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The Perils of a Dual Mandate*

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Abstract

We study the implications of a 'dual mandate' of price and output stability in a heterogeneous agent New Keynesian economy where fiscal policy is set in nominal terms. Specifically, the government controls the quantity of nominal debt, enabling price level determination independently of the interest rate trajectory (Hagedorn, 2021). Our findings indicate that under an inflation-targeting regime, price level determinacy is often the exception than the norm when the central bank pursues a dual mandate. The dynamics of government spending emerge as a crucial driver of this result. To address this challenge, we show that possible solutions include price level targeting and stabilizing consumption inequality.

Keywords: heterogeneity, price-level targeting, inflation targeting, HANK

JEL Classification: E12, E52, E58, E63.

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1 Introduction

The efficacy of price stability as the sole objective of monetary policy has been a topic of considerable debate in the context of macroeconomic stabilization. The concept of 'divine coincidence'—whereby stabilizing inflation inherently stabilizes economic activity—does not hold in various scenarios, leading to a significant trade-off between inflation and output stabilization. Evidence suggests that central banks could improve welfare by addressing real economic fluctuations (Rosengren, 2014; Debortoli *et al.*, 2019). Consequently, the 'dual mandate' of price stability and maximum sustainable employment has been embraced by the Federal Reserve and other central banks (Meyer, 2001; Svensson, 2014). Recent literature on optimal policy in Heterogeneous Agent New Keynesian (HANK) models further questions the divine coincidence result (Acharya *et al.*, 2023; Dávila and Schaab, 2023).

In this paper, we analyze the stability of a heterogeneous agent New Keynesian (HANK) economy with a central bank following a dual mandate of price and output stability. Our model integrates household heterogeneity, incomplete markets, and positive liquidity in a tractable manner, following Bilbiie (2024).[1] Fiscal policy is operationalized as in Hagedorn (2021) in nominal terms, with the government determining the fiscal stance through the management of nominal debt levels. As a result, nominal taxes automatically adjust to satisfy the nominal government's budget constraint for any price level.[2] By doing so, we introduce an additional channel whereby price level shifts impact aggregate demand via fiscal variables (Hagedorn, 2018). This complements the standard channel where fluctuations in public debt stock influence real aggregate demand (Barro, 1974).

In this economy, we find that an equilibrium under a Taylor rule targeting inflation and output fluctuations is not saddle path stable. We show that this crucially depends on the evolution of government spending. When the fiscal authority sets public spending in nominal terms, movements in the price level make real government spending endogenous even if nominal spending is held

[1]See Debortoli and Galí (2024) for a detailed analysis of merits and limitations of simple tractable two-agents heterogeneous New Keynesian models in replicating the aggregate predictions of fully fledged quantitative HANK models.

[2]This aligns with what happens in practice, as governments set their budget in nominal terms (Hagedorn *et al.*, 2019a).

constant. Specifically, a central bank responding to output fluctuations leads to instability if the nominal interest rate's reaction to inflation exceeds a one-to-one ratio (the *Taylor Principle*) and to indeterminacy otherwise.

The intuition of these results is as follows. Assuming that the fiscal authority keeps the quantity of nominal debt and spending constant, an increase in the price level (due to an increase in aggregate demand, for example) makes real debt and spending decrease over time. If we consider a persistent increase in the price level, this will necessarily also exert downward pressure on real taxes, independent of what happens to nominal taxes. A decline in real taxes has a positive effect on real consumption. On the other hand, households also face contrasting effects on their consumption from the substitution effect of higher interest rates (due to high prices) and the precautionary motive from idiosyncratic risk. As a result, real consumption will not increase one-to-one with the fall in real taxes. If real spending is declining and consumption does not increase enough to offset this, real activity might start declining.

In this situation, a dual mandate creates a complex balancing act for the central bank, as it must simultaneously respond to inflation and economic activity. Persistent inflation pushes nominal interest rates up, yet lower economic activity coupled with high savings demand from households signals a need for lower interest rates to balance goods and asset markets. This dual response dilutes the impact of interest rate hikes, unlike a single-mandate policy focusing solely on inflation. As real government spending keeps declining, aggregate demand fails to keep pace. To further highlight its crucial role, we show that if we remove public spending from the model, this destabilizing effect on aggregate demand is absent, implying a one-to-one relationship between real output and consumption.

We first present these results numerically, followed by a series of robustness and sensitivity checks. Allowing for real spending to be exogenous instead of nominal spending, the model remains saddle path stable under a dual mandate, resembling the version of the model without government spending. If real spending is held constant, changes in the price level affect only real output and consumption, contrasting with the endogenous effect seen when government spending is expressed in nominal terms. We then show that the addition of interest rate smoothing, different specifications of the fiscal rules, and the steady-state level of the debt-to-GDP ratio do not change our results. Furthermore, we demonstrate that allowing for different degrees of heterogeneity, idiosyncratic risk, and fiscal/profit redistribution also does not affect our results.

We also show that the stability issue with a dual mandate carries over even in a simplified version of the model where we assume myopic firms, no steady-state consumption inequality, and zero government spending in steady state to obtain closed-form expressions for stability analysis. This simplified model also allows us to gain intuition on the results.

The final part of this work proposes two possible solutions to the problem we highlight. We show that targeting the price level or consumption inequality can ensure determinacy in the presence of a dual mandate. Why is that? Standard inflation targeting is a growth target and not a level target. By generating history dependence in the policy rule, price level targeting can stabilize the economy. Instead, by stabilizing consumption inequality, the central bank allows consumption demand to be more stable in response to a real tax cut. This reduces the need to lower interest rates to clear demand and, therefore, the trade-off faced by the central bank under a dual mandate.

Related Literature Our research intersects with the extensive body of literature on equilibrium determinacy in New Keynesian models featuring rational expectations. It particularly focuses on the interplay between monetary and fiscal policy, agent heterogeneity, and market incompleteness.

Bilbiie (2008) and Galí *et al.* (2004) are the first to study the stability properties of heterogeneous agents NK models in a two-agent setup (TANK). They show that when asset market participation is restricted enough, i.e., the proportion of hand-to-mouth agents in the economy is relatively high, the Taylor principle gets inverted such that the central bank needs to follow a passive rule to ensure a unique equilibrium. The HANK literature has shown that achieving determinacy of equilibria in these economies is more challenging compared to RANK (Acharya and Dogra, 2020; Bilbiie, 2024; Ravn and Sterk, 2021). The key to this result lies in the cyclicality of income risk. If income risk is countercyclical, then the Taylor principle is not sufficient to ensure determinacy, and the central bank must follow a more aggressive stabilization of inflation to do so. None of them, however, focuses on a dual mandate for the central bank, which is our aim here. On the other hand, we abstract from income risk cyclicality here. Hagedorn (2023) studies local determinacy in incomplete market models with nominal fiscal policy. He focuses, however, mostly on constant fiscal/monetary policy and on monetary rules without a dual mandate.

Moreover, our work builds on Hagedorn (2021), which shows that the demand for nominal bonds in incomplete-market economies combined with a nominal debt supply rule leads to price-level determinacy independently of any interest rate rule. Determinacy is attained by allowing nominal taxes to adjust automatically to ensure that the government budget constraint is satisfied for any price level. The central bank then sets freely the nominal interest rate that clears the bond market, with no need to respond to any endogenous variable. Hence, the price level can be uniquely determined jointly by monetary and fiscal policy, even when the interest rate is exogenously chosen. Heterogeneity and incomplete markets are crucial as they generate precautionary savings demand. The specification of fiscal policy in nominal terms is a critical component of this outcome, as it ties the equilibrium to the price level, enabling unique determination. This mechanism contrasts with the traditional approach in macroeconomic models, where fiscal policy is usually specified in real terms. Crucially, this setup is also different from the Fiscal Theory of the Price Level, which assumes that the government budget constraint must be satisfied by only one price level.

Hagedorn (2021)'s Demand Theory of the Price Level is a powerful result. It broadens the set of monetary and fiscal policies that can be evaluated in macroeconomic models. This theory has already been employed in various contexts; for instance, Hagedorn *et al.* (2019b) used it to solve the *Forward Guidance puzzle* in incomplete markets models, while Hagedorn *et al.* (2019a) demonstrated that the magnitude of the fiscal multiplier is positively correlated with the degree of price stickiness. Importantly, they showed that this correlation is contingent on the specific combination of fiscal and monetary policies employed in such models.

Differently from us, Hagedorn *et al.* (2019b) focused on Taylor rules without a dual mandate, whereas Hagedorn *et al.* (2019a) on a price-level (*Wicksellian*) rule with a dual mandate. An analytical application of Hagedorn (2021)'s result is also provided in Bilbiie (2024). Here, we borrow his tractable HANK setup with positive liquidity and extend it by considering the presence of government spending in the model, which turns out to be crucial in driving the results.

This paper is organized as follows: Section 2 describes the model. Section 3 starts by studying numerically the stability of the model economy and presenting the robustness of the results. It then derives closed-form expressions based on a simplified version of the model and discusses the intuition behind the results. Section 4 discusses possible solutions to the dual mandate stability problem, and Section 5 concludes.

2 Model

The model is based on the tractable-HANK model developed in Bilbiie (2024). The key features of this setup include aggregate demand amplification through household heterogeneity, precautionary savings, and government-provided liquidity. We build upon the model's variant with liquidity in positive net supply and specify the fiscal side in nominal terms as in Hagedorn (2021). Throughout the model exposition, we use superscripts to indicate real (R) and nominal (N) variables as well as agent-specific variables defined below.

2.1 Households

A unit mass of households $j \in [0, 1]$ discount the future at rate β, derive utility from real consumption, $C_t^{R,j}$, and disutility from labor supply, N_t^j. They have access to two assets that differ in their liquidity: a government-issued riskless 'nominal' bond, B_t^N, which gives a nominal gross return $R_t^N > 0$ and is liquid, and illiquid shares in monopolistically competitive firms. The difference in liquidity implies that out of the two assets, only bonds can be used for self-insurance purposes. Households can freely adjust their portfolio, participate infrequently in financial markets, and receive dividends from firms when they do. Otherwise, they only receive the payoff from previously accumulated bonds. Household heterogeneity is introduced as follows. Type H agents do not save, but they make an optimal labor supply decision determining their wage income. Type S agents instead supply labor, save in bonds, and also receive profits.

The idiosyncratic uncertainty in the model can be explained as follows. There are two states, S for 'savers' (participants) and H for 'hand-to-mouth' (non-participants) households with exogenous switching between states. The change of state follows a Markov chain: the probability to stay type S is s, and the probability of staying type H is h, with transition probabilities $(1-s)$ and $(1-h)$, respectively.

The share of hand-to-mouth households thus evolves according to $\lambda_{t+1} = h\lambda_t + (1-s)(1-\lambda_t)$. The focus is on stationary equilibria, which implies the mass of H, denoted by λ, is the unconditional probability of being hand-to-mouth, which is given as:

$$\lambda = \frac{1-s}{2-s-h}. \tag{1}$$

Accordingly, the mass of S is $(1-\lambda)$. For stationarity, it is required that $s \geq (1-h)$, i.e., the probability of remaining in state S is larger than the probability of moving to state S.

The two states of the households can be thought of as being 'islands', with all savers on the S island and all hand-to-mouth agents on the H island. At the beginning of the period, the family head pools all the resources of the respective island and makes the symmetric consumption-saving decision for each household on the island after the aggregate shocks are revealed. Once these decisions are made, households are informed of their next period state, and they have to move to the corresponding state's island while only taking bonds with them.[3] No transfers can be made to any household once their next period state or the idiosyncratic shock is revealed. In this sense, there is 'full insurance within type' (due to the symmetric consumption-saving choice) but 'limited insurance across types'. This assumption of insurance is key to attaining a tractable representation.

Let's define $B_{t+1}^{R,j}$ as the per-capita, real beginning-of-period-$t+1$ nominal bonds on the island $j = S, H$, after the consumption-saving choice, and also after changing state and pooling. $Z_t^{R,j}$ is the per-capita, end-of-period-t real values after the consumption-saving choice but before agents

[3]Therefore, each period, a measure $(1-h)\lambda$ leaves the H island to become S next period, while a measure λh stays there. Similarly, a measure $(1-s)(1-\lambda)$ leaves the S island to move to the H island, while $s(1-\lambda)$ stay on S.

switch islands. Denote $\mathbf{B}^{R,j}_{t+1}$, $j = S, H$ as the aggregate real bond holdings of the entire island. We, therefore, have the following relationships for bond holdings:

$$\mathbf{B}^{R,S}_{t+1} = (1-\lambda)B^{R,S}_{t+1} = (1-\lambda)sZ^{R,S}_{t+1} + \lambda(1-h)Z^{R,H}_{t+1}, \tag{2}$$

$$\mathbf{B}^{R,H}_{t+1} = \lambda B^{R,H}_{t+1} = (1-\lambda)(1-s)Z^{R,S}_{t+1} + \lambda h Z^{R,H}_{t+1}. \tag{3}$$

So, the per-capita flow across islands, after re-scaling with relative population masses, $(1-\lambda)$ and λ respectively, and using (1), are:

$$B^{R,S}_{t+1} = sZ^{R,S}_{t+1} + (1-s)Z^{R,H}_{t+1}, \tag{4}$$

$$B^{R,H}_{t+1} = (1-h)Z^{R,S}_{t+1} + hZ^{R,H}_{t+1}. \tag{5}$$

With B^{R}_{t+1} as the aggregate real value of the bonds in the economy, the bond market clearing requires

$$B^{R}_{t+1} = \lambda Z^{R,H}_{t+1} + (1-\lambda)Z^{R,S}_{t+1}. \tag{6}$$

Since we know the H agents do not save i.e. $Z^{R,H}_{t+1} = 0$, the bond market clearing condition implies

$$B^{R}_{t+1} = (1-\lambda)Z^{R,S}_{t+1}. \tag{7}$$

So, the savers hold all the bonds next period. Given this, combining equation (4) with (7), we get the real holdings of the S island as

$$B^{R,S}_{t+1} = sZ^{R,S}_{t+1} = \frac{s}{1-\lambda}B^{R}_{t+1}. \tag{8}$$

Combining equation (5) with (7) and using (4), we get the following real holdings of the H island:

$$B^{R,H}_{t+1} = (1-h)Z^{R,S}_{t+1} = \frac{1-h}{1-\lambda}B^{R}_{t+1} = \frac{1-s}{\lambda}B^{R}_{t+1}. \tag{9}$$

Due to bonds being liquid, they make it to the H island, so a proportion $\frac{1-s}{\lambda}$ of the payoff from the bonds accrues to the next period's H households. Stocks are illiquid and never leave the S island, so we do not have to keep track of them.

By definition, and given their predetermined nature, we have that $Z_{t+1}^{R,j} = \frac{Z_{t+1}^{N,j}}{P_t}$ and $B_{t+1}^{R,j} = \frac{B_{t+1}^{N,j}}{P_t}$. Therefore, all the bond flow relationships presented in (2)-(9) also hold in nominal terms.

