merely to follow from a whole complex theory.

It is important to keep invocations apart from predictions or factual claims. As in the case of definitions the quoting of a single sentence out of context often leaves undertermined the question of whether a supposedly factual claim is intended or whether the intention is to invoke the necessity of the claim in question. Particularly the sentence about the evaporation of water, or a sentence such as "If fission occurs in an atom of uranium, free electrons must be produced.", will in different contexts serve equally well as a prediction and as an invocation. In the case of the prediction the force of the "must" or "necessarily" is that all

other alternatives must be discounted*, and the claim may be said to be about the phenomena concerned; the case we are interested in is one where the questioning of the claim amounts to questioning the relevance or validity of a particular theory in the present context.

Less scientific discussions also incorporate invocations based on theory, or theory-like background assumptions which are assumed to be agreed in the discussion. There too, one encounters difficulties in deciding whether a claim is intended merely to recall the theory or a consequence of it, or to make some factual prediction; "You can't do that without breaking the law." or "You must use caster sugar if you're to make real Madeira cake." are cases where the claims made may on the one hand recall the law and the authentic receipe, or on the other hand merely give gloomy prognostications of the outcome of a contemplated course of action. Only in the former case is a necessity invoked, and then it is the status of the law or recipe concerned relative to the discussion, which will give us our basis for deciding if the necessity invoked was acceptable.

The various bases upon which we may found an invocation of necessity may be called "the grounds" of invocations of necessity. These grounds have in common that some theory, doctrine, assumption, or definition has been explicitly or implicitly agreed as a basis for the position being expounded. Grounds may range from simple definitions to complex theories and from the practically unchallengeable laws of logic to purely ad hoc agreements such as that in a

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*We discuss the notion of alternatives in the second part of Chapter VIII.

given discussion whales will be regarded as fish. Their importance lies in that they absolve one from any need to give further evidence for claims involved in or deducible from them, and the invocation of necessity is a claim which represents something as having earned exemption from the normal requirements of substantiation in view of the purported existence of such grounds.

The Relativity of Acceptability

The exploration of circumstances in which invocations of necessity are to be found acceptable has brought us no nearer a LIN which is an extension of KC on lines similar to KCP. All that has now been achieved is to show more clearly how we should set about deciding whether such an assertion is acceptable; we have not been able to display the assertion itself as being a complex of parts which separately have relevance to the factors determining the acceptability of the assertion.

What we have managed to show is that the basis for deciding that an invocation of necessity is acceptable does not lie in our judgement of the interrelations among elements whose identity and purported interrelationship is vouchsafed us by virtue of our understanding of the invocation as it is formulated; there is here only one element, viz. the theory, definition, etc. which forms the real or chimerical grounding for the invocation; and the one and only relationship is that of this grounding's being adequate in the context in which the invocation is made. If the grounding to which our understanding of

the invocation leads us, does in our opinion have the necessary degree of adequacy in the context concerned, then and just then shall we say that the invocation is acceptable.

This analysis of the notion of "acceptability" as applying to invocations of necessity, has as a direct consequence the impossibility of the task we have set ourselves. A logic must be general in the sense that the deductions whose validity it displays will be valid in all contexts; farther it must rest its validity concept upon valuations whose assignability is determinable independent of context. It has just been pointed out that the assignment of the valuations suitable for invocations of necessity and proposed as those upon which the validity concept for LIN would be based is crucially dependent upon context.

Only if "context" were in this case somehow ambiguous, would there be an escape from this difficulty. It does not seem however that there is such an ambiguity, although the term is somewhat imprecise. The desideratum for a logic is that its applicability be general - viz. that its tenets should apply no matter what it is that is being talked about in the discourse to which it is applied. Yet in the case of the "acceptability" of an invocation of necessity, it is precisely "what it is that is being talked about" which is of crucial importance. The case is to be distinguished from that of claims such as "It is now raining" which appear (or may be said to appear) to be true in one context and false in another - such 'truth-value changes' are not dependent upon the subject matter of a

discourse but upon the speaker's position in the world; the analogy would be proper if for instance "It is now raining" had different truth values when put into discourses about sport and about the weather, at the same time and place.

Another ground for doubting whether the independence of context claimed for KC is really so essential might be seen in the obvious difficulty of symbolizing a claim such as "The lion is fierce." without knowing whether the context is one of a discussion about the lion as a species or a discussion of one particular lion. It is in fact only by understanding the context enough to make this decision that we are able to decide how the claim should be symbolized in the predicate calculus.

But this still does not provide a counterexample for the rule that the value assignments in a logic must be independent of context. All it shows is that individual sentences need not always be fully understandable without some knowledge of their context (or "point" in other cases); once they are understood however, all the necessary information is at hand for us to discern the factors that ought to influence our judgement of the proper truth-value to assign, and the further step of abstracting from all but those factors which influence the truth assignment, to give a logical form for the claim, is a step which is in no way influenced by the context or "what is being talked about". The case which would correspond to this as regards invocations of necessity, would be not the standard sort of case but

rather one in which only knowledge of the context could help us determine which of two possible groundings for a given invocation was in fact the one which the speaker had in mind (e.g. the bald invocation "Improving the roads must penalise pedestrianism." scarcely in itself shows whether it is meant to be based on a theory of the effects of diverting expenditure from pathways to roads, or a theory of the encouragement of road travel being a disincentive to walking; the context might help us decide which grounding was implicit, but this influence of context is quite apart from that which is involved when we have to determine whether the grounding in question is appropriate to the context and the invocation acceptable as a move in the argumentation).

The primary conclusion of this discussion is then, that "acceptability" while being on the one hand the valuation of invocations of necessity most suitable for leading us to a useful logic of them, is nonetheless not a possible basis for any "logic" in the normal sense of a system, independent of context, showing the validity of valid deductions.

Let us now return to the "assertions of necessity". Further definition of this class of utterances has been promised and is now due. Our argument here is not that the invocations of necessity and the assertions of necessity between them include all utterances which can reasonably be characterised as "saying it to be necessary that ..." something is the case. We shall indeed in Chapter VII discuss

another type of sentence which falls under such a characterisation, and which seems to be a candidate for separate consideration. The point of the present discussion is to establish that both assertions of necessity and invocations of necessity are bona fide and distinct types of utterance. The bona fides of the invocations of necessity have been established, and we must now establish those of the assertions, and must show that the two are distinct.

An assertion of necessity is, at the most general level, an utterance which says something to be necessary by talking about necessity rather than by talking about the thing which we are to take to be necessary. The sort of talk about necessity involved is, further, purely descriptive and hence susceptible of valuation as either true or false (but not both).

Let us suppose first of all that the reader cannot bring himself to believe that such talk exists at all. Our best means of answering him is with an example:

"In Newtonian physics it is necessary that bodies have mass." The utterance quoted is clearly not primarily about bodies or mass. It is our view that it is an assertion of necessity and to be seen as about necessity.

Yet one may also doubt whether it is primarily about necessity; one would perhaps feel more inclined to say that it was about Newtonian physics. This amendment we readily accept, pointing out that the complete omission of the words "it is necessary that" would

leave an utterance which can well bear the same meaning as the quoted one. In accepting this however we do not abandon our claim that the sentence is about necessity.

The utterance is in fact about the necessities of Newtonian physics. It is analogous to utterances such as:

"In Newtonian physics it is not assumed that bodies have size." This in turn is about Newtonian physics, but in particular about the things not assumed in Newtonian physics.

In the first example given we could omit the words "In Newtonian physics", and again be left with a plausible utterance. Now were the remaining words to be uttered in the course of a discussion about the necessities of Newtonian physics, then the meaning and force of the abbreviated form would coincide with those of the original example. Here, then, although there was no reference to Newtonian physics we would be talking about necessities of Newtonian physics, and just as previously the objection could be made that in this context the utterance rendered as "It is necessary that bodies have mass." was not primarily about necessity, but rather is primarily about Newtonian physics. Similarly in the same context "It is assumed that bodies have size" would be about Newtonian physics rather than about assumptions, - at least primarily.

In a specialized context such as that of discussions of the basis of Newtonian physics, its assumptions and necessities, we shall be content to say that an assertion of necessity is an utterance

which is both about necessity and about the subject matter of discussion (taking that in this case as being Newtonian physics). One could quibble about the complication of deciding whether the subject-matter was simply Newtonian physics or was necessities of Newtonian physics - but if it were the former then the explanation given will serve, and if the latter then we say simply that assertions of necessity say something about the necessities of a given field. The examples make clear that the utterance concerned is the sort which could display the speaker's knowledge or ignorance about Newtonian physics, and not the sort which tries to induce us to adopt Newtonian physics or to alter the bases for that which we call Newtonian physics.

The case is less straightforward when no specialised context is obvious. "It is necessary that bodies have mass." can be uttered in other contexts too. We are sure that not all these contexts are ones in which an alleged agreement or prior definition is being invoked; sometimes the speaker is making an ordinary assertion which he and his hearers regard as being either true or false, and as showing his ignorance or knowledge. But what is he ignorant of, if he is ignorant, or knowledgeable about, if he is knowledgeable? We propose to say that he is talking about necessities, if he uses such a phrase to mean something either true or false in a context where no other subject-matter (such as Newtonian physics in our example) is obviously under discussion. We take no position on the

question of whether there are any cases where absolutely no other subject-matter apart from necessity "sec" can be found; sufficient that the idea of such as case be brought forward. Our talk of assertions of necessity as being about necessity is to be read therefore as meaning "in most cases about the necessities of some subject-matter but at least conceivably about necessities of all subject-matters - that is of no particular ones."

The stock example used by many philosophers may well be regarded as an assertion of necessity:

'It is necessary that bachelors are male.

This utterance, regarded divorced from any context, seems to be one which is intended rather as a true/false statement (and in practice a true one) than as some attempt at persuading us to stick to some rule for the use of words or some theory whose technical terms are involved. The fact that it is seen out of context presents us with the seeming dilemma that it must either be about necessity or be about bachelors and malehood; since it quite plainly tells us nothing new about bachelors or malehood, and so long as we know the meanings of words cannot tell us anything new, we are led to feel that it is about necessity. It is the existence of a bona fide class of utterances such as this which we are accepting when we talk of assertions of necessity; under this description fall both the assertions which from the context can be seen to have a subject-matter more specific than "necessity", and those which have no such specific

subject-matter and are just seen as about necessity but still intended to be true or false.

Supposing that the bona fides of assertions of necessity as a subject for serious discussion have now been established, it remains for us to show that they are distinct from the other types of utterance discussed, and hence we turn now to the distinction between them and invocations of necessity.

The word-sequence chosen to express an invocation of necessity need not be different from that chosen to express an assertion of necessity. "It is necessary that time be measurable." is a form of words which in one case might be used to invoke a theory-based or agreement-based necessity, and in another to report on what is or is not necessary (in particular as a means of reporting on the necessities in the discussion where the same words were used as an invocation, it would be peculiarly appropriate). It is natural to ask "Is the assertion of necessity not merely a repetition of the invocation, but a repetition which lacks the persuasive intentions of the original?"

Our answer to this question is that the assertion may well be a repetition of the same words as the invocation had used, and the fact that the same words are used is clearly not pure chance, but the assertion is not a "mere" repetition in any sense which would give a basis for questioning our taking assertion and invocation as "distinct" cases each meriting separate philosophical consideration.

To argue from identity of wording to identity of significance (from the point of view of the seeker for a new logic) is, as we have seen, unattractive since it must meet the immediate objection that a sentence such as 'The lion is fierce.' may bear two logically different significances in its different uses.

A more subtle approach would be to argue that there are not two distinct significances which can be put upon the same set of words here, but just one significance for the set of words, and two different ways of regarding* what is being said. Thus what is said is in both cases the same, but in the case of what has been called an "invocation" of necessity the hearer takes up the attitude that he is being called upon to accept as binding for his own usage a rule being stated by the speaker; in the case of the assertion the speaker states a rule just as before but the hearer takes this statement as an attempt at conveying information rather than as a persuasive utterance. However our groundwork in Chapters IV and V allows us to despatch any such argument rapidly. Clearly the supposed difference in the attitude of the hearer is sufficient to induce him to apply to the words in question quite different and exclusive sets of valuations according to which attitude he takes. For our purposes as logicians it ceases to matter whether there is

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*We employ the term "regarding" to avoid committing ourselves at the beginning of this argument to the view that two logically distinct approaches are involved - such as we would appear to do if we used the term "understanding" which would more normally express our meaning.

some further basic unity amongst the set of words to which he takes these attitudes - we are interested in the valuations which proceed from these attitudes in combination with the form of words concerned. The case is merely analgous to that of different ways of referring to the same thing, and we should not be discouraged from examining the logics of proper names and of descriptions as (at least initially) separate objects of interest merely because someone points out to us that whatever can be named can be described and every description can be adapted to serve as a name. If the possible underlying identity of the bearer of a name with that described by a description does not hinder our exploring the logic of the two types of speech, then the postulated underlying identity of the significance or meaning of what is said in a necessity-invocation and in a necessity-assertion should not deter us from examining these two types of utterance as separate objects of logical interest.

Another reaction to the proposed distinction is that although there are some utterances whose normative force is so pronounced that we would no longer wish to regard them as either true or false, still the sentences which are here referred to as assertions of necessity cannot be a distinct group because although apparently true or false they have at least normative over-tones. Here the whole basis of the distinction we have made is being called in question; it is not denied that the invocations of necessity are normative and the assertions of necessity are true or false - but it is denied that these

two categorisations of utterances are mutually exclusive. In answering this objection we in effect begin our substantive discussion of assertions of necessity.

Suppose we adjudge someone's assertion 'It is necessary that oestropods are marine creatures.' as true. By doing so openly in the course of a discussion we appear to commit ourselves to accepting all future assertions to the effect that oestropods are marine creatures and all logical consequences of these, without any request for further (e.g. empirical) justification. Had we however openly accepted an assertion such as 'It is necessary in relativity physics that time is measurable.' then we would appear to have made a commitment to accepting that time is measurable only if the current discussion accepted relativity physics as unchallengeable, (i.e. purported to accept it). The contrast between these two cases seems to call in question our classification of both the forms of speech as being 'assertions of necessity', and the fact that in both cases a certain commitment to accept that which is said to be necessary follows upon acceptance of the quoted utterances is in turn a challenge to the view that assertions and invocations of necessity can be kept as mutually exclusive characterisations of utterances.

The view that straightforward assertions such as "It is necessary that oestropods are marine creatures" can be classified along with those utterances which refer to things being "necessary in ..." or being necessary relative to some context or frame of reference, can

none the less still be maintained. We say merely that the unqualified form is unqualified because it is intended to be generally relevant - that is to apply in all contexts, including the present one. It then becomes possible to see the problem of what commitments we accept when accepting an assertion of necessity as being strictly analogous in the two cases: the general assertion of necessity is treated just like the relativised one when that latter occurs in a context relative to which the necessity is asserted to hold.

That acceptance of an assertion of necessity does commit one to something need not be in doubt; the question is whether what one is committed to is in any way connected with or overlaps that which one accepts when accepting a similarly phrased invocation of necessity. Thus when we accept the necessity of oestropods being marine creatures (that is adjudge an assertion to this effect to be true) are we accepting just the same as or at least part of what we should be accepting were we to accept an invocation of the necessity of oestropods being marine creatures?

In the case of assertions of necessity which make explicit reference to contexts where a stated subject matter is dealt with, there is little problem. That is to say our acceptance of the assertion phrased as "In relativity physics time is necessarily measurable." involves our undertaking a certain commitment, but that commitment is about what is or is not "necessary in relativity

physics". So long as the context in which we encounter the assertion does not pretend to be a discussion whose presuppositions are just those of necessity physics, then we are not by our acceptance of the assertion even committed to accepting the assertion that time is always measurable.

Sometimes, of course, an assertion of necessity occurs in a context such as that to which it itself refers: thus the assertion discussed above might in fact be made during a discussion which was clearly devoted to a consideration of time from a relativity theory standpoint. In such cases it can be difficult to distinguish an assertion of necessity from an invocation, but we typify the assertion by saying that it pretends only to be true or false and is uttered (normally) in an expository tone of voice as opposed to the plaintive or slightly aggrieved tone of voice which is normal in the case of invocations of necessity. Since it pretends only to be true or false, this utterance is not open to rejoinders such as "One cannot just assume that to be true." or "That is confusing the issue"; the only appropriate objection to what is said would be "That is not so." accompanied by some talk about the doctrines of relativity physics. Clearly such talk about the doctrines of relativity physics is not to the point in a discussion which we have described as being <u>about time</u> from a relativity theory standpoint - the standpoint is assumed and should not itself be explored. None the less the odd sentence of exposition of the presuppositions of a discussion

is in practice frequently to be found, and it behoves us to explain the apparent commitment which it can carry.

By accepting an assertion of necessity about what is necessary in relativity physics we commit ourselves only to the view that that thing is indeed necessary in relativity physics: however if we happen to be engaged in a discussion which takes relativity physics as a starting point, then we shall be doing something very odd if, after accepting the assertion of necessity, we proceed to contest an invocation of the necessity of that very thing which was <u>asserted</u> to be necessary. This oddness arises because the truth of its being necessary in relativity physics that a certain thing is the case implies that in a discussion starting from that theory the invocation of that things being necessarily the case is justifiable. The implication which causes this to be odd is not however one which is to be discovered in the discussion based on the theory, nor is it possible to make the inference from "It is necessary that ...." to "It is odd for me to deny that ..." The implication is quite straightforwardly "It is odd for someone who has accepted that a given thing is necessary in a particular theory, to proceed to object that in that theory the given thing is not necessary."

The statement just given leaves open exactly how the objection referred to is expressed. One way of expressing it would be to make another assertion - i.e. a true or false statement - saying that it was also <u>not</u> necessary in the theory concerned that ... was

the case. A second way would be to accept the relevance of the theory to a discussion and yet there deny that thing's being necessarily the case. This last is the situation we are considering, and the oddity emerges here as an oddity of behaviour. A close analogy exists between this case and such odd remarks as the advice "Follow my example in never giving advice." We behave in fact as though we did not believe what we ourselves have said, and this is odd because we committed ourselves to its being the case by saying so, and thus our behaviour seems to indicate that what we had said was not in good faith.

Here we have attempted to explain informally what is odd about such a situation. More formally, we may say that it is clear that no explicit contradiction can arise in the situation described. We have characterised the assertion of necessity as only admitting of rejection on factual grounds, e.g. by saying "It is not so, the theory says that ....", yet our previous discussion of invocations of necessity has indicated that a challenge based on facts can never succeed against an invocation of necessity since it is in effect a proposal on how a discussion should be conducted or a theory should be stated. Conversely the objection which can be brought against the invocation of necessity is one which indicates the unacceptability of a proposed way of speaking, and since its unacceptability is never evidence that a thing is not the case, we may safely say that the rejection of an invocation of necessity is not formally speaking a

contradiction of the assertion of necessity, even though uttered by the same person in the following breath.

A further means of differentiating the assertions and the invocations of necessity is to consider the extent to which they are dependent upon their context for acceptability. This is of course closely related to the difference in valuations, since as we have asserted of the invocations the backing for the valuation consists in a consideration of the context, while on the other hand it is a commonplace of logical theory that the valuations "true" and "false" have a unique application to one and the same utterance irrespective of the context in which it is considered.

Our approach in the case of assertions of necessity can best be shown by an excursion into the problem of sentences whose truth values vary from context to context. Although we have just said that no utterances can acquire different truth-values simply by being set in different contexts, such a claim made about sentences would not be universally acceptable. The most frequently employed counterexamples are sentences like "It is raining" or "The 'Times' has come." which when uttered on different occasions evince sometimes reactions of agreement and sometimes reactions of disagreement - from precisely the same, otherwise sane and consistent, hearers.

We take a sentence to be a unit of spoken or written discourse. It is a physical unit in the sense that it has a beginning and an end and contains a definite collection of parts (words) in a prescribed order. It is not however "physical" in the sense that the

occurrence of the same sentence in today's newspaper as occurred in yesterday's press release is evidence that the sentence has moved from one place and time to another. It is a necessary condition for being a sentence that the thing concerned is a discernable structure which may be observed in various physical phenomena.* Sentences are used by people to say things (normally). What a person says by means of a sentence is not necessarily always the same even though he chooses the same sentence to say it. Equally, he may say the same thing by means of different sentences. We use the term "utterance" for the content of what a given person says on some particular occasion by means of a sentence (sometimes also for the act of 'uttering' the sentence). Hence we may say that a single sentence may be used on different occasions to make different utterances, while the same utterance may be made by means of different sentences on the same or different occasions. The term "utterance" is used here in a sense little different from that in which many writers have used the term "proposition"; our preference for "utterance" is based on the fact that "proposition" has not always been used in this sense, and on the experience that it is less easy to forget the connection between an "utterance" and the people

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*We accept Quine's elaboration of this sort of description (69OT ;195). The structure may in practice not happen to occur; some sentences are in fact never formulated but we are able to talk of them as sentences only insofar as we recognise that there is a structure of the sort described which could occur and be the observable token of that sentence.

involved in communication, than to do so when talking of "proposition"s. This last point is perhaps only due to the absence of a use of the verb"propose" and a noun "propositioner" analogous to the verb "utter" and the noun "utterer", but it is none the less much less tempting to seek to find some one-to-one relationship between sentences and utterances than to seek one between sentences and propositions - forgetting the use aspect.

When we agree with what someone says we are apt to say that what was said is "true". Many writers - among them Quine - talk as if it were the sentence by means of which the person concerned said what he said, which is characterised as true. This is the root of the problem of inconstant truth-values, for the one sentence can be used to make various utterances and these may, quite independently of one another, evoke our agreement or disagreement. Yet it is perfectly clear that what a person says at a given time and place should evoke in us at each time and place only one reaction, and that that reaction can be enshrined in one of the valuations "true" and "false" without any risk whatever that it will later turn out that we should not then and there have reacted to that utterance in the way described. This, taken with the fact that we should in normal informal logical consistency be bound to react to any other sentence used to make the same utterance, by giving just the same valuation, leads us to conclude that the truth-valuation is better described as being assigned to an utterance rather than to the sentence by which that utterance happens to be made.

With this, however not everything has yet been said about the valuation of sentences. It can fairly be objected that in order to form a judgement about a sentence written on a piece of paper, we need not always know the context or the author or any of the factors normally relevant to forming judgements about sentences such as "It is raining.". The sentence as such, it sometimes appears, has a meaning and we can on the basis thereof decide whether the sentence is true or false.

Our rejoinder to this objection is straightforward. A given utterance can be formulated in many ways - each a different sentence. According to the occasion we choose a sentence for making the utterance we wish to. There are certain "occasions" or contexts, such as writing on a loose sheet of paper which someone else is intended to find 'by chance', where we cannot assume anything about the attitude or state of mind of the person with whom we are to communicate except that he knows or can decipher the language concerned. In such a case we can still choose a sentence which will allow us to make the utterance concerned. Sentences which are suitable for use in such cases are those which most clearly have an unambiguous meaning of "their own" by reference to which we might decide that they were true or false as sentences. But of course the ability to formulate sentences which are minimally dependent upon context for the communication of the utterances we wish to make, has a corrollary; viz. the person who hears or reads such a sentence must be able to

understand the utterance which was intended without having to rely upon knowledge of the authorship or context of the particular use of the sentence which he encounters. This is to say that the sentence is suitable for making a given utterance unambiguously without aid from the context. This is the reason that we can assign a truth value when we encounter such a sentence. We should not say that there are some sentences which have fixed truth-values and others which do not; we should rather say that there are some sentences which are highly ambiguous - or can be used to make a wide variety of utterances - and some which are minimally so, and that since every utterance can have only one truth value there are some sentences which are not suitable for making utterances of any but a predictable truth-value. We shall not discuss this matter in more detail, but it casts some light on what in our system plays the same sort of role as the "eternal sentence" in Quine's (69CT;193ff).

Bearing in mind the danger of confusing evaluation of utterances with evaluation of sentences which are merely the normal vehicle of particular utterances, we may return to the influence of context on assertions and invocations of necessity. We have maintained that a consideration of the context is essential to the assignment of a valuation to the utterance made by an invocation of necessity; we shall now maintain that a consideration of the context is not necessary for the assignment of a valuation to an assertion of necessity.

