ADJUSTMENT TO EQUILIBRIUM AND OPTIMAL

INTERVENTION IN FIX-PRICE ECONOMIES

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to Lina and my parents

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This thesis is devoted to my wife Lina for my happiness and to my parents for their support.

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INTRODUCTION

This thesis is a study of certain issues in the context of Hicksian, fix-price economies. These are simply described as economies where prices remain rigid temporarily but they move from period to period.

During the period where prices remain rigid, equilibrium is achieved through quantity adjustments. In most of the models proposed so far – see Benassy [5], Dreze [17] etc.– this adjustment is conceived as a tatonnement process; no exchange takes place until the system arrives at a fix-price equilibrium. These are better interpreted as equilibrium theories; they provide a satisfactory explanation of how quantity constraints support the fix-price equilibrium allocation but they do not tell us much outside it.

In the first chapter of this Dissertation we consider a non-tatonnement adjustment process for a pure-exchange fix-price economy. We argue that a realistic approach is to assume that exchange takes place in a completely decentralized environment for which the assumption that only one side of the market is rationed is not imposed. In fact, we consider a bilateral, gradient-like exchange process: Agents are engaged in bilateral trade giving rise to utility non-decreasing allocations. During this process, actual net trades are feasible for the economy, satisfy the budget constraints and are at most equal and of the same sign to trade offers (voluntary exchange). Trade offers have been formulated subject to perceived constraints on trading opportunities.

We show that under some regularity assumptions, this process brings the economy to a Pareto efficient allocation relative to the vector of fixed prices and perceived constraints.

As far as the technical aspects of this chapter are concerned, we have modelled the exchange process as a feasible directions algorithm (see Zangwill [50]). This algorithm has already been applied to planning theory (see Otsuki [41]) but not in informationally decentralized environments such as the one considered here.

In the second chapter we consider a model of interperiod price adjustment. It is assumed that prices are evolving according to the size of effective excess demands and our purpose is to show that this is a stable mechanism. The results rest on the assumption that quantity adjustments have taken place before prices start moving. It is accepted that this is an unsatisfactory aspect of this process, and we also suggest a price adjustment mechanism in the spirit of the first chapter where prices and quantities may adjust together.

On the other hand, our model in the second chapter has a richer structure than that in the first: In fact, it is a <u>non-tatonnement</u> process in the context of world economies and the results cast some light on the monetary approach to the balance of payments. We propose a method to deal with the problem of regime switching which arises in similar contexts (see Eckalbar [18], Laroque [31]), based on the theory of non-linear direct control systems (see Lefschetz [32]). In fact, we argue that the feedback effects due to rationing are such as to allow for an application of the Lyapounov method.

In the last two chapters we give the government an active role: In chapter three the government needs revenue to correct the distortions behind price rigidities and to finance public expenditure. We argue that the theory of Distortions and Welfare is still the suitable framework to discuss these matters. An optimal policy calls for a tax-cum-subsidy in the source of the distortion while optimal taxes should be imposed to raise revenue. If distortions cannot be fully corrected, the fact that some markets may clear by quantity rationing should be included in the constraints of the problem.

In this chapter, the specification of price rigidities is important. The popular assumption according to which nominal prices are exogenously rigid cannot be made here since the government can choose tax-rates to affect prices. In the model we consider we assume that there is an institutionally set real wage constraint and that the price of a non-traded good is fixed by a profit maximizing monopolist.

The last chapter is based on the author's M.Phil. thesis - see [46] - and is an effort to put the New Cambridge propositions in an equilibrium fix price model. Since it is clear that the theory of optimum where instruments are chosen optimally under no constraints is not the environment the policy-maker in the New Cambridge economy has in mind, we assume that he only seeks for partial Welfare improvements in an environment where he can only use a limited number of instruments with a limited range of application. In this case it is shown that some Welfare background can be given to his ranking of alternative trade policies.

Chapter I

A NON-TATONNEMENT PROCESS UNDER FIXED PRICES

A. Overview

The subject of this chapter is the study of the allocation of resources in a pure exchange economy during a period where prices remain rigid at an arbitrary level.

The first question the reader may ask is why such a study is undertaken. There already exists a substantial quantity of pioneering theoretical work following that of J.Dreze [17] and J.P.Benassy [5] and a new effort bears the risk of being superfluous.

We hope that the present chapter offers some insight into the theory of quantity adjustments which differs, according to our present knowledge of the literature, from what has already been said.

The basic models which have been proposed so far assume that prices p are rigid over a short period and investigate a <u>tatonnement</u> process on quantities. The crucial role is played by effective demands. In simple terms, these are the quantities individuals decide to trade in one market having taken into account the fact that some of their trades in othermarkets may be rationed.

Formally, the effective demands of agent h is the vector of net trades which maximizes his utility subject to his income constraint and the vectors of upper and lower bounds $\bar{c}_h, \underline{c}_h$ which represent his perceived constraints on his purchases and sales respectively. It is crucial whether an agent takes into account constraints in <u>all</u> markets when forming his net offer of good i. If he does not take into account constraints on good i, we have the Benassy definition of effective demand. On the other hand, if he takes into account all constraints, including those on good i, we have the Dreze definition.

Let us see how equilibrium is defined and achieved under the two different definitions.

According to the Benassy formulation – see [5] – agents express their effective demands to all markets. Right from the beginning we notice the following : Since they ignore constraints on good i when they formulate their demand for it they eventually may find themselves unable to finance their effective demand vector. Although this is an unsatisfactory aspect of Benassy's model, it is not of our immediate concern.

A 'rationing function' transforms effective demands into feasible trades. This function depends upon the specific exchange process and satisfies the following hypotheses:

(i) Trade is voluntary.

(ii) Actual trades balance.

(iii) Markets are 'frictionless', i.e. those in the 'short' side of the markets realize their demands.

Benassy assumes that a comparison between agents' net offers (effective demands) and their realized trades generates subjective perceived quantity constraints in the following way. If an agent h realizes his effective demand $\tilde{C}_n^h$ for good n, he perceives opportunities for more trade in this direction. If he fails to realize it, he perceives a constraint equal to the actual trade he achieves, which is determined through the rationing function and satisfies the three conditions we have already mentioned .

Equilibrium is a 'fixed' point in the following recursive tatonnementprocess: At time t, agents express their net offers $\tilde{C}(t)$. These generate a set of perceived constraints $\bar{C}(t), \underline{C}(t)$. On the basis of $\bar{C}(t)$, $\underline{C}(t)$, a new set of effective demands $\tilde{C}(t+1)$ is formulated. When $\tilde{C}(t+1) = \tilde{C}(t)$, equilibrium has been achieved.

Obviously, what is required for equilibrium is the reproduction of effective demands $\tilde{C}(t)$ and not their coincidence with the realized transactions $C(t)$. The existence of a fixed point in this process is guaranteed since effective demands are bounded both from below -from the initial endowments since we are in a pure-exchange economy- and from above -an agent cannot demand more than he is able to pay for- and since the feasibility set of agents are convex and continuous in $\bar{C}, \underline{C}$.

In Dreze's formulation, each agent perceives a signal $S = (P, \underline{C}, \bar{C})$. All goods are subject to lower and upper bounds. An equilibrium is a set of bounds and consumption vectors C such that

(i) $\sum_h C^h = \sum_h C_o^h$, where C_o^h is the initial endowment of agent h

(ii) Only one side of the market is rationed.

(iii) For all h, C^h maximizes agent's h utility subject to his feasibility set.

(iv) No quantity rationing is allowed unless price rigidities are binding.

Equilibrium can be conceived as a fixed point of the following tatonnement (At the end of this Chapter - see Appendix - we will interpret it as an explicit dynamic process): Agent h receives a signal $S = (P, \underline{C}, \bar{C})$ and announces his effective demand $C_n^h(P, \underline{C}, \bar{C})$ for good n. Then, an auctioneer computes the excess demand for n over all h and adjusts bounds according to the following rule: He raises $\underline{C}_n$ if n is in excess supply or lowers $\bar{C}_n$ if n is in excess demand. Then, a new offer $C_n^h(P, \underline{C}, \bar{C})$ is announced and the procedure continues until total excess demands are zero. Dreze has assumed that rationing is uniform to all agents. All that matters is aggregate rationing and not the specific allocation of shortages. With a different rationing scheme, a different equilibrium could emerge.

We should add that in Dreze's formulation we are not told how quantity constraints bound effective demands away from equilibrium. The tatonnement on quantities we have just described is nothing more than a dynamic interpretation of the mapping used by Dreze to prove the existence of a fixed point. Another unsatisfactory aspect of Dreze's model is that no disparity is produced between demand and supply since agents are rationed in the messages they can send. The 'size' of disequilibrium cannot be computed from the description of equilibrium and the pressure for price changes cannot be measured.

In Benassy's formulation such a disparity is possible, but as we have already noticed, this may be artificial. Since the formulation of effective demands is defined by a separate procedure for each commodity and the bounds for the commodity in question are deleted, there is no guarantee that these effective demands reflect actions that would be profitable for individuals. Benassy's model raises two more issues which are crucial to the understanding of fix-price models. First, the specification of the rationing scheme, which is also related to the "short" side of the market property. Second, the role attributed to money.

A rationing scheme for agent h is a mapping from the effective demands expressed in markets by all agents to his actual transactions. A distinction exists between manipulable and non-manipulable rationing schemes. A manipulable scheme is an increasing function of agent's h trade offers while a non-manipulable one is independent of them. Manipulable schemes are not consistent with the attainability of quantity constrained equilibria. In riskless environments, agents can calculate the amount of trades offered which would ensure that the actual net trades they can achieve are equal to their true (i.e. desired) net offers. Hence, any outcome can be achieved by distorting the net demands they announce.

Examples of manipulable and non-manipulable schemes are the uniform rationing in Dreze's model (non-manipulable) and various deterministic proportional rationing schemes (manipulable).

A way out of the problem of manipulable schemes is to assume stochastic rationing, i.e. that actual transactions are stochastic. In this case, it may not be profitable for an agent to distort his offers, since the actual outcome bears the risk of being undesirable.

A model with stochastic rationing is that of J.Green [23] . Green assumes that an individual's realization of trades is a random function of his own effective demands and the aggregate effective demands and supplies in the markets. This random function satisfies the following conditions:

(i) Trade is voluntary with probability one.

(ii) For all effective demand vectors, actual trade balances (on average).

(iii) The realization stochastic functions have the same distribution for all agents (uniform rationing).

The condition that only the "long" side of the market is rationed is missing. This is an important departure from existing models; the reason is that exchange is highly decentralized. Markets are not conceived as central clearing houses as in traditional models and there may be some unsatisfied demand coexisting with unsatisfied supply. During the exchange process the extent of this mutual imbalance will be reduced. This is certainly an improvement over the previous models which implicitly assume the existence of highly organized markets in the sense that a tatonnement on quantities takes place until feasibility is reached.

Other main points in Green's model are that acceptable rationing schemes, i.e. schemes satisfying conditions (i - iii) above, may impose strong restrictions on functional forms and that existence of equilibrium is problematic.

Another way out of the problem of manipulability of schemes is to assume endogenous price changes. In Hahn's model [24] agents have conjectures concerning the prices at which they can realize trades. A desire to break a constraint implies a need to offer to sell at a price different than the prevailing one. We will say more about price changes in the second chapter of this thesis.

Let us now come to the question about money. In Benassy's model, it is claimed that it has a very important role to play. In other words it appears as a necessary condition for a quantity constrained equilibrium. This view seems to arise from the idea that all trades must be made against money. The medium of exchange is modelled as an <u>additional</u> constraint to agents' feasibility sets.

Unfortunately this idea is in sharp contrast to the actual role of money, i.e. that of decentralization of exchange . Money does not shrink the feasibility sets; on the contrary, it enables trade and minimizes the number of periods needed to locate an efficient distribution of commodities by decentralized voluntary exchange. This point has been made forcefully by Ostroy [40]. See also Drazen [16].

In multi-good economies, money is not necessary for the existence of quantity constrained equilibria. The non-existence of <u>complete</u> markets (in the sense of perfect matching processes) and decentralized exchange are sufficient conditions for the formation of quantity constraints.

B. A GRADIENT-LIKE PROCESS ON QUANTITIES

1. Introduction

The point of departure for our model is the fact that existing theories on quantity rationing do not describe satisfactorily the behaviour of the system out of equilibrium. The tatonnement on quantities coupled with the "short" side rule already discussed is not a realistic exchange process since it requires a very well organized set of markets. In that case, one may ask whether such an organized market mechanism is consistent with persistent disequlibrium.

We think that a more realistic process can be described as follows. Agents start by expressing effective demands $\tilde{C}$ to the commodity markets. However, these are not like the traditional clearing houses where offers are matched perfectly, but like Green's formulations - see [23] - where individuals can simply meet with each other and exchange goods in a completely decentralized way. Actual trades are given as the outcome of a gradient process in individual utilities where everyone tries to achieve the "best" allocation C_h given the others' similar behaviour and the system's feasibility constraints. During the process, actual trades are at most equal to trade offers and of the same sign (voluntary exchange) and satisfy the budget constraints for the given, and fixed, price vector p. If an efficient allocation of commodities has been found, no incentive for more trade exists. In other words we look for the existence of an efficient allocation (and more important, of how to locate it via decentralized exchange) for this multi-decision process. However, since we require that actual trades balance for all goods over all agents this allocation will be an equilibrium as well.

2. Assumptions and Description of the economy

For the rest of the chapter the following assumptions will hold: We are in a pure exchange economy with H agents ($h = 1,2, \ldots H$) and N goods ($i = 1, \ldots N$). The vector of initial endowments will be denoted by C^o, where $C^o \in R_+^{HN}$. Prices will be fixed at an arbitrary level and will be denoted by the (1XN) vector p. Preferences will be represented by strictly concave and at least twice continuously differentiable utility functions $U_h(C_h)$.

At the beginning of the period, a price vector p is announced. As a result agents formulate subjective quantity constraints $\underline{C}_h$, $\bar{C}_h$ with $(\underline{C}_h, \bar{C}_h) \in (R^N \times R^N)$ and $\bar{C}_h \geq 0 \geq \underline{C}_h$. These constraints represent their beliefs about lower and upper bounds on their net trades. (There is no need to assume that $\underline{C}_h, \bar{C}_h$ are the same for all agents). The utility maximizing consumption vectors $\tilde{C}_h$ are given as solutions of the following programs:

"
$$\max_{C_h} \quad U_h(C_h)$$

$$\text{subject to:} \quad p(C_h - C_h^o) \leq 0,$$

$$\underline{C}_h \leq C_h - C_h^o \leq \bar{C}_h, \qquad (1)$$

$$h = 1,2, \ldots, H."$$

It is clear that agents' optimal trade offers $\tilde{C}_h - C_h^o$ are not necessarily feasible: $\sum_{h=1}^{H} (\tilde{C}_h - C_h^o) \neq 0$. Hence, a realization scheme (i.e. feasible allocations $C_h, \forall_h$) must be found.

We think that allocations which are points of Edgeworth-like processes give the most natural and intuitively appealing scheme: Agents, while striving to achieve their most preferred allocations $\tilde{C}_h$, will only be engaged in utility non-decreasing net trades which are feasible for the economy, satisfy the budget constraints and are at most equal and of the same sign to the optimal net trades $\tilde{C}_h - C_h^o$. Formally, exchange will take place at $C^o \in R_+^{HN}$ if and only if there exists a new allocation $C \in R_+^{HN}$ such that

(i) $U_h(C_h) \geq U_h(C_h^o)$ $\forall h$ and

$U_h(C_h) > U_h(C_h^o)$ for some h.

(ii) $pC_h \leq pC_h^o$, $\forall h$.

(iii) $|C_h - C_h^o| \leq |\tilde{C}_h - C_h^o|$, component wise,

$(C_{hi} - C_{hi}^o)(\tilde{C}_{hi} - C_{hi}^o) \geq 0$, $\forall h,i$, where $\tilde{C}_h$ is the solution to the program "$\underset{C_h}{\text{Max}}.U_h(C_h)$, subject to

$\underline{C}_h \leq C_h - C_h^o \leq \bar{C}_h$, $pC_h \leq pC_h^o$ ".

(iv) $\sum_{h=1}^{H} C_h \leq \sum_{h=1}^{H} C_h^o$

Condition (iii) requires that actual trade is voluntary, i.e. at most equal and of the same sign to trade offers.

Since trade is decentralized, it would not make sense to impose the condition that only the "long" side of the market is rationed. As we have said, markets are not like the traditional perfect matching processes and it is quite natural that unsatisfied demand may coexist with unsatisfied supply. In Dreze's model [17] this condition is necessary in order to avoid trivial equilibria. On the other hand, our model does not require it since trade will take place (subject to some regularity assumptions) until a Pareto efficient allocation relative to the vector of fixed prices and perceived constraints is achieved.

In effect, we would like our exchange process to converge to an allocation which solves the problem:

"Find $C \in R_{+}^{HN}$, in order to maximize, in a Paretian sense, the vector $W = [U_1(C_1), \ldots, U_H(C_H)]$ subject to (i), (ii), (iii) and (iv), given p, C^o, $\underline{C}_h$, $\bar{C}_h$, $\forall h$." (2)

In other words, we would like to assimilate the exchange process to an algorithm for program (2), which satisfies the following properties:

(a) all constraints are satisfied <u>at each</u> moment t (feasibility)

(b) all individual utilities are non-decreasing (monotonicity)

(c) if a finite sequence of allocations $[C^t]$ is generated, its last element is a Pareto efficient point for program (2). If an infinite sequence of allocations is generated, any convergent subsequence of it (there must be at least one since C lies in a compact set) must converge to a Pareto efficient point (convergence).

Since we have assumed that exchange is completely decentralized in the sense that the economy does not possess any perfect matching processes of the tatonnement type the algorithm must also satisfy condition d (decentralization):

(d) There is no central agency, real (like a Central Planning Bureau) or fictitious (like the traditional auctioneer) which can compute an efficient allocation for program (2): Since we have a vector-valued criterion and require that trades must satisfy agents' budget constraints and the conditions about voluntary exchange, algorithms like those used in models for Socialist Economies (see Heal [26]) will not be informationally decentralized in our context (See Hurwitz [28]).

However, we need to specify exactly the notion of decentralized exchange . What we mean by decentralized trade is, in fact, bilateral trade; we implicitly assume that multilateral trades are infinitely costly and that bilateral trade is costless. This kind of trade is consistent with the maximum amount of decentralization since each trading pair does not need to know anything more about the economy except its own characteristics and the vector of fixed prices.

Of course, it is an unsatisfactory aspect of the model that, within the unit period, agents will not alter their terms of trade. On this point we follow the conventional literature in that prices remain fixed within the period concerned. This assumption makes the analysis easier but it does not mean that it is not possible to have price changes within the model of this chapter. In fact we are going to suggest a rule for price changes in the spirit of the model.

More specifically, we are going to assume that agents can only meet pairwisely and are engaged in voluntary, feasible, and mutually profitable trade. It will be of much convenience if we model this behaviour as if agents in each pair first determine a "direction" (a systematic definition of the direction vector will be given below) of mutually profitable and feasible trade and, if such a direction exists, agree on how much they will adjust their endowments along it. There is no loss of generality in this assumption since there exists a one-to-one correspondence between continuous and discrete versions of gradient processes.

In terms of the usual (2 x 2) diagram, this is depicted as follows:

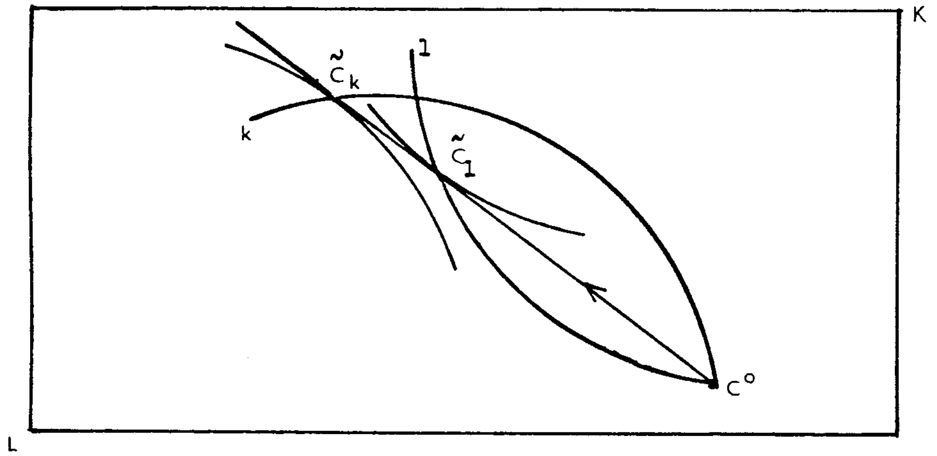

FIG. 1

In figure 1, the initial allocation is C^o. The optimal allocation for agent k is $\tilde{C}_k$ while the corresponding allocation for agent 1 is $\tilde{C}_1$.

The optimal offers do not balance. However, a direction for feasible, voluntary and mutually profitable trade is given by the arrow shown in the figure. Any allocation between C^o and $\tilde{C}_1$ satisfies our conditions. In fact, since we consider a (2 x 2) case, $\tilde{C}_1$ will be the efficient allocation and, trivially, only the long side of the market will be rationed.

We will say that a <u>bilateral trade move</u> takes place when the actual allocation is located between C_o and $\tilde{C}_1$. If agents adjust their endowments along the feasible direction until the efficient allocation $\tilde{C}_1$ is achieved, we say that an <u>optimizing</u> bilateral trade move takes place. This terminology has been borrowed from Feldman (see [19]).

Let us now define the direction vector in a more systematic way.

3. The direction vector: A definition

Let U(.) a real valued continuous fucntion defined on a subset X of the Euclidean N dimensional space E^N and C^o a point belonging to X. We say $d \in E^N$ is a <u>feasible direction</u> at $C^o \in X$ if for some $\delta > 0$ we have $(C^o + sd) \in X$, $\forall s$ with $0 < s \leq \delta$. If, in addition, $U(C^o) < U(C^o + sd)$, $0 < s \leq \delta$, we say that d is <u>usable</u> for U(.) at C^o. It follows (from Taylor's theorem) that if U(.) is continuously differentiable and d is feasible then d is usable at C^o if and only if $\nabla U(C^o)d > 0$, where $\nabla U(C^o)$ is the <u>gradient</u> vector of U(.) at C^o. By assumption $\nabla U(C^o)$ will be a row vector:

$$\nabla U(C^o) \equiv \left(\frac{\partial U(C)}{\partial C_1}, \ \ldots \ , \ \frac{\partial U(C)}{\partial C_n} \right) \Bigg|_{C = C^o} ;$$

d will be a column vector.

If there is no usable direction for U(.) at $C^o \in X$, C^o is a "stationary" allocation. If U(.) is concave and X convex, a stationary allocation will be optimal as well (see D.Topkis and A.Veinott, [47]). In our program (2), $C \in R_+^{HN}$, $d \in R^{HN}$; the constraints are all concave in the state variables C_{hi} (in fact, condition (iii) can be written as a system of two vector- inequalities which are truly linear in C_h according to whether agent h is a buyer or a seller of each of the goods). The maximand is a vector of strictly concave functions. Since an efficient solution can be obtained by maximizing any of the H utilities subject to the constraints of program (2), the fact that our maximand is vector-valued does not create any particular problem since we have assumed that all $U_h(C_h)$ are concave functions.

4. Implementation of the algorithm through bilateral exchange

As we have assumed, exchange will only take place bilaterally. We will also assume that agents in each pair can trade in all goods simultaneously: It is common in the literature — see Ostroy [40] — to postulate the existence of fictitious middlemen (one for each pair with no possibility of communication with those of other pairs; otherwise we would violate the condition of decentralized exchange) who are aware of the agents' offers in various markets and of their characteristics. In fact they know nothing more than the trading partners reveal to them. Trade takes place as if they do not exist.

Voluntary and mutually profitable exchange implies that whenever two agents k,l meet with each other, they order their middleman to solve the following program:

"Maximize, á la Pareto, the vector $\left[U_1(C_1), U_k(C_k)\right]$

$$\text{subject to: } U_h\ (C_h) \geq U_h(C_h^o) \quad h = k,l,\ C_k + C_l \leq C_k^o + C_l^o,$$
$$p(C_k - C_k^o) \leq 0,\ (C_{hi} - C_{hi}^o)(\tilde{C}_{hi} - C_{hi}^o) \geq 0 \quad \forall i,\ h = k,l, \tag{3}$$
$$|C_h - C_h^o| \leq |\tilde{C}_h - C_h^o| \quad h = k,l,\ C_k, C_l \in R_+^N \text{"}$$

The budget of agent l is implicit in (3).

The middleman should also respect the initial values of agent s' utilities in the sense that he is not permitted to produce any allocation, even temporary, which generates utilities which are smaller than the initial ones.

Turning back to Figure 1, the middleman will try to make a bilateral trade move through the following steps:

(a) Given the initial (sub) allocation (C_k^o, C_l^o), he determines a usuable direction $d_{(k,l)} = (d_k, d_l)$

for program (3).

(b) If a usable driection exists, he adjusts endowments along it according to a step of adjustment s so that a new feasible suballocation is obtained: $C^1_{(k,l)} = C^o_{(k,l)} + sd_{(k,l)}$ for $s > 0$. If such a direction does not exist, (C_k^o, C_l^o) is an efficient suballocation. To find a usable direction consider the following procedure (we will explain it immediately after):

Define $\sigma_{k,l} \equiv \text{Min}\ \{\nabla U_k(C_k)d_k,\quad \nabla U_l(C_l)d_l\}$

Then, for any compact subset M of the Euclidean 2N-dimensional space with the origin contained in its interior, solve the program:

$$
\begin{aligned}
&\text{"Max. } \sigma_{k,l} \\
\text{subject to:} \quad & (d_k, d_l) \in M, \\
& \nabla U_k(C_k) d_k \geq \sigma_{k,l}, \quad \nabla U_l(C_l) d_l \geq \sigma_{k,l}, \qquad (4) \\
& -p d_k \geq 0, \quad d_{ki} + d_{li} \leq 0 \quad \forall i, \\
& (\tilde{C}_{hi} - C^o_{hi}) \, d_{hi} \geq 0 \quad \forall i, \ h = k,l."
\end{aligned}
$$

The condition $(d_k, d_l) \in M$ involves no loss of generality. It is posed to guarantee a solution to the problem.

The $\sigma_{k,l}$ which solves program (4) must be non-negative, since any negative solution is no better than $\sigma_{k,l} = 0$. If $\sigma_{k,l} > 0$, the resulting (d_k, d_l) is a <u>usable</u> direction. This follows from (I) and (II) below:

(I) $\nabla U_h(C_h) d_h \geq \sigma_{k,l} > 0$, $h = k,l$

(II) The resulting (d_k, d_l) is feasible: For instance, take the budget constraint $p(C^o_k - C_k) \geq 0$. The direction d_k satisfies $-pd_k \geq 0$ (see (4)).

Hence for $\lambda > 0$, the allocation $C^1_k = C^o_k + \lambda d_k$ satisfies agents' k budget, since

$$p(C'_k - C^o_k) = p(C^o_k + \lambda d_k - C^o_k) = p\lambda d_k \leq 0$$

Using this argument for all constraints, we conclude that (d_k, d_l) is feasible. Any adjustment along it will constitute a bilateral trade move since it is also usable.

The idea behind program (4) is quite natural; the middleman moves along the gradient vectors in a way that he is always in the feasibility set. He certainly respects the initial values of utilities, since $\nabla U_k(C_k)d_k > 0$ if and only if

$$\{U_k(C_k^o + \lambda d_k) - U_k(C_k^o)\} / \lambda > 0, \text{ for } \lambda > 0.$$

(This follows from the definition of the gradient vector and Taylor's theorem). As long as both utilities are increasing and the constraints are satisfied, trade will continue. Obviously, exchange will cease when at least one of the agents in the pair does not wish to engage in any more trade along any of the feasible directions. In fact, the following lemma tells us what happens if $\sigma_{k,1} = 0$. The argument is standard (see Zangwill [50]).

Lemma 1: $\sigma_{k,1} = 0$ if and only if $C_{k,1}$ is an efficient (sub) allocation.

Proof

Since $\sigma_{k.1} \equiv \min.\{\nabla U_k(C_k)d_k, \nabla U_1(C_1)d_1\}$, $\sigma_{k.1} = 0$ if and only if the following system has no solution:

$$\begin{aligned} &\nabla U_k(C_k)\, d_k > 0 \\ &\nabla U_1(C_1)d_1 > 0 \qquad\qquad (5) \\ &-pd_k \geq 0 \\ &-(d_k + d_1) \geq 0, \quad \text{component by component,} \\ &(\tilde{C}_{hi} - C_{hi}^o)d_{hi} \geq 0 \quad h = k,1, \quad \forall\ i. \end{aligned}$$

By Gordan's theorem (see Bazaraa and Shetty [4]), (5) has no solution if and only if a non-negative vector w exists such that the following (vector) equation holds:

$$A'w = 0 \quad , \qquad\qquad (6)$$

where A' is the transpose of A, which is defined as:

$$A = \begin{bmatrix}
\frac{\partial U_K}{\partial C_{K1}}, \ldots, \frac{\partial U_K}{\partial C_{Kn}}, 0, \ldots\ldots, 0 \\[2ex]
0, \ldots, 0, \frac{\partial U_1}{\partial C_{11}}, \ldots, \frac{\partial U_1}{\partial C_{1n}} \\[2ex]
-p_1, \ldots, -p_n, 0, \ldots \ldots, 0 \\
-1, 0 \;.\;. 0, -1, 0, \ldots, 0 \\
0, -1, 0, \ldots 0, 0, -1, 0, \ldots, 0 \\
\vdots \qquad\qquad \vdots \quad \vdots \qquad\qquad \vdots \\
0, 0, \ldots, -1, 0, 0, \ldots, -1 \\
\tilde{C}_{K1} - C^{o}_{K1}, \ldots, \tilde{C}_{Kn} - C^{o}_{Kn}, 0, \ldots \ldots, 0 \\
0, \ldots, 0, \tilde{C}_{11} - C^{o}_{11}, \ldots, \tilde{C}_{1n} - C^{o}_{1n}
\end{bmatrix}$$

It is now easy to ver ify that (6) gives the Kuhn-Tucker conditions for an efficient solution to program (3), where $w > 0$ is the vector of Lagrangian multipliers; see Smale [43] ||.

In program (4) it is implicitly assumed that allocations are strictly positive; otherwise we should have imposed more restrictions on the set of feasible directions, namely $d_{hi} > 0$ for $C^o_{hi} = 0$. This simply says that the middleman never allocates negative quantities to any of the agents. This condition can be related to the literature: It has been said that the existence of a "money" commodity (in the sense that it is always held in strictly positive quantities by all agents) with a positive marginal utility opens up possibilities for trade which otherwise would remain closed (see Feldman [19], Arrow and Hahn [1]). For instance, assume that agent k would like to buy some of good n while agent l would like to sell some of it: $d_{kn} > 0$, $d_{ln} < 0$. Moreover for all goods $i \neq n$, m both agents are either buyers or sellers, i.e. $d_{ki} \cdot d_{li} > 0$; also $C^o_{km} = 0$. Even if $d_{lm} \geq 0$ agent k cannot exchange good m for good n since $d_{km} \geq 0$.

If $C^o_{km} > 0$ and $d_{km} \leq 0$ at C^o_{km}, exchange would take place and would be profitable for both agents.

Let us now return to program (4). Assume that the solution to (4) is positive, i.e. $\sigma_{k,l} > 0$. By virtue of the previous discussion, the middleman will adjust agents' endowments along the usable direction (d_k, d_l) until a new allocation is found. Naturally, the step of adjustment s must be 'acceptable'; An efficient, acceptable step may be found as a solution to the following program: (The procedure is quite natural and needs no explanation).

$$\text{"}\max_{s} \; U_k(C_k^o + sd_k)$$

$$\text{subject to:} \quad s > 0, \tag{7}$$

$$U_l(C_l^o + sd_l) - U_l(C_l^o) \geq 0,$$

$$s|d_h| \leq |\tilde{C}_h - C_h^o|, \; h = k,l \text{"}$$

The other constraints are satisfied for any $s > 0$ because of linearity; see the corresponding constraints in (4). Also, any λ satisfying $0 < \lambda \leq s$ may be used as a feasible step of adjustment.

After such a step has been found, the new suballocation will be $(C_k,C_l)^1 = (C_k,C_l)^o + s(d_k,d_l)$. The move from $(C_k,C_l)^o$ to $(C_k,C_l)^1$ is a bilateral trade move. Moreover if $(C_k,C_l)^1$ is an efficient suballocation (in the sense that there is no usable direction at $(C_k,C_l)^1$ for agents k,l when they are trading among themselves) the middleman has achieved an optimizing move for agents k,l.

Such a process takes place for all pairs (k,l), k = 1,2,H; l = 1,2,..H. We are interested in the following question: If all possible pairs of agents can be formulated, where does the sequence of bilateral trade moves bring the economy? Our intuition suggests that it should bring it to an allocation x which is pairwise efficient, i.e. it is such that no other allocation y can be achieved by bilateral voluntary exchange which is preferred to x by at least a pair of agents and no agent in any pair enjoys less utility at y than at x.

Before we discuss this matter formally, we impose the following condition (R)– see Feldman [19]– which ensures that all agents can meet and form trading pairs so that all mutually advantageous opportunities are being exploited.

