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What a Puzzle! Unravelling Why UK Phillips Curves were Unstable

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Abstract

The UK relationship between nominal wage inflation and the unemployment rate is unstable. Over sub-periods of the last 160 years of turbulent data, Phillips curve slopes range from strongly negative, slightly negative, flat, slightly positive and strongly positive. Our constant-parameter congruent model of real wages explains these instabilities, yet also implies a constant negative relationship between nominal wage inflation and the unemployment rate when corrected by its regressors. Disentangling these effects reveals that structural breaks in the real-wage model's variables do not explain the instabilities, which instead occur during sub-periods when some of its explanatory variables are insignificant.

I. Introduction

The Phillips curve in various formulations underpins many macroeconomic models and is often used by policymakers as a framework for possible trade-offs between inflation and unemployment. However, the instability over time in such Phillips curves is well known and well documented by both academics and policymakers: see Del Negro *et al.* (2020), Do and Spanos (2024) and Haldane and Quah (1999) for the former and Powell (2019) and Cunliffe (2017) for the latter, both from a US and UK perspective.[1] Phillips (1958) related changes in nominal wages (W_t) to the unemployment rate ($U_{r,t}$) as both are labour market variables, but Jacobson, Vredin, and Warne (1998) doubt there is any link and

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[1]Bill Phillips, whose amazing life is recounted by Alan Bollard (2016), is gratefully remembered by Hendry as his tutor at LSE in 1966–67 who successfully guided him through his initial struggles with econometrics. Bill and Denis Sargan ran an inspiring workshop for MSc students where they debated many econometric issues including nominal vs. real wage equations: see Hendry and Mizon (2000) and Hendry (2003), respectively.

recent variants use price inflation (see Forder, 2014 and Hoover, 2015, for historical perspectives).

Over five sub-periods of annual 1860–2021 UK time series, the slopes of the original Phillips (1958) curve, range from strongly negative, slightly negative, flat, slightly positive and strongly positive. The sub-periods are the original Phillips' sample 1860–1913; World War I (WWI) to the end of WWII, which also included the post-war UK collapse, General Strike, and the 'Great Depression'; 1946–80, post-war recovery till the end of the oil crisis; 1981–2011 which was the sample end in Castle and Hendry (2014); and 2011–21 which includes Brexit, the Covid-19 pandemic lockdowns and the UK government's furlough scheme to prevent excessive unemployment. The shifts prompt four puzzles:

Puzzle 1: what caused Phillips curve slopes to change so much?
Puzzle 2: why is the Phillips curve over the very turbulent period 1914–1945 similar to that over the relatively stable 1860–1913 which Phillips originally used?
Puzzle 3: can a constant-parameter real-wage model successfully encompass a shifting nominal wage change equation?
Puzzle 4: yet also entail a marginal relationship between nominal wage inflation and the unemployment rate constant across all the sub-periods?

Explanations for the instability of the Phillips curve often focus on changing inflation expectations.[2] In this paper however, to disentangle the causes of the sub-sample shifts, we derive the nominal-wage inflation–unemployment rate relation from the real-wage model in Castle and Hendry (2009, 2014) and use that transformation to demonstrate that its fitted values for nominal wage changes closely replicate the Phillips curves shifts for every sub-period. Thus, its additional conditioning variables explain the shifts, a successful mis-specification encompassing finding (see Hendry, 1995, Ch. 14, and Hendry and Nielsen, 2007, Ch. 13).

Sub-sample analysis is used to disentangle what is causing the instability in the nominal-wage inflation-unemployment rate relation. Usually, shifts in omitted relevant explanatory variables are the cause, but in sub-periods where the Phillips curve slope is positive, many of the real-wage model regressors are insignificant. However, full-sample coefficient estimates of that model can be imposed on the insignificant coefficients without loss confirming its constancy. Hence, it is not structural breaks in the real-wage model's regressors that explain the instabilities, but the absence of their effects in sub-samples. Thus, co-breaking between regressors that are significant and those that are not does not occur in these sub-samples, leading to the shifts in the Phillips curves.

The structure of the paper is as follows. Section II illustrates the instability over sub-samples of the original Phillips (1958) relation. Section III derives the nominal-wage inflation–unemployment relation from the real-wage model in Castle, Hendry, and Martinez (2023), and section IV analyses its sub-period implications, testing the validity of conditioning on the unemployment rate. Section V examines the sub-sample equations to ascertain what caused the shifts, and applies break detection to the sub-samples' regressors to determine if co-breaking occurs. The underlying relationship between

[2]Non-invariance of expectations-based new-Keynesian Phillips curves (NKPCs) is found by Castle *et al.* (2014).

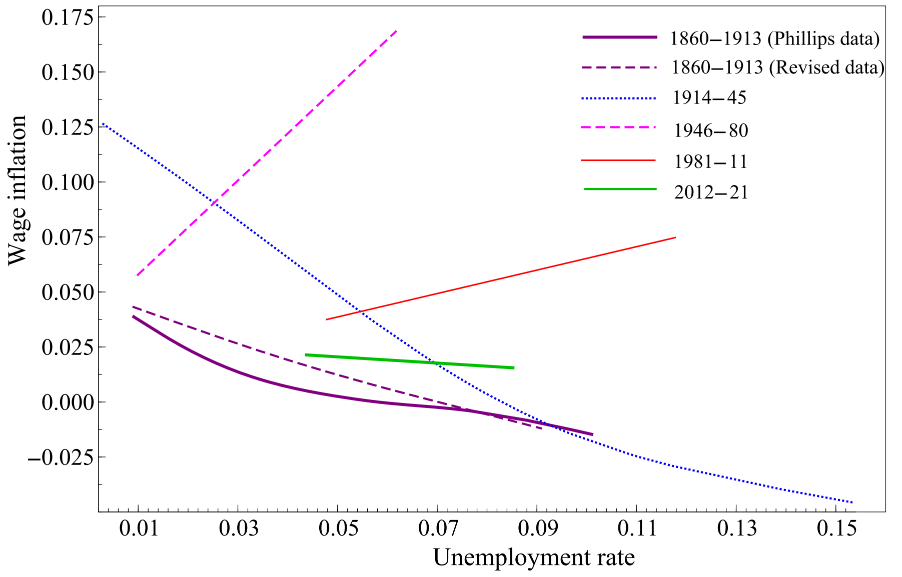

Figure 1. Shifts in the nominal wage inflation-unemployment rate relation: spline estimates over sub-samples using UK annual data [Colour figure can be viewed at wileyonlinelibrary.com]

nominal wage inflation and the unemployment rate conditional on the other real-wage regressors is shown to have essentially the same slope in every sub-period. Section VI concludes, and Appendix A provides definitions and sources of the data series used and Appendix B examines the validity of the conditioning variables by building a statistically adequate approximating model of the Phillips curve and testing for the significance of the explanatory variables within the approximating model.

