Effects of Labour and Credit Market Frictions on Shock Propagation, Allocation, and Recoveries



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Abstract

This thesis contains three papers studying the consequences of labour and credit market frictions on the efficient allocation of resources and business cycle dynamics. A common theme of the three papers is that they focus on markets, where some market elements can be captured well with search frictions. The first paper estimates the extent to which aggregate job creation is driven by non-fundamental noise rather than fundamental news shocks employing maximum likelihood estimation of a DSGE model with search and matching in the labour market. The paper finds noise to play no role in aggregate job creation. Motivated by survey data on income growth expectations and labour market flow data, the second paper develops a mechanism where the information acquisition about idiosyncratic match quality is pro-cyclical. The mechanism, which is integrated into a directed search dynamic block recursive equilibrium model allows for a parsimonious explanation of observed survey data, labour market transition rates and finds counter-cyclical labour market mismatch. The third paper finds that the share of non-performing bank loans in total bank loans can be viewed as a mirror image of capital misallocation in many developed countries. The model developed from this observation shows that search frictions in used capital markets may drive non-performing, as will forbearance incentives. The paper estimates the extent to which changes in forbearance incentives and in the efficiency of used capital were responsible for variation in the share of non-performing loans. The introduction and conclusion of the thesis explain common themes and point to further applications of the content in the papers.

To my mother Andrea,
my late father Robert,
and my late grandfather Kurt,
who taught me arithmetic early so that I would become a more worthy opponent playing the card game "Schnapsen" with him.

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Chapter 1
Introduction

The recent recession induced by the Covid-19 pandemic has brought many questions about the depth and length of the economic downturn and recovery again to the forefront. Do the improvements in job creation and hiring mean the economy is recovering? Why did people start quitting their jobs as the recovery took hold? Why are used durable goods like cars rising in price and what consequences will the imposed loan forbearance over the length of the pandemic and the subsequent increase in non-performing loans have on the economy? Even though this thesis was commenced before the outbreak of the pandemic in December of 2019, and written before the data capturing its economic impact is finalised, it may offer answers and shed some light on an array of topics that have arisen as the consequences of this most recent global economic shock unfold.

In tackling these and other questions the three chapters of this thesis follow a common thread in their study of the interplay of different kind of real frictions on agent decision-making and the ensuing dynamic stochastic general equilibrium. At the heart of the models in these chapters is a search and matching problem capturing heterogeinities in the creation of jobs or the creation of new loans with underlying collateral. This search problem creates a forward looking value function due to the real costs of search. In the models, optimal policies regarding the match are then influenced by further frictions affecting the response of agents to changes in the aggregate state of the economy. The frictions are motivated by current theory or by data and the objective in the chapters is to estimate the friction if possible and understand its impact on the propagation of the aggregate shock.

The first chapter, ”No Noise in Job Creation”, tackles the question of whether the aggregate behaviour of job creation is based on a precise information set about the future value that a new position creates. Costly search frictions, like any other hiring frictions, create a forward-looking problem for firms meaning their decision to hire is based in part on

news affecting the expected future value of a match. The noise affecting the precision of these expectation changes will influence hiring behaviour given changes in match productivity and thereby allow for its identification and estimation. Integrating two methods proposed in the literature in (Blanchard et al., 2013) and (Forni et al., 2017) to identify such information frictions in the agent choice of consumption in a Diamond-Mortensen-Pissarides style search model in the spirit of (Pissarides, 2000) the chapter estimates the noise in the fundamentals underlying the job creation decision by representative firms. As the title of the chapter conveys, this noise is found to be small when estimating on United States labour market data. Thus aggregate job creation in the United States behaves as if firms are well and precisely about the current and future value of job match, and non-fundamental expectation shocks are rare. This means that barring any surprise shocks, improvements in job creation following the Covid-19 recession indeed are strong indicators for the improvement of labour market fundamentals.

While the first chapter shows information frictions regarding aggregate fundamentals in the labour market are small this is unlikely to be the case for idiosyncratic fundamentals. These information frictions regarding idiosyncratic fundamentals are what the second chapter of this thesis "Information Speed Bumps in the Labour Market" focuses on.

Motivated by the observation that the dispersion in income growth expectations is procyclical while reported uncertainty by workers is counter-cyclical the second chapter develops a mechanism where the aggregate fundamentals influence the amount of information acquisition about idiosyncratic fundamentals. This mechanism can explain the reduced dispersion of income growth expectations in recessions as the result of less information acquisition due to less updating of the unknown idiosyncratic fundamental estimated by agents. Reported uncertainty is captured as the increased posterior about the idiosyncratic state. Generating a meaningful information acquisition process requires persistent idiosyncratic states. For this reason, the model presented in this chapter adopts directed competitive search. The model with persistently heterogeneous matches can then be solved and simulated by computing a directed search dynamic block recursive equilibrium of the type first described in (Menzio and Shi, 2010a), (Menzio and Shi, 2010b), and (Menzio and Shi, 2011). The information speed bumps created by the model mechanism during recessions can contribute to explaining reduced transition rates in recessions as action margins are hit less often due to the reduced volatility of shocks to estimated idiosyncratic fundamentals. Furthermore, the chapter finds that the inclusion of this mechanism creates additional mismatch during recessions. Finally, the chapter

argues that matching models do not capture the data well when uncertainty is modelled as an increase in the volatility of idiosyncratic shocks positively correlated with the aggregate state. They should rather model reported worker uncertainty as a decrease in information about the state of their job match, which means that the volatility of idiosyncratic shocks to match fundamentals decreases when the recession hits and only increases well into the recovery.

While not discussed in this chapter, the view of the model on increased quits in the third quarter of 2021 as the economic recovery from the Covid-19 recession took off is a working off of the increased posterior about the quality of a job match. Less information flow during the recession caused by the pandemic led to this increased posterior about the quality of their job match for many workers. Increasing information flow during the recovery then leads to frequent updates about the estimated quality of the job match, leading many workers to conclude that they are no longer well matched.

In the third chapter "Non-Performing Loans and Inefficient Capital Reallocation", a model with search frictions captures the decision-making problem of banks providing capital in the form of corporate credit with underlying collateral to firms. When firms turn out to be too unproductive to pay the interest on these loans the bank would ideally like to foreclose the loan and recall the capital. However, the capital provided has been specified in the form of the collateral and has become used, and the bank's decision on whether to foreclose the loan depends on the search frictions in used capital markets which will determine the value of the used collateral. This match efficiency in used capital markets, together with forbearance incentives defined as real cost banks face when foreclosing a loan will drive the share of non-performing loans in total loans and capital misallocation.

The chapter shows empirically that the share of non-performing loans in total loans and the efficient allocation of capital are closely linked using moments computed from micro-data provided by the Competitiveness Research network and aggregate time-series data. The model with search frictions in capital markets is then developed to match counter-cyclical non-performing loan shares and capital allocations. The paper shows that the thriving literature on used capital reallocation ((Eisfeldt and Shi, 2018) and (Lanteri, 2018)) can aid in understanding the problem of high non-performing loan shares. Forbearance incentives and strong search frictions in used capital markets are both shown to drive the share in non-performing loans. The prediction derived from the model that forbearance incentives will increase the price of used capital, while search frictions will

decrease it, is used in the chapter to propose a new identification to separate changes in search frictions from forbearance incentives. The chapter estimates which frictions may have driven the variance in the share of non-performing loans in different European economies and argues that most economies with a share of non-performing loans saw used capital markets become more inefficient during a recession, while in Italy banks faced strong counter-cyclical forbearance incentives.

From the model presented in the third chapter, it can be deduced that the large price increases in used capital goods in 2021 [1] can to some extend be explained by the extensive loan moratoria passed during the Covid-19 recession. These moratoria can be viewed as a strong increase in forbearance incentives leading to a cut in the supply of used capital, and thereby driving the price.

Following this introduction, this thesis presents "No Noise in Job Creation" in the first chapter following this introduction. After this chapter are the chapters "Information Speed Bumps in the Labour Market" and "Non-Performing Loans and Inefficient Capital Reallocation". Each of these three chapters is followed by a chapter appendix. Finally, chapter five concludes.

[1] e.g. : Article in *The Economist* on 14 July, 2021 "The used-car market sent American inflation soaring in June".

Chapter 2

No Noise in Job Creation

Abstract

Building on the recent literature of expectation driven business cycles called news shocks, this paper implements a framework to estimate to which extent job creation is driven by news and noise shocks. Both shocks will drive hiring in positive anticipation in a model with hiring frictions, as such frictions introduce a forward-looking component in the job creation decision. Contrary to news shocks, noise shocks would look in the data like spontaneous increases in labour demand, which cannot be justified by the productivity of employed workers. A higher noise component in news would further lead to slower job creation following positive signals about future labour productivity as agents cannot be sure about these positive signals being the result of fundamentals. The paper estimates the extent to which aggregate job creation is driven by non-fundamental noise about the productivity process, rather than by fundamental news shocks. For this purpose, a DSGE model featuring exogenous growth around a random search employment equilibrium is developed and estimated. Further, a structural VAR model using an alternative identification of noise based on an observed signal is estimated. Both find that it is unlikely that noise plays a significant role in aggregate hiring and job creation in the United States labour market. The result shows that series measuring labour market decision-making are more reliable predictors of rationally expected future labour productivity than series relying on consumption or investment measures and that aggregate labour market decision-making changing due to non-fundamental noise is unlikely.

Keywords: *News and Noise, Search and Matching, Labour Markets, Business Cycles*

JEL Classification: *E2; E24; E32; E37.*

2.1 Introduction

A consequence of costly job creation is that the decision of a firm on whether to employ a worker has a substantial forward-looking component. When deciding on how much to spend on hiring a new worker, the firm's management will look to the current and the expected future value that the new hire would bring to the firm. These added values will usually have to outweigh the costs that come with hiring, training, paying, and integrating the new worker in the production process. Similarly, a prospective worker deciding whether to take a job will weigh the benefits of the job, namely the wage, and the future career and income prospects, against the opportunity cost of not being able to devote her time to other productive activities. A consequence of this forward-looking component is that expectation shifts may drive job creation, whether these shifts are of a fundamental nature or due to noise. This paper estimates the role noise plays in these expectation shifts in the labour market by constructing an equilibrium unemployment model with search frictions in the spirit of (Pissarides, 2000) and adapting the methods for estimating the importance of noise in consumption decision-making developed in (Blanchard et al., 2013). For robustness, the paper also estimates noise in job creation by using an alternative identification method for noise in a structural VAR model developed in (Forni et al., 2017). To the author's knowledge, this paper is the first to offer an estimation of the extent to which fundamental and non-fundamental expectations shifts drive job creation.

While the literature has found that a substantial part of the movements in aggregate consumption is driven by non-fundamental noise rather than fundamental news about productivity, this paper finds that noise plays no significant role in job creation decisions. This means forward-looking agents face substantial information frictions when making consumption or investment choices based on expectations about the future, but similar information frictions play no significant role in job creation decisions. Thus variables capturing hiring behaviour are reliable indicators of agents expectations about fundamental labour productivity, and vice versa expectations are strong predictors of expected hiring activity. Concretely, the DSGE model finds that while reasonable model calibrations would allow for noise to make up to 24 % of the forecast error variance of employment to expectation shocks in the first quarter, maximum likelihood estimation shows that noise is likely to make up less than 0.1%. The structural VAR model confirms the result and shows that noise only makes up a small part of the forecast error variance of employment and the job-finding rate.

Optimal inter-temporal hiring decisions are at the heart of the Mortensen-Pissarides search and matching model ((Mortensen and Pissarides, 1994), (Pissarides, 2000)), where they are captured with the job creation condition. Matching frictions thereby provide another channel in business cycles models, besides the conventional consumption Euler equations, through which the present equilibrium may be affected by shifts in the expectations over future economic fundamentals, commonly referred to as news shocks. Empirically it has been shown that accounting for this channel by combining matching frictions, or other adjustment costs to labour input, with news shocks in a business cycle model, greatly improves the match of the model with the data ((Den Haan and Kaltenbrunner, 2009), (Jaimovich and Rebelo, 2009), and (Zanetti and Theodoridis, 2016)). A reason for this is that theoretical real business cycle models with search and matching frictions are able to produce the Pigou cycles that seem to be present in the data ((Beaudry and Portier, 2006), (Den Haan and Kaltenbrunner, 2009), (Krusell and McKay, 2010); see section 2.2 for a definition of a "Pigou" cycle). Thus a good part of aggregate job creation seems to occur in response to shifts in anticipations about future productivity, and this fact is reflected in the visible interest of investors and policymakers in new releases of the US employment report, and especially the changes in the number of non-farm workers on payroll.

Noise shocks could be considered wrong news, which lead agents to erroneously adjust their expectations over future productivity. As both aggregate consumption and aggregate job creation depend to an extent on the anticipated state of future economic fundamentals, recent findings on consumption being driven by noise raise the question of whether some fluctuations in the labour market can also be attributed to noise rather than news. Such noise shocks would look in the data like spontaneous changes in labour demand and hiring activity, which cannot be justified by current or future productivity changes of the employed worker. A large noise component in the information available to agents would further mean that expectations about future productivity are less precise, leading labour market participants to adjust their behaviour more gradually in response to labour productivity changes. The empirical question answered by this paper is how precise are agents' aggregate expectations about the value that employment relationships will provide, and thereby how much information about future fundamentals is contained in current labour market data.

For the purpose of answering this question, a DSGE model with search frictions is developed in which variations in job creation, and thereby employment and the number

of open vacancies, depend on news or noise about the future. This is achieved by combining a framework in which agents receive noisy information over future productivity growth as developed in (Lorenzoni, 2009) and (Lorenzoni, 2011), and (Blanchard et al., 2013) with a search and matching model of the labour market. The intuition of this model is the following. Agents receive a noisy signal providing them with information about future labour productivity growth. As there are labour market frictions, firms take a forward-looking approach to hiring. The signal provides firms with information about the expected relative value of opening a vacancy. Similarly, workers receive information about the value of an employment relationship, which will inform their wage negotiation. Agents know through long-term observation both how volatile labour productivity growth is and how noisy the signal is. These distributional parameters will influence the extent to which rational agents trust the information about the future received through the signal, and thus the speed and strength with which the agents will react to the information obtained via the signal. Concretely, in this case, the amount of additional vacancies firms open will depend on the parameters of the productivity and noise innovations. Since we have information about both actual labour productivity and labour market outcomes, as well as the optimal response chosen by firms and workers given their productivity expectations, we can gain information about the nature of the signal driving expectations without the need to observe this signal. If the behaviour of agents anticipates future movements in the productivity process accurately and speedily then the signal is not very noisy, while if the behaviour of agents leads to fluctuations in labour market outcomes without movements in the productivity series and vice versa, then noise would be identified as being an important driver of the labour market on the aggregate level.

The setup of the paper allows for both estimating noise regarding shifts in the long-term fundamental surplus of matches in the labour market, as well as temporary increases in the match surplus. Furthermore, different standard calibrations of an equilibrium unemployment model following (Shimer, 2005a) and (Hagedorn and Manovskii, 2008) are employed. The simulated model shows then that there exist calibrations consistent with US employment moments, where a substantial part of the forecast error variance in the employment rate one quarter ahead is caused by noise. However, bringing the model to the data and estimating the parameters of the news and noise process by maximizing their likelihood, yields the result that the volatility of the noise shock is very small. The results of this estimation suggest that when agents receive information about the future

productivity of a job match, then this information is very precise and noise plays only a small if any, role in employment fluctuations.

An analysis employing the techniques suggested in (Forni et al., 2017) confirms the result of the DSGE model. (Forni et al., 2017) suggest to identify structural noise and productivity shocks in a VAR model with the help of a variable that acts as a proxy for the actual signal on future productivity observed by the agents. The idea of the structural VAR model in this paper is to take the insights about the behaviour of the labour market obtained in the DSGE model and set up auto-regressive equations which capture the expected reaction of the job-finding rate and the employment rate with respect to news and noise shocks. By introducing additionally a variable to measure expectations and thereby act as a proxy for the signal, the model assumptions are used to identify news and noise shocks. The results of the structural VAR model show that a majority of fluctuations in the labour market is found to be caused by correctly anticipated or surprise fluctuations in productivity. Of the anticipated fluctuations, on average more than 90 % of the forecast error variance in the job-finding rate and in the employment rate in the first four quarters are found to be caused by news, while less than 10% are due to noise confirming the results of the DSGE model.

This paper is structured in six parts. The first part situates the paper within the literature by providing an overview of the recent advancements in the research on news and noise shocks. In the second part, a DSGE model with search frictions for estimating the news and noise components in the labour market is presented. In the third part, the data used is presented and the DSGE model is calibrated, simulated and estimated. The fourth part compares the results of the maximum likelihood estimation in the third part to the results of a structural VAR following the identification of noise by means of dynamic rotations of the reduced form residuals proposed in (Forni et al., 2017). Section five discusses the interpretation of the result. Section six concludes and relates the findings for the labour market to the findings for aggregate consumption.

2.2 Relation to the literature

The paper follows an extensive research programme studying how shocks to expected future productivity drive present agent behaviour and market outcomes. It focuses on the labour market as this market has been shown to be both enabler and driver of these outcomes and picks up the results of the news literature that search and matching fric-

tions, or other labour market frictions that create labour adjustment costs, create Pigou cycles in response to news shocks, which match the data well. Due to the empirical results produced by this literature and described in the next paragraphs, it is taken as given throughout this paper that hiring in labour market depends to a large extent on anticipations about the future productivity of the hired workers.

The renewed interest of macro-economists in news shocks can be traced back to Beaudry and Portier's seminal 2004 and 2006 papers. (Beaudry and Portier, 2004) proposed a model, which aimed to explain boom and recession cycles as the result of agents' difficulty in anticipating future needs for capital in production. This required diverging from the standard RBC model as *"such models are incapable of generating Pigou cycles, that is they are incapable of generating equilibrium paths in which: (i) a forecast of future technological improvement first leads to a boom defined as an increase in aggregate output, employment, investment and consumption, and (ii) the realization that a forecast is too optimistic leads to a recession defined as a fall in all the same aggregate quantities."*((Beaudry and Portier, 2004), p. 1185). (Beaudry and Portier, 2006) shows that the data supports the existence of Pigou cycles, and that models capable of producing these cycles are necessary for modelling the effect of anticipation changes correctly. The search and matching model employed in this paper can produce such Pigou cycles.

This empirical finding led to the search of theoretical models that could actually produce Pigou cycles in response to news shocks. To produce such cycles in an RBC model, (Jaimovich and Rebelo, 2009) suggest changing the utility function of the representative agents, including variable capital utilization, as well as adjustment costs for factor inputs. It finds that including labour adjustment costs in the model helps in generating aggregate co-movement in response to news shocks for a much wider range of parameters. The reason for this is that labour adjustment costs provide an increased incentive for smoothing labour input and thereby lead to a strong increase in labour supply in response to a news shock.

This effect of building up labour input in response to an expected productivity increase is also present in a model that takes a flow perspective of the labour market. This is used in (Den Haan and Kaltenbrunner, 2009), which adapts a search and matching model of the labour market such that it produces Pigou cycles. The model consists of endogenous entry in the job market, a labour market with matching frictions, and an otherwise frictionless capital market. Both the cost of finding a worker for a job as well as the cap-

ital for the job itself are interpreted as investment. As the flow constraints of the model are such that both current employment and current capital are predetermined state variables, the model cannot produce a Pigou cycle behaviour in the first period of a news shock as total output remains constant. In order to get a sufficiently volatile response of employment in the face of news shocks, (Den Haan and Kaltenbrunner, 2009) furthermore introduce sticky wages and set the bargaining power of the worker higher than the level of Pareto-optimality. With this set-up, they manage to produce Pigou cycles, which match the empirical moments closely. (Krusell and McKay, 2010) point out that even a simpler Mortensen-Pissarides search and matching model produces Pigou cycles, if the focus is on the periods following a news shock and the cost of finding a worker is interpreted as investment. (Zanetti and Theodoridis, 2016) show that an RBC model enriched with conventional search and matching frictions, variable capital utilization, and investment adjustment costs does well in matching the data. An estimation of the model on aggregate and labour market data shows that news shocks to labour market variables such as the job destruction rate and the matching productivity actually decrease the fit of the model with the data, while news shocks to aggregate productivity increase the fit with the data. Furthermore, the estimation shows that most short-run fluctuations are due to surprise shocks, while long-run fluctuations are caused by news shocks.

The papers above usually assume that the productivity series is subject to two types of shocks. These are news shocks and surprise shocks. News shocks determine the future productivity path, while surprise shocks in the current period may lead to sudden corrections. This assumption has the advantage of simplicity, however, it comes with some empirical and normative drawbacks. There may exist, for instance, a third type of shock that is not identified by the econometrician in this setup. This could be a shock, which shifts expectations, but contains no actual news on the future values of productivity. These kinds of shocks are interpreted by some authors as Keynesian animal spirits, following (Keynes, 1936). A more sober interpretation may be to view these shocks as noise in the news that agents receive over future productivity.

Not accounting for the noise shocks has the empirical drawback that it may lead to an overestimation of the importance of surprise shocks, or news shocks depending on the setup and identifiers used for estimating the shocks. For example, if every expectation shift is seen as a news shock, then the only possibility to correct for erroneous news is a surprise shock. Similarly, if noise leads to similar impulse responses as news in the short run, then a two shock model in a noisy environment will overestimate the importance of

news in driving business cycles. On the normative side the following dichotomy arises. On the one hand, if agents receive news and are afterwards subject to surprise shocks, which correct for the news received at an earlier time, then the choices made by the agents at all times must be viewed as optimal. On the other hand, if agents are making choices based on false news, then an all-knowing social planner could improve on the choices made by rational agents. Whether noise is an important source of fluctuations has thus become an important question in the recent literature on news.

(Lorenzoni, 2009) and (Lorenzoni, 2011) develop a setup for the productivity process that can be used for estimating the importance of noise shocks in business cycle fluctuations. The level of productivity is split up in a non-stationary permanent part and a stationary temporary part. Agents receive a noisy signal over the size of the permanent part and form expectations about future productivity growth based on this signal. These assumptions about the productivity process are also made in (Blanchard et al., 2013) and in the model in this paper (see section 3.1). (Blanchard et al., 2013) show that an econometrician who does not have information about the actual signal will be unable to recover this signal, and therefore also the basis for the agents' decision-making, with a structural vector autoregression. However, with a method of matching moments it will be possible to estimate the parameters of the assumed productivity and noise processes. Even more, with a maximum likelihood estimation and employment of the Kalman smoother, it is possible to estimate the parameters of the process and recover to some extent the shocks. Applying this method first on a simple model of productivity and consumption, and then in a DSGE model, the authors estimate that the volatility of noise and of temporary productivity shocks to be similar in size, which speaks for a significant role of noise in business cycle fluctuations.

Responding to the finding of (Blanchard et al., 2013) regarding the identification of noise shocks to the agent's information set without the signal variable, (Forni et al., 2017) show that if one changes the assumptions on the noisy signal slightly, a structural VAR model will be able to recover the correct shocks from the agents' decision-making. If one assumes that agents receive a noisy signal, where after a finite number of periods the agents learn exactly what part of the signal has been noise and what part has been news, then imposing the logical restriction that the noise does not affect actual productivity is sufficient to identify and recover the correct shocks. Like (Barsky and Sims, 2012) their estimation approach requires the choice of the series that serves as the signal. Furthermore, noise can only be separated from news up to the point where agents are able to separate noise

from news as well, thus not at the most recent data points. Using US potential output as the productivity series and expected business conditions within the next twelve months as a signal, the authors find that more than half of the fluctuations of GDP are driven by noise and news. Noise is an essential part of these fluctuations and accounts for approximately 30-40% of the forecast error variance in the short run. Thus the authors conclude that noisy expectations of future fundamentals should be considered a major source of business cycle fluctuations.

The literature has thus arrived at the conclusion that consumption over the business cycle is likely to be driven to a large extent by noise. This also means that a good part of investment is likely to be driven to some extent by noise, leaving a limited set of variables to policymakers to judge the state of the economy. This paper shows that by employing similar techniques, labour market variables are not found to be driven to the same extent by noise.

2.3 The model

2.3.1 Firm-worker match productivity

This first Section 2.3.1 describes the fundamental match productivity process, which will define the output of all firm-worker matches in the economy. The next Section 2.3.1 specifies the information set, while the last Section 2.3.1 describes the solutions to the information problem. The agent Kalman filter presented is the focus of the estimation determining the quality of the information content about future labour productivity growth held by agents.

Productivity fundamentals

The model takes the labour productivity process formulated in (Blanchard et al., 2013) and applies it to a labour market with equilibrium unemployment. The observed individual match productivity process a in equation (2.1), is taken to be the flow product of a match and consist of an unobserved permanent component x and an unobserved temporary component z.

$$a_t = x_t + z_t \tag{2.1}$$

The difference between the permanent component and the temporary component are

assumed to be stationary processes. The process x can be viewed as capturing permanent changes in production technology, while z captures short-term productivity deviations. e_t and η_t are independently, identically, and normally distributed exogenous shocks with mean 0 and constant and known variances σ_ϵ^2 and σ_η^2.

$$\Delta x_t = \rho^x \Delta x_{t-1} + \epsilon_t \tag{2.2}$$

$$z_t = \rho^z z_{t-1} + \eta_t \tag{2.3}$$

The simplifying assumption is made that no information over the magnitude or direction of future productivity is contained in the productivity process a itself. To achieve this one has to choose the parameters of the underlying processes in such a way that the permanent and temporary component are impossible to disentangle as the future productivity path in response to shocks in opposite directions to the components cancel each other out. The product of a worker a_t then appears to be a random walk. These properties are achieved by choosing the following parameter relations for the permanent and temporary component following (Lorenzoni, 2011) and (Blanchard et al., 2013).

$$\rho^x = \rho^z = \rho \tag{2.4}$$

$$\rho\sigma_\epsilon^2 = (1-\rho)^2\sigma_\eta^2 \tag{2.5}$$

Information

Since only the present and past joint realizations of the permanent and temporary match productivity components $a, a_{t-1}, a_{t-2}, a_{t-3}, ...$ are part of the agents' information set in period t, it is impossible for agents to tell whether future match productivity will grow or decline just from observing the productivity process. All information that may help agents disentangle the permanent from the temporary process and thereby predict future match productivity growth is summarised in a signal in equation (2.6).

All agents in receive a noisy signal s_t informing them about the share of the permanent component in total match productivity, which allows for expected productivity growth forecasts. ν_t is independently, identically, and normally distributed noise shocks with

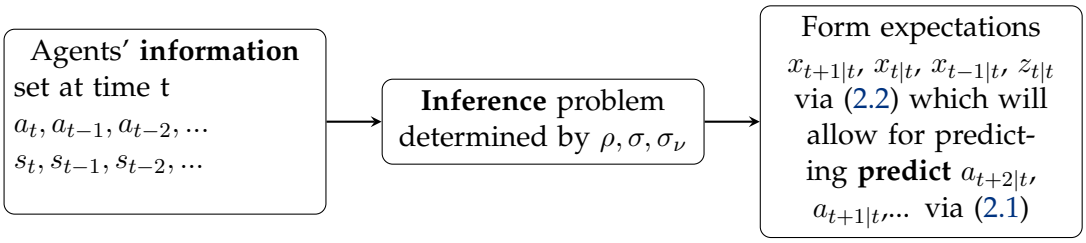


Figure 2.1: Information extraction problem

mean 0 and constant and known variance σ_ν^2.

$$s_t = x_t + \nu_t \tag{2.6}$$

Forming expectations over future match productivity

All agents have the same information set and are assumed to know the distributions of shocks and form the productivity process takes. The agents information set consists of all past and present productivity realisations and past and present signals. Assuming agents observe the processes and signals over a long time this information problem is optimally resolved by rational agents with a converged Kalman filter.

If agents wouldn't receive an informative signal then the choice of parameters $\rho^x, \rho^z, \sigma_\epsilon^2, \sigma_\eta^2$ would have the consequence that expected future productivity would be equal to current productivity. Thus $E_t(a_{t+s}) = a_t$ for all $s \geq 1$ as the path of productivity is observed to be a random walk. However, agents receive a present signal. After convergence of the Kalman filter, there is no information in past signals beyond the best guesses derived past expected values of the permanent (x) and temporary (z) component of worker productivity a .

Due to agents receiving a signal over the permanent component of productivity x, and thus over the future growth of productivity to be expected, a signal extraction problem evolves. As in (Blanchard et al., 2013) this information extraction problem is solved with a typical agent Kalman filter closer described in Appendix 2.A. Each element of the Kalman gain will be a function defined by the parameters $\sigma_\epsilon, \sigma_\nu, \rho^x$, and ρ^z.

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \end{bmatrix} = K(\sigma_\epsilon, \sigma_\nu, \rho^x, \rho^z) \tag{2.7}$$

The objective of the maximum likelihood estimation will be to estimate the parameters defining the Kalman gain in equation (2.7). Based on the estimated values of $x_{t|t}, x_{t-1|t}, z_{t|t}$ agents form their expectations over future productivity growth in each period.

$$a_t = x_{t|t} + z_{t|t}$$

$$E_t(a_{t+1}) = (1+\rho^x)x_{t|t} - \rho^x x_{t-1|t} + \rho^z z_{t|t}$$

$$E_t(a_{t+2}) = [(1+\rho^x)^2 - \rho^x]x_{t|t} - (1+\rho^x)\rho^x x_{t-1|t} + (\rho^z)^2 z_{t|t}$$

This convereges in the long-run to equation (2.8).

$$E_t(a_{t+\infty}) = \frac{x_{t|t} - \rho^x x_{t-1|t}}{1-\rho^x} \tag{2.8}$$

2.3.2 Household

There exists a large representative household, whose members maximize the present value of future expected utility and the instant utility function has a constant-elasticity of substitution specification.

$$\max_{g_t, c_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\zeta}}{1-\zeta} \tag{2.9}$$

subject to a budget constraint

$$c_t + g_t = (w_t n_t + d_t + b_t u_t + r_{t-1} g_{t-1}) = (a_t n_t - \kappa_t v_t + u_t b_t + g_t) \tag{2.10}$$

Here β is the discount factor, g_t are one period bonds r_{t-1} is the interest rate on bonds of the past period set in period $t-1$ and received in period t. The interest rate is defined as the equilibrium rate at which household members would be willing to lend to each other. w_t is the real wage, b_t is the value of household production for an unemployed worker, and d_t is the profit created by firms. The part after the second equal sign in the budget constraint follows from the aggregate resource constraint $w_t n_t + d_t = (a_t n_t - \kappa_t v_t)$.

As usual the first order conditions lead to the expected shadow value of the period budget constraint being $c_t^{-\zeta} = \mu_t$, where μ is the Lagrangian multiplier on the budget constraint. The inter-temporal Euler equation for bonds is $\mu_t = \beta r_t E_t(\mu_{t+1})$. We can rewrite

this as equation (2.11).

$$\frac{1}{r_t} = \beta \frac{E_t(\mu_{t+1})}{\mu_t} \tag{2.11}$$

2.3.3 Production

Firms are assumed to be identical and to produce goods in period t according to $\prod_t = a_t n_t^{1-\alpha}$. In order to produce, firms have to hire workers in a labour market with matching frictions. The number of firms and workers that meet each other in every period is determined by the matching function $m(v_t, u_t) = m v_t^{1-\xi} u_t^{\xi}$.[1] m is a linear matching productivity parameter, v_t is the number of posted vacancies in a given period, and $u_t = 1 - (1 - \lambda) n_{t-1}$ is the number of unemployed before matching occurs in a given period. Jobs are destroyed at the constant rate λ following the argument in (Shimer, 2012). Furthermore, it costs a firm κ_t to post a vacancy.

Firms are assumed to be small enough to take the matching probability $\frac{m(v_t,u_t)}{v_t} = q(\theta_t)$ as given. $\theta_t = \frac{v_t}{u_t}$ is a measure of labour market tightness. Firms are also assumed to be hiring in markets large enough compared to the number of workers they employ that the law of motion for employees for an individual firm corresponds to the law of motion of the labour market as a whole. Finally, the law of motion for the rate of employment is $n_t = (1-\lambda) n_{t-1} + m(v_t, u_t)$. Note that workers will take up production in the same period that they are hired. Furthermore, workers can be separated and re-employed in the same period, which means that there are more workers searching for a job in a current period than the number of unemployed $(1 - n_{t-1})$ in the previous period.

Firms will have to decide about how many workers to hire workers based on their expectations on present and future labour productivity. The firm's management then chooses v and n to maximize the value of expected profits discounted by the expected utility contribution of the production value, which in a model with a bond market would equal the interest rate.

$$\max_{v_t, n_t} E_t \sum_{t=0}^{\infty} \beta^t \mu_t (a_t n_t^{1-\alpha} - w_t n_t - \kappa_t v_t) \tag{2.12}$$

[1] I also use an alternative specification of the matching function $m(v_t, u_t) = \frac{v_t u_t}{(v_t^l + u_t^l)^{\frac{1}{l}}}$, which has the advantage of guaranteeing transition probabilities between 0 and 1 for any positive values of v and u, but requires bargaining power dependent on tightness $theta$ to fulfill the Hosios condition.

This value is maximised subject to the constraint in equation (2.13).

$$n_t = (1-\lambda)n_{t-1} + v_t q(\theta_t) \tag{2.13}$$

ζ_t is the Lagrange multiplier put on the constraint in equation (2.13). Combining the first order condition for v_t, $\zeta_t q(\theta_t) - \mu_t \kappa_t = 0$, with the first order condition for n_t, $\mu_t(1-\alpha)a_t n_t^{-\alpha} - \zeta_t + (1-\lambda)\beta E_t(\zeta_{t+1}) = 0$ yields the job creation condition where the expected cost of hiring a new worker equals the benefit of hiring a new worker. ζ is the Lagrange multiplier on the law of motion for the individual firms.

$$\frac{\kappa_t}{q(\theta_t)} = (1-\alpha)a_t n_t^{-\alpha} - w_t + \beta(1-\lambda)E_t[\frac{\mu_{t+1}}{\mu_t}\frac{\kappa_t}{q(\theta_{t+1})}] \tag{2.14}$$

We can simplify (2.14) by substituting for r_t with (2.11). Firms and workers split the expected surplus of a successful match according to a Nash bargaining protocol with the firm's bargaining strength being π. The negotiated wage is in equation (2.15).

$$w_t = \pi b_t + (1-\pi)[(1-\alpha)a_t n_t^{-\alpha} + (1-\lambda)\frac{1}{r_t}E_t\kappa_{t+1}\theta_{t+1}] \tag{2.15}$$

Substituting equation (2.14) back into the job creation condition yields the job creation condition as a function of productivity, tightness, the outside value and the vacancy positing cost.

$$\frac{\kappa_t}{q(\theta_t)} = \pi((1-\alpha)a_t n_t^{-\alpha} - b_t) + (1-\lambda)\frac{1}{r_t}E_t[\frac{\kappa_t}{q(\theta_{t+1})}) - (1-\pi)\kappa_{t+1}\theta_{t+1}] \tag{2.16}$$

2.3.4 Equilibrium job creation and integrated match productivity

Equation (2.2) shows that the choice for the match productivity process is integrated to capture periods of expected match productivity growth with the permanent component. Combining this with the job creation condition in equation (2.16) provides two possible interpretations for the model. In the first interpretation, x captures labour productivity changes due to permanent shifts in the surplus provided by a match. This interpretation of x is referred to as the fundamental surplus (FS) interpretation in this paper. The second interpretation of x is to capture permanent changes in labour productivity leading to temporary deviations of the match product due to the catch-up of other factors defining the value of a match, concretely in this model the cost of posting a vacancy and the

replacement rate. This allows for a constant long-run unemployment equilibrium with non-constant long-run productivity. This interpretation of x is referred to as the match productivity (MP) interpretation in this paper.

Note that the MP interpretation nests the FS interpretation for values of $\gamma_s = 0$ for all s, but both interpretations are important as they are able to capture news and noise regarding different features of the labour market. Therefore both versions of the model are implemented and estimated. However, as noise is a more potent factor in the MP interpretation simulation and estimation results of the MP interpretation will be presented in Section 2.4, while estimation results capturing the noise component in the FS interpretation will be presented in Appendix 2.C.

Fundamental surplus interpretation (FS)

The first interpretation is that the permanent component x captures structural changes in the labour market, permanently changing the fundamental surplus as defined in (Ljungqvist and Sargent, 2017) generated by matched employed over unmatched unemployed. In this case we can choose a constant value for $\kappa_t = \kappa$ and $\beta_t = \beta$. This interpretation allows for capturing periods of different employment volatility in response to fundamental shocks and the expectation and noise around permanent unemployment equilibrium shifts.

Match productivity interpretation (MP)

Alternatively, we can interpret x as a variable capturing labour productivity growth, with this labour productivity growth leading to a long-run stable Beveridge curve. A permanent increase in labour productivity would then permanently increase consumption and output, but would only lead to a temporary increase in employment. Thus it would be correct to remove the unit root from the job creation condition in equation (2.16) by introducing other variables which lead to a de-trending similar to (Schmitt-Grohé and Uribe, 2012). It turns out that for the unit root to be removed from the fundamental surplus and to introduce a long-run unemployment equilibrium it is required that the cost of vacancy creation κ_t and the outside value to a match b_t catch up with match productivity a_t in the long-run. Arguments for a relatively stable job creation condition over time follow from the evidence regarding the stable Beveridge curve in (Martellini and Menzio, 2020).

We can then interpret the permanent labour productivity component similarly to a supply shock in (Blanchard and Quah, 1993). Using these long-run restrictions on a VAR(2)

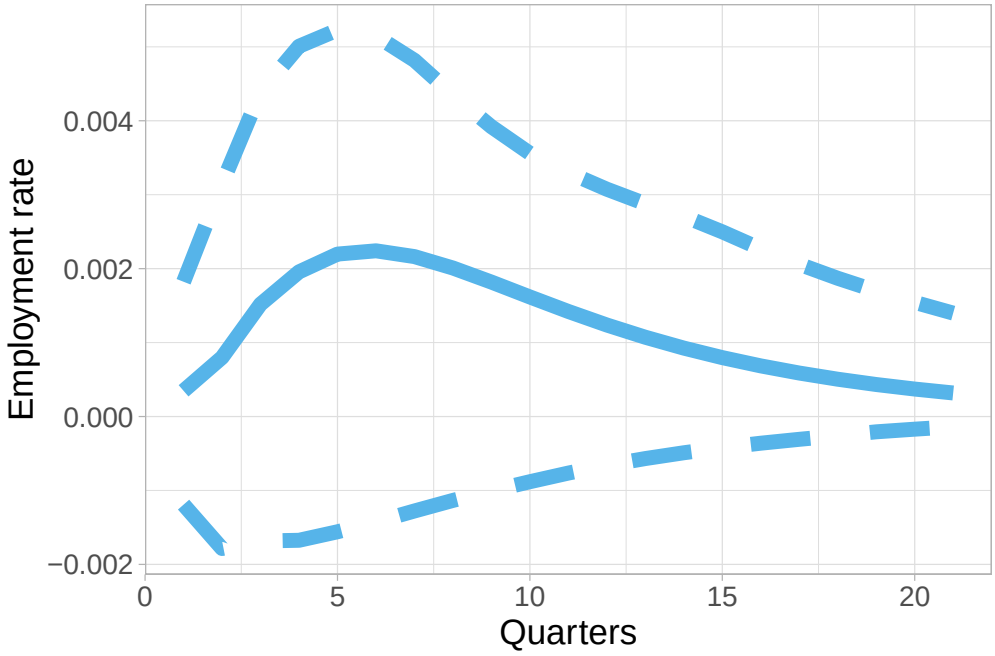


Figure 2.2: Response of the employment rate to labour productivity [OPHNFB] identified using long run restrictions following Blanchard and Quah (1993).

model with changes to labour productivity [FRED: OPHNFB] and the employment rate [FRED: 1-UNRATE] yields a positive temporary response of employment to a long-run labour productivity as shown in Figure 2.2. The model is supposed to create a similar response in the employment rate via increased job creation when output per worker permanently rises. It is proven that Proposition 1 holds, namely that when parameters κ_t and β_t depend on aggregate contemporary variables, such as output per worker, consumption, or wages, then a change in output per worker won't affect job creation.

Proposition 1. *Cost of vacancies κ_t and unemployment benefits b_t which are proportional to contemporary variables such as output per worker, consumption, output, or wages in a plain Diamond-Mortensen-Pissarides random search DSGE model lead to labour productivity changes leaving the employment rate unaffected.*

Proof. Assume $\kappa_t = \psi_1 a_t$ and $b_t = \psi_2 a_t$. Here ψ_1 and ψ_2 are parameters, which could stand for κ and b. For simplicity set $\alpha = 0$ and $\zeta = 1$. Inserting these in the job creation condition means the job creation condition becomes equation (2.17).

$$\frac{\psi_1}{q(\theta_t)} = \pi((1-\alpha)n_t^{-\alpha} - \psi_2) + (1-\lambda)E_t[\frac{1}{r_t}\frac{a_{t+1}}{a_t}[\frac{\psi_1}{q(\theta_{t+1})}) - (1-\pi)\psi_1\theta_{t+1}]] \tag{2.17}$$

Note that $\frac{1}{r_t}\frac{a_{t+1}}{a_t} = \frac{c_t}{c_{t+1}}\frac{a_{t+1}}{a_t}$ and $c_t = a_t(n_t - \psi_1 v_t + \psi_2(1-n_t))$. Thus all changes and expected changes to the product of labour cancel out. Equivalent results can be achieved when using output per worker where $y_t = a_t\frac{n_t+\psi_2(1-n_t)}{n_t}$ or wages $w_t = a_t[\pi\psi_2 + (1-\pi)[1+(1-\lambda)\psi_1 E_t(\theta_{t+1})]$. □

To achieve the theoretical result of employment growth as a result of growth in output per worker, consistent with Figure 2.2 and Okun's law, which continues to fit the data (Ball, Leigh, and Loungani, 2013), it is necessary for current labour productivity to temporarily increase in proportion to vacancy cost and the unemployment benefit. Making both series dependent on past labour productivity realisations achieves this result with $\kappa = \kappa\prod_{s=1}^{L} a_{t-s}^{\gamma_s}$ and $b_t = b\prod_{s=1}^{L} a_{t-s}^{\gamma_s}$. The parameters γ_s control the importance of each lag. Assume $\sum_{s=1}^{L}\gamma_s = 1$, which is necessary for a stable equilibrium to exist. Dividing the job creation condition by $\prod_{s=1}^{L} a_{t-s}^{\gamma_s}$ shows that job creation and employment rate changes will depend on relative labour productivity in (2.18).

$$\Delta n_t \propto \frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}} \tag{2.18}$$

We can rewrite then after taking logs this relative process as a sum of γ_s weighted changes.

$$\log(\frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}) = (\sum_{s=1}^{L}\gamma_s)\Delta a_t + (\sum_{s=2}^{L}\gamma_s)\Delta a_{t-1} ... + \gamma_s \Delta a_{t-s}$$

A regression of employment rate changes on past and present changes in output per worker my then reveal the relative importance of each lag. These regressions are shown in Table 2.1 and suggest that only the past two lags matter. Based on the model of the first column their relative size of γ_1 and γ_2 should be sat at $\gamma_1 = 0.697$ and $\gamma_2 = 0.303$. These values will be used in the calibration of the model with the labour productivity interpretation (MP) below.

2.3.5 Aggregate equilibrium and labour productivity as a hiring driving force

Market clearing in this simple setting requires that in this economy everything produced will be consumed.

$$c_t = a_t[n_t^{1-\alpha} - \kappa_t v_t + b_t(1-n_t)] \tag{2.19}$$

	Dependent variable:		
	$\Delta_Employment_rate$		
	(1)	(2)	(3)
$\Delta_Output_per_worker$	0.194^{***}	0.171^{***}	0.194^{***}
	(0.065)	(0.066)	(0.064)
$lag(\Delta_Output_per_worker)$	0.149^{**}	0.161^{**}	0.149^{**}
	(0.065)	(0.064)	(0.064)
$lag(\Delta_Output_per_worker, 2)$	0.088	0.094	0.087
	(0.065)	(0.066)	(0.064)
$lag(\Delta_Output_per_worker, 3)$		0.012	
		(0.064)	
$lag(\Delta_Output_per_worker, 4)$		0.008	
		(0.064)	
$lag(\Delta_Employment_rate)$	-0.052	-0.081	-0.052
	(0.063)	(0.064)	(0.063)
$lag(\Delta_Employment_rate, 2)$	-0.002	-0.019	
	(0.064)	(0.065)	
Constant	-0.002^{***}	-0.002^{**}	-0.002^{***}
	(0.001)	(0.001)	(0.001)
Observations	255	253	255
R^2	0.064	0.065	0.064
Adjusted R^2	0.045	0.039	0.049
Residual Std. Error	0.008 (df = 249)	0.008 (df = 245)	0.008 (df = 250)
F Statistic	3.406^{***} (df = 5; 249)	2.447^{**} (df = 7; 245)	4.274^{***} (df = 4; 250)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 2.1: Ordinary least square regressions of changes to the employment rate [FRED: 1-UNRATE] on it's past values as well as past and present labour productivity [FRED: OPHNFB] measures.

For both the FS interpretation and the MP interpretation this means that increases in the permanent integrated component x lead to permanent increases in the level of consumption. This is straightforward for the FS model but less straightforward for the MP interpretation. To see this for the MP interpretation assume for simplicity that $\prod_{s=1}^{L} a_{t-s}^{\gamma_s} = a_{t-L}$. This is the case when all but one value of γ_j are 0, and serves purely for making the illustration below easier and less heavy on notation. Further define $P_t = \frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}$. Equation (2.19) shows that, while long-run unemployment has an equilibrium level in this model in contrast to the simple model in (Blanchard et al., 2013), the qualitative long-run outcome of consumption in both models is the same as long as $n^* + bu^* > \kappa v^*$ in the steady-state. This inequality has to be fulfilled for any reasonable calibration of the search and matching model as otherwise, the matching process would cost more resources than the economy produces. Absent any permanent productivity shocks, the value long-run consumption will converge to $c_\infty = (n^* + bu^* - \kappa v^*)x_\infty = (n^* + bu^* - \kappa v^*)(\frac{x_t - \rho x_{t-1}}{1-\rho})$.

The remaining equilibrium equations for the five variables: $c_t, n_t, u_t, \theta_t, v_t$ are:

$$c_t/a_{t-L} = p_t n_t - \kappa v_t + b(1 - n_t) \tag{2.20}$$

$$\frac{\kappa}{q(\theta_t)} = \pi(P_t - b) + \beta(1-\lambda)E_t[P_{t-L+1}\frac{c_{t+1}^{-\zeta}}{c_t^{-\zeta}}(\frac{\kappa}{q(\theta_{t+1})} - (1-\pi)\kappa\theta_{t+1})] \tag{2.21}$$

$$n_t = (1-\lambda)n_{t-1} + v_t q(\theta_t) \tag{2.22}$$

$$u_t = 1 - (1-\lambda)n_{t-1} \tag{2.23}$$

$$\theta_t = \frac{v_t}{u_t} \tag{2.24}$$

Labour productivity as a hiring driving force

A closer examination of these equations shows that their equilibrium path is fully determined by the parameters $\beta, \lambda, \kappa, \pi, b, \zeta$, the value of employment in the previous period n_{t-1}, and the $L-1$ past, the expected current, as well as the expected future values of the product of a job $a_{t-L+1}, ..., a_{t-1}, E_t a_t, E_t a_{t+1}, ...$ The model can be linearised (see Appendix 2.A) in the form where the optimal policy resulting in current labour market tightness is a function of the exogenous parameters (summarised by ψ_1 and psi_2) and the

expected path of future expected productivity and cost changes.

$$\hat{\theta}_t = \psi_1 = \psi_1 E_t \sum_{s=0}^{\infty} \psi_2^s \hat{P}_{t+s} \tag{2.25}$$

Here $\hat{P}_t$ is is the linearisation of $P_t = \frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}$ with all $\gamma_s = 0$ for the FS interpretation.

2.3.6 State space form

Combining the equations from Section 2.3.1, Section 2.3.5 and Appendix 2.A allows representing the linearised model in a state-space form. The model has 7+2(L-1) state variables ($n_{t-1}, x_{t-1|t-1}, x_{t-2|t-1}, z_{t-1|t-1}, x_{t-1}, x_{t-2}, z_{t-1}, ..., x_{t-L}, z_{t-L}$) and one endogenous choice variable (θ).

$$AE_t \begin{bmatrix} \theta_{t+1} \\ n_t \\ x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \\ x_t \\ x_{t-1} \\ z_t \\ \dots \\ x_{t-L+1} \\ z_{t-L+1} \end{bmatrix} = B \begin{bmatrix} \theta_t \\ n_{t-1} \\ x_{t-1|t} \\ x_{t-2|t} \\ z_{t-1|t} \\ x_{t-1} \\ x_{t-2} \\ z_{t-1} \\ \dots \\ x_{t-L} \\ z_{t-L} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \eta_t \\ \nu_t \end{bmatrix} \tag{2.26}$$

The first two rows and columns of matrices A and B are given by the approximated market clearing equations. Meanwhile, the lower right parts of the matrices are a combination of the shocks driving the actual productivity series, which in turn together with the noise shock define the expectations about future productivity growth via the agent Kalman filter. The values in this lower part will be a function of the parameters of the productivity and noise process σ_ϵ, σ_ν, ρ^x and ρ^z. Estimating these parameters is the central aim of this model. As an example, in the case where $L = 1$ the number of state variables collapses to 7. The concrete implementation of this model is described in Appendix 2.E.

2.4 Empirical evaluation of the model

The first subsection of this section describes the data used to empirically evaluate the model, while the subsequent subsections present calibration, simulation, and estimation results.

2.4.1 Data

The parameters defining the variance of news σ_ϵ and noise σ_ν in the labour market as well as the persistence of shocks ρ are estimated below by maximizing the likelihood given that the data was produced by a model with these specifications. An advantage of this empirical strategy is that it is not necessary for the econometrician to find a series that is informative about the signal that agents receive about future productivity. Instead one can base the estimation of the noise term on the observed outcomes in the labour market. Given that the aim of the model is to estimate the extent to which news and noise on future worker productivity influences labour market decision making, the parameters will only be identified if the model is estimated based on a series that informs about the productivity developments and a series that informs about aggregate labour market outcomes following these productivity changes. If worker productivity changes precede and accompany hiring then these movements in the labour market series are best explained as the result of surprise or news shocks. On the other hand, if workers are hired while their productivity in comparison to the outside value of employment remains unchanged, then these hires are most likely the result of noise. For the productivity series, changes in real output per worker is chosen. For labour market outcomes, both the job-finding rate ($p(\theta_t)$), and the unemployment rate may be good variables for identifying the parameters of the shocks. Estimating based on unemployment outcomes has the advantage that the series is directly observed and the potential measurement error is therefore less. On the other hand, estimating a perturbed model based on transition probabilities has the advantage of ensuring that the probabilities are within the bounds 0 and 1, as required by the theoretical setup of the model. However, transition probabilities are not directly observed and have to be constructed which may lead to a bias or a larger measurement error of the series. As there are advantages and disadvantages to either series the model is estimated alternatively based on unemployment, based on the job-finding rate.

The concrete series for labour productivity changes used is the output per worker series

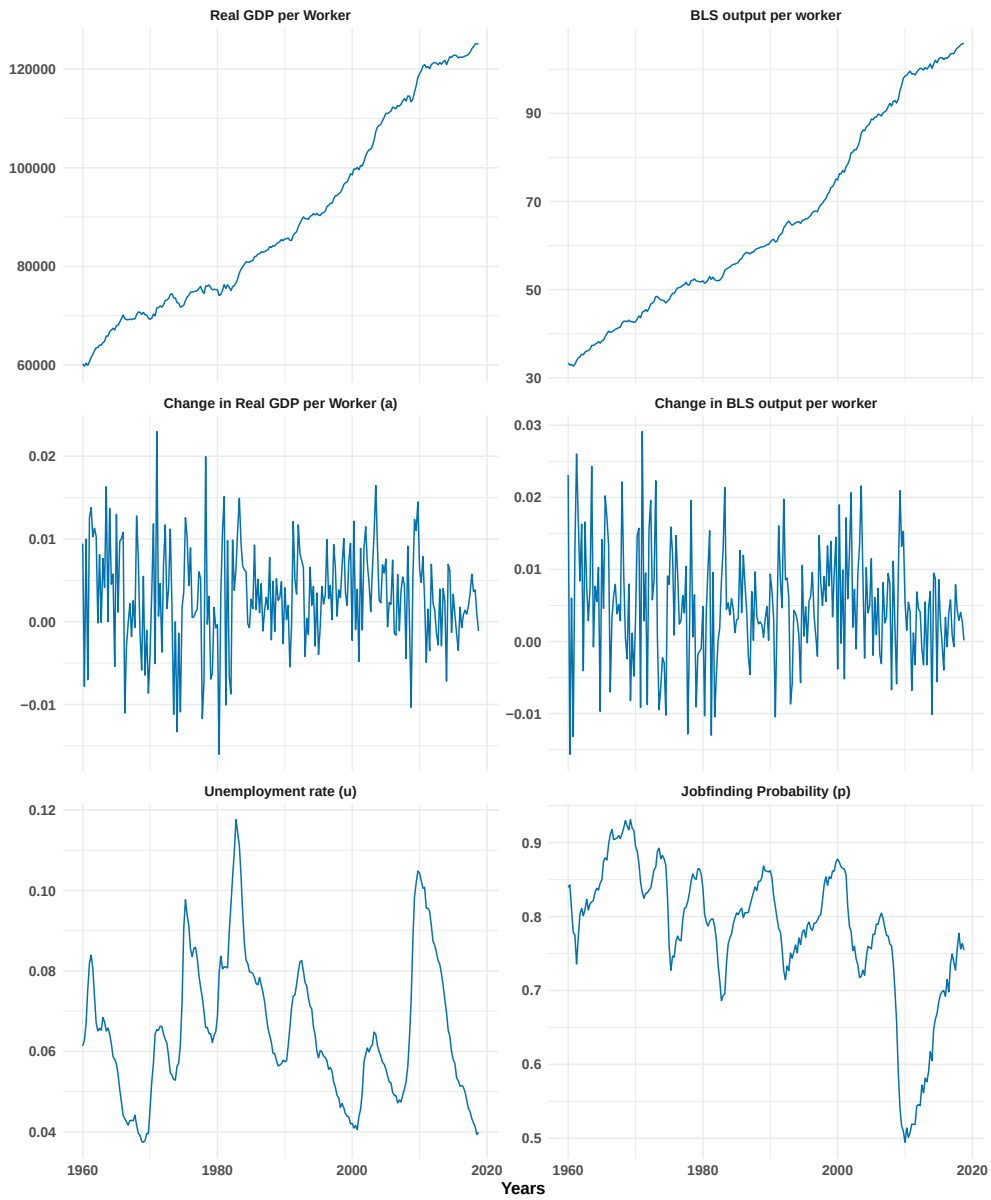


Figure 2.3: Data series used in the maximum likelihood estimation of the DSGE model. Source: Federal Reserve Economic Data (FRED)

computed by the BLS with the FRED code OPHNFB. The series is similar to the raw real output per worker series as can be seen in Figure 2.3. The unemployment rate is taken from the data, and the job-finding probability p is computed from monthly data and then aggregated to quarterly data following (Shimer, 2005a). Further descriptions and discussions about the data used can be found Appendix 2.B.

2.4.2 Calibration of the model

The model is calibrated to reflect employment and job-finding rates in the US labour market. Based on monthly data between 1949 and the end of 2019 the average unemployment rate for the US was 6.5%. The average monthly job-finding probability was

0.41 (see section 4.1). If we assume that the mean is a close reflection of the steady-state equilibrium rate then it follows that $u = \frac{\lambda}{\lambda+p(\theta)}$, and that 0.0262 is a good guess for the monthly separation rate - a value that is in agreement with (Hagedorn and Manovskii, 2008). The quarterly job-finding rate is then $1-(1-0.41)^3 = 0.79$. Taking the assumed law of transition in the model, this corresponds to a quarterly separation rate λ of 0.269. Thus about half of the jobs are created only in the past six months. This may seem like a high value, but under the model assumptions, 81% of the workers are rematched within the same period in steady-state and are thus productive. The 26.9 % destroyed jobs, therefore, include job to job transitions. The observed quarterly flow from employment to unemployment is only 5%, which is broadly in line with the empirically estimated values found in the literature. (Den Haan, Ramey, and Watson, 2000) find that the quarterly vacancy filling rate for the second quarter of 1972 to the 4th quarter 1988 was 0.71. The quarterly job-finding rate for this sample was on average 0.81. The values can therefore be assumed to be close to the steady-state. steady-state labour market tightness is then estimated to be $\theta = \frac{p(\theta)}{q(\theta)} = 1.141$.

Calculating the rate of vacancy postings as a proportion of the labour force in as measured by the JOLTS for the available sample (December 2000 - December 2017) yields 0.02647. Dividing this by the estimate for the unemployment rate for the sample yields a labour market tightness of 0.425. Dividing the sample job-finding rate 0.328 by this result yields a monthly vacancy filling rate of 0.77. The quarterly job-finding rate is then 0.696 and the vacancy rate 0.988. Given the assumption of the matching function one can then estimate the parameters of the constant elasticities of the transition functions as $\frac{p(\theta)-p(\theta')}{q(\theta')-q(\theta)}\theta \approx \frac{1-\xi}{\xi}$ Then the estimate for the matching elasticity is $\xi \approx 0.7356$. This value is close to the value used in (Shimer, 2005a), which is 0.7. I choose 0.7 as a reasonable parameter for the elasticity of the vacancy filling. Based on this result the estimated value of the matching productivity parameter is $m = 0.695$. (Hagedorn and Manovskii, 2008) estimate that the cost of posting a vacancy costs 47.4% of the weekly product of labour. The quarterly product of labour in the steady-state is normalized to 1 in the model. This means a cost of vacancy posting κ of 0.0362. Given this, the value of home production should be set to $b = 0.8825$ to match the unemployment moment. Worker bargaining power is chosen such that it is socially efficient, which requires that it fulfils the Hosios condition ((Hosios, 1990)) $\xi = 1-\pi$. For robustness, I employ two different calibrations which are matching the labour market moments equally well. The first is the (Shimer, 2005a) calibration which states $b = 0.4$ and uses chooses a value of κ to match unemployment. In

Parameter	Value	Description
β	0.99	Discount rate.
α	0.66	Labour share.
ζ	1	Inter-temporal elasticity of substitution
m	0.77	Parameter of the matching function (match productivity)
κ	see Table 2.3	Cost of posting vacancies for firms.
b	see Table 2.3	Replacement rate for unemployed.
ξ	0.7	Parameter determining matching frictions, following (Lubik, 2009) and (Petrongolo and Pissarides, 2001)
λ	0.269	Chance of exogenous match separation.
ρ	0.9	Chance of exogenous match separation.
σ_ϵ^2	0.0001	Volatility of the permanent component.
σ_ν^2	0.0121	Volatility of noise.

Table 2.2: Parameters for the simulation.

Replacement rate b			Vacancy posting cost κ		
Baseline	Shimer	Intermediate	Baseline	Shimer	Intermediate
0.8825	0.4	0.6	0.0365	0.144	0.1

Table 2.3: The parameters varied for robustness for the simulated impulse response.

this case the appropriate value of $\kappa = 0.144$. Finally, I employ an intermediate calibration between these two extremes with the replacement rate $b = 0.6$ and the vacancy posting cost $\kappa = 0.1$. These three calibrations are all able to produce a mean unemployment rate of 6.5% and a mean job-finding rate of 0.79. The calibration is summarised in Table 2.2 and Table 2.3.

