

K.L. Johnson and Contact Mechanics

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## 1. Preface

The Proceedings of the I.Mech.E. date back to 1847 and have appeared in various forms over the years. For those interested in the history of the Proceedings, a ‘timeline’ summary is provided by the current publishers, Sage, on their website [1]. In 1959, coincidentally when the second of the authors was born, the Journal of Mechanical Engineering Science was founded as a separate journal published by the Institution. In 1983, as part of a reorganisation of the Institution’s Journals, it became Part C of the proceedings. In this form it has continued to the present day, although publication transferred to Sage in 2010. Hence, in 2008 the journal was approaching its fiftieth anniversary. The chairman of the editorial board, as it was then constituted, Prof. Duncan Dowson FRS<sup>1</sup>, came up with the idea of arranging some personal contributions from those who had been material in the journal’s success. In the end not many such articles appeared, but it was in response to this initiative that the authors spoke informally to Ken Johnson about his life and work<sup>2</sup>. Typically modest in his approach, Ken was reluctant to see the article published during his lifetime, and so it has remained in the ‘bottom drawer of the desk’ ever since. But now, following Prof. Johnson’s passing in September 2015 we thought it appropriate to publish our brief article with minor modifications, and hope that it will serve as a memorial to the enormous contribution he made to his chosen field of study.

## 2. Introduction

The year 2009 saw the fiftieth anniversary of the first issue of the Journal of Mechanical Engineering Science. It was also 54 years since Prof K.L. Johnson FRS published his first paper on contact mechanics [2], and that event marked the start of a half century of prodigious development of this subject, substantially with him at the helm. Ken died in the autumn of 2015, and it seems an appropriate opportunity to review the progress which has been made in contact mechanics over the last fifty years or so, and to highlight his significant contributions to the subject during that time.

The term ‘contact mechanics’ is not precisely defined, but we may sensibly think of it as the study of all the phenomena associated with the interaction between solid bodies pressed or held in contact: it includes a knowledge of the contact stress field, a characterisation of the material response both elastically and plastically, deduction of the effects of surface roughness, of lubrication, of friction, and an understanding of

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<sup>1</sup> Professor Dowson was also the co-author of a paper in the first issue of the journal.

<sup>2</sup> Much of this article is based on a discussion with Ken, held in Cambridge on 16<sup>th</sup> June 2008.

the physics of the interaction forces between the surfaces. Ken Johnson has contributed to all of these aspects of the problem, and in the last of these fields his efforts have been seminal. In this article we will review some of these contributions; the coverage will emphasise some areas more than others, not because Ken's contributions have been uneven, but because our own knowledge is not as encyclopaedic as it might be. We begin by looking at the state of the subject when Ken was setting out on his journey of scientific exploration, and his early contributions.

### 3. Dry, Stationary Contacts

The first problem one learns about when studying contact mechanics is very often the Hertz solution to the nature of contact when two elastic bodies whose surface profiles are described by second order mathematical surfaces are pressed into contact. The solution was an incredible *tour de force* in 1882, and the insight Hertz showed in developing a contact stress solution when so relatively little was available to him is impressive. The most noteworthy assumption was the idealisation of each body as a 'half space', and this apparently sweeping simplification in fact has proved far more acceptable than one might anticipate in very many contact problems [3]. Hertz found the contact 'law' (the description of the size and shape of the contact, together with the contact pressure distribution), but under conditions of normal applied load only. If two bodies, pressed together to form a Hertz contact, are slid over each other in the presence of friction, the contact pressure is substantially unchanged. However, questions arose about what happened when such contacts carried shear forces insufficient to cause sliding. In the 1920s Carter examined the problem of contact between a locomotive driving wheel and a rail [4]. He was able to explain how the contact patch was split into a region which was stuck and a region in which there was slip, where the slip displacement between the two bodies was extremely small and controlled by local elasticity. In the following decade Cattaneo [5] examined the problem of a stationary contact between spheres, subject to shear, in which he showed that there would be a central disk of stick and a slip 'annulus' where interfacial slip occurred, again with only a tiny relative slip displacement. Unfortunately, Cattaneo published his work in an obscure journal, just before the outbreak of the Second World War and his work did not come to the attention of the wider academic community until much later. Indeed credit for the first solution of this type of partial slip problem is often given to Mindlin [6], who published his analysis over a decade later. The observations of Carter and Cattaneo showed that, even though there was no gross rigid-body movement between the contacting bodies, at a local scale particles at the surface were taken through a dissipative hysteresis loop.