(R) Pairs are formulated in an endless, rotating sequence; this means that agent 1 forms a pair with agent 2, then agent 3, ... then agent H; then, agent 2 with agent 3, then agent 4, ... then agent H; ...; agent (H-1) with agent H. Then, the round starts from the beginning and repeats ad infinitum. Obviously we want to impose such a condition: If one pair of agents is forever prevented from trading, convergence to a pairwise optimal allocation becomes problematical. (However, see Madden [35]). In fact (R) is a strong condition and the results will not change even if a much broader definition about the trading pattern is adopted. (All we need is a sequence of rounds of predetermined maximum length).

We can imagine that each bilateral trade move is an iterative step of the algorithm defined by $C^t = C^{t-1} + S^{t-1}d^{t-1}$, where $C^t \in R_+^{HN}$ is the sequence of allocations, d^t is the sequence of usable directions (see program (4)) and S^t is the sequence of adjustment steps (see program (7)). At each iteration t, i.e. in every bilateral trade move, at most 2N new elements of C^t are formulated since there are only two agents involved. The remaining N(H-2) elements of C^t are the corresponding elements of C^{t-1}. At this point we must make clear that this assumption is a consequence of the pattern of exchange we have adopted, i.e. that of a rotating sequence. If a broad definition had been adopted, we could still have assigned each bilateral trade move to an iteration of the algorithm; in that case, we should not interpret this assignment as a requirement that only two agents can meet at a time while the others are standing idle.

We will now show that the following proposition holds.

Proposition 1:

The rotating sequence of bilateral trade moves brings the economy to a pairwise efficient allocation.

Proof: Since we have assigned each bilateral trade move to an iteration of the algorithm $C^t = C^{t-1} + \delta^{t-1} d^{t-1}$, we must show that $\{C^t\}$ converges to a pairwise efficient allocation.

The sequence $\{C^t\}$ is in a compact set since we are in a pure exchange economy. Hence, it must have at least one convergent subsequence $\{C^{t(\tau)}\}$.

There are two cases:

(a) If $\{C^t\}$ is finite, its last element C^L must satisfy $\sigma_{k.l} = 0$, all k,l. Then, C^L is a pairwise efficient allocation (see Lemma 1).

(b) Now assume that $\{C^t\}$ is an infinite sequence. If C is a limit point of the subsequence $\{C^{t(\tau)}\}$, we must show that C is a pairwise efficient allocation. This will be shown by contradiction. Suppose C is not a pairwise efficient allocation. By Lemma 1, there must exist at least one pair of agents, say (k,l), such that

$$\nabla U_k(C_k)d_k > 0$$

$$\nabla U_1(C_1)d_1 > 0 \qquad (8)$$

$$-pd_k \geq 0$$

$$-(d_k + d_1) \geq 0$$

$$(\tilde{C}_{hi} - C_{hi})d_{hi} \geq 0, \quad h = k,1 \ , \quad i = 1, \ldots, N.$$

Without loss of generality assume

$$\nabla U_k(C_k)d_k = \min \{\nabla U_k(C_k)d_k, \ \nabla U_1(C_1)d_1\} = \quad \varepsilon > 0.$$

Now consider (d_k, d_1). This vector belongs, by assumption, to a compact set M. Hence, we know that the sequence $\{d^{t(\tau)}_{k,1}\}$ which corresponds to $\{C^{t(\tau)}_{k,1}\}$, must have at least one convergent subsequence. At this point we will make use of the following result:

<u>Lemma:</u> The set of usable directions $D_{k,1}(.)$ which solve program (4) is an upper-hemi continuous correspondence of (C_k, C_1)".

For the proof of this result see Zangwill [50] . (One should perhaps try to understand it better by assimilating the direction vector to the famili ar notion of demand correspondence in consumer theory). The consequence of this lemma is the following: If $d_{k,1}$ is the limit of $\{d^{t(\tau)}_{k,1}\}$, then $d_{k,1} \in D(C_{k,1})$.

Since $U_k(.)$, $U_1(.)$ are at least twice continuously differentiable functions and $d_{k,1} \in D(C_{k,1})$, there exists $\lambda > 0$ and an iteration number $\bar{T}$ large enough such that for all $T > \bar{T}$:

$$\nabla U_k(C_k^T + sd_k^T)d_k^T \;>\; \frac{\varepsilon}{2} \quad , \tag{9}$$

$$\nabla U_1(C_1^T + sd_1^T)\; d_1^T > \frac{\varepsilon}{2} \quad , s \in [o,\lambda].$$

Also, from the last three inequalities in (8) for $C^T_{k,1}$, $d^T_{k,1}$ and condition $|C_h - C_h^T| \leq |\tilde{C}_h - C_h^T|$, h = k,1, the allocation $C^T_{k,1} + \mu d^T_{k,1}$ is feasible for $\mu \in (o,\delta]$, some $\delta > 0$.

Now we are at a point T where a usable direction of adjustment for agents k,1 exists; hence, a step of adjustment will be determined. From program (7) we know that for at least one agent the step will be optimal. Assume, without loss of generality, that it is optimal for agent k. Then it must be the case that

$$U_k(C_k^{T+1}) \geq U_k(C_k^T + \rho d_K^T),$$

$$\text{where } \rho = \max\,(\lambda,\delta) \tag{10}$$

By the mean value theorem,

$$U_k(C_k^T + \rho d_k^T) = U_k(C_k^T) + \rho\nabla U_k(C_k^T + \theta\rho d_k^T)d_k^T \;,\quad o\leq\theta \leq 1 \tag{11}$$

From (10), (11) and (9) we obtain:

$$U_k(C_k^{T+1}) \geq U_k(C_k^T + \rho d_k^T) = U_k(C_k^T) + \rho\nabla U_k(C_k^T + \theta\rho d_k^T)d_k^T \geq U_k(C_k^T) + \rho\frac{\varepsilon}{2}. \tag{12}$$

In the limit, $C_k^{T+1} = C_k^T = C_k$, and (12) implies $o \geq \frac{\varepsilon}{2}\rho$, which cannot be true since $\frac{\varepsilon}{2}\rho > o$ ||

<u>Remark</u>: The idea of the proof is to show that there cannot exist sequences c^t, d^t which satisfy the conditions

(a) $c^t \to c$

(b) $d^t \to d$

(c) $\nabla U_h(c)d > 0$

(d) $c^t + \lambda d^t$ feasible for all c^t, d^t in the sequence and $\lambda > 0$.

In the first part we have constructed such a pair of sequences and then we have arrived at a contradiction. This idea is well known since Zangwill [50] . See also Bazaraa and Shetty [4] .

We should also mention the role played by strict concavity; we have used the relation ($\sigma_{(k,l)} = 0$ all $(k,l) \Longleftrightarrow$ pairwise efficiency). In the following diagram this does not hold:

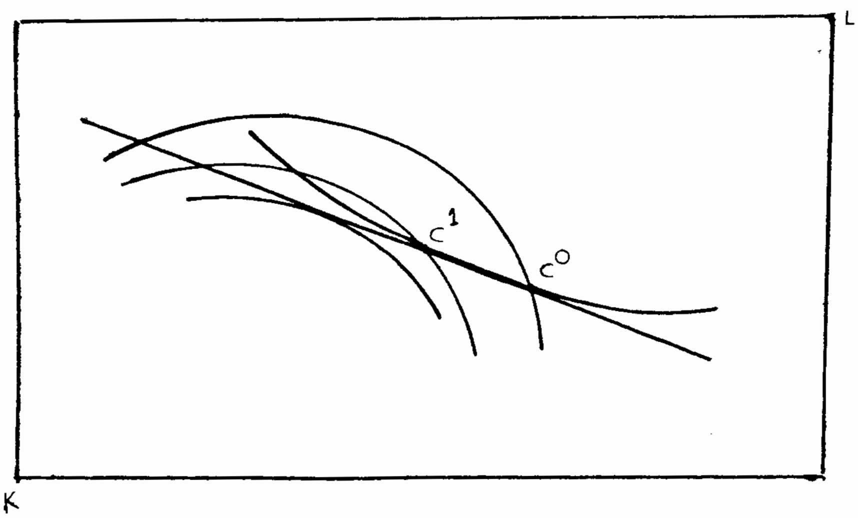

FIG 2

In figure 2, agent's k preferences are not strictly convex. At c^o, $\sigma_{(k,l)} \equiv \min\{\nabla U_k(c_k^o)d_k, \nabla U_l(c_l^o)d_l\} = \nabla U_k(c_k^o)d_k = 0$, but at c^1 agent l is better off than at c^o while agent k is indifferent.

Feldman [19] and Madden [35] have proved results similar to our proposition 1 for simpler economies. In Feldman [19], the only constraint is resource availability while in Madden [35] agents are also contrained by their budgets. In our economy exchange must also be voluntary, in the sense that actual trades must be at most equal and of the same sign to trade offers which, in turn, have been formulated subject to the vectors of fixed prices, perceived constraints and budget balance.

We have not shown yet that the pattern of decentralized exchange we have adopted brings the economy to a Pareto efficient allocation relative to the given fixed prices and perceived constraints; however, intuition suggests that such a conclusion is implied by pairwise efficiency.

Pairwise efficiency implies that Lemma 1 holds for all (k,l), i.e. for no pair is there any incentive for a further move. Formally,

Pairwise efficiency implies $\sigma_{k,l} = 0$ for all (k,l), or – see Lemma 1, page 21 – that none of the following systems has a solution:

$$\nabla U_k(C_k)d_k > 0 \quad , \tag{13}$$

$$\nabla U_l(C_l)d_l > 0 \quad ,$$

$$-pd_k \geq 0 \quad ,$$

$$d_k + d_l \leq 0 \quad ,$$

$$(\tilde{C}_{hi} - C_{hi}) \quad d_{hi} > 0, \quad h = k, l, \forall i, \ \underline{\text{for all } (k,l)}$$

This is equivalent to claiming that equations (6) (see page 22) hold <u>for all (k,l)</u>: (We have again assumed that endowments are strictly positive for all agents)

$$w^j_{1k} \nabla U_k(C_k) + w^j_{1l} \nabla U_l(C_l) - w^j_{2k} \nabla (pC_k) +$$

$$- w^j_3 \nabla (C_k + C_l) + \sum_i w^j_{4k-i} (\tilde{C}_{ki} - C_{ki}) + \tag{14}$$

$$+ \sum_i w^j_{4l-i} (\tilde{C}_{li} - C_{li}) = 0, \qquad \forall k, l.$$

In (14), superscript j denotes the jth pair where, by assumption, it occurs between agents k and l. We can imagine that j runs over all possible permutations among the integers $k = 1, \ldots, H$; (14) holds $\forall$ j. Hence, summing over all j we obtain:

$$\sum_{k=1}^{H} \nabla U_k(C_k) \pi_{1k} - \sum_{k=1}^{H} \nabla (pC_k) \pi_{2k} +$$

$$- \sum_{k=1}^{H} \nabla (C_k) \pi_{3k} +$$

$$+ \sum_{k=1}^{H} \sum_{i=1}^{N} (\tilde{C}_{ki} - C_{ki}) \pi_{4k-i} = 0, \tag{15}$$

where $\sum_j w^j_{\lambda k} \equiv \pi_{\lambda k}, \quad \lambda = 1, 2, 3$

$$\sum_j w^j_{4k-i} \equiv \pi_{4k-i}, \; i = 1, \ldots N.$$

(15) gives the Kuhn-Tucker conditions for an efficient solution to our initial program (2) on page 14 ; π_{ik} are the Lagrange multipliers (see Smale [43]).

As a consequence of proposition 1 and (15) the following result holds:

Proposition 2: The rotating sequence of bilateral trade moves brings the economy to a Pareto efficient allocation relative to the vector of fixed prices and perceived constraints .

C. DISCUSSION

We have taken the view that an alternative to the _tatonnement_ theories on the allocation of resources in a fix-price, pure exchange economy may be provided by a gradient-like process in a completely decentralized environment. The idea is briefly described as follows:

Agents express their optimal trade offers which have been formulated subject to perceived constraints, wealth constraints and fixed prices. We require that the actual exchange process respects these offers in the sense that realized trades are at most equal and of the same sign as them. In addition, actual trade gives rise to utility non-decreasing allocations for all agents. An important aspect is the pattern of exchange. We have assumed that trade takes place bilaterally not only because this is a realistic assumption but also, it is consistent with the maximum amount of decentralization. We think that "disequilibrium" issues are better understood in decentralized environments and in the absence of highly organized markets. It should be noticed that in such environments the "short" side of the market assumption has no place during the exchange process.

Our main result is that a scheme for resource allocation (a "realization" scheme) which is defined as a limit point of such an Edgeworth type bilateral exchange process is efficient relative to the vector of fixed prices and perceived constraints. In other words, it maximizes in a Paretian sense the vector of utilities subject to feasibility of resources, budget constraints, and voluntary exchange.

Perhaps it would be interesting to see how this result can be related to the literature. In similar studies, the authors are trying to prove the existence of "some" equilibrium. Benassy [5] requires that effective demands should reproduce themselves at the equilibrium. Dreze [17], proves the existence of a vector of quantity constraints which, if perceived by agents, produces equality between supply and demand. Green [23], and Heller and Starr [27] are trying to prove the existence of an equilibrium in effective demands where those are chosen such as to maximize the utility of the actual trades given the realization scheme. (In Green's model where the consequences of the agents' offers are random and the assumption that only one side of the market is rationed is not made, the existence of equilibrium is problematic.)

In all these models, agents have no incentive to choose a different action when the system is at equilibrium. In our study, the limit points of our gradient-like exchange process provide no incentive for a further move, given the pattern of exchange. In this case, they can be conceived as equilibrium allocations as well.

Morever, the question of manipulability of the rationing scheme becomes unimportant. The nature of decentralized exchange is such that at the limiting allocation, the relation between an agent's initial offer and his actual trade cannot be predicted. Since no deterministic relation exists between trade offers and actual allocations, there is no reason for distorting preferences in order to achieve a better outcome.

We have not yet explained why subjective perceived constraints $\bar{C},\underline{C}$ remain constant during the exchange process. An (unsatisfactory) answer could be that our exchange process is nothing more than a mathematical device to locate an efficient solution to program (2). Since our purpose is to describe a real exchange process, we need a convincing answer to this question. In fact, this is not difficult to find. If, during the exchange process, an agent h is constrained to trade less than he wishes for good i $(|C_{hi}^{(t)} - C_{hi}^{(t-1)}| < |\tilde{C}_{hi} - C_{hi}^{(t-1)}|)$ he does not revise his perceived constraint $\bar{C}_{hi}$ (i.e. his conjecture on his trade opportunities) on the grounds that in the next trading round he may find a partner who is wishing to satisfy his net offer. On the other hand, if he does satisfy his net offer $(|C_{hi}^{(t)} - C_{hi}^{(t-1)}| = |\tilde{C}_{hi} - C_{hi}^{(t-1)}|)$ he feels that he was rational in his guess about his trading opportunities and he does not revise it either. The question is what happens in the limit, i.e. when $t \to + \infty$. For $t > T$, where T is a large number assume that $|C_{hi}^{(t)} - C_{hi}^{(t-1)}| < |\tilde{C}_{hi} - C_{hi}^{(t-1)}|$. For $t \to \infty$, $C_{hi}^{t} = C_{hi}$ all t and agent h realizes that his trading opportunities have been exhausted; hence, he revises his constraints to make C_{hi} his optimal allocation.

In principle, it would not be impossible to modify our analysis to take into account possible revisions of the perceived constraints $\bar{C},\underline{C}$. For instance, when trade offers are realized we may assume, as Benassy does in [5], that agents revise their beliefs about trading opportunities: $\bar{C}_{hi}^{t}$, $\underline{C}_{hi}^{t}$ may remain constant as far as $|C_{hi}^{t} - C_{hi}^{t-1}| < |\tilde{C}_{hi} - C_{hi}^{t-1}|$ and will have a "kink" at the point where

$|C^t_{hi} - C^{t-1}_{hi}| = |\tilde{C}_{hi} - C^{t-1}_{hi}|$. Of course, Benassy's formulation is different since agents do not take into account constraints in all markets when they express their offers. In this respect our formulation also differs from those whereagents choose their offers in order to maximize the utility of their consequences, given the realization scheme, without any perceived bounds on their choice (see Heller and Starr [27] , Green [23]).

Another question is whether our model can be modified to include price adjustments as well. We postpone this discussion until the next chapter where some suggestions will be made.

Throughout the analysis, we describe a non-monetary exchange process. This does not mean that we ignore the importance of money. We simply do not think that it is a necessary counterpart to disequilibrium. Had we introduced money, it would have been problematical to use Edgeworth type adjustments. Agents may engage in speculative transactions implying a utility loss when they are made but eventually they are anticipated to produce a net gain. Much of our analysis depends on the fact that utilities are monotonically increasing and it would be hard to accommodate such speculative transactions.

Finally, we should add some words on the method of analysis. Since we require that actual trade is voluntary (i.e. at most equal and of the same sign to the trade offer) we have found it particularly appealing to model agents' behaviour as if they first determine a "direction" of mutually profitable exchange and then adjust their endowments along it for some "distance" S in a way that initial offers are not violated and the new allocations are feasible. This is nothing more than a discrete version of continuous gradient processes satisfying the condition of voluntary exchange.

APPENDIX:

A TATONNEMENT ON QUANTITIES

In Dreze [17] , we are told how quantity constraints and prices support the equilibrium allocation. We are not told what happens <u>outside</u> equilibrium, although it has been suggested that his method of proving the existence of equilibrium can also be interpreted as a tatonnement adjustment process on quantities. An exact parallel can be drawn between this case and the classical tatonnement on prices. To prove the existence of a competitive equilibrium, we usually try to prove the existence of a fix point of a mapping from prices to prices which mimicks the auctioneer's rule: Prices increase if there is excess demand and decrease if there is excess supply. In Dreze's model we can imagine an auctioneer who follows a similar rule. To fix ideas, consider the following pure exchange economy:

There are H agents and N goods. Agents will be denoted by the subscript h ($h = 1, \ldots H$) and goods by i ($i = 1, \ldots N$). Prices will form a vector $p \equiv (p_1, \ldots, p_N)$ and will be assumed fixed at an arbitrary level. (It does not make any difference if these are nominal or relative prices). Preferences will be represented by strictly concave utility functions. For simplicity we will also assume that goods are "specialized". This means that for each good i, agent h is <u>either</u> a net seller, <u>or</u> a net buyer. As a consequence, each agent's bounds on net trades can be represented by a single number $\bar{C}_{hi}$ for each good. We can assume that these numbers are positive: If agent h is a net seller for good i we can write $C^o_{hi} - C_{hi} \leq \bar{C}_{hi}$, where C^o_{hi} is the initial endowment and C_{hi} the desirable allocation; an increase in $\bar{C}_{hi}$ will reduce the "frustration" of agent h. The same applies in the case where h is a net buyer for good n, where the constraint on net trade will be denoted by $\bar{C}_{hn} \geq C_{hn} - C^o_{hn}$.

The adjustment on quantities may be described as follows: At each time t the auctioneer sends a signal S_h to each agent h, where $S_h = (p, \bar{C}_{h1}, \ldots \bar{C}_{hN})$. As a result each agent h expresses his net demand vector $Z_h = C_h(p,\bar{C}_h) - C^o_h$, where $C_h(p,\bar{C}_h)$ has been chosen optimally subject to income and upper bound constraints. Then, the auctioneer computes the <u>aggregate</u> excess demand C_{ai} for each good i, $C_{ai} = C_{ai}(p,\bar{C}_1 \ldots \bar{C}_H)$ and adjusts the upper bounds C_{hi} according to the following rule:

$$\dot{\bar{C}}_{hi} = \frac{d\bar{C}_{hi}}{dt} = K_{hi} \quad C_{ai}\,, \tag{A-1}$$

where K_{hi} satisfy the following conditions:

$$K_{hi} > 0, \qquad i \in S_h$$

$$K_{hi} < 0, \qquad i \in B_h$$

$$K_{hi} = 0 \ \text{ if } \bar{C}_{hi} = 0 \ \text{ and } \ C_{ai} > 0 \ (< 0),$$

$$i \in B_h(S_h).$$

$i \in B_h (i \in S_h)$ means that agent h is a net buyer (seller) of good i. Since all goods are specialized, $i \in B_h \Leftrightarrow i \notin S_h$. The adjustment rule (A-1) says that the auctioneer increases the bound on agent's h net trade of good i if h is a seller (buyer) of i and i is in excess demand (supply). Or, he reduces the bound if h is a buyer (seller) of i and i is in excess demand (supply). In order not to violate the condition $\bar{C}_{hi} \geq 0$, $\forall$ i,h, the auctioneer never reduces bounds in case that they are already zero. The requirement $\bar{C}_{hi}(t) \geq 0$ $\forall$i, h creates some problems for the system of differential equations (A-1): The adjustment functions are not always continuous in $\bar{C}_{hi}$. However, we can directly assume that any solution $\bar{C}_{hi}[t \mid \bar{C}_{hi}(t=0)]$ of (A-1) is uniquely determined by $\bar{C}_{hi}(t=0)$ and is continuous in $\bar{C}_{hi}(t=0)$. Also, for all admissible $\bar{C}_{hi}(t=0)$, the solution $C_{hi}[\bar{C}_{hi}(t=0)]$ is bounded. For a discussion on these assumptions, see Arrow and Hahn [1] . They are imposed to ensure a unique path $\bar{C}_{hi}(t)$ given the initial conditions, that all limit points are equilibria and convergence.

An interesting question is whether (A-1) is a stable rule of adjustment. The answer is not easy unless certain conditions are imposed on the excess demand functions $C_{ai}(\ )$. For instance, we could devise conditions to ensure that the Jacobian of $C_a(.)$ exhibits diagonal dominance. Another way out is to examine how far we can go by using the Negishi norm (i.e. the sum of indirect utilities)as a Lyapounov function.§ Agent's h offers are chosen such as to solve the following program:

$$
\text{" Max. } U_h(C_h) \quad \text{over } C_h
$$

$$
\begin{aligned}
\text{subject to:}\quad & p,\ C_h^o \text{ constant},\\
& pC_h \leq pC_h^o,\\
& C_{hi} - C_{hi}^o \leq \bar{C}_{hi}, \quad i \in B_h\\
& C_{hi}^o - C_{hi} \leq \bar{C}_{hi}, \quad i \in S_h \text{ "}
\end{aligned}
\qquad (A-2)
$$

(We have written the constraints in such a way that an increase in $\bar{C}_{hi}$ implies freer choice). (A-2) is a convex program ; the maximum value V_h of the criterion is a concave function of $(\bar{C}_{hi}, \ldots \bar{C}_{hN})$. Hence, from the Kuhn-Tucker argument, we obtain :

$$\Delta V_h \leq \mu_h \Delta \bar{C}_h, \tag{A-3}$$

where ΔV_h is the change in the maximum value of V_h given a change $\Delta \bar{C}_h$ in the upper bounds in the right hand side of the constraints in (A-2); μ_h is the vector of multipliers, the components of which (μ_{hi}) are non-negative and show complementary slackness with the corresponding constraints in (A-2). Dividing both sides of (A-3) by Δt and taking continuous changes we obtain

$$\dot{V}_h \leq \mu_h \dot{\bar{C}}_h \tag{A-4}$$

Taking into account the adjustment rule (A-1), (A-4) becomes:

$$\dot{V}_h \leq \sum_i \mu_{hi} K_{hi} C_{ai}(\ ) \tag{A-5}$$

It would be very convenient if we could show that the sum in the right hand side of (A-5) is non positive for all h. Let us postpone this for the moment and try to check first that V_h is a continuous function. In fact, continuity of V_h follows directly from the well-known results (I), (II) below:

(I) The correspondence $\gamma_h (p, \bar{C}_h)$ defined by all vectors C_h which satisfy the constraints in (A-3) is continuous.

(II) Let U(.) a continuous real valued function which is maximized on a set F(*). If g(*) is the value of the maximum of V(.) on F(*), and F(*) is continuous, then g(*) is continuous at *. For the proof of (I) see J. Dreze [17] . For (II) see G. Debreu (1959), "Theory of Value", page 19, Theorem (4).

In our program (A-3), $V_h(\bar{C}_h)$ is the indirect utility function g(*); it is defined on the continuous correspondence $\gamma_h(P,\bar{C}_h)$ and is, therefore, continuous.

Let us now go back to (A-5). We have said that it would be highly desirable to show that the sum in the right hand side of (A-5) is non-positive. It turns out that a sufficient condition for this is the well known property which holds at the fix -price equilibrium, that is the rule which says that at most one side of the market faces binding constraints.

To show that, let us pretend that this rule holds near equilibrium as well. More specifically we postulate : "Agents in the short side of the market do not face binding constraints in a neighbourhood close enough to the fix-price equilibrium." (S)

If agent h is a buyer of good j which is in excess demand, the term $\mu_{hj}\, C_{aj} K_{hj}$ in the sum $\sum_i \mu_{hi}\, K_{hi}\, C_{ai}$ is non positive since $C_{aj} \geq 0$, $\mu_{hj} \geq 0$ and $K_{hj} \leq 0$ according to (A-1). If agent h is a seller of good j' which is in excess demand, the term $\mu_{hj'} K_{hj'} C_{aj'}$ is zero since the constraint $C^o_{hj'} - C_{hj'} \leq \bar{C}_{hj'}$ is not binding according to our assumption, and as a consequence the shadow price $\mu_{hj'}$ is zero. Similarly if agent h is a buyer of good j" which is in excess supply, the constraint $C_{hj''} - C^o_{hj''} \leq \bar{C}_{hj''}$ is not binding and the shadow price $\mu_{hj''}$ will be zero.

Finally if agent h is a seller of good j* which is in excess supply, the term $\mu_{hj*}C_{aj*}K_{hj*}$ will be non-positive since $\mu_{hj*} > 0$, $C_{aj*} < 0$, $K_{hj*} > 0$ according to (A-1)|| .

Since $\dot{V}_h \leq 0$ and V_h is a continuous function in the state variables $\bar{C}_{hj}$, j=1, ... n, the function

$$W = \sum_{h=1}^{H} V_h$$

is a Lyapounov norm for system (A-1). Hence, (A-1) is a stable adjustment rule. Moreover if we assume local uniqueness of the fix-price equilibrium, this will be a locally stable equilibrium as well.

This result should not be interpreted as anything more than a simple suggestion. Since it is based on the (unjustified) assumption(S) above, it is bound to be a weak result.

Chapter Two

PRICE ADJUSTMENT AND ASSET ACCUMULATION

A. Overview

In this chapter we will discuss models of price adjustment and asset accumulation in a non-Walrasian framework. Our main task will be to study the stability of a specific price and asset adjustment rule.

An ideal situation would be to study price-adjustments in the context of the model presented in Chapter One. We know that at each date t, agent h's desirable direction of adjustment for good i is given by any λ_{hi} such that $\lambda_{hi}(\tilde{C}_{hi} - C^t_{hi}) > 0$. We can imagine that each middleman is not only responsible for making a bilateral trade move for the pair of agents he is assigned to, but he is also charged to report to a central authority these λ_{hi}'s.

A natural assumption is to postulate that $P^{t+1}_i - P^t_i = \sum_h \pi_h \lambda^t_{hi}$ where π_h are constants with $\sum_h \pi_h = 1$. This says no more than that the central authority adjusts prices according to the total "excess demand". When the new price vector p^{t+1} is announced, agents perceive new bounds $\bar{C}^{t+1}, \underline{C}^{t+1}$ on their trading opportunities and express new offers $\tilde{C}^{t+1}$. The exchange process is as in Chapter One.

Smale's results [44] may provide some help. Our model differs from his since we want the actual exchange process to respect trade offers, while in Smale's model agents do not express offers. They only want to move according to the gradient vector ignoring any quantity constraints.

It would be of much interest to examine the convergence properties of the exchange process presented in the previous chapter together with the price adjustment rule suggested above. A result similar to Proposition 1 on page 26 will be all what we want.

Not only technical considerations have prevented us from following this approach in this chapter. The main reason is that our pure exchange economy model is too abstract and we feel that, at this level, there is not much left to say apart from abstract results. Certainly we do not deny their importance; we simply feel that we will have more to say using more specific models with a richer structure.

The topic of price adjustment in non-Walrasian economies is very important not only for its own sake but because it helps us to rethink theories which, although they intend to explain dynamic phenomena such as the accumulation of assets, use arguments which are not sound unless they are applied only at equilibrium. The development of fix-price models has contributed to the understanding of "disequilibrium" issues and to the formulation of some ideas on how prices may change.

The first contributions on price adjustment in economies with trading out of equilibrium were included in the early non-tatonnement processes (see Arrow and Hahn [1] , chapter 13 for a survey of this literature). The main conclusion from these contributions is that models with trading out of equilibrium give stronger stability results than those which permit recontracting.

Our earlier remarks on the possibility of incorporating price changes into the first chapter of this thesis, may provide a suitable framework for including the formation of subjective quantity constraints into these non-tatonnement approaches.

Based on the notion of effective demands, the first effort for an endogenous price setting is that of Benassy [6] . In his model, firms set the prices of the goods they produce. The trading process is as follows: At the beginning of the period, a price vector is set. Then, quantity movements of a tatonnement type occur and a fix price quilibrium is established. Trade takes place, firms observe price and quantity signals, and re-estimate their perceived demand curves; then they change prices accordingly and so on. Equilbrium occurs when no firm wants to change its price.

The most important point in the model is that firms "clear" the markets. This is not an ad hoc assumption but a property which follows from profit maximization. For instance, if a firm is in the short side of the market for the good it produces it can increase the price of the good and still be able to carry out the same production and sales plan increasing its profit.

Throughout the model, there is the assumption that quantities adjust infinitely faster than prices. First, an intraperiod tatonnement adjustment on quantities occurs, and after a fix - price equilibrium has been established, prices change accordingly.

Drazen [16] suggests that since effective excess demands in Benassy's formulation are not reasonable indicators of the size of disequilibrium (as we have already noticed, Benassy's formulation permits agents to send offers which may violate their constraints) they do not present price setting agents with the information necessary to change prices.

We think that although the observation is correct it could be overcome by an idea proposed by Grandmont [22] : Grandmont argues that since the model reveals who the agents are who are constrained in commodity h at each fix-price equilibrium, this information becomes available to other traders. In this case they all know that there is a disequilibrium in some markets and have information on the number of agents who perceive binding constraints. As far as they also know the actual transactions which occur in each market (so that they can tell for sure which of Benassy's trade offers can, in fact, be financed) and provided that some markets are interpreted as future markets, the <u>size</u> of disequilibrium can also become known by virtue of the following arguments: If an agent is rationed in the market for good i in the current period, he will make an order in the future market (by making a downpayment) to obtain the good in some other unspecified date. In this case sellers have an idea of the size of disequilibrium by looking at the size of purchases in the corresponding future markets. In the absence of future markets, other indicators may be used such as the level of stocks of durable goods, unemployment benefits in the case of excess supply of labour etc.

Hahn's conjectural equilibria model [24] does not require that prices move only after the fix-price equilibrium has been established. Agents have conjectures concerning the prices at which they can realize trades. A conjecture C_h of agent h gives the price vector which he believes he must offer in order to trade in excess of his constraints in the direction of the desired excess. The conjecture is an increasing function of the desired excess: If an agent's actual transaction falls short of his desired offer for a good for which he is a buyer, then the greater the difference between those two quantities, the greater the price he believes he must offer in order to realize his desired offer. Agents maximize their utility by choosing price and quantity offers given their conjectures.

Hahn has shown that a conjectural equilibrium, i.e. an equilibrium where all markets clear and agents' conjectured prices are the market prices, exists. More important is that he is able to show that a conjectural economy has non-Walrasian equilibria. The reason behind this, is that conjecture functions are not everywhere differentiable: If they are differentiable the "short" side of the market property implies that conjectures are "flat", i.e. agents believe that they can trade as much as they want <u>without</u> offering a different price. In other words they are perfect competitors. (This line of reasoning is a verbal exposition of a method of proof due to Gale [21]).

The models by Benassy and Hahn mentioned above study the question of the existence of equilibrium. Of equal importance is to examine whether equilibrium is stable.

For, suppose that we specify some dynamic behaviour for the system and we find that prices always converge to the Walrasian equilibrium. Aren't we then inclined to conclude that sluggish wages and prices cause unemployment?

Unfortunately, the state of the art in this area is still unsatisfactory. There are conceptual and technical difficulties on how prices change in disequilibrium which have not been resolved yet. In the following paragraphs we provide a summary of some of the approaches suggested so far.

Laroque [30] has made a simple remark which may be used as a starting point for a game - the oretic approach to the topic. He argues that in a pure exchange economy with two durable goods, the dynamics of price adjustment can be interpreted as a struggle between agents in the opposite sides of the market to set the price. This happens because in a neighbourhood of a stable competitive equilibrium, agents in the short side of the market prefer the Walrasian allocation to the disequilibrium allocation, while, if trade actually takes place, there exists a disequilbrium allocation which is strictly preferred to the Walrasian allocation by all agents on the long side of the market.

In Figure 3, we present a geometric proof of this result using the familiar Edgeworth-box diagram.

Offer curves have the regular shape, so that the Walrasian equilibrium W is stable. The "short" side agent K prefers W than D because W is further up on his offer curve than D, while the "long" side agent L is better off at D than at W (see Fig. 3).