(Lubik, 2009)). However, the chosen matching elasticity of 0.7 is on the upper end of the matching elasticities suggested by (Petrongolo and Pissarides, 2001)

2.4.3 Simulated impulse response

First, it is important to note that a shock to the permanent component ϵ, a shock to the temporary component η, and a noise shock ν will have different impacts on current labour productivity a_t as well as actual a_{t+s} and expected $E_t(a_{t+s})$ paths of future labour productivity. Figure 2.4 plots the actual path of labour productivity a_{t+s} in black. Note that a positive shock to the permanent component ϵ will permanently lift $a = x + z$ as x is permanently increased, capturing depending on the interpretation increases in the match surplus or match productivity. Meanwhile, a shock η lifting z will only have a transitory effect, while a noise shock ν will leave actual a unchanged. The share of noise σ_ν^2 in the signal is varied between half and four times its baseline value with lighter blue lines representing larger values of σ_ν^2. The dashed lines show the expectation of long-run productivity $E_t(a_{t+\infty})$. Note that a shock to ϵ with higher noise will mean expectations are

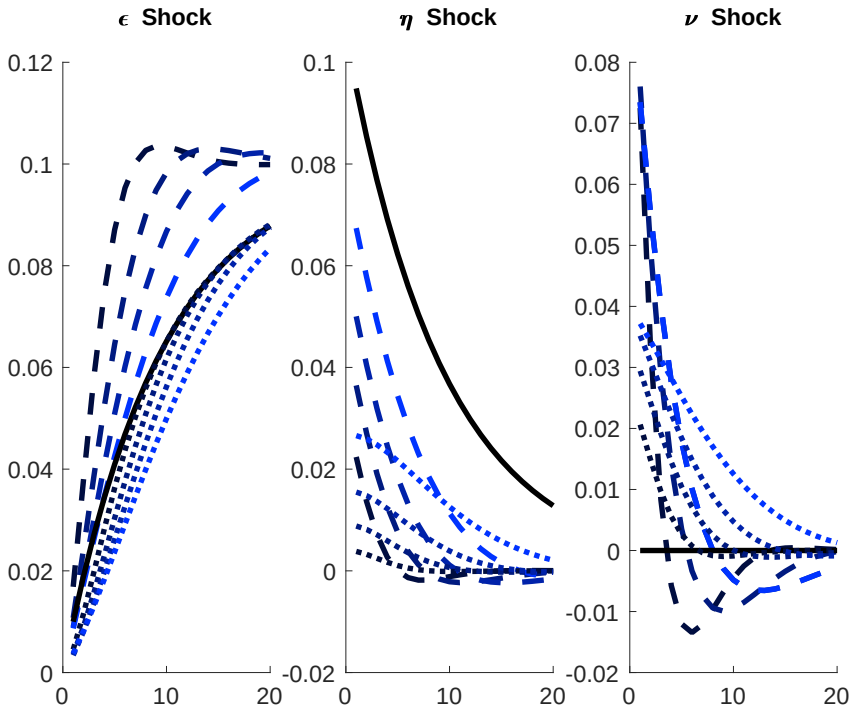


Figure 2.4: Comparison of a different strength of noise in the signal σ_ν^2 on the impulse response of actual a_t (black and solid), expected permanent productivity $x_{t|t}$ (dotted) and long-run expected productivity $E_t(a_{t+\infty})$ (dashed). Series with lighter blue represent larger values of σ_ν keeping σ_ϵ fixed.

slower upward adjusted. An η shock also means positive future expectations of $E_t(a_{t+\infty})$ especially when the noise is large. This is because a and the signal are the two observable series in the agent Kalman filter. When a rises due to an η shock agents may falsely conclude that the permanent component x has risen spelling a period of labour productivity growth rather than decline back to the original level. The noise shock leads to agents adjusting their expectations of long-run productivity rapidly upwards when noise is large and keeping them elevated. Finally, the dashed line shows the expectation of the current permanent component $x_{t|t}$. This is one of the state variables in the model. The simulations shock that a larger noise component will lead to a slower adjustment of expectations $x_{t|t}$ to actual x when it actually is shocked, and a larger falsely expected rise $x_{t|t}$ when it remains unchanged.

Figure 2.5 shows that a higher noise component will lead to a less aggressive hiring response to a rise in the permanent component x due to an ϵ shock in the MP version. Notably, employment will return to a long-run equilibrium following a shock. This is different in the FS version where an increase in the permanent component leads to a permanent increase in match surplus. This version is presented in Appendix 2.C. The employment response to shocks to the temporary component is very similar across noise

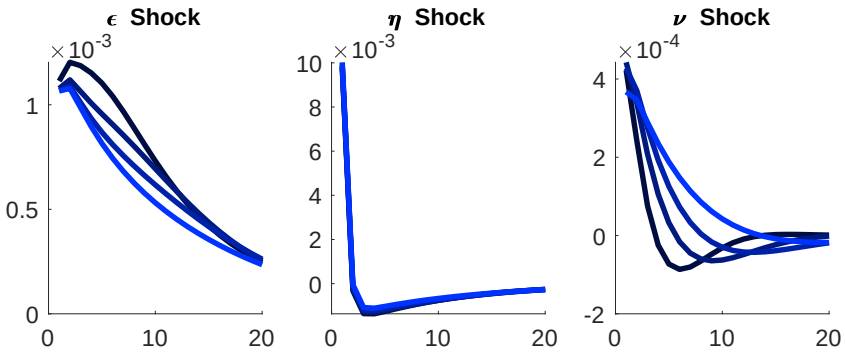


Figure 2.5: Comparison of a different strength of noise on the employment rate n with darker blue representing a smaller noise component for the MP interpretation.

levels. Meanwhile, Figure 2.5 shows that a higher noise component can significantly drive the hiring rate with noise shocks potentially leading to a significant contribution in the short run variance of the employment rate. The maximum likelihood estimation makes use of these different responses of productivity and employment rates in Figure 2.4 and Figure 2.5 to estimate the extent to which noise plays a role in aggregate job creation.

The impulse response functions of other variables of the model have been simulated with the parameters in Table 2.2, which result in an equilibrium employment rate of 93.5% in equilibrium. Similar to (Den Haan et al., 2000) the payments necessary to create a new job (κ) are interpreted as investment. The Figures below show the impulse response functions of labour productivity (a), employment (n_t), the job-finding rate ($p(\theta_t)$), consumption (c_t), labour market tightness (θ_t), and investment ($\kappa_t v_t$) for the different type of shocks employing the MP interpretations. Figures with the FS interpretation can be found in Appendix 2.C. The different colours represent the three different calibrations with the main calibration being blue, the calibration following (Shimer, 2005a) in green and the intermediate calibration in orange. The variance of the permanent component and the noise shock have been set to equal levels.

Figure 2.6 shows the impulse response function to a 1% shock to the permanent component x_t, while Figure 2.7 shows the same for z_t and Figure 2.8 for s_t as a result of a noise shock ν_t. The top graph shows the FS implementation, while the bottom the MP implementation. Both are estimated in Section 2.4.4.

A shock ϵ to the permanent component x of a

Figure 2.6 shows that a shock to the permanent component resulting in positive news about current and future productivity growth can capture the Pigou cycles in response to

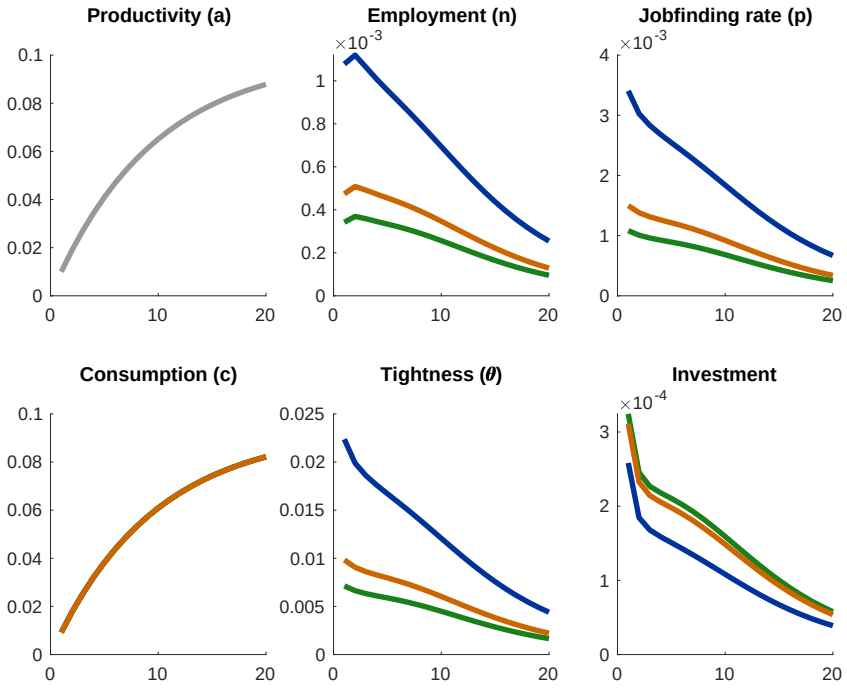


Figure 2.6: Responses to a shock to the permanent component (ϵ) in the MP implementation of the model. The baseline calibration is in blue, while the Shimer calibration is green, and the intermediate calibration is in orange.

news shocks found in the data. As a is rising employment and job creation and investment shares will temporarily be elevated, while total consumption will increase permanently. The baseline calibration will lead to a stronger response due to the higher responsiveness of the match surplus to shocks as would be expected given the discussion in (Ljungqvist and Sargent, 2017). The reason for this responsiveness is that the steady-state match surplus is smaller in the baseline calibration than in the Shimer or intermediate calibration. The increase in unemployment as growth stagnates produced by this model is coherent with the empirical observations that led to the formulation of Okun's law ((Okun, 1963)). In this process of declining employment, productivity growth will stagnate, but consumption growth will continue due to marginal increases in productivity and due to the decrease in investment and as fewer vacancies are being posted.

A shock η to the temporary component z of a

With regard to a shock to the temporary component z_t in period 0, a similar forces are at work as shown in Figure 2.7. In period 0 a surprise shock increases worker productivity and thereby output and consumption. Given the higher observed productivity value, agents form expectations over the permanent component and therefore the long-run value of a filled vacancy. Even though agents receive no signal that the permanent

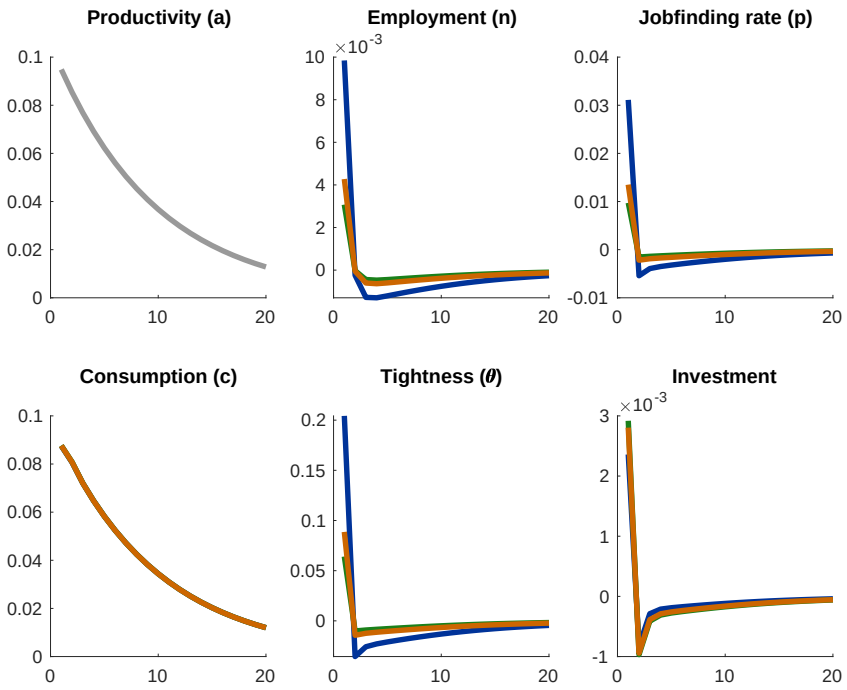


Figure 2.7: Responses to a shock to the temporary component (η) in the MP implementation of the model. The baseline calibration is in blue, while the Shimer calibration is green, and the intermediate calibration is in orange.

component has increased, they assume that the signal might have been drowned out by noise especially when the noise component is large. This may lead to elevated hiring beyond the initial period as long-run productivity expectations rise. In the MP implementation, the catch-up of the cost of posting a vacancy and the replacement rate will lead to the value of a match rapidly declining making the rise in employment in response to the shock brief. In the FS implementation in Appendix 2.C the temporary shock will lead to a temporary increase in the match surplus making the response similar to the responses to a permanent shock in Figure 2.6 in the MP implementation. Note that with higher noise it becomes harder and harder for firms to separate the temporary from the permanent increase, which would lead to both impulse responses becoming flatter.

A shock ν to the noise in signal s

Figure 2.8 finally shows the impulse response functions caused by a noise shock. Actual worker productivity remains flat, but the noise shifts the expectation over the size of the long-run component x_t. The reason that agents do not observe the supposed shock to x_t in an increase in a_t could be due to a negative shock to the temporary component η_t occurring at the same time. Consequently, due to the change in expectations about the growth in the product that a worker-firm relationship will produce the firm's management will

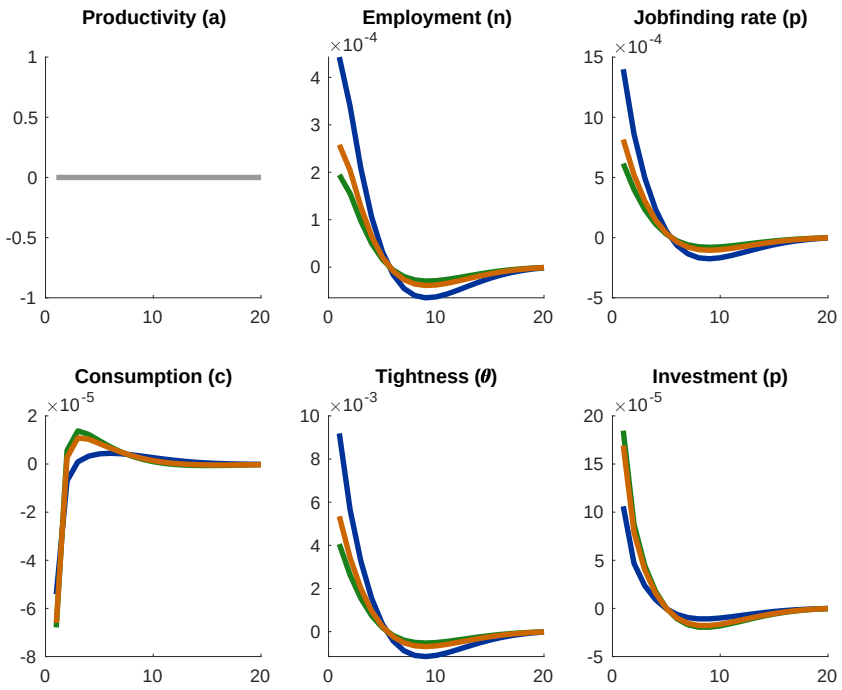


Figure 2.8: Responses to a shock to the noise shock (ν) in the MP implementation of the model. The baseline calibration is in blue, while the Shimer calibration is green, and the intermediate calibration is in orange.

increase job postings. This will lead to a positive response of employment, output, investment and the job-finding rate in the period of the noise shock. Consumption will fall in the very short-run. The reason for this is a combination of efficient matching as the Hosios condition is fulfilled, and the increase in investment necessary to create the new jobs. As agents become consequently more certain in the following periods that the shock was noise, and the expected value of long-run productivity even falls below the equilibrium value before returning to equilibrium, firms drastically reduce hiring leading to a decrease in employment and an increase in consumption.

Interestingly, for both the MP and FS implementation (Appendix 2.C) the reaction to a noise shock are similar. This is because the economy actually does not move in terms of fundamentals from its current state in either case and hiring and consumption movements are due to information misleading agents.

Variance decomposition of the simulation

The variance decomposition in Table 2.4 shows that under the baseline calibration noise may contribute substantially to employment fluctuations due to expected improvements in labour productivity in the short-run. The impact of noise is stronger in calibrations with lower b and higher κ as a higher cost of posting vacancies will increase the benefit

Calibration	*Quarters*	1	2	3	4
	Match productivity interpretation				
Baseline	Noise (ν)	14.4300 %	8.38 %	3.68 %	1.08 %
	Permanent (e)	85.57 %	91.62 %	96.32 %	98.92 %
Intermediate	Noise (ν)	22.84 %	13.86 %	6.26 %	1.87 %
	Permanent (e)	77.16 %	86.14 %	93.74 %	98.13 %
Shimer	Noise (ν)	24.47 %	14.99 %	6.82 %	2.05 %
	Permanent (e)	75.53 %	85.01 %	93.18 %	97.95 %

Table 2.4: Relative variance decomposition of noise and expected permanent labour productivity or fundamental surplus improvements with regard to the employment rate based on a simulation with a large noise impact.

of firms searching before expected labour productivity increases are realised. A quicker catching up and reduction of the fundamental surplus will lead to noise becoming a more substantial driver in the short-run for the same reasons. Noise has been chosen to have close to maximal impact in this calibration at $\sigma_\nu^2 = 0.0121$, which is achieved at $\frac{1-\rho}{\sqrt{\rho}}$. The simulation shows that noise can in the short run be a substantial driver of job creation. Under the MP interpretation with a long-run unemployment equilibrium noise contributes up to 24.47% to the short-run variation of employment relative to news from ϵ shocks. Noise impacts may even be higher if the assumption about current productivity knowledge is dropped and the signal and past observations determine current productivity expectations and firms hire based on these expectations. These simulations and the variance decomposition including the temporary shock can be found in Appendix 2.C.

2.4.4 Estimation of the model

The maximum likelihood estimation of the model is based on the demeaned productivity series for output per worker, and the unemployment or job-finding rate as described in Section 2.4.1. Even though there are three innovations using the Kalman smoother estimation will only be successful based on two observable series at once as there are only two independent innovations in the form of productivity and noise shocks due to the parameter restrictions on the temporary and permanent components z and x. Only the parameters of the news and noise processes are estimated, while the labour market parameters are set via the calibration above. Two series cannot be expected to contain enough information to identify parameters such as the matching cost κ or b together with the shock parameters. For this reason, instead the three calibrations of the model presented in the simulation are estimated. The noise component around x is estimated both for the FS and MP implementation.

Figure 2.9 shows the results of the maximum likelihood estimation for the MP imple-

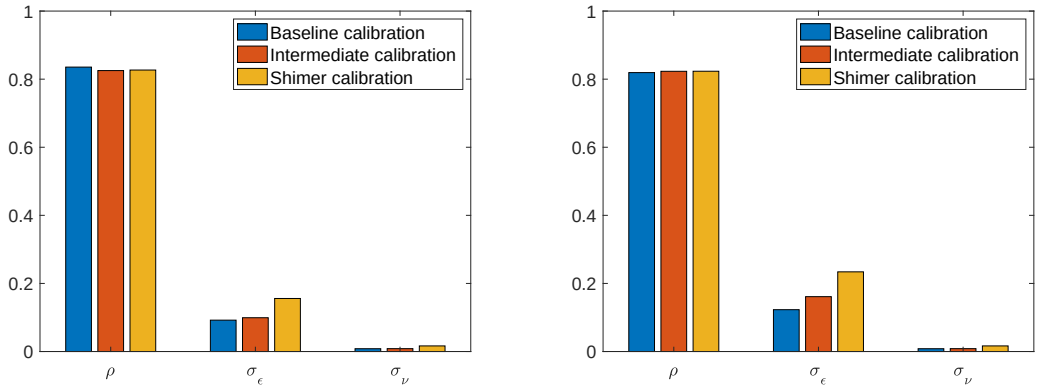


Figure 2.9: Results of the maximum likelihood estimation of the model in the MP implementation. Estimation results based on the unemployment rate are on the left, while estimation based on the job-finding rate are on the right.

Parameter estimated:	MP (Δa_t & Δu_t)	MP (Δa_t & $\Delta p(\theta_t)$)
ρ	0.8252 [72.5]	0.8193 [73.8]
σ_e	0.0922 [19.4]	0.1229 [23.7]
σ_ν	0.0083 [2.1]	0.0083 [0.6]

Table 2.5: Baseline estimated news and noise parameters by implementation with the series based on which the ML estimation was done in round brackets and the t-values in square brackets.

mentation. The results of the FS implementation of the estimation are in Appendix 2.C. Table 2.5 shows the results and table Table 2.6 shows the corresponding variance decomposition for the estimation based on unemployment changes. The estimations of both implementations merely differ in terms of the size of volatility of σ_ϵ, which is to be expected as a calibration with lower b requires a higher volatility of structural shocks (Shimer puzzle). Noise is much smaller than σ_ϵ in any of the estimations meaning it is from every perspective and with every calibration an unlikely driver of employment changes and job creation. This holds true both in the FS implementation and in the MP implementation.

The estimations suggest that on the occasions on which agents receive information about the future, the information is usually precise. Thus, agents react rapidly to changes in the expected future productivity of a worker-firm relationship. There is little to no noise in the signal on future worker productivity and barring surprise productivity shocks, the output that a worker will contribute to the economy is close to perfectly anticipated

Calibration	*Quarters*	1	2	3	4
	Match productivity interpretation				
Baseline	Noise (ν)	0.1 %	0.05 %	0 %	0 %
	Permanent (e)	99.9 %	99.95 %	100 %	100 %

Table 2.6: Estimation based of relative variance decomposition of noise and expected permanent labour productivity or fundamental surplus improvements with regard to the employment rate.

by agents. Fluctuations in employment are found to be likely the result of news and surprise shocks with noise shocks playing only a limited or no role in the labour market. It is thereby shown to be unlikely that spontaneous changes in labour demand are a significant driver of aggregate job creation.

2.5 A structural VAR estimation of noise in job creation

(Forni et al., 2017) suggest alterations to the assumptions taken in (Blanchard et al., 2013) to study the reaction of consumption to noise and news in a VAR model. If these assumptions hold, then it is possible to identify noise shocks with a structural VAR model. The proposed identification is implemented in this section as a method of further examining the plausibility of the results of the DSGE model present in previous sections. The most important change in the assumptions in (Forni et al., 2017) is that agents learn with certainty after a number of periods whether a past signal was news or noise. In contrast, in (Blanchard et al., 2013) agents only ever know with an increasing probability the true nature of past shocks. If agents are able to retrospectively identify noise shocks, then the econometrician will also be able to identify the shocks from the data reflecting the choices of the agents, provided that some reliable instrument for the signal based on which agent expectations are formed is available to the econometrician.

2.5.1 Relating the VAR model to the DSGE model

The process determining worker productivity is assumed to be a random walk with a drift τ.

$$a_t = a_{t-1} + \tau + \epsilon_{t-S} \tag{2.27}$$

ϵ_{t-S} is a news shock determined by a finite number of periods S in the past.

This process is related but not the same as the productivity process described in Section 2.3.1, where $a_t = x_t + z_t = (1+\rho)x_{t-1} - \rho x_{t-2} + \rho z_{t-1} + e_t + \eta_t = \rho a_{t-1} + x_{t-1} - \rho x_{t-2} + e_t + \eta_t$. If ρ is either 0 or 1 and in the second case the permanent component is not subject to shocks then the properties of the two processes are similar and the only difference is the timing of the signal. In the first case the result would be $a_t = x_t = x_{t-1} + \epsilon_{t-S}$, where $e_t + \eta_t = \epsilon_{t-S}$. In the second case $a_t = a_{t-1} + \eta_t = \epsilon_{t-S} = L^S \epsilon_t$.

As in Section 2.3.1 in each period agents are assumed to observe worker productivity a_t

and to receive a noisy signal over future innovations as shown in equation (2.28).

$$s_t = \epsilon_t + \nu_t \tag{2.28}$$

There is assumed to exist a cointegrated relationship between the present discounted utility value of a filled vacancy and the value of unemployment. This renders the relative value of a filled vacancy P_t a stationary process, which is assumed to be the de-trended process a_t.

$$\Delta a_t = \Delta a_t - \tau = L^S \epsilon_t \tag{2.29}$$

The agents are assumed to know that both ϵ_t and ν_t are mean zero normally distributed and uncorrelated with each other and with previous and future realizations. The future expected values of productivity are then simply projections from e_{t-S} on s_{t-S}. Thus if I_t is the information set of agents at period t then productivity changes in the future can be extracted from the signal according to equation (2.30).

$$E(\Delta P_{t+1}|I_t) = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\nu^2} s_{t-S} \tag{2.30}$$

It follows that the expected long-run change in the value of productivity is the sum of current productivity and the projections from the at time t available signals on future productivity shocks. This is described by equation (2.31).

$$E(P_{t+\infty} - P_t|I_t) = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\nu^2} \sum_{i=0}^{S} s_{t-i} \tag{2.31}$$

Given the equilibrium described in Section 2.3.5 and the state space form discussed in Section 2.3.6 it becomes straightforward to form equations for approximating the labour market described above. Both labour market tightness and the job-finding rate will be a function of the equilibrium value plus deviations to the employment equilibrium in the previous period plus expected deviations of the relative value of the firm-worker relationship, plus an error term to capture exogenous shocks such as shocks to matching

productivity.

$$\theta_t = \theta^* + \phi_{0,1}(n^* - n_{t-1}) + E_t \sum_{i=0}^{\infty} \phi_{0,2+i} \Delta p_{t+1+i} + e_{3,t} \tag{2.32}$$

Labour market tightness can be proxied for by the observed job-finding rate.

$$p(\theta_t) = p(\theta^*) + n^* + \phi_{1,1}(n^* - n_{t-1}) + E_t \sum_{i=0}^{\infty} \phi_{1,2+i} \Delta p_{t+1+i} + e_{3,t} \tag{2.33}$$

Finally using equation (2.34) in equation (2.33).

$$p(\theta_t) = p(\theta^*) + n^* + \phi_{1,1}(n^* - n_{t-1}) + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\nu^2} \sum_{i=0}^{S} \phi_{1,2+i} L^i s_t \tag{2.34}$$

Here θ^* and n^* are constants and expected to be the steady-state values when no productivity shocks occur. $e_{3,t}$ captures other shocks to the job-finding rate such as shocks to matching productivity. Finally, the law of motion of employment, or of unemployment can be captured by the equation (2.35).

$$n_t = n^* + \phi_{2,1}(n^* - n_{t-1}) + \phi_{2,2}(p(\theta^*) - p(\theta_t)) + e_{4,t} \tag{2.35}$$

The structural VAR system is then found in equation (2.36).

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{-\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_\nu^2} \sum_{i=0}^{S} \phi_{1,2+i} L^i & 1 & 0 \\ 0 & 0 & \phi_{2,2} & 1 \end{bmatrix} \begin{bmatrix} \Delta P_t \\ s_t \\ p(\theta_t) \\ n_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ p(\theta^*) + \phi_{1,1} n^* \\ (1+\phi_{2,1}) n^* + \phi_{2,2} p(\theta^*) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_{1,1} \\ 0 & 0 & 0 & -\phi_{2,1} \end{bmatrix} \begin{bmatrix} \Delta P_{t-1} \\ s_{t-1} \\ p(\theta_{t-1}) \\ n_{t-1} \end{bmatrix} + \begin{bmatrix} L^S & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \nu_t \\ e_{3,t} \\ e_{4,t} \end{bmatrix} \tag{2.36}$$

This model has a clear ordering. The job-finding rate will affect employment contemporaneously, but not vice versa. This ordering is used in the short-run restrictions imposed in Section 2.5.2.

2.5.2 Identification and estimation of the effect of noise

To estimate the model the difference of the two labour market series $p(\theta_t)$ and (n_t) is taken. Further, an instrument is chosen to measure the signal that is not observed directly. (Barsky and Sims, 2012) have suggested using the series in the Surveys of Consumers by

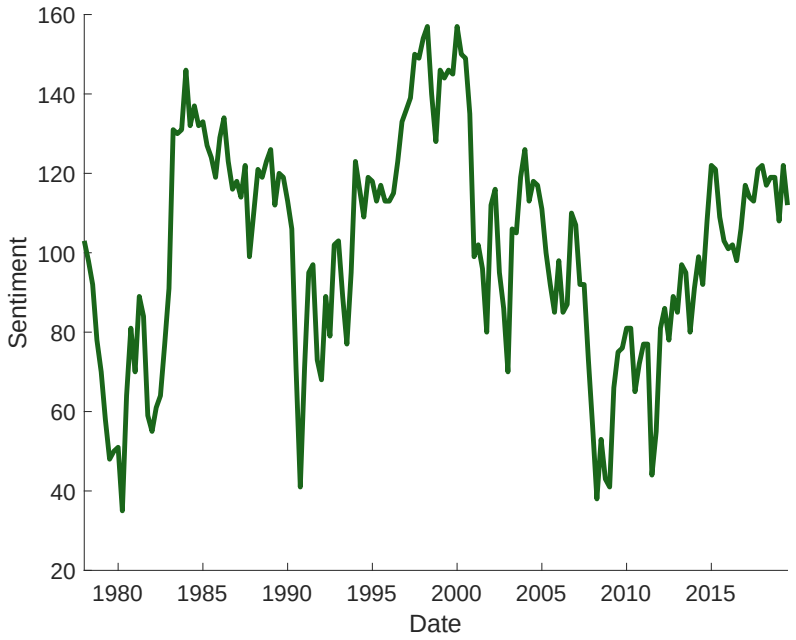


Figure 2.10: Surveys of Consumers by the University of Michigan index capturing business conditions for the next year.

the University of Michigan as instrument for future productivity expectations. The paper follows this suggestion with the chosen instrument z_t being the expectation of business conditions in a year from the Surveys of Consumers. The original question for the data series is "*And how about a year from now, do you expect that in the country as a whole, business conditions will be better, or worse than they are at present, or just about the same?*". Given this question, it is reasonable to assume that S=4, thus the measured signal informs about the news shock four quarters ahead. The signal series is shown in Figure 2.10. The series for output per worker, the job-finding rate and the employment rate are the same as the ones used for the estimation of the DSGE model and described in Section 2.4.1.

$$\begin{bmatrix} \Delta P_t \\ z_t \\ \Delta p(\theta_t) \\ \Delta n_t \end{bmatrix} = \sum_{i=1}^{4} A_i \begin{bmatrix} \Delta P_{t-i} \\ z_{t-i} \\ \Delta p(\theta_{t-i}) \\ \Delta n_{t-i} \end{bmatrix} + \begin{bmatrix} u_t \\ s_t \\ x_{3,t} \\ x_{4,t} \end{bmatrix} \tag{2.37}$$

The empirical strategy for identifying noise follows (Forni et al., 2017). It is assumed that there exists a fundamental representation of the system and a simple VAR with a four period lag is run to find the reduced form estimates as shown in equation (2.37). Here u_t is a surprise shock. s_t is a change in the signal. Meanwhile x_3 and x_4 are separate shocks that may also affect the observed variables and may be a combination of the shocks e_3 and

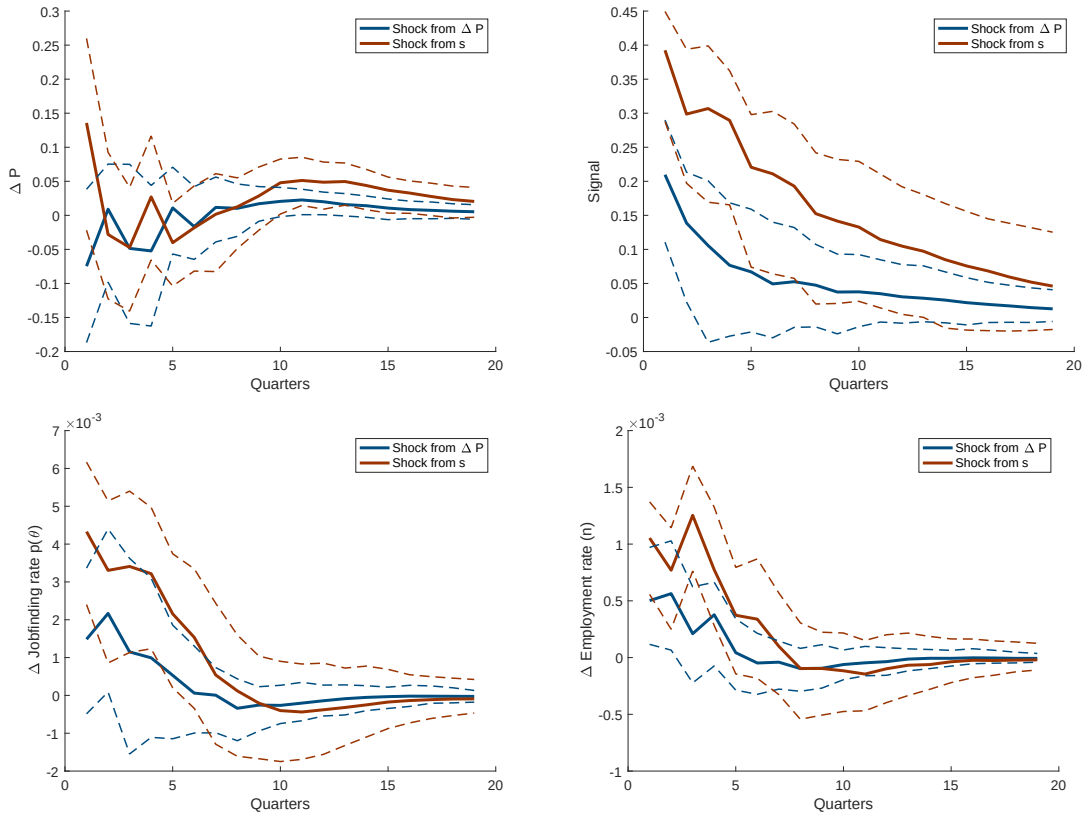


Figure 2.11: Impulse responses to one standard deviation shocks to productivity (blue) and the signal (red) from the quarter following the shock on the change in productivity, the signal, and the changes in the job finding and employment rate. Dashed lines show 95 % confidence intervals.

e_4 described above. The moving average representation is then computed as in equation (2.38) and impulse response functions of the model are computed by imposing short-run restrictions keeping the original ordering.

$$\begin{bmatrix} \Delta p_t \\ z_t \\ p(\theta_t) \\ n_t \end{bmatrix} = \sum_{i=0}^{\infty} \tilde{A}_i L^i \begin{bmatrix} u_t \\ s_t \\ x_{3,t} \\ x_{4,t} \end{bmatrix} \tag{2.38}$$

The impulse response functions of the model with short-run restrictions are are shown in Section 2.5.2 from the first period following the shock. As we would hope the signal predates the change in productivity and is briefly followed by a likely change in ΔP. Both positive shocks also lead to increases in the signal series, as well as increases in the job-finding rate and the employment rate as would be expected. Note that the signal shock seems to be having a stronger effect suggesting a possibly strong news or noise component in hiring.

(Forni et al., 2017) suggest that the structural shocks ϵ_t and v_t for fundamental news and non-fundamental noise can be recovered from the shocks of the reduced form representation u_t and s_t by using the structural assumptions made regarding the signal and rotating the VAR residuals to productivity and the signal instrument accordingly. Naturally, this method assumes that the structural assumptions in Section 2.5.1 correctly describe the actual process. The relation between the structural shocks and the fundamental shocks is described by equation (2.39).

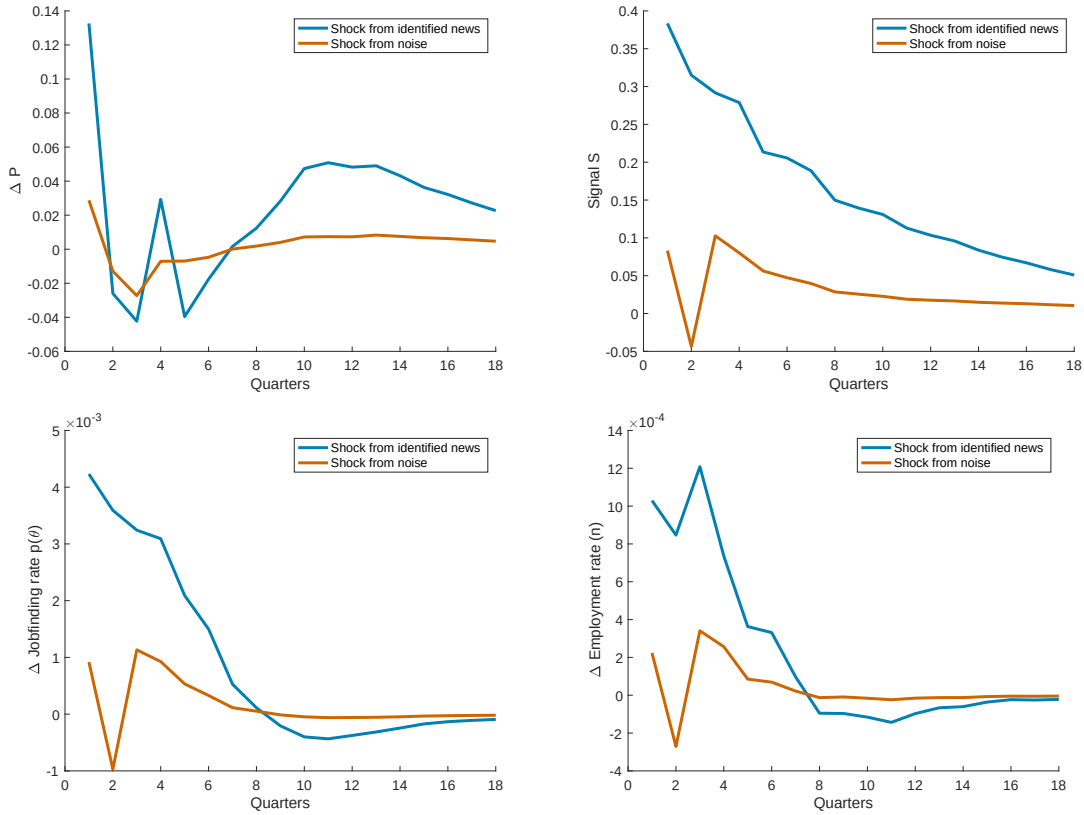
$$\begin{bmatrix} \epsilon_t \\ \nu_t \end{bmatrix} = \begin{bmatrix} b(L)\frac{\sigma_\nu}{\sigma_s} & -b(L)\frac{\sigma_\epsilon}{\sigma_s} \\ \frac{\sigma_\epsilon}{\sigma_s} & \frac{\sigma_\nu}{\sigma_s} \end{bmatrix} \begin{bmatrix} u_t \\ s_t \end{bmatrix} \tag{2.39}$$

Here $b(L)$ is computed from the roots within the unit circle smaller than one in modulus of the polynomial estimate of the second column in the first row of $\tilde{A}$ described in detail (Forni et al., 2017). σ_s is the result of the volatility of the signal with $\sigma_s^2 = \sigma_\epsilon^2 + \sigma_\nu^2$. An estimation of $\frac{\sigma_\epsilon}{\sigma_s}$ and $\frac{\sigma_\nu}{\sigma_s}$ is achieved in the following way. As explained in (Forni et al., 2017) an estimate of $\frac{\hat{\sigma}_\epsilon}{\hat{\sigma}_\nu}$ can be gotten from dividing the estimated responses of the productivity series to surprise and signal shocks at the assumed lag length by each other. Thus $\frac{\hat{\sigma}_\epsilon}{\hat{\sigma}_\nu} = \tilde{B}_{1,2}/\hat{B}_{1,1}$, where $\tilde{B} = \tilde{A}_5$ to account for the four period lag between the signal and the quarter predicted and the subscripts of $tildeB$ capture the matrix elements. Due to the trigonometric relationship $\frac{\sigma_\epsilon^2}{\sigma_s^2} + \frac{\sigma_\nu^2}{\sigma_s^2} = 1$ The estimates for are then $\frac{\sigma_\epsilon}{\sigma_s} = sin(arctan(\frac{\hat{\sigma}_\epsilon}{\hat{\sigma}_\nu}))$ and $\frac{\sigma_\nu}{\sigma_s} = cos(arctan(\frac{\hat{\sigma}_\epsilon}{\hat{\sigma}_\nu}))$. The results in the current case are $\frac{\hat{\sigma}_\epsilon}{\hat{\sigma}_\nu} = 0.977$ and $\frac{\hat{\sigma}_\epsilon}{\hat{\sigma}_\nu} = 0.212$. Finally, (Forni et al., 2017) suggest to transform the fundamental representation to the structural representation by post-multiplying the MA process in equation (2.38) with the matrix C in equation (2.40).

$$C = \begin{bmatrix} b(L)\frac{\sigma_\nu}{\sigma_s} & -b(L)\frac{\sigma_\epsilon}{\sigma_s} & 0 & 0 \\ \frac{\sigma_\epsilon}{\sigma_s} & \frac{\sigma_\nu}{\sigma_s} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.40}$$

The impulse response functions of the identified news and noise shocks are presented in Figure 2.12. The method is successful in limiting the response of output per worker to noise as can be seen from the first graph. Overall the response of the signal also matches the expected result from a news and noise shock, if there is a relatively small noise component. Of course, the brief negative dip of the signal to a noise shock is unexpected. A

Figure 2.12: Impulse responses to a 1% standard deviation shock ϵ_t (blue), v_t(black)



reason for this could be that the assumption that news and noise shocks are uncorrelated is too strong. Another problem could be that surprise shocks are mixed into the signal, and while the structural identification focuses on news and noise these could make the exact separation of the two more difficult. Nevertheless, both the job-finding rate and employment respond much stronger to the identified news than to the identified noise shocks.

The result of this empirical exercise suggests similarly to the DSGE model a limited importance of noise shocks on the labour market. A conditional variance decomposition of the forecast errors suggests that four quarters ahead the majority of fluctuations in the job finding rate and the employment rate are caused by news rather than noise thereby confirming the results of the DSGE estimation that there is little noise driving the US labour market. The slightly stronger noise component in the variance decomposition in Table 2.7 may be due to the less precise separation of noise, news and, surprise shocks with the instruments at hand and the slightly different definitions of the shocks in the VAR compared to the DSGE model.

Series	*Quarters ahead*	1	2	3	4
Δ Job finding rate ($p(\theta)$)	Noise (ν)	4.49 %	6.79 %	10.87 %	8.24 %
	News (e)	95.51 %	93.21 %	89.13 %	91.76 %
Δ Employment rate (n)	Noise (ν)	4.49 %	9.29 %	7.36 %	10.85 %
	News (e)	95.51 %	90.71 %	92.64 %	89.15 %

Table 2.7: Variance decomposition of changes to the job-finding and employment rate due to identified noise to news.

2.6 Interpretation of the result

This paper is as a first exploration into the question of whether agents' erroneous expectations over the expected surplus product of a job match are an important factor in changes in job creation on the aggregate. It finds in multiple setups that non-fundamental noise shocks play a very limited role in job creation, contrary to consumption and investment choices.

The robustness of the result is surprising and there are three interpretations possible regarding it.

- **Agents are well informed** about future labour productivity growth. This is the simplest explanation derived directly from the model. While there are surprises it may be that the reason agents on the aggregate seem better informed about job creation than consumption is that the decision-makers are different. A good part of labour market decision-making is done by firms who may be better informed about the productivity of jobs they open than consumers about their future incomes when making consumption and investment choices.
- **Self-fulfilling expectations** lead to expectations always matching labour productivity. In this case, the expectation of labour productivity rising or falling will lead to labour productivity fundamentally rising or falling. This is the case when there are strong aggregate complementarities of hiring, which for instance may be accompanied by increased demand or increased capital investment. Both may then lead to labour productivity moving in the direction of expectations in a quarter leading to no role in non-fundamental hiring movements and a quick aggregate reaction of hiring to labour productivity changes.
- **There is no forward-looking component in hiring decisions** due to very low hiring frictions or too high turnover. This explanation does not seem to be coherent with the data, given that even in the very conservative calibration used with low vacancy posting cost oriented on (Hagedorn and Manovskii, 2008) and a high ex-

ogenous separations noise can potentially play a significant role. However, from the different calibrations, it is clear that higher vacancy posting costs will lead to a higher role of noise, as will lower exogenous separations as both strengthen the importance of the forward-looking elements in the job-creation condition of the (Mortensen and Pissarides, 1994) random search model.

Of the three interpretations, the first and second are argued to be more likely, and the truth may be a combination of both. The results in this paper if either of these two interpretations is correct mean that variables capturing hiring behaviour are reliable indicators of agents expectations about fundamental labour productivity, and vice versa expectations are strong predictors of expected hiring activity. This robust result should be used to inform policy-making and merit continued high attention of central banks on labour market activity, possibly over investment or consumption activity when determining the state and future expectations for the economy.

2.7 Conclusion

This paper implements a DSGE model with information frictions and estimates the extent of noise agents face when making job creation decisions, which are based on their expectations about future aggregate labour productivity growth. Further, a strategy to identify noise in a VAR model is used for robustness to investigate the same question.

Both approaches find that noise is unlikely to play a significant role in employment and job creation in the United States labour market on the aggregate. Firms and workers react as if they are well informed about the present discounted value of a job match by observing current productivity, and signals on future innovations in aggregate worker productivity are found to largely contain news. Variables capturing hiring behaviour, therefore, are found to be reliable indicators of agents expectations about fundamental labour productivity, and vice versa expectations are strong predictors of expected hiring activity. This may explain why investors and financial markets react strongly to surprises in the latest release of the employment report as shown in Figure 2.13. A recent empirical evaluation of this phenomenon can be found, for instance in (Bauer, 2014).

The results of this paper cast the interesting question on where the noise that has been found in aggregate consumption decisions by agents in (Blanchard and Quah, 1993) and (Forni et al., 2017) is mirrored on the supply side. If aggregate labour market decision-making is largely unaffected by noise around future productivity expectations, then noisy

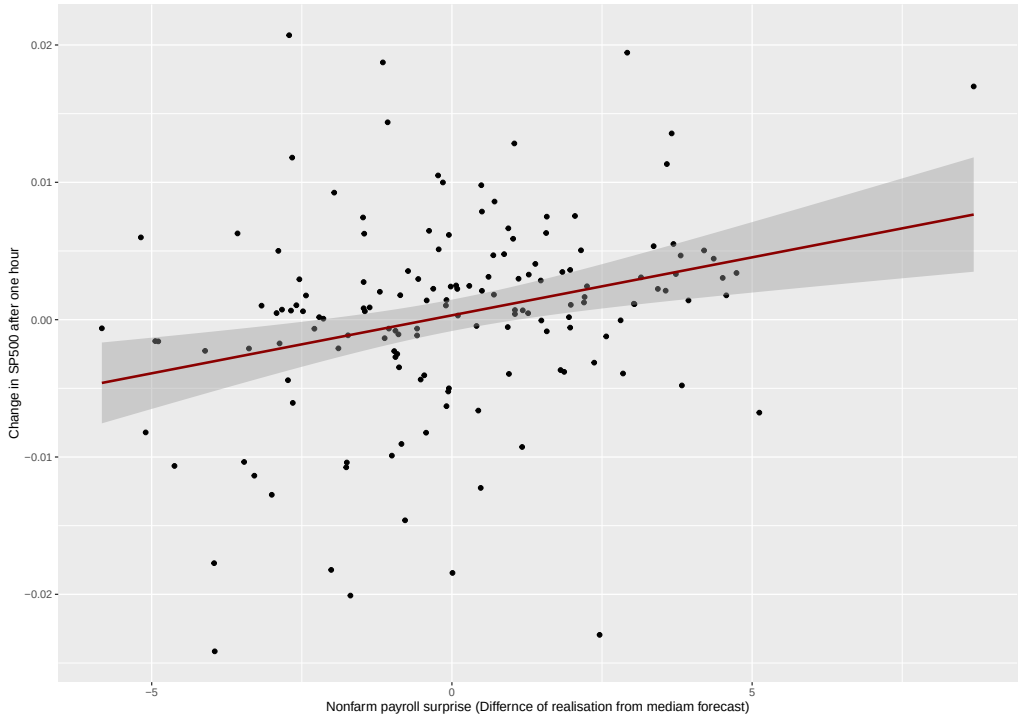


Figure 2.13: Effect of payroll surprises on the SP 500. The datasource is Bloomberg.

consumption choice regarding future productivity expectations is likely to be reflected in the capital input side of production. The results in this paper do not mean that there could not be information frictions regarding choices between idiosyncratic sectors or professions, or affecting decision-making in individual matches, but are for the behaviour of decision-making and job creation on the aggregate.[2] Overall, the robust result of this paper shows that aggregate variables capturing the choices taken on the labour market should be at the centre of attention of policymakers when determining the current state and future expectations for the economy.

[2]Information frictions between sectors and professions may be estimated with the extension to the DSGE model proposed in Appendix 2.D.

Appendix 2.A Model derivations

The agent Kalman filter

The agent Kalman filter is developed in the following way. Agents know that there is an unobserved process of the form in equation (2.41), but only observe only a_t and s_t which are however driven by the same shocks according to equation (2.42).

$$\begin{bmatrix} x_t \\ x_{t-1} \\ z_t \end{bmatrix} = \begin{bmatrix} 1+\rho^x & -\rho^x & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho^z \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_t \\ \eta_t \\ \nu_t \end{bmatrix} \tag{2.41}$$

$$\begin{bmatrix} a_t \\ s_t \end{bmatrix} = \begin{bmatrix} (1+\rho^x) & -\rho^x & \rho^z \\ (1+\rho^x) & -\rho^x & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_t \\ \eta_t \\ \nu_t \end{bmatrix} \tag{2.42}$$

Finally, expectations on the permanent and temporary component of a_t have to add up to a_t and the signal is a combination of news and noise as shown in equation (2.43).

$$\begin{bmatrix} a_t \\ s_t \end{bmatrix} = D \begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} + \begin{bmatrix} 0 \\ \nu_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} + \begin{bmatrix} 0 \\ \nu_t \end{bmatrix} \tag{2.43}$$

Given these identities, the result is the agent Kalman filter is found in equation (2.44).

$$\begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} = \begin{bmatrix} 1+\rho^x & -\rho^x & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho^z \end{bmatrix} (I - KD) \begin{bmatrix} x_{t-1|t-1} \\ x_{t-2|t-1} \\ z_{t-1|t-1} \end{bmatrix} + K \begin{bmatrix} a_t \\ s_t \end{bmatrix} \tag{2.44}$$

Where I is the identity matrix, K is the converged Kalman gain and D a 2x3 matrix. Given that the covariance matrix of the shocks is positive semi-definite, as the three shocks e, η andν are independent with positive finite variance the Kalman gain will converge (Anderson, McGrattan, Hansen, and Sargent, 1996).

With equations (2.42), (2.43) , and (2.44) the expectations about future productivity growth can be described as the result of current expectations and the fundamental shocks e, η, and ν.

Linearising the dynamic equilibrium

Market clearing and the first order conditions require that the aggregate constraint of the economy is given by the consumption equation. Assume for simplicity $\alpha = 0$ and define $P = \frac{a_t}{\Pi_{s=1}^{L} a_{t-s}^{\gamma_s}}$.

$$c_t = a_t n_t - a_{t-L}(\kappa v_t - b(1 - n_t)) \tag{2.45}$$

$$\frac{\kappa}{q(\theta_t)} = \pi(E_t P_t - b) + \beta(1-\lambda)E_t[P_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}(\frac{\kappa}{q(\theta_{t+1})} - (1-\pi)\kappa\theta_{t+1})] \tag{2.46}$$

$$n_t = (1-\lambda)n_{t-1} + v_t q(\theta_t) \tag{2.47}$$

$$u_t = 1 - (1-\lambda)n_{t-1} \tag{2.48}$$

$$\theta_t = \frac{v_t}{u_t} \tag{2.49}$$

The steady-state value of a worker-firm relationship relative to the value of unemployment is $p = 1$. The steady-state of a is subject to change if the permanent component is shocked due to the unit root process. For the approximation here the steady-state of a is normalized at 1. Approximating the above equation at this steady-state with a first order log-linearisation yields:

$$\hat{c}_t = \frac{n}{c}[\hat{P}_t + \hat{n}_t] - \frac{\kappa v}{c}\hat{v}_t - \frac{bn}{c}\hat{n}_t + \hat{a}_{t-L} \tag{2.50}$$

$$\hat{\theta}_t = \psi_1 E_t \hat{P}_t + \psi_2 E_t \hat{\theta}_{t+1} + \psi_3(E_t\hat{c}_t - E_t\hat{c}_{t+1}) \tag{2.51}$$

With $\psi_1 = \frac{\pi m}{\kappa\xi\theta^{\xi}}$, $\psi_2 = \beta(1-\lambda)(1 - \frac{m(1-\pi)}{\xi}\theta^{1-\xi})$, and $\psi_3 = \sigma\beta(1-\lambda)\frac{(1-m(1-\pi)\theta^{1-\xi})}{\xi} = \sigma(\psi_2 + \beta(1-\lambda)\frac{1-\xi}{\xi})$

$$\hat{n}_t = (1-\lambda)\hat{n}_{t-1} + \lambda(\hat{v}_t - \xi\hat{\theta}_t) = \frac{(1-n)(1-\lambda)}{1-(1-\lambda)n}\hat{n}_{t-1} + \lambda(1-\xi)\hat{\theta}_t \tag{2.52}$$

$$\hat{u}_t = \frac{(1-\lambda)n}{1-(1-\lambda)n}\hat{n}_{t-1} \tag{2.53}$$

$$\hat{\theta}_t = \hat{v}_t \ - \hat{u}_t \tag{2.54}$$

If one assumes that $\sigma = 0$, thus that agents have a linear instant utility function it becomes straightforward to show that vacancy postings only depend on last period's employment and the expected productivity path.

$$\hat{\theta}_t = \psi_1 E_t \hat{P}_t + \psi_2 \psi_1 E_t \hat{P}_{t+1} + \psi_2^2 E_t \hat{\theta}_{t+2} = \psi_1 E_t \sum_{s=0}^{\infty} \psi_2^s \hat{P}_{t+s} \tag{2.55}$$

$$\hat{v}_t = \psi_1 E_t \sum_{s=0}^{\infty} \psi_2^s \hat{P}_{t+s} + \frac{(1-\lambda)n}{1-(1-\lambda)n}\hat{n}_{t-1} \tag{2.56}$$

This result is generalized to values of $\sigma \geq 0$ by inserting equation (2.50) and (2.52), and the resulting equations into (2.51) substituting out for $\hat{c}_t$, $\hat{c}_{t+1}$.$\hat{c}_{t+2}$,... . However, the resulting equation for $\hat{\theta}_t$ as a function n_{t-1} and past, current, and future values of a_t does not have a closed form representation.

Note that even if agents knew current period productivity, thus if a_t and p_t would be part of their information set in period t, hiring in the current period will still depend on the future productivity expectations $E_t p_{t+1}, E_t p_{t+2}, ...$ and would thereby still be affected by news and noise besides observed shocks to current productivity. To emphasize the potential effect of news and noise the arguably more realistic modelling choice has been made to exclude the precise value of current aggregate productivity from the agents' information set.

Appendix 2.B Data description

The label of the series in question in the federal reserve database (FRED) is presented in square brackets during this section. **Worker Productivity (a)**
The product that a new worker-firm relation would create is the key driving factor in a search and matching model. This product p_t is assumed to depend in this model on the path of productivity a_t. The realized real output per worker could be used as a proxy for productivity a_t in this model as due to the modelling assumptions the marginal product

of labour equals the current level of productivity.

This series would be ideal if real output Y_t is the product of $A_t N_t$, where $A_t = exp(a_t)$ and $N_t = n_t L_t$. L_t is the total labour force. In this case $log(\frac{Y_t}{N_t}) = a_t$. Looking at the data a_t is clearly an integrated process.

On the other hand, if the aggregate real output of the economy is actually better modeled by some Cobb-Douglas form $Y_t = A_t N_t^{1-\alpha} K_t^{\alpha}$ then the product contributed by taking on another average worker on a continuum of workers is $\frac{Y_t}{N_t} = A_t(\frac{K_t}{N_t})^{\alpha} = \tilde{A} N_t^{-\alpha}$. This explains the generalisation using $n^{1-\alpha}$ in the model. In this case movements in a capture both TFP and capital changes. K_t is some measure of productive capital and mpl_t is the marginal product of labour. Once one focuses on the changes over time $\frac{mpl_t}{mpl_{t-1}} = \frac{\frac{Y_t}{N_t}}{\frac{Y_{t-1}}{N_{t-1}}}$. Then $\Delta a_t + \Delta k_t = (log(\frac{Y_t}{N_t} - log(\frac{Y_{t-1}}{N_{t-1}}))$, where $k_t = \alpha log \frac{K_t}{N_t}$ is capital per active worker.

From the equation $\Delta log(\frac{Y_t}{N_t}) = \Delta a_t + \Delta k_t$ it is clear that under the alternative Cobb-Douglas assumptions for production, capital per employed worker not changing as a result of employment activity resulting from future expectation shifts is a critical assumption for the identification of productivity news and noise shocks when estimating the model parameters based on this series. This assumption holds if capital fully depreciates after the end of each period and capital input is chosen by firms once agents observe the productivity realization a_{t+1}. On the other hand, if these assumptions do not hold and capital per worker were to increase as a result of a positive expectation shift, then the productivity of hired workers would be, to some extent the result of a self-fulfilling prophecy. This would weaken the ability of the model to separate news from noise.

In the latter case, the amount of capital resources per worker going into production would correlate with noise and thus worker productivity would react and not stay flat in response to a noise shock. In this case, the identifying qualities of the series for separating productivity noise from news would be lost. The series then would only be useful in identifying noise in the labour market to the extent that this noise leads to hiring, but does not lead to marginal product of labour adjustments. Thus the series would be $\Delta log(\frac{Y_t}{N_t}) = \Delta a_t + \Delta k_t = \Delta \tilde{a}_t$ This would still be a useful estimation, but it is important to note the qualitative difference of noise in both cases. In the second case noise is the result of misinformation over the path of $\tilde{a}_t = a_t + k_t$, where k_t is a function of the expected path. It is still possible for agents to be misinformed about this path, but as the misinformation has an effect on k_t identification is weaker and limits the power of the es-

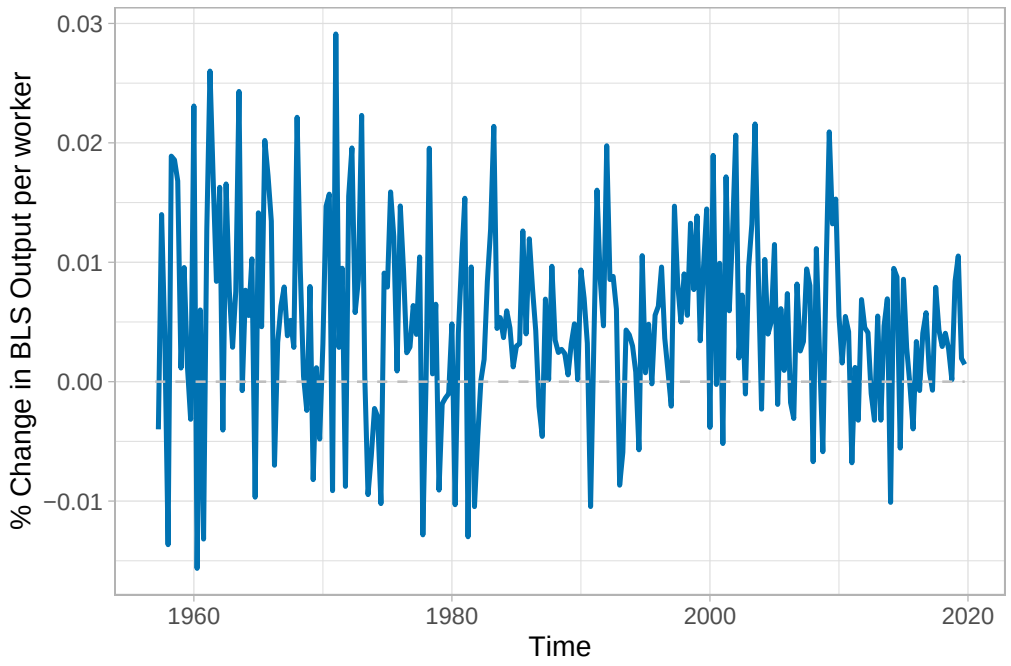


Figure 2.14: Changes in Labour productivity Δa_t based on Output per Worker [OPHNFB] computed by the bureau of labour statistics

timation. This problem may be addressed in an extension of the model, which explicitly takes optimal choices of capital input into account.

One can test whether the random walk assumption of the model is plausible for the output per worker series a_t. This is done by testing the hypothesis that for the AR(1) process $\Delta a_t = \mu_a + \rho_a \Delta a_{t-1} + u_t$ the estimated coefficient $\hat{\rho}_a = 0$. An augmented Dickey-Fuller test reveals t-values of -10.739 and -10.075 for Δ *Output per worker based on the BLS index* and Δ *Real output per worker* respectively. This means that the random walk assumption for labour productivity a cannot be rejected. This confirms the assumptions taken in Section 2.3.1.