We must however explain that this does not mean that the sentence by means of which we express an assertion of necessity can a always be understood without a consideration of its context. Such sentences will, like those just considered, be in varying degrees "all-purpose", and only a very few will be so specialised as to be useful for expressing only one assertion of necessity. Accordingly our argument that an assertion of necessity can be assigned a valuation independently of context is not to be read as implying that a particular sentence such as "It is necessary that oestropods are marine creatures." can be assigned a valuation before we have considered its context. It is not the sentence to which the valuation must be assigned, but the utterance which is made by that sentence. It is particularly common in the case of assertions of necessity that the context plays a large role in how we interpret a given sentence. We must decide therefore whether the talk of oestropods is to be understood as relating to necessity for some particular contexts, or for all contexts, or for the present context in particular. Only once we know that, shall we have understood the utterance that is being made, and only then can the question of assigning a valuation arise. The case is however, then simple. It can be that the utterance refers to contexts which happen to include the present one, but this presents no particular problems as long as another context may be devised in which an utterance synonymous to the one under consideration occurs, but which is not referred to by anything said

in that utterance. In such cases we may be sure that the valuation assignable to the original utterance is just that which we should assign to the postulated synonym, and that valuation can be determined without reference to the context in which the sentence used to make it occurs.

One case however, remains. Some sentences appear to make assertions of necessity which are of such generality that every sentence must fall in a context to which the assertion refers. Thus "It is necessary that bachelors are male." when this is taken as a report about necessities and one which is merely true, is used to make an utterance which can hardly be described as stating that the masculinity of bachelors was a necessity of any particular field or science. We should much more probably say that the sentence was normally used to say that this was necessary in every context and science. But if that is so, then willy nilly every context in which the sentence could occur is referred to in the utterance which is made by means of that sentence.

In such a case we have something more than a mere dependence of the meaning upon the context in which a sentence occurs; we have a genuine need to take this context (among others) into consideration in order to reach our conclusion about whether the utterance made is true or false. There are two obvious ways of avoiding this difficulty; first, we could argue that this type of dependence on context is significantly different from that which we discerned in

the case of invocations of necessity, and secondly, we could argue that utterances of the sort characterised are a philosophers' fiction and neither occur in ordinary speech nor would have any use there. While we feel that both approaches have their attractions, it is most economical to confine ourselves to the former.

The reason why the assertions of necessity which do not refer to particular fields of discussion or types of background, seem to require a consideration of their own context as a preliminary to the assignment to them of a valuation, is that they lack the restrictions inherent in other assertions of necessity. When we say that something is necessary in relativity physics, then the person who wishes to pronounce on the truth of our assertion need consider no fields other than relativity physics in order to reach his conclusion. That is to say, the specification of the type of discourse for which the necessity is alleged to hold imposes a restriction on the search we need to conduct in order to verify the assertion. When no such specification is made then the search is unrestricted and must include all possible contexts and types of discourse.

This lack of restrictions as to where we should seek evidence for the truth or falsity of what is said, is shared with those utterances normally dealt with by KC. Thus the straightforward assertion "Oestropods are marine creatures." is itself one which admits of confirmation or disconfirmation in an unrestricted range of experiences. There is nothing which we as sentient beings can

experience or witness which is in principle ruled out as evidence for or against oestropods being marine creatures. In particular the sentence used to make the assertion concerned can in principle itself form evidence for or against the assertion, and so can the context in which the assertion was made.

Principle is of course not practice. We shall not be on the lookout for evidence about oestropods when we are doing something which we regard as abstract algebra. In practice, we have sufficient self-awareness and insight into our own categorisations to know that activity which we think of as abstract algebra is activity which will yield no experiences that we should be prepared to accept as evidence for assertions about oestropods being or not being marine creatures. This is however a practical question; it is based upon reflection about how the world is - reflection which yields conclusions just as disconfirmable as that oestropods are marine creatures; this reflection can equally be described as "semantical" insofar as it presents a viewpoint where we consider the world as one whose categories and properties are known and examine the terminology which we 'happen' to use in order to represent that world to ourselves and others. That is, we have at some stage reached a conclusion which can be represented as (a) that abstract algebra and marine (or all) biology are so different that any apparent evidence about biological questions which we encounter is either based on some misunderstanding or else not gained from

genuine study of abstract algebra, or (b) any evidence about biology which is to be gained from the world in the course of our study of abstract algebra is misclassified if it is taken as abstract algebra rather than biological observation. In an analogous way the categories to which we assign various experiences and pursuits help us to determine which evidence has to be taken into account when deciding whether a given assertion is true, and which experience (or evidence) need not be considered at all.

Just as there are many assertions whose truth or falsity can - in this practical way - be determined without any consideration of language or its uses, there are also a certain number of assertions which are implicitly or explicitly about language, or which at least leave open the possibility that evidence gained from the study of language and its uses might help to determine the truth-value which ought to be assigned to them. The fact that an investigation of language may be relevant here does not separate these assertions from the others in principle, because we say that all assertions which are true or false can in principle be confirmed or disconfirmed by observations of any aspect of the world and language and its uses are very clearly aspects of the world which we can observe with the necessary objectivity. What we can accept is that there are some assertions which in practice can be assigned their truth-value on the basis of a consideration of language, even though in principle any observable phenomenon in the world might be evidence for their

truth or falsity. An obvious example of such an assertion would be "'Bachelor' is a common word."; in verifying this assertion we look first to the sentences used in a given language and not to the world "at large".

Let us now return to our problem of the assertions of necessity. There were those assertions which dealt with what was necessary in given specific contexts or discussions, and there were those which lacked any such restriction on their applicability. Of the first sort there were those which dealt with contexts or discussions other than that in which the assertion itself happened to occur; contrasted with these were the assertions which seemed to deal with a context just like that in which they themselves were stated. We argued that the place where a thing is stated does not affect <u>what it is</u> that is stated (though it perhaps influences how we say it, and how we are understood) and that the two types of assertion could be treated alike because it would always be possible to find a way of saying just the same as was said in the second case but saying it in a context to which the (paraphrasing) assertion itself did not apply. Our remaining problem was that the assertions which are not focussed on one sort of context or discussion appear to deal with all possible contexts, and hence it is in principle not possible to paraphrase such an assertion in a context to which it has no applicability.

Our rejoinder to this problem is that this situation is not a genuine difficulty. The reference of a given assertion to itself or to its own context makes us look in a particular place for a confirmation or disconfirmation of what is said, and hence it could have been possible to confuse the way in which we were thus directed to the context of an assertion of necessity with the way in which we are directed to the context of an invocation of necessity when seeking a valuation. But in the case of the generally-stated assertions of necessity there is no reason to look at that particular context rather than others; it is a case where 'things said' is the field in which we expect to find the evidence, but not one in which the actual context of the assertion itself is in any way specially relevant.

The case is totally different when we consider invocations of necessity. It is the particular context in which a given invocation is made which is the one and only relevant basis for our making a well-founded valuation. If in talking about relativity physics one invokes the necessity of time's being measurable, then no amount of evidence that time's being measurable follows from the presuppositions of other discussions or from the axioms of other theories can bear any relation to the acceptability of the invocation. Yet the assertion of the same necessity in the same context is not peculiarly bound to the context in this way. One may well doubt whether it is to be understood in the fully general sense which we postulated to

be possible, or in some sense which restricts the claim to certain contexts and discussions, but what is clear is that where the claim is restricted to certain contexts and discussions it will be these that we must examine and it is irrelevant whether the assertion itself occurs in one of them, and if there is no particular context referred to or intended then we shall be at liberty to seek confirmation anywhere at all, whether it is in the context in which the assertion itself was uttered or in any other; recapitulating, one might look to any phenomenon whatever for evidence but we trust our classifications enough to look only to those things which we classify as linguistic.

Apodeictic Deduction I

The relationship of assertions of necessity to invocations of necessity is probably the most interesting of all problems about the 'logic' of necessity. It revolves around two principles. The first is that if something may be truly said to be necessary then one is entitled to invoke the necessity of that thing's being so. The second is an analogue of Hume's objection that one cannot deduce an "ought" from an "is", viz. it remains to be shown how one can deduce a normative utterance such as an invocation of necessity from a purported factual one such as an assertion of necessity; we might phrase this as the problem of deriving "acceptable" from "accepted".

We shall not contest the first principle, as it is stated above. Instead we suggest that one's entitlement to invoke the necessity is

not a logically guaranteed entitlement, but is constituted merely by the fact that one would in the circumstances described not be being self-consistent were one to entertain the idea that one was not safe to invoke the necessity concerned.

What we claim is that when a given thing is truly alleged to be necessary - in terms of an assertion of necessity - then we are entitled to invoke the necessity of that thing. For any particular assertion of necessity it will be up to us to make up our minds if it is indeed true; supposing we think it is, what are we accepting? According to the discussion above we are accepting that a particular group (explicit or implicit) accepts for a particular context (again, explicit or implicit) the invocation of the necessity referred to. Where one talks here of assertions of necessity in general then the group and context concerned can be taken to be so extensive as to be no real restriction; the person judging whether the assertion of necessity is true, and his next utterances, will at any rate be included.

Clearly if one judges that an assertion of necessity of this general form is true, and then hesitates to invoke the necessity concerned, one is hesitating to do something which according to one's own belief can cause no objection. If one objects to someone else's invocation of that necessity then, by providing an instance of non-acceptance of the invocation of the necessity in question, one actually demonstrates in practice that what one believed true is not

not true. This situation occurs in all contexts where one participates oneself in a discussion of the group referred to, in the context referred to.

The matter is closely bound up with reference by the assertion of necessity to behaviour such as the making, accepting or rejecting of the invocation of necessity by the same person who believes in the truth of the assertion of necessity. No great problem arises when the assertion of necessity is a restricted one such as that made by saying that it is necessary for taxonomists that oestropods are marine creatures; were we to believe this to be true, then there would still be no compulsion for us to accept invocations of the necessity of oestropods being marine creatures unless we regarded ourselves as taxonomists. The fact of our regarding ourselves as taxonomists or not, is not a matter which can be allowed to influence the logical relations between pairs of our utterances. The compulsion which we would feel to accept the invocation in question, should be pointed out to us that we were* taxonomists, would, further, be a compulsion based on the need to do nothing which would cast doubt on our previous statement.

In a related way the situation of a third party who believes some assertion to be true, and yet by his actions provides evidence that it is not, helps to show the relationship between assertions and invocations of necessity. We may take this case in even

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*More plausibly, it might be suggested that we ought to be talking as taxonomists

greater generality, also building an analogy to the case under discussion.

Smith believes that the first word of his next utterance will be one of ten letters. He none the less uses as the first word of his next utterance a word of less than ten letters, and his belief is thereby shown to have been false. In this case there is barely a temptation to say that his utterance contradicted his belief in a logical sense. Even were he to have uttered the belief categorically, the two utterances would not be logically contradictory.

Again, suppose that it has been said that Smith's next utterance will be a true sentence, and suppose that Smith himself believes this. None the less his next sentence turns out to be a false one. Accordingly what was said about that sentence was false and Smith's belief was wrong. But there is again no question of there being a relationship of logical contradictoriness between the sentence uttered by Smith and either Smith's belief or the assertion about his utterance.

The case is yet clearer when the belief that Smith's next utterance will be a true sentence is held, not by Smith, but by someone else. A statement by that other person of his belief is in no way contradictory to Smith's actual next utterance. Nor is it possible for that statement merely by referring to, and saying something about it to imply Smith's utterance. That utterance which Smith in practice makes will have a content of its own, and depending upon what that content is it will be possible to decide if there are any logical relations between Smith's utterance and any other statements; it is certain

that no logical relation is guaranteed by the very fact of that utterance being the one referred to in another utterance.

The case which we have to consider is not that of an assertion about a particular easily identified utterance which Smith is going to make at a specified time. It is an assertion about the disposition of a group of which Smith is a member, to reiterate or at least accept a certain utterance. Thus things which members of the group say will definitely be relevant, but are not directly referred to. It is again evident here that the assertion believed true (the assertion of necessity, here) neither logically implies nor logically conflicts with the utterances which we have to consider (the invocations of necessity and the acceptances, rejections or reiterations of them), but is instead verified by reference to these utterances - such verification taking into due account the meaning of what is said, since the meaning is in this case crucial.

To sum up briefly, we hold that an assertion of necessity can only be true if certain invocations of necessity are acceptable; this follows from the meaning of the assertion of necessity, and a consideration of acceptability of the relevant invocations of necessity is crucial to any decision on whether the assertion is true. The utterances which form the means by which the invocation of necessity is shown to be acceptable or not are in this case evidence, and acceptance or non-acceptance which they convey is

neither implied nor contradicted* by the assertion of necessity which is being verified.

An explanation leading to the conclusion outlined above is shown to be necessary by the Hume-type objection to any supposition that a deduction process is involved. It is hard to come farther than merely restating that objection more fully: no evidence seems available to indicate that we may reasonably infer from the observation that it is generally, or even universally, accepted that a given thing is necessarily so, to the conclusion that it is acceptable to say that that thing is necessarily so. Nor is the fact that we ourselves believe it to be accepted by various people - even by us ourselves - a sufficient ground for concluding that an invocation of the necessity in question would be acceptable. What has been pointed out above should show that no difficulty is created by the invalidity of the attempt to deduce an invocation of necessity (or its acceptability) from an assertion of necessity; the relationship between the two is kept unproblematical so long as we recognise that we ourselves may be implicitly referred to in the assertion of necessity, and that our own behaviour can therefore belie our words, thus creating an apparent compulsion upon us to accept invocations which we have said we would accept. Where we do not accept what we have said we would, the penalty is not self-contradiction but simply

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*"Logically" contradicted; it is of course denied by the assertion that the invocation is unacceptable.

falsity of what we alleged (though in a situation which may lead others to a radical mistrust of our future utterances, since we are expected to know what we do and do not accept).

Apodeictic Deduction II

That which is necessary is undeniably the case. This is a statement of what appears to be the principle behind the much-vaunted deducibility of an unprefixed symbol from a symbol prefixed by the strong modality, in many modal logics. The existence of such a deducibility is demanded by Lukasiewicz as a condition for calling a system a "basic modal logic" (538K) and almost all writers take it as a basic feature for their systems. The principle, stated as above, is innocuous; but we shall argue that it does not justify the use frequently made of it, and in particular that there is no case for including a theorem of the 'CΓpp'* type in a modal logic.

Discussion of this point is apt to be complicated by confusion of invocations of necessity with assertions of necessity, and, where the distinction between them is made, belief in the deducibility of the invocations from corresponding assertions - as just discussed. It should be reiterated then that the reasoning which we take to be behind acceptance of 'CLpp' and analogous formulae as theorems is one which admits that when a certain thing has correctly been claimed to be necessary then all those deductions which would have followed

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*Łukasiewicz's formulation.

from the truth of that thing are in the given context also deducible. Thus in a discussion about oestropods one might quite reasonably deduce that these creatures are cold blooded, on the basis of the fact that marine creatures are cold-blooded and the allegation (accepted at least in the discussion concerned) that it was necessary that oestropods were marine creatures.

In a sense this type of "apodeictic deduction" is the objective envisaged by the paradigm for talk of necessities. It would in principle be possible to confine ourselves to discussion of a sort of attribute "necessity" which could be possessed in varying degrees and possession of which was guaranteed in the conclusions of valid arguments whose premisses also possessed it, etc; but such a closed system is of little interest compared to the study of that use of the notion of necessity which allows us to influence the factual* conclusions reached in a given situation by means of a consideration of what is and is not necessary. Unless there is some form of deducibility from necessities to facts it seems impossible that consideration of what is necessary can even absolve us from empirical testing of every imaginable absurd idea.

Regarding this case as putatively one of "deducibility" we start from a reasonably clear idea of what the conclusion of the deduction step is. When the necessity is properly allged to hold, then just those extra things are deducible in the context which would

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*We mean of course "phrased as the statement of a fact".

have been deducible had someone truly stated the corresponding factual claim. That is to say that from something's being necessary we have been able to make just those deductions which could have been made from that thing's being the case, (we return later to the definition of just what the "something" which can at one time be necessary and at another factual, can be). Although the factual claim is not always explicitly stated we may represent the case without loss of any possible validity of deduction as one in which the factual claim itself is deduced from the corresponding necessity. In doing this we follow the implicit doctrine of those who wish to represent the "deduction" in symbols by "CLpp", for the use of the 'p' both in the 'Lp' and the final 'p' for the formula is presumably to be taken as indicating that the "same proposition" is in one case said to be necessary and in the other deduced.

The premiss of the "deduction" seems clearly intended to be an invocation of necessity. A certain suspicion may remain that it could be an assertion of necessity, but it will be observed that the cases where this is at all plausible are precisely those cases where the suspicion of a deducibility relation between invocation and assertion of necessity is strongest. That is, when the assertion of necessity refers to a context such as that in which it occurs, or when it is so absolute and general that it appears to be relevant

to all contexts including ipso facto the present one. Even for those who do not accept the arguments put forward against any idea that invocations of necessity are deducible from assertions, it should therefore be acceptable if we take the premiss involved as an invocation, since they will feel able to derive this, our premiss, from their own premiss in cases where that is an assertion of necessity. There remains only the possibility of holding that certain assertions of necessity do imply the corresponding assertions of fact but that the implication relation from the assertion of necessity to the corresponding invocation of necessity or from that to the assertion of fact does not hold.

This last view would be held, for instance, by someone who thought that the truth of its being necessary is relativity physics that time is measurable implied the truth of time's being measurable, but that time's being measurable could not be deduced from an accepted invocation of the necessity of this, - perhaps because nothing at all can stand in a relation of deducibility to an invocation, or else because the invocation is not automatically acceptable just because the assertion of necessity is true. It is hard to find arguments against such a view, but perhaps harder still to find proponents of it. We are in sympathy with the view that an invocation cannot be premiss or conclusion in a "deduction" of the sort which normal logics are suitable for symbolizing, but it seems to us

to go far too far if one then seeks to save the rule that what is necessary is true, by postulating a direct deducibility of the fact from the assertion of its being necessary. Such a manoeuvre can only work if there is one definite final list of the true assertions of necessity (which are true in all contexts at all times in such a way that contradictory assertions of necessity are never held to be simultaneously true) or if the truth of assertions varies from one occasion in which they are used to another. Neither of these alternatives seems acceptable and we shall now show how the relationship between invocations of necessity and statements of fact can be clarified so as to leave the relationship between necessity and fact unproblematic, but without requiring acceptance of one of these alternatives, or of the theoremhood of 'CLpp' in logics which we might wish to apply to arguments in which a necessity is used to help reach a factual conclusion.

Our argument here is closely analogous to that used to explain the connection between assertions and invocations of necessity. We assert that where an invocation of necessity is acceptable the making of any statement which contradicts, implies, or is implied by the statement of that which is invoked as necessary, does not amount to saying anything which is open to empirical checking. The person who asserts as fact something which has been agreed to be necessary is clearly wasting his time; he need only have referred to the agreement in order to save himself any commitment to the production of evidence

for those who wish to challenge him. He is wasting his time, however, in an interesting way, for no challenge to the "factualness" of what he has claimed can be admitted in the same argument. Any such challenge would constitute evidence that the invocation of the necessity in question was not acceptable in the context concerned, and hence the addition of such a challenge to a context in which by supposition the invocation is acceptable, must be ruled out. And if the denial of a given utterance cannot according to the description* of that context be a part of a given context, then one can hardly say of the utterance itself that it is laid open to empirical challenge when it is stated in such a context. The person who actually makes the statement concerned may well believe that he is laying himself open to challenges on empirical grounds, but if we are correct in saying that his utterance is made in a context where the matter concerned may acceptably be invoked as necessary, then no such

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*The 'description' spoken of is given by us when we talk of this as a discussion in which the necessity involved was (acceptably) invoked; the occurrence of a challenge to the invocation breaks the continuity of such a discussion and presents us with the choice of either (a) excluding such challenges (and denials of things whose necessity has been invoked) thus preserving the integrity of the discussion as one where the cited invocation is acceptable, or (b) allowing our claim that the invocation is acceptable to lapse. Which of these courses we adopt will depend on whether the challenge which occurs convinces us that we have misjudged the basic assumptions of the discussion or whether we feel that the challenge itself is founded on a misunderstanding of, or an attempt to change, these assumptions.

challenge on empirical grounds is conceivable. In short, if it is true that the invocation concerned is acceptable then there can be no question of the corresponding "factual" claim being challenged in that context.

What has just been argued for, is something very different from the type of conclusion which would be required to support the acceptance of 'CLpp' as a theorem in a logical system applicable to such a context. The argument for this latter view would have to establish that whatever the values assigned to the invocation of a necessity and to the corresponding statement of "fact", the claim that there is (some sort of) implication from the invocation to the statement has to be an acceptable claim. To produce evidence for such a conclusion we have to contemplate cases where in one and the same context a certain thing is acceptably invoked to be necessary yet truly stated not to be the case. In our view it is impossible to contemplate such contexts because the description given is contradictory; the question of whether particular implications hold in such contexts does not, therefore, arise.

Our conclusion is not however a denial of the existence of any relationship between invocations of necessity and the corresponding "factual" claims. We fully accept that there is a relationship, but deny that it is a logical relationship between the two utterances concerned. The acceptance of an invocation of necessity is an activity of a particular language-using individual; the

acceptability of the invocation arises when we as objective observers take a certain normative standpoint to that activity. We may also take up a standpoint to certain utterances - we say that these say things which are 'true'. The situation which we have to consider when relating invocations to the corresponding factual assertions is one in which we have a standpoint about (i) the facts, and (ii) what every reasonable language-using third party ought to do. What this third party ought to do is, of course, to assent to certain statements about the facts, and not entertain other statements; and one such third party is, of course, usually identified with oneself. The reason why there is a link between the invocation and the statement of "fact", is that the invocation is acceptable if and only if every reasonable person in the context concerned ought to assent to the statement of "fact" without any further evidence and ought to refuse to entertain anything contradictory to the statement of "fact" in question. So long as we are concerned to show ourselves to be "reasonable" we shall just have to display the appropriate linquistic behaviour in every case where we have pronounced an invocation of necessity acceptable.

It bears reiterating here that as in the case of the connection between assertions of necessity and invocations of necessity, failure to act "reasonably" is not similar to the rejection of logical laws or to inability to understand the meanings of words; it is a case of making self-defeating utterances which leave the hearer in doubt

as to what one is trying to say. Insofar as someone makes a 'true-or-false' statement about what is necessary (an assertion of necessity) then his own acceptance or rejection of "factual" claims that what was said to be necessary in the case in the context(s) concerned will constitute evidence* for or against his assertion of necessity; insofar as his utterance is normative - i.e. proposing or reiterating some convention or theory which has convention or theory which has consequences for what can and cannot be said - (an invocation of necessity) then his own acceptance or rejection of the corresponding "factual" claims demonstrates his willingness to abide by the convention he has proposed.

Symbolizing the Invocation of a Necessity

To conclude this chapter let us examine how best one can symbolize the standard cases of what we have been discussing - viz. of purportedly coherent and consistent deductions leading to the sort of conclusions normally encountered in deductions symbolized by KC formulae, but including in addition to the type of assertions normally symbolized by KC formulae one or more invocations of necessity and claims apparently deduced from these.

We have already considered the relationship between assertions and invocation of necessity, and here we shall confine ourselves to invocations where these occur as the "black sheep" of the arguments

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*It will in this case be evidence of a more indirect nature than in the example of the assertion of necessity and the invocation of necessity. Now we shall have direct evidence that the person is not abiding by the convention which the invocation of necessity seeks to establish, and hence (indirect) evidence that the convention is not abided by and thus does not exist.

(as we shall call them) to be symbolized. In Part 2 we have learnt that it is of primary importance in choosing our means of symbolizing an argument is the valuation which is of most interest (taking into account the ease with which we can discern a "system" in the possession of that valuation by the elements of the argument, and the relevance the valuation has to any property of the argument as a whole, such as validity). Here the most interesting valuation seems no other than that which yielded KC, viz. that of truth or falsity, and we shall hope to find an analogue of validity as that concept applies in KC symbolizations.

Invocations of necessity cannot however be simply classified as true or false. One solution which has been considered briefly is that of extending the notions of "true" and "false" so that they included respectively "acceptable" and "not acceptable" as these valuations apply to invocations of necessity. Such a move allows us to apply KC itself directly to an argument of the type under consideration. It has however the great disadvantage that an acceptable invocation of a given necessity is not symbolically distinct from some claim which is merely true; certainly we may say that the conclusion of a valid argument form is "acceptable" if the premisses are, but we cannot see which of the premisses purport to be empirical and which are invocations of necessities which have (perhaps) been agreed upon purely ad hoc.