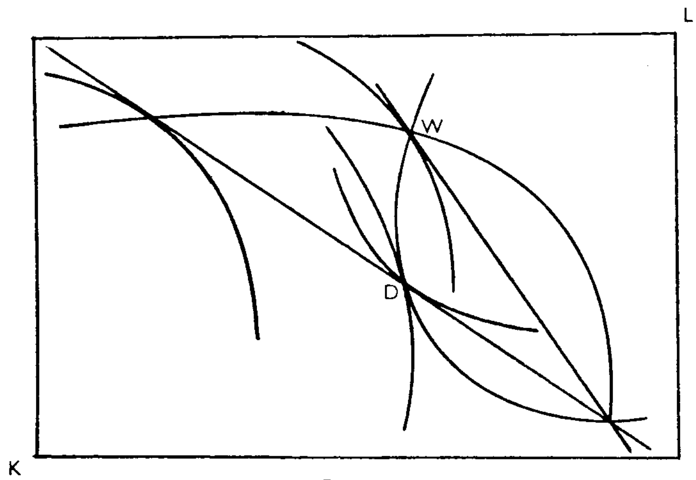

Fig. 3

A more common approach is to assume that the pressure on prices is given as a function of the aggregate effective demands and construct dynamic systems based on this assumption. This is the path followed by Varian [48] . The most important feature of his model is that he introduces an equation to describe the adjustment of expectations which are used as a state variable together with prices. This permits his system to produce not only Walrasian equilibria. The stability results it produces are even more surprising. He argues that the Walrasian equilibrium is unstable while the non-Walrasian one is stable. His argument is as follows. If, at the Walrasian equilibrium, expectations are perturbed downwards, firms will lower both their demand for labour and their supply of output. The price of output will not change because the demand for it will be reduced as a result of a fall in labour income. Also the excess supply in the labour market causes a fall in the real wage and, as a consequence, a further fall in demand. This last effect willmake firms even more pessimistic and they will revise their sales expectations downwards.

At the non-Walrasian equilibrium firms have specific expectations on their sales and we ask what happens if there is an upward shift on them; Varian argues that since the price is greater than the marginal cost at the non-Walrasian equilibrium, the value of consumption supply will increase more than income. This will narrow the gap between expected and actual demand, producing a stabilizing effect to the initial perturbation of sales expectations.

Our opinion is that the specification of this model is a bit unusual. The above argument on what happens at the Walrasian equilibrium is "Keynesian" in nature, while the implication of being at this point is that agents believe that they can transact as much as they want at the current prices i.e. they do not perceive any bounds on their net trades. In Varian's reasoning ("firms will lower their demand for labour after expectations are perturbed downwards") there is a presumption that at the Walrasian equilibrium the demand for labour function depends not only on relative prices but on quantity signals as well.

We should also mention the technique introduced by Varian to overcome the non-differentiability problem which arises because of regime switching. (We will later explain this problem in detail since it is crucial for the model which will be discussed in the next section). He has constructed a <u>differential</u> system which coincides with the real one at all equilibrium points and has used the Poincaré-Hopf index theorem to show that there must exist non-Walrasian equilibria as a consequence of the fact that all Walrasian equilibria have index - 1 (The sum of the indices of all equilibria is zero according to the index theorem).

Eckalbar [18] studies a dynamic system similar to that of Varian i.e. a system where the size of disequilibrium is given by effective excess demands and expectations appear as a state variable along with the real wage. His stability results are based on Lyapounov theory, and he overcomes the non-differentia - bility problem by a direct modification of this theory. Eckalbar is able to show that his system is quasi-globally stable but, as he admits, his method does not seem to work with more than two goods. In the first part of his paper he proves a result which provides a solution to the non-differentiability problem for some special cases. More specifically he shows that dynamic systems which admit the <u>same</u> Lyapounov function for any of the regimes on which they are defined are stable.

Fisher [20] works with a model similar in spirit to Arrow and Hahn ([1], Chapter 14). He introduces spillover effects which are due to quantity constraints and proves the stability of a price and quantity adjustment process. The specification of his model avoids the non-differentiability problem.

In fact, the way effective demands are specified overcomes the problem of regime switching.

Our opinion is that thishappens at the expense of a very strong assumption which is used throughout Fisher's model. More specifically it is assumed that agent h's target excess demands, which are defined as the net trades which maximize his utility subject only to his budget constraint , are of the same sign as the total effective excess demands.

Fisher calls this assumption "a modified Hahn process". As he admits right from the beginning, this is a very strong version of the Hah n assumption which simply asserts that in economies with proper markets willing buyers can always find willing sellers so that everyone's net effective demand is of the same sign as aggregate net effective demand after trade has taken place.

It is not only this assumption that seems very strong to us: Agents in Fisher's model are totally unaware of disequilibrium when forming their target net trades. We suspect that it would not be an easy task to show that the Negishi norm is a Lyapounov function for Fisher's adjustment processes if agents were aware of disequilibrium when forming their optimal offers.

Laroque [31] is working on the stability of a price adjustment process which is also based on the assumption that prices change according to the size of effective excess demands. His purpose is to tackle in a more systematic way the problem of non-differentiability which arises because of regime switching. His main proposition provides a correction to an earlier result obtained by Veendorp [49] in his pioneering effort to construct a stability theory of a price adjustment rule based on the above assumption. In fact, Veendorp's arguments were not fully correct because of regime switching.

Liviatan [34] has constructed a dynamic system for a small open economy under fixed exchange rates where the stock of money is used as a state variable along with the price of the non-traded good . His purpose is to modify the monetary approach to the balance of payments: While this is intended to be a dynamic theory, it is conducted as if prices clear all markets instantaneously.

If prices do not adjust instantaneously to their Walrasian equilibrium values, the monetary approach is misleading since it ignores the spill-over effects from one market to another; if we do take into account spill-over effects the relevant measure of the "pressure" on the state variables is not the notional excess demands but the effective ones.

Liviatan's effort is remarkable in taking into account this effect. Unfortunately, his results are not sufficient to prove the stability of the specie flow-cum-price adjustment mechanism. Laroque's criticism on Veendorp's model mentioned above, applies to Liviatan's model as well. In fact, Liviatan claims that his adjustment process is stable because the partial derivatives of effective excess demands with respect to the state variables are such as to make the matrices (.... of the linearized version of his system) stable in all regimes, i.e. having negative real roots. The problem with thisargument is that it does not take into account those issues which arise at the borders between regimes. More specifically, in dynamic systems where the motion of the state variables is not governed by the same set of differential equations everywhere, trajectories may move back and forth between regimes without ever approaching some steady state point.

The methods of studying the asymptotic stability properties of these systems have yet to be explored and the models by Varian, Eckalbar and Laroque mentioned above are indicative of the modifications needed in order to be able to conduct a proper analysis.

This is exactly what we will also try to do for the rest of this chapter. On this purpose we will first work with a model similar to that of Liviatan [34] . As we have noticed above, this model contains the money stock as a state variable and it will be of much interest if we can obtain a rigorous stability result. Then we will relax the assumption that the economy is small and will try to prove the stability of the specie flow-cum-price mechanism in a world of two countries of comparable size. In this case not only the non-traded good market will clear by quantity rationing; the markets for traded goods will also clear in this way and money will flow according to the value of the actual trade surplus which will differ from the value of the effective excess supply for traded goods.

B. PRICE ADJUSTMENT AND ASSET ACCUMULATION IN OPEN ECONOMIES

(a) The case of a small economy

1. Introduction

It is now widely accepted that there are no formal problems in discussing the balance of payments without incorporating explicitly financial assets. We simply need to distinguish goods available at different dates as different commodities and express prices on a comparable basis. The price vector is substituted for a set of present value price vectors and imports are a set of purchase contracts less sales contracts. Using this approach, our model in chapter one could be extended to capture issues in international trade as well. See Dixit and Norman [15] for a detailed discussion.

This kind of analysis is conducted underthe assumption that there is a set of perfect forward markets. Also, there are no payments in the usual sense; instead, there are claims for specific goods at specific dates.

A theory of the balance of payments which claims to be capable of explaining the real world should rather incorporate explicitly financial assets which can be associated with international payments. For this reason we prefer to work in the context of a Hicksian economy where money will be the only financial asset.

It would not be difficult to incorporate into the model more than one asset. We rather prefer to keep the analysis as simple as possible since our purpose is to examine the foundations of the adjustment mechanism. This does not mean that we underestimate the importance of many assets: With uncertainty, the characteristics of assets play a major role in the description of equilibrium. Moreover, any model which is constructed for policy purposes should incorporate interest rates and capital flows.

In a Hicksian economy, money is held for the sake of commodities it will buy in spot markets at later dates. At each date agents have access to spot markets for commodities and money. In fact, if we assume that agents expect some specific price vector to prevail in the future (i.e. if they have point expectations) money demand can be assimilated to future expenditure on a composite commodity.

According to the monetary approach to the balance of payments, the adjustment mechanism through which international equilibrium is achieved in a world with many national currencies and fixed exchange rates does not differ from the specie-flow mechanism described by D.Hume: There will be an inflow of money in a country at a rate equal to its trade surplus or an outflow at a rate equal to its trade deficit. In this context, a trade deficit is not necessarily a manifestation of real disequilibrium. By virtue of the argument made above on the nature of money demand in the context of a Hicksian economy, a trade deficit can simply reflect international differences in the demand for future versus present consumption and is nothing more than a manifestation of a short-run equilibrium with trade in present and future consumption.

If we now allow for the existence of non-traded goods the prices of which are rigid temporarily so that domestic markets clear by quantity rationing, a trade surplus or deficit may also reflect a spill-over effect from domestic markets.

The purpose of the first part of this section is to study an adjustment mechanism where the state variables are the stock of money and the price of the non-traded good. These variables evolve according to the size of excess effective demands. The stock of money will evolve according to the value of the effective excess supply for traded goods: Since we assume a small open economy the effective excess supply is always realized. Also, the price of the non-traded good will evolve according to the size of the excess effective demand for this good. We will be interested in the asymptotic stability properties of such a mechanism. Let us now describe formally our economy.

2. Assumptions and Description of the notional system

In the economy there are the following goods: A non-traded good denoted by the subscript N with nominal price P. Nominal domestic fiat money the quantity of which is $\bar{M}$. A traded good, denoted by subscript T, with international price fixed at unity. (Since the economy is small, it faces fixed world prices.

Hence, we can aggregate all traded goods in a composite good T the price of which is 1). Given that the exchange rate R will also be assumed fixed, the domestic nominal price of the traded good will be fixed at R.

The two goods will be produced domestically according to a well-behaved transformation curve $F(y_N, y_T) = 0$. The model does not take into account the labour market explicitly, although our analysis can incorporate it under the assumption that production takes place in separate sectors: In the traded good (non-traded good) sector, firms produce the traded good (non-traded good) using labour as input. Under this assumption we could also consider the case of disequilibrium in the labour market. Now, the only source of disequilibrium may come from the non-traded good market.

Of course, as it is usually done, we can reinterpret the model as a reduced form of another model including labour, the market of which being always cleared in the background.

The domestic economy will have a single consumer (or many similar consumers) who demand the two goods and money. As we have explained in the introduction, money demand in this context represents future demand for goods. Formally, preferences will be represented by a well behaved utility function U. This can be considered as a function derived from an <u>intertemporal</u> utility maximization procedure, defined on the traded good, the non-traded good and money. We will not consider the details of this derivation here as it is already a standard procedure (see Grandmont [22]).

The reader may observe that we have totally neglected the intertemporal character of production. This is done by assuming that firms hold no stocks. The analysis would become much more complicated with an intertemporal production plan and we would rather prefer to avoid it.

It will be useful if we first derive the notional excess demand functions for this economy. By definition, the notional demand (supply) functions are derived from utility (profit) maximization subject to the income (production possibilities) constraint only.

According to our assumptions, the notional supply functions will be found as the solution to the following program

$$\text{" } \max_{(Y_N, Y_T)} \quad PY_N + RY_T \tag{16}$$

$$\text{subject to:} \quad F(Y_N, Y_T) = 0 \text{ "},$$

where $\Pi \equiv PY_N + RY_T$ is the national income function. Solving (16) we find the supply functions

$$\begin{aligned} Y_T &= Y_T(P,R) \\ Y_N &= Y_N(P,R) \end{aligned} \tag{17}$$

where Y_T, Y_N are homogeneous of degree zero in P,R.

Also, notional demand functions can be derived as the solution to the program

1.

$$\text{"} \max_{(M, C_N, C_T)} \quad U(C_N, C_T, \tfrac{M}{f}) \tag{18}$$

subject to:

$$PC_N + RC_T + M = PY_N + RY_T + \bar{M} \text{ "}$$

where U(.) is a (derived) well-behaved utility function and f is a function homogeneous of degree 1 in P,R. Solving (18) we find the notional demand functions:

$$C_N = C_N(P,R,\bar{M}) \qquad (19)$$

$$C_T = C_T(P,R,\bar{M})$$

$$M = M(P,R,\bar{M})$$

where C_N, C_T are functions homogenous of degree zero in $P,R,\bar{M}$ and M is homogenous of degree one in $P,R,\bar{M}$.

Combining (19) and (17), we derive the notional excess demand functions:

$$E_N \equiv C_N - Y_N = E_N(P,R,\bar{M}) \qquad (20)$$

$$E_T \equiv C_T - Y_T = E_T(P,R,\bar{M})$$

$$E_M \equiv M - \bar{M} = E_M(P,R,\bar{M})$$

E_N, E_T are functions homogeneous of degree zero in $P,R,\bar{M}$, while E_M is a function homogeneous of degree one in $P,R,\bar{M}$. Since R will be constant throughout the analysis, we can divide all nominal variables by R and obtain the modified excess demand functions:

$$E_N = E_N(p,m) \qquad (20')$$

$$E_T = E_T(p,m)$$

$$E_m \equiv \frac{E_M}{R} = E_m(p,m), \text{ where } p \equiv \frac{P}{R}, \; m \equiv \frac{\bar{M}}{R}.$$

In the <u>steady-state</u> equilibrium, $E_N = E_T = E_m = 0$. In most of the models in the monetarist tradition it is assumed that the non-traded good market is cleared <u>instantaneously</u> through a flexible p: $E_N = 0$. The <u>short-run</u> equilibrium is defined for a given $\bar{M}$ and $E_N = 0$. The adjustment mechanism through which the steady-state equilibrium is achieved is given by $\dot{m} = -E_T$, $E_N(p,m) = 0$. From the overall budget constraint, it can be verified that $- E_T = E_m$; therefore actual money accumulation coincides with the desired one. Essentially, the adjustment mechanism is described by a single differential equation in one state variable (m).

The monetary approach runs into serious problems when the assumption of instantaneous market clearing of the non-traded good market is relaxed. Assuming that the speed of adjustment is unity, the adjustment mechanism is usually described as

$$\dot{p} = E_N(p,m) \tag{21}$$

$$\dot{m} = E_T(p,m)$$

At best, (21) can be described as a tatonnement system under the recontract assumption where the auctioneer not only adjusts p according to the size of the notional excess demand for the non-traded good but he also taxes (subsidizes) money balances when the value of the notional excess supply for the traded good is negative (positive). Exchange will take place only at the steady-state equilibrium ($\dot{p} = \dot{m} = 0$).

Without the recontract assumption the specification of the system is not correct. When trade is permitted out of equilibrium, there may be unsatisfied demand (or supply) in the non-traded good market and this will affect the money and traded good markets.

As we have already discussed in the overview of this Chapter, Liviatan's model [34] is a first effort toward a modification of the monetary approach to take into account these spill-over effects. For the reasons we have already explained, Liviatan's results are not sufficient to prove the stability of the modified adjustment process. We remind the reader that Liviatan has shown that the matrices of the linearized version of the modified adjustment mechanism are stable in all regimes. This is not a sufficient condition for stability, since the problem of regime switching which arises in this context is ignored.

Before we proceed to define the effective excess demand functions and study the stability of the motion of the effective system, we impose the following conditions:

"The matrix $A \equiv \begin{bmatrix} E_{Np} & E_{Nm} \\ -E_{Tp} & -E_{Tm} \end{bmatrix}$ evaluated at the steady-state **(22)**

equilibrium (m^*,p^*) has negative diagonal and positive off diagonal elements such that $[p^* \; 1] \, A < 0$".

Condition (22) guarantees the stability of the tatonnement system (21) around the Walrasian equilibrium (p^*,m^*). Since we are interested in the destabilizing influence (if any) of spill-over effects this assumption seems quite natural. Assumption (22) says that goods are gross substitutes. This, together with Walras's law imply $(p^* \; 1)A < 0$ which, in turn, implies that A has a quasi-dominant diagonal. Laroque [31] and Eckalbar [18] also use an assumption similar to (22).

3. Quantity constraints and effective demands

We will first describe the allocation of commodities at each date t for a more general economy and, as a particular case, we will apply it to our model. This is done for later reference; since we are going to extend our results to more complicated models than the present one this generalization is necessary. See also Laroque [31] .

At each date t, prices and the total quantity of money are fixed. Markets clear by quantity rationing in such a way that no one is forced to exchange more than he wants (voluntary exchange) and allprofitable opportunities for trade are exhausted. We can think of this allocation mechanism as a tatonnement process on quantities or as a non-tatonnment one similar to that described in chapter one of this thesis. We must keep in mind that during the period of quantity adjustments prices remain fixed.

Since we require that all profitable opportunities for trade are exhausted, only one side of each market (either supply or demand) can be rationed at the fix price equilibrium. This property does not necessarily hold during the process of quantity adjustment for the reasons we have already explained in the previous chapter. Hence, at each fix-price equilibrium, we can define the state of the market for good i by $S_i > 0$, $S_i = 0$, $S_i < 0$ according to whether supply is constrained, the market clears or demand is constrained.

The aggregate constrained excess demand may be defined by the usual maximization and aggregation procedure where the net trade of the rationed agent h on market i, is constrained to be equal to the ration $\bar{C}_{hi}$, given the state of the markets.

The effective demand of agent h on market i is defined through the usual maximization procedure under the assumption that the quantity constraint on market i, $\bar{C}_{hi}$ does not operate.

Following Laroque [31], we will assume that at prices (p,m) near the Walrasian prices (p^*, m^*) the fix - price equilibria are locally unique. This implies that the state of the markets $(S_1, \dots S_N)$ and the quantity constraints $\bar{C}_{hi}$ which support the fix - price allocations can be solved as functions of the state variables (p,m): $S_i = S_i(p,m)$, $\bar{C}_{hi} = \bar{C}_{hi}(p,m)$, $\forall$ h, i.

4. Price and money stock dynamics

Let us now come back to our model of a small-open economy. The only market which may clear by quantity rationing is the non-traded good market. Hence, at each date t and for given (p,m) the non-traded good market may be in excess demand $(S_N < 0)$, excess supply $(S_N > 0)$ or it may be cleared $(S_N = 0)$. Also, the effective excess demand for each good can be unambiguously defined according to what we have already said in the previous paragraph.

The evolution of the price p of the non-traded good through time is assumed to be determined by the size of the effective excess demand for the non-traded good $\tilde{E}_N$ (.):

$$\dot{p} = \tilde{E}_N^{S_N(p,m)}(\cdot) \tag{23}$$

The superscript S_N (p,m) denotes the fact that $\tilde{E}_N$ will depend upon the state of the market for the non-traded good. We will be more specific about the effective excess demand $\tilde{E}_N^{S_N}(\cdot)$ in a moment. The evolution of the money stock m will be determined by the value of the trade surplus. Since our economy is small, the effective excess supply for the traded good will always be realized. Hence:

$$\dot{m} = -\tilde{E}_T^{S_N(p,m)}(\cdot) \tag{24}$$

The differential equations (23) and (24) will now determine the evolution of the state variables (p, m). As far as equation (23) is concerned, this is an ad-hoc assumption, in the sense that it has no theoretical justification. (However, see the discussion at the end of the chapter). As we have already seen, there are some alternative assumptions but these are still suggestions rather than anything else.

Moreover, we assume that tastes and technical possibilities are stationary over time, so that the same effective excess demand functions $\tilde{E}_N^{S(.)}(\cdot)$, $\tilde{E}_T^{S(.)}(\cdot)$ will characterise the economy at each date.

We will now specify precisely the effective excess demand functions in each of the possible states of the non-traded good market.

(I) $S_N\ (p,m) < 0$ (Demand is rationed)

We will first find the effective excess demand for the traded good. In this regime, the effective demand for the traded good $\tilde{C}_T(.)$, which is also the realized demand by virtue of the small economy assumption, is given by the usual maximization procedure (see program (18)) subject to the additional constraint that the demand for the non-traded good is rationed at a level $\bar{C}_N$. Formally, the representative consumer solves the program

$$" \underset{(C_T,\ m)}{\text{Max.}} \quad U\left[C_N, C_T, \frac{m}{f}\right] \qquad \text{Subject to:} \quad C_N = \bar{C}_N, \quad pC_N + C_T + m = \bar{m} + pY_N + Y_T \ " \tag{25}$$

The demand for the traded good will be given by

$$\tilde{C}_T = \tilde{C}_T\ (p,\bar{m},\ \bar{C}_N) \tag{26}$$

We know that if we put the ration $\bar{C}_N$ at the level given by the 'notional' demand for the non-traded good $C_N\ (p,\bar{m})$, the effective demand for the traded good will become equal to the notional one.

$$C_T\ (p,m) = \tilde{C}_T\ (p,\bar{m},\ C_N\ (p,m)) \tag{27}$$

Hence, we can approximate $\tilde{C}_T\ (p,\bar{m},\ \bar{C}_n)$ as

$$\tilde{C}_T(p,\bar{m},\bar{C}_N) = C_T(p,m) + \frac{\partial \tilde{C}_T}{\partial \bar{C}_N} \left[\bar{C}_N - C_N(p,m)\right], \qquad (28)$$

2.

We can interpret $-\left[\bar{C}_N - C_N(p,m)\right]$ as the size of "frustration" in the non-traded good market; since only the non-traded good market clears by quantity rationing, the effective demand for the non-traded good in the regime defined by $S_N(p,m) < 0$ is the notional demand for this good; hence we are justified in interpreting $-\left[\bar{C}_N - C_N(p,m)\right]$ as the size of frustration.

In (28) the term $\frac{\partial \tilde{C}_T}{\partial \bar{C}_N} \left[\bar{C}_N - C_N(p,m)\right]$ is the spill-over effect from the non-traded good market. We assume $\frac{\partial \tilde{C}_T}{\partial \bar{C}_N} < 0$. This means that a relaxation (tightening) of the ration which is imposed on the demand for the non-traded good causes a reduction (an increase) in the demand for the traded good. In fact, this assumption, which implies that goods are substitutes in disequilibrium, is a necessary condition for the local uniqueness of the fix-price equilibria (see Laroque [31]).

The "short" side of the market property implies that the realized demand for the non-traded good $\bar{C}_N$ is equal to the notional supply for this good $Y_N(p)$. Hence, rewriting equation (28) we obtain:

$$\tilde{C}_T(p, m, \bar{C}_N) = C_T(p,m) + \frac{\partial \tilde{C}_T}{\partial \bar{C}_N} \left[Y_N(p) - C_N(p,m)\right] \qquad (29)$$

Now, it becomes clear how the unsatisfied demand in the non-traded good market spills over to the demand for the traded good.

The effective supply for the traded good $\tilde{y}_T$ is equal to the notional supply $y_T(p)$ since no constraint operates in the production side of the economy in this specific regime except from technical possibilities. Hence the effective excess supply for the traded good, which is also the realized trade surplus by virtue of the small economy assumption, is given by:

$$-\tilde{E}_T \equiv \tilde{y}_T(p) - \tilde{C}_T(p,m) = y_T(p) - C_T(p,m) - \frac{\partial \tilde{C}_T}{\partial \bar{C}_N}\left[y_N(p) - C_N(p,m)\right] =$$

$$= -E_T(p,m) - \frac{\partial \tilde{C}_T}{\partial \bar{C}_N}\left[y_N(p) - C_N(p,m)\right] \qquad (30)$$

where E_T (p,m) is the notional excess demand for the traded good.

The next step is to find the effective excess demand for the non-traded good. The effective demand for the non-traded good is given by the usual maximization procedure under the assumption that the constraint in the market for this good does not operate. Since no other constraint exists apart from the income constraint the effective demand for the non-traded good $\tilde{C}_N$ will be equal to the notional demand C_N (p,m).

The effective supply for the non-traded good $\tilde{y}_N$ is also given by the notional supply $y_N(p)$ since no constraint operates in the production side of the economy apart from technical possibilities. Thus the effective excess demand for the non-traded good is equal to the notional excess demand.

$$\tilde{E}_N = E_N\,(p,m) \qquad (31)$$

Equations (30) and (31) define the effective excess demand system when demand for the non-traded good is rationed ($S_N(p,m) < 0$). We will now define the effective excess demand system in the regime of rationed supply ($S_N(p,m) > 0$).

II. <u>$S_N(p,m) > 0$ (Supply is rationed).</u>

In this regime, the effective demand for the non-traded good $\tilde{C}_N$ is given by the usual utility maximization procedure given that "profit" income is the maximum value $\bar{\pi}(p,\bar{Y}_N)$ of the program :

$$" \underset{(Y_N,\ Y_T)}{\text{Max.}} \quad pY_N + Y_T$$

$$\text{subject to:} \quad F(Y_N,Y_T) = 0$$

$$Y_N = \bar{Y}_N \quad ". \tag{32}$$

The constraint $Y_N = \bar{Y}_N$ denotes the fact that supply of the non-traded good is rationed. Hence, the effective demand for the non-traded good $\tilde{C}_N$ is given as a solution to:

$$" \underset{(C_N,\ C_T,\ m)}{\text{Max.}} \quad U\ (C_N,\ C_T,\ \frac{m}{f}) \tag{33}$$

$$\text{subject to:} \qquad pC_N + C_T + m = \bar{m} + \bar{\pi}(p,Y_N)" .$$

Since the maximum value of (32) is no greater than the 'notional' $\pi(\cdot)$ (defined as $\pi \equiv \text{Max}\ (pY_N + Y_T)|F(Y_N,Y_T) = 0$) and assuming that all goods are normal we must have:

$$\tilde{C}_N(p,m) \leq C_N(p,m) \tag{34}$$

where $C_N(p,m)$ is the notional demand for the non-traded good defined in (19).

Approximating $\tilde{C}_N$ we obtain:

$$\tilde{C}_N(p,m,\bar{\pi}) = C_N(p,m,\pi(p)) + \frac{\partial \tilde{C}_N}{\partial \pi}(\bar{\pi}-\pi(p)) =$$

$$= C_N(p,m) + \frac{\partial C_N}{\partial \pi}(\bar{\pi}-\pi(p)) \tag{35}$$

where $\frac{\partial \tilde{C}_N}{\partial \pi}$ is the marginal propensity to consume the non-traded good. Formula (35) will be used in the study of the dynamics of the system.

The effective supply of the non-traded good $\tilde{Y}_N$ is given by the usual profit maximization condition, ignoring the ration in the market. Since no other ration operates, $\tilde{Y}_N$ will be equal to the notional supply $Y_N(p)$. Thus, the effective excess demand for the non-traded good is:

$$\tilde{E}_N = \tilde{C}_N - \tilde{Y}_N = \tilde{C}_N - Y_N(p) = C_N(p,m) - Y_N(p) + \frac{\partial \tilde{C}_N}{\partial \pi}(\bar{\pi}-\pi) =$$

$$= E_N(p,m) - \frac{\partial \tilde{C}_N}{\partial \pi}(\pi - \bar{\pi}) \tag{36}$$

We now turn to find the effective excess demand for the traded good.

By an argument similar to that preceding formulae (34), (35), the effective demand for the traded good $\tilde{C}_T$ is no greater than the notional demand $C_T(p,m)$: $\tilde{C}_T \leq C_T(p,m)$. Using the same approximation we obtain

$$\tilde{C}_T \simeq C_T(p,m) + \frac{\partial \tilde{C}_T}{\partial \pi} (\bar{\pi} - \pi) \tag{37}$$

The effective supply for the traded good $\tilde{Y}_T$ is given as a solution to program (32):

$$\tilde{Y}_T = \tilde{Y}_T(p, \bar{Y}_N) \tag{38}$$

Approximating again we have

$$\tilde{Y}_T = Y_T(p) + \frac{\partial \tilde{Y}_T}{\partial Y_N} (\bar{Y}_N - Y_N(p)), \tag{39}$$

where $Y_N(p)$ is the notional supply for the non-traded good. By the "short" side of the market property, the actual supply $\bar{Y}_N$ is equal to the effective demand $\tilde{C}_N$. Also, the derivative $\partial \tilde{Y}_T/\partial Y_N$ taken along the transformation curve is negative. From equations (37), (39) we derive the effective excess supply for the traded good:

$$-\tilde{E}_T \equiv \tilde{Y}_T - \tilde{C}_T \simeq -E_T(p,m) + \frac{\partial \tilde{Y}_T}{\partial Y_N} (\tilde{C}_N - Y_N(p)) - \frac{\partial \tilde{C}_T}{\partial \pi} (\bar{\pi} - \pi), \tag{40}$$

where $-E_T(p,m)$ is the notional excess supply for the traded good. Going one step further, we can write $\bar{\pi} - \pi \simeq \frac{\partial \bar{\pi}}{\partial Y_N} (\bar{Y}_N - Y_N(p)) = \frac{\partial \bar{\pi}}{\partial Y_N} (\tilde{C}_N - Y_N(p))$. So we can rewrite equation (40) as:

$$-\tilde{E}_T = -E_T(p,m) - \left[Y_N(p) - \tilde{C}_N\right]\left[\frac{\partial \tilde{Y}_T}{\partial Y_N} - \frac{\partial \tilde{C}_T}{\partial \pi}\frac{\partial \bar{\pi}}{\partial Y_N}\right] \tag{41}$$

Equations (36) and (41) define the effective excess demand system when supply for the non-traded good is rationed. These, together with (30), (31) specify precisely the functional form of the effective excess demand system under the two possible states of the non-traded good market. (When the non-traded good market clears ($S_N(p,m) = 0$), the flow of money will be given by the notional excess supply for the traded good: $\dot{m} = -E_T(p,m)$).

Thus, we can re-write the dynamic system described by equations (23) and (24) as:

$$\dot{p} = E_N(p,m) - \left\langle \begin{array}{ll} \dfrac{\partial \tilde{C}_N}{\partial \pi}\dfrac{\partial \bar{\pi}}{\partial Y_n}(Y_N(p) - \tilde{C}_N), & S_N(p,m) > 0 \\ 0, & S_N(p,m) \leq 0 \end{array} \right\rangle ,$$

$$\dot{m} = -E_T(p,m) + \left\langle \begin{array}{ll} \left[\dfrac{\partial C_T}{\partial \pi}\dfrac{\partial \pi}{\partial Y_N} - \dfrac{\partial \tilde{Y}_T}{\partial Y_n}\right]\left[Y_N(p) - \tilde{C}_N\right], & S_N(p,m) > 0 \\ 0, & S_N(p,m) = 0 \\ -\dfrac{\partial \tilde{C}_T}{\partial \bar{C}_N}\left[Y_N(p) - C_N(p,m)\right], & S_N(p,m) < 0 \end{array} \right\rangle \tag{42}$$

Before we proceed to study the stability properties of system (42) we will refer to a fundamental result in the stability theory of non-linear control systems. (See S. Lefschetz [32] , W. Hahn [25]).

Appendix I: Stability theory of non-linear direct control systems

Consider the system:

$$\dot{x} = Ax - b\phi(s)$$

$$s = c'x \qquad (43)$$

where A is a (n x n) stable matrix (i.e. a matrix with negative real roots), b, c are constant n vectors and ϕ a piece-wisely continuous scalar function which satisfies the so-called "sectoral" condition

$$o \leq s\,\phi(s) < + \infty \qquad (44)$$

Condition (44) says that the graph of ϕ in the (ϕ-s) plane should never enter the second and fourth quadrants (See Figure 4):

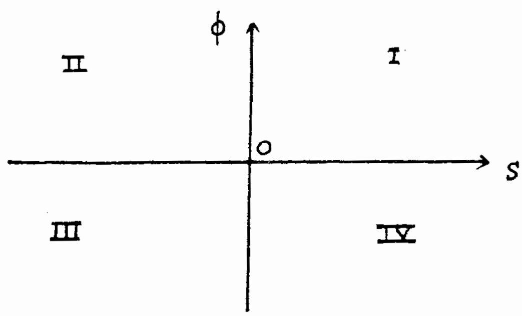

Figure 4

Assume that the origin is the only critical point of system (43). That is, the only solution of

$$Ax - \phi(c'x)\,b = 0 \text{ is, by assumption, } x = 0$$

Now define $\Phi(s) \equiv \int_o^s \phi(s)\,ds$. From (44), $\Phi(s) \geq 0$. We want to find a Lyapounov norm for system (43). On this purpose, consider the function

$$V = x'Bx + \Phi(s), \tag{45}$$

where B is implicitly defined by the (Lyapounov) relation

$$A'B + BA = -C \tag{46}$$

where C is a positive definite matrix and A' denotes the transpose of A. A standard result in Lyapounov theory says that the matrix B defined by (46), is positive definite. Since $\Phi(s) \geq 0$, it must be the case that $V > 0$, $x \neq 0$. It is also clear that V is continuous. We must also show that $\dot{V} = \dot{x}'Bx + xB\dot{x} + \dot{s}\phi(s) < 0$. It turns out that a sufficient condition for this, is to impose

$$c'b = d'C^{-1}d, \text{ where } d = Bb - \tfrac{1}{2} A'c \tag{47}$$

(see S. Lefschetz [32] pages 40, 41).

Before we state the theorem, we should discuss two relevant issues. Some authors assume $\Phi(s) \to +\infty$ as $s \to \pm\infty$ in addition to the "sectoral" condition (44). This condition is imposed to guarantee that every solution $x(t), s(t) \to o$ as $t \to +\infty$. We can ignore this assumption if we can directly assume that $x(t)$, $s(t)$ are bounded for all t. In economic applications we usually assume that prices are bounded without further comment. Also, as a consequence of the boundedness of the economy, excess supply will also be bounded. In any case, divergence of the integral $\Phi(s)$ as $s \to \pm\infty$ is not needed even if we do not directly assume boundedness of $[x(t), s(t)]$ (On this point see S. Lefschetz [32], page 30).

