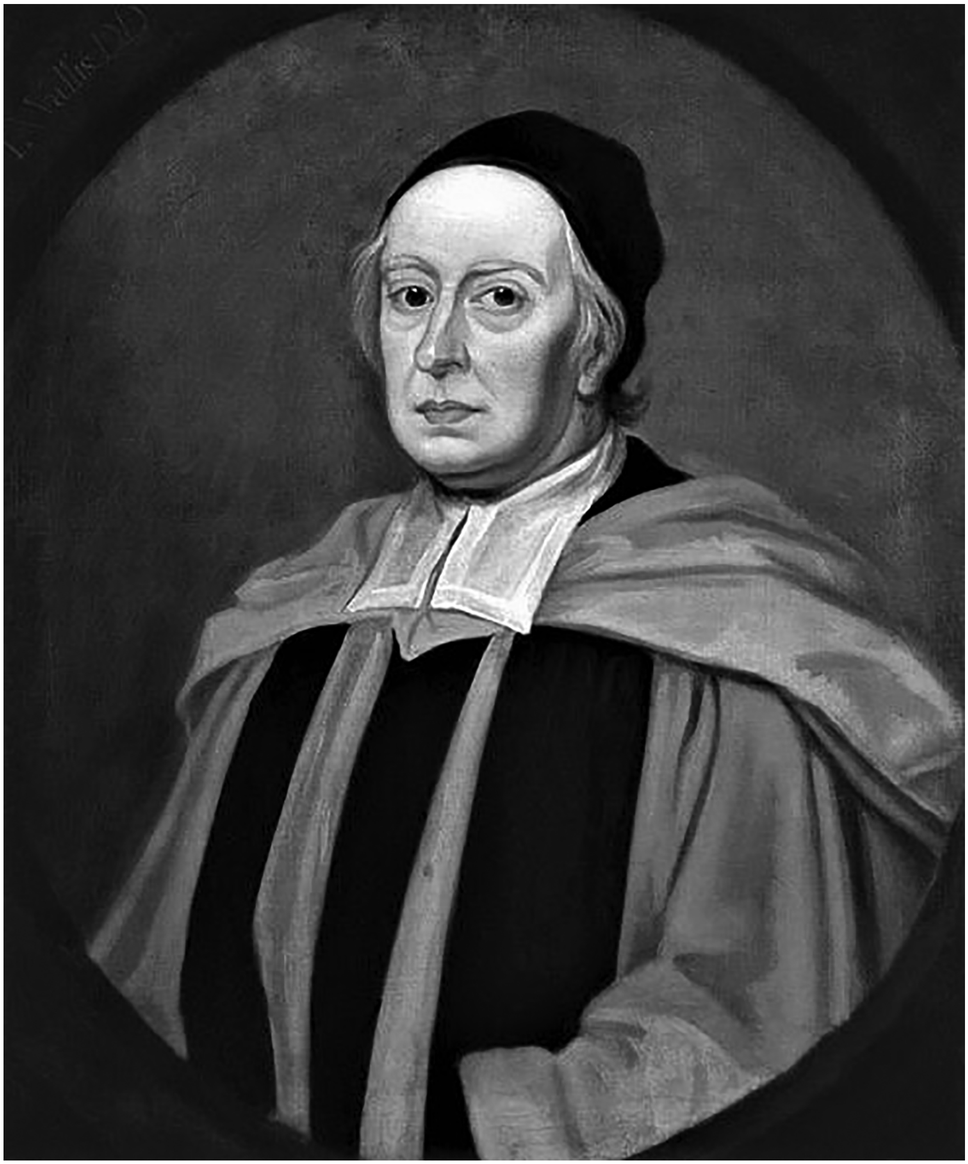


Portrait of John Wallis by Godfrey Kneller (*c.*1702). Samuel Pepys commissioned and paid for this portrait and presented it to Oxford, where it now hangs in the Examination Schools.

CHAPTER 2

John Wallis

PHILIP BEELEY AND BENJAMIN WARDHAUGH

Reading, writing, and doing mathematics in turbulent times, John Wallis (1616–1703) became the Savilian Professor of Geometry in unpromising circumstances, but held that position for longer than any other. Taking seriously the founder's injunctions to study, edit, and publish the ancient mathematical texts, as well as to teach mathematics, he also enjoyed a long career as a robust and combative mathematical author. In this chapter we consider Wallis's achievements as a reader, writer, and shaper of mathematics in the early modern world.

Introduction

In the long history of the Savilian professorships at Oxford, John Wallis's tenure of the geometry chair is unique. Not only has he been the longest serving incumbent – in all, he occupied the position for fifty-four years, from 1649 to 1703 – but no other holder of the post arrived in the way that he did.

Following the purge by the Parliamentary Commissioners of members of the University of Oxford who were deemed to have been too loyal to the old regime, both of the Savilian chairs were vacant by the time that England was declared a republic in May 1649. Wallis, whose services to Parliament as unofficial codebreaker had been both exceptional and successful, and who had made no secret of his desire to embark upon an academic career (preferably in the mathematical sciences), was elected to a Savilian professorship

Philip Beeley and Benjamin Wardhaugh, *John Wallis*. In: *Oxford's Savilian Professors of Geometry*.
Edited by Robin Wilson, Oxford University Press. © Oxford University Press (2022). DOI: 10.1093/oso/9780198869030.003.0002

by the Parliamentary Visitors on 21 June 1649. Allowed the liberty of choosing which chair he preferred, Wallis 'made choise of the Geometry professor the place out of which Dr Turner had been formerly ejected & the place consequently voyd'.[1]

It might seem to us today that, prior to his election as Savilian professor, Wallis had little mathematical experience and no public reputation as a mathematician, his previous appointments having been exclusively theological. As he admitted:[2]

> In the year 1649 I removed to *Oxford*, being then *Publick Professor of Geometry*, of the Foundation of *Sr. Henry Savile*. And *Mathematicks* which had before been a pleasing Diversion, was now to be my serious Study.

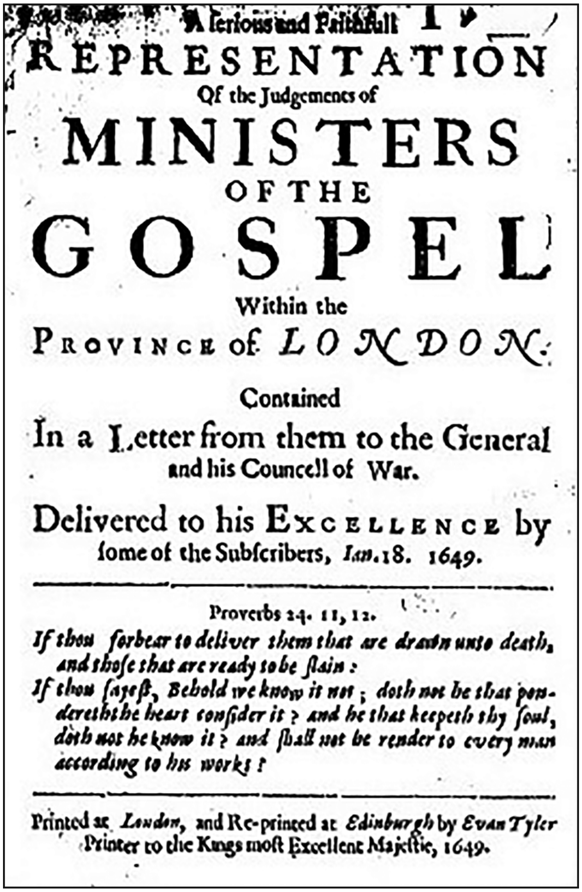
Although he was clearly mathematically talented, questions of political expediency were of overriding importance. Retrospectively, his election to the geometry chair was a remarkable stroke of good fortune, both for mathematics and for Oxford, as will become clear.

Born in 1616, during the reign of King James I, John Wallis experienced the Civil Wars, the Commonwealth, and the Protectorate. Following the Restoration of the monarchy in 1660, he became a leading member of the newly established Royal Society, while continuing to fulfil his professorial duties in Oxford.

He also continued to serve government as the country's leading decipherer, becoming holder of the first such official post, shortly before his death in 1703 during the reign of Queen Anne. As Wallis explained, his survival strategy through the vicissitudes of political life in England was to adopt a course of moderation, but this should not be taken to imply quiescence. It did not, for example, prevent him from adding his name to a pamphlet delivered on 18 January 1649 by more than fifty London ministers of the cloth, denouncing the purging of Parliament by the army and the trial and execution of King Charles I.[3] Nor did it prevent him from becoming embroiled in innumerable scientific disputes – most notably with Thomas Hobbes and with contemporary French mathematicians such as Gilles Personne de Roberval, Blaise Pascal, and Pierre Fermat. As so often with Wallis, his claim to moderation is largely true, but it pays for us to dig deeper.

Early years

We begin with John Wallis's studies at the University of Cambridge in the 1630s. As an undergraduate he attended Emmanuel College, a well-known centre of Puritanism at



A serious and Faithfull
REPRESENTATION
Of the Judgements of
MINISTERS
OF THE
GOSPEL
Within the
PROVINCE of LONDON.
Contained
In a Letter from them to the General
and his Councell of War.
Delivered to his EXCELLENCE by
some of the Subscribers, Jan. 18. 1649.

Proverbs 24. 11, 12.
If thou forbear to deliver them that are drawn unto death, and those that are ready to be slain:
If thou sayest, Behold we know it not; doth not he that pondereth the heart consider it? and he that keepeth thy soul, doth not he know it? and shall not he render to every man according to his works?

Printed at *London*, and Re-printed at *Edinburgh* by *Evan Tyler* Printer to the Kings most Excellent Majestie, 1649.