Given this, the optimization problem of the family head is given by:

$$\mathbb{W}\left(\frac{B_t^{N,S}}{P_{t-1}}, \frac{B_t^{N,H}}{P_{t-1}}, \Omega_t^R\right) = \max_{\left\{C_t^{R,S}, C_t^{R,H}, Z_{t+1}^{R,S}, Z_{t+1}^{R,H}, \Omega_{t+1}^R\right\}} \left[(1-\lambda)U(C_t^{R,S}, N_t^S) + \lambda U(C_t^{R,H}, N_t^H)\right] + \beta E_t \mathbb{W}\left(\frac{B_{t+1}^{N,S}}{P_t}, \frac{B_{t+1}^{N,H}}{P_t}, \Omega_{t+1}^R\right)$$

subject to:

$$C_t^{R,S} + Z_{t+1}^{R,S} + V_t \frac{\Omega_{t+1}^R}{1-\lambda} = W_t^R N_t^S + R_{t-1}^N \frac{P_{t-1}}{P_t} \frac{B_t^{N,S}}{P_{t-1}} + \frac{\Omega_t^R}{1-\lambda}(V_t + (1-\tau^D) D_t^R) - \frac{T_t^{N,S}}{P_t}, \tag{10}$$

$$C_t^{R,H} + Z_{t+1}^{R,H} = W_t^R N_t^H + R_{t-1}^N \frac{P_{t-1}}{P_t} \frac{B_t^{N,H}}{P_{t-1}} + \frac{\tau^D D_t^R}{\lambda} - \frac{T_t^{N,H}}{P_t}, \tag{11}$$

$$Z_{t+1}^{R,S}, Z_{t+1}^{R,H} \geq 0, \tag{12}$$

as well as laws of motion for bond flows (equations (4) and (5)).

To highlight the role of nominal fiscal policy, in the above optimization problem, we have explicitly left fiscal variables in nominal terms divided by the price level (P_t) while everything else is in real terms. Specifically, W_t^R is the real wage, R_{t-1}^N is the nominal interest rate earned on bonds, Ω_t^R is the portfolio of shares with price V_t held by S households who receive real dividends D_t^R. The possibility of redistribution of these dividends is determined by the constant tax on profits τ^D. Further, $T_t^{N,j}$ with $j = S, H$, are per-capita lumpsum nominal taxes on the respective island with

$\lambda T_t^{N,H} = \alpha T_t^N$ and $(1-\lambda)T_t^{N,S} = (1-\alpha)T_t^N$, where T_t^N is the aggregate nominal lumpsum taxes and α is the share of aggregate taxes paid by the H agent while $1-\alpha$ the share of the S agent.

The Kuhn-Tucker equations of the optimization problem are presented next. The set of equations governing the bond-holding decision of the S island are:

$$U'(C_t^{R,S}) \geq \beta E_t \left\{ R_{t+1} \left[sU'(C_{t+1}^{R,S}) + (1-s)U'(C_{t+1}^{R,H}) \right] \right\}, \tag{13}$$

$$\text{and } \; 0 = Z_{t+1}^{R,S} \left[U'(C_t^{R,S}) - \beta E_t \left\{ R_{t+1} \left[sU'(C_{t+1}^{R,S}) + (1-s)U'(C_{t+1}^{R,H}) \right] \right\} \right], \tag{14}$$

where we use the Fisher relation, $R_t = R_{t-1}^N \frac{P_{t-1}}{P_t}$. The set of equations governing the bond-holding decision of the H island are:

$$U'(C_t^{R,H}) \geq \beta E_t \left\{ R_{t+1} \left[(1-h)U'(C_{t+1}^{R,S}) + hU'(C_{t+1}^{R,H}) \right] \right\} \tag{15}$$

$$\text{and } \; 0 = Z_{t+1}^{R,H} \left[U'(C_t^{R,H}) - \beta E_t \left\{ R_{t+1} \left[(1-h)U'(C_{t+1}^{R,S}) + hU'(C_{t+1}^{R,H}) \right] \right\} \right]. \tag{16}$$

The equation corresponding to illiquid shares is:

$$U'(C_t^{R,S}) \geq \beta E_t \left\{ \frac{V_{t+1} + (1-\tau^D)D_{t+1}^R}{V_t} U'(C_{t+1}^{R,S}) \right\}; \tag{17}$$

with $\Omega_{t+1}^R = \Omega_t^R = (1-\lambda)^{-1}$. The bond Euler equations are of the Bewely-Huggett-Aiyagari form, as is standard in incomplete-markets models. The Euler equation of S agents (13) highlights the self-insurance motive of these agents. Here, $(1-s)$ is the probability of switching to a bad state next period, which generates a precautionary demand for bonds.[4] To get a simple equilibrium representation, the focus is on equilibria where the Euler equation of S agents is satisfied with a strict equality, while the one of H agents has a strict inequality.[5] There is no self-insurance motive

[4]With $s = 1$, i.e., the case of permanent idiosyncratic shocks, we get the Euler equation of the Savers/Ricardian agents in the TANK Model, as in Bilbiie (2008).

[5]Bilbiie (2024) rationalizes H agents' binding constraint as follows: This could be thought as a liquidity or impatience shock which makes H agents consume more today, or their average income in that state is lower enough than in the S state, or a technological constraint prevents them from accessing any asset markets, etc.

for stocks, for they cannot be carried to the H state. So their Euler equation, (17), is isomorphic to the representative-agent framework, determining the price V_t.

In the rest of the paper, the household's utility function takes the form:

$$U(C_t^{R,j}, N_t^j) = \frac{(C_t^{R,j})^{1-\sigma}}{1-\sigma} - \nu \frac{(N_t^j)^{1+\phi}}{1+\phi}, \tag{18}$$

where σ is the inverse of the intertemporal elasticity of substitution, ϕ is the inverse of Frisch elasticity of labor supply, and ν weights the disutility of working. We introduce monopolistic competition in the firm's sector by assuming that households consume an aggregate basket of goods $k \in [0,1]$, with constant elasticity of substitution $\eta > 1$: $C_t^R = \left(\int_0^1 C_t^R(k)^{\frac{1-\eta}{\eta}} dk\right)^{\frac{\eta}{\eta-1}}$. This yields the standard demand functions $C_t^R(k) = \left(\frac{P_t(k)}{Pt}\right)^{-\eta} C_t^R$ and the aggregate price index of the economy is $P_t^{1-\eta} = \int_0^1 P_t(k)^{1-\eta} dk$.

From the optimization problem above, the labor supply condition for each household type is $\nu\phi n_t^j = w_t^R - \sigma c_t^{R,j}$, where variables without time subscripts represent steady-state values and variables in lowercase are in log-linear deviations from their steady state i.e. $c_t^{R,j} = \frac{C_t^{R,j} - C^j}{C^j}$.[6] Following Bilbiie (2024), hours from the two types of agents are pooled into a labor union. This implies that $N_t^j = N_t$ and the log-linearised aggregate labour supply decision is $\nu\phi n_t = w_t^R - \sigma c_t^R$. The household's intertemporal decisions are captured by the following (log-linearized) self-insurance equation for the savers:

$$c_t^{R,S} = \frac{(1-s)\Gamma^\sigma}{(1-s)\Gamma^\sigma + s} E_t c_{t+1}^{R,H} + \frac{s}{(1-s)\Gamma^\sigma + s} E_t c_{t+1}^{R,S} - \sigma^{-1}(r_t^N - E_t \pi_{t+1}), \tag{19}$$

where $\pi_{t+1} = p_{t+1} - p_t$, and $\Gamma \equiv \frac{C^S}{C^H}$ is steady-state consumption inequality.

[6]The full non-linear model and steady-state equations are listed in Appendix A and the linearized model equations are listed in Appendix B. We normalize $P = 1$ in steady state therefore, we remove R and N superscripts from steady state variables as real and nominal variables are the same.

2.2 Firms

There is a continuum of monopolistically competitive firms producing differentiated goods $Y_t^R(k)$ using labor $N_t(k)$ according to a constant-returns production function $Y_t^R(k) = N_t(k)$. The final goods sector is competitive and aggregates these differentiated intermediate goods according to a CES technology, and the demand function for their output takes the form $Y_t^R(k) = \left(\frac{P_t(k)}{P_t}\right)^{-\eta} Y_t^R$.

Pricing decisions of these firms face a quadratic cost of adjustment as in Rotemberg (1982), parameterized by ξ. Cost-minimization implies that real marginal cost, MC_t^R, is equal to the real wage W_t^R, where the real wage is common to all firms. The flow of profits for producer k is given by:

$$D_t^R(k) = P_t(k)\left(\frac{P_t(k)}{P_t}\right)^{-\eta} Y_t^R - W_t^R N_t(k) - \frac{\xi}{2}\left(\frac{P_t(k)}{P_{t-1}(k)} - 1\right)^2 P_t Y_t^R. \tag{20}$$

Each period, firms choose the price to maximize the expected present discounted value of profits discounted by the savers's stochastic discount factor. In equilibrium, all firms behave identically, and the optimality condition for price-setting is given by:

$$(1-\eta) + \eta MC_t^R \; - \Pi_t \, \xi \; (\Pi_t - 1) + \beta \frac{\xi \, E_t \Pi_{t+1} \frac{U'(C_{t+1}^{R,S})}{U'(C_t^{R,S})} \; (E_t \Pi_{t+1} - 1) \; Y_{t+1}^R}{Y_t^R} = 0,$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$. This price-setting behavior implies the following (log-linearised) New Keynesian Philips Curve (NKPC):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \phi y_t^R + \kappa \sigma c_t^R, \tag{21}$$

where $\kappa = \frac{\eta - 1}{\xi}$.

2.3 Fiscal policy

The fiscal side of the model is specified in nominal terms as in Hagedorn (2021). The government spends a nominal amount, G_t^N, levies lump-sum nominal taxes, T_t^N, and supplies nominal liquid bonds, B_t^N. By definition real government spending is $G_t^R = \frac{G_t^N}{P_t}$, real bond holdings are $B_t^R = \frac{B_t^N}{P_t}$, and real taxes, $T_t^R = \frac{T_t^N}{P_t}$.[7] Given that the fiscal rule followed by the fiscal authority will also be specified in nominal terms and that spending is exogenous, either nominal taxes or nominal debt need to adjust to ensure that the following nominal government's budget constraint is satisfied

$$B_{t+1}^N = R_{t-1}^N B_t^N + G_t^N - T_t^N. \tag{22}$$

Note that this nominal constraint is satisfied for all price levels. Hence, this is different from the Fiscal Theory of the Price Level which assumes that the government budget constraint must be satisfied by only one price level.

In what follows, we assume that the fiscal authority sets fiscal policy by choosing the level of nominal debt.

The log-linearized fiscal block of the model is therefore represented by three equations. The government's budget constraint

$$b_{t+1}^N = R(b_t^N + r_{t-1}^N) + \frac{G}{B} g_t^N - \frac{T}{B} t_t^N, \tag{23}$$

The fiscal rule[8]

$$b_{t+1}^N = \Phi_{BB} b_t^N + \Phi_{BP} p_t + \Phi_{BG} g_t^N + \varepsilon_t^B, \tag{24}$$

and the following $AR(1)$ process for nominal spending

$$g_t^N = \rho_g g_{t-1}^N + \varepsilon_t^G. \tag{25}$$

[7]In the baseline calibration we assume no exogenous redistribution across agents via taxes. This means that $\alpha = \lambda$ and therefore $T_t^{N,H} = T_t^{N,S} = T_t^N$.

[8]In our baseline analysis, we will restrict this fiscal rule to be $b_{t+1}^N = 0$ following Bilbiie (2024). In section C, we provide robustness, allowing for the general rule presented here.

2.4 Market clearing

Aggregation of consumption and labor supply between the two types of households gives:

$$C_t^R = \lambda C_t^{R,H} + (1-\lambda) C_t^{R,S}, \tag{26}$$

$$N_t = \lambda N_t^H + (1-\lambda) N_t^S. \tag{27}$$

The resource constraint of the economy is $Y_t^R = C_t^R + \frac{G_t^N}{P_t}$, which in log-linear form is:

$$y_t^R = \frac{C}{Y} c_t^R + \frac{G}{Y}(g_t^N - p_t). \tag{28}$$

2.5 Monetary policy

The central bank sets the nominal interest rate following a dual mandate. It does so by following a Taylor type rule with $\Phi_\pi, \Phi_y \geq 0$ as the inflation and output coefficients:

$$r_t^N \quad = \Phi_\pi \pi_t + \Phi_y y_t^R + \varepsilon_t^M, \tag{29}$$

where ε_t^M is an iid monetary policy shock.

2.6 Log-linearized model

We solve the model around a zero net inflation steady state ($\Pi = 1$) and normalize the price level to 1 ($P = 1$). Consumption inequality in steady state arises naturally in this model because of the differences in two sources of income between the two types of agents: i) profits for the savers and ii) different bond holdings between the two islands.[9]

Combining the type S household's Euler equation (19) with the budget constraint of type H agents (11) and the resource constraint (28), we obtain the log-linearized aggregate Euler/IS equation

[9]Given that both agents work the same amount of hours, $N^S = N^H = 1/3$, both types of agents get the same labor income in steady state.

Table 1: Linearized equilibrium conditions

Description	Equation
IS equation	equation (30)
Philips curve	$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\phi C_y + \sigma\right) c_t^R + \kappa\phi G_y (g_t^N - p_t)$
Government budget constraint	$b_{t+1}^N = R(b_t^N + r_{t-1}^N) + \frac{G}{B} g_t^N - \frac{T}{B} t_t^N$
Debt rule rule	$b_{t+1}^N = \Phi_{BP} p_t + \Phi_{BB} b_t^N + \Phi_{BG} g_t^N + \varepsilon_{b^N}^B$
Evolution of prices	$p_t = \pi_t + p_{t-1}$
Government spending shock	$g_t^N = \rho g_{t-1}^N + \varepsilon_t^G$
Taylor rule	$r_t^N = \Phi_\pi \pi_t + \Phi_y (C_y c_t^R + G_y (g_t^N - p_t)) + \varepsilon_t^M$

Notes: This table lists the log-linearized equations of the model. The full set of non-linear conditions is presented in Table A1.

of the economy

$$\begin{aligned} c_t^R = {} & \delta E_t c_{t+1}^R + A_1 \left[\lambda \left(g_t^N - p_t\right) + \zeta \left(E_t g_{t+1}^N - E_t p_{t+1}\right)\right] + \\ & + A_2 \left[\lambda \left(t_t^N - p_t\right) + \zeta \left(E_t t_{t+1}^N - E_t p_{t+1}\right)\right] \\ & + A_3 \left[\lambda \left(b_t^N + r_{t-1}^N - p_t\right) + \zeta \left(E_t b_{t+1}^N + r_t^N - E_t p_{t+1}\right)\right] - A_4 (r_t^N - E_t \pi_{t+1}). \end{aligned} \tag{30}$$

The above equation's composite parameters $\delta, A_1, A_2, A_3, A_4$ and ζ are presented in Appendix B. The log-linear version of the model is made of seven equilibrium conditions jointly determining the evolutions of the following variables: $\{\pi_t, c_t^R, p_t, g_t^N, r_t^N, t_t^N, b_t^N\}$. These are summarized in Table 1. The steady state of the model is presented in Appendix A.

3 Stability Analysis

In this section, we perform numerical analysis to examine the stability properties of the model. The main focus of this exercise is on the value of the coefficients of the monetary policy rule.

