

Design and Application of Novel Metamaterial Elements and Configurations

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by

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Abstract

Metamaterials are artificially constructed “materials” which are formed from arrays of engineered elements. By designing individual elements as well as their interactions, the propagation of electromagnetic (EM) waves within the structure can be manipulated, so that new responses can be realised which may not be found in nature.

The subject of this research concerns the study of miniaturised elements with strong EM responses so that the constructed metamaterial can better approximate an ordinary low-loss material. The research involves designing miniaturised magnetic resonators operating in the microwave frequency range. A novel resonator prototypes, so-called “helical resonators”, have been successfully designed and fabricated whose physical sizes can fall below 1% of the free space wavelength at resonance. This contributes to a size reduction of 90% compared with previously published work. In addition, an analytical model has been developed, so that the resonance parameters of a helical resonator have explicit expressions. In particular, a constant optimal metallic fill ratio is demonstrated to exist, which can achieve a minimum resonant frequency and a maximum miniaturisation for any given outmost

dimension of the element. The accuracy of the model has been verified by both simulation and experiment. The frequency responses of fabricated helical elements were measured using a vector network analyser and a pair of small loop non-resonant dipole probes, and the parameters were extracted using the phase frequency fit method which proves to have the best accuracy and robustness.

The properties of a regular square array of helical resonators are subsequently investigated, which can be regarded as a two-dimensional metamaterial. A relevant analytical model has been developed, which characterises the array as an equivalent sheet with surface current distributions, rather than an artificial medium with finite thickness. The relation between the macroscopic EM fields and the small scale properties of individual helical resonators are then established. In particular, the helical resonators are observed to be inherently chiral, thus the assembled interlocking array exhibits dichroism. The transmission coefficients for the circular EM waves with two different polarisation states have been derived, which have been verified by simulation and measurement results as well. In addition, it has been theoretically demonstrated that the resonator elements and their spacings can be engineered, so that the circular EM wave with one particular polarisation state can be totally attenuated around the element resonance, while the other state suffers negligible attenuation. A quadratic relation between the optimal array spacing and the elements' quality factor has been demonstrated.

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List of Publications

- [1] J. Zhu, T. Hao, C. J. Stevens, and D. J. Edwards, "Optimal design of miniaturized thin-film helical resonators," *Applied Physics Letters*, vol. 93, p. 234105, 2008.

- [2] J. Zhu, T. Hao, C. J. Stevens, and D. J. Edwards, "Investigation of vertical spiral resonators for low frequency metamaterial design", 2nd International Congress on Advanced Electromagnetic Materials in Microwaves and Optics, Pamplona, Spain, September 2008.

List of Acronyms

1D	one dimensional
2D	two dimensional
3D	three dimensional
BC-SRR	broadside coupled split ring resonator
BW	backward wave
DNG	double negative materials
EM	electromagnetic
GSTC	generalised sheet transition condition
IR	infrared
LHM	left handed materials
MI	magneto-inductive
NRI	negative refractive index
PCB	printed circuit board
SNR	signal-to-noise ratio
SRR	split ring resonator
TL	transmission line
UV	ultraviolet

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Chapter 1

Introduction

1.1 General Background of Double Negative Metamaterials

Ordinary materials consist of atoms and molecules. When electromagnetic (EM) waves enter a material, the electric and magnetic fields may change the charge distribution within the material, and induce electric and magnetic dipole moments inside the comprising elements. The responses of a material to the external EM wave radiation are macroscopically depicted by two parameters, namely, relative dielectric permittivity ϵ_r and relative magnetic permeability μ_r . These two parameters characterise the collective contributions from the excited dipole moments, and usually do not depend on the size or the shape of the material sample, i.e., they are homogeneous and inherent properties of a material. Generally speaking, ϵ_r and μ_r are both positive for most ordinary materials.

Metamaterials are artificially constructed "materials" which are formed from arrays of engineered elements [1]-[8]. Their properties are gained from structure at levels much

larger than the atomic scale, i.e., the elements are big enough to be fabricated into complex shapes. Each element simulates the behaviour of a molecule in an ordinary dielectric in that it exhibits a dipole moment, and the collective effect of these microscopic inhomogeneities within the structure contributes to effective macroscopic behaviour. In addition, it is generally acknowledged that artificially engineered periodic microstructures, where the wavelength of operation is much longer than the composition units and the periodicity, can be described as effective media when depicting electromagnetic phenomena at a macroscopic level [9]. In general, the EM properties of such structures can be expressed in terms of effective permittivity and permeability. By tailoring individual elements as well as their interactions, the propagation of EM waves within the structure can be manipulated, so that new responses can be realised which may not be found in nature.

The most common type of metamaterials refers to structures which show both negative permittivity and negative permeability. The properties of such materials were first investigated theoretically by the Russian physicist Veselago [10], who predicted that a material with both negative ϵ_r and μ_r could have extraordinary optical phenomena. However, his work was largely ignored until Pendry analysed several periodic configuration of conductor arrays, and envisaged the first practical realisation of double negative metamaterial [11]-[13]. This research field was revived when Smith and Shultz succeeded in the experimental demonstration using straight conducting wires to yield negative values of permittivity ϵ_r , and split ring resonators (SRRs) to generate negative μ_r [14]-[17]. A transmission passband of EM wave with negative index of refraction was observed over a finite bandwidth in the microwave region. Since then, there has been an ever expanding interest in theoretical investigation and actual realisation of metamaterials with improved performance.

The simultaneous negative permittivity and permeability for metamaterials lead to unique properties which are never encountered in natural materials. Already back in 1968, Veselago predicted a number of remarkable characteristics for wave propagation in double negative metamaterials. Since the refractive index is also negative, propagation of EM wave described by the wave vector $\vec{k}$ is contrary to the direction of energy flow. This can result in various unusual phenomena including the reversal of Snell's law, reversal of Doppler effect, backward Cerenkov radiation, etc [10]. Furthermore, Pendry has theoretically suggested that a slab of metamaterial with $\epsilon_r = \mu_r = -1$ can act as a 'perfect lens' [18], i.e., both the propagating and evanescent spatial Fourier components of an object can be ideally reconstructed at the image plane, and focusing beyond the Abbe diffraction limit can thus be realised when the lens locates in the near field of the object.

Due to the various aspects of extraordinary attributes exhibited, the metamaterials are known by several other names. The commonly used terms include: double negative materials (DNG) - signifying the simultaneous negative values for both permittivity and permeability; left-handed materials (LHM) - highlighting the presence of left-handed $(\vec{E}, \vec{H}, \vec{k})$ triplet; backward wave materials (BW) - emphasising the opposite phase and group velocities; and negative refractive index materials (NRI) - stressing the negative index of refraction for these materials.

The realisation of metamaterials is of tremendous technological growth potential. The current trend is towards the application of these revolutionary artificial materials to utilise their unusual behaviour and the ability of tailoring material parameters in a controlled manner [20]-[35]. The possible practical application areas include those in aerospace, telecommunication, biomedical engineering, information technology, etc.

1.2 Theoretical Interpretation and Actual Implementation of Metamaterials

The study on the impact of material parameters on EM wave propagation lead to the prediction of different types of materials. Fig. 1-1 shows the permittivity–permeability ($\epsilon_r - \mu_r$) and refractive index (n) diagram. Ordinary materials having both positive ϵ_r and μ_r allow the propagation of EM waves, in which the electric field $\vec{E}$, magnetic field $\vec{H}$ and wave vector $\vec{k}$ form a right-handed triplet. In the second and fourth quadrants, either the permittivity or permeability is negative and $\epsilon_r\mu_r < 0$. EM waves cannot effectively propagate, and only evanescent waves exist inside such media. When negative ϵ_r is combined with negative μ_r , as the case in the third quadrant, EM waves can in theory traverse such materials in the form of backward waves, i.e., wave vector $\vec{k}$ has reversed direction as $\vec{E} \times \vec{H}$ [4],[20]. Thus the third quadrant is also known as the left-handed media region, where the refractive index have real negative values.