The productivity series on which the estimation is based is presented in Figure 2.14. This series is used in (Shimer, 2005a). An alternative, yet very similar measure for output per worker is the series constructed by dividing real Output[GDPC1] by the number of workers Worker[PAYEMS]. This series is very similar as can be seen in Figure 2.15. Finally, (Forni et al., 2017) use potential output. Potential output per worker is significantly less volatile. The changes of the series can be seen in Figure 2.16. While it can be seen as a problematic series as it accounts for the economies employment potential as well the series provides a sense of the current marginal product of hiring a worker given that the economy is not at potential.

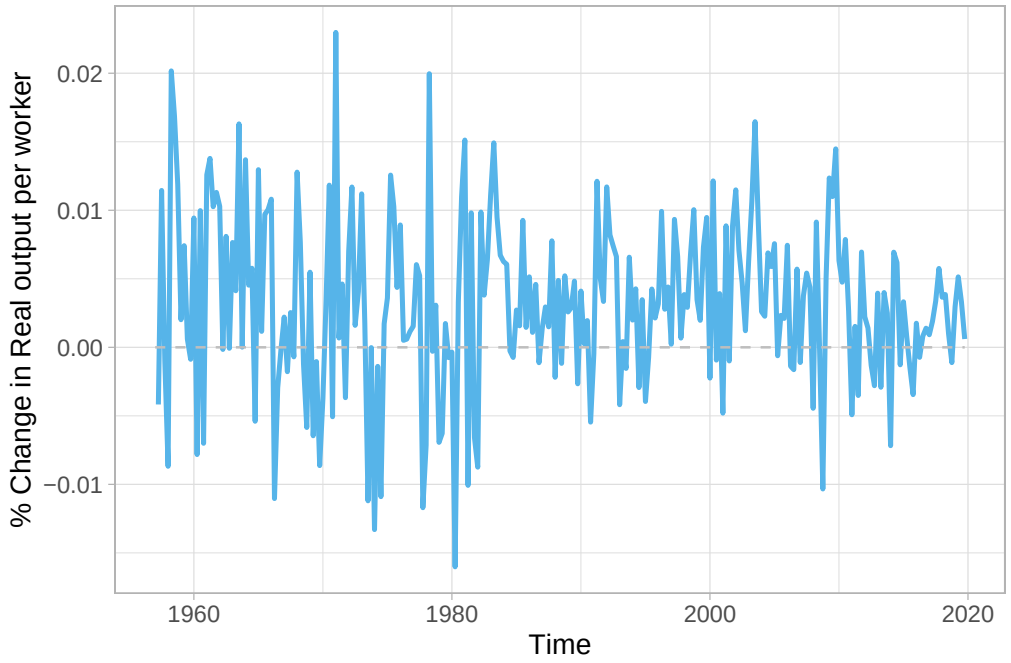


Figure 2.15: Changes in Labour productivity Δa_t based on Output[GDPC1] per Worker[PAYEMS]

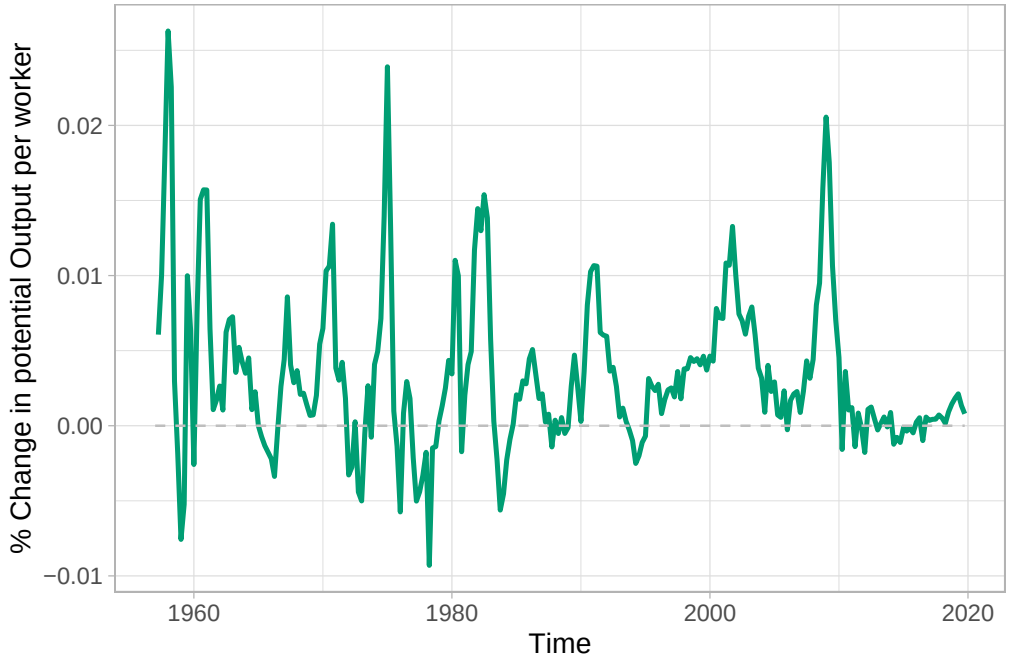


Figure 2.16: Changes in Labour productivity Δa_t based on potential Output[GDPPOT] per Worker[PAYEMS]

Job-finding rate ($p(\theta)$)

To find the job-finding rate I employ the procedure suggested in (Shimer, 2005a). I update Shimer's estimation of the job-finding-rate and by also imposing the 10% correction for the presumed level shift of the short-term unemployed series from the beginning of 1994. This means estimating the job-finding rate by rewriting the law of motion of unemployment $U_t = U_{t-1} + S_t - f_t U_{t-1}$ to

$$f_t = 1 - \frac{U_t - S_t}{U_{t-1}}$$

, where S_t is the number of separated workers who didn't succeed to find a job in period t. The number of people unemployed less than a month acts as a proxy for S_t. The unemployment monthly unemployment level [UNEMPLOY] and the number of short-term unemployed [UEMPLT5] is retrieved from the FRED based on the data by the Bureau of Labour Statistics.

Admittedly, this estimation of the job-finding rate makes a lot of assumptions, that are not going to hold in reality. Among other things it presumes, similar to the model presented above, that long-term unemployed workers have the same probability in finding a job as recently unemployed workers. An assumption that is clearly rejected by empirical investigations. The monthly job-finding rate estimated by this procedure is plotted in Figure 2.17.

Due to a lack of data on monthly productivity for the sample these transition probabilities have to be transformed to quarterly transition probabilities. The transformation is done by calculating the probability of the worker of not finding a job in the past three months and subtracting this from 1. $p(\theta_t) = 1 - (1 - f_t)(1 - f_{t-1})(1 - f_{t-2})$

This series in Figure 2.18 seems to be downward trending. Such a negative trend in the job-finding rate could be explained by a negatively trending parameter of matching productivity. Matching productivity is assumed constant in the model. For this reason, the model is estimated based on the demeaned changes in the job-finding rate, rather than the levels.

Unemployment ($1 - n_t$)

The unemployment series is based on the data available from the Bureau of Labour Statistics [UNEMPLOY/ (UNEMPLOY+PAYEMS)]. As for the job-finding rate, in order to avoid the estimation being influenced by not modelled trends, the estimation is based

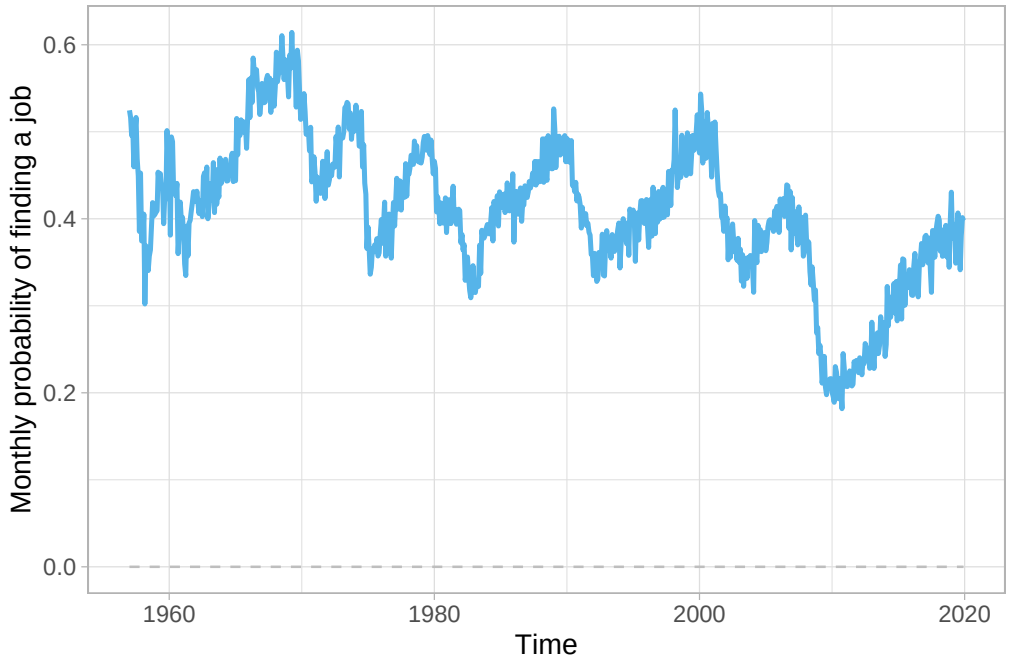


Figure 2.17: Monthly job-finding Rate f_t

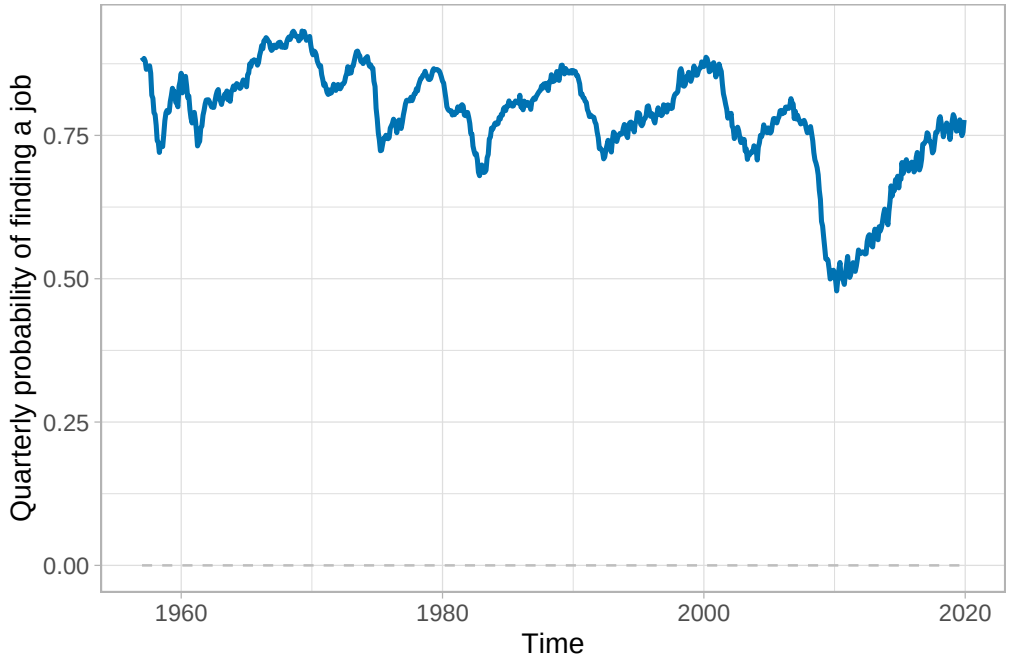


Figure 2.18: Quarterly job-finding Rate p(θ_t)

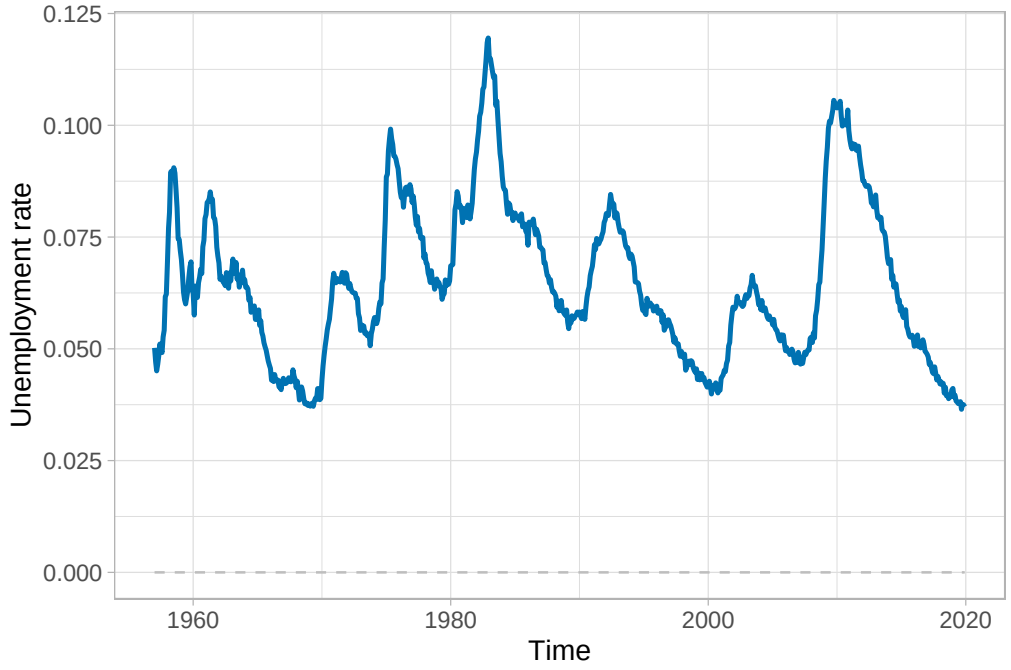


Figure 2.19: Unemployment rate.

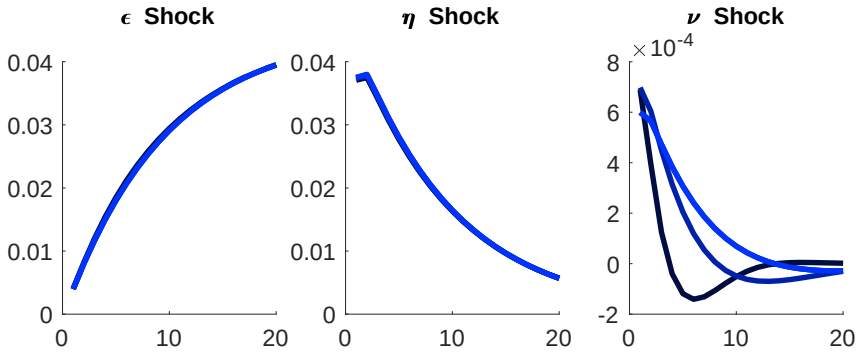


Figure 2.20: Comparison of a different strength of noise on the employment rate n with darker blue representing smaller noise for the FS implementation of the model.

on the demeaned changes of the unemployment rate rather than the series levels.

Appendix 2.C Additional simulation and estimation results

Simulation of the model in the FS implementation

Figure 2.20 shows the FS response of employment to the three type of shocks. In this case, an ϵ shock will lead to a permanent increase in the employment rate as the fundamental surplus of a match rises.

In the FS implementation, the impulse response function to a temporary shock is shown in Figure 2.21. Due to temporary productivity shocks being expected to have some persis-

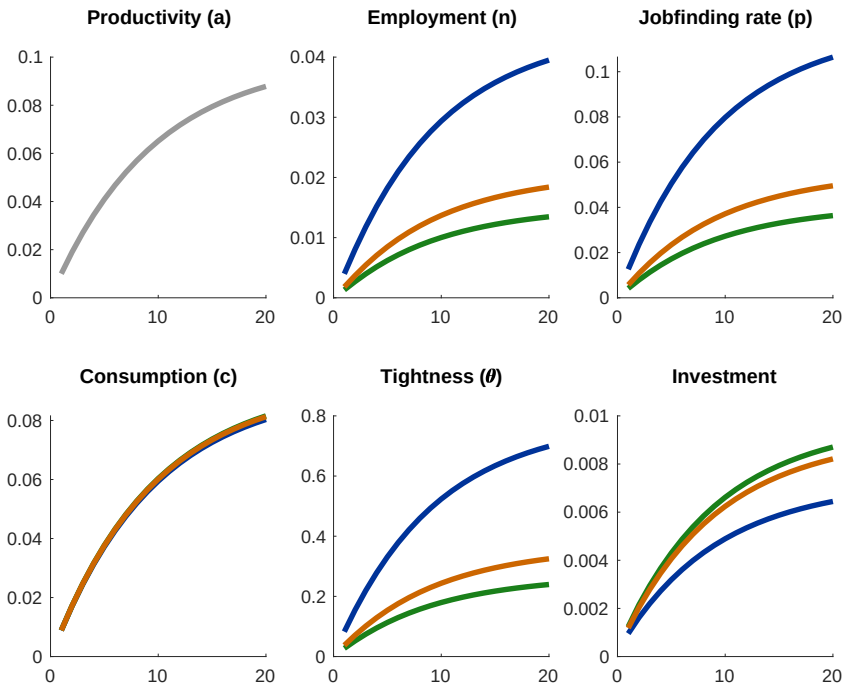


Figure 2.21: Responses to a shock to the permanent component (ϵ) in the FS implementation of the model. The baseline calibration is in blue, while the Shimer calibration is green, and the intermediate calibration is in orange.

tence as given by ρ combined with the possibility that the increase in a_t was actually the result of an increase of the permanent productivity component, firms will increase hiring. The fact that a temporary increase also temporarily increases expectations in long-run a can be seen in Figure 2.4. The impulse responses are similar to the impulse response of a persistent technology shock in a conventional RBC model where the income effect outweighs the wealth effect leading to increased labour demand. Note that with higher noise it becomes harder and harder for firms to separate the temporary from the permanent increase, which would lead to both impulse responses becoming flatter.

Under the setup under the FS implementation in Table 2.8, where permanent shocks to labour productivity permanently increase the match surplus noise may still contribute up to 5.55 % of the short run variance in the employment rate,

Full variance decomposition

Table 2.9 shows the variance decomposition of all three shocks. Furthermore, simulations are shown for the case that agents have to predict labour productivity for the current period allowing noise to fully drive the short-run variance. In this now-casting scenario

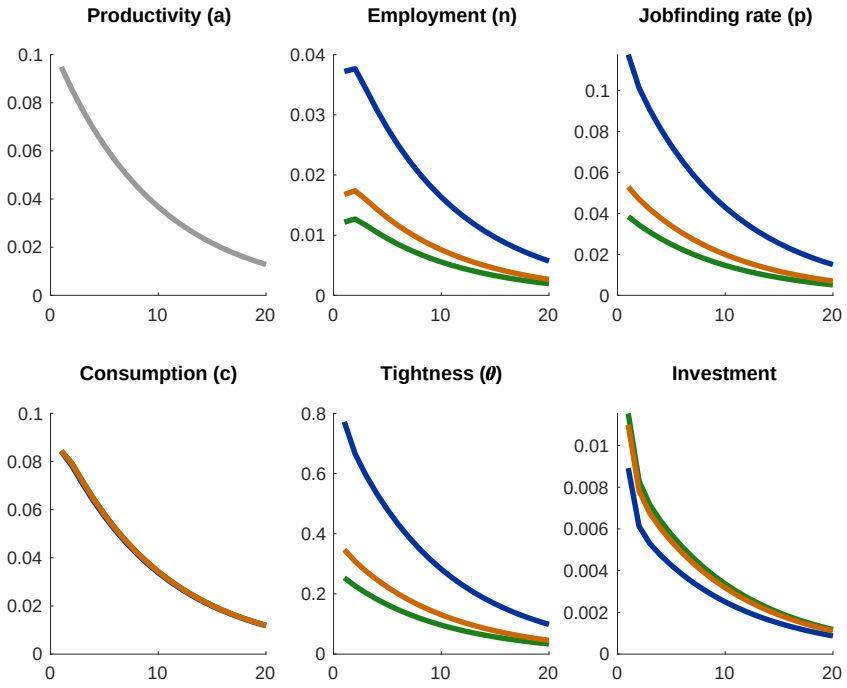


Figure 2.22: Responses to a shock to the temporary component (η) in the FS implementation of the model. The baseline calibration is in blue, while the Shimer calibration is green, and the intermediate calibration is in orange.

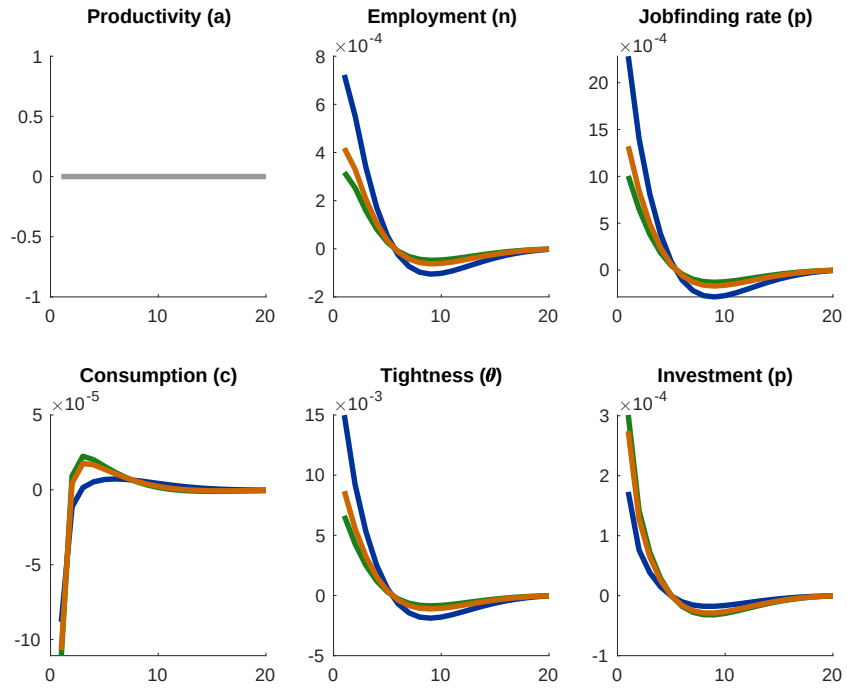


Figure 2.23: Responses to a shock to the noise shock (ν) in the FS implementation of the model. The baseline calibration is in blue, while the Shimer calibration is green, and the intermediate calibration is in orange.

Calibration	*Quarters*	1	2	3	4
	Fundamental surplus interpretation				
Baseline	Noise (ν)	3.18 %	0.47 %	0.08 %	0.01 %
	Permanent (e) %	96.82 %	99.53 %	99.92 %	99.99 %
Intermediate	Noise (ν)	5.1 %	0.79 %	0.14 %	0.02 %
	Permanent (e) %	94.9 %	99.21 %	99.86 %	99.98 %
Shimer	Noise (ν)	5.55 %	0.87 %	0.16 %	0.03 %
	Permanent (e) %	94.45 %	99.13 %	99.84 %	99.97 %

Table 2.8: Relative variance decomposition of noise and expected permanent labour productivity or fundamental surplus improvements with regard to the employment rate based on a simulation with a large noise impact

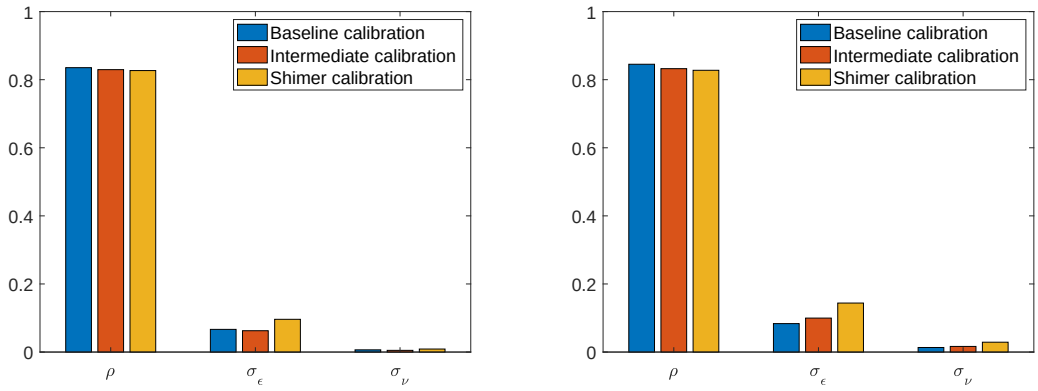


Figure 2.24: Results of the ML estimation of the model in the FS implementation. Estimation results based on the unemployment rate are on the left, while estimation based on the job-finding rate are on the right.

the job creation condition changes to equation (2.57).

$$\frac{\kappa}{q(\theta_t)} = E_t[\pi(P_t - b) + \beta(1-\lambda)P_{t-L+1}\frac{c_{t+1}^{-\zeta}}{c_t^{-\zeta}}(\frac{\kappa}{q(\theta_{t+1})} - (1-\pi)\kappa\theta_{t+1})] \tag{2.57}$$

Estimation results of the FS implementation of the model

Figure Figure 2.24 shows the estimated parameters for the FS implementation for estimations based on the unemployment and estimations based on the job finding series combined with productivity. Table 2.10 and Table 2.11 show the results of the maximum likelihood estimation and the relative variance decomposition for the FS implementation with the baseline calibration.

Appendix 2.D Extension of the model

The model can be extended to study the noise in relative match productivity between sectors or professions. This can provide an estimation of the extent to which homogeneous workers are reluctant to switch sectors due to information frictions.

Calibration	*Quarters*	1	2	3	4
	Fundamental surplus interpretation				
Baseline	Noise (ν)	1.56 %	0.37 %	0.07 %	0.01 %
	Temporary (η)	51 %	21.12 %	9.43 %	4.85 %
	Permanent (e)	47.45 %	78.51 %	90.5 %	95.14 %
Intermediate	Noise (ν)	2.54 %	0.62 %	0.13 %	0.02 %
	Temporary (η)	50.26 %	21.26 %	9.51 %	4.87 %
	Permanent (e)	47.2 %	78.11 %	90.36 %	95.11 %
Shimer	Noise (ν)	2.77 %	0.69 %	0.14 %	0.02 %
	Temporary (η) %	50.09 %	21.31 %	9.54 %	4.88 %
	Permanent (e)	47.14 %	78.01 %	90.32 %	95.1 %
	Match productivity interpretation				
Baseline	Noise (ν)	7.67 %	8.38 %	3.62 %	1.06 %
	Temporary (η)	46.84 %	0.05 %	1.69 %	1.99 %
	Permanent (e)	45.49 %	91.57 %	94.69 %	96.95 %
Intermediate	Noise (ν)	12.87 %	13.85 %	6.15 %	1.83 %
	Temporary (η)	43.65 %	0.01 %	1.73 %	2.16 %
	Permanent (e)	43.48 %	86.13 %	92.12 %	96.01 %
Shimer	Noise (ν)	13.95 %	14.99 %	6.71 %	2.01 %
	Temporary (η) %	43.01 %	0 %	1.72 %	2.19 %
	Permanent (e)	43.05 %	85.01 %	91.58 %	95.8 %
	Fundamental surplus interpretation, predicting $E_t(P_t)$				
Baseline	Noise (ν)	97.52 %	82.46 %	40.74 %	8.64 %
	Temporary (η)	0.35 %	0.39 %	0.28 %	0.12 %
	Permanent (e)	2.13 %	17.15 %	58.97 %	91.24 %
Intermediate	Noise (ν)	94.74 %	73.03 %	30.26 %	5.94 %
	Temporary (η)	1.34 %	1.55 %	1.17 %	0.59 %
	Permanent (e)	3.93 %	25.43 %	68.57 %	93.47 %
Shimer	Noise (ν)	92.44 %	66.73 %	25.04 %	4.74 %
	Temporary (η) %	2.26 %	2.53 %	1.81 %	0.92 %
	Permanent (e)	5.3 %	30.74 %	73.15 %	94.35 %
	Match productivity interpretation, predicting $E_t(P_t)$				
Baseline	Noise (ν)	99.57 %	96.56 %	82.25 %	38.46 %
	Temporary (η)	0.21 %	1.47 %	3.63 %	6.38 %
	Permanent (e)	0.22 %	1.97 %	14.12 %	55.16 %
Intermediate	Noise (ν)	99.53 %	96.2 %	81.51 %	38.62 %
	Temporary (η)	0.1 %	1.39 %	3.53 %	6.18 %
	Permanent (e)	0.38 %	2.41 %	14.96 %	55.2 %
Shimer	Noise (ν)	99.46 %	95.88 %	80.66 %	37.97 %
	Temporary (η) %	0.04 %	1.36 %	3.51 %	6.08 %
	Permanent (e)	0.5 %	2.77 %	15.83 %	55.95 %

Table 2.9: Relative variance decomposition of noise, expected temporary or permanent match productivity or fundamental surplus improvements with regard to the employment rate based on a simulation with a large noise impact

Table 2.10: Baseline estimated news and noise parameters by implementation with the series based on which the ML estimation was done in round brackets and the t-values in square brackets

Parameter estimated:	FS (Δa_t & Δu_t)	FS (Δa_t & $\Delta p(\theta_t)$)
ρ	0.8353 [75.9]	0.8453 [85.8]
σ_e	0.0666 [22.9]	0.0836 [2.5]
σ_ν	0.0065 [2.3204]	0.0134 [2.1]

Table 2.11: Estimation based of relative variance decomposition of noise and expected permanent labour productivity or fundamental surplus improvements with regard to the employment rate.

Calibration	*Quarters*	1	2	3	4
	Fundamental surplus interpretation				
Baseline	Noise (ν)	0.03 %	0 %	0 %	0 %
	Permanent (e) %	99.97 %	100 %	100 %	100 %

At the heart of these extensions is unemployed workers choosing to search in sector j such that their value of unemployment V^u is maximised as in equation (2.58).

$$V_t^u = b + p(\theta_t) \max_j [V_j^w - E_t(\mu_{t+t} V_{t+1}^u)] \tag{2.58}$$

In this case the relative output per worker is $\tilde{a}_{j,t} = \frac{a_{j,t}}{\bar{a}_t}$, where $\bar{a}_t = \sum_{j=1}^{J} n_{j,t}^{1-\alpha} a_{j,t}$ is the mean output per worker. The job creation condition for each sector is then a result of employment in the sector and expected labour productivity. Sectors with high employment will have high labour productivity. In equilibrium, the wages and match product of homogeneous workers will equalise. Shocks to relative productivity of one sector will lead to workers switching sectors. Large information frictions will delay the extent of these switches as workers will be afraid the increases in productivity could be temporary or non-fundamental.

$$\frac{\kappa}{q(\theta_{j,t})} = \int_{z_{s,t}}^{\bar{z}} [\pi(z_s \tilde{a}_{j,t} n_j^{-\alpha} - b) + \beta(1-\lambda) E_t [\frac{c_{t+1}^{-\zeta}}{c_t^{-\zeta}} (\frac{\kappa}{q(\theta_{j,t+1})} - (1-\pi)\kappa\theta_{j,t+1})]] h(z) dz \tag{2.59}$$

This setting will allow for a straightforward implementation via perturbation. Endogenous separation is introduced via idiosyncratic match productivity z_s in this setting to allow sectors to adjust and keep the match product in all sectors above the benefit from unemployment. Assuming a common information friction σ_ν sectoral wages, vacancies or hiring movements and output for homogeneous workers are sufficient to estimate the extent of information frictions. Homogeneity can be achieved by accounting for observable worker differences. The information frictions will introduce temporary mismatch driven by information frictions in the labour market as workers decide whether to switch industry or occupation, based on their expectation of an increase in match productivity in a sector being temporary or permanent.

Appendix 2.E Dynare implementation for the DSGE model

Model file

The model file below implements an equilibrium where it is possible to switch between the fundamental and non-fundamental interpretations. The model file shows how alternative expected paths may be implemented in Dynare, which are separate from the actual

productivity of the economy. It may be a useful guide for setting up a perturbed model with expectation driven decisions, where the expectations are different than the actual paths of the exogenous variables. The model is set up to compute the expected path of the future state n, x and z, and thereby determine the hiring decisions at present. Expectations of the converged Kalman filter are computed in the steady state file and passed as unchanging variables states K11 to K32 into the model file. In this way the estimation command is able to estimate these variables. The actual productivity path and signal will gradually lead to an update in expectations vie the Kalman gain variable K11 to K32.

```
close all;
%%%%%%%%%%%%%%%%%%%%%%%%
% Dynare markup macro settings
%%%%%%%%%%%%%%%%%%%%%%%%

%Define approximation length into the future
@#define len = 20

%Set to 1 if ML estimation is desired
@#define est = 0

%Set to 1 if Fundamental interpretation
@#define fund = 0

%Define length of lag of ARMA for cost
@#define len_ar = 3

%%%%%%%%%%%%%%%%%%%%%%%%
% Endogenous Variables
%%%%%%%%%%%%%%%%%%%%%%%%
var n, v, u, c, theta, Investment, wage, Output, a

@#for t in 1:len
n@{t} v@{t} u@{t} t@{t} a@{t} c@{t}
@#endfor

@#for i in 1:len+1
past_@{i}
@#endfor

aLR,
@#for i in 1:3
K@{i}1 K@{i}2
@#for j in 1:3
IKFA@{i}@{j}
@#endfor
@#endfor
Da,Dn, djf, du, Consumption dcons Unemployment Jobfindingrate
dx x z s prod dprod xtt xt1t ztt;

%%%%%%%%%%%%%%%%%%%%%%%%
% Exogenous Variables
%%%%%%%%%%%%%%%%%%%%%%%%
varexo eta epsilon nu;

%%%%%%%%%%%%%%%%%%%%%%%%
% Parameters
%%%%%%%%%%%%%%%%%%%%%%%%

parameters p_beta, sigma,lmb,xi,ppi,kap, b, m,
rho sig_u sig_nu exp_length p_alpha
```

```
@#for i in 1:len_ar+1
p_gamma_@{i}
@#endfor
;

load parameterfile;
set_param_value('m',pa.m);
set_param_value('p_beta',pa.beta);
set_param_value('sigma',pa.sigma);
set_param_value('lmb',pa.lambda);
set_param_value('xi',pa.xi);
set_param_value('ppi',pa.pi);
set_param_value('kap',pa.kappa);
set_param_value('b',pa.b);
set_param_value('sig_u',pa.sig_u);
set_param_value('rho',pa.rho);
set_param_value('sig_nu',pa.sig_nu);
set_param_value('p_alpha',pa.alpha);

exp_length = @{len};
@#if fund == 0
p_gamma_1 = 0 ;
p_gamma_2 = 0.6957742;
p_gamma_3 = 0.3042258;
p_gamma_4 = 0 ;
p_gamma_5 = 0;
p_gamma_6 = 0;
@#endif

% When all gamma are set to 0 no catchup processes influence the model
@#if fund == 1
p_gamma_1 = 0;
p_gamma_2 = 0;
p_gamma_3 = 0;
p_gamma_4 = 0;
p_gamma_5 = 0;
p_gamma_6 = 0;
@#endif

%%%%%%%%%%%%%%%%%%%%%%
% Model
%%%%%%%%%%%%%%%%%%%%%%
model;

/*IKFA-- K-- are actually the values of the converged agent Kalman filter-
these are estimated in the steady state file from the parameters rho, sig_u, and sig_nu */
@#for i in 1:3
K@{i}1=K@{i}1(-1);
K@{i}2=K@{i}2(-1);
@#for j in 1:3
IKFA@{i}@{j} =IKFA@{i}@{j}(-1);
@#endfor
@#endfor

/*Productivity process*/
dx = rho*dx(-1) + (1-rho)*sig_u*epsilon;
dx = x - x(-1);
z = rho*z(-1) + (rho^0.5)*sig_u*eta;
prod = x + z;
dprod = prod - prod(-1);
dcons = c - c(-1);

/*Signal*/
s = x + sig_nu*nu;

/*Solving time t productivity expectation witht the agent Kalman filter values*/
```

```
xtt =    IKFA11*xtt(-1) + IKFA12*xt1t(-1) + IKFA13*ztt(-1) + K11*prod + K12*s;
xt1t =  IKFA21*xtt(-1) + IKFA22*xt1t(-1) + IKFA23*ztt(-1) + K21*prod + K22*s;
ztt =  IKFA31*xtt(-1) + IKFA32*xt1t(-1) + IKFA33*ztt(-1) + K31*prod + K32*s;

/*******************************/
/*Actual model equations*/
/*******************************/
n= (1-lmb)*n(-1)+m*v^(1-xi)*u^xi;
u=1-(1-lmb)*n(-1);
theta=v/u;
theta^xi*kap/m= ppi*(prod/past_1/n^p_alpha-b)+(1-lmb)*
p_beta*(c^sigma/ c1^sigma)*(a/prod)*(kap *t1^xi/m - (1-ppi) * kap*t1);
c= n*prod-kap*v+b*(1-n);
Investment=kap*v;
Consumption=n*prod -kap*v+b*(1-n);

/*******************************/
/* This part is to to find the agents future expected path for the endogenous variables
based on the current expectations about the permanent and temporary component of productivity*/
/*******************************/

/*Computing the expected productivity path based on the current expected values of productivity*/
a=xtt*(1+rho)-rho*xt1t+rho*ztt;
a1=(a-rho*ztt)*(1+rho)-rho*xtt+rho^2*ztt;
a2=(a1-rho^2*ztt)*(1+rho)-rho*(a-rho*ztt)+rho^3*ztt;
@#for t in 3:len
a@{t}=(a@{t-1}-rho^@{t}*ztt)*(1+rho)-rho*(a@{t-2}-rho^@{t-1}*ztt)+rho^@{t+1}*ztt;
@#endfor
aLR = xtt + (xtt-xt1t)*(rho/(1-rho));
Da=aLR-a@{len};

% Past labour productivity realisations
@#for t in 1:len+1
past_@{t}=  p_gamma_1 +
@#for j in 2:len_ar+1
@#if j== t+1
prod
@#endif
@#if j==t
a
@#endif
@#if j>t+1
prod(@{t-j+1})
@#endif
@#if j<t
a@{t-j}
@#endif
^p_gamma_@{j}*
@#endfor
1
;
@#endfor

/*Expected employment and pre-matching unemployment*/
n1= (1-lmb)*n+m*v1^(1-xi)*u1^xi;
u1=1-(1-lmb)*n;
@#for t in 2:len
n@{t}= (1-lmb)*n@{t-1}+m*v@{t}^(1-xi)*u@{t}^xi;
u@{t}=1-(1-lmb)*n@{t-1};
@#endfor

/*Expected theta and consumption*/
@#for t in 1:len
t@{t}=v@{t}/u@{t};
@#endfor
```

```
c1= n1* a -kap*v1+b*(1-n1);
@#for t in 2:len
c@{t}= n@{t}* a@{t} -kap*v@{t}+b*(1-n@{t});
@#endfor

/*Expected Job creation*/
@#for t in 1:len-1
t@{t}^xi*kap/m= ppi*(a@{t}/past_@{t+1}/n@{t}^p_alpha-b)+(1-lmb)*
p_beta*(c@{t}^sigma/ c@{t+1}^sigma)*(a@{t+1}/past_@{t})*(kap *t@{t+1}^xi/m - (1-ppi) * kap*t@{t+1});
@#endfor
t@{len}^xi*kap/m=
ppi*(a@{len}/past_@{len+1}/n@{len}^p_alpha-b)+(1-lmb)*
p_beta*(c@{len}^sigma/ c@{len}(+1)^sigma)*(a@{len}/past_@{len})*
(kap*t@{len}(+1)^xi/m - (1-ppi) * kap*t@{len}(+1));

/*Computing other variables of interest and integrating observation variables into the model*/
wage=b*ppi+(1-ppi)*(a+p_beta*(1-lmb)*c/c1*kap*t1);
Jobfindingrate=m*theta^(1-xi);
Unemployment=1-n;
Output= prod*n+b*(1-n);
Dn=n-n(-1);
djf=Jobfindingrate-Jobfindingrate(-1);
du=Unemployment-Unemployment(-1);
end;

steady;

shocks;
var eta; stderr 1;
var epsilon; stderr 1;
var nu;  stderr 1;
end;

model_diagnostics;
check;

stoch_simul(irf = 20, order =1, Periods=10000) Output Consumption
n Jobfindingrate  djf Dn Da a prod xtt aLR x Investment theta wage du;

/*Estimation part*/

@#if est == 1
estimated_params;
rho, 0.8,0,1;
sig_u, 0.006,0, 10;
sig_nu, 0.006,0, 10;
end;
varobs dprod djf;
estimation(datafile=Est_data1,prefilter=1,lik_init=2,first_obs=1);
shock_decomposition dprod djf;
@#endif
```

Steady state file

```
function [ys,params,check] = RS_steadystate(ys,exo,M_,options_)
% function [ys,params,check] = NK_baseline_steadystate(ys,exo,M_,options_)
% computes the steady state for the NK_baseline.mod and uses a numerical
% solver to do so
% Inputs:
%   - ys         [vector] vector of initial values for the steady state of
%                    the endogenous variables
%   - exo        [vector] vector of values for the exogenous variables
%   - M_         [structure] Dynare model structure
%   - options    [structure] Dynare options structure
%
% Output:
%   - ys         [vector] vector of steady state values for the the endogenous variables
%   - params     [vector] vector of parameter values
%   - check      [scalar] set to 0 if steady state computation worked and to
%                     1 of not (allows to impose restrictions on parameters)
% read out parameters to access them with their name
NumberOfParameters = M_.param_nbr;
for ii = 1:NumberOfParameters
  paramname = M_.param_names{ii};
  eval([ paramname ' = M_.params(' int2str(ii) ');']);
end
% initialize indicator
check = 0;
%% END OF THE FIRST MODEL INDEPENDENT BLOCK.
%% THIS BLOCK IS MODEL SPECIFIC.

%Finding the labour market steady state
load parameterfile;
y0=[2,0.5];
[y,fval]=fsolve(@(y) [ (ppi*(1/y(2)^pa.alpha- b)- kap /m *(1-p_beta*(1-lmb))* y(1)^xi)/(kap*p_beta*(1-lmb)*(1-ppi))-y
      (1),(m*y(1)^(1-xi))/(lmb+(1-lmb)*m*y(1)^(1-xi))-y(2)],y0);
theta=y(1);
n=y(2);
u=1-(1-lmb)*n;
v=theta*u;
c= n -kap*v+b*(1-n);
Investment=kap*v;
Unemployment=1-n;
Output=n+b*Unemployment;
Jobfindingrate=m*theta^(1-xi);

%Setting the productivity processes steady state
z=0;x=1;dx=0;s=1;
prod=1;
dprod=0;
a=1; aLR=1; Da =0;
xtt=1; xt1t=1; ztt=0;

%Steady states of the agent's expected path of the endogenous variables
for i = 1:exp_length+10
    eval(sprintf('c%d= %s',i,'c;'));
    eval(sprintf('n%d= %s',i,'n;'));
    eval(sprintf('v%d= %s',i,'v;'));
    eval(sprintf('u%d= %s',i,'u;'));
    eval(sprintf('a%d= %s',i,'a;'));
    eval(sprintf('t%d= %s',i,'theta;'));
    eval(sprintf('past_%d= %s',i,'1;'));
end

%Steady states for other variables of interest
wage=b*ppi+(1-ppi)*(a+p_beta*(1-lmb)*c/c1*kap*t1); Consumption=c; Dn=0; du=0; djf=0; dcons=0; DUnemployment=0; DLYpN
      =0; DLYPotpL=0; DConsumption=0;
```

```
% Agent Kalman filter
A = [ 1+rho -rho 0 ; 1 0 0 ; 0 0 rho ];
B = [ (1-rho)*sig_u 0 0 ; 0 0 0 ; 0 0 (rho^.5)*sig_u ];
S_U = [ 1 0 0 ; 0 0 0 ; 0 0 1 ];
F = [ 1 0 1 ; 1 0 0 ];
G = [0 0; 0 sig_nu ];
S_V = [0 0; 0 1 ];
Ome = eye(3,3);
for iter = 1:1000;
    K = Ome*F'*inv(F*Ome*F'+G*S_V*G');
    Ome = A*(Ome-K*F*Ome)*A' + B*S_U*B';
end;
IKFA = (eye(3,3) - K*F)*A;

%Converged Kalman filter results
IKFA11= IKFA(1,1); IKFA12= IKFA(1,2); IKFA13= IKFA(1,3); IKFA21= IKFA(2,1); IKFA22= IKFA(2,2); IKFA23= IKFA(2,3);
      IKFA31= IKFA(3,1); IKFA32= IKFA(3,2); IKFA33= IKFA(3,3);
K11= K(1,1); K12= K(1,2); K21= K(2,1); K22= K(2,2); K31= K(3,1); K32= K(3,2);

%% END MODEL SPECIFIC BLOCK
params=NaN(NumberOfParameters,1);
for iter = 1:length(M_.params) %update parameters set in the file
  eval([ 'params(' num2str(iter) ') = ' M_.param_names{iter} ';' ])
end

NumberOfEndogenousVariables = M_.orig_endo_nbr; %auxiliary variables are set automatically
for ii = 1:NumberOfEndogenousVariables
  varname = M_.endo_names{ii};
  eval(['ys(' int2str(ii) ') = ' varname ';']);
end
```

Chapter 3

Information Speed Bumps in the Labour Market

Abstract

Survey data from the Michigan Survey of Consumers reveals that uncertainty perceived by workers is counter-cyclical, while the dispersion in expected changes to their income is pro-cyclical. Flows between unemployment and employment, as well as between jobs show that labour market fluidity is strongly pro-cyclical. A structural VAR model suggests that both uncertainty and productivity drive this labour market sclerosis marked by fewer transitions between jobs and from jobs to unemployment. However, the conventional way of implementing uncertainty in a labour market model by increasing the volatility of productivity shocks, cannot capture this cyclical sclerosis induced by uncertainty. Increasing the volatility of productivity shocks leads to growth option effects in a worker's output, which will ultimately result in more rather than less labour market fluidity. This paper develops a model where the increased uncertainty measured in the labour market is the result of higher information frictions in recessions, rather than a direct increase in the volatility of shocks. Instead, reduced information flows lead to an increase in the posterior about a job's fundamentals, which is interpreted as worker uncertainty and will at a later point during the recovery result in more volatile updates of expected productivity. During the period of reduced information flows action margins are less likely to be hit meaning fewer transitions occur. The paper shows that cyclical information flows can parsimoniously reconcile survey measures of worker uncertainty with data on job transition decisions. "Information speed bumps" meaning reduced information flows in the labour market during downturns can contribute to explaining labour market sclerosis and the breakdown of job ladders by leading to a by 18% to 33 % increased contraction of the hiring rate depending on other labour market frictions. Finally, information dynamics also induce worker mismatch during and after the recession and can contribute to understanding gradual recoveries.

Keywords: *Labour market misallocation, uncertainty, job ladders, information frictions, search and matching, directed search block recursive equilibrium*
JEL Classification: *E24; E27; E32.*

3.1 Introduction

What is the current value produced in this job and how has it changed from yesterday's? This kind of question is important to any worker, and any profit-maximising firm, but a question that neither workers nor their managers can usually answer with ease or precision. The worker would like to know the answer to this question in order to know which wage to ask for, and whether staying with the current employer offers better career prospects than could be found somewhere else. Similarly, the firm's management would like to know the answer to the question because it is decisive in determining whether it is still profitable to employ the worker and what benefits to offer the worker to entice her to continue doing her current job, or whether a superior strategy is to encourage the dissolution of the employment relationship. To some extent, the worker and the firm are always uncertain about the exact value of an employment relationship, and this uncertainty may vary over the business cycle.

Survey data from the Michigan Survey of Consumers reveals that uncertainty perceived by workers is counter-cyclical, while the dispersion in expected changes to their income is pro-cyclical. These observations are at odds with conventional modelling of uncertainty, which is usually argued to lead to increases in the dispersion in fundamental shocks. At the same time, flows between unemployment and employment, as well as between jobs show that labour market fluidity is strongly pro-cyclical. Uncertainty perceived by workers is shown to reduce labour market fluidity in a VAR with short-run restrictions rather than increase it, a result that is also at odds with conventional modelling approached of uncertainty as increasing the dispersion in fundamental shocks to employer-employee relationships would make the labour market more fluid.

The model presented in this paper captures these empirical observations by reinterpreting the uncertainty measured in the Survey of Consumers as workers being uncertain about the current quality of their job match. In this way, reported uncertainty is captured in the model via the posterior about the quality of a job match, rather than the volatility of idiosyncratic shocks. This subtle distinction can explain the decrease in expected income changes observed in the survey, even though uncertainty increases when less is learned about a job in a downturn. This means the expected quality of the match is less likely to change during the period of reduced information, while the posterior and uncertainty increase. Using the reduction in the dispersion of income change expectations to identify the magnitude of changes to information flows the model can then contribute to

explaining the puzzling behaviour of sclerotic job flows during a downturn, as reduced information can cause a larger contraction of the hiring rate 18% to 33 %, depending on the model calibration. The mechanism causing this reduction in the hiring rate is the following. Reduced information acquisition about job values leads to jobs being perceived as more similar. This will lead to fewer discrete decisions about switching jobs as the incentives to do so decline. Finally, an additional result predicted by this model is that counter-cyclical information frictions will also lead to additional labour mismatch during recessions, which is gradually resolved by the recovery, increased information, and the reemergence job flows.

Economists have widely discussed the causes and consequences of uncertainty during recessions in recent years. While uncertainty does not have a precise definition, a common theme in most work is that in uncertain periods a decline in aggregate productivity is accompanied by a rise in the volatility of shocks to an economic series, resulting in uncertainty. For a firm, this means a recession is accompanied by a rise in the volatility of shocks to its output, profits, sales, or to demand for its products (Bloom, 2014). The causes of this increase in volatility may be manifold, from the breakdown of production networks to changes in downstream demand, or sudden fluctuations in input cost. Workers may similarly be affected by an increase in uncertainty, as their career prospects, job location, tasks, the composition of their coworkers, and benefits may be affected by more volatile shocks.

This paper argues that for studying worker-firm matches and the resulting labour market dynamics, this uncertainty increase should be modelled as a deterioration of the information flow about the fundamental quality of the firm-worker match, and not as an increase in the volatility of shocks hitting the employment relationship and mirroring the impact of uncertainty on firm output. The difference between firm output and the firm-worker relationship is that the former is relatively easy to measure by firms, and optimality problems regarding it usually involve decision-making about continuous quantities. If there is noise affecting this decision-making problem, then by the law of large numbers it will have little effect as output quantities increase. Meanwhile, the quality of the match between a worker and a firm is hard to measure and the decision-making about a match is in every period a discrete problem. Optimal decision-making for the worker firm match is driven by the value of outside options relative to the match, resulting in a question for the worker and the firm on whether it is optimal to continue the match or to sever the relationship due to better alternative options for either side. The worker will seek

to find the state that delivers her the highest long-run utility, while the firm will seek to find the best worker for a position once it has decided to open it. For these reasons, time-varying information flows and noise about the fundamental quality of a job match may be much more important for modelling decisions about firm-worker relationships than for modelling firm decision-making regarding the optimal production problem.

There are three reasons for arguing that there is a decline in information about a job match during recessions, which will qualitatively have a similar effect. The first explanation is the decline in the volume of information containing activities such as declining hours of work, short-term work arrangements and fewer active workers. The second explanation is that there is less information in current match output about persistent idiosyncratic match productivity during a downturn due to very temporary deviations such as sudden non-persistent demand volatility, transitory policy shocks and fiscal stimulus, as well as supply and demand changes due to network re-orderings. For instance, (Engbom, 2021) proposes information loss in recessions due to a temporary rise in non-committed job applications for firms. The third explanation is that an employer-employee pair facing a convex cost of acquiring information will be optimally choosing to devote fewer resources to the information problem in down-times when the joint realisation of the match is less valuable to the parties. This explanation of endogenous information acquisition is further explored in Appendix 3.F.

All explanations lead to slower information acquirement about current match quality and to a widening of the posterior of agents' beliefs about their idiosyncratic match fundamentals, which can help explain more sclerotic labour markets. This reduction in state transitions has been termed a breakdown in job ladders and is well documented in (Moscarini and Postel-Vinay, 2016) and (Haltiwanger et al., 2018) in the US. It has also been shown to exist with regard to career ladders in the UK (Carrillo-Tudela et al., 2016). While there may be many reasons for this phenomenon and cyclical match efficiency ((Sedláček, 2014) and (Acharya and Wee, 2020)) is certainly an important part of the story, this paper explores another mechanism, which has allocative effects. On the aggregate, decreased information will lead in the short-run to a smaller variance in the quality of worker firm job matches, which may contribute to explaining pro-cyclical residual wage volatility as documented in (Morin, 2019). This paper is the first that offers a mechanism, which parsimoniously links the experienced uncertainty of workers in their jobs during a downturn and recovery to the decline in state transition decisions during this time.

The reduced likelihood of reaching action margins following a negative shock cannot be delivered by modelling the impact of uncertainty on the labour market via an increase in the volatility of shocks. Papers that have investigated uncertainty via an increase in the volatility of micro or macro shocks to match productivity have found exactly this result. The seminal paper in this category is (Schaal, 2017). The paper implements a (Hopenhayn, 1992) style model similar with time-varying uncertainty similar to (Bloom et al., 2018) in a directed search block-recursive framework. The modelling strategy allows for testing the importance real option effects may have on job flows, but it also creates Hartman-Abel-Ohm, and growth option effects. Due to the latter effects dominating, an increase in idiosyncratic volatility leads to action margins more frequently being hit, and labour markets becoming more fluid during times of uncertainty. As a result, the model produces a good fit of the unemployment series, but the general equilibrium version of the model produces impulse responses of flow measures to uncertainty shocks that are counterfactual to the data. Other papers in this category increase the volatility of aggregate shocks (Lin, 2018) and (Guglielminetti, 2016), which may lead to the unemployment series behaving correctly in response to an uncertainty shock as the job destruction margin is the only action margin being hit more often at the beginning of a recession, but have no notion of between job transitions. All these papers moreover assume that expected fundamentals of a job match become more volatile when a recession hits, which would imply that income should become more volatile too.

Ideas similar to the ones in this paper are discussed in the presentation of empirical data in (Schreft et al., 2005), but without a model. (Baley et al., 2019) present a similar dynamic-block recursive model focusing on information loss due to career switching, and found an increase in uncertainty and mismatch driven by workers choosing new career paths. The difference to the model presented here is that the mismatch is only driven by workers switching jobs and not by continuing relations. Furthermore, the idea about measures of uncertainty being caused by decreasing information flows, and an increase in the posterior about fundamentals is also a driving force in (David, Hopenhayn, and Venkateswaran, 2016) and in (Fajgelbaum et al., 2017). However, in both papers, the focus is not on the labour market, an area where I argue that the effect should be especially pronounced due to the frequency of discrete decisions.

In the next section, I will first present data showing that expected fundamentals about a job match are becoming less volatile in recessions and that uncertainty for workers rises persistently in a recession, while the fluidity of the labour market declines. These facts

are difficult to explain with the conventional modelling strategies. I will then present a simple structural VAR model which shows that employee uncertainty declines and the dispersion of expected income changes increases when the economy is hit by a positive short-run output shock, while increases in uncertainty and decreases in dispersion are going hand in hand with match action margins being hit less often, leading to declining hiring and separation rates. In the third section, I will present key mechanisms in stylised examples to build intuition for the heterogeneous agent block recursive model that follows. Section four then presents a block-recursive model as proposed in (Menzio and Shi, 2010a) with the aggregate state affecting information flows. The model is capable of capturing both job-to-job as well as job to unemployment and unemployment to job transitions. Finally, section five presents the results from simulating the block-recursive model and shows that accounting for cyclical information frictions in the labour market leads to an additional decrease in the efficient allocation of workers, which however subsides once the economy recovers. Section 6 concludes.

3.2 Empirical evidence

The data presented in this section outlines three empirical facts. First, the data suggests that the distribution of shocks hitting a job match is pro-cyclical. Second, surveys show that workers experience increased uncertainty during a downturn. Third, as worker uncertainty rises and the distribution of shocks hitting a match shrinks, action margins are hit less often and the share of state transitions of workers from one job to another, or to and from unemployment collapse. These three facts can be parsimoniously explained with cyclically declining information flows about the job match. This hypothesis will be explained below following the presentation of the data and before the development of the structural vector auto-regression, which confirms that the outlined relations correspond to the data.

3.2.1 Labour market uncertainty is counter-cyclical, but the idiosyncratic volatility of shocks to the job match is pro-cyclical

Ideally, the cyclically varying information hypothesis developed in this paper would be tested directly with a time series of survey questions focusing on the individual worker's or firm's perceptions about the value of their match paired with match product outcomes. Survey questions providing information of this sort would be akin to:

- *What value-added do you think your employee produces?*
- *What value-added do you think your employee will produce next period?*
- *What value do you think is produced by you in your job?*
- *What value do you think will be produced in your job next period?*

Surprisingly, even though the value of a match is often at the centre of models of the labour market, no such questions are to my knowledge found in any longitudinal surveys. Thus the information and uncertainty state of a worker has to be inferred more indirectly from related survey questions. However, for the data shown below it is argued that a decline in the information about match quality during downturns can parsimoniously explain the observed decrease in the volatility of the shocks hitting to idiosyncratic match quality, rising uncertainty perceived by workers and a temporary labour market sclerosis. The following paragraphs describe data, while the VAR in the next section shows dynamic relations between variables.

The most similar data to the questions above is found in the Michigan Survey of Consumers. Concretely the dispersion to the question posed to survey participants *"By about what percent do you expect your income to (increase/decrease) during the next 12 months?"* offers insights. The dispersion in expected change to income is presented in Figure 3.1. Assuming survey participants receive most of their income from wages [1], and that wages w are in some way proportional to match quality z meaning $w \propto z$, both of which are common assumptions in the literature and backed by standard modelling approaches, we can conclude that the dispersion in the expected shocks to idiosyncratic match quality is strongly pro-cyclical. These expectations seem to be in line with the estimations of residual wage dispersion, which is equally found to be pro-cyclical with a lag (Morin, 2019). Replicated Figures from this paper can be found in Appendix 3.A.

Dispersion in wages and expected changes to worker incomes or wages, which can be seen as a measure of the changes in perceived match quality are therefore shown to behave significantly different than other dispersion measures in crisis. The uncertainty reported by workers and shown in the next paragraphs does not translate to a higher dispersion in the predicted wage growth of the individual worker. Observations in the Survey of Consumer Expectations by the Federal Reserve of New York further strengthen this conclusion. The survey shows that workers judging the probability that their guess

[1] The shape of the series remains qualitatively unchanged filtering for lower or higher-income households and respondents at working age.

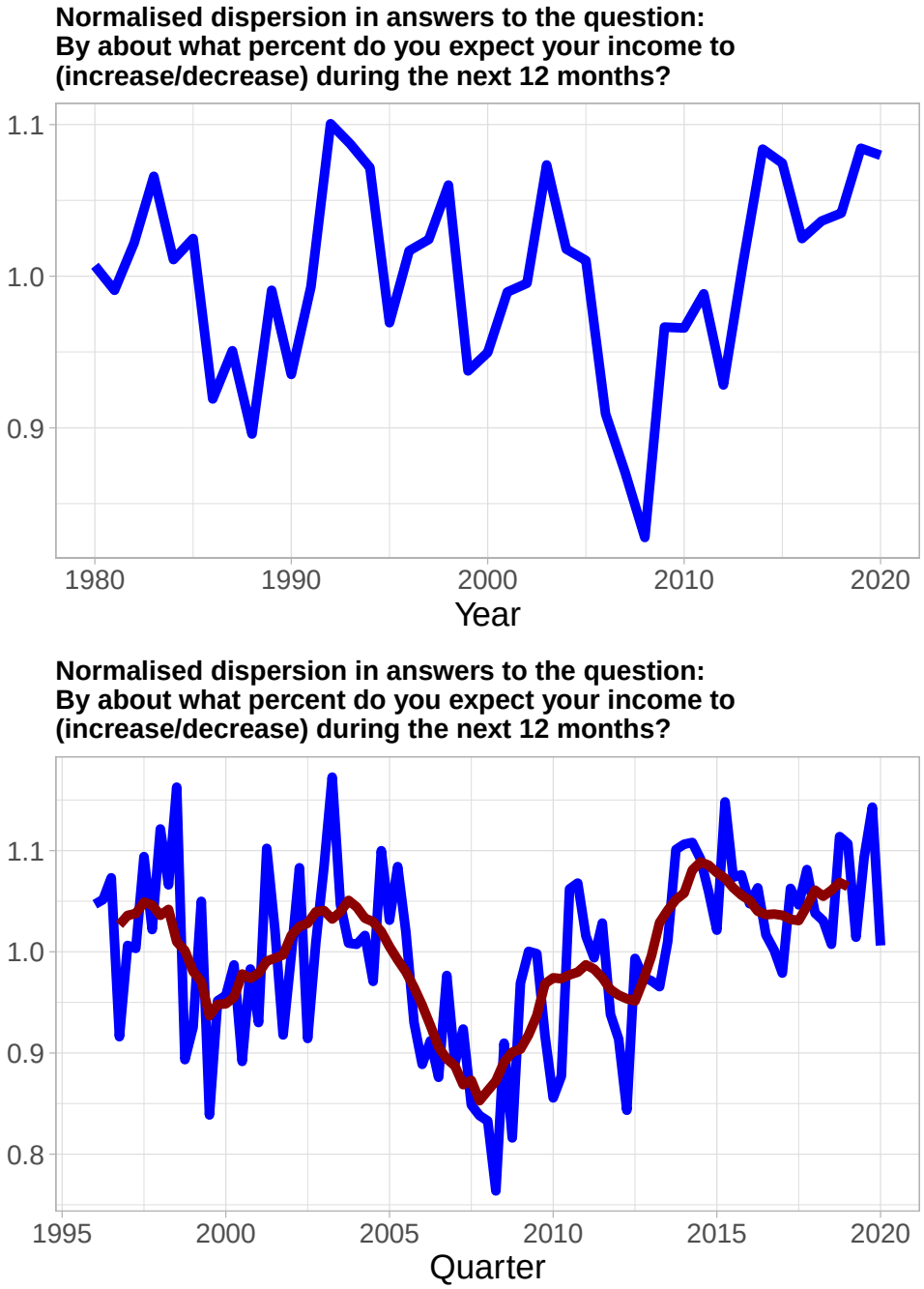


Figure 3.1: Normalised dispersion in the expected percent change in personal or household income during the next year from the Michigan Survey of Consumers. The dispersion is normalised by dividing through the mean dispersion over the sample. The top graph shows the series at an annual frequency which is available for a longer time period, while the bottom graph shows the series at a quarterly frequency. The dark-red line shows a moving average over two years.