The 1649 pamphlet to which John Wallis appended his signature.

the time, where his tutors included Benjamin Whichcote and Thomas Horton, and his student contemporaries included Ralph Cudworth, Jeremiah Horrocks, and John Worthington. Wallis pursued the standard undergraduate curriculum, but demonstrated an interest in the new philosophy by attending lectures by Francis Glisson on 'speculative physics' and anatomy. Later, he would claim that the mathematical sciences were all but completely neglected in England's two universities at that time: the following passage in his autobiography has often been quoted:[4]

For Mathematicks, (at that time, with us) were scarce looked upon as Accademical studies, but rather Mechanical; as the business of Traders, Merchants, Seamen, Carpenters, Surveyors of Lands, or the like; and perhaps some Almanak-makers in London.

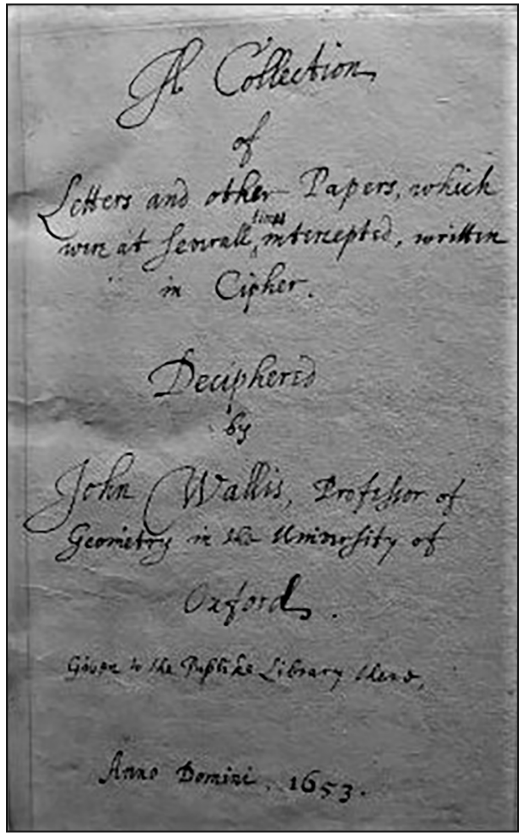
This was clearly an overstatement on his part, for he also recalled having engaged while at Cambridge in 'Astronomy and Geography (as parts of Natural Philosophy) and . . . other parts of the Mathematicks'.

By this time, the two Savilian chairs already existed in Oxford, and we know that most of the colleges at the two universities offered instruction in mathematics, albeit to varying levels. At Cambridge, Wallis excelled in logic; two logical theses that he defended publicly were later appended to his *Institutio Logicae* (Foundation of Logic) of 1687, which became a standard text in that discipline and went through five editions up to 1729.[5]

Unable to gain a fellowship at Emmanuel College, Wallis left Cambridge in 1640, having graduated with a BA degree in 1636/7 and an MA degree shortly before he departed. In the same year he entered holy orders, ordained by Walter Curll, Bishop of Winchester and a close supporter of Archbishop Laud. He subsequently spent formative years in private chaplaincy, first in the Puritan household of the Darley family in Yorkshire. While there, he produced his first significant publication, a philosophical tract entitled *Truth Tried* (1643), which was conceived as a response to Robert Greville's widely circulated and discussed *Nature of Truth* of 1640. Much of this work he rejected, yet he subscribed to Greville's admiration for the pansophic writings of Jan Amos Comenius.

Thereafter, Wallis served as chaplain to Lady Mary Vere, an influential Puritan widow who resided partly in London, partly in the county of Essex. Through her, he was able to establish useful contacts in Parliamentary circles, but the most important of these came about by accident in 1642, when his innate skill in codebreaking was discovered. A visiting Parliamentary army chaplain challenged him to try his hand at an intercepted Royalist cipher. Wallis's unexpected success led to his becoming a prize asset to the Parliamentary cause during the Civil Wars, and subsequently to the Cromwellian government – and, as we have seen, it ultimately secured his professional future at Oxford.[6]

A more immediate result of the favour in which Wallis now found himself was his being presented with a sequestrated living in London and being made secretary or scribe to the Westminster Assembly of Divines, charged with reforming the Church of England. Alongside carrying out his ecclesiastical duties to the satisfaction of those around him, he used his presence in the metropolis to pursue further his interests in the new philosophy. In particular, he began attending regular scientific meetings emanating from Gresham College with like-minded men such as John Wilkins, Jonathan Goddard, Theodore Haak, Samuel Foster, and his former teacher Francis Glisson. Topics in the mathematical sciences, physiology, and all branches of natural philosophy were discussed at these

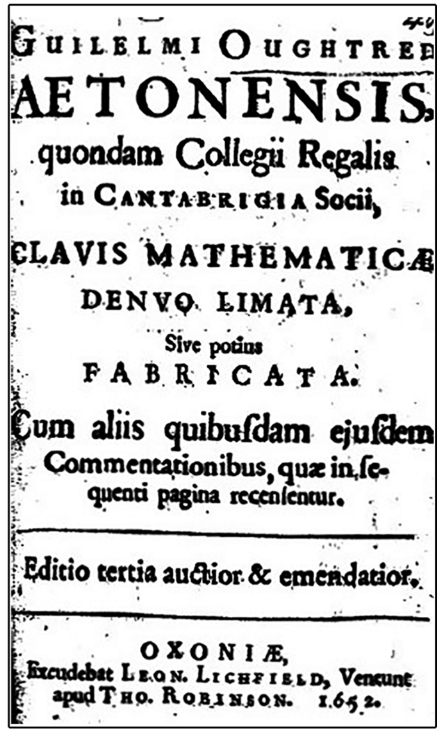

A Collection
of
Letters and other Papers, which
were at severall times intercepted, written
in Cipher.
Deciphered
by
John Wallis, Professor of
Geometry in the University of
Oxford.
Given to the Publike Library there,
Anno Domini. 1653.

The title page of a collection of deciphered letters, many of them connected with the Presbyterian Conspiracy, which Wallis deposited in the Bodleian Library in 1653.

meetings, and occasional experiments were carried out. Wallis would always consider these London meetings, which were probably initiated by Haak in 1645 and which clearly made a great impression on him, to have been the true origin of the Royal Society, founded officially fifteen years later.[7]

Wallis as Savilian Professor

About a year before his election to the Savilian professorship, Wallis had begun to occupy himself seriously with the study of mathematics. We know that he worked his way systematically through William Oughtred's widely read *Clavis Mathematicae* (The Key to Mathematics), first printed in 1631 and significantly expanded in subsequent editions,[8] and developed an early interest in algebraic equations. In this sense largely self-taught, Wallis soon began to conduct investigations of his own on the nature of cubic equations,


GUILELMI OUGHTREDI
AETONENSIS,
quondam Collegii Regalis
in CANTABRIGIA Socii,
CLAVIS MATHEMATICÆ
DENUO LIMATA,
Sive potius
FABRICATA.
Cum aliis quibusdam ejusdem
Commentationibus, quæ in se-
quenti pagina recensentur.

Editio tertia auctior & emendatior.

OXONIÆ,
Excudebat LEON. LICHFIELD, Veneunt
apud THO. ROBINSON. 1652.

(*Left*) William Oughtred (1575–1660).
(*Right*) This 1652 edition of Oughtred's *Clavis Mathematicae* received editorial input from John Wallis.

and happened independently upon Cardan's rules for solving them.[9] Indeed, he achieved such competence that John Smith, Platonist and University Lecturer in mathematics at Cambridge, sought his advice on difficult passages in Descartes's *Géométrie*, to which Wallis had turned soon after completing his study of Oughtred. Here again we see evidence of his early desire to pursue an academic career. Smith was a Fellow of Queens' College, and Wallis was also made a Fellow there in 1644, but had to resign his Fellowship in the following year after his marriage to Susanna Glyde.

Wallis prepared for his future calling in another way, too. Early in 1649 he initiated a scientific correspondence with Johannes Hevelius after the intelligencer and educational reformer Samuel Hartlib had lent him a copy of the Danzig astronomer's study of the Moon's surface, the *Selenographia*, which had been published two years earlier. Wallis had considerable contact with Hartlib around this time, and Jonathan Goddard, a member of Hartlib's circle who knew Wallis through the scientific meetings that they attended together in London, had drawn his attention to the importance of Hevelius's work. Goddard would soon follow Wallis to Oxford, where he was appointed Warden of Henry

Savile's old college, Merton. Interestingly, Wallis's first letter to Hevelius, written just over a month before his election to the Savilian professorship, already contains strong pointers in that direction. Taking note of the enormous labour required in the calculation of astronomical tables, he offered to assist Hevelius. At the same time, he conceded that there were competing pressures to carry out the theological tasks for which he was paid. He would have left Hevelius and others in no doubt about which he would rather be doing.[10]

Buoyed by such strong motivations, Wallis's progress in mathematics was stupendous. Within the space of a few years he produced a series of significant mathematical works, as we now discover.

De Sectionibus Conicis Tractatus (Treatise on Conic Sections)

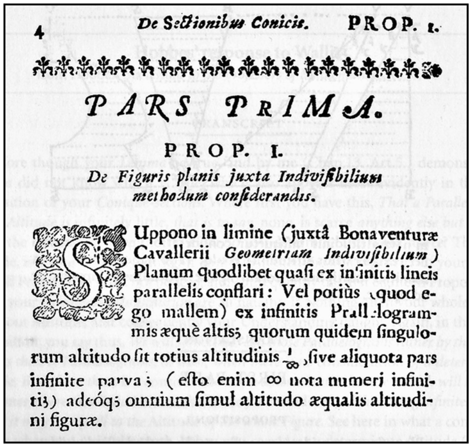
In this work of 1655 John Wallis treated the curves analytically, defining the parabola, the ellipse, and the hyperbola by means of algebraic equations, rather than as slices of a cone.[11] Here he spoke explicitly about treating an old topic by a new method – indeed, about liberating conics from a consideration of the geometrical object. In the second half of this book, he proceeded to find equations of tangents and other properties of the conic sections.