The macroscopic parameters of metamaterials are largely determined by the properties of the comprising elements and their interactions, therefore extensive research have been conducted on this area. With the aid of the theoretical frame developed by Pendry, researchers at the University of California, San Diego (UCSD) produced the first double negative metamaterial prototype working in the microwave range as depicted in Fig. 1-2(a). The composite metamaterial was comprised of a regular two-dimensional array of metallic rods stacking interlaced with another array of split ring resonators (SRRs). The former accounts for negative permittivity within a relatively broad frequency bandwidth, while the latter produces a narrowband response of negative permeability around element resonance, as depicted by the dip in the measured transmission response in Fig. 1-2(b). The appearance of a passband within the previous stopband indicates that the negative

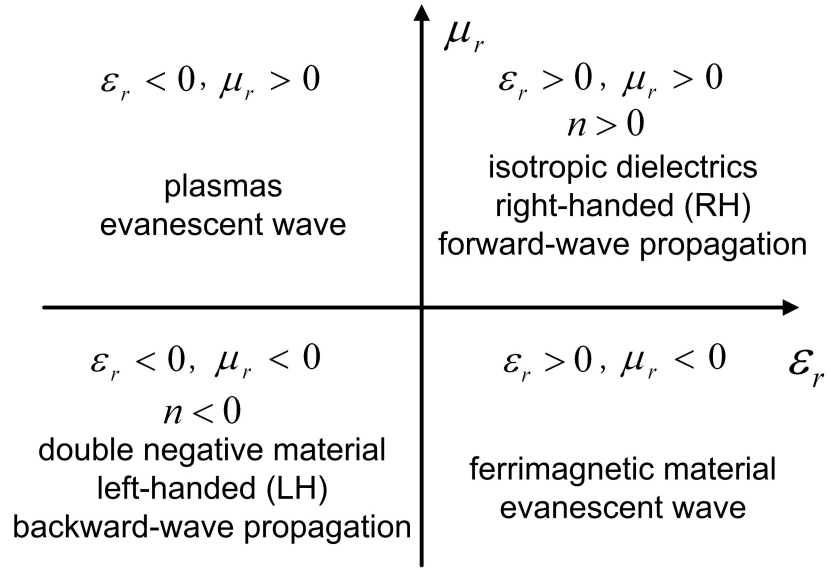
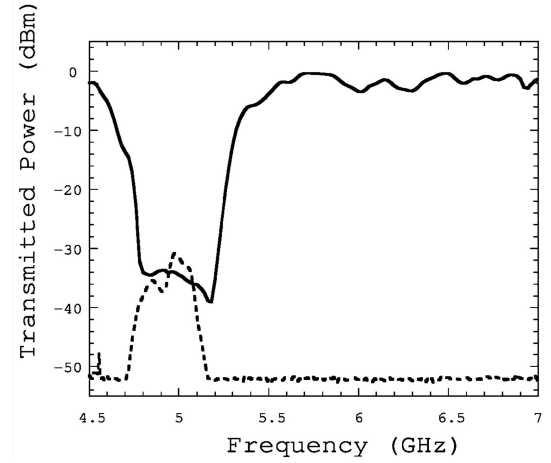


Figure 1-1: Four quadrant diagram of permittivity-permeability ($\epsilon_r - \mu_r$) and refractive index (n) [19].



(a)



(b)

Figure 1-2: (a) The first fabricated double negative metamaterial prototype working in the microwave range by stacking both SRR array and wire array interlaced. (b) Measured transmitted power of the split ring resonator (SRR) array alone with regular lattice of 8.0 mm (solid line) and the metamaterial sample by adding wires uniformly between SRRs (dashed line) [14].

μ_r of the SRR array has combined with the negative ϵ_r of the wire array to allow the propagation of EM wave, which results in a composite metamaterial with double negative material parameters simultaneously. As can be seen, the transmission loss is extremely high. Later the UCSD team experimentally demonstrated the negative refraction for such a material using a prism [15]-[17].

The principles of achieving negative values of permittivity and permeability in the above prototype will be discussed as follows.

1.2.1 Principle of Realisation of Negative Permittivity

Artificial materials with negative equivalent permittivity can be realised using plasma or its analogy. Plasma generally refers to an electrically neutral medium composed of both positive and negative charges, of which at least one charge type is mobile. In a solid, the negative charges of the conduction electrons are balanced by an equal concentration of positive charge of the ion cores. The plasma has a dielectric function of the following form [36]:

$$\epsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + j\gamma)} \quad (1.1)$$

The plasma frequency ω_p is typically in the ultraviolet or visible region of the spectrum for ordinary metals. Around the plasma frequency, a collective oscillation of electron density occurs which is defined as plasmon. γ is a damping term representing dissipation of the plasmon's energy. The equivalent permittivity ϵ_r is essentially negative below the plasma frequency ω_p , at least down to frequencies comparable to γ . At lower frequencies below the near infrared, the dielectric function becomes imaginary due to large dissipation.

In order to achieve negative values of permittivity at microwave frequencies, Pendry

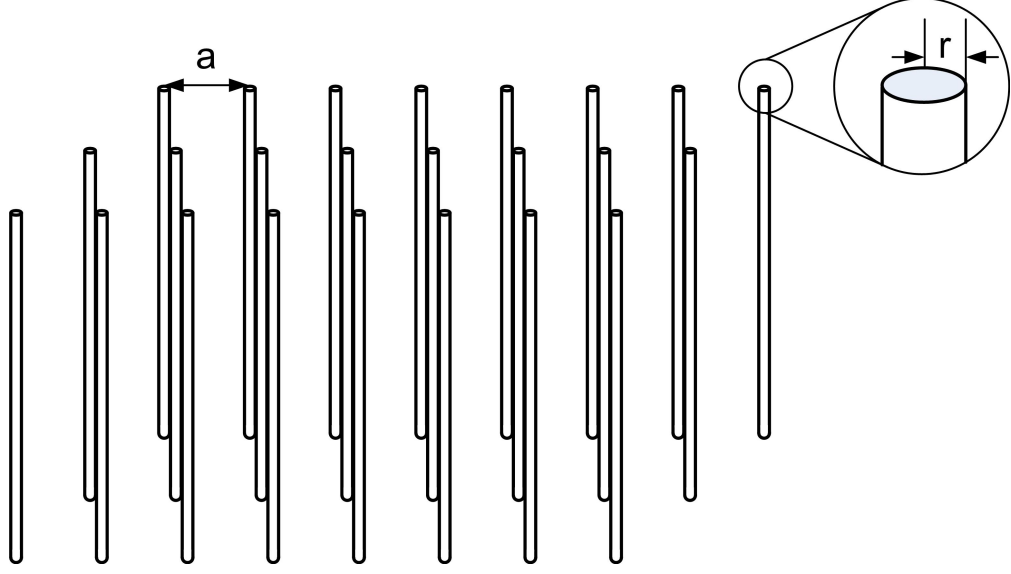


Figure 1-3: Periodic configuration of thin metallic wires having negative equivalent permittivity in the microwave frequency range.

proposed a mechanism for reducing the plasma frequency using periodic structures of very thin metallic wires. The plasma frequency of the thin cylindrical metallic wire array (as shown in Fig. 1-3) is given as

$$\omega_p = \frac{c_0}{a} \sqrt{\frac{2\pi}{\ln(a/r)}} \quad (1.2)$$

where c_0 is the speed of EM wave in free space, a is the lattice parameter and r is the radius of the wires.

In the thin wire case ($r \ll a$), the corresponding wavelength for ω_p is much larger than both a and r . Therefore, such an array of thin wires behaves as effectively homogeneous diluted plasma with plasma frequency within the microwave range.

1.2.2 Principle of Realisation of Negative Permeability

The realisation of an artificial material with negative equivalent permittivity in the microwave frequency range has been demonstrated. The negative value can exist within a relatively large frequency bandwidth. Consequently, it would be of interest to explore the possibility of achieving negative permeability in a similar way. However, the solution is not so simple.