The second issue has to do with the "sectoral" condition (44): Some authors use another sectoral condition instead

$$\text{"}\; s\phi(s) > 0, \quad s \neq 0 \quad \text{and} \quad \phi(s) = 0, \quad s = 0 \;\text{"} \tag{44'}$$

(44') is stronger than (44) since it excludes certain characteristic functions ϕ: Consider the following diagram:

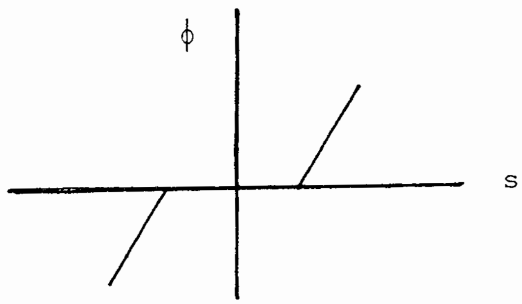

Figure 5

It is clear that the graph of ϕ in Figure 5 does not satisfy (44') although it satisfies (44). In fact, when condition (44') is imposed, it is the kind of discontinuity exhibited in Figure 5 that is excluded. However, characteristic functions ϕ similar to that of Figure 5 do not create any difficulties as far as direct control systems are concerned (i.e. systems similar to (43)). The following theorem (T) is now due (see Lefschetz [32]).

(T): If the origin is the only critical point of system (43), conditions (44) and (47) are sufficient for asymptotic stability of this system.

Theorem (T) applies to systems with a vector valued characteristic function ϕ. Consider the system

$$\dot{x} = Ax + Gu$$

$$u = f(v), \; v = Hx, \tag{48}$$

where A is a (nxn) stable matrix, G and H are constant (nxr), (rxn) matrices, u, v and f(v) are r vectors. Each component f_h of f satisfy (44), i.e. $0 \leq f_h(v)v_h < +\infty$, $\forall h$.

We will again assume that x = 0 is the only solution to $Ax + Gf(Hx) = 0$. Define $\Phi(v) = \int_0^v f(v)dv$, where the integral is taken along the ray L from the origin to the point v. That is, if σ is a parameter along the ray with value σ at v then $\Phi(v) = \int_0^{\sigma} f[v(\sigma)] \frac{dv}{d\sigma} d\sigma$. Since each component of f satisfies (44), $\Phi(v) \geq 0$. Hence, the function $V = x'Bx + \Phi(v)$, with B defined as before, is positive for $x \neq 0$ and continuous. We must also make $\dot{V} = \dot{x}'Bx + xB\dot{x} + \dot{v}f(v) < 0$ if V is to qualify as a Lyapounov norm for system (48). In fact, if we impose a condition similar to (47) above we can guarantee that $-\dot{V} > 0$. On this purpose, define $HA \equiv H_o$, $HG \equiv R_o$, $G'B + \frac{1}{2} H_o \equiv K_o$ and set:

$$\Delta_s^o \equiv \det \begin{pmatrix} C & -K_s^o \\ -K_s^{o\prime} & -R_s \end{pmatrix}, \tag{49}$$

where C is the positive definite matrix which satisfies $A'B + BA = -C$, $K_s^o = (k_{ij}^o)$, $R_s = (R_{ij}^o)$, $i,j \leq s$.

Now, condition (47) should be replaced by the Sylvester conditions which guarantee $-\dot{V} \geq 0$ (see S. Lefschetz [32] pages 57, 58 for the proof):

$$\Delta_1 = 0, \quad \ldots\ldots, \quad \Delta_r = 0 \tag{50}$$

5. Stability of the price and money stock adjustment mechanism

Let us go back to system (42). We will be interested in the stability of the "linearized" version of (42) near the steady-state equilibrium (p^*,m^*). For simplicity, we will assume $(p^*, m^*) = (0, 0)$. Taking into account condition (22) on page 62, system (42) will now be equivalent to

$$\dot{x} = Ax + Gf, \tag{51}$$

where $x = (p,m)$, $G = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $f = (f_1, f_2)$,

$$f_1 = \begin{cases} \dfrac{\partial C_N}{\partial \pi} \dfrac{\partial \pi}{\partial Y_n} \left[Y_n(p) - \tilde{C}_N \right], & S_N(p,m) > 0 \\[2ex] 0 \qquad , & S_N(p,m) \leq 0 \end{cases} ,$$

$$f_2 = \begin{cases} \left[\dfrac{\partial C_T}{\partial \pi} \dfrac{\partial \pi}{\partial Y_n} - \dfrac{\partial \tilde{Y}_T}{\partial Y_n} \right] (Y_N(p) - \tilde{C}_N), & S_N(p,m) > 0 \\[2ex] 0 \qquad , & S_N(p,m) \quad 0 \\[2ex] \dfrac{-\partial \tilde{C}_T}{\partial \bar{C}_N} \left[Y_N(p) - C_N(p,m) \right], & S_N(p,m) < 0 \end{cases}$$

By condition (22), A is a stable (2 x 2) matrix. Each component of f will satisfy the sectoral condition (44) as an immediate consequence of the principle "no one is forced to exchange more than he wants" (P). To see this, notice first that the effective excess supply for the non-traded good is given by $[Y_N(p) - \tilde{C}_N]$ when supply is constrained $(S_N(p,m) > 0)$ and by $[Y_N(p) - C_N(p,m)]$ when $S_N(p,m) < 0$. This has been shown in paragraph 4: see equations (31), (36).

Since we have assumed that effective excess supply gives the sign and measure of "disequilibrium", the principle (P) becomes " $S_N(p,m)$ has the same sign to effective excess supply". Thus, $0 \leq f_1 \quad S_N(p,m) < +\infty$, $0 \leq f_2 \cdot S_N(p,m) < +\infty$, as a consequence of (P) and

$$\frac{\partial C_N}{\partial \pi} \frac{\partial \pi}{\partial Y_N} > 0, \qquad \frac{\partial C_T}{\partial \pi} \frac{\partial \pi}{\partial Y_N} - \frac{\partial Y_T}{\partial Y_N} > 0.$$

It remains to consider the Sylvester conditions (50) for system (51). In fact, the Sylvester conditions are not so awkward as they look. Move specifically, the matrix G canbe chosen suitably (instead of taken as given) as a consequence of the fact that we can multiply f_1, f_2 by any positive numbers without violating the "sectoral" conditions (44). After such a matrix $\tilde{G} = G$ has been chosen (we can see below how we can choose it suitably), system (51) becomes

$$\dot{x} = Ax + \tilde{G}\,\tilde{f}, \tag{52}$$

where $\tilde{f} = (\tilde{f}_1, \tilde{f}_2)$, $\tilde{G} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} g_{11} & 0 \\ 0 & g_{22} \end{pmatrix}$, $\tilde{f}_1 = \dfrac{f_1}{g_{11}}$;

$$\tilde{f}_2 = \frac{f_2}{g_{22}} \quad , \quad g_{11}, g_{22} > 0; \quad f_1, f_2 \text{ are as in (51)}$$

In system (52), $n = r = 2$ and the Sylvester conditions which are imposed to guarantee its stability are

$$\Delta_1 = 0, \quad \Delta_2 = 0 \tag{53}$$

where Δ_1, Δ_2 are defined in (49). Our purpose is to choose (2 x 2) matrices $B = (B_{ij})$, $C = (C_{ij})$, $G = (g_{ij})$ such that: $A'B + BA = -C$, C positive definite, $\Delta_1 = \Delta_2 = 0$, $g_{11} > 0$, $g_{12} = 0 = g_{21}$, $g_{22} > 0$. For instance, such a choice may be implemented as a solution of the following program : For any $\tau \neq 0$,

$$\text{"}\max_{(B_{ij}),\,(C_{ij}),\,(g_{ij})} \quad \tau' B \tau$$

$$\underline{\text{subject to:}} \tag{54}$$

$$C_{11} > 0,$$

$$C_{11}\ C_{22} - C_{21}C_{12} > 0,$$

$$g_{11} > 0, \quad g_{12} = g_{21} = 0, \quad g_{22} > 0,$$

$$\Delta_1 = \Delta_2 = 0,$$

$$A'B + BA = -C\text{"}.$$

The conditions $C_{11} > 0$, $C_{11}C_{22} - C_{21}C_{12} > 0$ guarantee that C is a positive definite matrix. As a consequence of the Lyapounov equation $(A'B + BA = -C)$ and since A is stable, B is positive definite as well. The conditions $\Delta_1 = \Delta_2 = 0$ are the Sylvester conditons; these are equations which involve g_{ij}, B_{ij}, C_{ij} and the elements of matrices A and H (see (49)). In our economy, the matrix H is a singular (2 x 2) matrix as a consequence of the fact that only one market clears by quantity rationing; its first row $(n_{11}\ \ n_{12})$ is the vector of the coefficients of the linear approximation of the parameter S_N, which represents the "state" of the non-traded good market around the steady-state equilibrium $(p^*, m^*) = (0, 0)$. At the end of this chapter (see 'Remark', p. 98) we discuss in detail why it is legitimate to approximate S_N by a linear function of the state variables (p, m).

Assume now that (54) has been solved. As a consequence the matrix $\tilde{G}$ in (52) has been completely determined. The function

$$V = x' Bx + \int_0^{S_N} (\tilde{f}_1 + \tilde{f}_2) d_{S_N} \tag{55}$$

will be a Lyapounov norm for (52): Since B, C, $\tilde{G}$ have been chosen to satisfy the Lyapounov equation $A'B + BA = -C$ and the Sylvester conditions $\Delta_1 = \Delta_2 = 0$ for given A and H, and since $\tilde{f}_1$, $\tilde{f}_2$ satisfy the sectoral condition (44), V is a Lyapounov norm by virtue of the results of the previous section (Appendix I). Hence (52) is stable.

What we have shown is that the price and money stock adjustment mechanism described by the system of differential equations (23) and (24) is stable <u>in a neighbourhood</u> of the steady-state equilibrium. The result is for local stability only, since we have linearized all excess demand functions. However, provided that the "higher order" terms of the Taylor expansions of the excess demand functions satisfy certain regularity conditions, the same method can be used to prove global stability[3] (see Letov [33]).

(b) A world economy with two "large" countries

1. Introduction

In this section we will study the price and money stock adjustment mechanism when the assumption of a small economy is dropped: Although it is very convenient to assume that an economy can exchange as much as it wants at constant world prices, this is not always realistic. The adjustment to an international equilibrium in a world economy consisting of large countries cannot be explained by the model of the previous section since the size of the trade surplus of each individual country does not only determine the accumulation of assets; it may also affect world prices. In this context we should also consider the possibility of quantity rationing in the traded good markets. Models with instantaneous market clearing are not suitable for explaining numerous practical issues. For instance, one may ask why E.E.C. countries find it rational to discuss the possibility of quotas on Japanese cars, why persistent excess supply prevails for plenty of Third World products etc.

Our task will be to set up a model where all prices are endogenous. In the short-run prices will be rigid at an arbitrary level and quantity adjustments will take place. After a short-run equilibrium has been established, prices and the money stocks will evolve according to the size of effective excess demands and the value of trade surpluses respectively. (The world money stock will be constant). We will again be interested in the asymptotic stability of this process. In what follows we will describe the model in detail.

2. Assumptions and description of the notional economy.

Our world economy will contain two countries. These will be denoted by superscripts 1 and 2. Country 1 will produce and consume a non-traded good N^1 and a traded good T. It also has a stock of nominal domestic money $\bar{M}^1$. As it is the case with the model considered in (a), demand for money will represent future expenditure. Similarly, country 2 will produce and consume a non-traded good N^2 and the same traded good T. There will also be domestic nominal money $\bar{M}^2$. The price of $N^1(N^2)$ will be $p_1(p_2)$. Since the exchange rate R will be assumed fixed (R = 1) throughout the analysis, there will be a single price p for the traded good T in both countries: $p_T^1 = p_T^2 \cdot R = p$. All prices will be expressed in the currencies of the corresponding countries.

We will assume that production takes place according to well behaved transformation curves: $F_1(Y_N^1, Y_T^1) = 0$ for country 1, $F_2(Y_N^2, Y_T^2) = 0$ for country 2.

Each country will have a single consumer (or many similar consumers). Their preferences will be represented by well-behaved utility functions U_1, U_2. As we have explained in the previous section (a), these are derived functions from an intertemporal maximization procedure.

Following the same arguments as before, we can derive the notional excess demand functions

$$E_N^i = C_N^i\ (p,\ p_i,\ \bar{M}^i + p_i y_N^i + p y_T^i) - y_N^i\ (p,\ p_i)$$

$$E_T^i = C_T^i\ (p,\ p_i,\ \bar{M}^i + p_i y_N^i + p y_T^i) - y_T^i\ (p, p_i)$$

$$E_M^i = M^i\ (p,\ p_i,\ \bar{M}^i + p_i y_N^i + p y_T^i) - \bar{M}^i$$

$$i = 1,\ 2 \qquad (56)$$

The steady-state equilibrium vector (p^*, $p_1{}^*$, $p_2{}^*$, $\bar{M}^1{}^*$, $\bar{M}^2{}^*$) will be the solution of the following system:

$$E_N{}^i = 0, \quad i = 1,2$$

$$E_T{}^1 + E_T{}^2 = 0 \qquad (57)$$

$$E_M{}^1 = 0$$

$$\bar{M}^1 + \bar{M}^2 = \bar{M}$$

The last equation of (57) is the definition of the world money stock $\bar{M}$ taking into account that the exchange rate R is unity. Because of the budget constraint for each economy, the equation $E_M{}^1 = 0$ in (57) can be written $-pE_T{}^1 = 0$. Also, the overall budget constraint implies $E_M{}^1 + E_M{}^2 = 0$. (We omit the derivation of these results since it is a standard procedure). If markets do not clear instantaneously, we must specify the adjustment mechanism through which the steady state equilibrium will be achieved. We have already discussed the weakness of the monetary approach to the balance of payments in section (a). Now the situation is even worse: We must take into account the fact that the market for the traded good may clear by quantity rationing. As a consequence, it must be the case that for at least one country the desired excess supply for traded goods may not be realized and the accumulation of money will be determined by the value of the realized (not the effective) trade surplus.

3. Quantity constraints and effective demands

At each date t, the description of allocation of resources follows the logic of the fix- price method, i.e. markets clear by quantity rationing. We will not repeat the description in detail since we have already done so in (a), (see Section 3, page 63).

We are again interested in the interperiod adjustment process, where prices evolve according to the size of effective excess demands and the money stock according to the value of the realized trade surplus. Before we specify this dynamic process, we will derive some basic identities which hold at every date t. In what follows, all quantities will correspond to actual trades.

The budget constraints of economies 1 and 2 are:

$$p_1 C_N^1 + pC_T^1 + M^1 = p_1 Y_N^1 + pY_T^1 + \bar{M}^1$$

$$p_2 C_N^2 + pC_T^2 + M^2 = p_2 y_N^2 + pY_T^2 + \bar{M}^2 \qquad (58)$$

Since all markets clear by quantity rationing at each date t, we have

$$C_1^N = Y_1^N, \quad C_2^N = Y_2^N$$

$$C_1^T + C_2^T = Y_1^T + Y_2^T \tag{59}$$

From (58), (59) we obtain:

$$M^1 - \bar{M}^1 = p(y_T^1 - C_T^1), \quad M^2 - \bar{M}^2 = p(y_T^2 - C_T^2),$$

$$M^1 + M^2 - \bar{M}^1 - \bar{M}^2 = 0 \tag{60}$$

4. Price and money stock dynamics

As in section (a), we will assume that the evolution of the non-traded good prices is determined by the size of effective excess demands. Similarly, the price of the traded good T will be determined by the size of the world effective excess demand. The stock of money in each individual country will evolve according to the value of the actual trade surplus. Since (60) holds at each date t, we only need to consider the accumulation of money for one country.

Formally,

$$\begin{aligned}
\dot{p} &= \tilde{E}_T^1\ (.) + \tilde{E}_T^2\ (.) \\
\dot{p}_1 &= \tilde{E}_N^1\ (.) \\
\dot{p}_2 &= \tilde{E}_N^2\ (.) \\
\dot{\bar{M}}^1 &= -pE_T^1\ (.) = \tilde{M}^1(.) - \bar{M}^1 = \tilde{E}_M(.)
\end{aligned} \tag{61}$$

In (61), $\tilde{E}$ denotes excess effective demands and E denotes realized excess demands. We have also normalized all adjustment speeds to unity. This is not such an innocuous assumption in this context (see Laroque [31]); we impose it, following most of the authors in the field, to simplify the analysis.

Now, the specific functional form of effective demands in each of the possible combinations of regimes must be found. In country 1, the possible states of the non-traded good market are $S_N^1 > 0$ (supply is rationed), $S_N^1 < 0$ (demand is rationed), $S_N^1 = 0$ (the market clears). The same notation will apply to country 2. The possible states of the world market for the traded good T will be similarly described by $S_T > 0$, $S_T < 0$, $S_T = 0$.

Before we proceed to the detailed specification of effective demands in each of the possible regimes we ought to clarify the following point: Since agents (i.e. the representative consumers or firms) may now be rationed in more than one markets, the measure of "frustration" of agent h in market i which is produced because of unsatisfied demands (or supplies) will be given by $(\bar{C}_i^h - \tilde{C}_i^h)$, where $\bar{C}_i^h$ is the realized demand (or supply) and $\tilde{C}_i^h$ is the effective demand (or supply). In the previous section (a), only one market could clear by quantity rationing and the measure of frustration was the difference between realized and notional demands.

(1). We will start by specifying $\tilde{E}_N^1$ when demand for N^1 and T are rationed: S_N^1, $S_T < 0$. (It is understood that everything applies to country 2 as well). Since the derivation is the same as before (see section (a)) we will omit the details of the procedure. The effective demand $\tilde{C}_N^1$ for the non-traded good N^1

is given by the usual maximization procedure under the assumption that the ration in the non-traded good market does not operate. We write

$$\tilde{C}_N^{\,1} = \tilde{C}_N^{\,1} \;(p, p_1, \bar{M}^1, \bar{C}_T^1) \tag{62}$$

where $\bar{C}_T^1$ is the ration imposed on the traded good market. We do not need to worry about how the rationed commodity is allocated between countries; our analysis can accommodate any rationing scheme provided that exchange is voluntary.[4] Is it now possible to express $\tilde{C}_N^{\,1}$ as the sum of the <u>notional</u> demand for N^1 plus the spill-over effect from the traded good market? Sometimes econometricians express effective demands as follows: (see Ito [29] subnote 9 for a brief survey of the various specifications):

$$\tilde{C}_N^{\,1} = C_N^{\,1}(.) + \lambda \left[\bar{C}_T^1 - \tilde{C}_T^1 \right] , \tag{63}$$

where $C_N^{\,1}(.)$ is the notional demand and $\lambda \left[\bar{C}_T^1 - \tilde{C}_T^1\right]$ is the spill-over effect from the rationed commodity. Formula (63) is what we are looking for; what remains is to find a justification for it. As we have already noticed, in section (a) we could measure the size of frustration by $\bar{C} - C(.)$, where $C(.)$ was the notional demand, as a consequence of the fact that only one market could be in disequilibrium. In the present case, the corresponding measure is given by the difference between the realized demands $\bar{C}$ and <u>effective</u> demands $\tilde{C}$. From (62), if we expand $\tilde{C}_N^{\,1}$ "around" the effective demand $\tilde{C}_T^{\,1}$ and take the first-order approximation we obtain:

$$\tilde{C}^1_N = \tilde{C}^1_N (p, p_1, \bar{M}^1, \tilde{C}_T{}^1) + \frac{\partial \tilde{C}_N{}^1}{\partial C_T{}^1} (\bar{C}_T{}^1 - \tilde{C}_T{}^1) \qquad (64)$$

The first term in the right hand side of (64) is the effective demand for N^1, when the ration in the traded good market has been put equal to the effective (i.e. the desired) demand for the traded good. In effect, no ration operates since, by construction, the ration in the non-traded good market has already been deleted in the formulation of the effective demand for this good; therefore $\tilde{C}_N{}^1 (p, p_1, \bar{M}^1, \tilde{C}_T{}^1)$ is the notional demand for N^1, $C_N{}^1(.)$ and (64) may now be written as

$$\tilde{C}_N{}^1 = C_N{}^1(.) + \frac{\partial \tilde{C}_N}{\partial \bar{C}_T{}^1} (\bar{C}_T{}^1 - \tilde{C}_T{}^1) \qquad (65)$$

which is similar to (63), with $\lambda = \partial \tilde{C}_N / \partial \bar{C}_T{}^1$.§ The effective supply for N^1, $\tilde{Y}_N{}^1$ is equal to the notional supply $Y_N{}^1(p, p_1)$ since in this regime ($S_N^1 < 0$, $S_T^1 < 0$) no ration operates on production. Hence, the effective excess demand is given by

$$\tilde{E}^1_N = \tilde{C}_N{}^1 - \tilde{y}_N{}^1 = C^1_N(.) - Y^1_N(.) + \frac{\partial \tilde{C}_N}{\partial \bar{C}^1_T} (\bar{C}_T{}^1 - \tilde{C}_T{}^1) =$$

$$= E^1_N(.) - \frac{\partial \tilde{C}_N}{\partial \bar{C}_T{}^1} (\tilde{C}^1_T - \bar{C}_T{}^1). \qquad (66)$$

In (66), $E^1_N(.)$ is the notional excess demand for N^1; it has already been defined in (56). The term $\partial \tilde{C}_N / \partial \bar{C}_T^1$ is negative since, by assumption, a loosening of the ration $\bar{C}_T{}^1$ (i.e. an increase in $\bar{C}_T{}^1$) causes a reduction in the demand for all other commodities. By voluntary exchange, $\tilde{C}_T{}^1 - \bar{C}_T{}^1 \geq 0$. Hence:

$$\frac{\partial \tilde{C}_N}{\partial \bar{C}_T} \; (\tilde{C}^1_T - \bar{C}^1_T) \; \leq \; 0 \; . \tag{67}$$

(2). We will now specify $\tilde{E}^1_N$ when the state of the markets are described by $S^1_N < 0$, $S_T > 0$ (i.e. when the demand for the non-traded good and the supply for the traded good are rationed).

Since supply for the traded good is rationed, the profit income distributed to the consumer of country 1 is less than or equal to the notional profits at the same prices:

$$\bar{\pi}(p, p_1, \bar{Y}^1_T) \; \leq \; \pi(p, p_1) \tag{68}$$

Notice that this will hold as an equality in case that country 1 gets absolute priority in the disposition of sales in the world market. Taking into account (68), the effective demand for N^1 is

$$\tilde{C}_N{}^1 \; = \; \tilde{C}_N \; (p, p_1, \bar{M}^1, \; \bar{\pi}) \tag{69}$$

We can again approximate it as:

$$\tilde{C}_N{}^1 \; = \; C_N \; (p, p_1, \bar{M}^1, \; \pi(p, p_1)) \; + \; \frac{\partial \tilde{C}_N}{\partial \pi} \; (\bar{\pi} - \pi) . \tag{70}$$

The effective supply for the non-traded good $\tilde{y}_N{}^1$ solves the program: "Max. $(py^1_T + p_1 y_N{}^1)$ s.t. $F_1(y_T, y_N{}^1) = 0$, $y_T = \bar{y}_T$". The solution $\tilde{y}_N{}^1 = \tilde{y}_N{}^1 \; (p, p_1, \bar{y}_T{}^1)$ can be approximated as:

$$\tilde{y}_N{}^1 = y_N{}^1 \; (\cdot) \; + \; \frac{\partial \tilde{y}^1_N}{\partial \bar{y}^1_T} \left[\bar{y}^1_T \; - \; y_T{}^1 \; (\cdot) \right] . \tag{71}$$

where $y_N^1(\cdot)$, $y_T^1(\cdot)$ are the notional supply functions.

From (70), (71) we can derive the effective excess demand $\tilde{E}_N^1$:

$$\tilde{E}_N = E_N^1(\cdot) - \left[\frac{\partial \tilde{C}_N}{\partial \pi} \frac{\partial \pi}{\partial \bar{y}_T^1} - \frac{\partial \tilde{y}_N^1}{\partial \bar{y}_T^1}\right] \cdot \left(y_T^1(\cdot) - \bar{y}_T^1\right) \tag{72}$$

In (72) we have used the approximation

$$\bar{\pi} = \pi(\cdot) + \frac{\partial \bar{\pi}}{\partial \bar{y}_T^1}\left[\bar{y}_T^1 - y_T^1(\cdot)\right], \text{where } \pi(\cdot) \text{ is} \tag{73}$$

the notional profit function.

---

(3) Next, we examine the case $S_N^1 > 0$, $S_T > 0$

(supply is rationed in both markets).

The effective demand for N^1, $\tilde{C}_N^1$, is

$$\tilde{C}_N^1(p, p_1, M^1, \bar{\bar{\pi}}) = C_N^1(\cdot) + \frac{\partial \tilde{C}_N^1}{\partial \pi}(\bar{\bar{\pi}} - \pi), \tag{74}$$

where π is notional profits and $C_N^1(\cdot)$ the notional demand for N^1; $\bar{\bar{\pi}} = \bar{\bar{\pi}}(p, p_1, \bar{y}_T^1, \bar{y}_N^1)$

The effective supply for N^1, $\tilde{y}_N^1$, is

$$\tilde{y}_N^1(p, p_1, \bar{y}_T^1) = y_N^1(\cdot) + \frac{\partial \tilde{y}_N^1}{\partial \bar{y}_T^1}(\bar{y}_T^1 - \tilde{y}_T^1). \tag{75}$$

Approximating $\bar{\bar{\pi}} - \pi$ by $\frac{\partial \bar{\bar{\pi}}}{\partial \bar{y}_T^1}(\bar{y}_T^1 - \tilde{y}_T^1) + \frac{\partial \bar{\bar{\pi}}}{\partial \bar{y}_N^1}(\bar{y}_N^1 - \tilde{y}_N^1)$

we have: $\tilde{E}_N^1 = E_N^1(\cdot) + \frac{\partial \tilde{C}_N}{\partial \pi}(\bar{\bar{\pi}} - \pi) - \frac{\partial \tilde{y}_N}{\partial y_T}(\bar{y}_T^1 - \tilde{y}_T^1) =$

$$= E_N^{1}(\cdot) - (\tilde{y}_T^{\,1} - y_T^{\,1}) \left\{ \frac{\partial \pi}{\partial \bar{Y}_1^{\,T}} \quad \frac{\partial \tilde{C}_N^{\,1}}{\partial \pi} - \frac{\partial \tilde{Y}_N^{\,1}}{\partial \bar{y}_T^{\,1}} \right\} +$$

$$- \frac{\partial \tilde{C}_N}{\partial \pi} \quad \frac{\partial \bar{\bar{\pi}}}{\partial \bar{Y}_N^{\,1}} \quad \left(\tilde{y}_N^{\,1} - \bar{y}_N^{\,1} \right). \qquad (76)$$

(4): $S_N^{\,1} > 0, \quad S_T < 0.$

(a): $$\tilde{C}_N^{\,1} = \tilde{C}_N^{\,1}(P, P_1, \bar{M}^1, \bar{\pi}, \bar{C}_T^1) = C_N^1(\cdot) + \frac{\partial \tilde{C}_N^{\,1}}{\partial \pi}(\bar{\pi} - \pi) + \frac{\partial \hat{C}_N^{\,1}}{\partial C_T^{\,1}}(C_T^1 - \tilde{C}_T^1) =$$

$$= C_N^1(\cdot) + \frac{\partial \tilde{C}_N^{\,1}}{\partial \pi} \frac{\partial \pi}{\partial \bar{Y}_N^1} \left\{ \bar{Y}_N^1 - Y_N^1(\cdot) \right\} + \frac{\partial \tilde{C}_N^1}{\partial C_T^1} (C_T^1 - \tilde{C}_T^1) , \qquad (77)$$

(b): $$\tilde{Y}_N^{\,1} = Y_N^{\,1}(\cdot) \quad . \qquad (78)$$

Hence,

$$\tilde{E}_N^{\,1} = E_N^{\,1}(\cdot) - \frac{\partial \tilde{C}_N^1}{\partial \pi} \frac{\partial \pi}{\partial Y_N^{\,1}} \left(\tilde{Y}_N^1 - \bar{Y}_N^1 \right) - \frac{\partial \tilde{C}_N^1}{\partial C_T^1} \left(\tilde{C}_T^1 - C_T^1 \right). \qquad (79)$$

Equations (66), (72), (76), (79) specify precisely the effective excess demand function for N^1, $\tilde{E}_N^{\,1}(\cdot)$, under the possible states of the markets. [We have not considered the case $S_T = 0$ or (and) $S_N^{\,1} = 0$ although we can easily accommodate it as this will become clear in a moment]. By symmetry, $\tilde{E}_N^{\,2}(\cdot)$ is specified in the same way. Hence, equations $\dot{P}_i = \tilde{E}_N^{\,i}(\cdot)$, $i=1,2$ in (61) can be written as

$$\dot{P}_i = \tilde{E}_N^{\,i}(\cdot) = E_N^{\,i}(\cdot) - \left\{ \begin{array}{ll} \left[\dfrac{\partial \tilde{C}_N^{\,i}}{\partial \pi} \dfrac{\partial \bar{\pi}}{\partial y_T} - \dfrac{\partial y_N^{\,i}}{\partial Y_T}\right]\left[\tilde{y}_T^{\,i} - \bar{y}_T^{\,i}\right], & S_T > 0 \\ 0 & , S_T = 0 \\ \dfrac{\partial \tilde{C}_N}{\partial \bar{C}_T} (\tilde{C}_T^{\,i} - \bar{C}_T^{\,i}) & , S_T < 0 \end{array} \right\} +$$

$$- \left\{ \begin{array}{ll} \dfrac{\partial \tilde{C}_N}{\partial \pi} \dfrac{\partial \bar{\pi}}{\bar{y}_N^{\,1}} \left[\tilde{y}_N^{\,1} - \bar{y}_N^{\,1}\right] & , S_N^{\,i} > 0 \\ 0 & , S_N^{\,i} = 0 \\ 0 & , S_N^{\,i} < 0 \end{array} \right\},$$

$$i = 1,2 . \tag{80}$$

We will now specify the money stock adjustment equation for country 1 under the possible states of the markets. From (61), $\dot{M}^1 = p(y_T^{\,1} - C_T^{\,1}) = \tilde{E}_M^{\,1}(\cdot)$, where $p(y_T^{\,1} - C_T^{\,1})$ is the value of the realized trade surplus for country 1.