Greater progress can be made by posing the question "Why should one invoke a necessity rather than make the corresponding factual claim?" The answer to this; it seems to us, is that one does this because one is either unable to, or unwilling to defend the factual claim with empirical evidence. One wishes by means of the gambit of invoking a necessity to put the matter said to be necessary into a class separate from that of the 'mere' factual claims, and in particular one wishes to put it into a class where it is no longer vulnerable to any attack based on empirical evidence. This fact makes it clear why the lumping of "true" and "acceptable" together is so unattractive.

The solution to our problem is found if we can find a device whereby certain of the formulae used by KC to symbolize an argument can be given a privileged status such that nothing contradictory to them can be part of the same (valid) argument and that what they assert to be the case can at any stage be reiterated. Such a solution is not hard to find. The additional postulates added to the axioms of KC in order to derive some conclusion by the method of conditional proof play just such a role. We have, of course, as good logicians been thoroughly conditioned always to "discharge" the assumptions made in this way and to take as the conclusion of such a deduction the implication from the assumptions to the derived formula. It is however evident that if we omit this last step and take as the conclusion the derived formula on its own, then the

conclusion reached is valid so long as the assumptions are true. The formula derived can thus be validly deduced in any context where the assumptions concerned may be made; and thus we are presented with a context-relative notion of validity, such as might have been expected if we consider that the acceptability of invocations of necessity is also a context-relative matter.

We may illustrate the matter with a practical example. In the following formulae we symbolize the invocation of necessity "Oestropods are marine creatures" by "P", the factual claim "It is not the case that while oestropods are marine creatures (is true), it is false that oestropods are cold-blooded" by "P⊃Q", and "Oestropods are cold-blooded" by "Q":

Postulate:	P	(necessity established)
1.	P⊃Q	
2.	P	from postulate (necessity invoked)
3.	Q	from 1.2 by modus ponens

Our contention is that the above sequence shows the appropriate manner to symbolize an argument in which from the assertion symbolized as "P⊃Q", the conclusion symbolized as "Q" is derived with the help of an invocation of the necessity of oestropods being marine creatures.

It is worth noting that this approach illuminates the question of how the context of a given invocation of necessity is to be characterised. We see that the list of the postulates or assumptions which we should use to symbolize the argument in question provides an exhaustive spécification of the logically significant characteristics of that context - that is, of the factors which affect what can be deduced in that context. To say this may not be of practical help, but it illuminates the position of simple KC symbolizations as being symbolizations of arguments where the context is taken to contain no bases for invocations of necessity - i.e. no assumptions. Hence, of course, simple KC symbolization is 'context-free'.

CHAPTER VII

NATURAL NECESSITY

It has been noted (p.326) that a problem arises when we ask ourselves just what it is that is in one case asserted to be necessary, in another case invoked as necessary, and in yet another stated to be the case. The distinction between sentences and utterances has already been made; but here there seems to be a need for yet another type of entity - to which various utterances can be related.

The case is not, however, one of finding a lowest common denominator for a group of utterances. It is not normally the case that an arbitrarily chosen group of utterances all either assert to be necessary, invoke as necessary, state as a fact, or have some other such relation to, some one thing. The matter is the other way round - for any "thing" of the sort in which we are interested it will be possible to construct a group of utterances with members which state it to be a fact, assert it to be necessary, etc. We wish therefore to clarify our notion of this type of "thing" in such a way that we can represent utterances in terms of (i) the 'thing' involved, and (ii) the relationship to that 'thing' (i.e. whether the thing is stated as necessary, as fact, etc.).

This is a brief sketch of what we have to look for if we are to provide the necessary underpinning for the fragmented account of "saying that (something) is necessary" which we gave in Chapter VI. In the course of our discussion of this topic we shall digress to

consider a third type of utterance which qualifies for the description "says that (something) is necessary"; after having examined this type of utterance - which we regard as the most important and interesting remaining type - we shall turn again to the problem of what it is that can be sometimes necessary and sometimes true.

Continua

Let us give an example of the problem: In the course of discussion John says that it is necessary that punishment is relatable to a specified deed, and remembering our early training in moral philosophy we assent to this and use the statement that punishment is relatable to a specified deed in order to argue to a given factual conclusion. It is agreed that we have at first accepted an invocation of necessity and then made an assertion of what that invocation claimed to be necessary; but how can it be known that what we asserted is in fact the same as what was claimed to be necessary? It is perfectly conceivable that the words used in the assertion bore no obvious relationship to words occurring in the invocation, and further, even had they done so this would not have established the point unless we knew, (i) that these words in the two contexts were being used to talk about X_1 and X_2 which were in fact one and the same thing, and (ii) the two sentences had been so phrased that while the first invoked the necessity of what was talked about in it, the second asserted (factually) what was talked about in it.

In the arguments of Chapter VI the contrasting natures of invocations of necessity and factual claims, not being crucial to our arguments, were taken as commonplaces requiring no special defence. When presented with the present problem, however, one is at once aware that much of its apparent intractability springs from our feeling that factual claims and invocations of necessity are such radically different types of utterance that there is little more for us to say than that we 'just know' that given pairs of them are "about the same thing". The question accordingly arises whether we are right to think that the two types of utterance are so radically different, and we are drawn to consider the attempts of philosophers such as Waismann and Quine (for different reasons, neither the same as ours here) to assimilate utterances which are used to state facts ('contingent' utterances) and those which claim things to be necessary (leaving aside whether these are intended to be invocations or assertions). These authors have not, of course, been concerned with the relationship between utterances of the different types which are about the same thing, but they both wish to establish that there is a "continuum" of statements ranging from the factual to the modal, and if this were so it could well have consequences for our problem.

The attractiveness of the doctrine of a continuum lies in its appearing to provide us with the opportunity to say that there is only one "thing said" and that this is in the one case invoked as

necessary and in the other asserted as fact. Such a view is not explicitly contained in either of the doctrines referred to but does not seem to be ruled out by what the writers say. Let us therefore discuss the actual doctrines more closely and examine the merits of the continuum theory as such.

The view put forward by Waismann (in Principles of Linguistic Philosophy) is that we should classify "nothing is red and green all over simultaneously" along with "1 + 1 = 2" as an example of a rule for the use of expressions in the language.* In the latter case we are concerned with a substitution rule, while in the former we see a rule permitting the inference from "... is red" and "... is green" to "... is not also simultaneously green" and "... is not also simultaneously red" respectively.

There is no doubt that a sentence such as "Nothing is red and green all over simultaneously." can be used to state a rule of the language - or recall a definition or invoke a necessity - however Waismann's discussion of how this can be so is what interests us. He suggests, in Chapter III, that there is a continuum between uses of sentences as rules (of philosophical "grammar" in his terminology) and uses of sentences to state facts. From this he argues that at least some case can be made for "synthetic a priori" proposition as being ones drawn from a particular part of the middle regions of

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*To the extent that Waismann sees an ordinary indicative sentence such as this as stating a rule, we count him as an ally in our argument that many "normal" indicative sentences far from being truth-functional are covert expressions of views about definitions - i.e. necessity-invocations.

this continuum; in the absence of other reasons for separating off this part of the continuum, however, he is inclined to think that we should simply persuade the a priorists to look on their pet examples as a sub-class of the "rules" which Waismann has just described.

The difficulty about adhering to this continuum view, can be seen plainly when we examine a sentence whose status is unclear:

(a) "Summer is warmer than winter."

Sentence (a) can be used either to state a contingent fact (if fact it be) or else to establish a rule about usage (in pursuance of which people in the northern hemisphere say that July is in the summer and December in the winter, while those in the southern hemisphere say the reverse). Not only is there a very substantial difference in the force the sentence acquires when used in these two ways, but it is also clear that as regards its making either an a priori, or an empirical claim the two uses must lie very far apart upon Waismann's continuum.

At this point there are two approaches; on the one hand we may consider the sentence or on the other we may consider what the sentence is in the two different cases used to say (which for this discussion we shall call the two "propositions" it can be used to express). Suppose our continuum is to be seen as consisting of sentences. A sentence which has uses both to state rules of philosophic grammar and to make empirically checkable statements, must lie on the part of the continuum where the borderline between

empirical and a priori comes. Yet the forces of this sentence in its two uses are so dissimilar that one feels one should put it on two different parts of the continuum simultaneously. To be more specific it seems that Waismann's own example "Nothing is both red and green all over simultaneously" should on the one hand be on the more a priori side of (a) because it is less concerned with empirical fact than (a) when interpreted as stating a fact, and yet should be on the empirical side of (a) when that sentence is intepreted as giving a rule for using the terms "summer" and "winter" - because Waismann's sentence deals with a less essential aspect of redness and greenness than the aspect of summer and winter with which (a) deals.

To put the matter on a more theoretical footing we can say that there is no clear means of seeing how one would judge whether one given sentence is farther or less far along the continuum in a given direction than is another sentence; there are sentences of which we readily say that they are unambiguously empirical and others equally unambiguously a priori, but the notion of a continuum requires that we should have some parameter such that we can say that the value that the parameter takes in the case of given things determines where these things should be placed upon the continuum; Waismann provides no such parameter, values of which might be determined for given sentences.

Nor is the matter easier if we take the philosophically more satisfying course of considering as separate elements the two

propositions which the sentence (a) can be used to express. The lack of a parameter is here all the more evident. What is there which is in measure x present in the empirical proposition and in measure y present in the a priori proposition, and which by the measures in which it is present in all other propositions can determine the place they should occupy in a continuum? The untenability of the continuum hypothesis shows when we ask for an example of a proposition which is one unit more a priori than the empirical proposition expressed by (a) - the unit may be arbitrary and either defined in terms of proportions of the total range or as an absolute. No means is suggested by which we might be able to check whether a candidate for this status was really qualified; indeed only the characteristic of being presented in a more or less norm-stating fashion seems to give much help at all. Let us leave Waismann and turn to Quine, who with a different universe for his continuum has at least apparently a parameter to allow us to determine position upon the continuum.

The Quinean continuum* is populated by "statements"; these we shall take as being mappable into the "propositions" spoken of above - it will remain open whether it is necessary to consider more than one "statement" in order to explain a single "proposition". The parameter which determines position on the continuum is variously described, but perhaps most definitively given as "the relative likelihood, in practice, of our choosing one statement rather than

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*We shall discuss the doctrine of "Two Dogmas of Empiricism" as being among the most uncluttered expositions of the viewpoint.

another for revision in the event of recalcitrant experience"; the first-line candidates for revision are the most empirical and Quine characterizes them as "especially germane to particular experiences".

Quine is not concerned here with the distinction between rule-giving and fact-stating uses of language such as interests Waismann. The fact that Quine has begun his piece by talking about necessary truth and definitions might lead one to expect otherwise, but it can be seen from his opening words in § 6 that he is talking about the normal use of language for reporting, or describing in a non-normative* fashion:

> "The totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the profoundest laws of atomic physics or even of pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges. ... A conflict with experience at the periphery occasions adjustments in the interior of the field. Truth values have to be redistributed over some of our statements."

Quine's most provocative doctrine is that the "logical laws" are simply further statements of this system and as such are statements which are also in principle open to revision and liable to have their truth-value changed in the course of some redistribution of truth-values following a particularly traumatic 'conflict with experience.'

If we were never confronted with statements other than those purporting to be factual, Quine's arguments would be highly convincing.

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*With a striving towards objectivity: it is an open question if this can be perfectly achieved.

We do of course work on the basis of theories as we go about the business of living; and according to our theories there are more and less important doctrines. When we are only concerned with communicating facts there will naturally be a universe of statements in which some instantiate doctrines far more important in our theories than others. We can then devise a continuum into which our statements can be fitted according to the importance to our personal body of theory which is possessed by the various generalizations these statements instantiate. This seems to us the basic truth in the Quinean doctrine: Quine himself prefers to talk of "relative likelihood" of our revising statements - this relative likelihood is for us the more publicly observable outward manifestation of the importance accorded to a general doctrine which the particular statement concerned is taken as instantiating (Quine skirts round the problem of explaining why the truth-value of a generalization should be revised in the face of recalcitrant experience which has led one to revise the truth-value accorded to a statement instantiating that generalization).

Further support - of a highly practical nature - for the view that a continuum, - or more accurately a system, can be devised to embrace the relative "entrenchedness" of the factual statements we make, is to be found in Nicholas Rescher's book Hypothetical Reasoning (711A) and we shall not waste time arguing that what is done there is possible. The problem with what Quine says is

whether it in fact gives any help to us in explaining the relationship between invocations of necessity and assertions of necessity and in particular how it is that we can apprehend as one and the same (the "necessity") that which is at the one time invoked and at the other time asserted.

Quine seems to us to be right and to have a convincing argument for the following points:

(i) That statements of what it is that logical laws, necessary truths, scientific theories, and observation reports say to be logically necessary, necessary, in accordance with the laws of nature, or simply true, respectively, are all susceptible of valuation within the "true"-"false" range of valuations, and as such can be symbolized simultaneously in KC and KCa.

(ii) That our readiness to alter our valuations of particular statements from these classes when we have experiences which we regard as inconsistent with what our body of knowledge and belief would lead us to expect, is a variable by reference to which we can arrange such statements along a continuum.

What Quine does not manage to explain or illuminate are:

(iii) What valuations are appropriate for logical laws, necessary truths, etc., when stated in a norm-giving fashion.

(iv) Whether the fact that if an individual is to be consistent he must assent to the statement of what it is that his own necessity-invocations have presented as necessary, is a basis for the

symbolization of a logical relationship between the invocation and the statement, (what we have discussed in Chapter VI under the heading Apodeictic Deduction II).

The principal point of introducing a discussion of Quine at this time is to show that his sort of continuum theory both differs from one which puts invocations and assertions* on the same continuum and provides a theory of the relative strength of different assertions which is in fact helpful to those who wish to make a split between invocations and assertions. That the Quienean theory is adequate we leave to Quine to defend further; that it deals exclusively with assertions of necessity, insofar as it deals with necessity-claims, has been argued above; and that the existence of such a satisfactory explanation of the readiness with which we change our assessment of some assertions and reluctance with which we change that of others is helpful to those who wish to make a clear dichotomy between invocation and assertion of necessity should be obvious from the fact that there is no longer a need to see the reason why a given assertion is more sacrosanct than others in some 'invocation'-like status which it has over and above or besides its "true"-or-"false" status.

No matter what arguments are put forward to the effect that any old assertion can be backed up by a corresponding invocation which would make the assertion necessarily true, there will always remain

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*The reference to "assertions" here and in the rest of this section is to assertions of fact not to assertions of necessity.

the highly persuasive counter-gambit of citing some very sacrosanct assertion and rhetorically asking how a randomly chosen assertion could be given this degree of respectability simply by the enunciation of some formula known as an "invocation of necessity"; it is only those assertions which are backed up by real necessities - to which, the argument might run, there are no conceivable alternatives - that have this great degree of sanctity. These are the assertions of which one can say "It is necessary that p." and in doing so make a statement which is at once classifiable under the categories "true" and "false" and, simultaneously, cannot be conceived to be false (or in stronger versions of the doctrine "is also necessary itself").

To meet this type of gambit it is important to be able to give an account of why certain assertions are more readily given up than others. Quine's theory gives the necessary account, explaining this in terms of the interrelatedness of our beliefs about the world and the structuring of these beliefs into theory. When we recognise the importance of theories it becomes easy to see why randomly chosen assertions do not easily acquire the same persuasiveness as necessary truths, which basic assertions of theories current in our culture do - even though we may explicitly invoke the necessity of what is said in the randomly chosen assertion. We are thus able to hold the view that invocations are quite distinct from assertions, and yet preserve the view that some assertions are bound to occupy a privileged place as relatively less open to revision in the face of new experience,

and as such of course be much easier to defend when proposed as necessary truths for given discussions.

Scientific musts

It is at this point helpful to indulge in the promised digression from the subject of what it is that is necessary, and to examine the notions of necessity met with in scientific contexts. This topic is clearly of importance; it is appropriate to discuss it here because the discussion clarifies the relationship between the various things said both about necessity and about what is necessary, and because Quine's doctrines provide a suitable starting point for the discussion.

The topics to be discussed here are:

(i) The use of "must" by the scientist who, reasoning as he experiments, expresses a conclusion by saying "That must be how it happens!".

(ii) The question of whether subjunctive conditionals should be regarded as a sub-class of modal assertions, and if so what the equivalent assertions of a more obviously modal nature are.

(iii) The relationship between the modality in sentences such as: "What goes up must come down" and "If butter is heated to 90°C then it must melt"
and the modality in the types of utterance we discuss elsewhere.

When experimenting, the good scientist is not merely making observations; he is indulging in intelligent observation. He tends

to think more in terms of what "must" happen than in terms of what does actually happen. For, were he only to think in terms of what does happen he would run the risk of being misled by purely fortuitous regularities in the phenomena he is observing. The scientist, in short, is seeking to sort out the lawlike - scientifically important - generalities from the purely fortuitous ones. But here we must explain why it is so appropriate to express this in terms of words like "must" which are typical of modal utterances.

It is salutary to notice that the same scientist who, when in the laboratory discovering a given phenomenon says "That's how it must happen", says, when he is writing about the phenomenon in a journal later, that the thing <u>does</u> happen in such-and-such a fashion. Why should be make this shift? The reason seems to us to lie in the presuppositions present in the two situations.

Let us suppose that there is a phenomenon X which requires further scientific explanation. The scientist at his work bench begins by reproducing the phenomenon a few times with various attendant circumstances. He casts about for analogies upon the basis of which he might be able to construct a hypothesis to the effect that alteration of certain variables in the situation would have a predictable effect on the reproduction of the phenomenon. At some stage he gets an idea which he feels helps to explain just what is happening in the problematic situation, and he says that this <u>must</u> be what

happens. What he means to say in fact is that he has considered and rejected all alternative mechanisms by which he might explain the production of this phenomenon - there remains only one possibility which must be the real explanation.

The presuppositions inherent in the experimental situation may be summed up as including (i) the scientific background beliefs of the scientist (which he presumably shares with his colleagues) and (ii) evidence and definitions adequate to vouchsafe and delineate the occurence of the phenomenon under examination; it is also clear that in this situation the following are lacking, (iii) an established theory or concept of the mechanism by which the occurrence of the problematic phenomenon is to be explained, and (iv) a thoroughly exhaustive set of results of tests of any hypothesis which, to the mind of the scientist concerned, adequately explains the phenomenon.

By contrast, when the scientist is writing his conclusion up in a learned journal he sets them in the context of a theory, he fits the theory into the scientific background which he assumes his readers to have, and he assembles a set of results of tests which he believes gives exhaustive verification of his hypothesis. Now what would be the reaction if in the second context he were to write that the mechanism producing the phenomenon in question "must" be that set out in his theory? One would react by asking whether the fact that the theory as a consistent part of an edifice of scientific theory and as having been subjected to exhaustive tests, predicted such a mechanism

producing this phenomenon were 'not enough' for him to say without qualification that there was such a mechanism and that it did have the described effect. By putting in a "must" the scientist raises in our minds the questions "What forces one to this conclusion?" and "What is inadequate about the normal arguments tending to this effect, that they have to be bolstered up?"

In the experimental situation the postulated lack of a definite theory, and the need to work with an assortment of predictions on the basis of general scientific background and evidence incompletely supporting various hypotheses, forces the scientist to reason with himself to talk in terms of what "can" and "must" happen. Only in the case of detailed observation where there is no shadow of doubt about the interpretation of what he sees, is he able to say "does". His use of "must" is therefore to be distinguished from that whereby someone attempts to evade the necessity to prove what he is claiming: the scientist in this position says "must" because he sees no alternative* to the statement about the facts which he is making.

An explanation in terms of "seeing no alternative" does not as such get us much farther. We must still ask how it is that the scientist is able to rule out alternatives. Clearly some alternatives are ruled out because experiments he has done with an aim to testing the alternative have had a definite negative outcome. But no number of experimental results can assure him that there remains only one further possibility; in order to know this he must know

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*We return to this topic in The Modality of Uniqueness (Chapter VIII).

what the totality of possibilities is, and that is not itself given in the form of an experimental result.

In order to reach his conclusion that there is no alternative but for things to happen in a given way (and to reach this conclusion short of by actually observing how the things happen) the scientist must rely upon theory. It may be nothing more than the very general theoretic background of what we know as science - e.g. conservation laws, or eschewing of explanation in terms of psychic influences - but without a theoretical postulate he can never reach the conclusion of only one alternative, if only because he cannot ever rule out additional alternatives such as divine intervention and systematic hallucinations.

In this the scientist is no exception; no-one else can reach the conclusion that no alternative to a given effect exists unless he too has some means of enumerating the alternatives that have to be taken into account; nor is it merely trivial to describe such a means of enumerating alternatives a "theory". But we are considering the scientist because since the time of Hume and Kant there has existed the question of whether there is a particular type of relationship - sometimes called "necessary connection" - between cause and effect which it is the business of the scientist to discover and describe. With this background it is particularly important to investigate the use of modal-like words by scientists in order to guard ourselves against accusations of having ignored a particular

category of necessities, viz. those necessities which exist in nature ('natural necessities') and are the subject matter of science.

The explanation, then, of the muttered "must" in the laboratory, is that it arises in the process of weighing off theory against incomplete data, and is to be paraphrased as saying that there is no alternative to the conclusion entertained. The evidence for this is strengthened by the fact that in writing up exactly the same conclusion the same scientist, when addressing himself to an audience which shares his background, will tend to present a theory from which the conclusion may be <u>deduced</u> in the normal fashion and will speak further in terms of "is" and "does".

The weighing of data against theory which is spoken of here is only likely to lead to a conclusion phrased in terms of this type of "must" if the theoretical background which the scientist is regarding as beyond question is inadequate to yield an unequivocal prediction about the phenomenon being investigated; it is in this situation that the scientist is forced to a hypothesis on the basis of which he is able to rule out all but one of the possible outcomes. There are in fact two cases; the first is that in which a hypothesis is adopted in order to allow an unequivocal prediction of what will happen in a given case; the second is that in which the reproducibility or representativity of a given result or measurement is assumed (perhaps along with some theoretical hypotheses) in order to allow one to state a definite doctrine by means of which existing

theories can be extended to cover the phenomenon concerned. These cases correspond to the conclusion "That is what must happen.", and "That must be how the effect occurs." The two processes complement each other; one may begin by making an hypothesis about what will happen, then test the occurrence of the effect, and finally take the test results as being the reproducible representative evidence which forces one to a certain conclusion about the interrelation of phenomena. Equally well one may start from casual (or even imaginary) observations, assume these to be reproducible outcomes of crucial experiments, and then proceed to construct the only possible extension of existing theories (or as much of them as can be saved), with or without the aid of further assumptions; here again the newly extended theory can then be turned into the hypothesis and new experiments conducted to validate it.

In considering the significance of this type of expression for our conclusions about the most suitable means of symbolizing utterances which say things to be necessary, we may take it that this type of "must" typifies conclusions which are based upon assumptions which have not yet been well enough examined for them to rank as common-places (one might even say "facts") of science.

Natural Necessity and Counterfactuals

When the evidence for a scientific theory has been assembled and its assumptions substantiated to the satisfaction of the scientific community the theory acquires respectability and comes to be used as

a basis for making predictions about events. Where sufficient detail about the situation is known particular theories may allow us to make concrete predictions about what is going to happen; if we wish to state such predictions in a manner which commits us only to acceptance of the theory, and not of particular claims about the situation being dealt with, then we shall probably make the prediction by saying that if certain named circumstances pertain then the predicted effect will result, or certain other things dealt with in the theory will be the case. An example of such a prediction about effects might be:

> "If in a frictionless environment one moving billiard ball strikes a second stationary billiard ball, then that second ball will move."

Another:

> "If in normal kitchen conditions butter is heated to 150°F, it melts."

The type of utterances just characterised deal with what in other centuries were usually referred to as "causes and effects", a subject popularised and related to necessity by Hume* and Kant**. Hume in particular spends much time on the supposed 'necessary connexion' between cause and effect. Our treatment of necessity could not hope to be adequate if it gave no attention to this subject,

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*Treatise I.iii.2ff; Enquiry IV, Part II.