The relative magnetic permeability of most ordinary materials are equal to unity, and the magnetic responses of most natural materials begin to tail off at frequencies above gigahertz range. Those materials that remain active are often heavy, and may not have very desirable mechanical properties. On the other hand, artificially engineered materials can be employed to achieve negative equivalent permeability even at high frequencies. By incorporating suitable microstructure, materials can be fabricated from non-magnetic components, that still show magnetic responses when they interact with EM waves of the correct frequency. Their weight is normally quite small, which provides an additional advantage. However, due to the lack of magnetic analogue to a good electrical conductor, one cannot directly imitate the procedure as in § 1.2.1 and realise negative permeability through "magnetic plasma".

Pendry introduced a variety of microstructures built from conducting sheets. When grouped into a periodic array, they were shown to exhibit effective permeability which can be tuned to values not accessible in natural materials [13]. Fig. 1-4 shows the layout of a split ring resonator (SRR) as being employed in UCSD's prototype, which consists of two planar concentric metallic rings and each has a split with opposite orientations. The transient current and charge distributions illustrate that the current on the two rings circulate in the same direction, thus the concentric rings jointly act as an inductive

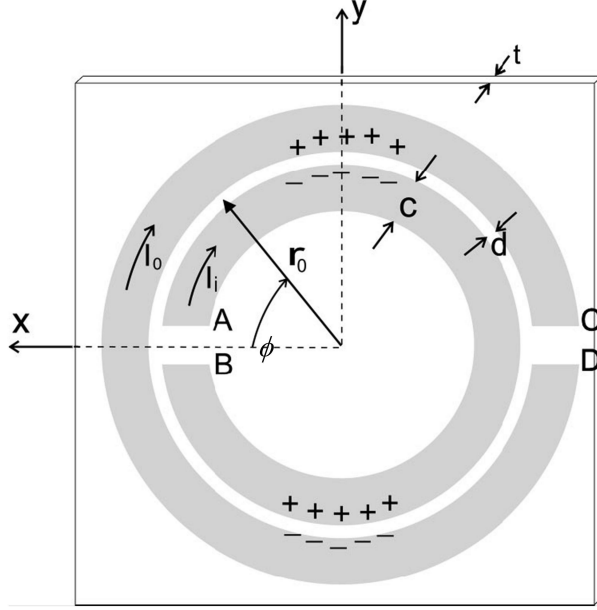


Figure 1-4: Layout of a split ring resonator (SRR) with quasi-static transient current and charge distributions [37].

loop. The charges accumulate at both sides of the gap between the rings, which serves as a distributed capacitance. If all the gaps are absent so that the SRR degenerates into a single metallic loop, it is well known that the induced current in such a loop tends to oppose the change in the external magnetic flux, which exhibits diamagnetic response. However, the diamagnetism is generally too weak to affect the effective material permeability if artificial materials are built from these loops.

The introduction of capacitive elements into the structure plays a key role in enhancing the magnetic response of artificially designed materials, so that the effective permeability can significantly deviate from unity. The combination of inductive loop and capacitive gap makes the SRR unit to have resonant behaviour. When being approximated by a lumped LC resonant circuit fed by external magnetic flux, the phase of current circulating around the element follows that of the excitation before resonance

while reverses after resonance. Therefore both paramagnetic and diamagnetic behaviour can be realised. In addition, the self capacitance of an SRR element is much larger than that of a single split ring with similar size due to the edge coupled capacitance between the rings, thus the SRR unit can resonate at a wavelength much larger than its diameter, which makes the effective medium approach applicable, i.e., the SRR array will respond to external EM fields homogeneously on the scale of wavelength.

The SRR element, as shown in Fig. 1-4, can only respond to external magnetic field perpendicular to its plane while has no response to excitations in other directions, thus exhibits anisotropic behaviour. In order to realise an isotropic material having negative permeability among all dimensions, Pendry proposed a three-dimensional symmetry unit by placing planar SRRs on each side of a cubic cell [13]. However, the actual fabrication process for a lattice structure based on such unit cells will be quite cumbersome.

In order to better approximate a low loss uniform material, the elements of metamaterials should have physical sizes much smaller than the operating wavelength. The typical size of SRR, however, is around a tenth of the free space wavelength at resonance [38], which is just within the limit of effective medium description. Therefore the assembled metamaterials lack homogeneity, and the consequent significant diffraction within the structure makes its EM properties inconsistent with the estimations using effective medium theory [39]. In addition, elements with such large size have substantial radiation loss, which even exceed the conductive loss [38]. Therefore miniaturised low loss magnetic metamaterial elements with strong responses to external excitation are aimed to be developed, which is the bottleneck of the metamaterial design.

1.3 Investigation of Bulk Metamaterials

Ever since the successful implementation of double negative metamaterials, there has been a booming interest in characterising their EM properties, and exploring how to design the elements and lattice structures so as to achieve desired macroscopic material characteristics [40]-[65]. A common practice when designing or analysing metamaterials consisting of three-dimensional lattice of elements is to view the whole structure as an effective uniform dielectric with equivalent ϵ_r and μ_r . These equivalent parameters can be retrieved from the measured scattering parameters (reflection and transmission coefficients) from a slab of metamaterial under test, which is a well established method for similar parameter extraction on ordinary materials [66]. By employing magnetically/electrically self resonant particles, a wide range of effective material parameters can be obtained.

The validity of such a method on characterising composite metamaterial, however, is left to be questioned. First, since most metamaterial structures contain only a finite number of regularly cascaded parallel layers of particles, their thickness can only be integer multiple of the layer spacing, which introduces ambiguity to the retrieved material parameters. The ambiguity can theoretically be suppressed by reducing the sample thickness, but then the boundary effect will become significant, and the extracted parameters tend to be dependent on the thickness of material samples [40],[59]-[61]. This is in contrast to those inherent parameters for ordinary dielectrics. In addition, for those metamaterials that are based on resonating elements, the magnitude of the effective parameters can become quite large near resonance, which can make the thickness of the composite artificial material comparable to the equivalent dielectric wavelength.

So far realisable metamaterial samples quite often have only a few layers of particles

across the sample thickness, partly because the transmission loss of EM wave through the material is still quite significant. It can then be envisaged that the transmission properties of such thin layer structure can be accurately calculated as the cascade of those of two-dimensional planar array of elements, which can be developed from the transition conditions of an equivalent sheet with certain electric and magnetic surface current distributions. This idea has been well established from the research on frequency selective surfaces [49], [60]-[64], and can thus be employed to guide the accurate characterisation of thin layer artificial materials rather than using effective medium theory.

1.4 Thesis Structure

This thesis is organised as follows.

In Chapter 2, a new type of miniaturised magnetic metamaterial elements, the so-called "helical resonators", will be thoroughly investigated. The self capacitance of elements is significantly increased by adopting the broadside coupled configuration comparing with planar edge coupled SRR, thus the physical size is dramatically reduced. By employing thin-film substrate with a thickness of $30\text{ }\mu\text{m}$, helical resonators with outmost dimensions below 1% of the free space wavelength at resonance have been successfully designed and fabricated. This means 90% size reduction comparing with that of a typical SRR, which offers significant potential in building metamaterial with better homogeneity.

In addition, an analytical model is developed from quasi-static circuit analysis. The explicit expressions for equivalent circuit parameters are derived, thus the dependence of element resonance parameters including resonant frequency and quality factor Q on its physical dimensions can be accurately estimated, and some unexpected phenomena can

be revealed. For example, it will be illustrated that a roughly constant optimal dimension ratio independent of outmost size exists for both circular and square helical resonators, which can achieve the maximum level of miniaturisation.

The quality factor Q of magnetic resonators is a dominant factor which determines the frequency dependent complex permeability of the assembled artificial media. In order to accurately model its behaviour, the losses induced from both conductor, dielectric substrate and radiation are all incorporated into the analysis, and the element dimensions are modified to reflect the actual spatially nonuniform conduction current distribution at resonance. Finally, the effect of miniaturisation on the Q of helical resonators will be investigated.