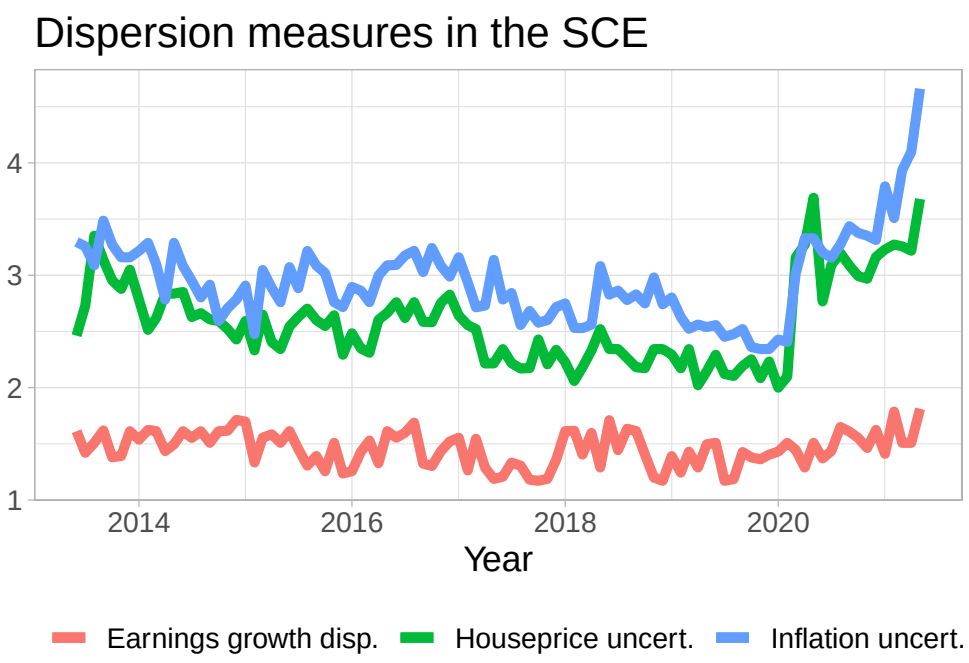


Figure 3.2: "Uncertainty" or about inflation and wage growth from the Survey of Consumer Expectations

for their wage growth next year is correct do not show more uncertainty with the beginning of the Covid-crisis. This is different to other questions in the survey where workers are asked about the probability that their expectation of inflation or house price changes is correct. These "Uncertainty" measures are shown in Figure 3.2. Thus the cycle also does not lead to more dispersion about the expected change in wage changes.

While the dispersion of shocks to match quality seems to decline during a recession, uncertainty reported by workers increases. Figure 3.3 shows equally weighed the judgement of consumers about the state of buying conditions for durable, vehicles, and houses with regard to uncertainty. A high value of this index means buying conditions are considered bad by more consumers than unusual due to uncertainty, while a low index means they are perceived as good. This means the uncertainty faced by firm establishments as for instance estimated in (Bloom et al., 2018), behaves significantly different from the uncertainty for individual workers in job matches. Firms experiencing uncertainty face a higher volatility of idiosyncratic shocks during a downturn, while workers experiencing uncertainty are confronted with the opposite development with regard to the volatility of shocks to the job match.

Finally, Figure 3.4 presents the probability survey participants assign to job loss into the future. The question stated to workers is *"During the next five years, what do you think the chances are that you (or your husband/wife) will lose a job they wanted to keep?"*. The data is

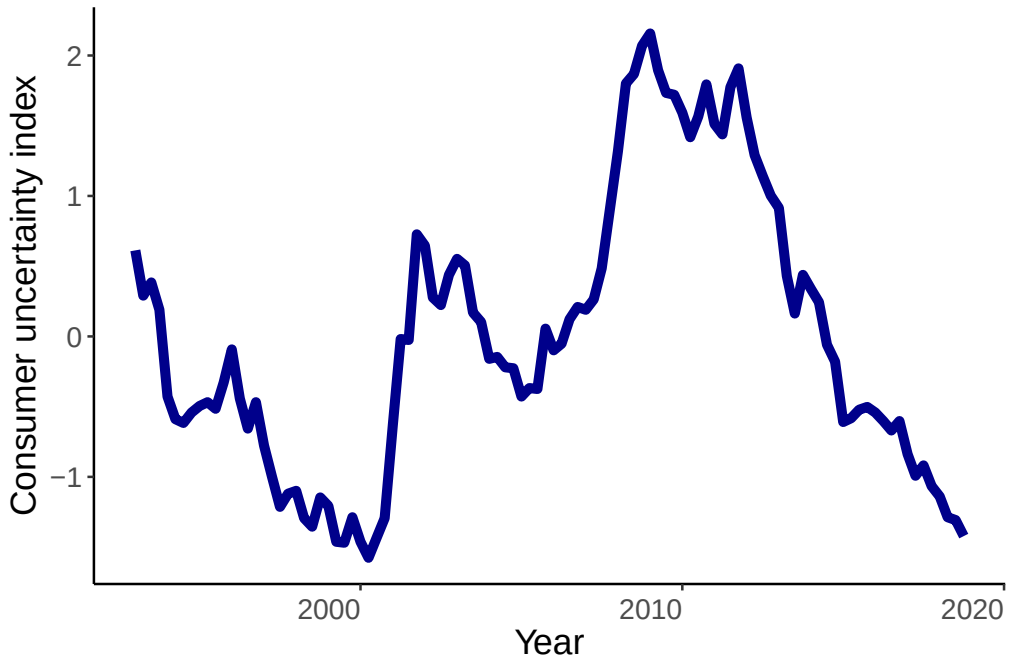


Figure 3.3: Quarterly data from the Michigan Survey of Consumers - index computed from weighing equally indexes asking whether it is a bad time for investments due to uncertainty

filtered to only account for respondents of working age. The shape of the series remains qualitatively the same when filtering only for responses of survey participants without a partner. A way to interpret this graph is to conclude that the certainty of workers to remain in their current job match over the next five years is counter-cyclical with some persistence. This is surprising as uncertainty episodes experienced by firms and in capital markets are usually brief. Meanwhile, a recession seems to leave a certainty scar for several periods into the recovery for workers.

3.2.2 Labour market decision-making

It is evident from the labour market data that action margins for a job match are hit less often in a downturn. This means that fewer discrete decisions are taken in the labour market as the dispersion of shocks hitting job matches decreases and perceived worker uncertainty rises. Figure 3.5 shows the hiring rate and separations rate decline during the early 2000 and the great recession.

The hiring rate includes transitions of workers from a job to another, as well as transitions from unemployment into a job. Similarly, separations includes layoffs and quits into unemployment as well as quits to take up another job. As can be seen the hires and separations series move largely in tandem with most separations being due to hires and vice versa, as layoffs only spike during the beginning of a downturn. Data from the

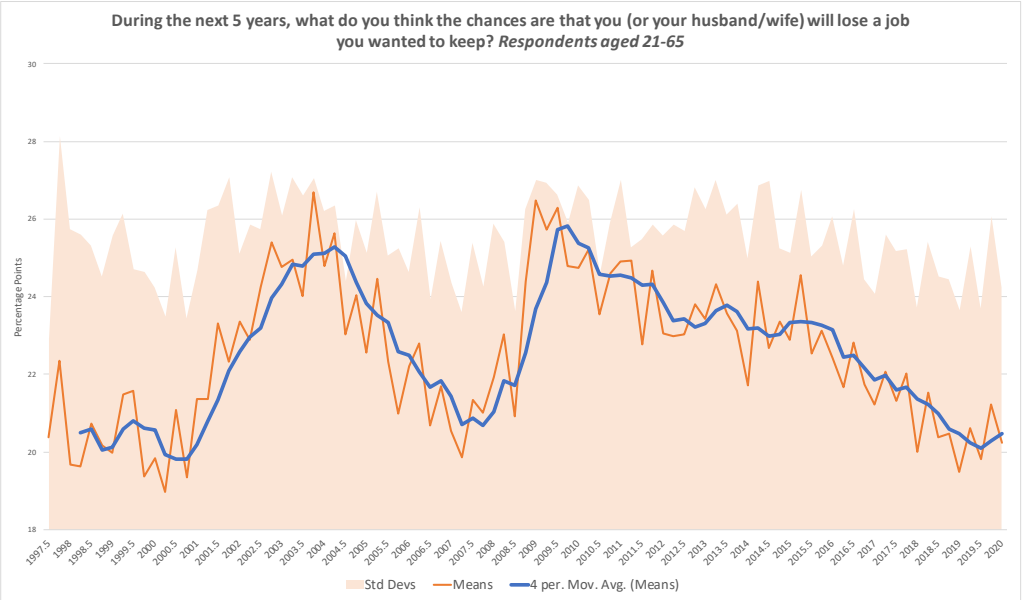


Figure 3.4: Quarterly data about the expected probability of an employee or their partner being laid off extracted from the Michigan Survey of Consumers

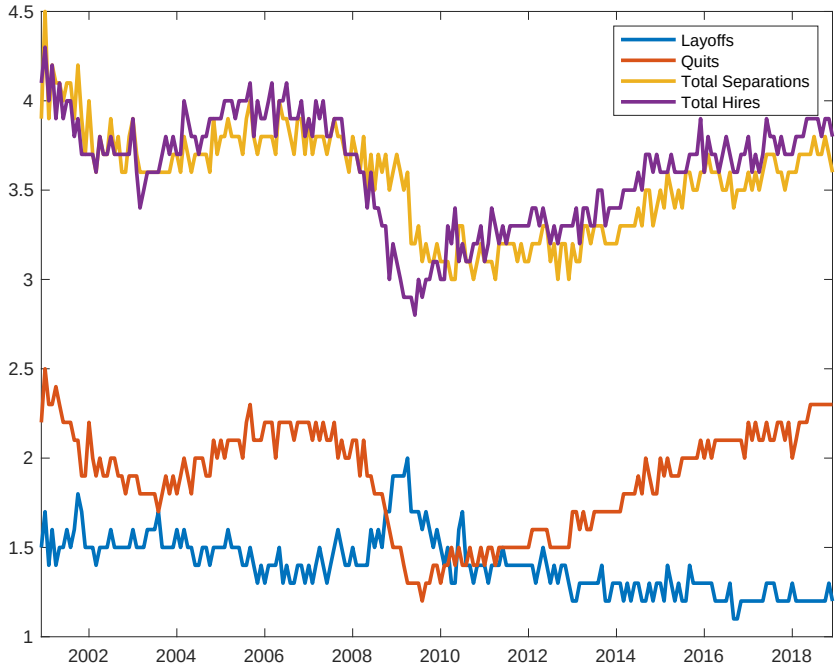


Figure 3.5: Data from the Bureau of Labour Statistics JOLTS survey

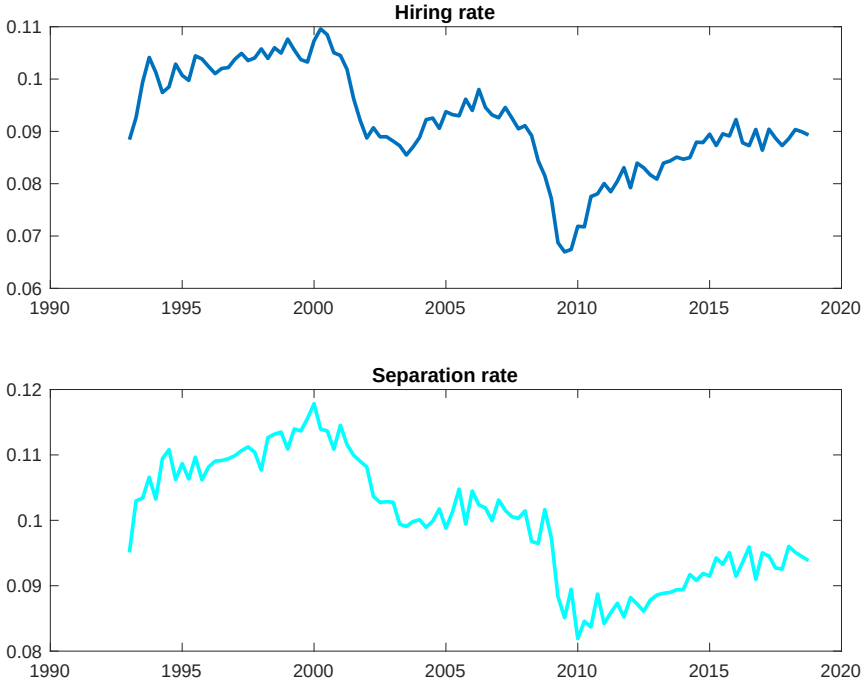


Figure 3.6: Hiring and separation rates from the Quarterly workforce indicators

Quarterly Workforce Indicators in Figure 3.6 shows a very similar picture for the hiring and separation rate as the JOLTS data. The decline in actions taken in the labour market results in a breakdown in job ladders as documented in (Haltiwanger et al., 2018) and a breakdown in career ladders as described in (Carrillo-Tudela et al., 2016).

3.2.3 Interpreting the observations

The data presented in the previous section shows that the dispersion of shocks hitting a job match and labour market transitions are pro-cyclical, while experienced worker uncertainty is counter-cyclical. The hypothesis postulated in this paper to explain this is that the aggregate state of the economy also influences the information flow of agents about the quality of their job match. A reduced flow of information during a downturn will lead to a reduced change in the expected value of a job match, as periods with a low aggregate state will be less informative. This can explain a reduced dispersion in the perceived shocks hitting the estimated expected value of a job match, assuming fundamentals of the job match actually change at a constant rate while information flows are cyclical. Further, a reduced information flow during downturns would lead to an increased posterior about the estimated expected value of a match as changes to the fundamental job match quality may happen during times when little information is available. This may explain

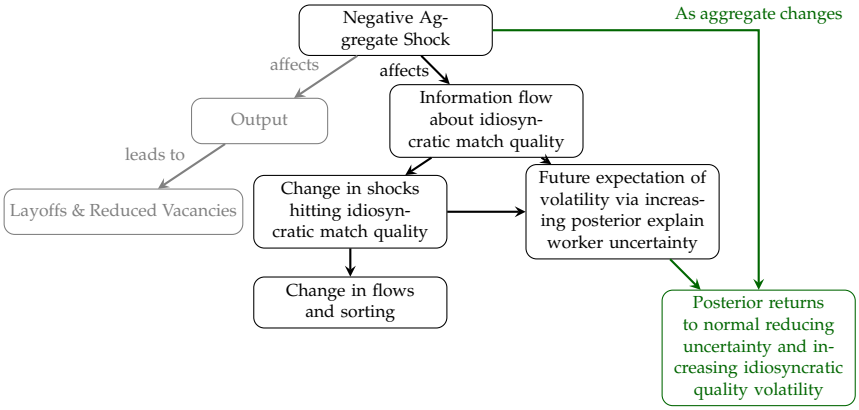


Figure 3.7: Illustration of the mechanism

the increase in reported worker uncertainty following a downturn. Worker uncertainty is then interpreted as uncertainty about where the job match ranks relative to other jobs and whether its fundamentals would still justify a continuation of the match. Finally, reduced information flows will lead to a less frequent hitting of action margins as the estimated expected value of job matches is moved less during less informative periods. This may explain in part the exceptional breakdown in discrete labour market decision-making during a downturn as action margins are hit less often when idiosyncratic shocks become less volatile. Figure 3.7 illustrates this mechanism.

3.2.4 VAR

To test if the proposed dynamics are consistent with the data a simple structural vector auto-regression with short-run restrictions is estimated from the data. The hypothesis provides a natural ordering for the structure imposed on the unrestricted vector auto-regression. A shock to aggregate worker productivity will affect the information flow, uncertainty, and labour market transitions. As a result, the variable is ordered first. The variable used to measure worker output is the log of real output per worker (Fred code [OPHNFB]) computed by the Bureau of Labour Statistics. As Figure 3.7 shows the next variable in the ordering should be the dispersion of shocks hitting the job match. Dispersion should become smaller when a negative shock hits and larger when a positive shock hits due to the hypothesised changes in the flow of information. The measure used is the quarterly dispersion measure for expected changes to household income (Figure 3.1) lagged by four periods to match the expectation with the output and uncertainty realisation. A reduction in dispersion will be the result of a decreased information flow leading to the measure of uncertainty being ordered next in the structural VAR. The uncertainty series used is the series presented in Figure 3.3 and is ordered next. Finally, the hiring

and separations rates are ordered last and show the impact the previous variables have on labour market transitions. The ordering last assumes that labour market transitions will have no contemporaneous effect on the other variables. All series are filtered for business cycles with a Hodrick-Prescott filter calibrated appropriately for quarterly data.

The vector showing the ordering of the VAR model is summarised by Y_t and presented in equation (3.1). The reduced form VAR equation estimated can be found in equation (3.2), while Appendix 3.A contains the results of the unrestricted estimation of the model and the results once short-run restrictions are imposed. The lag length is chosen with the Bayesian information criteria. As the quarterly series only consists of 84 observations narrow confidence bands for high valued confidence intervals are hard to achieve. To provide a more informative graph on the likely impulse response, besides the median impulse response, confidence intervals are computed via bootstrapping at the levels 0.66 and 0.95 to illustrate an area where the impulse response is more likely than not. In all Figures, the confidence band for the 0.66 level is plotted with dark-red lines, while the confidence band for 0.95 is plotted with light-red lines.

$$Y_t = \begin{bmatrix} \text{Output per worker} \\ \text{Dispersion} \\ \text{Uncertainty index} \\ \text{Hiring rate} \\ \text{Separations rate} \end{bmatrix} \tag{3.1}$$

$$Y_t = Constant + \beta Y_{t-1} + Shocks \tag{3.2}$$

The hypothesis outlined in Figure 3.7 predicts that a rise in output per worker will increase the flow of information and thereby lead to a higher dispersion of shocks as well as an increase in reported uncertainty. This predicted response is indeed shown by Figure 3.8. Dispersion increases and uncertainty declines in response to an output per worker shock. Figure 3.9 shows the response of output per worker on the detrended hiring and separations rate controlling for dispersion and uncertainty. As predicted, a positive output per worker shock is shown to lead to increased labour market fluidity, while a downturn will lead to sclerosis and fewer discrete decisions taken leading to fewer transitions in job match states. Hiring rates are rising immediately in response to a positive shock while separations rates may drop briefly as matches become on average

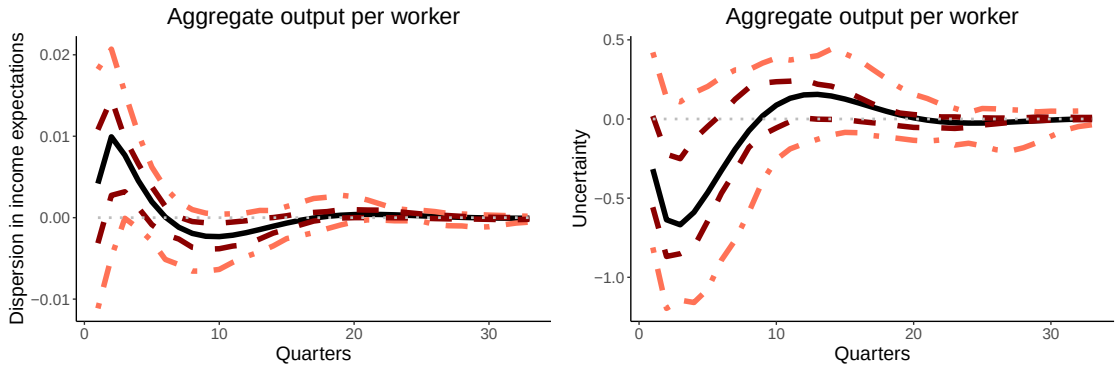


Figure 3.8: Results from the structural VAR using short-run identification of the output per worker on the dispersion of income growth and consumer uncertainty. The black line shows the identified impulse response while the confidence intervals are created via bootstrapping at the confidence levels 0.66 (dark red) and 0.95 (light red).

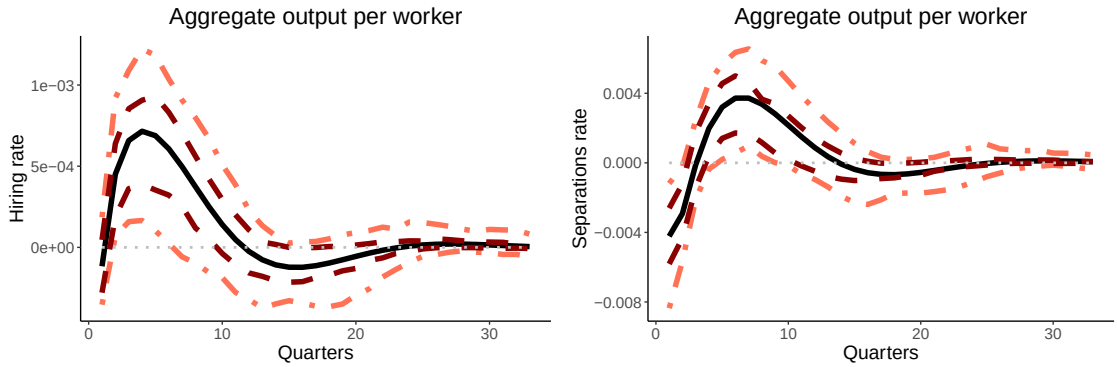


Figure 3.9: Results from the structural VAR using short-run identification of the output per worker on the hiring and separations rates from the quarterly workforce indicators. The black line shows the identified impulse response while the confidence intervals are created via bootstrapping at the confidence levels 0.66 (dark red) and 0.95 (light red).

more productive, but do rise shortly thereafter due to more frequent transitions. Finally, Figure 3.10 shows the response of the hiring rate to dispersion and uncertainty shocks. An increase in dispersion is accompanied by an increase in the hiring rate, while an increase in the uncertainty experienced by consumers reduces the hiring rate. Both these results are consistent with the hypothesis in Figure 3.7, if the uncertainty shock is also capturing a decline in information flows which was not as well captured by dispersion movements.

3.3 Model mechanisms

This paper proposes that the aggregate state of the economy affects the matches in the labour market through two channels. The conventional channel runs via the productivity of the match. The additional channel proposed works via the information content regarding the productivity of a specific match. It is proposed that recessions reduce this information content received by inducing noise in the individual information extraction problem of a match leading to jobs becoming more similar as expectations about the pro-

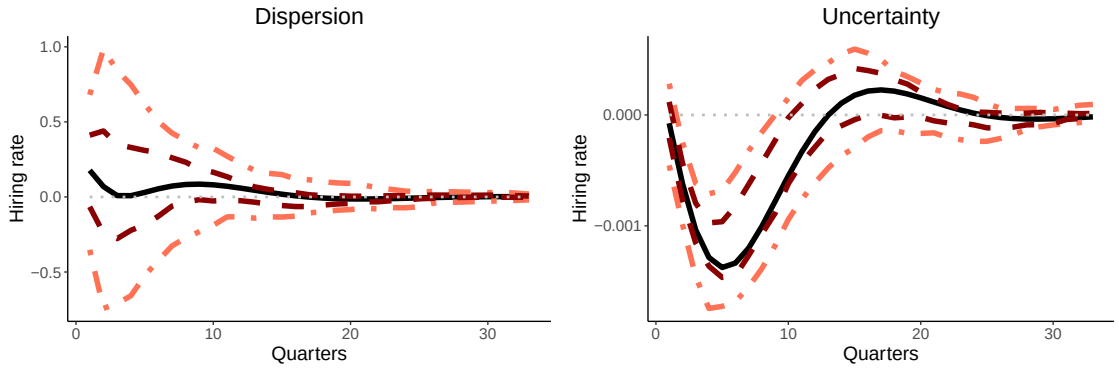
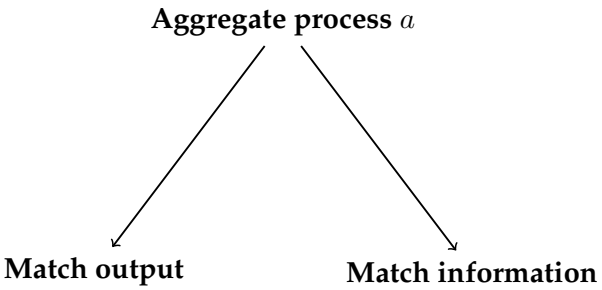


Figure 3.10: Results from the structural VAR using short-run identification of a shock to dispersion of shocks to income (left) or uncertainty of consumers (right) on the hiring rate. The black line shows the identified impulse response while the confidence intervals are created via bootstrapping at the confidence levels 0.66 (dark red) and 0.95 (light red).

ductivity of a match change by less, and regress to the mean. This then leads to reduced churn in the labour market during recessions and recoveries as action margins are hit less often. This section presents several simplified models to make the forces driving the dynamic block recursive directed search model clearer. The section starts with the information extraction problem and the behaviour of the agent Kalman filter, and then develops a random search model to illustrate the impact of noise on action margins. Finally, the section presents intuition for endogenising the individual information flow and the result that cyclical information flows may endogenously arise when there are convex information extraction costs.



To keep the model as simple as possible the idiosyncratic quality of a match is summarised with the variable z_i

$$z_{i,t} = \rho_z z_{i,t-1} + \zeta_{i,t} \qquad \zeta_t \sim N(0, \sigma_z) \tag{3.3}$$

The information problem is introduced via the assumption that the idiosyncratic match quality can never be directly observed but is inferred from past estimates of the match quality, and the current observation of output produced by a match. These together lead to an expectation about the idiosyncratic quality of the match $\hat{z}_i$, which is the variable

based on which all labour market decisions, from the firm vacancy posting over the wage to state transitions are made. Idiosyncratic output $y_{i,t}$ is the main variable informing $\hat{z}_i$ and is the result of the idiosyncratic quality of the match, a noise term ν which's dispersion is a function of the aggregate state, and the aggregate state itself.

$$y_{i,t} = f(a_t + \nu_i(a_t) + z_{i,t}) \quad \nu_i \sim N(0, \sigma_\nu(a_t)),\ z_i \sim N(\mu_z, \sigma_z) \tag{3.4}$$

One needs to make assumptions about the functional form of $\sigma_\nu(a_t)$. The suggested functional form is given in equation (3.5) and ensures that the variance of noise is positive for any realisation of the aggregate state ($\sigma_\nu(a_t) : \mathbb{R} \to \mathbb{R}^+$).

$$\sigma_\nu(a_t) = \zeta_1\, \sigma_z \exp(-\zeta_2 a_t) \tag{3.5}$$

In equation (3.5) ζ_1 controls the level of information in a period of observation while ζ_2 controls the elasticity of information to noise.

It is assumed that agents solve the information problem rationally with a Kalman filter. Section 3.3.1 builds intuition on how the agent Kalman filter works in this case, while the Section 3.3.2 describes a random search model where the information problem is reduced to a single period.

3.3.1 Agent Kalman filter with time-varying noise

Estimating the auto-regressive process z_t with a signal y_t subject to time-varying noise $\sigma_{\nu,t}$ leads to the posterior of the Kalman filter not to converge in a standard fashion. It is rather a state variable describing the extent to which the realisation of actual z is unknown around expectation $\hat{z}$, and will be interpreted as capturing the reported uncertainty of workers in a survey. The system of equations describing the Kalman filter is in (3.6).

$$\begin{aligned} \hat{z}_{i,t+1} &= (1 - K_t)\hat{z}_{i,t} + K_t s_t \\ K_t &= \rho_z \frac{p_t \sigma_{\nu,t}}{p_t + \sigma_{\nu,t}} \\ p_{t+1} &= \rho_z K_t + \sigma_z \end{aligned} \tag{3.6}$$

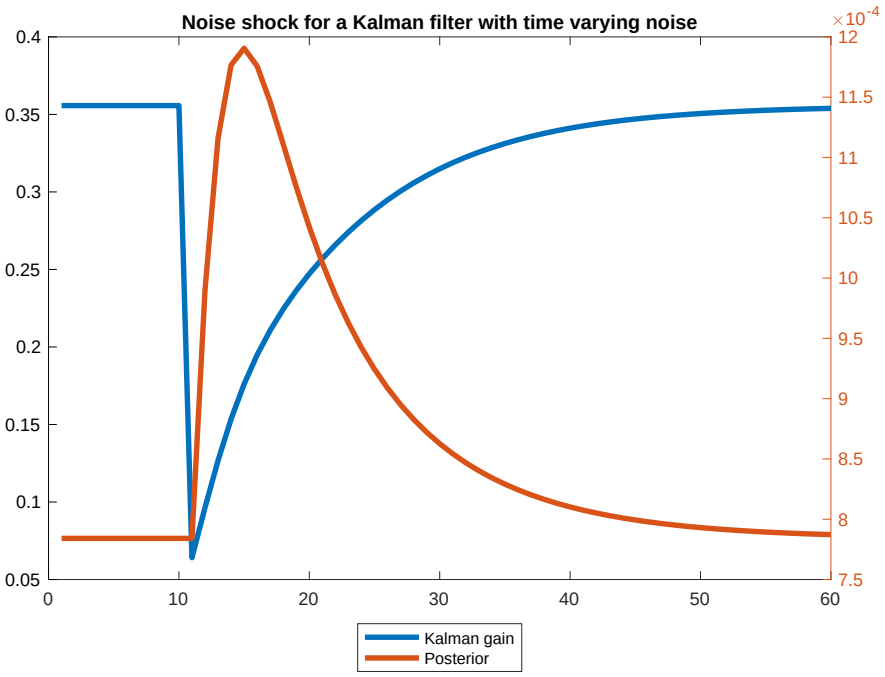


Figure 3.11: Noise shock on a converged Kalman gain and the posterior

Increased noise in a downturn means that the Kalman gain of each period will decrease, while the posterior capturing uncertainty will mechanically increase. Figure 3.11 shows the response to a 10% shock to a persistent aggregate process and a noise function with parameters $\zeta_1 = 3$ and $\zeta_2 = 20$. Simulating multiple idiosyncratic auto-regressive processes z shows that rising noise will make each of the processes resemble more the other as they regress to the mean. The lesser information about each stochastic auto-regressive process z reduces the variance of the idiosyncratic expectations about the processes $\hat{z}$. In the model, this is the driver of information-driven mismatch during a downturn and an explanation of the pro-cyclicality of residual wage dispersion as described in (Morin, 2019). Figure 3.12 presents such a simulation with the same parameterisation as in Figure 3.11.

The intuition provided by Figure 3.12 aids in understanding the block recursive model presented in Section 3.4. The rest of the current section will focus on presenting a simple random search model where noise varies with the aggregate state. It becomes solvable in closed form equations by assuming that $\rho_z = 0$ and thereby limiting the model to intra-period heterogeneity.

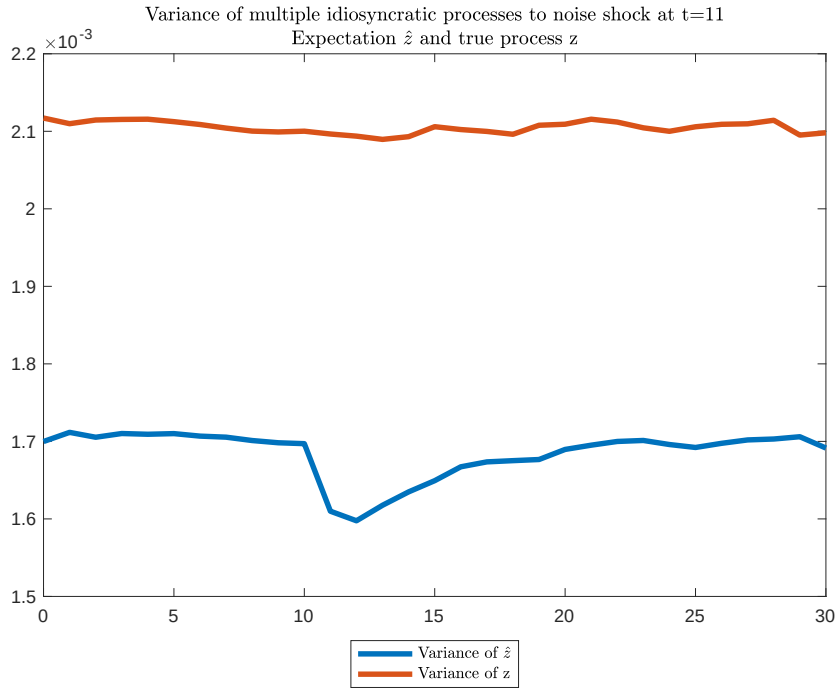


Figure 3.12: Simulation of a noise shock at t=11 of the variance and variance of the expectation 10^4 processes z

3.3.2 Random Search model

The model in this section illustrates the key mechanism of how information frictions lead to smaller labour market flows in a simple random search and matching structure without idiosyncratic persistence or on-the-job search. This means that any heterogeneity in the model is merely within period and any transition is only between employment and unemployment. As a result, some of the main features, especially the medium run effects of prolonged noise on sorting and state transitions cannot be studied. Further transitions are only from unemployment to employment and vice versa and not from job-to-job. Nevertheless, a useful intuition of the effect of noise about the idiosyncratic productivity component can be provided with regard to reaching the job destruction action margin and with regard to changes in allocative efficiency.

Productivity process

The output produced by a worker firm match i consists of an idiosyncratic component z_i and an aggregate component a. Both final output $y_{i,t}$ and $z_{i,t}$ are unknown to agents at the beginning of a period. The final output is the sum of the aggregate and idiosyncratic

components. Next period values denoted $t+1$ are expectations.

$$y_{i,t} = a_t + z_{i,t} \tag{3.7}$$

Workers and firms are assumed to only observe a and to receive a noisy signal about the productivity of a match at the beginning of each period.

$$s_{i,t} = z_{i,t} + \nu_{i,t} \qquad \nu_i \sim N(0, \sigma_{\nu,t}), z_i \sim N(\mu_z, \sigma_z) \tag{3.8}$$

$\nu_{i,t}$ is the noise component around the idiosyncratic match value. Firms and workers estimate the productivity of a match via the signal extraction mechanism in equation (3.9).

$$\hat{z}_{i,t} = \mu_z + \frac{\sigma_z}{\sigma_z + \sigma_{\nu,t}}(s_{i,t} - \mu_z) \tag{3.9}$$

While the idiosyncratic component z is assumed non-persistent both the aggregate productivity component a, and the induced noise component of the signal σ_ν are allowed to be persistent.

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \qquad \epsilon_a \sim N(0, \sigma_{\epsilon_a}) \tag{3.10}$$

$$\sigma_{\nu,t} = \zeta_1 \, \sigma_z \, \exp(\, \zeta_2 \, a_t \,) \tag{3.11}$$

Random search model with endogenous destruction

An unemployed worker's value function is in equation (3.12).

$$U_t = b + p_t \beta \int_{\tilde{s}_{t+1}}^{\infty} (W_{i,t+1} - U_{t+1}) h(s) \, ds + \beta U_{t+1} \tag{3.12}$$

b captures the replacement rate of unemployment, p_t the probability of matching with an open vacancy, β the discount value and $W_{i,t+1}$ the value function of a job match to the worker. $h(s)$ is the probability density function of the signal about idiosyncratic match productivity s, which will be normally distributed. An employed worker receives a wage that is determined by the expected value of production and thus her signal $w(s_{i,t})$. The

functional form chosen for the match probabilities p_t summarises matching frictions and depends on labour market tightness θ_t. It is the result of a conventional Cobb-Douglas matching function with $p_t = m\theta_t^{1-\xi}$ and the probability of a firm finding a worker q_t being $q_t = m\theta_t^{-\xi}$.

$$W_{i,t} = w(s_{i,t}) + \beta \int_{\tilde{s}_{t+1}}^{\infty} (W_{i,t+1} - U_{t+1}) h(s)\, ds + \beta U_{t+1} \tag{3.13}$$

A firm expects that an existing match will produce a value determined by the signal and the future value of a match. The expected present discounted value $J_{i,t}(s_{i,t})$ of an idiosyncratic match is given by equation (3.14).

$$J_{i,t}(s_{i,t}) = a_t + \mu_z + \frac{\sigma_z}{\sigma_z + \sigma_{\nu,t}}(s_{i,t} - \mu_z) - w(s_{i,t}) + \beta \int_{\tilde{s}_{t+1}}^{\infty} (J_{t+1} - V_{t+1})\, h(s)\, ds \tag{3.14}$$

Firms are assumed to only receive a signal about the quality of the worker they have hired after matching. Combining this with the free entry condition yields the familiar condition that the expected cost of a new match $\frac{\kappa}{q_t}$ to the firm equals the firm's expected benefit.

$$\frac{\kappa}{q_t} = \beta \int_{\tilde{s}_{t+1}}^{\infty} J_{i,t+1}(s_{i,t+1})\, h(s)\, ds \tag{3.15}$$

Wages

As conventional in the literature wages are assumed to be determined via the Nash bargaining procedure with the worker's bargaining power being η. The firm is assumed to take on any risk from production not turning out as expected. A simple reasoning for this could be a marginally higher risk aversion of the worker in comparison to the firm. Firms are further assumed to have deep pockets.

Wages are then:

$$w_{i,t} = \eta[a_t + \mu_z + \frac{\sigma_z}{\sigma_z + \sigma_{\nu,t}}(s_{i,t} - \mu_z) + \kappa\theta_t] + (1 - \eta)b \tag{3.16}$$

Action Margin

It is clear that the match should rationally be severed upon receipt of a too negative signal. This defines the cutoff signal at a level where $J_{i,t}(s_{i,t}) = 0$.

$$J_{i,t}(\tilde{s}_t) = 0 = (1-\eta)[a_t + \mu_z + \frac{\sigma_z}{\sigma_z + \sigma_{\nu,t}}(s_{i,t} - \mu_z) - b] + \frac{\kappa}{q_t} - \eta\kappa\theta_t \tag{3.17}$$

$$(\tilde{s}_t - \mu_z)\frac{\sigma_z}{\sigma_z + \sigma_{\nu,t}} = b - [\frac{\kappa}{q_t} - \eta\kappa\theta_t]\frac{1}{1-\eta} - \mu_z - a_t \tag{3.18}$$

$$\tilde{s}_t = \mu_z + \frac{\sigma_z + \sigma_{\nu,t}}{\sigma_z}[b - [\frac{\kappa}{q_t} - \eta\kappa\theta_t]\frac{1}{1-\eta} - \mu_z - a_t] \tag{3.19}$$

Finding the action margin allows for calculating the expected value of output produced by a worker, job creation, and job destruction and thus employment transition. The accepted mean value of the signal is given in equation (3.20).

$$\bar{s}_t = \mu_z + (\sigma_z + \sigma_{\nu,t})^{0.5}\frac{\phi(\frac{\mu_z - \tilde{s}_t}{(\sigma_z+\sigma_{\nu,t})^{0.5}})}{\Phi(\frac{\mu_z - \tilde{s}_t}{(\sigma_z+\sigma_{\nu,t})^{0.5}})} \tag{3.20}$$

From this equation, it is straightforward to find the expected value of output produced by a worker $\bar{z}_t$ in equation (3.21).

$$\bar{z}_t = \mu_z + \frac{\sigma_z}{(\sigma_z + \sigma_{\nu,t})^{0.5}}\frac{\phi(\frac{\mu_z - \tilde{s}_t}{(\sigma_z+\sigma_{\nu,t})^{0.5}})}{\Phi(\frac{\mu_z - \tilde{s}_t}{(\sigma_z+\sigma_{\nu,t})^{0.5}})} \tag{3.21}$$

Note $\frac{\mu_z - \tilde{s}_t}{(\sigma_z+\sigma_{\nu,t})^{0.5}} = [a_t + \mu_z - b + \frac{1}{1-\eta}[\frac{\kappa}{q_t} - \eta\kappa\theta_t]]\frac{(\sigma_z+\sigma_{\nu,t})^{0.5}}{\sigma_z}$. As the Mills ratio is monotonically increasing it's inverse is monotonically decreasing meaning that an increase in noise will lead to $\bar{z}_t$ decreasing.

Finally, the action margin defines the amount of severances in each period as $\Phi(\frac{\tilde{s} - \mu_z}{(\sigma_z+\sigma_{\nu,t})^{0.5}}) = \Phi([b - \mu_z - \frac{1}{1-\eta}[\frac{\kappa}{q_t} - \eta\kappa\theta_t]]\frac{(\sigma_z+\sigma_{\nu,t})^{0.5}}{\sigma_z})$. It is straightforward to see that the amount of job destruction is decreasing in the noise in the system. The reason is that more and less productive matches become harder to separate.

Law of Motion

The only persistent state between periods in this economy besides the aggregate process is the share of workers in employment.

$$n_t = [1 - \Phi(\frac{\tilde{s} - \mu_z}{(\sigma_z + \sigma_{\nu,t})^{0.5}})][n_{t-1} + m_{t-1}] = [1 - \Phi(\frac{\tilde{s} - \mu_z}{(\sigma_z + \sigma_{\nu,t})^{0.5}})][n_{t-1} + v_{t-1}q_{t-1}] \quad (3.22)$$

In the labour matching market tightness is defined is the conventional way as vacancies over unemployed $\theta_t = \frac{v_t}{u_t}$, $u_t = 1 - n_{t-1}$. The probability of a match is defined by the function $q(\theta)$, which is a decreasing function of tightness.

Simulation

The steady-state of the model is presented in Appendix 3.C. The model is perturbed at the second order around the steady-state, and simulated. The model calibration is chosen close to (Shimer, 2005b) and is presented in Table 3.1. Figure 3.13 shows the impulse responses for different values of ζ_2. ζ_2 defines the elasticity of noise in the idiosyncratic signal to aggregate productivity movements.

The simulations show that rising noise around the idiosyncratic component induced by a lower aggregate state reduces labour market churn and increases labour market sclerosis in a downturn. With increasing ζ_2 meaning a higher transmission of the aggregate process on noise, the response of vacancies and flows is for a much longer time subdued due. Vacancies and tightness are reduced by the lower mean worker productivity, which is the result of less effective sorting. It also suppresses the expected present discounted value of a match.

However, these simulations lead for higher values of ζ_2 to a counter-factual response in employment due to the separation rate falling more than the hiring rate. The non-persistence of z_i means that matches are becoming immediately more similar, reducing separations. The hiring rate will not drop as much because, while lesser sorting means the expected value of a match to the firm falls, destruction is becoming less likely feathering off some of the value loss. At the limit with infinite noise, as long as the value of a match is higher than the value of unemployment, this economy would reach a 100% employment rate due to lack of any sorting. This model also cannot account for is job-to-job transitions and sorting as these are meaningless as long as match productivity is non-

Parameter	Value	Description
β	0.99	Discount rate.
ρ_a	0.878	Persistence of the aggregate process.
σ_a	0.0004	Volatility of shocks to the aggregate process.
σ_z	0.75	Volatility of the idiosyncratic process.
ζ_1	2	$\frac{\sigma_z}{\sigma_\nu+\sigma_z} = \frac{1}{3}$ information share in every period
ζ_2	tbd	Parameter determining the elasticity of noise
κ	0.2	Cost of posting vacancies for firms.
b	0.4	Replacement rate for unemployed.
δ	0.01	Chance of exogenous match separation.
m	1	Parameter of the matching function (match productivity)
ξ	0.72	Parameter determining matching frictions
η	0.72	Bargaining power in the Nash bargain

Table 3.1: Calibration for the random search model

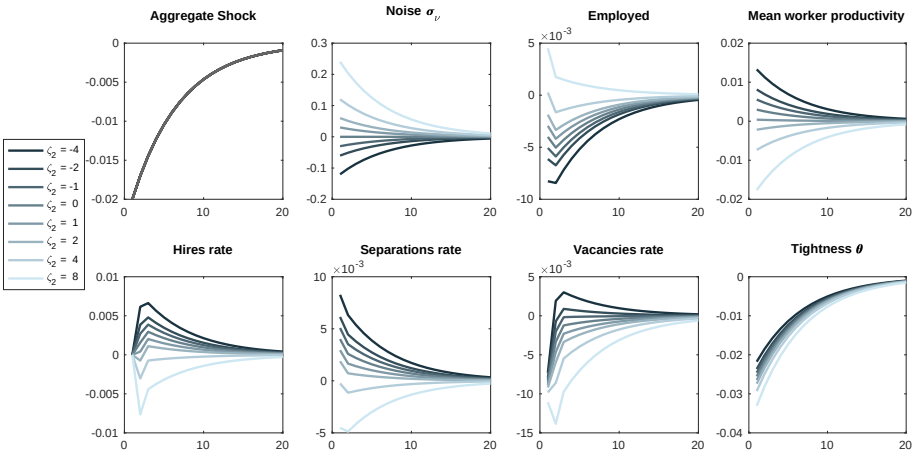


Figure 3.13: Impulse response functions of the perturbed model with an aggregate shock a affecting information at different strengths defined by ζ_2.

persistent. This model should therefore only be understood as building intuition, while the full directed search model with idiosyncratic persistence addressing these issues is presented in Section 3.4.

3.3.3 Endogenising the information flow

The change in information over the business cycle Section 3.3.2 can be endogenised with a costly information model. For this purpose, assume a convex spending function $c(x) \in [0, \infty)$ determining the precision with which the match product can be observed. More spending x will mean smaller noise, which will be a decreasing function of x. This can be thought of as the cost of hiring outside consultants or organising internal human resource activities to judge the value of a match. It can also be thought of as the firm's cost of monitoring the worker, and the workers cost of informing herself about outside opportunities. Assuming spending on information x is chosen in every period the match-

ing parties solve the problem in equation (3.23) to determine the expected value of a job match $\bar{J}$. Unless otherwise stated, parameters capture the same values as in Section 3.3.2.

$$\bar{J}_t = \max_{x_t}[\int_{\tilde{s}_t}^{\infty} J(s_i, x_t)\, h(s)\, ds - c(x_t)] \tag{3.23}$$

This leads to the first order condition in equation (3.24).

$$\frac{d}{dx}[\int_{\tilde{s}_t}^{\infty} J(s_i, x_t)\, h(s)\, ds] = \frac{d}{dx} c(x_t) \tag{3.24}$$

Note using equations in Section 3.3.2 can write $\int_{\tilde{s}_t}^{\infty} J(s_i, x_t)\, h(s)\, ds = \Phi(\frac{\tilde{s}-\mu_z}{(\sigma_z+\sigma_{\nu,x_t})^{0.5}})[(a_t + \mu_z - b)(1-\eta) - \eta\kappa\theta_t + (1-\eta)\frac{\kappa}{q_t}] + \phi(\frac{\tilde{s}_t-\mu_z}{(\sigma_z+\sigma_{\nu,x_t})^{0.5}})\frac{\sigma_z}{(\sigma_z+\sigma_{\nu,x_t})^{0.5}}$ and $\frac{\tilde{s}_t-\mu_z}{(\sigma_z+\sigma_{\nu,x_t})^{0.5}} = [b - \mu_z - \frac{1}{1-\eta}[\frac{\kappa}{q_t} - \eta\kappa\theta_t]]\frac{(\sigma_z+\sigma_{\nu,x_t})^{0.5}}{\sigma_z}$. Providing a convex cost function $c(x_t) = \exp(\zeta_4 x_t)$ and a function for the reduction of noise is $\sigma_\nu(x_t) \in [0, \infty)$, $\sigma_{\nu,x_t} = \exp(-\zeta_3(x_t + zeta_4))$ allows for computation of the resources spent on information x_t given aggregate state a_t. Here ζ_3 is a parameter controlling the elasticity of noise to the spending decision, while $zeta_4$ allows for calibration of noise to the appropriate level.

Figure 3.14 shows the steady-state spending on information, the resulting noise, endogenous job destruction, and the average value of a filled vacancy when aggregate component a is varied. The parameter choice for the information choice problem is $\zeta_3 = 1.25$ and $\zeta_4 = 6$. The figures show that higher aggregate match productivities for all matches mean that more resources will be devoted to reducing noise around the idiosyncratic productivity of a job match. The reason for this is that the benefit from successfully separating bad from good matches increases, as the right graph shows. While there is more job destruction showing a more fluid labour market, the expected benefit of a successful match increases. This result shows that an appropriate calibration allows for the hypothesis of counter-cyclically increasing noise to be straightforwardly endogenised. For simplicity, the model presented in Section 3.4 takes noise to be counter-cyclically increasing as a given without solving and computing an endogenised problem. However, the results of the perturbed impulse responses of this model and a block recursive model with endogenised noise in Appendix 3.E show that this computational simplification can be thought of as the endogenous result of rationally behaving agents facing a costly information problem.

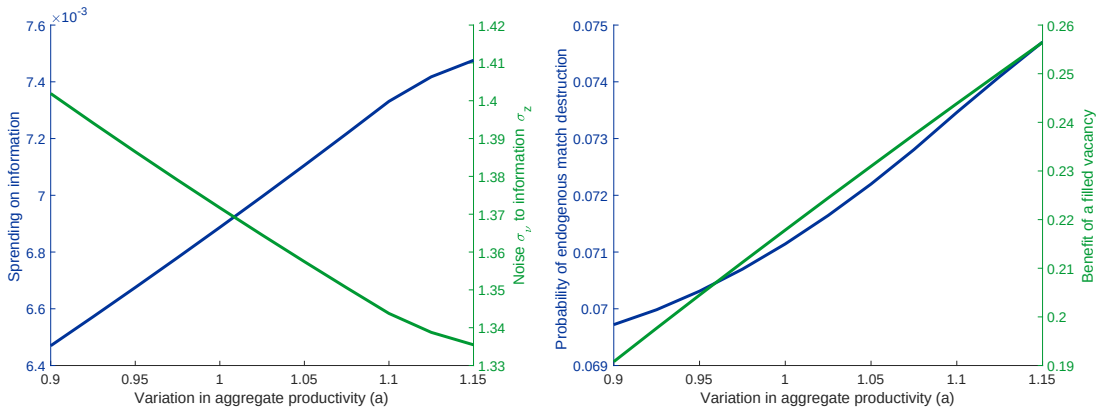


Figure 3.14: Comparative spending on information, noise, endogenous job destruction, and the average value of a filled vacancy when aggregate productivity is varied.

3.4 Dynamic directed search block recursive model

In a random search model with inter-period heterogeneity the policy function of each agent in each idiosyncratic state, whether in a match or without a match, will depend on the policy functions of all other agents in all other states. This makes solving and simulating a random search model with persistent idiosyncrasies of matches and aggregate risk a computationally intensive exercise requiring approximations of the form proposed in (Krusell and Smith, 1998). In contrast, changing the assumption on how search in the market works from random to directed competitive search allows for a dynamic block recursive equilibrium solution. (Menzio and Shi, 2010a) and (Menzio and Shi, 2011) describe, and prove that there exists an efficient block recursive equilibrium (BRE) when agents are assumed to be able to direct their search towards partially complete contracts offering different expected utilities. In the resulting general labour market equilibrium, the optimal policy functions of each agent in each state only depend on the aggregate state of the economy, and not on the distribution of the idiosyncratic states of the agents. The model in this section makes use of this result in order to combine persistent idiosyncratic productivity with an idiosyncratic expected state of the match and aggregate processes describing aggregate productivity and the posterior of all agents in each state.

3.4.1 Exogenous processes

As in the model in Section 3.3.2, there is an aggregate auto-regressive process driving the productivity of each match. This process is given in equation (3.25) and is assumed to be

driven by mean zero normally distributed shocks ϵ_a with variance σ_a.

$$a' = \rho_a a + \epsilon_a \tag{3.25}$$

Additionally, each match's productivity is determined by an idiosyncratic auto-regressive process z_i.

$$z_i' = \rho_z z_i + \epsilon_z \qquad \epsilon_z \sim N(\mu_z, \sigma_z) \tag{3.26}$$

z_i is only indirectly observed via the match's output.

3.4.2 Labour market with information frictions

The labour market consists of ex-ante homogeneous firms and homogeneous workers. Thus the model aims to capture the idiosyncratic dispersion that is unexplained by permanent firm and worker characteristics, such as education, firm size, or industry choice, and to explore to what extent the unexplained residual can be explained by cyclical information frictions.

All firms are assumed to have a large information set over types of workers that they may hire, hence there is only one aggregate information state of the economy defined by the information flow of past periods. This is a simplification and in truth, information states would differ as explored in Appendix 3.E. When information states all experience pro-cyclical information flows then this modelling approach significantly facilitates computation as it means that all employed workers have the same measure of the posterior at all times and the posterior becomes simply another aggregate object. The model with endogenised information flow in the appendix delivers similar dynamics for the information state thus justifying the simplification.

Newly hired workers will start their career at a firm at a position of average expected productivity $\bar{z}$ given their characteristics, but this productivity may stochastically change according to the auto-regressive process outlined above. The idea is to capture wage increases and career progression for an ex-ante homogeneous worker during their tenure at a firm, and their decision to either search and find a better job, or to leave the match for unemployment, and the decision of the firm to lay off the worker if it learns that the match productivity is too low given the wages promised to the worker. This assumption

puts the focus of labour market dynamics on decision-making after the initial matching process is resolved and new uncertainty is created due to possible shocks to the match over time. Thus the focus of the model is not on a (Jovanovic, 1979) process following the initial job match, but rather on the changes that follow once the initial matching problem is resolved. While the model is open to implementing Jovanovic-type learning as well at additional computational cost as shown in Appendix 3.E, recent arguments for this initial learning becoming less important (Mercan, 2017) may justify not opening these hiring frictions and treating them in this case as part of the matching friction.

The idiosyncratic productivity process of a match is assumed to be unobserved. It thus requires solving a signal extraction process for agents to come up with an expectation $\hat{z}_i$ about idiosyncratic productivity. The information flow in a period received by the matching parties is determined by the size of aggregate noise caused by the aggregate process via the same parameterised equation for noise as described in Section 3.3.2.

$$\sigma_\nu = \zeta_1 \sigma_z \exp(-\zeta_2 a) \tag{3.27}$$

Defining $\hat{z}_i$ as the expected level of idiosyncratic productivity, and assuming that rational agents know the distributions of $\epsilon_{z,t}$ and noise ν_t, and that these distributions are normal the information extraction problem can then be formulated as a Kalman filter with time-varying noise levels as described in Section 3.3.1.

$$\hat{z}_i' = \rho_z \hat{z}_i + K(\sigma_\nu, p)(s_i - \hat{z}_i) \tag{3.28}$$

Here $K(.)$ is the Kalman gain determined by the level of noise and the posterior p_t, while $(s_{i,t} - \hat{z}_{i,t})$ is the new information content provided by another observation.

The posterior state of all matches will evolve as noise varies. The state of this aggregate posterior about the actual state of an idiosyncratic match is interpreted as the worker uncertainty recorded by the surveys. It tells the worker and the firm that their current expectation about the productivity of the job match may be false and the range of expected true values. The evolution of the posterior, and therefore uncertainty is captured by equation (3.29).

$$p' = \rho_z^2 p \frac{\sigma_\nu}{p + \sigma_\nu} + \sigma_z \tag{3.29}$$

3.4.3 Contracts and search equilibrium

As is conventional in directed competitive search models, firms post contracts in every period proposing a utility split between the worker and the firm. The proposed contracts offer to provide the worker with a present discounted expected utility x. Firms cannot renege on this expected utility, but employment contracts are state-dependent.

During the search stage in every period, firms decide how many vacancies to create and where to locate them in terms of promised utility x. Creating a vacancy costs firms κ. Since firms are free to enter any sub-markets, a firm's optimal strategy requires choosing a market such that in this market equilibrium expected profits equal expected cost. This free entry condition in every sub-market leads to labour market tightness in every sub-market being a function of the utility promise in the sub-market, the value of a filled vacancy given successful search and the parameters defining the vacancy creation costs and the search frictions.

$$\kappa \geq q(\theta(x,\psi))J(x,\psi,\bar{z}) \tag{3.30}$$

That the idiosyncratic starting value of each vacancy is a given value $\bar{z}$ is provided by the assumption that workers are homogeneous at the start of each job, but is not a computational requirement and can easily be relaxed to any form of stochastic matching. Match probabilities $q(\theta(x,\psi)$ and $p(\theta(x,\psi)$ are a function of the aggregate state and tightness, with the respective CES functional forms $q(\theta(x,\psi)) = (1+\theta_{x,\psi}^{\lambda_e})^{\frac{-1}{\lambda_e}}$ and $p(\theta(x,\psi)) = \theta_{x,\psi}(1+\theta_{x,\psi}^{\lambda_e})^{\frac{-1}{\lambda_e}}$. ψ summarises the aggregate state combinations of the aggregate state and the posterior $\psi = [a,p]$.

3.4.4 Unemployed worker value function

Each worker receives instantaneous utility according to the function $u()$, which is weakly concave, continuous, and strictly monotonically increasing. The chosen instantaneous utility function in the model is the logarithm $\log()$. Workers discount the future with β. An unemployed worker receives the unemployment benefit b, while an employed worker receives a wage. Unemployed workers may with probability λ_u choose sub-market x to search for employment for the next period. x summarises the expected utility a firm commits to deliver to the worker in an employment contract in the sub-market, should the worker be successful in finding a firm in that market. The probability of meeting a firm in

the given sub-market is determined by the function $p(\theta(x, \psi))$, where ψ summarises the aggregate state. In this case, the aggregate state is given by the combination of the aggregate productivity state a, and the posterior or match uncertainty state p. An unemployed worker's value function is in equation (3.31).

$$V_u(\psi) = u(b) + \beta E_{\psi'}[V_u(\psi') + \lambda_u \max_{x_u}[p(\theta(x_u, \psi'))(x_u - V_u(\psi'))]] \tag{3.31}$$

x_u is the sub-market unemployed workers optimally choose to search in given the aggregate state. An employed worker, who has been given the opportunity to search on-the-job will solve a similar policy problem. In this case, the worker will choose a sub-market x to search in, given the firm's promise of delivering utility V' to him if she stays in the current job. Here R is the solution to the optimal search problem for a worker with a job at promised utility provision V'.

$$R = \max_{x_{V'}}[p(\theta(x_{V'}, \psi'))(x_{V'} - V'(\psi'))] \tag{3.32}$$

3.4.5 Firm value function

The firm offers a dynamic employment contract promising utility x at the beginning of the creation of each match. $x = V$, which provided that the match survives, is the remaining present discounted utility promised to the worker. Firms are assumed to be bound to deliver the expected promised utility, not to deliver the actual utility to the worker. Each contract specified by the firm sets out the wage dynamically based on the state of the match z, the aggregate state of the economy ψ, and the promised expected delivery of remaining utility V' given state realisations, as well as the probability of destroying the job given the realised state d'_i. The firm will aim to maximise its own expected profit from the match subject to these constraints, and by choosing a state-dependent contract that specifies policies regarding the current wage w, the probability of severing the worker at the end of the period conditional on the state realisations d', and the promised utility left to deliver tomorrow's V'. The state of each job match can be fully described by the current promised utility V, the aggregate state of the economy ψ, and the idiosyncratic state $\hat{z}$. For simplicity expected match output is assumed to be a simple exponential function of a and $\hat{z}$. The value of a job match to the firm $J(V, \hat{z}, \psi)$ is the result of choosing optimal

state-contingent policies w d' and V' as shown in equation (3.33).

$$J(V,\hat{z},\psi) = \max_{w,d',V'}\{\exp(a+\hat{z}) - w + \beta E_{(\hat{z}',\psi')}[(1-d')(1-\lambda_e p(\theta(x_{V'},\psi'))J(V',\hat{z}',\psi')]\} \tag{3.33}$$

λ_e describes an exogenous probability with which the worker participates in on-the-job search. δ is an exogenous probability of the match being dissolved. As can be seen in equation (3.32) the firm can choose in which location of the market the matched worker will search for a job next period, and hence the probability of the worker leaving by varying V'. The firm will choose V', as other parts of the contract optimally.