We turn, now, to the state of aircraft development towards the end of the Second World War. Gas turbines had just been invented, but there remained a significant use for propeller-driven military aircraft, and a growing problem with the replacement of wooden, fixed-pitch propellers by aluminium alloy variable pitch propellers was the presence of self-excited vibration. Aluminium had much less internal damping than wood, but the pitch adjustment, made through a roller bearing assembly was a possible important point of energy dissipation. And it was in this world that Ken Johnson found himself when he went to work for Rotol Airscrews, a U.K. propeller manufacturer. Kenneth Langstreth Johnson had been born in 1925 in Barrow-in-Furness, then in Lancashire, now a part of Cumbria. Ken became a pupil at Barrow

Grammar School, where his father, a Cambridge graduate, was a schoolmaster. Ken had hoped to follow in his father's footsteps by applying to Cambridge, but his plans were disrupted by the war, and instead he accepted a State Scholarship on an accelerated course to read Mechanical Engineering at Manchester. Ken graduated with a first class degree after little over two years, and set off to Gloucester for his first job with Rotol.

Clearly, to make progress in quantifying the details of the energy dissipated in a very complicated multiple contact such as a propeller bearing, it is necessary to understand in detail what is happening at a single contact. So, after a few years with Rotol, Ken decided that he would carry out a detailed investigation. Hence, in 1949 he went to start a PhD at what was then the College of Technology in Manchester (later to become UMIST) to carry out fundamental work on this phenomenon. At about the same time another, completely separate but closely related investigation was being carried out in the Bell Laboratories, three thousand miles away. The motivation here was totally different; it was to understand very much better the way granular aggregates behaved, a substantial driving force being the further development of carbon-granule microphones. Experiments were carried out by Duffy and Mindlin [7] on arrays of close-packed spheres, but the initial calculations on individual contacts were led by Mindlin [6]. He published the basic form of the solution in 1949, but went on, with several research students, to carry out a comprehensive analysis of the partial slip problem between contacting spheres, with a range of contact loading histories, and an excellent review of this was compiled by one of Mindlin's students, Deresiewicz [8]. As noted above, the problem studied was essentially similar to that investigated by Cattaneo, but Mindlin had not seen Cattaneo's paper, and he also extended the results considerably. Indeed it was a very powerful group of people who were working on this and related problems with Mindlin at the time; those who made further contributions to contact problems included Goodman, who subsequently visited Johnson when the latter had secured a lectureship at Cambridge. Earlier, Goodman's appointment at Minnesota started a whole line of research into the field of contact mechanics, principally through his student Keer [9], who secured a position at Northwestern. This University became another important centre for contact research in the 1960s, and collaboration with Mura (who contributed basic elasticity) together with expertise in dislocation theory (through Dundurs), brought considerable insight into a range of contact cracking problems. Burton, who was also there, brought in thermoelastic considerations, and those in the next generation include Bryant, who was strongly influenced by Burton, and has established an important group at Austin, Texas, and Farris, who subsequently had an influential group at Purdue<sup>3</sup>. Farris contributed substantially to the continuation of fretting fatigue studies initiated three generations earlier, and was at the forefront of applying these studies to problems in aircraft propulsion. Of course the principal application has now moved on from propellers to gas turbines.

It was against this background that Ken Johnson started his work in contact mechanics in Manchester. Motivated by the propeller issues he had experienced at Rotol, Ken carried out the first, simple experiments on partial slip in which a ball was pressed onto a flat plate, and subject to oscillatory shear. The results showed an annulus of fretting damage surrounding a central undamaged disk, in accordance with

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<sup>3</sup> Farris has since moved and is now Dean of Engineering at Rutgers University.

