From Physical to Metaphysical Necessity

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Let *Nomological Bound* be the thesis that there is nothing objectively possible beyond what is physically possible. Nomological Bound has struck many as a live hypothesis. Nevertheless, in this article I provide a novel argument against it. Yet even though I claim that Nomological Bound is false, I argue that the boundaries of objective possibility can still be characterized intimately in terms of physical necessity. This is philosophically significant, for on a natural understanding it constitutes the powerful anti-sceptical result that those who believe in physical necessity should not harbour any scepticism towards merely metaphysical possibilities.

Where do the boundaries of objective possibility lie? The question is notoriously vexed, but in *Naming and Necessity* Kripke entertains a particularly bold answer to it:

> Physical necessity *might* turn out to be necessity in the highest degree. But that's a question which I don't wish to prejudge. … It might be that when something's physically necessary, it always is necessary *tout court*. (Kripke, 1980, p. 99)

Call this conjecture *Nomological Bound*, the thesis that nothing is objectively possible beyond what is physically possible. Although Nomological Bound is a radical thesis about modal reality, it strikes many as a live hypothesis. For example, in a similar vein to Kripke, Tim Maudlin (2007, pp. 187–8) states that it is an 'important open question whether metaphysical possibility extends more widely than physical possibility' (see also Edgington 2004). Indeed, Nomological Bound is a natural way of capturing a familiar scepticism towards merely metaphysical possibilities which are forbidden by our physical laws.

Nomological Bound has received surprisingly little philosophical scrutiny. Nevertheless, in this article I provide a novel argument against it. The argument is general, and exploits predictions which various theories of laws yield about the logic of physical necessity. However, although Nomological Bound is false, I argue that the boundaries of objective possibility can still be characterized in terms of physical necessity. This is philosophically significant, for it

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demonstrates that what is metaphysically possible can be characterized in terms of physical possibility. Consequently, those who indulge in talk of physical necessity are under substantial pressure not to harbour any scepticism towards merely metaphysical possibilities.

I first sharpen the notion of physical necessity in terms of its connection to laws and make various novel observations about its logic (§1). Using a natural higher-order logic, I then situate physical necessity in a general modal framework (§2). I then use this framework to refine what I have described informally as the 'boundaries' of 'objective' possibility, and undermine Nomological Bound. Nevertheless, I argue that physical necessity can still be used to characterize where the boundaries of objective possibility lie (§3). Finally, I consider various responses to my arguments, including how they function according to less mainstream accounts of laws (§4).

1. Physical necessity

1.1 Models-based characterization

Physical necessity is appealed to throughout metaphysics and the philosophy of science. In these two areas, one extremely popular idea is that physical necessity can, and perhaps should, be characterized in terms of the models of a world's laws of nature. In this section, I refine this characterization and highlight various ramifications it has for the logic of physical necessity.

According to a rough statement of the characterization, the physical possibility of a proposition at a world consists in there being some model of that world's laws of nature which models that proposition as being true. In other words, physical possibility simply tracks the models of ultimate physical theory, which can themselves be viewed as solutions of the laws of nature. As Tim Maudlin describes it evocatively:

> The [physically] possible worlds consistent with a set of laws are described by the models of a theory that formulates those laws. . . . Once we have the set of models, physical necessity is easily defined in the usual way, using the models as mutually accessible possible worlds. . . . The content of the laws can be expressed without modal notions, and suffices to determine a class of models. The models can then be treated as 'possible worlds' in the usual way, and so provide truth conditions for claims about nomic [i.e. physical] possibility and necessity. The laws themselves, of course, turn out to be nomically [i.e. physically] necessary, since they obtain in all the models. (Maudlin 2007, pp. 14–17)

Crucially, I shall use ‘laws’ to refer to the propositions *p* such that it is a law that *p*; ‘law’ is not used to refer to the proposition that *p is a law* or *p is physically necessary*. I take this to be in line with standard practice, since the theory which Maudlin states ‘formulates’ the laws is a presumably a *physical* theory with no modal vocabulary in its signature. Moreover, Maudlin is explicit that the content of the laws can be expressed without modal notions, which rules out modal expressions like ‘it is a law that’ and ‘it is physically necessary that’.

Despite its popularity, this *models-based characterization* invites a number of immediate questions. What is it for the physically possible worlds to be ‘consistent’ with a set of laws? Can the models-based characterization dispense with that appeal to consistency? Moreover, what is it to ‘treat’ or ‘regard’ the models of a theory—mere set-theoretic constructs—as possible worlds? These questions can be answered by a more careful schematic statement of the models-based characterization.

Model-World ϕ is physically possible at a world just in case $\ulcorner\phi\urcorner$ is true in some model of the laws of nature.

In this schema, instances of $\ulcorner\phi\urcorner$ are sentences of some formal language with the capacity to express the mathematics required by ultimate physics. For instance, the formal language might be that of first-order ZFCU, in which physical vocabulary is permitted in instances of the separation and replacement schemas, or a variant of the higher-order language used in §2 below.

It is not always appreciated that a restriction is required on the models of the laws of nature to which Model-World appeals, for it may be that a single proposition is expressed by distinct sentences of the language which differ in their overall model-theoretic status. For example, a proposition such as *Hesperus is Phosphorus* may well be expressed by the distinct sentences ‘Hesperus is Phosphorus’ and ‘Hesperus is Hesperus’. In order to remedy this issue, let *S* be the set of formulae $\ulcorner\phi\leftrightarrow\psi\urcorner$ such that the proposition expressed by ϕ is the same as that expressed by ψ. More carefully, the right bijunct of Model-World should then read ‘$\ulcorner\phi\urcorner$ is true in some model of the laws of nature $+S$’. I shall leave this implicit in the official statement of the principle.[1]

[1] There is a related issue regarding essentialist theses. Unless further restrictions are placed on the class of models, a sentence like ‘Bertrand Russell is a photocopier’ will be true in certain models, which is perhaps not a physical possibility. Thus one might require that these essentialist theses are added to *S*. This already raises a difficult question for advocates of

Nevertheless, although Model-World addresses the initial questions, the schema still has a crucial limitation. The issue is that models of physical theories are designed to interpret theories with quite limited signatures, and are not used to interpret 'higher-level' theories, such as those of taxation or neuroscience. To put the point less technically, the models of physical theories do not settle the truth of sentences which do not concern ultimate physics, such as sentences about social goods or neural circuits.[2] These of course also include sentences stating whether a proposition is a law of nature or physically necessary. More generally, since the models are quite austere set-theoretic constructs, they are not informationally rich enough to serve as proxy for possible worlds in the sense of maximally consistent propositions. However, a more sophisticated version of Model-World can overcome this limitation. According to this alternative, each model is associated with at least one *world proposition* which describes the physical state that set-theoretic construct is intended to model. Here, a world proposition *describes a model* just in case a proposition ϕ about the fundamental physical state is true according to the world proposition whenever $\ulcorner\phi\urcorner$ is true in the model.[3] A more sophisticated version of the schema may then be stated as follows.

Model-World^{+} p is physically possible at a world just in case p is true according to some world proposition associated with a model of the laws of nature.

The more sophisticated Model-World^{+} is a plausible refinement of the popular but informal models-based characterization that Maudlin describes. When discussing this principle, 'world proposition' will often be abbreviated as 'world', and 'true according to a world

Nomological Bound: if Model-World can be used to characterize physical necessity, why do more general classes of models not also characterize broader species of objective modality too, like metaphysical necessity? See Williamson (2016, pp. 462–3) for a related argument. Thanks to Peter Fritz for discussion here.

[2] This point is only a model-theoretic one: I am not suggesting that such 'higher-level' theories do not reduce (in the sense of Wallace 2012, pp. 53–8) to ultimate physical theory.

