

Aspects of Supersymmetric Phenomenology.

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ABSTRACT:

The current state of supersymmetric models of elementary particle physics is reviewed. Full calculations of the differential cross sections for the production of squarks and gluinos are performed, and the results folded with parton distributions to give total cross sections for pp and $\bar{p}p$ processes. The gluino production rate is used together with an analysis of its decay, taking proper account of the transverse momentum distribution of the decay products, to derive limits on the gluino mass from existing beam dump experiments; this limit is rather weak, typically $m_{\tilde{g}} > 3 \text{ GeV}$ (the exact results depend on the details of the supersymmetric model).

The production of squarks and sleptons in ep processes is considered, allowing for gaugino-Higgsino mixing, and the prospects for using these processes to extract information about the gaugino-Higgsino mixing parameters are discussed. Finally we consider the production of squarks or sleptons in association with a photino; although of order α^3 in perturbation theory, this process has a characteristic signature and may be where squarks or sleptons are first discovered if they are too heavy to be pair produced at LEP.



For Lianne.

I would like to thank all those people who have helped to make my time in Oxford so enjoyable; in particular my supervisor Chris Llewellyn Smith for his continual insight, help and encouragement, my family and all my friends, especially Lianne for her constant love and support.

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...Daß ich erkenne, was die Welt
Im Innersten zusammenhält,
Schau'alle Wirkenskraft und Samen,
Und tu'nicht mehr in Worten kramen.

O sähst du, voller Mondenshein,
Zum letztenmal auf meine Pein,
Den ich so manche Mitternacht
An diesen Pult herangewacht:
Dann über Büchern und Papier,
Trübsel'ger Freund, ershciest du mir!
Ach! konnt'ich doch auf Bergeshohn
In deinem liebe Lichte gehn,
Um Bergshohle mit Geistern schweben,
Auf Wiesen in deinem Dämmer weben,
Von allem Wissensqualm entladen,
In deinem Tau gesund mich baden!

(Goethe, *Faust*.)

Chapter 1. Introduction.

1. General Introduction and Plan of Thesis.

It has been known for some time [1-3] that there exists a new class of symmetry transformations which are the only ones which have not been exploited in previous models of elementary particle physics. For this reason if no other, supersymmetry (or SUSY) seemed worthy of investigation, and the stage has now been reached where there exist many plausible models [4] which use SUSY and which predict a plethora of new particles, many of which are not far beyond the reach of experiments now running or being planned.

As yet however, there is no experimental evidence to suggest that nature has chosen to use SUSY in the ways that the model-builders have proposed, and the multiplicity of models now appearing reflect the many ingenious attempts to satisfy existing phenomenological bounds whilst retaining the advantageous aspects of SUSY discussed in section 3.

In this thesis we examine possible methods of detecting SUSY in the most direct way, by production of some of the predicted new particles at accelerators. There are no firm predictions for the masses of these particles, except that many of them must lie below 1 TeV or else much of the motivation for SUSY is lost, and that some of them should lie around, or even below, M_W . We therefore treat the particle masses as free parameters and consider what will be seen in the coming

generation of accelerator experiments if the relevant particles are light enough.

The rest of this chapter is devoted to a brief introduction to SUSY, the motivation for making models incorporating this new symmetry, a brief review of the classes of model that have resulted, their phenomenological predictions, and the existing bounds on the existence of supersymmetric particles.

In chapter 2 we calculate the cross sections for hadroproduction of SUSY particles, and examine the signatures for these events in $\bar{p}p$ collider experiments. In chapter 3, the cross section for gluino production derived in chapter 2 is coupled with a new analysis of possible gluino decay mechanisms to provide a limit on the gluino mass from existing beam dump experiments.

In chapter 4 we turn to the production of squarks and sleptons in ep collisions and examine the influence of gaugino-Higgsino mixing on these cross sections and how the measurement of these cross sections may lead to information about the masses and couplings of the exchanged charginos and neutralinos. Finally, in chapter 5, we examine the process $e^+e^- \rightarrow q\bar{q}\tilde{\gamma}$, which although of a higher order in perturbation theory than the processes normally considered, has a characteristic signature, and may be the process at which squarks are first discovered if they are too heavy to be pair produced at LEP.

Note on nomenclature; throughout this thesis we keep to the nomenclature which seems to have become most widespread. The superpartners of the known bosons are identified by the name of that boson with the suffix -ino, whilst scalar partners of

fermions are identified by the prefix s-. The symbols for the new particles are created by the addition of a tilde ($\sim$). Thus we have, for example, gluino $\tilde{g}$ and squark $\tilde{q}$.

The charged and neutral mass eigenstates that result from the mixing between gauginos and Higgsinos are called charginos and neutralinos and are identified by the symbol χ .

The convention for masses is less well established but scalar masses are generally represented by μ and fermion masses by m or M .

2. What is Supersymmetry?

A supersymmetry transformation differs fundamentally from the more familiar gauge and Lorentz transformations in that it is parameterised by anticommuting (Grassmann) variables, and the generator of the transformation has a spinorial character. An important property of these generators is that, together with the generators of the Poincaré group, they form a closed algebra involving both commutators and anticommutators (a Graded Lie Algebra)[5].

In particular, if we write the generators as two-component Weyl spinors Q_α , $\bar{Q}_{\dot{\alpha}}$, then

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu$$

so that the anticommutator of two SUSY transformations is a translation.

It is also possible for the SUSY generators to carry an internal index, leading to 'extended' supersymmetries. Since the operation of a SUSY generator on a state will change its helicity by half a unit, we must not have more than eight SUSY generators if we do not wish to have states with helicity > 2 .

Maximally extended (N=8) SUSY has been widely discussed in the literature, as has the possibility of gauging the SUSY transformation leading to a natural link with gravity ('supergravity') [6], but there have been many problems with this programme, particularly in identifying the fields in this model with the known fields and their superpartners without invoking another level of compositeness [7]. For phenomenological model building it is generally assumed that

any higher symmetry is broken to a global N=1 SUSY somewhere below the Planck scale.

By application of a SUSY transformation to a field we can generate those fields which are related to it by supersymmetry. Such a collection of fields is known as a supermultiplet, and can conveniently be expressed as a single entity, a superfield, which is a function in superspace, which contains not only ordinary spatial coordinates x_μ , but also two-component Grassmann coordinates θ and $\bar{\theta}$. The superfield can be expanded in θ and $\bar{\theta}$, the expansion terminating at the $\theta\theta\bar{\theta}\bar{\theta}$ term due to the anticommuting nature of θ and $\bar{\theta}$, each coefficient being one of the component fields of the supermultiplet.

If we start with a single complex scalar field A, we will generate a supermultiplet which contains a Weyl fermion ψ and an 'auxilliary field' F. The auxilliary field does not represent a physical particle for it has dimension 2 and no propagator term for it can occur in a renormalisable Lagrangian, thus it can be eliminated using its equations of motion. Such a supermultiplet is called a chiral supermultiplet and the corresponding chiral superfield is;

$$\Phi = A + \sqrt{2}\theta\psi + \theta\theta F$$

Similarly if we start with a vector field v_μ , repeated applications of a SUSY transformation generate a vector supermultiplet which can be regarded as an expansion of the vector superfield*

$$V = -\theta\sigma^\mu\bar{\theta}v_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\theta\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D$$

where D is again a non-propagating auxilliary field, and λ is a Weyl spinor.

* Footnote on next page.

Note in each case that the number of physical degrees of freedom in the fermionic and bosonic sectors are equal, thus the superpartners of the gauge bosons are Majorana particles. The Dirac fermions, which represent matter fields, and which must be assigned to a chiral supermultiplet as they are not self conjugate, have two scalar partners, which may be regarded as partners to the left- and right- handed components of the Dirac field or as scalar and pseudoscalar, though in general neither of these representations will be mass eigenstates.

In order to construct a Lagrangian that has any hope of bringing about a marriage between SUSY and the well explored low-energy phenomenology of the real world, the Lagrangian must not only be supersymmetric but must also contain the three classes of interaction which are the basis of the highly successful standard model. These are i) the gauge kinetic term and cubic and quartic self-interactions amongst the gauge fields, ii) kinetic terms for the matter and Higgs fields and their gauge interactions, and iii) Yukawa interactions amongst the matter and Higgs fields and Higgs masses and self-interactions. It turns out that once these have been written down in a supersymmetric way there exists surprisingly little freedom in the interactions between the new fields, for the supersymmetric analogues of all the interactions in the first two of the above categories are completely determined by the gauge coupling.

* This form for a vector superfield implies a particular choice of gauge in superspace, the Wess-Zumino gauge [2]. This gauge choice still admits all gauge transformations in ordinary space, and is satisfactory as it can be shown that the gauge invariant interactions written down later in this section are invariant under all gauge transformations, including those which take us away from the Wess-Zumino gauge.

The gauge kinetic term in a supersymmetric Lagrangian, the analogue of $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$, is most compactly written in superfield notation as $\frac{1}{4}(W^\alpha W_\alpha|_{\theta\theta} + \text{h.c.})$,^{*} where

$$W_\alpha = \frac{-1}{4g^2}\bar{D}\bar{D}e^{-gV}D_\alpha e^{gV}$$

with V a vector superfield and $-iD_\alpha$ being the fermionic analogue of $-i\partial_\mu$ in that just as the latter generates translations in ordinary space, the former generates translations in superspace;

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\theta^{\dot{\alpha}}\partial_\mu.$$

The appearance of the exponential of a field in a Lagrangian may appear alarmingly non-renormalisable, but it must be remembered that this is a superfield, and treating the exponential as a power series, it is clear that the anticommuting properties of superspace result in only renormalisable interactions being generated, including exactly those normally found in non-Abelian gauge theories.^{**}

The kinetic and gauge terms for the matter fields, the analogue of $i\bar{\psi}\not{\partial}\psi$, take the form

$$\Phi^\dagger e^{gV}\Phi|_{\theta\theta\bar{\theta}\bar{\theta}}$$

This once again produces the interactions we require if we are to reconstruct a conventional gauge theory of fermions, and also completely fixes the gauge interactions of their scalar superpartners.

^{*} The notation $|_{\theta\theta}$ means that only the $\theta\theta$ component of the expression is to be taken. This projection can be achieved by an appropriate integration over superspace.

^{**} The proof of this relies on the invariance of the action under supergauge transformations, and the use of the Wess-Zumino gauge which removes that term in the vector superfield which has no powers of θ .

It is only when we come to the non-gauge sector of the theory that we have any freedom in the interactions of the new particles, for the supersymmetric lagrangian includes a 'superpotential'

$$F(\Phi) = (a_i \Phi_i + b_{ij} \Phi_i \Phi_j + c_{ijk} \Phi_i \Phi_j \Phi_k) |_{\theta\theta} + \text{h.c.}$$

Note that only Φ and not $\Phi^\dagger$ enters the superpotential, this leads to an immediate difference between the standard model and its minimal supersymmetric extension in that we now need to introduce two Higgs superfields, with opposite hypercharges, in order to be able to give masses to both the up and the down quarks, whereas in the standard model a single Higgs field and its conjugate sufficed.

It is not possible to have, say, the scalar lepton doublet in the rôle of the second Higgs multiplet, as this leads to baryon decay at a disastrous rate [8,9]. A second Higgs superfield is also needed to cancel ABJ anomalies generated by the Higgsinos [8].

The $a_i \Phi_i$ term can of course only exist if Φ_i is a gauge singlet, and so is not applicable to any of the superfields we would expect to find in the minimal extension to the standard model. Some models however have used a superheavy singlet superfield as the starting point for a system of gauge and supersymmetry breaking, this is discussed in more detail in section 4 of this chapter.

The superpotential leads to a fermion interaction term;

$$\bar{\psi}_i \psi_j \frac{\partial^2 F}{\partial A_i \partial A_j} = b_{ij} \bar{\psi}_i \psi_j + c_{ijk} A_k \bar{\psi}_i \psi_j$$

i.e. the conventional fermion mass terms and Yukawa interactions are generated as well as interactions amongst bosons of the form:

$$\left| \frac{\partial P}{\partial A_i} \right|^2 = |b_{ij}A_j + c_{ijk}A_jA_k|^2$$

In addition there are quartic scalar self-interactions generated by the gauge term, which take the form

$g^2 |A_i \underline{T} A_i|^2$ where T is the generator of the gauge group appropriate to the superfield Φ_i .

Note that we have not yet said anything about the mechanism by which supersymmetry is broken. As it stands, our Lagrangian is supersymmetric, and unless it can be shown that it spontaneously breaks supersymmetry, or unless we add terms which break supersymmetry explicitly, we will have the obviously unsatisfactory situation of every known particle acquiring a superpartner of exactly the same mass.

Achieving a satisfactory SUSY breaking scheme has produced a great many models, and methods of SUSY breaking and the resultant classes of model are discussed in section 4.

3. The Advantages of Low Energy SUSY.

Although it would seem natural to expect theoretical physics to expend some effort in pursuit of any new idea, this alone does not account for the great interest that has been shown in SUSY in spite of the total lack of experimental evidence. SUSY does however offer a solution to the hierarchy problem, or to be more exact, the second hierarchy problem [10].

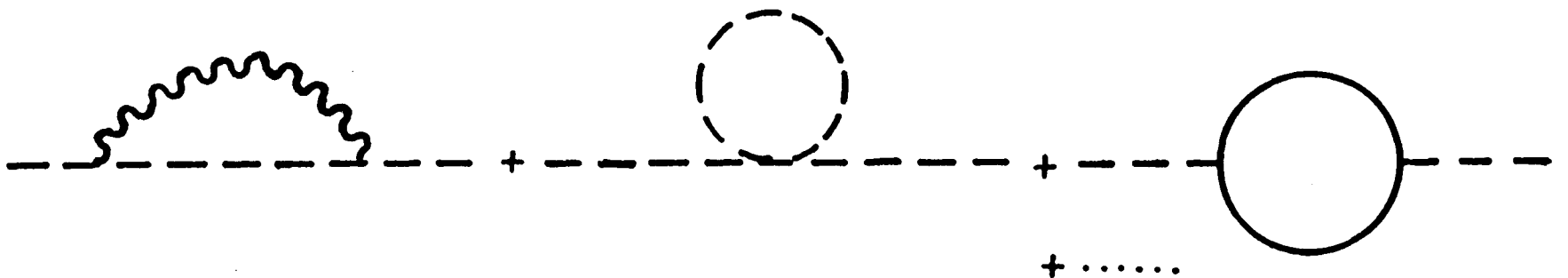
Not everyone concedes that this problem is really a problem. It has to do with the stability of scalar masses, in particular the Higgs doublet of the standard model, in the face of radiative corrections from some superheavy mass scale, e.g. the GUT scale, which are vastly greater than the original light particle mass. Even if we do not believe in GUTs, we must presumably one day face the problem of radiative corrections arising from physics at the Planck scale.

One is of course free to believe that "God doesn't do perturbation theory", and that the only problem to be understood is why the ratio M_W/M_X is so large in the first place (for which the parameters in the Lagrangian need to be fine-tuned at tree level), rather than the need for it to be continually fine-tuned as we consider higher and higher orders of perturbation theory. This is the first hierarchy problem, to which SUSY does not of itself make any new contribution.

Returning to the second hierarchy problem, the alternative solution was to postulate that the scalars are composites, which must, if the problem is to be solved, manifest itself at

a scale $O(M_W/\alpha)$. This is the Technicolour scenario [11], which runs into problems due to the non-observation of the Technipion, which is a pseudo-Goldstone boson, the analogue of the pion in QCD, and which was thus predicted to be light, most probably around 1.5 GeV, although some model builders have been able to push the Technipion mass up to 40 GeV [12].

The only alternative is to postulate an exact, or very nearly exact, cancellation between radiative corrections. To see how this comes about, consider the 1-loop corrections in an SU(5) GUT to the mass of the light Higgs boson from a heavy gauge boson, a heavy Higgs boson and a heavy fermion (which does not appear in the minimal SU(5) GUT).



Next remember that the fermion loop of the last diagram will contribute to the mass correction with the opposite sign to the other diagrams. We can see that it is possible to arrange the couplings of the new heavy fermion so that the quadratic divergences cancel. Provided that the masses of the heavy particles are nearly equal, we can also arrange for the radiative corrections to the light Higgs boson to be small compared to its mass, although these mass corrections derive from a scale many orders of magnitude larger. This is just the magic that supersymmetry performs, and is the basis of the non-renormalisation theorems [2,13,14], which show that for a wide class of SUSY models, including those in which SUSY is broken spontaneously, or explicitly by 'soft' breaking terms

in the Lagrangian, there are no quadratic divergences and that all logarithmic divergences can be absorbed by wavefunction renormalisations.

We know that SUSY must be broken, but we can now see that if this cancellation of radiative corrections is not to be spoilt, the splittings between members of a supermultiplet must not exceed about 1 TeV.*

If it is true that supersymmetry can help solve the fine-tuning problem it must show itself at quite low energies. That is not to say that the fundamental breakdown of the symmetry occurs at such low energies, indeed many models have been produced in which SUSY breakdown starts at far higher energies, in a hidden sector, and all subsequent mass splittings are generated radiatively.

The different classes of model are discussed in the next section, but for the moment we note only that if supersymmetry is to solve the fine-tuning problem there must exist new physics at the TeV scale and most probably, according to many models, quite a way below that scale.

* As it stands, the argument presented above implies only that the superheavy gauge bosons and their partners must be nearly degenerate. Consideration of other radiative corrections to the light Higgs mass shows that the addition of any new particle which has a mass too far from its superpartner will upset the cancellations.

4. Classes of Supersymmetric Model.

We have already described in section 2 the extent to which a supersymmetric Lagrangian is constrained by the requirement of reproducing all the ordinary interactions in accordance with the gauge principle. The model-builder's only freedom lies in the superpotential, using this he (or she) has to break SUSY and produce an acceptable particle spectrum.

Remembering the algebra of the SUSY generators, in particular their link with translation generators, and that the Hamiltonian is itself the generator of time translations, we find;

$$H = \frac{1}{4}(Q_1\bar{Q}_1 + Q_2\bar{Q}_2 + \bar{Q}_1Q_1 + \bar{Q}_2Q_2).$$

Hence we see that $\langle 0|H|0\rangle \geq 0$, and if we have a ground state for which $\langle 0|H|0\rangle = 0$ that ground state is supersymmetric, for then $Q|0\rangle = \bar{Q}|0\rangle = 0$. But if $\langle 0|H|0\rangle > 0$, we have succeeded in spontaneously breaking supersymmetry.

The simplest way of achieving spontaneous SUSY breakdown was discovered by Fayet and Iliopoulos [15], who observed that the $\theta\theta\bar{\theta}\bar{\theta}$ component of a vector superfield is a supersymmetric invariant, and if it is in a $U(1)$ gauge group it is also gauge invariant. Such models are called D-type models as the $\theta\theta\bar{\theta}\bar{\theta}$ component of a vector superfield is also known as the D term. This term creates an additional term ξ^α in the scalar potential

$$V = \sum_{\alpha} \frac{g_{\alpha}^2}{2} |A_i T^{\alpha} A_i + \xi^{\alpha}|^2 + \left[\left| \frac{\partial P}{\partial A_i} \right|^2 + \text{h.c.} \right]$$

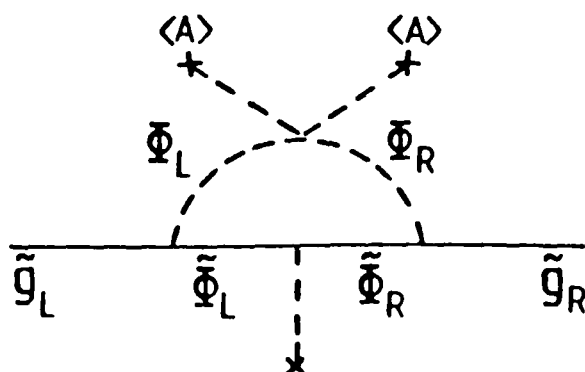
from which, once ξ has been eliminated by its equation of motion, we find $\langle 0|V|0\rangle > 0$, causing spontaneous SUSY

breakdown. Unfortunately it is not possible to use the $U(1)$ hypercharge group in this rôle as this leads to the obviously unacceptable result that one of the scalar partners of each matter fermion is lighter than the fermion itself (the Dimopoulos Georgi theorem)[8]. Models have been built using a separate axial $\tilde{U}(1)$ group [16], but it is difficult to avoid anomalies whilst maintaining SUSY breaking, a large number of new fields have to be introduced to cancel the anomalies and the model not only looks ugly, but the gauge couplings tend to evolve so as to become large well before the grand unification (or Planck) scale. A model has been built [17] in which the SUSY breaking scale is large enough to push the $\tilde{U}(1)$ gauge boson mass up to the Planck mass, and perhaps then we need no longer worry about anomalies.

A second type of spontaneous SUSY breakdown is based on the idea of O'Raifeartaigh [17] that it is possible to contrive a superpotential which leads directly to spontaneous supersymmetry breakdown. Such 'F-type' models are not natural in the sense that many couplings could exist which are arbitrarily set to zero, but these are protected from radiative corrections by the non-renormalisation theorems. In a typical model there are gauge singlet superfields A , B , C which break SUSY via a superpotential [18,19]

$$P = \lambda_1 A^2 B + \lambda_2 (A^2 - M^2) C.$$

The superfield A can then couple [20,21] to a gauge non-singlet chiral superfield Φ , which can in turn induce a gluino mass as shown:



$$m_{\tilde{g}} = \frac{\alpha_s \lambda_3^2 \lambda_4 (A^2 - M^2) C}{64\pi^3}$$

where λ_3 and λ_4 are Yukawa couplings. For a reasonable value for these couplings and $\langle A \rangle \sim 10^{15}$ GeV, the gluino mass is a few TeV with squarks deriving their masses from gluino radiative corrections. In a more devious version of this model [20,21] it is possible to push the SUSY breaking scale up to the Planck scale.

Such models tend to predict a mass spectrum which is typically $\tilde{H}^0 < \tilde{H}^\pm < \tilde{t} < W < \tilde{B} < \tilde{W} < \tilde{q} < \tilde{g}$ (few TeV) [20-22]

Attempts have also been made to have F-type models in which the potential in the superheavy sector is flat at tree level, the exact minimum being determined by radiative corrections. These are the inverted, or geometric, hierarchy models [23]. Unfortunately these models run into problems due to the large number of unwanted light particles that have to be introduced, leading to running couplings that both set $M_X > M_P$, making unification without gravity inconsistent, and become large so that perturbative renormalisation group equations are unreliable [24]. Furthermore, even though these problems can be circumvented, there appear to be problems with cosmological baryosynthesis [24].

The difficulties in obtaining a satisfactory breaking pattern for global SUSY, coupled with the gauge dogma which suggests that the interesting properties of local symmetries should be fully exploited, has led to a consideration of N=1 global SUSY as a consequence of the breakdown at high energy of some local SUSY theory via the super-Higgs effect [25].

The supergravity action contains the usual superpotential along with kinetic terms for the vector and chiral superfields, which may be modified to produce more than the minimal terms described in section 2 [25,26]. These extra terms may, for example, give rise to gaugino masses.

The resultant scalar potential takes the form

$$V = \exp\left[\frac{|A|^2}{M_P^2}\right] \left[\sum_i \left| \frac{\partial P}{\partial A_i} + \frac{A_i^*}{M_P^2} P(\Phi) \right|^2 - \frac{3}{M_P^2} |P(\Phi)|^2 \right] + D \text{ terms}$$

In addition to the terms expected in global SUSY there are non-renormalisable terms which vanish as the Planck mass $M_P \rightarrow \infty$, which have been suggested as the source of fermion masses [27,28], baryon decay [27,29], the gauge hierarchy [30] etc.

It has been shown [31] that supersymmetric gauge theories embedded in N=1 supergravity can generate 'soft' supersymmetry breaking terms. These are terms which explicitly violate SUSY but which do not spoil the non-renormalisation theorems, and which were added to the Lagrangian by the authors of some early D-type models [8,9], without any theoretical motivation, to escape the Dimopoulos Georgi theorem.

The potential of these supergravity models can be simplified by taking the limit $M_P \rightarrow \infty$ with $m_{3/2}$ fixed, where $m_{3/2}$ is the gravitino^{*} mass, related to the scale of SUSY breaking, Λ_{SS} by $m_{3/2} = \Lambda_{SS}^2/M_P$.

In the minimal supergravity model, all the scalars receive masses of $m_{3/2}$, [32] though models can be arranged [33] to give scalar masses only at the 1-loop level. Gaugino masses are also typically $O(m_{3/2})$ and are generated by radiative corrections

^{*}The gravitino is the spin 3/2 superpartner of the spin 2 graviton, and acquires its mass in the super-Higgs effect by eating a spin 1/2 Goldstino. The phenomenology of gravitinos and Goldstinos is discussed in section 5.

if they are not present at tree level.

The full scalar potential typically takes the form

$$V = \sum_i \left| \frac{\partial P}{\partial A_i} \right|^2 + \frac{1}{2} \sum_{\alpha} g_{\alpha}^2 \sum_i |A_i T^{\alpha} A_i|^2 + m_{3/2}^2 \sum_i |A_i|^2 \\ + \sum_i \left[A m_{3/2} P_3(\Phi) + \text{h.c.} \right]$$

Where A is a constant dependent on the details of the supergravity action and P_3 is the trilinear part of the superpotential. An interesting feature of this potential is that it is possible to bring about $SU(2) \times U(1)$ symmetry breaking using radiative corrections to the Higgs mass by a sufficiently heavy top quark mass [34]*. Although the simplest forms of this model may be in trouble if the recent speculation [35] about the discovery of the top quark at UAl with a mass of around 35 GeV proves correct, it seems possible [36,37] that the model can survive without drastic alterations.