(1). $S_N^{\,1} < 0, S_T < 0$. Approximating $\tilde{E}_M^{\,1}(\cdot)$"around" $\tilde{C}_N^{\,1}$, $\tilde{C}_T^{\,1}$ we obtain:

$$\tilde{E}_M^{\,1} = \tilde{M}^1 (\bar{M}^1, p, p_1, \bar{C}_N^{\,1}, \bar{C}_T^{\,1}) - \bar{M}^1 = E_M^{\,1}(\cdot) + \frac{\partial \tilde{M}^1}{\partial \bar{C}_N^{\,1}} (\bar{C}_N^{\,1} - \tilde{C}_N^{\,1}) +$$

$$\frac{\partial \tilde{M}^1}{\partial \bar{C}_T^{\,1}} (C_T^{\,1} - \tilde{C}_T^{\,1}) \tag{81}$$

(2). $S_N^{\ 1} < 0, \quad S_T > 0$

$$\tilde{M}^1 = \tilde{M}^1(\bar{M}^1, p, p_1, \bar{C}_N^{\ 1}, \bar{\pi}) \cong M^1(\cdot) + \frac{\partial \tilde{M}^1}{\partial \bar{C}_N^{\ 1}}(\bar{C}_N^{\ 1} - \tilde{C}_N^{\ 1}) + \frac{\partial M^1}{\partial \pi}(\bar{\pi} - \pi). \quad (82)$$

Approximating $\bar{\pi} - \pi$ by $\frac{\partial \bar{\pi}}{\partial y_T^{\ 1}}(\bar{y}_T^{\ 1} - \tilde{y}_T^{\ 1})$ we obtain

$$\tilde{E}_M^{\ 1} = E_M^{\ 1}(\cdot) + \frac{\partial \tilde{M}^1}{\partial C_N^{\ 1}}(\bar{C}_N^{\ 1} - \tilde{C}_N^{\ 1}) + \frac{\partial \tilde{M}^1}{\partial \pi}\frac{\partial \bar{\pi}}{\partial y_T^{\ 1}}(\bar{y}_T^{\ 1} - \tilde{y}_T^{\ 1}) \quad (83)$$

(3). $S_N^{\ 1} > 0, S_T > 0$

$$\tilde{M}^1 = \tilde{M}^1(p, p_1, \bar{\bar{\pi}}, \bar{M}^1) \cong M^1(\cdot) + \frac{\partial \tilde{M}^1}{\partial \pi}(\bar{\bar{\pi}} - \pi) \quad (84)$$

Approximating $\bar{\bar{\pi}} - \pi$ by $\frac{\partial \bar{\bar{\pi}}}{\partial \bar{y}_T^{\ 1}}(\bar{y}_T^{\ 1} - \tilde{y}_T) + \frac{\partial \bar{\bar{\pi}}}{\partial \bar{y}_N}(\bar{y}_N^{\ 1} - \tilde{y}_N^{\ 1})$

we obtain

$$\tilde{E}_M^{\ 1} = E_M^{\ 1}(\cdot) + \frac{\partial \tilde{M}^1}{\partial \pi}\frac{\partial \bar{\bar{\pi}}}{\partial y_T^{\ 1}}(\bar{y}_T^{\ 1} - \tilde{y}_T) + \frac{\partial \hat{M}^1}{\partial \pi}\frac{\partial \bar{\bar{\pi}}}{\partial y_N^{\ 1}}(\bar{y}_N^{\ 1} - \tilde{y}_N^{\ 1}). \quad (85)$$

(4). $S_N^{\ 1} > 0, \quad S_T < 0$

$$\tilde{M}_1 = \tilde{M}_1(p, p_1, \bar{M}_1, \bar{\pi}, \bar{C}_T^{\ 1}) = M_1(\cdot) + \frac{\partial \tilde{M}_1}{\partial C_T^{\ 1}}(\bar{C}_T^{\ 1} - \tilde{C}_T^{\ 1}) + \frac{\partial \tilde{M}_1}{\partial \pi}(\bar{\pi} - \pi), \quad (86)$$

Hence,

$$\tilde{E}_M^{\ 1} = E_M^{\ 1}(\cdot) + \frac{\partial M^1}{\partial\, C_T^{\ 1}} (C_T - \tilde{C}_T) + \frac{\partial M^1}{\partial \pi} \frac{\partial \pi}{\partial y_N^{\ 1}} (y_N^{\ 1} - \tilde{y}_N^{\ 1}) \, . \qquad (87)$$

From (81), (83), (85), (87) we can write the money stock adjustment equation as

$$\dot{M}^{\ 1} = \tilde{E}_M^{\ 1}(\cdot) =$$

$$= E_M(\cdot) - \left\langle \begin{array}{ll} \dfrac{\partial \tilde{M}^1}{\partial \pi} \dfrac{\partial \bar{\pi}}{\partial y_T^{\ 1}} (\tilde{y}_T^{\ 1} - \bar{y}_T) & , S_T > 0 \\ 0 & , S_T = 0 \\ \dfrac{\partial \tilde{M}^1}{\partial \bar{C}_T^{\ 1}} (\tilde{C}_T - \bar{C}_T) & , S_T < 0 \end{array} \right\rangle - \left\langle \begin{array}{ll} \dfrac{\partial \tilde{M}^1}{\partial \pi} \dfrac{\partial \bar{\pi}}{\partial y_N^{\ 1}} (\tilde{y}_N^{\ 1} - \bar{y}_N^{\ 1}) & , S_N^{\ 1} > 0 \\ 0 & , S_N^{\ 1} = 0 \\ \dfrac{\partial \tilde{M}^1}{\partial \bar{C}_N^{\ 1}} (\tilde{C}_N^{\ 1} - \bar{C}_N^{\ 1}) & , S_N^{\ 1} < 0 \end{array} \right\rangle \qquad (88)$$

It remains to specify the equation which gives the rate of adjustment of the traded good price. From (61)

$$\dot{p} = \tilde{E}_T^{\ 1}(\cdot) + \tilde{E}_T^{\ 2}(\cdot) .$$

It is not now necessary to repeat the arguments since it can be easily verified that they will be symmetric to those used to specify $\tilde{E}_N^{\ i}(\cdot)$, $i = 1,2$ under the possible states of the markets. Hence, taking into account (80), we can write

$$\dot{p} = \sum_{i=1}^{2} E_T^{\ i}(\cdot) - \left\langle \begin{array}{ll} \left[\dfrac{\partial \tilde{C}_T^{\ 1}}{\partial \pi^1} \dfrac{1}{\dfrac{\partial \bar{\pi}}{\partial y_N^{\ 1}}} - \dfrac{\partial \tilde{y}_T^{\ 1}}{\partial y_N^{\ 1}} \right] \left[\tilde{y}_N^{\ 1} - y_N^{\ 1} \right] & , S_N^{\ 1} > 0 \\ 0 & , S_N^{\ 1} = 0 \\ \dfrac{\partial \tilde{C}_T^{\ 1}}{\partial C_N^{\ 1}} \left(\tilde{C}_N^{\ 1} - C_N^{\ 1} \right) & , S_N^{\ 1} < 0 \end{array} \right\rangle +$$

$$
-\left\langle \begin{array}{ll}
\sum_{i=1}^{2} \dfrac{\partial \tilde{C}_T^{\,i}}{\partial \pi^i} \dfrac{\partial \bar{\pi}^{\,i}}{\partial y_T^{\,i}} \left[\tilde{y}_T^{\,i} - \bar{y}_T^{\,i} \right], & s_T > 0 \\
0 & , s_T = 0 \\
0 & , s_T \leq 0
\end{array} \right\rangle +
$$

$$
-\left\langle \begin{array}{ll}
\left[\dfrac{\partial \tilde{C}_T^{\,2}}{\partial \pi} \dfrac{\partial \bar{\pi}}{\partial y_N^{\,2}} - \dfrac{\partial \tilde{y}_T^{\,2}}{\partial y_N^{\,2}} \right] \left[\tilde{y}_N^{\,2} - \bar{y}_N^{\,2} \right] & , s_N^{\,2} > 0 \\
0 & , s_N^{\,2} = 0 \\
\dfrac{\partial \tilde{C}_T^{\,2}}{\partial C_N^{\,2}} \left(\tilde{C}_N^{\,2} - \bar{C}_N^{\,2} \right) & , s_N^{\,2} < 0
\end{array} \right\rangle \qquad (89)
$$

The study of the stability properties of the system described by (80), (88), (89) is exactly the same to that of section (a) (see (42)). Hence, the next section will be devoted to a discussion of the issues involved rather than to a detailed analysis.

5. Stability of the price and money stock adjustment mechanism.

We are interested in showing that the system of the differential equations (80), (88), (89) is stable. More specifically, we will show that the linearized version of this system around the steady-state equilibrium is stable. Denoting by x the vector of the state variables $(p_1, p_2, \bar{M}^1, p)$ and normalizing their steady-state equilibrium values by $x^* = 0$, we can write (80), (88), (89) as

$$\dot{x} = A^* x + G^* f^*, \qquad (90)$$

where A^*, G^*, f^* are specified as follows:

A^* is the matrix of partial derivatives of the notional excess demand functions, evaluated at the steady-state equilibrium $x^* = 0$. We will again assume that A^* is a stable matrix: Since we are interested in the destabilizing influence (if any) of spill-over effects, this is a natural assumption. We will have more to say about that at the end of the chapter. However, we have already worked with a similar assumption; see (22).

f^* is the vector of the piece-wise linear functions f_r, $r=1,\ldots,9$ defined as follows:

f_1 (f_3) is the second term in the right hand side of equation (80) for $i = 1$ ($i = 2$);

f_2(f_4) is the third term in the right hand side of (80) for $i = 1$ ($i = 2$);

f_5(f_6) is the second (third) term in the r.h.s. of (88);

f_7 is the third term in the r.h.s. of (89);

f_8(f_9) is the second (fourth) term in the r.h.s. of (80).

It can be easily verified that each f_r, $r=1,\ldots,9$ satisfies the "sectoral" condition (44). In fact, this follows from the following assumptions:

(1) Voluntary exchange: This implies that actual demand (supply) is at most equal to the effective demand (supply), i.e.

$\bar{c}_i \leq \tilde{c}_i(\cdot)$, $\bar{y}_i \leq \tilde{y}_i(\cdot)$

(2) Consumers treat all goods as substitutes in disequilibrium. This implies that a loosening of the ration on good j causes a reduction in the demand of all other goods, including money:

$$\frac{\partial \tilde{c}_i}{\partial \bar{c}_j} < 0, \quad \forall\ i,j.$$

G^* is a (4x9) matrix defined as follows:

$$G^* = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

System (90) is similar to (51); hence we can apply exactly the same arguments as before to show that it is stable. Instead of repeating the same analysis we would rather prefer to illuminate some technical points by examining, step by step, the method used to bring the effective demand system into the form (90).

First we have derived the effective demand and supply functions in each regime. Then we have approximated them by the formula:

"Effective demand" $\simeq$ "notional demand" + "spill-over effects"

In the third step, we linearized the notional demand and supply functions near the steady-state values of the state variables (i.e. prices and the money stock). We have denoted by A^* the matrix of partial derivatives of the notional excess demand functions evaluated at the steady-state equilibrium.

The spill-over effects from the various markets (denoted by f^* in (90)) are of the following type: When demand for good i is rationed, the unsatisfied demand spills-over to the other markets creating an additional pressure on prices.

When supply is rationed the effects are not only felt in the production side; demand is also affected through profit income. Hence, when supply is rationed, the quantities of goods demanded will be less than the corresponding notional quantities <u>for the same values of the state variables</u>.

In fact, the functions f_r, $r=1, .., 9$ satisfy the sectoral condition (44) by virtue of the above arguments and (1)-(2) above. This must already be quite clear from what has been said so far.

Since the matrix A^* has been assumed to be stable and each element of $f^* = (f_1, ...f_9)$ satisfy (44), we can apply the same stability analysis to system (90) as before (see (51)).

REMARK

System (90) is similar to a non-linear direct control system whose characteristic functions are $(f_1,...,f_9)$ and the control parameters are (S_T, S_N^1, S_N^2). More specifically, we treat the numerical values of the excess effective supplies as parameters. In fact, we are only interested in the sign of S_T, S_N^1, S_N^2 as far as the states of the markets are concerned. In this sense we can legitimately approximate the "state" of the markets by linear functions of the state variables x: These functions are such that their numerical values for a given value of x have the same sign to the true numerical value of the excess effective supplies for the same value of x.

The above discussion helps us to understand why in system (51) (section (a)) we have approximated S_N by a linear function $n_{11}\, p + n_{12} m = nx$ of the state variables $x = (p,m)$: See program (54) and the subsequent discussion. Moreover, since only one market can be in disequilibrium in the economy described by (51), (n_{11}, n_{12}) can be taken equal to the partial derivatives of the notional excess supply for the non-traded good taken at the steady-state point. This follows since the numerical value of the notional excess supply for N has always the same sign to the effective excess supply for good N, S_N, for given values of the state variables. This will not be true if more than one markets clears by quantity rationing.

In system (90) we have three parameters (S_N^1, S_N^2, S_T) which we approximate by linear functions; then we can apply the same analysis as in the case with system (51) to show that (90) is stable. (Strictly speaking, we only need to "linearize" $\dot{S}_N$ in (51), and ($\dot{S}_N^1$, $\dot{S}_N^2$, $\dot{S}_T$) in (90). For example, let us take (51): From (55),

$$\dot{V} = \dot{x}'Bx + xB\dot{x} + \dot{S}_N\tilde{f}_1 + \dot{S}_N\tilde{f}_2 \qquad (91)$$

The Sylvester conditions (see (53)) for the negative-definitiveness of $\dot{V}$ are sufficient conditions for the positive-definitiveness of $-\dot{V}$. Their derivation is straightforward after we substitute $\dot{x}$ for $(Ax + \tilde{G}\tilde{f})$, $\dot{S}_N$ for $n\dot{x} = n(Ax + Gf)$, and take into account the Lyapounov relation $A'B + BA = -C$, which defines the matrix B).

Finally, we should add some words on the assumption that A* is stable. As in section (a), this assumption is crucial in the stability analysis; without it, we cannot be sure that V (see (55)) is a Lyapounov norm for system (51). Unfortunately, the conditions which are usually imposed on the notional excess demand functions to make A a stable matrix are <u>ad hoc</u> assumptions which do not follow from primitive concepts. On the other hand, these are just sufficient conditions: For instance, gross substitutability or diagonal dominance are not necessary conditions for stability, although they are sufficient.

C. DISCUSSION

In this chapter we study the dynamics of prices and the money stock in open economies. In particular, we study a non-tatonnement adjustment mechanism according to which the interperiod adjustment of prices is determined by excess effective demands and the money stock evolves according to the value of the realized trade surplus. We have shown that this mechanism is stable not only for "small" economies but for "large" economies as well. Here, a "large" economy is an economy which may affect world prices and face rations in world markets.

The assumptions needed for this result are: (a) the matrix A of the partial derivatives of the notional excess demand functions evaluated at equilibrium is stable. (b) Goods, including money, are substitutes in disequilibrium, in the sense that a loosening of the ration on good j causes a reduction in the demand for all other goods. Moreover, since effective demands give the "pressure" on prices we have to assume that intraperiod fix-price equilibria are locally unique, in the sense that prices determine uniquely the state of the markets and the quantity constraints at each fix price equilibrium. However, local uniqueness of the fix-price equilibria follows from the assumption about substitutability of goods (see Laroque [31]).

The assumption on the stability of the matrix of the partial derivatives of the notional excess demand functions is undoubtedly strong. However, all the short-run propositions[5] of the monetary approach to the balance of payments are based on it, in the sense that none of its comparative statics results will be valid without local stability (see Dixit and Norman [15]).

Those authors who have worked under the assumption that prices evolve according to the size of excess effective demands have also made use of this assumption (see, for instance, Eckalbar [18] , Laroque [31]).

The essense of our stability result is that the highly unrealistic assumption of instantaneous market clearing which is imposed by those working on the monetary approach to the balance of payments is not needed. On the other hand, those who do not impose this assumption but assume that prices evolve according to the size of the <u>notional</u> excess demands , do not describe a real exchange process since they ignore the effects of rationing due to unsatisfied demands. At best, they describe a tatonnement process under the recontract assumption where the auctioneer not only adjusts prices, but he also transfers purchasing power (money) from one country to another according to the value of the notional trade surpluses. No exchange in domestic or foreign goods is permitted unless the system is in its steady-state equilibrium.

N. Liviatan [34] was the first to try and modify the monetary approach along these lines and our present chapter owes much to his model. But, as we have already noticed, his results are not sufficient to prove the stability of the adjustment mechanism. We hope that our method offers a suitable framework for the investigation of the issues involved in models with regime switching and it could be considered in relation to the methods introduced by Eckalbar [18] , Laroque [31] , Veendorp [49] , Varian [48] etc.

An informal discussion of the dynamics of prices and the money stock in an open economy under short-run price rigidities is also contained in Dixit [14] . Dixit's model incorporates a labour market explicitly which clears by quantity rationing. Although our model does not have labour, we can easily introduce a labour market provided that we make the assumption that full-employment prevails at all-times. (See Note 6, p. 105a for a brief discussion about rationing in the labor mark.) In particular, our model in section (b) where large economies of comparable size are involved, could be used to provide an equilibrium framework to Keynesian approaches to the balance of payments. Import and export functions which appear in Keynesian models can be considered as specific rationing rules which allocate traded goods to individual countries at each fix-price equilibrium.

Before we close the discussion we should try to relate the assumption which postulates that the interperiod evolution of prices is determined by the size of excess effective demands to the equilibrium theories of Benassy [6] and Hahn [24] . As we have already noticed (see the overview in the beginning of the present chapter), in Benassy's monopolistic equilibrium model firms find it in their best interests to change prices to clear the markets. More specifically, the pricing rule is based on the difference between the actual and desired offers; now, the "short side" of the market property implies that the actual transactions are the effective offers of those in the "short" side. Hence our price adjustment mechanism can be considered as a surrogate of a process with endogenous price setting. The same applies to Hahn's model . Moreover, Hahn's economy may have competitive equilibria while Benassy's can only have monopolistic equilibria. We have assumed that the adjustment mechanisms studied in this chapter have a unique steady-state equilibrium, which is the Walrasian equilibrium. This is an arbitrary assumption. In particular, one may relate Hahn's 'kinked' conjecture functions to the non-differentiability of effective demands due to regime switching. Kinked conjecture functions are responsible for the existence of equilibria other than Walrasian in Hahn's economy and we may ask whether our adjustment mechanism may admit equilibrium points other than the Walrasian one. We have not gone very far with this question, although we can suggest a way out by speculating on the fact that our adjustment mechanism can be assimilated to a non-linear direct control system (see Appendix I on page 73). As it is known (see Lefschetz [32] , page 41) the conditions for absolute stability(i.e. for a unique stable critical point) of direct control systems may be quite strong. This can be shown by a simple example (see Appendix II, page 104).

Appendix II: Non-linear control systems

Consider the non-linear direct control system

$$\dot{x} = Ax - b\phi(S), \quad S = C'x \tag{92}$$

where ϕ satisfies the sectoral condition $o \leq \phi(S)S < +\infty$, A is a stable matrix and b a constant vector. We assume that the "state" system

$$\dot{x} = Ax, \tag{93}$$

has a unique critical point $x = 0$ and we want to know under which conditions $x = 0$ is a unique critical point for (92) as well.

The critical points of system (92) solve the equations $Ax - b\phi(C'x) = 0$. We can write them as

$$\sum_{i=1}^{n} a_{ki}x_i + b_k \mu = 0, \quad k = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} c_i x_i = S \tag{94}$$

where $\mu \equiv \phi(S)$, (a_{ki}) is a typical element of A, b_k is the k^{th} component of b and c_i is the i^{th} component of C. Treating μ as an independent variable, the solution to (94) is

$$\mu = Ms, \quad x_i = N_i S \tag{95}$$

$$\text{where } M = \frac{\det A}{\det D}, \quad D = \left[\begin{array}{ccc|c} & & & b_1 \\ & & & \vdots \\ & A & & b_n \\ \hline C_1, & \ldots & C_n & o \end{array}\right].$$

The scalars N_i are defined similarly to M. We assume $\det D \neq 0$; since $\mu = \phi(S)$, we must have

$$sM = \phi(S) \tag{96}$$

If the origin (x = 0) is to be the only critical point of (92) the only solution to (96) must be S = 0. As it is clear from the following diagram, this is not always the case.

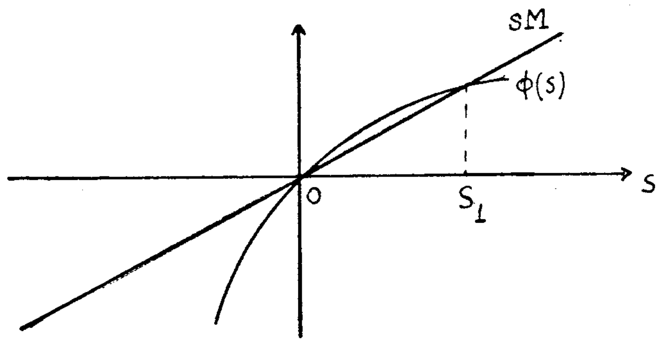

Figure 6.

In the above diagram, we assume $M > 0$; $\phi(S)$ satisfies the sectoral condition $o \leq \phi(S)\, S < +\infty$. The equation $[sM = \phi(S)]$ has two distinct solutions $S = 0$, $S = S_1 > 0$ and the original system will have more than one equilibria. (For a detailed analysis on the uniqueness of equilibrium see Letov [33], Lefschetz [32]).

Notes

(1) The inclusion of real balances in the utility function implies that expectations of future prices are unit-elastic with respect to current prices.

(2) In an article published after the submission of this thesis - see Bibliography - Deaton & Muellbauer have shown that (28) is always justified locally. If it is also assumed that preferences are such as to generate linearity in the rationed demands, (28) is globally valid as well.

(3) See note (2).

(4) Any realization scheme (i.e. any rule allocating the rationed traded good between countries) is consistent with our model provided that the conditions of voluntary exchange and non-manipulability of the scheme are satisfied. For instance, if there is excess demand for the traded good, $\bar{C}_T^{\,1}$ will be some function of the total effective supply $(\tilde{y}_T^{\,1} + \tilde{y}_T^{\,2})$, satisfying the above two conditions.

(5) We talk about the short-run propositions of the monetary approach; the steady-state results require only homogeneity.

(6) We could explicitly incorporate a labour market in the model without altering our results provided that this market always clears by a flexible wage rate. If we allow for quantity rationing in the labour market we should make additional assumptions in order to make sure that the spill-over effects from the labour market satisfy our sectoral condition (44).

CHAPTER III

OPTIMAL GOVERNMENT INTERVENTION UNDER PRICE RIGIDITIES

A. OVERVIEW

In the previous two chapters we have examined economies where the government has not been given any explicit role. The analysis did not have normative implications in the sense that no policy suggestions have been made. Now, we will be more concerned with normative theory.§ Many economists have directed their efforts in using simple equilibrium models for analyzing macroeconomic issues. Those in the Keynesian tradition prefer the fix-price equilibrium framework where some markets clear by quantity rationing than flex-price models where prices move very fast to clear all markets. On the contrary, authors in the monetarist tradition prefer to work with flex-price models; to them, unemployment, if any, is not involuntary but rather supply determined. Others have gone even further in postulating that all movements in unemployment represent shifts in the level of the natural rate of unemployment and advocate that no corrective measure should be undertaken.

We rather share the view of those economists who believe that fix-price models are closer to reality in explaining the short-run effects of government action on macroeconomic variables. To assume that prices move infinitely fast to clear all markets is an extreme idealization of how the economic system works which may produce misleading results when it is used for economic policy purposes.

It is one thing to construct abstract models to demonstrate the efficiency properties of the competitive equilibrium and another thing to use simple flex-price models to derive comparative statics results and then pretend that "this is what is going to happen in practice". To us, the logic of using models where prices clear all markets instantaneously in order to explain what is going on in a real world where friction is rather the rule than the exception does not seem clear at all.

On the other hand, macroeconomists in the Keynesian tradition use the fix-price framework in a rather ad hoc way and they almost never go further than saying that any policy which causes an increase in employment is desirable: They usually analyze the effects of fiscal and monetary policies and although they sometimes suggest a ranking of certain policy actions, the welfare background on which they base their judgements is not made explicit. Also, they do not seem to take into account the full costs of these actions, in the sense that more efficient tools may be available for an increase in employment than those suggested. For instance, one may ask why trade taxes should be used in a small open economy when the financing of government activities, which may include the payment of wage subsidies, can be achieved by consumption taxes or, ideally, by non-distortionary taxation (if this is available).

Closely related to this issue is the specification of the kind of price rigidity. The Keynesian assumption which is also used in the recent equilibrium models developed by Barro and Grossman [3], Malinvaud [36] and others, is that the nominal wage rate is rigid.

The model of the economy that Keynes had in mind was a model where wages (and sometimes the interest rate) were inflexible downwards but prices were moving to clear the markets. Malinvaud [36] and Barro and Grossman [3] assume that both the nominal wage rate and commodity prices are exogenously fixed and all markets clear by quantity rationing.

Numerous explanations have been given for the rigidity of nominal wages and prices. For some time arguments based on informational inadequacies of the market mechanism were very popular but our relative inability to incorporate them rationally in full equilibrium models has cast some doubt on whether they are operationally useful. In the next few paragraphs we briefly comment on some of them since we cannot afford to ignore them completely.§ We will first discuss a model of price rigidities based on the assumption of costly search. According to this, each consumer has imperfect information about prices and other characteristics of commodities supplied by different shops. Moreover he has a "reservation" level concerning the combination "price-quality" below which he does not wish to buy.

Now assume that a shop increases its price. As a result, it is expected that it will lose sales not only because each of the consumers who continue to buy from that shop will buy less, but also because many others find that the increase in price brings the new price-quality combination above their reservation level and decide to search for another store.

Let us see what is expected to happen if that shop reduces its price. Those who buy from the shop will buy more but those who buy from other shops will not know about the price change unless by chance. Because of this asymmetry, a kink in the demand curve of the shop will occur with a corresponding vertical part in the marginal revenue curve. Hence, provided that a reduction in aggregate demand causes a proportional reduction in the demand that each shop faces, it is quite possible that the firm will find it profitable to meet the reduction in demand without lowering its price but by reducing its sales.

Another argument based on screening theory has been proposed as an explanation of wage rigidity. According to this, productivity is a function of the wage rate. This is justified as follows: Individuals have reservation wages; the more productive an individual is, the higher his reservation wage. Hence, the average productivity of a worker is expected to be greater at a higher wage because there is a better mix of applicants. Moreover, when the wage rate is too low, quit rates are expected to be high and given the training costs for new-comers, the effective cost per worker is high.

Since productivity is a function of the wage rate, each firm will wish to minimize the wage paid per productivity unit. The wage rate that solves this minimization program is called the efficiency wage and will be paid to workers independently of the conditions which prevail in the labour market. Thus, it is quite possible that demand for labour falls short of the supply for it at the specific level of the efficiency wage rate and workers who offer to work at a lower wage rate may not find a job because firms infer that they are less productive. This is an example of unemployment equilibrium.

The above arguments provide some explanation of wage and price rigidities ; as we have said before, to incorporate issues like informational inadequacies or screening devices into (dis)equilibrium models with endogenous price setting is a difficult task. On top of this, the analysis of specific macroeconomic issues via equilibrium models will not be possible unless the structure of these models is simple enough to permit definite results.

To assume exogenous fixing of nominal wages and prices may provide an easy way out of the problem, but for models where governments are allowed to vary the level of prices via tax rates, that makes little sense. As far as only "small" variations in the government instruments are allowed (as it is the case with the comparative statics exercises conducted in Barro and Grossman [3] , Malinvaud [36] and others), exogenously rigid nominal wages is not such a bad assumption. It is not the same with models where governments pursue optimal policies choosing freely the instruments they have at their disposal. As we have already said, by varying the tax rates the government may affect the real wage irrespective of the level of the nominal wage. Since only relative prices matter, the government may thus achieve any allocation it wants. A correct specification for models of this kind is to assume real wage rigidities and(or) monopolistic pricing. Real wage rigidities may come from institutional reasons; for instance, wage indexation may be part of a government's election campaign as recent experience has shown. Alternatively, trade unions may exert monopolistic power in the labour market; in this case the wage rate may be functionally related to the price level as a result of rational choice. Also, commodity prices may be determined by producers who exert monopolistic power in the commodity markets. In fact, many econometric estimations show that in the short-run, changes in the prices of manufactured goods are explained satisfactorily by models which assume a constant mark-up above costs. These findings are consistent with rational behaviour: For instance, we could explain them by assuming a monopolist facing a constant elasticity demand curve with elasticity greater than unity (see p. 158a, n. 2)

By specifying the model in this way, we create a truly Hicksian fix-price world; the price-system is self contained and any changes in quantities do not feed back to prices. Although this offers an acceptable explanation of quantity rationing equilibria, it is rather a different model from what economists grown up in the Keynesian tradition had in mind when they were talking about unemployment and other relevant issues. Pessimistic expectations, the liquidity trap, speculation in financial markets and numerous other issues have not been mentioned at all. This omission reflects the difficulty of incorporating them into equilibrium models, and the even greater difficulty in constructing models where the government follows an optimum policy accepting that individuals may have different expectations about the future, that may enter in speculative transactions etc.

A simple example may demonstrate the difficulties encountered in modelling such macroeconomic issues in the framework of the theory of optimum: An old Keynesian argument suggests that the wage rate may be correct and low asset prices due to pessimistic expectations are the cause of unemployment. Assume now that expectations turn out to be false. How shall we specify the welfare criterion function for the government in this case? The answer is not easy as the policy which maximizes some ex ante welfare criterion will be different than the policy which maximizes ex post welfare. In particular, false expectations make the problem even bigger: How shall we judge a policy of interest rates subsidization if those investors who had foreseen bad days were in fact, wrong?

As we have already mentioned, according to the traditional Keynesian macroeconomists any policy which causes an increase in employment is desirable. In fact, it can be argued that their welfare criterion can be represented by a lexicographic social preference ordering where first priority is given to certain targets, and employment is one of them. The objection is that such an ordering does not have a choice-theoretic basis; it may also lead to wasteful use of resources. Incidentally, we notice that not only Keynesians base their policies on some form of target achievement; monetarists also pursue target policies. The recent debate on economic policy where much of the discussion has to do with the ability of the authorities to keep within the announced target of money supply growth makes that perfectly clear.

Due to the difficulties mentioned above about modelling Keynes's elaborate theory in a utilitarian framework where the government follows optimal policies, we will stick to simple fix-price models where the real wage is fixed and the commodity prices will be either exogenous (as in the case of a small-open economy facing constant world prices), or, in the case of non-traded goods, will be determined by a domestic monopolist. Although it would be possible to incorporate some financial assets (see Barro and Grossman [3]) we prefer not to do so since a satisfactory approach with many assets should be based explicitly on uncertainty and this would complicate matters considerably.

Kevin Roberts [42] has constructed a model where fiscal policies are examined in the light of a utilitarian framework. The economy is one with a wage rate higher than the competitive wage and completely flexible commodity prices. He considers only one regime, that is the regime where excess supply prevails in the labour market and the commodity markets clear . Within this regime, he considers various fiscal activities and determines shadow prices for public projects.

In the next sections we will work with a rather different model. We will assume a small-open economy where its <u>domestic</u> markets clear by quantity rationing due to price rigidity. The government's policy instruments will consist of commodity and profit taxes which can be chosen under no restriction. In this economy, revenue is needed for the correction of those distortions which lie behind price rigidities and the financing of certain government activities. For simplicity, we will assume that there is a fixed requirement for public expenditure consisting of the provision of a vector g of traded goods. Alternatively, we could consider a public sector choosing its project s optimally subject to a production possibilities curve.

The simplifying assumption about the provision of a vector of traded goods has been borrowed from Bliss [7] ; in his model, the revenue for financing public expenditure could only come through trade taxes. In the case where only tariffs are available to the government, we should also consider their role as an unemployment reducing instrument in an economy where the labour market clears by quantity rationing due to real wage rigidities.

In what follows the government will choose its control instruments in order to maximize some social welfare function subject to the constraints of the system. The framework will be essentially the same as that of Diamond and Mirrlees [13] Dasgupta and Stiglitz [12] and Bliss [7] .

B. OPTIMAL TAXES TO RAISE REVENUE IN A SMALL OPEN ECONOMY UNDER A MINIMUM WAGE FLOOR

1. Introduction

The aim of this chapter is to bring together the theory of distortions and welfare, the theory of equilibrium under quantity constraints and the elaborate theory of optimum taxation as it has evolved after the work of Diamond and Mirrlees [13]

The theory of distortions and welfare (see, among others, Corden [8]) examines the problems which arise due to various deviations from competitive behaviour and suggests the best way to deal with them. It is shown that the first-best policy is to use Pigouvian taxes or subsidies to counteract the distortions directly (at "their source"), while the revenue for these subsidies should come from non-distortionary taxation. If such non-distortionary taxes are not available, the (second-best) theory of welfare deals with the question of determining the "best" set of distortions to have, given that some of the first-best optimality conditions do not hold. In fact, the theory of optimum taxation, developed under the assumption that the government cannot impose lump-sum taxes to finance its expenditure, gives a solution to this problem.

In the models we are going to discuss, the distortions will have to do with non-competitive pricing. In particular we will assume that there is an institutional constraint in the labour market which takes the form of a minimum wage floor. Also, when we come to consider the case of an open economy producing both traded and non-traded goods, we will assume that the non traded commodity is produced by a profit maximizing monopolist.

As far as the government is concerned, it will be simply assumed that there is a need for the provision of a public good which is defined as a fixed vector g of traded goods. We could also consider the case where the government chooses its projects optimally subject to a production function. As we have already mentioned, we will stick to the above simplifying assumption since the more general case does not involve any novelty as far as the issues we want to stress are concerned (However, see K. Roberts [42]).

The government will not be able to impose lump-sum taxes not for any other reason but because the analysis would have been almost trivial otherwise; however, 100 per cent profit taxes will be allowed. It can use commodity taxes (subsidies) in such a way as to maximize some social welfare function subject to the economy's feasibility and institutional constraints.

Throughout the analysis we assume that the economy is small. This assumption allows us to separate the welfare effects of concern here from the terms of trade effects.

The rest of the chapter will be organised as follows:

We will first examine the desirability of production efficiency under excess labour supply. As already known, all production sectors of the economy should face the same relative prices when taxes are chosen optimally and all profits are taxed away.
This result has been demonstrated under the assumption of perfect market clearing (see Diamond and Mirrlees [13] , Dasgupta and Stiglitz [11]) and it would be interesting to examine whether it

continues to hold under unemployment. In particular, is the argument which suggests that tariffs should be used to reduce unemployment correct? If, as it is expected, the argument is wrong on the basis of efficiency considerations, one may ask what are the best subsidies to give to producers to make them employ more labour; should they be uniform or otherwise?

Next, we try to determine the structure of optimum taxes. This question is particularly important in that case where profit taxes, even if they are set at 100 per cent, are not sufficient to finance public expenditure and the correction of distortions. Within this context welfare considerations are not always possible; for instance, if the real wage is set "too high" or (and) the level of public expenditure is similarly high, the use of Lagrangian multipliers may not be valid as known from Kuhn-Tucker theory. Incidentally this observation can be related to a practical issue, that is the generation of inflationary pressures in economies where there is wage indexation. Although our model will not contain money or other financial assets and will be essentially static, we can identify the case where the constraint qualification is not satisfied with the practical cases where governments cannot finance all public expenditure through taxation (or by other means which immediately reduce the purchasing power of other sectors of the economy) and print money instead.

In the third section we will study a model where the government can only levy trade taxes on commodities but no consumption or production taxes and subsidies. In this case, the only instrument to reduce unemployment will be trade taxes (tariffs) and the solution of the government's optimization program will determine how much this should be reduced, taking into account all other effects and constraints.

Finally, we will consider the case where the domestic economy contains a non-traded good, with its price being fixed by a profit maximizing monopolist, along with traded goods and labour. In particular, if the only input to production of the non-traded good is labour and the elasticity of the demand curve for the non-traded good is constant and greater than unity, the monopolist will fix the price of the non-traded good at a constant mark-up over labour costs.