But his prime concern at this time was already with questions of quadrature – that is, finding areas. Indeed, in the very first proposition of his tract he introduced Cavalieri's method of indivisibles as a means of determining the areas enclosed by curves – or more accurately, the ratios of these areas to those of inscribed or circumscribed rectangles, the former being conceived as of infinitely small altitude. In fact, along with a number of his contemporaries, Wallis had not drawn this method from reading Cavalieri, but instead from the more widely distributed interpretation of Cavalieri's method by Evangelista Torricelli: we know that in the early 1650s Wallis systematically worked his way through the 1644 edition of Torricelli's *Opera Geometrica*.[12]

A notable feature of Wallis's book on conic sections is his introduction of new notation. Here we find the first appearances of the symbol ∞ for 'infinity' and the symbol $\geq$ for 'greater than or equal to'.

Arithmetica Infinitorum (Arithmetic of Infinities)

Wallis sought to proceed further with quadratures in another work that appeared shortly thereafter, his *Arithmetica Infinitorum* of 1656;[13] this would become his most widely read mathematical publication and constituted an important stage in the emerging field of



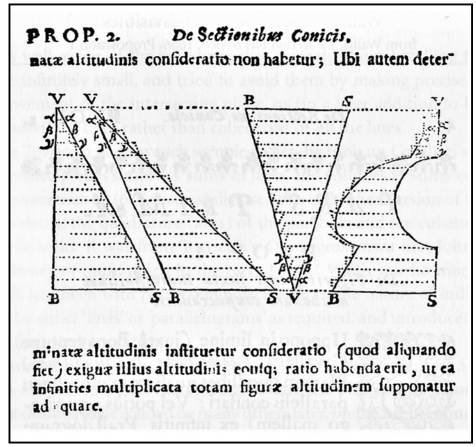
4 *De Sectionibus Conicis.* PROP. 1.

PARS PRIMA.

PROP. I.

De Figuris planis juxta Indivisibilium methodum considerandis.

Suppono in limine (juxtà Bonaventuræ Cavallerii *Geometriam Indivisibilium*) Planum quodlibet quasi ex infinitis lineis parallelis conflari: Vel potiùs (quod ego mallem) ex infinitis Parallelogrammis æquè altis; quorum quidem singulorum altitudo sit totius altitudinis $\frac{1}{\infty}$, sive aliquota pars infinite parva; (esto enim ∞ nota numeri infiniti;) adeóq; omnium simul altitudo æqualis altitudini figuræ.



PROP. 2. *De Sectionibus Conicis.*

natæ altitudinis consideratio non habetur; Ubi autem deter-

minatæ altitudinis instituetur consideratio (quod aliquando fiet) exiguæ illius altitudinis eoníq; ratio habenda erit, ut ea infinities multiplicata totam figuræ altitudinem supponatur adæquare.

Two extracts from *De Sectionibus Conicis Tractatus* of 1655, exhibiting the symbol for infinity and Wallis's use of Cavalieri's method.

analysis. Newton studied it in depth and profitably, but Leibniz came to it late during his sojourn in Paris from 1672 to 1676 after he had already developed the foundations of his infinitesimal calculus.

Instead of using the algebraic techniques that he had developed in *De Sectionibus Conicis*, Wallis now adopted an arithmetical approach to the method of indivisibles. This was where he was perhaps most innovative, for Torricelli had proceeded geometrically, supposing that a plane figure was made up from an infinite collection of lines deemed to compose it, and in similar fashion that a solid was made up from an infinite collection of planes or surfaces.

Wallis saw how the necessary summations could be carried out arithmetically, by finding the sums of increasing sequences of terms such as an arithmetical progression (in the case of a triangle), or of squares or square roots (when determining the area of a parabola); Wallis's sequences began at 0 and had a finite number of terms. Decisively, when dealing with different cases, he moved from dealing with a finite number of steps to infinitely many, corresponding to the supposed composition of areas or volumes by infinitely many indivisibles. Thus, his treatment of quadratures and cubatures came to depend on the summation of infinite sequences of arithmetic infinitesimals, thereby replacing Cavalieri's geometry of indivisibles by his own arithmetic of infinities.

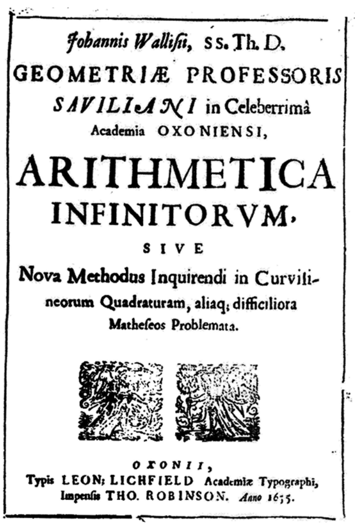
Wallis sought a general formula for the area under a curve of the form $y = x^n$, already known when n is a positive integer, and he extended the result to fractional and negative exponents. Starting with sequences of simple powers, he achieved his goal of finding the area inside a circle by making increasingly sophisticated interpolations. To this end he constructed tables of powers – an approach that was clearly inspired by his work as a codebreaker where there had often been a question of recognizing numerical regularities and patterns – and he arrived at the following infinite fraction for 4/π, which he denoted by □; this is now known as *Wallis's formula*:

$$\frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \text{etc.}}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \text{etc.}}$$

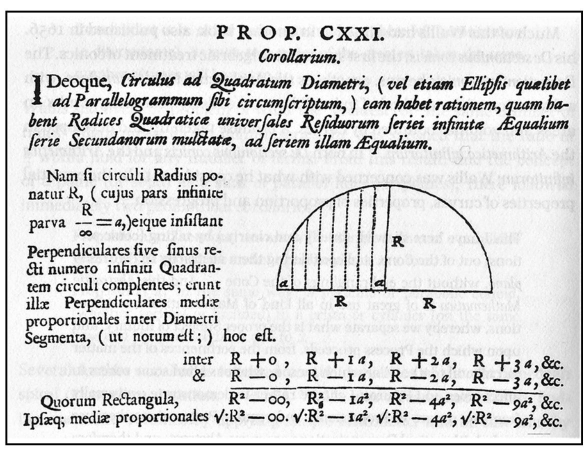
While Wallis's derivation of this product was justly criticized, the result was confirmed by his pupil William Brouncker, who used a more rigorous method based on continued fractions.

As the Savilian Professor of Geometry, Wallis was required to deliver public lectures (open to all members of the University) on Euclid's *Elements*, the *Conics* of Apollonius, and all the known books of Archimedes. Alongside these foundational tasks, deeply rooted in the humanist tradition embodied by Henry Savile, there was to be instruction in other more accessible topics, such as theoretical and practical arithmetic, practical geometry or geodesy, mechanics, and music.

Only in respect of Euclid are we afforded any immediate insight into the form that Wallis gave to his lectures, for there survives a complete transcript in his hand, covering the years 1651 to 1652.[14] Interestingly, these were not introductory lectures, but focused instead on questions of interpretation based on the rich tradition of Euclidean


Johannis Wallisii, SS. Th. D.
GEOMETRIÆ PROFESSORIS
SAVILIANI in Celeberrimâ
Academia OXONIENSI,
ARITHMETICA
INFINITORUM,
SIVE
Nova Methodus Inquirendi in Curvilineorum Quadraturam, aliaq; difficiliora Matheseos Problemata.

OXONII,
Typis LEON: LICHFIELD Academiæ Typographi,
Impensis THO. ROBINSON. *Anno* 1655.


PROP. CXXI.
Corollarium.

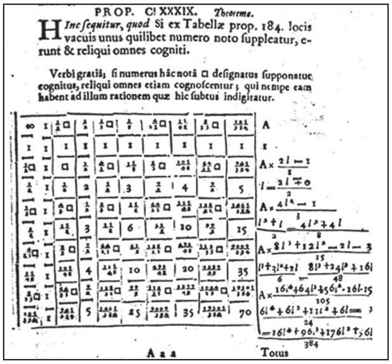
IDeoque, *Circulus ad Quadratum Diametri, (vel etiam Ellipsis quælibet ad Parallelogrammum sibi circumscriptum,) eam habet rationem, quam habent Radices Quadraticæ universales Residuorum seriei infinitæ Æqualium serie Secundanorum mulctatæ, ad seriem illam Æqualium.*

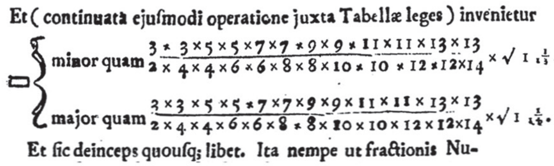
Nam si circuli Radius ponatur R, (cujus pars infinite parva $\frac{R}{\infty} = a$,) eique insistant Perpendiculares sive sinus recti numero infiniti Quadrantem circuli complentes; erunt illæ Perpendiculares mediæ proportionales inter Diametri Segmenta, (ut notum est;) hoc est.

inter R + 0, R + 1a, R + 2a, R + 3a, &c.
& R − 0, R − 1a, R − 2a, R − 3a, &c.

Quorum Rectangula, R² − 00, R² − 1a², R² − 4a², R² − 9a², &c.
Ipsæq; mediæ proportionales √:R² − 00. √:R² − 1a². √:R² − 4a², √:R² − 9a², &c.