It is also possible to build supergravity models [38] which break SUSY by an analogue of the Coleman Weinberg mechanism [39], in which the SUSY breaking scale is determined via dimensional transmutation from radiative corrections to a potential which is flat at tree level, leading to a gravitino mass $\exp(-O(1)/g^2) M_p$.

In any case it seems that supergravity offers the best possibility yet of building a model in which the masses of all the new particles are derived from a single scale, the gravitino mass, which can easily be arranged to lie in the phenomenologically desired range with

$$20 \text{ GeV} \lesssim m_{\tilde{q}}, m_{\tilde{l}} = O(m_{3/2}) \lesssim O(1) \text{ TeV}$$

* This idea was previously used by the same authors in a very similar way in an F-type model of global SUSY breaking [21].

For completeness we also note that models have been widely discussed in the literature [40] in which SUSY is broken dynamically by an interaction ('supercolour') which becomes strong at the TeV scale. It is not clear however [41] that such a mechanism actually can break supersymmetry.

5. Supersymmetric Phenomenology and Existing Constraints on Supersymmetric Models.

In virtually all of the SUSY models so far constructed there has been a discrete symmetry known as R-parity, which can be taken as positive for the known particles, and negative for their new superpartners. This R-parity is a remnant of a continuous global symmetry, R-symmetry [42], which is broken when the gluino acquires a mass. (The possibility that the gluino may be massless is discussed in chapter 3.)

The motivation for retaining R-parity comes from a desire to avoid terms such as $D_R^C D_R^C U_R^C$ in the superpotential. (U_R^C and D_R^C are the left handed chiral supermultiplets containing the charge conjugates of the right handed up and down quarks.) Such a term leads to a dimension 4 operator with $\Delta B=1$, which will mediate baryon decay at a disastrous level.

The consequence of R-parity conservation for the processes discussed in this thesis is that the new particles can only be produced in pairs, and that the lightest of these new particles will be stable.

The lightest new stable particle is generally believed to be neutral. In the D-type models of SUSY breaking which were initially popular it was generally taken to be a Goldstino, the phenomenology of which is discussed below, but in F-type models, and models deriving from supergravity, it is believed to be a photino, or more generally, the lightest neutralino.

Recently work has been done on an interesting new type of model [43] in which R-parity is broken spontaneously by the scalar τ -neutrino radiatively developing a vacuum expectation

value in much the same way as the Higgs boson can. It is not yet clear what the immediate consequences of present-day phenomenology will be for this model. τ -lepton number is violated for example, leading to immediate constraints on the mixing between the lepton and gauge sectors, but if the model can survive such tests to the point where it is sensible to consider the production of new particles in the ways it predicts, the whole mass of existing calculations of SUSY phenomenology may well have to be scrapped as they are based entirely on the assumption that R-parity is conserved. This thesis is as guilty in this respect as all the rest.

The couplings of the particles that we are interested in for phenomenological purposes are generally gauge couplings, and so are already determined, leaving only masses to be treated as parameters in our calculations, or to be selected from the model of our choice. The one exception to this is the Goldstino, which is the Goldstone fermion of global supersymmetry breaking, and so will be massless. Its couplings can be determined from supercurrent algebra [13,15,44] in a way highly analagous to the way that the pion-nucleon coupling is determined by the Goldberger-Treimann relation [45] in traditional current algebra. The Goldstino coupling is $\Delta m^2/\Lambda_{SS}^2$, where Λ_{SS} is the SUSY breaking scale and Δm^2 is the mass² splitting between the members of the supermultiplet. In the F-type model Λ_{SS} is large compared to the supermultiplet splittings, so that the Goldstino essentially decouples, but in D-type models it can play an important role as the lightest SUSY particle [46,47]. In supergravity derived theories the Goldstino is eaten in the super-Higgs mechanism [25] to

provide the massive degrees of freedom to the previously massless gravitino, but its couplings remain the same. Since the SUSY breaking scale in such models is generally much larger than the masses of the relevant new particles, the gravitino can be ignored for phenomenological purposes.

In this thesis we shall assume that the Goldstino/gravitino does not play a significant role in phenomenology, except in chapter 3 where we are interested in the possibility of a light (a few GeV) gluino. Since gluinos are generally rather massive in the other classes of model, it is appropriate there to consider the possibility of a D-type model with a phenomenologically relevant Goldstino.

Important phenomenological constraints on model building come from the K_L - K_S mass difference and other flavour changing neutral current processes [48]. There is no guarantee that the SUSY analogue of the Cabibbo-Kobayashi-Maskawa matrix will be equal to the non-SUSY matrix. To ensure that the K_L - K_S mass difference is not disastrously altered, it is required that the squarks be nearly degenerate. This happens naturally in some D-type models [49] and in minimal supergravity models [32], but in any case it suggests that we will reach threshold for the production of all flavours of squarks virtually simultaneously, leading to a large jump in R^* once that threshold is reached.

Whatever the exact hierarchy of particle masses, almost all models have a light neutral particle as the lightest SUSY particle into which all others will decay.

* R is the cross section ratio $\frac{\sigma(e^+e^- \rightarrow \text{all})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$.

Limits on neutral particles come from cosmology [50] which excludes the mass range between 200 eV and 0.5 GeV for the photino. Direct searches for photinos in accelerator experiments have been suggested in the literature [48,51] but no results have yet been published. Some weak limits on the mass of a light stable gluino have also been derived from searches [52] for exotic heavy isotopes, which also seem to rule out charged stable SUSY particles up to 1 TeV and down to an abundance much lower than that which would be expected from their cosmological production rate. Gluino mass limits will be discussed in more detail in chapter 3.

The direct production of squarks and sleptons has been much discussed in the literature [53] and the best existing limits come from e^+e^- annihilation at PETRA [54] and suggest that there are no charged SUSY particles below about 20 GeV.

Many other supersymmetric production processes have been suggested, the common signature for all of them is missing energy and momentum carried away by the lightest SUSY particle, mimicking neutrino events. Three of these processes are discussed in detail in this thesis, and many of them are reviewed in ref.[55]. Once again however, we add the caveat that if R-parity is broken things may be very different, although if the level of R-parity breaking is small, as it may have to be to satisfy existing constraints, the lightest SUSY particles may live long enough to leave a track in a detector.

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Chapter 2. Hadroproduction of SUSY Particles^{*}.

1. Introduction.

In this chapter we consider the hadroproduction of SUSY particles. This subject was first investigated by Fayet and Farrar [1], who were interested in the possibility of the existence of very light gluinos. Previous discussions centered on the hadroproduction of the gluino because it was expected to be much lighter than the squark. However the squark is much lighter than the gluino in models which have recently become popular (see chapter 1). Our major purpose is to provide comprehensive formulae for the production of gluinos and squarks in pp and $\bar{p}p$ collisions. The only model-dependent elements of these calculations are the masses of the SUSY particles. According to some models they are too heavy to be produced at an appreciable rate even at the $\bar{p}p$ collider but we think it wise to keep an open mind. It is well worthwhile attempting to obtain experimental limits whatever the current theoretical fashion.

Unfortunately the only signature for SUSY hadrons, discussed in section 2, is missing energy and missing p_T . To work out what limits could be reached according to our cross section formulae needs a Monte Carlo programme tailored to a particular experiment^{**}. We therefore give explicit formulae

^{*} The results in this chapter and in chapter 3 have been published in Nucl. Phys. B213(1983)223, with an erratum in Nucl. Phys. B223(1983)542. The figures here have been redrawn to take account of the corrections published in the erratum.

^{**} For some recent results of this work see refs. [2,13].

for differential production cross sections which we hope our experimental colleagues will use.

All our calculations are based on a fusion model in which we calculate the cross section for production by quarks and gluons which we then fold with the appropriate momentum distributions. Aspersions have recently been cast on this model which gives a charm production cross section much smaller than that observed at ISR energies. (for a review see ref [3]). However we would not expect the fusion model to work for $c\bar{c}$ production at the ISR, for the reasons given in section 7 of ref.[4]. The use of perturbative QCD requires factorisation into a cross section, controlled by short-distance physics, and distribution functions which absorb the long-distance effects. In $gg \rightarrow c\bar{c}$ the minimum momentum transfer is M_c^2 at threshold so this is a short-distance process, in so far as M_c^2 is large. As $s \rightarrow \infty$, however, $|t_{\min}|$ behaves as M_c^4/s , long distances come into play and the process is completely outside the domain of perturbative QCD. Allowing $|t_{\min}|$ as low as 0.5 GeV^2 and using typical x 's for the gluons we find that the fusion model should only work for $\sqrt{s} < 20 \text{ GeV}$ in which region it is not in contradiction with the data [3]. Technically only moments of σ in M_c^2/s are described by QCD [4], and these moments are controlled by the threshold region*.

For processes such as gluon + gluon $\rightarrow$ gluino + gluino we can use the formula

$$|t_{\min}| = \left\{ -M^2 + \frac{1}{2}\hat{s} - \frac{1}{2}\sqrt{\hat{s}^2 - 4M^2\hat{s}} \right\},$$

*Similar criticisms of the fusion model have recently been expressed by Mazzanti and Wada [5].

where $\sqrt{\hat{s}} = \sqrt{xx's}$ is the c.m. energy in the subprocess, to calculate the lowest energy for which the fusion model might work. Requiring $|t_{\min}| > 0.5 \text{ GeV}^2$ we obtain:

$$\begin{aligned} \sqrt{s} = 20 \text{ GeV}, \quad x = x' = 0.1 &\rightarrow M > 0.96 \text{ GeV} \\ &x = x' = 0.2 \rightarrow M > 1.5 \text{ GeV} \\ \sqrt{s} = 60 \text{ GeV}, \quad x = x' = 0.1 &\rightarrow M > 2.0 \text{ GeV} \\ &x = x' = 0.2 \rightarrow M > 2.9 \text{ GeV} \\ \sqrt{s} = 540 \text{ GeV}, \quad x = x' = 0.1 &\rightarrow M > 6.2 \text{ GeV} \\ &x = x' = 0.2 \rightarrow M > 8.7 \text{ GeV} \end{aligned}$$

With $|t_{\min}| > 1 \text{ GeV}^2$, the values of $M_{\min}$ are all increased by a factor of about 1.2. We conclude that the fusion model is probably not completely unreliable for $M > 1.5, 3, 9 \text{ GeV}$ at SPS, ISR and collider energies respectively. Experience with charm strongly suggests, but does not prove, that if the fusion model is unjustifiably used for lower masses it will underestimate cross sections by a large factor.

The rest of this chapter is organised as follows. In section 2 we briefly summarise relevant properties of gluinos and squarks and ideas about their decays. Section 3 is devoted to gluino pair production. In section 4 we discuss the production of squarks and the joint production of squarks and gluinos. Our conclusions are summarised in section 5.

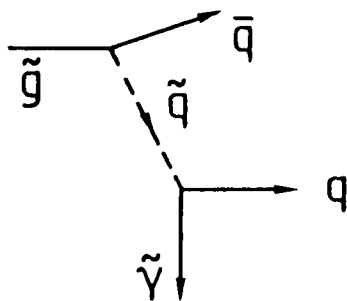
2. Some Properties of Gluinos and Squarks.

In supersymmetric theories there is a Majorana Gluino ($\tilde{g}$) corresponding to every gluon and a scalar and pseudoscalar quark, which we call generically squarks ($\tilde{q}$), corresponding to every quark. The interactions of $\tilde{q}$ and $\tilde{g}$ with gluons are entirely fixed by gauge invariance. The only new strong interaction is a $q\tilde{q}\tilde{g}$ coupling which is given by

$$\mathcal{L}_{\text{int}} = g \bar{q} \frac{1}{2} \lambda^a \tilde{g}^a \tilde{q}_S + g \bar{q} \frac{1}{2} \lambda^a \gamma_5 \tilde{g}^a \tilde{q}_P + \text{h.c.}$$

We assume for simplicity that the scalar and pseudoscalar squarks are degenerate; in general the mass eigenstates are not parity eigenstates but the splitting and mixing are constrained by limits on $\Delta S=0$ processes (see Chapter 1).

In D-type models, the gluino is expected to be much lighter than the squarks, which decay rapidly to quark + Goldstino ($\tilde{G}$). The gluino decays to $g + \tilde{G}$ or $q\tilde{q}\tilde{\gamma}$ via:



The lifetime for $\tilde{g} \rightarrow g + \tilde{G}$ can be calculated using supercurrent algebra [6] and is

$$\tau = 1.1 \times 10^{-15} \left[\frac{\Lambda_{\text{SS}}}{M_Z} \right]^4 \left[\frac{1 \text{ GeV}}{M} \right]^5 \text{ sec},$$

where M is the gluino mass and Λ_{SS} the supersymmetry breaking scale. For $0.09 \Lambda_{\text{SS}} > m_{\tilde{q}}$ the decay to $q\tilde{q}\tilde{\gamma}$ dominates with lifetime

$$\tau = 1.1 \times 10^{-11} \left[\frac{\mu}{M_W} \right]^4 \left[\frac{1 \text{ GeV}}{M} \right]^5 \text{ sec},$$

where μ is the squark mass.

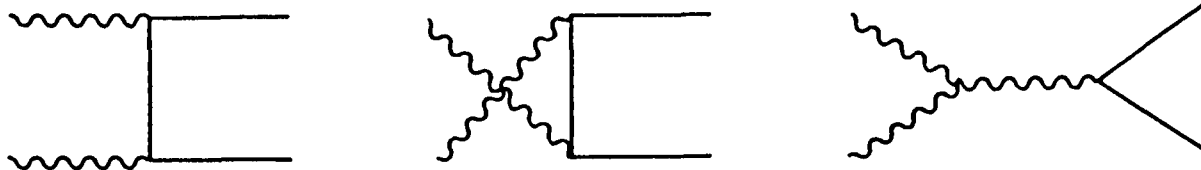
Kane and Leveille [6] have used the limits on the production of relatively long-lived hadrons to set the limit $\Lambda_{SS} M^{-1.5} \leq 1000 \text{ GeV}^{-1/2}$. The inverse of the decay process in which $\tilde{G}$ and $\tilde{\gamma}$ interact with gluons or quarks in matter can be calculated. The cross sections turn out [1,6] to be similar to neutrino cross sections. It may be possible to detect $\tilde{G}$ and $\tilde{\gamma}$ in beam dump experiments as discussed in chapter 3, but otherwise the only signature for SUSY production is missing energy and p_T .

In F-type models the decays of $\tilde{q}$ or $\tilde{g}$ to the ground state x^0 may frequently proceed via a second $x^{0'}$ which decays to x^0 and a lepton-antilepton pair or via $x^\pm$ which decays to x^0 (or $x^{0'}$) + $e\nu$ (or $\mu\nu$, $\tau\nu$). This provides excellent signatures but, alas, it is currently thought that $\mu \approx (\text{few}) \times M_W$ and $M = \text{few TeV}$ in these models making production impossible. An unfashionable possibility, which also leads to cascades involving leptons, is that the SUSY ground state is a scalar neutrino. Finally the gluino could be the lightest SUSY particle (although it is hard to see how it could be lighter than the photino) in which case there would be stable neutral strongly interacting gluinoballs ($\tilde{g}\tilde{g}$).

In our subsequent remarks about the detection of gluinos and squarks we assume that they decay to $\tilde{G}$ and $\tilde{\gamma}$. If, contrary to current expectations, the decays frequently involve leptons, even if $\tilde{g}$ and $\tilde{q}$ are relatively light, the prospects for finding them will obviously be much better.

3. Gluino Production.

The major contribution to gluino production in pp and $p\bar{p}$ collisions turns out to come from gluon fusion:



This process has recently been studied by Kane and Leveille [6] and our numerical results agree well with theirs allowing for a factor of $1/2$, due to the fact that gluinos are Majorana particles, which they omitted. However as they did not publish details of their calculation, we give explicit formulae, which can be obtained from well-known results [7] for heavy quark production by altering the colour factors appropriately and allowing for the difference of Dirac and Majorana particles [8]. We find

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = & \frac{9\pi\alpha_s^2}{4s^2} \left\{ \frac{2(M^2-t)(M^2-u)}{s^2} + \frac{(M^2-t)(M^2-u) - 2M^2(M^2+t)}{(M^2-t)^2} \right. \\ & + \frac{(M^2-t)(M^2-u) - 2M^2(M^2+u)}{(M^2-u)^2} + \frac{M^2(s-4M^2)}{(M^2-t)(M^2-u)} \\ & \left. - \frac{(M^2-t)(M^2-u) + M^2(u-t)}{s(M^2-t)} - \frac{(M^2-t)(M^2-u) + M^2(t-u)}{s(M^2-u)} \right\}, \end{aligned}$$

where M is the gluino mass.

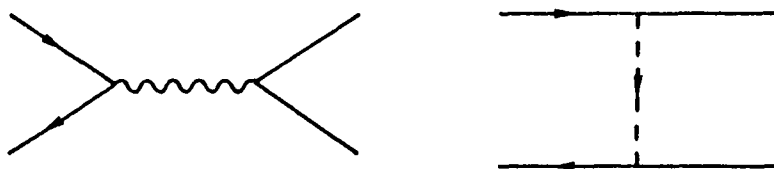
Integrating and dividing by 2 to avoid double counting of identical gluinos, we obtain

$$\hat{\sigma} = \frac{3\pi\alpha_s^2}{4s} \left[3 \left(1 + \frac{4M^2}{s} - \frac{4M^4}{s^2} \right) \ln \frac{1+x}{1-x} - \left(4 + \frac{17M^2}{s} \right) x \right],$$

where $x = \sqrt{1-4M^2/s}$.

Folding this expression with the the Gluon distributions of Glück et al. [9] evaluated at a scale $\hat{s}/2$ with $\alpha_s = \alpha_s(\hat{s}/2)$, where $\sqrt{\hat{s}}$ is the c.m. energy for $gg \rightarrow \tilde{g}\tilde{g}$, we obtained the results in figure 1. We found that the QCD parameterisation of the gluon distribution due to the CDHS group gives essentially identical results*. Almost indistinguishable results are obtained at $\sqrt{s} = 28$ GeV using the gluon distribution $3(1-x)^5x^{-1}$. We also tried the much softer gluon distribution of Owens and Reya [12] which behaves roughly as $(1-x)^6x^{-1}$ at low energy, with the results shown in figure 2. As expected, this gives much smaller cross sections for small s/M^2 , which probes large x . Subsequently we only give results for the Glück et al. distributions but it should be remembered that although softer gluon distributions are disfavoured they are not totally excluded and the cross sections we obtain may be too large.

Gluinos may also be produced in $q\bar{q}$ annihilation through s-channel gluon exchange or t-channel squark exchange:



This leads to

$$\frac{d\hat{\sigma}}{dt} = \frac{8\pi\alpha_s^2}{9s^2} \left\{ \frac{4}{3} \left(\frac{M^2-t}{\mu^2-t} \right)^2 + \frac{4}{3} \left(\frac{M^2-u}{\mu^2-u} \right)^2 + \frac{3}{s^2} \left[(M^2-t)^2 + (M^2-u)^2 + 2M^2s \right] \right. \\ \left. - \frac{3(M^2-t)^2}{s(\mu^2-t)} - \frac{3(M^2-u)^2}{s(\mu^2-u)} \right\},$$

μ being the mass of the squark.

*Kane and Leveille [6] used the distribution of Baier et al. [11] which was also found to give very similar results.

The cross section for this process is

$$\hat{\sigma} = \frac{4\pi\alpha_s^2}{3s} \left[\chi \left(\frac{5}{9} + \frac{2\Delta}{s} + \frac{4M^2}{3s} + \frac{8}{9} \frac{\Delta^2}{\Delta^2 + \mu^2 s} \right) + \ln \frac{2\Delta + s(1-\chi)}{2\Delta + s(1+\chi)} \left(\frac{2\Delta^2}{s^2} + \frac{16\Delta}{9s} + \frac{2M^2}{s} - \frac{2}{9} \frac{M^2}{2\Delta + s} \right) \right],$$

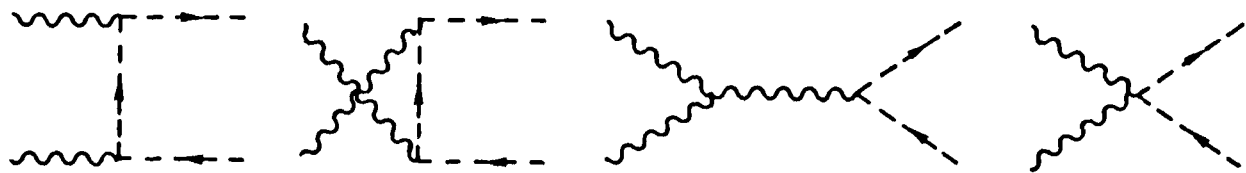
where $\Delta = \mu^2 - M^2$, $\chi = \sqrt{1 - 4M^2/s}$.

We have calculated this contribution to $\bar{p}p$ scattering using the parton distributions of Glück et al. [9]. Adding the result for $gg \rightarrow \tilde{g}\tilde{g}$ we obtained the total $\bar{p}p \rightarrow \tilde{g}\tilde{g}$ cross section in figure 3. Comparison with figure 1 shows that, as in heavy quark production, gluon fusion is the dominant process, contributing 75% or more, except close to threshold where the flatter quark distribution causes quark fusion to become relatively more important. Consequently the results are very insensitive to μ and a change from 25 to 50 GeV only changes σ by a few percent. In the case of pp the $q\bar{q}$ annihilation process is negligible and the cross section is given by figure 1.

4. Production of Squarks.

There are a number of ways in which squarks can be produced*. In presenting the results for differential cross sections below, we have assumed that it is not possible to distinguish experimentally between scalar and pseudoscalar (or alternatively left- and right- handed) squarks, nor between squarks of different flavour, but that squark and antisquark can be distinguished. In practice this is presumably unrealistic. $d\sigma/dt$ for events with $\tilde{q}$ and $\bar{\tilde{q}}$ detected is given by adding a second contribution with $t \leftrightarrow u$.

For gluon fusion producing squarks we have:



leading to a differential cross section

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = & \frac{F\pi\alpha_s^2}{2s^2} \left\{ \frac{1}{3} \left[\frac{\mu^2+t}{\mu^2-t} \right]^2 + \frac{1}{3} \left[\frac{\mu^2+u}{\mu^2-u} \right]^2 + \frac{3}{32s^2} (8s(4\mu^2-s) + 4(u-t)^2) \right. \\ & + \frac{7}{12} - \frac{1}{48} \frac{(4\mu^2-s)^2}{(\mu^2-t)(\mu^2-u)} \\ & + \frac{3}{32} \frac{1}{s(\mu^2-t)} [(t-u)(4\mu^2+4t-s) - 2(\mu^2-u)(6\mu^2+2t-s)] \\ & + \frac{3}{32} \frac{1}{s(\mu^2-u)} [(u-t)(4\mu^2+4u-s) - 2(\mu^2-t)(6\mu^2+2u-s)] \\ & \left. + \frac{7}{96} \frac{1}{(\mu^2-t)} [4\mu^2+4t-s] + \frac{7}{96} \frac{1}{(\mu^2-u)} [4\mu^2+4u-s] \right\}, \end{aligned}$$

where F is the number of flavours of degenerate squarks.

* Squark production has also been studied by Kane and Leveille (private communication).

Integrating this,

$$\hat{\sigma} = \frac{F\pi\alpha_s^2}{3s} \left[\left(\frac{5}{8} + \frac{31}{4} \frac{\mu^2}{s} \right) \zeta + \left(4 + \frac{\mu^2}{s} \right) \frac{\mu^2}{s} \ln \frac{1-\zeta}{1+\zeta} \right],$$

where $\zeta = \sqrt{1-4\mu^2/s}$.

There is also the possibility of two quarks, or a quark and an antiquark scattering. If the two quarks have different flavours the processes are:



For these processes the differential cross section is

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = \frac{4\pi\alpha_s^2}{9s^2} & \left\{ \frac{1}{(M^2-t)^2} [s(M^2-t) - (\mu^2-t)^2] \right. \\ & \left. + \delta \frac{1}{(M^2-u)^2} [s(M^2-u) - (\mu^2-u)^2] \right\}, \end{aligned}$$

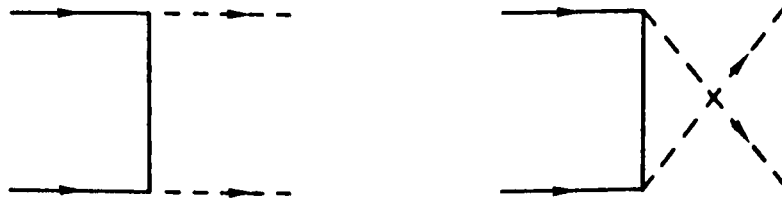
$$\text{where } \delta = \begin{cases} 0, & \text{for } \bar{q}q \text{ scattering} \\ 1, & \text{for } qq \text{ scattering} \end{cases}$$

this difference being due to our assumption about the experimental discrimination between particle and antiparticle.