3.1 Calibration

The calibration of the remaining parameters of the model follows mainly from Bilbiie (2024) and Bilbiie *et al.* (2023), and is summarized in Table 2. We assume log-utility ($\sigma = 1$) and unitary Frisch elasticity ($\phi = 1$). The elasticity of substitution between varieties (η) is calibrated to imply a 20% price markup in the economy. The share of H households (λ) is set to 20% in line with the empirical evidence on average marginal propensity to consume out of transitory income shocks (Johnson *et al.*, 2006; Parker *et al.*, 2013; Kaplan *et al.*, 2014). The probability of staying saver is set to 98.7% as in Bilbiie *et al.* (2023).[10] These last two numbers imply a probability of staying hand-to-mouth of $\approx 95\%$. We calibrate the steady-state share of government spending in output (20%) and the annual debt-to-output ratio (57%) to match the average for the US economy from 1984 to 2018. Rotemberg price adjustment costs are calibrated to match a frequency of price adjustment of 3.5 quarters. Taxes on profits are set to zero ($\tau^D = 0$), and we assume no exogenous redistribution across agents via taxes ($\alpha = \lambda$).[11] In line with Bilbiie (2024)'s application of Hagedorn (2021)'s Demand Theory of the Price Level, in the baseline calibration, we assume that the government chooses the quantity of nominal debt such that $b^N_{t+1} = 0$ and hence $b^R_{t+1} = -p_t$. This implies $\Phi_{BP} = \Phi_{BB} = \Phi_{BG} = 0$.[12] Finally, we will allow for the coefficients in the monetary policy rules to be $\Phi_\pi \in (0, 3)$ and $\Phi_y \in (0, 1)$ when checking for the stability of the economy.

[10] They calibrate it to match the cross-sectional standard deviation and kurtosis of the annual change in log individual income reported by Guvenen *et al.* (2021).

[11] We will relax these assumptions and present robustness checks in Appendix C.

[12] We will present robustness checks involving a more general fiscal rule in Appendix C.

Table 2: Parameters calibration

Parameter		Value
Discount factor	β	0.99
Inverse IES	σ	1
Inverse Frisch elasticity of labour supply	ϕ	1
Elasticity of substitution between varieties	η	6
Share of hand-to-mouth agents	λ	0.2
Probability of staying saver	s	0.987
Coefficient of Π in Taylor rule	Φ_π	$\in (0, 3)$
Taxes of profits	τ^D	0
Share of lumpsum taxes paid by the H agents	α	λ
Coefficient of Π in Taylor rule	Φ_π	$\in (0, 3)$
Coefficient of Y in Taylor rule	Φ_y	$\in (0, 1)$
Coefficient of B_t^N in fiscal rule	Φ_{BB}	0
Coefficient of P_t in fiscal rule	Φ_{BP}	0
Coefficient of G_t^N in fiscal rule	Φ_{BG}	0
Steady-state price level	P	1
Steady-state hours	$\bar{N}$	0.33
Steady-state nominal debt-to-GDP ratio	B_y	0.57
Steady-state nominal government spending-to-GDP ratio	G^N/Y	0.2
Rotemberg price adjustment parameter	ξ	42.68

Notes: This table shows the parameter values used to produce the figures in the main text, and in the Appendices C and D.

3.2 Numerical results

Solving for the Blanchard-Kahn conditions of the model, Figure 1a displays the regions in (Φ_π, Φ_y) space associated with either a locally unique and stable equilibrium, indeterminacy or instability. This figure shows the main message and contribution of the paper. We find that all values of the sensitivity of the nominal rate to inflation (Φ_π) are associated with a unique and saddle-path-stable equilibrium as long as the coefficient on output is $\Phi_y = 0$. Therefore, in line with the results in Hagedorn (2021), in this economy, the Taylor Principle is not a necessary condition for stability. What is novel here is that a dual mandate for the central bank is incompatible with a unique and locally stable equilibrium. When the central bank responds to fluctuations in real activity ($\Phi_y > 0$), this leads to multiple equilibria if monetary policy is *passive* ($\Phi_\pi \leq 1$), and otherwise to instability if monetary policy is *active* ($\Phi_\pi > 1$).

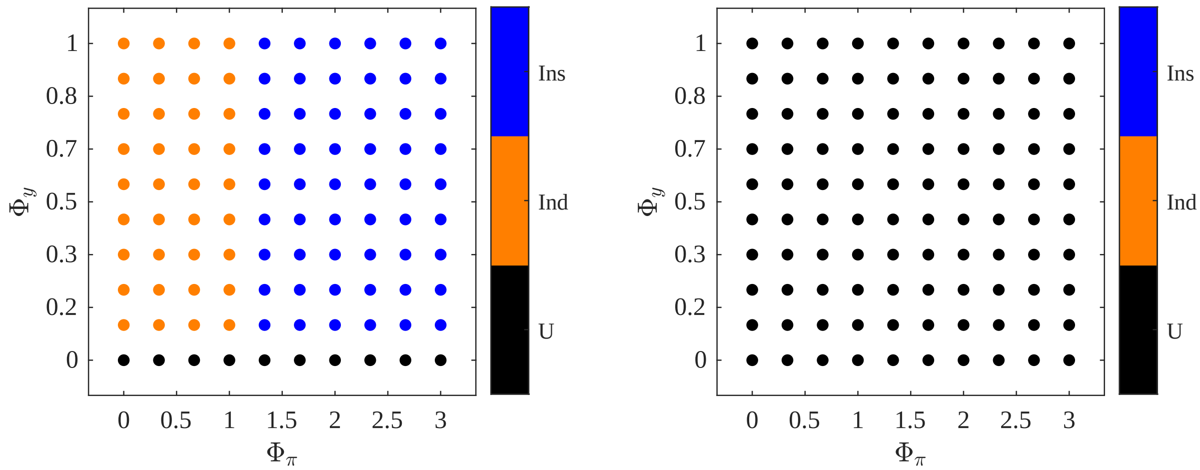
To gain intuition, let's consider a version of the model economy that does not feature government spending. In this economy, Bilbiie (2024) has shown that when the government chooses the quantity of nominal debt as in Hagedorn (2021), the price level is determined even without an interest rate rule. Without government spending, income now equals consumption, i.e., $g_t^N - p_t = 0$ and $y_t^R = c_t^R$, and fiscal policy is determined only by bonds and taxes, i.e. $b_{t+1}^N = R(b_t^N + r_{t-1}^N) - \frac{T}{B} t_t^N$. Figure 1b shows that, in this case, the model is always stable for all acceptable parameter values under a dual mandate. These results point toward a crucial role for the presence (or not) of government spending in driving the stability properties of the model under analysis. Price level determinacy seems to be the exception rather than the norm in an economy with public spending where fiscal policy is specified in nominal terms, and the central bank follows a dual mandate. What is the role of spending in driving this result? When government spending is set in nominal terms, changes in the price level render real government spending endogenous even if nominal spending is constant. On the contrary, when public spending is set in real terms, this does not happen. In section 3.4, we provide a detailed explanation of the mechanism behind these results in a simplified version of the model. But first, we present robustness checks of the numerical results presented so far.

3.2.1 Robustness

In this subsection, we now briefly discuss the robustness and sensitivity analysis of our numerical results. All the details and relevant figures are relegated to Appendix C.

We start by looking at what happens if we keep debt and taxes expressed in nominal terms, but we model real spending as an AR(1) process instead. We find that the model behaves as in the case without government spending. This result is in line with our intuition regarding the endogenous movements in real spending driven by movements in the price level in the economy where nominal spending is exogenous.

Figure 1: Stability region for different values of Taylor Rule parameters Φ_π and Φ_y



(a) Baseline model

(b) Model without government spending

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

Next, we show that adding interest rate smoothing to the Taylor rule does not affect the results. We then proceed to analyze the sensitivity of the results to different parameterizations and specifications of the fiscal rule for nominal debt. Adding nominal debt inertia or allowing it to respond to either the price level or nominal or real government spending does not affect the stability of the model. Allowing the quantity of nominal debt to respond to the debt-to-GDP ratio or changing the steady state debt-to-GDP ratio also does not affect our results.

Next, we analyze the sensitivity of our results to different degrees of idiosyncratic risk, heterogeneity, and fiscal/profit redistribution. Regarding the latter, Hagedorn (2023) recently demonstrated that local determinacy in incomplete market models is very sensitive to the distribution of taxes and/or profits. Numerical simulations, however, show that the shortcomings related to a dual mandate we present here are robust across these dimensions.

Finally, in Appendix D we show that the model produces standard impulse responses to fiscal and monetary policy shocks as well as a sunspot shock using the methodology proposed by Bianchi and Nicolò (2021).

3.3 Analytical Results

In this section, following Bilbiie (2024), we further simplify the model economy to get closed-form solutions. We impose that government spending is zero in the steady state ($G = 0$), and therefore, we log-linearize it around the steady state of the output. With this change, the linearized resource constraint of the economy becomes

$$y_t^R = c_t^R + (\tilde{g}_t^N - p_t), \tag{31}$$

where $\tilde{g}_t^N = \left(\frac{G_t^N - G}{Y}\right)$.

Keeping nominal debt still fixed at its steady-state value ($b_t^N = 0$ for all t), the government budget constraint, expressed in nominal terms, now reads:

$$t_t^N = \frac{RB_y}{T_y} r_{t-1}^N + \frac{\tilde{g}_t^N}{T_y}, \tag{32}$$

where $T_y \equiv \frac{T}{Y}$.

We then proceed by imposing no consumption inequality in steady state ($\Gamma \equiv \frac{C^S}{C^H} = 1$). This is achieved by assuming the presence of a steady state transfer from savers to hand-to-mouth, taken as given by the agents, which implies *full consumption insurance* in steady-state $C^S = C^H$, and hence $\Gamma = 1$.[13] Importantly, this only implies that consumption is equalized in steady state. Income is not, and therefore we still maintain positive liquidity in steady state. Hence, this does not preclude a well-defined demand for bonds needed to retain the Hagedorn (2021) demand theory of the price level. We also still assume no profits nor fiscal redistribution between agents.

Furthermore, we simplify the supply side by assuming that firms are myopic, which delivers a

[13] See Appendix B.5 in Bilbiie (2024).

static New Keynesian Philips curve. This implies $\beta = 0$ in equation (21):[14]

$$\pi_t = p_t - p_{t-1} = \kappa\left(\phi + \sigma\right) c_t^R + \kappa\phi(\tilde{g}_t^N - p_t). \tag{33}$$

After these simplifications, the system can be summarised by two equations. The first equation is the aggregate demand/IS equation:

$$X_1 E_t p_{t+1} + X_2 p_t + X_3 p_{t-1} + X_4 r_t^N + X_5 r_{t-1}^N = X^*, \tag{34}$$

where X^* denotes exogenous terms and the rest of the composite coefficients is presented in Appendix E. The second equation is the monetary policy rule, where we now assume that the central bank targets expected inflation at $t+1$ with a coefficient equal to 1:

$$r_t^N = E_t \pi_{t+1} + \Phi_y E_t y_{t+1}^R + \varepsilon_t^M. \tag{35}$$

Substituting the forward-looking Taylor rule (35) in the IS equation (34), we obtain a second-order difference equation in the price level:

$$Q_2 E_t p_{t+1} + Q_1 p_t + Q_0 p_{t-1} = Q^*, \tag{36}$$

[14] We use equation (28) in (21) and set $\beta = 0$.

where Q^* denotes exogenous terms and the other composite coefficients are:

$$
\begin{aligned}
Q_2 &= \frac{-2\kappa\lambda(\lambda+s-1)\left(B_y-\beta T_y\right)+(\kappa-1)\Phi_y\left(\beta(\lambda-1)\lambda+B_y(\lambda+s-1)^2\right)}{2\beta\kappa(\lambda-1)\lambda} \\
&+ \frac{\beta\lambda(-(\kappa-3)(\lambda-1)-2(\kappa-1)s)}{2\beta\kappa(\lambda-1)\lambda}, \\
Q_1 &= \frac{B_y\left(2\kappa\left(2\lambda^2+2\lambda(s-1)+(s-1)^2\right)+(\lambda+s-1)\Phi_y(-(\kappa-2)\lambda+s-1)\right)}{2\beta\kappa(\lambda-1)\lambda} \\
&+ \frac{\beta\lambda\left((\kappa-6)\lambda+\kappa-2s-2\kappa\lambda T_y+(\lambda-1)\Phi_y+4\right)}{2\beta\kappa(\lambda-1)\lambda}, \\
Q_0 &= \frac{\beta(3\lambda-1)-B_y(\lambda+s-1)\left(2\kappa+\Phi_y\right)}{2\beta\kappa(\lambda-1)}.
\end{aligned}
$$

Having simplified the model, we can now examine its stability properties analytically using the characteristic polynomial associated with this second-order difference equation. Calling $J(x)$ the characteristic polynomial, the necessary and sufficient condition for determinacy is $J(1)J(-1) < 0$ (Woodford, 2003). We will now check when this condition is satisfied for values of the model's parameters inside the acceptable regions summarized in Table 3. We consider values of the discount factor (β) between 0 and 1, the slope of the Phillips curve (κ) between 0 and 0.25, the probability of staying on the savers' island (s) between 0 and 1, debt to GDP ratio (B_y) between 0 and 1, and the response to output in the monetary policy rule (Φ_y) greater than or equal to 0.

The acceptable domain for λ is instead restricted to be greater than or equal than $(1-s)$ and less than a threshold $\lambda_j^\star$. We will discuss each of these in turn. To see why we need $\lambda \geq (1-s)$ consider the limiting case of $s = 1-h = 1-\lambda$. This implies that idiosyncratic shocks in the model are iid. That is, being S or H tomorrow is independent of whether one is S or H today. So, $(1-s) \leq \lambda$ or $s \geq (1-h)$ is intuitive and ensures stationarity as the probability to stay a saver is larger than the probability to become one (Bilbiie, 2024). On the other hand, λ needs to be strictly lower than a threshold $\lambda_j^\star$. This represents the value of λ where the slope of the aggregate demand curve of the model switches signs. When $\lambda > \lambda_j^\star$ the model is in the *Inverted Aggregate Demand Logic* region, see Bilbiie (2008). In the model without positive liquidity and abstracting from fiscal policy, this threshold would be equal to the one in Bilbiie (2024): $\chi^{-1} = (1+\varphi)^{-1}$.[15] However, in the presence of positive liquidity and fiscal policy variables entering the aggregate demand curve, this threshold becomes a complex function of the parameters in Table 3 and will change across the different specifications considered.[16]

Table 3: Acceptable domain of parameters

Parameter	Restriction
Discount factor	$0 < \beta < 1$
Slope of the NKPC	$0 < \kappa < 0.25$
Probability to stay type S	$0 < s < 1$
Steady-state debt to GDP ratio	$0 < B_y < 1$
Monetary policy rule coefficient on output	$\Phi_y \geq 0$
Mass of type H	$(1-s) \leq \lambda < \lambda_j^{\star} < 1$

Notes: This table shows the acceptable domain of the parameters used in the evaluation of the characteristic polynomial of the different model specifications considered.

Proposition 1 ***Determinacy under dual mandate***: *When the central bank follows a Taylor rule with a dual mandate, the tractable HANK model summarized by equation (36) has a determinate, locally unique rational expectations equilibrium if and only if (as long as $\lambda < \lambda_T^{\star}$):*

$$\Phi_y = 0.$$

The proof is in Appendix E1. This proposition is perfectly in line with the numerical results presented in Figure 1a and implies that stability can only be obtained with $\Phi_y = 0$ if $\lambda < \lambda_T^{\star}$.[17] That is, in the region of the parameter space where the aggregate demand of the economy has the standard negative slope (*Standard Aggregate Demand Logic* (SADL), Bilbiie (2008)), determinacy cannot be

[15] This is equivalent to the one in Bilbiie (2024) assuming zero taxes on profits as we do here.