(*Left*) The title page of John Wallis's *Arithmetica Infinitorum*, published in 1656.
(*Right*) Proposition 121, in which Wallis investigates the area inside a circle.



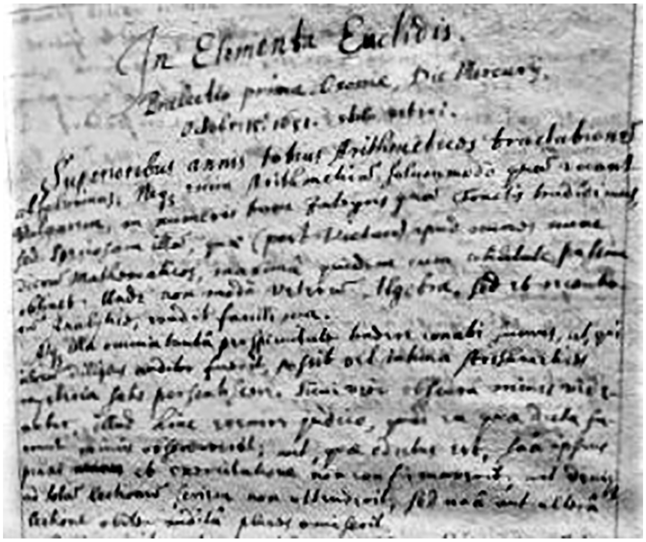


(*Left*) A table from the *Arithmetica Infinitorum*, involving the number □ = 4/π.
(*Right*) Wallis's upper and lower estimates for □ = 4/π.

commentary. Over a number of days, Wallis addressed philosophical topics such as the 'Quaestio de certitudine mathematicarum' (Question of the certainty of mathematics) or a debate over the nature of the angle of contact between two curves. It remains an open question as to how these lectures would have coordinated with college tuition, which was often based on plain editions of the first six books of the *Elements* or on such compendia as Pierre Gautruche's *Mathematicae Totius* (The Whole of Mathematics) of 1668.

Throughout the 17th century, mathematical teaching and learning remained patchy at Oxford and Cambridge. Although there were pockets of excellence that involved committed and able tutors and keen students who went on to become mathematicians of national or international reputation, the majority of those who studied at the English universities were indifferent to mathematics, had little contact with the subject, and acquired little of it by the time they graduated.

Wallis seems to have done little to change this. We search in vain for his students, or for reminiscences of his teaching; the manuscript text of his lectures on Euclid has survived, but we have nothing to indicate that he applied unusual energy to his teaching duties or that he undertook them with more success than his predecessors. But with another of Wallis's early publications we find ourselves in a slightly better position.


In Elementa Euclidis.

John Wallis's handwritten lectures on Euclid's *Elements.*

Mathesis Universalis, sive Opus Arithmeticum (General Mathematics, or Arithmetical Work)

John Wallis's *Mathesis Universalis* of 1657 is an elementary introduction to arithmetic.[15] Clearly the product of lectures that he delivered in his early years as Savilian professor, it is notable in that he traces the history of mathematical notation from antiquity to the present in both the Eastern and Western traditions. This reflects the important role that he ascribed to the history of mathematics in his writings, and also his approach to scientific problems in general:[16]

And (herein as in other studies) I made it my business to examine things to the bottom; and reduce effects to their first principles and original causes. Thereby the better to understand the true ground of what hath been delivered to us from the Antients, and to make further improvements of it.

Wallis dedicated his *Mathesis Universalis* to four of his most powerful Oxford contemporaries: Gerard Langbaine (Provost of The Queen's College), Henry Wilkinson (Canon of Christ Church and Lady Margaret Professor of Divinity), John Wilkins (Warden of Wadham College), and Jonathan Goddard (Warden of Merton College). Like Wallis and Goddard, Wilkins had been intruded by the Parliamentary Visitors, and was now the guiding spirit of Oxford's philosophical club which met regularly in his lodgings in Wadham College. In his dedication Wallis again declared his commitment to the

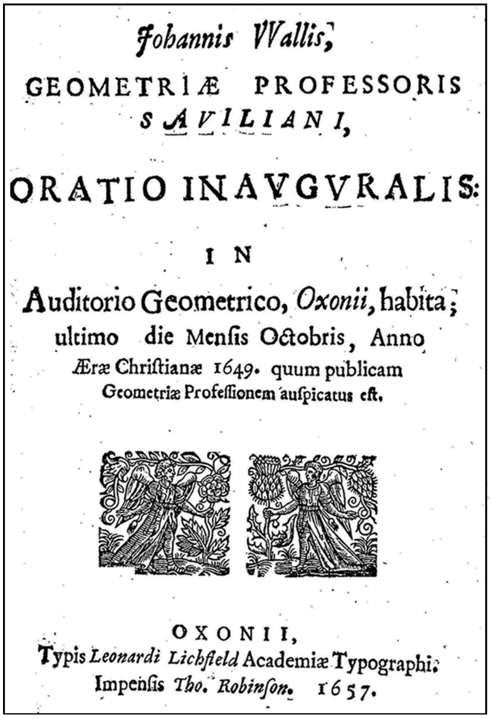
Savilian statutes in nurturing (and above all in promoting) mathematical studies – seeking, for example, to show how some propositions in Book II of Euclid's *Elements* can be effortlessly demonstrated in a purely algebraic manner.[17]

For Henry Savile there had been two blemishes in Euclid's *Elements*. One of these was the fifth postulate (otherwise referred to as the 'parallel postulate'), while the other was the fifth definition of Book VI.[18] Savile charged his successors with addressing these, and Wallis did not neglect to do so. On the evening of 11 July 1663 he delivered a lecture in Oxford on these topics, in which he argued persuasively that Euclid's fifth postulate could be derived from his other axioms. His argument was based on his deduction that if there were a geometry in which the first four postulates hold, but not the fifth, then any two geometrical objects with the same shape (such as two squares) must necessarily have the same size. Since this is clearly nonsensical, he claimed, no such geometry can exist.[19] This attempt by Wallis to derive the parallel postulate as a theorem was to become a notable episode in the history of 'non-Euclidean geometries', later to be investigated by Johann Heinrich Lambert and Girolamo Saccheri. It was not until the 19th century that such strange geometries were studied seriously as consistent mathematical structures in their own right.

Correspondence and controversy

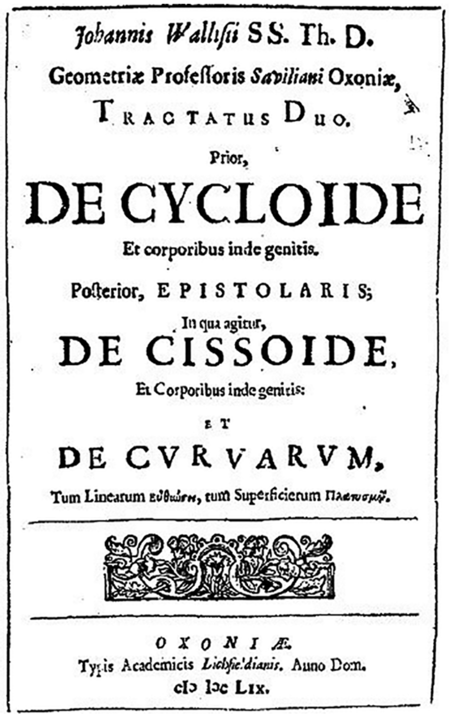
Like no other English mathematician, Wallis grasped his new opportunities for promoting the growth of scientific knowledge through the free exchange of ideas. In his inaugural lecture as Savilian professor, delivered on 31 October 1649 in the Geometry lecture room of the Schools Quadrangle, he presented an overview of the mathematical tradition from the beginning of civilization, and suggested that it was much easier for him and his contemporaries to add to the discoveries of the Greeks than it had been for them to make their discoveries in the first place. His justification for this claim was the variety of means of communication that were now available, 'especially since it is now possible through the benefit of printing for ideas to be communicated across Europe which in those days were almost restricted to Athens'.[20] Alongside print, Wallis wrote many letters, soon adding scholars like Frans van Schooten, Christiaan Huygens, and Pierre Gassendi to his network of correspondents.

But there were disadvantages to such prominence. In coming to personify England's late emergence in discourse on the mathematical sciences, Wallis soon found himself drawn into controversies with mathematicians on the Continent – especially in France – who sought to test his capabilities and the originality of his ideas. Political and



Johannis VVallis,
GEOMETRIÆ PROFESSORIS
SAVILIANI,
ORATIO INAVGVRALIS:
IN
Auditorio Geometrico, *Oxonii,* habita;
ultimo die Menſis Octobris, Anno
Æræ Chriſtianæ 1649. quum publicam
Geometriæ Profeſſionem auſpicatus eſt.

OXONII,
Typis *Leonardi Lichfield* Academiæ Typographi:
Impenſis *Tho. Robinſon.* 1657.



Johannis Wallıſii SS. Th. D.
Geometriæ Profeſſoris *Saviliani* Oxoniæ,
TRACTATUS DUO.
Prior,
DE CYCLOIDE
Et corporibus inde genitis.
Poſterior, EPISTOLARIS;
In qua agitur,
DE CISSOIDE,
Et Corporibus inde genitis:
ET
DE CVRVARVM,
Tum Linearum εὐθύνσει, tum Superficierum πλατυσμῷ.