The resonance parameters estimated from analytical circuit model are in good correspondence with the numerical results using commercial simulation software, as well as experimental measurement. Chapter 3 will illustrate the details of how to retrieve these parameters from the recorded responses. Especially, measurement of the transmission spectrum were conducted using a vector network analyser and two small loop dipoles as the magnetic probes, and the parameters were extracted using the nonlinear least-squares fit to the phase of S_{21} data versus frequency. This method has been demonstrated to have the best performance in terms of both precision and robustness when measuring microwave cavity resonators, and will be re-examined for accuracy when being applied to the current experiment configuration by comparing with other methods. In addition, the impact of the coupling strength of probes to the resonator under test on the accuracy of parameter extraction will be investigated.

After thoroughly examined the properties of individual elements, the performance of two-dimensional (2D) array of helical resonators will be subsequently studied in Chapter

4. A relevant analytical model will be developed, which characterises the array as an equivalent sheet with electric and magnetic surface current distributions, rather than an artificial medium with finite thickness. The relation between the macroscopic EM fields and the small scale properties of individual helical resonators, as well as the interactions between elements, will then be established.

The helical resonators are inherently chiral, thus the assembled 2D interlocking array exhibits dichroism around element resonance, i.e., left- and right-handed circularly polarised EM waves experience different levels of attenuation when passing through the array. The analytical expressions of transmission coefficients for the circular waves have been derived, which depend on the properties of individual chiral elements and their density of distribution. The results of the analytical model will be compared with those from simulation and measurement as well. In addition, it will be theoretically demonstrated that the resonator elements and their spacings can be engineered, so that circular EM wave with one particular polarisation state can be totally attenuated around the element resonance when traversing the chiral array, while the other experiences negligible variation. The relation between the optimal spacing of array and the corresponding quality factor of element will be revealed.

Finally, Chapter 5 will present summary and conclusions of the entire thesis, which are followed by discussions on possible future works as well as applications of the research outcome stated in this thesis.

Chapter 2

Analytical Modelling of Miniaturised Helical Resonators

It has been demonstrated that the SRR prototype has various inherent faults, especially the relatively large element size, which makes it unsuitable for assembling uniform low loss isotropic metamaterials. Therefore many other element prototypes have been proposed so as to make improvements [67]-[75]. For example, by moving one ring in a SRR to the other side of substrate, the coupling mechanism between the two rings evolves from planar edge coupling to greater broadside coupling, as illustrated in Fig. 2-1. Thus the formed "broadside coupled SRR (BC-SRR)" has much larger self-capacitance than an ordinary SRR with roughly the same size, which contributes to significant reduction in its resonant frequency [37],[67].

In this chapter the so-called "helical resonator", which is a type of miniaturised magnetic metamaterial element that employs a broadside coupled two-turn helix structure,

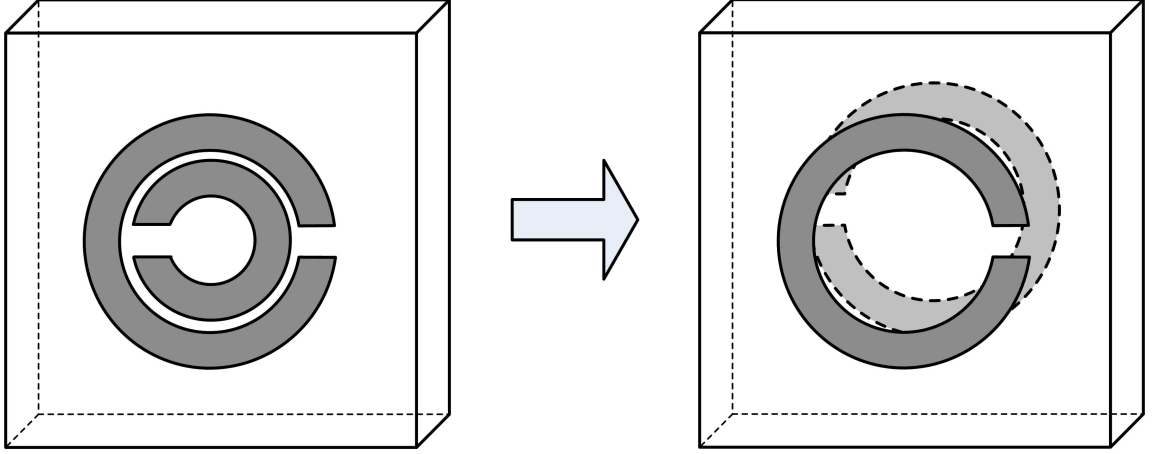


Figure 2-1: Evolvement from edge-coupled split ring resonator (SRR) to broadside-coupled alternative (BC-SRR).

will be thoroughly investigated. An equivalent quasi-static RLC circuit model will be developed in § 2.1 having analytically derived circuit parameters, so that the dependence of element properties including resonant frequency and quality factor Q on its physical dimensions can be accurately estimated. In § 2.2, it will be illustrated that a roughly constant optimal metallic fill ratio independent of diameter exists for helical resonators, which can achieve the maximum level of miniaturisation. The quasi-static analysis is valid since the physical size of element is much smaller comparing with the free space wavelength at element resonance. For example, by employing thin-film substrate with a thickness of $30\text{ }\mu\text{m}$, helical resonators with outmost dimension below 1% of the free space wavelength at resonance have been successfully fabricated.

The quality factor Q of magnetic resonators, which is defined as the ratio of the total stored energy and those dissipated per cycle of oscillation, is a key feature describing the frequency dependent complex permeability of the assembled artificial media. In § 2.3 an analytical approach for predicting the Q of helical resonators working in the microwave frequency range will be provided, which is also applicable to other miniaturised magnetic

resonators with a broadside coupled configuration. The losses induced from both conductor, dielectric substrate and radiation are all incorporated into the analysis. The element dimensions are modified to reflect the actual spatially nonuniform conduction current distribution at resonance, so that the accuracy of estimation of circuit parameters, especially conductor loss, can be significantly improved. Finally, the effect of miniaturisation on the Q of helical resonators will be investigated in § 2.4.

The analytical model developed in this chapter can provide results which are in good coincidence with both numerical simulation and experiment, thus can be utilised as a reliable starting point in the design of miniaturised magnetic resonating elements. These can be used to fabricate a bulk metamaterial with good homogeneity and low loss, as well as other electrically small devices including antennas, filters, etc.

2.1 Helical Resonator and Quasi-Static Analysis

The general structures of both circular and square helical resonators are plotted in Fig. 2-2. Two concentric metallic split rings with the same dimensions (outer diameter/side d_{out} , strip width w) are printed on both sides of a dielectric slab with a thickness of h . A conducting via through the slab connects the two rings into a broadside-coupled version of two-turn spiral. The relative dielectric constant and loss tangent of the substrate are denoted by ϵ_r and $\tan \theta$ respectively. Through numerical simulation, it has been demonstrated by researchers in our group that the helical resonator has superb characteristics including high miniaturisation level, small radiation loss and extremely high quality factor [76]. In this section, an analytical equivalent circuit model will be built based on quasi-static analysis.