This gives a total cross section for either of these processes,

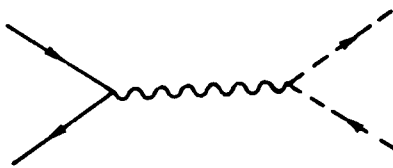
$$\hat{\sigma} = \frac{4\pi\alpha_s^2}{9s^2} \left[\left(1 - \frac{2\Delta}{s} \right) \ln \frac{s(1+\zeta)-2\Delta}{s(1-\zeta)-2\Delta} - \zeta \left(1 + \frac{\Delta^2}{\Delta^2 + M^2 s} \right) \right].$$

In the case of qq scattering with quarks of the same flavour there is also an interference term in the amplitude:



$$\begin{aligned} \frac{d\hat{\sigma}}{dt} &= \frac{4\pi\alpha_s^2}{9s^2} \left\{ \frac{1}{(M^2-t)^2} [s(M^2-t) - (\mu^2-t)^2] \right. \\ &\quad \left. + \frac{1}{(M^2-u)^2} [s(M^2-u) - (\mu^2-u)^2] - \frac{2}{3} \frac{M^2 s}{(M^2-t)(M^2-u)} \right\}, \\ \hat{\sigma} &= \frac{4\pi\alpha_s^2}{9s} \left[\left[1 - \frac{2\Delta}{s} + \frac{2}{3} \frac{M^2}{(2\Delta-s)} \right] \ln \frac{s(1+\zeta)-2\Delta}{s(1-\zeta)-2\Delta} - \zeta \left[1 + \frac{\Delta^2}{\Delta^2 + M^2 s} \right] \right]. \end{aligned}$$

In the case of $q\bar{q}$ collisions with quark and antiquark having the same flavour, there is the extra possibility of gluon exchange:



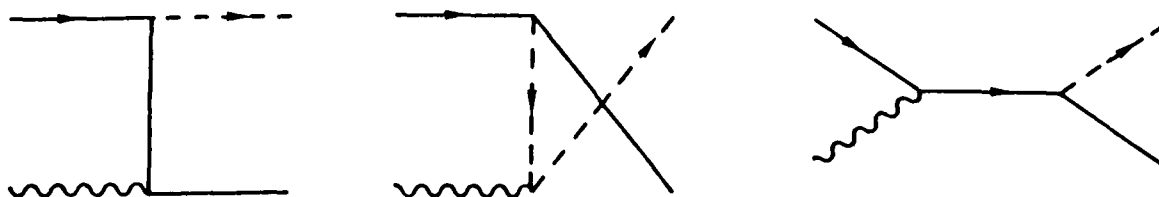
Then if there are F degenerate flavours of squark,

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} &= \frac{2\pi\alpha_s^2}{9s^2} \left\{ \frac{2}{(M^2-t)^2} [s(M^2-t) - (\mu^2-t)^2] \right. \\ &\quad \left. - \frac{2}{3} \frac{1}{s(M^2-t)} [(\mu^2-t)(u-t) + s(\mu^2+t)] \right. \\ &\quad \left. + \frac{F}{s^2} [s(s-4\mu^2) - (u-t)^2] \right\}, \\ \hat{\sigma} &= \frac{4\pi\alpha_s^2}{9s} \left[\left[1 - \frac{2\Delta}{s} - \frac{2\Delta^2}{3s^2} - \frac{2M^2}{3s} \right] \ln \frac{s(1+\zeta)-2\Delta}{s(1-\zeta)-2\Delta} \right. \\ &\quad \left. + \left[\frac{F-2}{3} - \frac{2\Delta}{3s} - \frac{4F\mu^2}{3s} - \frac{\Delta^2}{\Delta^2 + M^2 s} \right] \zeta \right]. \end{aligned}$$

Using these formulae we have calculated the total cross section for the production of squarks in pp and $p\bar{p}$ collisions assuming three flavours of degenerate squarks; near degeneracy

occurs naturally in many models and seems to be required by the absence of flavour changing neutral currents (see chapter 1). Our results are shown in figures 4 and 5. We do not show σ for pp collisions which is only very slightly less than for $\bar{p}p$ (by not much more than the thickness of the lines in figure 5). Comparing figures 4 and 5, we see that gluon fusion is dominant at $\sqrt{s} = 28$ GeV as expected, since a gluino mass of 25 GeV suppresses the gluino exchange diagram significantly at these energies. At $\sqrt{s} = 1800$ GeV and 540 GeV, where M is relatively smaller, gluon fusion dominates at very small μ^2/s but quark fusion takes over as μ^2/s increases because the quark distribution is flatter. For these values of $\sqrt{s}$, increasing M from 25 to 50 GeV always has a small effect (30% at most, generally much less) because it only changes the quark fusion cross section significantly at very small μ^2/s where it is unimportant compared to gluon fusion. This is because M enters through $(M^2-t)^{-1}$ or $(M^2-u)^{-1}$ and $|t_{\min}|$, $|u_{\min}|$ increase with increasing μ^2/s making the value of M^2 become less important.

There is also the possibility of a gluon and a quark scattering into a gluino and squark, although since most SUSY models have one of these particles much lighter than the other, acquiring its mass only through radiative corrections, this seems unlikely to be the first process to be observed. The relevant diagrams are:



The differential cross section is:

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = & \frac{4\pi\alpha_s^2}{s^2} \left\{ \frac{1}{9s}(M^2-u) - \frac{1}{9(\mu^2-t)^2}(M^2-t)(\mu^2+t) \right. \\ & + \frac{1}{4(M^2-u)^2} \left[(M^2-u)(M^2-t) + (3M^2-u)(\mu^2-u) \right] \\ & - \frac{1}{144s(\mu^2-t)} \left[s(\mu^2+M^2+2t) + \Delta(4M^2-4t-s) \right] \\ & + \frac{1}{4s(M^2-u)} \left[(\mu^2-u)(\Delta-s)-M^2s \right] + \frac{1}{16(\mu^2-t)(M^2-u)} \\ & \times \left[(\mu^2-u)(2t+u+M^2) - (M^2-t)(2\mu^2+M^2+u) \right. \\ & \left. \left. + (M^2-u)(2\mu^2-2u-s) \right] \right\}. \end{aligned}$$

Integrating we obtain

$$\begin{aligned} \hat{\sigma} = & \frac{4\pi\alpha_s^2}{s} \left[\ln \frac{s(1-\eta)-\Delta}{s(1+\eta)-\Delta} \left[-\frac{1}{4} + \frac{1}{2} \frac{\Delta}{s} \left(1 - \frac{\mu^2}{s} \right) \right] \right. \\ & + \ln \frac{s(1-\eta)+\Delta}{s(1+\eta)+\Delta} \left[\frac{\Delta}{9s} \left(2 - \frac{5}{2} \frac{\mu^2}{s} - \frac{2M^2}{s} \right) \right] \\ & \left. + \eta \left[-\frac{7}{36} + \frac{8}{9} \frac{\Delta}{s} \right] \right], \end{aligned}$$

$$\eta = \sqrt{1 - \frac{2(\mu^2+M^2)}{s} + \left(\frac{\Delta}{s} \right)^2}.$$

The results are shown in figure 6. We see that the cross sections are intermediate between those for $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{q}(\bar{q})$ production. The dominance of $\tilde{g}\tilde{g}$ production is due to the large colour factor associated with the colour octet gluinos.

We have collected our results for all these processes in figures 7 and 8 for ISR and collider energies. The cross sections are quite large for gluino masses not yet ruled out by experiment. The extent to which the limits can be improved can only be discovered by Monte Carlo calculations for particular

experiments, for which we hope our results will prove useful.

5. Conclusions.

It seems clear that the limits on the mass of squarks and gluinos can be improved significantly by careful searches for events with missing p_T at the ISR and $\bar{p}p$ collider. We hope that our formulae will prove useful in this endeavour. A recent result from $\bar{p}p$ collider data [13] suggests that the gluino and squark masses must be greater than about 40 GeV, such improvements in the limit on the gluino mass are a severe embarrassment for D-type models in which very light gluinos are expected.

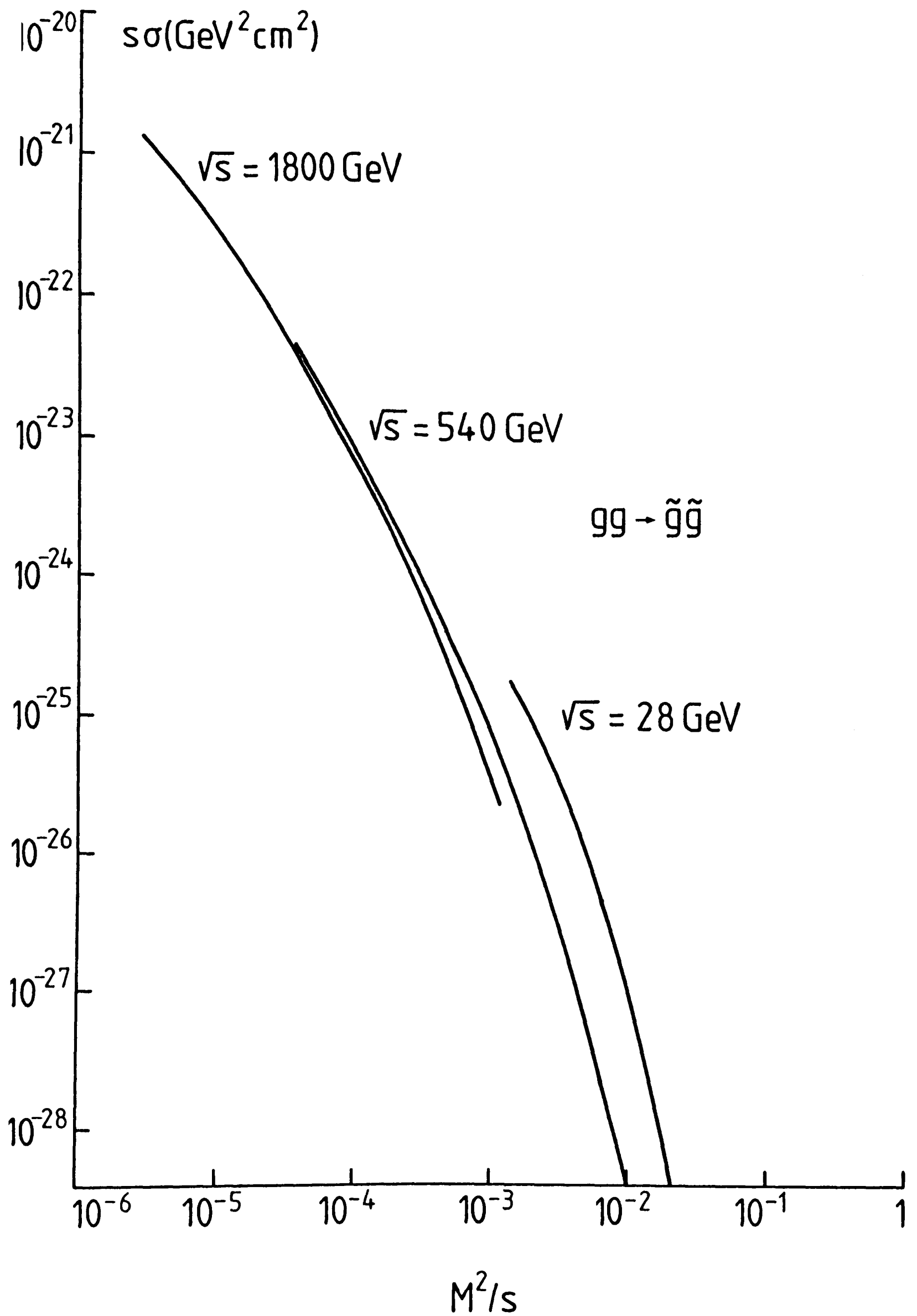


Fig. 1. Cross section for gluino production via $gg \rightarrow \tilde{g}\tilde{g}$ obtained using the gluon distribution of Glück et al. [9].

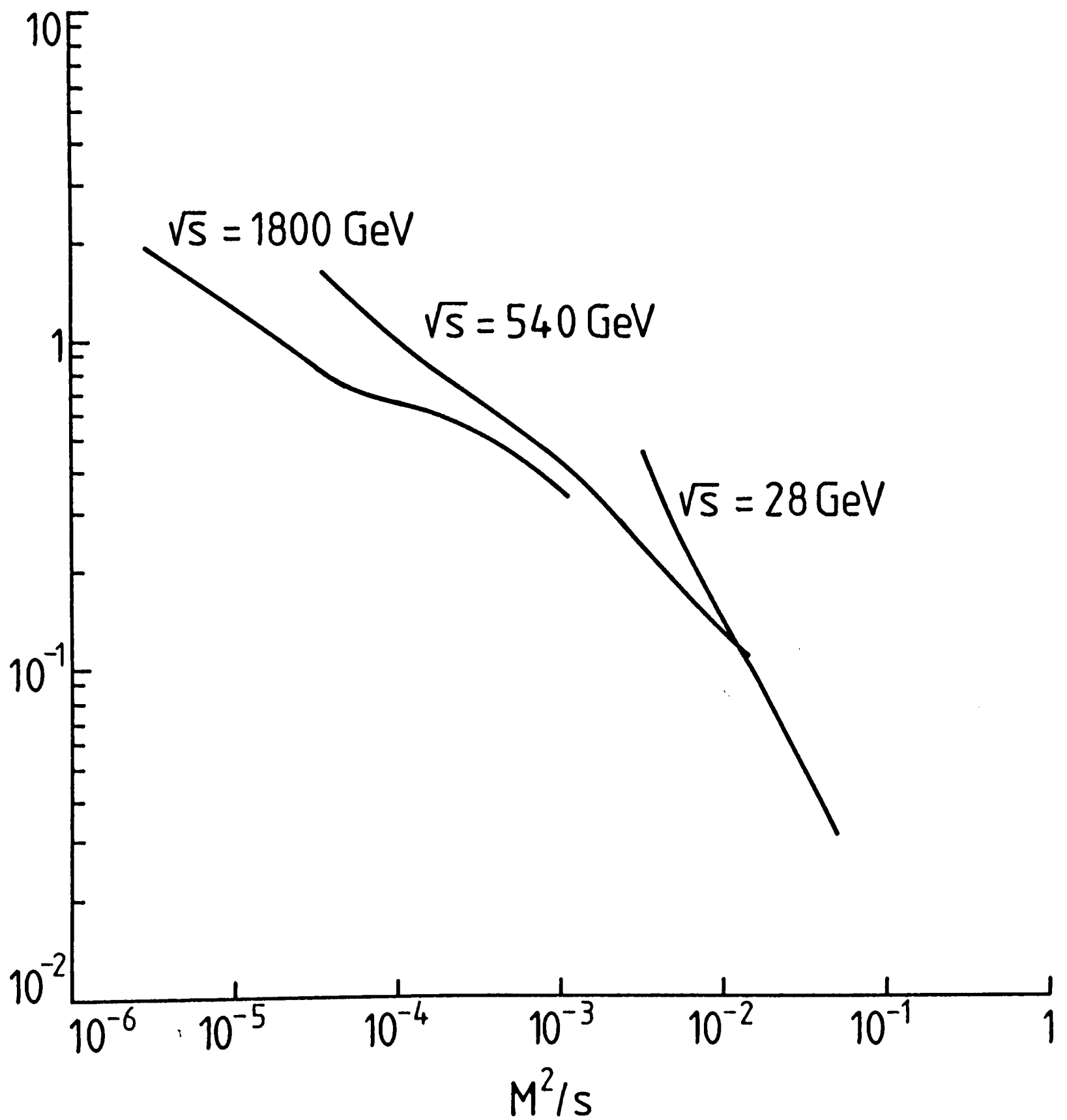


Fig. 2. Cross section for gluon fusion production of gluinos obtained using the gluon distribution of Owens and Reya [12] divided by that obtained using the distribution of Glück et al.

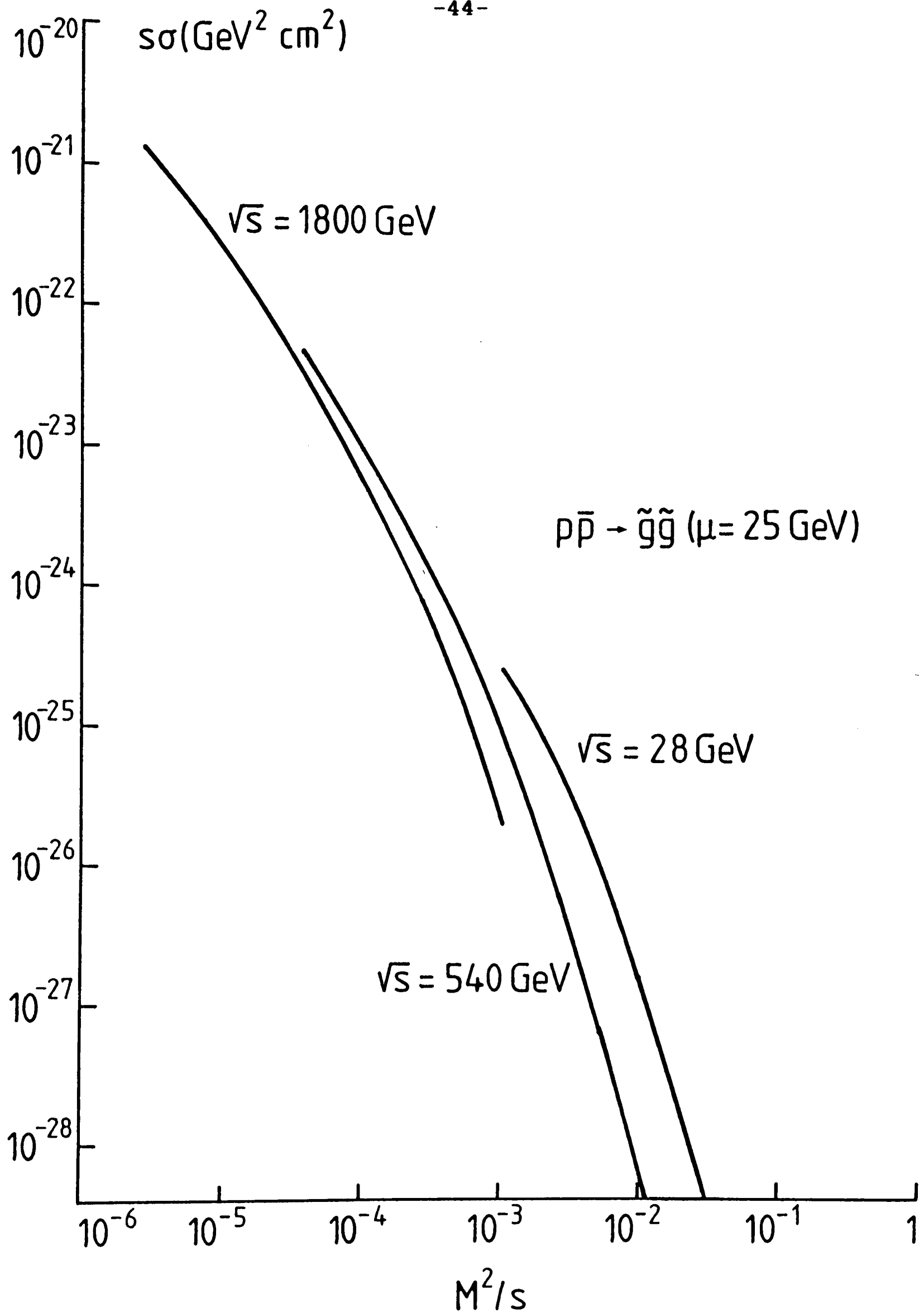


Fig. 3. Cross section for $\bar{p}p \rightarrow$ gluinos for $\mu = 25 \text{ GeV}$ (results change by less than 10% for $\mu = 50 \text{ GeV}$), using the parton distributions of Glück et al.

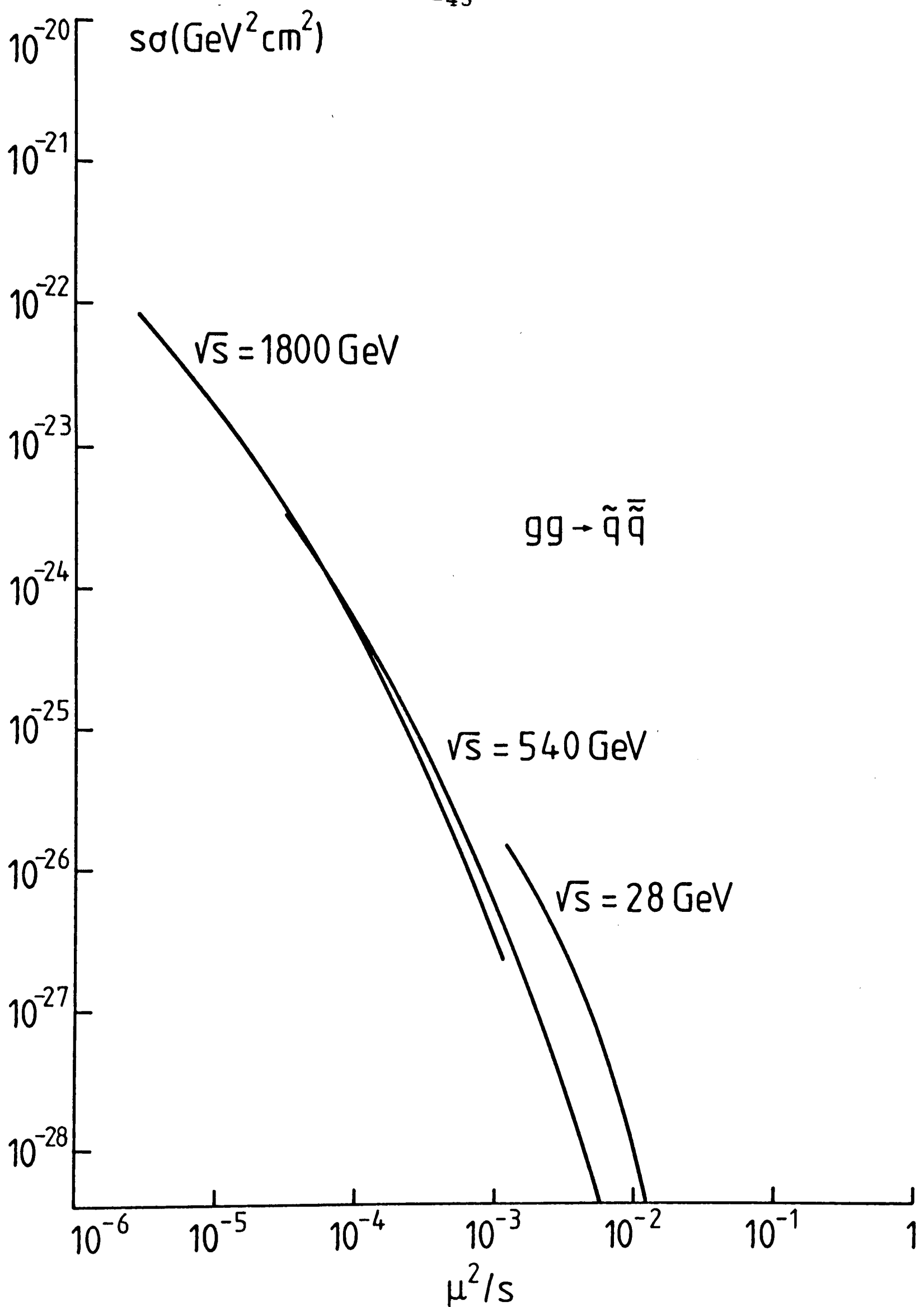


Fig. 4. Cross section for squark production via $gg \rightarrow \tilde{q}\tilde{q}$, assuming three flavours of light squark, using the gluon distribution of Glück et al.

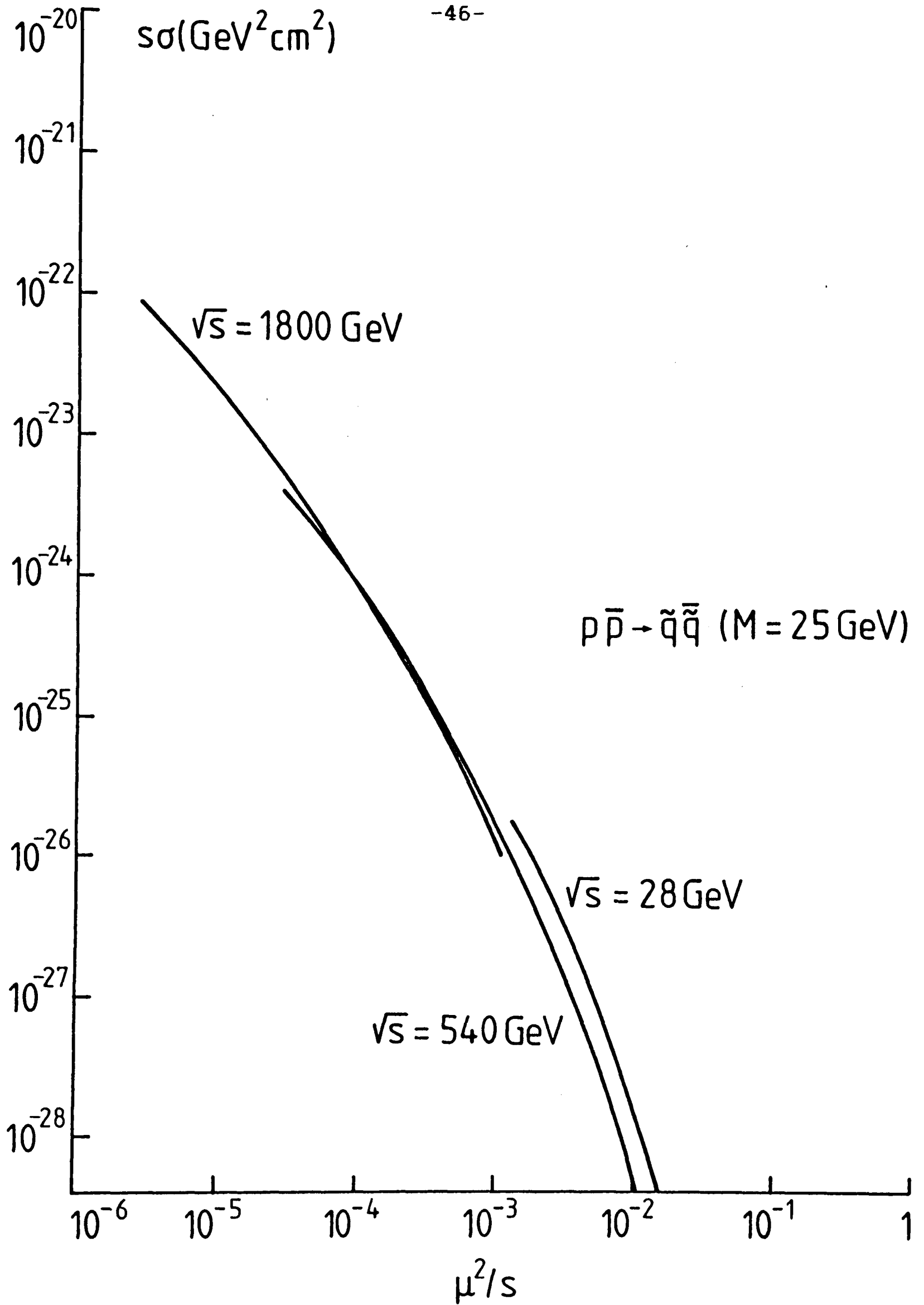


Fig. 5. Cross section for $p\bar{p} \rightarrow$ squarks, assuming three flavours of light squark, for $M = 25 \text{ GeV}$ (results change by less than 30% for $M = 50 \text{ GeV}$), using the parton distribution of Glück et al.