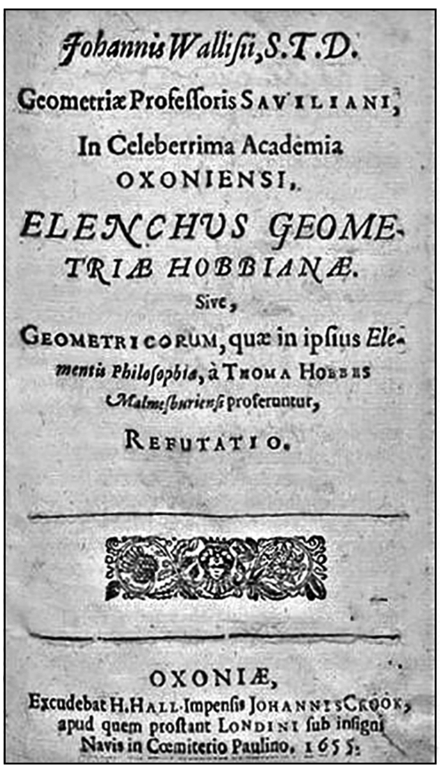
OXONIÆ.
Typis Academicis *Lichfieldianis.* Anno Dom.
CIƆ IƆC LIX.

(*Left*) John Wallis's inaugural lecture as Savilian Professor of Geometry took place in October 1649.
(*Right*) His 1659 tract on the cycloid, in which he presented his solution to questions posed by Pascal.

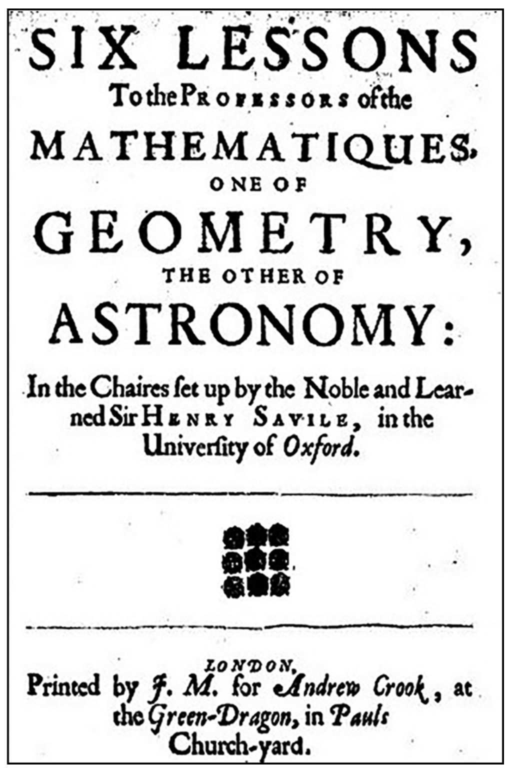
military rivalries between the two nations also helped to fuel a quick succession of disputes with Roberval (over the rectification of the semi-cubical parabola, $ay^2 = x^3$), with Pascal (over the quadrature and cubature of the cycloid), and with Fermat (over questions in number theory).[21] Ultimately, such disputes would serve to dent his belief in the value of open scientific communication. They would also contribute to his unfavourable perception of Leibniz in the priority dispute over the discovery of the calculus, even though his own behaviour was certainly not always above reproach.

There were other disputes, too, but none came to overshadow Wallis's professional career so much as those with the philosopher Thomas Hobbes, which lasted for almost a quarter of a century. Wallis's Oxford colleague Seth Ward, the Savilian Professor of Astronomy, had accused Hobbes of plagiarizing the surviving manuscripts of Thomas Harriot's disciple Walter Warner, and had challenged him to provide evidence of his own mathematical discoveries. Wallis joined the assault on a thinker who was widely suspected in England's intellectual circles of espousing radical materialism, while Hobbes, for his part, had friends in high places, including the King, whom he had instructed in geometry as a young man in Parisian exile.

In his *Elenchus Geometriae Hobbianae* (Examination of Hobbesian Geometry) of 1655, Wallis set out to display to a wider scholarly audience the ineptitude of the solutions to ancient problems such as the quadrature of the circle and the duplication of the cube, which Hobbes had published shortly beforehand in his *De Corpore* (On Body). Hobbes responded in the following year with his acrimonious *Six Lessons to the Professors of the Mathematiques, one of Geometry, the other of Astronomy: In the Chaires set up by the Noble and Learned Sir Henry Savile, in the University of Oxford*. This work attacked both Wallis and Ward, to which Wallis in turn wrote a reply entitled *Due Correction for Mr Hobbes*. This intellectual war, in which belittlement of the opponent was increasingly used as a rhetorical tool, did more harm than credit to both men and continued on and off until Hobbes's death in 1679.[22]

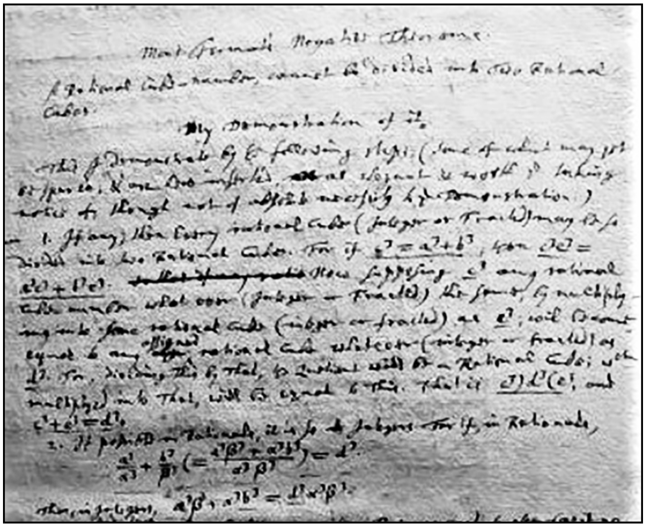

Johannis Wallisii, S.T.D.
Geometriæ Professoris SAVILIANI,
In Celeberrima Academia
OXONIENSI,
ELENCHUS GEOMETRIÆ HOBBIANÆ.
Sive,
GEOMETRICORUM, quæ in ipsius Elementis Philosophiæ, à THOMA HOBBES Malmesburiensi proferuntur,
REFUTATIO.

OXONIÆ,
Excudebat H. HALL. Impensis JOHANNIS CROOK, apud quem prostant LONDINI sub insigni Navis in Cœmiterio Paulino. 1655.


SIX LESSONS
To the PROFESSORS of the
MATHEMATIQUES.
ONE OF
GEOMETRY,
THE OTHER OF
ASTRONOMY:
In the Chaires set up by the Noble and Learned Sir HENRY SAVILE, in the University of Oxford.

LONDON,
Printed by J. M. for Andrew Crook, at the Green-Dragon, in Pauls Church-yard.

(*Left*) John Wallis's *Elenchus Geometriae Hobbianae*.
(*Right*) Thomas Hobbes's *Six Lessons to the Professors of the Mathematiques*.



John Wallis's manuscript on Fermat's negative theorem: 'A rational cube cannot be divided into two rational cubes'.

John Wallis was always keen to make his mark beyond Britain's shores, and his work as a linguist was conceived directly with this aim in view. In 1653 he published his *Grammatica Linguae Anglicanae* (Grammar of the English Language), a largely traditional grammar with minor innovations, produced with the core purpose of providing an easy introduction to the English language for foreigners, at a time when the language was scarcely read abroad. He prefixed this work with what he considered to be a universal treatise on speech, *De Loquela*, which later served him as a model for the practical instruction of deaf mute individuals. But here again Wallis soon found himself in conflict, this time with the English clergyman and music theorist William Holder, who felt that his own efforts at such instruction had been disregarded. What made this dispute all the more difficult was that the two protagonists were both highly respected and scientifically active members of the Royal Society. It therefore raised questions about the very principles for which that institution stood.[23]

As revealed earlier through his work on logic, Wallis was a master of reasoned argument who repeatedly put his evident skills in dispute and refutation to use in his academic endeavours. He also evidently considered it important, and proper for him as Savilian professor, to play a part in policing the borders and the content of mathematics.

One telling example is the case of the Danish scholar Marcus Meibom, who in the early 1650s had prepared Greek and Latin editions of the entire corpus of Greek musical texts (barring three added later by Wallis); he was then in his 20s, a precocious, careful, and by most standards, brilliant scholar. In the wake of this musical editing project he took it upon himself to write a book-length account of mathematical ratios, an active subject of research that was of much relevance to the study of music, which often involved the elaborate consideration of the ratios of lengths of sounding strings. His book on ratios took the form of a dialogue whose participants included Euclid himself; Meibom's main point was that the proper manipulation and combination of ratios was being carried out wrongly by some of his contemporaries.

The merit of Meibom's arguments is moot, although they were not obviously absurd and his position concerning ratios was defensible. But his book, which appeared in 1655, raised the wrath of several mathematicians, including that of Wallis who proceeded to write *Adversus Marci Meibomii, De Proportionibus Dialogum Tractatus Elencticus* (Didactic Treatise against Marcus Meibom's Dialogue on Ratios). It was in print by 1657 and could scarcely have been more offensive about the book's alleged absurdities – at one point, Wallis said he initially thought Meibom was joking.[24]


Johannis Wallisii SS. Th. D.
Geometriæ Professoris Saviliani,
In Celeberrima Academia
OXONIENSI;
ADVERSUS
MARCI MEIBOMII,
De Proportionibus Dialogum,
Tractatus Elencticus.
OXONII,
Typis Leonard. Lichfield Academiæ Typographi.
Impensis Tho. Robinson. 1657.

John Wallis's *Adversus Marci Meibomii, De Proportionibus Dialogum Tractatus Elencticus* of 1657.