Mindlin and Cattaneo's predictions of the stick-slip regime. These results were later reported in Johnson's first paper in 1955 [2], which has become very well-known. He was initially unaware of Mindlin's work, but came across his paper in the Library and was pleased to find that the results fitted the predicted curves almost exactly (once a missing factor of two had been identified; Johnson's experiment naturally had two contact surfaces, whereas some of Mindlin's equations applied to only one of the two contacting bodies). Ken wrote to Ray Mindlin and they subsequently corresponded on the problem. Indeed, Ken believed it was he who may have pointed out to Mindlin that the notion of using a superposition and scaling procedure to determine the size of the stick region had occurred to Cattaneo more than ten years earlier. Ken can remember Mindlin saying that he rather lost interest in the problem after discovering Cattaneo's paper.

At around this time Johnson bought a copy of what would then have been a newly published book on a rapidly developing subject: Bowden and Tabor's *The Friction, and Lubrication of Solids* [10]. In 1953, David Tabor came to Manchester to give a talk to some engineers working in industry and Ken was introduced to him after the lecture. Ken recalls how Tabor insisted on going down to the basement to see his apparatus and appeared to be impressed by the work he saw. Tabor suggested to Johnson that 'You must get your Prof to pay for you to come to Cambridge'. So began a friendship and professional collaboration which was to last until Tabor's death in 2005. Johnson did visit Cambridge a little while later, so presumably his Head of Department must have been persuaded by Tabor's advice. Ken recalls his surprise at finding Tabor's group essentially 'working in a corridor'. A research student of Tabor's at the time was Jim Greenwood, who was working on problems involving elastic hysteresis. Jim's apparatus had to be lifted from the table so that Johnson and Tabor could sit down for a scientific discussion. Johnson's relationship with Greenwood was, of course, also to develop from that first meeting into a lifelong friendship.

#### 4. Cambridge

Ken Johnson secured an appointment at Cambridge University Engineering Department in 1954, bringing to an end his five year appointment at Manchester. His Ph.D. was not, however, completed until after the move. The thesis itself was handed in just one day before Ken married Dorothy. This really was a time of forming lifelong relationships. There remained the matter of the viva-voce examination, which was held in Manchester. Tabor had been appointed as external examiner and both candidate and examiner travelled up together by train from Cambridge for the occasion. Ken recalls Tabor reading the thesis, whilst he read the Manchester Guardian, interrupted by technical questions from Tabor, who then realised they should properly wait until the viva itself. Despite this slightly unorthodox start, all went well once they reached their destination and Johnson was duly awarded his Ph.D.

When Ken arrived in Cambridge Prof J. F. Baker, still extremely well known for his contributions to structural theory, and, in particular, to photoelastic analysis, was head of department. The head of the mechanics group was Alan Percival, who had just been elected bursar of Jesus College and, presumably because he was being 'stretched

both ways', needed help with college supervisions<sup>4</sup>. He therefore approached Johnson who became a college lecturer at Jesus, and was subsequently appointed to a Fellowship in 1957. Here, he taught generations of undergraduates, including the current Master, Ian White, and Norman Fleck, well known for his later contributions to solid mechanics.

Johnson's early work at Cambridge remained in the area of frictional contacts in partial slip, but it is noteworthy that in 1958 Johnson published, in the *British Journal of Physics* [11], a paper, less than two pages long, in which he made the first allusion to the effect of cohesive forces, and to which we will return later in this article. Interestingly, Ken's conclusion at the time was that 'adhesion is physically impossible'. As we will see later, he was happy to revise his views later in his scientific career when new evidence came to light. Initially, Ken's work in Cambridge built on his work on the Cattaneo-Mindlin contact by applying similar reasoning to rolling contact problems, producing some very elegant enduring results on tractive rolling with and without spin, investigated both analytically (with economy of mathematics but great physical insight) and through carefully controlled experiments which continued to be the hallmarks of Johnson's investigations throughout his career. One particular apparatus involved three balls held between parallel plates. It was found that when one plate was rotated with respect to the other the balls crept outwards, for either possible direction of rotation. This led to the conclusion that rolling with spin produced a transverse force. Ken was later to discover that a similar phenomenon was well known in the automotive industry in the context of contact between the tyre and the road (and known as camber thrust). However this was the first time that the phenomenon had been comprehensively investigated in the context of rolling element bearings. Looking back, Ken remarked that this work took time, energy, and was great fun. Mindlin admired this work but did not contribute himself to the analysis of these problems, and it was Kalker (who, in the late 1950s, had completed his PhD under de Pater), who collaborated and contributed to these studies. The motivation was partly provided by the railway industry, and the phenomenon of 'hunting' in the motion of a wheelset, together with creep in tractive rolling contact.