**Essay on the Introduction of Negative Magnitudes into Philosophy and other writings, especially Critique of Pure Reason where the 'category' of Causality is discussed.

and the possibility that the pursuit of scientific knowledge yields a special sort of 'thing said' which admits of being called 'necessary' in a sense of 'necessary' other than those we have so far discussed.

The term "natural necessity" seems an apt one for characterising any such special sort of thing. This is a term which is out of fashion at the moment and which is relatively little used in every-day speech even though most people would think they knew roughly what was meant by it. We shall take it to be applicable very generally to utterances such as those quoted, so long as the subject matter dealt with can be described as "natural" phenomena and what is said is, or is a particular application of, some law or principle which is part of the corpus of knowledge which we have accumulated about "nature".

That a given utterance states a natural necessity may be made explicit by prefixing to it the words "It is naturally necessary that ..."

Hence:

> "It is naturally necessary that if in normal kitchen conditions butter is heated to 150°F, it melts."

or "It is naturally necessary that what goes up comes down."

Clearly then, whether the mention of the fact is explicit or implicit, this sort of utterance can be classed along with the others as one by means of which something is said to be necessary. The principal question which arises here is whether we should assimilate this type of utterance to the assertions of necessity - which have been portrayed

as typically about linguistic behaviour, or to the invocations of necessity - which have been denied to be straightforwardly "true"-or-"false". One's preliminary temptation is to say that such utterances as we now have to deal with are definitely of the "true"-or-"false" variety, but clearly not about linguistic behaviour, and that though they are based on a background of theory they are not "mere" proposals about how one should argue in a particular case. If it turns out that they fit so badly into the dichotomy we have built up, then a third category will have to be created for them.

The paradigm case of a natural necessity has been depicted as being that of a particular prediction based on an accepted theory of science and stated in such a way that the detailed information necessary for applying the theory in question is presented in the antecedent of a hypothetical sentence. A more general type of utterance has also been introduced, viz. "What goes up must come down." Let us now examine how wide a range of utterances can be classed as stating natural necessities.

Since our paradigm is stated as a conditional, a natural candidate for consideration as a possible form of natural necessity claiming utterance is the "lawlike" statement. This predicate "lawlike" is applied to statements when we consider that the subjunctive conditional generated from them would be acceptable. (Cf. N. Goodman (362D) and many others) "Lawlike" and the related term "projectible" are used in the philosophy of science to typify

utterances not with respect to the way in which people do react to them, but with respect to the way in which people ought to react to them. The statement of something which is in fact a law of nature is therefore a lawlike statement whether or not anyone knows this, and conversely the fact that people accept certain subjunctive conditionals on the basis of generalizations which they believe to state laws of nature is not a sufficient reason for saying that these generalizations are in fact lawlike. We are concerned with a range of utterances which are "like" laws of nature in that they can yield acceptable subjunctive conditionals and not in that they are treated as if they were laws. The fact that - just as with the ascription of acceptability to invocations of necessity - we may in any and every case turn out to be wrong in our belief that a given utterance is lawlike, does not detract from the importance of showing the relationship of this concept to others and examining what follows from the supposition that a given utterance is lawlike.

The stock example of an utterance which is not lawlike is that normally made in the sentence:

"Everything in my pocket on VE day was silver."

That this is no lawlike utterance will be clear to those who see that we cannot on the basis of this evidence infer that if P had been in my pocket on VE day, P would have been silver. We do however feel inclined to accept on the basis of the utterances cited earlier that if P goes up, then P also comes down, and that if P is butter heated

under normal kitchen conditions to 150°F, then P melts.

It seems a relatively straightforward matter that when some utterance is a candidate for the description "lawlike", the matter can be settled in the affirmative whenever it is acceptable to say that what the utterance says is naturally necessary. Thus were we to become convinced that it is a matter of natural necessity that everything in my pocket on VE day was silver, then the corresponding subjunctive conditionals would ipso facto be acceptable (otherwise they would constitute counterexamples to the "necessity") and it would be established that the utterance concerned was lawlike. This conclusion is hardly surprising since the "law" referred to in "lawlike" is a "law of nature" and our notion of "natural necessity" applies to just those utterances which behave like natural laws.

That lawlike statements can also be characterised as naturally necessary is less clear; we purport to establish no more than that this is likely and that the two concepts are closely related. A strong argument is to be found in a consideration of the respective borderline cases of "lawlike" statements and natural necessities. An example of such a statement would be a generalization about the effects of aspirin*, or about the association between smoking and lung cancer. We are faced with the problem of whether these statements are of the sort which allows us to generate acceptable

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*We are supposing that no mechanism is yet known to explain the working of aspirin but that its effects are (statistically) well-attested. There is thus indubitable and persistent constant conjunction but no theory to give the basis for a "casual" explanation.

subjunctive conditionals on the basis of them, (or in Goodman's term to "project" applications of them). Since these statements do not rest upon any theory which explains the mechanism whereby the effect is produced, we may be unwilling to accord the title of being "naturally necessary" to them - and with them, probably, to the statement that what goes up comes down. On the other hand there is something to be said for drawing the boundaries of natural necessity a little wider than the realms of fully developed mechanistic theories and if we are content to do this the first group of statements to be accorded the honour will be those generalizations which we see as lying on the borderline, at this point. This situation is analogous to that which we face when dealing with the lawlikeness of such statements.

A good reason for assimilating the notion of natural necessity to that of lawlikeness, can be found in the idea (imported from the previous Chapter) that a necessity is something which can serve as an assumption for the current substantive discussion. A <u>natural</u> necessity will, then, be one to be assumed in discussions about the course of nature. Such discussions are very vulnerable to the intrusion of non-lawlike statements, however: suppose that we have a sequence of respectable lawlike generalizations as premisses for a syllogistic type of argument (Barbara, for instance) - it is clear that the conclusion is lawlike if the premisses were; but if one of the premisses were to have been a mere fortuitous generalization

of the non-lawlike type, then the conclusion - though syllogistically validly obtained - need not be lawlike, and all subjective conditionals based on it or on conclusions based on it, will be suspect. If we are to draw subjunctive conditionals as conclusions in such an argument we must be sure that all the premisses leading to a generalization upon which such a conditional is based, are lawlike.

That natural necessities are lawlike entails that they may be used in such arguments; that our lawlike statements be regarded as natural necessities becomes attractive as soon as we consider a move such as creating an assumption-set for every such discussion. By identifying the two groups we can kill two birds with one stone; in the first place we say that the class of lawlike statements is just that class which is contained in or logically derivable from our assumption-set for discussions about the course of nature (or more prosaically for "scientific discussions"); on the other hand we say that natural necessities are just those utterances used to make explicit statements of matters in such an assumption-set.

It should not however be imagined that this assimilation of lawlike statements and utterances which state natural necessities is a non-controversial matter. Its acceptance involves departures from traditional doctrine which are even more radical than those outlined in the previous Chapter; and for this reason some will perhaps prefer to identify in the statement of a natural necessity - whether or not this is taken as identical with a "lawlike" statement - a

third type of "saying that something is necessary".

Facts and Artifacts

One of the most basic doctrines of 20th century linguistic philosophy (perhaps one which is seldom explicitly stated, though) is that there is a fundamental difference between talk about things and talk about talk or words. Speech and the words we use are seen as human artifacts which can be taken as objects of study and can be talked about in the same way as one talks about any other sort of object of study - e.g. stones or psychological complexes. Yet it is simultaneously accepted by many philosophers that "semantic ascent" can be a helpful philosophical tool; this semantic ascent works out in practice* as a procedure by means of which a quasi-metaphysical discussion of some subject-matter whose existence may even be in question, is conducted not by means of statements about that subject-matter itself but by means of (frequently normative) statements about language and in particular the language which would be appropriate for talking about the subject matter which is of interest.

In Word and Object (§56) Quine has suggested that the device of semantic ascent does not enable us to escape from the realm of empirical fact into some sphere where arguments may be conducted a priori taking into account only the meanings of words; in particular he points to highly abstract scientific theories such as relativity as examples of subject matter which although testable is most

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*Its theoretical significance is expounded by Carnap (the originator of the particular expression 'semantic ascent') and Quine, among others.

conveniently expounded in the "formal" mode of speech (i.e. making use of the device of semantic ascent). Our own researches tend to substantiate this view - certainly if we are to treat lawlike utterances and natural necessity as cases which should be fitted into the schema of the previous Chapter.

The support which our arguments lend here can be seen best by considering the question "If lawlike statements are also a species of utterances stating that something is necessary, what is there left which can be pronounced "true" or "false" in the simple old-fashioned way?" This is a problem because when the lawlike statements are used in an argument about natural phenomena we have supposed them to be functioning as invocations of (natural) necessity; thus while they are being established they are mere hypotheses, and when established they at once get incorporated into the assumptions for arguments. It seems that only the fortuitous generalizations and those (perhaps imaginary) utterances which report experience "raw" and uninterpreted in the light of scientific theory, are left as candidate for the valuation 'true'. On the other hand we have substantially increased the range of our predicate 'acceptable' by applying it to uses of lawlike statements as well as to the invocations of other necessities.

Exploring the assimilation of natural laws and necessities farther, we should expect that invocations of natural laws will be assigned their acceptability value on the basis of whether the natural

law in question is well-grounded for the particular discussion. As will be remembered, we supposed that the grounding which would be appropriate to a necessity of the normal sort would be some convention, theory or definition which was to be taken as being applicable to the context. Clearly natural laws can be grounded in 'theory' but it will be a more difficult question to determine just what theories are to be taken as assumed in a wide-ranging discussion of natural phenomena.

In practice however this problem is readily solved. Some discussions are clearly such that we take it that the findings of science are not to be assumed - we are discussing what is in principle possible; but there are also others where the current state of scientific knowledge is to be taken as given. This latter assumption is no more than that of <u>all</u> currently accepted theories of science. This may seem strange when we consider that "science" has accepted many conflicting theories through the ages and that in certain fields there is no one accepted theory to this day; yet a brief consideration of practice reveals that where it is not clear which theory is the scientifically accepted one, neither is included in the set being assumed - as witness the immediate question to someone who tries to invoke one of the laws laid down by such a suspect theory - "That may appear to be generally so, but can we assume that it is a matter of regularity in nature before the theory backing it up is fully attested?"

There is, thus, we are supposing, a type of talk in which we are supposed to take all the discoveries of science as being assumptions of the sort which can give rise to invocations (of natural necessity) and to validly derived subjunctive conditionals. This supposition creates a group of groundings for necessity invocations which are 'empirical' in the sense that they are among the class of assertions whose truth is argued about on the basis of facts and observations (the argument taking place, of course, in contexts other than those which assume the matter which it is desired to establish the truth of). This is in no way inconsistent with what we said of invocations of necessity in the previous chapter. However it is still possible on the basis of what is said there to hold that the groundings of acceptable necessity invocations are (or ought to be) exclusively (a) ad hoc assumptions which enjoy their status of being uncontestable on a strictly temporary basis, (requiring no special probability of truth in order to attain it and deriving no special aura of trustworthiness outside the context for which they are assumptions) and (b) matters which we cannot imagine to be otherwise and which merit the title "analytic".

With scientific theory, however, the case is different. Here the inclusion of a theory in the assumption-set for a discussion is directly linked to the fact of its having been tested and adjudged 'true' (as we may continue to style the positive outcome of a verification test). Conversely if some theory is a member of the

assumption-sets for normal discussions on the course of nature, then we can in practice take it that on testing this theory will turn out to be true... at least according to currently accepted ideas of the appropriate tests and of the appropriate criteria for interpreting their results.

The fact that something is in one context verified or discovered and in another is used as part of the assumption-set for a discussion, does not show that we 'verify assumptions' or 'discover assumptions'. Assumptions are not verified as such - it is more a case that some assumptions turn out to be things which we have verified or can verify. We may therefore separate the role of the theory - and the lawlike statement - as something which is verified, and as something which is an assumption with the help of which we can draw certain conclusions.

By incorporating the natural necessity (as theory or lawlike statement) into our system of groundings by reference to which an invocation of necessity may be made acceptable, we provide ourselves with a whole spectrum of groundings. This spectrum incorporates both laws of logic and assumptions of things which are simply not true; amongst the groundings which consist of the assumption of something true, are both assumptions of the truth of observation statements involving a minimum of theoretical committment, and assumptions of highly abstract scientific theories, such as that of relativity.

Semantic Ascent

What then of semantic ascent? Quine's apology for this method in philosophy cites the difficulties of seriously arguing about the existence of miles without talking about the word 'mile' and suggests that semantic ascent helps in philosophy because "The strategy is one of ascending to a common part of two fundamentally disparate conceptual schemes, the better to discuss the disparate foundations." (op.cit; 272).

It is hard to see how any discussion about the word 'mile' and the contexts of it which "are useful and for what purposes" is going to be of any avail. If party A does not believe in the existence of miles then he is not going to be won over by talking about the use of the word 'mile'; it will of course depend upon whether he thinks it is ever useful to refer to things which do not exist, but we may presume that he will have little or no use for the word so long as he does not believe in the thing.

References to human behaviour including the use of words and sentences are frequent enough in our normal speech. We can and do use the device of quotation in such cases, and we talk about words - such as 'mile'. Further it is plausible that people who do not agree about matters of fact which require complex demonstration, will at least be in agreement about matters concerning words, since words are things they meet everyday and have ample opportunity to find out about. These are the factors indicating that Quine may be

on the right path.

On the other hand as stated above (p.363) we should distinguish two ways of talking about words, their meanings, and uses. It is possible to talk 'factually' - that is, treating the linguistic phenomena of which we have experience as being open to the same sort of description, explanation and reporting as facts of nature. But it is also possible to talk about words and particularly their meanings and uses from the point of view of the normative lexicographer, (or as a philosopher). We may pose a dichotomy regarding the argument about miles: if a statement about the relationship of the word 'mile' to certain distances, measurements etc. is taken by both participants as being 'factual' then it is probable that they will be able to agree on some such statement since they probably both observe roughly the same about how people do use the word concerned - their philosophical disagreement, however, remains; the non-believer still has no good reason to join the mass in their use of what he has all along regarded as misleading terminology (or worse). On the other hand, if the participants concern themselves with how we should interdefine our words and how we should use language to refer to the world, then it is highly unlikely that they will reach agreement since one of them believes in a world which contains miles and the other does not believe in such a world - even their attempt to agree that we should avoid referring to miles is likely to be stranded in a wrangle about how we are to state this agreement.

Exactly what we are to say about semantic ascent will depend on whether we regard it as a shift from talk about things to talk about how words are used, or talk about how they should be used. Our view is that the shift to talk about how words are used, furthers agreement among people but leads to an agreement which is irrelevant to the original issue; and that shifting to talk about how words should be used preserves relevance to the original topic but, as could be expected, does not remove the original disagreement. It is the latter shift with which we concern ourselves here.

While disagreeing with Quine about the value of semantic ascent as a panacea for philosophical disagreement, we feel that it is a valuable tool for clarifying just what the point at issue is. This fact is amply demonstrated by the examples which Quine himself gives; we shall confine ourselves to the application of semantic ascent to invocations of necessity - including what we have styled "natural" necessities.

Referential Opacity

For Quine one of the vices of "modal" contexts is that they are, in his terms, 'referentially opaque'. By this he means principally that there is a breakdown of salva veritate substitutability of terms between which the identity relation holds. His second, clearly connected, complaint about such contexts is that when we try to introduce quantification it becomes necessary to abandon the supposition that the variables of quantification vary over objects;

that is we cannot maintain that different instances of such variables may correspond with intensionally different references to the same thing, when we consider interpretations of our formula.

Ruth Barcan Marcus, defending her own promulgation of a quantified modal logic, takes the view that certain different levels of identity relation have to be admitted into logics of this sort and that substitutivity of identity only holds within a given level of identity. On further interrogation her view of quantification is different from Quine's (555M) and the replacements for the variables in her sentence frames turn out to be referring expressions rather than the designata of referring expressions*.

It is unhelpful of Quine to bend this case to fit his terminology of referential opacity. He clearly recognises the connection between:

(a) Necessarily $9 > 4$.

and (b) $(9 > 4)$ is analytic.

However - seemingly in the interests of the universal applicability of the notion of referential opacity - he immediately draws the conclusion that the punctuation round "9 > 4" ought to be quotation marks, and that this case is a standard one of a paradox arising

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*This discussion is retained so far as possible in the original terms, despite the strangeness which occasionally results - e.g. replacement of variables by <u>things in</u> sentence frames. Our acceptance of sentence-frames is to be seen in the light of comments on Quine's <u>regimentation</u> of ordinary speech. (pp. 309-310).

because of the presence (here implicit) of quotation marks or expressions equivalent to quoted expressions.

While he may be correct in concluding that quotation marks round the "9 > 4" are called for, we need not draw the conclusion that this is a standard case of referential opacity; in fact, as we shall show, there are many ways of using quotation marks and an analysis of which use is here appropriate leads to a better argument against Mrs. Marcus, and to a 'gradualism' which dispenses with the hard and fast distinction between "about words" and "about things" - at least so far as invocations of necessity are concerned.

Consider the following utterances:

1. John said, "Winter is colder than summer."
2. "Winter is colder than summer" is analytic.
3. It is necessary that winter is colder than summer.
4. John said "Mbiglu zerlr."
5. "Mbiglu zerlr." is analytic.
6. It is necessary that mbiglu zerlr.

In examples 1 and 4 certain words supposedly uttered by John are put down on the page and enclosed in quotation marks; in the case of 4 the words concerned are nonsensical in English and probably in all other languages, but we are clearly entitled to represent the noises which John made and insofar as he made these noises as linguistic activity we may say that he said whatever it is we manage to write down. We are in case 4 perfectly at liberty to quote nonsense or unintelligible sounds.

Taking 4 as our paradigm we might be tempted to accept that 5 was also perfectly permissible. Certainly one can conceive of a use of the predicate "is analytic" which would be in place here; in such a case we should gain information about the analyticity of a sentence* (if it be such) in some unknown language. If we recall our terminology of assertions and invocations of necessity, this case is to be likened to that of the assertion of necessity, and indeed insofar as the form "x is analytic" and the form "It is necessary that x." are interchangeable we take the latter as an assertion of necessity whenever the quoted expression is treated this way.

However we need not and probably should not interpret 'is analytic' in such a way. Quine's concern is with the type of talk about necessity which allows us to make deductions about facts other than those of linguistic behaviour or semantics; it is the sort of talk about necessity which is relevant to "the thing" said to be necessary. And this means that we have to consider the invocation of necessity and not just the assertion. Further it is Quine's contention that 2 and 3, and presumably 5 and 6 are at least so closely related to one another that we can assert the biconditional of the pairs.

Example 6 however presents a serious problem. It is not an intelligible utterance of any language known to us. In particular we are not in a position to assent to claims that if 6 is true then

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*In its standard use, one presumes

something else is the case; since we do not know what type of linguistic entity 6 is we cannot go so far as assuming that it makes sense to suppose it to be true or false - it might turn out to be an exclamation, or a very complicated way of clearing one's throat.

If utterances of the type of 2 and 5 are to form any basis for clarifying the meaning of utterances like 3, then we shall have to accept that there is a use of quotation marks (possibly only in conjunction with certain words such as "is analytic", but still a genuine use) such that 5 is unintelligible in the same respects as 6 is. This is in itself enough to establish that we cannot assume the device of quotation to be unambiguous.

So long as we only have one type of quotation to consider, we can simple-mindedly identify sentences involving quotation marks as being "about words" and the rest as being "about things". Having two and perhaps more types of quotation presents the problem of whether some cases of quotation are more properly "about words" than others, and whether the division is in fact a hard and fast one.

Examples 1 and 4 are clearly about words in a trivial way (taking it to follow from their susceptibility to being represented by letters of the alphabet that the shapes "mliglu" and "zerlr" are words). Also about words, are utterances such as "The third sentence is analytic" or the sentences 2 and 5 uttered in such a way that they are interchangeable with assertions of necessity. We shall not pursue the question of whether these two cases are the same.

That 3 is also in a sense about words can be seen by considering the utterance:

> 7. That one of two seasons is the warmer dictates that that season rather than the other should be called 'summer' and that the other should rather be called 'winter' (insofar as it is desired to apply this pair of terms to the two seasons concerned).

While 7 is not claimed to be entirely synonymous with 3, it is clearly very closely related and may be taken as at least as good a representation of 3 as a Quinean "regimented translation". 7 is about words; it is indeed about specific words whereas the utterances previously discussed were about groups of words and dealt with properties of the groups as much as with the individual words.

Where 3 is an invocation of necessity it can be represented as about the use of specific words. It is natural to ask whether any invocation of necessity can be so represented, but the matter is apt to rest purely on personal insight. We have already referred to the close connection between invocations of necessity and definitions. It would not be correct to exclude invocations whose grounding is something other than an agreed definition, but we may well say that since our basis for taking invocations of necessity as one class irrespective of the variety of groundings on which they could be based, was the fact that they all behaved like the invocations grounded on definitions; accordingly we may assume that they can for the purposes of the present discussion be taken as being as much about words as are definitions. Examples to support this view can be

found in Chapter VI; a consideration tending to support this view is that when an invocation of necessity is called in question the discussion which results almost invariably (so far as it is to the point) deals with words and what we should take as built into the meanings of the terms to be used - even in cases such as "What goes up must come down" the point of argument at once becomes whether "down" must mean back to the earth's surface, etc.

If we take it that invocations of necessity are about words - the specific words which would be quoted in a paraphrase such as 7 is for 3. - then we can treat the examples which Quine regards as referentially opaque, in a different way. Let us consider two examples:

8a. Necessarily $9 > 4$.

8b. Necessarily the number of the planets > 4.

9a. The evening star is the evening star, necessarily.

9b. The evening star is the morning star, necessarily.

Quine took it that referential opacity arose here because even though the number of the planets <u>is</u> 9 and the morning star <u>is</u> the evening star the use of these identities as a basis for substitution should be ruled out because the rest of the sentences 8a and 9a apart from the word "necessarily" should be seen as within sets of quotation marks.

While agreeing that substitution is not possible and that the

reason is to be found by considering quotation, we do not accept Quine's point. If we consider the identity statement which Quine uses to justify his transfer from 9a to 9b, viz.

10. evening star = morning star

and the paraphrases of 9a and 9b, viz.

11a. A thing which is referred to by the words "evening star" should also be taken to be being referred to by the words "evening star".

11b. A thing which is referred to by the words "morning star" should also be taken to be being referred to by the words "evening star".

then it is clear that the identity stated in 10 is not that of the terms which have to be substituted to yield 11b from 11a. Instead we should have had to have

12. "evening star" = "morning star".

Analogously we can argue that 8b is not properly obtained from 8a by a process of substitution because the identity which would be required to justify such a substitution is not that of 9 with the number of the planets but that of "9" with "the number of the planets".

This explanation relies on our denying the truth of 12 and the corresponding identity claim for "9" and "the number of the planets". It is however not entirely clear what 12 is; we have taken it is an abbreviated version of some utterance. A good candidate for this utterance is the claim the two quoted expressions are synonymous; this claim is in the cases cited a false one. Further, whenever it is false the substitution of one term concerned for the other will

yield paradoxes even though both have the same reference. And further it may be taken that if two terms have the same reference and may invariably be substituted one for the other in invocations of necessity, without causing paradox then the two terms are synonymous.

Are Invocations of Necessity about Words?

The question with which we opened this Chapter was that of what it is that can at one time be asserted to be necessary, at another invoked as necessary, and at another simply stated to be true. It was promised that discussions of the "scientific must" and of natural necessity would help us on the way to a solution of this problem as well as being in themselves relevant to the topic of necessity. From talk of natural necessity and lawlike statements we have progressed to a consideration of the contrast between utterances which are about things and those which are about words.

The attack made on Quine's example of referential opacity in modal contexts, is that Quine has mistaken a claim about words for one about things. Because of this confusion he bases his substitution upon an identity relation which, even if properly expressed is irrelevant; the relevant identity relationship would be between words seen as words (thus including the quotation marks around them) rather than between words in use (referring to things). We have however based our objection upon the fact that the utterances with which Quine is concerned lend themselves to paraphrasing in such a

way that particular terms are displayed in quotation marks and something is said about how they should be used. More interesting than a defeat of Quine's argument is the question whether all the utterances which we are tempted to lump together as invocations of necessity can be paraphrased in this way as utterances about words.