The firm's maximisation problem is subject to the following constraints. The first is an incentive compatibility constraint meaning that the worker will choose to sever the match when the promised utility of the period is below the utility of unemployment.

$$d = [\delta \ if \ V_u(\psi') \leq V' + \lambda_e \max_{x_{V'}}[p(\theta(x_{V'},\psi'))(x_{V'} - V'(\psi'))];\ 1 \ else] \tag{3.34}$$

The second constraint is the promise-keeping constraint in equation (3.35), which constrains the firm choice of V', because the firm has to deliver the promised utility of V set out in the contract to the worker in expectation.

$$V = u(w) + \beta E_{(\hat{z}',\psi')}[(1-d')[V' + \lambda_e p(\theta(x_{V'},\psi'))(x_{V'} - V'(\psi'))] + d'V_u(\psi')] \tag{3.35}$$

3.4.6 Block recursive equilibrium

Given this setup, it is possible to compute a dynamic directed search BRE. This is defined by a set of recursively consistent policy functions for the dynamic contract, θ, x, m, U, J, and R, which depend only on the aggregate state of the economy summarised by ψ and not on the distribution of agents over states. Computation relies on finding a fixed point for $J = TJ$, where solving the associated policy problem of agents in each state, together with the transition laws provides for the fixed point operator T. The algorithm describing the computation of the equilibrium is described in detail in Appendix 3.F.

3.4.7 Calibration

The calibration of the model targets the data moments of the hiring rate and the unemployment rate. The exogenous probability of job destruction δ is calibrated to be approximately $1/50$ for every quarter resulting in about 7.8% of job matches being exogenously destroyed in a year due to other reasons than finding a new job, firing, or an unemployment choice. This corresponds to the share of employees exiting employment to retire, or due to disability (Frazis, 2017) so the model can be seen as replacing these workers by new young unemployed workers. The outside value of unemployment b is oriented on (Shimer, 2005b) calibration, which is based on the benefits received by unemployed workers. The idiosyncratic productivity process is assumed to be very persistent, while the aggregate process is oriented to match the properties of the cyclical component of output per worker. Information frictions are assumed high, as the persistence of uncertainty is high for the individual match, with a single quarter only revealing a third of the actual state of the process. Finally, the baseline level of noise ζ_1 elasticity of noise ζ_2 to the aggregate process is chosen such that the dispersion of $hatz$ replicates the movements in dispersion about next periods income growth in Figure 3.1. These dispersion measures mean the model should target updates to the estimated idiosyncratic component to vary in dispersion between 0.87 and 1.13 by targeting an appropriate Kalman gain when the aggregate process is low and high. This is achieved by setting $\zeta_1 = 0.12$ and $\zeta_2 = 5.4$.

The probability with which workers may participate in on the job search λ_e, the matching parameter l and the vacancy posting costs κ are set to two different calibrations to match moments. While l is oriented on (Menzio and Shi, 2011) in one calibration κ is close to the calibration in (Shimer, 2005b) in the other. λ_e is set to 0.25, l to 0.75 and κ to 0.15 in calibration 1 , and λ_e is set to 0.4, l to 0.25 and κ to 0.01 in calibration 2.

The unconditional moments targeted via the calibration are the separation and hiring rate, which is 0.092 in the quarterly workforce indicators, and the unemployment rate at 0.0614 for the same period. The model moments are produced via the simulation in the baseline calibration and presented together with the data in Table 3.3. As can be seen, both calibrations deliver moments that are, if not exact then close to the data. Calibrating the model to match the simulated moments exactly is time-consuming due to the coarseness of the grid required to keep the computing time within bounds, and the non-linear response of the model to parameter variation. However, the two presented calibrations deliver successfully and robustly moments close to the data.

Parameter	Value	Description
β	0.987	Quarterly discount rate.
ρ_a	0.878	Persistence of the aggregate process.
σ_a	0.0001	Volatility of shocks to the aggregate process.
ρ_z	0.95	Persistence of the idiosyncratic process.
σ_z	0.004	Volatility of the idiosyncratic process.
ζ_1	0.012	$\frac{\sigma_z}{\sigma_\nu+\sigma_z} = \frac{1}{4}$ information share in every period
ζ_2	5.4	Kalman gains vary +/- 10 % from income dispersion changes.
b	0.385	Replacement rate for unemployed.
δ	0.02	Chance of exogenous match separation.
	Calibrations 1 & 2	
l	0.75 & 0.25	Parameter determining matching frictions.
λ_e	0.25 & 0.4	Probability employed can participate in on the job search.
κ	0.15 & 0.01	Cost of posting vacancies for firms.

Table 3.2: Calibration of the model.

	Mean Hires Rate	Mean Unemployment rate
Data	0.092	0.0614
Model Calibration 1	0.116	0.066
Model Calibration 2	0.094	0.065

Table 3.3: Moments

3.4.8 The solved BRE

The solved model produces the value of unemployment, the value of a filled vacancy, and policy functions for each state combination (a, p, z, V). In Figure 3.15 the posterior is fixed at its median state. The values are shown for calibration 1, but equivalent graphs for calibration 2 can be found in Appendix 3.D. As would be expected, the graphs show that the value of a vacancy for a firm in the block recursive equilibrium increases in the idiosyncratic and aggregate states, and when a smaller amount of present discounted utility V is promised to the worker. As Figure 3.16 shows this a higher value of a filled job match translates into higher tightness in the respective market sub-market $x = V$. At the same time, higher utility promises to workers lead to higher and pro-cyclical wages. Finally, Figure 3.17 shows that firms will choose a policy of promising higher utilities rather than paying present wages when the aggregate state is low. Meanwhile, workers will search for better contracts as their current contract V increases, but also as the aggregate state increases.

Not all states in the model will actually see large numbers of workers. Some may not be searched for, or some may be below the incentive compatibility constraint as the delivered utility by the firm is below the value of unemployment given the aggregate states a and p. These choices to search in the solved model are displayed in Figure 3.18. The value of the aggregate state is increasing, while the posterior is increasing and then decreasing

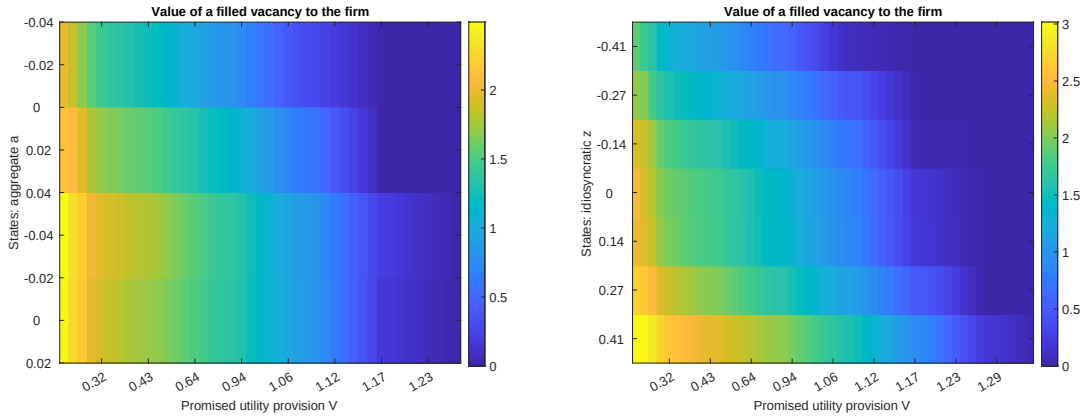


Figure 3.15: The value of a match to the firm depending on the states of the match. The state of the posterior p are fixed to it's median state. Further, in the left graph the idiosyncratic state z is at it's median state, in the right graph the aggregate state a.

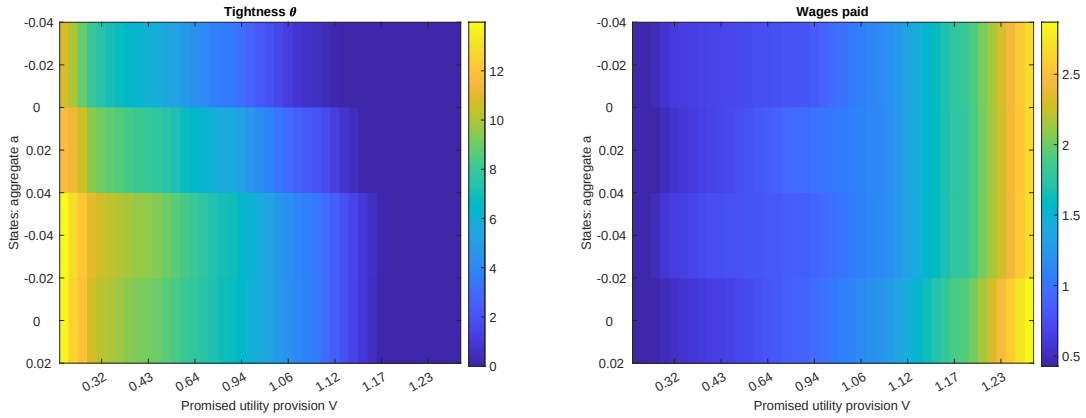


Figure 3.16: Left: Tightness for each sub-market $x = V$ keeping the state of the posterior p fixed to it's median state. Right: Wages paid in each state keeping the state of the posterior p, and the idiosyncratic state z fixed to their median state.

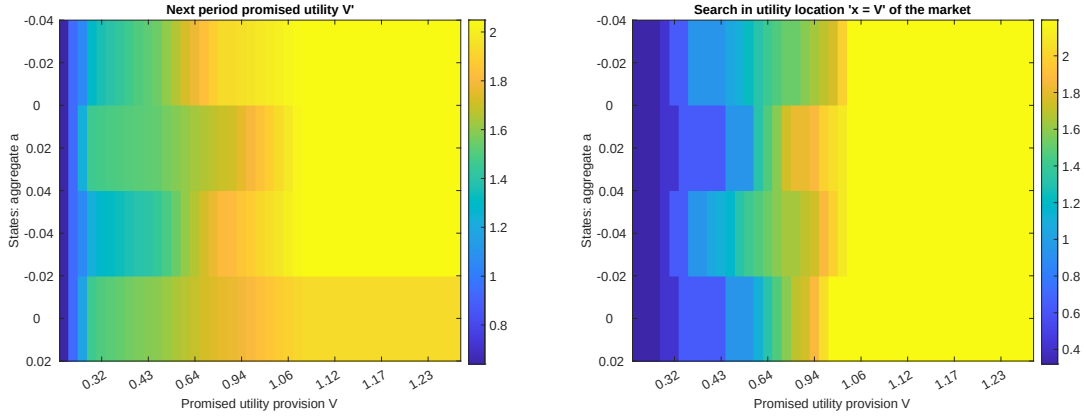


Figure 3.17: Left: Next period's promised utility provision V' keeping the state of the posterior p, and the idiosyncratic state z fixed to their median state. Right: Chosen search location $x' = V'$ for each worker on the job given their aggregate and utility state, keeping the state of the posterior p, and the idiosyncratic state z fixed to their median state.

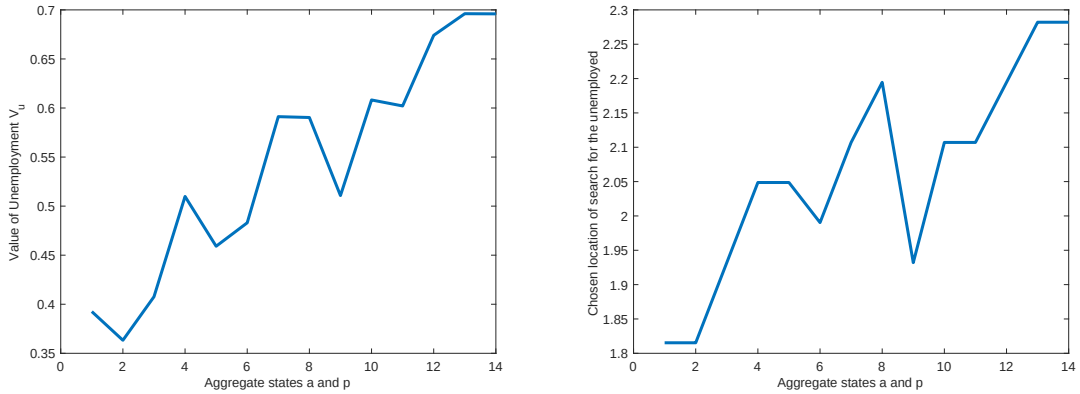


Figure 3.18: Left: Value of unemployment for the aggregate state space combinations a and p. Right: Chosen search location $x'_u = V'$ Value of unemployment for the aggregate state space combinations a and p.

in these graphs.

The dynamics of the solved model in both calibrations are displayed in Figure 3.19. These responses are generated by simulating the Markov Chain of the aggregate states of the model multiple times and computing a mean response, starting in the lowest aggregate state and median posterior state to simulate a shock. The responses show that the posterior rises, and the dispersion of shocks to the expected idiosyncratic match productivity $\hat{z}$ decreases. In calibration one the negative shock will lead to rapid job destruction, followed by a higher number of unemployed hired. This response is quantitatively weaker in the second calibration, possibly due to the lower job-finding rates in the model due to higher labour market frictions. In both cases, the model replicates the large decline in job to job transitions observed in the data.

3.4.9 Simulation

In Section 3.4.8 impulse response functions of the model were computed by simulating the coarse model multiple times. This may not be the best way to compare the model to the data, as both the data and the model are presumably generated with non-linear processes, but the data is analysed using a linear vector auto-regression with short-run restrictions. Hence arguably the best way to compare the model to data is to similarly estimate a linear vector auto-regression from simulated data and impose short-run restrictions to identify the behaviour of conditional moments. In Figure 3.20 the two calibrations are compared to the data using this approach. The figures show that the model is able to capture key dynamics of the data well in both calibrations.

The model ordering is the same as for the observed data. The aggregate state is ordered

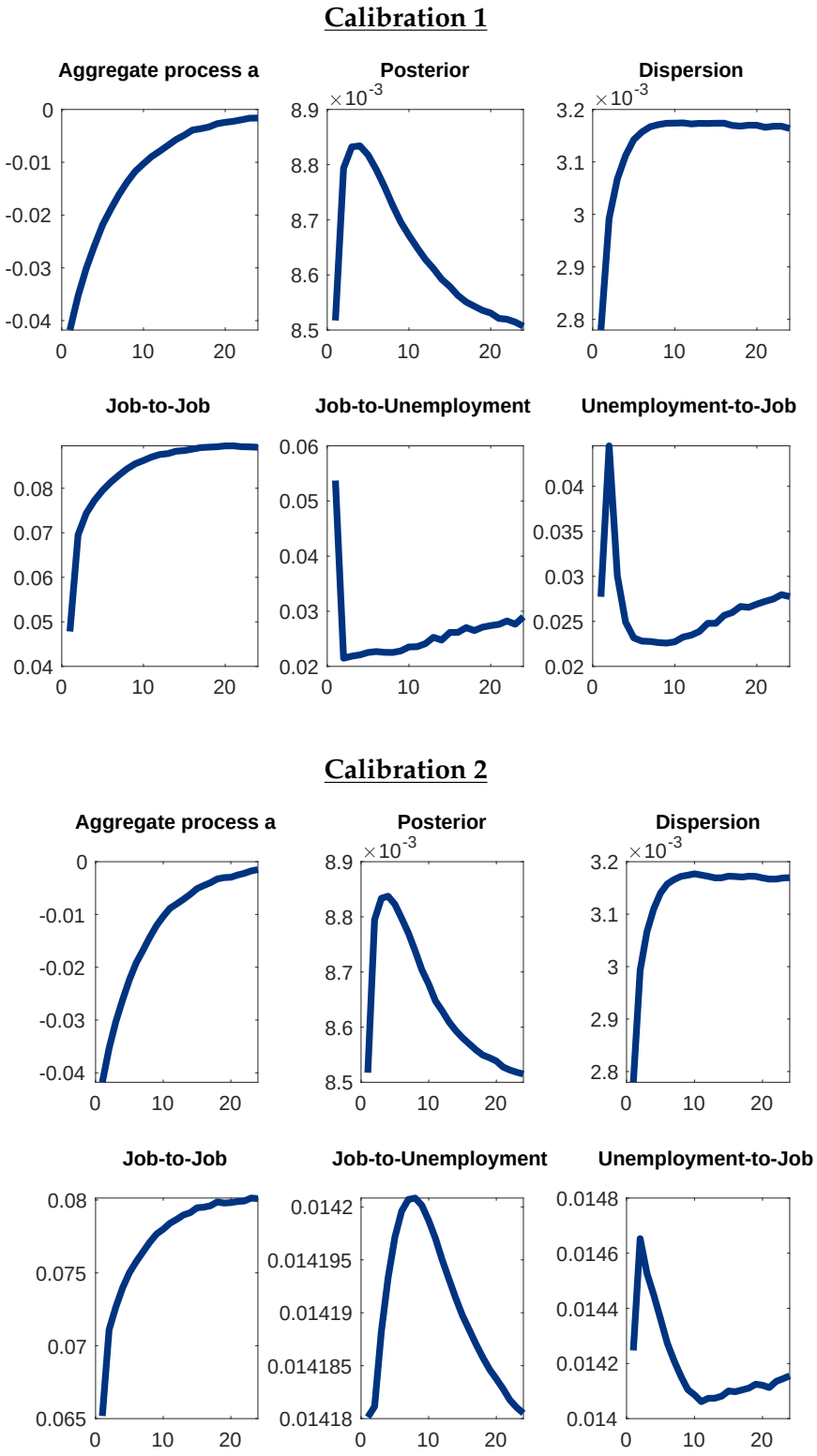


Figure 3.19: Mean dynamics of the model starting from the lowest state a and median p.

first. Next in the ordering is the dispersion in changes to observed $\hat{z}$, which is compared to the expected dispersion in incomes in Figure 3.8. Next uncertainty is compared to the uncertainty index in Figure 3.3. Last, the hiring rate is ordered to capture state transitions. The model produces similar, albeit in parts overly reactive dynamics for the expected dispersion of income changes. The model further matches the dynamics of uncertainty experienced by consumers, well when the posterior of information about a job match is interpreted as this worker or consumer uncertainty. Furthermore, the hiring rate dynamics accounting for uncertainty and dispersion are well matched. The model reacts a bit more abruptly to the shock than in the data, especially in calibration two. This may be due to the assumptions of a homogeneous posterior for all workers leading to an abrupt change in the information state and dispersion. This may be due to frictions in changing jobs such as notice periods, which may also explain the too strong early response of hiring rates. The model with endogenous information acquisition in Appendix 3.E is able to deliver a smoother transition. Overall, the simulation shows that in contrast to volatility increases in shocks to idiosyncratic fundamentals, counter-cyclical information flows lead to with the data consistent dynamics for the dispersion in idiosyncratic match productivity shocks, uncertainty and labour market flow rates.

The impact of cyclical information frictions relative to no information frictions is shown in Figure 3.21 and Figure 3.22. Again, the model is simulated in response to an aggregate shock. It can be seen that the hiring and separations rates react stronger in case of a positive aggregate shock, and will fall more in case of a negative aggregate shock when there are cyclical information frictions. The reason is that a slower speed of information acquisition about match productivity will mean that shocks to the expected quality of a match $\hat{z}$ will become less dispersed, or as can be seen in the graph more dispersed when the aggregate shock is positive. The increase in the dispersion of shocks results in action margins being hit more often. In the case of calibration one, this leads to an 18 % larger stronger response in the hiring rate at the maximum, while in calibration 2 cyclical information frictions cause a 33 % higher response at the maximum. Mean idiosyncratic match productivity in a standard random search labour market would decline when a positive aggregate shock hits the economy due to less stringent sorting of bad matches (see for example Figure 3.26 in Appendix 3.B). This effect is reduced in calibration 2 and even turned around in calibration 1 due to cyclical information frictions. These frictions create an inability of firms and workers to discern bad from good matches leading to a decline in mean idiosyncratic productivity when information frictions rise. Thus depending on

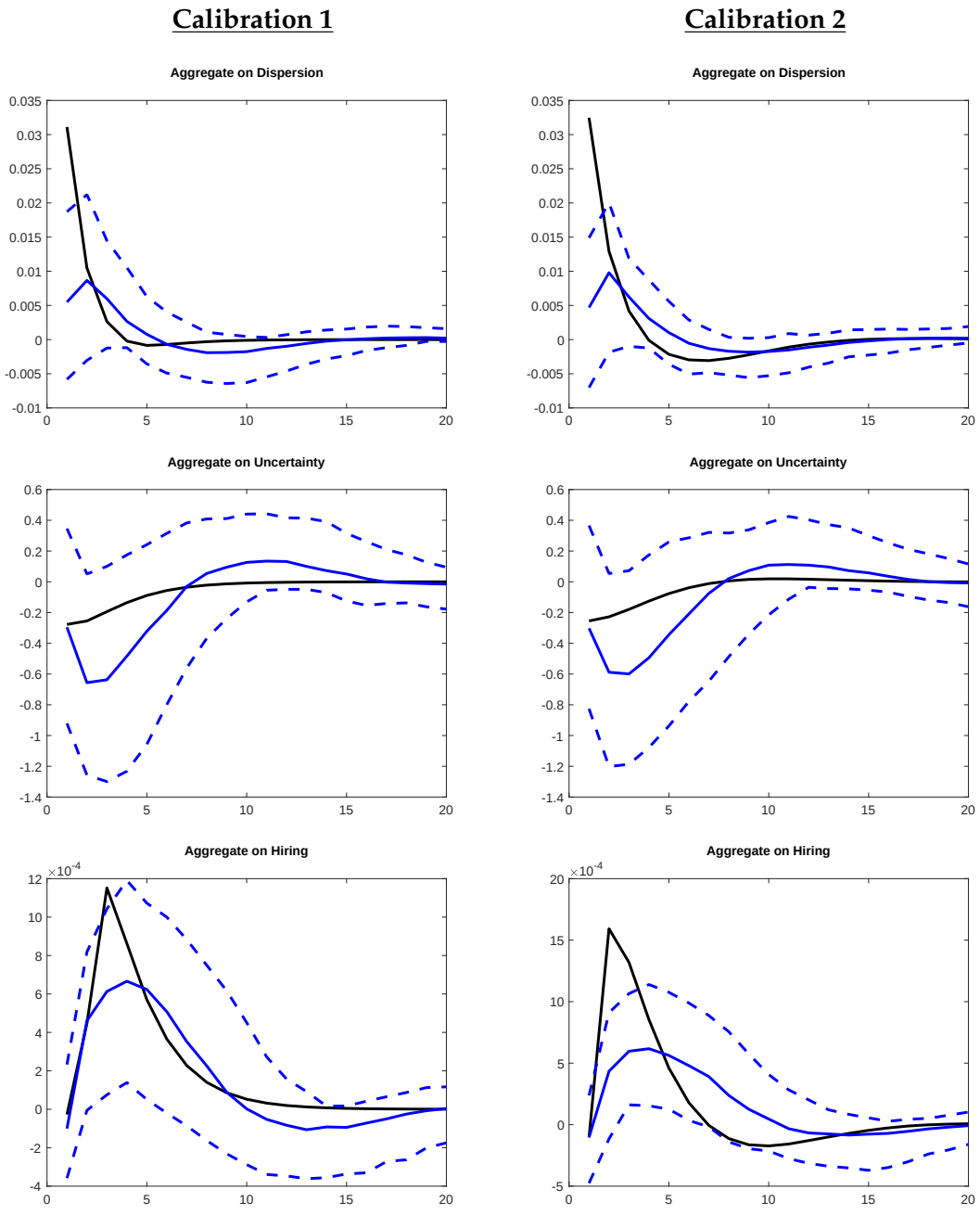


Figure 3.20: Comparison of the impulse responses of a VAR with short-run restrictions on simulated data generated by the model with $\zeta_2 = 5.4$ in both calibrations (black) with observed data (blue) for a positive aggregate shock.

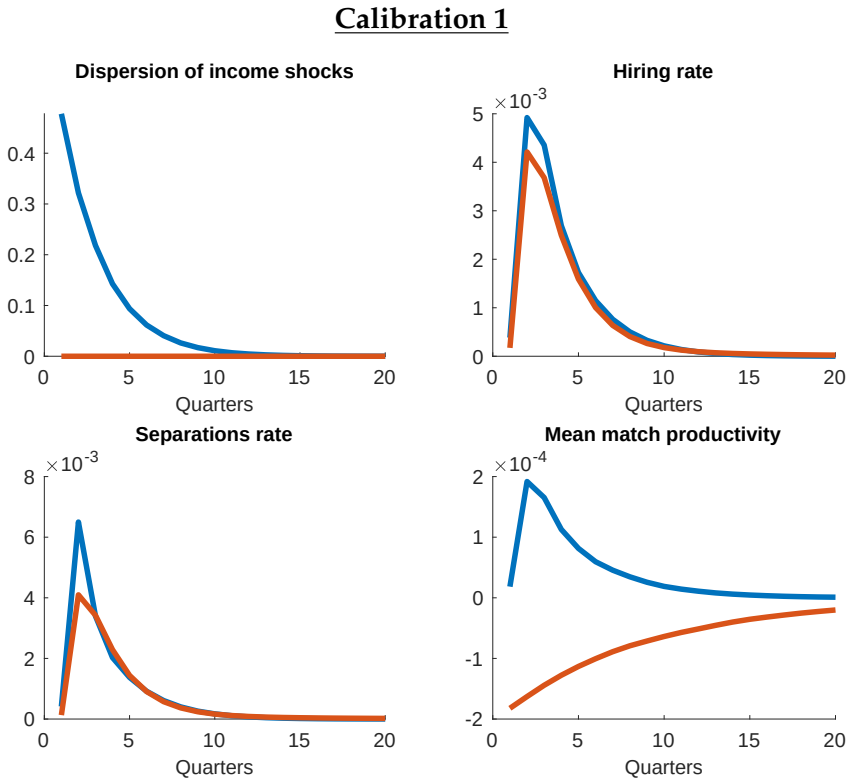


Figure 3.21: Simulated impulse responses of the model with and without cyclical information frictions for a positive aggregate shock. Impulse responses of the model with no cyclical information change $\zeta_2 = 0$ are in red, while impulse responses of the model with cyclical information frictions are in red $\zeta_2 = 5.4$.

the strength of the information frictions and properties of the labour market information frictions are able to explain a substantial increase in labour market mismatch during a downturn. This mismatch sorts itself once the economy improves.

3.4.10 The forces at work

The random search model in Section 3.3.2 and the dynamic directed search BRE have in common that a downturn is accompanied by a decrease in the dispersion of shocks to expected match productivity, while a boom will increase them. The decrease in dispersion during a downturn leads to the expected value of a match remaining closer to the mean value of a match, meaning the value of the match is unlikely to fall below the value of unemployment. This means it is less likely to violate the incentive compatibility constraint in equation (3.34) in the block recursive model. This will reduce labour market churn from jobs to unemployment, and will make the quality jobs matches harder to discern. Depending on the optimal job search behaviour on-the-job reduced information will also reduce on-the-job search in the BRE. This is the case when workers in more similar matches choose to search in locations of the market with a lower probability of finding a match, than a mass of more dispersed matched workers. An appropriate calibration can

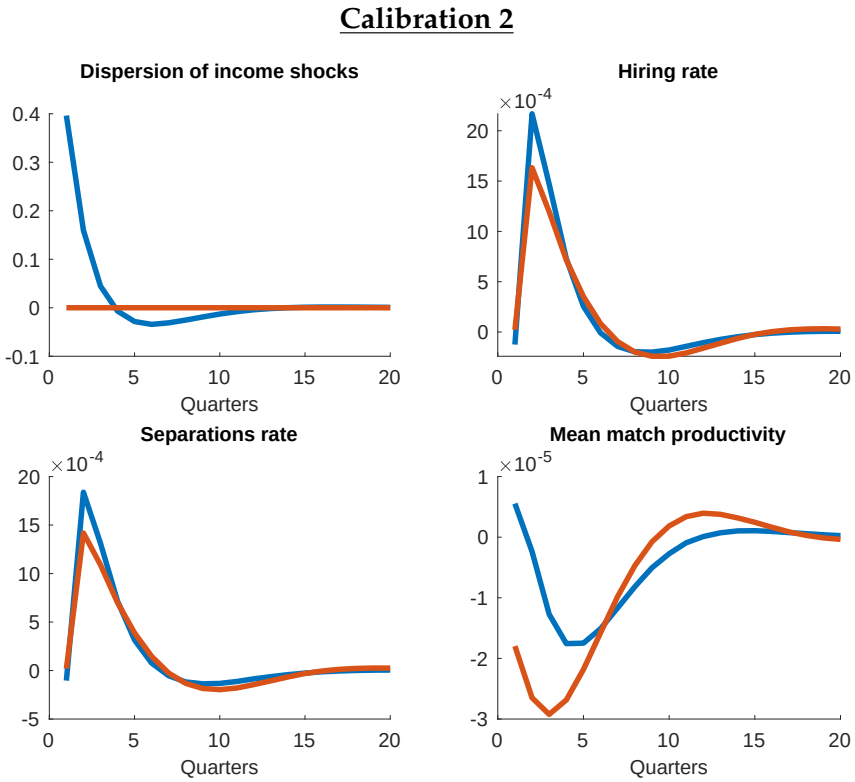


Figure 3.22: Simulated impulse responses of the model with and without cyclical information frictions for a positive aggregate shock. Impulse responses of the model with no cyclical information change $\zeta_2 = 0$ are in red, while impulse responses of the model with cyclical information frictions are in red $\zeta_2 = 5.4$.

ensure this is the case, but it is also expected search behaviour as changing to a median match is of less benefit to the worker when jobs are less dispersed. Only a large mass of well-matched workers moving closer to being median matched workers and engaging in search behaviour that outweighs the result of fewer bad matches seeking to change jobs could negate this result. This is not the case in the calibrations presented thus cyclical information flows cause a decline in labour market fluidity.

Declining match information will at the same time for the same reasons lead to a lower mean quality of matches in the BRE as in the random search model in Section 3.3.2. Declining information flows mean the expected value of idiosyncratic productivity of all workers $\hat{z}$ becomes less dispersed. However, this means firms and workers make mistakes and matches that should actually be destroyed, or matches survive where the worker should engage in more active behaviour to find a new opportunity. This creates the information mismatch effect.

3.5 Conclusion

The paper first shows that employment state transitions are pro-cyclical, as is the dispersion of the expectation of consumers regarding income shocks, while their perceived uncertainty is counter-cyclical. Assuming consumers are mostly workers receiving their main income from wages these observations result in the conclusion that modelling uncertainty in the labour market by increasing the volatility of shocks to idiosyncratic match quality produces results that are inconsistent with survey data and measures of employment state transition rates. The paper then proceeds in showing that introducing cyclical information frictions, and defining uncertainty as the posterior about match quality, may improve the match of flow rates, and may provide a coherent and parsimonious explanation for survey responses showing a pro-cyclical dispersion in income growth expectations and counter-cyclical uncertainty of consumers and workers during the recession and recovery. To implement persistent measures of idiosyncratic match quality and create meaningful information and flow dynamics the paper then presents a dynamic block-recursive equilibrium model.

The model shows that in interacting with other labour market frictions, counter-cyclical information frictions may reduce labour market transitions and drive mismatch during recessions, leading to more productive booms and deeper more unproductive recessions. If the current model were to be integrated into a framework with precautionary savings, as for instance found in (Ravn and Sterk, 2017), the uncertainty experienced by workers in the model could lead to an even further propagation and feedback of the downturn as a high posterior of the job match may cause workers to fear unemployment and save, even when the actual transitions to unemployment remain moderate.[2] Finally, the paper shows that some mismatch in the labour market will resolve itself when an improving aggregate state is accompanied by an improving information state making labour market sorting more efficient.

Appendix 3.A Additional Empirics

Wage dispersion

Figure 3.23 shows residual wage dispersion as a result of running a Mincerian wage equation on the data provided by the ASEC supplement of the CPS following (Lemieux,

[2] Elevated posteriors about the fundamental value of a job match may explain the prolonged risk experienced by workers regarding their job loss in Figure 3.4.

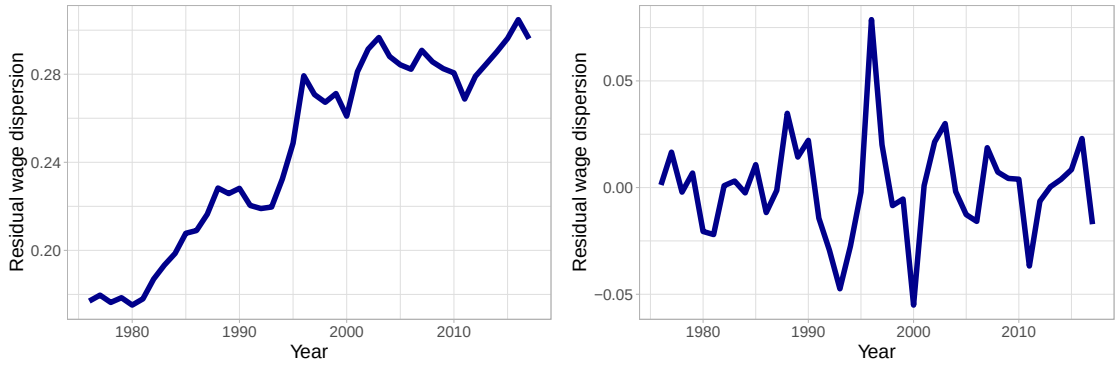


Figure 3.23: Left: Residual wage dispersion. Right: deviations of the series from a HP trend with annual parameter. The series are computed from CPS microdata from the ASEC supplement. The residual of wage dispersion are those differences in wages of male workers that cannot be attributed to other controls in a Mincerian wage equation, such as industry age or education.

2006) and (Morin, 2019). The data has been downloaded from IPUMS. The series shows that wage dispersion is pro-cyclical, similar to income growth expectations. (Morin, 2019) shows that this is due to an adjustment at the right or upper side of the wage distribution.

Worker sentiments and satisfaction from the General Social Survey

While only an indirect measure of information, questions in the general social survey regarding job satisfaction and pay fairness also tell a story, which can be interpreted sensibly in the light of cyclically varying information frictions. Figure 3.24 shows the weighed share of workers responding they are dissatisfied with their job. The general pattern is that dissatisfaction falls in recessions, and rises in the recovery. Volatility increases at the start of a recession would suggest the opposite. An increase in the volatility of shocks would intuitively put more workers on a margin where they are dissatisfied with their job at the start of the recession, and workers would become universally less dissatisfied as the recovery proceeds. In contrast, a possible way to explain the data is that the recession will lead to information frictions making it harder for workers to realise that they are in a bad match in the years 2000, 2002, and 2008. Meanwhile, during the recovery in 2004, 2010, and 2014 the posterior about a job match is declining due to the recovery leading to disproportional information release and a high proportion of workers realising that their match is not a good fit and starting to look more intensely for a new job. This story can be corroborated to some extent by the share of workers considering their wage to be fair considering their work Figure 3.25. 2010 sticks out as the year when a larger than average share of workers considered their wages lower than what they deserve.

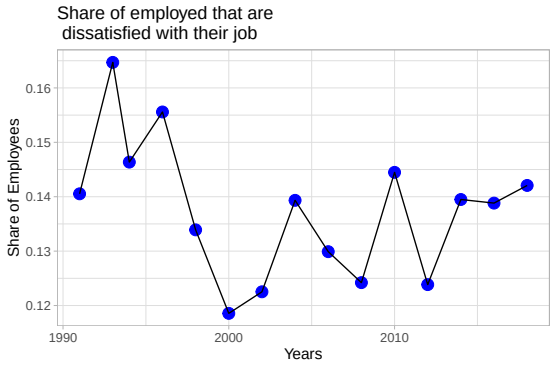


Figure 3.24: Weighed share of employed workers responding they are dissatisfied with their job from the general social survey. Survey observations are blue dots. The survey is conducted in even years.

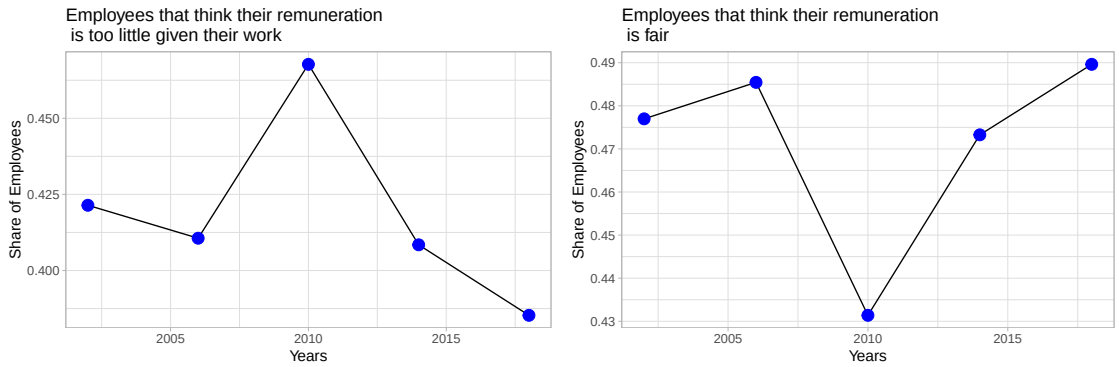


Figure 3.25: Weighed share of employed workers responding their remuneration is lower than what they deserve (left panel), or what they deserve (right panel), from the general social survey. Survey observations are blue dots. The survey question is set every four years since 2002

VAR estimations

The structural vector autoregressive model with short run restrictions is described in equation (3.36). y is the vector of variables. A (Table 3.5) is the matrix ordering the variable A^* (Table 3.4) the reduced form estimation matrix and B (Table 3.6) the restricted co-variance matrix.

$$Ay_t = AA^* y_{t-1} + B\epsilon_t \tag{3.36}$$

Variables	*Reduced form estimation AR (1)*				
	Output per worker	Dispersion	Uncertainty	Hiring rate	Separations rate
Output per worker (1 Lag)	0.355***	6.620	−337.660	0.493***	−0.611
	(0.106)	(6.406)	(243.660)	(0.161)	(1.323)
Dispersion (1 Lag)	0.003	0.176	−2.317	0.001	0.006
	(0.002)	(0.114)	(4.355)	(0.003)	(0.024)
Uncertainty (1 Lag)	−0.0001**	−0.001	0.891***	−0.0002***	−0.0001
	(0.00003)	(0.002)	(0.077)	(0.0001)	(0.0004)
Hiring rate (1 Lag)	−0.203***	1.377	131.994	0.710***	3.093***
	(0.050)	(3.018)	(114.793)	(0.076)	(0.623)
Separations rate (1 Lag)	−0.009	−0.386	−9.599	0.020	0.459***
	(0.008)	(0.481)	(18.290)	(0.012)	(0.099)
Constant	−0.00000	0.004	−0.213	0.00001	0.001
	(0.0002)	(0.015)	(0.572)	(0.0004)	(0.003)
Trend	0.00000	−0.0001	0.004	−0.00000	−0.00001
	(0.00000)	(0.0003)	(0.011)	(0.00001)	(0.0001)
Observations					84
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 3.4: Vector auto-regression estimation results A^*

	Output per worker	Dispersion	Uncertainty	Hiring rate	Separations rate
Output per worker (1 Lag)	1	0	0	0	0
Dispersion (1 Lag)	−3.813	1	0	0	0
Uncertainty (1 Lag)	298.588	−2.711	1	0	0
Hiring rate (1 Lag)	0.131	−0.004	0.00003	1	0
Separations rate (1 Lag)	3.707	0.042	0.0002	−0.345	1

Table 3.5: Results of the SVAR with short run restrictions: A Matrix

	Output per worker	Dispersion	Uncertainty	Hiring rate	Separations rate
Output per worker (1 Lag)	0.001	0	0	0	0
Dispersion (1 Lag)	0	0.067	0	0	0
Uncertainty (1 Lag)	0	0	2.517	0	0
Hiring rate (1 Lag)	0	0	0	0.002	0
Separations rate (1 Lag)	0	0	0	0	0.013

Table 3.6: Results of the SVAR with short run restrictions: B Matrix

Appendix 3.B Idiosyncratic volatility shock

An increase in idiosyncratic volatility in the standard random search model

Figure 3.9 and Figure 3.10 show the output gap and and consumer uncertainty affecting the hiring and separation rates. Both rates show an increase to positive shocks to the output gap and a decline in uncertainty rising. These responses are showing that the labour market is becoming more sclerotic during a downturn and during an increase in uncertainty are not reflected in standard search models, where uncertainty is modelled via an increase in the volatility of shocks. This is demonstrated with the perturbed impulse response functions of an equilibrium unemployment model with endogenous match destruction in Figure 3.26. A rise in idiosyncratic volatility can be seen as strongly affecting the expected value of a job match via growth option effects. It leads to less employment but increased churn of firm-worker matches meaning rising flows. (Riegler et al., 2014) finds similar effects using different uncertainty series and looking at their effect on the job-finding rate. The difference in the separation rate is due to separations in (Riegler et al., 2014) only being calculated implicitly as the result changes into unemployment. Meanwhile, the separation rate here is directly taken from the quarterly workforce indicators and includes all kind of reasons for match severance, including job to job switches. Thus the typical choice of modelling uncertainty via an increase in idiosyncratic volatility of match productivity does not seem to work well for an equilibrium unemployment model with search frictions.

Appendix 3.C Steady-state of the perturbed model

The steady-state of employment can be found by assuming no shocks to the aggregate component. In this case the aggregate process $a = 0$ and noise is $\sigma_\nu = \zeta_1 \sigma_z$.

$$n = \frac{1 - \Phi_s}{\Phi_s} m \tag{3.37}$$

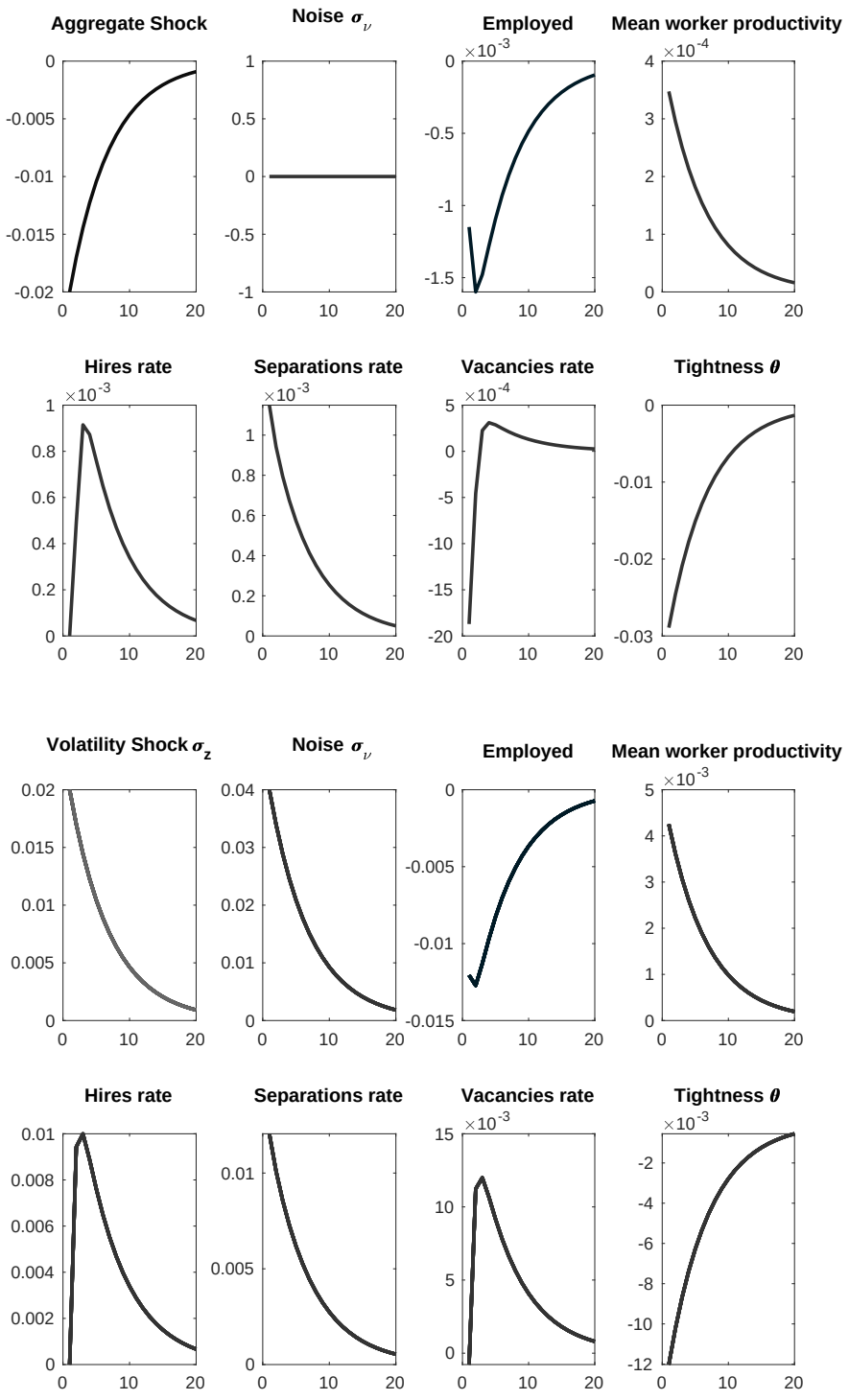


Figure 3.26: Top: Impulse response functions to a 1% productivity shock. Bottom: Impulse response functions to a 1% volatility shock. The model is a classic equilibrium unemployment model with endogenous job destruction as in (Mortensenand Pissarides, 1994) model in the (Shimer, 2005) calibration. The underlying model is formulated in Section 3.3.2. In the lower panel the volatility shock and noise are made to rise proportionally meaning that overall volatility of shocks to the job match is increasing, while the information share stays constant at $\frac{1}{3}$.

$$u = 1 - n \tag{3.38}$$

$$m = up \tag{3.39}$$

$$p = \theta(1 + \theta^l)^{-\frac{1}{l}} \tag{3.40}$$

$$n = \frac{p(1 - \Phi_s)}{\Phi_s + p(1 - \Phi_s)} \tag{3.41}$$

$$\frac{\kappa}{q}(1/\Phi_s - \beta) = [(\bar{z} - b)(1 - \eta) - \eta\kappa\theta] \tag{3.42}$$

$$\Phi_s = \Phi([(1 - \eta)[\mu_z - b] + \frac{\kappa}{q_t} - \eta\kappa\theta]\frac{(\sigma_z + \sigma_\nu)^{0.5}}{\sigma_z}) \tag{3.43}$$

$$\Phi_s = \Phi([(1 - \eta)[\mu_z - b] + \frac{\kappa}{q_t} - \eta\kappa\theta]\frac{(\sigma_z + \sigma_\nu)^{0.5}}{\sigma_z}) \tag{3.44}$$

$$\bar{z} = \mu_z + \frac{\sigma_z}{(\sigma_z + \sigma_\nu)^{0.5}}\frac{\phi_s}{\Phi_s} \tag{3.45}$$

$$\frac{\kappa}{\beta q} = \Phi_s[(\mu_z - b)(1 - \eta) - \eta\kappa\theta + \frac{\kappa}{q}] + (1 - \eta)\frac{\sigma_z}{(\sigma_z + \sigma_\nu)^{0.5}}\phi_s \tag{3.46}$$

Appendix 3.D Block recursive solutions for the second calibration

Figure 3.27 to Figure 3.29 show properties of the solved model in the second calibration. The model with lower κ exhibits higher tightness, but also lower utilities provided to workers as job creation is less costly for firms.

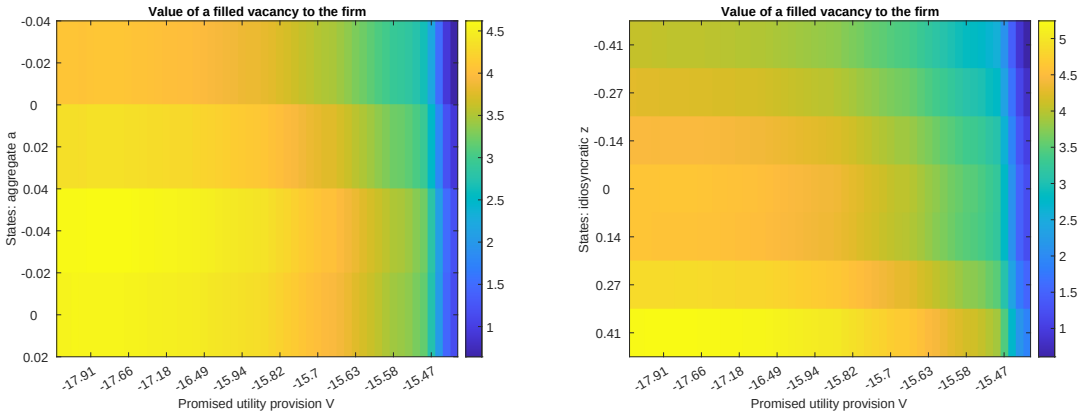


Figure 3.27: The value of a match to the firm depending on the states of the match. The state of the posterior p are fixed to it's median state. Further, in the left graph the idiosyncratic state z is at it's median state, in the right graph the aggregate state a.

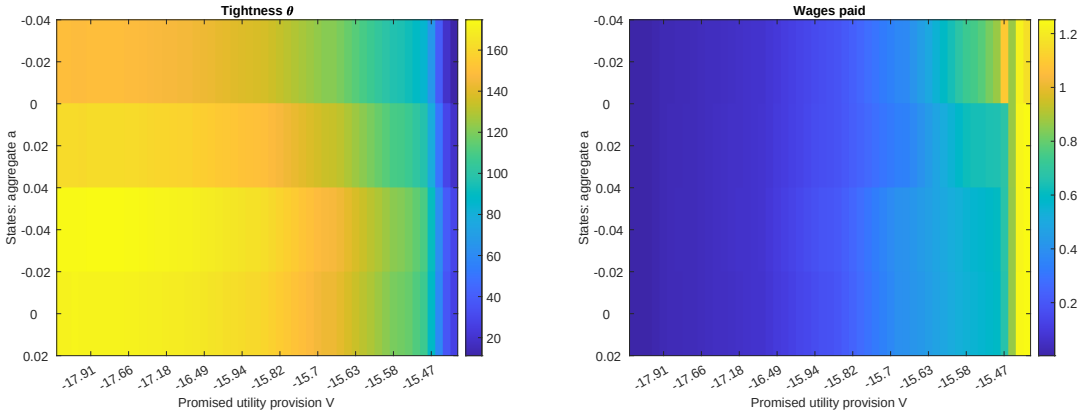


Figure 3.28: Left: Tightness for each sub-market $x = V$ keeping the state of the posterior p fixed to it's median state. Right: Wages paid in each state keeping the state of the posterior p, and the idiosyncratic state z fixed to their median state.

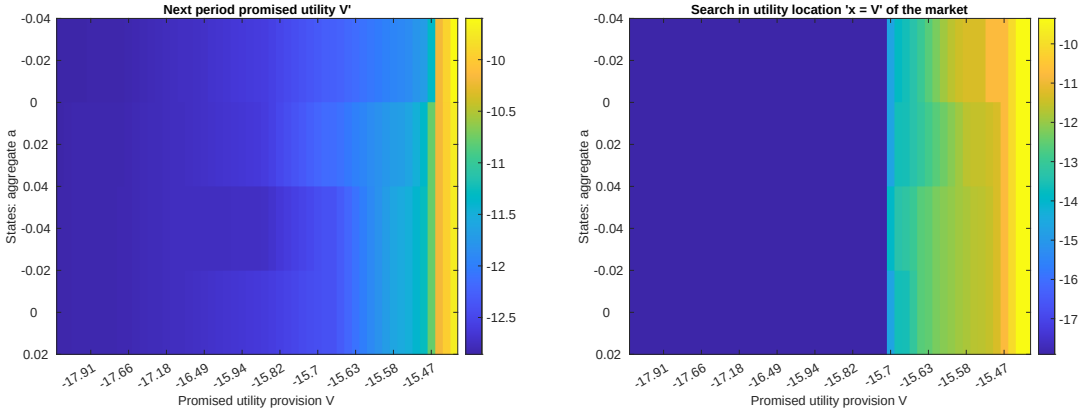


Figure 3.29: Left: Next period's promised utility provision V' keeping the state of the posterior p, and the idiosyncratic state z fixed to their median state. Right: Chosen search location $x' = V'$ for each worker on the job given their aggregate and utility state, keeping the state of the posterior p, and the idiosyncratic state z fixed to their median state.

Appendix 3.E Endogenised information acquisition

Random search model

Impulse response functions for the random search model with endogenous information acquisition presented in Section 3.3.3 are shown in Figure 3.30. As can be seen, the calibration allows for a strong reduction of noise as a result of a positive shock to aggregate productivity leading to a more fluid labour market. Mean idiosyncratic productivity rises. The reaction of noise is so strong that employment falls due to more active sorting for the same reason as in the random search model without endogenous information acquisition. This counterfactual result can be corrected for in a model with persistent idiosyncratic match productivities as presented in Appendix 3.E.

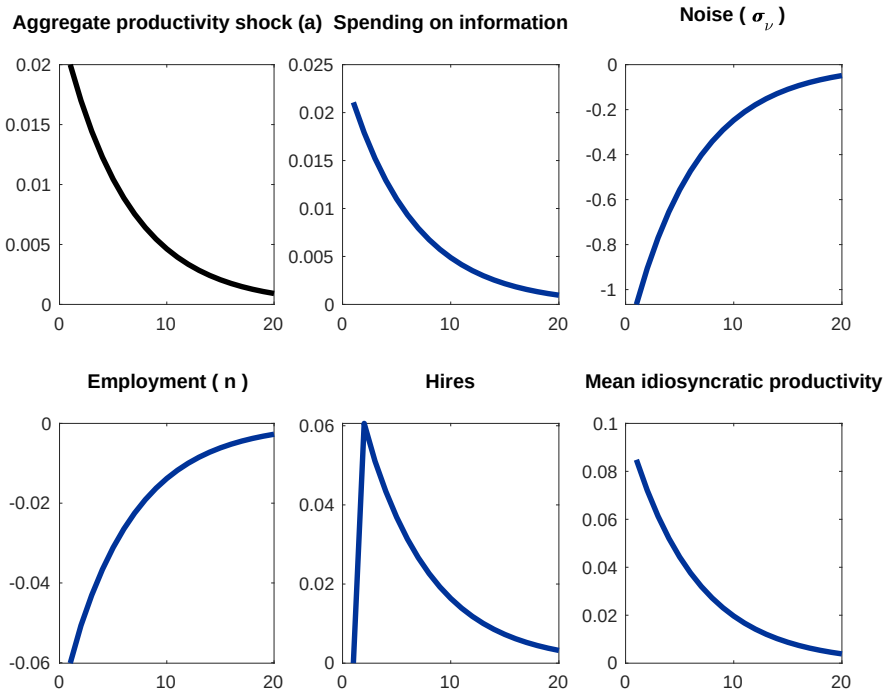


Figure 3.30: Impulse response function from the perturbed model to a positive aggregate shock.

Block recursive model

Cyclical information acquisition can be the result of an endogenous choice in the model presented in Section 3.4. This endogenisation is similar to the proposed endogenisation in Section 3.3.3. Assume that part of the contract that a firm offers contains a state-contingent choice on acquiring information about the idiosyncratic process by paying a cost c. In this case, next period's match posterior p_i' is an idiosyncratic rather than an

aggregate state and the expected range of transitions is a choice variable $E'_{\hat{z}_i}$, while this period's posterior p_i changes from being an aggregate state variable to being an idiosyncratic state variable as well.

$$J(V,\hat{z},p,a) = \max_{w,d',V',c',p',E_{\hat{z}'}} \{\exp(y+\hat{z})-w-c+\beta E_a[E_{\hat{z}'}[(1-d')(1-\lambda_e p(\theta(x_{V'},a'))J(V',\hat{z}',a',p')]]\} \tag{3.47}$$

This optimisation is subject to the incentive compatibility constraint in equation (3.48) and the promise-keeping constraint in equation (3.49).

$$d = [\delta \ if \ V_u(a') \leq V'(a') + \lambda_e p(\theta(x_{V'},a'))(x_{V'} - V'(a')); \ 1 \ else] \tag{3.48}$$

$$V = u(w)+\beta E_a[E_{\hat{z}'}[(1-d'(\hat{z}',p',a'))[V'(a')+\lambda_e \max_{x_{V'}}\{p(\theta(x_{V'},a'))(x_{V'}-V'(a'))]]+d'V_u(a')\}] \tag{3.49}$$

Further the optimisation is subject to the information and idiosyncratic productivity transition laws in equation (3.50), (3.51), and (3.52).

$$p' = \rho_z^2 p \frac{\sigma_\nu(c)}{p+\sigma_\nu(c)} + \sigma_z \tag{3.50}$$

$$\sigma_\nu = \zeta_1 \exp(\zeta_2 c) \tag{3.51}$$

$$\hat{z}' = \rho_z \hat{z}_i + K(\Sigma_2(c_t))(s-\hat{z}) \tag{3.52}$$

The model is else solved similarly to the model where the information state is an aggregate state.

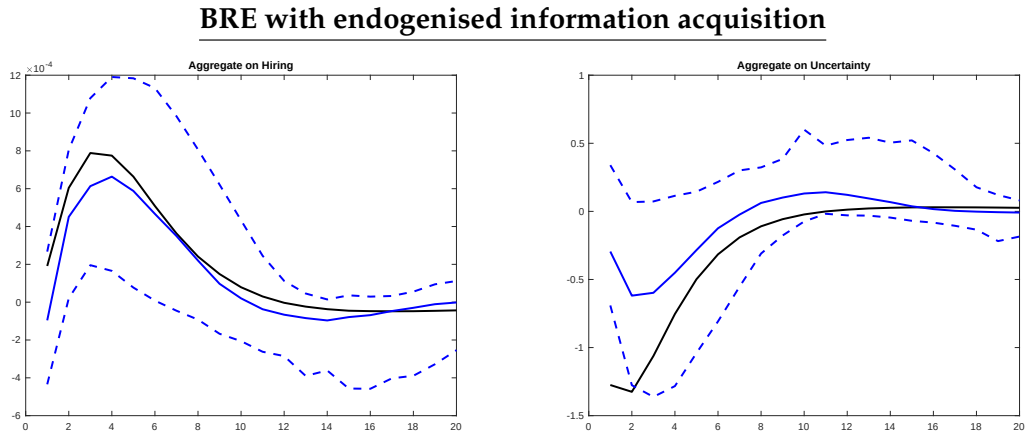


Figure 3.31: Comparison of the impulse responses of a VAR with short run restrictions on simulated data generated by the model with endogenised information acquisition (black) with observed data (blue) for a positive aggregate shock.

Appendix 3.F Computation, simulation, and evaluation of the dynamic directed search BRE model

The following process pertains to computing the equilibrium of the dynamic BRE described inSection 3.4.

1. First the joint distribution of $a, p, \hat{z}$ are discretised and approximated with transition matrices using the method following (Tauchen, 1986). A useful step includes eliminating at this stage all transitions which are impossible to occur. This saves computing time in the BRE as irrelevant transitions are not computed. Finally the joint implications of $a, p \rightarrow \hat{z}'$ are computed using an altered Tauchen method where the approximating grid points of $\hat{z}$ remain the same, but the transition matrix for $\hat{z}$ changes given the joint aggregate state of a and p.

2. Finding the optimal state-contingent contract is not straightforward as the problem is non-linear and the appropriate grid to approximate utility splits is unknown. A useful simplification is to use the promise-keeping constraint in equation (3.49) to substitute for wages. This leaves match destruction d and the promise of next period's utility V', which will determine the workers' search behaviour as choice variables for the firm. Then follow the following process.