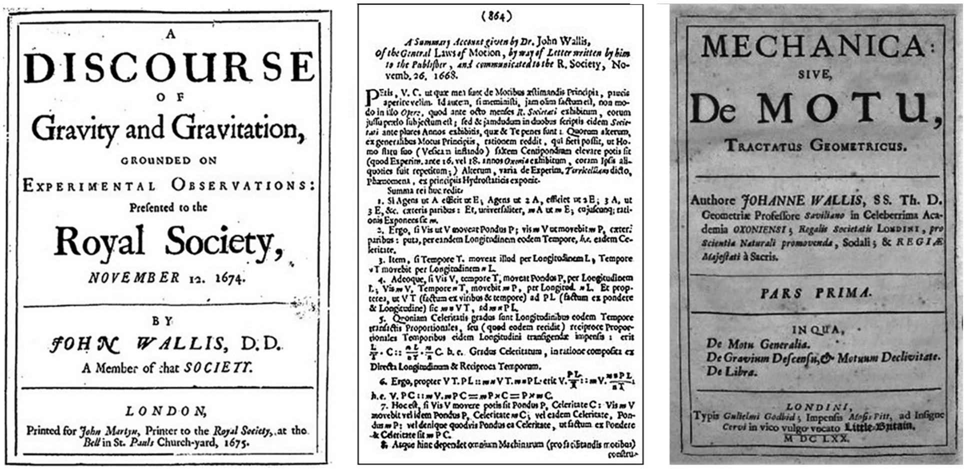
There is an air of a storm in a teacup about Meibom's book on ratios and the four responses that it elicited, but such things mattered in an era before peer review or printed book notices. Meibom never published on mathematics again; indeed, Wallis's disapproval in particular seems to have ended his attempt to enter the international community of publishing mathematicians. As with his more celebrated dispute with Hobbes, Wallis gave the strong impression that he considered it his role both to defend and to define the discipline of mathematics, pronouncing on who was allowed to participate and who was not. No one seems to have disputed his right to exercise that judgement, even though some would disagree with the judgements themselves.

Wallis and the Royal Society

Wallis relied on correspondence to keep abreast of developments in the scientific world, and to play an active part in discussions that were taking place at a considerable distance away in London. Henry Oldenburg, the Royal Society's corresponding secretary, was a decisive figure in this respect, communicating Wallis's frequent letters on mathematical and natural philosophical topics to members attending the weekly meetings and acting as an intermediary in subsequent exchanges. In return, Oldenburg supplied the Savilian professor regularly with the latest news, gleaned from his own extensive correspondence, while also occasionally sending him materials for perusal and assessment.[25] Alongside John Pell and John Collins, Wallis served as Oldenburg's foremost adviser on mathematical topics.

Despite being based in Oxford, Wallis was therefore able to become one of the most active members of the Royal Society, right through to the end of his life, and on more than one occasion he was called upon to help revive the institution when periods of non-participation of members threatened its collapse.[26] Some seventy articles or book reviews published in the Society's *Philosophical Transactions* are either under his name or can be reliably ascribed to him as author. Most are on mathematical topics, but taken as a whole they reflect the full breadth of his scientific interests: thus, alongside papers on the method of tangents or the true division of the meridians in a sea-chart, we find observational accounts of the tides in Kent (the county from which he hailed) or on the suspension of quicksilver in the Torricellian tube. He delivered numerous learned contributions that sought to explain natural phenomena based on experimental observation, including an important discourse on gravity, published in 1674.[27]

Wallis also contributed to a Royal Society debate on the laws of motion in 1668–69, in which his 'summary account of the general laws of motion' appeared alongside


A DISCOURSE OF Gravity and Gravitation, GROUNDED ON EXPERIMENTAL OBSERVATIONS: Presented to the Royal Society, NOVEMBER 12. 1674.

BY JOHN WALLIS, D.D. A Member of that SOCIETY.

LONDON, Printed for John Martyn, Printer to the Royal Society, at the Bell in St. Pauls Church-yard, 1675.

(864)

A Summary Account given by Dr. John Wallis, of the General Laws of Motion, by way of Letter written by him to the Publisher, and communicated to the R. Society, November. 26. 1668.

MECHANICA: SIVE, De MOTU, TRACTATUS GEOMETRICUS.

Authore JOHANNE WALLIS, SS. Th. D. Geometriæ Professore Saviliano in Celeberrima Academia OXONIENSI; Regiæ Societatis LONDINI, pro Scientia Naturali promovenda, Sodali; & REGIÆ Majestati à Sacris.

PARS PRIMA.

IN QUA, De Motu Generalia. De Gravium Descensu, & Motuum Declivitate. De Libra.

LONDINI, Typis Gulielmi Godbid; Impensis Mosis Pitt, ad Insigne Cervi in vico vulgo vocato Little-Britain. M DC LXX.

(*Left*) Wallis's *A Discourse of Gravity and Gravitation, Grounded on Experimental Observations*, presented to the Royal Society on 12 November 1674.
(*Centre*) His 'A summary account of the general laws of motion'.
(*Right*) His *Mechanica* of 1670–71.

those of Christopher Wren and (somewhat later) of Christiaan Huygens. Wallis derived these laws while he was working on his three-part *Mechanica: sive, De Motu, Tractatus Geometricus* (Mechanics: or a Geometrical Treatise on Motion) of 1670–71, published under the auspices of the Royal Society. This was a milestone in the mathematization of mechanics, containing a large section on the determination of centres of gravity. Moreover, it attracted the attention of Leibniz, who studied it in detail while in Paris, and who concurred with Wallis on his principle of the proportionality between effects and their sufficient cause, citing this as representing the gateway from mathematics to physics.[28]

Wallis had earlier been called upon by the Royal Society to review Leibniz's youthful tract *Hypothesis Physica Nova* (New Physical Hypothesis) of 1671. His enthusiastic account, in which he drew attention to considerable areas of agreement with his own ideas, contributed decisively to Leibniz's election to the Royal Society in 1673. Significantly, when discussing the causes of gravity and elasticity, for which Leibniz had proposed mechanistic explanations, Wallis took recourse to fundamental ideas on the nature of scientific truth, which again reflected his self-ascribed moderation:[29]

I am not one to rush in as an umpire where there is so great diversity of opinion. The question must be left to time and the arguments of the learned on either side. Indeed, almost the same thing happens with the swings of pendulums; after many oscillations on either side, at last they come to rest in the perpendicular.

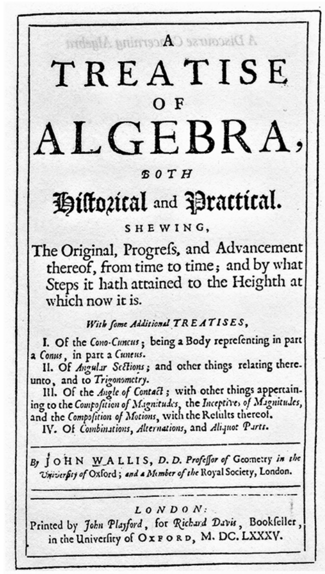
We have seen this with the Copernican hypothesis which, though known to the ancients, lay buried so long that it was regarded as new; and although it had the strongest possible support it did not at once prevail but was attacked by different persons in different ways and bitterly disputed, until in the end through the ascendancy of reason over authority it was so universally acknowledged that virtually no one with any knowledge of the matter has any doubt about it, except those swayed by the Cardinals' decree.

Historical studies and classical editions

Moderation was less in evidence in Wallis's *Treatise of Algebra, both Historical and Practical* of 1685, comprising one hundred short chapters that ranged over algebra and its history.[30] As a history of algebra, the book was the first of its kind and included useful and informative discussions on methods of quadrature and on infinite series, as well as on the developing concept of mathematical proof. However, his *Treatise* was heavily biased toward English contributions to algebra, especially from Harriot and Pell, and in it Wallis set out, not for the first time, the unsubstantiated and false claim that Descartes had plagiarized the former in his *Géométrie*.[31]

One topic of interest in the *Treatise* was Wallis's geometrical construction for the square root of a quantity bc, when b and c are both positive numbers.[32] His construction involved drawing a circle with diameter AC of length $b + c$ and constructing a perpendicular from the point at distance b from A; the length of this perpendicular is then the required square root of bc. Wallis subsequently attempted to modify his process (not entirely satisfactorily) so as to construct the square root of bc when b and c have opposite signs, thereby hinting at the idea of an imaginary number.

Original mathematical publications were only part of Wallis's output in the discipline. In the mid-1660s he had been tasked by the Royal Society with collecting and selecting for publication the letters and papers of the Liverpool astronomer Jeremiah Horrocks, his prematurely deceased contemporary at Cambridge; this was his first foray into mathematical editing, but it almost failed for lack of money.[33] Horrocks's astronomical observations and ideas were rightly judged important enough to devote real effort to their presentation in print, and it was Wallis who undertook much of the work of arranging the texts for publication, also receiving assistance from John Collins and the astronomer John Flamsteed. Eventually, his edition of the posthumous works appeared in 1673 and included short pieces by William Crabtree and Flamsteed.

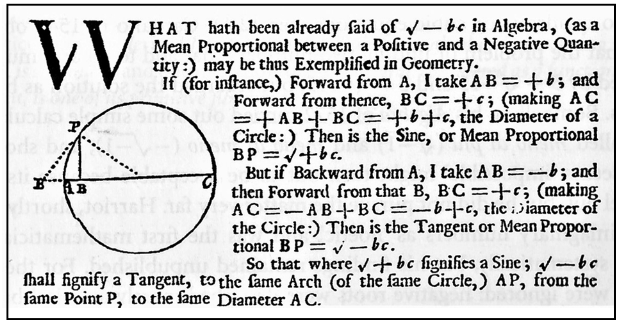

A
TREATISE
OF
ALGEBRA,
BOTH
Hiſtorical and Practical.
SHEWING,
The Original, Progreſs, and Advancement thereof, from time to time; and by what Steps it hath attained to the Heighth at which now it is.