II. The instability of the Phillips Curve

Phillips defined nominal wage inflation, Dw, as $Dw = 0.5(W_{t+1} - W_{t-1})/W_t$, whereas we use the change in $\log(W_t)$ (Δw_t, where lower case represent logs).[3] Castle and Hendry (2009) replicate his findings. To illustrate the instability of the relationship between nominal wage inflation and the unemployment rate, Figure 1 records the five sub-sample estimates of the Phillips curve over 1860–2021. Since Phillips conducted his study, pre-WWI data on unemployment have been substantially revised by Boyer and Hatton (2002). Our pre-WWI results using the revised data are close to those Phillips reported but do not have the curvature that Phillips observed: the dashed purple line fits a cubic spline to the revised data and is linear compared to the convex spline function that Phillips found in the solid purple line. In the subsequent analysis we use the revised unemployment rate data.

[3] There is a difference in timing of the peaks and troughs, but the two series are highly correlated: see Hendry, 2001. Phillips was aware that discrete approximations created moving-average errors (see Phillips, 2000), but in 1958 these were nearly impossible to estimate. He also knew that the 'loops' around his long-run relation represented dynamic adjustments, so calculated his equation from subsets of unemployment levels within which the average over a business cycle should be close to zero (see Desai, 1975, and Gilbert, 1976).

UK wage changes and the unemployment rate have varied hugely over the past century and half as shown by the time series plots of Δw_t and $U_{r,t}$ in Figure 2, with the sub-periods in Figure 1 marked, using shading for the two world wars. Their very different patterns suggest a solution to Puzzle 1, that the original Phillips curve would not be constant across the next four sub-samples. Sub-sample analysis using the linear regression:

$$\widehat{\Delta w_t} = \widehat{a} + \widehat{b} U_{r,t}, \tag{1}$$

results in sub-sample coefficient estimates $\{\widehat{b}\}$ with their heteroscedasticity and autocorrelation consistent (HAC) standard errors (see Andrews, 1991) shown in Table 1.

Figure 3a–d shows forecasts of Δw_t given $U_{r,t}$ from each period for the next, so (a) is forecasting 1914–45 from the model estimated using Phillips original period, etc. The forecasts in Panels (a) and (c) are manifestly awful, and while those in (b) and (d) are closer to the outcomes, the forecast errors are highly autocorrelated.

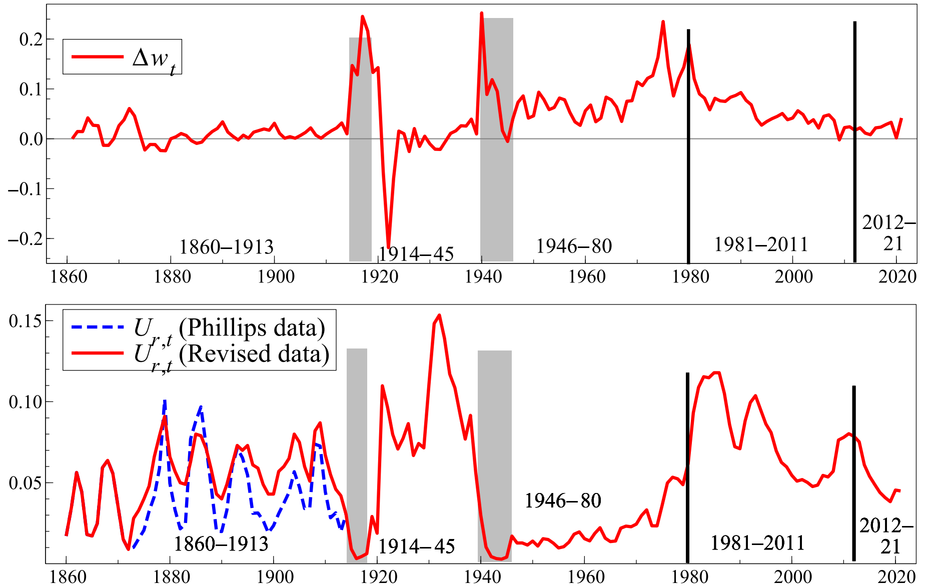

Figure 2. UK time series of Δw_t and $U_{r,t}$ with the two World Wars shaded and the other sub-samples shown by vertical lines [Colour figure can be viewed at wileyonlinelibrary.com]

Table 1
Estimates and HAC SEs of $\widehat{b}$ *in the regression* $\widehat{\Delta w_t} = \widehat{a} + \widehat{b} U_{r,t}$

	$\widehat{b}$	HAC SEs
1861–1913	−0.67	0.10
1914–45	−1.30	0.41
1946–80	2.14	0.36
1981–2011	0.53	0.13
2012–21	−0.27	0.15