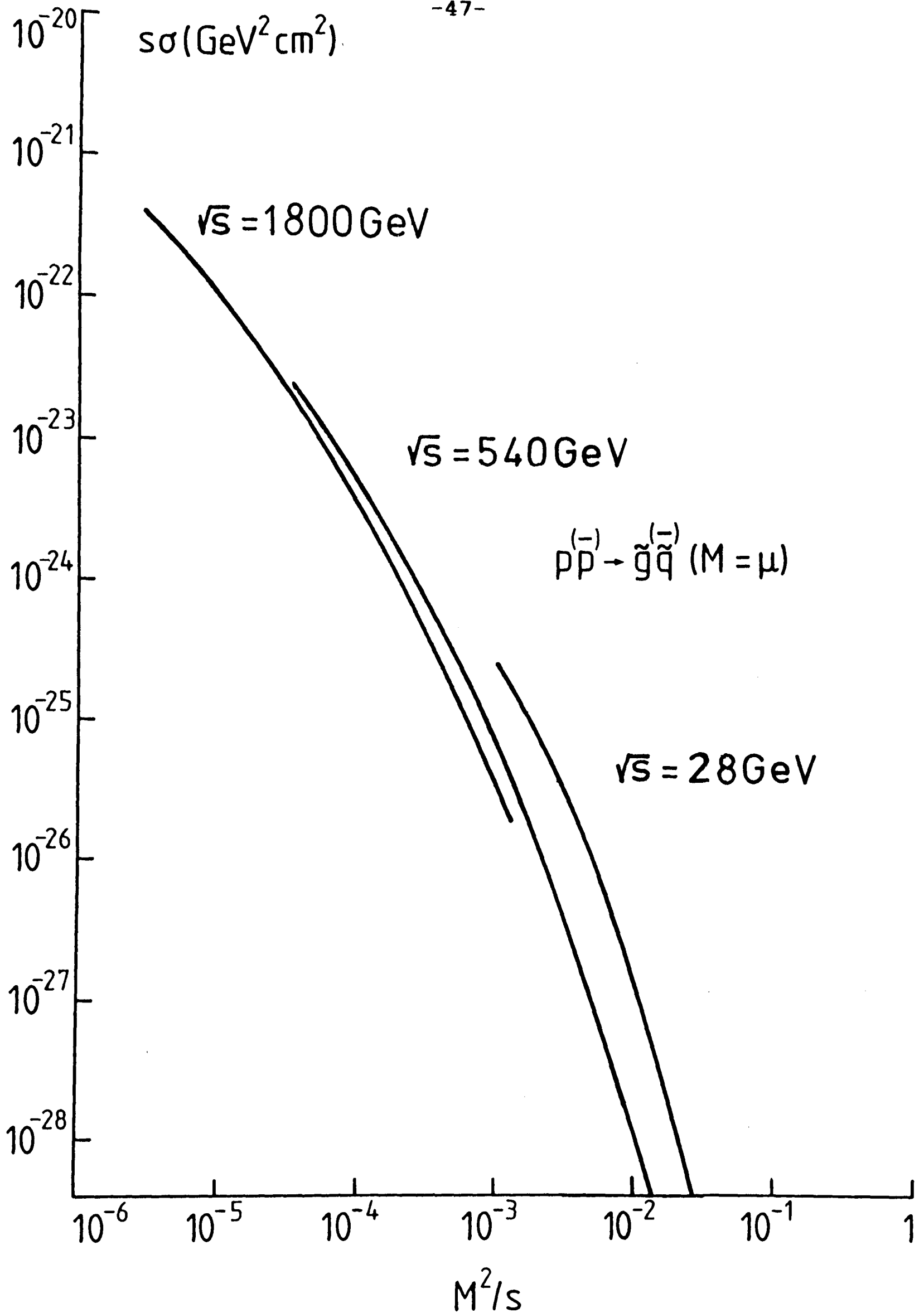


Fig. 6. Cross section for pp or $\bar{p}p \rightarrow \text{gluino} + (\text{anti})\text{squark}$, assuming these have equal mass, using the parton distributions of Glück et al.

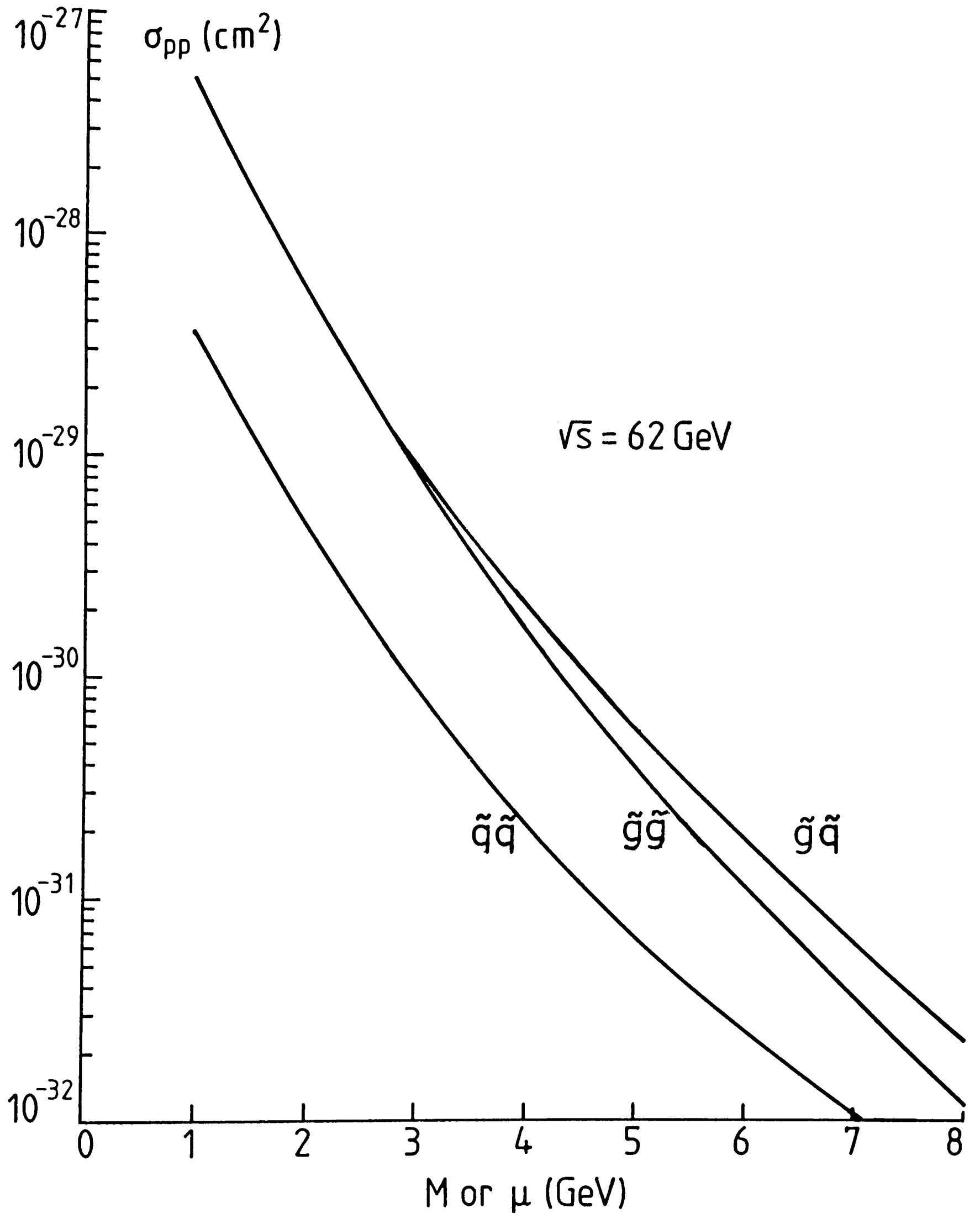


Fig. 7. pp production cross sections as a function of M or μ at $\sqrt{s} = 62$ GeV, calculated using the parton distributions of Glück et al. $\sigma(-q\bar{q})$ and $\sigma(-g\bar{g})$ were calculated using $M = 25$ GeV and $\mu = 25$ GeV respectively, but the results are insensitive to these masses. $\sigma(-q\bar{q})$ was calculated with $M=\mu$.

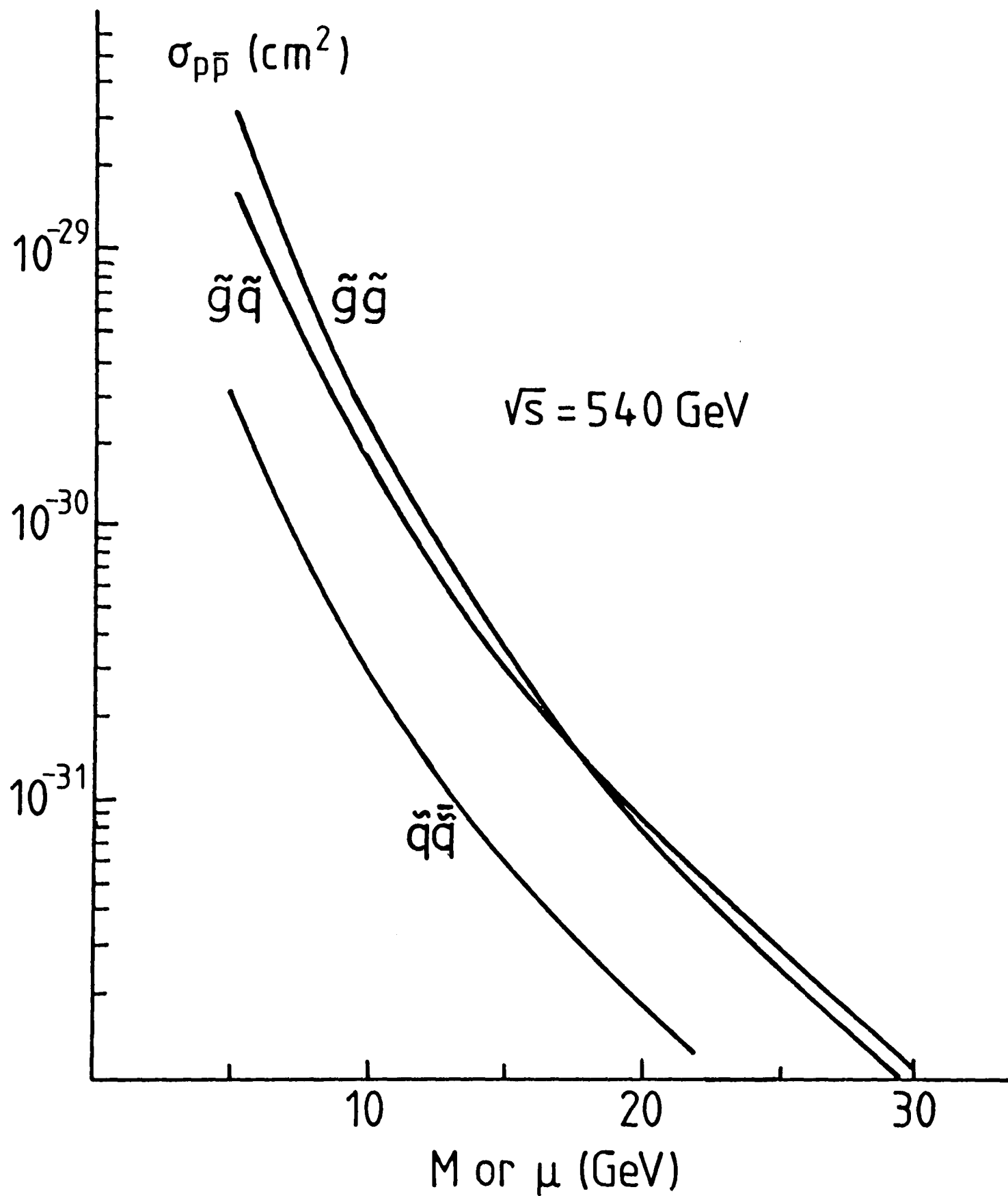


Fig. 8. pp production cross sections as a function of M or μ at $\sqrt{s} = 540 \text{ GeV}$, calculated using the parton distributions of Glück et al. $\sigma(\tilde{q}\tilde{q})$ and $\sigma(\tilde{g}\tilde{g})$ were calculated using $M = 50 \text{ GeV}$ and $\mu = 25 \text{ GeV}$ respectively, but the results are insensitive to these masses. $\sigma(\tilde{g}\tilde{q})$ was calculated with $M = \mu$.

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Chapter 3. The Search for Gluinos in Beam Dump Experiments.

1. Introduction.

In this chapter we use the cross section for gluino production, calculated in chapter 2, to set limits on the gluino mass from existing beam dump experiments.

Earlier attempts to place limits on SUSY particle masses centered on the gluino for two reasons; firstly in the D-type models then popular the gluino was predicted to be light, perhaps even massless [1], and secondly an unexpected, neutral, strongly interacting particle provided an incentive to take a closer look at data from current experiments in order to put a stronger bound on its mass.

The decays of gluinos have been discussed briefly in the previous chapter (for more details see ref.[2]), and the lifetimes have been used to exclude the possibility of a very light, long lived, gluino by searches for events with tracks which do not point back to the primary interaction vertex [3].

If the gluino is lighter than about 2 GeV we have to face the vexed question of the extent to which its binding with other coloured particles to form a colourless hadron will alter the effective mass of the particle produced in experiments. Various techniques have been used in attempts to calculate the masses of supersymmetric hadrons [4,5], and it is still not completely clear that a limit of 2-6 GeV on the gluinoball mass, as found in this chapter, completely excludes a massless gluino [6], though beam contamination searches [4,7] rule out

long lived ($\tau > 10^{-8}$ sec.) charged and neutral SUSY hadrons in the mass range between about 1.5 GeV and 10 GeV. However, if the gluino were very light, it is difficult to imagine that the charged SUSY hadrons could be much heavier than the neutral ones [4,5], and the limits are then much more stringent. In any case, if the gluinos really are this light, there will be enormous uncertainties in the parton model calculations of the gluino production rate (see chapter 2).

For more massive gluinos we are probably safe in assuming that the mass of the gluinoball exceeds that of the gluino by about 1 GeV.

In the rest of this chapter we concentrate on the limits that can be obtained from beam dump experiments. Kane and Leveille [3] set limits on the gluino mass using the published limits on the total production $\times$ interaction cross section for axions. Unfortunately this is incorrect as the decay products of gluinos will be produced with much larger p_T than axions, which were assumed to be produced like pions. We have corrected the limits, which are unfortunately much weakened. In section 2 we review some of the relevant beam dump experiments and in section 3 we use the results of these experiments to extract limits on the gluino mass. Our conclusions are summarised in section 4.

2. The Beam Dump Experiments.

The experiments most relevant to gluino searches are those which have been performed using the 400 GeV proton beam of the CERN SPS, which is 'dumped' into a thick copper target. There is then several hundred metres of shielding before the detector.

The purpose of such an experiment is to examine the sources of 'prompt' neutrinos coming from the semi-leptonic decays of short lived particles, in particular mesons containing charmed quarks. The more conventional sources of neutrinos from the decay of π and K mesons are greatly reduced by these mesons having their energy degraded by collisions within the dump before they decay.

In addition to prompt neutrinos, any other penetrating particle which was produced in the initial proton-nucleon collision, or in a rapid ($<10^{-11}$ sec.) decay of particles produced in these collisions could be detected.

A number of beam dump experiments have been performed by various collaborations using various detectors, we review here some of those from which relevant limits on the gluino mass might be extracted.

One of the early experiments was performed using the Gargamelle detector [8]. Analysing the results in terms of the expected weak interactions of neutrinos, after corrections for detector inefficiencies, 26.9 charged current (CC) events compared to 5.1 candidate neutral current (NC) events were found where, within the framework of the standard model, 6.2

would have been expected. The collaboration do not quote any errors on these results, but with such small event numbers, a typical statistical error might lead to the number of NC events being 5.1 ± 2.3 , leading to a maximum excess of NC events of about 1.3 compared to the prediction of the standard model.

Another early experiment was performed by the CDHS collaboration [9]. The CDHS detector can only directly detect muons, and not electrons, thus events without a muon may either be produced by ν_e or $\bar{\nu}_e$ (NC or CC) or by a genuine neutral current event involving ν_μ or $\bar{\nu}_\mu$. However the shower development in the detector has been analysed, and from this the total number of events involving electrons can be found, and hence the total number of genuine NC events. They find that their results are consistent with a simple model for the production of $D\bar{D}$ pairs. However they also state that "Up to 65 out of the 130 excess muonless events [i.e. after the subtraction of expected muonic neutral current interactions and all known sources of ν_e] could be purely hadronic showers and due to axion interactions." Presumably then this number of events could also have been due to prompt Goldstinos or photinos from gluino decay.

A later CDHS experiment [10] does not perform this analysis (by this time axion searches had proved fruitless) and merely assumes the NC/CC ratio predicted by the standard model to differentiate between muonless events produced by muon and electron neutrinos.

The experiment performed by the CHARM collaboration [11] in its main analysis again assumes the standard model NC/CC ratio in deciding how many muonless events are genuine NC events,

and how many are CC events with an electron. An interesting feature of this experiment is the significantly large number of low energy ($E_{vis} < 20$ GeV) muonless events in excess of the number predicted by the usual $\bar{D}D$ production and decay model. They have performed a measurement of the shower profile to detect the $e^\pm$ CC events directly and find that the results of this analysis are in agreement with their model of $\bar{D}D$ production. This indicates that the excess events are genuine neutral current events. Cabbibo, Farrar and Maiani [12] have suggested that these events are indeed due to Goldstinos or photinos, but it seems difficult to explain the energy distribution of such events, which in any case are not present in the latest CHARM experiments [13].

The most interesting experiments from the point of gluino searches are the two performed by the BEBC group [14]. By using a heavy liquid bubble chamber, electrons are identified unambiguously and an accurate value for the NC/CC ratio is obtained. The first of these two experiments set a constraint on axion production of $\sigma_{prod} \sigma_{int} (axion) < 2 \times 10^{-67} \text{cm}^4$.

They report an NC/CC ratio of 0.42 ± 0.17 when they would expect 0.324 ± 0.006 . The second BEBC experiment has rather better statistics and reports an NC/CC ratio of 0.28 ± 0.07 .

From these experiments, we can extract limits on the number of excess NC candidate events which could be attributed to photino or Goldstino interactions. These limits are expressed in terms of event numbers per tonne of detector per 10^{18} protons on target (abbreviated to p.o.t.), normalised to all detectors being at the position of BEBC (820m downstream of the target).

Experiment	Low Energy Cut (GeV)	Event Number Limit (events/t. 10^{18} p.o.t.)	
		(1σ)	(2σ)
BEBC I	15	1.71	2.80
BEBC II	10	0.29	0.77
CDHS I	20	0.38 (no errors quoted)	
Gargamelle	10	0.43	1.3

The best limit on non-observation of excess NC events is seen to come from the later BEBC experiment. This is to be expected as this experiment had a relatively large total event number and a straightforward analysis of the data. The CDHS experiment has a particularly large total number of events due to the large mass of its iron detector (465 tonnes, compared to 12.5 tonnes for BEBC), however it suffers from the need for involved data analysis to distinguish the $e^\pm$ CC events. The results we present below are based on the BEBC data.

The CHARM collaboration has subsequently published a similar analysis of its data [15] assuming that the photino decay dominates; their conclusions are very similar to our own. The results of an FNAL beam dump search for gluinos have also recently been published [16]. The results are similar to the CHARM results, but consistently exclude gluino masses approximately 50% greater than those excluded by CHARM for the same squark mass.

3. Calculation and Results.

To obtain a limit on the gluino mass from beam dump experiments is simply a matter of folding the differential cross section with the probability of the decay product (isotropic in the gluino rest frame) being produced at the appropriate angle to reach the detector, and with the probability that it interacts in the detector.

In the case of the two body decay of a gluino producing a Goldstino, the calculation is simplified as the energy of the emerging Goldstino is determined as a function of the Mandelstam variable t of the initial collision;

$$E_G = \frac{M^2 x E_{\text{beam}}}{M^2 - t}$$

where M is the gluino mass. The expected number of events is given by

$$N_{\text{event}} = \frac{N_{\text{beam}}}{\Sigma} \int_{4M^2/s}^1 dx G(x) \int_{4M^2/xs}^1 dx' G(x') \int_{t_{\min}}^{t_{\max}} dt \\ \times \frac{d\hat{\sigma}}{dt} \frac{4x^2 E_{\text{beam}} M^2}{(M^2 - t)^2} \frac{N_{\text{det}}}{4\pi D^2} \sigma_{\text{int}}(t).$$

The factor $\frac{4x^2 E_{\text{beam}} M^2}{(M^2 - t)^2}$ is the probability per unit solid angle that the decay product will be emitted in the right direction to reach the detector, N_{beam} is the number of incident protons of laboratory energy E_{beam} , Σ is the total proton-nucleon cross section at that energy. D is the distance from the beam dump to the detector, which contains N_{det} nucleons, and σ_{int} is the interaction cross section for a Goldstino, for which Kane and Leveille [3] give

$$\sigma_{\text{int}} = \frac{\pi M^2}{\Lambda_{\text{SS}}^4} \frac{M^2}{s_G} G(M^2/s_G)$$

where s_G is the (c.m. energy)² of the Goldstino - detector nucleon system and $G(x)$ is the gluon distribution function evaluated here at $x = M^2/s_G$.

The case of the three body decay to a photino is rather more complicated, as the photino energy is no longer completely determined by the kinematics of the gluino production process. The event number is now

$$N_{\text{event}} = \frac{N_{\text{beam}} N_{\text{det}}}{\Sigma D^2} \int_{4M^2/s}^1 dx G(x) \int_{4M^2/xs}^1 dx' G(x') \int_{t_{\text{min}}}^{t_{\text{max}}} dt \\ \times \frac{d\hat{\sigma}}{dt} \int_0^{E_{\tilde{\gamma}}^{\text{max}}(t)} dE_{\tilde{\gamma}} \frac{2E_{\tilde{\gamma}}^2 (M^2 - t)}{\pi x E_{\text{beam}} M^6} \left[3M^2 - \frac{2E_{\tilde{\gamma}} (M^2 - t)}{xE_{\text{beam}}} \right] \sigma_{\text{int}}(E_{\tilde{\gamma}}),$$

where [15]

$$\sigma_{\text{int}} = \frac{32}{9} \frac{\pi \alpha \alpha_s}{\mu^4} s_{\tilde{\gamma}}^2 \Sigma \int_0^1 e_q^2 (xs_{\tilde{\gamma}} - M^2)^2 (xs_{\tilde{\gamma}} + M^2/8) \frac{q(x)}{x^2} dx,$$

where $q(x)$ is the quark distribution function for a quark of charge e_q (in units where the electron charge is unity), and the sum runs over all quarks and antiquarks in the nucleon. (Note that the formula given for $\sigma_{\text{int}}(E_{\tilde{\gamma}})$ in ref [3] is incorrect.)

A computer program was written to calculate the expected event numbers for each decay mode, taking a rough account of the low energy cut of the detector. This was done by using the criterion that the energy of the particle arriving at the detector must be greater than the low energy cut of the detector. This will produce an overestimate of the number of events that would be expected, since some particles will satisfy this criterion and yet deposit less than the required

amount of visible energy in the detector, the balance being carried away by the secondary photino or Goldstino which escapes the detector. The results of our computation were extremely insensitive to the value chosen for the low energy cut, changing by less than 5% when the cut was changed from the 10 GeV appropriate to the BEBC experiment to 2 GeV.