Invocations grounded on definitions can be seen as about words without difficulty; those based on specific ad hoc assumptions are less obviously open to this treatment - especially when the assumption is about some observable phenomenon. But the inclusion of natural necessities as a species of invocation of necessity casts the most doubt upon the proposal since it is the natural necessities (including the laws of nature) which embody our discoveries about the world, and this type of knowledge about the world is practically a paradigm case of something about things rather than about words.

Let us consider some further examples:

12. It is necessary that the airpressure in the chamber, C, increases as the crankshaft, A, rotates.

13. It is necessary that the position and velocity of a body cannot both be determined with arbitrary precision for one and the same instant.

14. It is necessary that psychotics suffer from hallucinations.

At first sight these utterances are not about words. We may indeed use Quine's device of examining the effect of substitution of equivalent references, as a means of bringing out the sense in which the utterances are not about words.

In 12 it is immaterial how we refer to the chamber and the crankshaft - at least if we judge in terms of salva veritate substitutability. So long as the hearer understands that A and C are intended we do not change the truth value of what is said even if we replace "chamber" by "coffin" "cavity" or "chamber pot" and "crankshaft" by "spindle", "sceptre" or "psychokinetic channel".

Example 13 is more abstract. It deals with the sort of matters which according to Quine invite semantic ascent. None the less it is difficult to see it as about words, if we consider the question of salva veritate substitutability. If, for example, one holds that the mass of a body is directly related to its velocity and vice versa in such a way that the two parameters are merely different ways of measuring the same variable*, then one can - without any risk that one's view of the truth-value of the utterance will change - substitute "mass" for "velocity". "Mass" and "velocity" would in such a case remain intensionally distinct, yet despite this there seems no reason to reassess the truth value after such a substitution.

These examples seem heartening for those who wish laws of nature also to be "true"-or-"false" claims about things rather than words. However they rest on the test of salva veritate substitutability (a phenomenon which we, like Quine, have left to the insight of the reader), and if one holds that laws of nature are not "true"-or-"false" in the first place then it is somewhat inconsistent to invoke such a notion.

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*we mean here "source of variability".

Our current supposition that laws of nature can be treated like invocations of necessity, forces us to seek some other type of substitu-ability test. The natural one would be that of conservation of acceptability-value under substitution of alternative referring expressions. Let us re-examine the two cases considered, this time bearing in mind that they are assigned a value such as "acceptable" and purport to invoke (in practice this amounts to reiterating or applying) some law of nature which is assumed for the discussion.

Example 12 is the most difficult to conceive of as being an invocation of necessity; none the less it is possible to see it as such. Scientific theory rarely deals with particular items such as "A" and "C", but we may consider this as a natural necessity which purports to apply theory to a particular case. The indication of the elements of that case - presumably with the help of a sketch on which "A" and "C" are depicted - is not of any special interest to us; more important is whether the substitution for terms drawn from the presumed background theory of terms which though having the same extension as these in terms of which the theory would be stated are not intensionally equivalent can be done without affecting the acceptability of the invocation.

That this cannot be done is simple to argue in principle. If some term happens to have the same extension as a term involved in some theory which is assumed for a given discussion it will not be acceptable to substitute that term for the theory's term when applying

the theory in the course of that discussion unless the extensional identity concerned is based on something more than pure chance. For example we may consider it to be the case that the air-pressure is always and everywhere the same as the resistance to implosion of bodies immersed in air. That we consider the airpressure in chamber C to be equally well referred to by the phrase "the resistance to implosion of the walls of the chamber, C," is not sufficient justification for our substituting this latter phrase and expecting the acceptability-value of the resulting utterance to be the same as that of 12. (always remembering of course that we are dealing with purported applications of theory). In order to assure ourselves that there cannot be a change in acceptability-value, we must confirm that there is another theoretical assumption made in the same (consistent) assumption set, which lays down that airpressure is correlative with resistance to implosion in such a way that either description can be used to refer to one and the same phenomenon. Even more clearly in the case of 13, when we judge the utterance as acceptable or not by considering whether it is a reiteration of or application of some assumed law, then we cannot rely on the identity of mass and velocity unless that identity is stated in some farther assumption.

This is to say that the fact that two terms refer to one and the same phenomenon is not sufficient to justify our substitution of one of the terms for the other where that other term was used to

state a given theory. This is not as paradoxical as it may seem; we are not saying that theories deal with terms for things instead of with the things themselves. We are saying rather that if a theory does not recognise two terms as referring to one and the same thing then it is not legitimate to substitute one term for the other when applying the theory (or restating it). In normal scientific discourse it is hard to gather evidence for or against this view since the technical terms are deliberately kept as unambiguous and distinct as possible, with the result that few synonyms exist; further, due to the tendency to unify as great a range of phenomena as possible in the explanatory field of a given theory, it is seldom that the same phenomena can be referred to by terms which are extensionally identical but of different intension.

The less developed sciences provide the best examples for this case as we can see from utterance 14. We take the sentence here to be being used not to state the definition or part of the definition of a psychotic, but as being used to state some tenent of a particular psychological theory. The acceptability of the utterance depends therefore upon whether one is entitled to assume that particular psychological theory for the context in question. We now assume that the term 'zz' happens to apply to all and only hallucinations (as defined in the psychological theory concerned), but not to be synonymous with the term "hallucination" as used in the theory. May we substitute "zz"?

"zz" may be a technical term of the psychological theory concerned, if so the possibilities of substitution will be determined by the theory; in particular the universal substitutability of one term for another in a given theory amounts to their synonymy (as technical terms) and in view of this such a case is of no interest for us. Where the terms are not synonymous within the theory the substitution may lead to an utterance which is not one of the tenets of the theory, and hence one which is not acceptable in a context where only that theory is assumed. Similarly if the term "zz" is wholly unknown in the theory (as for example "nut-case" probably would be in the theory on which 14 is based) then the result of the substitution is again an utterance which cannot be acceptable in a context for which that theory is the only relevant assumption, since it makes what is now an empirical claim, but purports to base that upon the theoretical background to the discussion.

It may have been established that invocations of necessity are referentially opaque with respect to acceptability-value, but we have not <u>ipso facto</u> broken down the distinction between being about words and being about things. Nor is it our intention to break it down entirely; we are concerned with the uses of sentences and with the fact that what at one time is stated as a law of nature is at other times confirmed or observed to be the case. We assert that when being used as lawlike statements sentences can be seen as about words, while when used as reports of observation the very same

sentences are to be regarded as about things.

A paraphrase analogous to that which 7 provided for 3, can very easily be described for the cases where a theory is invoked. We take it that scientific theories are susceptible of being explained and the meanings of their technical terms stated. The paraphrase desired is one which states that the technical terms of the theory concerned (quoting those which occur in the original utterance) should be used in the way the theory uses them and should have the interrelation that an explanation of the theory would attribute to them.

Observation and assumption

It is not our purpose to go deeper into the philosophy of science here. But at least a prima facie case has been made for the view that in scientific talk the same thing can be talked of in two different ways:

(i) it can be stated to be the case - in an assertion which we value as "true" or "false".

(ii) it can be taken as given - in the reiteration of or an application of a theoretical principle supposedly assumed for the purposes of the discussion; in which case we treat it like an invocation of necessity, assigning it a valuation "acceptable" if indeed that principle should have been taken as assumed.

This contrast can thus be made for a very wide variety of things which we say, and presses us more strongly to give an explanation of what "thing" it is which is talked of in these different ways.

Philosophy usually consists of a procedure whereby one looks intently at a familiar phenomenon or expression until it becomes unfamiliar - just as a common word begins to sound strange if repeated monotonously long enough - followed by a stage in which the phenomenon or expression is rehabilitated with the help of an analysis of its interrelation with the rest of our experience. This process is only possible if not everything is made unfamiliar at once, and we, unlike Descartes, have no indubitable truth which we can exempt from scrutiny. The basic undefined concept which we choose to question no further is that of a "characterisation".

The notion of a characterisation comes close to Kant's idea of a judgement; it plays a role here which is similar to that played by the notion of a "propositional attitude" in Quine's earlier writing. A characterisation may be described as a product of our ability to form a communicable means of referring to our environment as we claim to experience it. There is no question that the experience which we claim to have had needs to be possible in order for there to be a characterisation dealing with it; all that is necessary is that we are able to frame references to the environment which communicate to others sufficient information about what we are claiming for them to distinguish this claim from others. It is a further philosophical question whether there are experiences which we can imagine but not communicate, or whether we can ever describe genuinely impossible experiences. Finally, a characterisation

is not "true" or "false" or "acceptable" or "unacceptable"; insofar as a valuation is appropriate for it, it can be called "successful" or "unsuccessful", but this is a doubtful matter for it is hard to imagine what an unsuccessful characterisation is like or to know whether animals or humans of sub-normal intelligence have infinite (or near infinite) numbers of unsuccessful characterisations, rather than simply being unable to communicate their own view* of what they experience. The reason why we do not have proper valuations for characterisations is simply that they lie one stage earlier than utterances of any sort; they correspond to utterances in a way similar to that in which concepts correspond to words.

The positing of characterisations only saves the production of an account of what it is that is in common among the utterances which are used to say the same thing in different ways - at the expense of introducing a need to show how these various types of utterance are related to characterisations. We have to maintain for instance, that one bases both factual claims and invocations of necessity on underlying characterisations. We must state whether every characterisation is the basis for both a factual claim and an invocation of necessity, and if not why not. And, if possible, we

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*We are not unaware of the apparent pleonasm here: the word "communicate" is normally restricted to deliberate acts which lead others to knowledge, but we have added the words "their own view" here to make absolutely clear that we are not concerned with behaviour which may be <u>evidence</u> leading others to know what a given creature does experience.

should indicate some field independent of that with which we are currently concerned, in which the positing of characterisations and various types of utterance based upon them seems likely to have particular explanatory value.

The significance of introducing the notion of a characterisation lies in the fact that we can now refer concretely to the human ability to find words for a situation or state of affairs, without making any presupposition about how these words are to be <u>used</u>. Such an ability is analgous to that which other writers postulate, of being able to identify particular words as suitable for use as a means of referring to certain objects or situations; those words are then available as 'labels' for concepts. When speaking of words in this way one is apt to think of each word as having a meaning - independently of particular cases in which it is used. In fact, such a supposition is not necessary; one need only suppose it to be a human ability to select <u>an appropriate</u> word for the situations in which one is interested, without that human being necessarily choosing to use the word in question. Being unable to give an account of any procedure whereby the ability to form sentences can be explained in terms of the ability to choose appropriate words, we elect to postulate the ability to form sentences; this we do by means of the notion of a characterisation. The notion will be explained further by the use we make of it.

We may base either a factual claim or an assumption upon a given characterisation. Where an assumption has been based upon a certain characterisation then a reiteration of that assumption or an invocation of necessity based upon that assumption is an utterance dealing with "the same thing" as the factual claim which could be been based upon the given characterisation. This is the 'solution' of the problem posed at the beginning of this Chapter. Its suitability will be shown by considering the case of characterisations arising in scientific enquiry - in the broadest sense.

The apparatus which we have now constructed allows us an elegant explanation of the problem of confirmation. We no longer seek a logical relationship between evidence and law of nature; instead we accept that certain factual reports are instantiations of a certain generalisation and seek in the number and variety of the factual reports taken into account a sufficient case for making - in other contexts - the assumption which can be generated from that characterisation on which that generalisation is based. When the assumption is made, the 'instantiations' become applications to particular cases, and since we are dealing with a case where the law is assumed to hold, there is no question of testing the "truth" of these utterances; they are acceptable or unacceptable according to our judgement of what one would be entitled to include in the assumption-set for the current discussion.

It is a purely prudential decision that we make when we decide that a hypothesis has been confirmed; we decide in fact to assume the regularity in question as holding in all generality, hence-forward. The extensive literature on confirmation relationships is not hereby rendered beside the point; it is displayed as dealing with the correct way of going about this decision process. It may be that one should try to disconfirm a hypothesis; it may be that one should attempt to fill in the variables in some equation derived from probability theory, or perhaps one should use Mill's methods; the question remains an interesting one. We are however freed from the idea that there must be a logical relationship between a factual claim and a law of nature; this is because the factual claim involved is an instantiation of the statement of fact (the generalisation) based on a given characterisation whereas, although based on the same characterisation, the law of nature is the (statement of) the assumption based thereon.

The phenomenon remarked upon above, viz. that an empirical generalisation when entertained as an assumption does give us a basis for deriving subjunctive conditionals, is explained. The act of assuming a (real or unreal) constant conjunction or empirical generalization to hold unrestrictedly is nothing other than making a law of nature of it. Similarly when we dethrone a law of nature it tends to remain an empirical generalization applying to a wide domain. Finally, in the sort of discussion where evidence is being

sifted for possible disconfirmation of a theory - thus where it is (perhaps temporarily) deprived of its status as accepted law of nature, it is illegitimate to derive from the theory the subjunctive conditionals which normally can be derived from it when it is taken as accepted: to do so would be petitio principii. In fine, when being confirmed the theory is not assumed and not a law of nature, when assumed, a generalization is like a law of nature, and when behaving like a law of nature and giving rise to subjunctive conditionals, a generalization is not open to confirmation or disconfirmation: it is in our terms, assumed.

Quine's dictum that there is just what we say there is, can be stated more precisely in this context. What we say there is - that is what we are committed to the existence of - are those things about which we make factual claims. We are not committed to the existence of those things which we merely suppose to be involved in various relationships. Thus in a theory about electrons there is no committment to the existence of electrons; it is after all only a theory about how electrons would behave. None the less, in order to gain any benefit from a theory we must apply it and in doing so we inevitably decide that the terms in the theory - such as "electron" - apply to certain observable or inferable entities. That is to say we make factual claims involving the technical terms of the theory and it is only with the help of these that the theory - via its subjunctive conditionals for example - provides us with a means

of making predictions. A theory about Pegasus, then, is not ruled out as a theory; a normal person would probably not entertain it as a "scientific" theory because it would be unlikely to have had sufficient confirmation; however a person who did entertain it could do so without committing himself to the existence of Pegasus - it would, as it were, be a part of his definition of Pegasus - but as soon as he wishes to use the theory to help him reach a conclusion about something which ought to be observable, he is bound to commit himself to the existence of Pegasus because he is otherwise unable to introduce the appropriate description of the circumstances to which the theory is being applied.

The pursuit of abstract reasonings about sets is clearly a respectable one; so long as we remain abstract, making suppositions and displaying the interactions of their consequences, we shall enter into no committment to the existence of sets. But as soon as we state that something is a set, then we are committed to the existence of sets. What we are committed to, is of course an entity satisfying the suppositions we have made about sets in our theory; hence if our theory is inconsistent or leads to undecidable propositions, then we commit ourselves to the existence of things which cannot exist or things about which unverifiable claims can be made. That we do so is our own business, and our neighbour need not listen if he is dissatisfied; it is up to us not to talk about things as sets if we do not have a satisfactory definition of that concept. But

on the other hand belief in a contradictory theory does not commit us to (saying) that 'contradictory' things exist - talk about such impossible things begins only when we use the theory.

Hume is correct then, in saying that we can never claim to have observed anything more than a constant conjunction, but that we must behave as though the world was governed by natural necessity. Our explanation of this has been to say that talk about natural necessity simply is not the sort of talk which comes in for valuation as being "true" or "false"; it is talk which we accept or not according to our view of what assumptions should be made. That we must make some assumptions is clear since only with the help of assumptions - we have argued - can we derive subjunctive conditionals and thereby say what is connected to what. On the other hand when we set out to say something "true" or "false" we naturally do not assume that thing from the outset; it can of course be that we use a theoretical term in the course of saying something "factual" - but this commits us to the existence of things of the sort described by the theory, and we must assure ourselves that we want to undertake this committment. If we do, then it is up to us: that is why we can observe electrons, instances of causation and statistical "facts" but do not observe causation in general and, in the absence of the necessary theories, cannot yet say that we observe aspirin killing pain, or cigarettes causing lung cancer. Only when the theory is acceptable are these last two "facts" or "things" that we shall be prepared to say we observe.

In this Chapter we have taken the 'invocation of necessity' - 'assertion of necessity' contrast developed in Chapter VI and extended it to the case of natural necessity. With the help of discussions which became necessary in this field we have been led to a postulate upon which we have based the intuition of "sameness" which we have about pairs of these utterances and certain factual claims. The extension of the theory of Chapter VI has been undertaken because the theory itself casts interesting light on the problems of the field concerned, and because the applicability - though sometimes forced - to this other field reinforces the credibility of the theory itself. We are not of the view, however, that this one theory is or ought to be sufficient to explain all uses of the term "necessary" or "must", or to cover every utterance of which it can correctly be said "The utterer said that it was necessary that it was so." We shall therefore examine in Chapter VIII various other interesting types of necessity, applying our criterion of how an utterance can be valued, as a measure of how it can be symbolized in a "logic", and shall conclude by summarising the significance of what has been said for the study of modal talk in general.

CHAPTER VIII

ALTERNATIVE POINTS OF DEPARTURE

In this Chapter we turn our attention to two other topics which have a bearing upon the symbolization of necessity either directly or because of links between other modalities and that of necessity. We conclude by summarising what has been achieved in these three chapters in the way of a reappraisal of the bases upon which logics of modal notions such as necessity should rest. It is not our intention in this Chapter to use the theory developed in Chapters VI and VII as a means for explaining the various matters considered; the theory itself did not claim absolute generality and cannot deal with all the problems which can be raised; it is therefore preferable to argue certain of the points on ground which lies closer to that on which they have previously been discussed. Where it is appropriate, of course, reference to the theory developed above will be made.

The Evidential Interpretation of Modal Distinctions

Here we examine the doctrine that the basic use of statements employing the "logical" and the "epistemic" modalities (as opposed to the moral, for instance) is to make an otherwise normal claim in circumstances where unusual evidence conditions exist and deserve to be indicated. It is thus suggested that the speaker would have liked to make a straightforward categorical* assertion, but that his assessment of the evidence at his disposal influenced him into making

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*In what follows we use this term as illustrated here.

his assertion in a different way or "mode".

Thus instead of,

(a) "It will rain tomorrow."

we meet,

(b_1) "It must rain tomorrow."

(b_2) "I believe it will rain tomorrow."

(b_3) "It will probably rain tomorrow."

As always we are on the lookout for a basis upon which we can construct a logic. It is therefore of less interest to us that this approach provides an escape from doctrines which postulate special types of propositions which are asserted in such cases, or which take it that we have to deal with predications of properties to the names of propositions. We are concerned with whether there is in the notion that the appropriateness of such types of utterance is based upon the evidence which the speaker has for the corresponding categorical assertion, any "valuation" upon which a logic could be founded.

Let us, in contrast to our procedure in previous discussions, attempt to write a symbolic expression representing what is being claimed - hereby we can at least pin down the claim with the help of familiar notions, though not of course thereby showing that it can be the basis of an interesting system.

We take it that the theory deals with a range of modal expressions; these we represent as 'M_i's. Farther, we take the

categorical expression which one would have liked to utter, as 'c', and introduce 'e' for the categorical factual claim which sums up all the available evidence for 'c's being true. What seems to be asserted by the theory is that there is a relationship between the appropriateness of a particular M_i and the result obtained by considering the pair (e,c). The consideration of (e,c) may be taken to yield a result represented as 'f(e,c)' in each case; hence where f(e,c) has a value in the range corresponding to a given M_i then that M_i will be the appropriate one to use. For each M_i, then, we require a "corresponding range"; such a corresponding range of values is represented as the set "$\underline{M_i}$'". We take the theory in its weakest form to be representable as:

For any i, if f(e,c) ε M_i' in a given case, then $M_i(c)$ is assertable. A stronger form of the theory, depicting the claim about the evidence as being interchangeable with the modalised claim about the conclusion, is:

$$M_i(c) \equiv f(e,c) \varepsilon \underline{M_i}'$$

The symbols which have now been introduced require further consideration. We shall try to show the merits and limitations of the evidential interpretation of modality by examining some of them. We do not however, propose to question the validity or aptness of the symbolism "$M_i(c)$" since we may take this straightforwardly as a designation of our explicandum - viz. an utterance such as is made

in (b_1), (b_2) or (b_3) where a modal expression is obvious and a corresponding categorical assertion can easily be identified. We ourselves had detected similar elements in necessity claims in previous chapters, and we take it on trust that they may be discerned here without philosophical impropriety.

Of much more interest is the symbol "$\underline{M_i}$'". It is essential to the doctrine in either of its forms that there is a specific range of values appropriate to each of the various modal expressions, "M_i". A naive version, for example, might hold that the various modal expressions should be mapped onto ranges of values between 0 and 1 - taking these as the limits of probability values - with say, "possibly" corresponding to values greater than 0 but less than or equal to 0.5, and "probably" corresponding to those between 0.5 and 1.

The difficulty which the theory is bound to face is that of showing that the ranges chosen as corresponding to the given modal expressions are not just arbitrary ones. Take a simple analogy; we enunciate the theory that when the value of the expression $F(X,\underline{a})$ lies in the range $(0 \leqslant \text{value} < \infty)$ then $\underline{a}$ may be asserted: it then turns out that to evaluate the expression '$F(X,\underline{a})$' we must calculate the inverse of the number 1 if the disjunction of X and $\underline{a}$ is true, and of the number 0 if that disjunction is false, while X is a contradiction. But here nothing at all has been explained in the theory; it is merely claimed that $\underline{a}$ is assertible if the

disjunction of a and a contradiction is true; we must seek defence against the rejoinder that the evidential interpretation in an analogous fashion shows no more than that when something is rendered possible, probable, or certain by the evidence, it is possible, probable, or certain. The fact that ranges are brought into the picture is not as such any proof that we are avoiding such traps. What would be of interest would be a demonstration that the various modal expressions can be ordered in a certain way, and that this corresponds to some ordering appropriate for values of f(e,c) - a matter which is demonstrable only if some structure can be defined such that both the modalities and the values of f(e,c) can be mapped onto the various distinct (ordered) elements it contains.

That is to say that even given an ordering of the modalities we should come close to a vindication of the evidential interpretation if we could show that the expression f(e,c) could be understood in such a way that it yielded values with the same relative ordering as the modalities which would be appropriate in the various cases. Further the evaluation of f(e,c) and the ordering of the values obtained, should be independent of the decision as to which modality is appropriate. Failing such a defence, the interpretation remains vulnerable to the charge that it merely proposes the replacement of somewhat vague talk using modal terms by equally vague talk about evidence without showing how or why the two sorts of talk are related.

The particular calibration of the scale "M_i'" which we shall consider is that of "strength". Two commonplaces may be cited as justification of this choice. First, consider two persons confronted with what purports to be evidence for a particular conclusion; often enough they reach different conclusions about what they can say - as may be illustrated by the examples given above (p.396), if we take them as the reactions of different persons to the sight of thunder clouds. A very natural way of describing what happens in the case of such disagreements is to say that the persons concerned did not all find the evidence equally strong. Secondly we may cite the case of regret at have made an unguarded claim, or at not having dared to make a claim as downrightly as we really thought was called for: cases where we say afterwards that we have spoken "too strongly" or "too weakly", and where the formulation of what we did in fact say typically involves a modal expression. Strength then, is a concept applied both to utterances employing modal expressions, and to the evidence upon which we base conclusions about the world. If there is one scale onto which we can project both modal expressions and assessments of the evidence-conclusion couplets referred to above, it may be expected to be or be closely related to one of strength.

Committments

Each modal expression, M_i, corresponds to some part of the scale of strengths which we postulate. That part defines the set of all members of the "M_i'" corresponding to that M_i. Our first

task is to describe how the various modal expressions can be compared with one another in strength so as to allow us to fix the parts of the scale which correspond to them.

When a claim that one makes is too strong, one is liable to be shown wrong. What is shown to be wrong is the "literal" content of what was said; by qualifying the defeated claim as having been "too" strong we indicate that there was a claim of the same sort which would have had just the right strength, and would have been invulnerable to the attack which was brought (or invulnerable to falsification by events). If we take it that our concern here is with the logical aspects of things said and that these logical aspects do not extend beyond what may broadly be termed the "commitments" undertaken by assenting to or stating a given thing, then we may conclude that the claim of the proper strength involves no commitments which are vulnerable in a logical* sense. The claim which is too strong, by contrast, clearly involves some commitments which are vulnerable to attacks of a potentially interesting kind. Making too strong a claim, may be characterised then, for present purposes, as undertaking vulnerable commitments on a given subject in addition to (or instead of) invulnerable ones.