In most of what follows it will be assumed that the home economy contains a single consumer; mainly, but not exclusively, we consider issues which arise because of efficiency considerations only. This is clearly an unsatisfactory assumption especially when it is related to the real wage constraint. In a single consumer economy the competitive allocation is the optimal allocation and one may ask whether a minimum wage floor can be set on a rational basis. We have side-stepped this question by assuming an institutionally set minimum wage floor, although the idea of institutional constraints does not seem very convincing either particularly in a utilitarian context with rational agents. For instance, suppose that the government had included wage indexation in its election campaign promises. If the electorate consisted of similar consumers-workers who were rational and aware of the properties of the competitive equilibrium, this government would not be in office today. However, with different consumers a rationally set minimum wage floor can be justified. In this framework we must be careful in defining the source of difference among citizens. The popular simplifying assumption which distinguishes consumers according to whether they are fully employed or totally unemployed cannot be made here, since it implies inefficient rationing of the available amount of employment. A more suitable distinction could be to consider two classes of citizens, one of them earning income from selling labour alone, the other from the ownership of a fixed factor of production, say land.

2. Description of the economy

The first model to consider will be the model of a small-open economy in which only traded goods are produced. The only non-traded good will be labour which is supplied by a single (representative) consumer in a fixed amount L^S.

The preferences of the representative consumer will be represented by a well behaved utility function defined over the consumption vector of traded goods:

$$U = U(c), \tag{97}$$

where c is the vector of consumed traded goods. If we denote by q the vector of consumer prices for the traded goods and by Y the income of the consumer, consumer behaviour will be described by the program:

$$\text{"Max. } U(c)\text{, subject to: } qc = Y.\text{"} \tag{98}$$

Income Y will consist of the (net of profit tax) profit income Π plus earnings from the selling of labour. If $\bar{\omega}$ is the wage rate received by the consumer and $\bar{L}$ the amount of labour <u>actually sold</u>, $Y = \Pi + \bar{\omega}\bar{L}$. The solution to (98) is denoted by

$$V(q,Y) \quad ; \tag{99}$$

$V(\cdot)$ is the consumer's indirect utility function.

The production sector will consist of m private firms. For simplicity, each firm will produce according to a strictly concave production function $F^j(\cdot) = 0$, defined over the production vector (y^j, L^j):

$$F^j(y^j, L^j) = 0, \tag{100}$$

where L^j is the amount of labour input used by firm j, while each element y_i^j of the vector y^j will denote the output of good i by the $j^{\underline{th}}$ producer. If y_i^j is an input, $y_i^j < 0$. Denoting producer prices for the traded goods by the vector p^j and the price for the labour input by w^j, $j = 1, .., m$, we can describe producer behaviour by the m programs:

$$\text{"Max. } (p^j y^j - \omega^j L^j), \text{ s.t. } F^j(y^j, L^j) = 0, \quad j = 1, .., m. \text{"} \tag{101}$$

The solution to (101) is denoted by $\pi^j(P^j, \omega^j)$, $j = 1, .., m$. Each $\pi^j(P^j, \omega^j)$ is uniquely determined by (P^j, ω^j) since F^j is strictly concave, $\forall_j$.

The government will have a fixed requirement for public expenditure which will consist of a vector g of traded goods. Hence, if we denote by e the vector of world prices, the government will need to raise revenue $R = eg$.

Finally, there will be an institutional constraint in the economy according to which the consumer wage $\bar{\omega}$, is no less than an index based on (after-tax) consumer prices q:

$$\bar{\omega} \geq \bar{\omega}(q) \tag{102}$$

If $\bar{\omega}(q)$ is a linear homogeneous function, equation (102) defines the minimum expenditure needed to sustain a certain utility level. In this case, $\bar{\omega} \geq \bar{\omega}(q)$ is a real wage constraint.

For the moment we assume that producers face the minimum wage $\bar{\omega}$ $(\bar{\omega} = \omega^j, \forall_j)$ and that $\bar{\omega}$ exceeds the wage rate which clears the labour market. Hence, the quantity of labour actually sold $\bar{L}$ will be the quantity of labour demanded by the m private producers, i.e. the consumer will be rationed to sell less than $\tilde{L}^S$ units of labour-units where $\tilde{L}^s$ is his perfectly inelastic labour supply.

The government's aim is to choose consumer prices $\bar{\omega}$,q, production prices p^j and profit taxes in order to maximize the representative consumer's utility, subject to the balance of trade constraint, the revenue requirement for public expenditure, the institutional minimum wage constraint and the feasibility constraints for the economy. For simplicity, it will be assumed that profits are taxed at 100 per cent. In fact, if profits are not sufficient to cover all government expenditure $(eg > \sum_j \pi^j)$, it will be optimal to tax them at 100 per cent, (see Dasgupta and Stiglitz [11]). Moreover, we should make clear that choosing $\bar{\omega}$ does not imply that the government can tax labour supply. It rather means that it chooses $\bar{\omega}$ to satisfy the wage constraint. Since we have assumed that no lump-sum taxes (other than profit taxes) are allowed, it would not make sense to tax the exogenously fixed labour supply.

Before we write the government's objective formally, we should first derive the various demand and supply functions.

From (98),

$$C = C(q, \bar{\omega}, \bar{L}), \tag{103}$$

where $\bar{L}$ is less than L^s since $\bar{\omega}$ exceeds the market clearing wage rate by assumption. In fact

$$\bar{L} = \sum_j L^j < L^s. \tag{104}$$

In (103) there is no profit income since profits are taxed at 100 per cent. Also, an increase in $\bar{\omega}$ will only have income effects. (However, this is always the case since labour supply is exogenously fixed).

From (101),

$$y^j = y^j(p^j, \omega^j), \quad L^j = L^j(p^j, \omega^j) \tag{105}$$

Since, by assumption, $\bar{\omega} = \omega^j$, $j = 1, .., m$,

$$y^j = y^j(p^j, \bar{\omega}), \quad L^j = L^j(p^j, \bar{\omega}). \tag{106}$$

In fact, $\bar{\omega} = \omega^j$, $\forall_j$ is equivalent to saying that the untaxed numeraire good is labour. In this case $\bar{\omega}$ is fixed and the institutional constraint says that consumer prices satisfy the condition $\bar{\omega} \geq \bar{\omega}(q)$ for a fixed $\bar{\omega}$. In most of what follows we will not choose labour as the untaxed numeraire since it will be easier for the arguments to go through by being able to subsidize the producer wage. However, the choice of the numeraire does not matter (since only relative prices matter) and a subsidy to the producer wage will have the same effects on producer decisions as a uniform subsidy to producer prices.

Now, the government will choose the m+1 vectors $p^j, (j=1,\ldots,m)$ and q in order to maximize the indirect utility function (99) subject to the institutional constraint (102) for a fixed $\bar{\omega}$, the balance of trade constraint

$$e \left[\sum_j y^j(p^j,\bar{\omega}) - C(q,\bar{\omega},\bar{L}) - g \right] \geq 0, \tag{107}$$

and the fact that the labour market clears by quantity rationing:

$$\bar{L} = \sum_j L^j(p^j,\bar{\omega}). \tag{108}$$

The production functions have already been taken into account in deriving supply functions. However, we prefer to keep them as explicit constraints

$$F^j \quad [L^j(p^j,\bar{\omega}),\ y^j(p^j,\bar{\omega})] = 0, \tag{109}$$

$$j = 1, \ldots, m.$$

Also, (107) and the overall budget for the economy imply that the solution of this program guarantees the sufficiency of revenue for the financing of government expenditure. To see this, imagine that the government buys y^j from producers at prices p^j and sells them in the world markets at prices e. Then it buys the consumer demand vector c from world markets at prices e and sells it to the consumer at prices q. Finally, it buys the vector g of public expenditure at world market prices e and taxes all private profits π^j. The total profits on these operations are:

$$P = \sum_j (e - p^j)y^j + (q - e)c - eg + \sum_j \pi^j. \tag{110}$$

The budget of the representative consumer and (108) imply:

$$qc = \bar{\omega}\bar{L} = \sum_j \bar{\omega}L^j \tag{111}$$

Also, $$\sum_j \pi^j = \sum_j \{p^j y^j - \bar{\omega}L^j\} \tag{112}$$

Taking into account (111), (112) and (107), (110) implies:

$$P \geq \sum_j e\, y^j - \bar{\omega}(\sum_j L^j - \sum_j L^j) - ec - eg \geq 0 \tag{113}$$

It is clear from (110) that P is the total revenue from taxes on consumption, production and profits minus the amount needed to finance public expenditure; (113) implies that P is non-negative. ||

This argument also makes clear that the government's objective can be alternatively stated as follows:

"Max. $V(q, \bar{\omega}(q), \bar{L})$

$$\text{s.t. } \sum_j [e - p^j]\, y^j(p^j, \bar{\omega}) + [q - e]c(q, \bar{\omega}(q), \bar{L}) + \sum_j \pi^j \geq eg \text{"} \tag{114}$$

We know from Kuhn-Tucker theory that in order to be able to apply the first-order conditions for an optimum solution to (114) we must guarantee that the so-called constraint qualification holds. In fact, there are several alternative ways to ensure that this condition holds (see Bazaraa and Shetty [4] , Diamond and Mirrlees [13]). In our framework it will be illuminating to use the Slater condition (see Bazaraa and Shetty [4]). This condition says that the constraint qualification will be satisfied if there exist values of the control variables q, p^j, for which the constraint in (114) holds with strong inequality. (Or, if there exist consumption and production taxes such that the revenue obtained is strictly greater than the sum needed to finance that amount of public expenditure which cannot be bought by profits alone). Formally, the constraint qualification will hold if $\exists q, p^j$:

$$\sum_j [e-p^j]y^j(p^j,\bar{w}) + [q-e]c(q,\bar{w}(q),\bar{L}) > eg - \sum_j \pi^j \tag{115}$$

Let us now examine in detail the structure of commodity taxation. The first question to ask is about production efficiency.

3. Should private producers face world prices under excess labour supply? [1]

The government's objective is described by the following program:

"Max. $\quad V(q, \bar{w}, \bar{L})$

$(\bar{w}, q, p^j, \bar{L})$

s.t: $\quad (\xi): \quad \bar{w} \geq \bar{w}(q),$

$$(\mu): \quad e\Big[\sum_j y^j(p^j, \bar{w}) - c(q,\bar{w},\bar{L}) - g\Big] \geq 0, \tag{116}$$

$$(\lambda): \quad \bar{L} \leq \sum_j L^j(p^j, \bar{w}),$$

$$(\nu_j): \quad F^j\left[y^j(p^j, \bar{w}), L^j(p^j, \bar{w})\right] = 0,$$

$$j = 1, \ldots, m$$

The Lagrangian for (116) is:

$$\Phi = V(q,\bar{w},\bar{L}) + \xi(\bar{w} - \bar{w}(q)) + \mu e\Big[\sum_j y^j(p^j,\bar{w}) - c(q,\bar{w}, \bar{L}) - g\Big] + \tag{117}$$
$$+ \lambda\Big[\sum_j L^j(p^j, \bar{w}) - \bar{L}\Big] + \sum_j \nu_j F^j\left[y^j(p^j,\bar{w}), L^j(p^j,\bar{w})\right]$$

where ξ, μ, λ, ν_j, $j=1,\ldots m$ are the shadow prices for the corresponding constraints in (116) with their usual meaning. As we have already noticed, the constraints $F^j = 0$, $j=1,\ldots m$ are not needed since they are implicitly contained in the supply functions. However, we retain them in this section because the arguments are easier to follow. Provided that the constraint qualification holds (see condition (115)), and assuming interior solutions and differentiability, the necessary conditions for an optimum of (116) are found after we equate the derivatives of Φ with respect to the state variables to zero. However, since the demand functions lack any convexity properties there is no reason why the necessary conditions should imply global maximization.

On this problem see Diamond and Mirrlees [13] .

From now on, first derivatives will be denoted by subscripts. For instance, $L^j_{p^j_i} = \frac{\partial L^j}{\partial p^j_i}$. Also, the notation $L^j_{p^j}$ will denote the N vector $[L^j_{p^j_1}, \ldots, L^j_{p^j_N}]$. Similarly if y^j and p^j are both vectors, $y^j_{p^j}$ will denote a matrix with typical element $(y^{jk}_{p^{jl}})$, where y^{jk} is the k^{th} element of y^j and p^{jl} is the l^{th} element of p^j .

Hence, assuming interior solutions, the first order conditions with respect to p^j and q for an optimum of (116) are:

$$\Phi_{p^j} = 0, \ j=1, \ldots m, \ \Phi_q = 0. \tag{118}$$

Let us examine the first set of equations, that is $\Phi_{p^j} = 0$, j=1,...m. By differentiating (117) with respect to (w.r.t) p^j we obtain:

$$\Phi_{p^j} = \mu e y^j_{p^j} + \lambda L^j_{p^j} + \nu_j \left[F^j_y \ y^j_{p^j} + F^j_L \ L^j_{p^j}\right] = 0, \tag{119}$$

$$j = (1, \ldots, m) \ ;$$

rearranging terms,

$$\Phi_{p^j} = L^j_{p^j} \left[\lambda + \nu_j F^j_L\right] + \left[\mu e + \nu_j F^j_y\right] y^j_{p^j} = 0, \tag{120}$$

$$j = (1, \ldots, m)$$

From the production functions,

$$F^j_y \dot{y}^j_{p^j} + F^j_L \dot{L}^j_{p^j} = 0, \quad j = (1, \ldots, m),$$

Hence, (120) becomes

$$\Phi_{p^j} = \left[-\lambda F^j_y + F_L \mu e\right] y^j_p = 0 \qquad (121)$$

Assuming that $y^j_{p^j}$ is a non-singular matrix and taking into account the profit maximizing conditions $(p^j/\bar{w}) = (F^j_{y^j}/F^j_{L^j})$, $j = (1, \ldots m)$, (121) implies

$$\frac{\mu e}{\lambda} = \frac{F^j_{y^j}}{F^j_{L^j}} = \frac{p^j}{\bar{w}}, \quad j = (1, \ldots, m), \qquad (122)$$

Taking into account that labour is the numeraire good, (122) says that all private producers should face world prices when evaluating their projects, and no taxes or tariffs are allowed on intermediate goods. In other words, a single price vector should apply to the production sector as a whole: Production efficiency is desirable. Had we incorporated a public sector's production function, this result would imply that the suitable shadow prices for the evaluation of public projects would be world prices.

The presence of excess labour supply does not make any difference as far as efficiency in production is concerned. This result says that it is desirable to produce on the frontiers of the production possibility curve which is constructed under the constraint that labour employed is less than the planned labour supply. In the following diagram, T_1T_2 is the full-employment production possibility curve of a small open economy, producing only two traded goods, while $T_1'T_2'$ is the production possibility curve for a given amount of employment $\bar{L}$ which is less than the fixed labour supply L^s.

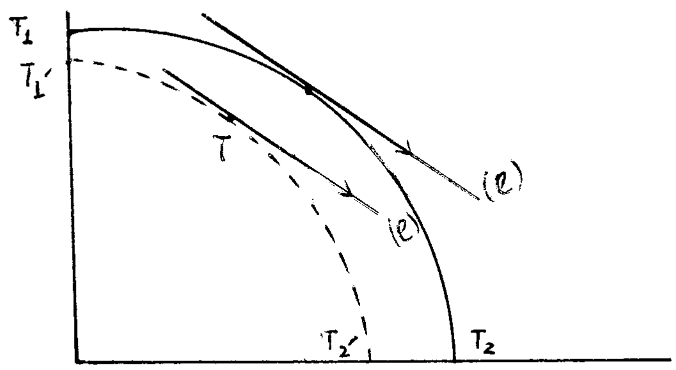

Figure 7.

Efficiency in production (including the foreign sector) implies that production should take place at T where the value of home production is maximized at world prices (e).

In fact, the intuitive explanation of the desirability of production efficiency (relative to the "shrunk-in" production possibilities) is exactly the same to that of Diamond and Mirrlees [13] : For, assume that even 100 per cent profit taxes are not sufficient to finance public expenditure (notice that in our model there always exist positive profits since we have assumed strictly concave production functions in order to be able to work with functions rather than correspondences) and some distortionary taxes ought to be imposed. If the optimum solution for program (116) occurs in the interior of the production set for given $\bar{L}$, "small" changes in the consumption prices of goods would result in feasible demands. But a reduction of the price of a consumption good increases welfare. Hence, at the optimum solution, production must take place on the frontiers of the production set.§ In the case of a many-person economy, it is known that production efficiency may not be desirable on the basis of distributional considerations. As we have already suggested, a suitable characteristic for distinguishing citizens could be the source of their income. Suppose that there is a two-class society consisting of those who earn income by selling labour alone and those whose only income source is profit income. Moreover, if there exists unemployment this is allocated efficiently among workers. It is obvious that 100 per cent profit taxes are not desirable on the basis of distributional considerations. But with less than 100 percent profit taxes, it is known that production efficiency is not generally desirable.§ In fact, if profit income enters the indirect utility function, the Lagrangian lo ses its "separability" properties. For instance, in (116) the structure of the Lagrangian Φ makes the first-order conditions in production coincide with the corresponding conditions of the first-best problem, where it is known that the marginal rate of transformation between any two goods must be equal for all firms and also equal to the world prices. With profit income in the indirect utility function, this property ceases to hold: Production will enter the objective function and there will be no subset of first-order conditions which will coincide with the corresponding conditions of the first-best problem.

In a sense, production prices enter the objective function in (116) through labour demand, but "separability" still obtains. To see this clearly, let us consider a simpler economy than that in (116) by aggregating all domestic producers together. Now, there will only be two sectors: The domestic and the world sectors.

Let us assume that constraint (λ) in (116) is binding, so that $\bar{L} = L(p, \bar{w})$. Labour demand will enter the indirect utility function, so that the program can be written

$$\text{"Max.} \quad V[q,\ w(q),\ L(P,w)],\ \text{s.t.}$$
$$(\mu):\quad e[y(p,w) - c(q,w(q),\ L(p,w)) - g] \geq 0\ \text{"} \tag{123}$$

The first-order conditions w.r.t. p are:

$$V_L\, L_p + \mu e\ [y_p - C_L\, L_p\ ] = 0 \tag{124}$$

Since $F_y y_p + F_L L_p = 0$, (124) becomes

$$\left[F_y\ (\mu\, eC_L - V_L\ ) + \mu e\, F_L \right]\ y_p = 0 \tag{125}$$

assuming again that y_p is non-singular, (125) implies

$$\frac{F_y}{F_L} = \frac{p}{\bar{w}} = \left[\frac{\mu}{V_L - \mu eC_L} \right] e \equiv ke \tag{126}$$

i.e. domestic producer prices must be proportional to world prices (labour is the numeraire good). Incidentally, we notice that condition (126) is the same as (122): In fact, the scalar λ which appears in (122) is the shadow wage rate (see (116)); λ measures the increase in social welfare when an additional unit of labour is demanded by producers. By differentiating the Lagrangian Φ (see (116)) w.r.t. $\bar{L}$ we obtain

$$\Phi_{\bar{L}} = V_{\bar{L}} - \mu e C_{\bar{L}} - \lambda = 0, \text{ or, } \lambda = \bar{V}_L - \mu e C_{\bar{L}} . \qquad (127)$$

Hence, (122) is exactly the same as (126). (127) says that the increase in social welfare caused by an additional unit of labour demand is equal to the marginal utility of income ($V_{\bar{L}}$) minus the social (dis)utility caused by a reduction in the value of trade surplus which is due to increased demand for traded goods. (An increase in the demand for labour has a positive income effect on consumption: $C_{\bar{L}} > 0$).

Going back to our discussion, we were saying that production efficiency may not be desirable on distributional considerations. Unemployment alone is not a justification for discrimination among producers, including the foreign sector.§ Another issue should be clarified at this point: Production inefficiency does not necessarily imply that tariffs should be used when domestic taxes are still available. For, assume that profits are not taxed away and consider program (123). This is modified to read

$$\text{" Max.} \quad V(q, w, L(p,w), \pi(p,w)), \text{ subject to:}$$

$$(\mu): \quad e\left[y(p,w) - c(q,w,L(p,w),\pi) - g\right] \geq 0 \qquad (128)$$

The budget constraint is $qc = py$; as a consequence of it the constraint in (128) can be written as:

$$ey - ec = ey - py - ec + qc = y(e-p) + (q-e)c \geq eg \qquad (129)$$

where $e-p = t'$ are production taxes and $q-e = t$ consumption taxes.

Instead of solving (128), we maximize

$$V(q,\ w,\ L\ (p,w),\ \pi(p,w)),\ \text{s.t.} \qquad (129)$$

Using the well-known result $\pi_p = y$, the first order conditions for a maximum of this program are

$$V_L L_p + V_\pi\ y + \mu\ [y_p\ (e-p) - y + C_\pi (q-e)\ (L_p + y)\ ] = 0. \quad (130)$$

It can be verified that the term $V_\pi y$ prevents production efficiency (i.e. it is not true that producers should face world prices). On the other hand there is no reason why (130) should be satisfied by q=p or, what amounts to the same thing, by imposing trade taxes and not consumption and production taxes at different rates. (We owe this argument to Bliss [7]).

So far, we have shown that in a one consumer economy where 100 per cent profit taxes are imposed and commodity taxes are chosen optimally, production efficiency is desirable for a given level of employment $\bar{L}$. In particular, if the government uses the revenue from profit taxes to subsidize domestic producers so that they employ more labour, the optimum subsidy ought to be a uniform ad-valorem subsidy to all producers, i.e. domestic producers should always face world relative prices.

The next question to consider is what kind of taxes should the government impose to finance its public expenditure. The analysis would be trivial in the case where profit taxes were sufficient to buy the vector g at world prices: Then, no (distortionary) consumption taxes should be imposed since all revenue may come from profit taxes (The issue of managerial effort is a different one and we ignore it completely). The more interesting case arises when profit taxes are not sufficient to finance public expenditure and some distortionary consumption taxes should be imposed. Now, one may ask at what level of employment should those taxes be imposed. Well, since profits are always positive in our model (we have assumed strictly concave production functions) it is desirable to subsidize the producers' real wage until they employ the whole amount of labour supply. For, assume that the government stops sort of full employment and imposes consumption taxes. At this point, a small reduction in consumption prices (i.e. in tax-rates) will reduce the miminum wage through the real wage equation and will enable producers to employ more labour. On the other hand, the tax-base will increase since an increase in labour demand will have a positive income effect on consumption. Moreover, welfare will also increase because of lower consumption prices and higher employment. The new higher demands will be feasible since more labour is employed enabling more production. Hence, the optimum cannot occur at any other level of employment than full employment.

The above argument brings together the explanation of the desirability of production efficiency provided by Diamond and Mirrlees [13], and the so-called "free-lunch" policy for an increase in employment in economies with wage indexation: According to the advocates of the "free-lunch" policy (see Corden [9]) a reduction in tax rates allows trade unions to ask for a lower wage, giving incentive to producers to employ more labour. An increase in labour income enables consumers to spend more and the government will be able to increase its revenue through a larger tax-base.

In fact, this policy, which is also a piece of "orthodox" Keynesian thinking, may provide an answer to the question posed to those who advocate expansionary policies without inflationary effects, that is to the question of the method of financing expansion: The revenue may come through an increased tax-base which is due to the reduction of unemployment.

4. Optimal consumption taxes under a minimum wage floor in a one consumer economy

We will consider the case of a real wage constraint. A uniform consumption tax will bring no revenue because of homogeneity of the demand functions $c(q, \bar{w}L)$ and of the wage function $\bar{w} = \bar{w}(q)$. In what follows, one of the traded goods will serve as the untaxed numeraire(τ). All other traded goods can be taxed freely and the government can also subsidize the producers' wage so that consumer's and producers' wages will differ. According to the arguments above the government will wish to subsidize the producer wage until full employment is achieved. Hence, optimal consumption taxes will be found as a solution of the following program :(For simplicity, we have aggregated all domestic producers into one)

$$\text{"}\max_{(w^f,q,p)} \quad V(q,\ w(q)\ \tilde{L}^s), \ \text{subject to :}$$

$$(\mu):\quad e\left[y(w^f,p) - c(q, w(q)\tilde{L}^s) - g\right] \geq 0$$

$$(\lambda):\quad \tilde{L}^s = L(w^f,p), \quad \text{"} \tag{131}$$

where $\tilde{L}^s$ is the perfectly inelastic labour supply and w^f is the market clearing wage rate. (The role of the government in the labour market is that of an auctioneer). Now, it can be easily verified that production efficiency is desirable, so that producer prices p should be chosen proportional to world prices: $p = \lambda e$. Since world prices e are fixed, choosing q to solve (131) is equivalent to choosing consumption taxes t: $t = q-e$. Program (131) may not have a solution. Let us examine the government's operations in detail: We can again imagine that the government buys the consumer demand vector c from world markets at prices e and sells it to consumers at prices q. Similarly it buys the vector of domestic supply y and sells it at world prices e. Finally it buys the total amount of labour supply $\tilde{L}^s$ at a wage rate $\bar{w} = \bar{w}(q)$ and sells it to producers at the full employment wage rate w^f. The net revenue from these operations plus profit taxes must be sufficient to cover public expenditure (eg). In other words,

$$(q-e)\ c + ey - ey + [w^f - w(q)]\tilde{L}^s + \pi \geq eg, \text{ or}$$

$$\pi + (q-e)\ c \geq eg + [w(q) - w^f]\tilde{L}^s \qquad (132)$$

The left hand side of (132) is the revenue from profit and consumption taxes and the right hand side the funds needed to subsidize the producer's wage and to buy the vector g of public expenditure. It is clear that not all levels of public expenditure and (or) a minimum wage are feasible.

Since $\pi = ey - w^f\tilde{L}^s$, $qc = w(q)\tilde{L}^s$, $\tilde{L}^s = L(w^f, e)$, (132) is equivalent to:

$$ey - w^f \tilde{L}^s + w(q)\tilde{L}^s - ec \geq eg + w(q)\tilde{L}^s - w^f \tilde{L}^s, \qquad or,$$

$$e\left\{ y(w^f, e) - c\left[q, w(q)L(w^f, e)\right] - g \right\} \geq 0 \qquad (133)$$

which shows that (132) is equivalent to the constraint in (131). A sufficient condition which ensures that the constraint qualification holds for (131) is to assume that g and w(q) are such that there exist a q for which (133) holds with strict inequality. Imposing this assumption, we can solve (131) by applying the Lagrangean method. Assuming interior solutions, the first-order conditions for an optimum solution are:

$$-\frac{ac}{\mu} + \frac{a}{\mu} w_q \tilde{L}^S - (q-t)\left[C_q + C_I W_q \tilde{L}^S\right] = 0 \qquad (134)$$

where $a = \partial V/\partial I$ is the marginal utility of income and C_I is the income effect on consumption. Since labour supply is perfectly inelastic, an increase in the consumer's wage has only income effects. The Hicks-Slutsky equation in matrix form is

$$C_q = (C_q)_{\bar{U}} - C_I C \qquad (135)$$

where $(C_q)_{\bar{U}}$ is the (negative semi-definite) substitution matrix. Hence, (134) becomes

$$-\frac{a}{\mu} c + \frac{aw_q}{\mu}\tilde{L}^S - q(c_q)_{\bar{U}} + qC_I C - qC_I W_q \tilde{L}^S - tC_I C + tC_I w_q \tilde{L}^S = -t\,(C_q)_{\bar{U}}, \qquad (136)$$

Since $q(C_q)_{\bar{U}} \equiv 0$, $qC_I \equiv 1$, (136) may be written as

$$C\left(-\frac{a}{\mu} + 1 - tC_i\right) - W_q\tilde{L}^s\left(-\frac{a}{\mu} + 1 - tC_I\right) = -t(C_q)_{\bar{U}} \qquad (137)$$

Assuming that $(C_q)_{\bar{U}}$ is non-singular,

$$t = \gamma\,(C - W_q\tilde{L}^s)\,(C_q)_{\bar{U}}^{-1} \qquad (138)$$

where $\gamma \equiv -(1 - \frac{a}{\mu} - tC_I)$; $(C_q)_{\bar{U}}^{-1}$ is the inverse of $(C_q)_{\bar{U}}$. (138) gives the optimum taxation formula. In fact we will recognize most of its terms if we rewrite (137) as:

$$\sum_j \frac{t_j}{C_i}\left(\frac{\partial C_i}{\partial q_j}\right)_{\bar{U}} = \gamma\left(1 - \frac{\partial w}{\partial q_i}\;\frac{\tilde{L}^s}{C_i}\right), \quad \forall_{i \neq \tau} \qquad (138)$$

Without the wage constraint, (138) would imply that for small taxes the consumption of all goods should have been reduced along the compensated demand curve by the same amount from what it would have been at world prices. The formula in (138) takes into account the minimum wage constraint. Denoting by n_i the elasticity of the minimum wage with respect to q_i ($n_i \equiv \partial \ln w/\partial \ln q_i$), we can rewrite (138) as

$$\sum_j \frac{t_j}{C_i}\left(\frac{\partial C_i}{\partial q_j}\right)_{\bar{U}} = \gamma\left(1 - n_i\,\frac{I}{q_i C_i}\right), \quad \forall_{i \neq \tau} \qquad (139)$$

which implies that the relative reduction in the consumption of good i is inversely proportional to the share of good i in the budget of the (representative) consumer, weighted by the elasticity of the minimum wage with respect to the price of good i. In the case where n_i is proportional to the share of good i in the budget, i.e. $n_i = k\,(q_i C_i/I)\ \forall_i$, formula (139) implies that the consumption of all goods should be reduced by the same amount.

$$\sum_j \frac{t_j}{C_i}\left(\frac{\partial C_i}{\partial q_j}\right)_{\bar{V}} = \gamma\,(1-k), \quad i \neq \tau\ . \tag{140}$$

Incidentally, $-\gamma$ can be interpreted as the benefit expressed in terms of foreign exchange, from being able to switch to lump-sum taxation (see Atkinson and Stiglitz [2] , page 373 for a detailed discussion).

(b) Two Consumers

As we have already noticed, a one consumer economy is not a very suitable framework for minimum wage constraints. A more reasonable assumption to make is to consider a two consumer economy, where the first consumer earns income from selling labour alone and the other from profit shares.

Since the only source of income for the second consumer is profit income, we will assume that the government does not impose profit taxes at all. This is only a simplifying assumption: Since revenue could only come from distortionary taxation, some profit taxes would generally be desirable.

We will again assume that the minimum wage constraint is binding (in the sense that it exceeds the competitive wage rate) and the labour market clears by quantity rationing under laissez-faire. Since profits are untaxed, the government's revenue for the purchase of the vector of public expenditure and the subsidization of the producer's wage rate may only come through consumption and production taxes. The government will now wish to maximize a social welfare function $\nu[V_1(q,I_1), V_2(q,I_2)]$ subject to the constraint: $0 \leq e\,[y(w^f,p)-g-C^1(q,I_1)-C^2(q,I_2)]$ by choosing consumer prices q, producer prices p, the worker's wage rate $\bar{w}$ subject to the minimum wage constraint $\bar{w} \geq \bar{w}(q)$ and the producer's wage w^f in such a way that the labour market clears at full employment. In the above formulation, $I_1 = \bar{w}\tilde{L}^s$ is the income of the first consumer ("worker") and $I_2 = py-w^f\tilde{L}^s$ that of the second. Formally, the government's objective is described by the following program :

"Max. $\nu\,[V_1(q,\bar{w}(q)\tilde{L}^s),\; V_2(q,py-w^f\tilde{L}^s)]$, subject to :

$$(\mu):\quad 0 \leq e\,[y(w^f,p) - g - C^1(q,\bar{w}(q)\tilde{L}^s) - C^2(q,py-w^f\tilde{L}^s)] \qquad (141)$$ ".

By the usual argument, the above program is equivalent to maximizing the social welfare function ν subject to the requirement that the revenue from consumption and production taxes should be sufficient to buy the vector of public expenditure g and to subsidize the producer wage in such a way that all labour is employed. Assuming that the constraint qualification holds, the first-order conditions with respect to q and p give the set of optimum taxes. We immediately notice that the existence of profit income in the indirect utility function of the second consumer implies that producer prices p will not generally be proportional to world prices e for the reasons we have explained before. Defining $\frac{\partial \nu}{\partial V_1} = \nu_1$, $\frac{\partial \nu}{\partial V_2} = \nu_2$, $\frac{\partial V_1}{\partial I_1} = a_1$, $\frac{\partial V_2}{\partial I_2} = a_2$ the first order conditions w.r.t. q are:

$$\frac{\nu_1}{\mu}\left[-a_1C^1 + a_1w_q\tilde{L}^s\right] - \frac{\nu_2}{\mu}a_2C^2 - (q-t)\left[C_q^{\ 1} + C_q^{\ 2} + C_I^{\ 1}W_q\tilde{L}^s\right] = 0 \qquad (142)$$

Using the Hicks-Slutsky equations

$C_q^{\ 1} = (C_q^{\ 1})_{\bar{\upsilon}} - C_I^{\ 1}C^1, C_q^{\ 2} = (C_q^{\ 2})_{\bar{\upsilon}} - C_I^2C^2$, it can be verified that (142) gives:

$$t = \left[\gamma_1 (C^1 - w_q\tilde{L}^s) + \gamma_2 C^2\right]\left[\sum_{h=1}^{2}(C_q^{\ h})_{\bar{\upsilon}}\right]^{-1} \qquad (143)$$

where it has been assumed that the substitution matrices $(C_q^{\ h})_{\bar{\upsilon}}$ h=1,2 are non-singular; $\gamma_1 = -1+\frac{b_1}{\mu}+tC_I^1$, $\gamma_2 = -1+\frac{b_2}{\mu}+tC_I^2$. The scalar $\frac{b_1}{\mu}$ $(\frac{b_2}{\mu})$ is the social marginal utility of income of the first (second) consumer in terms of the foreign exchange constraint.