We give the lower limits on the gluino mass as a function of the SUSY breaking scale (Goldstino decay) or squark mass (photino decay) in figures 1 and 2.

4. Conclusions.

The limits on the gluino mass which can be obtained from existing beam dump data for reasonable values of Λ_{SS} and μ are very weak. It seems certain that stronger constraints can be found from ISR and $\bar{p}p$ collider experiments, both from searches for tracks which do not point back to the original interaction vertex, and for events with missing energy.

These conclusions, which were first reached by Kane and Leveille [3], are now even stronger since, unfortunately, we see that their limits are considerably weakened by taking the angular distribution in gluino production into account. It might be possible [3] to improve the limit by making a p_T^{vis} cut to decrease the neutral current background, but it seems that more experiments with better statistics and an emphasis on measuring the NC/CC ratio would be needed if experimental uncertainties are to be reduced to a level where a useful limit can be found. In this context a design for a 20 TeV beam dump experiment is discussed in ref.[17].

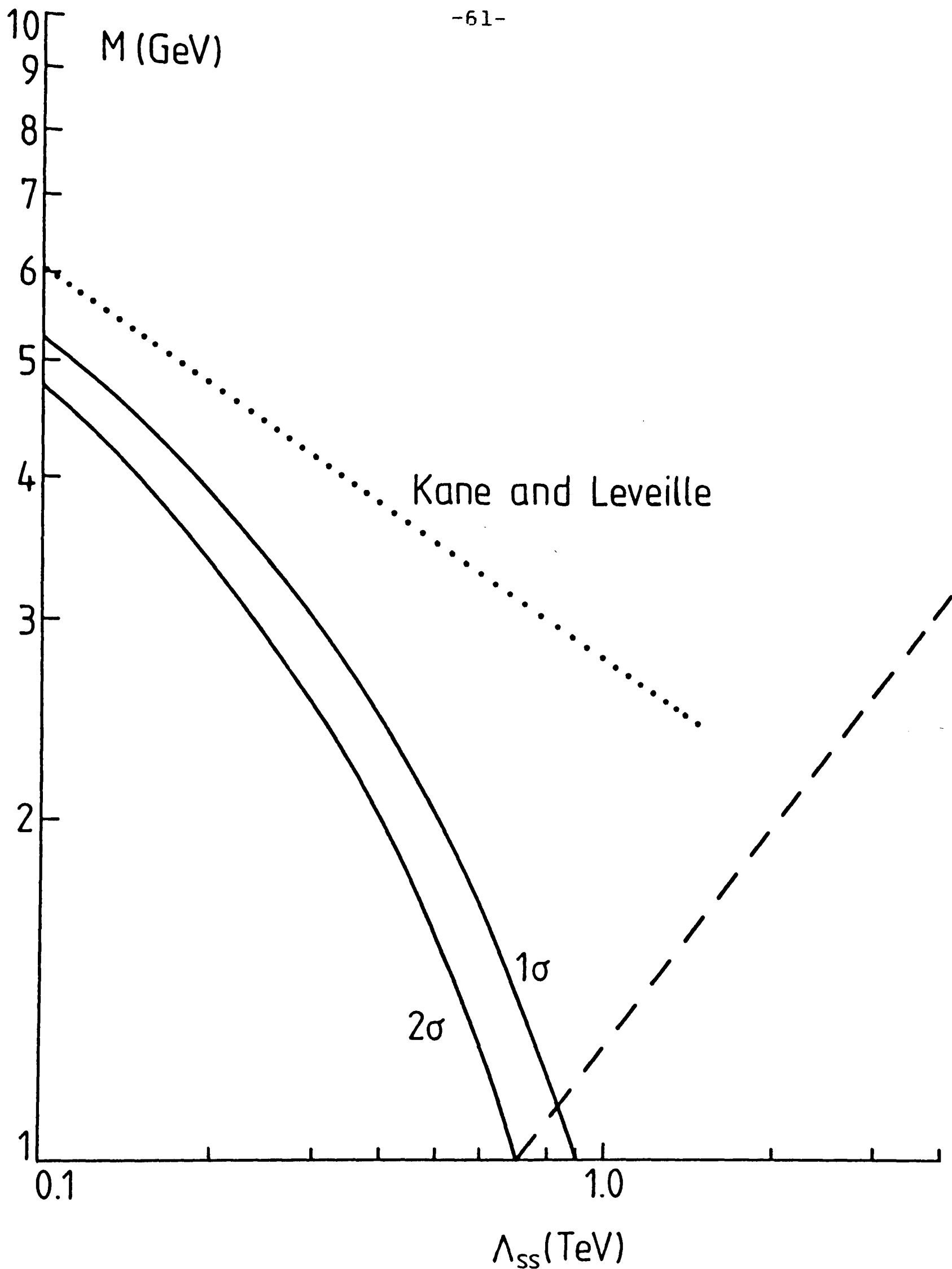


Fig. 1. Lower limits on M as a function of Λ_{SS} valid for $\mu > 0.09\Lambda_{SS}$. Solid lines: our analysis. Dotted lines: the previous bound derived in ref.[3] assuming gluinos produced with the same angular distribution as pions. The requirement that the gluino lifetime is not too long excludes the region to the right of the dashed curve [3].

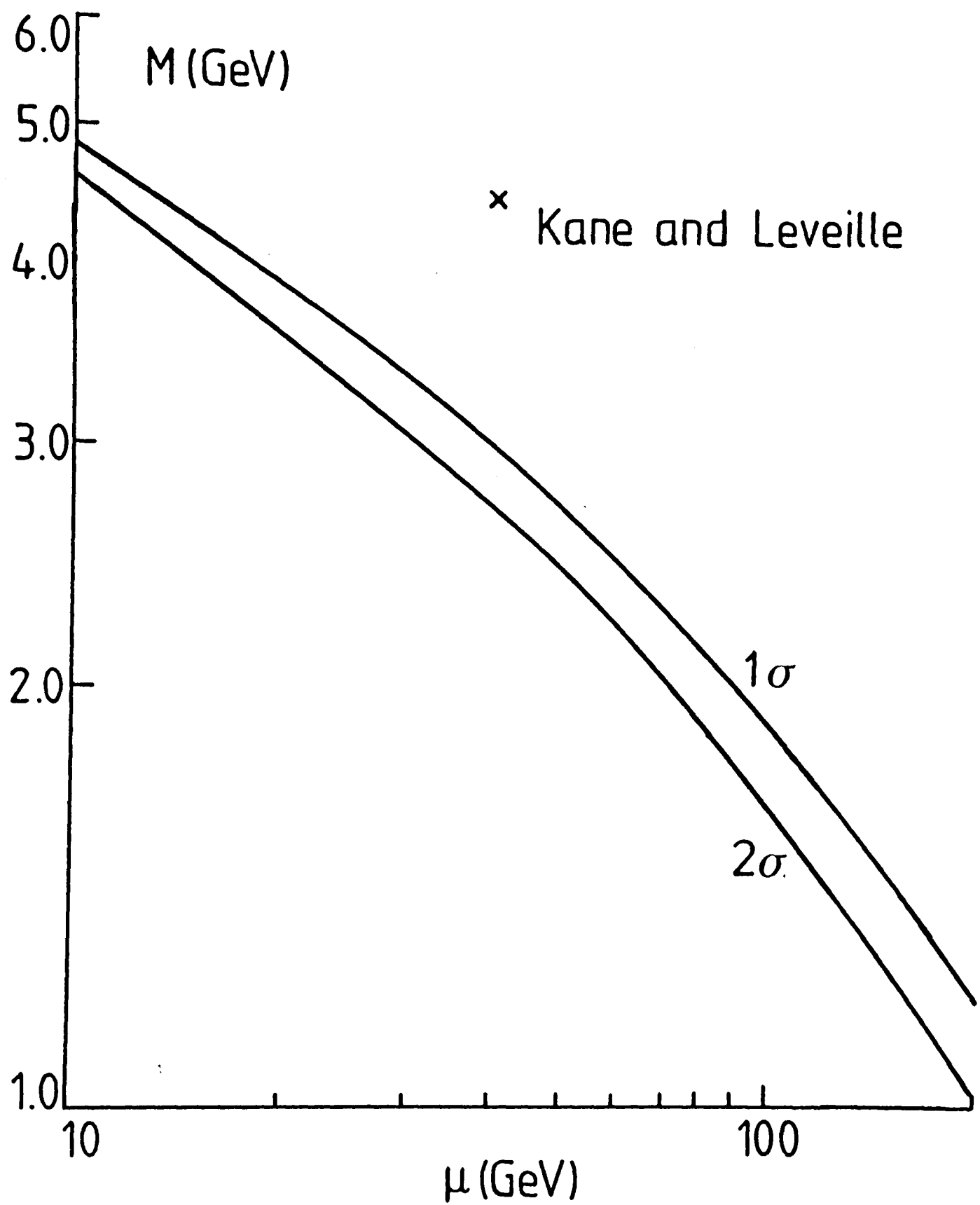


Fig. 2. Lower limits on M as a function of μ valid for $\mu < 0.09\Lambda_{\text{ss}}$. The limit derived in ref.[3] assuming $\mu = M_W/2$ and gluinos produced with the same angular distribution as pions is also shown.

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Chapter 4. The Gaugino-Higgsino Mixing Matrix and the ep Production of Squarks and Sleptons.

1. Introduction.

The cross sections for ep production of squarks and sleptons by Wino, Zino and photino exchange have been calculated by Jones and Llewellyn Smith [1]. In that calculation the simplifying assumption was made that there was no mixing between the gauginos and the Higgsinos [2].

The consequences of this are rather severe, for in the simplest SUSY models the mass terms in the charged sector take the form

$$\begin{bmatrix} \tilde{W}^+ & \tilde{H}_1^+ \end{bmatrix} \begin{bmatrix} 0 & g_2 v_2 \\ g_2 v_1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{W}^- \\ \tilde{H}_2^- \end{bmatrix},$$

so that there is a great deal of mixing, particularly if $v_1 \sim v_2$ as we would expect (v_1 and v_2 are the v.e.v's of the two scalar Higgs fields).

We assume throughout that the two v.e.v's are equal. There seems to be no good reason for this to be the case, but it simplifies the calculation in models which already have a large number of mass parameters for which values must be assumed.

If $v_1 = v_2$ then the masses of the charginos are $\pm M_W$. The signs are important, for the amplitude for a process with an electron and quark in the initial state is

$$\frac{m_1 \cos\phi \cos\theta}{t-m_1^2} + \frac{m_2 \sin\phi \sin\theta}{t-m_2^2},$$

where m_1 and m_2 are the chargino masses and θ and ϕ the mixing angles between the two chargino eigenstates and the Winos and

Higgsinos. If $v_1 = v_2$ we can diagonalise the mass matrix with a single rotation, and so we may choose $\theta = \phi$, but then all the sines and cosines are equal to $1/\sqrt{2}$, and the opposite signs attached to the eigenmasses cause the amplitude to vanish.

There is no such cancellation for the case where the incident particles have opposite helicity, hence while the electron - valence quark cross section found in ref.[1] would vanish in this limit, the positron - valence quark result (which was much smaller) would be unaffected. Note however that both cross sections had a considerable contribution from sea-quarks, which contains both helicity combinations. Calculations with this simple mass matrix produced an e^- cross section down by almost an order of magnitude on the previous result.

2. Calculation and Results.

The formula for the charged current cross section is

$$\sigma^{AB} = \left[\frac{e}{\sqrt{2} \sin\theta_W} \right]^4 (\cos^4\theta \hat{\sigma}^{AB}(\mu_1) + \sin^4\theta \hat{\sigma}^{AB}(\mu_2) + 2 \cos^2\theta \sin^2\theta \hat{\sigma}_{\text{int}}^{AB}(\mu_1, \mu_2)),$$

where $\hat{\sigma}^{AB}$ and $\hat{\sigma}_{\text{int}}^{AB}$ are as in equations (3) and (5) of ref.[1], and θ is the rotation needed to diagonalise the mass matrix. The cross section for neutral currents is as given in equation (4) of ref.[1], except that the sum now runs over the four neutralino eigenstates, and the couplings of equation (7) should be multiplied by the square of the mixing amplitude to the Wino or Bino.

Recently, supersymmetric models have become fashionable in which the diagonal elements of the chargino matrix are non-zero. Following Ellis et al.[3], we parameterise the mass matrix as

$$\begin{bmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & -\epsilon \end{bmatrix}.$$

We then consider various values of the parameters, consistent with the existing limits from cosmology and PEP/PETRA limits on light charged particles [3], and calculate the charged current cross sections.

In addition we calculate the neutral current cross sections, diagonalising the neutralino mass matrix;

$$[\tilde{W}_3, \tilde{B}^0, \tilde{H}_1^0, \tilde{H}_2^0] \begin{bmatrix} M_2 & 0 & -g_2 v_1/\sqrt{2} & g_2 v_1/\sqrt{2} \\ 0 & M_1 & g_1 v_1/\sqrt{2} & -g_1 v_2/\sqrt{2} \\ -g_2 v_1/\sqrt{2} & g_1 v_1/\sqrt{2} & 0 & \epsilon \\ g_2 v_2/\sqrt{2} & -g_1 v_1/\sqrt{2} & \epsilon & 0 \end{bmatrix} \begin{bmatrix} \tilde{W}_3 \\ \tilde{B}^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{bmatrix}.$$

Once again we assume that $v_1 = v_2$, and further assume that our low energy, supersymmetric, $SU(2) \times U(1)$ model is the consequence of an $SU(5)$ GUT, in which case we expect [3]

$$M_1 = \frac{5}{3} \frac{\alpha_1}{\alpha_2} M_2.$$

Grinstein et al.[4] suggest that the three interesting limits for the parameters ϵ and M_2 are the "Higgsino" limit $\epsilon \rightarrow 0$, where the lightest new particle is predominantly a Higgsino, the "Wino" limit $M_2 \rightarrow 0$, and the "Wiggsino" limit, where both parameters are small. We consider each of these limits to the extent that they are consistent with cosmological bounds, and as there is no reason why either of these parameters should be small, we also consider a fourth set of values; $M_2 = 120$ GeV, $|\epsilon| = 100$ GeV, consistent with the existing bounds, and implying no particularly light -inos.

In addition we are free to choose $\epsilon \gtrless 0$, so we have eight sets of parameters for which we calculate the charged and neutral current cross sections.

The values of M_2 and ϵ representative of the other three limits are:

Higgsino limit; $M_2 = 80$ GeV, $\epsilon = \pm 33$ GeV

Wino limit; $M_2 = 3$ GeV, $\epsilon = \pm 80$ GeV

Wiggsino limit; $M_2 = 3$ GeV, $\epsilon = \pm 1.6$ GeV

In comparing our results against those of ref.[1] we discovered an error in the computer program used in the previous calculation, in that the parton distributions had incorrectly been evaluated at $Q^2 = \frac{1}{2}(s - \mu_1^2 - \mu_2^2)$ rather than $\frac{1}{2}(\hat{s} - \mu_1^2 - \mu_2^2)$, where μ_1 and μ_2 are the final state particle masses. This error led to a too rapidly evolving sea-quark distribution, which

accounts for their results being slightly larger than our equivalent computation at high energies (figure 1). The discrepancies at lower energies are due entirely to the difficulties in transferring values from the linear scale used in ref.[1] to our logarithmic scale when the cross sections are small.

The largest and smallest charged current cross sections we found among our eight parameter sets are plotted in figure 2. These correspond to: No limit, $\epsilon > 0$ (smallest σ_{e+}); Wiggsino limit (smallest σ_{e-} , no significant dependence on the sign of ϵ); and no limit, $\epsilon < 0$ (largest σ_{e+} and σ_{e-}).

We have assumed that the lepton beams are unpolarised. When σ_{e-} is at its minimum σ_{e+} is also rather small, but the converse is not true, for the maximum value of σ_{e+} corresponds to a value of σ_{e-} within a factor of 2 of its maximum (see table 1).

The extreme results for neutral currents are similarly presented in figure 3, where the smallest cross sections for both e^+ and e^- were produced by the "no limit" set of parameters with $\epsilon > 0$, and the largest by the Wiggsino limit with $\epsilon > 0$.

The cross sections for all eight sets of parameters are given in table 1 for $s = 10^5 \text{ GeV}^2$, the value appropriate to the projected HERA ep collider, and assuming final state particles to each have a mass of 40 GeV. Table 1 also shows the results for a squark mass of 20 GeV and a slepton mass of 60 GeV, or vice versa, the two masses enter symmetrically. Table 2 shows the results for squark and slepton masses of 20 + 20 GeV and 80 + 80 GeV, and in figure 4 we plot the charged current cross

sections for mass matrices which are representative of the maximum and minimum values of the cross section over the mass range considered. It was not possible to produce such a plot for neutral currents as the mass matrix which gives the extremal cross section varies rather strongly with the final state particle masses, but the neutral current cross sections were always found to lie around the upper end of the range of the charged current cross sections.

Throughout these calculations we have used the parton distributions of Glück, Hoffman and Reya [5].

Note that the cross sections presented in ref.[1] are quite similar to our maximal predictions, which exceed our smallest cross sections by about an order of magnitude, thus those results present a view of the prospects for finding these particles which is optimistic, but not wildly so.

It has been suggested [6] that these events with their characteristic signatures [1,6,7] could be detected at HERA down to a cross section of 0.1 pbn (10^{-37} cm²), and even if this is over-optimistic by as much as an order of magnitude, there is still a good chance of finding evidence for these particles if the masses are in the range we have assumed.

In particular, if the cross sections for charged and neutral current processes could be measured for various polarisations and charges of the lepton beam, it might be possible to learn something about the gaugino mass matrix once the masses of the final state squarks and sleptons had been fixed by the energy dependence of the cross sections, which are presumably quite close to threshold.

Since the contribution from sea quarks is unaffected by the charge or helicity of the lepton beam, sensitivity is improved by looking at the difference between two cross sections. This yields four independent pieces of data (one from charged currents, three from neutral currents) from which it ought to be possible to eliminate a fair region of the (ϵ, M_2) plane. As an example of this technique we present some typical contour plots of cross section differences in figure 5. In each plot we have kept the same contour interval ($0.375 \times 10^{-35} \text{ cm}^2$).

For a significant fraction of the domain we have considered, the charged current cross section is most sensitive to the exchanged particle mass matrix parameters, though this may be outweighed by difficulties in its measurement if, as predicted in most models, sleptons (squarks) decay to leptons (quarks) plus a light neutralino, as there will be three neutral particles in a charged current final state, compared to two for neutral currents.

3. Concluding Remarks.

Provided SUSY particles are light enough for sleptons and squarks to be produced at a reasonable rate in an accelerator, a reasonably accurate measurement of the cross section will yield not only masses for the squarks and sleptons produced, but also a good deal of information about the mass matrices of the Winos and Higgsinos. This task will be simplified if the event rate is sufficient for the masses of the exchanged particles to be inferred from the angular distribution of the events, for example by an analysis of the missing p_T distribution, leaving only the couplings to be determined.

Table 1.

Cross sections for charged current (CC) and neutral current (NC) processes at $s = 10^5 \text{ GeV}^2$ with final state particle masses of 40 GeV. (Figures in brackets are for a squark mass of 20 GeV and a slepton mass of 60 GeV or vice versa.)

Limit	M_2 (GeV)	ϵ (GeV)	σ_{e-CC}	σ_{e+CC}	σ_{e-NC}	σ_{e+NC}
$(\times 10^{-35} \text{ cm}^2)$						
None	120	100	0.397 (0.381)	0.081 (0.076)	0.465 (0.459)	0.265 (0.230)
	120	-100	0.753 (0.782)	0.268 (0.262)	0.509 (0.386)	0.457 (0.324)
Wino	3	80	0.473 (0.491)	0.256 (0.246)	0.537 (0.613)	0.773 (0.846)
	3	-80	0.465 (0.488)	0.273 (0.264)	0.538 (0.602)	0.866 (0.940)
Higgsino	80	33	0.230 (0.219)	0.086 (0.079)	0.524 (0.534)	0.394 (0.386)
	80	-33	0.124 (0.122)	0.157 (0.150)	0.474 (0.384)	0.738 (0.324)
Wiggsino	3	1.6	0.036 (0.032)	0.115 (0.104)	0.490 (0.758)	0.703 (0.988)
	3	-1.6	0.035 (0.031)	0.115 (0.104)	0.689 (0.563)	0.221 (0.767)

Table 2.

Cross sections for charged current (CC) and neutral current (NC) processes at $s = 10^5 \text{ GeV}^2$ with final state particle masses of 20 GeV. (Figures in brackets are for final state particle masses of 80 GeV.)

Limit	M_2 (GeV)	ϵ (GeV)	σ_{e^-CC}	σ_{e^+CC}	σ_{e^-NC}	σ_{e^+NC}
$(\times 10^{-35} \text{ cm}^2)$						
None	120	100	1.05 (0.043)	0.359 (0.003)	1.64 (0.003)	1.13 (0.013)
	120	-100	3.63 (0.046)	1.66 (0.006)	2.43 (0.031)	2.11 (0.027)
Wino	3	80	2.11 (0.018)	1.26 (0.009)	5.27 (0.011)	5.85 (0.063)
	3	-80	2.45 (0.015)	1.51 (0.009)	5.24 (0.011)	6.33 (0.031)
Higgsino	80	33	0.602 (0.025)	0.299 (0.004)	2.20 (0.027)	1.76 (0.018)
	80	-33	0.584 (0.012)	0.678 (0.004)	2.23 (0.023)	3.06 (0.038)
Wiggsino	3	1.6	0.152 (0.007)	0.323 (0.006)	5.86 (0.019)	6.50 (0.031)
	3	-1.6	0.151 (0.019)	0.326 (0.006)	4.98 (0.009)	5.56 (0.024)

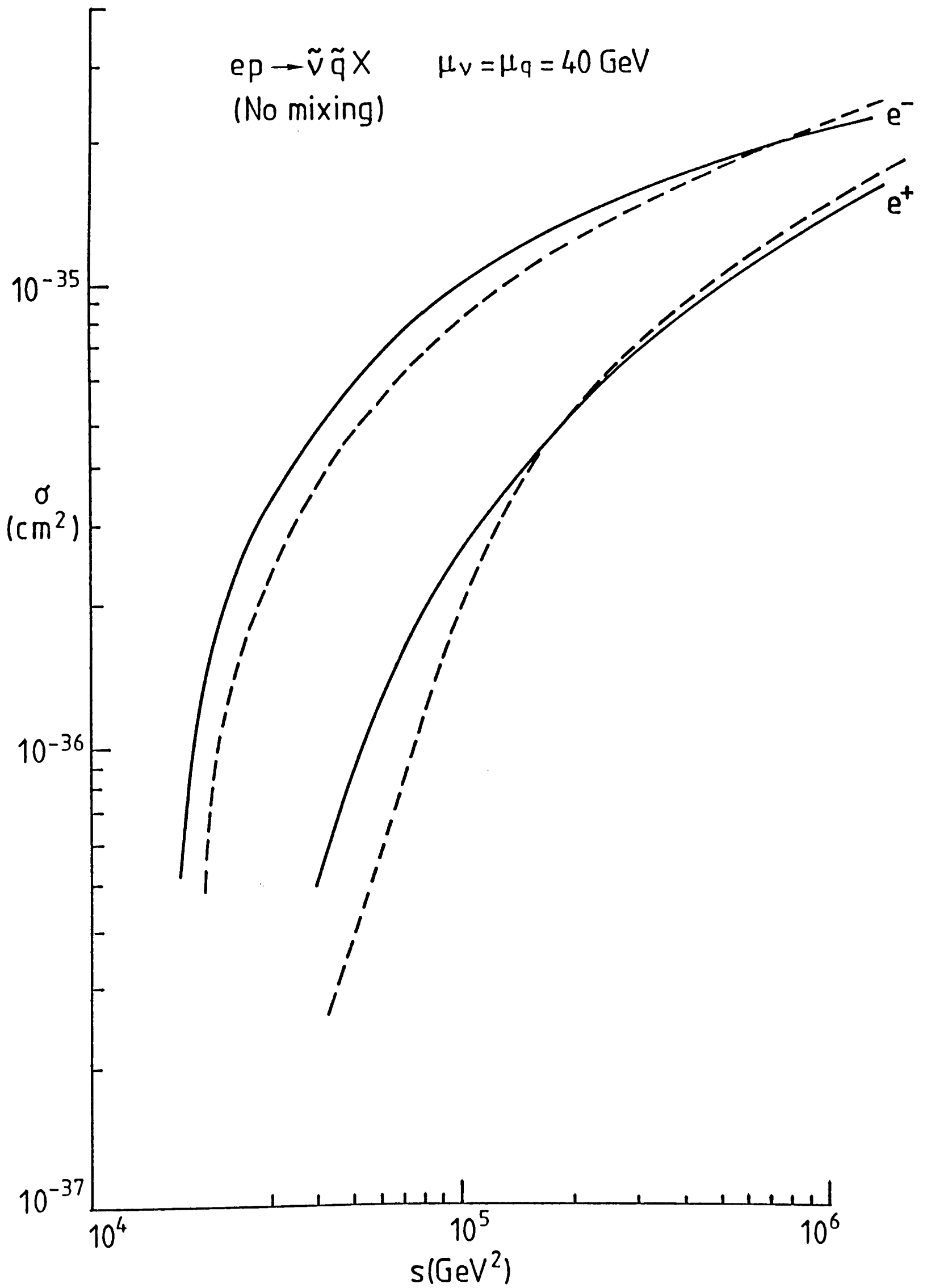


Fig. 1. Cross section for charged current processes assuming no gaugino-Higgsino mixing. The broken lines are the results from ref.[1]. $\mu_e = \mu_q = 40 \text{ GeV}$, unpolarised beams.