[3] Usually, a world proposition is defined as a proposition which is metaphysically possible and metaphysically necessitates every proposition or its negation; see Fine (1970, p. 339; 1977, Technical Appendix). A proposition is then true according to a world proposition just in case it is metaphysically necessitated by it. Advocates of Nomological Bound who are wary of metaphysical necessity may rephrase such definitions in terms of the operator *having every finite iteration of physical necessity* (see §3), which is an S4 operator in the strongest theory considered below.

proposition' will be abbreviated as 'true at a world'. It is also worth highlighting that there may be cases in which at least one world proposition is associated with two different models. For example, the two models may be isomorphic or related by 'gauge transformations', which would indicate that the models do not differ in any physically significant respect.[4] Moreover, some strong physicalists may even maintain that each model of a world's laws has at most one world proposition associated with it. Keeping this in mind, I shall still refer plurally to the world propositions associated with a given model.

1.2 Laws

Given Model-World^{+}, physical necessity is intimately connected with the laws of nature. However, reductionist and non-reductionist conceptions of laws have different and sometimes opposing criteria of lawhood. In rough terms, reductionist conceptions all endorse the claim that the particular matters of fact of a world completely determine that world's laws, and non-reductionist conceptions reject this claim. It is crucial, then, to explore whether different theories of lawhood generate different verdicts about what is physically necessary at a given world.

Since David Lewis's (1983, pp. 366–8) Humean conception of laws is the best developed, most popular reductionist theory, I shall choose it as a representative of reductionist views. For Lewis, to be a law of nature at a world is to be expressed by a certain universal generalization which belongs to the best systematization of that world's particular matters of fact. A basic constraint on this system is that it must not prove anything false about the particular matters of fact it systematizes. However, Lewis also requires that the axiomatization must strike the optimal balance between simplicity and strength, where these notions are characterized measure-theoretically with respect to a canonical language defined in terms of Lewis's notion of naturalness. Lewis's Humean theory of laws may, then, be summarized as follows:

Humean A regularity ϕ is a law of nature if and only if $\ulcorner\phi\urcorner$ is a theorem of the simplest and strongest axiomatization of particular matters of fact in the canonical language.

[4] According to Weatherall (2016, pp. 1047–8), the presence of gauge transformations between models of a theory which are not isomorphisms indicate that (the particular formulation of) the theory has 'excess structure'.

In contrast to the reductionist market, the landscape of non-reductionist views is vast. Non-reductionist views are, however, unified in their denial of lawhood supervening on the non-nomic. For example, Marc Lange (2009) and John Carroll (2008) reject the Humean's supervenience claim, but still analyse laws in terms of different respective nomic notions. Famously, David Armstrong (1983), Fred Dretske (1977) and Michael Tooley (1977) also reject the attempted Humean reduction, and analyse laws in terms of a 'contingent necessitation' relation holding between universals. More recently, Tim Maudlin (2007) has taken lawhood to be a simply primitive status of certain true propositions, and not something which is to be non-trivially analysed in either nomic or non-nomic terms. I shall choose Maudlin's primitivist view as the representative of non-reductionism in what follows, for it is the simplest such view and all points made about it will apply to the other non-reductionist theories.

Having identified two representative theories of lawhood, we can see that different theories of lawhood generate different verdicts about what is physically necessary at a world given Model-World^{+}.[5] The crucial point is that the two opposing theories of laws will disagree on whether every world proposition associated with a model of a world's laws will agree on which propositions are *laws*. Given the Humean claim that the laws of a world supervene on its ultimate physical facts, for them every world proposition associated with a model will agree on which propositions are laws since they agree on the facts about ultimate physics. In contrast, on primitivist conceptions of laws, each model will be associated with a class of world propositions which agree on the particular matters of fact but differ as to the laws. Indeed, in effect Maudlin himself is explicit about this fact in his influential criticism of the Humean theory of laws (2007, pp. 67–8). In that criticism, he argues that two different sets of laws can have models with the same physical state; in particular, he points out that empty Minkowski spacetime is a vacuum solution to the field equations of general relativity and of other theories of gravity. In

[5] Some advocates of Humeanism might simply refuse to theorize in terms of physical necessity and thus reject Model-World^{+}, perhaps due to a more general scepticism about modal notions. Since my central argument concerns those who already believe in physical necessity, such Humeans are not its intended target. Note, however, that Lewis himself did not take this view, and was willing to theorize about 'the logic of iterated nomological [i.e. physical] necessity' (1986, p. 20). (Of course, in the setting of his concrete modal realism, such logical questions reduce to questions about relations between spatiotemporally disconnected spacetimes.) Thanks to an anonymous reviewer for raising this issue.

order to advance this criticism, the primitivist cannot think that models of scientific theories determine what the laws are at all world propositions associated with those models. Hence they must think that each model of ultimate physics is associated with a class of world propositions with varying laws. Thus, even with the strictures of Model-World^{+} in place, the two representative theories of laws will disagree on the space of physical possibilities from a given world. This difference will become relevant at certain points.

1.3 Logic

In the literature on physical necessity, one finds multiple further characterizations of physical necessity that are incompatible with one another. Three prominent examples are as follows:[6]

PN A world *v* is physically possible from a world *w* if and only if every law of *w* is true at *v*.

PN* A world *v* is physically possible from a world *w* if and only if every law of *w* is a law at *v*.

PN† A world *v* is physically possible from a world *w* if and only if *v* has the exact same laws as *w*.

On the assumption that there are no false laws, it is straightforward to see that PN-necessity implies PN*-necessity, which in turn implies PN†-necessity, despite none of the converse implications being true.[7] Indeed, one can appreciate their non-equivalence by noting that each characterization validates a different logic for physical necessity. Again on the assumption that all laws are true, PN† validates an S5 logic for physical necessity, PN* validates an S4 logic for physical necessity, and PN validates merely a T logic for physical necessity.

[6] Hale and Leech (2017) appeal to PN, and reject various definitions of physical possibility solely on the basis that they are inconsistent with it, which suggests that PN is fairly well-entrenched. Lewis (1986, p. 20) and van Fraassen (1989, pp. 44–5) also endorse PN. By contrast, John Carroll (1994, p. 18 n. 8) assumes the PN† conception without argument, and Tim Maudlin's (2007, pp. 67–8) argument against Humeanism strongly suggests that he endorses the PN* gloss at the very least. Schneider (2007, p. 314) presumes without argument that PN† must be the account of physical possibility endorsed by non-reductionists about laws; this is also suggested by Hall (2015, pp. 31–2), who, however, argues that the Humean should endorse PN. In effect, Fine (2005, p. 242) assumes the PN* conception too, but he notes that it is incompatible with reductionism about laws (2005, p. 244).

[7] Marc Lange (2000, ch. 6; 2009) and Boris Kment (2006) have suggested that some actual laws of nature are false. However, although nothing in what follows requires all laws to be true, it is a plausible working assumption and simplifies the discussion in certain respects.

The presence of these competing characterizations raises an immediate question: which, if any, of them are licensed by Model-World^{+}? The first substantive premiss in my overall argument is that Model-World^{+} licenses PN since it generates counterexamples to the 4 axiom for physical necessity (where $\blacksquare$ expresses physical necessity):

$$4_{\blacksquare} \quad \blacksquare p \rightarrow \blacksquare\blacksquare p$$

In particular, both reductionist and non-reductionist conceptions of laws admit counterexamples to $4_{\blacksquare}$ in the presence of Model-World^{+}.

First consider the reductionist conception of laws, of which the Humean view was the representative. As argued in §1.2, Humeans will take each world associated with a given model to agree on which propositions are laws, for Humeans take the fundamental physical state of a world—captured by the model—to determine its laws completely. However, there will be worlds based on models of the laws in which those laws do not belong to the best systematization of particular matters of fact at those worlds. This may arise for various reasons. For example, consider the law that all like-charged particles repel, and a world based on a model of that law in which all like-charged particles *do* repel but in which there are extremely few pairs of like-charged particles owing to bizarre initial conditions.[8] It is difficult to see why this seemingly accidental generalization—that all like-charged particles repel—would be a theorem of the best systematization of that world's particular matters of fact. Similarly, surely not all generalizations of the form 'all *F*s are *G*' are laws at worlds where they are *vacuously* true. However, when Humeanism is combined with Model-World^{+}, such worlds will be physically possible from worlds where it is a law that all *F*s are *G*, provided this does not conflict with any other of the laws. Humeans, then, will recognize counterexamples to $4_{\blacksquare}$.[9]

Let me emphasize that I am not claiming that it is physically impossible for there to be vacuously true laws. As Tooley (1977, p. 669) and Loewer (1996, p. 187) both argue, there might be cases of vacuously true laws. Instead, my claim is merely that not all vacuously true

[8] Nathan Salmon (1989, pp. 8–9) discusses a similar counterexample to $4_{\blacksquare}$. There, he also presents a counterexample to the **B** axiom for physical modality, $p \rightarrow \blacksquare\blacklozenge p$.