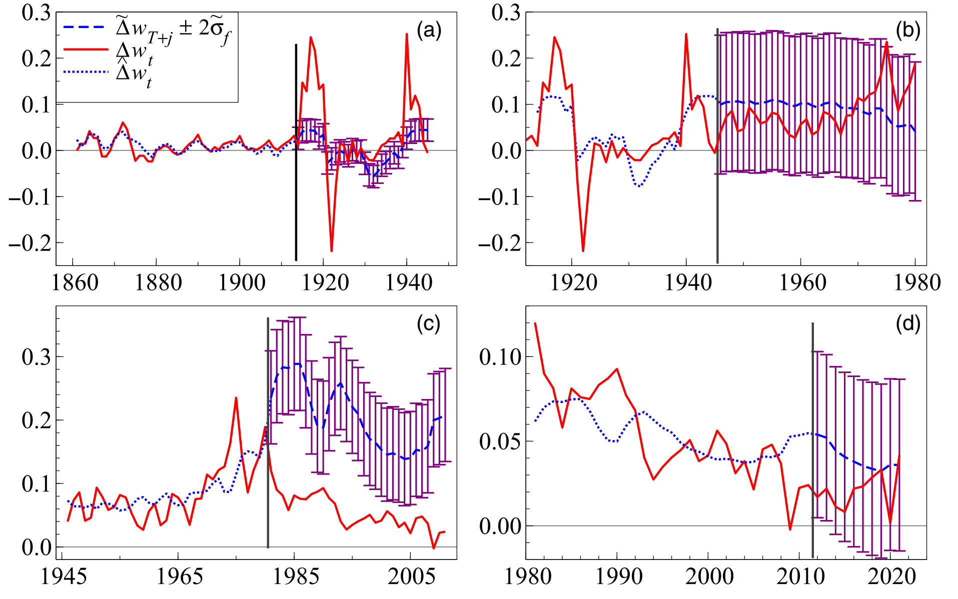

Figure 3. Forecasts of wage inflation from the unemployment rate with $\pm 2\tilde{\sigma}_f$, shown as error bars (allowing for parameter estimation uncertainty), over (a) 1914–45 from 1861 to 1913; (b) 1946–80 from 1914 to 1945; (c) 1981–2011 from 1946 to 1980; (d) 2012–21 from 1981 to 2011 [Colour figure can be viewed at wileyonlinelibrary.com]

As the sub-sample sizes vary from 53 to 10 observations, estimation uncertainty is a potential concern. We test this by estimating the encompassing model:

$$\widehat{\Delta w}_t = \sum_{j=1}^{5} \left\{ \widehat{a}_j I_j + \widehat{b}_j \left(U_{r,t} \times I_j \right) \right\}, \tag{2}$$

where I_j is a dummy variable taking the value 1 for all observations in sub-sample j, corresponding to the five sub-samples defined in Table 1. Testing all pairwise comparisons $\mathsf{H}_0: \widehat{b}_j = \widehat{b}_i, \ \forall j = 1, \ldots, 5, \ j \neq i$, results in 10 restrictions which are jointly rejected despite multiple testing [$\chi^2(10) = 65.8^{**}$; P-value $= 0.00$], so the point estimates vary due to shifts rather than estimation uncertainty. This is supported by the results of step indicator saturation (Castle *et al.*, 2015) applied to the constant parameter model which detects level shifts at similar points to our sub-sample breaks. These shifts are then interacted with U_r to obtain sub-sample slope parameters using data-based break detection methods.

Figure 4 provides a more detailed visual description of Figure 1, recording each observation of Δw_t and $U_{r,t}$ by its date. The colour coding represents the five sub-samples, with the lines corresponding to the estimated linear relationships in Table 1. While Figure 1 shows cubic splines graphically fitted to the data given the nonlinear relation in Phillips (1958), these are close to the linear sub-sample estimated regressions in Figure 4, confirming coefficients of $U_{r,t}$ change substantially over time. The figure highlights 'outliers' from wars, oil crises, price controls, indexation and the 'Great Depression' in 'boxes' as they derive from different extraneous causes at different times. However, the

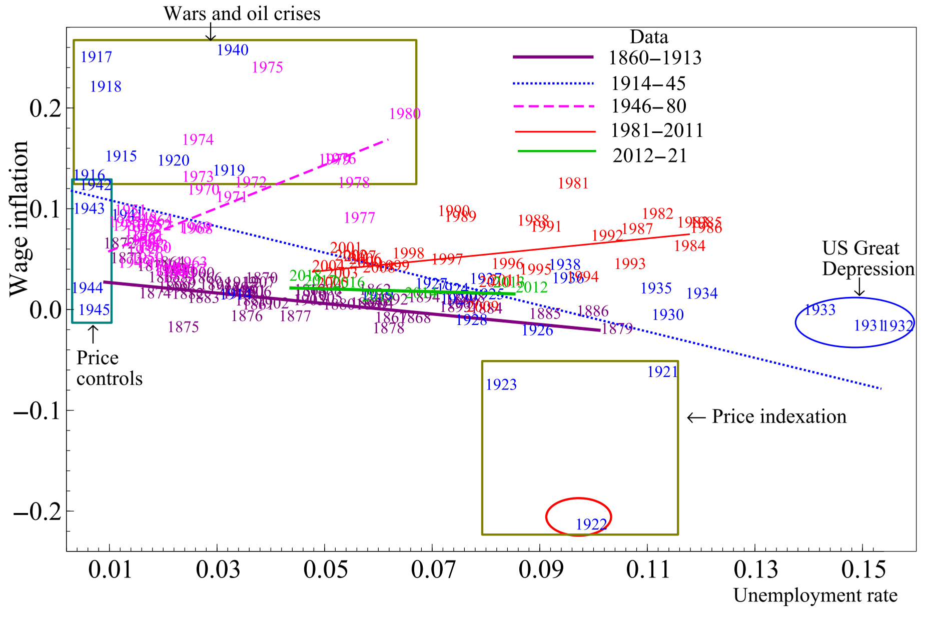

Figure 4. Shifts in wage inflation-unemployment rate relation: Colours correspond to sub-samples, dates denote observed data points, lines fit linear regressions to the corresponding sub-sample data and boxes highlight outliers due to specific labelled events [Colour figure can be viewed at wileyonlinelibrary.com]

forecast failures in Figure 3 are clearly not just due to those extreme events, nor do they align with the slope shifts. To reconcile these results with a constant Phillips curve relationship, we next derive a nominal wage inflation–unemployment rate relation from our real-wage model.

III. Deriving a Phillips curve from our real-wage model

The natural explanation of such unstable estimates is that relevant variables not included in the model experience shifts. If all excluded variables were stationary and maintained a constant correlation with U_r, its coefficient would be constant despite the omissions. In stationary data, observational equivalence can be a problem, as there are potentially many different combinations of additional explanatory variables that could give rise to a statistically adequate model. One approach would be to model the error process from (1) but this imposes parameter restrictions on the observable process, resulting in inconsistent estimators: see McGuirk and Spanos (2009). Alternatively, missing explanatory variables could be added to (1) with the aim of finding a statistically adequate model. Such a specific-to-general approach is problematic as model comparisons are infeasible (Spanos, 2006) and inference is misleading (see Spanos, 2019, p. 646, for an example).