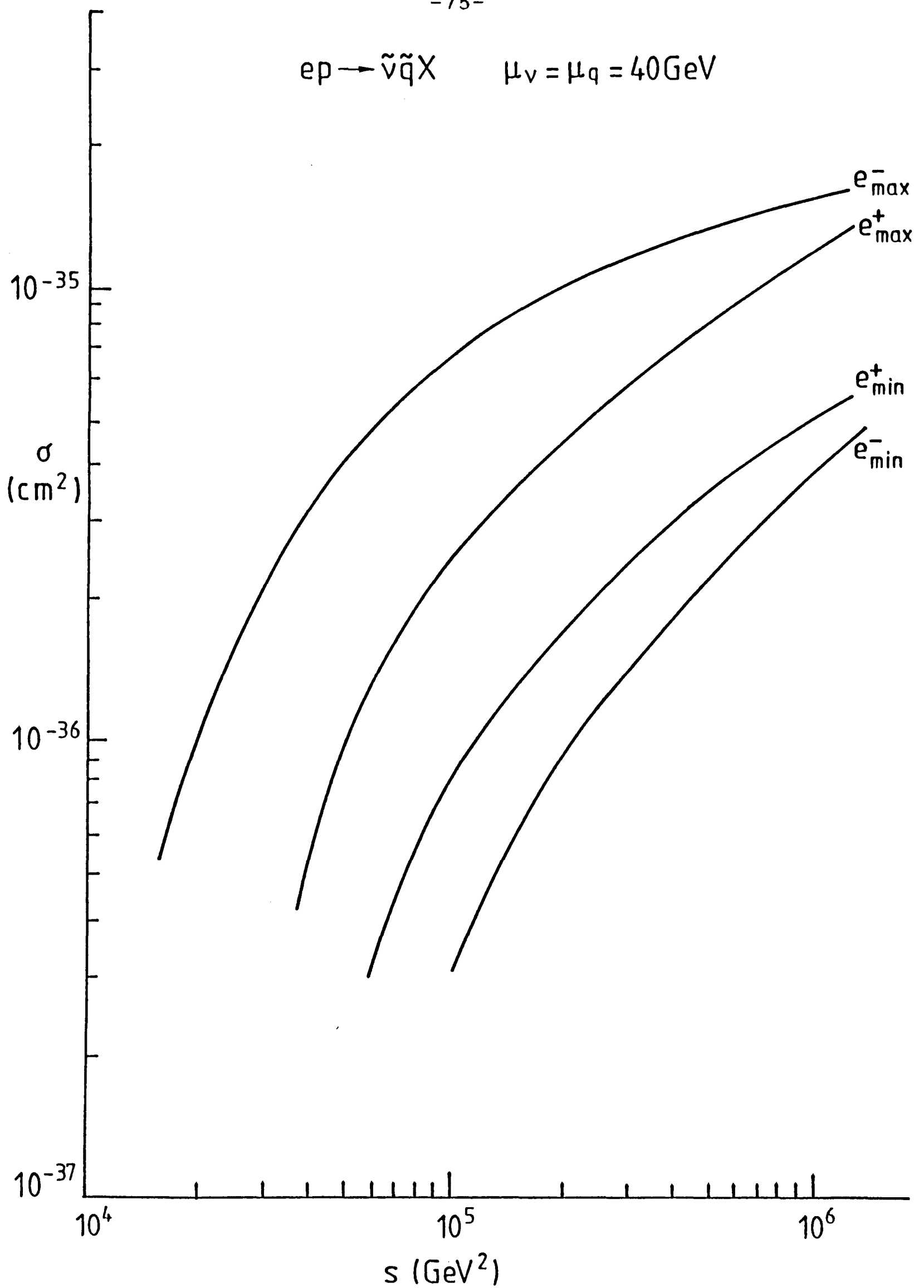


Fig. 2. Extremal values of the charged current cross section found amongst our eight mass matrix parameter sets. $\mu_e = \mu_q = 40 \text{ GeV}$, unpolarised beams.

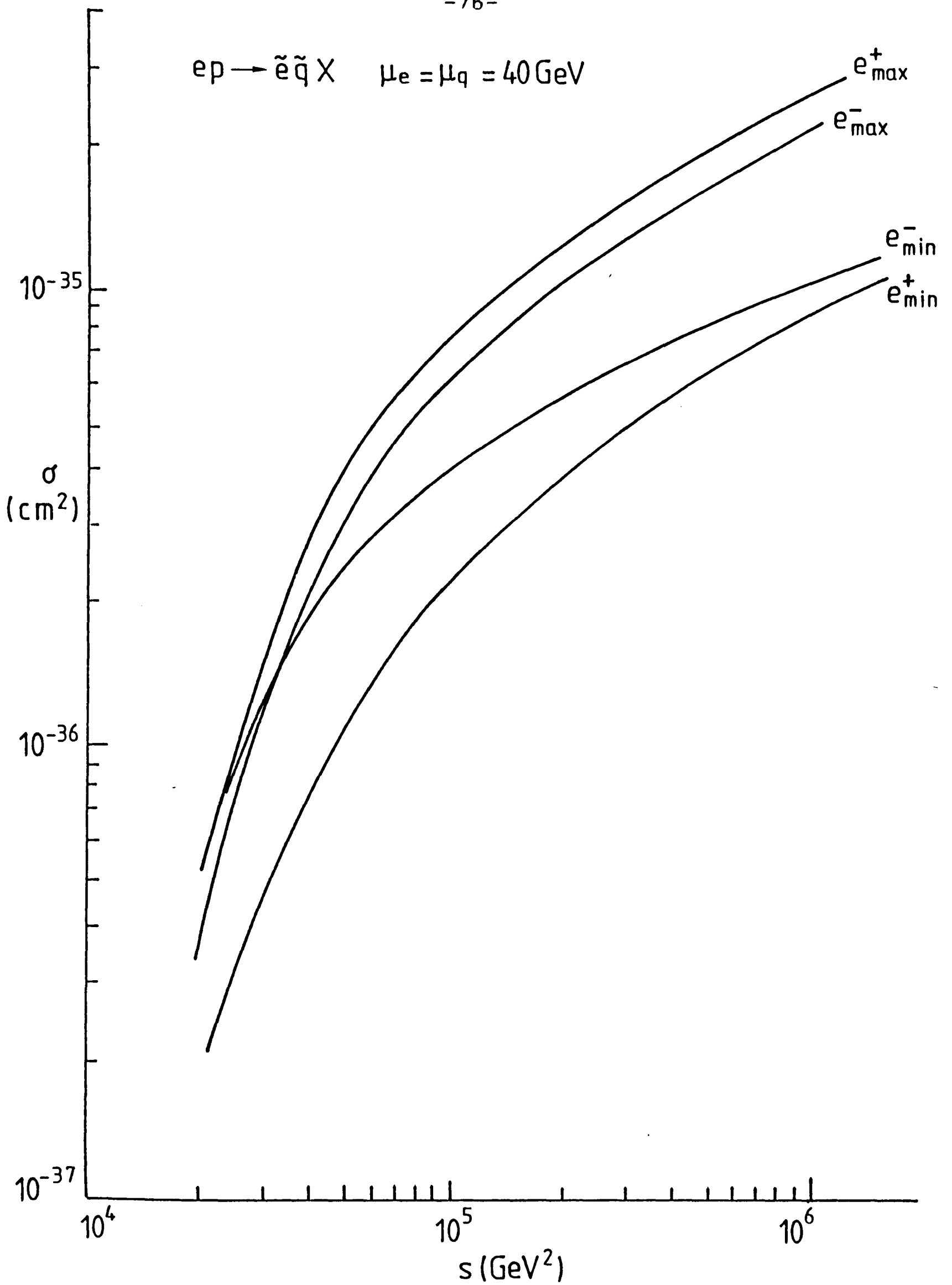


Fig. 3. Extremal values of the neutral current cross section found amongst our eight mass matrix parameter sets. $\mu_e = \mu_q = 40 \text{ GeV}$, unpolarised beams.

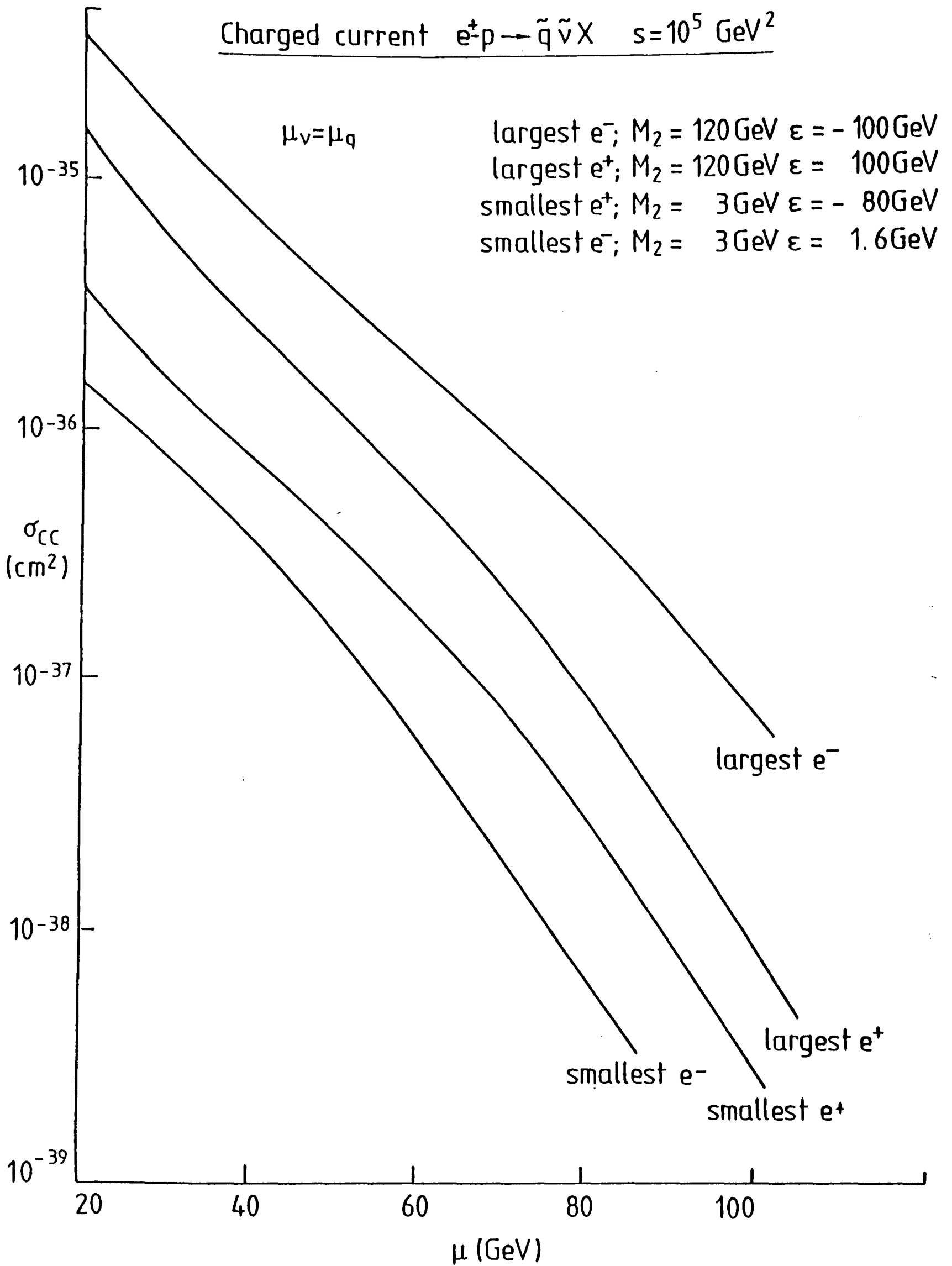


Fig. 4. Extremal values of the charged current cross section found amongst our eight mass matrix parameter sets. $s = 10^5 \text{ GeV}^2$, $\mu_e = \mu_q$, unpolarised beams.

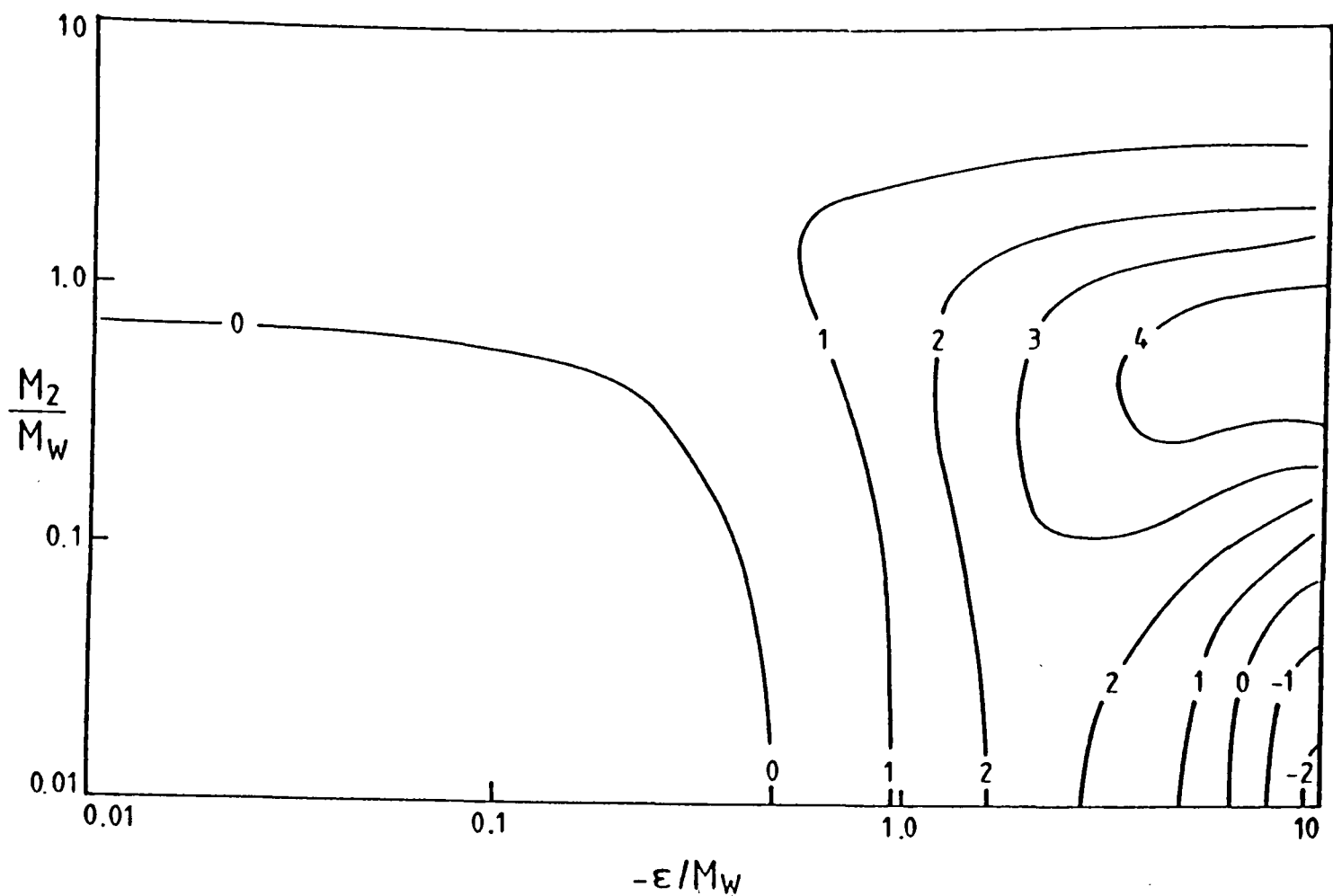


Fig.5ai

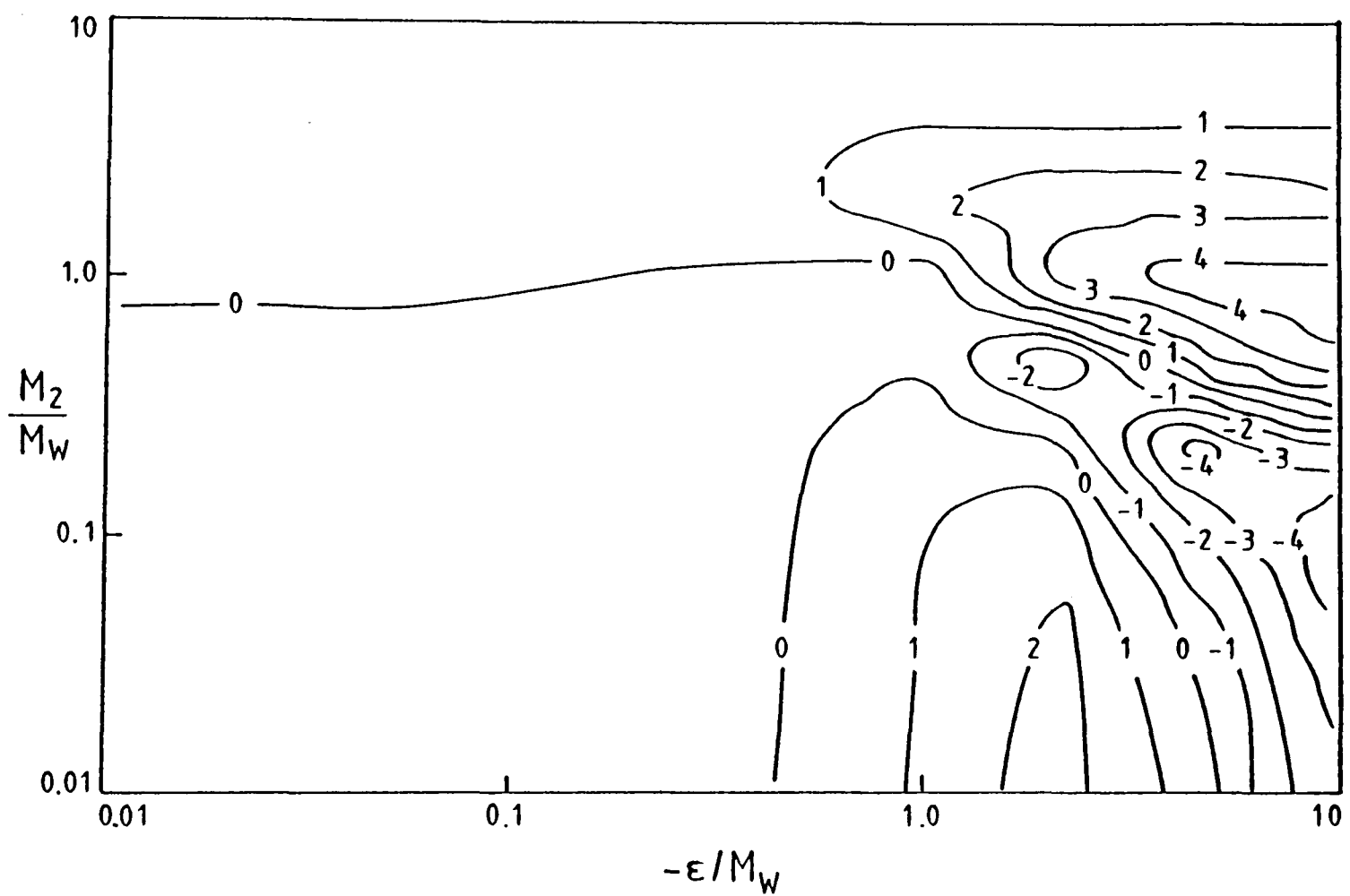


Fig.5aii

Fig. 5a. Contour plots of cross section differences in the (ϵ, M_2) plane. $s = 10^5 \text{ GeV}^2$, $\mu_e = \mu_q = 40 \text{ GeV}$. Contour separation $3.75 \times 10^{-36} \text{ cm}^2$. Charged current, LH electron - RH positron. i) $\epsilon > 0$ ii) $\epsilon < 0$.

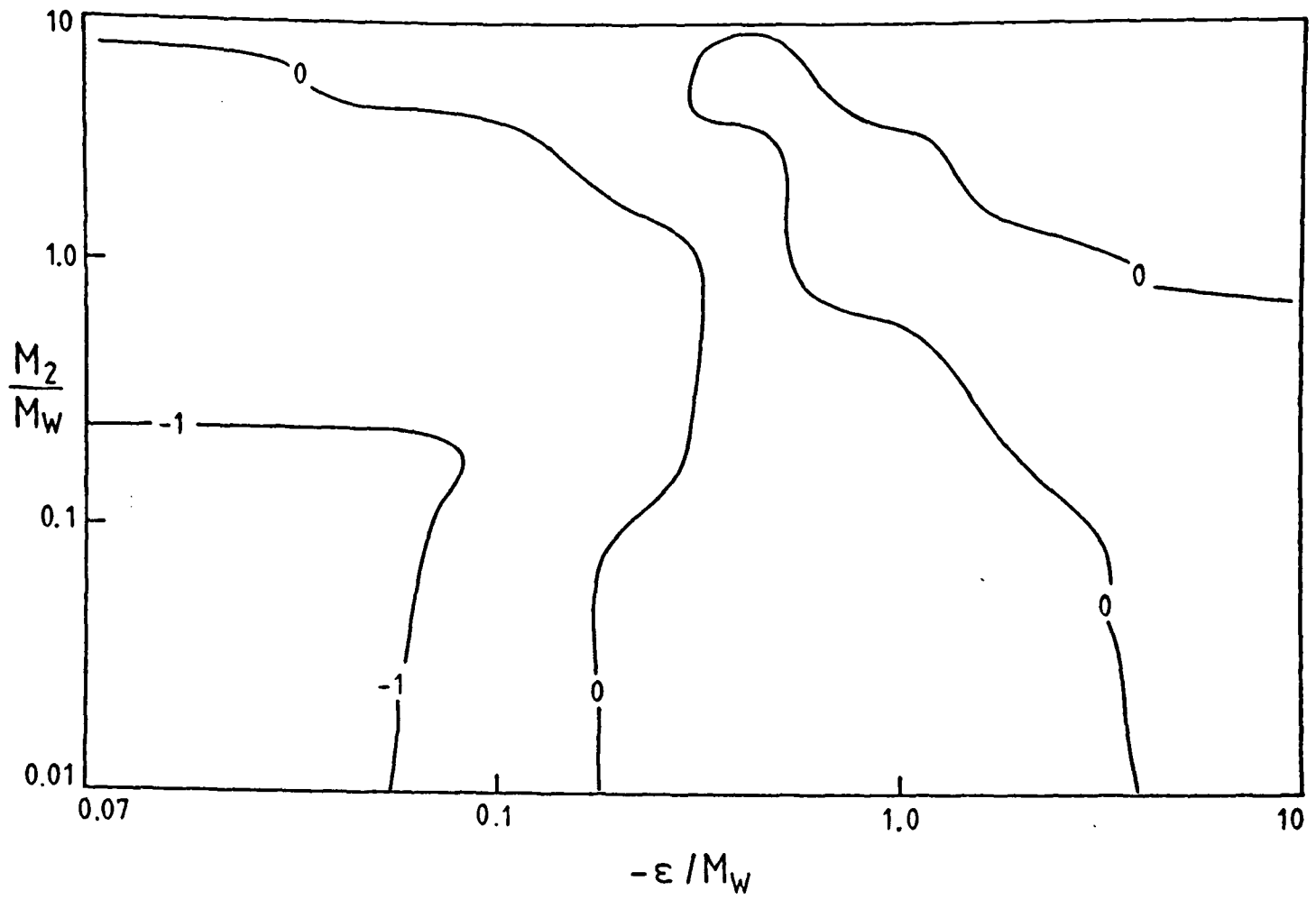


Fig.5bi

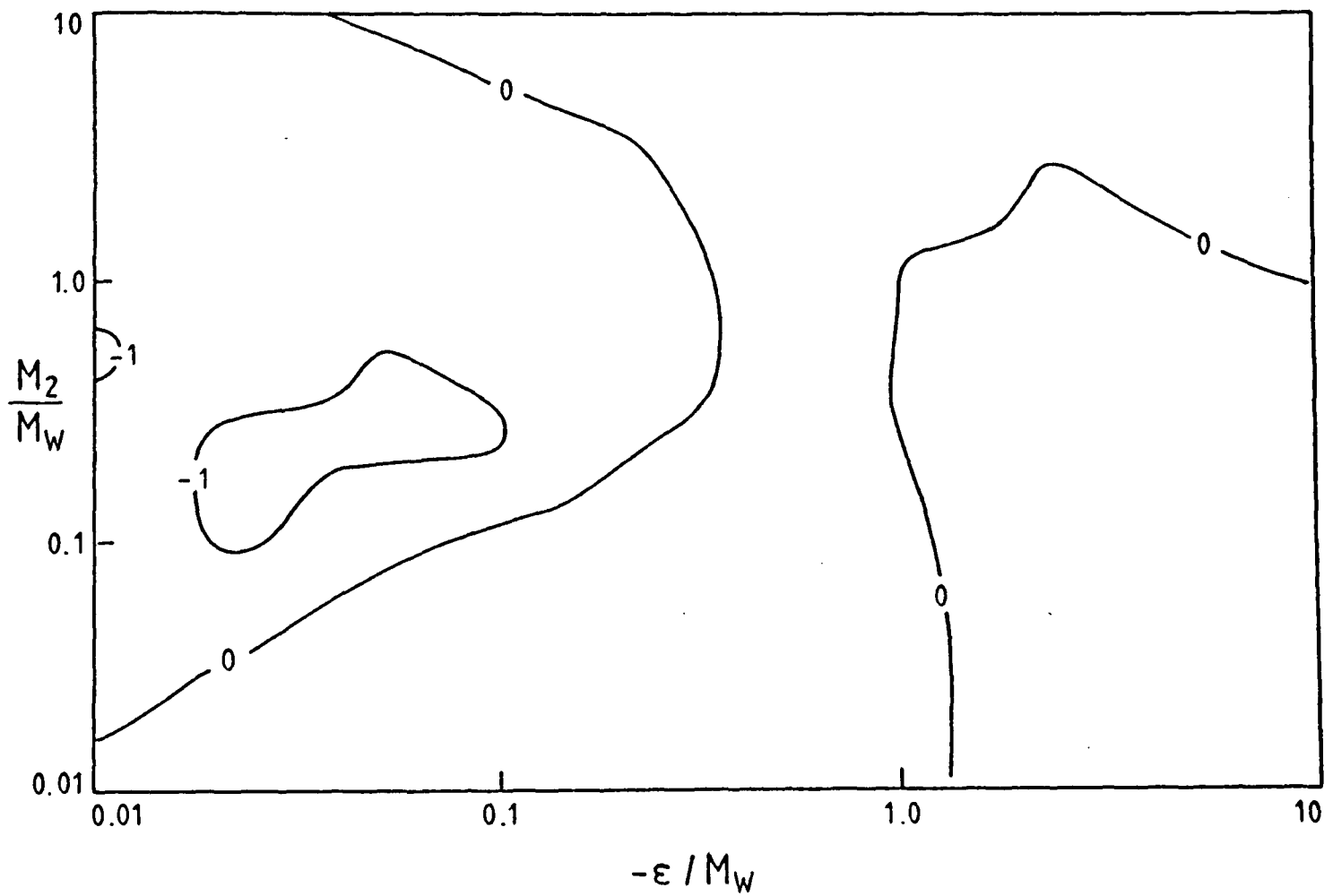


Fig.5bii

Fig. 5b. Contour plots of cross section differences in the (ϵ, M_2) plane. $s = 10^5 \text{ GeV}^2$, $\mu_e = \mu_q = 40 \text{ GeV}$. Contour separation $3.75 \times 10^{-36} \text{ cm}^2$.