Johnson's first research student at Cambridge was John O'Connor, who arrived in 1958. The work they tackled together took a deeper look at damage under partial slip beneath a stationary contact, or 'fretting'. O'Connor went on subsequently to develop a very widely respected form of replacement knee joint [12], but, before doing so, he spent time (in 1961 and 1962) at Minnesota, where he was influenced by Goodman, and met Keer, before taking up a post at Oxford. In the late 1960s O'Connor carried out some extremely influential work in developing well-controlled fretting and fretting-fatigue tests for both incomplete and complete geometries (both representing considerable advances in the subject, and where there was also significant analytical progress). A new line of research was thereby opened up which has proved fruitful for two of the authors of the current paper, starting in the mid-1980s.

Another line of enquiry for Johnson concerned the behaviour of incomplete contacts where there was 'coupling', again under partial slip. The origin of the coupling was

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<sup>4</sup> By coincidence, the first author currently finds himself in a similar position.

either elastic or geometrical mismatch (e.g. because one of the components had a surface layer or 'tyre'). This led to the development of a numerical technique for representation of surface tractions relying on the use of a triangular element of traction (or a hexagonal pyramid in three dimensions), so that a piecewise linear representation of the traction distribution could easily be made [13, 14], a technique which has since been adapted and used by a number of researchers, including ourselves [15, 16]. This work required numerical solution and therefore involved the use of digital computers, then only just becoming available in university departments.

## 5. Plastic Contact Problems

In the early 1960s, Johnson's reputation was already developing quickly. He collaborated with Haines at Bristol and Ollerton and others at Nottingham, who were carrying out investigations into basic elastic contact fields using photoelastic methods. This collaboration prompted Johnson to look further at some basic considerations of the strength of a Hertzian type contact. Evaluation of the elastic limit is straightforward, but this led Ken on to thinking about limited plastic flow under rolling conditions. What followed was some extremely elegant analysis of small strain plastic flow, all conducted without a finite element in sight. As Ken remarked: "We didn't have finite element programmes in those days, so we had to think, instead". Stimulation was also provided by a visit to Brown University in the early 1960s when fundamental research into the basics of plasticity was in its heyday, and Drucker and Symonds were making major inroads into understanding the underlying mechanics.

In many ways, the paper with Merwin, published in 1963 [17] exemplified the thinking behind so much of Johnson's approach: the paper concerns the state of residual stress established during rolling of a cylinder over an elastic-plastic half-plane, at loads moderately above the elastic limit. Because the plastic region is fully confined by elastic material, Merwin and Johnson argued that the strains, throughout, would be approximated very well by the extended elastic strains. With this kinematic assumption, the Prandtl-Reuss rules could be tracked through relatively straightforwardly, even with the very limited computing power available to them at the time, and the results found have proved very valuable. At higher loads, and in the presence of a tractive shearing force, surface plastic creep occurs, and this was also subsequently measured experimentally [18].

Johnson's work on basic problems in contact mechanics continued apace. Not content with work on confined plasticity problems, he tackled limit state type problems, too, all without recourse to purely numerical techniques. An early, typically elegant calculation was conducted in 1963, in which Johnson looked at the state of stress on the axis of loading beneath a spherical indenter pressed into a body, until a limit state was achieved, i.e. a hardness test was conducted. Using very straightforward kinematic assumptions Johnson was again able to solve the Prandtl-Reuss rules exactly, and hence to demonstrate that, during removal of the indenter, there will always be some reversed plastic flow, rather than a purely elastic recovery as had previously usually been assumed. This work was published as a short note in *Nature* [19]. Further work on the nature of indentation testing followed, giving clear descriptions of the responses of solids under elastic, elastic-plastic, and limit state loading [20]. Ken continued to maintain an interest in these most basic of contact

problems, publishing a further study of indentation testing, albeit now using numerical methods, in the mid-1980s [21].