[9] Lewis (1986, p. 20) himself recognizes that $4_{\blacksquare}$ fails on his view; see also Roberts (1998, p. 433). Based on similar considerations, Marc Lange (2009, p. 198) observes that Humeans will reject the 4-like 'axiom' that if it is a law that *p*, then it is a law that it is a law that *p*.

generalizations are laws. And this must be admitted even by advocates of vacuously true laws, for if 'all *F*s are *G*' is vacuously true, so is 'all *F*s are not-*G*'. But were both of these propositions laws, it would follow that in all physically possible worlds (even where there are *F*s), all *F*s are *G* and not-*G*. Indeed, Lewis's Humean theory of lawhood is widely regarded as an improvement on crude Millian regularity theory in so far as it does not classify all vacuously true universal generalizations as laws.

Now consider the non-reductionist conception of laws, of which the primitivist view was the representative. As emphasized in §1.2, on the primitivist conception of laws each model of a scientific theory will not be associated with a unique physically possible world, but rather with a class of such worlds. All such worlds will share the same particular matters of fact, but many will differ in their laws. After all, according to the primitivist, the particular matters of fact at a world simply do not determine that world's laws. That is to say, for each true proposition the entirety of the non-nomic facts does not determine whether that proposition is a law. Given this lack of determination, the primitivist must recognize that for each law there will be worlds based on models of the laws in which that law is true but not a law. As a result, the primitivist will admit the above type of counterexample to 4■ too.

Certain primitivists might object that not all of these worlds are *physically* possible. But this requires a modification to Model-World^{+}, by identifying a proper subclass of the worlds associated with the models of the laws to be physically possible—perhaps those worlds whose laws include the actual laws. However, it is not clear that there is a principled reason, especially not one stemming from scientific practice, which warrants that modification. Moreover, let me be utterly clear that, ultimately, whether the species of necessity which conforms to Model-World^{+} is *really* physical necessity will be beside the point: my central arguments will only require that *some* notion of necessity is characterized by Model-World^{+}, regardless of whether it is called 'physical necessity' or not.[10] And indeed the primitivist is under considerable pressure to admit belief in such a necessity for their own theoretical purposes, for they are obliged to explain what they mean by the failure of the laws to supervene on particular matters of fact.

[10] Given the many widespread competing characterizations of physical necessity, it is not unreasonable to think that 'physical necessity', a philosophical term of art, is vague, and none of the proposed characterizations deserve the status of being its determinate semantic value.

Yet since supervenience is understood modally, the primitivist must employ some notion of necessity according to which it is possible for the particular matters of fact to be as they actually are but for some actual law to fail to be a law. Indeed, even if the notion of 'primitive' laws is not ultimately understood in terms of supervenience, the primitivist position is partly motivated by cases of nomic variation between non-nomically indiscernible possibilities. Deferring to Maudlin's canonical statement of primitivism, '[l]aws are ontological primitives at least in that two worlds could differ in their laws but not in any observable respect' (2007, p. 17).

Returning to the main thread, I conclude that Model-World$^+$ licenses PN instead of PN* and PN†. (I suppress the minor qualification discussed in the previous paragraph for convenience.) Model-World$^+$ thus licenses exactly a T logic for physical necessity, or at least some modality closely related to it. This constitutes a central observation about the logic of physical necessity to which I shall appeal regularly in what follows.

2. Objective possibilities

If we are to argue against Nomological Bound, the thesis must first be refined. To do so, I shall introduce a general modal framework within the setting of a higher-order language. In addition, I shall use this framework to argue that there is a precise sense in which physical necessity can be used to characterize the boundaries of objective possibility.

The higher-order language is that of the simply typed λ-calculus. In such languages, each term has a syntactic type which determines which expressions it can be combined with to produce which terms of different types. In the particular language I shall use, there is a single 'base' type *t*, the type of formula expressions. The set of types *Typ* is then the least set containing t such that if $\sigma, \tau \in Typ$, then $\sigma \to \tau \in Typ$ too. The signature on which our language is based is $\Sigma = \{\to\} \cup \{\forall_\sigma : \sigma \in Typ\} \cup \{\blacksquare\} \cup \{\mathcal{O}\}$, where $\to$ has type $t \to (t \to t)$, $\forall_\sigma$ has type $(\sigma \to t) \to t$, $\blacksquare$ has type $t \to t$, and $\mathcal{O}$ has type $(t \to t) \to t$. Informally, one can think of the conditional as a function from propositions to a function from propositions to propositions, and universal quantifiers for entities of type σ as functions from 'monadic properties' of type σ entities to propositions (the proposition that everything of type σ has the monadic property in question). On their intended interpretation, $\blacksquare$ expresses physical necessity, a function from propositions to propositions, and $\mathcal{O}$ the property (of propositional operators) of being an objective necessity operator, which will be explicated shortly. The other connectives such as $\neg$ and $\wedge$ are introduced in the usual ways. Moreover, I adopt the standard conventions for abbreviations, for instance, writing $\forall_\sigma \lambda x.\phi$ as $\forall x\phi$ where x has type σ and $\exists x\phi$ for $\neg\forall x\neg\phi$, and using lower-case p and q for variables of type t. For an expression α and type σ, $\alpha : \sigma$ is used to abbreviate the claim that α is of type σ. Finally, as is typical, identity for a given type, $=_\sigma$, is defined as higher-order indiscernibility, $\lambda x \lambda y \forall Z(Zx \leftrightarrow Zy)$, where $x, y : \sigma$ and $Z : \sigma \to t$.

In this language, I state the core theory which serves as backdrop to the central argument. In informal terms, the core theory is a classical logic and quantification theory with a principle of $\beta\eta$ equivalence governing the λ device. The core theory will also prove a slightly generalized version of the view that propositions form a Boolean algebra under the usual truth-functional operations. This theory is dubbed *Booleanism* (B), and is axiomatized by Dorr (2016) and Bacon (2018, 2020) in various ways.

PC All instances of propositional tautologies.

MP From A and $A \to B$ infer B.

Gen From $A \to B$ infer $A \to \forall_\sigma x B$ when x does not occur free in A.

UI $\forall_\sigma x A \to A[t/x]$ (where $t : \sigma$ and is substitutable for x in ϕ).

$\beta\eta$ $A \leftrightarrow B$ whenever $A : t$ and $B : t$ are $\beta\eta$ equivalent.[11]

RE $A =_t B$, whenever $A \leftrightarrow B$ is provable from these axioms and rules

RE is primarily responsible for the theory's Boolean character. Whenever there is a tautological equivalence between two sentences, such as p and $p \wedge p$, RE permits one to convert it into a familiar Boolean identity, $p = (p \wedge p)$. Moreover, one can even prove identities between Boolean identities, for instance $(p = (p \wedge p)) = (p = \neg\neg p)$.

[11] Terms are $\beta\eta$ equivalent just in case one is obtained from the other by a sequence of substitutions of sub-terms which are either:

Immediately β*-equivalent*: of the form $(\lambda x\phi)a$ and $\phi[a/x]$, where a is substitutable for x in ϕ and $\phi[a/x]$ is the result of uniformly substituting a for x in ϕ, or

Immediately η*-equivalent*: of the form $\lambda x.Fx$ and F, where x is not free in F.

Finally, Booleanism proves that there is a unique tautologous proposition, an extremely helpful feature in what follows.