Neutral current, LH positron - RH electron. i) $\epsilon > 0$ ii) $\epsilon < 0$.

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Chapter 5. The Associated Production of Photinos and Squarks or Sleptons in e^+e^- Collisions.

1. Introduction.

The prospects for finding evidence for SUSY at LEP have been widely discussed [1]. Interest has naturally centered on the pair production of squarks and sleptons as these processes are of low order in perturbation theory and have a distinct signature. It may be however, that even the lightest scalars are too heavy to be pair produced at LEP energies, but it is possible that they could be produced singly in association with some other SUSY particle such as a photino*.

The pair production of a photino with a tagged photon (figure 1) has also been discussed in the literature [2], but this process has a substantial standard model background from neutrino pair production (figure 2), which must be understood and subtracted before any conclusions about new physics can be reached.

The process studied here, $e^+e^- \rightarrow q\bar{q}\tilde{\gamma}$ is of order α^3 rather than α^2 , but has the advantage that its characteristic signature of two hadron jets with missing p_T and no lepton in the final state has a standard model background of $O(\alpha^4)$ neutrino pair production (e.g. figure 3)**. A similar process, $e^+e^- \rightarrow q\bar{q}\tilde{g}$ has also been studied in the literature [4], but

* We use the term photino to refer to the lightest neutral fermion. In general this will be a mixture of Wino, Bino and the two Higgsinos [3].

** Footnote on next page.

although the cross section is larger for the same masses, the gluino is generally predicted to be rather heavier than the photino, and in models currently fashionable the gluino is often heavier than the squark (see chapter 1) making this process a poor second to squark pair production.

Another related process is the leptonic one, $e^+e^- \rightarrow l\tilde{l}\tilde{\gamma}$ where l may be any of the e, μ, τ leptons (the results for electron + selectron will be different to those for μ or τ production as there will be extra t - and u - channel diagrams).

We give results for $e^+e^- \rightarrow \mu\tilde{\mu}\tilde{\gamma}$ (at very high energies the lepton pair in the final state leads a large standard model background from W -boson pair production and subsequent leptonic decay).

Both $e^+e^- \rightarrow \mu\tilde{\mu}\tilde{\gamma}$ and $e^+e^- \rightarrow q\tilde{q}\tilde{\gamma}$ will have a background from the pair production of τ -leptons, since this process is understood, it can be subtracted. It can also greatly be reduced by the application of suitable cuts; at high energies the opening angle for the τ decay products is usually very small, and if a decay product is emitted at a large angle to the direction of the parent τ it will have a very low energy, so the background to the leptonic final state can be substantially reduced by a cut removing events with low energy leptons and events in which the leptons are nearly back to back. The small opening angle will cause the semi-leptonic τ decays to look like 2-jet rather than 4-jet events, but here a cut to remove back to back events should efficiently remove the background of 2-jet events with substantial missing p_T .

** The possibility of bremsstrahlung emission of an on-shell Z -boson and its decay to neutrinos can be eliminated if the charged tracks can be measured with sufficient accuracy to reconstruct the missing energy and momentum completely.

2. Calculation and Results.

There are five diagrams which contribute to the process $e^+e^- \rightarrow q\bar{q}\tilde{\gamma}$ (figure 4). The calculation of the matrix element is straightforward if rather long-winded. The only subtlety concerns the Majorana nature of the neutralinos [5].

Note that diagram 3 of figure 4 arises because of the Higgsino component of the neutralino, which couples to the Z-boson.

In diagrams 3 and 4 we take the left- and right- handed scalar electrons to be degenerate.

The analytic results of the calculation are presented in the appendices. In presenting a numerical analysis of these results we make the following assumptions:

- i) There are two light generations which are essentially degenerate. Although all the squarks are expected to be nearly degenerate (see chapter 1), this process is only interesting at energies where squarks are too massive to be pair produced. At such energies the extra masses of the quarks in the third generation may well reduce the cross section significantly for that generation.
- ii) We assume left- and right- handed squarks are degenerate.
- iii) To avoid the singularity at the Z-pole we have to assume a value for the Z width [6]. We have taken this as 3 GeV, the value predicted by the standard model for three light generations [7].
- iv) We note that there is an asymmetry about the beam

direction, which is not apparent from the cross sections and p_T distributions presented here. It would, of course, be useful when it comes to eliminating backgrounds from other processes with different asymmetries.

We have performed numerical integrations over phase space for energies (per beam) of 20 GeV, $M_Z/2$ and 100 GeV. We took the squark and slepton masses to be equal at 1.2 or 1.4 times the beam energy (the cross section falls very rapidly above this).

For the neutralino masses and mixing angles we took six of the eight choices described in chapter 4. The "no limit" case described there is of no interest to us here as it does not contain a light neutralino.

The results are presented in table 1 for $e^+e^- \rightarrow q\bar{q}\tilde{\gamma}$ and in table 2 for $e^+e^- \rightarrow \mu\bar{\mu}\tilde{\gamma}$. The cross sections for the leptonic final state are rather smaller as there is no colour factor, and we have considered only one generation of leptons.

The cross sections for the Wiggsino limit are particularly small off the Z-pole. This is due to destructive interference between the two almost degenerate neutralinos that occur in this limit. Note that these Wiggsino limit cross sections can effectively be doubled, as the results presented are for the lightest neutralino, in the Wiggsino limit the next lightest neutralino is very nearly as light and so will be produced at a similar rate.

We have also analysed the missing p_T distributions for two cases of interest (both with $\mu_q = 1.2 E$). These are the Wiggsino limit, $\epsilon < 0$ at $E = M_Z/2$, and the Wino limit, $\epsilon < 0$ for $E = 100$ GeV, which correspond to the largest cross sections found at each energy.

The missing p_T plots (figure 5) assume that the squark decays to a quark plus photino isotropically in the squark rest frame, and were produced by a simple Monte Carlo program which generated 40 000 random events, calculated the probability of each event from the matrix element, and the missing p_T . As a check on the accuracy of the Monte Carlo, the total cross section it produced was compared with the value found from a Gauss' rule integration routine, the two agreed to well within the expected errors (a few percent).

3. Conclusions.

We note that although the cross section is rather small, the signature is good, and there is a fair chance of observing the process if the masses are light enough for it to occur on the Z-pole. If higher energies are required then luminosities of at least $10^{32} \text{ cm}^{-2}\text{s}^{-1}$ will be required before a worthwhile event rate can be achieved.

Table 1.

Total cross section for $e^+e^- \rightarrow q\bar{q}\tilde{\gamma}$.

$\mu_q = \mu_e = 1.2E$. (Figures in brackets are for $\mu_e = \mu_q = 1.4E$.)

All cross sections are in cm^2 . The largest and smallest (non-zero) values for each parameter set are marked with an asterisk (*). Zero entries arise where the lightest neutralino is too massive to be produced in association with a squark.

Limit	M_2 (GeV)	ϵ (GeV)	E=20GeV	E= $M_2/2$	E=100GeV
Wino	3	80	4.56×10^{-37} (1.02×10^{-37}) *	3.00×10^{-36} (1.45×10^{-37}) *	3.21×10^{-38} (4.80×10^{-39}) *
	3	-80	4.68×10^{-37} * (9.48×10^{-38})	3.03×10^{-36} (1.55×10^{-37})	3.30×10^{-38} * (4.59×10^{-39})
Higgsino	80	33	0 (0)	1.89×10^{-38} * (0)	4.17×10^{-39} * (2.68×10^{-40}) *
	80	-33	0 (0)	2.07×10^{-38} (0)	4.17×10^{-39} * (7.05×10^{-40})
Wiggsino	3	1.6	1.77×10^{-40} * (4.26×10^{-41}) *	1.09×10^{-34} (4.41×10^{-35})	7.83×10^{-39} (1.61×10^{-39})
	3	-1.6	1.09×10^{-39} (2.51×10^{-40})	2.07×10^{-34} * (7.86×10^{-35}) *	4.17×10^{-39} * (7.47×10^{-40})

Table 2.

Total cross section for $e^+e^- \rightarrow \mu\mu\gamma$.

$\mu_\mu = \mu_e = 1.2E$. (Figures in brackets are for $\mu_e = \mu_\mu = 1.4E$.)

All cross sections are in cm^2 . The largest and smallest (non-zero) values for each parameter set are marked with an asterisk (*). Zero entries arise where the lightest neutralino is too massive to be produced in association with a smuon.

Limit	M_2 (GeV)	ϵ (GeV)	E=20GeV	E= $M_2/2$	E=100GeV
Wino	3	80	$1.35 \times 10^{-37} *$ (2.22×10^{-38}) *	5.50×10^{-37} (3.06×10^{-38}) *	$1.06 \times 10^{-38} *$ (2.14×10^{-39}) *
	3	-80	1.29×10^{-37} (1.93×10^{-38})	5.37×10^{-37} (3.13×10^{-38})	$1.06 \times 10^{-38} *$ (2.13×10^{-39})
Higgsino	80	33	0 (0)	$1.60 \times 10^{-39} *$ (0)	1.66×10^{-40} (1.33×10^{-41}) *
	80	-33	0 (0)	$1.43 \times 10^{-39} *$ (0)	1.66×10^{-40} (1.94×10^{-40})
Wiggsino	3	1.6	5.13×10^{-41} (1.19×10^{-41})	8.93×10^{-36} (3.38×10^{-36})	1.91×10^{-40} (3.43×10^{-41})
	3	-1.6	$4.94 \times 10^{-41} *$ (1.13×10^{-41}) *	$9.66 \times 10^{-36} *$ (3.66×10^{-36}) *	$1.92 \times 10^{-40} *$ (3.44×10^{-41})

Appendix A.

Feynman Rules for the Vertices in the Calculation of
 $e^+e^- \rightarrow q\bar{q}\gamma$.

The vertices occurring in this calculation are listed below together with the symbols used for them in appendix B. The values of those vertices which do not occur in the standard model are also tabulated.

A complete set of Feynman rules can be found in appendix C of ref.[1]. (Note that the suffices 1 and 2 are transposed in equation C77 of that reference.)

$$\begin{aligned}
 & \text{Vertex 1: } e^+e^- \text{ vertex} \quad \text{Diagram: } e^- \text{ (incoming), } e^+ \text{ (outgoing), } \gamma, Z \text{ (wavy)} \\
 & \quad \quad \quad = (A_e^{\gamma, Z} + B_e^{\gamma, Z} \gamma_5) \gamma_\mu \\
 & \text{Vertex 2: } q_{L,R} \text{ vertex} \quad \text{Diagram: } q_{L,R} \text{ (incoming), } q_{L,R} \text{ (outgoing), } \gamma, Z \text{ (wavy)} \\
 & \quad \quad \quad = (A_{q_{L,R}}^{\gamma, Z} + B_{q_{L,R}}^{\gamma, Z} \gamma_5) \gamma_\mu \\
 & \text{Vertex 3: } \chi_i^0 \text{ vertex} \quad \text{Diagram: } \chi_i^0 \text{ (incoming), } \tilde{p} \text{ (outgoing), } Z \text{ (wavy)} \\
 & \quad \quad \quad = H_i \gamma_5 \gamma_\mu
 \end{aligned}$$

$$\text{Diagram: } q_{L,R} \text{ and } \bar{q}_{L,R} \text{ annihilation into } \gamma, Z \text{ with vertex factor } c_{q_{L,R}}^{\gamma,Z} (p_1 + p_2)_\mu$$

$$\text{Diagram: } q_{L,R} \text{ and } \bar{q}_{L,R} \text{ annihilation into } \chi_i^0 \text{ with vertex factor } D_{q_{L,R}}^i (1 \mp \gamma_5)$$

[$D_{q_{L,R}} (1 \mp \gamma_5)$ is used to denote the coupling to the photino.]

$$\text{Diagram: } e_{L,R} \text{ and } \bar{e}_{L,R} \text{ annihilation into } \chi_i^0 \text{ with vertex factor } D_{e_{L,R}}^i (1 \mp \gamma_5)$$

[$D_{e_{L,R}} (1 \mp \gamma_5)$ is used to denote the coupling to the photino.]

$$c_{u_{L,R}}^{\gamma} = -\frac{2ie}{3}$$

$$c_{d_{L,R}}^{\gamma} = \frac{ie}{3}$$

$$c_{u_L}^Z = \frac{ig}{2\cos\theta_W} \left(-1 + \frac{4}{3} \sin^2\theta_W\right)$$

$$c_{d_L}^Z = \frac{ig}{2\cos\theta_W} \left(1 - \frac{2}{3} \sin^2\theta_W\right)$$

$$c_{u_R}^Z = \frac{2ig}{3} \frac{\sin^2\theta_W}{\cos\theta_W}$$

$$c_{d_R}^Z = \frac{-ig}{3} \frac{\sin^2\theta_W}{\cos\theta_W}$$

$$D_{u_L}^i = \frac{ig}{\sqrt{2}} \left(N_{i1} + \frac{1}{3} \tan\theta_W N_{i2}\right)$$

$$D_{d_L}^i = \frac{ig}{\sqrt{2}} (-N_{i1} + \frac{1}{3} \tan\theta_W N_{i2})$$

$$D_{u_R}^i = -i\sqrt{2} \frac{2}{3} g \tan\theta_W N_{i2}$$

$$D_{d_R}^i = i \sqrt{\frac{2}{3}} g \tan\theta_W N_{i2}$$

$$D_{e_L}^i = \frac{ig}{\sqrt{2}} (N_{i1} - \tan\theta_W N_{i2})$$

$$D_{e}^i = i\sqrt{2} g \tan\theta_W N_{i2}$$

N_{i1} and N_{i2} are the mixing amplitude of the i^{th} neutralino to the Wino and Bino respectively.

Appendix B.

Calculation of the cross section for $e^+e^- \rightarrow q\bar{q}\tilde{\gamma}$.

We present below the analytic forms for $M(i,j)$, the product of the j^{th} Feynman diagram with the complex conjugate of the i^{th} Feynman diagram, so that the matrix element squared is given by

$$|M|^2 = \sum_{i,j} M(i,j), \quad \text{with } M(j,i) = M(i,j)^*.$$

The differential cross section is

$$\frac{d^4\sigma}{dE_3 d(\cos\theta_3) d(\cos\theta_4) d\phi_{34}} = \frac{3}{4(4\pi)^4} |M|^2 \frac{|p_3||p_4|}{E_5}$$

where $|p_4|$ and E_5 are fixed by the kinematics of the process.

p_1 and p_2 are the 4-momenta of the initial electron and positron respectively, whilst the momenta of the squark, photino and quark are p_3 , p_4 and p_5 .

θ_3 and θ_4 are the angles of the squark and photino to the electron beam direction and ϕ_{34} is the angle between the squark and photino in the plane perpendicular to the beam axis.

The masses of the squark and selectron are μ_q and μ_e , whilst the mass of the i^{th} neutralino eigenstate is m_i . The mass of the final state photino (the lightest neutralino) is written as m_χ .

The formulae given are for the production of left handed squark and right handed antiquark. A factor of two should be included for particle $\leftrightarrow$ antiparticle.

The matrix element for left $\leftrightarrow$ right can be found by writing L for R everywhere and reversing the signs of H^i and all the B's.

$$M(1,1) = 4 \sum_{i,j=\gamma,Z} C_{qL}^{j*} C_{qL}^i |D_{qL}|^2 \frac{(A_e^{j*} A_e^i + B_e^{j*} B_e^i)}{(s-M_i^2)(s-M_j^2)[(p_4+p_5)^2-\mu_q^2]}^2$$

$$p_4 \cdot p_5 \left\{ (4p_3 \cdot p_1 - s)(4p_3 \cdot p_2 - s) - s[4\mu_q^2 - 4(p_1+p_2) \cdot p_3 + s] \right\}.$$

$$M(1,2) = 4 \sum_{i,j=\gamma,Z} \frac{|D_{qL}|^2 C_{qL}^{j*} (A_{qL}^i + B_{qL}^i)}{(p_4+p_3)^2 [(p_4+p_5)^2-\mu_q^2] (s-M_j^2)(s-M_i^2)}$$

$$\left\{ (A_e^{j*} A_e^i + B_e^{j*} B_e^i) \right.$$

$$\left[\left[(4p_3 \cdot p_1 - s)(p_3 \cdot p_2 p_4 \cdot p_5 - p_4 \cdot p_2 p_3 \cdot p_5 + p_5 \cdot p_2 (m_\chi^2 + p_4 \cdot p_3)) \right] \right.$$

$$+ [p_1 \leftrightarrow p_2] + s[p_4 \cdot p_5 (\mu_q^2 - 2p_3 \cdot p_4) + p_5 \cdot p_3 m_\chi^2] \left. \right]$$

$$+ 2 (A_e^{j*} B_e^i + B_e^{j*} A_e^i)$$

$$\left[\left[s(p_1 \cdot p_5 p_3 \cdot p_4 - p_1 \cdot p_4 p_3 \cdot p_5) + 2p_3 \cdot (p_1+p_2) p_1 \cdot p_4 p_2 \cdot p_5 \right] \right.$$

$$\left. \left. - [p_1 \leftrightarrow p_2] \right] \right\}.$$

$$\begin{aligned}
 M(1,3) = & -4 \sum_{\substack{j=\gamma, Z \\ i=1,4}} \frac{C_{qL}^{j*} D_{qL}^{*} H^i D_{qL}^i (A_e^{j*} A_e^Z + B_e^{j*} B_e^Z)}{(s-M_j^2)(s-M_Z^2)[(p_4+p_5)^2-\mu_q^2][(p_3+p_5)^2-m_i^2]} \\
 & \left\{ (A_e^{j*} A_e^Z + B_e^{j*} B_e^Z) \right. \\
 & \left[[4p_1 \cdot p_3 (-p_2 \cdot p_5 (-m_i m_\chi + p_3 \cdot p_5) + p_2 \cdot p_3 p_4 \cdot p_5 + p_2 \cdot p_4 p_5 \cdot p_3) \right. \\
 & \quad \left. + [p_1 \leftrightarrow p_2] - 2s p_5 \cdot p_3 (p_3 \cdot p_4 - p_3 \cdot p_5 - m_i m_\chi) \right] \\
 & \left. + 2(A_e^{j*} B_e^Z + B_e^{j*} A_e^Z) \right. \\
 & \left[[s(p_1 \cdot p_4 p_3 \cdot p_5 - p_1 \cdot p_5 p_4 \cdot p_3) + 2p_3 \cdot (p_1 + p_2) p_1 \cdot p_5 p_2 \cdot p_4] \right. \\
 & \quad \left. \left. - [p_1 \leftrightarrow p_2] \right] \right\}.
 \end{aligned}$$

$$\begin{aligned}
 M(1,4a) = & \sum_{\substack{j=\gamma, Z \\ i=1,4}} \frac{C_{qL}^{j*} D_{qL}^{*} D_{qL}^i (D_{eL}^i D_{eR} + D_{eR}^i D_{eL})}{(s-M_j^2)[(p_4+p_5)^2-\mu_q^2][(p_3+p_5)^2-m_i^2][(p_2+p_5)^2-\mu_e^2]} \\
 & \frac{1}{2} \left\{ (A_e^{j*} + B_e^{j*}) m_\chi m_i (2p_1 \cdot p_3 p_2 \cdot p_5 + 2p_1 \cdot p_5 p_2 \cdot p_3 - s p_3 \cdot p_5) \right. \\
 & \quad + (A_e^{j*} - B_e^{j*}) [\mu_q^2 (2p_1 \cdot p_5 (s - p_2 \cdot p_3) - s p_3 \cdot p_5 - 2p_1 \cdot p_3 p_2 \cdot p_5) \\
 & \quad \left. - 2p_3 \cdot p_1 (2p_3 \cdot p_2 p_1 \cdot p_5 + p_5 \cdot p_3 (s - 4p_2 \cdot p_3) - 2p_3 \cdot p_1 p_2 \cdot p_5) \right] \left. \right\}.
 \end{aligned}$$

$M(1,4b)$ can be found by taking $M(1,4a)$ interchanging p_1 and p_2 replacing B_e^{j*} by $-B_e^{j*}$ and multiplying by an overall factor of -1 .

$$\begin{aligned}
 M(2,2) = & 8 \sum_{i,j=\gamma,Z} \frac{|D_{qL}|^2 (A_{qL}^{j*} + B_{qL}^{j*}) (A_{qL}^i + B_{qL}^i)}{(s-M_j^2)(s-M_i^2)[(p_4+p_3)^2]^2} \\
 & \left\{ (A_e^{j*} A_e^i + B_e^{j*} B_e^i) \left[[2m_\chi^2 s p_1 \cdot p_5 + s p_5 \cdot p_4 p_3 \cdot p_5 \right. \right. \\
 & \quad + 2p_1 \cdot p_5 (p_5 \cdot p_4 p_3 \cdot p_2 + p_5 \cdot p_2 p_4 \cdot p_3 - p_4 \cdot p_2 p_5 \cdot p_3) \\
 & \quad \left. \left. + [p_1 \leftrightarrow p_2] \right] \right. \\
 & \quad \left. + (A_e^{j*} B_e^i + A_e^i B_e^{j*}) \left[[p_1 \cdot p_5 (p_2 \cdot p_3 (2m_\chi^2 + p_3 \cdot p_4)) \right. \right. \\
 & \quad \left. \left. + p_2 \cdot p_4 (m_\chi^2 - \mu_q^2)] - [p_1 \leftrightarrow p_2] \right] \right\}.
 \end{aligned}$$

$$\begin{aligned}
 M(2,3) = & -2 \sum_{\substack{j=\gamma,Z \\ i=1,4}} \frac{D_{qL}^* D_{qL}^i H^i (A_{qL}^{j*} + B_{qL}^{j*})}{(s-M_i^2)(s-M_Z^2)(p_4+p_3)^2[(p_3+p_5)^2-m_i^2]} \\
 & \left\{ 4(A_e^{j*} A_e^Z + B_e^{j*} B_e^Z) \right. \\
 & \quad \left[m_\chi(m_\chi+m_i)(2p_5 \cdot p_1 p_2 \cdot (p_4+p_3) + (p_1 \leftrightarrow p_2) + s(p_4+p_3) \cdot p_5) \right. \\
 & \quad - 2p_3 \cdot p_4 (p_5 \cdot p_1 p_3 \cdot p_2 + p_5 \cdot p_2 p_1 \cdot p_3 - s p_5 \cdot p_3) \\
 & \quad - p_5 \cdot p_4 (4p_3 \cdot p_1 p_3 \cdot p_2 - \mu_q^2 s) \\
 & \quad \left. \left. + p_5 \cdot p_3 (2p_4 \cdot p_1 p_3 \cdot p_2 + 2p_4 \cdot p_2 p_3 \cdot p_1 - s p_4 \cdot p_3) \right] \right. \\
 & \quad \left. + (A_e^{j*} B_e^Z + B_e^{j*} A_e^Z) \right. \\
 & \quad \left[[2m_i m_\chi s p_1 \cdot p_5 + 4(m_\chi^2 + \mu_q^2) p_5 \cdot p_1 p_4 \cdot p_2 \right. \\
 & \quad - 4p_3 \cdot p_4 p_5 \cdot p_1 p_3 \cdot p_2 - s(p_1 \cdot p_3 p_5 \cdot p_3 - \mu_q^2 p_1 \cdot p_5) \\
 & \quad \left. \left. + 2(p_1+p_2) \cdot p_3 p_1 \cdot p_5 p_2 \cdot p_3] - [p_1 \leftrightarrow p_2] \right] \right\}.
 \end{aligned}$$

$$M(2,4a) = \sum_{\substack{j=\gamma, Z \\ i=1,4}} \frac{D_{qL}^* D_{qL}^i (A_{qL}^{j*} - B_{qL}^{j*}) (D_{eL}^i D_{eR} + D_{eL}^i D_{eR})}{2(s-M_j^2)(p_4+p_5)^2[(p_3+p_5)^2-m_i^2][(p_2-p_4)^2-\mu_e^2]}$$

$$\begin{aligned} & \left\{ (A_e^{j*} - B_e^{j*}) [4p_1 \cdot p_3 (p_2 \cdot p_4 p_3 \cdot p_5 + p_2 \cdot p_5 p_4 \cdot p_3 - p_2 \cdot p_3 p_4 \cdot p_5) \right. \\ & \quad - \mu_q^2 (2p_2 \cdot p_4 p_1 \cdot p_5 + 2p_2 \cdot p_5 p_4 \cdot p_1 - s p_4 \cdot p_5) \\ & \quad \left. + m_\chi^2 (s p_3 \cdot p_5 + 2p_2 \cdot p_5 p_1 \cdot p_3 - 2p_2 \cdot p_3 p_1 \cdot p_5) \right] \\ & \quad + 2(A_e^{j*} + B_e^{j*}) m_i m_\chi p_5 \cdot p_1 (s - 2p_2 \cdot p_5) \Big\}. \end{aligned}$$