The idea of "too many" or "too few" commitments can also be used to explain the notion that some claims are too weak. The practical significance of making too weak a claim, is that one is

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*That is, they are not vulnerable to attacks which employ only admitted truths and formal consequences of them; they may well, however, be vulnerable to allegations of pointlessness, understatement, or being based on unjustified inductions.

unable to establish things which one wants to establish and which could have been established had one used the stronger claim which was in order. Rephrasing this, we may say that one who makes too weak a claim has failed to undertake commitments which in fact were invulnerable (and might have been of use to him).

Even though we have used the vague term "commitment" and have not attempted to expand upon it, difficulties in this account of strength can be seen. If the modal expression "It is necessary that.." is to be dealt with in this way, it will be difficult to include the straightforward categorical assertion as well. For what are the commitments carried by the necessity claim but not by the categorical? In Chapters VI and VII we have countenanced the idea that the necessity claim actually carries less (or different) commitments than the categorical, but even apart from that type of objection we are forced into a dilemma. On the one hand we might take the claim as being analogous to "It is true that..." and say that the commitments involved amounted to those of the categorical plus the claim <u>that</u> the categorical was necessary; this however will not do in the context of an attempt to clarify the meaning of the modal terms. On the other hand we might argue that the claim was to be taken as a unit not as the predication of some property of a sentence, but then we meet the problem that, as pointed out in Chapter VI, acceptance of the claim leads to just the same consequences (and no more) as does acceptance of the corresponding

categorical claim. It is also worth noting that if instead of looking at the consequences of an utterance as these are normally dealt with in current logics, we look to the content of what is said (an option which we have left open by keeping the term "commitment" vague) then the philosophical saw that a tautology has no content (or that it conveys no information) leads us to the position that a claim that a particular tautology is necessary has the commitments of that which is claimed to be necessary - viz. none, since that has no content - plus a few commitments of its own, and that the corresponding claims regarding the probability or possibility of the matter in question are either (a) stronger than the categorical (assuming they make <u>some</u> commitment, or (b) equal in strength to the categorical (assuming they like it have <u>no</u> commitments) or (c) in the remarkable position of carrying a negative number of commitments. While (a) and (b) are counterintuitive, (c) is still less attractive, and it would seem wise to stick to commitments in the conventional sense, or to develop a different theory of the tautology.

None the less despite the difficulties in fitting in all the interesting modal expressions, it seems that there is at least a basis for ordering a number of them in terms of strength as measured by the extra commitments undertaken when one uses a stronger rather than a weaker modal expression. With this as working basis, at least we turn now to the notion of strength as it applies to

the calculation of f(e,c).

Strength of Evidence

We have used 'c' to represent a statement of the conclusion and 'e' to represent a statement of everything which is to be taken into account as evidence for that conclusion; both "e" and "c" are thus as near pure descriptive utterances as we can get. In the majority of cases where the subject matter dealt with is of a scientifically systematised nature or is part of everyday experiences we can say at once whether "e" is sufficient to establish conclusion "c", and if not, whether much more evidence or just a little more is needed. Again, normally, there is a large measure of agreement among different people on the extent to which given evidence justifies a given conclusion (though of course as pointed out above, this is by no means always the case).

It is worth recalling some points of a general nature relating to this case. First, since the utterances used to state 'c' and 'e' are as far as possible descriptive and thus not specifically labelled as being evidence or conclusion, we shall meet the same sentences being used in different contexts to state both evidence and conclusion - 'It is raining' is suitable for use in either role, and only by considering the particular situation in which it is used can we identify it as "e" or "c". Secondly, "e" will often be a conjunction of various statements and we must be careful to ensure that only statements which can be regarded as evidence get into the

conjunction; neither statements which logically imply "c", nor statements to the effect that other statements which are in "e" imply "c", are evidence for "c", even though their truth along with that of the other (genuine) evidence statements may put us in a position to say that "c" is true. We must be careful of avoiding such statements in "e" if we are to remain objective on the question of whether the relationship f(e,c) can be given a value a priori.

It is not accidental that our fashion of writing the result of an assessment of the strength of evidence "e" for conclusion "c", is "f(e,c)". This form has been chosen for its echoes of confirmation theory discussions. If on the basis of the evidence, "e", we can say that the conclusions "c", is the case and if that conclusion is of the sort which interests the confirmation theorists, then we turn out to have been dealing with a typical instance of the confirmation problem. Conversely we might extend the range of question posed for confirmation theory to include not only "how can we ascertain that a lawlike generalization is the case?", but also "How can we know that a lawlike generalization is probably the case" etc. In short, the evidential interpretation of modal distinctions allows us to present the question of when each of the variously modalized claims is acceptable on the analogy of the problem of confirmation. An approach to confirmation theory beginning with this more general problem might also be worth conducting.

Both this relationship to confirmation theory and a consideration of Carnap's view that probability can be determined a priori (207F)

lead us to pose the question "Is the value of f(e,c) decidable a priori?" It would seem that if Carnap is right we have to conclude at least that if the value falls in the range corresponding to "It is probable that..." then that value is a priori determinable.

We have noted that it is often a widely agreed matter whether a given thing can be said to be, for instance, probable on the available evidence, and further that the investigation of the matter is not one which requires further experiment or even "wait and see" techniques before we can state our own conclusion. These two factors seem to support the idea that a thing's being probable is an "objective" matter and that - since there is no need to collect evidence - it is "a priori" in at least a broad sense.

None the less, a person's decision that on evidence, "e", a certain conclusion, "c", is probable (or that the value of f(e,c) falls in the strength range appropriate to probability) is not one which is independent of that person's views about the facts. In particular his views about the laws of nature will have a strong influence upon his assessment of whether "c" is certain, probable, or only possible on the evidence "e". It is just because of this third influence on the value of f(e,c) that we have cases where two well-informed observers can have honest differences about what is or is not probable. Not only laws of nature but also other principles or beliefs can lead to such differences; for example our trusty "What goes up comes down." or "MacFadzean generally has kippers if they're on the breakfast menu.". As a general term for

this type of background against which assessments of probability, possibility, etc., (and fact) are made, we may introduce the term "presuppositions". It seems then that we shall only achieve complete intersubjective agreement for assessments of F(p,e,c) which gives the value of f(e,c) for a given set of presuppositions, "p".

Rather than say that valuations of f(e,c) can be regarded as a priori, we should say that the explicit addition to "e" of all presuppositions involved allows us to reach an objective valuation without consideration of any matters not stated in the evidence, presuppositions or conclusion. Even this is rather different from the normal notion of being a priori; it is usual to take a priori as opposed to empirical, yet the evidence and presuppositions used in reaching valuations of f(e,c) may be riddled with statements making factual claims that could be experimentally checked, and the reliability of the modal expression ultimately derived can be attacked via an attack on the evidence or presuppositions used in deciding that that was the appropriate modality.

This factor gives rise to one of the persistent conundrums about probability and possibility. There are those who say that it is an objective matter whether something can be said to be possible (or probable) or not; they will perhaps admit that a change can come in our judgement of such matters as a result of some new discovery or in the light of the event itself, but they maintain that what at a given moment is possible, is just what a correct assessment

of the evidence shows to be so. On the other hand there are those who say that the matter dealt with is or is not the case - definitely, even though it may now still be in the future - and that therefore a statement to the effect that that matter is (or will be) just as it in fact is (or will be), must be at least as acceptable as a claim representing the conclusion which would be reached about that matter, taking all at present available evidence into account.

From these two positions we seem to be able to derive the following paradox:

(i) At time t all the available evidence indicates that event E will not take place.

(ii) That evidence is strong enough to justify a "probably".

(iii) Event E, is, none the less, going to happen.

(iv) From (i) and (ii) we may conclude that E will probably not take place.

(v) From (iii) we may conclude that E will probably take place (or that it will take place).

(vi) The conclusions drawn in (v) and (iv) are contradictory.

Our solution to the conundrum is that although (vi) is true, the two conclusions are not drawn on the basis of the same presuppositions. The "c" with which we are concerned is "event E will not take place." From (i) and (ii) we discover that there is a set of presuppositions, "p", such that F(p,e,c) has a value in the strength range corresponding to "probable". However anything which can assure us of (iii) cannot be present in the "p" and "e" involved in such a case;

for, if that thing were part of the evidence it would itself be conclusive evidence - otherwise we can never know that E is going to happen - and if it were part of the presupposition set then even though the evidence tended to suggest that E would not occur we should still not conclude that it would probably not occur - rather, because of the supposition set we should conclude that it would occur despite the appearances to contrary. Thus as soon as we suppose that the fact of E's going to happen is taken account of in either the evidence or the presuppositions, we shall not conclude that the event E will probably not happen, no matter how much other evidence of opposite tendency we consider. The paradoxical effect is reached by confusing different semantical levels: we, as spectators, note that observations (i) and (ii) can be made about the evidence available to a person, P, and we conclude that person P is in a position to reach the conclusion that E will probably not take place. Only, however, when we put ourselves in the position of P shall we reach that conclusion. We know better, though, for we know (iii) to be true. We shall not, therefore put ourselves in the position of P, even though we may accept that all the facts which constitute P's evidence are exactly as he supposes them, and that all the theories which he employs in his presuppositions are valid. What we say is that E probably will take place, even though the evidence is against it.

This last type of remark is of much importance to the evidential theory for without the recognition of the role of presuppositions in helping us to interpret the facts which constitute the evidence, it would seem that it was inconsistent both to advocate the evidential interpretation and to ascribe any meaning at all to claims that things were probable despite the evidence being against them. We see however that this need mean nothing more than that the speaker possesses knowledge which is not strictly speaking <u>evidence</u>, but which leads him to a different interpretation of the facts or to a different conclusion about "c". Others may find it better to claim that in such cases we should say that the evidence only <u>seems</u> to point the other way; but we find this far-fetched, particularly in the case of those who have knowledge of the facts and wish to say that things have turned out surprisingly "given the evidence".

Although we have been able to go along with the idea of interpreting the value of f(e,c) as a strength and have clarified the procedure for giving the evidence-conclusion pair a valuation, we must leave this topic without giving a satisfactory account of how the various "strengths" are to be ordered, and whether the calibration of this continuum of strengths is identical to that of the continuum of strengths appropriate to the modal expressions.

It seems that for some cases the set of valuations assigned to f(e,c) should be mapped onto values of a variable which we might

describe as "the extent to which 'c' is confirmed by 'e'." On the basis of our discussion of presuppositions we may conjecture that determination of the extent to which a given lawlike statement is confirmed by other statements is also a matter which is not independent of the presuppositions present, and further, that in the advance of science confirmation behaves "dynamically"* - that is, the presuppositions used in deciding what degree of confirmation to diagnose in a given case will include matters whose status has been decided in earlier confirmations.

The notion of "strength" is certainly a familiar enough one in ordinary speech. Relative to a given conclusion various evidence is - in quite natural speech - characterised as having various strengths, and we say without risk of misunderstanding that given evidence is "not strong enough" to establish a given conclusion. But for the present purposes we should have to find a unit of strength and preferably to discern some natural way of marking the borderline between a strength suitable for saying "possibly" and one suitable for saying "probably" etc. It does not seem to us that the notion of evidence is one which allows any farther analysis in terms of units of evidence so that we could, for example, say that the more there were the stronger the evidence. It was difficult enough to imagine enumerating "commitments" but in the case of evidence it seems to us - intuitively - that the basic way of

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*The term is used more in the systems theory sense than in Kant's.

enumerating or measuring is in terms of strength and that attempts to reduce strength to something more fundamental raise more questions than they answer.

Despite our inability to found an extensive explanatory structure upon it, we feel that the evidential interpretation of modality casts a refreshing light on the subject. If one considers necessity in particular it is noteworthy that whether one regards necessity as being the modality appropriate to cases where the evidence available is perfect, or as being the modality appropriate to cases where evidence is as such irrelevant (or regards these two evidence situations as characteristic for "natural" and "logical" necessity respectively), the evidential interpretation as extended in the foregoing pages is consistent with the ideas we have brought forward in Chapters VI and VII. The introduction of the notion of presuppositions - which was itself necessary as a defence for the evidential interpretation - equips the evidential interpretation with something corresponding to the theoretical background and/or definitions and conventions in the assumption-set of our own theory. It is only when two speakers have the same theoretical background (i.e. presuppositions) that they can be expected - on given evidence, if evidence is required - to reach the same conclusions about what is necessary. Where for natural necessities we have assumed laws of nature to be in our assumption-sets, the evidential interpretation will have just these laws of nature amongst its presuppositions,

and where for other "Logical" necessities we have assumed definitions or conventions employed to have been included in the assumption-set, just the same definitions or conventions will have to be stated amongst the presuppositions required to reach the modal utterance according to the evidential interpretation. One obvious difference which arises is that we have not had any cause to introduce the notion of evidence when considering necessity - in effect we were able to make do with only presuppositions; it would be paradoxical, to say the least, to hold that for the case of at least one modality the evidential interpretation of modalities explained the appropriateness of the modality without reference to evidence at all; there is thus certainly this difference - the evidential interpretation requires us at least to consider whether evidence is relevant to the conclusion to be reached; our own theory does not. The reason for this is probably only that the evidential interpretation is concerned with justifying the choice of a form of words while our theory deals with assigning and value to it - hence we need not consider how the form of words was reached.

Only, Alternatives and Possibilities

A particularly interesting approach to necessity is that of paraphrasing expressions employing terms such as "necessary" and "must" by expressions indicating the absence of alternatives. We now turn our attention to this topic, addressing ourselves in particular to the word 'only' which is frequently to be found in such paraphrases and to the question of whether the words

"Possibility" and "alternative" which are also frequently to be found in them should be regarded as referring to "things". In doing so we cast some light on the uses of the phrase "can only".

We may take as a preliminary example the utterance "Pianists necessarily have hands." The suggestion of lack of alternatives creeps into many of the most natural paraphrases:

"You can't be a pianist unless you have hands."

"Only if you have hands can you be a pianist."

"For being a pianist there is no alternative to having hands."

Even more convincing illustrations can be given if we consider "natural necessity" claims such as "Necessarily a current tends to flow across a potential difference." We say here quite naturally that there is no alternative to the current's behaving this way, and as a variant of "no alternative" we can almost always use the words "the only possibility" in slightly different sentence-structure:

> "The only possibility is for a current to tend to flow across a potential difference."
>
> "The only possibility for playing the piano is to have hands."

These examples raise a further question viz. "Is it justifiable to reify 'alternatives' and 'possibilities' as we here appear to?"

Let us begin by examining whether we can eliminate the word "only", confining our study to the other expressions mentioned. This seems an attractive idea because, as will have been observed, when "only" is used in a paraphrase of some claim that a thing is

necessary, it is almost invariably accompanied by other words which either are thought of as modal in the traditional sense, or are terms which we shall in any case have to discuss.

Even cases where no other modal word is obvious can often be shown to be ones in which it would be more felicitous to use 'only' along with such a modal term. For example, "Only those wearing badges will get in." is as it stands ambiguous; the use of it which is appropriate for conveying that it is necessary to wear a badge if one is to get in, is better rendered by saying "one can only get in if wearing a badge." - where the "can" is one of the words traditionally associated with modality.

In the field of talk about human activity a more troublesome case arises. Starting from "I'll have to take that book back to the library." we first produce the paraphrase "The only thing for me to do is to take that book back to the library." This is a case where "only" occurs on its own, but it is at once clear that we can substitute "possibility" for "thing ... to do" giving "The only possibility for me is to take that book back to the library.". We call this troublesome, not because it is difficult to get at the formulation which employs "possibility" as well as "only", but because we are left with a problem which is probably just as great as that of showing that "Only" is not the source of the modality in such cases. This problem is that of explaining <u>why</u> "possibility" should be equivalent to "thing ... to do". This question will come on the order again when we turn to the question of the

reification of possibilities; for the moment let us try to survey the variety cases which arise when we try to use the phrase "only possibility" as standard in paraphrasing necessity and related notions.

At this stage a set of examples is in order. These are given first in their form as claims that something is necessary or "must" happen:

A. What goes up must come down.

B. It was necessary to take avoiding action.

C. Playing with fire necessarily leads to getting burnt.

D. It is necessary to wear a badge to get in.

E. That is not necessarily the case.

F. That is a necessary truth.

G. Psychotics necessarily have delusions. (by definition)

H. Psychotics necessarily have delusions. (it is an inevitable consequence of their being psychotic.).

(The form of words used for G and H is ambiguous. As indicated, G is used to say something which is so by definition, whereas what is said in H is purportedly based upon discoveries. Although we do not presuppose this, it may be remarked that the cases correspond to those of necessities 'proper' and 'natural necessities' as dealt with in Chapters VI and VII. The ambiguity is resolved when we consider the paraphrases).

The search for reformulations in terms of "only possibility" yields the following list - all in some sense understandable, though with widely varying degrees of naturalness:

A1. The only possibility for something which goes up is to come down.

B1. The only possibility (for me/one) was to take avoiding action.

C1. The only possibility is that playing with fire will lead to getting burnt.

D1. The only possibility of getting in is to wear a badge.

E1. That is not the only possibility.

F1. The only possibility is that that is true.

G1. The only possibility for someone being (called a) psychotic is his having delusions, (or 'if he has delusions').

H1. The only possibility is that someone who is psychotic will have delusions.

or H1'. The only possibility for someone who is psychotic is that he will have delusions.

The relationship between "possible" and "can" suggests the use of another phrase - "can only". This turns out to be somewhat less powerful*;

A2. If something goes up it can only come down.

B2. One could only take avoiding action.

C2. Playing with fire can only lead to getting burnt.

D2. One can only get in if wearing a badge.

E2. That only that can be the case, is not so.

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*Or perhaps in another sense "more powerful"; it has a range of dissimilar idiomatic uses which interfere with our aim of finding a general paraphrasing tool. It is thus "strong" in use.

F2. That can only be true.

G2. People can only be psychotic if they have delusions.

H2. People who are psychotic can only have delusions.

It is clear that "can only" yields a far less satisfactory group of paraphrases. This may be ascribed to the fact that it is a more colloquial phrase than "the only possibility" and as such as a limited range of set uses; where no such set use is appropriate to a given member of the list A - H, it is only by means of considerable effort that we can reach any paraphrase in terms of "can only" at all. Further, from these examples it is clear that "can only" is itself ambiguous. On the one hand it can be used to say that a given <u>thing</u> (typically a physical object) is of restricted usefulness, powers or incidence. On the other hand it can be used to point out one constant aspect which, no matter how other features may vary, will never be absent from, or untrue of, what is being spoken about. Thus one might in the first way speak of a vehicle as being so specialised that it <u>can only</u> run on tracks; while in the second sense we use A2 to say that everything which goes up will, among other things and independently of what else happens to it, come down.

If we are to retain the meaning of the originals we shall have to understand the "can only" in A2, B2, C2, F2 and H2 in this last way. D2 makes wearing a badge a condition for getting in - but does not say that one is forced to <u>go</u> in, i.e. does not say that it

is bound to be part of the future life history of any "one" wearing a badge that they do get in. G2 is similar to D2, except that it deals with the conditions for being called a psychotic (correctly) rather than those for becoming one. G2 and D2 might then loosely be said to give "logical conditions" and "practical conditions" respectively. A third type of 'can only' - that which specifies conditions should thus be added to our list.

To maintain at least some semblance of naturalness we have split the "can" from the "only" in E2; what we intend E2 to say is roughly "That 'what is the case' can only include that, is not so" where "can only" has the meaning ascribed to it in A2. The difficulty in finding a phrasing for E2 may be traced to the behaviour of "the case" - in practice to "be the case" seems nothing different from being part of "what is the case" It is possible in any case to fit the "can only" of E2 into the same class as that of A2 and we are left with three types of case:

(i) "can only" in the enumeration of the totality of the restricted possibilities.

(ii) "can only" in the specification of a possibility which is inevitably realised.

(iii) "can only" (normally in conjunction with "if") in the stating of conditions for something's meriting a given description or something's coming into a certain state.

Thus for at least these cases of "only" used in a modal way we can provide an account of what is being said, by making reference to "possibilities" - which we shall have to discuss in any case - or

to "conditions" - which for reasons of space we shall leave aside. We have not however eliminated the "only" completely; we have focussed upon its role in the phrase "the only possibility" and we shall now consider that phrase and the related one "the only alternative", seeking, by means of a consideration of how one enumerates possibilities and alternatives, simultaneous solutions to both the problem of "only" and that of the justifiability of reifying the two notions.

Not Possibly Not

Our discussion has centred on the possibility of paraphrasing talk about necessities by means of talk about the "only possibility" Another proposal for paraphrasing such talk is that we should substitute "It is not possible that .... not ..." for the "necessarily" from which we began. This traditional paraphrase is enshrined in most "modal logics" and its presence as an equivalence is taken by Łukasiewicz as one of the preconditions for a system's being a basic modal logic (538K). Our motive for considering the formulation here is both that it is another possible paraphrase for necessity and that it does not carry the ontologically committed tone of the phrase "the only possibility", and a comparison of the two may be helpful in deciding the second of our two questions.

Let us take some examples:

K. It is necessary that all bachelors are male.

L. It is necessary that winter is colder than summer.

Ka. The only possibility is that all bachelors are male.

La. The only possibility is that winter is colder than summer.

Kb. It is not possible that not all bachelors are male.

Lb. It is not possible that winter not be colder than summer.

The question now arises whether the a and b forms are the same. This we can approach by means of a quasi-symbolization of the type Quine frequently uses. First we transform the b forms into synonymous forms which talk of "possibilities"; that is, for "It is not possible that" we write "There is no possibility that". Further we turn the a form from "The only possibility is" into "There is only one possibility, viz. that". From these two forms we may derive the symbolic expressions:

a' $\sim (\exists x)$ (x is the possibility that... not...)

b' $(\exists x)$ ((x is the possibility that...) and

$(\forall y)$ ((y is the possibility that...) $\equiv$ (y = x)))

Here '=' is used to indicate identity between the individual constants concerned; we shall not pursue the symbolization to the point where it becomes important which order our quasi-predicate calculus has.

The 'not' which remains in a' is clearly of logical import but it is difficult to take account of without translating into symbols.

It certainly cannot be represented by any normal truth-functional symbol (such as tilde) in a position governing the whole phrase "(x is the possibility that..)"; to put a tilde there for example would lead to the conversion of a' into "$(\forall x)$ (x is the possibility that...)" and in general we may say that a symbolization of this "not" in the position immediately following the quantifier can hardly yield the desired result since the negation involved does not at all apply to the ascription of the stated identity to 'x' which ascription must remain unaffected whether or not the rest of the expression involves a "not". To deal with this objection we must introduce the symbolization of the "not" at the point in the expression where it actually occurs. We suppose the "...." to be replaceable by the letter 'p' and thus obtain:

a'' $\sim (\exists x)$ (x is the possibility that $\sim$ p)

b'' $(\exists x)$ (x is the possibility that p) and $(\forall y)$((y is the possibility that p) $\equiv$ (y = x)).

Now even supposing it to be permissible to symbolize 'z is the possibility that q' as 'Pzq' in such a way that 'q' continued to behave as a normal propositional variable, still we obtain from a'' and b'' only

a''' $\sim (\exists x)$ (Px p)

b''' $(\exists x)$ (Pxp) and $(\forall y)$ ((Pyp) $\equiv$ (y=x))

There remains a discrepancy between the two formulae. For example a''' is not inconsistent with "$(\exists x)(\exists y)$ ((Pxq) (Pyq) $\sim$(y=x))"

even though this is explicitly in conflict with b''' as can be seen by reading it "There are several, non-identical, possibilities that ...."

What we have attempted to explore with the aid of this naive quasi-symbolization are the results of following the most naive interpretation of the phrases "It is not possible that ... not" and "The only possibility is...". The two ought to have been equivalent, but even after substantial concessions to rigour we have reached formulations which quite clearly are not equivalent. The failure of this enterprise may of course lie on us; it is therefore heartening to note that we have reduced yet another form of paraphrase for necessity (the "not possibly not" type) to terms of "possibilities". So long as this reduction does not contain the error which has prevented us finding the two paraphrases equivalent, we shall have even more reason to seek an answer to the remaining question - which may be phrased as "What is a possibility?". In fact our answer to this question will help to provide the reason for the failure of the attempt to show the two paraphrases equivalent in terms of the normal calculus.

What is a possibility?

One talks about "possibilities" in the plural; we have seen that "the only possibility" is an important concept and that the existence of "no possibility" can also be seen as important. Further it seems permissible to talk of "a possibility". There

remains one locution which we do not use in the full range of uses which other nouns generate, viz. "the possibility".