The assumption of Booleanism is of course contentious. However, Booleanism's primary benefit in what follows is that it permits a very simple definition of what it is to be a necessity, and none of the theory's more surprising consequences will play an indispensable argumentative role. For example, Bacon (2018) demonstrates that the theory B proves that there is a strictest necessity, which is naturally understood as a propositional species of logical necessity.[12] This species of necessity can be defined as *being identical with the only tautology* as follows:

Definition 1 (Tautology) $\top := \forall p(p \to p)$

Definition 2 (Logical necessity) $L := \lambda p(\top = p)$

This consequence of the theory B in particular will not play a central role in any of the following arguments. (Indeed, none of the theories considered below will even prove that logical necessity is an objective necessity.) Furthermore, many of the background results about modalities can be recreated in a grain-neutral setting at the cost of further complexity (cf. Bacon and Zeng, forthcoming).

Within the setting of the theory B, one can define more carefully what it is for a propositional operator to be a *necessity*. In rough terms, the necessities will be exactly those operators whose logic includes a rule of necessitation. To state this definition, one first defines a *weak necessity* to be a propositional operator which applies to the tautologous proposition, and a *necessity* to be a propositional operator which applies to the tautologous proposition as a matter of every weak necessity.[13]

Definition 3 (Weak necessity) $Nec^- := \lambda X.X\top$, where X: $t \to t$

Definition 4 (Necessity) $Nec := \lambda X \forall Y(Nec^-(Y) \to YX\top)$, where $X, Y : t \to t$

In intuitive terms, then, a necessity may be thought of as a propositional operator whose logic includes a rule of necessitation, for since

[12] In other work, Bacon (2020) cements this understanding via an axiom which guarantees that this strictest necessity behaves as one would expect logical necessity to.

[13] Definitions 3–5 are from Bacon (2018). The arguments in what follows are compatible with alternative conceptions of 'necessity', for example, one on which a necessity must apply to the tautologous proposition as a matter of every objective necessity; see Roberts (MS). However, I use Bacon's definition for reasons of familiarity.

the operator L is obviously a weak necessity, the result of applying any necessity X to $\top$ is itself $\top$, a theorem of the logic.

With the necessities clearly demarcated, one can characterize a natural ordering of them according to their *broadness*. This notion of broadness might be familiar from examples: logical necessity is broader than the necessity *it is the case that* (i.e. $\lambda p.p$) since, as a matter of every necessity, everything which is logically necessary is the case.[14] To this end, we define the broadness ordering on necessities as follows (to ease readability, I adopt the convention of writing $\Phi(X)Y$ as $\Phi(X, Y)$):

Definition 5 (Broadness) $Br := \lambda Y \lambda Z \forall X(Nec(X) \rightarrow X \forall p(Yp \rightarrow Zp))$, where $X, Y, Z : t \rightarrow t$

The intuitive idea behind this definition is that a necessity Y is at least as broad as Z when it is a necessary matter—according to every necessity—that whatever is necessary according to Y is necessary according to Z. We say that X is *broader than* Y if and only if Y is as broad as Z and not vice versa.

The necessities are an extremely inclusive family. For all that has been said, a necessity does not have to imply truth, or even be a Kripke operator. Indeed, the necessities might even include various propositional operators like deontic necessity and determinacy. It is natural, then, to make further divisions amongst the necessities. Following an extant tradition, we shall distinguish between *objective and non-objective* necessities.[15] Put evocatively, the objective necessities are those which regard variation in worldly circumstance: what really can happen. By way of contrast, objective necessities are often juxtaposed with epistemic, psychological, doxastic, and intentional modalities, which instead concern variation along a broadly representational dimension. Indeed, perhaps some of these notions even fail to qualify as necessities in our sense. Moreover, it is typical to contrast the objective necessities with 'logical' or 'mathematical' modalities such as logical necessity and determinacy, understood as propositional operators and not as predicates of expressions. In addition, deontic

[14] Though entrenched, the use of the term 'broadness' to describe this relation has its drawbacks, for when necessity X is broader than necessity Y, Y applies to propositions to which X does not; the broadness metaphor pertains to the idea that when a necessity X is broader than necessity Y, X-possibility is more inclusive than Y-possibility.

[15] Williamson (2016) uses the terminology of 'objective' and 'non-objective' modalities; Kratzer (2012) and Rosen (2006) use different terminology to describe this same distinction.

and teleological necessities are not to be classified as objective in the intended sense either, for they concern not variation in worldly circumstance but rather normative requirements on how those circumstances ought to be. In a similar vein, Rosen (2006, p. 16) distinguishes between ‘real’ modalities, which he glosses as ‘alethic, non-epistemic, and sometimes substantive or synthetic’, and non-real modalities. Indeed, the objective necessities are the modalities expressed in natural language by expressions belonging to Kratzer’s (2012, p. 61) class of circumstantial modals. Examples of objective necessities include various familiar modalities, such as: technological possibility—what is possible given the current state of technology; biological possibility—what is possible given our current biological make-up; practical possibility—what is possible given our current means; and of course, physical possibility.[16] Although no explicit definition of the objective necessities has been offered, this characterization should suffice to isolate a unified family of modalities that is familiar to philosophical practice.

My central argument will appeal to some very modest conditions on the objective necessities. To this end, we require that the objective necessities are all necessities which obey the K axiom of modal logic as a matter of every objective necessity, and are closed under composition and conjunction.[17] We also impose the plausible requirements that the operators *it is the case that* (i.e. $\lambda p.p$) and physical necessity are objective necessities too.

Definition 6 (Kripke necessity)

$$K := \lambda X(Nec(X) \wedge \forall Y\ (\mathcal{O}\ (Y) \rightarrow\ Y\, \forall p \forall q(X(p \rightarrow q)\ \rightarrow Xp \rightarrow Xq)))$$

Kripke	$\forall X(\mathcal{O}(X) \rightarrow K(X))$
Truth	$\mathcal{O}(\lambda p.p)$
Basis	$\mathcal{O}(\blacksquare)$

[16] I doubt that there are totally precise, context-invariant notions of technological, biological and practical possibility. A more plausible view is one on which such notions are contextually variable, with the context supplying additional restricting information such as the laws of nature, or a certain time frame. Indeed, even within a single context, such restricted modals may still exhibit a reasonable amount of vagueness.

[17] See Williamson (2016) for similar principles within his algebraic setting. A more powerful and comprehensive axiomatization of the objective modalities is given in Roberts (MS). Certain axioms discussed there, concerning for instance the modal stability of the objective necessities, are not among the minimal principles required by the following arguments, and so are omitted here.

Composition $\forall X \forall Y(\mathcal{O}(X) \wedge \mathcal{O}(Y) \rightarrow \mathcal{O}(\lambda p.XYp))$

Conjunction $\forall X \forall Y(\mathcal{O}(X) \wedge \mathcal{O}(Y) \rightarrow \mathcal{O}(\lambda p(Xp \wedge Yp)))$

These basic principles should strike one as plausible. Amongst other things, they guarantee the plausible consequences that if physical necessity and biological necessity are objective necessities, then so are *physically necessary biological necessity* (by Composition) and *being both physically and biologically necessary* (by Conjunction).

Supplementing these principles, we also require that the operator which applies all finite iterations of an objective necessity to a proposition is itself an objective necessity. The force of this principle stems from the fact that all finite iterations of an objective necessity are objective necessities by Composition. Thus there would appear to be little basis to exclude an operator which merely applies all those objective necessities from being objective itself. To state this principle in the object-language, we first define the *closure* of a given propositional operator. This is done in two stages. First, an operator Y_2 is defined to be an iteration of the operator Y_1 when any property which applies to *it is the case that* and is closed under composition with Y_1 also applies to Y_2. For example, $\blacksquare\blacksquare$ is an iteration of $\blacksquare$ because any property (of propositional operators) which applies to *it is the case that* and is closed under composition with $\blacksquare$ applies to $\blacksquare\blacksquare$. Second, the closure of an operator is defined to be a function from that operator to the propositional operator which applies all of the initial operator's finite iterations.