[16] $\lambda_j^{\star}$ is model-specific with $j = (T, \tilde{T})$ where these represent, respectively, the baseline model and the model without government spending. See equations (E7) and (E13) for their relative expressions.

[17] The threshold $\lambda_T^{\star}$ is presented in equation (E7).

attained if the central bank responds to movements in output.[18]

3.3.1 The role of nominal government spending

Given the discussion in the previous section, to isolate the role of government spending for local determinacy, we now revisit the previous proposition in a version of the model where we impose $\tilde{g}_t^N - p_t = 0$, i.e., no government spending in the model.

With this assumption, the IS equation (36) summarizing the model becomes

$$F_2 E_t p_{t+1} + F_1 p_t + F_0 p_{t-1} = F^*, \tag{37}$$

where F^* denotes exogenous terms and the other composite coefficients are:

$$\begin{aligned}
F_2 &= \frac{\beta\lambda\left(3\lambda + 2\kappa(\lambda+s-1)T_y + 2s - (\lambda-1)\Phi_y - 3\right) - B_y(\lambda+s-1)\left(2\kappa\lambda + (\lambda+s-1)\Phi_y\right)}{2\beta\kappa(\lambda-1)\lambda}, \\
F_1 &= \frac{2\kappa B_y\left(2(\lambda-1)\lambda + s^2 + 2(\lambda-1)s + 1\right) - 2\beta\lambda\left(3\lambda + s + \kappa\lambda T_y - 2\right)}{2\beta\kappa(\lambda-1)\lambda} \\
&+ \frac{\Phi_y\left(\beta(\lambda-1)\lambda + B_y(\lambda+s-1)(2\lambda+s-1)\right)}{2\beta\kappa(\lambda-1)\lambda}, \\
F_0 &= \frac{\beta(3\lambda-1) - B_y(\lambda+s-1)\left(2\kappa+\Phi_y\right)}{2\beta\kappa(\lambda-1)}.
\end{aligned}$$

Proposition 2 ***Determinacy under dual mandate without government spending***: *When the central bank follows a Taylor rule with a dual mandate, the tractable HANK model summarized by equation* (37) *has a determinate, locally unique rational expectations equilibrium (as long as* $\lambda < \lambda^{\star}_{\tilde{T}}$*):*

$$\forall \Phi_y \geq 0.$$

The proof is in Appendix E2. This proposition confirms the numerical results presented in Figure 1b. Removing government spending from the model makes the price level locally determinate for

[18] It can be shown that if $\lambda > \lambda^{\star}_{T}$, the region of the parameter space where the model displays an *Inverted Aggregate Demand Logic* (IADL). Determinacy can be obtained with $\Phi_y > 0$. See Appendix E for details.

any value of $\Phi_y \geq 0$ in the SADL region.[19] In sum, this section confirmed that the numerical results presented in section 3.2 carry over in a simplified version of the model where stability analysis can be analyzed using closed-form solutions. Let's now turn to the discussion of the intuition behind these results.

3.4 Intuition

We showed, numerically and analytically, that price level determinacy in the economy under analysis is the exception rather than the norm when the central bank has a dual mandate of prices and output stabilization. Why is that? And, why does the presence (or not) of government spending affect price level determinacy so much in this model? In this section, we provide answers and intuition to these questions using the (log-linear) simplified version of the model presented in the previous section.

An economy is locally determinate if a unique equilibrium exists and any purely belief-driven equilibria are ruled out. As such, determinacy analysis does not require considering the role of shocks, and the economy is determinate if the steady-state price level constitutes the unique equilibrium. However, it is easier to grasp intuition by considering the role of shocks, which is what we will do here.

Let's start from the resource constraint of the economy: $y_t^R = c_t^R + (\tilde{g}_t^N - p_t)$. Suppose there is an increase in economic activity due, for example, to a demand shock. This leads to an increase in hours, lower markups (because of sticky prices), and persistently high inflation. Given that nominal debt and nominal government spending are constant, following the simple nominal fiscal rules, their real counterparts will persistently decline. The government budget constraint expressed in nominal terms implies that nominal taxes will be proportionally following the (lagged) movement in the nominal interest rate. From (23), keeping $\tilde{g}_t^N = b_t^N = b_{t+1}^N = 0$, we have: $t_t^N = \frac{RB_y}{T_y} r_{t-1}^N$.

We know that high prices will lead the central bank to raise nominal rates, which will push up nominal taxes. On impact, the effect on real tax revenues, $(t_t^N - p_t)$, depends on the relative strength of the movements in nominal taxes and prices. However, as real spending $(\tilde{g}_t^N - p_t)$ keeps declining,

[19] The threshold $\lambda_{\tilde{T}}^{\star}$ is presented in equation (E13).

this will put downward pressure on real taxes $(t_t^N - p_t)$, which will eventually start declining as well. This will have a positive income effect on real consumption. On the other hand, households will face a substitution effect from higher interest rates plus will also increase savings due to the precautionary savings motive. The overall effect on real consumption is ambiguous, but it is clear that it would not increase one-to-one with a fall in real taxes. So overall, we may end up with lower actual economic activity as private demand and public demand are not perfect substitutes. This might already wipe out some of the initial increase in economic activity without any intervention. The endogenous response of real government spending is the key to this mechanism. If the strength of this mechanism is large enough to push economic activity below the steady state level, the persistent decline in output requires the central bank, with $\Phi_y > 0$, to lower interest rates to stimulate demand and discourage savings.

Moving to the asset market and assuming nominal debt is constant, a rise in prices would imply a lower value of real debt and households' savings $(b_t^N - p_{t-1})$, which would increase demand for savings, requiring lower interest rates to discourage this. In sum, while increasing prices (and inflation) require higher interest rates, lower goods demand and high saving demand call for lower interest rates to clear the goods and the asset market.

We know that in the presence of this trade-off, when the central bank responds to both inflation and economic activity in the Taylor rule, the increase in the nominal rates will be less than in the case where it only targets inflation. This will keep p_t high, pushing real government spending $(\tilde{g}_t^N - p_t)$ down, and aggregate demand may not keep up. Hence, output stabilization would not be possible, leading to instability. In the model without public spending, the resource constraint is $y_t^R = c_t^R$, and therefore the destabilizing effect coming from it vanishes.

4 Solutions

In this section, we now switch back to the full model used in the numerical analysis in section 3.2 and propose two alternatives that a central bank could resort to in pursuit of a dual mandate. These include targeting the price level instead of inflation, and stabilizing consumption inequality. We will discuss how these alternatives can lead to price level determinacy in the model.

Price Level Targeting (PLT) / Wicksellian Rule

A key distinction between price level and inflation targeting lies in the 'history dependence' of the central bank's strategy to achieve price stability. PLT requires the central bank to undertake corrective measures in response to unexpected inflationary shocks, thus leading to future inflation rates that are either below or above the target, depending on the nature of the shock. Conversely, inflation targeting regards deviations from the target as bygones, focusing on achieving an average, on-target inflation over time. We will consider the former now. Suppose the central bank targets the price level following a Wicksellian rule:

$$r_t^N = \Phi_p p_t + \Phi_y y_t^R + \varepsilon_t^M, \tag{38}$$

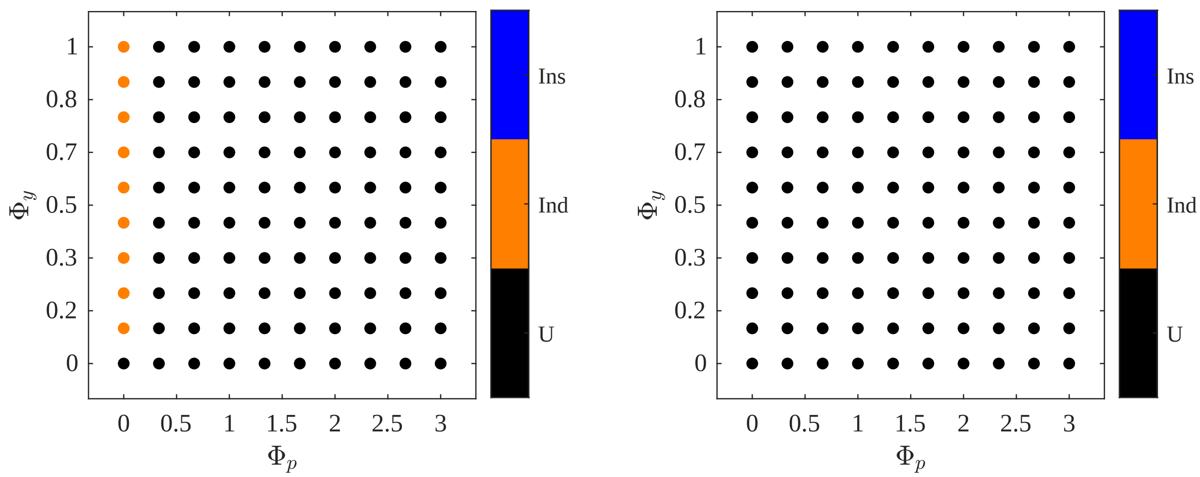
where Φ_p is the coefficient for the price-level and $\Phi_p, \Phi_y \geq 0$.

The rest of the equilibrium conditions of the model are the same as the ones presented in Table 1. Examining the stability properties with the rule in equation (38) in Figure 2a, we find that there is no constraint on responding to output as long as the coefficient on the price level, Φ_p, is positive. This result is in line with what Woodford (2003) shows for the textbook RANK model. Moreover, Bilbiie (2024) finds that a Wicksellian rule of the form $r_t^N = \Phi_p p_t$ gives determinacy in tractable HANK economies even in the case of countercyclical inequality. However, he abstracts from positive liquidity and a dual mandate for the central bank. In the model without government spending, the results echo the ones with inflation targeting, as shown in Figure 2b.

Responding to consumption inequality

Acharya *et al.* (2023) and Bilbiie (2024) show that optimal monetary policy in HANK economies is different from the RANK case because monetary policy can affect consumption inequality.

Figure 2: Wicksellian Rule: Stability region for different values of Φ_p and Φ_y

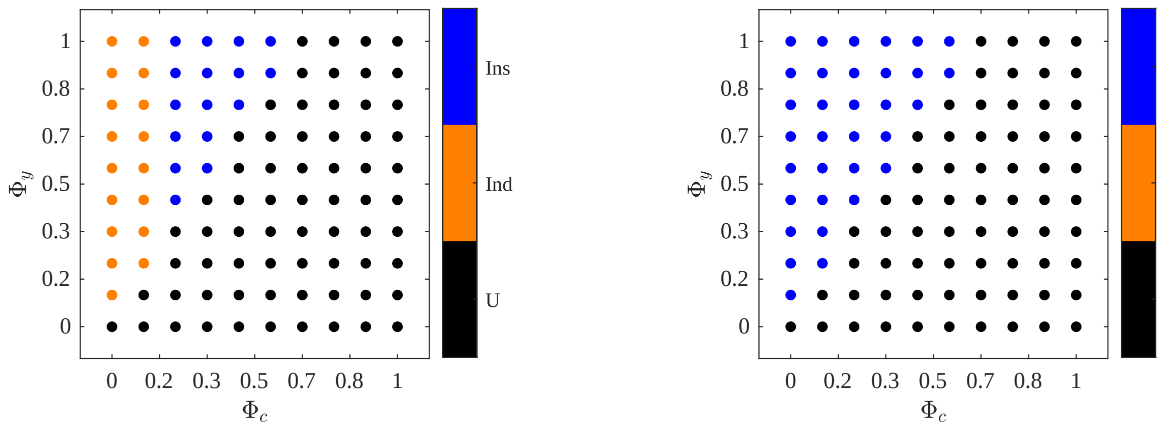


(a) Baseline model

(b) Model without government spending

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A with a monetary policy rule as in equation (38) is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

Figure 3: Taylor rule augmented with consumption inequality: Stability region for different values of Φ_π, Φ_y and Φ_c



(a) $\Phi_\Pi = 0.5$

(b) $\Phi_\Pi = 1.5$

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A with monetary policy rule in equation (39) is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

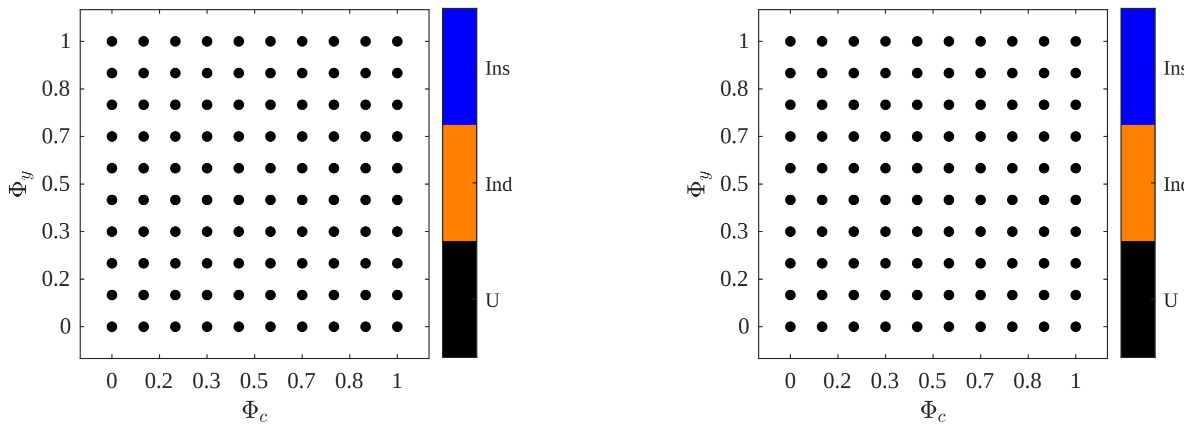
Therefore, in the presence of consumption risk arising from both idiosyncratic and aggregate shocks, it is optimal for the central bank to stabilize consumption inequality. Therefore, we augment our inflation-targeting Taylor rule to allow the central bank to respond to consumption inequality:

$$r_t^N = \Phi_\Pi \pi_t + \Phi_y y_t^R + \Phi_c \gamma_t + \varepsilon_t^M, \tag{39}$$

where $\Phi_c \geq 0$ is the coefficient on consumption inequality, $\gamma_t = c_t^{R,S} - c_t^{R,H}$.

Once again, the rest of the model is as in Table 1. Figures 3a and 3b now show numerical results for the stability of the model for different values of Φ_y and Φ_c keeping the response to inflation equal to $\Phi_\pi = 0.5$ and $\Phi_\pi = 1.5$, respectively. We find that responding to consumption inequality increases the area of price level determinacy of the model, consistent with a positive response to output in the Taylor rule. Stronger stabilization of consumption inequality is compatible with stronger responses to real activity. In the model without government spending, as in all cases considered so far, the price level is always determinate (Figures 4a and 4b).

Figure 4: Taylor rule augmented with consumption inequality in the model without government spending: Stability region for different values of Φ_π, Φ_y and Φ_c



(a) $\Phi_\Pi = 0.5$

(b) $\Phi_\Pi = 1.5$

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A with monetary policy rule in equation (39) is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

What is the rationale behind the two solutions proposed here?

