

Komaravolu Chandrasekharan (1920-2017)

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Komaravolu Chandrasekharan was born on November 21, 1920, in Machilipatnam, India. He obtained his M.A. in Mathematics from the University of Madras through Presidency College in 1943 and his Ph.D in 1946. He was a student of Ananda Rau, who worked with Hardy on "Summability" and was a contemporary of Ramanujan in Cambridge. Marshall Stone met Chandrasekharan in Madras and was so impressed by the young man that he arranged for him to go in 1946 to the Institute of Advanced Study in Princeton, to work as assistant to Hermann Weyl. During his Princeton years, Chandrasekharan had an extensive collaboration with Salomon Bochner, which resulted in papers and a book in analysis centering around Fourier transforms. In 1949 Homi Bhabha, founder of the Tata Institute of Fundamental Research (TIFR), Bombay, invited him to join the School of Mathematics of TIFR. Chandrasekharan became a full professor there in 1952. The current success of this institute owes a great deal to his skills as an organizer and administrator.

Another significant contribution of Chandrasekharan to mathematics in India was his work as chief editor of the Journal of the Indian Mathematical Society (1950-1958) and of the Mathematics Student (1953-1958). He encouraged promising young Indian mathematicians and invited well-known mathematicians, like Hermann Weyl and André Weil to publish there.

In 1964 Chandrasekharan was invited to ETH (in Zurich) for the summer term. A year later he took up a professorship there, and remained until his retirement in 1988. The ETH archives show that the mathematics department was interested in hiring someone internationally prominent, who might bring in new ideas. (At the time all the full professors were former ETH students.)

Chandrasekharan was a dedicated teacher and mentor. Several books, discussed below, grew out of his lecture courses at ETH. He had a total of 12 doctoral students, some from his time in India, and some from ETH: Anton Good, Luzuis Grünenfelder, Henri Joris, Hans Keller, Max-Albert Knus, Mudumbai Narasimhan, Raghavan Narasimhan, Manual Ojanguren, Conjeevaram Seshadri, Albert Stadler, John Steinig, and Peter Thurnheer.

Chandrasekharan had a long and important association with the International Mathematical Union (IMU). He was a member of the Executive Committee (1955-1978), Secretary (1961-1966), and President (1971-1974). He was also a Vice-President (1963-1966) and Secretary-General (1966-1970) of the International Council of Scientific Unions (ICSU). He was a member of the Editorial

Board (?) of Acta Arithmetica from ?? to ??

Komaravolu Chandrasekharan died on April 13, 2017 in Zurich.

Chandrasekharan was the author (or co-author) of 75 reviewed publications, including 51 research papers. The latter include joint papers with R.P. Boas Jr., S. Bochner, A. Good, H. Joris, S. Mandelbrojt, S. Minakshisundaram, R. Narasimhan, and O. Szász.

He wrote 8 well-known books: Fourier Transforms (with S. Bochner), Typical Means (with S. Minakshisundaram), Introduction to Analytic Number Theory, Arithmetical Functions, Elliptic Functions, Classical Fourier Transforms, A Course on Topological Groups, and A Course on Integration Theory. Of these, “Introduction to Analytic Number Theory” and “Arithmetical Functions” will be the ones most familiar to number theorists. Paradoxically, the former contains a lot of basic theory concerning arithmetic functions, while the latter primarily uses analytic methods! Both volumes are masterpieces of exposition, leading the reader very efficiently to the key results in the area.

Chandrasekharan’s early research (1940 – 1955) focused on logic and analysis, and in summability theory and Fourier series in particular. His first works in number theory ([35], [36], [39], [40]), written jointly with Bochner in one case and Mandelbrojt in others, can be viewed as extensions of Hamburger’s theorem. They describe solutions of the functional equation

$$\pi^{-s/2}\Gamma(\frac{1}{2}s)\phi(s) = \pi^{-(\delta-s)/2}\Gamma(\frac{1}{2}(\delta-s))\psi(\delta-s),$$

where ϕ and ψ are analytic continuations of appropriate generalized Dirichlet series.

The next group of articles ([41], [42], [43], [45], [46], [48]), written by Chandrasekharan with his student Narasimhan, are his most widely cited research papers. They consider the sum function for coefficients of generalized Dirichlet series satisfying suitable functional equations. Both ordinary sums and Riesz means are examined, and both O -estimates and Ω -theorems are given for the error terms that arise. In addition the mean square of the error term is also considered. Despite the generality of the setting for these theorems, they recapture many of the classical estimates for sums of arithmetic functions, and provide a number of new bounds.

Also highly influential was the paper [47] on the approximate functional equation for generalized Dirichlet series, which again was written jointly with Narasimhan. The result is quite complicated to state, as is not surprising given its remarkable degree of generality, but the paper convincingly shows how familiar properties of the Riemann Zeta-function may often be extended to a much larger family of functions. In this way Chandrasekharan’s work clearly foreshadowed our current viewpoint on the natural setting in which to view the Riemann Zeta-function.

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