Definition 7 (Iteration)

$It := \lambda Y_1 \lambda Y_2 \forall X(X\lambda p.p \rightarrow (\forall Y_3(XY_3 \rightarrow X\lambda p.Y_1 Y_3 p) \rightarrow XY_2))$, where $Y_1, Y_2, Y_3 : t \rightarrow t$ and $X : (t \rightarrow t) \rightarrow t$

Definition 8 (Closure operator)

$Cl := \lambda Y \lambda p. \forall X(It(Y)X \rightarrow Xp)$, where $Y, X : t \rightarrow t$

Closure $\forall X(\mathcal{O}(X) \rightarrow \mathcal{O}(Cl(X)))$

Put informally, the closure of a necessity operator X is the operator *having every finite iteration of X*. Finally, it is plausible that these principles about the objective necessities are not merely true, but also capture how the objective necessities behave as a matter of every objective necessity. Thus let BO be the system which results from adding these principles to the theory B and closing it under *modus ponens*, universal generalization, and the modest rule that if $\phi \in$ BO then $\forall X(\mathcal{O}(X) \rightarrow X\phi) \in$ BO.

Importantly, I wish to highlight that one can characterize a notion of broadness *as far as the objective necessities are concerned*, in addition to the unqualified notion of broadness above. According to this notion, a notion of necessity Y is *as objectively broad as* Z (for short, *as o-broad as*) precisely when it is a matter of every *objective* necessity that Y-necessity implies Z-necessity.

Definition 9 (Objective broadness)

$Br^{\mathcal{O}} := \lambda Y \lambda Z \forall X(\mathcal{O}(X) \rightarrow X \forall p\ (Yp \rightarrow Zp))$, where $X, Y, Z : t \rightarrow t$

In the presence of the Kripke axiom, broadness implies o-broadness. For all that has been said, however, it might be the case that some objective necessity is as o-broad as another objective necessity despite not being as broad as it *simpliciter*. To appreciate why, it helps to notice that since logical possibility is a species of necessity, broadness *simpliciter* will be a very demanding relation if logical possibility is anywhere near as liberal as it is assumed to be. For example, it does not seem logically impossible for there to be practically necessary propositions that are not physically necessary, which implies that physical necessity is not broader than practical necessity. Similarly, it does not seem logically impossible for metaphysical necessity to fail to imply truth, in which case metaphysical necessity is not broader than truth (i.e. $\lambda p.p$); for related discussion, see Bacon (2020, pp. 553-554). Nevertheless, we have no reason to think that such logical possibilities fall within the realm of objective possibility, in which case physical possibility may still be as o-broad as practical possibility, and metaphysical necessity may still imply truth as a matter of every objective necessity.[18] For such reasons, without further assumptions o-broadness will be the more philosophically interesting relation when one is studying different relations between objective necessities. Since the focus of what follows is on physical necessity's place in the objective necessities, I shall therefore focus on o-broadness. In particular, when talking about an *o-broadest objective necessity*, I shall mean a notion of objective necessity which is as o-broad as all objective necessities.

Definition 10 (O-broadest objective)

$BON^{\mathcal{O}} := \lambda Y(\mathcal{O}(Y) \wedge \forall X\ \ (\mathcal{O}(X) \rightarrow Br^{\mathcal{O}}(Y, X)))$, where $X, Y, Z : t \rightarrow t$

[18] In fact, Roberts (MS) presents an argument that logical necessity cannot be an objective necessity without collapsing the entire distinction between objective and non-objective necessities.

This allows us to state the thesis of Nomological Bound more carefully, as the claim that physical necessity is the o-broadest objective necessity:

Nomological Bound $BON^{\mathcal{O}}(\blacksquare)$

3. The boundaries of objective possibility

3.1 Nomological bridges

The two strands of the previous two sections join together to form an argument against Nomological Bound. Initially, I argued that physical necessity does not obey $4_{\blacksquare}$. However, from this observation it is immediate that physical necessity is not the o-broadest objective necessity, for the failure of $4_{\blacksquare}$ generates objectively possible counterexamples to the generalization $\forall p(\blacksquare p \rightarrow \blacksquare\blacksquare p)$. Yet by Composition, $\blacksquare\blacksquare$ is itself an objective necessity, so there is some objective necessity which physical necessity is not as o-broad as: it is objectively possible that there are physically possible possibilities which are not physically possible. Indeed, this is captured by the following theorem of BO (see appendix).

Theorem 1 $\vdash_{\mathrm{BO}} \neg\forall p(\blacksquare p \rightarrow \blacksquare\blacksquare p) \rightarrow \neg BON^{\mathcal{O}}(\blacksquare)$

Thus I conclude that physical necessity is not the broadest objective necessity and that Nomological Bound is false.

Let me highlight that even if certain primitivists maintain that whichever modality Model-World^{+} characterizes is not physical necessity, they are still susceptible to essentially the same argument. Indeed, if they maintain that physical necessity is either of the S4 or S5 modalities corresponding to PN* and PN† respectively then, as I argued previously, they still ought to recognize that *some* objective modality is characterized by Model-World^{+} which corresponds to PN. However, it should now be clear that PN-necessity is broader, and therefore o-broader, than both PN*-necessity and PN†-necessity. Thus neither PN*-necessity nor PN†-necessity (nor PN-necessity) circumscribes the boundaries of objective possibility.

Is any objective necessity an o-broadest objective necessity? Interestingly, BO is consistent with the claim that there is no such objective necessity. It would be natural, then, to investigate further closure conditions on the objective necessities which yield this consequence.[19] Here, however, I wish to adopt a slightly different approach to the question. My strategy will be to argue that Model-World$^+$ supports a particular principle about physical necessity, which, once added to BO, proves not only that there is such an objective necessity but that it can be characterized in terms of physical necessity; call this thesis *Nomological Bridges*. In slogan form, the principle states that it is a matter of objective necessity—in every sense—that no objectively possible world is isolated from all iterations of physical possibility. In other words, for every objective necessity *X*, it is a matter of every objective necessity that there is a necessity expressed by some iteration $\blacksquare^n$ of only $\blacksquare$ operators, such that $\blacksquare^n$-necessity implies *X*-necessity.

$$\textit{Anti-Isolation} \quad \forall X \forall Y(\mathcal{O}(X) \wedge \mathcal{O}(Y) \to Y \exists Z \forall p(It(\blacksquare)Z \wedge (Zp \to Xp)))$$

Let BOA be the system which results from adding Anti-Isolation to BO and closing it under *modus ponens*, universal generalization, and the rule that if $\phi \in$ BO, then $\forall X(\mathcal{O}(X) \to X\phi) \in$ BO. In this theory, there is a neat argument for Nomological Bridges.

Before I present this argument, however, Anti-Isolation needs motivating. One of the strongest reasons in its favour is that Model-World$^+$ motivates it by a *modal subtraction argument*. Take an arbitrary objective necessity *X*, and suppose that there is some *X*-possible world *w* not reachable from the actual world by chains of physical possibility. Recall that Model-World$^+$ licenses PN, and together they generate counterexamples to axiom $4_{\blacksquare}$. Now, one feature of the counterexamples to $4_{\blacksquare}$ (on both the Humean and primitivist theories) becomes salient: in each of them there was some physically possible world in which one of the actual laws fails to be a law and, crucially, in which no other laws replace it. Yet this subtraction can be iterated an arbitrarily large finite number of times. Thus it ought to be physically possibly … possible that all the actual laws fail to be laws and no laws replace them. Yet consider such a lawless world. From that world, *w* will be physically possible, since trivially the lawless world's laws will all be true at *w*. Thus, contrary to supposition, *w* is reachable from the actual world by chains of physical possibility. Since the subtraction did not depend on any feature peculiar to the actual world, the argument supports the objective necessitation of Anti-Isolation.

[19] This is undertaken by Williamson (2016) and Roberts (MS).

This argument for Anti-Isolation requires several pieces of immediate commentary. First, the argument assumes that there are no physically necessary connections between distinct laws. To be exact, the assumption is that for any laws, it is physically possible that one fails to be a law whilst the others maintain their status as such. (I do not claim that one law can be *false* whilst the others are laws.) Whilst this might be assumed in scientific practice, it would fail were the laws closed under various logical operations such as conjunction. However, the argument in fact requires only the weaker assumption that there is some core basis of laws which exhibit the required independence and from which all others can be derived. The argument for lawless worlds could then be run in terms of these core laws.