Let's start with PLT. On the one hand, we have that a standard Taylor rule with inflation targeting is not consistent with price level determinacy in the presence of a dual mandate. On the other hand, a price-level rule does not present the same issue. What can explain this divergence? With PLT, "bygones are not bygones" as a higher price level would make the central bank raise interest rates to achieve the target path of prices in the next period, ensuring the stability of real government spending. In the baseline case, the focus is on inflation, which is a growth target and not a level target and implies a different level of prices. By generating history dependence in the central bank's response, PLT helps introduce a level target that is consistent with price level determinacy in the model.

Let's now look into the case of responding to consumption inequality. Consumption inequality in this economy is decreasing in the level of idiosyncratic risk. By reducing fluctuations in consumption inequality, the central bank also stabilizes aggregate consumption demand in response to a real tax cut (cf. section 3.4). This, in turn, lowers the interest rate reduction required to clear the asset market. As a result, the trade-off faced by a central bank following a dual mandate is relatively milder, allowing it to be stricter in stabilizing inflation. This opens up the possibility of responding to output while ensuring stability of real government spending.

This is linked to the precautionary savings motive in the economy. It is the precautionary savings motive driven by market incompleteness that generates a (nontrivial) demand for bonds in this economy (Hagedorn (2021)). Absent this, public demand and private demand are perfect substitutes. A sunspot increase in prices would lower government spending, which would be made up for via higher consumption and output. When this is coupled with an increase in the interest rate by the central bank, as required by higher prices and output, the central bank can prevent the initial sunspot increase from being justified, in line with the standard Taylor Principle logic.

5 Conclusion

This paper has demonstrated that under a 'dual mandate' of price and output stability, price level determinacy is the exception rather than the norm in a heterogeneous agent New Keynesian (HANK) model with nominal fiscal policy. Our findings highlight the critical role of government spending dynamics. When nominal spending is exogenous, movements in the price level make real spending endogenous, leading to a destabilizing effect on real activity, which creates a trade-off between prices and output stabilization for the central bank.

Building on Bilbiie (2024)'s tractable HANK framework and incorporating the demand theory of the price level of Hagedorn (2021), we started by investigating the model's stability properties numerically. We then presented a series of robustness checks, showing that the results are robust to different specifications of fiscal rules and calibration of key parameters, including the degree of heterogeneity in the economy and the redistribution of profits and taxes. We also showed that the shortcomings of a dual mandate carry over to a simplified version of the model where stability can be analyzed using closed-form solutions.

Finally, we proposed two possible solutions to restore price level determinacy when the central bank needs to stabilize both prices and output fluctuations. This can be achieved by targeting the price level directly and/or consumption inequality. A price level rule introduces history dependence and stabilizes real government spending directly. Responding to consumption inequality keeps aggregate demand steady, and therefore, it indirectly reduces the destabilizing effects coming from endogenous real spending.

Future research using this setup could focus on relaxing the assumption of constant income/consumption risk and/or computing optimal policy.

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Appendix

A Non-linear model and steady state equations

Table A1: Non-linear model

Description	Equation
Marginal utility of consumption	$U'(C_t^{R,j}) = (C_t^{R,j})^{-\sigma}, \, j = S, H$
Euler equation	$U'(C_t^{R,S}) = \beta \, E_t \left\{ R_{t+1} \left(s \, U'(C_{t+1}^{R,S}) + (1-s) \; U'(C_{t+1}^{R,H}) \right) \right\}$
Marginal utility of labour	$U'(N_t^j) = -\nu \, (N_t^j)^{\phi}, \quad j = S, H$
Labour	$N_t^j = N_t, \quad j = S, H$
Aggregate Labour supply	$W_t^R = \frac{\nu \, N_t^{\phi}}{(C_t^R)^{-\sigma}}$
Law of motion of bonds, S	$\mathbf{B}_{t+1}^{R,S} = (1-\lambda) B_{t+1}^{R,S} = (1-\lambda) s Z_{t+1}^{R,S} + \lambda (1-h) Z_{t+1}^{R,H}$
Law of motion of bonds, H	$\mathbf{B}_{t+1}^{R,H} = \lambda B_{t+1}^{R,H} = (1-\lambda)(1-s) Z_{t+1}^{R,S} + \lambda h Z_{t+1}^{R,H}$
Budget constraint, S	$C_t^{R,S} + Z_{t+1}^{R,S} + \frac{V_t \Omega_{t+1}^R}{1-\lambda} = W_t^R N_t^S + R_{t-1}^N \frac{B_t^{N,S}}{P_t} + \frac{\Omega_t^R}{1-\lambda}(V_t + (1-\tau^D) D_t^R) - \frac{T_t^{N,S}}{P_t}$
Budget constraint, H	$C_t^{R,H} + Z_{t+1}^{R,H} = W_t^R N_t^H + R_{t-1}^N \frac{B_t^{N,H}}{P_t} + \frac{\tau^D D_t^R}{\lambda} - \frac{T_t^{N,H}}{P_t}$

Notes: This table (continued to the next page) lists the full set of non-linear equations of the model described in Section 2 of the main text.

Table A1: Non-linear model (contd.)

Description	Equation
Bond holdings, *H*	$Z_t^{R,H} = 0$
Production function	$Y_t^R = N_t$
Real Marginal costs	$MC_t^R = W_t^R$
Real Profits	$D_t^R = Y_t^R - W_t^R N_t$
Fisher equation	$R_t = R_{t-1}^N \frac{P_{t-1}}{P_t}$
Inflation	$\Pi_t = \frac{P_t}{P_{t-1}}$
Philips curve	$1 - \eta + \eta MC_t^R \; - \Pi_t \, \xi \; (\Pi_t - 1) + \beta \frac{\xi \, E_t \Pi_{t+1} \frac{U'(C_{t+1}^{R,S})}{U'(C_t^{R,S})} (E_t \Pi_{t+1} - 1) \, Y_{t+1}^R}{Y_t^R} = 0$
Government's budget constraint	$B_{t+1}^N = R_{t-1}^N B_t^N + G_t^N - T_t^N$
Government spending	$\log\left(\frac{G_t^N}{G^N}\right) = \rho_g \log\left(\frac{G_{t-1}^N}{G^N}\right) + \varepsilon_t^G$
Aggregate consumption	$C_t^R = \lambda \, C_t^{R,H} + (1 - \lambda) \, C_t^{R,S}$
Aggregate debt	$B_{t+1}^R = \lambda Z_{t+1}^{R,H} + (1 - \lambda) Z_{t+1}^{R,S}$
Resource Constraint	$Y_t^R = C_t^R + \frac{G_t^N}{P_t}$
Tax, *S*	$\lambda T_t^{N,S} = \; \alpha T_t^N$
Tax, *H*	$\lambda T_t^{N,H} = \; \alpha T_t^N$
Debt Rule	$\log\left(\frac{B_{t+1}^N}{B^N}\right) = \Phi_{BB} \log\left(\frac{B^N{}_t}{B^N}\right) + \Phi_{BP} \log\left(\frac{P_t}{P}\right) + \Phi_{BG} \log\left(\frac{G^N{}_t}{G}\right) + \varepsilon_t^B$
Taylor Rule	$log\left(\frac{R_t^N}{R^N}\right) = log\left(\frac{\Pi_t}{\Pi}\right)^{\Phi_\pi} + log\left(\frac{Y^R{}_t}{Y^R}\right)^{\Phi_y} + \varepsilon_t^M$

Notes: This table (continued from the previous page) lists the full set of non-linear equations of the model described in Section 2 of the main text.

We normalize the price level equal to 1 (P = 1), and therefore drop the real and nominal superscripts in steady state.

Table A3: Steady state model

$$P = 1$$
$$MC = \frac{\eta - 1}{\eta}$$
$$N^S = N^H = N = 1/3$$
$$Y = N$$
$$W = MC$$
$$C = (1 - G_y)Y$$
$$G = G_y Y$$
$$B = 4B_y Y$$
$$Z^H = 0$$
$$Z^S = \frac{B}{1 - \lambda}$$
$$B^H = (1 - \lambda)(1 - s)Z^S$$
$$B^S = s(1 - \lambda)Z^S$$

Considering R as unknown, we have:

$$T = RB + G - B$$
$$T^H = T$$
$$T^S = T$$
$$C^S = WN^S + RB^S - T^S - Z^S + \frac{1 - \tau^D}{1 - \lambda} D$$
$$C^H = WN^H + RB^H - T^H + \frac{\tau^D}{\lambda} D$$
$$\Gamma = \frac{C^S}{C^H}$$
$$C^R = \lambda C^H + (1 - \lambda)C^S$$

We find R by solving:

$$R = \frac{1}{\beta} \frac{1}{s + (1 - s)\Gamma^{\sigma}}$$

Notes: This table lists the equations used to solve for the steady state interest rate of the model described in Section 2 of the main text and summarised in Table A1.

B Log-linearized model aggregate IS Equation

1. IS equation

$$c_t^R = \delta E_t c_{t+1}^R + A_1 \left[\lambda \left(g_t^N - p_t\right) + \zeta \left(E_t g_{t+1}^N - E_t p_{t+1}\right)\right] + A_2 \left[\lambda \left(t_t^N - p_t\right) + \zeta \left(E_t t_{t+1}^N - E_t p_{t+1}\right)\right]$$
$$+ A_3 \left[\lambda \left(b_t^N + r_{t-1}^N - p_t\right) + \zeta \left(E_t b_{t+1}^N + r_t^N - E_t p_{t+1}\right)\right] - A_4 (r_t^N - E_t \pi_{t+1}), \tag{B1}$$

Define $\chi \equiv \frac{C^H}{Y}$ i.e. the steady-state share of HtM income in aggregate income, $q \equiv \frac{WN}{Y}$ as the steady-state labour share, and $G_y \equiv \frac{G}{Y}$.[20] Other coefficients are as follows:

$$\zeta = \frac{(\lambda - 1)(s-1)\Gamma^{\sigma+1} - \lambda s}{(1-s)\Gamma^{\sigma} + s}$$

$$A_1 = \frac{q(\phi+1)G_y}{-(\Gamma-1)\lambda\chi - \lambda q(\phi+1)C_y + \Gamma\chi + \lambda(-q)\sigma}$$

$$A_2 = \frac{-T_y}{-(\Gamma-1)\lambda\chi - \lambda q(\phi+1)C_y + \Gamma\chi + \lambda(-q)\sigma}$$

$$A_3 = \frac{R(1-s)B_y}{\lambda\left(-(\Gamma-1)\lambda\chi - \lambda q(\phi+1)C_y + \Gamma\chi + \lambda(-q)\sigma\right)},$$

$$A_4 = \frac{\Gamma(1-\lambda)\chi}{\sigma\left(-(\Gamma-1)\lambda\chi - \lambda q(\phi+1)C_y + \Gamma\chi + \lambda(-q)\sigma\right)},$$

$$\delta = \frac{\frac{(\lambda-1)q(s-1)\Gamma^{\sigma+1}\left((\phi+1)C_y+\sigma\right)}{(\Gamma-1)\lambda\chi+\lambda q(\phi+1)C_y-\Gamma\chi+\lambda q\sigma} - s}{(s-1)\Gamma^{\sigma} - s}.$$

[20]Given that $Y = N$ in steady state, $q = W = MC$ which is the inverse of the price markup.

2. Philips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\phi C_y + \sigma\right) c_t^R + \kappa \phi G_y (g_t^N - p_t)$$

3. Government budget constraint

$$b_{t+1}^N = R(b_t^N + r_{t-1}^N) + \frac{G}{B} g_t^N - \frac{T}{B} t_t^N$$

4. Debt rule

$$b_{t+1}^N = \Phi_{BB} b_t^N + \Phi_{BP} p_t + \Phi_{BG} g_t^N + \varepsilon_t^B$$

5. Evolution of prices

$$p_t = \pi_t + p_{t-1}$$

6. Government spending shock

$$g_t^N = \rho g_{t-1}^N + \varepsilon_t^G$$

7. Taylor rule

$$r_t^N = \Phi_\pi \pi_t + \Phi_y (C_y c_t^R + G_y (g_t^N - p_t)) + \varepsilon_t^M$$

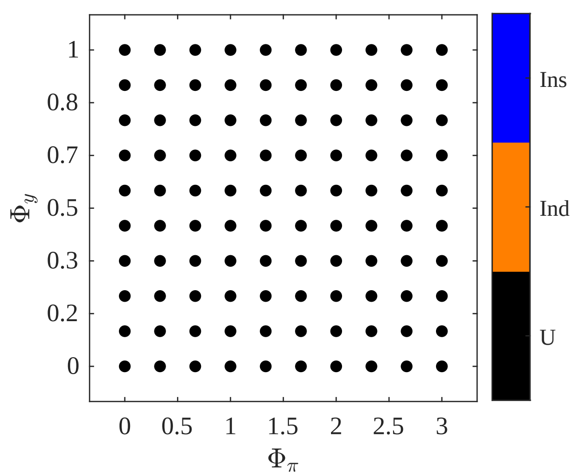
C Numerical results: Robustness and Sensitivity Analysis

In this section, we present sensitivity analysis and robustness of the results presented in section 3.2.

C1 Real Government Spending

We modify the model in Table 1 to include an AR(1) process for 'real' government spending instead of nominal spending.

Figure C1: Stability region for the model with real government spending for different values of Taylor Rule parameters Φ_π and Φ_y



Notes: The figure shows regions in parameter space where the equilibrium of the model summarized in Appendix A with AR(1) process for 'real' government spending is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

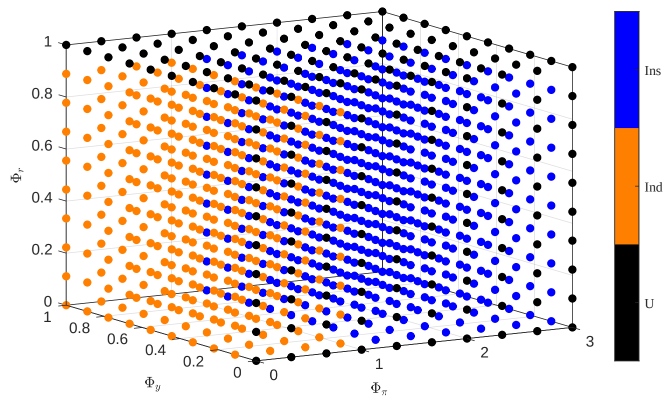
C2 Interest rate smoothing

Consider a Taylor rule of the form

$$r_t^N = \Phi_r r_{t-1}^N + \Phi_\pi \pi_t + \Phi_y y_t^R + \varepsilon_t^M, \tag{C1}$$

where Φ_r is the interest rate smoothing parameter.

Figure C2: Stability region for different values of Φ_r, Φ_π and Φ_y



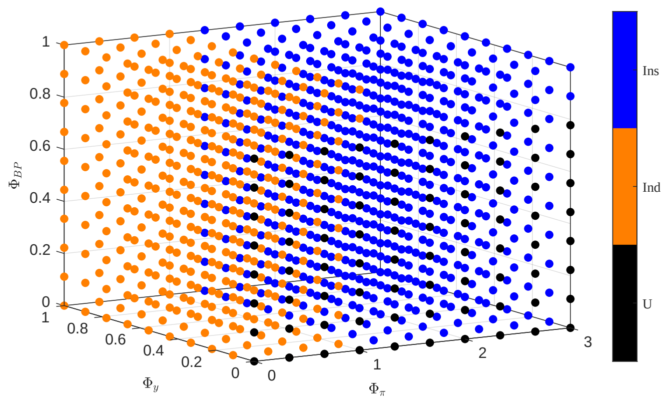
Notes: The figure shows regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the values of other parameters, which the plots is conditional on, see Table 2.