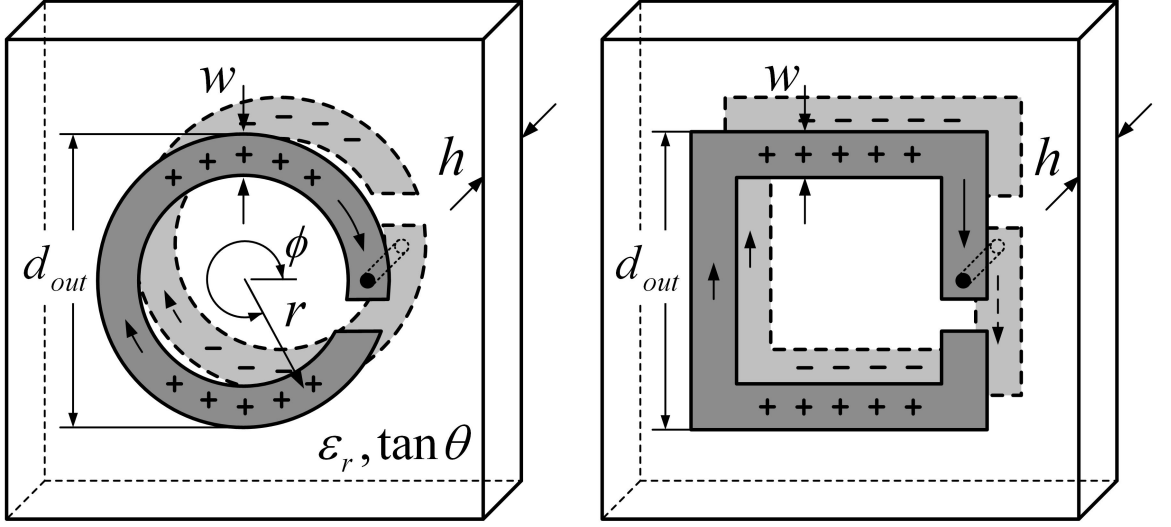


Figure 2-2: Layout of circular and square helical resonators. Grey zones with solid boundaries correspond to conductor patterns on the top of substrate while those with dashed boundaries correspond to conductor on the bottom level. The two rings in each helical resonator are connected by a metallic via. The snapshot of charge and current distributions on the resonators are also exhibited.

Generally speaking, the behaviour of an object when interacting with external incident EM waves can only be comprehensively characterised by solving the full-wave Maxwell equations with certain boundary conditions. When the element size is much smaller compared with the wavelength of propagation, however, the spatial variation of EM fields in the element are quite small and can be safely neglected, which enables the quasi-static analysis procedure (i.e., the spatial variation of magnetic field is induced by current excitation only, with the displacement current item $\partial D/\partial t$ removed from Maxwell equations).

The helical resonator can be excited when external magnetic flux penetrates the helix, which then behaves as a lumped RLC resonant circuit fed by the external magnetic field B_0 . Assuming that the incident magnetic field vector is along the normal axis of the

element, one can obtain the induced quasi-static electromotive force according to the law of electromagnetic induction [9]

$$\varepsilon = -\frac{\partial\psi}{\partial t} = -j\omega B_0 S \quad (2.1)$$

where ω is the angular frequency of the incident EM wave, and $e^{j\omega t}$ time harmonic has been assumed; S is the area enclosed by the element.

Using a circular helical resonator as an example, due to the quasi-static assumption, the distribution of voltage along the continuous metallic strip can be assumed to have a linear variation with the rotation angle ϕ as a first-order approximation, which can be plotted as in Fig. 2-3(a) (the effect of splits in each ring is neglected). As can be seen, the voltage difference between the strips on both sides of substrate is a constant value along the element circumference, which equals the above electromotive force. The loop currents flowing on both rings also have linear dependence on ϕ , which can be deduced to have the distribution as shown in Fig. 2-3(b). Such a distribution leads to an angle-invariant sum current flowing on both rings, which coincides with the rotational symmetry of the circular element structure. These conclusions also hold for square resonators since the variation in radial dimension can be safely neglected in the quasi-static regime.

The quasi-static equivalent RLC circuit for a helical resonator can be drawn as in Fig. 2-4, which is in series resonant form and incorporates loss mechanisms including conduction loss (reflected by R_c), dielectric dissipation (R_d) and radiation (R_r).

Due to the uniform current distribution assumption, R_c , R_r and L can be approximated by the electrical resistance, radiation resistance and inductance of a single planar conductive disk having the same dimensions as each split ring in a helical resonator. Meanwhile, R_d and C can be regarded to equal the distributed dissipation loss and ca-

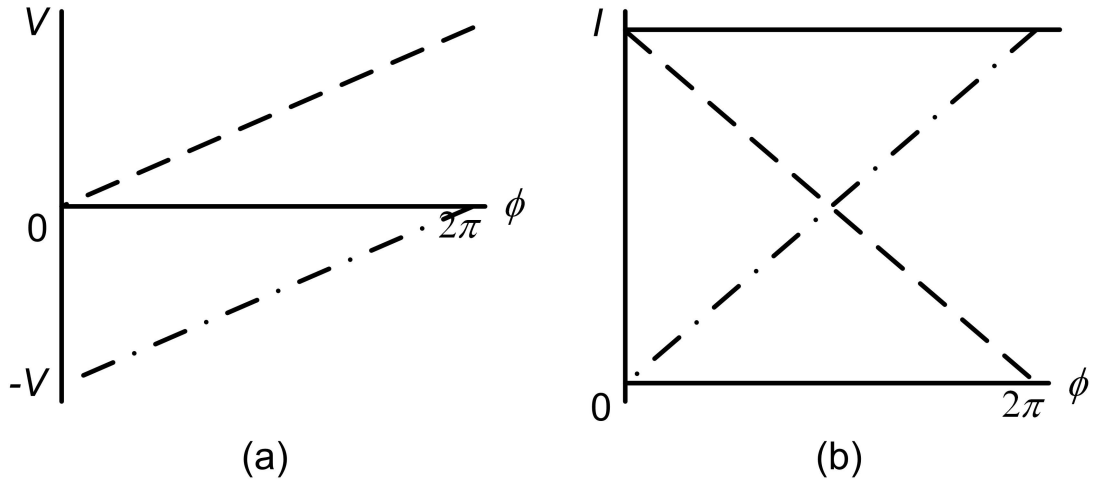


Figure 2-3: (a) The quasi-static voltage and (b) current intensity in a helical resonator as functions of the angular polar coordinate ϕ under first-order approximation. The dashed lines (- -) in both graphics correspond to those of the upper ring and the dashed and dotted lines (-.-) correspond to those of the lower ring. The continuous line in (b) stands for the total current flowing along the circumference of a helical resonator, i.e., the sum of the two lines.

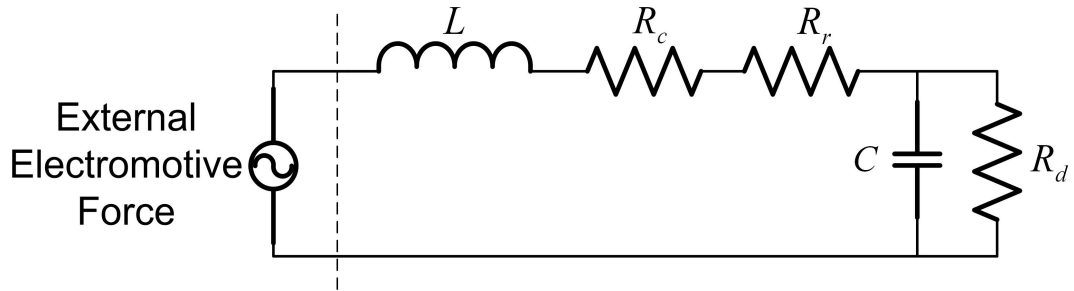


Figure 2-4: The quasi-static equivalent RLC circuit model of helical resonator.

capacitance of a capacitor formed by the broadside coupled metallic strips. The square helical resonator has a longer circumference, a larger inductance and capacitance and hence a lower resonant frequency than the circular version with the same dimensions, thus should be a better candidate for maximum miniaturisation purposes. However, the corners of a square resonator introduce additional inductance, which have to be incorporated into the modelling process.