Second, the argument also assumes that there are only finitely many laws. This may be uncontroversial, but again the argument could be run on the weaker assumption that there is a finite axiomatic basis from which all laws can be derived. Moreover, even supposing that there are infinitely many laws, Model-World^{+} would still support the claim that it is physically possible that some finite subset of those laws are the only laws. But from this possibility the lawless world could then be reached by a finite number of steps of physical possibility.

Third, given that *w* is objectively possible from the initial world, it seems unprincipled to deny that *w* is objectively possible from the lawless world, for it is difficult to see why the mere failure of a proposition to be a law would *prevent w* from being objectively possible. Moreover, in Williamson's (2016) more comprehensive axiomatization of the objective modalities, which is recreated in Roberts (MS), this scenario is formally ruled out. In Williamson's framework, if a proposition is possible according to some species of objective possibility, then as a matter of every objective necessity, that proposition is possible according to some species of objective possibility.

Fourth, recall Maudlin's plausible claim (from §1.1 above) that 'the content of the laws can be expressed without modal notions'. Given this non-modal character of laws, it will not be a law that a given proposition *p* is a law (or a law that it is a law that *p*, and so on). Nor will laws include propositions such as *It is physically necessary that it is a law that like-charged particles repel.* This prevents the laws themselves mandating that in all physically possible worlds a certain proposition is a law, which would block the argument for the accessibility of lawless worlds.

Fifth, it may be that some laws are objectively necessary in all senses. For example, Fine (2005, p. 242) suggests, in effect, that the generalization that all electrons have negative charge is both a law and objectively necessary in all senses of objective necessity. Moreover, Fine's discussion also suggests that he may take it to be similarly necessary that *it is a law* that all electrons have negative charge. Were there to be such laws, it may be thought that the above argument for Anti-Isolation could be resisted, for no objectively possible world would be lawless. However, this thought would be mistaken. Imagine that it is indeed objectively necessary in all senses that it is a law that all electrons have negative charge. In that case, no objectively possible world is lawless. Nonetheless, for those laws which are objectively contingent (in perhaps different senses of objective contingency), the above considerations could be used to argue for a physically possibly … possible world which lacked any contingent laws, a contingent-lawless world. Now, given that all remaining laws, if any, at that world would be objectively necessary in every sense, that world's laws would all be true at any objective possibility. Thus from that contingent-lawless world every objective possibility would be physically possible, and so Anti-Isolation would be true. As a consequence, Anti-Isolation is well-motivated even if certain laws are objectively necessary in every sense, and indeed laws in every objective sense of necessity too.

I shall consider other objections to the subtraction argument in §4. However, to return to the main thread, the argument within BOA for Nomological Bridges can be completed. First, notice that even in BO the closure of physical necessity is an objective necessity; that is, $\vdash_{\mathrm{BO}} \mathcal{O}(Cl(\blacksquare))$. Yet by Anti-Isolation, for every notion of objective necessity X it is objectively necessary in all senses that some iteration of $\blacksquare$ implies X. Thus since $Cl(\blacksquare)$ simply applies all iterations of $\blacksquare$ operators, $Cl(\blacksquare)$ is as o-broad as every objective necessity, and is thus an o-broadest objective necessity. Indeed, the formula $BON^{\mathcal{O}}(Cl(\blacksquare))$ is simply a theorem of BOA (see appendix).

Theorem 2 $\vdash_{\mathrm{BOA}} BON^{\mathcal{O}}(Cl(\blacksquare))$

Since $Cl(\blacksquare)$ is quite clearly characterized in terms of physical necessity, this theorem demonstrates that the informal thesis of Nomological Bridges is vindicated in BOA: in that system, the boundaries of objective possibility can be characterized in terms of physical necessity. On a natural understanding, then, the theorem constitutes the powerful anti-sceptical result that those who believe in physical necessity should not harbour any scepticism towards metaphysical necessity.[20]

Nevertheless, there are various subtleties surrounding this issue. Since metaphysical necessity is often just defined as the o-broadest objective necessity, it is tempting to understand Theorem 2 as the result that $Cl(\blacksquare)$ just is metaphysical necessity. However, BOA does not prove that there is a *unique* o-broadest objective necessity: there might be multiple objective necessities which characterize the boundaries of objective possibility.[21] Despite this, the result requires those who believe in physical necessity to recognize merely metaphysical possibilities which are forbidden by our physical laws. Yet it is difficult to imagine anyone admitting this whilst still harbouring scepticism about the notion of metaphysical necessity itself.

3.2 A shorter bridge?

There is a simpler yet more controversial argument for Nomological Bridges. Earlier I argued that the lawless world is reachable from any objectively possible world by a finite number of steps of physical possibility. For, from any given world, it is physically possible that one law fails and is not replaced by any other law. However, more controversially, one might think that from a given objectively possible world it is physically possible that *every* law at that world fails to be a law and no one of them is replaced. In that case, the lawless world would be simply physically possible from every objectively possible world. Thus, since every objectively possible world is trivially physically possible from the lawless world, every objectively possible world would be physically possibly possible from every objectively possible world. In other words, $\blacksquare\blacksquare$ would be the o-broadest objective necessity.

On this conception, although 4 would fail $\blacksquare$, the following weaker condition would not:

$$4^-_{\blacksquare} \quad \blacksquare\blacksquare p \rightarrow \blacksquare\blacksquare\blacksquare p$$

If $4^-_{\blacksquare}$ and $T_{\blacksquare}$ (that is, $\blacksquare p \rightarrow p$) are added as axioms to BO, the twofold and threefold iterations of physical necessity become necessarily

[20] From this result it is easy to see that the closure of physical necessity, unlike physical necessity itself, admits the objective necessitation of the 4 axiom in BOA, that is, $\vdash_{BOA} \forall X(\mathcal{O}(X) \rightarrow X(Cl(\blacksquare)p \rightarrow Cl(\blacksquare)Cl(\blacksquare)p))$. Note, however, that this is not guaranteed in BO.

[21] The failure of a similar uniqueness claim is discussed in detail in Roberts (MS).

equivalent in all senses of objective necessity (recall the objective necessitation condition from §2). Thus the twofold iteration of physical necessity would circumscribe the boundaries of objective possibility, and it would be objectively necessary in all senses that a proposition is metaphysically possible if and only if it is simply physically possibly possible.

This route is more controversial than the initial one, for it requires that there is always a world based on a model of a theory in which all its laws fail to be laws, despite of course being true. Yet this claim is hostage to empirical fortune, and is therefore a more contentious working assumption. (The idea seems less contentious on a primitivist view of laws, since then the laws do not supervene on the particular matters of fact. For Humeans, however, the second route does appear genuinely more speculative.) In contrast, with the initial route there is far less risk in claiming that there is a world based on a model of a scientific theory in which at least one of the actual laws fails to be a law. Thus, although the second route offers a more speculative path, there is a more circuitous, less risky way to establish Nomological Bridges.

4. Objections and alternatives

4.1 Alternative conceptions of physical necessity

In order to resist the central arguments above, some opponents might take the somewhat radical stance of denying that Model-World$^+$ succeeds in introducing *any* objective necessity whatsoever, regardless of whether it is physical necessity or not. If those who adopt this stance believe in physical necessity, they must reject the traditional theoretical role of physical necessity in terms of its connection to models of a world's laws. Indeed, it would be natural for this particular opponent to adopt either the PN* or PN† conception of physical necessity discussed in §1.3 above.

Perhaps the only extant view with this character has been developed in detail by Alastair Wilson (2020), a view he dubs *quantum modal realism* (QMR). Wilson develops QMR in the context of the Everettian interpretation of quantum mechanics as the thesis that all and only the objective possibilities are given by the Everett multiverse. According to this view, physical necessity meets the PN† conception and thus conforms to an S5 logic (Wilson, 2020, pp. 37–40).

Moreover, crucially, Wilson's QMR simply implies Nomological Bound; indeed, QMR is just one precise way in which the thesis could be realized. It is important, therefore, to recognize the limitations of my arguments: QMR is a principled hypothesis about the metaphysics of objective possibility which undercuts key assumptions to which my arguments appeal. For those who do not antecedently accept Wilson's thesis, however, the arguments against Nomological Bound and for Nomological Bridges are much harder to resist.