With ſome Additional TREATISES,

I. Of the *Cono-Cuneus*; being a Body repreſenting in part a *Conus*, in part a *Cuneus*.
II. Of *Angular Sections*; and other things relating thereunto, and to *Trigonometry*.
III. Of the *Angle of Contact*; with other things appertaining to the *Compoſition of Magnitudes*, the *Inceptives of Magnitudes*, and the *Compoſition of Motions*, with the Reſults thereof.
IV. Of *Combinations*, *Alternations*, and *Aliquot Parts*.

By JOHN WALLIS, *D. D. Profeſſor of* Geometry *in the Univerſity of* Oxford; *and a Member of the* Royal Society, London.

LONDON:
Printed by *John Playford*, for *Richard Davis*, Bookſeller, in the Univerſity of OXFORD, M. DC. LXXXV.



WHAT hath been already ſaid of $\surd - bc$ in Algebra, (as a Mean Proportional between a Poſitive and a Negative Quantity:) may be thus Exemplified in Geometry.

If (for inſtance,) Forward from A, I take $AB = +b$; and Forward from thence, $BC = +c$; (making $AC = +AB + BC = +b+c$, the Diameter of a Circle:) Then is the Sine, or Mean Proportional $BP = \surd + bc$.

But if Backward from A, I take $AB = -b$; and then Forward from that B, $BC = +c$; (making $AC = -AB + BC = -b+c$, the Diameter of the Circle:) Then is the Tangent or Mean Proportional $BP = \surd - bc$.

So that where $\surd + bc$ ſignifies a Sine; $\surd - bc$ ſhall ſignify a Tangent, to the ſame Arch (of the ſame Circle,) AP, from the ſame Point P, to the ſame Diameter AC.

(*Left*) John Wallis's *A Treatise of Algebra, both Historical and Practical* of 1685.
(*Right*) His construction of a square root, from *A Treatise of Algebra.*

In 1676 Wallis brought out an edition of Archimedes' 'Sand-reckoner' (*Arenarius*) and 'Measurement of the Circle' (*Dimensio Circuli*), together with Eutocius's commentary on the latter. Later, in 1688, there followed critical editions of other Greek texts, including Aristarchus's work on the size and distance of the Sun and the Moon, and a previously unknown fragment from Book 2 of Pappus's *Collection* that set new standards in English textual scholarship.[34]

Wallis was also involved with a long-running project in Oxford to produce a new edition of the works of Euclid. His colleague in the astronomical chair, Edward Bernard, assembled to this end a vast quantity of textual and annotation material in a somewhat chaotic manner, and sought unsuccessfully to find a publisher. When the project was restarted with Bernard's successor David Gregory as named editor, Wallis was tasked with making sure that this time everything went to plan, the magnificent edition eventually being published in 1703.[35]

In addition to all this, Wallis developed and maintained throughout his academic life an interest in the mathematical theory of music. This was another subject on which there were ancient Greek texts of great interest, most of which had been edited in their original language and translated into Latin by the middle of the 17th century. Music, or 'harmonics', was one of the seven liberal arts and remained notionally part of university arts curricula into the 17th century, being mentioned by name in the

Savilian statutes. Wallis evidently considered this subject part of his purview and, as well as writing letters and papers on such topics as the classical divisions of the scale, he also prepared editions of the three Greek texts that had not been included in Meibom's mid-17th century edition.[36] Thus in 1682 he brought out the three books of Claudius Ptolemy's *Harmonics* in Greek and Latin, with an extensive appended commentary.

Wallis reprinted this work in the final volume of his monumental *Opera Mathematica* (Mathematical Works), whose three volumes were published by Oxford's University Press at the Sheldonian Theatre between 1693 and 1699. Also included were an edition of Porphyry's musical commentary and an edition of the *Harmonics* of Manuel Bryenne. These were sophisticated and learned scholarly editions, bringing together the readings of eleven different manuscripts for the Ptolemy text, and reporting this evidence with a precision that was innovative in its day; indeed, they remained the best editions of the three texts until the 20th century.

Cryptographer

In earlier years, Wallis and Leibniz had corresponded through Oldenburg as intermediary, but in the 1690s, when they began to exchange letters directly, two themes were in the foreground: cryptography, and the priority dispute between Newton and Leibniz over the discovery of the calculus. Both were motivated by Leibniz's interests at the time, but they had also come to dominate Wallis's intellectual activity in his final years.

Wallis had been engaged by successive governments as a codebreaker, and this was doubtless a substantial reason for his remaining in post as Savilian Professor of Geometry throughout the Restoration and beyond. The regicide Thomas Scot, in an account of his own intelligencing activity provided shortly before his execution, vouched for Wallis's skill as a cryptographer, indicating the disinterested nature of his work and emphasizing his value to any future government:[37]

> The Kings transactions with the Presbyterian Ministers . . . were made knowne to mee . . . very much more by letters intercepted which commonly were every word & syllable in Cypher, and deciphered by a learned gentleman incomparably able that way, Doctor Wallis of Oxford (who never concerned himself in the matter, but only in the art & ingenuity); it is a jewell for a Princes use & service in that kind.

Johannis Wallis S. T. D.
Geometriæ Profeſſoris SAVILIANI, in Celeberrima
Academia OXONIENSI,

OPERUM MATHEMATICORUM

Volumen Tertium.

QUO CONTINENTUR

CLAUDII PTOLEMÆI
PORPHYRII
MANUELIS BRYENNII } Harmonica:

ARCHIMEDIS { Arenarius, & Dimenſio Circuli;

Cum EUTOCII Commentario:

ARISTARCHI SAMII, de Magnitudinibus & Diſtantiis
Solis & Lunæ, Liber:

PAPPI ALEXANDRINI, Libri Secundi Collectaneorum,
hactenus deſiderati, Fragmentum:

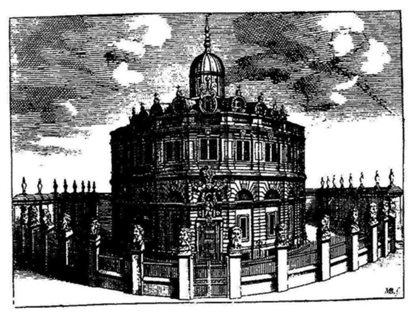
Græce & Latine Edita, cum Notis.

ACCEDUNT

EPISTOLÆ nonnullæ, rem Mathematicam ſpectantes;

ET

OPUSCULA quædam MISCELLANEA.



OXONIÆ,

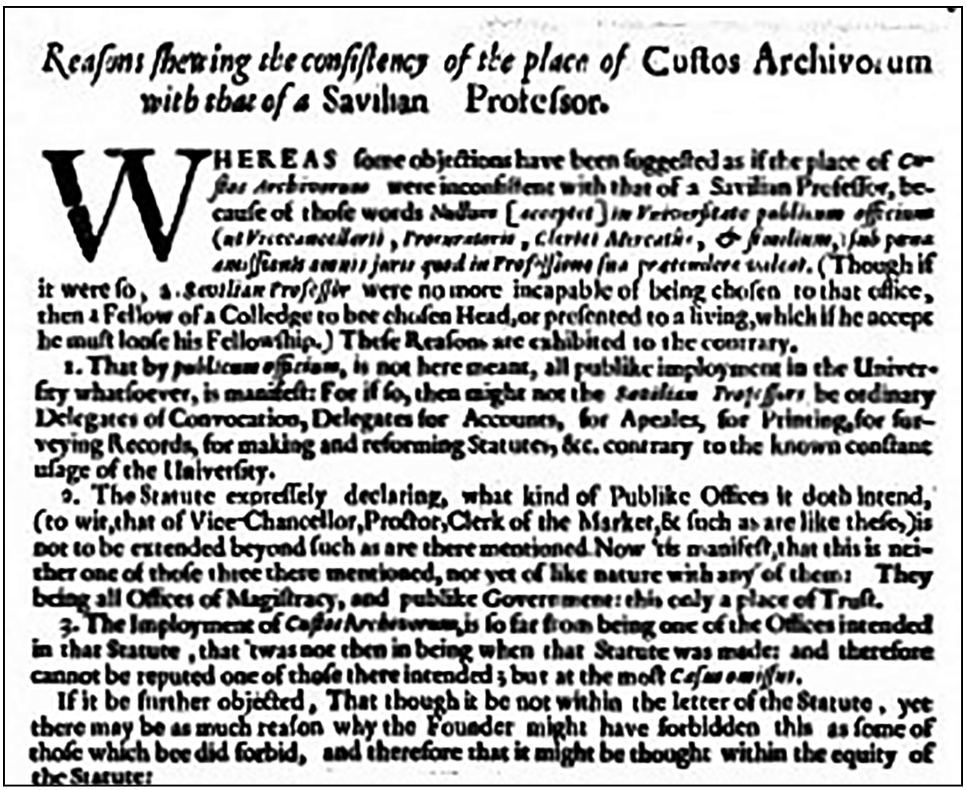
E THEATRO SHELDONIANO, An. Dom. MDCXCIX.

John Wallis's *Opera Mathematica*, published at the Sheldonian Theatre, attest to a remarkable career of promoting mathematical studies and publishing at Oxford. This third volume contained his contributions to the Oxford ancient texts project: his editions of works by Ptolemy, Archimedes, Aristarchus, Pappus, and others, and the musical items of Porphyry and Bryenne.