This section targets the miniaturisation of elements, thus all the losses will be neglected in the first instance in the following analysis of element resonance. The resonant frequency f_0 of a helical resonator is given by classical circuit analysis as

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad (2.2)$$

It is then of interest to compare helical resonators with BC-SRRs which also have broadside coupled configuration. Based on similar analysis procedure, the first-order approximation of angular voltage and current distributions on a BC-SRR can be deduced, with the graphs plotted in Fig. 2-5. As can be seen, the sum current is still uniform at any cross-section of element, while the direction of the constant voltage drop between the strips reverses at the two halves of resonator divided by the gaps. This essentially illustrates that the two capacitors formed by the broadside coupled metallic strips separated by the planar gaps are in series connection, while those in a helical resonator are in parallel connection where a constant voltage drop holds throughout the entire element circumference. Therefore it can be deduced that a helical resonator has the same quasi-static inductance L as that of a BC-SRR with the same physical dimensions, but four times capacitance C than the latter. These contribute to a reduction of roughly a half in the resonant frequency, which demonstrates that helical resonator is a better element

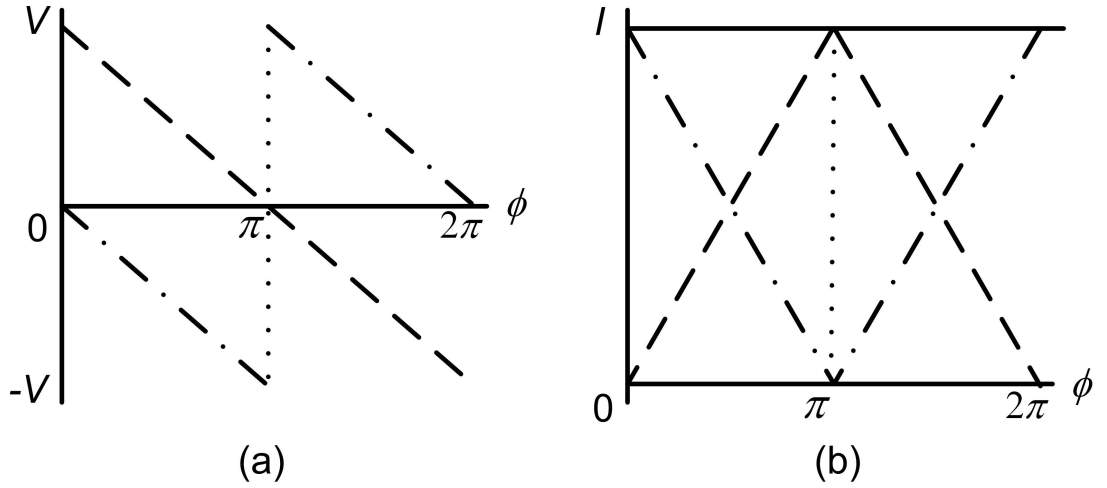


Figure 2-5: (a) The quasi-static voltage and (b) current intensity in a broadside coupled split ring resonator (BC-SRR) as functions of the angular polar coordinate ϕ under first-order approximation. The dashed lines (- -) in both graphics correspond to those of the upper ring and the dashed and dotted lines (-.-) correspond to those of the lower ring. The continuous line in (b) stands for the total current flowing along the circumference of a BC-SRR, i.e., the sum of the two lines.

prototype in terms of miniaturisation level than BC-SRR.

It needs to be pointed out that a helical resonator has another distinct "chiral" property that is exempt within a BC-SRR, which means it can also be excited by an electric field along its normal axis. This property will be fully investigated in Chapter 4 when analysing the macroscopic transmission of EM waves through a two-dimensional interlocking array assembled by helical elements.

In the following the influence of different geometric parameters on the resonant frequency of helical resonators will be theoretically investigated. The inductance is estimated by employing the accurate "modified Wheeler formula" which was originally developed for the total inductance of a planar spiral inductor by treating a conductive disk as a one-turn spiral [77], rather than numerically solving an infinite integral as was

used in [67]. The expression for the quasi-static inductance L of a helical resonator can thus be deduced as follows

$$L = k_1 \mu_0 \frac{2r}{1 + k_2 \frac{w}{2r}} = k_1 \mu_0 \frac{(d_{out} - w)^2}{d_{out} + (k_2 - 1) w} \quad (2.3)$$

where r , w and d_{out} are the geometric parameters as illustrated in Fig. 2-2. $k_{1,2}$ are constant coefficients dependent on the geometry of the structure. For circular rings,

$$k_1 = 1.96, k_2 = 2.81 \quad (2.4)$$

while for square rings,

$$k_1 = 2.34, k_2 = 2.75 \quad (2.5)$$

The expression for parallel plate capacitors is employed to estimate the global equivalent quasi-static capacitance of a helical resonator as follows

$$C = \frac{2\epsilon_0\epsilon_r g m r w}{h} = \frac{\epsilon_0\epsilon_r g m (d_{out} - w) w}{h} \quad (2.6)$$

where ϵ_r is the relative dielectric constant of substrate, g is a correction factor which accounts for the effect of the planar split on the total capacitance. When the splits are small compared with the total circumference, $g \approx 1$, but if they become larger then g is reduced. m is a geometry dependent factor. For circular resonators $m = \pi$, while for square resonators $m = 4$. The employment of parallel plate model implies that the fringe capacitances are insignificant contributions, which holds true for thin substrates with $h \ll w$. This is a reasonable assumption because the miniaturisation level of elements with broadside coupled configuration can be improved by reducing the substrate thickness, which coincides with the aim of this chapter.

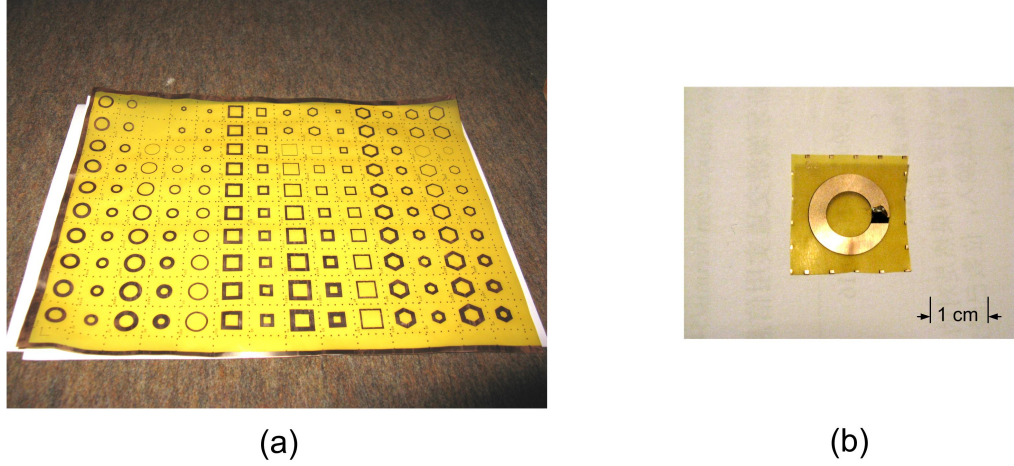


Figure 2-6: (a) Fabricated helical resonators with different sizes and shapes on a thin-film polyimide sheet with $\epsilon_r = 3.1$ and $h = 30 \mu\text{m}$. (b) Close-up of a circular helical resonator.

In order to verify the accuracy of the developed model, the resonant frequencies of helical resonators with different geometric parameters are calculated, and then compared with the results derived from commercial full-wave EM simulation software CST MicroStripesTM [78] as well as data retrieved from measurement of actual elements, as shown in Fig. 2-6. The details of numerical simulation and experimental measurement will be provided in Chapter 3. The helical resonators were fabricated altogether using standard optical lithography technique at the Photofabrication Unit, Department of Physics, University of Oxford. The substrate used was EspanexTM — an adhesiveless polyimide flexible laminate with copper cladding on both sides. The thickness of polyimide layer is $h = 30 \mu\text{m}$, while that of each copper layer is $t = 18 \mu\text{m}$. The relative dielectric constant and loss tangent of the dielectric are $\epsilon_r = 3.1$ and $\tan \theta = 0.006$. Initially the geometric parameters of helical elements are taken to be $r = 3.5 \text{ mm}$ and $w = 1 \text{ mm}$. Only one parameter is changed at a time, while the other remains constant.