When modelling long-run data that is non-stationary from distributional shifts, few possible alternatives exist. Here, to explain the shifts in the nominal wage growth–unemployment rate relation, we use the congruent, constant parameter model of real wages over 1862–2021 selected by a general-to-specific methodology, which

Table 2

Summary data definitions

$\Delta(w-p)_t$	Annual change in real wages
$\Delta(y-l)_t$	Annual change in labour productivity (output per worker)
e_t	Wage share $= (w-p-y-l-\widehat{\mu})_t$: real wages as a proportion of Output per worker de-meaned by the full-sample average $\widehat{\mu}$
Δp_t	Annual price inflation
$\widetilde{f}_t$	$= \frac{1}{0.88}\left(\left[1+\exp\left(-10\left(100(\Delta p_t)^2-0.2\right)\right)\right]^{-1}-1\right)$
$(\widetilde{f}_t \times \Delta p_t)$	The nonlinear wage-price spiral
$\Delta^2 p_t$	$\Delta p_t - \Delta p_{t-1}$
$(U_{r,t}-0.05)$	Unemployment rate de-meaned by full-sample average of 5% p.a.
$\Delta_2 U_{r,t}$	$U_{r,t} - U_{r,t-2}$
S_{xxxx}	Step indicator equal to 1 till the date *xxxx* and 0 after
I_{xxxx}	Impulse indicator variable equal to 1 for observation *xxxx* only
I_{WWII}	$= I_{1942} + I_{1943} - I_{1944} - I_{1945}$

initially allowed for many possible explanatory variables, first developed in Castle and Hendry (2009) building on Sargan (1964, 1971), and now given by:

$$\begin{aligned}\widehat{\Delta(w-p)}_t = &\underset{(0.04)}{0.40}\Delta(y-l)_t + \underset{(0.04)}{0.13}\Delta(y-l)_{t-1} - \underset{(0.03)}{0.18}e_{t-2} \\ &+ \underset{(0.07)}{0.41}(\widetilde{f}_t \times \Delta p_t) - \underset{(0.03)}{0.14}\Delta^2 p_t - \underset{(0.03)}{0.18}(U_{r,t}-0.05) \\ &+ \underset{(0.68)}{3.1}(U_{r,t}-0.05)^2 - \underset{(0.05)}{0.22}\Delta_2 U_{r,t} - \underset{(0.01)}{0.13}S_{1939} \\ &+ \underset{(0.02)}{0.18}S_{1940} - \underset{(0.01)}{0.07}S_{1941} + \underset{(0.002)}{0.02}S_{2012} - \underset{(0.01)}{0.05}I_{1916} \\ &- \underset{(0.01)}{0.05}I_{1977} + \underset{(0.01)}{0.03}I_{\mathrm{WWII}} \\ \widehat{\sigma} = 1.1\%\ \ &R^2 = 0.79\ \ F_{\mathrm{ar}}(2,137) = 0.25\ \ F_{\mathrm{arch}}(1,152) = 0.03 \\ \chi^2_{nd}(2) = 0.62\ \ &F_{\mathrm{Het}}(19,130) = 2.5^{**}\ \ F_{\mathrm{reset}}(2,137) = 2.34\ \ F_{\mathrm{nl}}(24,121) = 1.09\end{aligned} \tag{3}$$

Data definitions are given in Appendix A, but we list the variables here for convenience (see Table 2).

$\widetilde{f}_t$ is a logistic smooth transition function (see Luukkonen, Saikkonen, and Teräsvirta, 1988) where the scaling bounds the function between $[-1, 0]$.[4]

In (3), the short-run impact of changes in productivity on real wages ≈ 0.5 with rapid correction from the labour share of income of about 20% p.a. (the equilibrium correction, as $w-p$ and $y-l$ are cointegrated). There is a greater reaction of real wages to price inflation as it rises, inducing a wage-price spiral when inflation exceeds $\approx 8\%$. Rising unemployment reduces real-wage growth initially then raises it at high levels confirming unemployment is involuntary. The step shifts account for the real-wage growth rate doubling after WWII, possibly due to an increase in female labour force participation and

[4]Coefficient SEs are in parentheses (HAC in brackets), $\widehat{\sigma}$ is the residual SD, F_{ar} tests residual autocorrelation (see Godfrey, 1978), F_{arch} tests autoregressive conditional heteroscedasticity (see Engle, 1982), F_{het} tests residual heteroskedasticity (see White, 1980), $\chi^2_{\mathrm{nd}}(2)$ tests non-Normality (see Doornik and Hansen, 2008), F_{reset} tests nonlinearity (see Ramsey, 1969), F_{nl} also tests nonlinearity (see Castle and Hendry, 2010), and F_{chow} tests parameter constancy (see Chow, 1960). One star indicates test significance at 5%, two at 1%.

up-skilling during the war; and a downward shift from 2012, possibly structural changes in the economy post-financial crisis, neither explained by the variables in the model.

The model passes most diagnostic tests, is constant over the long sample, and also passes a test for super-exogeneity of its contemporaneous regressors. Castle and Hendry (2009) also investigated possible roles for Trade Union power (measured by membership), strikes, and unemployment benefits, all of which varied greatly over the 160 years, but found no significant impacts (e.g., (3) has no dummy for 1926 despite its large effects in Castle and Hendry, 2020).

Expressing (3) in terms of Δw_t in relation to $U_{r,t}$ plus other drivers shown as $[\cdot]$ yields:

$$\begin{aligned}\widehat{\Delta w_t} = &-0.71U_{r,t} + \big[3.1U_{r,t}^2 + 0.22U_{r,t-2} + 0.025 + 0.86\Delta p_t + 0.14\Delta p_{t-1} \\ &+0.41(\widetilde{f}_t \times \Delta p_t) + 0.41\Delta(y-l)_t + 0.13\Delta(y-l)_{t-1} - 0.18(w-p-y+l-\widehat{\mu})_{t-2} \\ &-0.13S_{1939} + 0.18S_{1940} - 0.07S_{1941} + 0.02S_{2012} - 0.05I_{1916} - 0.05I_{1977} + 0.03I_{\mathrm{WWII}}\big]\end{aligned} \quad (4)$$

The coefficient of Δp_t is carried over at unity from undoing $\Delta(w-p)_t$, so $0.86 = 1 - 0.14$ from (3), but is 0.96(0.08) if estimated unrestrictedly on the right-hand side of (3). $\widehat{x}_t$ is given by $[\cdot]$ in (4) which is used in equation (5) to partial out the effects of the additional explanatory variables.