We can also write the optimum tax formula in a more familiar form. Denoting $\delta^h = \frac{b_h}{\mu} + \sum_i t_i \frac{\partial C_i^h}{\partial I_h}$, h=1,2 and using the Hicks-Slutsky equations, it is easy to verify that (142) leads to:

$$\frac{\sum_j t_j \sum_h \left(\frac{\partial C_i^h}{\partial q_j}\right)_{\bar{\upsilon}}}{\sum C_i^h} = -\left[1 - \sum_h \frac{\delta^h}{2}\left(\frac{C_i^{\ h}}{\bar{C}_i}\right) + \gamma_1 n_i \frac{I^1}{q_i\bar{C}_i}\right], \qquad (143')$$

where $\bar{C}_i = \Sigma C_i^{\ h}/2$. The left hand side of (143') is a proportional reduction in the consumption of the i^{th} commodity along the compensated demand curve. The right hand side is the same as formula (12-55) of Atkinson and Stiglitz [2], augmented by the term $\gamma_1 n_i \frac{I^1}{q_i\bar{C}_i}$. Hence, it is not now sufficient to have the same "net" social marginal income valuation (δ^h) for both consumers or the same $C_i^{\ h}/\bar{C}_i$ ratio $(\forall i)$ in order to obtain the same proportional reduction in the consumption of all goods. In addition, the elasticity of the minimum wage with respect to the price of good i, weighted by the share of the "average" expenditure on good i in the income of the consumer-worker, must be the same for all i.

It remains to find production taxes. These will be found from the first-order conditions (for an optimum of (141) with respect to p. This is straightforward and we will omit it.

5. Trade Taxes in a one-consumer economy

It is sometimes argued (although not very convincingly) that administrative constraints allow the imposition of trade taxes only ($q = p$). This argument is more common in developing countries where the taxation of goods is easier at the borders than domestically. In our framework we can study the role of tariffs not only as a revenue seeking device but also as an instrument to be used to reduce unemployment. In the previous sections, the government could subsidize the producer wage to achieve full-employment. Now $q = p$ and unemployment can be reduced only through non-uniform trade taxes. A uniform trade tax on all traded goods ('devaluation') will have no effect on employment since the demand for labour is homogeneous of degree zero in w, p and w is homogeneous of degree one in $q(=p)$ through the wage equation.

To simplify the analysis we will assume that the government taxes profits at 100 percent. Its objective will now be described by the program :

"Max $\quad V(q,\bar{w},\ \bar{L})$, subject to :

(μ): $\quad e\left[y(q,\bar{w}) - C(q,\bar{w},\bar{L}) - g\right] \geq 0$

(λ): $\quad \bar{w} \geq w(q)$

(ν): $\quad \bar{L} \leq L\ (q,\bar{w})$. " $\qquad$ (144)

Taking into account that trade taxes (t) are equal to (q-e), the first order conditions for an optimum of (144) are

$$V_q + \mu(q-t)\ \left[y_q - C_q\right] - \lambda w_q + \nu L_q = 0 \qquad (145)$$

The shadow prices μ, λ , ν have the usual meaning and satisfy the conditions

$$V_{\bar{w}} + \mu e\ \left[y_{\bar{w}} - C_{\bar{w}}\right] + \lambda = 0 \qquad (146)$$

$$V_{\bar{L}} - \mu e\ C_{\bar{L}} - \nu = 0$$

Using the Hicks-Slutsky equation and the production constraint $F\ (y,L) = 0$ along with the profit maximizing conditions $(F_y/F_L) = (q/w)$, (145) can be written as:

$$0 = \frac{a}{\mu}\ c - \frac{\lambda}{\mu}\ w_q + \frac{\nu}{\mu}\ L_q - \bar{w}L_q - q(C_q)_{\bar{\upsilon}} + qC_I C - t_{y_q} + t(C_q)_{\bar{\upsilon}} - t_{C_I}C, \qquad (147)$$

or :

$$t.\left[y_q - (C_q)_{\bar{\upsilon}}\right] = -\gamma\ C - \frac{\lambda}{\mu}\ w_q + L_q\ \left(\frac{\nu}{\mu} - \bar{w}\right) \qquad (148)$$

where γ is the familiar formula $\left[\frac{a}{\mu} - 1 + tC_I C\right]$. The system of equations (148) can now be solved with respect to t assuming that $\left[y_q - (C_q)_{\bar{v}}\right]$ is non-singular. Alternatively, we can re-write (148) as follows:

$$\sum_j \frac{t_j}{\varepsilon_i}\left[\frac{\partial \varepsilon_i}{\partial q_j}\right]_{\bar{\nu}} = \frac{-\gamma C_i}{\varepsilon_i} + \left(\frac{\nu}{\mu} - \bar{w}\right)\frac{\partial L}{\partial q_i}\frac{1}{\varepsilon_i} - \frac{\lambda}{\mu}\frac{\partial w}{\partial q_i}\frac{1}{\varepsilon_i}, \quad (149)$$

where ε_i is defined as $y_i - c_i$. Equation (149) can be related to the usual Ramsey type formulae. For the moment, assume that the shadow wage rate in terms of foreign exchange (ν/μ) is equal to the commercial wage $\bar{w}$ and that there does not exist a minimum wage constraint. Then, the second and third terms in the right hand side of (149) will vanish, and (149) says that the relative reduction in excess demand is inversely proportional to the ratio of excess demand to consumption. (See Dasgupta and Stiglitz [11] formula 4.9). A corollary to this is that no tariffs should be imposed on non-consumed intermediaries. This does not hold in our model, since the elasticities of the labour demand with respect to the price of the various inputs will differ from zero. This is clear from the second term in the right hand side of (149) provided that $\bar{w} \neq (\nu/\mu)$.

6. One consumer and production of a non-traded good

We will now assume that a non-traded consumable good is produced in the economy by a domestic monopolist. For simplicity, it will not be used as an intermediary in the production of traded goods, but its production will require both labour and traded goods. In addition we will assume that the monopolist produces subject to a decreasing returns to scale production function. Although it is sometimes argued that monopolistic situations arise because of increasing returns to scale, we will continue to assume well-behaved production functions since decentralization is problematic in the opposite case.

Finally, we retain the assumptions that the economy is small, facing constant world prices e, and that public expenditure consists of the provision of a fixed vector g of traded goods.

The wage equation will now include the consumption price of the non-traded good q_N along with the vector of consumption prices of the traded goods q_T

$$\bar{w} \geq \bar{w} \ (q_N, q_T) \tag{150}$$

The monopolist will be perfect competitor in the labour and traded good markets (i.e. in those markets where he buys his inputs) but he will face a declining demand curve in the non-traded good market. Profit maximization implies that he will set his marginal cost MC (P_T, w, Y_N) equal to his marginal revenue MR (P_N, Y_N).

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$$MC \ (P_T, w, Y_N) = MR \ (P_N, Y_N) \tag{151}$$

where P_T, w, P_N are the producer prices of the traded goods, labour and the non-traded good.

The deviations from competitive behaviour in this economy is the minimum wage constraint and the existence of a monopolist who fixes the price of the non-traded good P_N to satisfy (151).

The government will be able to impose one hundred per cent profit taxes but no other lump-sum taxes. In fact if profit taxes are sufficient to cover public expenditure (e g) and correct distortions, all that the government has to do is to find the competitive (as opposed to the laissez-faire) wage rate and non-traded good price and subsidize the producers' wage rate and the monopolist's production of the non-traded good, until the Pareto efficient allocations are the profit maximizing choices of the private producers. Efficiency requires that all subsidies should be uniform to all producers.

More specifically, the government will buy from the consumer his fixed labour supply $\tilde{L}^s$ at the minimum wage $\bar{\omega}$ and sell it to domestic producers at the competitive wage w^c. Also, it will pay the monopolist a unit subsidy $u = P_N^C - MR(Y_N^C, P_N^C)$ to induce him to produce the competitive quantity Y_N^C; p_N^C is the competitive price of the non-traded good. The expenditure on these operations is $E = uY_N^C + (\bar{\omega} - \omega^c)\tilde{L}^s$, while the total public expenditure is E + eg. By assumption, profit taxes are sufficient to pay for (E + eg). § The more interesting case arises when profits are not sufficient to pay for (E + eg) and some distortionary taxes are needed. (It can be verified that a uniform consumption tax will bring no revenue; this follows from the consumer budget, the wage equation, and homogeneity of consumer demands). The government will now impose both profit and distortionary taxes, chosen optimally, to finance the provision of the public good and to subsidize the producers' wage and monopolist's production. On this purpose, consider the following (auxiliary) program:

$$\text{Max.}\quad V\left[q_T,\ q_N,\ \bar{\omega}\ (q_T,\ q_N)\ \tilde{L}^s\right] \quad \text{subject to :}$$

$$(\mu):\quad o \ \leq\ e\left\{ -g + y_T(P_T,\omega^C) - y^C_{N(T)}(P^C_N,P_T,\overset{C}{\omega}) - C^T\left[q_T,q_N,\bar{\omega}(q_T,q_N)\ \tilde{L}^s\right]\right\}$$

$$(\nu):\quad \tilde{L}^s = L_T(\omega^C,P_T) + L^C_N(\omega^C,P_N{}^C,P_T)$$

$$(\sigma):\quad Y^C_N\ (\omega^C,P^C_N,P_T)\ \geq\ C_N\left[q_N,q_T,\bar{\omega}(q_T,q_N)\ \tilde{L}^s\right]$$

$$(152)$$

In (152), the traded good production has been aggregated into one sector with aggregate supply vector $\left[y_T(P_T,\omega^C),\ L_T(P_T,\omega^C)\right]$;

(152) is an auxiliary program used by the government to find the competitive producer price p_N^C and the labour market clearing wage rate $\overset{C}{\omega}$, along with consumption prices q_T,q_N. The vector of the non-traded good sector supply functions

$\left[Y_N{}^C(\cdot),\ Y_{N(T)}{}^C(\cdot),\ L^C_N(\cdot)\right]$ has been found from the profit maximizing conditions of an auxiliary, competitive non-traded good sector. Notice that we have assumed that the non-traded good sector faces the same production prices (ω,p_T) as the traded good sector. In fact, we have deliberately done so since it can be easily verified that efficiency in production is desirable for the competitive economy described by (152). Also, taking into account the foreign sector, it follows that p_T must be proportional to the world prices of the traded goods e.

For the moment assume that (152) has been solved and that consumption prices (q_N, q_T) and production prices (ω^C, e, P_N^C) have been found. The competitive quantities $Y_N^C, \left[L_N^C(\cdot) + L_T(\cdot)\right]$ will be the profit maximizing choices for the (real) private sector if the monopolist receives a unit subsidy $u = p_N^C - MR(p_N^C, Y_N^C)$ and all producers face world prices e and the competitive wage ω^C. Of course, producer prices will now be determined simultaneously with consumption prices. § The government's revenue will come from profit and consumption taxes and must be sufficient to buy eg, to induce the monopolist to produce the competitive quantities and all producers to employ the available labour supply $\tilde{L}^s$. Formally, we must show the following relation:

$$\Pi_T + \Pi_N + (q_N - P_M)\, C_N + (q_T - e) C_T \geq eg + (\bar{\omega} - \overset{c}{\omega})\tilde{L}^s, \qquad (153)$$

where P_M is the price the monopolist is given to produce the competitive amount Y_N^C, π_T is the traded good sector's profits $(= ey_T - \overset{c}{w}L_T)$, Π_N is the monopolist's profits $(= P_M Y_N^C - \overset{c}{\omega}L_N - ey_{N(T)})$, while eg is the amount of public expenditure. Since $C_N = Y_N^C$, $\tilde{L} = L_T + L_N$ (from (152)) and $q_N C_N + q_T C_T = \bar{\omega}L$ (from the budget constraint), (153) is: $ey_T - \overset{c}{\omega}L_T + P_M Y_N^C - \overset{c}{\omega}L_N - ey_{N(T)} +$

$$\bar{w}\tilde{L}^s - P_M Y_N^C - ec_T \quad \geq \quad eg + \bar{\omega}L - \overset{c}{\omega}L_T - \overset{c}{\omega}L_N \quad . \qquad (154)$$

More simply, it suffices to show that $e(y_T - y_{N(T)} - C_T - g) \geq 0$. But this is true by construction (see (152)).

It remains to determine q_N, q_T or, in other words, consumption taxes t_N, t_T. Since world prices e are fixed, choosing q_T is equivalent to choosing t_T, according to $e = q_T - t_T$. As far as the non-traded good's consumption price q_N is concerned, this must satisfy the relation $q_N = \frac{\sigma}{\mu} + t_N$, where $\frac{\sigma}{\mu}$ is the shadow price of the non-traded good in terms of the foreign exchange constraint (see program (152)). To see this clearly, remember that the (untaxed) numeraire good is one of the traded goods, τ. Efficiency requires $\sigma/\mu = \frac{\partial F/\partial Y_N}{\partial F/ Y_\tau}$, as can be easily verified from (152) and the production functions. Consumption price q_N will now differ from the shadow price by the amount of the consumption tax t_N. We should also make clear that the monopolist (who fixes his price at a higher level than $\frac{\partial F_N/\partial Y_N}{\partial F_N/\partial Y_\tau}$) is subsidized by the government in order to produce the competitive quantity Y_N^C. The unit subsidy is $u = P_N^C - MR(P_N^C, Y_N^C)$, so that the monopolist faces a final price $\frac{\sigma}{\mu} + u$.

Assuming that the constraint qualification holds for (152), the first-order conditions with respect to q_T, q_N for an optimum solution will give the consumption taxes t_T, t_N. (From now on we will adopt the notation $\frac{\partial C_K}{\partial q_i}$ for partial derivatives and not the vector notation C_q of the previous sections, in order to avoid confusion with those lower letters which denote traded and non-traded goods).

From (152), the first order conditions with respect to q_i,

$\forall_i$ are:

$$-\frac{ac_i}{\mu} + \frac{a}{\mu}\frac{\partial w\tilde{L}^s}{\partial q_i} - \left[\frac{\partial c_N}{\partial q_i} + \frac{\partial c_N}{\partial I}\quad\frac{\partial w}{\partial q_i}\quad\tilde{L}^s\right](q_N - t_N) +$$

$$-\sum_{K\in T}(q_K - t_K)\left(\frac{\partial c_K}{\partial q_i} - \frac{\partial c_K}{\partial I}\quad\frac{\partial \bar{w}}{\partial q_i}\quad\tilde{L}^s\right) = 0 \tag{155}$$

To derive (155) we have used the following equations: $e_K = q_K - t_K \quad K\in T$, $\frac{\sigma}{\mu} = q_N - t_N$, $\frac{\partial v}{\partial q_i} = -aC_i$, $a \equiv \frac{\partial V}{\partial I}$, where T denotes the set of consumed traded goods. On rearranging terms we obtain

$$-\frac{aC_i}{\mu} + \frac{a}{\mu}\frac{\partial w}{\partial q_i}\tilde{L}^s - \sum_{j\in C} q_j\left(\frac{\partial c_j}{\partial q_i} + \frac{\partial c_j}{\partial I}\quad\frac{\partial \bar{w}}{\partial q_i}\quad\tilde{L}^s\right) +$$

$$+\sum_{j\in C} t_j\left(\frac{\partial c^j}{\partial q_i} + \frac{\partial c^j}{\partial I}\quad\frac{\partial \bar{w}}{\partial q_i}\quad\tilde{L}^s\right) = 0 \tag{156}$$

where C denotes the set of all consumed goods. The Hicks-Slutsky equation is

$$\frac{\partial c_j}{\partial q_i} = \left(\frac{\partial c_j}{\partial q_i}\right)_{\bar{v}} - c_i\frac{\partial c_j}{\partial I}, \tag{157}$$

where $(\partial c_j/\partial q_i)_{\bar{v}} = (\partial c_i/\partial q_j)_{\bar{v}}$. Homogeneity implies

$\sum_{j\in C} q_i\left(\frac{\partial c_i}{\partial q_j}\right)_{\bar{v}} = 0$. Hence, (156) reduces to

$$\sum_j \frac{t_j}{c_i}\left(\frac{\partial c_i}{\partial q_j}\right)_{\bar{v}} = \gamma\left(1 - n_i\frac{I}{q_i c_i}\right), \qquad \forall i \neq \tau \tag{158}$$

where $\gamma = \left[\frac{a}{\mu} - 1 + \sum t_j\frac{\partial c_j}{\partial I}\right]$, $n_i = \frac{\partial \ln\bar{w}}{\partial \ln q_i}$.

(158) is the well-known Ramsey rule augmented by the term $(n_i\bar{w}\bar{L}/q_i c_i)$. It says that taxes should be such that the relative reduction in

consumption of good i from what it would have been had shadow prices been charged is inversely proportional to the share of good i in the budget, weighted by the elasticity of the minimum wage with respect to the price of good i.

The fact that the monopolist fixes his price to equate marginal revenue to marginal cost does not prevent the government from achieving the competitive allocation since it can subsidize production from profit taxes. As it is clear from the analysis, all taxes and subsidies should be found together from the solution of the optimum program.

7. Unemployment and optimal trade taxes

In the previous section the government could subsidize production so that the fixing of the non-traded good price and the consumer's wage could be fully corrected. The monopolist was induced to produce the competitive quantity of the non-traded good through a suitable unit subsidy and producers were induced to employ the available labour supply through a wage subsidy.

We will now assume that trade taxes are the only available instruments in the hands of the government. (As we have said before, this assumption is controversial and it applies better to developing countries than to industrialized nations. We will consider it here to examine the case where the combination of real wage rigidity and monopolistic pricing gives rise to a regime of irremovable constraints.)

In other words the government will choose taxes to maximize social welfare subject to the additional constraint that it cannot drive a wedge between consumer's and producers prices. Thus, all agents will always face the institutionally fixed minimum wage $\bar{\omega}$ $(= \bar{\omega}(q_T, q_N))$, and the non-traded good market will clear by monopolistic pricing resulting in non-competitive allocations.

To simplify the subsequent analysis we will assume that the non-traded good is produced under constant returns to scale, and its production requires only one input, labour. We will also assume that the monopolist producing the non-traded good faces a declining demand curve with <u>constant</u> elasticity $|\varepsilon|>1$. Under those assumptions his profit maximizing behaviour will be described by the following program:

$$\underset{(q_N, Y_N, L_N)}{\text{Max.}} \; q_N Y_N - \bar{\omega} L_N$$

$$\text{s.t.} \qquad Y_N = C_N (q_N, q_T, \bar{\omega}\bar{L})$$

$$L_N = \frac{1}{r} C_N (q_N, q_T, \bar{\omega}\bar{L}), \tag{159}$$

where $C_N(\ )$ is the demand for the non-traded good, $\bar{L}$ is the total demand for labour by the monopolistic non-traded good sector and the competitive traded good sector and $1/r$ is the unit labour requirement for the production of the non-traded good. The first-order conditions for a solution of (159) amount to:

$$1 + \frac{\partial C_N}{\partial q_N} \frac{q_N}{C_N} = \frac{\bar{\omega}}{r} \frac{\partial C_N}{\partial q_N} \cdot \frac{q_N}{C_N} \frac{1}{q_N} . \tag{160}$$

According to our assumptions, $-\frac{\partial C_N}{\partial q_N} \frac{q_N}{C_N} \equiv |\varepsilon|>1$. Hence, the monopolist's pricing equation is:

$$q_N = \frac{\bar{\omega}}{r} \frac{\varepsilon}{1+\varepsilon}. \tag{161}$$

Calling $\frac{\varepsilon}{1+\varepsilon} \frac{1}{r} \equiv k$ (a constant), the pricing equation can be written as:

$$q_N = k\bar{\omega}, \tag{162}$$

(162) is an irremovable constraint for the government. If the institutionally set minimum wage $\bar{\omega}$ exceeds the market clearing wage rate, the labour market will clear by quantity rationing. In this case, the desirability of unemployment reduction and the need to raise revenue to finance public expenditure (we continue to assume a need for the provision of a vector g of traded goods) will be considered simultaneously. (This should be contrasted to section 6 where the government could first subsidize firms to employ all labour and then levy optimum taxes). For simplicity, we again assume that profits are taxed at 100 per cent. (Revenue cannot be used to subsidize production but we may assume that profits are used to finance part of the need for public expenditure).

The government can choose q_T (since world prices e are fixed , $q_T - \bar{e} = t$) , the domestic price q_N and the wage rate $\bar{\omega}$ (not in the sense that it can tax the non-traded good and labour but because it can choose them to satisfy the constraints $\bar{\omega} \geq \bar{\omega}(q_N, q_T)$, $q_N = k\bar{\omega}$). Its objective will be described by the program

$$\text{Max.} \quad V(q_T, q_N, \overline{\omega L}), \qquad \text{subject to} \quad :$$

$$(\mu): e\left\{C_T(q_T, q_N, \bar{\omega}\,\bar{L}) - Y_T(q_T, \bar{\omega}) + g\right\} \leq 0$$

$$(\nu): \quad \bar{L} = L_T(\bar{\omega}, q_T) + (1/r)\, C_N(q_N, q_T, \bar{\omega}\,\bar{L})$$

$$(\xi): \quad \bar{\omega} \geq \bar{\omega}(q_T, q_N) \tag{163}$$

$$(\zeta): \quad q_N = k\bar{\omega}$$

where μ, ν, ξ, ζ are the shadow prices corresponding to the constraints of (163). The condition that the non-traded good market clears is implicit in constraints (ζ), (ν). Since we have assumed a constant coefficient technology with a single non-produced input (labour) the non-substitution theorem implies that the shadow price of the non-traded good in terms of the foreign exchange constraint will be equal to $\nu/(\mu . r)$, where ν/μ is the shadow wage rate in terms of foreign exchange - see (163) - and (1/r) is the unit labour requirement for the production of the non-traded good.

The first-order conditions for an optimum of (163) are:

$$o = -\frac{\alpha}{\mu} c_i - \frac{\xi}{\mu} \frac{\partial \bar{\omega}}{\partial q_i} + \frac{\nu}{\mu}\left[\frac{\partial L_T}{\partial q_i} + \frac{1}{r}\frac{\partial c_N}{\partial q_i}\right] +$$

$$+ \sum_{K \in T}\left[q_K - t_K\right]\left[\frac{\partial Y_K}{\partial q_i} - \frac{\partial c_K}{\partial q_i}\right]. \qquad (164)$$

Using the well-known conditions

$$\sum_{K \in T} q_K \frac{\partial Y_K}{\partial q_i} = -\bar{\omega}\frac{\partial L_T}{\partial q_i}, \qquad \sum_{K \in T} q_K \frac{\partial c_K}{\partial q_i} + q_N \frac{\partial c_N}{\partial q_i} = -c_i,$$

$$\frac{\partial Y_K}{\partial q_i} = \frac{\partial Y_i}{\partial q_K}, \quad k, i \in T, \quad \left(\frac{\partial c_K}{\partial q_i}\right)_{\bar{\upsilon}} = \left(\frac{\partial c_i}{\partial q_K}\right)_{\bar{\upsilon}},$$

(164) may be written as follows:

$$\sum_{K \in T} \frac{t_K}{\varepsilon_i}\left(\frac{\partial \varepsilon_i}{\partial q_K}\right)_{\bar{\upsilon}} = \frac{1}{\varepsilon_i}\left\{ -\frac{\xi}{\mu}\frac{\partial \bar{\omega}}{\partial q_i} + \frac{\partial L_T}{\partial q_i}\left(\frac{\nu}{\mu} - \bar{\omega}\right) + \right.$$

$$\left. + \frac{\partial c_N}{\partial q_i}\left(q_n + \frac{1}{r}\frac{\nu}{\mu}\right) - \gamma c_i \right\},$$

where $\varepsilon_i = y_i - c_i$, $i \in T$ and $\gamma = -1 + \sum_T t_K \frac{\partial c_K}{\partial I} + \frac{a}{\mu}$. (165)

Formula (165) implies that tariffs should be such that the relative reduction in excess demand for the traded goods should depend not only on the term $-\frac{\gamma c_i}{\varepsilon i}$ (which can be identified with the distortion involved in the inability to raise revenue by other means than tariffs) but also on the effects that tariffs have on the demand for labour and the non-traded good as well as on the sensitivity of the real wage with respect to each price.

This is a very complicated tariff scheme and should be compared to the already complicated case where the only distortion has to do with the inability to raise revenue by means other than tariffs (see Dasgupta and Stiglitz [11], Bliss [7]):

$$\sum_{K \in T} \frac{t_K}{\varepsilon i} \left(\frac{\partial \varepsilon i}{\partial q_K} \right)_{\bar{U}} = -\gamma \frac{c_i}{\varepsilon i} + \sum_{K \in N} \left(P_K - \frac{\rho_K}{\mu} \right) \frac{\partial \varepsilon i}{\partial q_K} , \qquad (166)$$

where P_K, $K \in N$ are the private producer prices of the non-traded goods, and ρ_K/μ, $K \in N$ their shadow prices.

C. DISCUSSION

In this chapter we have considered the case where the government needs revenue to finance its activities under the assumptions that it cannot impose lump-sum taxes, that the real wage is rigid due to an institutionally set minimum wage constraint and the price of the non-traded good is being fixed at a level higher than its marginal cost.

The fact that domestic markets may clear by quantity rationing under laissez-faire does not present any particular problem provided the government may impose commodity taxes-cum-subsidies, financed by the revenue raised from profit taxes, to correct those distortions behind price rigidities. If profits are not sufficient to finance public expenditure as well, optimal consumption taxes should be imposed to raise revenue. Under the assumption that labour supply is exogenous, it is shown that those taxes should be imposed at full employment. The argument is similar to that used by those who advocate the "free lunch" solution to the problem of raising revenue to finance public expenditure under real wage rigidity: If some indirect taxes are cut, employers will be able to employ more labour because the real wage will be reduced. As a result of higher employment, consumption will increase enabling the government to raise revenue from a higher tax-base. This bonanza will cease to exist at full employment.

We have shown that unemployment is <u>not</u> a justification for discriminating among producers (i.e. for production inefficiency).

In particular, producers should face world relative prices at any level of employment $\bar{L}$. Inefficiency in production may be desirable on other grounds e.g. income distribution. However, inefficiency in production does not necessarily imply that tariffs should be imposed. Imposing optimal consumption and production taxes at different rates to raise revenue involves lower resource costs than tariffs.

Finally, if the government is unable to impose other than trade taxes (tariffs), the tax formula should take into account that domestic markets clear by quantity rationing. Now, the desirability of unemployment reduction will be determined through the optimum program, after all the effects and constraints have been taken into account.

Throughout the analysis we have assumed that public expenditure consists of the provision of a fixed vector of traded goods. Thus, we have avoided complications involved in crowding-out effects; also we have side-stepped the question of the optimal provision of public goods when the public sector acts subject to a production possibility function.

Although the results can be extended to the case where the government can finance its activities through money creation (see K. Roberts [42]), incorporating more than one assets into the model to permit financing through borrowing will not be satisfactory unless uncertainty is explicitly taken into account. This is not an easy task in the framework of the theory of optimal intervention for reasons that we have already explained. However, a proper model of optimal government intervention in a Keynesian environment should take into account uncertainty and be essentially dynamic.

NOTES

(1). This section is nothing more than an algebraic proof of the Diamond-Mirrlees theorem in an economy with less than full-employment due to a rigid real wage. The well-known intuitive argument about production efficiency - see page 129 - could be applied directly for any given quantity of labour demanded. The algebraic proof of the theorem helps us to understand that although production enters the indirect utility function due to rationing in the labour market, the Lagrangean remains 'separable' due to homogeneity - see discussion on page 130 - and the result follows.

(2). This is only a sufficient condition. There is no necessary connection between monopoly and prices being fixed for one period.

(3). It is assumed here that the monopolist never learns about his effective monopsony power.

Chapter Four

THE NEW CAMBRIDGE APPROACH: SUGGESTIONS FOR A PARTIAL WELFARE IMPROVEMENT UNDER LIMITED INSTRUMENT AVAILABILITY?

A. OVERVIEW

The New Cambridge macroeconomics is a theory of macroeconomic management developed in the last several years. Although its main propositions have aroused a lot of controversy, they are supported by many economists and influence a good number of politicians.

The two main theorems of this approach concern the relationship between fiscal policy and the balance of payments and the relative merits of protection and devaluation as alternative means for full employment (see Cripps and Godley [10]).

In its more extreme version, the first relationship stems from some (limited indeed) empirical evidence which show that (in Great Britain) the private sector spends exactly what it earns. Thus, the source of discrepancy between output and absorption is the public sector, and this implies that the only instrument to affect the trade balance is the budget deficit. Changes in the exchange rate will not have other than transient effects on it.

This is a very extreme version of the first theorem and is too implausible to be acceptable (see Spraos [45]). Incidentally, many economists have stressed the similarity of this result to the monetary approach to the balance of payments.

In its milder version, the first theorem is a proposition about the relative effectiveness of the budget deficit and the exchange rate to achieve full employment and external balance. It is claimed that there is evidence to believe that the right policy is to assign the exchange rate to internal balance and the budget deficit to external balance.

Two important corollaries can be inferred from the New Cambridge assignment principle. The first is that the possibility of conflict among countries increases if all conform to the New Cambridge prescriptions. Let us consider a two country example, where the foreign country has full employment and the home country suffers from unemployment. The home country will devalue to improve its internal balance according to the New Cambridge recommendations. To avoid unemployment, the foreign country will try to reverse the movement of the exchange rate while the home-country will resist such a reversal.

The second corrollary is that monetary policy does not play an independent role in the model. In the extreme version, the growth of net asset demand in the private sector is exactly matched by the generation of new assets in the same sector through new investment. This self-sufficiency implies that substitution between private and public sector securities is very limited, and there is no way that, in the aggregate, the stock of central bank money in the hands of the private sector can change (for a detailed discussion on these results see Spraos [45]).

The second theorem concerns the relative merits of adopting protection as an alternative to devaluation for full employment. More precisely, the New Cambridge method specifies three targets of policy, namely employment and output, the rate of inflation and the current balance of payments. The available instruments in the hands of the government whereby the above targets will be achieved are fiscal policy and the exchange rate. Protection is an alternative instrument to devaluation. In fact, under the New Cambridge model there is the presumption that the first two instruments are generally insufficient to achieve the three targets together,

because the Phillips curve relationship between inflation and unemployment is denied on empirical grounds.

It is claimed that protection is more effective than devaluation because it is less inflationary. The argument is the following: Keeping the trade deficit at a fixed level, suppose that an increase in output is to be achieved by fiscal expansion combined either with devaluation or with a tariff on imports. Devaluation will worsen the terms of trade and will induce a rise in export profit margins. As a result, devaluation will reduce the disposable real wage; this implies that nominal wages will have to increase: In fact, under the New Cambridge assumptions, money wage determination is the outcome of wage negotiations; negotiators bargain to establish an ex-ante real wage on the basis of prices and taxes that prevail at the time of the settlement. A worsening of the terms of trade, or (and) an increase of the share of profits in the national product will tend to reduce the disposable real wage. Hence, money wages will increase to reconcile the ex-ante real wage target to the disposable real wage.

On the contrary, the use of tariffs on imports as an alternative to devaluation is less inflationary because the disposable real wage will be higher due to the gains in the terms of trade and to the absence of any adverse effects on export profit margins.

Let us now examine the relevance of these arguments in the light of the theory of optimal intervention. First, as we have already noticed in the previous chapter, there is no theoretical justification in specifying the objectives of the government in target form, unless the highly unsatisfactory explanation of lexicographic preferences is adopted. Some people argue that preferences in target form can be justified on the grounds of uncertainty. There is something in this argument but in the New Cambridge approach there is no explicit reference to it.

Second, the instruments available in the hands of the government are not chosen optimally. For instance, since the wage rate is fixed by trade unions and the price of domestic expenditure is, by assumption, set as a mark-up on costs, full optimality would require subsidization of the producers' wage and the monopolist's production such that the competitive allocation would become the profit maximizing choice of the non-competitive agents. In fact, one would like to avoid resource costs by financing government activities through money creation. In this case medium term considerations should also be taken into account, the suitable maximand being an intertemporal social welfare function. § The use of tariffs in the New Cambridge model is justified because of the terms of trade effect. We already know from welfare theory that optimality requires tariffs in the case of an open economy which faces a declining demand schedule for its exports. Sometimes government action is not needed: If the export industry is a monopolist, its pricing policy is equivalent to imposing a tariff on the exports of an otherwise competitive industry. The possibility of retaliation complicates the situation but it does not necessarily rule out the use of tariffs as a device for improving the welfare of the home country.

The New Cambridge approach does not use the argument for tariffs on the above lines. The relation between the terms of trade and a tariff on imports appears rather as an identity than a specific proposition based on welfare considerations. Finally, tariffs are not chosen at any level that can be characterized as optimum .

Briefly, an optimum solution to the New Cambridge distorted economy would require: (1) Uniform wage subsidies to producers to employ the available labour supply and sales subsidies to make them produce the competitive quantities of goods. (2) The imposition of an optimum tariff to exploit the terms of trade, unless a private monopoly has already done so. (3) Devaluation to improve the balance of payments in case that such a correction is necessary. (4) Optimum selection of public projects and tax rates according to the government's social welfare function. In fact, the financing of public expenditure may be achieved through higher revenue from taxation due to the higher efficiency of the economy ("free lunch" possibility).

If a richer financial structure is included, the possibility of financing expenditure through borrowing from the public or even the Central Bank should be taken into account. In this case, medium term considerations should be added to the short-term effects.