We do of course sometimes talk about "the possibility" - "The possibility that he will come is remote." or "The possibility of his being delayed must not be forgotten." are quite standard cases. Yet if we consider that in other cases where we talk for instance, of "the only...." we can, without ceasing to make sense, drop the "only", then it will be clear that there is something odd about possibilities. For example:

(i) The only answer to our problem is to seek help.

(i)a The answer is to seek help.

(ii) The only possibility is that playing with fire will lead to getting burnt.

(ii)a The possibility is that playing with fire will lead to getting burnt.

(iii) That is not the only possibility.

(iii)a That is not the possibility.

While (i)a is a perfectly intelligible utterance, (ii)a and (iii)a are not. This despite the fact that the forms with "only" are intelligible (if not elegant) and that in both cases that which is talked about would be included in "the possibilities" for the situation involved.

In this respect possibility is worse off than probability. In (ii)a at least we should probably be able to make sense of the sentence resulting when "The probability is" is substituted for "The possibility is"; in such a case we would read the words as

equivalent to "It is probable". Such a reading however, does not fit in (iii)a and here it is as impossible to fit in "the probability" as it was to fit in "the possibility".

The difference between (ii) and (iii) is that in the former it is made explicit what we are talking about. In (iii) reference is made by means of "that" to something which may or may not be possible or probable, depending upon what it is that is being referred to. In (ii) a particular relationship is stated in sufficient detail for us to form a judgement on it, and because we normally take it that only one thing is probable at a time we are in a position to treat the utterance (ii)a as claiming to state what is probable. This approach gives us so little help in dealing with (iii)a that one is unlikely to begin with it except in the case where "that" is used in the specialized sense (normally stressed in speech) where "That is not what you're lookingfor!" is an indication that one can find what is sought either by investigating things very closely related to, or maximally dissimilar to, what one is at present investigating. In this last case the interpretation put on "That is not the probability." would be something like "What is probable in this case is something to which that is a clue, but not that itself." This is however to go too far in exploring the psychology of "putting a sense on" defective utterances. The point of the example is that "the possibility" cannot be fitted in in such cases because we normally accept that a whole range of things is possible in any

given situation and even if we know the situation we either do not know why this possibility is being picked out as specially "the" one, or else cannot on the mere basis of a statement of what is possible gain enough idea of the situation involved to be able to reach any sensible conclusion. Thus here again we are apt to abandon the utterance as a complete failure rather than rewrite it as "It is possible that.." In this we are correct for it is hard to imagine that anyone would characterise a possibility as <u>the</u> possibility if he regarded it as merely one of many. Similar considerations apply to "alternative" - although "the only alternative" is a pointful expression "the alternative" makes sense only relative to an explicit statement of situations to which an alternative is sought or as a reference back to something introduced more fully earlier.

Still all this does not rule out the other uses of "the possibility". We shall continue to talk about the possibility of a clerical error and so on. Now since the object of the present discussion is to clarify the extent to which a possibility is a 'thing' and should be reified, it will be helpful to compare the behaviour of our phrase "the only possibility" with that of phrases about everyday things, where "only" also appears. Consider then "The only child of that family is John." and "The only hope is that we shall reach land tonight." We may bring out our point about

"the possibility" by performing on utterances of the "the only possibility" type the same inversion required to turn these two sentences into ones using 'the child' and 'the hope' respectively:

(iv) The only child of that family is John.

(iv)b The child, John, of that family is the only one.

(v) The only hope is that we shall reach land tonight.

(v)b The hope that we shall reach land tonight is the only one.

(ii) The only possibility is that playing with fire will lead to getting burnt.

(ii)b The possibility that playing with fire will lead to getting burnt is the only one.

However strained (iv)b and (v)b, it is clear that (ii)b conjures up even stranger visions. It seems to suggest that out of all the possible possibilities the one referred to is the only one which is an actual possibility. The normal function of the definite article is to allow us to indicate a particular one of a range of things falling into the same category or under the same description. When we are dealing with what is possible in a given situation and in fact only one possibility exists - i.e. there is one thing which we can say to be "necessary" in the situation - there simply is no range of possibilities out of which one is indicated by the use of "the".

Category words

This characteristic of "possibility" is shared in varying degrees by other words, to which we shall refer as "category words".

Consideration of a further example with less philosophical deadwood clinging to it, will illustrate this notion. As has been pointed out, "necessary" can also frequently be dispensed with in favour of formulations using the phrase "no alternative". "Alternative" displays the same aversion to certain combinations with "the" as does "possibility", and a closer examination of it will clarify the notion of a category word.

For the most part "alternative" behaves as a normal noun. We talk naturally of alternatives "existing", of some being "better" or "worse", of there being "some" or "no" alternatives, and so on. This behaviour of the word might lead us to suppose that an alternative should be seen as a "thing" - like a house or a draught or an atmosphere. It will be readily agreed, however, that alternatives are not things which we, without any effort on our own part, are confronted with or discover; there are many things which are alternatives but each of these things is first experienced as something else ("what it <u>is</u>" one might say) before we recognise it as being "an alternative" or else is simply postulated by us as an intellectual construct out of things with which we are familiar (in which case the alternative may never be realised and the 'thing' which is described in answering the question "What <u>was</u> that alternative?" is one which never exists).

To express this point in yet another way we might say that we do not perceive or conceive a thing as being "an alternative" with

various other characteristics; instead we perceive or conceive of things each as what it is, and then, in a logically subsequent stage, dub certain things "alternatives". This is not to say that the stages are psychologically separate: it is a common enough experience to 'try to think of an alternative' - but the successful conclusion of such an attempt is thinking of a fully-fledged 'thing' which in addition is fit to be dubbed an alternative, rather than that we should start by thinking of some vague thing with no characteristics other than its being "an alternative" and then going on to fill in the detail of the other characteristics it should have.

Typical examples of things which do gain the title of being alternatives are things which can be chosen for - e.g. courses of action or the outcomes of courses of action, but not things as such, like "the moon"*. When a given course of action is described as an alternative we can always ask "what is the alternative?". The answer as we have seen is that it is nothing other than the course of action. And in general alternatives are just the things which are dubbed alternatives. If we persist in asking questions about all the things which are said to be alternatives we shall only learn more and more about the particular things involved, but will never learn anything about a supposed quality of "alternativeness"

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*It is interesting to note that the "no alternative" paraphrase of "necessary" breaks down for the case of "necessary being" because of this. The only sorts of thing to which there could be "no alternative" in this case would be (i) procuring the existence of the being, or (ii) saving the being from destruction; the being as such is not an object of choice.

which could be the basis of our identifying further things as alternatives. The question which we should be asking is not "What is an alternative?" but "Why should we call something an alternative?".

And to this question there is a straightforward uncontroversial answer; "To show that it should be considered in a certain way relative to other 'alternatives' when discussing the subject in question." That is, to call something an alternative is not to give any information about what it is, or to put us in a better position to identify it, but is to give it a place in the context of some decision-making procedure. We could say that the thing had been put into a "category" even though it had not been described in any way. The point of our discussion of "alternative" then, is to point out that there are such category words and that for them the question "What is a .....?" is out of place; the only relevant question is "Why do we call something a ...?".

"Possibility" is, we suggest, like "alternative", a category word, and the interesting question to ask is therefore "Why do we call something a possibility?". The inappropriateness of the question "What is a possibility?" is a warning to us that even though most of the things which can be said about "ordinary" things can be said about possibilities and even though we can to some extent count them, they are not things in the normal sense and a full reification of them is not called for.

Although possibilities are not to be reified as normal things the word "possibility" does interact with "only" in the way in

which words for things do. Thus we can compare "The only child of that family is John." and "The only possibility is that playing with fire will lead to getting burnt." but in doing so we must remember that 'being a possibility' is belonging to a category while 'being a child' is being a thing of a certain type. When we perform the inversion from (ii) to (ii)b we introduce the phrase "the only one", and probably because of this in combination with the use of the definite article applying to "possibility" the impression is made overwhelming that we are taking possibilities to be things.

In conclusion let us return to two cases which have arisen in the course of our discussion and which have not yet been fully explained. First is the use of "can only" to help state a restriction on the possibilities open to something under discussion. This case did not occur in our list of examples but we have cited as an instance:

"The railway engine can only run on tracks."

As pointed out this states not that the railway engine must, come what may, run on tracks, but that the railway engine if it is to run must have tracks. Talking in terms of "possibilities" one might say that the claim had to do not with a restriction on the possibilities for the railway engine but with a restriction on the possibilities for its running. This we can bring out by reorganising the sentence to give:

"The railway engine can run only on tracks."

or when dealing with the form using "possibility" by saying

"The only running possibility for the railway engine is on tracks."

Consideration of a few of the examples from our list will show that such inversions are not satisfactory for the other sorts of "can only";

A3. If something goes up it can come only down.

B3. One could take only avoiding action.

C3. Playing with fire can lead only to getting burnt.

The unsatisfactoriness of the inversion is, of course, strongly influenced by the verb in question. One "can come" in many ways and the restriction to 'down' is a substantial one which can hardly be seen as identical with the imposition on the thing concerned of the necessity that it at some time or another "come down". B3 seems to suggest that whatever action one decided to take would have turned out to be avoiding action, whereas B2 suggested that of all action open, only taking avoiding action should have been considered. The difference between C2 and C3 is of an even vaguer nature because of the vague nature of the verb "lead" - in fact all that is being said is that one thing inevitably follows another. None the less C3 unlike C2 would seem to be incompatible with playing with fire's leading to <u>both</u> amusement <u>and</u> getting burnt.

Suffice it to say that the displacement of "only" from its place next to "can" distorts the prima facie meaning of the sentence in the case of these "can only" sentences but not necessarily in the case of the railway engine example. This might lead us to try to reduce the latter cases to ones in which various types of possibility were dealt with (e.g. "running" possibility). Our classification of "possibility" as a category word makes us chary of applying adjectives to it, and we would suggest that rather than embark on this course (which leads in any case to odd-sounding results) one should take it that in cases where "can" is used as an auxiliary verb and the composite verb then used in conjunction with only the point of using the main verb in question is to indicate a restriction of the field in which possibilities are being considered. Thus in the cited example by saying "can run only" we indicate that we shall not concern ourselves with the whole field of possibilities for railway engines but shall confine the discussion to those relevant to the running of railway engines.

The second point which requires comment is that of the discrepancy found between "not possible ... not" and "the only possibility". It will be recalled that the difference between the two quasi-symbolizations was shown by a formula which was inconsistent with the result we had reached for the latter case but quite consistent with the formula based on "not possible ... not". The explanation in words which we gave for the formula in question was "There are

several, non-identical possibilities that ...". While we do not suggest that the quasi-symbolization is hereby shown to be on the right track, nor that the two types of paraphrase are identical, it should be noted that our discussion of "possibility" allows one to regard this difference much more lightly.

The notion of different non-identical possibilities is one which seems peculiarly empty if we take "possibility" to be a category word. We have seen that to call something a possibility is to put it into a certain category and not to indicate a reidentifiable member of a species of thing. Further when we talk of "possibilities" we adjoin to the word "possibility" a statement of that which is to be put into the category concerned, and hence when we say "there is possibility that ...." we are saying that "...." is to be put into the category "possible". Now the offending formula was interpreted as saying that there were several possibilities that "....". On the account just given, however, this amounts to saying no more that the one "...." should be put several times into the same category, "possible". It is difficult to feel that it matters much if the thing in question is counted as possible only once or many times. More seriously, category-words although behaving as ordinary nouns cannot be regarded as subject to the same rules about identity-claims as are other nouns.

Conclusion

In Part 1 we have suggested that the most popular of existing symbolizations of modal talk lacks, and probably cannot be given, a coherent interpretation in terms of individual utterances such as one would wish it to deal with. In Part 2 we have outlined the principles which should govern attempts to construct a logical system suitable for such an application - or for many others. In Part 3 we have attempted an ostensive definition of the "logical" treatment of the modalities. That is to say we have ourselves attacked certain problems concerned with necessity and examined how one could and should symbolize various types of utterance falling under the description "saying that something is necessary". We have concluded Part 3 with the discussion of certain interesting points which have come to light in the course of our preparation of the foregoing Chapters; both the evidential interpretation of modality and the category word account of "possibility" seem to us to suggest interesting bases upon which approaches to the symbolization of modal notions could be founded - independent of the account we ourselves have given in Chapters VI and VII. As stated at the outset it was not our purpose to present solutions, but rather to establish that the topic "Modalities and Formal Systems" is one which, given a more rigourous fundamental approach than has been usual, offers considerable scope for development. If we have

established that the problem is not merely one of adapting or complementing the Lewis systems to deal with the deontic modalities and if we have succeeded in showing some of the methods and directions for future development, we shall have been successful.

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(i) Lewis modal systems, or other important formal systems with pretensions to the title 'modal logic'.

(ii) Formal or semi-formal analysis of modalities in the broad sense - including temporal and juridical logics.

(iii) Topics discussed in the foregoing dissertation.

In view of the inevitably large overlap between this bibliography and that compiled by J. Dopp (published in Modal Logics, by R. Feys), the latter has been included in its entirety (in some places it has been corrected or amplified).

Entries are arranged in alphabetical sequence of the names of their authors, and publications of one author are arranged in sequence of publication. Each author has been assigned one or more three-digit numbers; each entry has a code consisting of such a number followed by a letter. References to the bibliography in the body of the dissertation are normally made by means of such codes plus, where appropriate, a page number (in the case of books page numbers always refer to the last edition referred to in the bibliography).

Entries have, as far as possible, been standardised. For articles the title, journal, volume, year, and pages are given in that order; where the article is a translation the original title and the translator's name are usually added to the title; where there are

co-authors these are listed in parentheses under the entry (and the article is listed again under the various co-authors). In the case of journals or series which occur frequently a three- or four-letter code is used, the year appears after the first comma and the pages after the second. The codes are listed overleaf.

In the case of books the sequence is similar save that in place of the journal and volume, the publisher and place of publication are shown. The titles of books are underlined, as are the names of journals not shown in code-form.

Codes used for Periodicals and Series

AJAT	Ajatus. (Helsinki)
AJPH	Australasian Journal of Philosophy.
AMJM	American Journal of Mathematics.
AMLG	Archiv für mathematische Logik und Grundlagenforschung.
AMPQ	American Philosophical Quarterly.
ANAL	Analysis.
APHF	Acta Philosophica Fennica.
ARPH	Archiv für Philosophie.
BAMS	Bulletin of the American Mathematical Society.
BJPS	British Journal for the Philosophy of Science.
DIAL	Dialectica.
DILG	Dialogue.
DNOI	Dianoia.
DOMS	Dominican Studies.
ERKS	Erkenntnis.
FCAV	Filosoficky Časopis. (Československé Akademie Ved).
FRAN	Fransiscan Studies.
FUNM	Fundamenta Mathematicae.
HATH	Harvard University, Graduate School of Arts and Sciences. Summaries of theses accepted in partial fulfilment of the requirements for the degree of Doctor in Philosophy.

IEUS International Encyclopaedia of Unified Science.

IJPH Indian Journal of Philosophy.

INDM Indagationes Mathematicae.

INQY Inquiry.

JCOS Journal of Computing Systems.

JHID Journal of the History of Ideas.

JPHI Journal of Philosophy.

JSL Journal of Symbolic Logic.

KNED Koninklijke Nederlandsche Akademie van Wetenschappen. Proceedings of the Section of Sciences.

LETA Logique et Analyse.

MANN Mathematische Annalen.

MARV Mathematical Review.

MESC Methodology and Science. (Netherlands)

METH Methodos.

MFMP Monatshefte für Mathematik und Physik.

MIND Mind. (New Series)

MONI Monist.

MSIN Acta Mathematica Sinica.

MULL Modern Uses of Logic in Law. (Yale)

OSAK Memoirs of the Osaka University of the Liberal Arts and Education. B. Natural Sciences.

OSMJ	Osaka Mathematical Journal.
PAMS	Proceedings of the American Mathematical Society.
PASS	Proceedings of the Aristotelian Society, Supplementary Vclumes.
PHIL	Philosophy.
PHQU	Philosophical Quarterly.
PHRN	Phronesis.
PHRV	Philosophical Review.
PHST	Philosophical Studies.
PPRE	Philosophy and Phenomenological Research.
PRAS	Proceedings of the Royal Aristotelian Society.
PRZF	Przegląd Filozoficzny.
PSCI	Philosophy of Science.
RATI	Ratio.
RDEM	Revue de Metaphysiques et Morales.
RIFI	Rivista di Filosofia.
RIFD	Rivista Internazionale di Filosofia del Diritto.
RMET	Review of Metaphysics.
RVDL	Revue Philosophique de Louvain.
RVUE	Revue Philosophique.
SCIA	Scientia.
SSIN	Scientia Sinica.
STLO	Studia Logica.

STPH. Studia Philosophica.

SYNT Synthese.

THEM Theoria. (Madrid)

THEO Theoria.

VOPR Voprosii Filosofii. (Moscow)

ZMLM Zeitschrift für Mathematische Logik und Grundlagen der Mathematik.

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APPENDIX

With the assistance of a grant from the Committee for Advanced Studies and facilities made available by Prof. W. L. van der Poel of the Technische Hogeschool in Delft, the following computer programme has been written. It is designed to test whether formulae of a Lewis-like system are or are not theorems, and uses the decision procedure in terms of truth-tables which is set out by H. S. Leonard in his article "Two-valued Truth-tables for Modal Functions." (515A).

The development of such a programme is of interest from two points of view. First it demonstrates the suitability of truth-table formulations for use in mechanical theorem-proving for Lewis-like systems. Recent work of G. J. Massey (563A and 563B) gives the prospect that the truth-table approach can be applied successfully to various Lewis systems. Further our own comments in Chapter III on n-dimensional truth-tables indicate another approach, and one which without computational assistance can be of little more than theoretical interest.

Secondly work on mechanical theorem-proving in the predicate calculus may be related to this approach. In Modal Logics (315S) R. Feys presents a formulation of modal logic which is parallel to predicate calculus. That formal approach may be complemented by the observation that the principal difference between the modal systems and predicate calculi (apart from the ability of object language variables to masquerade as variables of the modal meta-language)

lies in the fact that the individual variables of the predicate calculus may range over a universe of indefinite extent while the variables of the modal system (which are taken as ranging over formulae) have a range whose variability can be represented in terms of the sequences of truth-value assignments to the formulae onto. which they may be interpreted.

The basis of much current work on automatic theorem-proving for predicate calculus (cf. works cited below) is the application of Herbrand's theorem. This shows that if a contradiction is present in some set of formulae that fact will be demonstrable in a finite number of steps if we consider the instantiation of all variables in the set of formulae with respect to elements of the 'Herbrand universe' (that is the set of constants constructible from all variables and all constants which occur in the original set of formulae).

We may conjecture that a truth-table presentation of this sequence of instantiations can be developed, and that the value-assignment procedures of Leonard and Massey will turn out to be special cases of such a presentation. Should this turn out to be so, experience gained in the use of this type of programme will become relevant to the more general problem of theorem-proving (which despite the advance made as a result of A. Robinson's development of the "resolution" method, is still in need of a more powerful procedure for reducing the number of clearly unpromising instantiations which most programmes test in their search for a contradiction).

The Programme

The programme shown here is written in LISP 1.5 for the IBM 7090. Detailed understanding of it will present considerable difficulties for those unfamiliar with that programming language, but we set out below some notes on the most important features of LISP and it should be possible to gain a reasonable impression of the structure of the programme by reading the comments which have been inserted and consulting the notes we give.

The programme is executed by reading in cards on which are punched 'TRUTHTABLE' and a putative theorem. The latter should be in Polish notation, using only the connectives 'N', 'M', and 'K', should be enclosed in double parentheses, and should have a blank column between each successive pair of letters of the formula. Thus to test ' ◊ (p. ~ p)' one reads in:

TRUTHTABLE ((M K P N P))

'TRUTHTABLE' is a function which has been added to the normal range of LISP functions by means of the definition shown below. The definition uses another non-standard LISP function, 'HEADINGLIST', for which the definition is also shown. These two definitions constitute 'the programme'.

The general structure of the programme is straightforward. Execution of TRUTHTABLE causes use of the function HEADINGLIST which provides TRUTHTABLE with a list of 'headings' - i.e. all well-formed

parts of the well-formed formula to which TRUTHTABLE is being applied. TRUTHTABLE then considers the successive members of the list of headings, and according to whether these are new variables, or applications of operators to expressions already considered, extends an array called 'TABLE' to make it the desired truthtable for the headings so far considered. In view of the impossibility of arbitrarily extending arrays in LISP, the extension is accomplished by making an array 'TAB' which is the extension of 'TABLE', then redefining 'TABLE' and copying the contents of 'TAB' back into it; a more elegant solution to this problem will be required in future versions of the programme.

The programme has been tested in part on the IBM 7090 at Imperial College, London and in part on the IBM 360/65 at the Technische Hogeschool in Delft.

LISP

A complete description of the version of LISP used here can be found in LISP 1.5 Programmer's Manual and an index to that work is to be found in The Programming Language LISP (both listed in the references below). LISP systems for the IBM 360 series are available from the University of Waterloo and the University of Stanford, both of which publish manuals for the respective systems.

The listing below shows only the definitions of the two functions TRUTHTABLE and HEADINGLIST. In the definitions the name of the function is followed by '(LAMBDA', then the dummy-names for the

arguments given to the function - here 'WFF', which is listed, i.e. is between parentheses, - and finally the instructions for dealing with the argument or arguments. The instructions are given in the form of a 'PROG': that is, the word 'PROG' occurs as first member of a list in which it is followed by a list of the programme variables used, and then a sequence of instructions.

The instruction sequence includes both expressions surrounded by parentheses and isolated words. The latter are labels - points to which the programme can return in loops. The former are expressions which can be 'evaluated'. Such an expression within parentheses must begin with a function-name; this is followed by the arguments of that function. Arguments of a function may simply be variable letters, but they may also themselves be expressions which have to be evaluated, and expressions can thus be nested within expressions to an indefinite depth.

The values which variables may assume are the following: (i) numbers, (ii) truth, falsity or the empty list (which we refer to as 'NIL'), (iii) lists (possibly containing other lists as members). The following symbols have a constant meaning:

'T' is used for truth

'F' is used for falsity

'NIL' is used for the empty list (i.e. that with no elements)

each decimal numeral is used for the corresponding number.

In practice 'F' and 'NIL' are indistinguishable, but interchanging them complicates the understanding of programmes considerably.

Where a function has a variable as an argument, the function is applied to the value which that variable currently has. Where a function has a constant as an argument the constant value is that to which the function is applied. Hence if the variable 'VAR' has the value 500 at this moment, the execution of the instruction (PRINT VAR) will cause '500' to be printed. Because numerals are constants, the execution of (PRINT 500) will also cause '500' to be printed. On the other hand where 'VA' is a variable whose current value is the list (A B C), the result of executing (PRINT VA) will be the printing of '(A B C)', and the result of executing (PRINT (A B C)) will probably be an error because the argument of 'PRINT' is evaluated, and the evaluation of an expression in parentheses is accomplished by applying the first symbol to the remainder as if that first were a function; unless 'A' is a function this will be an error. A result of this way of working is, however, that the execution of (PRINT (ADD1 VAR)) will lead to '501' being printed, for 'ADD1' is a function and it gives a result 1 greater than the value of its argument. In order to print the name of a variable we have to prevent that name being evaluated; this is done by applying the function 'QUOTE' to it. Hence (PRINT (QUOTE VAR)) will lead to 'VAR' being printed.

The following is a list of the functions used in the programme shown:

ADD1 : The value of (ADD1 X) is the number 1 greater than the value of 'X'.

AND : The value of (AND X Y) is 'F' unless neither 'X' nor 'Y' is or evaluates to 'F' or 'NIL'. Execution of (AND X Y) causes both 'X' and 'Y' to be evaluated and where these are clauses this may have further consequences (e.g. alteration of the values of variables).

APPEND : The value of (APPEND LIST1 LIST2) is a list consisting of the members of the list 'LIST1' (if any) followed by the members of the list 'LIST2' (if any).

ARRAY : The clauses beginning with '(ARRAY ..' are used to create arrays. The use of arrays is described under 'TAB'.

CAR : The value of (CAR LIST) is the first member of the list 'LIST'.