First, the strip width w is varied, with the impact on element resonant frequencies

shown in Fig. 2-7(a). Smaller strip width results in a higher resonant frequency. Then, the mean radius r of the ring (or side for square element) is changed and the corresponding frequencies are depicted in Fig. 2-7(b). Larger rings have smaller resonant frequencies. The calculated resonant frequency data from the analytical model (f^{ana}), CST Micro-StripesTM simulation (f^{sim}) and experimental measurement (f^{mea}) are provided in Table 2.1 - 2.4 together with the relative discrepancies in terms of f^{mea}

$$\delta f^{ana/sim} = \frac{f^{mea} - f^{ana/sim}}{f^{mea}} \quad (2.7)$$

In both cases a circular resonator has a higher resonant frequency than its square counterpart with the same dimensions, which is in line with intuition. The data retrieved from experiment have reasonably good agreements with those of the quasi-static circuit model, although greater disagreements can be seen when the strip width is significant as in the right part of Fig. 2-7(a); while numerical simulation tends to slightly underestimate the resonance when the element dimension (either r or w) is small. The measured width and radius of the fabricated copper patterns are given in the tables as w^{mea} and r^{mea} respectively. The mean error in dimensions of all fabricated helical resonators is -0.012 mm, while the standard deviation is 0.011 mm. Therefore the fabrication process was highly accurate.

All of the helical resonators investigated have diameters which occupy only a few percent of the free space wavelength at resonance, thus a significant level of miniaturisation can be achieved by exploiting the broadside coupled structure with very thin dielectric layers.

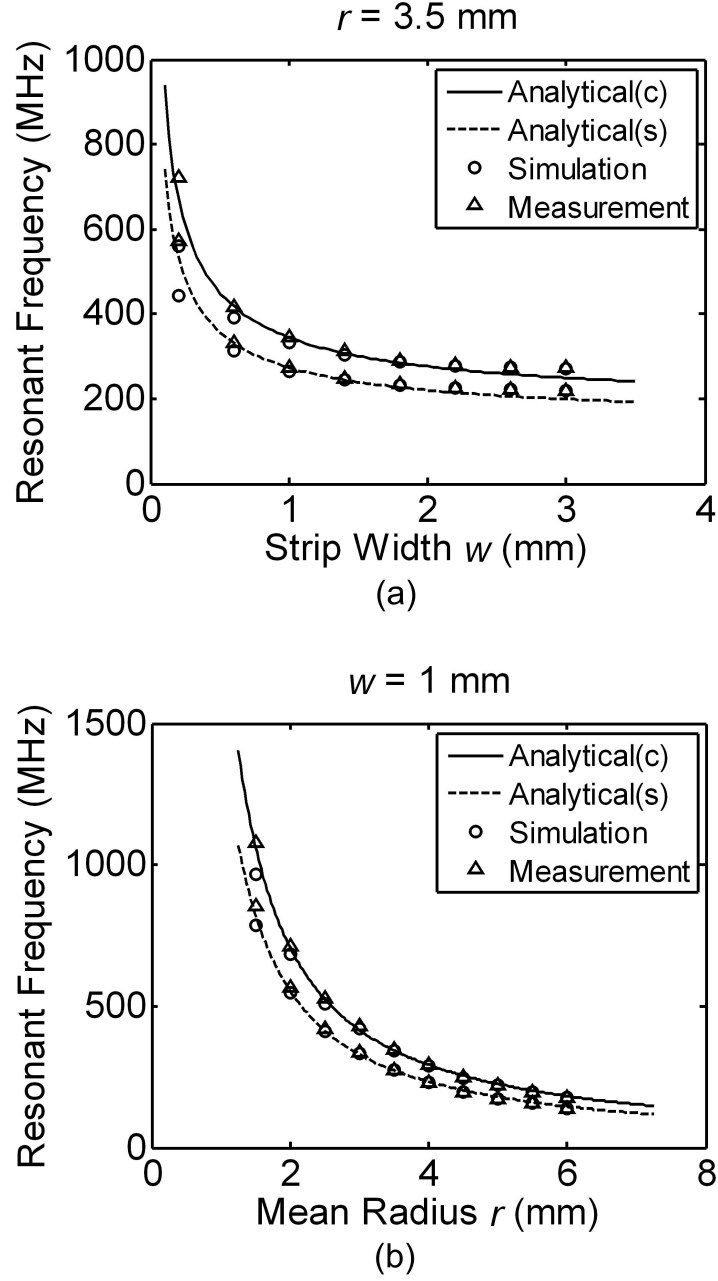


Figure 2-7: The impact of geometrical parameters (a) strip width w ; (b) mean radius/side r on the resonant frequency of both circular (solid line) and square (dashed line) helical resonators. Circles and upper triangles denote the numerical simulation and measured results. The relative dielectric constant and thickness of thin-film substrate is $\epsilon_r = 3.1$ and $h = 30 \mu\text{m}$, and the metallisation thickness is $t = 18 \mu\text{m}$.

Table 2.1: Comparison of the resonant frequencies of circular helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $r = 3.5 \text{ mm}$.

$w(\text{mm})$	$w^{mea}(\text{mm})$	$f^{ana}(\text{MHz})$	$f^{sim}(\text{MHz})$	$f^{mea}(\text{MHz})$	$\delta f^{ana}(\%)$	$\delta f^{sim}(\%)$
0.2	0.185	676.4	560.6	722.8	6.4	22.4
0.6	0.590	418.5	390.3	417.5	-0.2	6.5
1.0	1.000	344.5	332.7	346.5	0.6	4.0
1.4	1.385	307.4	302.4	311.7	1.4	3.0
1.8	1.790	284.7	285.9	291.8	2.4	2.0
2.2	2.180	269.2	277.8	281.9	4.5	1.5
2.6	2.580	258.9	272.9	275.8	6.1	1.1
3.0	2.990	249.4	271.7	273.2	8.7	0.5

Table 2.2: Comparison of the resonant frequencies of square helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $r = 3.5 \text{ mm}$.

$w(\text{mm})$	$w^{mea}(\text{mm})$	$f^{ana}(\text{MHz})$	$f^{sim}(\text{MHz})$	$f^{mea}(\text{MHz})$	$\delta f^{ana}(\%)$	$\delta f^{sim}(\%)$
0.2	0.190	536.0	444.5	574.1	6.6	22.6
0.6	0.595	332.3	312.4	334.3	0.6	6.6
1.0	0.990	273.9	264.9	275.6	0.6	3.9
1.4	1.390	244.6	246.3	249.7	2.0	1.4
1.8	1.775	226.8	233.6	234.1	3.1	0.2
2.2	2.190	214.6	225.5	225.8	5.0	0.1
2.6	2.585	205.8	221.6	222.1	7.3	0.2
3.0	2.975	199.1	220.7	220.8	9.8	0.0

Table 2.3: Comparison of the resonant frequencies of circular helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $w = 1 \text{ mm}$.

$r(\text{mm})$	$r^{mea}(\text{mm})$	$f^{ana}(\text{MHz})$	$f^{sim}(\text{MHz})$	$f^{mea}(\text{MHz})$	$\delta f^{ana}(\%)$	$\delta f^{sim}(\%)$
1.5	1.500	1063.4	967.7	1080.0	1.5	10.4
2.0	1.995	707.7	682.5	713.1	0.8	4.3
2.5	2.480	526.9	509.9	527.8	0.2	3.4
3.0	2.990	416.9	420.3	429.7	3.0	2.2
3.5	3.490	344.5	345.6	349.8	1.5	1.2
4.0	3.995	293.1	288.3	292.4	-0.2	1.4
4.5	4.490	254.9	248.3	250.8	-1.6	1.0
5.0	5.000	225.3	219.2	220.7	-2.1	0.7
5.5	5.490	201.8	196.3	197.1	-2.4	0.4
6.0	5.990	182.7	176.2	177.0	-3.2	0.5

Table 2.4: Comparison of the resonant frequencies of square helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $w = 1 \text{ mm}$.