C3 Fiscal Rule

C3.1 Nominal debt responding to the price level

We relax the assumption of $\Phi_{BP} = 0$ in the fiscal rule made so far.

Figure C3: Stability region for different values of Φ_{BP}, Φ_π and Φ_y

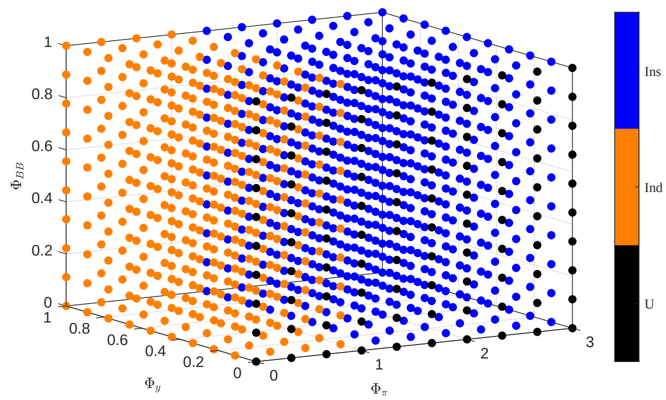


Notes: The figure shows regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

C3.2 Nominal debt smoothing

We relax the assumption of $\Phi_{BB} = 0$ in the fiscal rule made so far.

Figure C4: Stability region for different values of Φ_{BB}, Φ_{π} and Φ_{y}

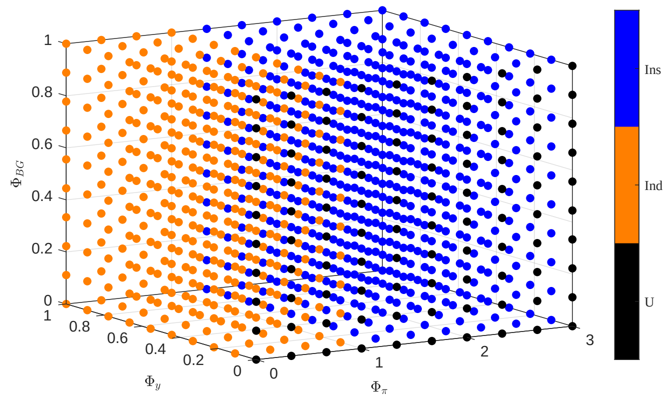


Notes: The figure shows regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

C3.3 Nominal debt responding to nominal spending

We relax the assumption of $\Phi_{BG} = 0$ in the fiscal rule made so far.

Figure C5: Stability region for different values of Φ_{BG}, Φ_π and Φ_y



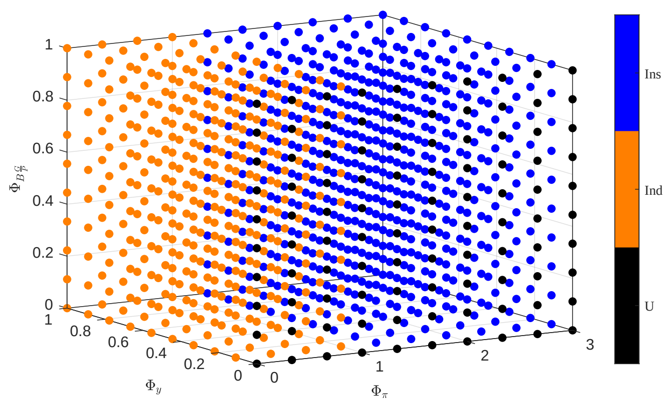
Notes: The figure shows regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

C3.4 Nominal debt responding to real spending

We now consider a debt rule where the nominal debt responds to real spending:

$$b^N_{t+1} = \Phi_{B\frac{G}{P}}(g^N_t - p_t) + \varepsilon^B_{b^N}.$$

Figure C6: Stability region for different values of $\Phi_{B\frac{G}{P}}$, Φ_π and Φ_y



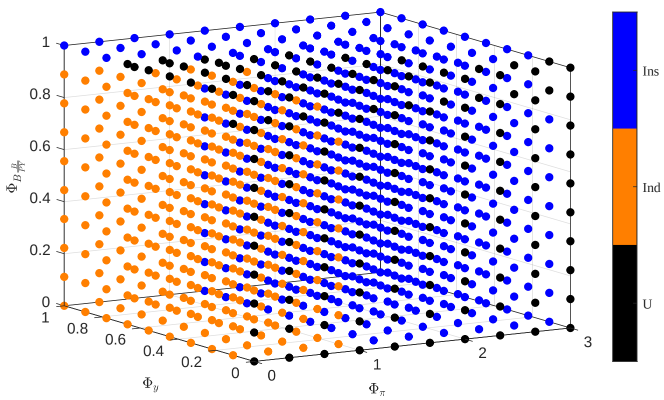
Notes: The figure shows regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

C3.5 Nominal debt responding to the Debt to GDP ratio

We now consider a debt rule where the nominal debt responds to the debt-to-GDP ratio

$$b_{t+1}^N = \Phi_{B\frac{B}{Y}} (b_t^N - p_t - y_t^R) + \varepsilon_{b^N}^B.$$

Figure C7: Stability region for different values of $\Phi_{B\frac{B}{Y}}$, Φ_π and Φ_y

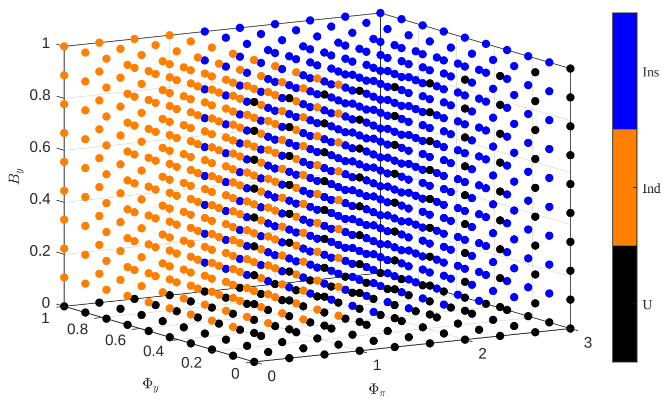


Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

C4 Robustness to debt to GDP ratios

We look at how the stability regions change with $B_y \in (0, 1)$.

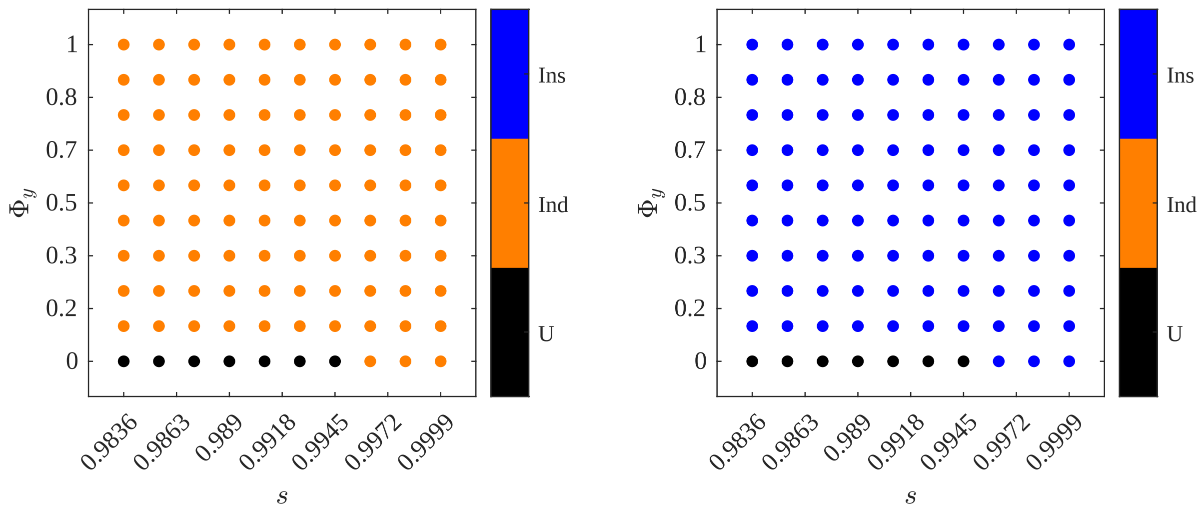
Figure C8: Stability region for different values of B_y, Φ_π and Φ_y



Notes: The figure shows regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the calibration, see Table 2.

C5 Robustness to risk, heterogeneity and redistribution

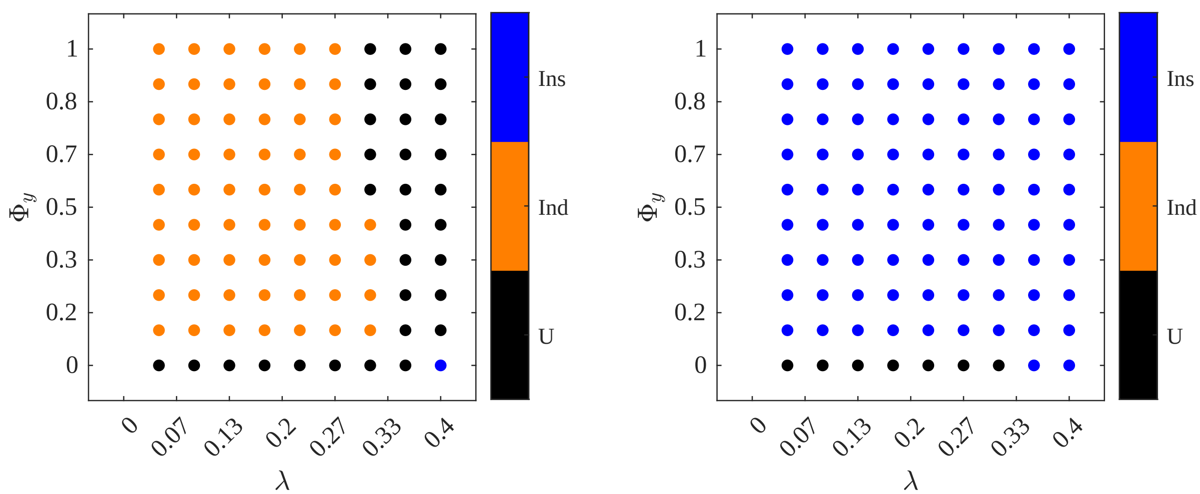
Figure C9: Stability region for different values of risk parameter, s, and Taylor Rule parameter Φ_y for alternative values of Φ_π



(a) $\Phi_\pi = 0.5$ (b) $\Phi_\pi = 1.5$

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the values of other parameters, which the plots are conditional on, see Table 2. Given the calibration, $s \geq 0.9836$ is the region where $R < \frac{1}{\beta}$ allowing for precautionary savings in steady state.

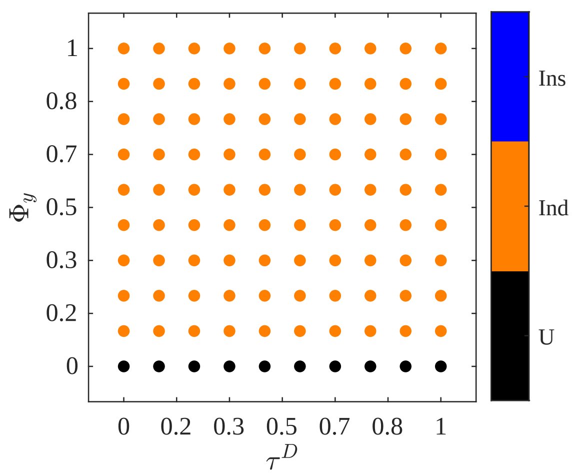
Figure C10: Stability region for different values of heterogeneity parameter, λ and Taylor Rule parameter Φ_y for alternative values of Φ_π



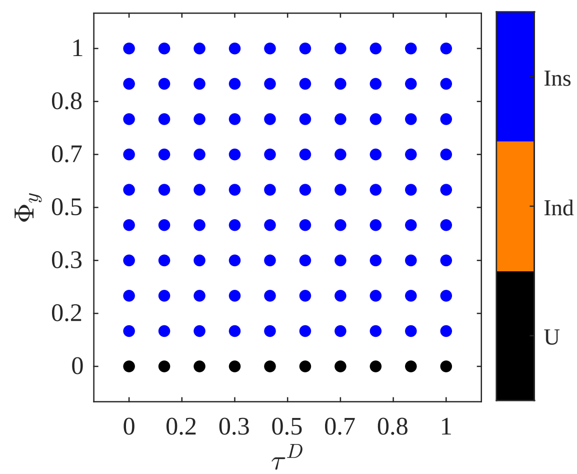
(a) $\Phi_\pi = 0.5$ (b) $\Phi_\pi = 1.5$

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the values of other parameters, which the plots are conditional on, see Table 2.

Figure C11: Stability region for different values of redistribution of profits parameter τ^D and Taylor Rule parameter Φ_y for alternative values of Φ_π



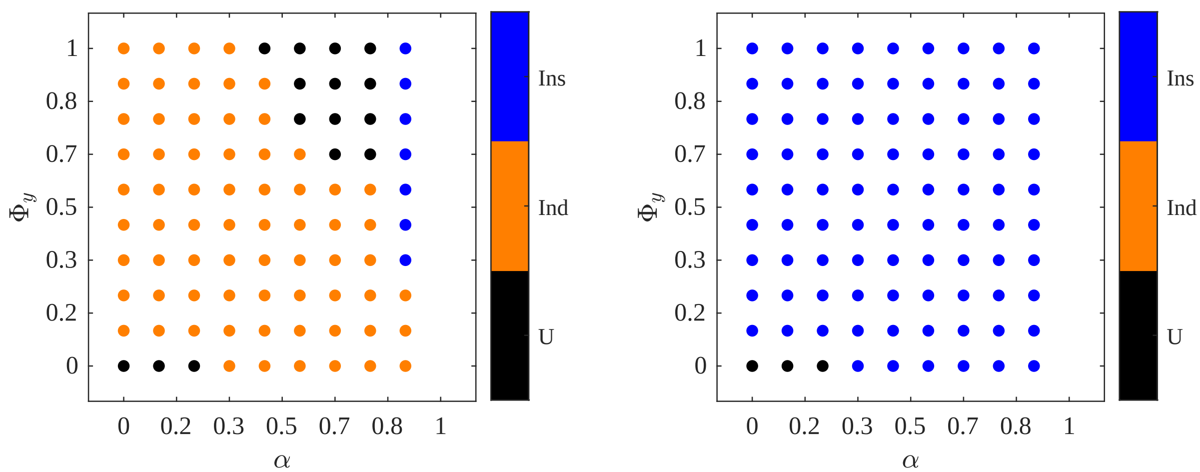
(a) $\Phi_\pi = 0.5$



(b) $\Phi_\pi = 1.5$

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighborhood of the steady-state. For details on the values of other parameters, which the plots are conditional on, see Table 2.

