Aspects of Categorical Physics: A category for modelling dependence relations and a generalised Entropy functor

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Abstract

Two applications of Category Theory are considered. The link between them is applications to Physics and more specifically to Entropy.

The first research chapter is broader in scope and not explicitly about Physics, although connections to Statistical Mechanics are made towards the end of the chapter. Matroids are abstract structures that describe dependence, and strong maps are certain structure-preserving functions between them with desirable properties. We examine properties of various categories of matroids and strong maps: we compute limits and colimits; we find free and cofree constructions of various subcategories; we examine factorisation structures, including a translation principle from geometric lattices; we find functors with convenient properties to/from vector spaces, multisets of vectors, geometric lattices, and graphs; we determine which widely used operations on matroids are functorial (these include deletion, contraction, series and parallel connection, and a simplification monad); lastly, we find a categorical characterisation of the greedy algorithm. In conclusion, this project determines which aspects of Matroid Theory are most and least conducive to categorical treatment.

The purpose of the second research chapter is to provide a categorical framework for generalising and unifying notions of Entropy in various settings, exploiting the fact that Entropy is a monotone subadditive function. A categorical characterisation of Entropy through a category of thermodynamical systems and adiabatic processes is found. A modelling perspective (adiabatic categories) that directly generalises an existing model is compared to an axiomatisation through topological and linear structures (topological weak semimodules), where the latter is based on a categorification of semimodules. Properties of each class of categories are examined; most notably a cancellation property of adiabatic categories generalising an existing result, and an adjunction between the categories of weak semimodules and symmetric monoidal categories. An adjunction between categories of adiabatic categories and topological weak semimodules is found. We examine in which cases each of these classes of categories constitutes a traced monoidal category. Lastly, examples of physical applications are provided. In conclusion, this project uncovers a way of, and makes progress towards, retrieving the statistical formulation of Entropy from simple axioms.