## 6. Lubrication

Ken Johnson's interests in lubricated contact date back to the late 1950s. F.T. Barwell had recently moved with his group from the National Physical Laboratory to the National Engineering Laboratory at East Kilbride. He became aware of Russian work in the late 1940s, which investigated the lubrication of surfaces under high pressure and formed the beginnings of the study of elastohydrodynamic lubrication. Aware that conventional hydrodynamic theory could not explain the lubrication of gears, Barwell wrote to a number of university departments in an attempt to stimulate research in this area. Among the recipients of this letter were Ken Johnson and Duncan Dowson (at the University of Leeds). Ken responded positively to this initiative, but regretted that he was unable to do very much as he did not currently have a student. Subsequently he recruited Vermeulen, but by now Dowson had started work on the problem. Ken went to see Dowson and concluded that the Leeds group had made such good progress that it was not worth working in that area. Hence, Vermeulen went on to work on dry contacts [22].

Thus it was not until the late 1960s that Johnson expanded his work from the study of dry contacts to the effects of lubrication. It was at a time of prodigious development of the subject, with many centres of research in the UK, notably at Imperial College, The University of Leeds and the University of Wales, Cardiff. Jefferis, who followed Merwin in studying limited plastic flow under rolling contact, now with shear, carried out his experimental work on a rolling disk machine. This work went on to investigate shearing in EHL films and led to Ken's first paper involving lubrication [23]. Of particular interest were the effects of non-Newtonian properties, and these were investigated by Rod Cameron, originally from Vancouver. After completing his doctorate he returned to Vancouver where Ken visited him and found him living on a boat in the harbour and making a living from supply teaching. Rod's real passion, however, was musical instruments and he went on to make a successful career as a manufacturer of reproduction flutes, putting into practice (as he put it) 'those things which I learnt in the lab'. Cameron was actually supervised by one of Ken's colleagues, Wiley Gregory (who later contributed to studies of rail corrugations and other track phenomena, together with Stuart Grassie). Johnson and Cameron carried out some measurements of traction in a lubricated twin disc experiment and found that the traction reduced at high sliding speeds. Significant attention was paid to understanding this phenomenon and the conclusion was reached that at high pressures the lubricant film can behave like a plastic solid with a critical flow stress [24].

Like many of the best scientists, Ken drew inspiration from the work of others in the field. This did not just apply to ideas; in the 1970s, Ken was hoping to build an apparatus which would measure all the components of force in a spinning contact. He had seen something appropriate during a visit to Swansea, although he thought it rather too compliant. Ken therefore arranged to borrow the equipment, which was used by Alan Roberts in his studies. As Ken had predicted, the apparatus proved insufficiently precise, and a stiffer version was later built by Joe Tevaarwerk as part of his doctoral work. Ken also started to work with Jim Greenwood after Jim returned to Cambridge in 1970, and a paper was published with S.Y. (Peter) Poon in 1971 [25].

This concentrated on the mixed lubrication regime where asperity contact was also important. Ken and Jim subsequently published a number of joint papers spanning a period of nearly 30 years. As with dry contacts, Ken's work on lubrication was characterised by well thought out and carefully executed experiments, combined with modelling based on physical understanding. A paper with Tevaarwerk, published in Proc. Roy. Soc. in 1977 is perhaps typical of this approach, by proposing a non-linear Maxwell model for the lubricant and combining this with supporting experimental results [26].

## 7. Railways [\[refs\]](#)

Railways have already figured in this narrative<sup>5</sup>. It is natural that the well-defined, heavily loaded contacts between rail vehicle wheels and rails should attract the attention of the analyst, because they are potentially a much more viable and tractable thing to study than, say, the motor car tyre, with all its complexities. But the interaction between railway vehicle wheels and the railhead is much more complex than one might think; there is the question of shearing traction distribution, originally solved for the plane steady state case, by Carter, and where the transient from the Cattaneo problem was solved by Kalker [27]. Similarly, there is the influence of the solid axle on steering and hunting, substantially again solved by Kalker [28], and there are the effects of inelastic behaviour, where Johnson made major inroads. For the plane, quasi-static idealisation of the problem, the work conducted with Merwin [17] and with Jefferis [23] went some way to explaining the phenomenon of ‘forward flow’, but rail corrugation is associated with dynamic coupling effects. It was, apparently, first observed when the West Coast Main Line was electrified, but its distance from Cambridge meant that scientific observations were usually made rather nearer Cambridge, at Huntingdon. Huntingdon is, of course, on the East Coast Main Line and the railway here was not at the time electrified. Whether the 36 cylinder<sup>6</sup> ‘Deltic’ locomotives, which served this route, caused the same problems as the electric rolling stock on the other side of the country is unclear. Apparently the research involved relaxing afternoons sitting in a deckchair at the trackside, measuring vibrations *in situ*, and there is a photograph of Ken and his colleagues involved in this activity on a particularly sunny afternoon. Alistair Gilchrist, one of the sponsors of the work at British Rail Research, saw the photograph and remarked that ‘it looks like a picnic’. Ken recalls that this was not too far from the truth, as someone was frequently sent a few hundred yards to the buffet at Huntingdon station for essential supplies.