It is clear that the above framework where instruments are chosen optimally is not the one that the policy maker in the New Cambridge economy has in mind. Perhaps it would be more suitable to examine the New Cambridge approach in the light of a model where only "small" steps are allowed and the use of instruments is limited. It is true that such a model can only be justified on some restrictive assumptions about what the policy-maker is permitted to do. However there are people who advocate that for a number of reasons the policy maker may be unable to make large changes in the structure of policy instruments. These reasons may include uncertainty, high costs, even ignorance. Under such a framework the sole ambition is to show what combinations of small variations in the use of the available instruments have a desirable effect, i.e. represent a partial welfare improvement.

In what follows, we will set up a simple model to study the New Cambridge instrument-target approach along these lines. More specifically, the proposition that protection is preferable to devaluation will be examined under the assumption that the available instruments are fiscal policy (this term will be specified below) and the exchange rate. Protection will be an alternative to the exchange rate.

B. The New Cambridge instrument-target approach as a comparative statics exercise in the context of a fix-price model

1. Introduction

We will work in the context of a Hicksian fix-price model where money is the only asset. (Such a model has also been considered in chapter two. In fact, in their formal interpretation of the New Cambridge model (see Cripps and Godley [10]) its initiators do not consider monetary influences since they claim that these have but a minor role in the United Kingdom. Also, they regard money supply as entirely endogenous, being fully determined by decisions on fiscal policy. These simplifications justify the use of a Hicksian model where money is the single financial asset. The role attributed to money supply by the New Cambridgists make things even more convenient. What they simply say is that the government makes sure that there is enough money to meet the demand for money balances. To put it another way, the government always chooses its money supply to finance its budget deficit. Of course, this implies that government action has intertemporal effects because issuing money at time (t) makes consumers richer at (t + 1). To avoid these complications, we can only pay attention to the results of government's action in a single date, assuming that the government neutralizes the intertemporal effects of its policy or that the future is heavily discounted. If medium term considerations are to be taken into account, the analysis should become explicitly dynamic (see K. Roberts [42]).

In the New Cambridge model, the term fiscal policy does not imply that taxes are chosen optimally in any sense. It is used in the traditional Keynesian sense to denote that a fiscal stimulus to the economy can be given by reducing the overall tax rate which is an average of direct and indirect taxes. In the model we are going to consider we will adopt the convention that fiscal policy consists of the decision by the government to buy a certain amount of a domestically produced good. In the context of a small-open economy with labour and non-traded goods the prices of which are rigid temporarily, this will have the same qualitative effects as reducing the overall tax-rate provided that it is the non-traded good that is bought by the government.

We now turn to specify the economy in more detail.

2. Assumptions and Description of the economy

We assume a small-open economy. There are the following goods: A consumed exportable good, denoted by the subscript e. An importable good which is not produced at home, denoted by i. A non-traded good which is produced and consumed at home, denoted by N. Labour supply L^s will be perfectly inelastic. In fact, we will assume that labour is supplied by a single consumer, and that there will be no other consumer in the economy. This assumption and the fact that the economy is assumed to be small imply that there will be no income distribution effects and that the terms of trade cannot be affected by the home country. A lot of simplification is achieved through this assumption but at the expense of departing from two crucial considerations of the New Cambridge model, i.e. the terms of trade effects and income distribution. Since the aim of the present chapter is to show how the New Cambridge approach can be rationalized in an explicit equilibrium framework and not to present a detailed survey of the New Cambridge ideas, we prefer to work with the above assumption.

Finally, there will be domestic money, the stock of which will be denoted by $\bar{m}$. As we have already explained in chapter two, the demand for money will represent demand for future expenditure.

3. Prices

The nominal price of the non-traded good P_N and the nominal wage rate w will be assumed rigid within the unit period: $P_N^t = \bar{P}_N$, $w^t = \bar{w}$.

This assumption is in accordance to the New Cambridge model which, in effect, corresponds to the regime of Keynesian unemployment in terms of the familiar Malinvaud (see [36]) classification. (Recall that Keynesian unemployment is characterized by excess supply in the labour and the non-traded good markets). In fact, output and employment in the New Cambridge model are determined by the usual Keynesian multiplier process in response to fiscal policy and export and import propensities.

When we come to discuss the inflationary effects of alternative trade policies, we will assume that $\bar{w}(\cdot)$ and $\bar{P}_N(\cdot)$ are determined as functions of the cost of living and production costs in the previous period.

The assumption of rigid nominal prices in the unit period <u>does</u> make sense since the government will not be allowed to make large changes in the disposable instruments altering the regime characteristic.

Since the economy is small, it faces constant world prices which we normalize to unity. We will denote the exchange rate by R (an increase in R will imply devaluation of the home currency), the tariff[1] on the exportable good by t_e and that on the importable good by t_i. Thus, the domestic price of the exportable good will be $P_E = (1 + t_e)R$ and that of the importable good $P_i = (1 + t_i)R$.

We assume that there are no taxes on the non-traded good and labour. In the New Cambridge model there are taxes on domestic expenditure and income, but they are not chosen optimally. As we have already noticed, tax rates are modelled as an instrument used to provide a fiscal stimulus to the economy via the traditional Keynesian process. The stimulus is provided by reducing the overall tax rate, which is an average of all direct and indirect taxes. In our model, this stimulus will be provided by the government decision to buy the non-traded good, while the money supply will be an accommodating variable as it is in the New Cambridge model.

4. Producers

There will be two sectors in the economy. In the non-traded good sector, labour will be employed to produce the non-traded good according to the following Cobb-Douglas production function:

$$y_N = L^k, \qquad k \leq 1 \tag{167}$$

where k is the elasticity of output with respect to labour. $k < 1$ implies decreasing returns.

Similarly, in the exportable good sector, labour will be employed to produce the exportable good according to the following production function

$$y_e = L^r, \qquad r \leq 1 \tag{168}$$

By virtue of the assumption of a small open economy, the exportable good sector will never be rationed in its sales.

On the contrary, the non-traded good sector may be rationed in its sales if domestic demand for the non-traded good falls short of the non-traded good sectors effective supply.

For instance, if the non-traded good sector is rationed to sell no more than $\bar{X}$ units of its product, where $\bar{X}$ is less than the sector's effective supply, (168) gives the sector's effective demand for labour:

$$\tilde{L}_N = \bar{X}^{\frac{1}{k}} \tag{169}$$

(169) should be contrasted with the sector's notional demand which is derived as the solution to the usual profit maximization procedure subject to the production function only:

"Max. $(\bar{P}_N Y_N - \bar{w}L)$ s.t. $Y_N = L^k$",

which gives:

$$L_N = \left(\frac{\bar{w}}{k\bar{P}_N}\right)^{1/k-1}, \qquad Y_N = \left(\frac{\bar{w}}{k\bar{P}_N}\right)^{K/K-1} \tag{170}$$

Similarly, the notional choices by the exportable good sector are given by:

$$L_e = \left(\frac{\bar{w}}{rp_e}\right)^{1/r-1}, \qquad y_e = \left(\frac{\bar{w}}{rp_e}\right)^{\frac{r}{r-1}} \tag{171}$$

5. The consumer

We assume that preferences are represented by a well behaved utility function $U(\cdot)$ defined over the non-traded good consumption, the exportable good consumption, the importable good consumption and money. As we have already discussed in chapter two, money represents future demand for goods. Labour supply L^s is exogenously fixed and does not enter the utility function. For simplicity, the utility function will be of the Cobb-Douglas form:

$$U = x_e^{a}\, x_i^{b}\, x_N^{c}\, m^{f} \qquad 2 \tag{172}$$

The notional demand for x_e, x_i, x_N and m is found by maximizing U subject to the budget constraint

$$P_N x_N + P_e x_e + P_i x_i + m = \bar{w}L^s + \bar{m}\,, \tag{173}$$

where $\bar{m}$ is the stock of money inherited from the past. There is no profit income in (173) because we assume that the government taxes profits at 100 per cent. In any case, since we are interested in the effects of policies within the unit period and profits are distributed at the end of the period, a relaxation of the 100 per cent profit taxes assumption is not harmful. By the usual maximization procedure, the notional demand functions are:

$$x_e = \frac{a(\bar{m} + \bar{w}L^s)}{AP_e}\,, \qquad x_i = \frac{b(\bar{m} + \bar{w}L^s)}{AP_i}\,, \tag{174}$$

$$x_N = \frac{c(\bar{m} + \bar{w}L^s)}{AP_N}\,, \qquad m = \frac{f(\bar{m} + \bar{w}L^s)}{A}\,,$$

where $A = a + b + c + f$.

In the case where the consumer is rationed, the effective demands will be derived by the usual maximization procedure provided that the feasibility set includes the ration constraints. For instance, if he is rationed to sell less than L^S units of labour, say $\bar{L}$, the effective demand functions for consumption goods and money will be given by (174) provided that we substitute $\bar{L}$ for L^S in formulae (174).

6. The government

As we have already discussed , the government may purchase the non-traded good N. The expenditure on this good will be denoted by $G = \bar{P}_N g$, where g is the volume purchased and $\bar{P}_N$ the (fixed) price of this good. It will also ensure that the balance of payments is kept at a fixed level. For simplicity, it will be assumed that the deficit is kept at zero. According to the New Cambridge alternative policies, the government will either impose tariffs keeping the exchange rate fixed, or it will allow the exchange rate to float and impose no tariffs.

Finally, the deficit in the government's budget will be financed through profit taxes and money creation.

7. Keynesian unemployment

In terms of the usual classification of regimes, the New Cambridge corresponds to the regime of Keynesian unemployment i.e. to the regime where the consumer is rationed in the labour market and the non-traded good sector is rationed in the non-traded good market. We have already discussed this matter in section 3 above. Government action will not, by assumption, change the regime-characteristic. This is consistent with the assumption that the policy maker is not allowed to induce "large" variations in the available instruments. Although our model may give rise to several regimes according to the fix-price constellation (see, among others, Malinvaud [36] , Muellbauer and

Portes [38] , Neary [39] , Dixit and Norman [15]) there are arguments according to which Keynesian Unemployment (generalized excess supply) and Suppressed Inflation (excess demand) are the most likely to appear in practice. For a detailed discussion see Malinvaud [36] , p.108-109. More specifically, since we neglect all supply side shocks such as technical progress and exogenous shifts in the terms of trade - these are quite reasonable assumptions provided that we concentrate our attention to short run analysis only - Classical Unemployment (the regime where only consumers are rationed) is not likely to appear. Moreover, ignoring stock accumulation in the production side and assuming that the non-traded good sector never gets priority [3] in the labour market under excess demand for labour, the regime where only firms are rationed ("Underconsumption") is not likely either. For a discussion of these two last assumptions see Muellbauer and Portes [38] and Neary [39] respectively.

Since Suppressed Inflation and Keynesian unemployment are "symmetric" regimes, concentrating our attention to Keynesian unemployment only is not very restrictive since most of the results will be qualitatively similar (but with opposite sign) to both regimes. Let us now write down the equations which characterize the Keynesian unemployment equilibrium.

(a) The labour market

Keynesian unemployment is characterized by excess supply in the labour and non-traded good markets. Hence, the quantity of labour actually sold in the labour market will be determined by the demand side:

$$\bar{L} = \tilde{L}_N(\cdot) + \left(\frac{\bar{w}}{rP_e}\right)^{\frac{1}{r-1}} , \qquad (175)$$

where $\tilde{L}_N(\cdot)$ is the effective demand for labour by the non-traded good sector and $(\bar{w}/rP_e)^{\frac{1}{r-1}}$ is the notional demand for labour by the exportable good sector (see 171). By virtue of the small economy assumption, the exportable good sector will never be rationed in its sales and its effective labour demand will coincide with its notional labour demand. $\tilde{L}_N(\cdot)$ will be specified below.

(b) The non- traded good market

Since excess supply prevails in the non-traded good market, the quantity sold will be similarly determined by the demand side:

$$\bar{X} = \frac{c(\bar{m} + \bar{w}\bar{L})}{A\bar{P}_N} + g, \tag{176}$$

where $c(\bar{m} + \bar{w}L)/A\bar{P}_N$ is the effective demand for the non-traded good by the representative consumer and g is the government demand for this good. In fact, the effective consumer demand is given by (174) where the amount of labour actually sold is given by $\bar{L}$ (see 175) rather than L^S. Now, we are able to specify the effective demand for labour by the non-traded good sector, $\tilde{L}_N(\cdot)$. From (169)

$$\tilde{L}_N = \bar{X}^{\frac{1}{k}}, \tag{177}$$

where $\bar{X}$ is determined by (176). Hence, (175) can be re-written as:

$$\bar{L} = \bar{X}^{\frac{1}{k}} + \left(\frac{\bar{w}}{rP_e}\right)^{\frac{1}{r-1}}. \tag{178}$$

In fact, we can write the two equilibrium conditions (176), (178) together:

$$\bar{L} = \left[\frac{C(\bar{m} + \bar{w}\bar{L})}{A\bar{P}_N} + g \right]^{\frac{1}{k}} + \left(\frac{\bar{w}}{rP_e} \right)^{\frac{1}{r-1}} . \tag{179}$$

(c) The money market

We have already assumed that the government will issue money to satisfy the effective demand for it. Since we will keep the balance of payments fixed, the flow of money will be equal to the governments' deficit:

$$\Delta\bar{m} = \bar{P}_N g_N - (\pi + t_e C_e + t_i C_i - t_e Y_e) \tag{180}$$

where π are the revenue from profit taxes and t_e, t_i are the tariff rates on the traded goods (if any).

The New Cambridge assertion that money supply is completely determined by decisions on fiscal policy amounts to choosing $\Delta\bar{m}$ to clear (180). By the overall budget constraint of the economy and taking into account (180), (175), (176), the condition that the money market is in equilibrium

$$m - \bar{m} = \Delta\bar{m} \tag{181}$$

is redundant.

(d) The balance of payments

The international payments' surplus B is equal to the value of excess supply of traded goods. Since we have normalized world prices at unity and the importable good is not produced at home, we have

$$B = y_e - \tilde{C}_e - \tilde{C}_i, \tag{182}$$

where $\tilde{C}_e$, $\tilde{C}_i$ are the effective demands for the exportable and importable goods respectively and y_e is the domestic supply for the exportable good (which coincides with the notional supply - see (171)).

From (171) and (174), B can be written:

$$B = \left(\frac{\bar{w}}{rp_e}\right)^{\frac{r}{r-1}} - \frac{a}{AP_e}(\bar{m} + \bar{w}\bar{L}) - \frac{b}{AP_i}(\bar{m} + \bar{w}\bar{L}), \tag{183}$$

where $\bar{L}$ is defined by the implicit function (179).

8. Comparative Statics

Conditions (179) and (183) will be used to examine the New Cambridge propositions. In accordance with the New Cambridge approach we will first examine the effects of fiscal policy to reduce unemployment when this is combined _either_ with the exchange rate _or_ with tariffs.

(a) Fiscal Policy and exchange rate floating

Assume that initially the trade surplus B is zero and that the aim of policy is to keep it at this level. Also, assume that there are no tariffs on the traded goods. This implies that the domestic prices of the traded goods are equal to the exchange rate R: $P_e = R$, $P_i = R$. The equilibrium conditions are:

$$\left(\frac{\bar{w}}{rR}\right)^{\frac{r}{r-1}} - \frac{a}{AR}(\bar{m} + \bar{w}\bar{L}) - \frac{b}{AR}(\bar{m} + \bar{w}\bar{L}) = 0, \tag{184}$$

for zero trade deficit, and

$$\bar{L} = \left[\frac{c}{\bar{AP}_N} (\bar{m} + \bar{w}\bar{L}) + g \right]^{\frac{1}{k}} + \left(\frac{\bar{w}}{rR} \right)^{\frac{1}{r-1}} , \qquad (185)$$

for the labour market to clear by quantity rationing. As we have already noticed, the non-traded good market equilibrium condition is contained in (185).

From (185), it is easily verified that an increase in the government's purchase of the non-traded good Δg will exert multiplier effects on employment $\bar{L}$. From (184), an increase in employment tends to worsen the trade deficit, calling for a devaluation to keep the deficit constant at zero level. Devaluation will also affect employment. From (185), an increase in R will reduce the real wage paid by the exportable good sector, causing an increase in the sector's labour demand. This 'second round' increase in employment calls for a new devaluation and so on. Assuming stability (we will discuss this issue later) the system will settle down at a new Keynesian unemployment equilibrium with higher employment and higher domestic prices for the traded goods due to currency depreciation.

Differentiating (184), (185) with respect to g while allowing the exchange rate to change to clear (184) we obtain :

$$\begin{bmatrix} r^* + \frac{a+b}{A}(\bar{L}+1) & -\frac{a+b}{A} \\ r^* & \frac{c^*}{k}\,\frac{c}{A} - 1 \end{bmatrix} \begin{bmatrix} \frac{dR}{dg} \\ \frac{d\bar{L}}{dg} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-c^*}{k} \end{bmatrix}, (186)$$

where $r^* = \frac{r}{1-r}^{\frac{1}{1-r}}$, $c^* = \left[1 + \frac{(1+\bar{L})c}{A}\right]^{\frac{1}{k}-1}$. We have

also normalized the initial exchange rate, government expenditure, money balances, nominal wage and the price of the non-traded good to unity. These normalizations are completely harmless (as it can be easily verified) and facilitate the interpretation considerably.

Assuming that the square matrix in (186) is non singular, we can solve this system with respect to the unknowns dR/dg, $d\bar{L}/dg$ to obtain

$$\frac{d\bar{L}}{dg} = \frac{c^*/k}{D}, \tag{187}$$

$$\frac{dR}{dg} = \frac{\frac{c^*}{k} \frac{a+b}{Ar^* + (a+b)(1+\bar{L})}}{D} \tag{188}$$

$$\text{where } D = 1 - \frac{c^*}{k}\frac{c}{A} - \frac{r^*(a+b)}{Ar^* + (a+b)(1+\bar{L})}. \tag{188$'$}$$

The numerators of (187), (188) are positive scalars. In fact, the numerator of (187) gives the immediate increase in employment after a unit increase in g. The denominator D defined in (188)$'$ can be assimilated to the Keynesian multiplier effect ($^1/_D$) which gives the 'second round' effects on employment due to income effects ($c^* c/kA$) and to the exchange rate change $\left[(a+b)r^*\right] / \left[Ar^* + (a+b)(1+\bar{L})\right]$.

Although, in principle, there is no reason why the case $D < 0$ should be excluded, we will follow the convention $D > 0$ which ensures that an increase in autonomous expenditure combined with a floating exchange rate has a positive effect on employment. In fact it can be easily verified that D > 0 gives the Routh-Hurwitz conditions for stability of the principle which assigns the exchange rate to 'external' balance and the budget deficit to 'internal' balance.

Adopting the traditional geometric interpretation of the assignment principle we can translate the condition D > 0 into the familiar condition which relates the slopes of the employment equation and the external balance locus in the $(\bar{L},R)$ space:

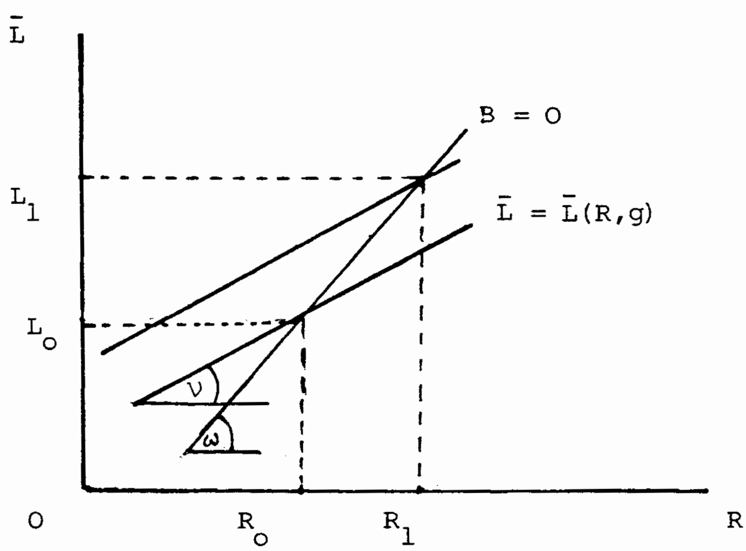

Fig. 8

In the above diagram ω approximates the slope of the locus of the (L,R) points which satisfy equation (184) and ν approximates the slope of the implicit function defined by (185). ($\bar{L} = \bar{L}(R)$ should not be compared to the traditional 'internal' balance locus). The stability condition D > 0 is equivalent to the condition that the slope of the B = 0 locus exceeds that of $L = L(R,g)$: $\nu < \omega$

From Fig.8 it is clear that an increase in g causes an upward shift in the schedule $\bar{L} = L(R)$. If $\nu > \omega$, the new equilibrium would have lower employment while the exchange rate would appreciate. For a detailed derivation of these results see the author's M.Phil. Thesis [46].

The reader may have noticed that we have used the 'conventional' assignment principle, that is the exchange rate to external balance and the government's budget to reduce unemployment, instead of the New Cambridge one. This is not crucial for two reasons: first, stability of the conventional assignment principle does not necessarily imply instability of the New Cambridge one. Second, the arguments of the New Cambridge initiators (see Cripps and Godley [10]) on the relative merits of the alternative Trade Policies do not depend on the New Cambridge assignment principle.

(b) Fiscal Policy combined with tariffs

In section (a), a fiscal expansion to increase employment has been combined with exchange rate floating to keep external balance. Now, we will use tariffs for external balance instead of the exchange rate. These will differ from devaluation because their effect is not 'neutral'. This means that the domestic price ratio of the traded goods differs from the world price ratio as a result of non-uniform trade taxes. To simplify the analysis we will assume that the exchange rate iskept fixed at unity. Furthermore, the importable good will be untaxed ($t_i = 0$) and anad-valorem tariff t_e on the domestically produced traded good will be imposed. It is clear that the domestic price ratio of the traded goods will differ from the world price ratio as a result of such a Trade Policy:

$$\frac{P_i}{P_e} = \frac{(1+t_i)}{(1+t_e)} \frac{R}{R} = \frac{1}{1+t_e} \neq 1 \quad .$$

The equilibrium conditions are similar to (184), (185). We reproduce them here to make clear how the tariff on the exportable good is imposed to keep external balance, while the exchange rate is kept fixed at unity and the importable good is left untaxed :

$$\left(\frac{\bar{w}}{rP_e}\right)^{\frac{r}{r-1}} - \frac{a}{AP_e}(\bar{m} + w\bar{L}) - \frac{b}{A}(\bar{m} + \bar{w}\bar{L}) = 0, \tag{189}$$

$$\bar{L} = \left[\frac{C}{A\bar{P}_N}(\bar{m} + \bar{w}\bar{L}) + g\right]^{\frac{1}{k}} + \left(\frac{w}{rP_e}\right)^{\frac{1}{r-1}}, \tag{190}$$

where $P_e = 1 + t_e$. Following the same arguments as before, we differentiate (189), (190) with respect to g while adjusting the tariff t_e to clear (189).

We again construct a system of two equations in the two unknowns $d\bar{L}/dg$, dPe/dg similar to (186) above. Solving it we obtain:

$$\frac{d\bar{L}}{dg}' = \frac{C^*/k}{D'}, \qquad \frac{dPe}{dg} = \frac{\frac{c^*}{k}\,\frac{a+b}{Ar^* + a(1+\bar{L})}}{D'}, \tag{191}$$

$$\text{where } D' = 1 - \frac{C^*}{k}\,\frac{C}{A} - \frac{r^*(a+b)}{Ar^*+a(1+\bar{L})} \tag{191$'$}$$

The solution has the same interpretation as before. Comparing (188)$'$, (191)$'$ we observe that $D > D'$, which implies that the magnitude of the multiplier is higher under the specific tariff policy we have followed here than a policy of floating the exchange rate: $(1/D) < (1/D')$. Hence, comparing (191), (187) we have

$$(d\bar{L}/dg) \quad < \quad (d\bar{L}/dg)' \qquad (192)$$

The explanation of this is not difficult to find. In the second case, the adjustment to balance of payments equilibrium is achieved via a tariff on the exportable good alone, which means that its domestic price has to increase more than in the floating exchange rate case where the price of the importable good was partly responsible for the restoration of external balance. Since the importable good is not produced at home, a change in its price will not have any effect on the demand for labour (this is also due to the Cobb-Douglas utility function which excludes cross-price effects) adding less to the multiplier process.

A similar result could hold even if we allow for home production of the second traded good. However, the tariff policy would have been much more complicated. A careful selection of trade taxes will depend on the elasticities of the demand for labour by each traded good sector and, in the case of a more general utility function than the one considered in this model, the cross-price effects between the traded and non-traded goods.

What we have shown is that a unit expansion in the government's budget results in a higher reduction in unemployment when it is combined with a tariff policy than an exchange rate policy to keep external balance. This result alone is not sufficient to prove the superiority of tariff policy, since the final change in welfare will also depend on prices.

(c) Welfare implications of the two alternative policies

To examine the Welfare implications of the alternative Trade Policies we should compare the induced changes in the utility of the (single) consumer. On this purpose, we first derive the indirect utility function by substituting (174) into (172), where the labour actually sold is $\bar{L}$ (defined in (179)) and not L^s.

$$V = M\left(\frac{1}{P_e}\right)^a \left(\frac{1}{P_i}\right)^b \left(\frac{1}{\bar{P}_N}\right)^c , \qquad (193)$$

$$\text{where } M = \left(\frac{am^*}{A}\right)^a \left(\frac{bm^*}{A}\right)^b \left(\frac{cm^*}{A}\right)^c \left(\frac{fm^*}{A}\right)^f, \qquad m^* = w\bar{L} + \bar{m}. \qquad (194)$$

Initially, with no tariffs and the exchange rate fixed at unity, the maximum level of utility was $V_o = M$, where not only world prices but domestic prices as well have been normalized to unity: $\bar{w} = 1$, $\bar{P}_N = 1$, $P_e = 1$, $P_i = 1$.

To examine the effects of the first set of policy choices, that is fiscal expansion combined with exchange rate floating, we write (193) as

$$V^F = M \cdot R^{-(a+b)} , \qquad (195)$$

since $P_i = P_e = R$, $\bar{P}_N = 1$.

The superscript F stands for floating. The change in Welfare induced by this set of policy is:

$$\frac{dV^F}{dg} = V^F_L \frac{dL}{dg} + V^F_R \frac{dR}{dg} = V^F_L \frac{dL}{dg} - (a+b)M\frac{dR}{dg} , \qquad (196)$$

where $\frac{dL}{dg}$, $\frac{dR}{dg}$ are given by (187), (188) respectively and the derivative V_R^F has been evaluated at the initial value $R \equiv 1$. (see (195)). Finally, V_L^F is the marginal utility of income.

To examine the effects of the second set of policy choices (fiscal expansion combined with a tariff on the exportable good) we write (193) as:

$$V^T = M\, P_e^{-a} \quad , \tag{197}$$

where the exchange rate, the domestic price of the importable good, the price of the non traded good and the wage rate have been normalized to unity. The superscript T stands for tariff policy.

The change in Welfare induced by this set of policy is:

$$\frac{dV^T}{dg} = V_L^T \frac{d\bar{L}'}{dg} + V_{P_e}^T \frac{dP_e}{dg} = V_L^T \frac{d\bar{L}'}{dg} - aM \frac{dP_e}{dg} \quad , \tag{198}$$

where $\frac{d\bar{L}'}{dg}$, $\frac{dP_e}{dg}$ are given by (191), and the derivative

$V_{P_e}^T$ has been evaluated at the initial value $P_e = 1$. Of course, $V_L^T = V_L^F = V_L$.

To compare the relative merits of the two alternative Trade Policies (devaluation or tariffs) we should find the sign of $\Delta \equiv \frac{dV^T}{dg} - \frac{dV^F}{dg}$.

From (198), (196), (191), (187), (188) we obtain

$$\Delta = V_L \left[\frac{d\bar{L}'}{dg} - \frac{d\bar{L}}{dg} \right] + M \frac{c^*}{k} (a+b) \left\{ \frac{a+b}{E\left[Ar^* + (a+b)(1+\bar{L})\right] - r^*(a+b)} + \right.$$

$$\left. - \frac{a}{E\left[Ar^* + a(1+\bar{L})\right] - r^*(a+b)} \right\} . \tag{199}$$

where $E \equiv 1 - \frac{C^*}{k} \frac{C}{A}$

From (192), the first term in the right hand side of (199) is positive. It can be also verified that the sign of the second term of (199) is given by the sign of the following sum, provided that the assignment principle under both policy alternatives is stable (we have discussed this matter before):

$$N = 1 - \frac{C^*}{k} \frac{C}{A} - \frac{(a+b)\,r^*}{Ar^*} \tag{200}$$

If N were negative, this would imply that a fiscal expansion combined with an export subsidy (tariff) would result in lower employment (i.e. the assignment principle would be unstable) in the case where the exportable good is not consumed at home (a = 0). This is clear from (191), (191)' above. But this contradicts our assumption that the assignment principle is stable. Hence

$$\Delta \equiv \frac{dV^T}{dg} - \frac{dV^F}{dg} > 0, \tag{201}$$

What we have shown is that fiscal expansion combined with tariffs is more effective than fiscal expansion and devaluation, provided that the assignment principle considered here is stable. Now, we can translate this result into the New Cambridge proposition which claims that a tariff policy is more effective than devaluation because it is less inflationary.

(d) Inflationary implications of the two alternative policies

As we have already discussed in the overview of this chapter, the rate of inflation in the New Cambridge model is determined by money wages, import prices and tax rates. Especially, money wage bargaining aims at a real wage target.

In our model, initial utility was V_o. As a result of fiscal expansion, utility becomes $(V_o + dV^F)$ if a policy of exchange rate floating is followed or $(V_o + dV^T)$ if tariffs are used as an alternative to floating. From (201), $V_o + dV^T > V_o + dV^F$. Hence, if money wages change to reconcile some ex ante utility target $\bar{V}$ to the ex post utility $(w^t - w^{t-1} = f(\bar{V} - V^{t-1}),\ f' > 0)$ a tariff policy is less inflationary than devaluation since it entails a higher ex post utility.

It should be noticed that our formulation has the merit that unemployment enters the "real-wage" target (through its effect on the indirect utility function) together with after-tax prices. Hence, the claim by the New Cambridgists that the Phillips-curve is rather positively sloped (in the sense that a lower rate of unemployment is associated with a slower rate of inflation) has a clear micro-economic foundation.

Taking into account the change in the price of the non-traded good, the above results on the inflationary implications of the alternative trade policies will be unchanged under the New Cambridge mark-up pricing equation. More specifically, if we assume constant returns to scale in the non traded good sector, marginal and average cost will be the same and the New Cambridge mark-up on historic costs equation will be

$$p^t = \lambda w^{t-1}, \tag{202}$$

which implies that the non-traded good price changes according to the following rule

$$p^t - p^{t-1} = \lambda(w^{t-1} - w^{t-2}) \tag{203}$$

Since the wage inflation determines the non-traded good price change, none of the above inflationary implications of the alternative trade policies would change. Of course, if we drop the assumption of constant returns to scale we should take into account the effect of output changes on marginal cost and thus on the mark-up equation.

Finally, we have not so far taken into account the fact that the government's budget deficit is financed through money creation. Under the New Cambridge assumptions, money supply does not have any independent influence on prices or on demand (see Cripps and Godley [10]). In our model this is not true unless we pay attention to the effects of policy at a single date only, assuming that the government neutralizes the effects the present policies may have on future dates or that the discount factor is negligible. If medium term considerations are to be seriously taken into account, the analysis should become essentially dynamic. For instance, the welfare criterion should be an intertemporal indirect utility function

$$W = V + \nu \tag{204}$$

where V is the present utility defined in (193) above and ν is the present value of future utility. As future utility will be affected if $\bar{m}$, w and P_N change now, we can write $\nu = \nu(\bar{m}, w, p_N)$.

We will not consider here the intertemporal effects of the alternative Trade Policies. However, the reader may consult section (3.4) of K. Roberts [42] for a detailed analysis of the arguments involved in considering such medium term effects in a somewhat similar context.

Before we close this chapter, we should refer to the views of J. Meade (see [37]) on the New Cambridge propositions. Although he seems to endorse the approach, he claims that the missing instrument from the New Cambridge model is an incomes policy to reduce the rate of wage inflation.

C. DISCUSSION

In this chapter we have tried to provide an equilibrium framework for analyzing the New Cambridge proposition on the relative merits of alternative Trade Policies. More specifically, we have argued that the theory of Welfare where instruments are chosen optimally and under no constraints may not be suitable to the New Cambridge model. An environment where the policy maker can only use a limited number of instruments with a limited range of application may be closer to what the New Cambridge initiators have in mind. If this is acceptable, the New Cambridge instrument-target approach can be given a background of welfare considerations in the sense that it can be conceived as a criterion which ranks alternative Trade Policies according to which gives the maximum desirable change in Welfare for a given change in the available instruments. In fact, we have shown that a fiscal expansion combined with tariffs is more effective than a fiscal expansion and a floating exchange rate in the sense that the induced desirable change in welfare is higher under the former policy than in the second. This result holds provided that the principle of assigning fiscal policy to reduce unemployment and trade policy to keep external balance is stable.

There is a serious objection to this specification. Although some justification can be given to the New Cambridge propositions along the above lines, one may ask why people with strong interventionist views should think in terms of models where the policy maker can only make "small" steps and not choose optimally the available policy instruments.

NOTES

(1). More precisely, a tariff on the exportable good is equivalent to a tax on imports of that good coupled with an equal subsidy on its exports. Although there is no loss of generality in talking about a 'tariff on the exportable good', the above clarification may help in the subsequent discussion.

(2). We include nominal and not real balances in equation (172) to avoid the complications of the response of future prices to present policies. This implies that expectations are inelastic or that agents believe in the temporary character of government policies.

(3). To avoid an Underconsumption regime we have to assume that the traded good sector is never rationed in the labour market.

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