Figure C12: Stability region for different values of transfers parameter α and Taylor Rule parameter Φ_y for alternative values of Φ_π



(a) $\Phi_\pi = 0.5$

(b) $\Phi_\pi = 1.5$

Notes: These figures show regions in parameter space where the equilibrium of the model summarized in Appendix A for alternate values of Φ_π is either: unique and stable (U - black), indeterminate (Ind - orange), or unstable (Ins - blue) in a neighbourhood of the steady-state. For details on the values of other parameters, which the plots are conditional on, see Table 2.

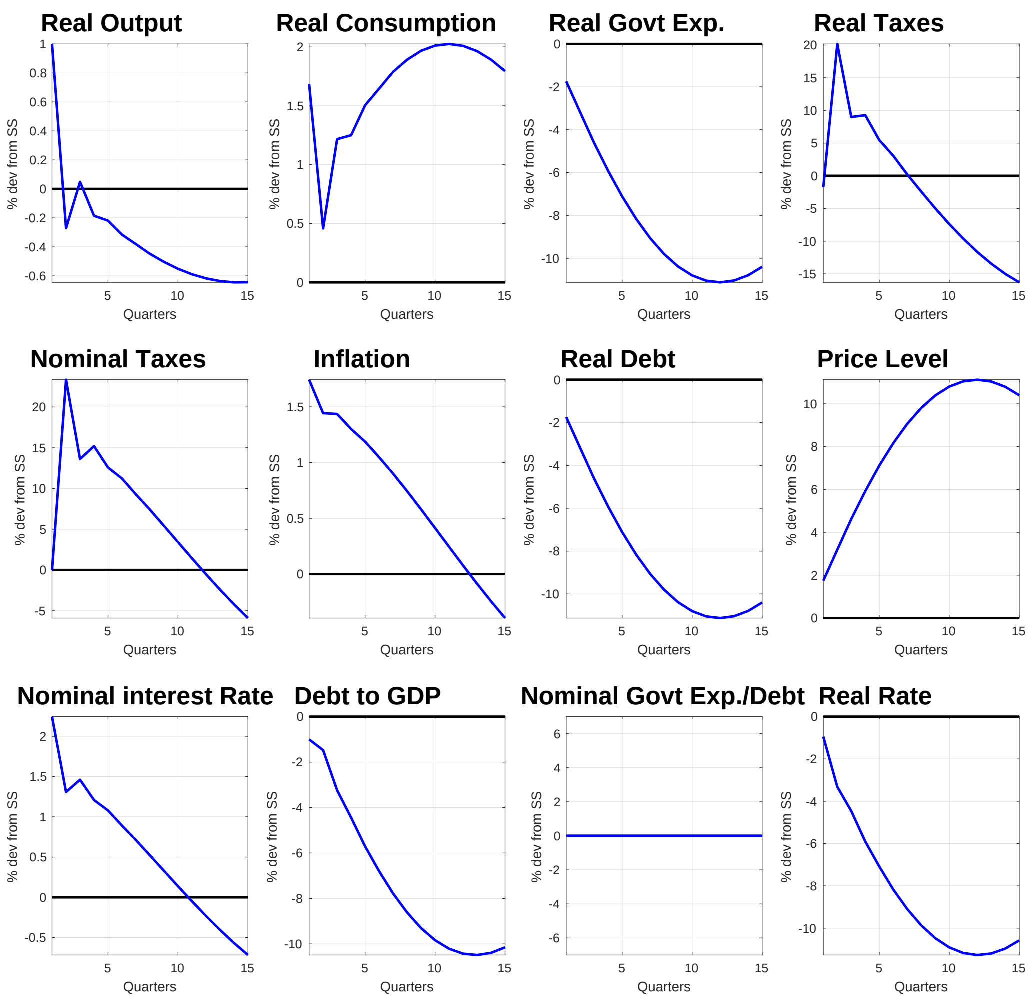
D Impluse responses functions

In this section, we present the impulse responses of the model summarized in Table 1 to a sunspot shock, a monetary shock, and a government spending shock respectively.

In Figure D1, we simulate a sunspot shock to demand based on the method proposed by Bianchi and Nicolò (2021). This method entails augmenting the state space of the system with an auxiliary exogenous equation that has an explosive root, which helps in satisfying the Blanchard-Kahn conditions of this otherwise indeterminate system. The indeterminacy comes through the standard channel when the Taylor Principle is not satisfied: if agents anticipate an increase in output without any change in fundamentals, and the economy experiences a sustained negative real rate, it provides a boost to consumption, making the initial sunspot increase to income self-fulfilling.

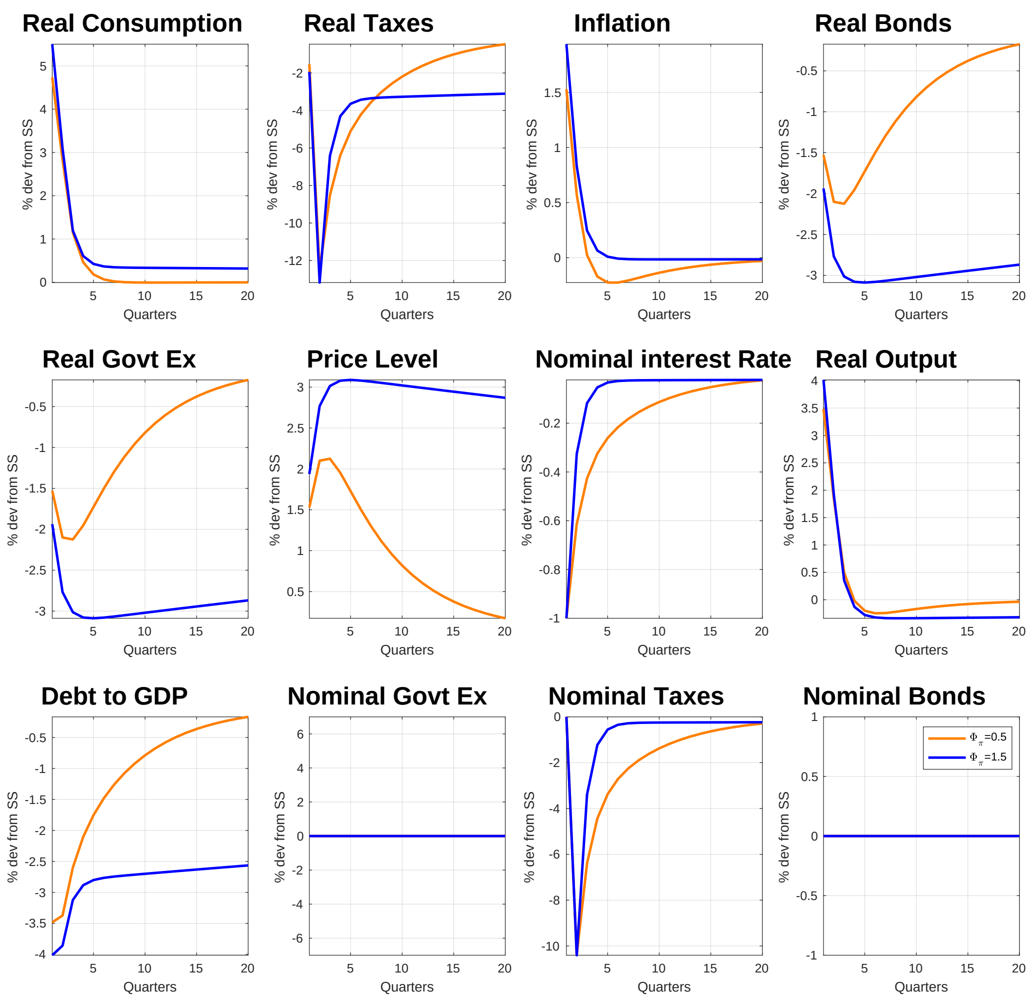
Figure D2 shows that in response to an expansionary monetary policy shock, the model presents standard dynamics. We see an increase in consumption, output, price level, and inflation, and a decline in real debt, taxes and government spending. In Figure D3, we simulate a government spending shock. We observe an increase in output, price level, and inflation, as expected. The nominal interest rate rises accordingly; the degree of associated rise is increasing in the size of the inflation level responsiveness coefficient in the Taylor rule.

Figure D1: Impluse response functions to a sunspot shock



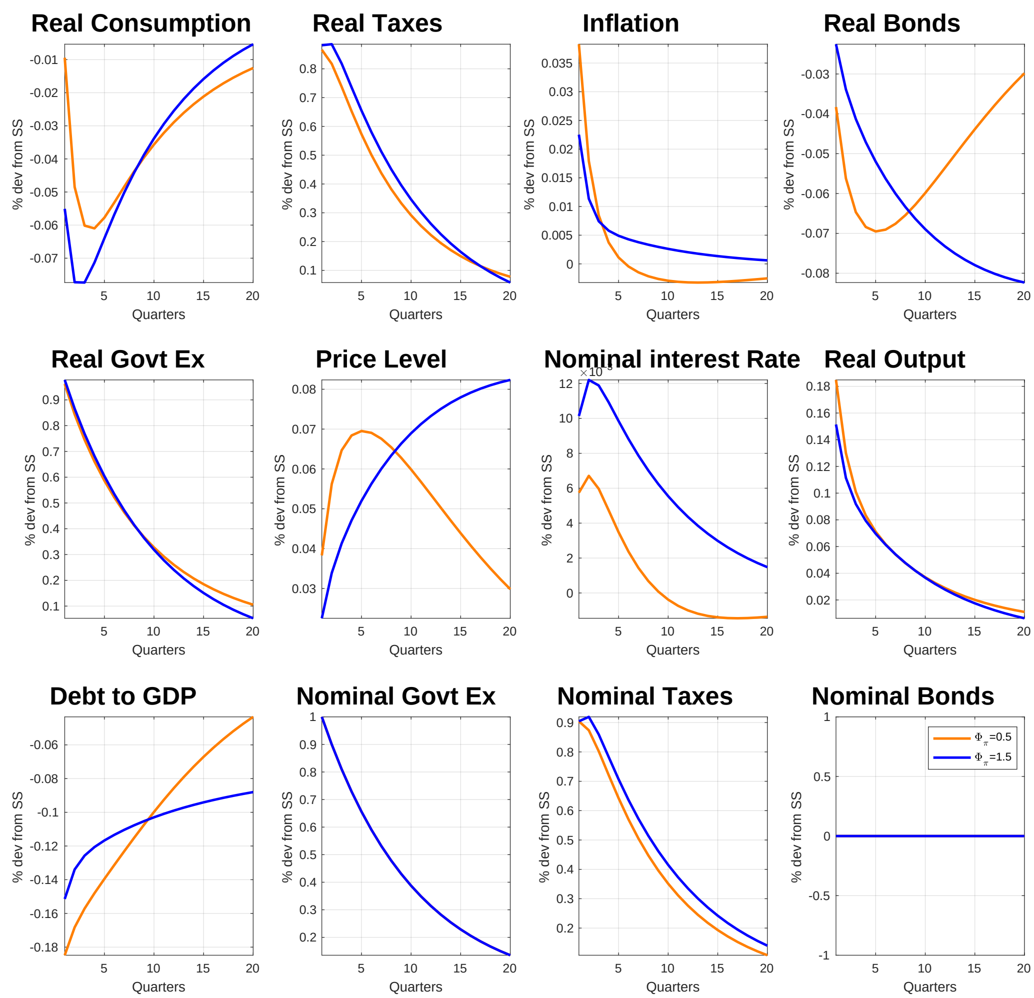
Notes: These figures are produced with $\Phi_\pi = 1$ and $\Phi_y = 0.5$ which generate indeterminacy (cfr. Figure 1a). The rest of the calibration is as in Table 2.

Figure D2: Impluse response functions to a Monetary shock



Notes: These figures are produced with Φ_π as in the legend of the plot and $\Phi_y = 0$. We also add interest rate smoothing in the Taylor rule = 0.7. The rest of the calibration is as in Table 2.

Figure D3: Impluse response functions to a Government spending shock



Notes: These figures are produced with Φ_π as in the legend of the plot and $\Phi_y = 0$. The rest of the calibration is as in Table 2.

E Simple model details

Here we present the details of the simple model analyzed in section 3.3 of the main text. The starting point is the full model in log-linear form summarised in Table 1.

As discussed in the main text, the assumption of myopic firms implies a static New Keynesian Phillips curve. This is obtained by using equation (28) in (21) and set $\beta = 0$, which implies:

$$\pi_t = p_t - p_{t-1} = \kappa\,(\phi + \sigma)\, c_t^R + \kappa\phi(\tilde{g}_t^N - p_t). \tag{E1}$$

Imposing zero steady-state government spending ($G = 0$) and that the fiscal authority fixes nominal debt at its steady-state value ($b_{t+1}^N = 0 = b_t^N$) simplifies the government budget constraint to

$$\frac{T_y}{B_y} t_t^N = R r_{t-1}^N + \frac{\tilde{g}_t^N}{B_y}.$$

These assumptions reduce the size of the system from seven to two equations

$$X_1 E_t p_{t+1} + X_2 p_t + X_3 p_{t-1} + X_4 r_t^N + X_5 r_{t-1}^N = X^*, \tag{E2}$$

and the monetary policy

$$r_t^N = \Phi_\pi \pi_t + \Phi_y E_t y_{t+1}^R + \varepsilon_t^M, \tag{E3}$$

where X^* denotes exogenous terms and the rest of the coefficients are given by:

$$
\begin{aligned}
X_1 &= \frac{\beta\lambda\left(\lambda(\kappa\sigma(\sigma+\phi-1)+\sigma+\phi+1)-\kappa\left(\sigma^2+\phi(s+\sigma)\right)+\sigma(\kappa-\kappa s+s-1)+(s-1)\phi-1\right)}{\beta\kappa(\lambda-1)\lambda(\sigma+\phi)} \\
&+ \frac{\kappa(\lambda+s-1)(\sigma+\phi)\left((s-1)B_y+\beta\lambda T_y\right)}{\beta\kappa(\lambda-1)\lambda(\sigma+\phi)}, \\
X_2 &= \frac{\beta(-\lambda(\sigma(\kappa(\sigma+\phi-1)+2)+2(\phi+1))+\kappa(\sigma(\sigma+\phi)+\phi)-s\sigma-s\phi+\sigma+\phi+2)}{\beta\kappa(\lambda-1)(\sigma+\phi)} \\
&+ \frac{\kappa(\sigma+\phi)\left(-\left((s-1)B_y\right)-\beta\lambda T_y\right)}{\beta\kappa(\lambda-1)(\sigma+\phi)}, \\
X_3 &= \frac{\frac{\lambda}{\lambda-1}+\frac{1}{\sigma+\phi}}{\kappa}, \\
X_4 &= -\frac{B_y(\lambda+s-1)^2}{\beta(\lambda-1)\lambda}-\sigma, \\
X_5 &= \frac{B_y(\lambda+s-1)}{\beta(\lambda-1)}.
\end{aligned}
$$

Finally we assume that the central bank targets the next period's inflation with a coefficient equal to 1, effectively choosing the real rate

$$r_t^N = E_t\pi_{t+1} + \Phi_y E_t y_{t+1}^R + \varepsilon_t^M. \tag{E4}$$

The model can now be represented by just one second-order difference equation in terms of the price level. This is equation (36) in the main text.