The accession of William III to the throne in 1689 brought with it an increased workload for supplying political and military intelligence, particularly for the allied German state of Brandenburg–Prussia. Leibniz, who openly described Wallis as Europe's greatest living cryptographer, sought unsuccessfully to persuade the English mathematician to divulge his methods as a way of enriching his philosophical programme of the 'art of discovery'. At the same time, Wallis used the publication of his *Opera Mathematica* in the 1690s not only to present examples of his cryptographic skill, but also to set out a rather tendentious factual record of what Newton had revealed to Leibniz of his method of fluxions in 1676. This, more than anything else, led to an irreparable breakdown in trust between the two men.[38]

Survivor

Wallis's professional career was a model of survival in turbulent times, an intricate construct of duties and responsibilities to the State and to his academic institution. He combined his wide-ranging scholarly endeavours with the assiduous defence of Oxford's ancient privileges through the office of *Custos Archivorum* (Keeper of the Archives), to



Reaſons ſhewing the conſiſtency of the place of Cuſtos Archivorum *with that of a* Savilian Profeſsor.

WHEREAS ſome objections have been ſuggeſted as if the place of *Cuſtos Archivorum* were inconſiſtent with that of a Savilian Profeſſor, becauſe of thoſe words *Nullum [acceptet] in Univerſitate publicum officium (ut Vicecancellarii, Procuratoris, Clerici Mercatûs, & ſimilium,) ſub pœna amittendi omnis juris quod in Profeſſione ſua pratendere valeat.* (Though if it were ſo, a *Savilian Profeſſor* were no more incapable of being choſen to that office, then a Fellow of a Colledge to bee choſen Head, or preſented to a living, which if he accept he muſt looſe his Fellowſhip.) Theſe Reaſons are exhibited to the contrary.

1. That by *publicum officium*, is not here meant, all publike imployment in the Univerſity whatſoever, is manifeſt: For if ſo, then might not the *Savilian Profeſſors* be ordinary Delegates of Convocation, Delegates for Accounts, for Apeales, for Printing, for ſurveying Records, for making and reforming Statutes, &c. contrary to the known conſtant uſage of the Univerſity.

2. The Statute expreſſely declaring, what kind of Publike Offices it doth intend, (to wit, that of Vice-Chancellor, Proctor, Clerk of the Market, & ſuch as are like theſe,) is not to be extended beyond ſuch as are there mentioned. Now 'tis manifeſt, that this is neither one of thoſe three there mentioned, nor yet of like nature with any of them: They being all Offices of Magiſtracy, and publike Government: this only a place of Truſt.

3. The Imployment of *Cuſtos Archivorum*, is ſo far from being one of the Offices intended in that Statute, that 'twas not then in being when that Statute was made: and therefore cannot be reputed one of thoſe there intended; but at the moſt *Caſus omiſſus*.

If it be further objected, That though it be not within the letter of the Statute, yet there may be as much reaſon why the Founder might have forbidden this as ſome of thoſe which hee did forbid, and therefore that it might be thought within the equity of the Statute:

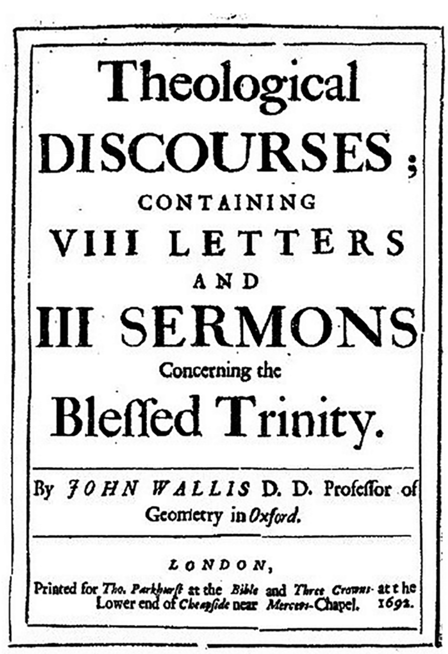
John Wallis's, 'Reasons Shewing the Consistency of the Place of Custos Archivorum with that of a Savilian Professor'.

which he had been elected through an act of strategic, but not wholly legitimate, foresight in 1658.[39]

Quite simply, Wallis soon made himself indispensable to the University, both academically and institutionally. He was one of the Royal Society's most active members, and was always at hand to provide support in times of crisis. He was an important adviser to churchmen on theological issues, and for historical and religious reasons fiercely opposed the introduction of the Gregorian calendar in England. Over the course of his lifetime he published numerous theological discourses and sermons, most of which he had delivered in the University Church of St Mary.[40]

Finally, he had been a reliable servant of government, ultimately becoming the country's first official decipherer. Wallis himself saw the key to his professional survival as having been his ability to avoid the extremes of contemporary politics, and at the end of his autobiography, *Pro Vita Sua*, penned for his friend Thomas Smith on 29 January 1696/7 (8 February 1697), he summed up his life and times in the following words:[41]

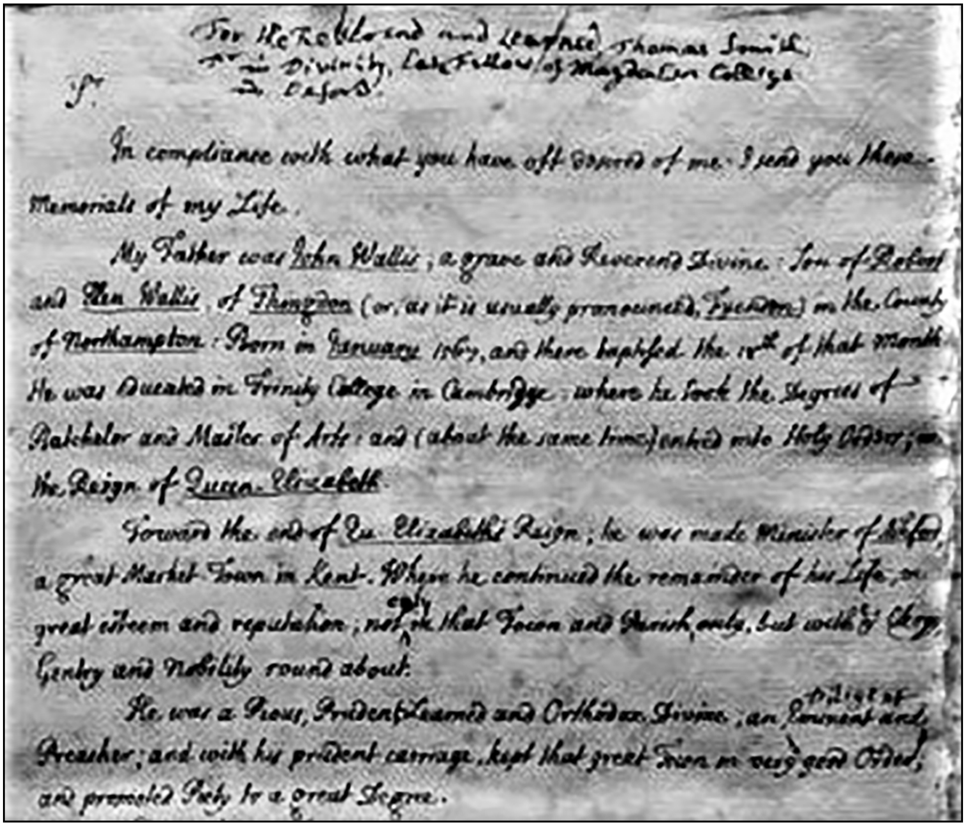
It hath been my Lot to live in a time, wherein have been many and great Changes and Alterations. It hath been my endeavour all along, to act by moderate Principles, between the Extremities on either hand, in a moderate compliance with the Powers in being, in those places, where it hath been my Lot to live, without the fierce and violent animosities usual in


Theological
DISCOURSES;
CONTAINING
VIII LETTERS
AND
III SERMONS
Concerning the
Bleſſed Trinity.

By *JOHN WALLIS* D. D. Profeſſor of Geometry in *Oxford.*

LONDON,
Printed for *Tho. Parkhurst* at the *Bible* and *Three Crowns* at the Lower end of *Cheapside* near *Mercers*-Chapel. 1692.

John Wallis's *Theological Discourses; containing VIII Letters and III Sermons Concerning the Blessed Trinity.*


For the Reverend and Learned Thomas Smith,
Dr in Divinity, late Fellow of Magdalen College
in Oxford.

Sr

In compliance with what you have oft desired of me: I send you these Memorials of my Life.

My Father was John Wallis; a grave and Reverend Divine: Son of Robert and Ellen Wallis, of Thengdon (or, as it is usually pronounced, Fynedon) in the County of Northampton: Born in January 1567, and there baptized the 19th of that Month. He was Educated in Trinity College in Cambridge: where he took the Degrees of Batchelor and Master of Arts: and (about the same time) entred into Holy Orders; in the Reign of Queen Elizabeth.

Toward the end of Qu. Elizabeths Reign; he was made Minister of Ashford, a great Market Town in Kent. Where he continued the remainder of his Life; in great esteem and reputation; not only in that Town and Parish, but with ye Clergy, Gentry and Nobility round about.

He was a Pious, Prudent, Learned and Orthodox Divine; an Eminent and diligent Preacher; and with his prudent carriage, kept that great Town in very good Order, and promoted Piety to a great Degree.

Letter from John Wallis to Thomas Smith on 29 January 1696/7.

such Cases, against all, that did not act just as I did, knowing that there were many worthy Persons engaged on either side. And willing whatever side was upmost, to promote (as I was able) any good design for the true Interest of Religion, of Learning, and the publick good; and ready to do good Offices, as there was Opportunity; And, if things could not be just, as I could wish, to make the best of what is: And hereby, (thro' Gods gracious Providence) have been able to live easy, and useful, though not Great.