CDR : The value of (CDR LIST) is a list containing all but the first member of the list 'LIST'. Where the list 'LIST' has only one member the value of (CDR LIST) is the fixed constant 'NIL'.

COND : A clause employing the function COND has as argument a list of 'implication pairs'. Thus (COND ((A X)(B Y)(C Z))) contains the three implication pairs (A X), (B Y), and (C Z). The first expressions in these pairs are evaluated one by one (with possible consequences for the values of variables) and as soon as the value of one such first expression is something other than 'F' (or 'NIL') the corresponding second expression is evaluated and the rest of the COND clause abandoned. Where no first expression in any implication pair evaluates to anything other than 'F' or 'NIL' the clause after the COND clause in the PROG is evaluated.

CONS : The value of (CONS X Y) - as used in the programme shown - is a list in which the value of 'X' precedes the elements (if any) of 'Y' (which is either a list or 'NIL').

EQ : The value of (EQ NUM1 NUM2) is 'F' unless its two arguments are or evaluate to the same number or letter.

EQUAL : The value of (EQUAL X Y) is 'F' unless its two arguments are or evaluate to the same expression (whether a list, a member of a list, or a number).

ERROR : Evaluation of (ERROR X) causes an error-condition which stops execution of the programme; the value of 'X' is printed.

GO : Execution of (GO LABEL) causes the clause immediately following the label shown here as 'LABEL' to be the next one executed.

GREATERP : The value of (GREATERP X NUM) is 'F' unless the value of 'X' is greater than the number which 'NUM' is or evaluates to.

LENGTH : The value of (LENGTH LIST) is the number of members in the list 'LIST'.

LESSP : The value of (LESSP X NUM) is 'F' unless the value of 'X' is less than the number which 'NUM' is or evaluates to.

MAP : The execution of a clause beginning with MAP causes the following action. The first argument is evaluated, and should be a list. The form (FUNCTION (LAMBDA (X) .....)) is taken as the definition of a function, and this is applied successively to the list obtained, and to the CDR of the last list to which the form has been applied, until the value of that CDR is 'NIL'. The clause is executed because of the effect it has on the values of variables; its own value is unimportant.

MEMBER : The value of (MEMBER X LIST) is 'F' unless the value of 'X' is an expression which occurs as a member of the list 'LIST'.

NOT : The value of (NOT X) is 'F' unless the value of 'X' is 'F' or 'NIL'.

NULL : The value of (NULL X) is 'F' unless the value of 'X' is 'NIL' or 'F'. (NOT and NULL are written differently only for intuitive convenience).

OR : The value of (OR X Y) is 'F' unless either 'X' or 'Y' is or evaluates to something other than 'F'.

MAX : The value of (MAX X Y) is the greater of the values of 'X' and 'Y'.

PLUS : The value of (PLUS X Y) is the number obtained by adding the values of 'X' and 'Y' together.

PRINT : The value of (PRINT X) is the same as that of 'X'. Execution of this clause causes 'X' to be evaluated - which may have consequences - and causes the value of 'X' to be printed.

RETURN : Execution of the clause (RETURN X) causes termination of the present programme (i.e. PROG). The function concerned is thereby determined as having evaluated to the value which 'X' has.

REVERSE : The value of (REVERSE LIST) is a list containing just the same members as the list 'LIST', but in the reverse order.

SETQ : Execution of (SETQ VAR X) causes the variable with the name "VAR" (not the variable to whose name 'VAR' evaluates - the 'Q' indicates that the first argument is 'QUOTE'd) to take the value to which 'X' evaluates (Where 'X' is 'NIL, 'F', 'T' or a number the variable takes the fixed constant in question as its value, of course.

SUB1 : The value of (SUB1 X) is the number 1 less than the value of 'X'.

TAB : 'TAB' is not, strictly speaking, a function. The value of (TAB X Y) is the element in the row bearing the number to which 'X' evaluates and the column bearing the number to which 'Y' evaluates. That same element is fixed to the value 123 by the clause (TAB (QUOTE SET) X Y 123).

TABLE : 'TABLE' is analogous to 'TAB'.

TERPRI : Evaluation of the clause (TERPRI) causes the printer to skip one line.

TRIM : The value of (TRIM LIST) is a list containing all different members of the list 'LIST' but without any duplicates.

ZEROP : The value of (ZEROP X) is 'F' unless 'X' is or evaluates to zero.

MAX : The value of (MAX X Y) is the greater of the values of 'X' and 'Y'.

```
(HEADINGLIST (LAMBDA (WFF)                                                  HEADL010
          (PROG (A FIRST REST CUR PARTS)                                    HEADL020
          (SETQ A (REVERSE WFF))                                            HEADL030
          (SETQ FIRST (LIST (CAR A)))                                       HEADL040
LOOP      (SETQ A (CDR A))                                                  HEADL050
          (SETQ PARTS (CONS FIRST PARTS))                                   HEADL060
          (COND ((NULL A) (COND ((EQUAL FIRST WFF)                          HEADL070
                                 (RETURN (REVERSE (TRIM PARTS))))           HEADL080
                ((NULL REST)(RETURN (ERROR (QUOTE HEADINGLIST))))           HEADL090
                (T (RETURN (ERROR (QUOTE HEADINGLIST-REST))))))))           HEADL100
          (SETQ CUR (CAR A))                                                HEADL110
          (COND ((EQ CUR (QUOTE K))(GO KON))                                HEADL120
                ((OR (EQ CUR (QUOTE M))(EQ CUR (QUOTE N)))(GO NEG)))        HEADL130
          (SETQ REST (CONS FIRST REST))                                     HEADL140
          (SETQ FIRST (LIST CUR))                                           HEADL150
          (GO LOOP)                                                         HEADL160
KON       (SETQ FIRST (APPEND (LIST CUR)(APPEND FIRST (CAR REST))))         HEADL170
          (SETQ REST (CDR REST))                                            HEADL180
          (GO LOOP)                                                         HEADL190
NEG       (SETQ FIRST (APPEND (LIST CUR) FIRST))                            HEADL200
          (GO LOOP)                          )))                            HEADL21*


(TRUTHTABLE (LAMBDA (WFF)                                                   TTBL0010
          (PROG (AA AB COL COUNR COUNS COUNT DBLLIST DB FA                  TTBL0020
           FA1 FAL FALS FALST FALSTDUM KVA KVB KVC                          TTBL0030
           KVD KVE L M1 MDUM MLIS MNO RCOUNT SVAR                           TTBL0035
           TR TRU TRUD UNAMUNO UMDUM UNM1 VA VB VC                          TTBL0040
           VD VD1 VD2 VD3 VE VF VG VH VJ VK VN )                            TTBL0050
C
C THE SINGLE ARGUMENT OF THE FUNCTION IS A WELL-FORMED
C FORMULA = WFF. THE PROGRAMME USES THE FORTY SIX VARIABLES LISTED
C
          (SETQ VA 1)                                                       TTBL0070
          (SETQ AA (HEADINGLIST WFF))                                       TTBL0090
START     (SETQ AB (CAR AA))                                                TTBL0100
          (SETQ VD L)                                                       TTBL0110
          (SETQ L (APPEND L (LIST AB)))                                     TTBL0115
          (SETQ VN VA)                                                      TTBL0120
          (SETQ VB (CDR AB))                                                TTBL0130
          (SETQ VC 0)                                                       TTBL0140
          (SETQ VE 0)                                                       TTBL0150
C
C BRANCH TO LUPO WHERE THE FIRST LETTER OF THE CURRENT HEADING IS M OR N
C WHERE IT IS K, BRANCH TO KONJ. OTHERWISE CONTINUE
C
          (COND ((OR (EQ (CAR AB)(QUOTE N))(EQ (CAR AB)(QUOTE M)))          TTBL0160
                 (GO LUPO))                                                 TTBL0165
                ((EQ (CAR AB)(QUOTE K))(GO KONJ)))                          TTBL0170
C
C A NEW VARIABLE HAS BEEN FOUND IN THE WFF AND DOUBLING TAKES PLACE
C
BILD      (COND ((ZEROP VN)(GO TABU)))                                      TTBL0180
          (SETQ COL (APPEND COL (LIST 2 0)))                                TTBL0190
          (SETQ DBLLIST (APPEND DBLLIST (LIST VE)))                         TTBL0200
          (SETQ VN (SUB1 VN))                                               TTBL0210
          (SETQ VE (ADD1 VE))                                               TTBL0220
          (GO BILD)                                                         TTBL0230
C
C THE HEADING - CAR VD - WHICH IS THE SAME AS THE CURRENT ONE LESS
C ITS INITIAL OPERATOR, IS SOUGHT. IT IS FOUND IN COLUMN VC
C
LUPO      (COND ((EQUAL VB (CAR VD))(GO LUOP)))                             TTBL0240
          (SETQ VC (ADD1 VC))                                               TTBL0250
          (SETQ VD (CDR VD))                                                TTBL0260
          (GO LUPO)                                                         TTBL0270
```

```
C
C BRANCH TO POSS IF THE OPERATOR WAS  M
C
LUOP      (COND ((EQUAL (CAR AB) (QUOTE M))(GO POSS)))                    TTBL0280
C
C CONSTRUCT A COL FOR THE NEGATION OF THE HEADING OF COLUMN VC
C
LUUP      (SETQ VB (TAB VE VC))                                           TTBL0290
          (SETQ COL (COND ((ZEROP VB)(APPEND COL (LIST 2)))               TTBL0300
                          (T (APPEND COL (LIST 0)))))                     TTBL0310
          (SETQ VE (ADD1 VE))                                             TTBL0350
          (COND ((EQUAL VE VA)(GO TABU))                                  TTBL0360
                (T (GO LUUP)))                                            TTBL0370
C
C LOCATE THE COLUMNS WHICH HAVE THE ARGUMENTS OF K AS HEADINGS
C
KONJ      (SETQ KVA 1))                                                   TTBL0380
          (SETQ KVB AB)                                                   TTBL0390
          (SETQ KVC NIL)                                                  TTBL0400
          (SETQ KVD NIL)                                                  TTBL0410
          (SETQ KVE NIL)                                                  TTBL0420
KLA       (SETQ KVB (CDR KVB))                                            TTBL0420
          (COND ((NULL KVB)(RETURN (ERROR CONJUNCTS)))                    TTBL0430
                ((ZEROP KVA)(GO KLC)))                                    TTBL0440
          (SETQ KVC (APPEND KVC (LIST (CAR KVB))))                        TTBL0450
          (COND ((EQ (CAR KVB)(QUOTE K))(GO KLB))                         TTBL0460
                ((OR (EQ (CAR KVB)(QUOTE M))(EQ (CAR KVB)(QUOTE N)))      TTBL0470
                                          (GO KLA))                       TTBL0480
                (T (SETQ KVA (SUB1 KVA))))                                TTBL0490
          (GO KLA)                                                        TTBL0500
KLB       (SETQ KVA (ADD1 KVA))                                           TTBL0510
          (GO KLA)                                                        TTBL0520
KLC       (COND ((NULL VD)(RETURN (ERROR EMPTYVD)))                       TTBL0530
                ((OR (EQUAL KVC (CAR VD))(EQUAL KVB (CAR VD)))(GO KLD)))TTBL0540
          (SETQ VC (ADD1 VC))                                             TTBL0550
          (SETQ VD (CDR VD))                                              TTBL0560
          (GO KLC)                                                        TTBL0570
KLD       (COND ((NULL KVD)(SETQ KVD VC))                                 TTBL0580
                (T (SETQ KVE VC)))                                        TTBL0590
          (COND ((NOT (NULL KVE))(GO KLE))                                TTBL0600
                ((EQUAL KVB KVC)(GO KLE)))                                TTBL0610
          (SETQ VC (ADD1 VC))                                             TTBL0620
          (SETQ VD (CDR VD))                                              TTBL0630
          (GO KLC)                                                        TTBL0640
KLE       (SETQ KVE VC)                                                   TTBL0650
KLE       (SETQ COL (COND ((OR (ZEROP (TAB VE KVD))(ZEROP (TAB VE KVE)))  TTBL0660
                              (APPEND COL (LIST 0)))                      TTBL0670
                (T (APPEND COL (LIST 2))))                                TTBL0680
          (SETQ VE (ADD1 VE))                                             TTBL0690
          (COND ((EQUAL VE VA) (GO TABU))                                 TTBL0700
                (T (GO KLE)))                                             TTBL0710
POSS      (COND ((NOT (ZEROP (TAB VE VC)))(GO POSTRU)))                   TTBL0720
C
C MLIS IS USED TO LIST THE COLUMNS WHICH HAVE AN 'M' AS FIRST
C LETTER OF THEIR HEADING
C
          (SETQ MNO 0)                                                    TTBL0740
          (SETQ VD3 VD)                                                   TTBL0750
MLISLUP   (COND ((EQUAL (CAAR VD3)(QUOTE M))(SETQ MLIS                    TTBL0755
                                          (APPEND MLIS (LIST MNO)))))     TTBL0756
          (SETQ VD3 (CDR VD3))                                            TTBL0760
          (COND ((NULL VD3) (GO REPORT)))                                 TTBL0770
          (SETQ MNO (ADD1 MNO))                                           TTBL0780
          (GO MLISLUP)                                                    TTBL0790
REPORT    (TERPRI)                                                        TTBL0800
          (TERPRI)                                                        TTBL0810
          (PRINT (QUOTE MLIS)) (PRINT MLIS)                               TTBL0820
          (PRIN1 VA) (PRINT (QUOTE ROWS))                                 TTBL0830
```

```
C
C WHERE NO HEADINGS WITH 'M' HAVE PREVIOUSLY OCCURRED
C BRANCH TO  CASED
C
        (COND ((NULL MLIS) (GO CASED)))                                    TTBL0840
C
C ADD THE NUMBER OF EVERY ROW (COUNR) TO TR IF IN COLUMN  VC
C THAT ROW SHOWS NO ZERO
C
        (SETQ COUNR -1)                                                    TTBL0850
LUPOS   (SETQ COUNR (ADD1 COUNR))                                          TTBL0860
        (COND ((LESSP COUNR VA)                                            TTBL0870
               (COND ((NOT (ZEROP (TAB COUNR VC)))(SETQ TR                 TTBL0880
                                    (APPEND TR (LIST COUNR))))             TTBL0890
                     (T (GO LUPOS))))                                      TTBL0900
              (T (GO TREORMED)))                                           TTBL0910
        (GO LUPOS)                                                         TTBL0920
C
C FA IS CONSTRUCTED ON THE BASIS OF THE COLUMNS GIVEN IN MLIS
C FA IS LIST OF LISTS
C
TREORMED (SETQ MDUM MLIS)                                                  TTBL0930
FAEIND  (SETQ FA (APPEND (LIST NIL) FA))                                   TTBL0940
        (SETQ COUNS 0)                                                     TTBL0950
FALIS   (COND ((LESSP COUNS VA)                                            TTBL0960
               (COND ((ZEROP (TAB COUNS (CAR MDUM)))(SETQ FA (APPEND       TTBL0970
                     (LIST (APPEND (CAR FA) (LIST COUNS)))(CDR FA))))      TTBL0980
                     (T (GO FADV))))                                       TTBL0990
              (T (GO MOFF)))                                               TTBL1000
FADV    (SETQ COUNS (ADD1 COUNS))                                          TTBL1010
        (GO FALIS)                                                         TTBL1020
MOFF    (COND ((NULL MDUM) (GO FALSFORM)))                                 TTBL1030
        (SETQ MDUM (CDR MDUM))                                             TTBL1040
        (GO FAEIND)                                                        TTBL1050
C
C TR AND FA ARE NOW FORMED  FALS MUST BE CONSTRUCTED
C
FALSFORM (SETQ FALST (REVERSE FA))                                         TTBL1060
        (SETQ FAL MLIS)                                                    TTBL1070
        (SETQ FALSTDUM FALST)                                              TTBL1080
XXX     (COND ((NULL FAL) (GO UP)))                                        TTBL1090
        (SETQ FA1 (CAR FAL))                                               TTBL1100
        (SETQ VD1 VD)                                                      TTBL1110
XX      (COND ((ZEROP FA1)(GO MFOUND)))                                    TTBL1120
        (SETQ FA1 (SUB1 FA1))                                              TTBL1130
        (SETQ VD1 (CDR VD1))                                               TTBL1140
        (COND ((NOT (NULL VD1)) (GO XX)))                                  TTBL1150
        (SETQ FAL (CDR FAL))                                               TTBL1160
        (SETQ FALSTDUM (CDR FALSTDUM))                                     TTBL1170
        (GO XXX)                                                           TTBL1180
MFOUND  (SETQ M1 (CAR VD1))                                                TTBL1190
        (SETQ UNM1 (CDR M1))                                               TTBL1200
        (SETQ UMDUM (APPEND UMDUM (LIST UNM1)))                            TTBL1205
        (SETQ UNAMUNO 0)                                                   TTBL1210
        (SETQ VD2 VD)                                                      TTBL1220
MM      (COND ((EQUAL UNM1 (CAR VD2))(GO UNMFIND)))                        TTBL1230
        (SETQ VD2 (CDR VD2))                                               TTBL1240
        (SETQ UNAMUNO (ADD1 UNAMUNO))                                      TTBL1250
        (COND ((NOT (NULL VD2)) (GO MM))                                   TTBL1260
              (T (RETURN (LIST (QUOTE $$$DEMODALISED PART NOT FOUND$)      TTBL1270
                               M1))))                                      TTBL1280
UNMFIND (SETQ RCOUNT 0)                                                    TTBL1290
NMN     (COND ((MEMBER RCOUNT (CAR FALSTDUM))                              TTBL1300
                    (COND ((NOT (ZEROP (TAB RCOUNT UNAMUNO)))(GO MMM))     TTBL1310
                          (T (AND (PRINT (QUOTE $$$REJECTED ROW NUMBER$    TTBL1320
                                  ))(PRINT RCOUNT))))))                    TTBL1330
```

```
          (SETQ RCOUNT (ADD1 RCOUNT))                                    TTBL1340
          (COND ((LESSP RCOUNT VA) (GO NMN))                             TTBL1350
                (T (GO NNN)))                                            TTBL1360
MMM       (SETQ FALS (APPEND FALS (LIST RCOUNT)))                        TTBL1370
          (SETQ RCOUNT (ADD1 RCOUNT))                                    TTBL1380
          (GO NMN)                                                       TTBL1390
NNN       (SETQ FALS (TRIM FALS))                                        TTBL1400
          (SETQ FAL (CDR FAL))                                           TTBL1410
          (SETQ FALSTDUM (CDR FALSTDUM))                                 TTBL1420
          (GO XXX)                                                       TTBL1430
UP         (SETQ SVAR TR)                                                TTBL1440
UP1        (COND ((NOT (MEMBER (CAR SVAR) FALS))(SETQ DB T)))            TTBL1450
           (COND (DB (GO DIC)))                                          TTBL1455
           (SETQ SVAR (CDR SVAR))                                        TTBL1456
           (COND ((NULL SVAR)(GO FARTHER)) (T (GO UP1)))                 TTBL1458
FARTHER    (SETQ COL (APPEND COL (LIST 0)))                              TTBL1470
          (GO BEG)                                                       TTBL1480
DIC       (SETQ MDUM MLIS)                                               TTBL1490
LODIC2    (COND ((NULL UMDUM)(GO TRUBILT)))                              TTBL1500
          (SETQ COUNT -1)                                                TTBL1510
          (SETQ VD1 VD)                                                  TTBL1520
LODIC     (SETQ COUNT (ADD1 COUNT))                                      TTBL1530
          (COND ((EQUAL (CAR UMDUM)(CAR VD1))(GO TRUBILDER)))            TTBL1540
          (SETQ VD1 (CDR VD1))                                           TTBL1550
          (COND ((NULL VD1) (RETURN (ERROR27))))                         TTBL1560
          (GO LODIC)                                                     TTBL1570
TRUBILDER       (SETQ TRU (APPEND TRU (LIST COUNT)))                     TTBL1580
          (SETQ UMDUM (CDR UMDUM))                                       TTBL1590
          (GO LODIC2)                                                    TTBL1600
C
C WHERE A MEMBER OF TRUD IS IN THE UNION OF TR AND FALS,
C COL IS EXTENDED WITH 2 ONLY. OTHERWISE IT IS CASE D.
C
TRUBILT   (SETQ TRUD TRU)                                                TTBL1610
TRUBILI   (COND ((NULL TRUD)(GO CASED))                                  TTBL1620
                ((MEMBER (CAR TRUD) (UNION TR FALS))( GO POSTRU))        TTBL1630
                (T (SETQ TRUD (CDR TRUD))))                              TTBL1640
          (GO TRUBILI)                                                   TTBL1650
C
C THIS IS CASE D: THE ROW WILL HAVE TO BE DOUBLED.  THE ROW NUMBER
C IS PUT IN DBLLIST AND COL IS EXTENDED WITH BOTH 2 AND 0.
C
CASED     (SETQ COL (APPEND COL (LIST 2 0)))                             TTBL1660
          (SETQ DBLLIST (APPEND DBLLIST (LIST VE)))                      TTBL1670
          (GO BEG)                                                       TTBL1680
POSTRU    (SETQ COL (APPEND COL (LIST 2)))                               TTBL1690
BEG       (SETQ VE (ADD1 VE))                                            TTBL1700
          (COND ((EQUAL VE VA)(GO TABU)))                                TTBL1710
          (GO POSS)                                                      TTBL1720
C
C BUILD 'TABLE' ON THE BASIS OF ANY 'TAB' WHICH EXISTS PLUS THE NEW
C COLUMN 'COL' AND 'DBLLIST', THE LIST OF ROWS TO BE DOUBLED.
C
TABU      (ARRAY (LIST (LIST (QUOTE TABLE)(LIST (LENGTH COL)(LENGTH L))  TTBL1730
                       (QUOTE LIST))))                                   TTBL1740
          (SETQ VE 0)                                                    TTBL1750
TLA       (SETQ VG 0)                                                    TTBL1760
          (SETQ VH 0)                                                    TTBL1770
          (COND ((ONEP (LENGTH L))(GO NUCOL)))                           TTBL1780
TLB       (COND ((MEMBER VH DBLLIST)(GO TLC))                            TTBL1790
                (T (TABLE (QUOTE SET)(TAB VH VE) VG VE)))                TTBL1800
          (SETQ VG (ADD1 VG))                                            TTBL1810
          (GO BAAK)                                                      TTBL1820
```

```
TLC   TLC((TABLE((SETZ (GETZ (SAT) VJ1&E) VKG VJ
          (TABLE((SETZ (GETZ (SAT) VJ1&E) VKA VJ
          (SETQ VSETOLMS 60 VJ) 2 VC)))
RAAK  RAAK(COND ((OMP (LMSSP(VSSSRUN' VAPP)
                (T (SETQ VSETAOMP (VAPP) VJ
          (GO TLOOO TLO)
TLE   TLE(SETQ VSETAOOVF VAPP) VE))
          (COND ((GENDAT(GOTHEBUMF (GEMBTHLEVP
          (GO TLFGO TLE
          (GO TLAPP TLA)
NUCOL NUCOSETQ VSETOAXAVG'2P) VG 2))
          (SETQ OSETDSPONLIST NIL)
          (SETQ VSETP VG 0)
          (PRINT(APPINTPAPTPAINT COLL)
NULP  NULP(COND ((CNOOL((CALL((GALPAGEAPPEST
          (TABLF((TABLEF(CSETT((SETQCCLAPVGCLE
          (SETQ VSETAOOVC VSPP) VG)))
          (SETQ OSETOCPBLCOCPP COLL))
          (GO NULOO NULP))
      C
C PREPCRPTPAPEOOTSS2TAESSEXHHEAEYTNGHTATH V
      C
      C
PRESTARPFSTART ASEMOOAAGOOE AA))
          (COND ((CNOOL (MALLAEMOMPEPTJEN T)))
          (ARRAY((ERRAY (LTSF (OUTSTE (TAOTE(LT
                                    (OUT
          (SETQ VJSETP VJ 3)
          (SETQ VKSTP VK 0)
PLL   PLL(COND ((OPENSA (VJFSAP) VJTAA)((OUTAPE (
          (T (GO (PLKOOO) PLK)))
          (SETQ VJSTOOOVJ VAPP) VJ))
          (GO PLUOO PLL)
PLK   PLK(SETQ VJSTOP VJ 0)
          (COND ((OLEASA (VJESSUAK (LSUBSTHLEV
                (T (GO (START) START)))
          (GO PLUOO PLL)      )))
```

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