$M(2,4b)$ can be obtained from $M(2,4a)$ in exactly the same way as $M(1,4b)$ was obtained from $M(1,4a)$.

$$M(3,3) = - \sum_{i,j=1,4} \frac{H_j^* D_{qL}^{j*} H_i^i D_{qL}^i}{(s-M_j^2)^2 [(p_3+p_5)^2-m_j^2] [(p_3+p_5)^2-m_i^2]}$$

$$\begin{aligned} & \left\{ (|A_e^Z|^2 + |B_e^Z|^2) [2(\mu_q^2 - m_j m_i) (p_1 \cdot p_4 p_2 \cdot p_5 + (p_1 \leftrightarrow p_2)) \right. \\ & \quad - (m_i + m_j) m_\chi s p_3 \cdot p_5 \\ & \quad \left. - 4p_3 \cdot p_5 (p_3 \cdot p_1 p_2 \cdot p_4 + (p_1 \leftrightarrow p_2)) \right] \\ & \quad + (A_e^Z B_e^{Z*} + B_e^Z A_e^{Z*}) \left[[m_i m_j p_5 \cdot p_2 p_4 \cdot p_1 + (m_i + m_j) m_\chi p_5 \cdot p_2 p_4 \cdot p_1 \right. \\ & \quad \left. \left. + 2p_3 \cdot p_5 p_3 \cdot p_1 p_4 \cdot p_5 \right] - [p_1 \leftrightarrow p_2] \right] \Big\}. \end{aligned}$$

$$\begin{aligned}
 M(3,4a) = & \sum_{i,j=1,4} \frac{H_j^* D_{qL}^{j*} D_{qL}^i (D_{eL}^i D_{eR} + D_{eR}^i D_{eL})}{(s-M_Z^2) [(p_3+p_5)^2 - m_j^2] [(p_3+p_5)^2 - m_i^2] [(p_2-p_4) - \mu_e^2]} \\
 & \frac{1}{2} \left\{ (A_e^{Z*} + B_e^{Z*}) [m_\chi m_j (sp_3 \cdot p_5 + 2p_1 \cdot p_3 p_2 \cdot p_5 - 2p_1 \cdot p_5 p_2 \cdot p_3) \right. \\
 & \quad + 4p_2 \cdot p_4 (2p_1 \cdot p_3 p_5 \cdot p_3 - \mu_q^2 p_1 \cdot p_5)] \\
 & \quad + (A_e^{Z*} - B_e^{Z*}) [m_\chi m_i (sp_3 \cdot p_5 - 2p_1 \cdot p_5 p_2 \cdot p_3 - 2p_1 \cdot p_3 p_2 \cdot p_5) \\
 & \quad \left. + 4m_i m_j p_2 \cdot p_4 p_1 \cdot p_5] \right\}.
 \end{aligned}$$

$M(3,4b)$ can be found from $M(3,4a)$ by interchanging p_1 and p_2 and replacing B_e^{Z*} by $-B_e^{Z*}$.

$$\begin{aligned}
 M(4a,4a) = & - \sum_{i,j=1,4} \frac{D_{qL}^{j*} D_{qL}^i (|D_{eR}|^2 + |D_{eL}|^2) (D_{eL}^i D_{eR}^{j*} + D_{eL}^i D_{eR}^{j*})}{[(p_3+p_5)^2 - m_i^2] [(p_3+p_5) - m_j^2] [(p_2-p_4)^2 - \mu_e^2]^2} \\
 & \frac{1}{2} p_2 \cdot p_4 [2p_1 \cdot p_2 p_5 \cdot p_3 + p_1 \cdot p_5 (m_i m_j - \mu_q^2)].
 \end{aligned}$$

$$\begin{aligned}
 M(4a, 4b) = & - \frac{1}{16} \sum_{i, j=1, 4} D_{q_L}^{j*} D_{q_L}^i \times \\
 & \frac{1}{[(p_3+p_5)^2 - m_j^2][(p_3+p_5)^2 - m_i^2][(p_1-p_4)^2 - \mu_e^2][(p_2-p_4)^2 - \mu_e^2]} \\
 & \left\{ (D_{e_L}^* D_{e_R}^{j*} + D_{e_R}^* D_{e_L}^{j*}) (D_{e_L} D_{e_R}^i + D_{e_R} D_{e_L}^*) m_\chi m_i s p_3 \cdot p_5 \right. \\
 & + (D_{e_L}^* D_{e_R}^{j*} + D_{e_R}^* D_{e_L}^{j*}) (D_{e_L} D_{e_R}^i + D_{e_R} D_{e_L}^i) \\
 & \left[m_i m_j (-s p_4 \cdot p_5 + 2 p_4 \cdot p_2 p_5 \cdot p_1 + 2 p_4 \cdot p_1 p_5 \cdot p_2) \right. \\
 & - \mu_q^2 [(s - 2 p_4 \cdot p_2) p_5 \cdot p_1 + (s - 2 p_4 \cdot p_1) p_2 \cdot p_5 + s p_3 \cdot p_5] \\
 & \left. \left. - 2 p_3 \cdot p_5 [(s - 2 p_4 \cdot p_2) p_3 \cdot p_1 + (s - 2 p_4 \cdot p_1) p_2 \cdot p_3 - s p_3 \cdot p_5] \right] \right\}
 \end{aligned}$$

$M(4b, 4b)$ can be found from $M(4a, 4a)$ by interchanging p_1 and p_2 .

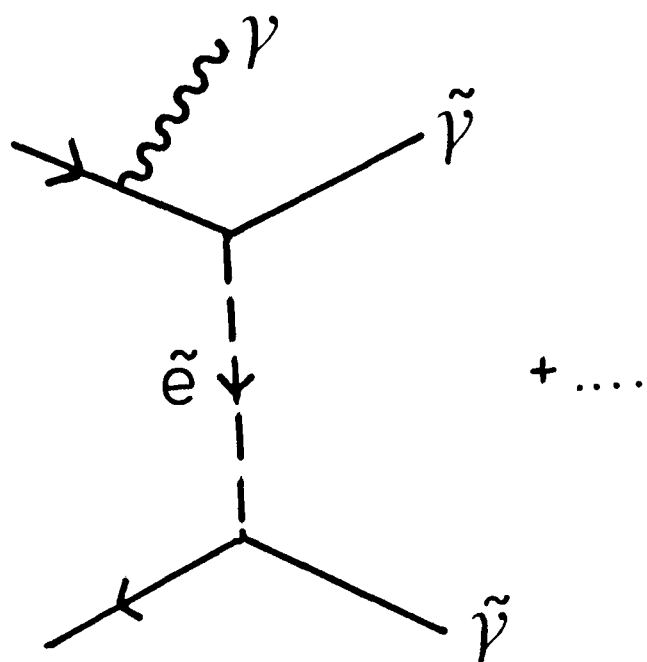


Fig. 1. The production of photinos in e^+e^- collisions with a tagged photon.

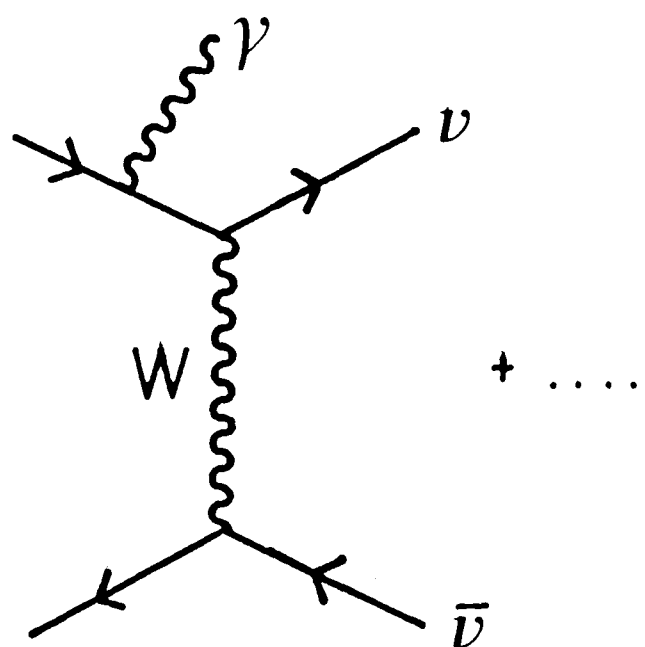


Fig. 2. Neutrino pair production with a tagged photon.

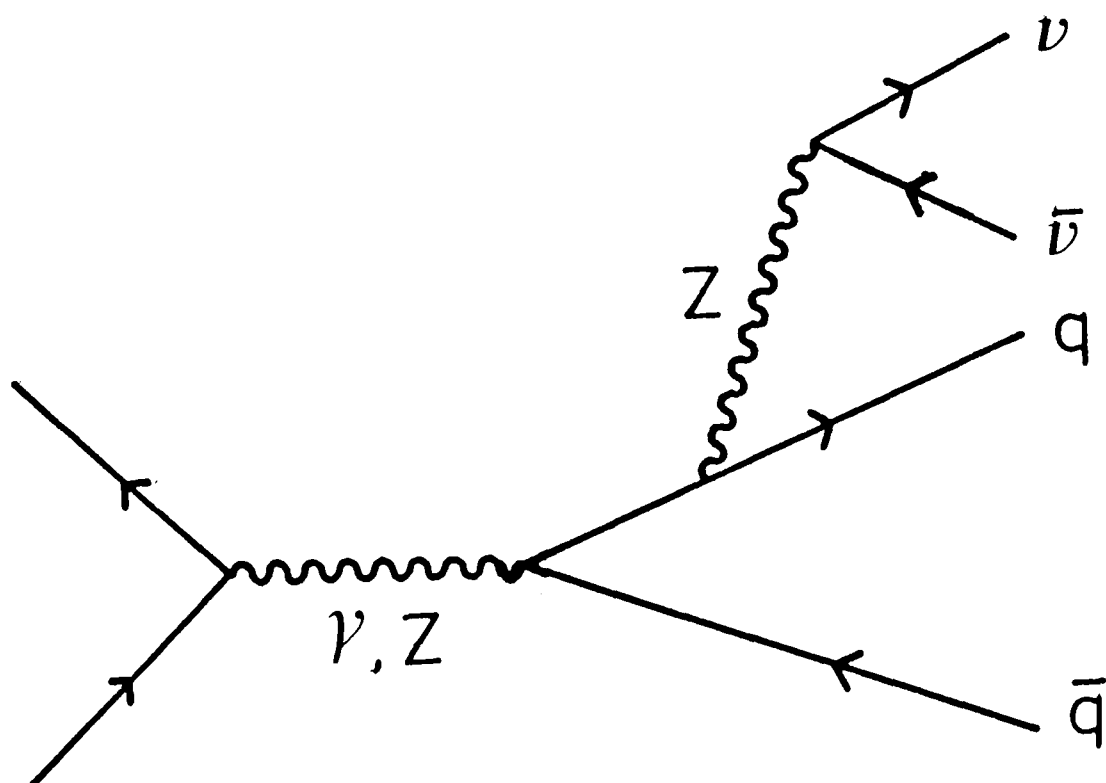
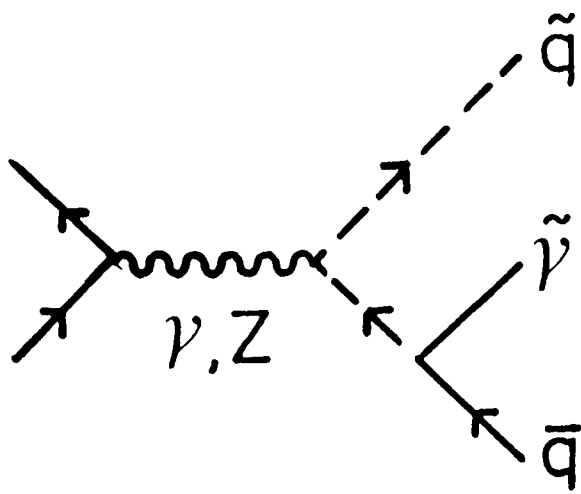
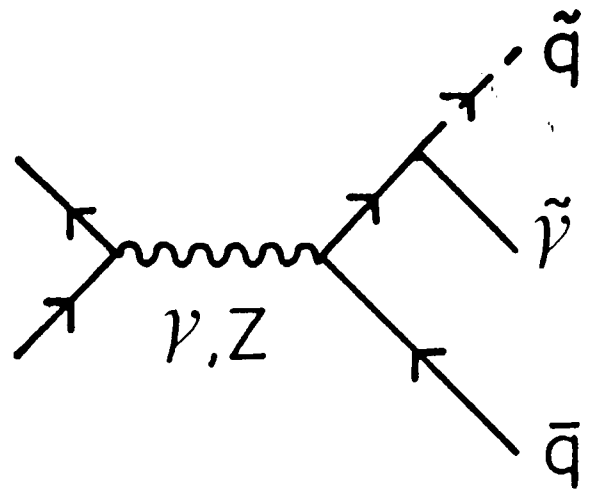


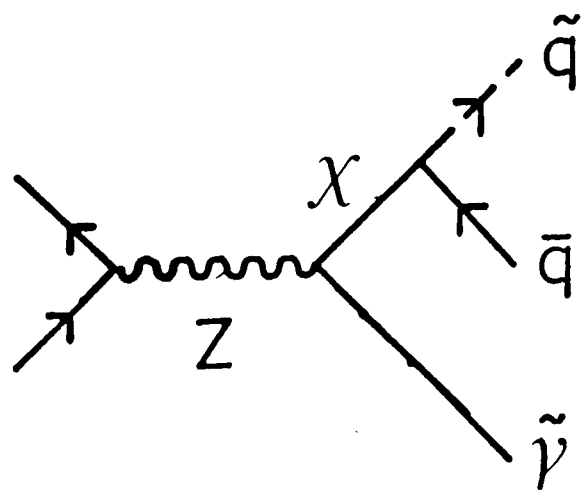
Fig. 3. Simultaneous production of a $q\bar{q}$ and a $\nu\bar{\nu}$ pair.



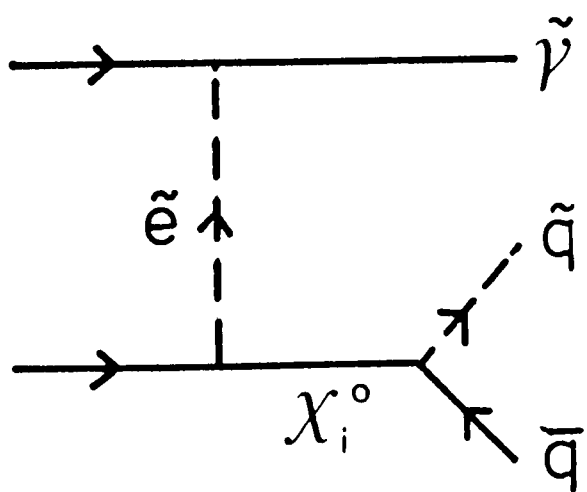
(1)



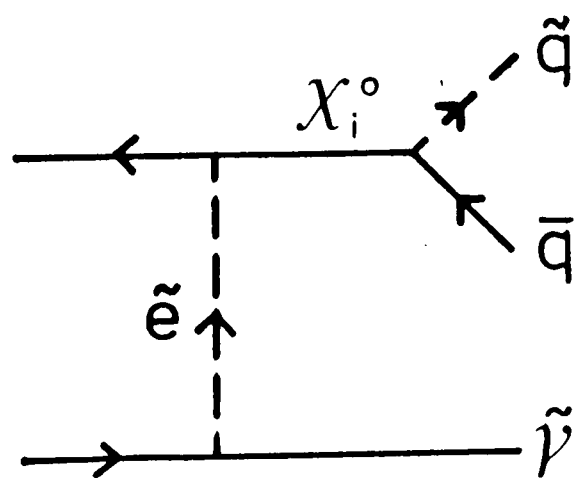
(2)



(3)



(4a)



(4b)

Fig. 4. The Feynman diagrams for $e^+e^- \rightarrow q\bar{q}\tilde{\nu}$.

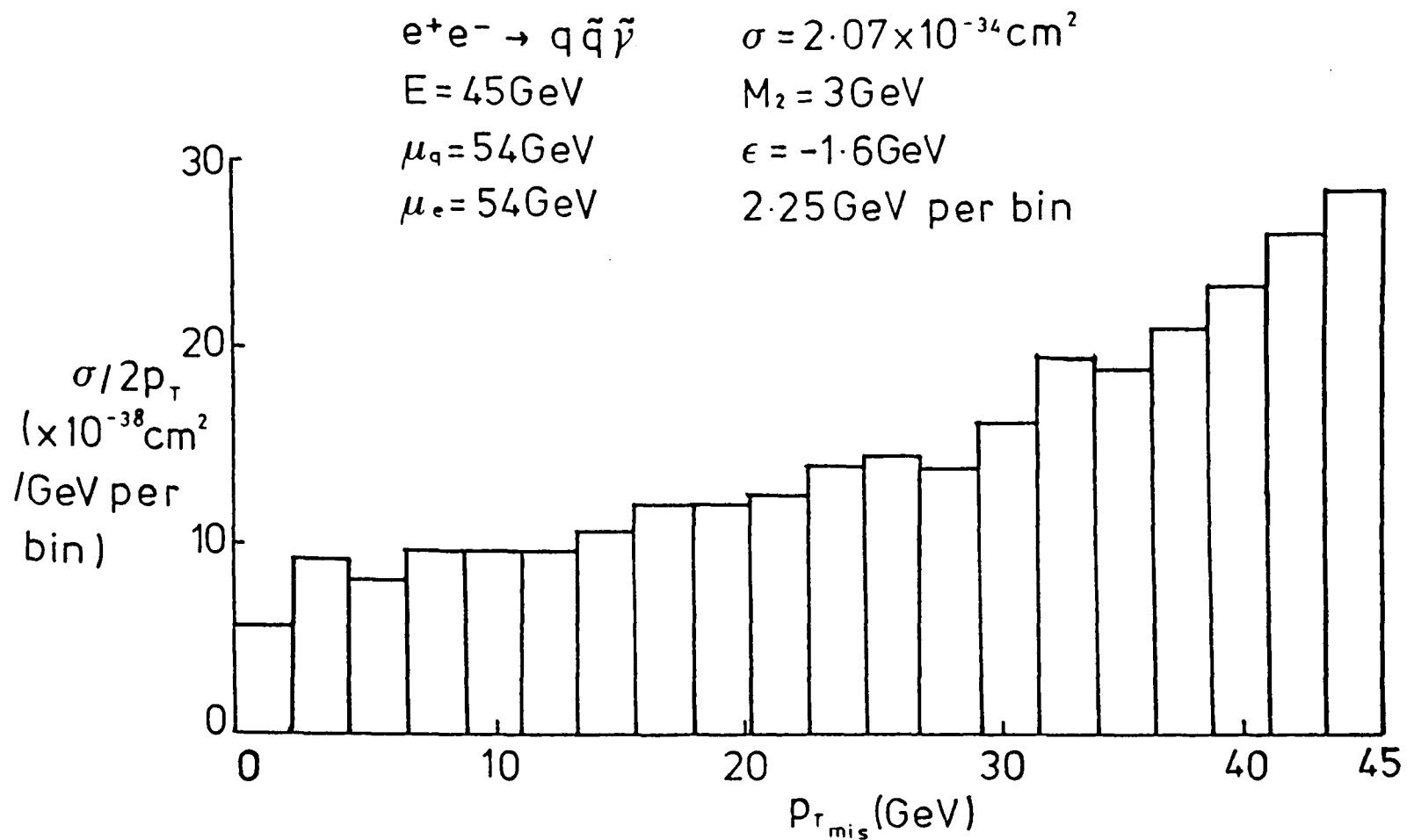


fig. 5a

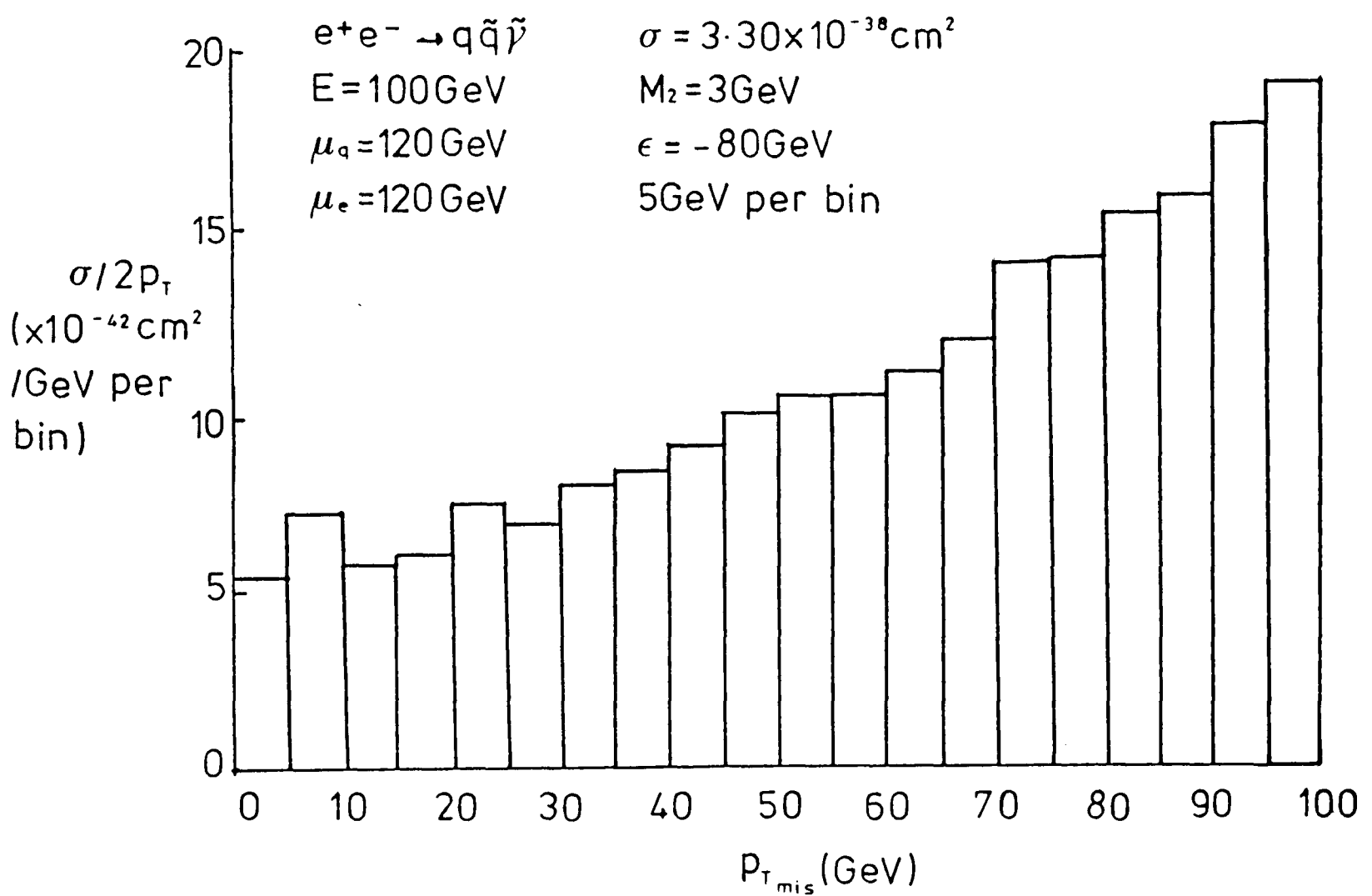


fig. 5b

Fig. 5. Missing p_T plots for $e^+e^- \rightarrow q\bar{q}\tilde{\gamma}$;
 a) $E = M_2/2$, Wiggsino limit, $\epsilon < 0$.
 b) $E = 100 \text{ GeV}$, Wino limit, $\epsilon < 0$.

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