Laboratory work on the corrugation phenomenon had been started by Gerry Hamilton, formerly at Aldermaston, but then at the University of Reading. Hamilton had already published, with Goodman, the first full solution to the state of elastic stress under a sliding sphere [29], but was also undertaking extensive studies in lubrication, and he had noticed the phenomenon of corrugation in a rolling disk machine when the disks were made from copper. Johnson thought that the process was excited by a ‘contact resonance’, and it was the trio of Grassie, Gregory and Johnson which made

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<sup>5</sup> Appropriately so, as two of the current authors started their careers as engineers with British Rail.

<sup>6</sup> Each locomotive had two engines, each with six banks of three cylinders in a ‘delta’ configuration (hence the name). Each cylinder had two opposing pistons, connected to three geared crankshafts at the vertices of the delta. Hence there were 72 pistons on each locomotive, and a change of piston rings was a substantial undertaking.



significant progress towards to a full solution of the problem. However, it remains in some respects enigmatic. The obvious questions were how occurrences of the phenomenon correlated with axle load and train speed, so heavily loaded track bearing ore traffic was strain gauged, and Grassie spent afternoons walking along Shap Bank (where, presumably the tractive effort would have been exceptionally severe) looking for evidence of corrugations. It was noted that the characteristic wavelength seemed not to vary much with average train speed, and bright spots were noted at Paddington station 'right up to the buffers'. This seemed inexplicable if it was, indeed, a dynamic phenomenon; until it was discovered that the rail was 'second hand', and had just been moved there! Rail corrugation also proved to be rather an English phenomenon - Scottish track further North along the West Coast Main Line seemed hardly to suffer, although the mechanical specification of the rail was identical. It was eventually discovered that the English rails were made by the Acid Bessemer process in Workington, whereas those North of the border were produced by the Basic Open Hearth process at Glengarnock. By the time that this was discovered, both process had been largely superseded by the Electric Open Hearth process. These rails also corrugated, but not as fast.

It was becoming clear that rail corrugation was a very complex phenomenon [30]. There did appear to be a link with wear properties, and very hard coatings were to be found on the crests of the corrugations. Freddy Barwell was by this time a professor at University College Swansea, but had spent a period as Director of Research for the British Railways Board. He felt it was associated with differential corrosion. Other links were made with gauge tolerance, and with the less forgiving nature of concrete sleepers. Some final experimental work was carried out on the Vancouver 'SkyTrain' metro system. Here driverless trains mean that the speed is known at all points and tractive forces are absent as the train is driven by a linear motor. Some interesting results were discovered, but were not fully applicable to more heavily loaded and higher speed main line operation. Overall rail corrugation remains a phenomenon with several possible causes, and without a full, compelling explanation.

## 8. JKR

We have already mentioned Johnson's enduring contributions to the study of indentation testing, at a very fundamental and basic level; work involving, in many cases, at least some plastic flow, and yet investigated without much recourse to full numerical schemes. But there are two classes of indentation tests in which there is little plastic flow, but which both require explanations beyond a classical contact solution based on a Hertzian contact. The first of those is when a sphere is pressed into a brittle or almost brittle material – usually glass – and, at a critical load, a shallow crack in the form of the frustum of a cone spontaneously forms. This is a subject which now produces many papers in the open press, but a key underlying element of the problem is the effect of elastic mismatch between the material of the indenter and the indented material on the interfacial traction distribution. The analytical problem of the nature of the surface traction distribution was addressed at the end of the 1960s by Spence, who produced a very elegant but rather complicated solution; but it was Johnson, together with O'Connor (by this stage at Oxford) and Woodward (a student of Mike Ashby's), who looked in detail at the additional surface tension developed [31] when the indenter was stiffer than the substrate.