   I Guess a grid and V for present discounted utility proposals in the market. Further, guess a value of a job match to firms given these proposals J.

   II Compute optimal tightness in each location given J using the free entry condition at each "present discounted utility proposal - idiosyncratic state - aggre-

gate state".

III Given tightness and utility locations $x = V$ compute optimal search behaviour R of a worker employed at utility V.

IV Compute the value of unemployment V^u given tightness and search behaviour finding the optimal location of search x_u for unemployed for each aggregate state.

V Given V, R, V^u, θ update the value of J by finding the optimal contract V' and d' given aggregate and idiosyncratic states. As this is a non-linear non-continuous problem this is the most computationally intensive part of the optimisation. The discrete decisions in d are solved with a costly max operator while the $continuous$ optimisation for V' is solved by interpolating on the grid and using a golden-section search algorithm. The expected value function J' is in the optimisation approximated with a Chebychev polynomial.

VI Update J, further update the grid V based on proposals V' to avoid computing locations in the market that firms would not propose but sever due to too high utility promises or too low meaning the incentive compatibility constraint of workers would no longer be fulfilled.

3. Solving the model is subject to a curse of dimensionality. There are four states in the model. An aggregate productivity state, an aggregate information state (idiosyncratic in the model with endogenous information), an idiosyncratic productivity state, and a utility state. The model is solved with a V grid of 53, a z grid of 7, and 5 aggregate states a, as well as 5 aggregate states p. Given the simplifications and cross-state reductions described above this results in solving the model for 5194 states. Using parallelisation of the 6 cores this takes approximately 15 minutes to solve in Matlab on a computer with 2.2 GHz 6-Core Intel Core i7 processor and 16 GB 2400 MHz DDR4 Ram.

4. The model is then simulated and moments are computed. BRE models are difficult to evaluate as the policy functions are not linearised and can thus not be as easily brought to the data as other macro models, which essentially replicate vector auto-regressive models. Most evaluation of the model has thus focused so far on computing the moments a simulated model produces, as impulse responses are hard to compare. It is proposed to evaluate the model additionally to the mo-

ments it produces, by evaluating the conditional moments of the data produced. The model is simulated by simulating the aggregate process. In a next step a vector auto-regression is run both on the actual and simulated data. A good version of the model should produce simulated data, which leads to similar impulse response functions from imposing short-run restrictions on the model simulated data as on the actual data.

Chapter 4

Non-Performing Loans and Inefficient Capital Reallocation

Abstract

A high share of non-performing bank loans in total bank loans (sNPL) has been shown to negatively affect aggregate investment and economic growth. While these empirical facts have been well established, neither the mechanisms causing the great cross-country heterogeneity in the sNPL, nor the channels through which they affect the real economy are well understood. A commonly invoked channel is that the sNPL leads to reduced credit supply. This paper first shows that focusing solely on this channel would provide an incomplete picture. Reduced credit supply implies higher rates of return to capital in economies with a higher non-performing loan burden as profitable projects are not met with sufficient credit supply. This can neither be observed in country cross-sections nor in time series data. The more important channel through which the sNPL affects the economy stems from the credit demand side with the sNPL providing a mirror image of real capital misallocation. The paper then proposes a structural model with search frictions in used capital markets that links the sNPL with frictions in capital reallocation to explain the observed sNPL and investment dynamics. The structural model shows that long and persistent sNPL increases in response to a negative shock, are either a symptom of a low option value of foreclosure for banks due to inefficient used capital markets or a symptom of forbearance incentives for banks due to balance sheet weaknesses and regulatory requirements. Both frictions are captured parsimoniously in the model. The fact that these frictions imply different impulse responses for capital prices is used to identify and estimate the extent to which they drive variation in the sNPL over the business cycle. Both inefficiencies lead to more misallocated capital and reduce the marginal product of fresh capital, thereby impacting credit creation. A higher sNPL following a negative shock, such as the Covid-19 pandemic, will lead to more prolonged output and investment below equilibrium in countries with less efficient used capital markets and larger increases in forbearance incentives. The tractable model can provide an explanation for observed sNPL and macroeconomic outcomes and presents an identification and estimation of the two identified drivers of the sNPL over the business cycle, used capital market inefficiencies and forbearance incentives.

Keywords: *Business cycles, Search frictions in Capital Markets, Investment, Capital allocation, Non-performing loans*
JEL Classification: *E2; E22; E30; E32; G33.*

"... At the end of 2016, the stock of gross NPLs in the EU banking sector was around € 1 trillion. This number, however, does not take into account the fact that that collateralised lending plays an important role in Europe. For example, including collateral and provisioning, the coverage of NPLs is, on average, 82% in the euro area ...

... The outstanding stock of NPLs is a consequence of cyclical and structural factors. First, the severe recession resulting from the global financial crisis led to a deterioration of the quality of banks' loan books... At the same time, structural weaknesses still persist. These include inadequate internal governance structures in banks, ineffective and costly debt recovery procedures in some Member States and misaligned incentives that prevent a quick resolution of NPLs."[1]

4.1 Introduction

The share of non-performing bank loans in total loans (sNPL) in a country has received a lot of attention by policymakers ever since the Global Financial Crisis in 2007 led to a persistent rise of the share in some countries. The recession induced by the Covid-19 pandemic is expected to lead to a similar rise (Kasinger et al., 2021). A high share of badly performing loans in total loans and subdued aggregate output go hand in hand. For good reasons, both have been suspected to be the cause of the other. The sNPL may increase when negative supply or demand shocks cause the economy to contract, causing previously profitable firms and solvent households to default on payments. On the other hand, a high sNPL may lead to more restrictive lending and investment keeping the economy subdued. This paper presents the behaviour of the sNPL over the business cycle and shows then that an inefficient capital allocation and a high sNPL go hand in hand. The paper then builds on existing and predominantly empirical literature on the connection between the sNPL and macroeconomic performance to develop a structural macroeconomic model capturing sNPL statics and dynamics. Based on the model, it presents new insights explaining the highly different reaction of the sNPL to macroeconomic shocks for different countries. Specifically, the model allows for an evaluation of the importance of forbearance incentives and used capital market efficiencies. Judging which of these fictions become more prevalent in an economy during a recession, provides a basis for formulating and prioritising the most effective policies to resolve NPLs across Europe.

This paper first shows that increases in the sNPL go hand in hand with reductions in real

[1] Speech by Mario Draghi, at the time President of the ECB and Chair of the European Systemic Risk Board, at the second annual conference of the ESRB, Frankfurt am Main, 21 September 2017

capital returns, and provides evidence of the sNPL being linked to real capital misallocation. In the next step, a structural model with search frictions in used capital markets is developed to show that the sNPL can be understood as a symptom of these capital market frictions which will encourage banks to maintain loans in low-quality matches. Concretely, the focus is on two kinds of frictions. The first is the relative ease with which used capital can be reused compared to fresh capital. This is the efficiency of used capital markets. The second friction is forbearance incentives which are defined to be real costs incurred by banks when calling a loan. Both forbearance incentives and inefficiencies in used capital markets are shown to be drivers of the sNPL in the model. The paper then uses the model to estimate how these frictions vary over the business cycle in different economies. The paper finds changes to the efficiency of used capital markets to be a more important driver of sNPL in countries with high sNPL, except for Italy where increases in forbearance incentives dominate. This means that the sNPL in these countries may be understood as a symptom of an economy's inability to efficiently reallocate capital from unproductive to productive use during recessions.

Even though non-performing loans have recently taken a center stage in the policy discussion in many countries, we know little about what the sNPL tell us about the state or dynamic response of a country's economy to macroeconomic shocks. Figure 4.1 shows that much like unemployment the sNPL is commonly cyclical, and remains persistently elevated following a recession. The right graph in Figure 4.1 shows that the speed of recovery from a recession by means of investment is correlated with the ability of a country to keep the sNPL low following the negative shock. This relationship did not change with the great increase in NPLs during the great recession. Recently, a growing empirical and policy-oriented literature has emphasized this negative correlation of the sNPL on consumption, investment, and more generally on the macroeconomic performance of a country. A non-exhaustive list of empirical and policy papers in this area include (Louzis et al., 2012), (Klein, 2013), (Beck et al., 2013), (Jassaud and Kang, 2015), and (Ari et al., 2020). (Balgova et al., 2016) argue using an event study approach that reducing the sNPL leads to an increase in real growth and investment. Even though it is difficult to determine the direction of causation between macroeconomic variables and sNPL, policy-making institutions have become concerned with the sNPL and started a lively discussion about approaches to reduce and prevent these loans from arising. Policy proposals focus on macro-prudential policies, asset management strategies, as well as faster default processes and capital reallocation in the form of liquidating collateral.

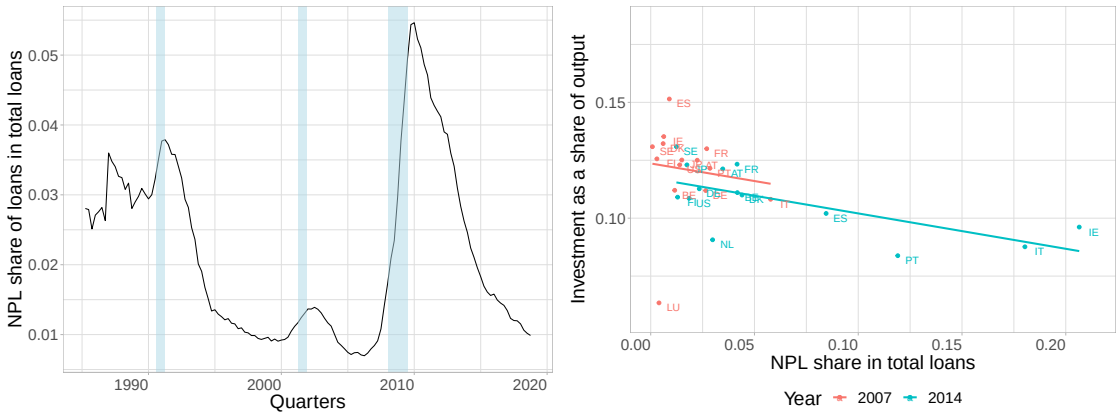


Figure 4.1: Left: NPLs as a share of total gross loans in the US from the FRED database. Right: Change in investment share of GDP and NPLs for OECD countries between 2007 and 2014. Investment calculated from the KLEMS database, sNPL from World bank and IMF data.

This policy discussion has so far largely focused on data and empirical models, while structural and business cycle models studying non-performing loan dynamics remain scarce. The general narrative for interpreting the empirical observations on sNPL has been that non-performing loans on a bank's balance sheet lead to lower bank profits and a regulatory need for higher cash reserves to compensate loan losses. Banks with a higher stock of non-performing loans on their balance sheet will then have less balance sheet space to lend out capital and thereby reduce credit supply.[2]. While the general intuition is compelling, it is difficult to find clear evidence that the lending behaviour of individual banks is differing due to their non-performing loan stocks ((Bredl, 2018) and (Accornero et al., 2017)). The cross-country comparison in this paper shows that this theorised reduction in credit supply cannot be the only channel through which the sNPL affects macroeconomic outcomes as it would imply higher real rates of return for capital in the long-run. The paradox of a policy discussion paired with a lack of structural models has been pointed out by a senior policymaker of the bank of Italy, which is due to the high stock of NPLs in Italy a major stakeholder in this discussion: *"To my knowledge, there is no clear theory suggesting that high volumes of NPLs impair the credit allocation mechanism."*[3].

The empirical part of this paper finds the sNPL to be strongly counter-cyclical. The sNPL correlates negatively with investment activity, aggregate returns on capital, capital prices, and capital reallocation. A VAR model with long-run restrictions (Blanchard and Quah, 1988) shows that positive supply shocks will lead to a reduction in the sNPL, while a

[2]See, for instance, https://www.bankingsupervision.europa.eu/banking/priorities/npl/html/index.en.html
[3]Paolo Angelini, at the time Deputy Director General for Financial Supervision and Regulation, Bank of Italy; VOX EU CEPR, 12 April 2018

rise in the sNPL has an ambiguous short-run effect on output. Meanwhile, short-run restricted VARs suggest that controlling for output, an increase in the sNPL will lead to reduced investment, reduced capital returns, and delayed capital reallocation. On the other hand, positive investment shocks have an ambiguous effect on the sNPL, while a capital return shock reduces them. A country cross-section using aggregate data and aggregated firm microdata shows that a higher sNPL is linked with lower aggregate returns on capital, lower investment, and a higher misallocation among firms.

The paper then presents a structural business cycle model where banks act as intermediaries for household lending to firms to match observed dynamic data and identify cross-country differences in sNPL outcomes as either the result of skewed bank forbearance incentives or inefficiently working markets for used capital. In the model, loans are provided with underlying collateral to firms for them to produce. The search and matching framework applied to bank-firm credit relationships combined with frameworks developed for capital reallocation ((Cao and Shi, 2017), (Ottonello, 2017), (Gopinath et al., 2017), (Lanteri, 2018), (Eisfeldt and Shi, 2018), and (Cui, Wright, Zhu et al., 2019)) is shown to be particularly useful for modelling the decision-making of banks on whether to foreclose or forbear a loan. It can parsimoniously capture heterogeneity in the quality of a loan and heterogeneity in capital allocation while allowing for a tractable model that can explain the sNPL, investment, and capital allocation dynamics. Loans may become non-performing in the model, upon which a bank must decide whether to foreclose the loan and reallocate the foreclosed collateral or to forbear the loan incurring real costs and hoping that the loan will become profitable in the future. This dynamic decision-making problem is modelled by assuming search frictions in used capital markets. This is a way to capture the fact that used capital has high asset specificity ((Bertola and Caballero, 1994) and (Kermani and Ma, 2020)) meaning it may hold a heterogeneous value for heterogeneous firms involving information and market search problems.

The bank's foreclosure decision will depend on the value of forbearance incentives and the efficiency of markets for the collateral. Forbearance incentives are regulatory or other frictions that result in banks incurring real economic losses when foreclosing a loan. Higher forbearance incentives and lower efficiency of used capital markets will both cause higher non-performing loan levels in equilibrium. However, they imply different dynamics regarding the price of capital and new investment activity. This allows using the response of countries to output shocks to judge the extent to which changes to

forbearance incentives or inefficient used capital markets drive sNPL over the cycle. The paper finds that in most European countries struggling with a high sNPL following the global financial crisis changes to the inefficiency of used capital markets is at fault. This result correlates well with the resolving insolvency scores from the World bank's doing business indicators which can be interpreted as a combination of asset specificity and used capital market efficiency.

Models with search frictions in credit markets have recently become more popular. The model presented in this paper builds on many of the insights of models from search frictions in labour markets. It is kept simple in a random search fashion with intra-period heterogeneity similar to (Mortensen and Pissarides, 1994). However, the model setup is chosen in such a way as to allow for persistent heterogeneity dynamic directed search block recursive equilibrium extensions of the type developed in (Menzio and Shi, 2010a) and (Menzio and Shi, 2010b). Papers introducing search in credit markets in a similar manner to this paper like (Beaubrun-Diant and Tripier, 2015), (Boualam, 2018) or (Cui et al., 2019) have mostly set up the models with firm search and bank free-entry, leading to a challenge in determining firm stocks or assuming less-intuitive fixed stocks of entrepreneurs which may gather financing. The paper also speaks to the recently re-emerging zombie firm literature (Caballero et al., 2008) and (Acharya et al., 2020), which is unsurprising as corporate non-performing loans are likely to have been granted to zombie firms. While not focusing explicitly on either the financial crisis or the financial accelerator literature (Bernanke, Gertler, and Gilchrist, 1999), this model can also be straightforwardly integrated in such models and may be used for studying the effect of unconventional central bank policies in models of the (Gertler and Karadi, 2011) type.

The next section of this paper presents the business cycle properties of non-performing loans and shows that the sNPL has an effect on the macro-economy beyond the pure credit supply channel. The third section presents the model focusing on bank foreclosure decisions and the consequences on the sNPL, capital reallocation, and capital productivity is presented. The model section first describes a simple partial equilibrium model, which serves to provide intuition for the main mechanism for the dynamic stochastic general equilibrium of the main model. Following this, the main model is presented featuring endogenous foreclosure decisions, search frictions in used capital markets and heterogeneity in the profitability of loans, and forbearance incentives. It is shown that the efficiency of used capital markets and forbearance incentives play a crucial role in determining the outside value of foreclosing capital versus forbearing capital. The model

is then calibrated to show that it can explain the correlations of the business cycle with sNPL dynamics. Furthermore, the variation of forbearance incentives and matching frictions in used capital markets over the business cycle is estimated using the proposed identification scheme. Finally, the fourth section concludes.

4.2 Business cycle properties of NPL shares in bank balance sheets

This section first presents the business cycle properties of the sNPL using aggregated US data for the period 1985 - 2018. US data on the sNPL and capital reallocation is available for a longer time period than for most other economies. The sNPL is shown to be counter-cyclical. Long-run restrictions suggest that output growth leads to a decline in the sNPL, while the opposite impact is ambiguous. Controlling for Real GDP short-run restrictions suggest a rise in the sNPL will drive down investment, increase delinquency rates, reduce property prices, and reduce capital returns.

The correlations in Table 4.1 present the aggregate business cycle properties of the sNPL. The series are downloaded from the federal reserve of St. Louis' database unless otherwise specified. The sNPL series describes the share of non-performing loans in total loans. Return on capital is calculated as the value-added accruing to capital over the capital stock calculated via a perpetual inventory method from capital formation and consumption. Investment is the GDP share of gross fixed capital formation, while capital prices are captured by the property price index. Reallocation is calculated similarly to (Eisfeldt and Rampini, 2006) as the sum of firm acquisitions of existing property plant or equipment over total firm assets. Firm data is retrieved from the WRDS Compustat database for the relevant period. The reallocation series which is only available at annual frequencies is linearly interpolated. All series are Hodrick-Prescott filtered at quarterly frequencies to highlight cyclical properties and remove trends.

Table 4.1 shows that the sNPL is counter-cyclical. The correlations further suggest that low returns on capital lead the sNPL, while the sNPL lead reductions in investment, reduced reallocation, and reduced property prices. Delinquency rates increase as the sNPL increase. The underlying series for Table 4.1 and the impulse response functions identified via short-run restrictions in the next subsection can be found in Appendix 4.B.

Variable	sNPL (-2)	sNPL (-1)	sNPL	sNPL (+1)	sNPL (+2)
Real GDP	-0.64	-0.67	-0.64	-0.56	-0.43
Return on capital	-0.34	-0.49	-0.60	-0.68	-0.73
Investment	-0.67	-0.75	-0.77	-0.74	-0.66
Reallocation	-0.51	-0.47	-0.41	-0.32	-0.20
Delinquency rates	0.75	0.81	0.81	0.77	0.68
Property prices	-0.79	-0.75	-0.69	-0.60	-0.49

Table 4.1: Business cycle properties of sNPL. sNPL, Real GDP, Delinquency rates, and property prices are downloaded from the FRED. Aggregate capital returns are calculated based on BEA data. Capital reallocation is calculated following (Eisfeldt and Rampini, 2006). All series are calculated as deviations from a quarterly Hodrick-Prescott trend. Property prices stand in as capital prices. Sources: FRED, BEA, WRDS

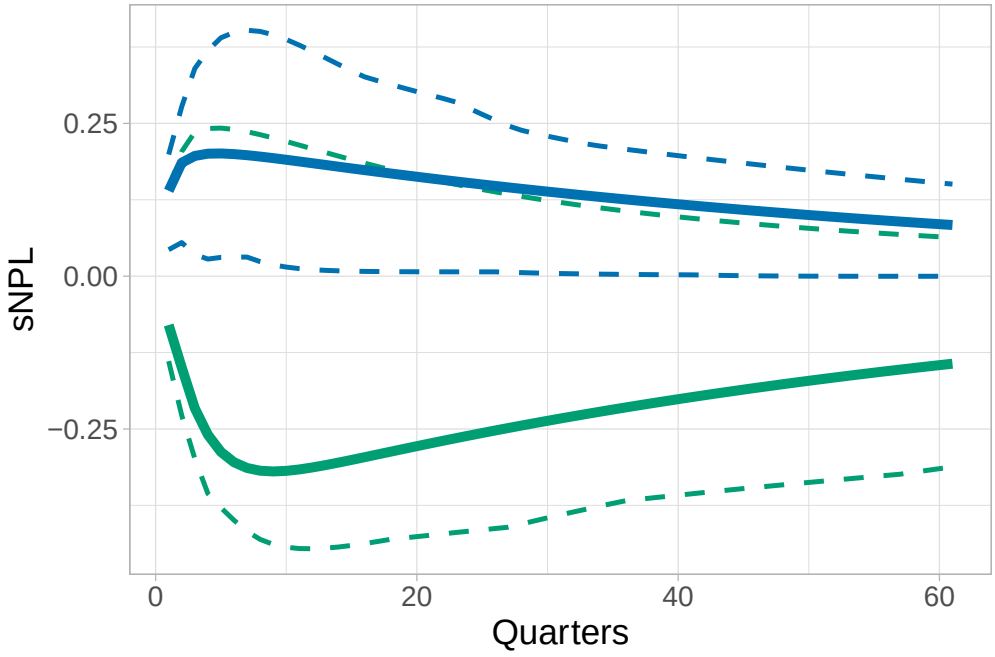


Figure 4.2: Impulse response of the sNPL via long-run identification. Green captures supply shocks (from real GDP) and blue demand shocks (from sNPL). Dashed lines show bootstrapped confidence intervals.

4.2.1 Shock identification and impulse responses

First, we are interested in the impact real output has on non-performing loans and vice versa. Given the long-run property of the sNPL in the US presented in Figure 4.1 to return to an equilibrium, and their similarity to unemployment it is reasonable to identify the effects of the sNPL on output and vice versa by assuming variation in them has only a temporary effect on output. Thus one can impose long-run restrictions of the type suggested by (Blanchard and Quah, 1988) to separate demand and supply shocks with the sNPL capturing demand. While the available thirty years of data are not enough to provide clearer confidence intervals for either of the series responding to shocks from the other Figure 4.2 suggests that it is very likely a rise in real GDP will reduce the sNPL.

It is therefore clear that controlling for real GDP is necessary to identify correct impulse responses for a change in the sNPL with relation to other variables that provide indications about capital reallocation, such as return on capital, reallocation flows, delinquency rates, or property prices. Using the suggested ordering from the correlations and real GDP ordered before non-performing loan rates short-run restrictions are imposed. The number of lags (L) is chosen using the Hannan–Quinn information criterion. ϵ is a vector of identified shocks. Importantly, the main reason for the imposition of short-run restrictions here is not to identify causal relations, but to explore the dynamic behaviour of the sNPL in relation to other variables. Causality in this paper is implied by the mechanisms in the structural model. The VAR models for the relevant variables are found in equations (4.1) and (4.2). Equation (4.1) is used for variables for which the correlation table implies that they lead NPL shares, while equation (4.2) is used for variables that lag them.

$$\begin{bmatrix} Var\ leading\ sNPL \\ Real\ GDP \\ sNPL \end{bmatrix} = A(L) \begin{bmatrix} Var\ leading\ sNPL(L) \\ Real\ GDP(L) \\ sNPL(L) \end{bmatrix} + B\epsilon \tag{4.1}$$

Figure 4.3 suggests the sNPL declining as capital returns increase, while the effect in the opposite direction is also negative meaning that a higher sNPL causes lower returns to capital while controlling for the impact of output. Meanwhile higher levels of the sNPL lead to lower investment as the graphs in Figure 4.4 show, and higher investment is accompanied by a reduction in the sNPL. This suggests that increased investment will result in a smaller share of weak loans driving down the sNPL. Capital reallocation flows are initially affected negatively as the sNPL rise as shown in Figure 4.5. Meanwhile, delinquency rates and the sNPL both affect each other positively as shown in Figure 4.6. This means that capital reallocation flows decline with a higher sNPL even though foreclosures by banks rise. This suggests a classic congestion mechanism inherent in markets with search frictions. Loans and their underlying collateral may become delinquent, but no immediate new productive use is found. Finally, Figure 4.7 shows that a higher sNPL will negatively affect the outside value of the underlying collateral capital as proxied for by property prices. Given the negative correlation of the sNPL with capital reallocation, this is an expected fact highlighted by the capital reallocation literature. Low capital prices are in this literature found to go hand in hand with low reallocation. Search frictions in used capital markets are useful in explaining this strong correlation of the capital

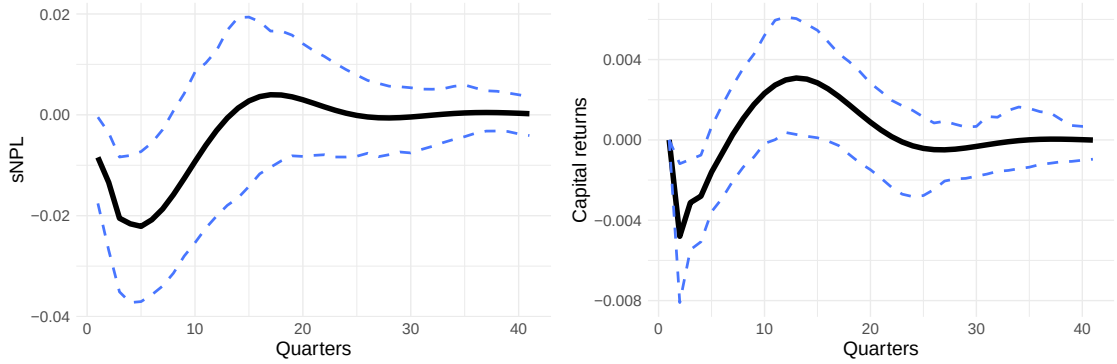


Figure 4.3: Impact of capital returns on the sNPL (left) and vice versa (right).

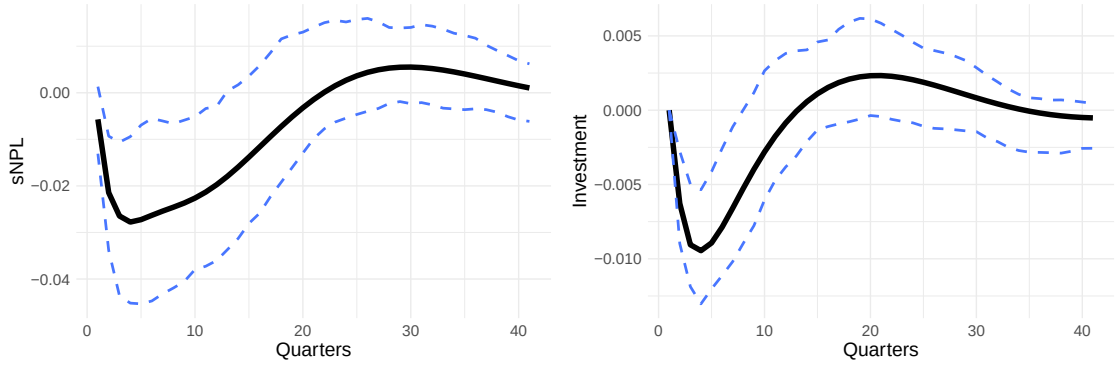


Figure 4.4: Impact of Investment on the sNPL (left) and vice versa (right).

price with capital reallocation.

$$\begin{bmatrix} Real\ GDP \\ sNPL \\ Var\ lagging\ sNPL \end{bmatrix} = A(L) \begin{bmatrix} Real\ GDP(L) \\ sNPL(L) \\ Var\ lagging\ sNPL(L) \end{bmatrix} + B\epsilon \tag{4.2}$$

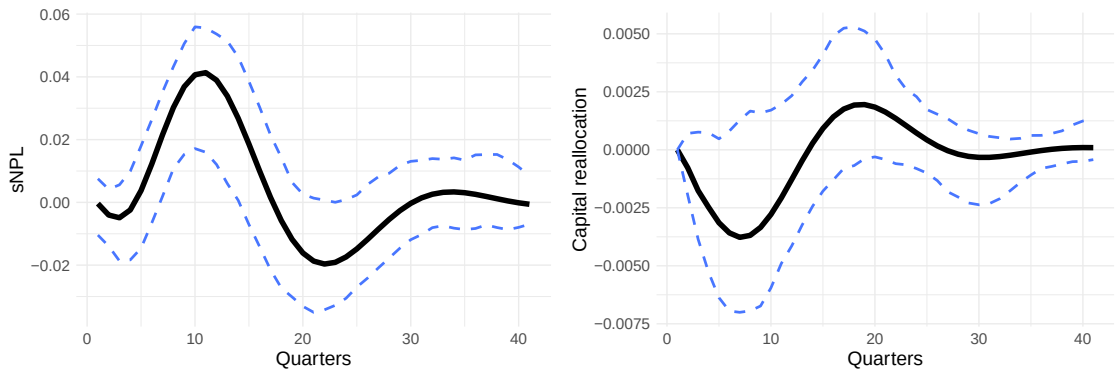


Figure 4.5: Impact of capital reallocation on the sNPL (left) and vice versa (right).

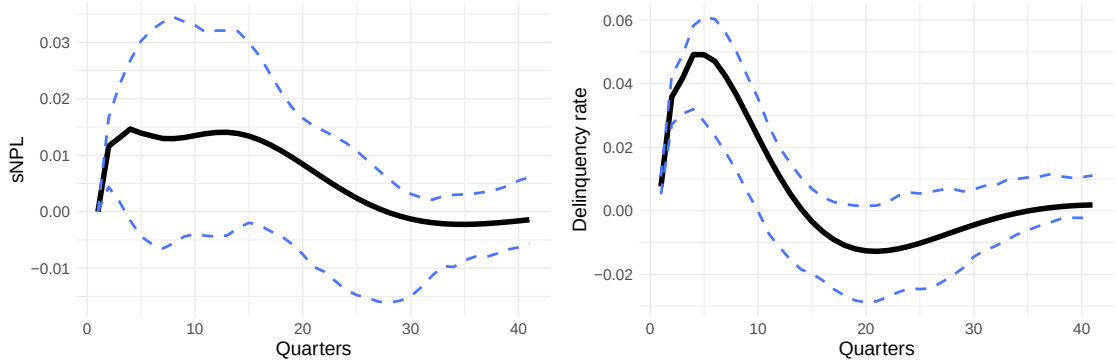


Figure 4.6: Impact of delinquency rates on the sNPL (left) and vice versa (right).

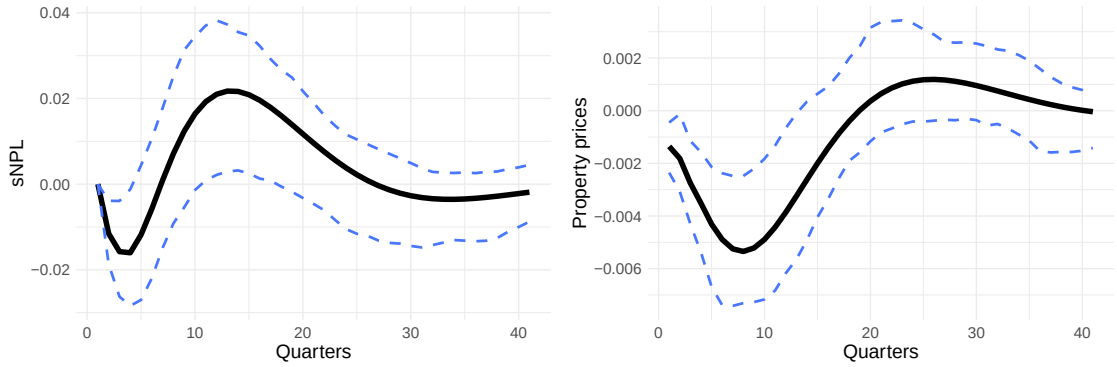


Figure 4.7: Impact of property prices on the sNPL (left) and vice versa (right).

4.2.2 Cross-country comparison showing that capital misallocation and the sNPL are connected

A lemma from the assumption that a high sNPL only reduces credit supply via the bank profit channel is that real returns of capital should increase. The reason for this is that in a frictionless credit market the price of capital should equal marginal returns to capital $r_k = MPK = r_l$. If the lending rate increases due to a reduction in credit supply then $r_k = MPK = r_l + u^{NPL}$. Thus reduced credit supply should lead to increased marginal real returns of capital due to unrealised opportunities.

However, Figure 4.8 shows that real capital returns seem at best negatively correlated with the sNPL in OECD countries for the data available. In the figure, the mean NPL share between 1995 - 2017 is plotted against the mean capital return over the same period. This result is robust to using other measures for the correlations such as the sNPL at the start of the dataset (start_npl), sNPL growth (d_npl), capital returns corrected by value-added growth (r_g_y), and their respective growth rates. The results of these simple regressions are in Table 4.2 and all correlation plots can be found in Appendix 4.B. The main takeaway from this section is that the credit supply channel cannot be the only

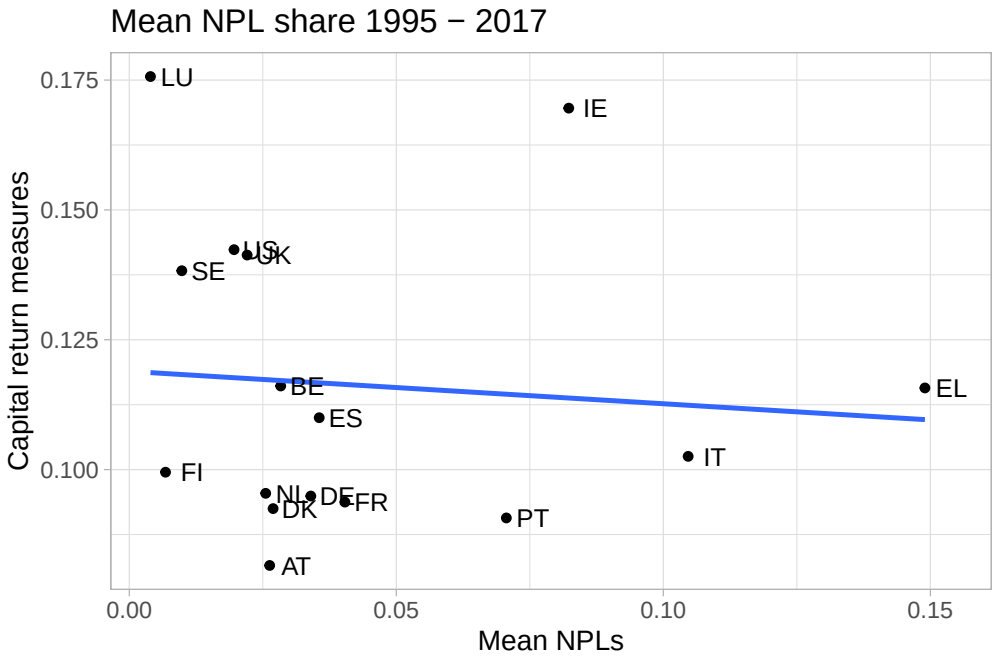


Figure 4.8: Capital return measures are from KLEMS data, while NPL measures are from merged world bank and IMF data.

way through which the sNPL affects other macroeconomic outcomes in general and capital productivity in specific. The structural model in the next section will argue that the results can be explained by the sNPL also indicating a reduction in marginal returns to capital in the economy via a misallocation channel $MP_{Misallocation}$. This can explain the equation before in very reduced form as $r_k + MP_{Misallocation} = r_l + u^{NPL}$. Thus a higher sNPL doesn't lead to higher capital returns, because higher NPL shares indicate that a larger part of the capital stock gets stuck in unproductive relationships and cannot escape these relationships due to market frictions.

Table 4.2: Regressions of capital returns on NPL variables

	Dependent variable:											
	r_k	r_g_y	d_r_k	dr_g_y	r_k	r_g_y	d_r_k	dr_g_y	r_k	r_g_y	d_r_k	dr_g_y
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
m_npl	-0.062	-0.053	-0.038	-0.027								
	(0.194)	(0.183)	(0.057)	(0.056)								
start_npl					-0.150	-0.128	-0.093*	-0.084				
					(0.189)	(0.178)	(0.051)	(0.051)				
d_npl									0.336	0.348	0.133	0.322
									(1.843)	(1.732)	(0.543)	(0.527)
Constant	0.119***	0.116***	-0.0001	0.0003	0.121***	0.118***	0.001	0.002	0.116***	0.113***	-0.002	-0.001
	(0.011)	(0.011)	(0.003)	(0.003)	(0.010)	(0.009)	(0.003)	(0.003)	(0.008)	(0.008)	(0.002)	(0.002)
Observations	16	16	16	16	16	16	16	16	16	16	16	16
R^2	0.007	0.006	0.032	0.016	0.043	0.035	0.189	0.160	0.002	0.003	0.004	0.026
Note:												*p<0.1; **p<0.05; ***p<0.01

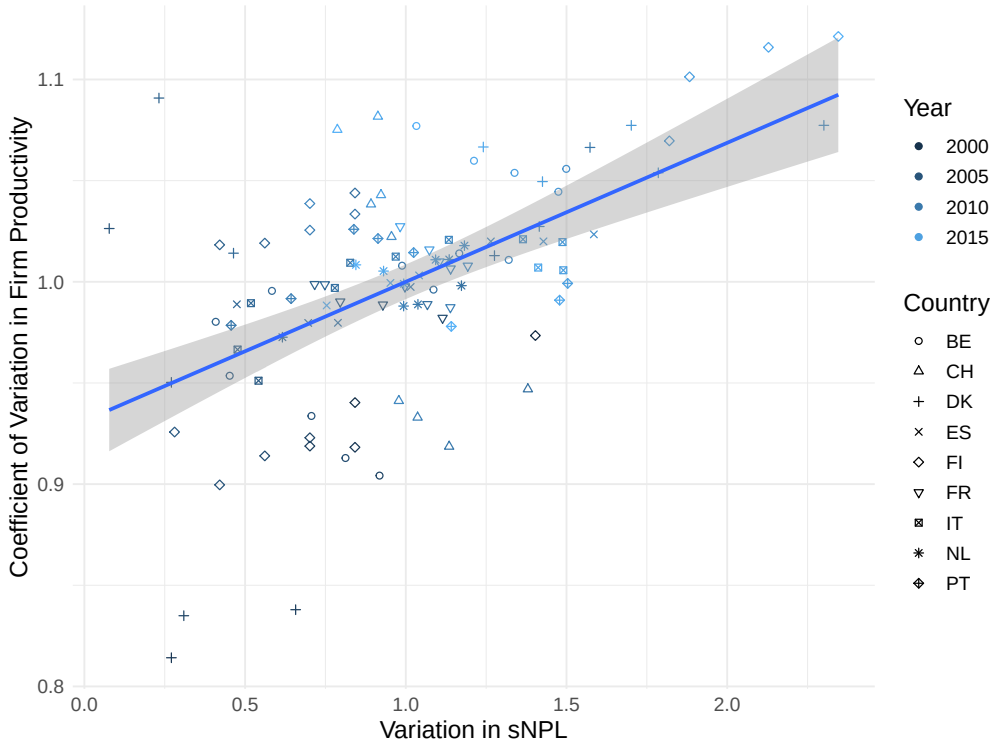


Figure 4.9: As the sNPL changes estimated MPK dispersion from C-D estimation by sector increases proportionally. Source: CompNet 7th vintage dataset for the TFP estimate and sNPL measures are from merged world bank and IMF data.

The result that misallocation rises when the sNPL increase beyond their country mean, which was drawn from aggregate data is confirmed by aggregated micro-data from the 7th Vintage CompNet dataset. Figure 4.9 presents the changes to the coefficient of variation for total factor productivity estimated by imposing a Cobb-Douglas production function on two-digit sectors and employing ordinary least squares estimates on the log of production inputs and value-added. The figure shows that the sNPL being above its mean value in a country is going hand in hand with a larger dispersion of firm productivity. The estimates for total factor productivity employing instead the method for total factor productivity estimation proposed in (Wooldridge, 2009) produce a similar result. They are presented together with further dispersion and misallocation estimates in Appendix 4.B. The structural model will capture this increase in misallocation via lower values of idiosyncratic capital productivity being accepted when the sNPL rises. Table 4.3 further confirms that it is not only output variation driving this variation in dispersion of firm productivity, but that variation in the sNPL significantly contributes to explaining it.

Finally, the variable for the suggested identification in this paper for the dynamic importance of forbearance incentives and frictions in used capital markets is the price of

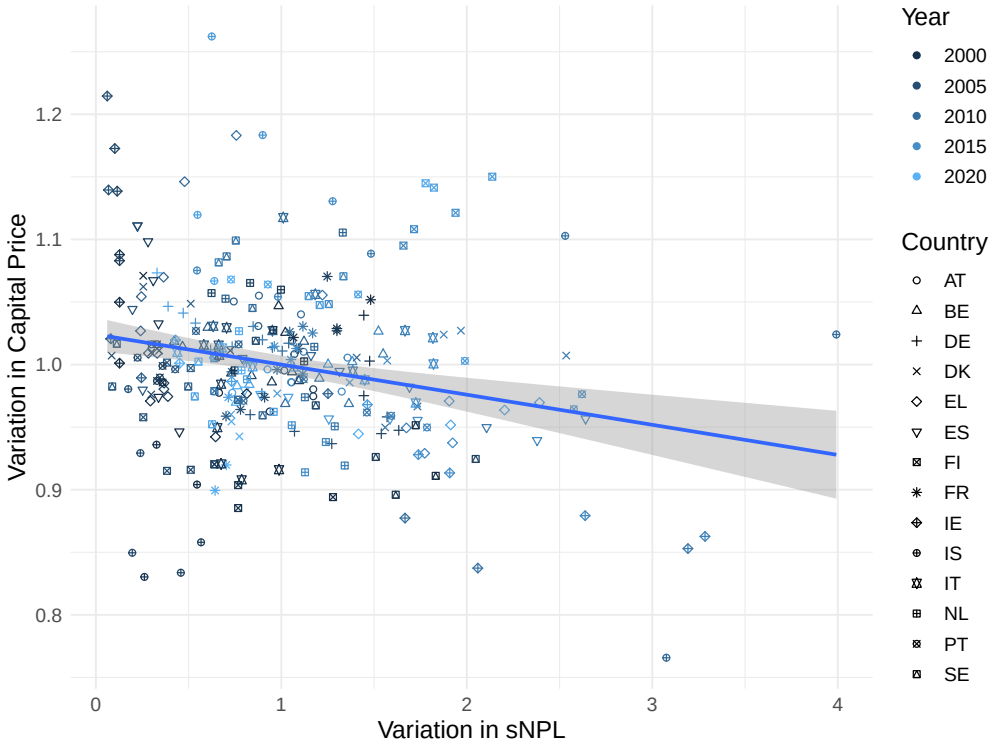


Figure 4.10: As sNPL increase capital prices fall. sNPL measures are from merged world bank and IMF data. The real price of capital is from Eurostat.

capital. Figure 4.10 and Table 4.3 show that this variable generally reacts ambiguously to increases in the sNPL, while in the US time series shows it to decline in sNPL increases in Figure 4.7. Overall the price of capital seems to be more likely to decrease than increase as non-performing loans increase beyond their mean. As will be shown in Section 4.3, capital becoming cheaper would be an unlikely result of forbearance incentives rising strongly and driving the sNPL. The structural model shows rising forbearance incentives will lead to more tight used capital markets, which would lead to an expected increase in the real price of capital, while higher frictions in used capital markets will lead to the opposite.

4.3 Model

The model assumes that capital markets experience matching frictions summarising similar heterogeneities as matching frictions in labour markets and that the aggregate product of capital experiences marginally diminishing returns, whether the capital is put to use or not. Modelling used capital with higher matching frictions than fresh capital is meant to capture the heterogeneities entrepreneurs who want to productively employ used and specified capital face when reusing this specified capital, for example, a specific type of

	Dependent variable:		
	Coefficient of Variation - OLS	Coefficient of Variation - Wooldridge	Capital Price
	(1)	(2)	(3)
NPL variation	0.083***	0.084***	-0.005
	(0.010)	(0.011)	(0.007)
Output variation	0.598**	0.634**	0.198
	(0.256)	(0.276)	(0.145)
Constant	0.917***	0.915***	1.003***
	(0.011)	(0.012)	(0.008)
Observations	85	85	203
R^2	0.453	0.425	0.014
Adjusted R^2	0.440	0.411	0.005
Residual Std. Error	0.039 (df = 82)	0.042 (df = 82)	0.048 (df = 200)
F Statistic	33.990*** (df = 2; 82)	30.277*** (df = 2; 82)	1.466 (df = 2; 200)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 4.3: Explanatory power of variation in the sNPL versus aggregate output variation on capital productivity dispersion and capital prices.

factory. If entrepreneurs were to receive fresh capital they could build the factory to any specification. An entrepreneur seeking to employ a reused factory may find an excellent and inexpensive fit, but may also spend a long time on search not finding a suitable production location. Capital experiencing marginally decreasing returns on the aggregate is a common assumption and can be micro-founded by unused real estate or factories occupying the most productive locations, or occupying other non-modelled resources such as labour performing maintenance tasks or engaging in production with foreclosed capital. As a consequence, each unit of capital exhibits a negative externality on all other units of capital.

The model is placed into a general equilibrium framework to assess the impact of frictions in used capital markets on the dynamics and long-run outcomes of reallocation, non-performing loans, and capital returns. Figure 4.11 sketches an overview of the workings of this general equilibrium model. Households provide fresh resources to create capital in the form of deposits to banks. Households own firms, which search for profitable opportunities to borrow this capital from banks, which then becomes a new loan with underlying collateral. Firms, which can be also viewed as entrepreneurs, make proposals for using the capital to banks, which are at the heart of the capital market. Banks provide capital in form of loans with underlying collateral to firms. Once firms become unproductive and can no longer pay a share of profits in the form of interest to banks in return for the financing provided, the banks have to decide whether to foreclose the loan and seize the collateral capital. If the bank decides on doing this it will seek to re-lend

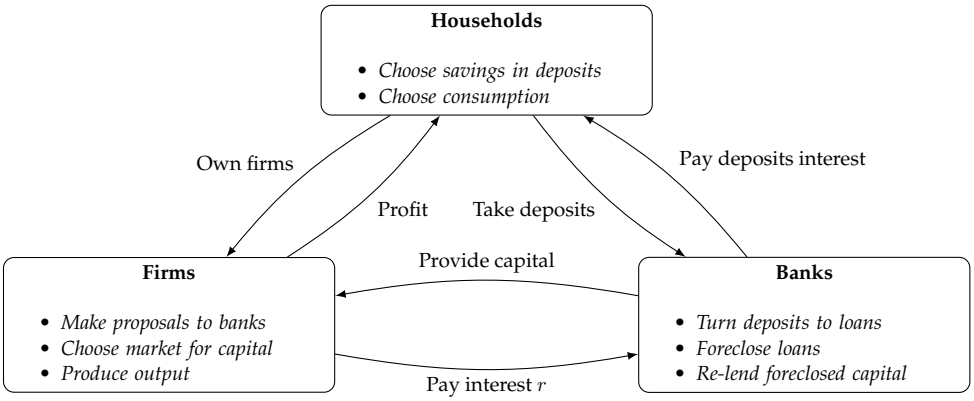


Figure 4.11: Model relations

the foreclosed capital to entrepreneurs. Banks pass any profits or losses generated with household deposits on to households. The capital markets with search frictions in this model are closed on the loan demand side via free entry by firms, while they are closed on the loan supply side via the deposit provision from households, which is derived from the intertemporal Euler equation.

The subsection (Section 4.3.1) in this section aims to illustrate the key mechanism, namely how the efficiency of used capital markets and forbearance incentives affect the bank forbearance or foreclosure decision, in a partial equilibrium model. The second subsection then presents the full dynamic stochastic general equilibrium model with endogenous foreclosure.

4.3.1 Partial equilibrium foreclosure decision

The purpose of this partial equilibrium model is to show that the loan foreclosure decision will be determined by the ability of the bank to reuse foreclosed capital productively and the cost it incurs in foreclosure. For illustration, very simple assumptions are taken leading to a closed-form solution. These are subsequently relaxed in the full model following this section.

Assume a model that only runs for one period. Banks come into the period with all capital lent to firms. For simplicity, banks learn about the productivity of lent capital units, decide whether to foreclose the loan and have the option in case of foreclosure to re-lend the capital within one period. Banks will aim to maximise profits by making an optimal foreclosure decision.

A firm matched with a bank will pay the bank a stochastic interest rate realisation $r(\epsilon)$, which depends on the firm's marginal productivity. $r(\epsilon)$ is assumed to be the stochastic

realisation of $\bar{r} - \epsilon$, where $\log(\epsilon)$ is assumed to be mean zero normally distributed with variance σ normalised to 1. Hence, the set of possible interest rate realisations for legacy loans is $(-\infty, \bar{r})$.

Let p be the probability for the bank of finding a new firm if it decides to foreclose a current loan. p is assumed to be a function of the match efficiency μ of the used capital market and of the tightness of this market θ. Tightness is the number of entrepreneurs offering a business plan g for a capital unit over the number of capital units searching to be matched s. $p(\mu, \theta)$ is a function with the properties $\frac{\partial p}{\partial \mu} \geq 0$ and $\frac{\partial p}{\partial \theta} \geq 0$. Let $V_u = [p(\mu, \theta)\bar{r} + (1 - p(\mu, \theta))b]$. Thus the reward of finding a new loan is to be matched at the productivity frontier $\bar{r}$. The bank will decide to foreclose a legacy loan when $V_m = r(\epsilon) < V_u$. The cutoff value $\tilde{\epsilon}$ at which a bank decides to foreclose a unit of capital is then given by (4.3). p is specified as the result of a Cobb-Douglas matching function with $p = \mu\theta^{0.5}$. Finally, there may be forbearance incentives for banks τ which would increase the benefit from keeping a beginning of period loan.

$$r(\tilde{\epsilon}) = \bar{r} - \tilde{\epsilon} = p(\mu, \theta)\bar{r} + (1 - p(\mu, \theta))b - \tau = p(\mu, \theta)[\bar{r} - b] + b - \tau \tag{4.3}$$

With an appropriate calibration of $b < 0$ to capture possible losses of capital due to depreciation it is then clear that values of $V_u < 0$ are possible depending on the productivity of used capital markets, the value of $\bar{r}$, and match probabilities $p(\mu, \theta)$. A reasonable assumption is to define an NPL cutoff where $r(\epsilon_{NPL}) = 0$, meaning those legacy loans that are maintained by banks even though $r(\epsilon) < 0$ are non-performing loans. The distribution presented in Figure 4.12 with the calibration with $\bar{r} = 1$, $b = -1$, $\mu = 0.3$, $\theta = 1$ and the distributional parameters for $\log(\epsilon)$ set to mean 0 and standard deviation 1 shows that there can be a significant mass on NPLs in this model with matching frictions.

The reason NPLs arise in this model is that foreclosing a loan doesn't necessarily mean for the bank that it will be able to re-lend the underlying capital to a new firm and gain $\bar{r}$. The bank may make a loss b on the capital instead as it stays idle, depreciates, or possibly requires maintenance costs or management by the bank. The probability of this negative event happening depends on match probabilities p. Low probabilities due to low used capital efficiencies will mean that the bank is willing to accept more and more negative interest rates increasing the NPL share in total loans given in equation (4.4), where $\Phi()$ is

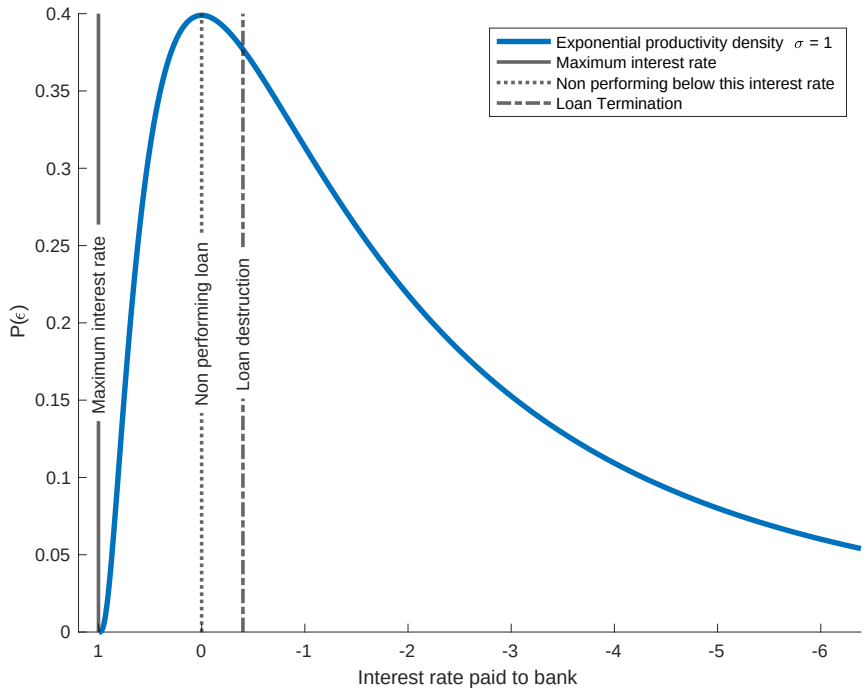


Figure 4.12: Simple illustration of a mass of NPLs in the distribution of existing loans

the cumulative distribution function of a normal distribution.

$$NPLshare = \frac{\Phi(\log(\tilde{\epsilon})) - \Phi(\log(\epsilon_{NPL}))}{\Phi(\log(\tilde{\epsilon})) + p[1 - \Phi(\log(\tilde{\epsilon}))]} \tag{4.4}$$

$\Phi(\log(\tilde{\epsilon}))$ captures is the share of surviving legacy loans while $p[1-\Phi(\log(\tilde{\epsilon}))]$ is the value of newly created loans. Mean expected interest rates r_m received by banks can be computed from the mills ratio as $r_m = \bar{r} - \exp(-\frac{\phi(\log(\tilde{\epsilon}))}{\Phi(\log(\tilde{\epsilon}))})$ and 50 % variation in parameters μ, θ, and $\bar{r}$ can be compared to provide an idea of the forces in the model. Here μ is an increase in used capital market efficiency, while θ is an increase in entrepreneurial activity, thus an increase in investment and $\bar{r}$ can be interpreted as a rise in total factor productivity lifting interest rates. The underlying outcomes of these variations on the partial equilibrium model are shown in Figure 4.13. These variations show that a higher value of foreclosure only leads to more loans being called, hence more reallocation if the increase is due to a rise in match probabilities, which happens when θ or μ increase. While increases in θ, which can be interpreted as rising capital demand, experience marginally diminishing returns, increases in match productivity exponentially increase mean interest rates and decrease NPL shares due to increased reallocation. When $\bar{r}$ rises reallocation falls as shown by the rise in surviving loans. The only reason NPLs fall, in this case, is

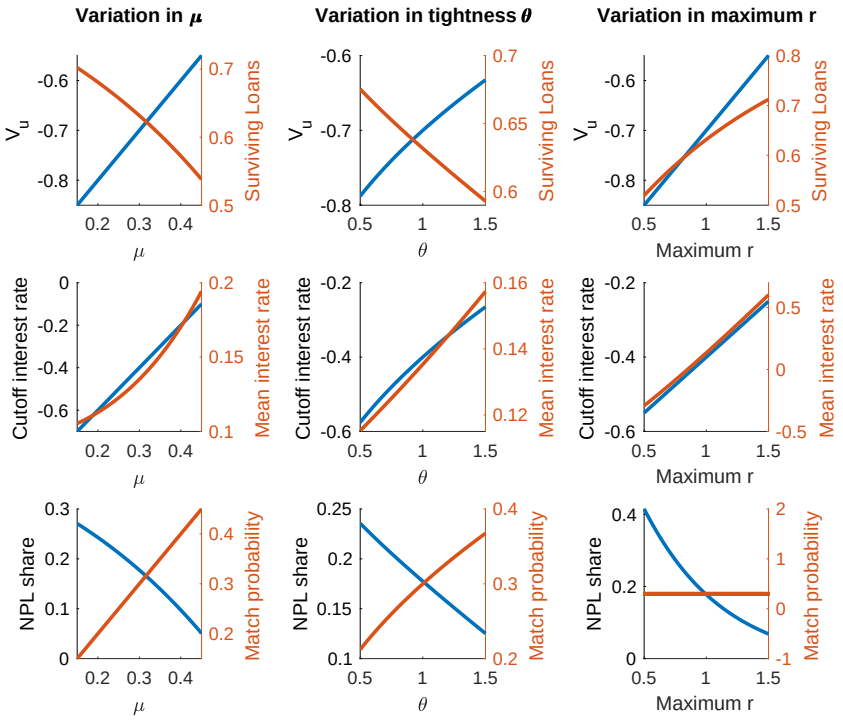


Figure 4.13: Comparison of the value of foreclosing a loan V_u, surviving legacy loans, the cutoff interest rate, the mean interest rate paid to banks, the share of NPLs in total loans and the match probability to variations in μ, θ, and $\bar{r}$. The blue line plots the values on the left axis, while the orange plots values of the right axis.

due to the rising denominator and the shift of the distribution, but not due to reallocation. This shows that match efficiency and reallocation go hand in hand. However, to study the business cycle properties of this mechanism it has to be included in the dynamic general equilibrium setting, which is done in the next section.

Figure 4.14 finally shows the comparison between an increase in the efficiency of used capital markets and a decrease in forbearance incentives. From the comparative statics, it is clear that both would lead to a similarly shaped decrease in the sNPL as both affect the cutoff condition for $r(\tilde{\epsilon})$. However, while a higher forbearance incentive leads to loans being kept due to the cost of dissolving the loan, a lower efficiency of used capital markets drives down the probability of rematching and thereby V_u. This ultimately will lead to less tight capital markets as more capital is on the market searching to be matched with a lesser likelihood of success. The dynamic general equilibrium setting can use this distinction to identify the importance of forbearance incentives in comparison to the efficiency of used capital markets using the behaviour of the observed price of capital following a shock.

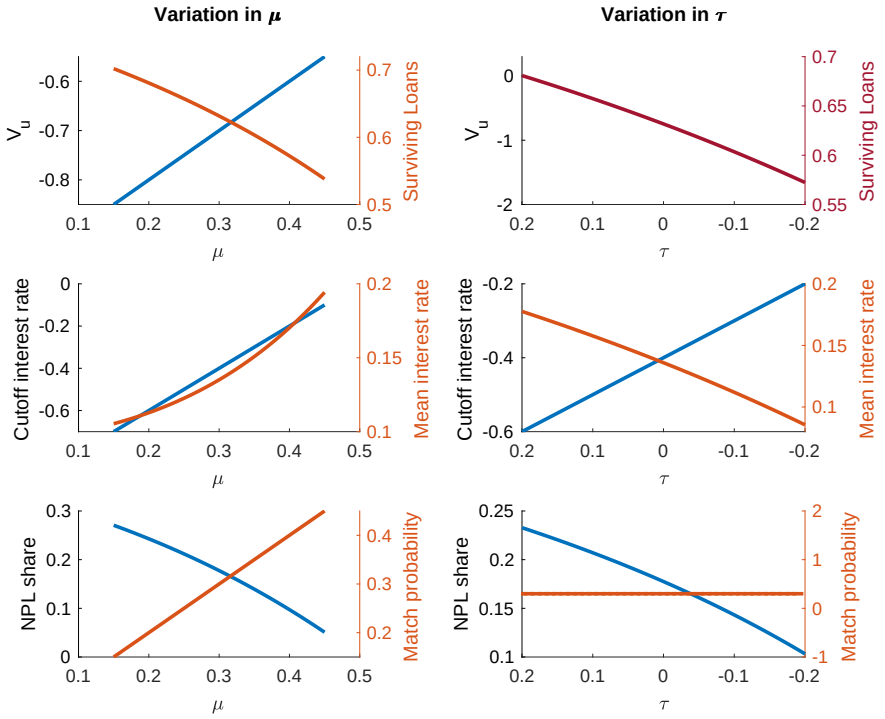


Figure 4.14: Comparison of the value of foreclosing a loan V_u, surviving legacy loans, the cutoff interest rate, the mean interest rate paid to banks, the share of NPLs in total loans and the match probability to variations in μ, τ. The blue line plots the values on the left axis, while the orange line plots the values of the right axis. The red line means the blue and orange lines are overlaid.