Although the model of $\Delta\widehat{(w-p)}$ is constant over the full-sample, we can try to derive the shifts in $\{\widehat{\Delta w_t}\}$ relative to $U_{r,t}$ from (4), and Figure 5 records these sub-period linear regressions. The dashed lines correspond to fitted nominal wage growth using (4) and the solid lines are from the sub-sample Phillips curves (1). The close match confirms that the regressors present in the real-wage model account for the instabilities in the simple Phillips' curve. Since the real-wage model is constant over the whole period

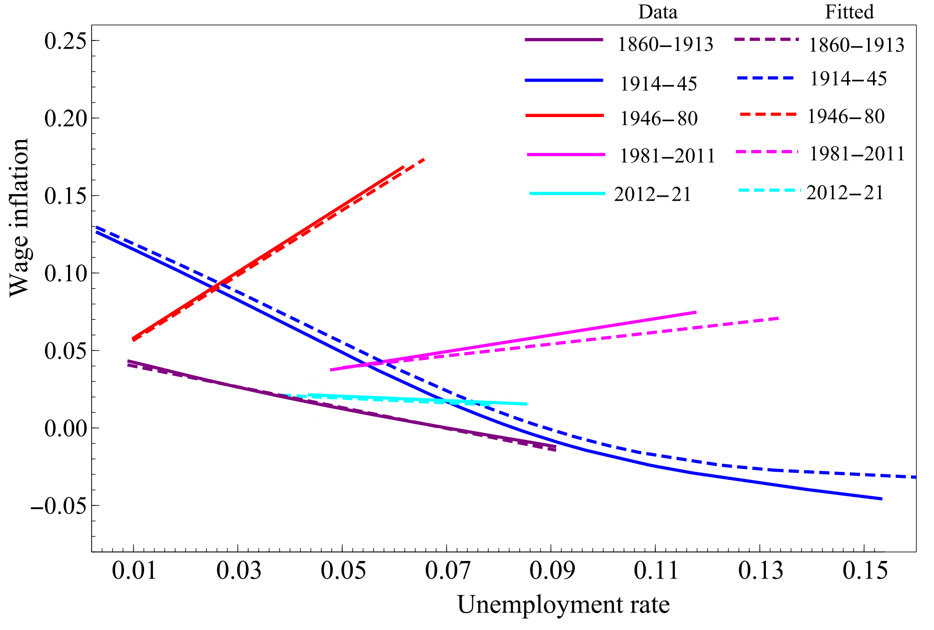

Figure 5. Comparing direct and derived Phillips curves [Colour figure can be viewed at wileyonlinelibrary.com]

$T = 1862-2021$, it successfully mis-specification encompasses the shifting Phillips curves, resolving Puzzle 3.

Puzzles 2 and 4 remain, so we now investigate the sub-period implications of (4).

IV. Sub-period implications

To evaluate if correcting for the additional drivers $\{\widehat{x}_t\}$ in (4), namely the sum of all influences on nominal wage growth from the real-wage model other than the unemployment rate, produces stable sub-sample Phillips curve estimates, we derive $(\widehat{\Delta w_t|\hat{x}_t})$ as the residuals from the full-sample regression of Δw_t on $\widehat{x}_t$, shown in (5) with HAC SEs:

$$\begin{aligned} \widehat{\Delta w_t} &= \underset{(0.007)}{-0.044} + \underset{(0.08)}{0.98}\widehat{x}_t \\ \widehat{\sigma} &= 2.6\% \;\; R^2 = 0.80 \;\; F_{\mathrm{ar}}(2{,}156) = 98^{**} \;\; F_{\mathrm{arch}}(1{,}158) = 85^{**} \\ \chi^2_{nd}(2) &= 3.3 \;\; F_{\mathrm{Het}}(2{,}157) = 2.13 \;\; F_{\mathrm{reset}}(2{,}156) = 7.3^{**} \end{aligned} \tag{5}$$

Thus, wage inflation is only corrected by a scalar which uses the same coefficients in all sub-periods, leading to the full-sample regression recorded in (6):

$$\begin{aligned} (\widehat{\Delta w_t|\hat{x}_t}) &= \underset{(0.002)}{0.039} - \underset{(0.025)}{0.73}\, U_{r,t} \\ \widehat{\sigma} &= 1.05\% \;\; R^2 = 0.84 \;\; F_{\mathrm{ar}}(2{,}156) = 0.23 \;\; F_{\mathrm{arch}}(1{,}158) = 0.03 \\ \chi^2_{nd}(2) &= 0.86 \;\; F_{\mathrm{Het}}(2{,}157) = 0.47 \;\; F_{\mathrm{reset}}(2{,}156) = 0.03 \end{aligned} \tag{6}$$

The model passes all diagnostic tests and results in a constant parameter model. The sub-period regressions from (6) are shown in Figure 6 and are nearly identical across all sub-periods with the full-sample regression. Table 3 reports the estimated coefficients of unemployment and their HAC standard errors, which stand in sharp contrast to the estimates in Table 1, resolving Puzzle 4.

The Frisch and Waugh (1933) theorem suggests that $U_{r,t}$ should also be corrected for $\widehat{x}_t$, but $U_{r,t}$ is essentially orthogonal to $\widehat{x}_t$ in the full sample. However, as shown in Table 4, $\widehat{x}_t$ is significantly correlated with $U_{r,t}$ in those sub-samples where the Phillips curve shifted. We explore the implications of this result in section V.

Valid conditioning in non-constant processes requires super exogeneity, see Engle, Hendry, and Richard (1983) and Engle and Hendry (1993). We test the validity of conditioning on $U_{r,t}$ in the regression of $(\widehat{\Delta w_t|\hat{x}_t})$ using the automated test of super-exogeneity in Hendry and Santos (2010). We find that the major shifts in $U_{r,t}$ do not enter the regression of $(\widehat{\Delta w_t|\hat{x}_t})$ on $U_{r,t}$ confirming $U_{r,t}$ is super exogenous in that model.[5] These results are consistent with the evidence in Castle and Hendry (2014) that most UK unemployment has been involuntary.

[5]Results are available on request.