E1 Proof of Proposition 1

The characteristic polynomial of equation (36) is:

$$J(x) = Q_2x^2 + Q_1x + Q_0. \tag{E5}$$

Evaluated at -1, $J(-1)$, we have:

$$J(-1) = \frac{\lambda\left(2\left(-3\kappa\lambda+\kappa+6\lambda+\frac{\kappa(2\lambda+s-1)}{\beta}-2\kappa s+2s-4\right)+(\kappa-2)(\lambda-1)\Phi_y\right)}{2\kappa(\lambda-1)\lambda} + \frac{B_y(2\lambda+s-1)\left((\kappa-2)(\lambda+s-1)\Phi_y-2\kappa(2\lambda+s-1)\right)}{\beta 2\kappa(\lambda-1)\lambda}. \tag{E6}$$

$\lambda < 1$ is enough to ensure that both denominators are negative. Hence the sign of $J(-1)$ depends on the sum of the two numerators. Given $0 < s < 1$, $\lambda \geq (1-s)$, $0 < B_y < 1$ and $0 < \kappa < 0.25$ the sum of the two numerators is < 0 if $\lambda < \lambda_T^\star$ where[21]

$$\lambda_T^\star = \frac{\Lambda_1^T + \sqrt{\Lambda_2^T}}{\Lambda_3^T}, \tag{E7}$$

and

$$\Lambda_1^T = \beta\left(-2\Phi_y+\kappa\left(\Phi_y+4s-2\right)-4s+8\right)-(s-1)\left(-6B_y\Phi_y+\kappa\left(3B_y\Phi_y-8B_y+2\right)\right),$$

$$\begin{aligned}\Lambda_2^T = {}&\beta^2\left(-2\Phi_y+\kappa\left(\Phi_y+4s-2\right)-4s+8\right)^2\\ &-2\beta(s-1)\left(4B_y\Phi_y\left(\Phi_y+s\left(2\Phi_y-6\right)\right)\right.\\ &+\kappa^2\left(s\left(2B_y\left(\Phi_y\left(\Phi_y-2\right)-4\right)+8\right)+\left(\Phi_y-2\right)\left(B_y\left(\Phi_y+4\right)+2\right)\right)\\ &\left.-4\kappa\left(B_y\left(\Phi_y^2+\Phi_y+s\left(\Phi_y\left(2\Phi_y-5\right)+4\right)+4\right)+\Phi_y+2s-4\right)\right)\\ &+(s-1)^2\left(B_y\Phi_y\left(\kappa-2\right)-2\kappa\right)^2,\end{aligned} \tag{E8}$$

$$\Lambda_3^T = -8B_y\Phi_y+2\beta\left(\Phi_y-6\right)(\kappa-2)+4\kappa\left(B_y\left(\Phi_y-4\right)+2\right).$$

Hence the sign of $J(-1)$ depends on (E7) which defines the threshold above which λ generates

[21] With $\lambda^\star$ being the largest root that solves the corresponding second order polynomial.

an IADL in this case.

The characteristic polynomial evaluated at 1, $J(1)$, is instead

$$J(1) = \frac{2\beta\lambda(1-s) + 2(s-1)B_y(-\beta\lambda + \lambda + s - 1)}{2\beta(\lambda-1)\lambda} + \frac{\Phi_y\left(\beta(\lambda-1)\lambda + (s-1)B_y(\lambda+s-1)\right)}{2\beta(\lambda-1)\lambda}. \tag{E9}$$

The sign of $J(1)$ depends on the value of Φ_y. The first term is unambiguously negative; the denominator is always negative and the numerator is always positive in the acceptable parameters range. Therefore the sign of $J(1)$ depends on the second term.

To get $J(1) < 0$ we need the second term to be either negative or smaller than the first one. But the latter is not true unless $\Phi_y = 0$. If $\Phi_y > 0$, it is easy to show that the second term in (E9) is larger than the first one. Now let's consider the conditions under which the second term in (E9) can be negative. For it to be negative we need the coefficient multiplying Φ_y in (E9) to be negative. Since the denominator is unambiguously negative, we have the following condition:

$$\beta(\lambda-1)\lambda + (s-1)B_y(\lambda+s-1) \geq 0. \tag{E10}$$

For (E10) to hold when $(1-s) \leq \lambda$, we require $B_y(\lambda+s-1) \leq (\lambda-1)\beta$. But this last condition is always violated following the acceptable parameters domain in Table 3. Hence $J(1) < 0$ if $\Phi = 0$, otherwise if $\Phi_y > 0$ we have $J(1) > 0$.

In the next subsection, we complement this analysis by plotting $J(-1)$ and $J(1)$ against different values of the parameters.

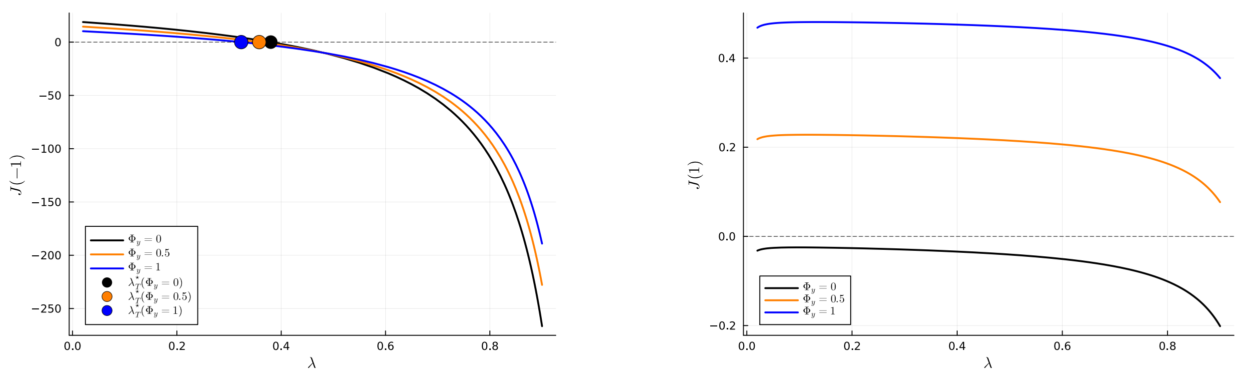
E1.1 Stability of the simple model with Taylor rule: Graphical representation

Here we complement the proof in the previous section by plotting the values of the characteristic polynomial for each case against different values of λ and Φ_y.

To produce all the plots we calibrate the rest of the parameters in line with what we use for the numerical analysis in Table 2 i.e. $\beta = 0.99$, $s = 0.98$, $\kappa = 0.11$ and $B_y = 0.57$.

Figure E1 plots the values of equations (E6) and (E9) against λ for $\Phi_y = [0, 0.5, 1]$. It also shows the corresponding value of $\lambda_T^\star$. Figure E2 plots $\lambda_T^\star$ for different values of Φ_y.

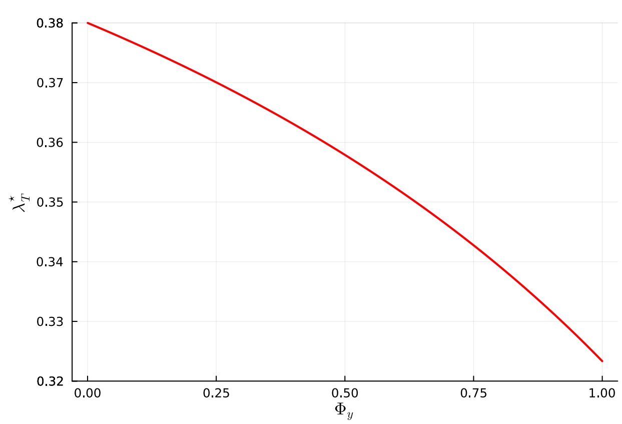
Figure E1: Taylor rule: Characteristic polynomial for different values of λ and Φ_y



(a) $J(-1)$ (b) $J(1)$

Notes: These figures plot the values of $J(-1)$ and $J(1)$ against λ and Φ_y. The rest of the parameters are calibrated as follows: $\beta = 0.99$, $s = 0.98$, $\kappa = 0.11$ and $B_y = 0.57$.

Figure E2: $\lambda_T^\star$ for different values of Φ_y.



Notes: This figure plots the values of $\lambda_T^\star$ for different values of Φ_y. The rest of the parameters are calibrated as follows: $\beta = 0.99$, $s = 0.98$, $\kappa = 0.11$ and $B_y = 0.57$.

E2 Proof of Proposition 2

The characteristic polynomial of the difference equation (37) is:

$$\tilde{J}(x) = F_2 x^2 + F_1 x + F_0. \tag{E11}$$

The characteristic polynomial evaluated at -1 is:

$$\begin{aligned}\tilde{J}(-1) = &\frac{2\beta\lambda(3\lambda + s - 2) - \kappa B_y(2\lambda + s - 1)(\beta\lambda + \lambda + s - 1)}{\beta\kappa(\lambda - 1)\lambda} \\ &+ \frac{\Phi_y\left(\beta\lambda(1-\lambda) + B_y(\lambda + s - 1)(-2\lambda - s + 1)\right)}{\beta\kappa(\lambda-1)\lambda}.\end{aligned} \tag{E12}$$

As for the previous case, $\lambda < 1$ is enough to ensure that both denominators are negative. Hence the sign of $\tilde{J}(-1)$ depends on the sum of the two numerators. Given $0 < s < 1$, $\lambda \geq (1-s)$, $0 < B_y < 1$ and $0 < \kappa < 0.25$ the sum of the two numerators is < 0 if $\lambda < \lambda^{\star}_{\tilde{T}}$ where

$$\lambda^{\star}_{\tilde{T}} = \frac{\Lambda_1^{\tilde{T}} + \sqrt{\Lambda_2^{\tilde{T}}}}{\Lambda_3^{\tilde{T}}}, \tag{E13}$$

and

$$\Lambda_1^{\tilde{T}} = 4\beta - 3B_y\kappa - \beta B_y\kappa - \beta\Phi_y - 3B_y\Phi_y - 2\beta s + 3B_y\kappa s + \beta B_y\kappa s + 3B_y\Phi_y s,$$

$$\begin{aligned}\Lambda_2^{\tilde{T}} = &(-4\beta + 3B_y\kappa + \beta B_y\kappa + \beta\Phi_y + 3B_y\Phi_y + 2\beta s - 3B_y\kappa s - \beta B_y\kappa s - 3B_y\Phi_y s)^2 - \\ &- 4(6\beta - 2B_y\kappa - 2\beta B_y\kappa - \beta\Phi_y - 2B_y\Phi_y)(-B_y\kappa - B_y\Phi_y + 2B_y\kappa s + 2B_y\Phi_y s - B_y\kappa s^2 - B_y\Phi_y s^2)\end{aligned} \tag{E14}$$

$$\Lambda_3^{\tilde{T}} = 2(6\beta - 2B_y\kappa - 2\beta B_y\kappa - \beta\Phi_y - 2B_y\Phi_y).$$

$$\tilde{J}(1) = \frac{(1-s)B_y(-\beta\lambda + \lambda + s - 1)}{\beta(1-\lambda)\lambda}. \tag{E15}$$

With $(1-s) \leq \lambda$, $\tilde{J}(1)$ is negative and it does not depend on Φ_y. Importantly this expression shows the importance of having positive steady-state liquidity. Without it, $B_y = 0$, $\tilde{J}(1)$ would be equal to 0 meaning that the model would not have a unique saddle-path stable solution.

Hence we get stability ($\tilde{J}(1)\tilde{J}(-1) < 0$) for all combinations of parameter values in Table 3 under SADL ($\lambda < \lambda^{\star}_{\tilde{T}}$).

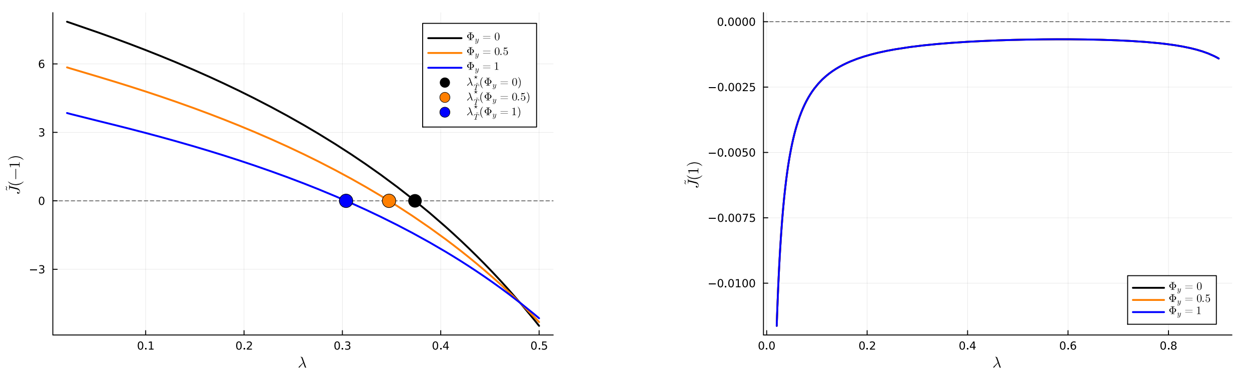
E2.1 Stability of the simple model without government spending with Taylor rule: Graphical representation

Here we complement the proof in the previous section by plotting the values of the characteristic polynomial for each case against different values of λ and Φ_y.

To produce all the plots we calibrate the rest of the parameters in line with what we use for the numerical analysis in Table 2 i.e. $\beta = 0.99$, $s = 0.98$, $\kappa = 0.11$ and $B_y = 0.57$.

Figure E3 plots the values of equations (E12) and (E14) against λ for $\Phi_y = [0, 0.5, 1]$. It also shows the corresponding value of $\lambda^{\star}_{\tilde{T}}$.

Figure E3: Taylor rule case - no G: Characteristic polynomial for different values of λ and Φ_y

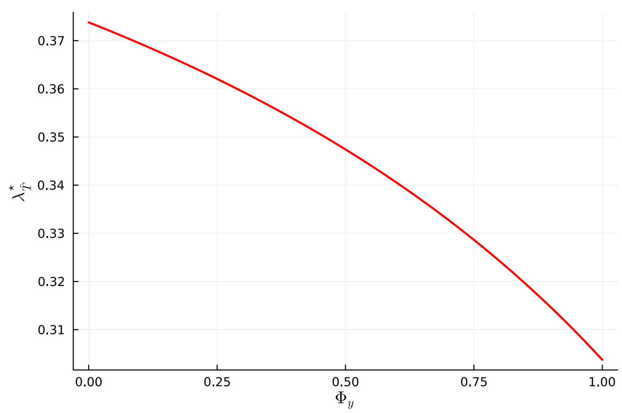


(a) $\tilde{J}(-1)$ (b) $\tilde{J}(1)$

Notes: These figures plot the values of $\tilde{J}(-1)$ and $\tilde{J}(1)$ against λ and Φ_y. The rest of the parameters are calibrated as follows: $\beta = 0.99$, $s = 0.98$, $\kappa = 0.11$ and $B_y = 0.57$.

Figure E4 plots $\lambda^{\star}_{\tilde{T}}$ for different values of Φ_y.

Figure E4: $\lambda^{\star}_{\tilde{T}}$ for different values of Φ_y.



Notes: This figure plots the values of $\lambda^{\star}_{\tilde{T}}$ for different values of Φ_y. The rest of the parameters are calibrated as follows: $\beta = 0.99$, $s = 0.98$, $\kappa = 0.11$ and $B_y = 0.57$.