4.3.2 General equilibrium model with endogenous loan foreclosure

The decision on whether to foreclose a non-performing loan, on which the partial equilibrium model of the previous section focused, can be integrated into a dynamic stochastic general equilibrium. The expected value of a capital unit underlying as collateral a loan for banks to entrepreneurs then creates an infinite value function, which depends on the ability of the entrepreneur to pay for the capital unit of the loan hence the entrepreneur's capital productivity. Other parts of the model are kept as simple as possible. Some kind of heterogeneity in the productivity of loans will be necessary to model non-performing loans. Non-performing loans are those where capital is below a certain productivity $z < \tilde{z}$, but above the bank's cutoff value z_s. Further, there is a certain level below which a bank chooses to foreclose the non-performing loans $\tilde{z}$. From this time on the loan will only produce the fixed value g and the only purpose will be to rematch the capital with another entrepreneur. Defining reasonable conditions for when to consider a loan as non-performing it will be shown that with matching frictions in used capital markets some loan productivities z fall in the interval $[\tilde{z}, \hat{z}]$, and thus there exists a share of loans that are non-performing but not foreclosed at all times.

4.3.3 Aggregate production function

Similar to (Ottonello, 2017), who distinguishes between several types of capital based on its "employment status", there are three types of capital stocks in the economy. These are matched capital in safe loans and in weak loans denoted by K_t^N and K_t^E and unmatched capital denoted by K_t^U. Matched capital is employed in firms in production. Matched capital is split into matched capital in safe loans K_t^N and capital in weaker loans where payment of interest rate is uncertain, possibly lower than the market rate, and which may become non-performing K_t^E. Unmatched capital is owned by banks and only produces output g, but still forms part of the capital stock. The aggregate capital stock in the economy is $K_t = K_t^N + K_t^L + K_t^U$. Firms produce output with capital units. Each capital unit can either be in a loan or be foreclosed and held by the bank. When a capital unit is in a fresh loan, which will not be foreclosed it produces output according to equation (4.5). When a capital unit is in a loan that may be foreclosed it produces output according to equation (4.6). When a capital unit is foreclosed it produces output according to equation (4.7).

$$\bar{y} = AK^{\alpha-1}\bar{z} \tag{4.5}$$

$$y_i = AK^{\alpha-1}z \tag{4.6}$$

$$y_i = g \tag{4.7}$$

The total aggregate capital stock negatively affects returns to the capital unit, but the distribution of capital over states does not. Employed capital K_t^E is the sum of all employed capital units K_i^L over all states $K_t^E = \sum_{i=0}^{Z} z_i K_i^L$. Total output is given by equation (4.8).

$$Y = \bar{y}K_t^N + \int_{z_s}^{\bar{z}} y(z)\, d(z) K^L + gK^U = AK^{\alpha-1}(\bar{z}K^N + \int_{z_s}^{\bar{z}} z\, d(z) K^L) + gK^U \tag{4.8}$$

Output may be used for consumption by the households, as well as investment into bank deposits to create more productive capital, or as a resource to set up a business plan proposal by the entrepreneur.

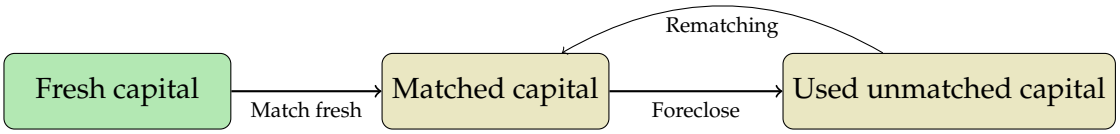


Figure 4.15: Life cycle of collateral underlying lent capital

K_u is the capital banks have foreclosed. This capital remains idle and only produces with g, which may also be chosen to be a negative output consuming resources. Banks seek to rematch foreclosed capital in secondary capital markets with productive entrepreneurs. The life-cycle of a physical capital unit underlying a bank loan is sketched in Figure 4.15. Fresh capital can be added to the existing capital stock via investment but matched and foreclosed unmatched capital remain in the economy. Matched capital changes exogenously with probabilities π^N and π^L from being matched in a safe or weak loan and vice versa. Capital switches states from weak loans to unmatched capital owned by banks according to the agent decision-making. The only way for physical capital to exit the economy is via depreciation.

4.3.4 Households

The economy is populated by a unit mass of identical households. Each household has an initial deposit wealth D_0. However, not all deposits are automatically turned to capital. Deposits may stay idle and remain bank cash reserves X. This means capital deposit values won't always suffer a depreciation in the same way as the underlying collateral values of capital do.

$$\max_{C_t, I_t} E_t(\sum_{s=0}^{\infty} \frac{C_{t+s}^{1-\sigma}}{1-\sigma}) \tag{4.9}$$

The maximisation problem is subject to a conventional budget constraint where consumption C and investment I equal firm profits Π_t and interest rates paid on bank deposits $\rho_t D_t$.

$$C_t + I_t = \Pi_t + \rho_t D_t \tag{4.10}$$

Deposits evolve according to equation (4.11) with the current value of $\delta_{d,t-1}$ taken as

given by the individual household and defined at a later time.

$$D_t = (1 - \delta_{d,t-1})D_{t-1} + I_{t-1} \tag{4.11}$$

This means the inter-temporal Euler equation is as is conventional a function of depreciation, interest rates ρ, as well as present and future consumption.

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta_{d,t+1} + \rho_{t+1})] \tag{4.12}$$

4.3.5 Credit market

Financial intermediaries (Banks)

Banks turn household deposits into physical capital units when they match with entrepreneurs making a convincing business proposal that receives financing. The rate at which proposals arrive is $p(\theta_{x,t-1})$, where $\theta_{x,t} = \frac{g_{x,t}}{s_{x,t}}$ denotes market tightness. $s_{x,t}$ is the share of deposits that may be lent out, which may be affected by regulatory policy $s_{x,t} = \psi_x x_t$. Denote deposits that have not yet been turned into capital with X_t. Fresh deposits evolve according to equation (4.13).

$$X_t = (1 - \delta_x)[X_{t-1} - p(\theta_{x,t-1})S_{t-1}] + I_{t-1} \tag{4.13}$$

A bank's present discounted value from a safely matched unit of capital in a match that was just created by lending out deposits or rematching used capital is denoted by $V_{\bar{z}}^{B,L}$. The present discounted value of a capital unit in a weak lending relationship is denoted by $V_z^{B,L}$. In both cases, z describes the idiosyncratic productivity of the capital being used. The present discounted value of a capital unit that is unused because the bank has foreclosed it is $V^{B,U}$. The present discounted value of a new and a weak lent unit are defined by equations (4.14) and (4.15). For simplicity of notation define the mean expected surplus-value of a weak loan match over a foreclosed loan next period as $\hat{V}_{\hat{z},t+1}^{B,L} = \int_{z_{z,t+1}}^{\bar{z}} (\hat{V}_{z,t+1}^{B,L} - V_{t+1}^{B,U} + \tau)\, d(z)$. τ captures a loan forbearance incentive for banks, who depending on the calibration may incur real costs when foreclosing a loan

and reducing the size of their balance sheet.

$$V_{\bar{z},t}^{B,N} = r_{\bar{z},t} + (1-\delta_k)E_t[\mu_{t+1}[\pi^N(V_{\bar{z},t+1}^{B,L} - V_{t+1}^{B,U}) + (1-\pi^N)(\hat{V}_{\hat{z},t+1}^{B,L} - \tau) + V_{t+1}^{B,U}]] \quad (4.14)$$

$r_{\bar{z}}$ denotes the interest rate paid by firms in safe loans, which will depend on the productivity of the capital underlying the loan, while $\mu_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\sigma}$ is the stochastic discount factor. δ_k is the depreciation rate of capital. π^N is the probability that a safe loan will remain safe, while it will turn into a weaker loan with probability $(1-\pi^N)$. In the case where $\pi^N = 0$ and $\pi^L = 1$ the model is similar to the interpretation of new loans being created by businesses at the technology frontier as is assumed in (Mortensen and Pissarides, 1994) for new jobs.

$$V_{z,t}^{B,L} = r_{z,t} + (1-\delta_k)E_t[\mu_{t+1}[(1-\pi^L)(V_{\bar{z},t+1}^{B,L} - V_{t+1}^{B,U}) + \pi^L(\hat{V}_{\hat{z},t+1}^{B,L} - \tau) + V_{t+1}^{B,U}]] \quad (4.15)$$

r_z denotes the interest rate, which will depend on the productivity of the capital underlying the loan, while $\mu_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\sigma}$ is the stochastic discount factor. δ_k is the depreciation rate of capital. The present discounted value of a foreclosed unit of capital is in equation (4.16).

$$V_t^{B,U} = g + (1-\delta_k)E_t[\mu_{t+1}(p(\theta_{u,t})[V_{\bar{z},t+1}^{B,L} - V_{t+1}^{B,U}] + V_{t+1}^{B,U})] \quad (4.16)$$

g is the period benefit banks receive or the period cost banks incur on a foreclosed capital unit. $p(\theta_u)$ is the probability with which a bank will find a new entrepreneur willing to take on the foreclosed capital unit. This probability will depend on market tightness in secondary capital markets θ_u and match productivity μ_u.

Firms

Firms submit proposals for funding to banks. Opening a proposal and presenting it to financial intermediaries comes at a cost κ_j. j denotes the possibility of the firm searching either in markets for fresh or markets for used capital. It is assumed that the expected benefit of receiving a used or a new capital unit is brought to the same value by market forces. This leads to the following equivalencies for allocating search between fresh and

used capital markets in equation (4.17).

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = \frac{\kappa_u}{q_u(\theta_{u,t})} = (1-\delta_k)E_t[\mu_{t+1}V^{E,N}_{\bar{z},t+1}]) \tag{4.17}$$

The value of a successful match to the firm is found in equation (4.19). The firm will produce the period output $A_t z K_t^{\alpha-1}$ with the collateral capital provided and pay interest rate $r_{z,t}$ to the bank in return for providing it. The future present discounted value of the collateral capital provided is $(1-\delta_k)E_t(\mu_{t+1}[V^{E,L}_{z,t+1}])$, which accounts for temporal discounting and capital depreciation. Again, for simplicity of notation define the mean expected surplus value of a weak loan match to the entrepreneur next period as $\hat{V}^{E,L}_{\hat{z},t+1} = \int_{z_{z,t+1}}^{\bar{z}} V^{E,L}_{z,t+1}\, d(z)$.

$$V^{E,N}_{\bar{z},t} = A_t K_t^{\alpha-1}\bar{z} - r_{\bar{z},t} + (1-\delta_k)E_t\left[\mu_{t+1}[\pi^N V^{E,N}_{\bar{z},t+1} + (1-\pi^N)\hat{V}^{E,L}_{\hat{z},t+1}]\right] \tag{4.18}$$

$$V^{E,L}_{z,t} = A_t K_t^{\alpha-1} z - r_{z,t} + (1-\delta_k)E_t\left[\mu_{t+1}[(1-\pi^L)V^{E,N}_{\bar{z},t+1} + \pi^L\hat{V}^{E,L}_{\hat{z},t+1}]\right] \tag{4.19}$$

Equilibrium interest rate

The equilibrium interest rate is determined via Nash bargaining between the bank and the firm. This delivers a common solution, though more complicated bargaining solutions may be implemented as well. Let η be the bargaining power of the bank.

$$\eta V^{E,L}_{\bar{z},t} = (1-\eta)(V^{B,L}_{\bar{z},t} - V^{B,U}_t) \tag{4.20}$$

$$\eta V^{E,L}_{z,t} = (1-\eta)(V^{B,L}_{z,t} - V^{B,U}_t + \tau) \tag{4.21}$$

The Nash bargaining solution for safe loans is in equation (4.20), while the solution for weak loans is in equation (4.21). For weak loans there is a possibility of separation, so continuation of the match forms part of the surplus for banks.

$$r_{\bar{z},t} = \eta[A_t K_t^{\alpha-1}\bar{z} + \kappa_u\theta_{u,t}] + (1-\eta)g \tag{4.22}$$

$$r_{z,t} = \eta[A_t K_t^{\alpha-1} z + \kappa\theta_{u,t}] + (1-\eta)[g - \tau(1-\beta\pi^L(1-\delta_k))] \tag{4.23}$$

Loan creation decision

Substituting equation (4.22) into equation (4.18) and combining it with the free entry condition for entrepreneurs in capital markets in equation (4.17) yields the loan creation condition in equation (4.24).

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = (1-\eta)(A_t K_t^{\alpha-1}\bar{z} - g) - \eta\kappa_u\theta_{u,t} + E_t\left[\pi^N \frac{\kappa_x}{q_x(\theta_{x,t+1})} + (1-\delta_k)(1-\pi^N)E_t(\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1})\right] \tag{4.24}$$

Foreclosure decision

A bank will only choose to foreclose capital when the benefit from the foreclosed capital exceeds the benefit from keeping the match. The benefit from foreclosing a loan is $V_t^{B,U}$, which is the real present discounted value of the used specified capital in the match. This foreclosed capital can be sold in frictionless financial markets and purchased by other banks or kept by the bank itself. In either case, the bank will choose not to foreclose a loan as long as (4.25) holds.

$$V_{z,t}^{B,L} - V_t^{B,U} + \tau > 0 \tag{4.25}$$

The foreclosure condition in equation (4.26) is the result of combining equation (4.15), (4.16), and (4.23).

$$0 = [(1-\eta)(A_t K_t^{\alpha-1} z_{s,t} - g + \tau(1-\beta\pi^L(1-\delta_k))) - \eta\kappa\theta_{u,t} + E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1}\right]] \tag{4.26}$$

$$z_{s,t} = K_t^{1-\alpha} A_t^{-1}[g - \tau(1-\beta\pi^L(1-\delta_k))) + \frac{\eta}{1-\eta}\kappa_u\theta_{u,t} - \frac{1}{1-\eta}E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}^{E,L}_{\hat{z},t+1}\right]] \tag{4.27}$$

The foreclosure condition implies that the productivity cutoff for weak loans will increase

with the total capital stock, as loans become less productive due to declining marginal returns. The cutoff decreases with higher productivity levels and decreases with lower levels of demand for newly built capital by entrepreneurs. Further, forbearance incentives will lead to lower productivities being accepted by banks before they foreclose the entrepreneur.

Transition laws

The total real value of existing capital units will equal the total value of real deposits at all times.

$$D_t = X_t + K_t^N + K_t^L + K_t^U \tag{4.28}$$

The transition law for bank cash is found in equation (4.29). Cash may be reduced by a small overhead taken by the bank δ_x as a storage fee and the capital $p(\theta_{x,t-1})S_{t-1}$ lent out to firms. X_t is increased by deposits made by households I_{t-1}.

$$X_t = (1-\delta_x)[X_{t-1} - p(\theta_{x,t-1})S_{t-1}] + I_{t-1} \tag{4.29}$$

The transition law for collateral capital in safe loans K^N is given by equation (4.30) and states that present capital is the not depreciated $(1-\delta_k)$ part of past safely lent capital K_{t-1}^N, which has also not been turned weak with probability π^N. Loans that have turned from weak to strong loans again $(1-\pi^L)K_{t-1}^L$ are added. Finally, the capital which has been successfully re-lent $p(\theta_{u,t-1})K_{t-1}^U$ plus the successfully added fresh capital built from cash $p(\theta_{x,t-1})S_{t-1}$ form also part of K_t^N.

$$K_t^N = (1-\delta_k)[\pi^N K_{t-1}^N + p(\theta_{u,t-1})K_{t-1}^U + p(\theta_{x,t-1})S_{t-1} + (1-\pi^L)K_{t-1}^L] \tag{4.30}$$

The transition law for collateral capital in weak loans K^L is given by equation (4.31) and states that present capital in weak loans is the not depreciated $(1-\delta_k)$ part of past lent capital in weak loans K_{t-1}^L, which has also not been foreclosed $(1-H(z_{s,t}))$ and not turned safe again with $(1-\pi^L)$. To this capital, the loans which have turned from safe to weak in the last period are added $(1-\pi^N)K_{t-1}^N$.

$$K_t^L = (1-\delta_k)(1-H(z_{s,t}))[\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N] \tag{4.31}$$

Here $H(z)$ is the cumulative distribution function of the stochastic variable z. The transition law for K^U in equation (4.32) is the not depreciated part $(1-\delta_k)$ of past unmatched capital plus foreclosed matched capital $H(z_{s,t})[K^L_{t-1} + p(\theta_{u,t-1})K^U_{t-1}]$ minus successfully re-matched foreclosed capital $p(\theta_{u,t-1})K^U_{t-1}$.

$$K^U_t = (1-\delta_k)[H(z_{s,t})(\pi^L K^L_{t-1} + (1-\pi^N)K^N_{t-1}) + (1-p(\theta_{u,t-1}))K^U_{t-1}] \tag{4.32}$$

Overall the transition law for deposits is then the addition of cash X, lent capital $K^N + K^L$, and foreclosed capital K^U.

$$X_t+K^N_t+K^L_t+K^U_t = (1-\delta_x)(X_{t-1}-p(\theta_{x,t-1})S_{t-1})+(1-\delta_k)[K^N_{t-1}+K^L_{t-1}+K^U_{t-1}+p(\theta_{x,t-1})S_{t-1}]+I_{t-1} \tag{4.33}$$

This transition allows for finding the deposit depreciation rate $\delta_{d,t}$ in every period in equation (4.34).

$$\delta_{d,t} = \delta_k \frac{K^N_{t-1} + K^L_{t-1} + K^U_{t-1} + p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} + \delta_x \frac{X_{t-1} - p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} \tag{4.34}$$

With the deposit depreciation rate, the aggregate deposit law of motion originally specified for the household is complete.

$$D_t = (1-\delta_{d,t})D_{t-1} + I_{t-1} \tag{4.35}$$

Interest rates

As expected, the interest rate paid to the household ρ_t depends on the state of the lent capital in safe $r_{\bar{z},t}$ and weak loans $r_{\hat{z},t}$ and the proportion of lent $\frac{K^N_t+K^L_t}{D_t}$ and foreclosed capital $\frac{K^U_t}{D_t}$ on the active side of the bank balance sheet versus deposits, the passive side.

$$\rho_t = \frac{r_{\bar{z},t}K^N_t + r_{\hat{z},t}K^L_t + gK^U_t}{D_t} \tag{4.36}$$

4.3.6 NPL cutoff

The sNPL is defined endogenously. This share will be loans that are no longer profitable for the bank. In this case, NPLs are defined as the share of total weak loans K^L where the interest rate paid to banks doesn't cover the long-run risk-less equilibrium rate paid to consumers. The idea is that a bank at this point would be better off if it could take back the capital unit provided with the specific loan without facing any frictions and transfer the capital into a fixed income asset providing the long-run risk-less rate.

$$r_{z,t} \leq \rho \tag{4.37}$$

We can thus find the real interest rate paid by firms to banks below which the loan turns non-performing by finding $\hat{z}$ as a function of equilibrium ρ.

$$r_{\hat{z},t} = \rho \tag{4.38}$$

4.3.7 Intra-period heterogeneity via the productivity distribution of loans

Each period the loan productivity of the capital in weak loans is drawn from a negative exponential distribution. The choice of an exponential distribution versus the log-normal distribution in the partial equilibrium model does not impact the model properties. The advantage of the exponential distribution over the log-normal is that it facilitates finding closed expressions for the equations above due to the closed-form expressions available for conditional expectations for this type of distribution. The idea of the negative exponential distribution is to assume that most loans are productive, but there is a subset in the tail that become costly to the banks in real terms due to being very unproductive. Thus there is a maximal value of z, which is $\bar{z}$. All capital in safe loans is employed at productivity level $\bar{z}$. Capital in weak loans may be employed at a productivity $z \in [\bar{z}, -\infty]$. This is the result of a realisation ζ of the exponential distribution that is subtracted from $\bar{z}$. The exponential distribution for ζ with the calibrated values is pictured in Figure 4.16.

$$z = \bar{z} - \zeta \tag{4.39}$$

In general, ζ is exponentially distributed with probability density function $\gamma \exp(-\gamma\zeta)$.

The properties of the exponential distribution allow for a closed form computation of

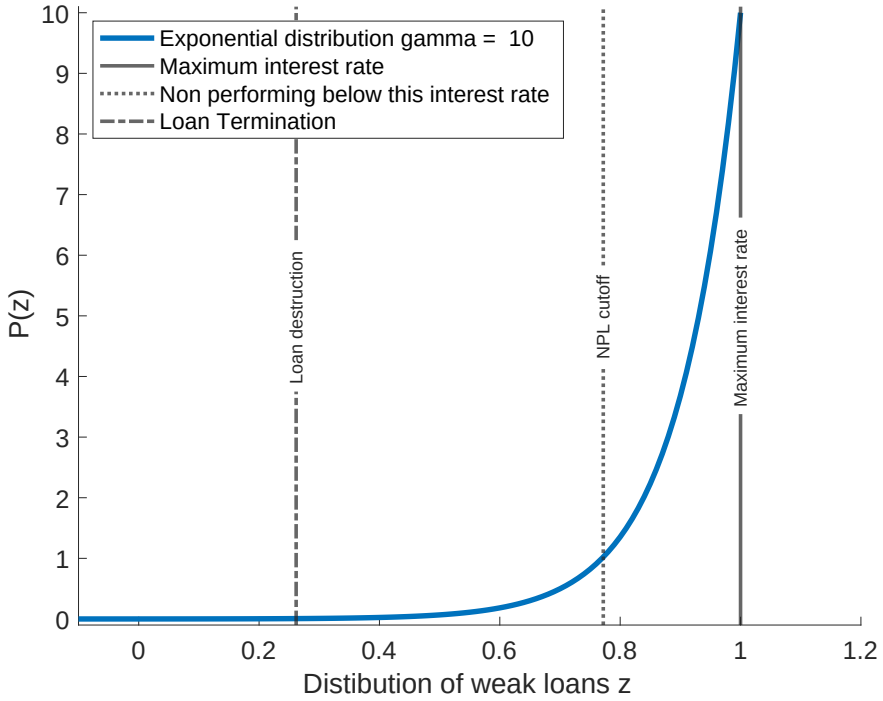


Figure 4.16: Distribution of the productivity of weak loans in the stochastic steady-state with $\gamma = 10$.

the mean value of not foreclosed weak loans $V_{z,t}^{\hat{E},L}$. Given a cutoff z_s, the cutoff of ζ will be $\zeta_s + \bar{z}$. This leads to a mean value of the exponential distribution, with γ as the distributional parameter of $-\zeta_s$, or transformed to z to a conditional expected value $z_s \in (-\infty, \bar{z}]$. We can transform a value of ζ to idiosyncratic productivity with equation (4.39). The probability of weak loans being foreclosed is then $H(z_s) = exp(-\gamma(\bar{z} - z_s))$. The mean productivity of a weak loan is then given by equation (4.40).

$$\hat{z} = \frac{\bar{z} - \frac{1-\gamma \exp(-\gamma[\bar{z}-z_{s,t}])(\bar{z}-z_{s,t})}{\gamma}}{1 - \exp(-\gamma[\bar{z} - z_{s,t}])} \tag{4.40}$$

4.3.8 Stochastic exogenous processes

There are three stochastic processes in this economy. Equation (4.41) shows the auto-regressive process capturing typical deviations in aggregate productivity. Equation (4.42) consists of an auto-regressive process capturing shocks to the current forbearance incentive with $t\hat{a}u = \tau_t - \pi^L(1-\delta_k))E_t(\mu_{t+1}\tau_{t+1})$. $\sigma a, \tau$, which is suspected to be less or equal to 0 captures forbearance incentives possibly rising in line with an aggregate shock. Similarly, equation (4.43) captures possible changes to the efficiency of used capital markets. It is possible that $\sigma a, \mu > 0$, especially when shocks to aggregate productivity affect the functioning of capital transactions due to other turmoil in the economy. Further, the

Dynamic competitive equilibrium: Exogenous processes

Stochastic aggregate process:

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t]$$

Stochastic forbearance cost process:

$$\hat{\tau}_t = (1 - \rho_\tau)\tau + \rho_\tau \hat{\tau}_{t-1} + \sigma a, \tau A_t + \epsilon_\tau]$$

Stochastic used capital market efficiency process:

$$\mu_{u,t} = (1 - \rho_\mu)\mu_u + \rho_\mu \mu_{u,t-1} + \sigma a, \mu A_t + \epsilon_\mu]$$

Highlighted Box 4.1: Competitive equilibrium: Exogenous processes

Cobb-Douglas matching function may not capture all changes in frictions in capital markets similar as in employment markets, where deep recessions have been shown to be accompanied by a significant decline in matching efficiency (Sedláček, 2014). Equally, it is likely that $\sigma a, \tau < 0$ as forbearance incentives for banks probably rise due to loan moratoria and balance sheet pressures in recessions.

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \tag{4.41}$$

$$\hat{\tau}_t = (1 - \rho_\tau)\tau + \rho_\tau \hat{\tau}_{t-1} + \sigma_{a,\tau}\epsilon_t + \epsilon_\tau] \tag{4.42}$$

$$\mu_{u,t} = (1 - \rho_\mu)\mu_u + \rho_\mu \mu_{u,t-1} + \sigma_{a,\mu}\epsilon_t + \epsilon_\mu] \tag{4.43}$$

4.3.9 Equilibrium

The competitive equilibrium of the economy can be summarised by the following equations in the highlighted boxes 4.1, 4.2, 4.3, and 4.4. The matching functions are specified in a Cobb Douglas form with the number of matches of new and foreclosed capital given as $M_x = \mu_x \psi X \theta_x^{1-\xi}$ and $M_u = \mu_u \psi K^U \theta_u^{1-\xi}$. $\hat{tau}$ is defined as $\hat{tau} = \tau(1 - \beta\pi^L(1 - \delta_k))$ capturing possibly dynamically changing forbearance incentives in the banking sector. The idea is that these incentives may increase when the economy is experiencing a negative shock. Similarly, the efficiency of used capital markets may decrease in economic crisis as market makers exit.

Dynamic competitive equilibrium: Household and aggregate

Euler equation:

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta_{d,t+1} + \rho_{t+1})]$$

Resource constraint:

$$C_t + I_t = A_t K_t^{\alpha-1}(\bar{z}K_t^N + \hat{z}K_t^L) + gK_t^U - \kappa_x\theta_{x,t}\psi X_t - \kappa_u\theta_u K_{u,t}$$

Deposit depreciation:

$$\delta_{d,t} = \delta_k \frac{K_{t-1}^N + K_{t-1}^L + K_{t-1}^U + p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} + \delta_x \frac{X_{t-1} - p(\theta_{x,t-1})S_{t-1}}{D_{t-1}}$$

Interest rate paid to households:

$$\rho_t = \frac{r_{\bar{z},t}K_t^N + r_{\hat{z},t}K_t^L + gK_t^U}{D_t}$$

Aggregate capital and deposits:

$$K_t = K_t^N + K_t^L + K_t^U, \quad D_t = X_t + K_t^N + K_t^L + K_t^U$$

Highlighted Box 4.2: Competitive equilibrium: Household and aggregate constraints

Dynamic competitive equilibrium: Credit markets

Loan creation condition:

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = E_t((1-\delta_k)\mu_{t+1}\left[(1-\eta)(A_{t+1}K_{t+1}^{\alpha-1}\bar{z} - g) - \eta\kappa_u\theta_{u,t+1} + \pi^N\frac{\kappa_x}{q_x(\theta_{x,t+1})} + (1-\pi^N)(1-\delta_k)\mu_{t+2}\hat{V}_{\hat{z},t+2}^{E,L}\right])$$

Loan foreclosure decision:

$$z_{s,t} = K_t^{1-\alpha}A_t^{-1}[g - \tau(1-\beta\pi^L(1-\delta_k)) + \frac{\eta}{1-\eta}\kappa_u\theta_{u,t} - \frac{1}{1-\eta}E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}_{\hat{z},t+1}^{E,L}\right]]$$

Mean productivity of a weak loan and foreclosure share of weak loans:

$$\hat{z} = \frac{\bar{z}\gamma - 1 + \exp(-\gamma[\bar{z} - z_{s,t}])(\gamma[\bar{z} - z_{s,t}] + 1)}{\gamma(1 - \exp(-\gamma[\bar{z} - z_{s,t}]))}, \quad H(z_{s,t}) = \exp(-\gamma[\bar{z} - z_{s,t}]))$$

Mean value of a weak loan:

$$\frac{\hat{V}_{\hat{z},t}^{E,L}}{(1-\exp(-\gamma(\bar{z}-z_{s,t})))} = [(1-\eta)(A_tK_t^{\alpha-1}\hat{z} - g + \tau(1-\beta\pi^L(1-\delta_k))) - \eta\kappa\theta_{u,t} + E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}_{\hat{z},t+1}^{E,L}\right]]$$

Match probabilities:

$$p(\theta_{u,t}) = \mu_u\theta_u^{1-\xi}, \quad p(\theta_{x,t}) = \mu_x\theta_x^{1-\xi}, \quad q(\theta_{u,t}) = \mu_u\theta_u^{-\xi}, \quad q(\theta_{x,t}) = \mu_x\theta_x^{-\xi}$$

Interest rates to banks:

$$r_{\bar{z},t} = \eta[A_tK_t^{\alpha-1}\bar{z} + \kappa_u\theta_{u,t}] + (1-\eta)g, \quad r_{z,t} = \eta[A_tK_t^{\alpha-1}z + \kappa\theta_{u,t}] + (1-\eta)[g - \tau(1-\beta\pi^L(1-\delta_k))]$$

Highlighted Box 4.3: Competitive equilibrium: Credit market decision-making

Dynamic competitive equilibrium: State transitions

Law of Motion for cash deposits:

$$X_t = (1-\delta_x)[X_{t-1} - p(\theta_{x,t-1})\psi X_{t-1}] + I_{t-1}$$

Law of Motion for capital in safe loans:

$$K_t^N = (1-\delta_k)[\pi^N K_{t-1}^N + p(\theta_{u,t-1})K_{t-1}^U + p(\theta_{x,t-1})S_{t-1} + (1-\pi^L)K_{t-1}^L]$$

Law of Motion for capital in weak loans:

$$K_t^L = (1-\delta_k)(1-H(z_{s,t}))[\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N]$$

Law of Motion for foreclosed capital:

$$K_t^U = (1-\delta_k)[H(z_{s,t})(\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N) + (1-p(\theta_{u,t-1}))K_{t-1}^U]$$

Highlighted Box 4.4: Competitive equilibrium: Laws of Motion

4.3.10 Comparative statics

The baseline calibration of the model parameters is set out in Table 4.4. Variations of this baseline calibration are shown in this comparative statics section. The choice of the matching functions for fresh unspecified and used foreclosed capital is made such that fresh capital markets are in the baseline three times as efficient in matching compared to specified used capital markets with $\mu_u = 0.2$, while $m_x = 0.6$. This corresponds to the finding in the literature that (Huljak, Martin, Moccero, and Pancaro, 2020) asset specificity on average leads to a mere recovery rate of 35 % for non-financial firms. The share of interest payments by non-financial firms compared to their profits between 1985 and 2018 in the US is around 0.297. This suggests banks being able to extract a share of around 0.3 from the joint surplus of the Nash bargain with firms. The bargaining power of banks η is set therefore set to 0.3, meaning that the matching parameter ξ is set to 0.7 to fulfill the (Hosios, 1990) condition.

κ_x and κ_u are both set to 0.01 keeping the cost for new proposals for capital from firms to banks at 1% of the loan value. g is chosen at $1-\beta(1-\delta_k)$ allowing for foreclosed capital to be perfectly mothballed so that the real cost of foreclosed capital are close to 0. The discount factor β is set to 0.99, while physical capital depreciation δ_k is set to 0.05, following (Cooley, 1995). The Cobb-Douglas exponent on capital in the economy α is set to 0.35 as is conventional. δ_x is set to 0.02 which can be considered a real cost of keeping the unmatched capital charged by the bank, but could also be set to $\delta_x > 0$ to obtain similar results. The maximum idiosyncratic productivity of a capital unit in a loan $\bar{z}$ is normalised to 1. The parameter of the exponential distribution is then chosen to calibrate endogenous loan foreclosure. A higher value will mean a steeper distribution

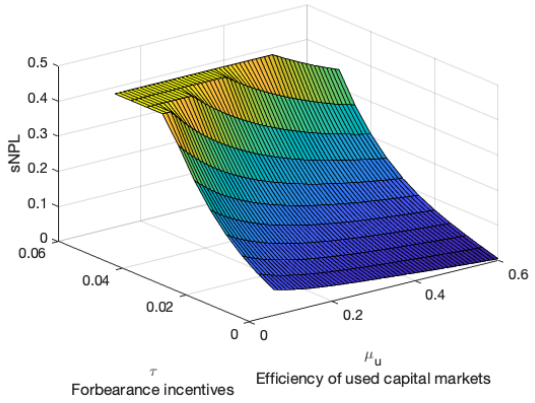
Parameter	Value	Description
β	0.99	Discount rate
ψ_x	0.95	Share of cash that can be lent
δ_k	0.05	Capital depreciation
δ_x	0.02	Real cash depreciation
κ_x	0.01	Proposal cost for new capital
κ_u	0.01	Proposal cost for used capital
m_x	0.6	Match productivity new capital
m_u	0.2	Match productivity for used capital
$\bar{z}$	1	Maximum loan productivity
γ	6	Exponential distribution parameter
α	0.35	Aggregate capital exponent
ξ	0.7	Parameter on matching function
η	0.3	Bargaining power of banks
τ	0	Possible forbearance incentives
g	0.0595	Production value of foreclosed capital
π^N	0.8	Probability of a safe loan staying safe in the next period
π^L	0.8	Probability of a weak loan staying weak in the next period
ρ_a	0.5220	Persistence of the aggregate process
ρ_τ	0.5220	Persistence of the forbearance process
ρ_μ	0.5220	Persistence of the process for used capital market efficiency
$\sigma_{a,\tau}$	0	Correlation of forbearance incentives with the aggregate process
$\sigma_{a,\mu}$	0	Correlation of used capital market efficiency with the aggregate process

Table 4.4: Baseline calibration

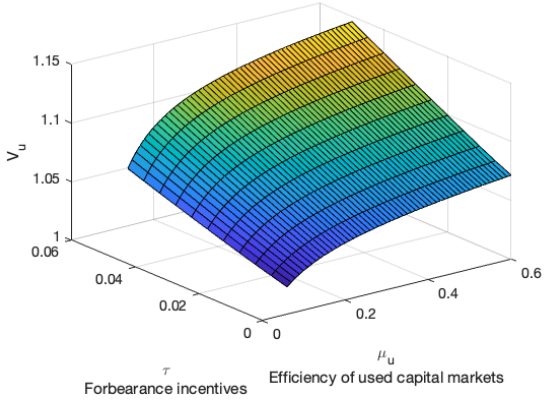
with more foreclosed loans in every period. In this case, a value of 10 means 3% of loans are foreclosed in every period. π^N is set to 0.8 meaning safe loans have a 20% chance of turning weak and, weak loans have a 20% chance of turning safe again due to $\pi^L = 0.8$. These choices are made to target a sNPL of 0.0448 corresponding to the mean sNPL of the 15 European economies over the period 1999-2017. Finally, for foreclosure incentives $\tau = 0$ in the baseline calibration.

The persistence of the aggregate process ρ_a is set to 0.5220 for an annual calibration. The other exogenous processes are assumed to be of similar persistence. In the baseline calibration $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ are set to 0 meaning that a shock to the aggregate process is not accompanied by increases in foreclosure costs or decreases in the efficiency of used capital markets. These parameters are then varied in the dynamic simulation to judge which one is more likely to capture the data.

The steady-state of the model is found with a non-linear solver solving the equations in Appendix 4.A. Figure 4.17a to Figure 4.18b show the comparative statics when varying the efficiency of a match in used capital markets and forbearance incentives similar to the partial equilibrium section. Clearly, as the match efficiency in used capital markets



(a) Comparative effects of τ and μ_u on sNPL.

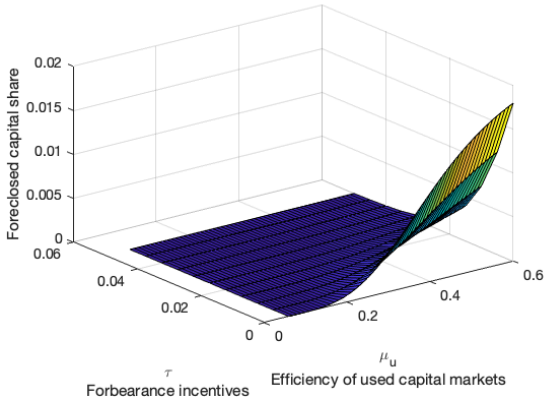


(b) Comparative effects of τ and μ_u on the value of foreclosed capital V^U

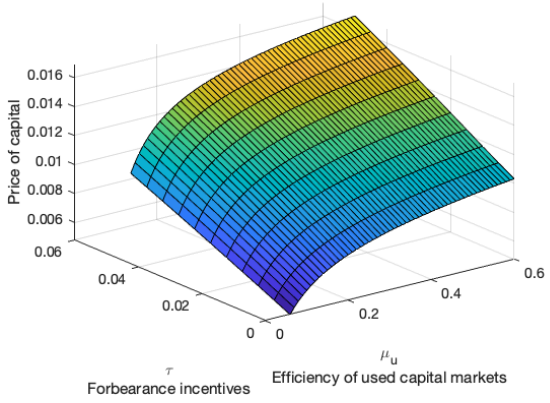
increases or forbearance incentives decrease the sNPL decreases as Figure 4.17a shows.

On the other hand, the steady-state value of foreclosed capital in Figure 4.17b rises when used capital markets are more efficient, but also when forbearance incentives are larger as these will decrease the overall steady-state capital stock and thereby negative externalities for each foreclosed capital unit. The amount of foreclosed capital decreases with an increase in forbearance incentives as Figure 4.18a shows. An increase in efficiency of used capital markets will lead more banks to foreclose loans, because of the improved prospect of re-lending foreclosed collateral productively.

Capital prices are not defined in this model. Nevertheless, as will become apparent in the section matching impulse response functions defining a variable that can capture the dynamics of capital prices is useful. It is argued that used capital market tightness θ_u should be seen as capturing recorded capital price dynamics. The reason is that the prices already capture existing capital, which is represented by the denominator of θ_u, and the numerator is representing capital demand. Assuming conventional demand and supply behaviour the price of capital should therefore be positively correlated to θ_u, which will be used as a proxy for the price of capital in the model. Forbearance incentives mean there is less capital on offer, which given similar demand will drive up the price of capital assumed to be proportional to capital market tightness. In steady-state, however, agents will account for the additional cost of foreclosure making the effect of forbearance incentives for loans on capital prices neutral. Meanwhile, higher demand in more efficient markets and equal supply will mean that capital prices increase as the efficiency of used capital markets increases as shown in Figure 4.18b. This shows that the dynamics of capital prices provide information to separate forbearance incentives from the efficiency of used capital in their impact on the sNPL.



(a) Comparative effects of τ and μ_u on the share of foreclosed capital.



(b) Comparative effects of τ and μ_u on capital prices.

4.3.11 Dynamics

The model is simulated from the steady-state via a second order perturbation. The simulated impulse response functions in this section show first that the model can replicate the dynamics of an aggregate output shock on sNPL. Further, the graphs show that the effect of a rise in the sNPL due to an increase in market frictions on investment, capital returns, and capital prices matches the impulse responses computed from the data. Finally, variations in the impulse response are presented when the steady-state value of frictions and the correlation of market friction shocks to output shocks are varied.

The simulated impulse response functions in Figure 4.19 show that the model can replicate the general correlations of non-performing loans and the business cycle. The model, which is calibrated for annual rates exhibits a large rise in the sNPL following an adverse shock. As expected investment and the return to capital also decline. In the current model the sNPL forms an endogenous outcome. Hence to observe the effect of a rise in sNPL keeping aggregate output stable frictions are shocked, which will drive sNPL. Figure 4.20 shows the response of sNPL rising as a result of the efficiency of used capital markets declining. The mean value of a weak non-foreclosed loan will decline, meaning that banks accept not to foreclose more loans with a lower value. This leads to misallocation rising, and the real return to capital declining as well as investment falling. Note that in this case the price of capital declines. Meanwhile, a $10^{-}3$ shock to the forbearance incentives will also increase the sNPL. Investment will fall due to the reduced value of a new loan, as will the real return to capital due to an increase in misallocation. However, tightness in used capital markets will rise, meaning there is less supply relative to demand for capital, which should lead to a proportional increase in the price of capital.

Figure 4.22 shows that the sNPL responds similarly if the aggregate shock has no effect

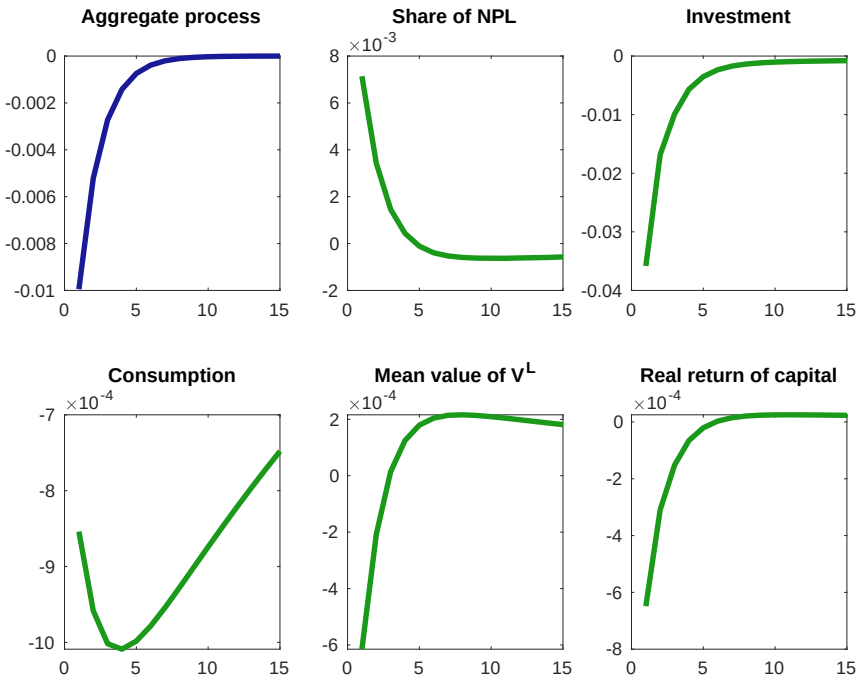


Figure 4.19: Impulse response to a 1% aggregate productivity shock.

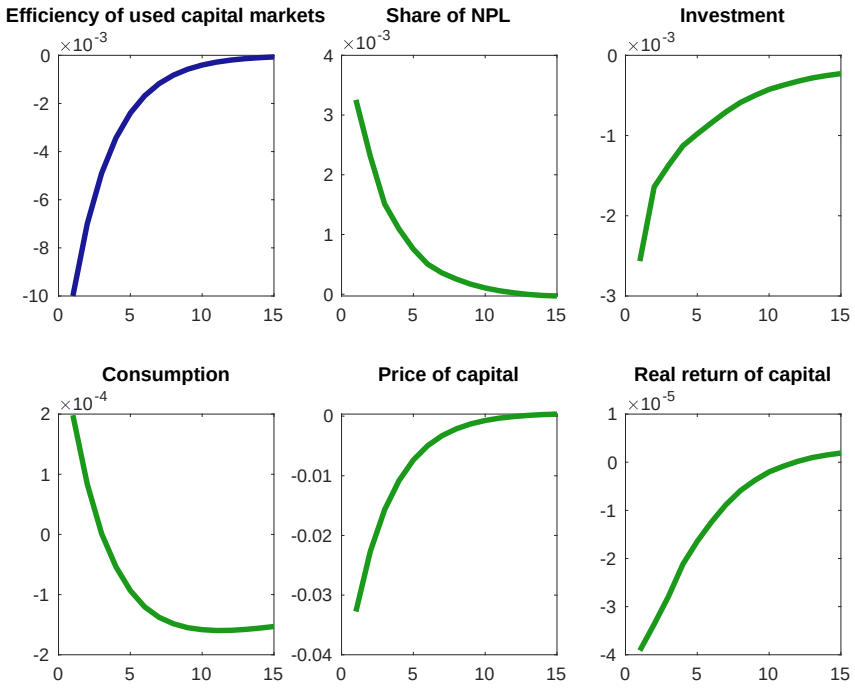


Figure 4.20: Impulse response to a 1% shock to the match productivity μ_u of used capital markets.

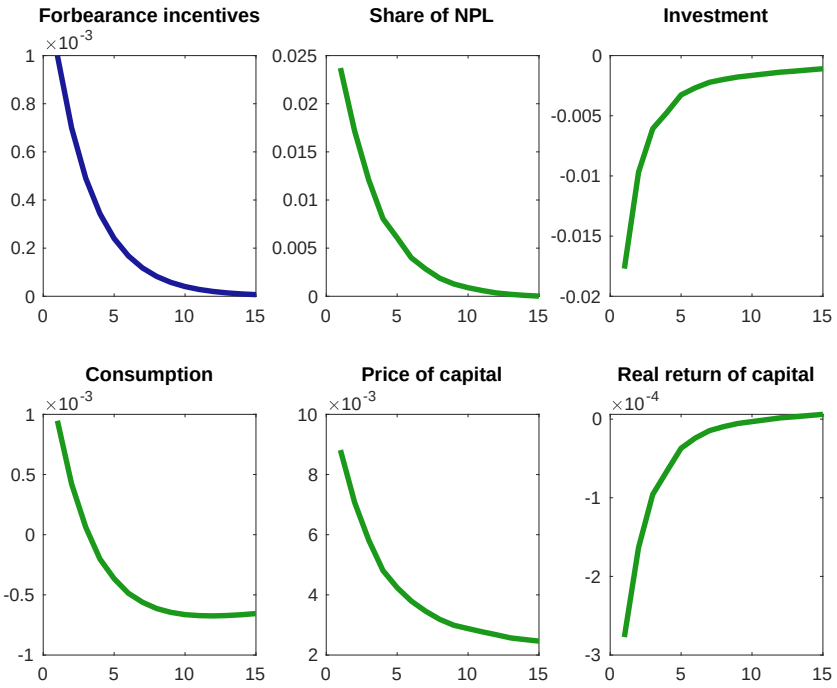


Figure 4.21: Impulse response to a $10^{-}3$ shock to forbearance incentives.

on $\mu_{u,t}$ and $\hat{\tau}_t$ and only the steady-state value is varied. In both cases, higher forbearance incentives or lower used capital market efficiency will lead to a higher increase of the sNPL in response to a negative aggregate shock. Nevertheless, banks get more restrictive as shown in Figure 4.23 in the type of weak loan they accept, but this doesn't make up for the number of loans that fall below the performing level as the productivity of all loans shifts down.

The purpose of Figure 4.24 to Figure 4.26 is to show that when varying towards $\sigma_{a,\tau} < 0$ or $\sigma_{a,\mu_u} > 0$ it is clear that a negative shock is likely to be paired with a shock to the effi-

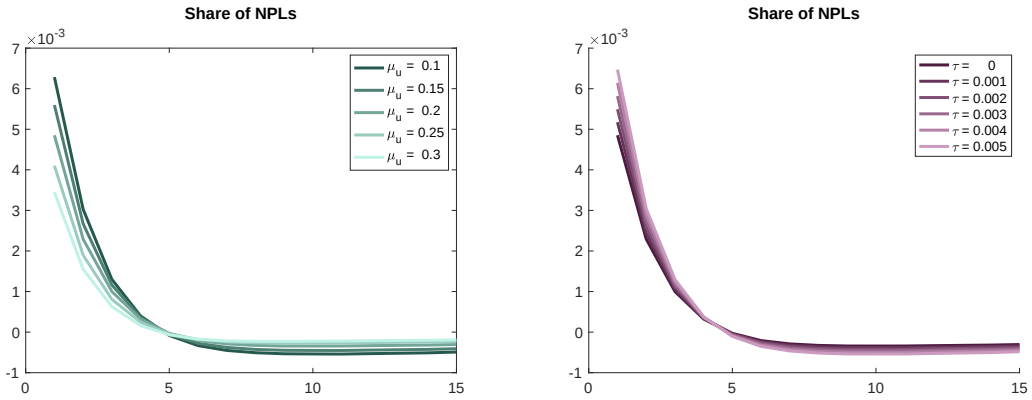


Figure 4.22: Dynamic responses to a negative 1% TFP shock (top row) and varying measures of μ_u (left) and $\hat{\tau}$ right.

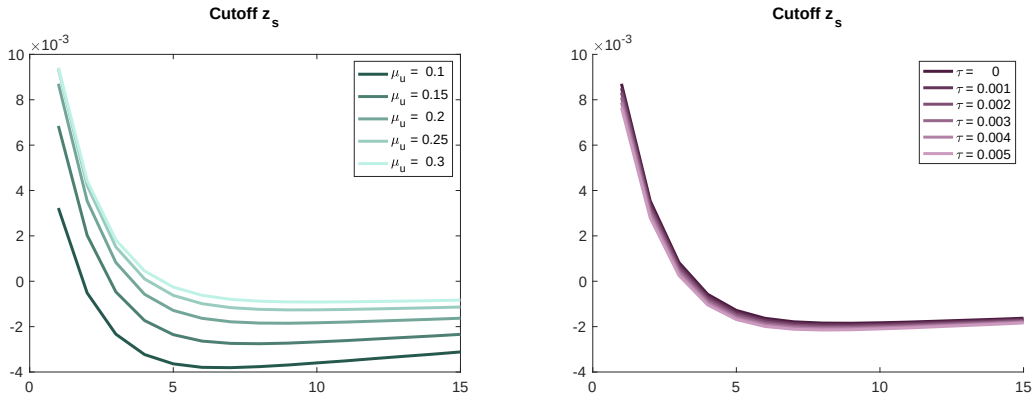


Figure 4.23: Dynamic responses to a negative 1% TFP shock (top row) and varying measures of μ_u (left) and $\hat{\tau}$ right.

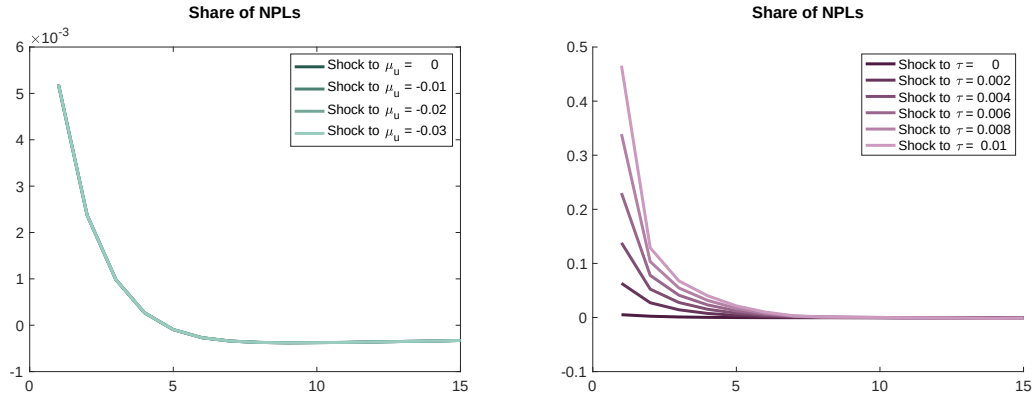


Figure 4.24: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the sNPL.

ciency of used capital markets if the price of capital is supposed to fall as observed in the data. The model shows in Figure 4.24 that even small increases in forbearance incentives or changes to the efficiency of used capital markets can have very large effects on the response of the sNPL. However, Figure 4.25 shows that even a relatively small increase in forbearance incentives may drive the price of capital up, as less used capital is supplied to the market. Figure 4.26 shows this decrease in the supply of used capital to the market is not there when μ_u is decreasing, but only when $\hat{\tau}$ is increasing as foreclosure is stopped via incentives not to foreclose and not via congestion in capital markets driving down the value of foreclosed capital. This shows that a low efficiency in used capital markets and a decline in recessions is a driver the sNPL more congruent with the observed response of the sNPL and capital prices.

To summarise, the model predicts that increasing the sNPL goes hand in hand with reduced capital market reallocation. This is a testable prediction. Looking at cross-country changes in countries' capital reallocation activity in Figure 4.9 it seems that the prediction holds. The model shows that both forbearance incentives and used capital market fric-

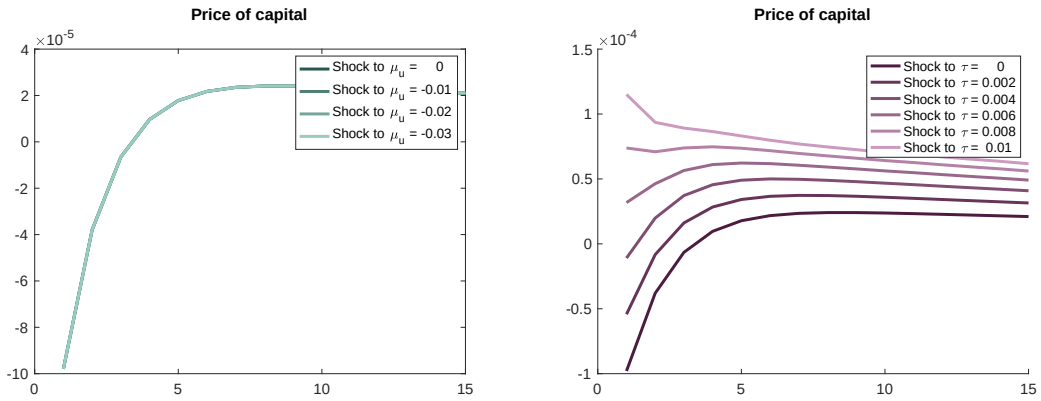


Figure 4.25: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the price of capital.

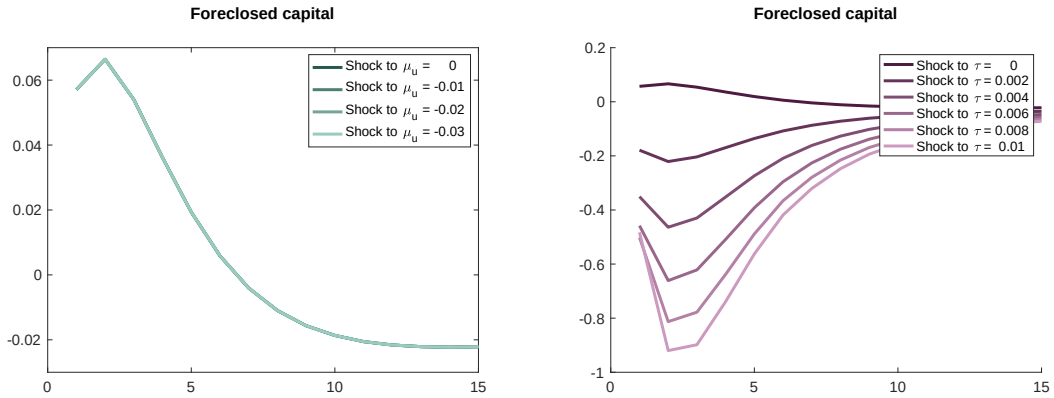


Figure 4.26: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the state of foreclosed capital seeking re-matching.

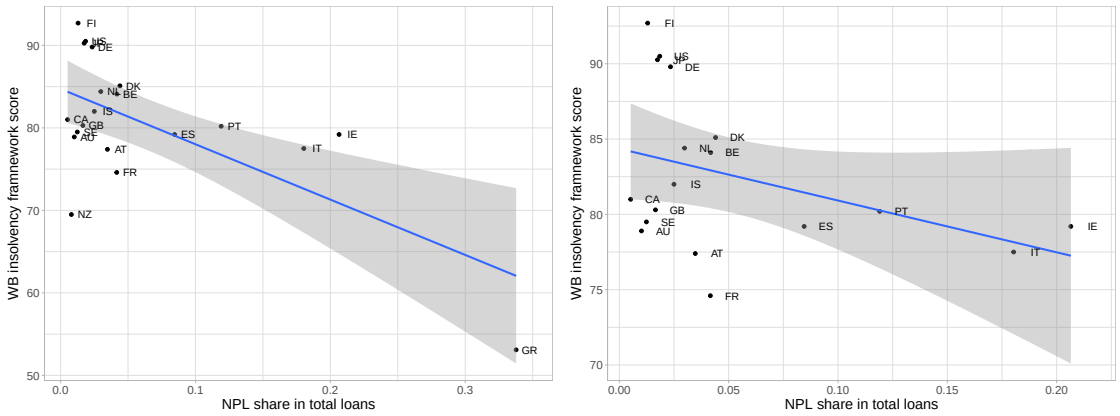


Figure 4.27: sNPL and May 2020 resolving insolvency scores. Source: IMF, World bank, World bank doing business indicators.

tions drive the sNPL. Figure 4.27 provides credit to this theoretical result by showing the World Bank insolvency scores from the ease of doing business indicators plotted against mean non-performing loans share for the sample of countries. We can see the difficulty in administering an insolvency process as real costs in foreclosing a loan or losses due to the low prices realised for foreclosed capital and the high matching frictions. The correlation in the two figures is negative meaning that a higher ease of winding up a company, which delivers a higher outside value to banks will lead to a lower sNPL. Both figures are based on the same data, but the second figure removes outliers by only plotting countries with an insolvency score larger than 70.

4.3.12 Estimating the change of used capital market matching efficiency and forbearance incentives over the cycle

Simplifying the exogenous processes as shown in equation (4.44) to (4.46) allows for estimating the correlations $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ of frictions and incentives with the aggregate process by exploiting the different dynamic response of changes to μ_u and $\hat{\tau}$ on capital prices and employing impulse response function matching. The impulse response function matching method employed follows the conventional approach described in (Fernández-Villaverde, Rubio-Ramírez, and Schorfheide, 2016). The estimated correlations can reveal to which extent the efficiency of used capital markets and forbearance incentives changed over the cycle.

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \tag{4.44}$$

$$\hat{\tau}_t = \tau + \sigma_{a,\tau} A_T \tag{4.45}$$

$$\mu_{u,t} = \mu_u + \sigma_{a,\mu} A_T] \tag{4.46}$$

The data used in this estimation are the per cent deviations from a Hodrick Prescott trend on annual data of the real value added to capture the actual output produced by an economy and of capital price indicators provided by Eurostat. Furthermore, the sNPL provided by the World Bank and IMF is used after removing their trend. The empirical impulse response to be matched is generated via the estimation of a vector auto-regression of order 1 and a constant. The unrestricted VAR is identified with short-run restrictions to identify an output shock on the three variables of interest. The impulse response of the variables and the model fit for each country can be found in Appendix 4.C.

The estimation of $\sigma_{a,\tau}$ is restricted to the space $\sigma_{a,\tau} \in (-\infty, 0]$ meaning that a positive output shock will always reduce forbearance incentives. $\sigma_{a,\mu}$ is not restricted as there are good arguments for used capital markets getting both more and less productive in a recession $\sigma_{a,\mu} \in (-\infty, \infty)$. The restriction on $\sigma_{a,\tau}$ is to discipline the estimation which has a limited number of observations available. However, note that it would be possible for the estimation to find no changes in τ. Figure 4.28 shows the estimates for several countries that were well matched by the model. It shows that in general, forbearance incentives and used capital market frictions both tend to increase with adverse shocks. A group of countries to the right experiences few changes of these frictions over the business cycle. Note that these are countries with low sNPL. Forbearance incentives increase most in Italy (IT). Other countries with high sNPL such as Greece (EL), Portugal (PT) and Ireland (IE) see both an increase in forbearance incentives paired with used capital markets becoming less efficient. The low levels of sNPL in France (FR) and Spain (ES) in the data are explained by used capital markets in these countries becoming more efficient in a recession, even though incentives for forbearance increase. For the United Kingdom (UK) and Denmark (DK) high estimates for the pro-cyclicality of capital market frictions are proposed by the model as an explanation to explain the highly cyclical variation of capital prices and sNPL, even though the mean sNPL is low in these countries.

Figure 4.29 shows the estimates of $\sigma_{a,\mu}$ plotted on the left axis and $\sigma_{a,\tau}$ plotted on the right

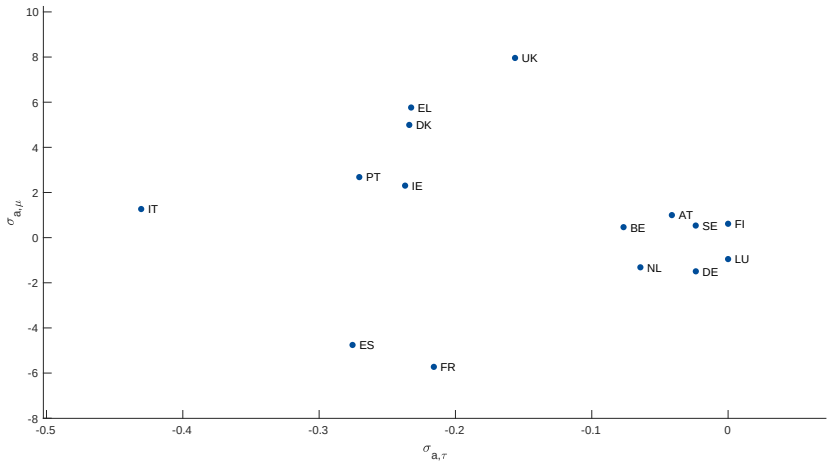


Figure 4.28: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes. A stronger reaction in capital market efficiency is correlated with a stronger reaction in forbearance incentives.