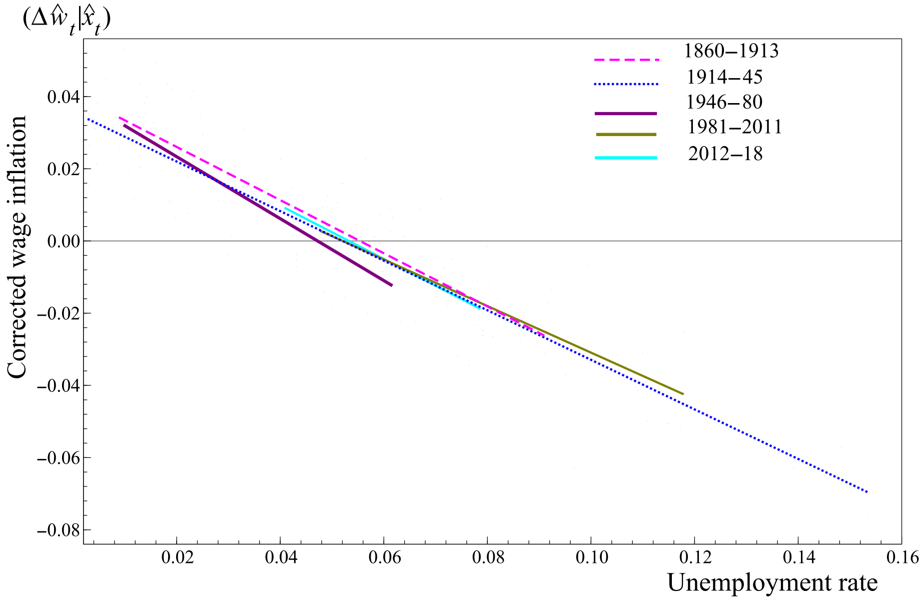

Figure 6. All sub-sample Phillips curves for Δw_t corrected for $\widehat{x}_t$ on U_r [Colour figure can be viewed at wileyonlinelibrary.com]

Table 3

Coefficients of $U_{r,t}$ and HAC SEs in the sub-sample $(\widehat{\Delta w_t|\hat{x}_t})$ regressions

	Coefficients	HAC SEs
1860–1913	−0.76	0.07
1914–45	−0.72	0.04
1946–80	−0.85	0.15
1981–2011	−0.67	0.07
2012–21	−0.84	0.11

Table 4

Coefficients of $\widehat{x}_t$ and their SEs with residual standard deviations $\widehat{\sigma}$ in the sub-sample $U_{r,t}$ regressions

	Coefficients	SEs	$\widehat{\sigma}$
1860–2021	0.002	0.05	0.033
1860–1913	0.11	0.18	0.019
1914–45	−0.23	0.11	0.045
1946–80	0.19	0.03	0.010
1981–2011	0.50	0.07	0.015
2012–21	1.01	0.30	0.010

V. Sub-sample equations: What caused the shifts?

To examine what changes were due to which variables and hence disentangle what is driving the instability, we decompose $\widehat{x}_t$ into its constituent parts, listed in the columns of Table 5, which records the regression coefficient values with $|t| \geq 2$ from fitting the

Table 5

Coefficients with $|t| \geq 2$ in sub-sample regressions for general model of $\Delta(w-p)_t$

Variable	$\Delta(y-l)_t$	$\Delta(y-l)_{t-1}$	$\Delta^2 p_t$	$\bar{U}_{r,t}$	$\bar{U}^2_{r,t}$	$\Delta_2 U_{r,t}$	$(\widetilde{f}_t \times \Delta p_t)$	e_{t-2}
1862–2021	0.41	0.13	−0.14	−0.18	3.1	−0.22	0.41	−0.18
1862–1913	0.18	0.17	−0.30	−0.20	3.3	−0.21	0.44	−0.10
1914–45	0.34	0	−0.12	−0.22	3.4	−0.38	0.65	0
1946–80	0.63	0.45	0	0	0	0	0	−0.41
1981–2011	0.49	0	−0.50	0	0	−0.47	0	0
2012–21	0.23	0.42	−1.1	0.16	32	0	0.47	−0.61

Table 6

Numbers of step shifts and outliers affecting the explanatory variables

Variable	$\Delta(y-l)_t$	$\Delta^2 p_t$	$\bar{U}_{r,t}$	$\bar{U}^2_{r,t}$	$\Delta_2 U_{r,t}$	$(\widetilde{f}_t \times \Delta p_t)$	e_t
Step indicators	4	8	5	16	11	9	12
Impulse indicators	0	5	0	1	1	3	1

Notes: IIS and SIS applied at $\alpha = 0.001$ with a forced constant over $T = 1862–2021$.

general model to sub-samples. Almost no mis-specification tests rejected in any of these sub-samples, but the final sample is too short to be reliable.

Cells marked with zeros are where parameter estimates have $|t|$-statistics less than 2. The first two sub-period coefficients are similar to the full sample despite the changing non-stationarity, resolving Puzzle 2. However, the next two differ in many respects: the impacts of $\bar{U}_{r,t}$ and $\bar{U}^2_{r,t}$ are then insignificant, as are the nonlinear inflation reactions. It is these *absent* impacts on wage inflation that explain the upward slopes, finally accounting for Puzzle 1.

Imposing the whole-sample coefficient estimates for the zero values over 1946–1980 in Table 5 yields an equation SE of $\widetilde{\sigma} = 1.17\%$ as against the unrestricted fit of $\widehat{\sigma} = 1.11\%$, with $\chi^2_{\text{rest}}(5) = 8.5$. Similarly, for 1981–2011, $\widetilde{\sigma} = 0.91\%$ vs. $\widehat{\sigma} = 0.98\%$ (the lower constrained value is an artefact of not counting restricted coefficients in the degrees of freedom) with $\chi^2_{\text{rest}}(6) = 3.7$. In neither case are the constraints rejected nor are any mis-specification tests significant for the constrained models, consistent with the overall constancy of the general model despite sub-sample estimate variations, confirming that the slope changes in the original Phillips curve are due to the lack of variability of the real-wage model regressors in the sub-samples.