A trickier issue is whether proponents of Alexander Bird's (2007) nomic essentialism should be described as endorsing Nomological Bound. According to nomic essentialism, the laws are true as a matter of metaphysical necessity, a thesis which Bird himself claims 'identifies nomic necessity with metaphysical necessity' (2007, p. 48). Nevertheless, nomic essentialists still recognize that even though the laws are metaphysically necessary, in many metaphysical possibilities they are merely vacuously satisfied, for the physical kinds they concern are not instantiated. Thus one might regard *the laws of a particular world* as those laws which concern any physical kind instantiated at that world. The salient question for nomic essentialists then becomes whether they recognize that Model-World$^+$ characterizes some species of objective necessity in terms of the models of a particular world's laws. Yet if they do, this species of objective necessity will be less o-broad than metaphysical necessity, and will also fulfil the typical theoretical role associated with physical necessity. As a consequence, there is one natural version of nomic essentialism according to which Nomological Bound is not a straightforward consequence of the theory.

4.2 Humean lawless worlds

In his discussion of Humeanism, Alexander Bird (2007, p. vii) suggests that the Humean cannot admit lawless worlds. Bird's argument is straightforward: no matter what the particular matters of fact, there will always be some best systematization of them, even if by absolute standards the systematization is utterly inelegant. If this is correct, the appeal to lawless worlds in motivating Nomological Bridges trades on the falsity of a well-established theory of laws, which severely limits the argument's scope.

Bird's suggestion, however, is far from obvious. First of all, even Lewis himself recognizes that it is optimistic to think that there will always be a best systematization of a given world. He thus makes the following proviso:

> [A law] is any regularity that earns inclusion in the ideal system. (*Or, in the case of ties, in every ideal system.*) (Lewis 1983, p. 41; my emphasis)

But is there any guarantee that the intersection of the ideal systems will always contain lawlike generalizations? To provide a toy example which casts doubt on this, suppose that there was some world which was nomically chaotic in that there are no highly general nomic regularities there. More specifically, where '$F, G, H, I, \ldots$' are predicates of the canonical language and '$a, b, \ldots$' canonical terms, none of which are co-referential, consider a world w in which the pattern of facts is given by simple binary conjunctions of the form $Fa \wedge Ga$, $Hb \wedge Ib$, and so on. There are different axiomatizations of how things are at w which are tied with one another with respect to simplicity and strength:[22]

S1 $Fa \wedge Ga,\ Hb \wedge Ib,\ \ldots$

S2 $\forall x((Fx \wedge Gx) \vee (Hx \wedge Ix) \vee \ldots)$

First, note that neither system appears at least as strong as the other. For starters, neither proof-theoretically includes the other. S1 will not prove S2's single axiom, since the conjunction of all its axioms does not prove S2's axiom. In order to secure that implication, one would need the universal generalization that $a, b, \ldots$ are all the things there are. But for the Humean, totality facts like these will not typically be categorized as laws: there ought to be physically possible worlds with different individuals. Indeed, as a consequence, the only regularities S1 contains will be trivial. Moreover, S2 will not prove any one of S1's axioms; from the generalization that everything is either such-and-such or so-and-so, one cannot derive *which* things are such-and-such and *which* are so-and-so. More generally, neither system looks to prove much more than the other in a measure-theoretic sense. Furthermore, on any reasonable measure, both systems will be of roughly the same complexity, for S1 is non-finitely axiomatizable and S2's single axiom contains infinitely many expressions. Yet the

[22] Although S2's single axiom contains a countably infinite disjunction, given Lewis's (1986, p. 61) suggestion of characterizing the comparative naturalness of properties in terms of their lengths of definitions from the perfectly natural, it would be surprising if his canonical language had a finitary syntax.

intersection of both systems will contain only tautologies and disjunctions of S1 and S2 theorems. However, as mentioned above, the only regularities of S1 are trivial generalizations, so the regularities in the intersection of both systems will be trivial, which deprives them of the explanatory capacity and relevance required of laws. Thus there will be no laws at this chaotic world.

Complementing this, Lewis (1994, p. 479) himself later came to the opinion that there would be no laws in the absence of a clearly best system:

> Likewise for the threat that two very systems are tied for best. . . . I used to say that the laws are then the theorems common to both systems, which could leave us with next to no laws. Now I'll admit that in this unfortunate case there would be no very good deservers of the name of laws.

Indeed, in his later work, Lewis seemed to require not just that laws belong to the best system, but that the laws need to be sufficiently explanatory as well.[23] On this conception, the Humean can easily recognize that nomic chaos might induce lawlessness.

4.3 Dead-end worlds

A final objection to Anti-Isolation concerns the objective possibility of *dead-end worlds*, objective possibilities at which physical necessity is materially equivalent to truth.[24] At such worlds, the only physically possible world is the world in question itself, so no modal subtraction argument can be run. Are such worlds genuine objective possibilities? The question leads to some speculation. For example, on the Humean theory of laws the objective possibility of dead-end worlds requires it to be objectively possible that the best systematization of the particular matters of fact has all truths as theorems. Of course, this would be an extremely strong system in the working measure-theoretic sense. But whether it would best balance the criteria of simplicity and strength is a speculative matter, and not obviously a point which we are able to extract from the Humean theory. As for the primitivist, their view—characterized simply by a denial of a supervenience claim—would appear to be silent on whether dead-end worlds are

[23] A theme of Sider (2020) is that this is a theory-independent constraint on laws. Thanks to Ted Sider and Jade Fletcher for highlighting Lewis's change of view to me.

[24] Thanks to Jeremy Goodman for raising this style of concern.

objectively possible. Nothing therefore precludes the primitivist from asserting that it is objectively possible that the true propositions are exactly the laws.

My preferred approach to this issue is not to resort to modal speculation, and simply consider the worst-case scenario—the objective possibility of dead-end worlds—and argue that ultimately they are no great concern. The crucial point is that even though at such worlds physical necessity may be materially equivalent to truth, many other objective necessities will not be. For example, most participants to the debate will be willing to recognize a notion of *nearby counterfactual necessity* as an objective necessity. (The point applies to other modalities such as, for example, biological necessity.) Given a counterfactual conditional $>$ obeying the Lewis semantics, one can characterize this notion of necessity as truth in all counterfactually close worlds.[25]

Definition 11 (Counterfactual necessity) $C\phi := \top > \phi$

However, on the standard Lewis semantics which requires merely weak centring on the world of evaluation, the above notion of counterfactual necessity is distinct from truth. Thus, at dead-end worlds, counterfactual necessity is not coextensive with physical necessity or any of its iterations. As a result, any non-actual counterfactual possibility will plausibly involve one of the actual laws being false, or at least not being a law. Consequently, if dead-end worlds are objectively possible, then although the o-broadest objective necessity of $Cl(\blacksquare)$ is not motivated by the subtraction argument, the o-broadest objective necessity of $\lambda p.(Cl(\blacksquare) \wedge Cp)$ is, which is an objective necessity by Conjunction. Thus, at dead-end worlds, although the only physical possibility might be that world, other non-dead-end worlds will be counterfactually possible and thus can be subject to the modal subtraction argument: if there are dead-end worlds, some nomological bridges might begin with a counterfactual plank. Although this picture is less elegant, it sustains the overarching philosophical point that what is metaphysically possible can be characterized in terms of familiar modalities, such as nearby counterfactual necessity and physical necessity.

[25] This definition does not rely on Lewis's claim that counterfactuals with metaphysically impossible antecedents are vacuously true, since with that claim one can arguably characterize a notion of metaphysical necessity; see Lewis (1973, p. 22) and Williamson (2007, ch. 5). The weak counterfactual conditional required to define counterfactual necessity as in Definition 11 falls well short of the counterfactual resources required to introduce a notion of metaphysical necessity.