The second kind of indentation which introduces unexpected results arises when the interface is such that the classical Signorini inequalities do not apply: specifically, there is the possibility that tensile direct tractions, originating from cohesion or adhesion are present, so that the size of the contact is no longer uniquely defined – it is history dependent – and is normally larger than that implied by a classical solution. Mention has already been made of Johnson's prescient paper of 1958 [11], but it is the 1971 paper, published jointly with Kendall and Roberts [32], which really describes the problem in detail, including both analysis and extremely elegant experimental work with mirror-finish rubber spheres, which really must go down as marking the beginning of the scientific investigation of this important subject. Even the 1971 paper may be regarded as prescient, in itself, because the phenomenon at that time was not much more than a curiosity, interesting mainly for its basic physics, and it was not until the development of the atomic force microscope that the effect became so important that half of the contact mechanics world seemed to want to contribute to an explanation. Johnson has continued his own investigations with original contributions in the 21st century [33].

Interestingly, Ken's research in the area of adhesion started from a practical application. David Tabor obtained a contract to investigate the causes of squealing in windscreen wipers. Alan Roberts joined him as a research student and started by addressing the simpler problem of the contact between a rubber ball and a flat glass sheet. The rubber specimens were obtained by casting into a mould formed by a good-quality concave lens. Roberts found that when the normal load was removed the rubber remained adhered to the glass with a well-defined radius of contact. At this point Kevin Kendall became involved in the work. Ken Johnson remembered Tabor introducing him at one of their regular Thursday morning meetings, with the words "I have a research student here who doesn't believe in Hertz". Kendall made the suggestion that the phenomenon was something to do with surface energy, but he was struggling to undertake any meaningful analysis. Kendall and Tabor wondered whether Ken had anything that he was able to contribute. Ken recalls that he went straight back to his 1958 paper, which had predicted the impossibility of adhesion, and included a surface energy term. The agreement with the experimental results was found to be very encouraging and Johnson, Kendall and Roberts wrote their landmark paper in 1971 [32]. The theory has become well-known as the JKR theory of adhesion, though not quite as well-known as a more recent occurrence of the same three initials. Ken was quite surprised at the number of Google hits for 'JKR', until he realised that most of them referred to the creator of Harry Potter..... Nevertheless, the original paper now has over 4000 citations, more than many scientists can expect from a lifetime's work.

## 9. Conclusion

This article was intended to give a brief summary of the contributions Professor Ken Johnson has made to the world of contact mechanics; a period of effort which happens to correspond with the life of this journal. It cannot be comprehensive, and greater emphasis has, inevitably, been given to those general areas where the authors have, themselves, a particular scientific interest. For anyone wishing to have a more systematic review of Johnson's work, the annotated compilation of papers edited by Kauzlarich and Williams [34] is strongly recommended. Of course, no discussion of Ken's work would be complete without mentioning his book 'Contact Mechanics',

published in 1985 [35], which contains a comprehensive introduction to the subject and has become the ‘bible’ for many a research student and professional engineer. As Ken himself says in the introduction, it is a ‘user’s book rather than a course text book’, and the collection and presentation of so much material in a comprehensive and clear manner represents a considerable achievement in itself.

Ken Johnson’s contributions have been profound, original, and enduring. The JKR model of adhesive contact will enter the ‘subject index’ of many texts on mechanics, and there is no more objective test of originality than that. Those of us who study the general subject of contact mechanics and its applications will continue to benefit for many years from the progress he has made, and to make use of the results he established. But, of course, we of this generation will also benefit from the pedagogy implied by following the procedures he used in obtaining those results – the careful and insightful use of clear approximations, and the elegant but economical use of mathematics, all underpinned by well thought-out and carefully performed experiments – from which we will continue to draw inspiration. Finally, it should be said that those of us who had more personal contact with Ken over the years came to appreciate not just his scientific and technical qualities. He was unfailingly interested in and supportive of the work of others in the field, particularly young researchers, and he and Dorothy made many friends from among those who interacted with Ken on a professional level.

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