As the real-wage model is constant over all sub-samples, shifts in the explanatory variables could cancel from co-breaking (Hendry and Massmann, 2007), reducing their variability which might account for their insignificance in some sub-samples. To test this hypothesis, we examine if the regressors in the real-wage model exhibit shifts that align. To detect outliers and shifts in each regressor, we apply impulse and step indicator saturation (IIS: Johansen and Nielsen, 2009; SIS: Castle *et al.*, 2015) at $\alpha = 0.001$ with just a forced constant. Table 6 reports the very different numbers of mean shifts and impulses selected, rejecting co-breaking and reinforcing the structural credentials of (3).[6]

[6]Their dates are available from the authors.

VI. Conclusion

The UK Phillips' curve relating changes in the log of nominal wages to the unemployment rate is unstable. Sub-period relationships can be strongly negative, slightly negative, flat, slightly positive and strongly positive in a time series from 1860 to 2021. Such outcomes prompted four puzzles, now resolved.

Puzzle 1: what caused Phillips curve slopes to change so much?

Partialling out the regressors other than the unemployment rate, $U_{r,t}$, in the real-wage model in section III from nominal wage inflation, Δw_t, the full-sample estimated coefficient linear combination of the regressors, $\widehat{x}_t$, was calculated. The correlation of $U_{r,t}$ with $\widehat{x}_t$ is insignificant in the full sample, but is significant in the sub-samples where the Phillips curve shifts. Estimating the real-wage model (3) for every sub-sample revealed some insignificant sub-sample coefficients, but imposing the full-sample estimated values on these produced constant equations with no deterioration in fit, identifying the culprits behind the instability in Puzzle 1 by their **absence**.

Puzzle 2: why is the Phillips curve over the very turbulent period 1914–45 similar to that over the stable 1860–1913 which Phillips originally used?

In the first two sub-periods, all the coefficients of the real-wage model (3) were significant and had similar magnitudes, resolving Puzzle 2 as to why the Phillips curve was not unstable over the most turbulent sample.

Puzzle 3: can a constant-parameter real–wage model successfully encompass a shifting nominal wage inflation equation? As shown in Figure 5, shifts in the sub-sample Phillips curves are matched by the implied outcomes of the constant parameter real-wage equation (3), a successful mis-specification encompassing outcome answering Puzzle 3.

Puzzle 4: yet can (3) also entail a marginal relationship between nominal wage inflation and the unemployment rate constant across all the sub-periods?

The constant real-wage equation (3) also entailed constant marginal nominal wage inflation equations shown in Figure 6 and in Table 3 with essentially the same downward slopes of between −0.67 to −0.85 explaining Puzzle 4.

As a check, in Appendix B a statistically adequate approximating model is developed to test the significant and comprehensive roles of the real-wage explanatory variables used in explaining the shifts.

Although the whole sample regression of Δw_t on $U_{r,t}$ delivers the same coefficient as in a much more general constant parameter equation, it is not a useful way to model an inflation–unemployment relation for economic policy. Instead, useful policy implications require taking account of the multivariate, nonlinear, dynamic, constant parameter relationship for real wages that encompasses the original Phillips curve formulation, interacting that with a price inflation model to determine the overall level and persistence of inflation, as in Castle *et al.* (2023).

Appendix A: Data definitions and sources

Y_t	=	real GDP, £million, 1985 prices	[6], p. 836, [5]a (1993), ONS code:YBHH at 2005 prices, [9].
P_t	=	implicit deflator of GDP, (1860 = 1)	[3], p. 836, [5]a (1993), ONS code: ABML, [9].
U_t	=	unemployment	[4], [5]c (1993), ONS code: MGSC.
$W\text{pop}_t$	=	working population	[4], [5]c (1993), ONS code: MGSF.
$U_{r,t}$	=	$U_t/W\text{pop}_t$ (unemployment rate, fraction)	
L_t	=	employment (= $W\text{pop}_t - U_t$)	[1], [5]
W_t	=	average weekly wage earnings	[7], [8], ONS code: LNMM
$W_{r,t}$	=	nominal wage rates	[2], [6], [8]
Δz_t	=	$(z_t - z_{t-1})$ for any variable z_t	
$\Delta^2 z_t$	=	$\Delta z_t - \Delta z_{t-1}$	

Sources:

[1] Shadman-Mehta (1995) (who cites Sleeman (1981) and Thomas (1984) as sources);

[2] Phillips (1958);

[3] Mitchell (1988);

[4] Feinstein (1972) and Boyer and Hatton (2002);

[5] Bean ((a) *Economic Trends Annual Supplements*, (b) *Annual Abstract of Statistics*, (c) *Department of Employment Gazette* and (d) *National Income and Expenditure*, as well as other sources cited here);

[6] Office for National Statistics, Blue Book;

[7] Crafts and Mills (1994);

[8] Feinstein (1990);

[9] ONS.

Hendry and Ericsson (1991) and Hendry (2001) provide detailed information about many of these series.

Appendix B: Checking the validity of the real-wage variables

The above results hinge on the constant parameter real-wage equation but could there be other explanations? To provide validity to our approach we establish a statistically adequate model of nominal wages using polynomial approximations and saturation estimators, and then test for the significance of the real-wage regressors within the approximating model, see Sargan (1964) and Spanos (2019), pp. 716–729, for an example. The approximating model relies on Weierstrass' approximation theorem along with impulse and step indicator saturation to model outliers and discontinuities.

We select a dynamic linear regression model using Autometrics (Doornik, 2009). We generate polynomials up to the fourth order of Δw and U_r and include lags 1 and 2 of the polynomials of Δw, and lags 0, 1 and 2 of the polynomials of U_r, along with a constant and trend. All the regressors are forced to enter the selected model when impulse and step saturation is applied at a significance of $\alpha = 0.001$. Then all regressors (polynomials and indicators) are selected at $\alpha = 0.05$: the final statistically adequate model over T =1863–2021 is:

$$\begin{aligned}
\Delta\widehat{w}_t = & \underset{(0.01)}{0.05} + \underset{(0.09)}{0.28}\Delta w_{t-1} + \underset{(0.69)}{2.55}\Delta w_{t-1}^2 + \underset{(2.01)}{5.94}\Delta w_{t-1}^3 - \underset{(14.3)}{39.9}\Delta w_{t-1}^4 - \underset{(6.87)}{39.5}U_{r,t}^2 \\
& + \underset{(2.63)}{10.8}U_{r,t-1}^2 + \underset{(102)}{502}U_{r,t}^3 - \underset{(17.8)}{55.3}U_{r,t-1}^3 - \underset{(403)}{1796}U_{r,t}^4 + \underset{(0.02)}{0.11}I_{1915} + \underset{(0.02)}{0.14}I_{1917} \\
& - \underset{(0.02)}{0.12}I_{1921} - \underset{(0.02)}{0.22}I_{1922} - \underset{(0.02)}{0.08}I_{1944} - \underset{(0.02)}{0.22}S_{1939} + \underset{(0.03)}{0.33}S_{1940} - \underset{(0.02)}{0.13}S_{1941} \\
& + \underset{(0.02)}{0.06}S_{1944} - \underset{(0.02)}{0.05}S_{1945} - \underset{(0.01)}{0.03}S_{1967} - \underset{(0.02)}{0.08}S_{1974} + \underset{(0.02)}{0.13}S_{1975} - \underset{(0.01)}{0.09}S_{1977} \\
& + \underset{(0.01)}{0.06}S_{1980}. \\
\widehat{\sigma} \; & = 1.51\% \;\; R^2 = 0.94 \;\; F_{\mathrm{ar}}(2,132) = 0.85 \;\; F_{\mathrm{arch}}(1,157) = 0.69 \\
\chi^2_{nd}(2) = & \; 0.99 \;\; F_{\mathrm{Het}}(21,128) = 1.11 \;\; F_{\mathrm{reset}}(2,132) = 1.36 \;\; F_{\mathrm{nl}}(27,107) = 1.57
\end{aligned} \tag{B1}$$

The dynamic linear approximating model (B1) forms the basis for selecting the real-wage explanatory variables as it is congruent, passing all diagnostic tests. If the proposed real-wage variables are statistically significant and render the approximating polynomials in (B1) insignificant then we can be confident that the selection process is reliable and the retained explanatory variables are relevant to explaining the instability in the Phillips curve. To do so, we augment (B1) with the regressors from the real-wage model listed in

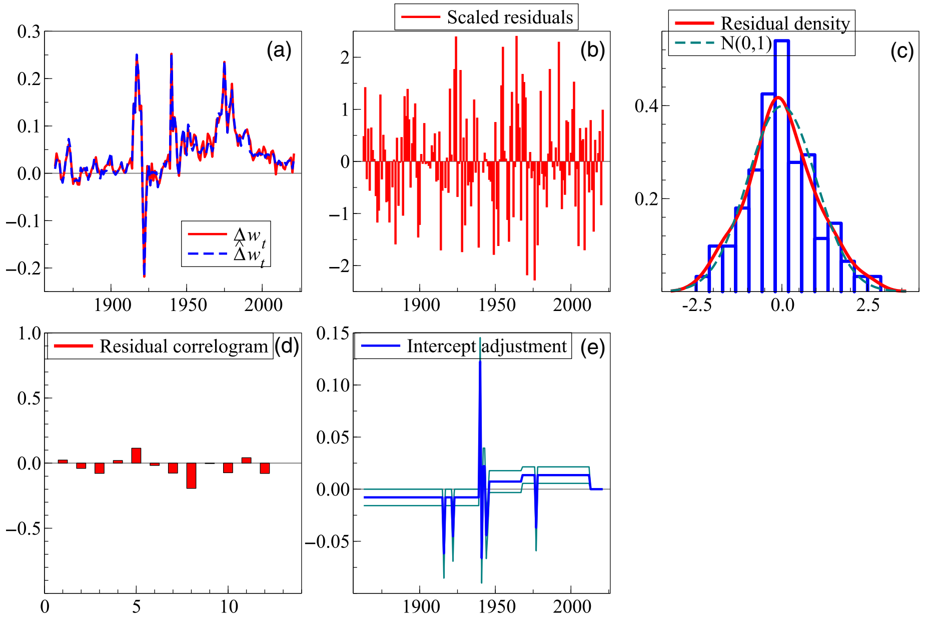

Figure B1. Equation (B2) model fit (panel (a)), residuals (panel (b)), residual density (panel (c)), autocorrelogram (panel (d)), and intercept adjustment (panel (e)) [Colour figure can be viewed at wileyonlinelibrary.com]

Table 5 and apply selection, imposing a coefficient of 1 on price inflation to directly map to the real-wage model, resulting in:

$$\begin{aligned}\widehat{\Delta w_t} = &\underset{(\cdots)}{1.0}\Delta p_t + \underset{(0.04)}{0.41}\Delta(y-l)_t + \underset{(0.04)}{0.15}\Delta(y-l)_{t-1} - \underset{(0.03)}{0.16}e_{t-2} \\ &+ \underset{(0.07)}{0.38}(\sim f_t \times \Delta p_t) - \underset{(0.03)}{0.15}\Delta^2 p_t - \underset{(0.03)}{0.17}(U_{r,t} - 0.05) \\ &+ \underset{(0.66)}{3.0}(U_{r,t} - 0.05)^2 - \underset{(0.05)}{0.18}\Delta_2 U_{r,t} - \underset{(0.01)}{0.13}S_{1939} \\ &+ \underset{(0.02)}{0.18}S_{1940} - \underset{(0.01)}{0.07}S_{1941} + \underset{(0.002)}{0.02}S_{2012} \\ &- \underset{(0.01)}{0.04}I_{1916} - \underset{(0.01)}{0.05}I_{1977} + \underset{(0.005)}{0.03}I_{\text{WWII}} - \underset{(0.01)}{0.04}I_{1922} \end{aligned} \tag{B2}$$

$$\hat{\sigma} = 1.1\% \;\; R^2 = 0.82 \;\; F_{\text{ar}}(2{,}142) = 0.07 \;\; F_{\text{arch}}(1{,}158) = 0.25$$

$$\chi^2_{nd}(2) = 0.61 \;\; F_{\text{Het}}(19{,}135) = 1.21 \;\; F_{\text{reset}}(2{,}142) = 1.29 \;\; F_{\text{nl}}(24{,}120) = 0.87$$

The selected model (B2) maintains statistical adequacy and is almost identical to the real-wage model reported in equation (3). The only additional significant term is the impulse indicator for 1922, which reflects an outlier not included in (3) and removes the significance of the heteroscedasticity test. The model fit, residuals, residual density and autocorrelogram, and intercept adjustment are recorded in Figure B1. From these results we conclude that the appropriate explanatory variables have been selected to deliver a statistically adequate model of nominal wages, and hence our inference regarding the explanations for the instability of the Phillips curve is justified.

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