5. Concluding remarks

I have argued that the orthodox characterization of physical necessity undermines Nomological Bound. Nevertheless, I have also argued that this characterization can be used to show that the boundaries of objective possibility can still be characterized intimately in terms of physical necessity. In this sense, I take my arguments to place significant theoretical pressure on those who reject that there are merely metaphysical possibilities but still indulge in talk of physical necessity. Consequently, those who are sceptical about merely metaphysical possibilities which are forbidden by our physical laws must either reimagine the notion of physical necessity or reject extremely modest principles about the objective necessities.[26]

6. Appendix

In this appendix, I provide axiomatic proofs of several claims from the main body. Elementary justifications are often omitted, and routine B derivations are indicated as such. For convenience, we note the following two theorems of B (cf. Bacon 2018):

Distribution $\vdash_B \forall_\sigma x(\phi \to \psi) \to (\forall_\sigma x\phi \to \forall_\sigma x\psi)$

L-Necessitation If $\vdash_B \phi$, then $\vdash_B L\phi$

Theorem 1. $\vdash_{BO} \neg\forall p(\blacksquare p \to \blacksquare\blacksquare p) \to \neg BON^{\mathcal{O}}(\blacksquare)$

(1) $\neg\forall p(\blacksquare p \to \blacksquare\blacksquare p) \to \neg(\lambda p.p)\forall p(\blacksquare p \to \blacksquare\blacksquare p)$ βη

(2) $\neg(\lambda p.p)\forall p(\blacksquare p \to \blacksquare\blacksquare p) \to (\mathcal{O}(\lambda p.p) \to \exists Y(\mathcal{O}(Y) \wedge \neg Y\forall p(\blacksquare p \to \blacksquare\blacksquare p)))$ B

(3) $\neg(\lambda p.p)\forall p(\blacksquare p \to \blacksquare\blacksquare p) \to \exists Y(\mathcal{O}(Y) \wedge \neg Y\forall p(\blacksquare p \to \blacksquare\blacksquare p)))$ 1, 2, Truth

(4) $\exists Y(\mathcal{O}(Y) \wedge \neg Y\forall p(\blacksquare p \to \blacksquare\blacksquare p)) \to (\mathcal{O}(\lambda p.\blacksquare\blacksquare p) \to \exists Y\exists Z(\mathcal{O}(Y) \wedge \mathcal{O}(Z) \wedge \neg Y\forall p(\blacksquare p \to Zp)))$ B

[26] Thanks to Ross Cameron, Neil Dewar, Andreas Ditter, Kit Fine, Jade Fletcher, Peter Fritz, Jeremy Goodman, Alex Kaiserman, Ofra Magidor, Jonathan Schaffer, Ted Sider, audiences in Nottingham, Oxford, Oslo, Toronto, and Worcester College, and two anonymous referees for *Mind*. Special thanks are due to Annina Loets, James Studd and Timothy Williamson for feedback on several drafts, and to Andrew Bacon for much discussion about these issues. This article was completed with the support of a grant from the Leverhulme Trust (ECF-2020-414).

(5) $\mathcal{O}(\lambda p.\blacksquare\blacksquare p)$ Basis, Composition

(6) $\exists Y(\mathcal{O}(Y)\wedge\neg Y\forall p(\blacksquare p\rightarrow\blacksquare\blacksquare p))\rightarrow\exists Y\exists Z(\mathcal{O}(Y)\wedge\mathcal{O}(Z)\wedge\neg Y\forall p(\blacksquare p\rightarrow Zp))$ 4, 5

(7) $\neg\forall p(\blacksquare p\rightarrow\blacksquare\blacksquare p)\rightarrow\exists Y\exists Z(\mathcal{O}(Y)\wedge\mathcal{O}(Z)\wedge\neg Y\forall p(\blacksquare p\rightarrow Zp))$ 1, 3, 6

Lemma $\vdash_{\mathrm{B}}$ $L(\forall p\exists Y(It(\blacksquare)Y\wedge Yp\rightarrow Xp)\rightarrow\forall p(Cl(\blacksquare)p\rightarrow Xp))$

(1) $(It(\blacksquare)Y\rightarrow Yp)\rightarrow((It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow Xp)$ PC

(2) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow\forall Y((It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow Xp)$ 1, Gen, Distribution

(3) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow((It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow Xp)$ 2, UI

(4) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow(\neg Xp\rightarrow\neg(It(\blacksquare)Y\wedge(Yp\rightarrow Xp)))$ 3, PC, MP

(5) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow\forall Y(\neg Xp\rightarrow\neg(It(\blacksquare)Y\wedge(Yp\rightarrow Xp)))$ 4, Gen

(6) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow(\neg Xp\rightarrow\forall Y\neg(It(\blacksquare)Y\wedge(Yp\rightarrow Xp)))$ 5, Gen

(7) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow(\neg\forall Y\neg(It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow\neg\neg Xp)$ 6, PC, MP

(8) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow(\neg\forall Y\neg(It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow Xp)$ 7, PC, MP

(9) $\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow(\exists Y(It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow Xp)$ 8, Def. $\exists$

(10) $\exists Y(It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow(\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow Xp)$ 9, PC, MP

(11) $\forall p\exists Y(It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow\forall p(\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow Xp)$ 10, Gen, Distribution

(12) $L(\forall p\exists Y(It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow\forall p(\forall Y(It(\blacksquare)Y\rightarrow Yp)\rightarrow Xp))$ 11, L-Necessitation

(13) $L(\forall p\exists Y(It(\blacksquare)Y\wedge(Yp\rightarrow Xp))\rightarrow\forall p(Cl(\blacksquare)p\rightarrow Xp))$ 12, Def. Cl, $\beta\eta$

Theorem 2. $\vdash_{\mathrm{BOA}}$ $BON^{\mathcal{O}}(Cl(\blacksquare))$

(1) $L(\forall p\exists Y(It(\blacksquare)Y\wedge Yp\rightarrow Xp)\rightarrow\forall p(Cl(\blacksquare)p\rightarrow Xp))$ Lemma

(2) $\forall Z(Z\top\rightarrow(Z(\forall p\exists Y(It(\blacksquare)Y\wedge Yp\rightarrow Xp)\rightarrow\forall p(Cl(\blacksquare)p\rightarrow Xp))))$ 1, Def. L

(3) $\forall Z(Nec^{-}(Z)\to(Z(\forall p\exists Y(It(\blacksquare)Y\wedge Yp\to Xp)\to\forall p(Cl(\blacksquare)p\to Xp))))$ — 2, Def. Nec^{-}

(4) $\forall Z(\mathrm{Nec}(Z)\to(Z(\forall p\exists Y(It(\blacksquare)Y\wedge Yp\to Xp)\to\forall p(Cl(\blacksquare)p\to Xp))))$ — 3, B

(5) $\forall Z(\mathcal{O}(Z)\to(Z(\forall p\exists Y(It(\blacksquare)Y\wedge Yp\to Xp)\to\forall p(Cl(\blacksquare)p\to Xp))))$ — 4, Kripke

(6) $\forall Z(\mathcal{O}(Z)\to(Z\forall p\exists Y(It(\blacksquare)Y\wedge Yp\to Xp)\to Z\forall p(Cl(\blacksquare)p\to Xp)))$ — 5, Kripke

(7) $\forall X(\mathcal{O}(X)\to(\forall Z(\mathcal{O}(Z)\to(Z\forall p\exists Y(It(\blacksquare)Y\wedge Yp\to Xp)\to Z\forall p(Cl(\blacksquare)p\to Xp)))))$ — 6, PC, Gen

(8) $\forall X(\mathcal{O}(X)\to(\forall Z(\mathcal{O}(Z)\to Z\forall p\exists Y(It(\blacksquare)Y\wedge Yp\to Xp))))$ — Anti-Isolation, PC, Kripke

(9) $\forall X(\mathcal{O}(X)\to(\forall Z(\mathcal{O}(Z)\to Z\forall p(Cl(\blacksquare)p\to Xp))))$ — 7, 8, PC, UI, MP, Gen

(10) $\forall X(\mathcal{O}(X)\to Br^{\mathcal{O}}(Cl(\blacksquare),X))$ — 9, Def. $Br^{\mathcal{O}}$

(11) $BON^{\mathcal{O}}(Cl(\blacksquare))$ — 10, Basis, Closure, Def. BON^{O}

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