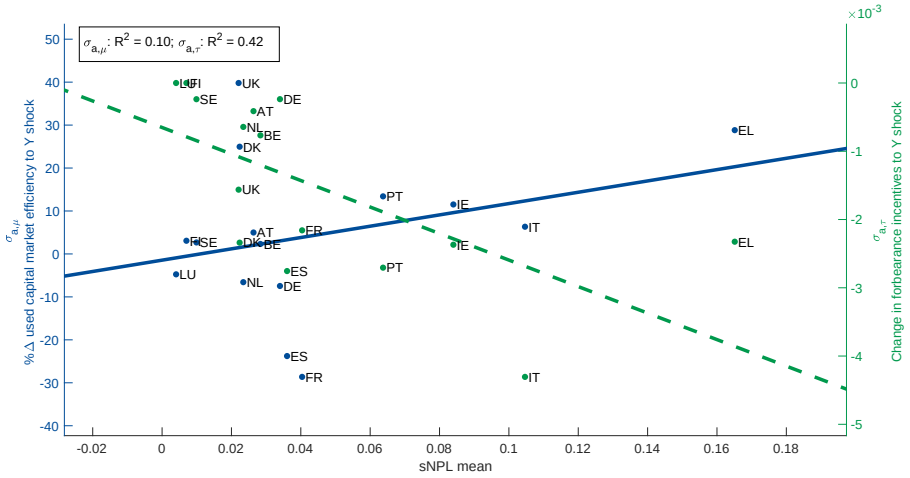


Figure 4.29: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes plotted against variation in NPLs.

axis against the mean sNPL of countries over the sample period. It is shown that both more pro-cyclical match efficiencies and counter-cyclical forbearance incentives are correlated with a higher mean sNPL. While the R^2 of $\sigma_{a,\mu}$ is low, estimated changes to capital market efficiencies in response to an aggregate shock are large. Meanwhile, forbearance incentive changes cannot be computed as percentage changes due to the assumption that forbearance incentives are 0 in equilibrium. However, as shown Section 4.3.11 even the small forbearance incentive increases estimated in the order of 10^-3 can lead to large sNPL responses. Figure 4.30 shows that the estimated weights can also contribute to explaining the variance of sNPL. A pro-cyclical capital market efficiency can explain comparatively more of cross-country differences than cross-country mean differences, while the explanatory power of forbearance incentives is reduced.

Overall, the estimates show that as expected both more pro-cyclical used capital market

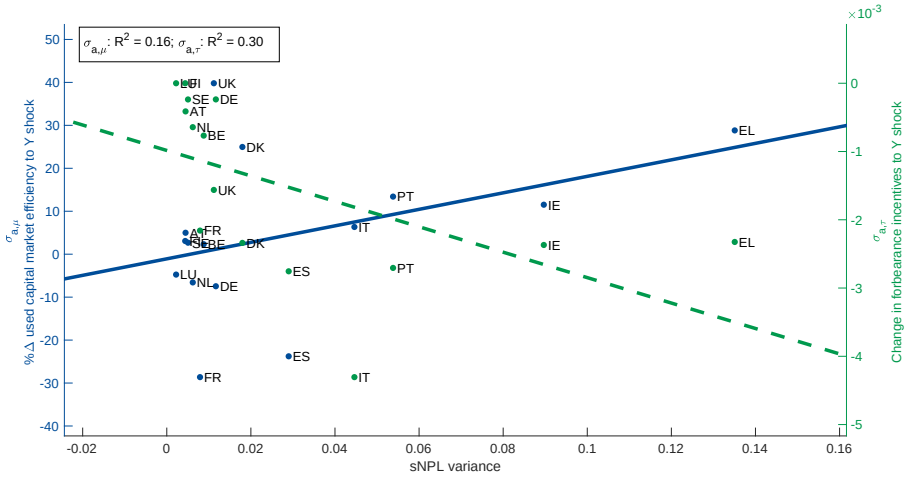


Figure 4.30: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes plotted against variation in NPLs.

efficiencies and counter-cyclical forbearance frictions contribute to explaining variation in the mean and variance of the sNPL between countries. Several countries with high mean sNPL seem to face declines in used capital market efficiency when adverse shocks hit the economy, making misallocation worse in these countries and increasing the sNPL.

4.4 Conclusion

This paper shows that the sNPL is counter-cyclical, and correlates with reduced investment, capital reallocation, returns to capital, and investment. Further, periods of a higher sNPL in a country are shown to be linked to higher misallocation. A structural model with search frictions in capital reallocation can produce the sNPL endogenously and capture the empirical relations presented in Section 4.2. The model presents two drivers for the sNPL forbearance incentives and frictions in used capital markets, which drive down the outside value of foreclosing a loan. The model can thereby combine recent advancements in the capital misallocation literature with empirical studies regarding the sNPL. The theoretical relations in the model imply that resolving non-performing loans is a matter of the efficiency of capital reallocation. This is defined by an economy's ability to bring used specified capital to new and more productive us.

The model further proposes a novel avenue to identify to what extent these two proposed frictions, which are found to drive the sNPL in a country, become worse over the cycle. The identification uses the different impulse response of capital prices to a worsening of the matching function and to an increase in forbearance incentives. The estimates show changes in both frictions to be important to explain the data in most countries, but

different countries show a different prevalence for some frictions. Countries with high sNPL over the sample period, are found to struggle more with a worsening of capital market efficiency, except Italy where bank forbearance seems to be prevalent.

The model is kept simple and tractable but is set up in such a way that it allows for being extended to capture persistent heterogeneity via a dynamic directed search block recursive equilibrium. In this way, the simple model can be expanded to allow for dealing with heterogeneity via a block recursive solution of the credit market as developed in (Menzio and Shi, 2010a). A solution would be possible in this general equilibrium setup because the aggregate return of a unit of capital remains uninfluenced by the distribution of states of capital in employed or unemployed capital linked to non-performing loans. With the block recursion persistent heterogeneous loan relationships, different types of firms and banks, refinancing, endogenous bankruptcies, and different policies to encourage solutions to non-performing loans may be handled. An example would be endogenising the empirical observation that banks with a weaker balance sheet also exhibit increased forbearance incentives (Bergant and Kockerols, 2018), or allowing for different firm and bank types which may enjoy differing levels of government loan support and insolvency protection. Finally, the tractability of the model allows for studying optimal policy towards creating or discouraging bank forbearance as well as encouraging the build-up of used capital markets or the conditions under which the founding of public asset management companies may be beneficial.

Appendix 4.A Steady-States

Exogenous foreclose

The steady-state can be solved around the steady-state of the exogenous processes where $a = 1$ and $\lambda_t = \lambda$.

$$\delta K = I \tag{4.47}$$

$$K_E = \lambda^{-1}[\frac{\delta}{1-\delta} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda)]K_U \tag{4.48}$$

Define $\psi = \lambda^{-1}[\frac{\delta}{1-\delta} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda)]$ so $K_E = \psi K_U$

$$\Pi = K_E[(1-\eta)[K_E + K_U]^{\alpha-1} - \eta p(1-\delta)(1-\lambda)\beta V_E)] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}K_U \tag{4.49}$$

$$\Pi = K_E[(1-\eta)K_E^{\alpha-1}[1+\psi^{-1}]^{\alpha-1} - \eta p(1-\delta)(1-\lambda)\beta V_E)] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}\psi^{-1}K_E \tag{4.50}$$

$$C = \Pi + (\rho - \delta)D = \Pi + (\beta^{-1} - 1)D \tag{4.51}$$

$$\beta^{-1} + \delta - 1 = \rho \tag{4.52}$$

$$r = (1+\psi^{-1})[\beta^{-1} + \delta - 1] \tag{4.53}$$

$$V_E = \frac{(1-\eta)[1+\psi^{-1}]^{\alpha-1}}{1-\beta(1-\delta)(1-\lambda)(1-\eta p)}K_E^{\alpha-1} \tag{4.54}$$

$$C = K_E^{\alpha}[1+\psi^{-1}]^{\alpha-1} - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}\psi^{-1}K_E - \delta K_E(1+\psi^{-1}) \tag{4.55}$$

$$r = \eta[[1+\psi^{-1}]^{\alpha-1}K_E^{\alpha-1} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}\beta\delta(1-\lambda)V_E] \tag{4.56}$$

$$\beta^{-1} + \delta - 1 = r\frac{1}{1+\psi^{-1}} \tag{4.57}$$

Use r to solve for K_E. With K_E solve for the other variables.

$$K_E^{1-\alpha} = \frac{\eta[1+\psi^{-1}]^{\alpha-1}}{r}[1 + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}\beta(1-\delta)(1-\lambda)\frac{1-\eta}{1-\beta(1-\delta)(1-\lambda)(1-\eta p)}] \tag{4.58}$$

Endogenous foreclose

Steady states are solved around the aggregate process $A = 1$.

$$1 = \beta[1 - \delta_d + \rho] \tag{4.59}$$

Creation condition

$$\frac{\kappa_x}{q_x(\theta_x)}(\frac{1}{\beta(1-\delta_k)} - \pi_N) = (1-\eta)[\bar{z}K^{\alpha-1} - g] - \eta\kappa\theta_u + \beta(1-\delta_k)(1-\pi^N)\hat{V}^E \tag{4.60}$$

$$\hat{V}^E = \frac{1 - H(z_s)}{1 - \beta(1-\delta_k)(1-H(z_s))\pi^L(1-\frac{\hat{z}}{z_s})}([(1-pa.eta)\hat{z}K^{\alpha-1} - g + \hat{\tau}] - \eta\kappa\theta_u + (1-\pi^L)\frac{\kappa_x}{q_x(\theta_x)}) \tag{4.61}$$

$$[\delta_x + p(\theta_x)\psi_x(1-\delta_x)]X = I \tag{4.62}$$

Interest to banks

$$r_{\bar{z}} = \eta[\bar{z}K_t^{\alpha-1} + \kappa\theta_u] + (1-\eta)g \tag{4.63}$$

$$r_{\hat{z}} = \eta[\hat{z}K_t^{\alpha-1} + \kappa\theta_u] + (1-\eta)(g - \hat{\tau}) \tag{4.64}$$

State steady states

K^L:

$$K^L = \frac{(1-\pi^N)(1-\delta_k).(1-H(z_s))}{1-(1-\delta_k)\pi^L(1-H(z_s))}X = \nu_1 X \tag{4.65}$$

K^U:

$$K^U = \frac{H(z_s)(1-\delta_k)(\nu_1\pi^L + (1-\pi^N))}{1-(1-\delta)_k)(1-p_u(\theta_u))}X = \nu_2 X \tag{4.66}$$

K^N:

$$K^N = \frac{(1-\delta_k)p_x(\theta_x)\psi_x}{1-(1-\delta_k)(\pi^N + (1-\pi^L)\nu_1 + \nu_2 p_u(\theta_u))}X = \nu_3 X \tag{4.67}$$

Foreclosure decision

$$z_s = K^{1-\alpha}[g + \frac{\eta}{1-\eta}\kappa_u\theta_u - \frac{1}{1-\eta}((1-\pi^L)\frac{\kappa_u}{q_u(\theta_u)} + \pi^L\hat{V}^E) - \hat{\tau}] \tag{4.68}$$

$$K^{\alpha-1} = [g + \frac{\eta}{1-\eta}\kappa_u\theta_u - \frac{1}{1-\eta}((1-\pi^L)\frac{\kappa_u}{q_u(\theta_u)} + \pi^L\hat{V}^E) - \hat{\tau}]\frac{1}{z_s} \tag{4.69}$$

δ_d"

$$\delta_d = \delta_k\frac{\nu_3 + \nu_3\nu_2 + \nu_3\nu_1 + p(\theta_x)\psi}{1 + \nu_3 + \nu_3\nu_2 + \nu_3\nu_1} + \delta_x\frac{1-p(\theta_x)\psi}{1 + \nu_3 + \nu_3\nu_2 + \nu_3\nu_1} \tag{4.70}$$

$$\rho = \frac{\nu_3 r_{\tilde{z}} + \nu_3\nu_1 r_{\hat{z}} + \nu_3\nu_2 g}{1 + \nu_3 + \nu_3\nu_2 + \nu_3\nu_1} \tag{4.71}$$

Appendix 4.B Additional data

The series underlying the correlations table in Section 4.2 are presented in Figure 4.31.

Developed countries with higher NPLs have lower returns on capital. This relation is strong since the NPL divergence with Great Recession.

The impulse response function of demand (sNPL) and supply (real GDP) shocks on Real GDP are shown in Figure 4.33.

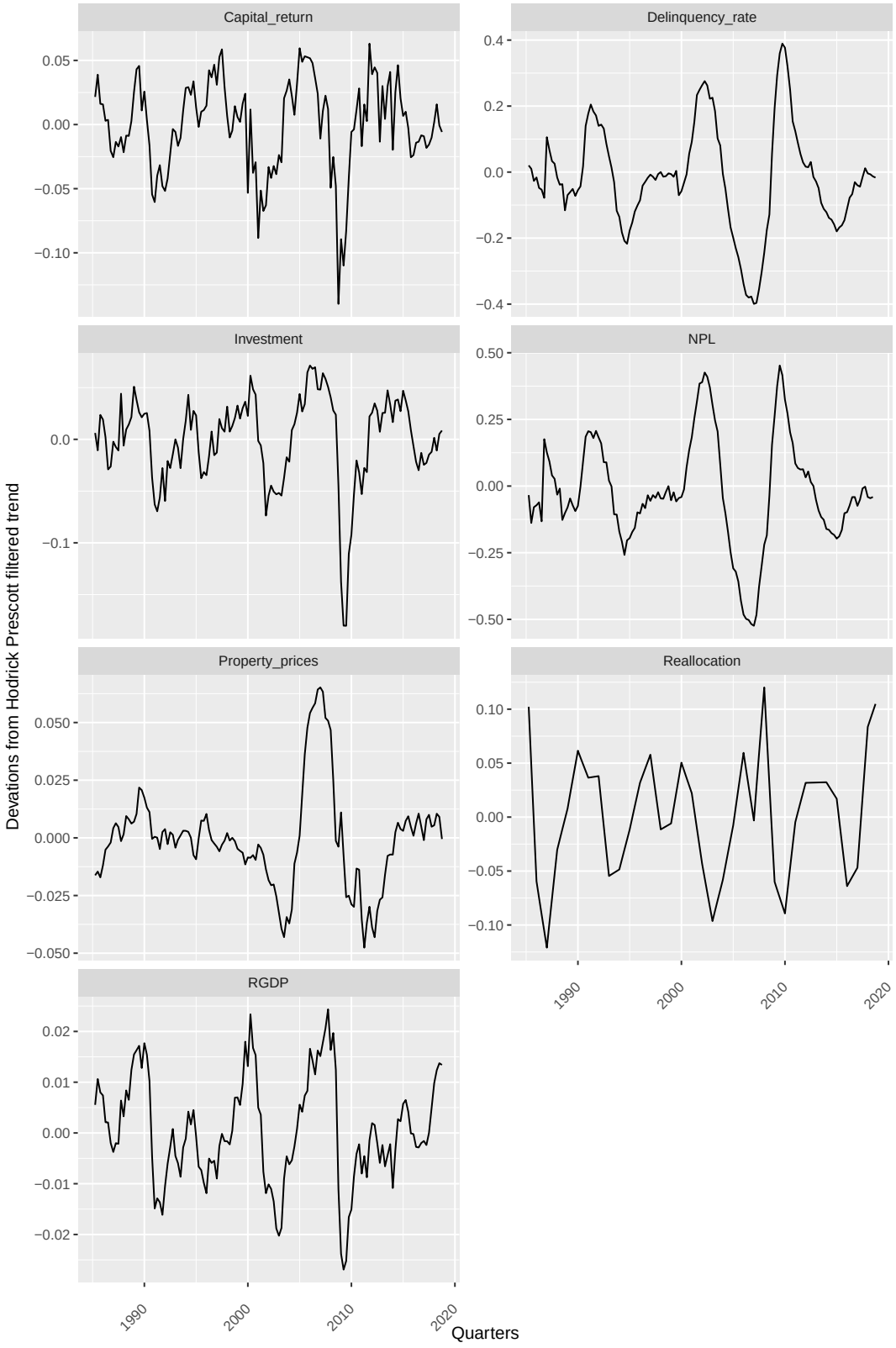


Figure 4.31: HP filtered US series

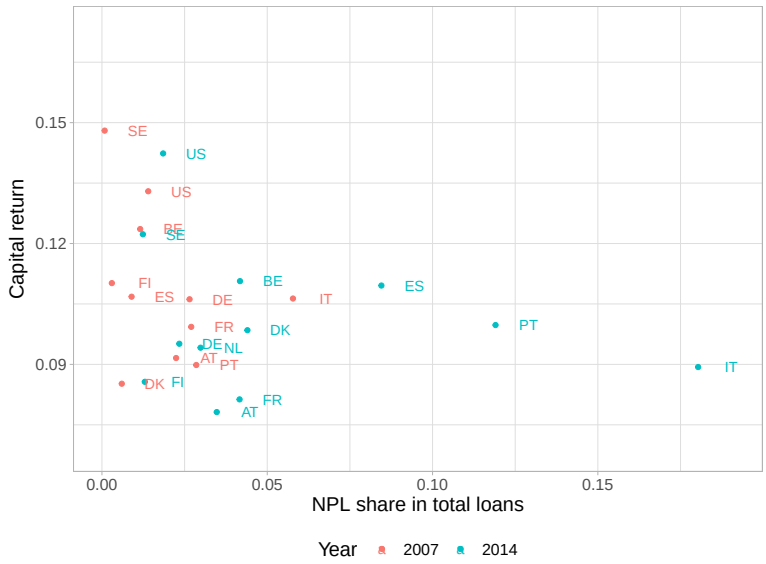


Figure 4.32: Returns on capital calculated from the KLEMS database, NPL ratios from World bank and IMF data

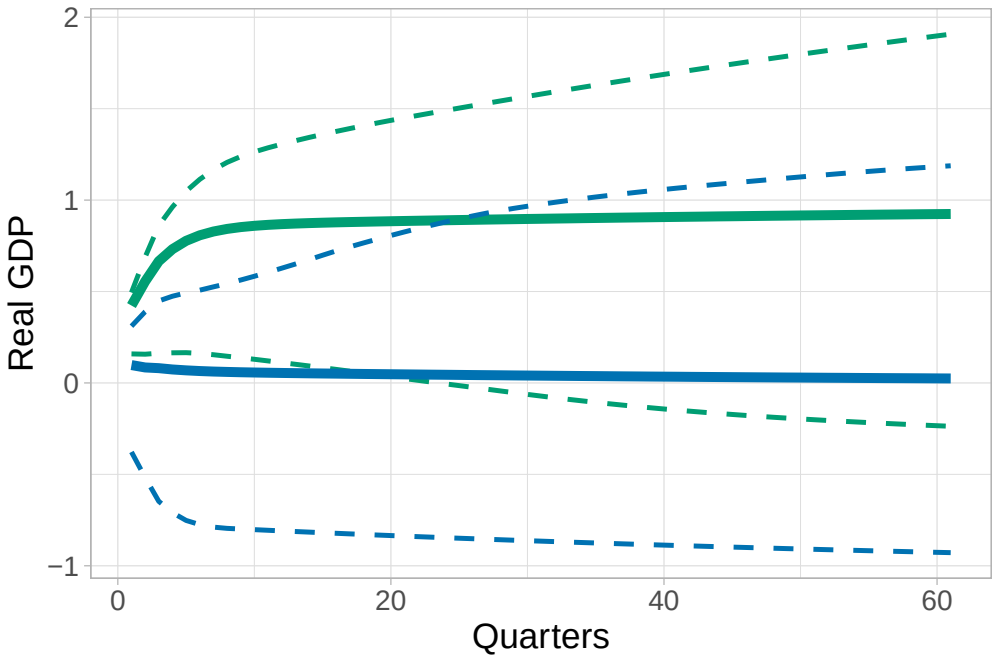


Figure 4.33: Green captures supply shocks (from real GDP) and blue demand shocks (from sNPL). Dashed lines show bootstrapped confidence intervals.

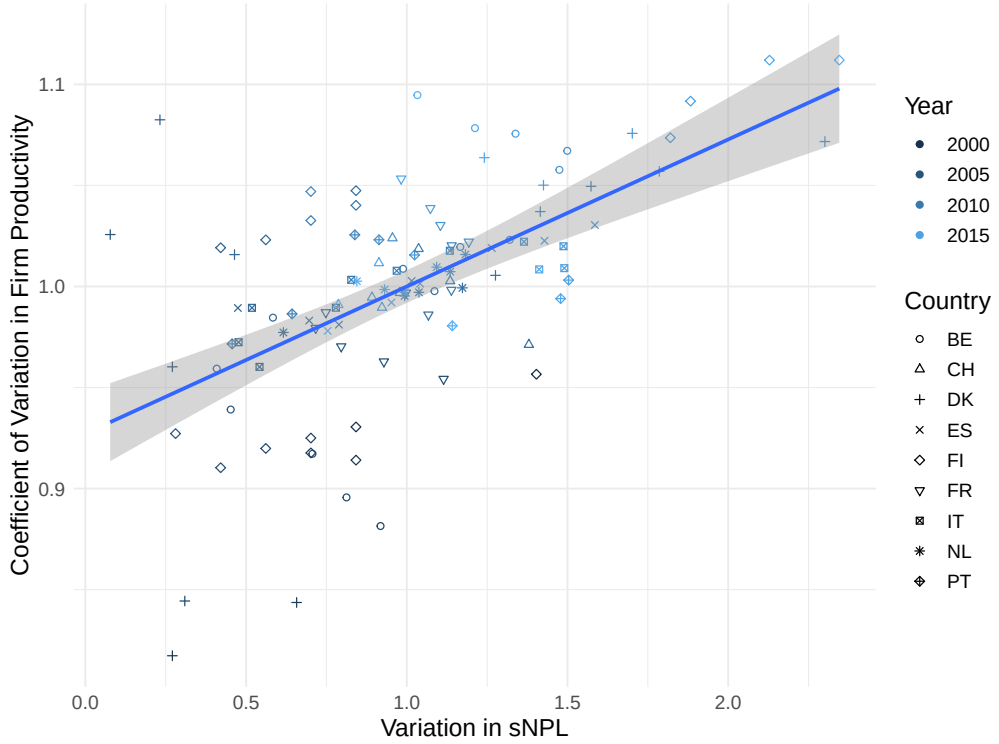


Figure 4.34: As the sNPL varies in a country estimated Coefficient of Variation in firm TFP estimated from a C-D production function imposed for each sector and following Wooldridge estimation increases proportionally. Source: CompNet 7th vintage dataset for TFP estimate and sNPL measures are from merged world bank and IMF data.

Other dispersion and misallocation measures

As the sNPL varies in a country estimated Coefficient of Variation in firm TFP estimated from a Cobb-Douglas production function imposed for each sector and following Wooldridge estimation (Wooldridge, 2009) increases proportionally. Figure 4.34 shows that dispersion increases as the sNPL increases.

Figure 4.35 shows how the co-variance of firm productivity and size following the estimated method in (Olley and Pakes, 1992) changes as the sNPL in a country varies.

Figure 4.36 shows how the dispersion marginal product of capital following (Kehrig et al., 2011) changes as the sNPL in a country varies. More information on the exact implementation of the method can be found in the user guide to the CompNet 7th vintage dataset (Deist, Amlung, Blyzniuk, Lang, Lalinskỳ, Inferrera, and Papagalli, 2020).

Finding in (Balgova et al., 2016)that NPL reduction increases investment

(Balgova et al., 2016) show that countries that reduce NPLs with asset management companies experience real investment and output growth following such periods compared to countries that don't.

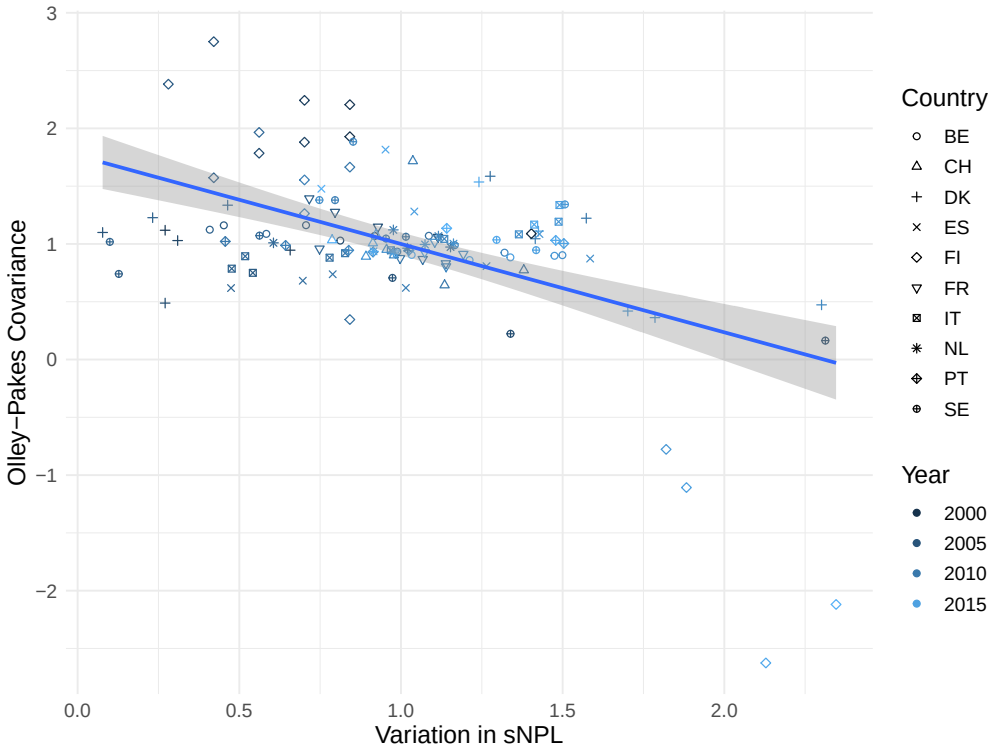


Figure 4.35: As the sNPL varies in a country estimated Olley-Pakes covariance declines. Source: CompNet 7th vintage dataset for Olley-Pakes estimate and sNPL measures are from merged world bank and IMF data.

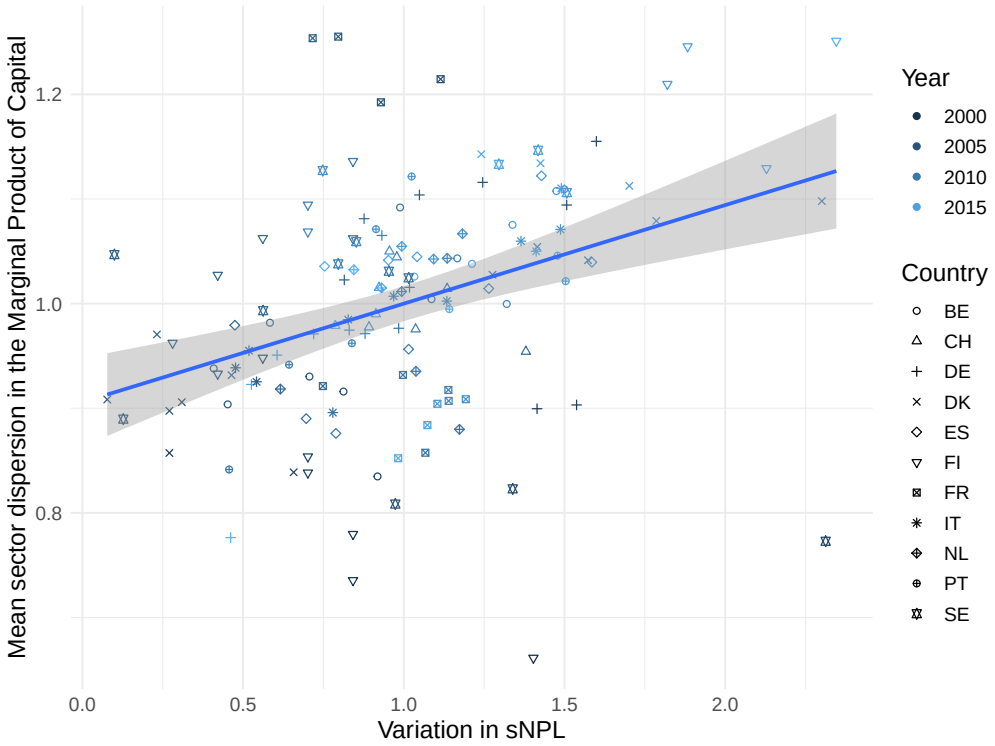


Figure 4.36: As the sNPL varies in a country estimated dispersion of marginal products of capital following Kehrig (2011) increases. Source: CompNet 7th vintage dataset for dispersion estimate and sNPL measures are from merged world bank and IMF data.

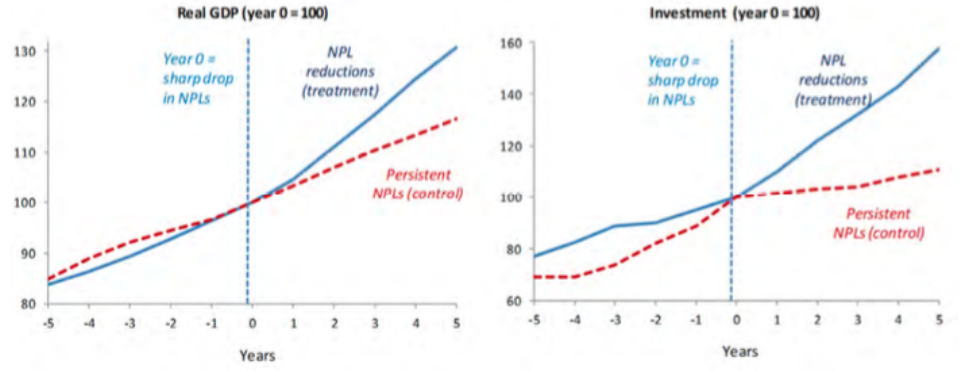


Figure 4.37: Increase in real output and investment following a reduction in NPLs via state intervention.

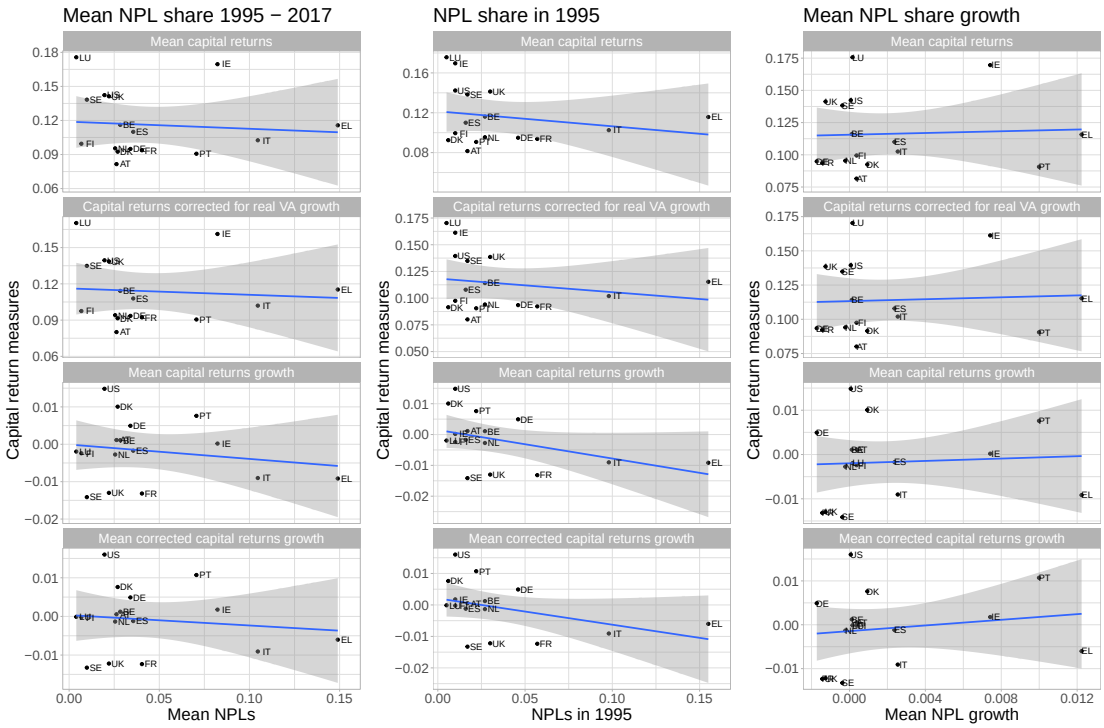


Figure 4.38: Mean, start and NPL share growth correlated with capital returns, capital return growth, and capital reuturns corrected for VA growth to proxy for TFP changes

A cross-country comparison between OECD economies shows that non-performing loans tend correlate negatively with average real capital returns.

Appendix 4.C Alternative model with exogenous loan foreclosure

The model with exogenous loan foreclosure is presented here to provide some straightforward intuition about the impact of sudden rises in unemployed capital on other aggregate variables such as investment consumption or capital productivity. Unemployed capital can be interpreted in this model both as rises in unmatched capital and exoge-

nous rises in non-performing loans. The model provides a structural explanation for the empirical observations in (Balgova et al., 2016) and other policy papers investigating the impact of non performing loan increases or reductions on the real economy. It shows that an increase in misallocated capital, which can here be interpreted as both non-performing loans and foreclosed loans, can have large and persistent real effects on the economy.

Aggregate output

There are two types of capital stocks in the economy:

- Employed or productive capital K_t^E linked to performing loans or non-performing loans.
- Unemployed capital meaning non-performing or foreclosed capital K_t^U.

Both together form the aggregate capital stock in the economy $K_t = K_t^E + K_t^U$. Unemployed capital provides a negative externality for employed capital as aggregate output is given by:

$$Y_t = A_t K_{E,t}(K_{E,t} + K_{U,t})^{\alpha-1} \tag{4.72}$$

A is a stochastic TFP process. We commonly assume that $\alpha < 1$. Note that this function simplifies to a standard RBC function when $K_{U,t} = 0$. [4] [5]

Aggregate output in this economy can be consumed, used for investment in further capital, or used for setting up firms with provided capital.

A is an exogenous auto-regressive process describing productivity.

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \tag{4.74}$$

[4] Then $Y_t = A_t K_{E,t}^{\alpha}$

[5] It is also straightforward to generalise the output function to foreclosed loans also producing

$$Y_t = [K_{E,t} + \zeta K_{U,t}] A_t (K_{E,t} + K_{U,t})^{\alpha-1} \tag{4.73}$$

where $\zeta < 1$ and small enough that the value of a unit of performing loan capital exceeds the value of a non-performing loan to the bank.

Savings problem of the individual household

The economy is populated by a unit mass of identical households. Each household has some initial deposit wealth D_0. Each household takes credit and capital markets as given when making the individual saving decision. Thus the saving decision is a result of expected interest rates paid on deposits leading to a typical savings problem leading and a conventional Euler equation defining the savings decision.

$$\max_{C_t, I_t} E_t(\sum_{s=0}^{\infty} \frac{C_{t+s}^{1-\sigma}}{1-\sigma}) \tag{4.75}$$

subject to a budget constraint:

$$C_t + I_t = \Pi_t + \rho_t D_t \tag{4.76}$$

Here Π are profits by firms, ρ is the interest rate paid by banks on deposits, D is the stock of deposits and I is investment.

$$D_t = (1-\delta)D_{t-1} + I_{t-1} \tag{4.77}$$

Deposits depreciate in value with δ in this simple model as they are identical to the value of the underlying collateral of loans. In the expanded model this will not be the case.

Solving the maximisation problem it is straightforward to find the inter-temporal Euler equation.

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1-\delta+\rho_{t+1})] \tag{4.78}$$

One can also summarize the stochastic discount factor for firms and banks in the following sections as μ.

$$\mu_{t+1} = \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \tag{4.79}$$

Financial market

For simplicity deposits are assumed to equal lent out capital in this model $D_t = K_t$, but this is relaxed in the next section. Introducing matching frictions for new capital as well leads to a state of matched loans, a state of foreclosed loans, and a state of deposits.

All capital is assumed to be held by banks and equals the value of deposits by the household. Employed capital is used by firms, which pay interest r_t on the capital borrowed. New investment builds the capital stock. Investment of fresh capital works without frictions. However, once the firm that first received the capital defaults the capital will become unemployed, something that can be interpreted as a non-performing loan.

Foreclosed capital has to be re-matched to an entrepreneur willing to transform it for her purposes for a cost. This argument is similar to the argument in (Lanteri, 2018), who has shown that it helps provide a micro-foundation for RBC models with capital adjustment costs, which are needed to match the pro-cyclical capital reallocation observed in the data. This heterogeneous process of matching new entrepreneurs with old capital underlying non-performing loans is modelled in reduced form via a matching function.

In order to set up a firm from used capital agents need to spend κ to present a business plan to a bank and agree on an interest rate. The business proposals B_t will be matched with cash deposits and available unemployed capital. This is a frictional process, which will be summed up by the matching function in equation (4.80).

$$M_t = mB_t^{1-\xi} K_{U,t}^{\xi} \tag{4.80}$$

The cost of creating a new unit of employed capital from output in the simple model without matching frictions for fresh capital can be normalised to 1. Thus as long as there is positive investment in the economy the cost of creating a unit of employed capital from unemployed capital must equal the cost of new.

$$\kappa\theta^{\xi} = m \tag{4.81}$$

$$B_t = (\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} \tag{4.82}$$

The cost of reemploying capital must be equal to the cost of creating fresh capital to the representative household. Substituting (4.82) in the matching function yields.

$$M_t = (\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t} \tag{4.83}$$

This means $\theta_t = \theta$ in the simple model due to the fixed cost of creating fresh capital.

Benefit of an average foreclosed unemployed or an employed capital unit

The expected benefit of a single capital unit invested in the banking sector is defined by the current level of employed and unemployed capital, and aggregate investment. In the following equations, δ is the depreciation rate of capital, while λ is the exogenous probability of the firm exiting production and the capital becoming a non-performing loan.

$$V^U(K_t) = p(1-\delta)(1-\lambda_t)E_t(\beta_{t+1}[V^L(IK_{t+1}) - V^U(K_{t+1})]) + (1-\delta)E_t[\beta_{t+1}V^U(K_{t+1})] \tag{4.84}$$

$$V^L(K_t) = r_t + (1-\delta)(1-\lambda_t)E_t(\beta_{t+1}[V^L(K_{t+1}) - V^U(K_{t+1})]) + (1-\delta)E_t[\beta_{t+1}V^U(I_{t+1}, K_{t+1})] \tag{4.85}$$

The value of lent capital in the surplus notation familiar from (Mortensen and Pissarides, 1994) is in equation (4.86).

$$V^L(K_t) - V^U(K_t) = r_t + (1-\delta)(1-\lambda_t)(1-p)E_t(\beta_{t+1}[V^L(K_{t+1}) - V^U(K_{t+1})]) \tag{4.86}$$

Entrepreneurs incur a cost κ for creating a business plan and will enter the market for used capital until the benefits of entering equal the cost of entering.

The benefit of a matched unit of capital to the firm is:

$$V^E(K_t) = A_t K_t^{\alpha-1} - r_t + (1-\delta)(1-\lambda_t)E_t(\beta_{t+1}V_{E,t+1}]) \tag{4.87}$$

Solving for the interest rate paid by firms to banks

The interest rate for lent capital can be found by assuming Nash bargaining between firms and banks, with the bank's bargaining power being η. The equilibrium interest rate

is then the result of the bargain.

$$r_t = \eta[A_t K_t^{\alpha-1} + p\beta_t(1-\delta)(1-\lambda_t)E_t(V_{E,t+1})] \tag{4.88}$$

This means the pooled interest rate ρ_t paid by the bank to the household for depositing a capital unit is defined by the share of productive capital in the overall capital stock, and the level of marginal returns per capital induced by the total capital stock.

$$\rho_t = r_t \frac{K_t^E}{K_t} \tag{4.89}$$

Transition equations for capital states

The transition equations follow from the household investment decision, and the exogenous firm destruction shock λ as well as the re-matching frictions for the market of used capital measured by non-performing loans.

$$K_{E,t} = (1-\delta)(1-\lambda_t)[K_{E,t-1} + (\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t-1}] + I_{t-1} \tag{4.90}$$

$$K_{U,t} = (1-\delta)[\lambda_t K_{E,t-1} + (1 - p(\theta_{t-1})(1-\lambda_t))K_{U,t-1}] \tag{4.91}$$

Solving the model

The dynamics of this simple model can be simulated by perturbing the model around the steady-state. This can be done with ease and allows for a quick enough simulation to estimate parameters of the model such as match efficiency, which may be useful when comparing the impact of aggregate shocks and winding up of non-performing loans of different countries.

Exogenous processes Assume for now an exogenous aggregate productivity process and an exogenous default rate. The default rate can be straightforwardly endogenised. Both processes are assumed to be auto-regressive.

The aggregate productivity process is in equation (4.92).

$$\log(a)_t = \log(a)_{t-1}\rho_a + \epsilon_{a,t} \tag{4.92}$$

The exogenous process of the default rate is in equation (4.93).

$$\lambda_t = \lambda(1-\rho_\lambda) + \lambda_{t-1}\rho_\lambda + \epsilon_{\lambda,t} \tag{4.93}$$

Equilibrium equations The following are equilibrium equations and constraints describing the dynamics of the economy.

- Budget constraint:

$$C_t + D_{t+1} = \Pi_t + [1-\delta+\rho_t]D_t \tag{4.94}$$

- Euler Equation

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1-\delta+\rho_{t+1})] \tag{4.95}$$

- Marginal input cost of investment equality

$$B_t = (\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} = \theta K_{U,t} \tag{4.96}$$

- The value of a fresh capital unit [6]

$$V^E(K_t) = (1-\eta)A_t K_t^{\alpha-1} + (1-\delta)(1-\lambda_t)(1-\eta p)E_t(\beta_{t+1}V_{E,t+1}]) \tag{4.97}$$

- Profits from firms passed on to the households

$$\Pi_t = K_{E,t}[(1-\eta)A_t K_t^{\alpha-1} - \eta p(1-\delta)(1-\lambda_t)\beta_{t+1}V_{E,t+1})] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} \tag{4.98}$$

- Transition equation of employed capital

$$K_{E,t} = (1-\delta)(1-\lambda_t)[K_{E,t-1} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t-1}] + I_{t-1} - \tag{4.99}$$

- Transition equation of unemployed capital

$$K_{U,t} = (1-\delta)[\lambda_t K_{E,t-1} + (1 - m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda_t))K_{U,t-1}] \tag{4.100}$$

[6] It is assumed that $V^E(K_t) > 1$, which is achieved with appropriate calibration. This ensures that investment is always positive, i.e. $I > 0$. For moderate shocks the evolving non-differentiability can be ignored or alternatively a penalty function for low investment can be introduced to ensure more accurate estimation.

- The transition function for the capital stock, which is in the simple version assumed to equal deposits.

$$K_t = (1 - \delta)K_{t-1} + I_{t-1} \tag{4.101}$$

- The interest rate paid by banks to households

$$\rho_t = r_t \frac{K_t^E}{K_t} \tag{4.102}$$

- The interest rate charged by banks to firms per lent capital unit

$$r_t = \eta[A_t K_t^{\alpha-1} + p\beta_t(1 - \delta)(1 - \lambda_t)E_t(V_{E,t+1})] \tag{4.103}$$

The steady-state equations can be found in Steady-States.

Simulated model

The model is simulated with $\beta = 0.99$, $\alpha = 0.66$, $m = 0.4$, $\xi = 0.8$, $\eta = 1 - \xi$, $\delta = 0.05$, $\lambda = 0.05$, and $\kappa = 0.1$. While the parameters may certainly need more calibration the dynamics of the model are nevertheless interesting. The simulated model shows that a brief rise in the default rate leads to a persistent decline of consumption and productive loans. Interest rates barely rise leading to a long recovery as investment does not rise to levels to recover destroyed productive capital due to the negative impact of non-performing loans remaining in the economy. This seems a compelling story for countries struggling with high foreclosed loan levels following financial crisis.

Meanwhile, the effect of aggregate productivity shocks is mooted due to the assumptions taking for the secondary capital market and non-endogenised defaults. These effects would change once endogenous loan calling is introduced, and search frictions are also introduced for fresh capital as the share of non-performing loans would not remain ”essentially” flat and hidden. Once an endogenous decision is introduced the evidently costly foreclosure result is endogenised. Banks will decide on foreclosure and thereby also decide to forbear foreclosing decisions leading to non performing loans.

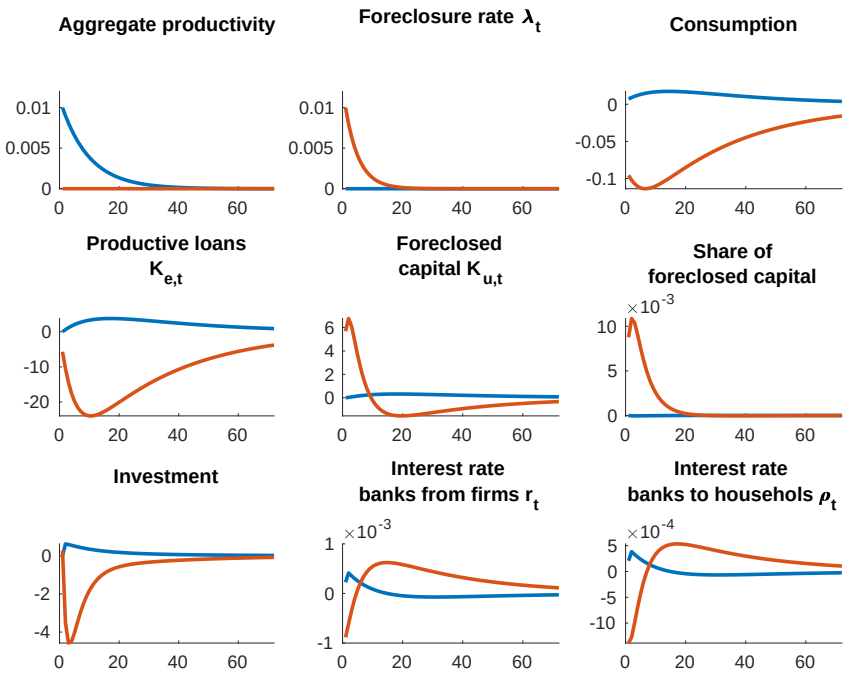


Figure 4.39: Impulse response functions for a 1% shock to aggregate productivity a_t (blue), and a 1% shock to the default rate λ_t (red).

Appendix 4.D Matched Impulse Response Functions

Figure 4.40 shows the result of the estimation with regard to the sNPL data variation provided for computing the empirical impulse responses. The difference to the raw variance of sNPL is that the variation is scaled to the theoretical model's steady state on which the estimation is based on. However, it can be seen that the results are similar to the explanatory power of sNPL variance, with changing forbearance explaining most of the variation, and capital market efficiency some. Appendix D NPL. A country's variation of sNPL is esymated to increase with pro-cyclical used capital market efficiencies and counter-cyclical forbearance incentives.

The impulse response functions of output, non-performing loans and value added to an output shock are plotted below. The model estimates the size of parameters $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$, which will determine the reaction of capital prices and NPL to the output shock and plots the model response in red in Figure 4.41 and Figure 4.43.The two digit code of the countries forms the title of each plot.

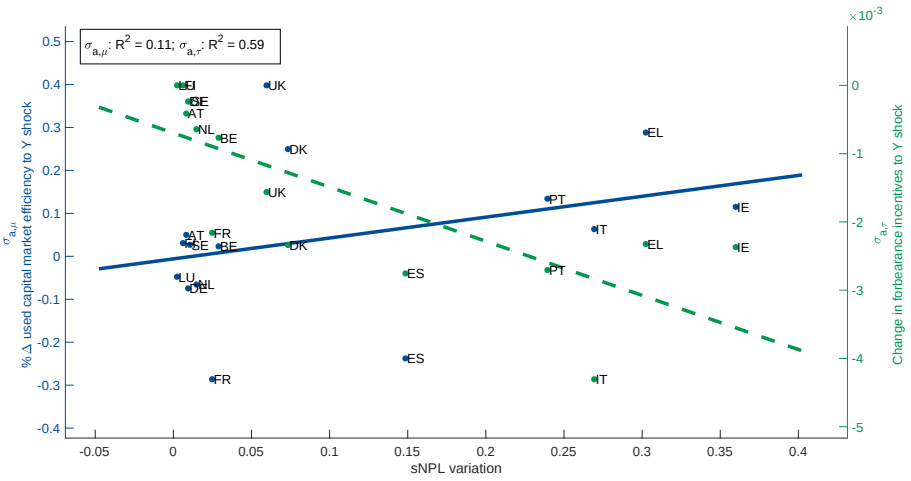


Figure 4.40: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes plotted against variation in NPLs.

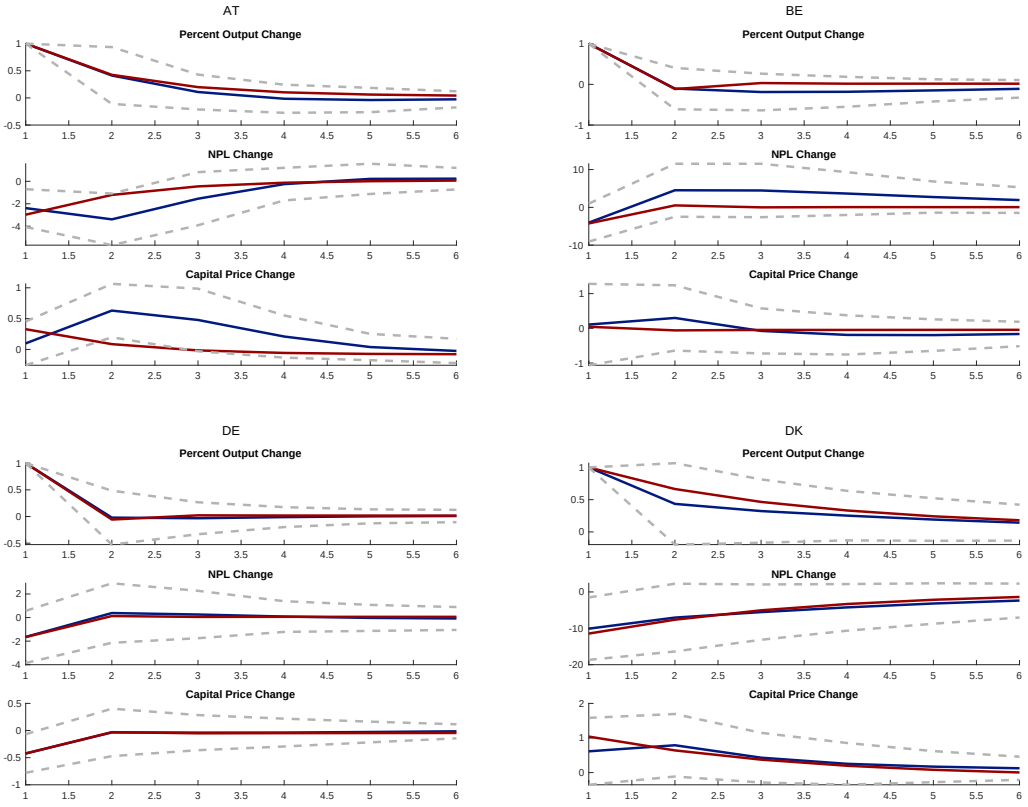


Figure 4.41: Short-run identified impulse responses of the variables to an output shock. The estimated model is plotted in red. Dashed lines give confidence intervals. The two-digit country code is above the plot.

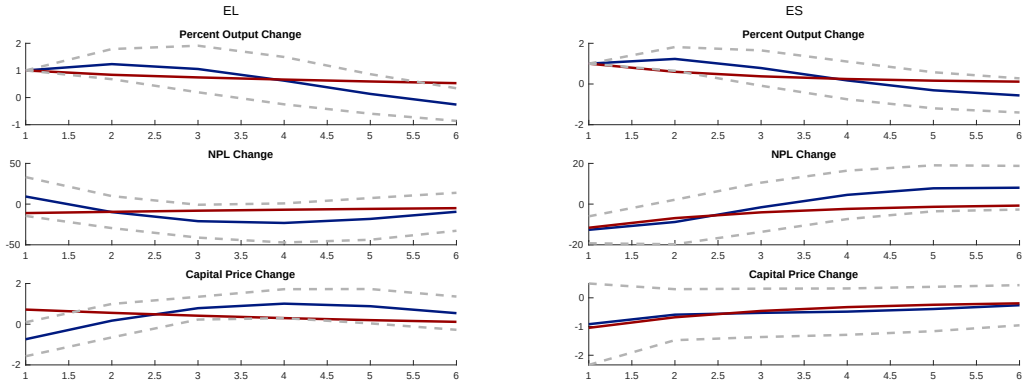


Figure 4.42: Short-run identified impulse responses of the variables to an output shock. The estimated model is plotted in red. Dashed lines give confidence intervals. The two-digit country code is above the plot.

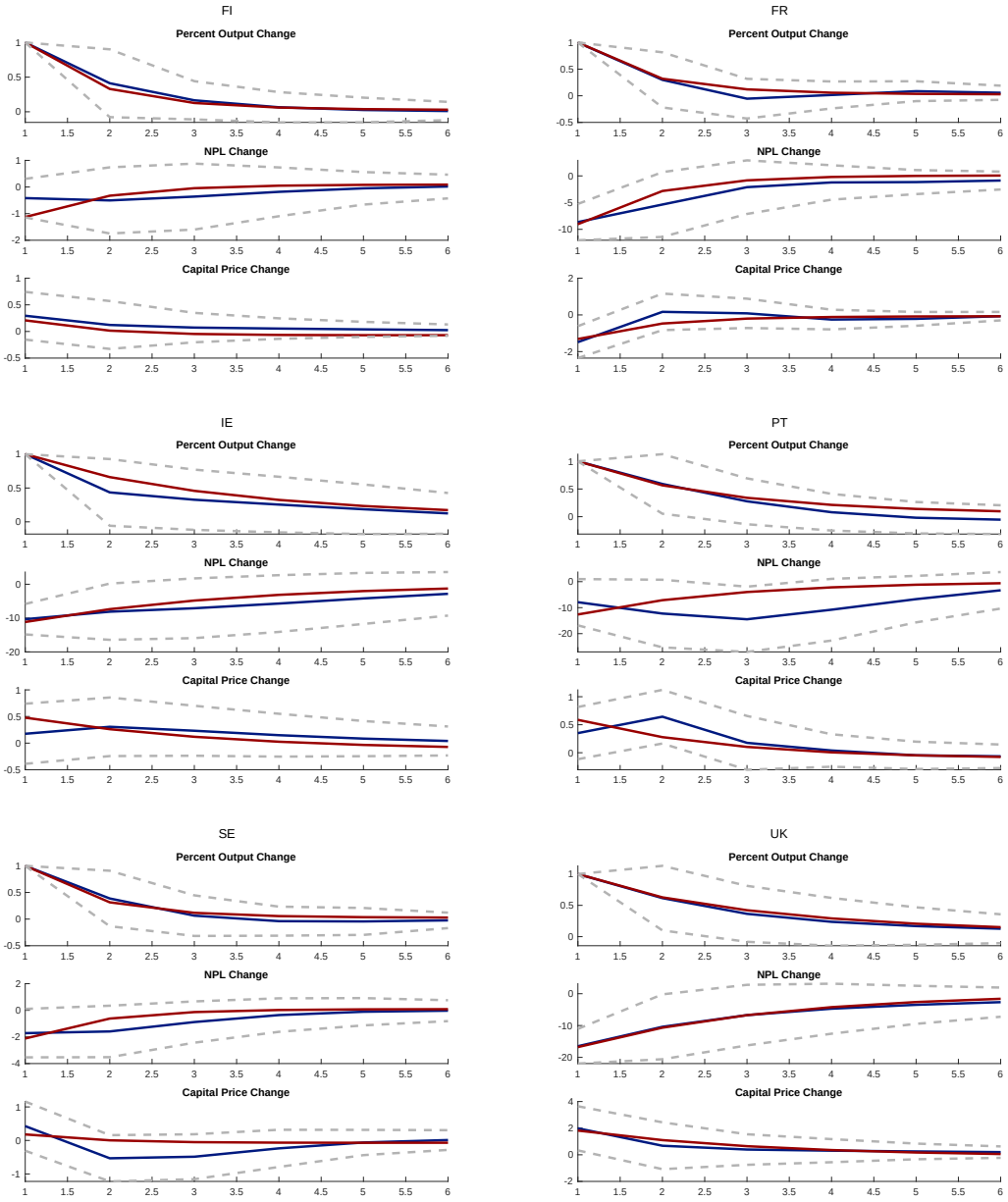


Figure 4.43: Short-run identified impulse responses of the variables to an output shock. The estimated model is plotted in red. Dashed lines give confidence intervals. The two-digit country code is above the plot.

Chapter 5
Conclusion

The three chapters of this thesis develop new macroeconomic models and show how these can yield concrete results.

The first chapter implements a DSGE model with search and matching and information frictions and estimates the extent of noise agents face when making job creation decisions, which are based on their expectations about future aggregate labour productivity growth. Further, a strategy to identify noise in a VAR model is used for robustness to investigate the same question with both approaches finding that noise is unlikely to play a significant role in employment and job creation in the United States labour market on the aggregate.

The second chapter shows that employment state transitions are pro-cyclical, as is the dispersion of the expectation of consumers regarding income shocks, while their perceived uncertainty is counter-cyclical. The paper then proceeds in showing that introducing cyclical information frictions, and defining uncertainty as the posterior about match quality, may provide a coherent and parsimonious explanation for survey responses showing a pro-cyclical dispersion in income growth expectations and counter-cyclical uncertainty of consumers and workers during the recession and recovery. Simulation of the dynamic directed search block recursive equilibrium model developed in the chapter shows that in interacting with other labour market frictions, counter-cyclical information frictions may reduce employment state transitions and drive mismatch during recessions, leading to more productive booms and deeper more unproductive recessions.

The third chapter shows that periods of a higher share of non-performing loans in a country are shown to be tightly linked to higher misallocation. A structural model with search frictions in capital reallocation can produce the non-performing loan shares endogenously and capture this and other empirical relations. The model evaluates two drivers for the share of non-performing loans, forbearance incentives and frictions in used capital markets, which drive down the outside value of foreclosure. The chapter

then proposes a novel avenue to identify to what extent these two frictions that drive non-performing loan shares in a country become worse over the cycle. It makes use of the different dynamics of capital prices caused by a worsening of the matching function in used capital markets and an increase in forbearance incentives to estimate these changes.

While the results in these chapters are self-contained, the models and mechanisms developed offer avenues for future research. The mechanism developed "No Noise in Job Creation" can be extended to estimate information frictions workers face in choosing sectors while relying only on sectoral or occupational employment and output data as outlined in the chapter appendix. This can explain labour mismatch and sluggish adjustment of labour to structural change due to information deficits about the most area with the best productivity prospects in a (Lucas and Prescott, 1978)-type model. As employer-employee matched data of the type described in (Abowd and Kramarz, 1999) becomes more widely available in the form of anonymised administrative data it will be possible to further estimate and evaluate the mechanism developed in "Information Speed Bumps in the Labour Market", especially if this data can be merged with surveys about expectations. The same holds for directly estimating and evaluating the frictions in "Non-performing Loans and Inefficient Capital Reallocation" with creditor-lender data such as the ECB's Ana Credit database for the Euro-zone (Israel, Damia, Bonci, and Watfe, 2017) or Tokyo Shoko Research data for Japan. Persistent heterogeneities in the data can be captured in the model developed in this chapter may be captured by implementing the mechanism in a dynamic directed search block recursive equilibrium. Finally, it is clear that the non-performing loans problem is closely related to the problem of zombie firms (Caballero et al., 2008). Integrating the mechanism in this chapter in a general equilibrium framework for firm creation and destruction following (Bilbiie, Ghironi, and Melitz, 2012) and (Hamano and Zanetti, 2017) can produce a tractable New Keynesian model capturing zombie lending, financial frictions, and firm dynamics.

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