$r(\text{mm})$	$r^{mea}(\text{mm})$	$f^{ana}(\text{MHz})$	$f^{sim}(\text{MHz})$	$f^{mea}(\text{MHz})$	$\delta f^{ana}(\%)$	$\delta f^{sim}(\%)$
1.5	1.495	822.1	785.8	856.0	4.0	8.2
2.0	2.000	554.8	546.1	565.9	2.0	3.5
2.5	2.485	415.3	413.3	419.2	0.9	1.4
3.0	2.990	330.6	334.6	337.7	2.1	0.9
3.5	3.485	273.9	273.9	275.8	0.7	0.7
4.0	3.990	233.5	231.5	231.8	-0.7	0.1
4.5	4.490	203.3	199.2	199.6	-1.9	0.2
5.0	4.995	179.9	175.1	175.3	-2.6	0.1
5.5	5.490	161.3	156.9	157.0	-2.7	0.1
6.0	5.985	146.1	141.0	141.0	-3.6	0.0

2.2 Optimal Design of Miniaturised Thin-Film Helical Resonators

After the initial analysis in § 2.1, it is consequently of interest to investigate the maximum miniaturisation level that can be achieved by helical resonators. Since it's more meaningful to refer to the outmost dimensions of resonators when discussing the achievable miniaturisation level, the outer diameter (or side for square elements) d_{out} instead of the average radius r (as depicted in Fig. 2-2) is used in this section. The metallic fill ratio of a helical resonator is defined as

$$\alpha = \frac{w}{d_{out}} \quad (2.8)$$

which reflects how much area enclosed by the resonator is covered by metal surface and has an upper limit of 0.5.

By incorporating the quasi-static expressions for inductance L (2.3) and capacitance C (2.6) into (2.2), the analytical expression for the frequency of fundamental mode res-

onance of a helical resonator has the following form

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{h [d_{out} + (k_2 - 1) w]}{k_1 \mu_0 \epsilon_0 \epsilon_r g m (d_{out} - w)^3 w}} \quad (2.9)$$

If the outmost dimension d_{out} and substrate thickness h remain fixed, whilst the strip width w varies, it can be shown using equations (2.3), (2.6) and (2.9) that the increase of w results in a monotonic decrease of the equivalent inductance and an similar increase of the capacitance, hence a unique optimum strip width w_{opt} ought to exist which can achieve a minimum resonant frequency for given d_{out} and h .

This optimal value can be obtained by finding the stationary point of resonant frequency f_0 as a function of the strip width w . Since the valid solution is unique, it also equals the point where the derivative of the product of inductance and capacitance to w is zero, which can be deduced as follows

$$\begin{aligned} \frac{\partial (LC)}{\partial w} &= 0 \\ \Rightarrow \frac{\partial}{\partial w} \left[\frac{(d_{out} - w)^3 w}{d_{out} + (k_2 - 1) w} \right] &= 0 \\ \Rightarrow \frac{(d_{out} - w)^2}{[d_{out} + (k_2 - 1) w]^2} [3(k_2 - 1) w^2 + 4d_{out} w - d_{out}^2] &= 0 \\ \Rightarrow \alpha_{opt} = \frac{w_{opt}}{d_{out}} = \frac{-2 + \sqrt{4 + 3(k_2 - 1)}}{3(k_2 - 1)} \end{aligned} \quad (2.10)$$

The derived optimal ratio is only a function of the coefficients in the Wheeler formula, thus is relevant with the geometry of element but independent of the detailed dimensions. The optimal ratios for both square and circular helical resonators are calculated to be approximately $\alpha_{opt} = 0.2$.

The normalised electrical size of element

$$\Lambda = \frac{d_{out}}{\lambda_0} \quad (2.11)$$

is a useful gauge of the miniaturisation level achieved and radiation loss incurred, where λ_0 is the free space wavelength corresponding to the element resonance. For an optimised resonator, the minimum resonant frequency and associated normalised electrical size are given by

$$f_{\min} = \frac{c_0}{2\pi} \sqrt{\frac{h [1 + (k_2 - 1) \alpha_{opt}]}{k_1 \epsilon_r g m (1 - \alpha_{opt})^3 \alpha_{opt} d_{out}^3}} \quad (2.12)$$

$$\Lambda_{\min} = \frac{1}{2\pi} \sqrt{\frac{h [1 + (k_2 - 1) \alpha_{opt}]}{k_1 \epsilon_r g m (1 - \alpha_{opt})^3 \alpha_{opt} d_{out}}} \propto h^{\frac{1}{2}}, d_{out}^{-\frac{1}{2}} \quad (2.13)$$

where $c_0 = 1/\sqrt{\mu_0 \epsilon_0}$ is the speed of EM wave propagation in free space. Therefore higher level of miniaturisation can be achieved by increasing the outmost dimension and reducing the substrate thickness of resonator. In this case, the use of very thin substrates is indicated which helps to support the assumption of quasi-static conditions.

In order to validate the existence of the optimal ratio, helical resonators of both circular and square shapes were fabricated for both $d_{out} = 8$ mm and 12 mm, and a range of strip widths. The thin-film substrate used was the same as in § 2.1. The resonant frequencies obtained from both measurements and simulations using CST MicroStripesTM are plotted in Fig. 2-8 together with the analytical predictions.

From Fig. 2-8 it is clear that there is an optimal ratio α_{opt} at which the resonant frequency is minimum for all sets of results and is relatively independent of the outermost dimension and shape of resonator, although its exact value is around 0.17 for experimental and numerical results which differs from the analytical prediction of 0.2. The resonant frequencies of elements estimated by all three means are again listed in Table 2.5 - 2.8,

Table 2.5: Comparison of the resonant frequencies of circular helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $d_{out} = 8 \text{ mm}$.

$w(\text{mm})$	w^{mea}	$f^{ana}(\text{MHz})$	$f^{ana'}$	f^{sim}	f^{mea}	$\delta f^{ana}(\%)$	$\delta f^{ana'}$	δf^{sim}
1.0	0.975	345.9	331.0	332.7	337.6	-2.5	2.0	1.5
1.2	1.205	335.8	326.5	328.3	329.3	-2.0	0.9	0.3
1.4	1.385	330.8	326.6	327.0	328.6	-0.7	0.6	0.5
1.6	1.590	329.6	330.5	334.3	333.1	1.1	0.8	-0.4
1.8	1.775	331.3	337.7	342.4	338.9	2.2	0.4	-1.0

Table 2.6: Comparison of the resonant frequencies of square helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $d_{out} = 8 \text{ mm}$.

$w(\text{mm})$	$w^{mea}(\text{mm})$	$f^{ana}(\text{MHz})$	$f^{sim}(\text{MHz})$	$f^{mea}(\text{MHz})$	$\delta f^{ana}(\%)$	$\delta f^{sim}(\%)$
1.0	1.000	274.7	268.2	275.5	0.3	2.6
1.2	1.195	266.9	266.0	272.2	1.9	2.3
1.4	1.380	263.1	267.0	271.1	3.0	1.5
1.6	1.595	262.3	270.7	272.5	3.7	0.7
1.8	1.785	263.8	277.7	277.5	4.9	-0.1

Table 2.7: Comparison of the resonant frequencies of circular helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $d_{out} = 12 \text{ mm}$.

$w(\text{mm})$	w^{mea}	$f^{ana}(\text{MHz})$	$f^{ana'}$	f^{sim}	f^{mea}	$\delta f^{ana}(\%)$	$\delta f^{ana'}$	δf^{sim}
1.6	1.600	181.1	174.4	173.4	173.1	-4.6	-0.8	-0.2
1.8	1.790	177.9	173.1	172.5	172.0	-3.4	-0.6	-0.3
2.0	1.990	175.9	172.8	172.3	172.0	-2.3	-0.5	-0.2
2.2	2.180	174.9	173.4	173.3	172.0	-1.7	-0.8	-0.8
2.4	2.385	174.6	174.9	175.0	173.7	-0.5	-0.7	-0.7

Table 2.8: Comparison of the resonant frequencies of square helical resonators from quasi-static circuit model, numerical simulation and measurement. $\epsilon_r = 3.1$, $h = 30 \mu\text{m}$, $d_{out} = 12 \text{ mm}$.

$w(\text{mm})$	$w^{mea}(\text{mm})$	$f^{ana}(\text{MHz})$	$f^{sim}(\text{MHz})$	$f^{mea}(\text{MHz})$	$\delta f^{ana}(\%)$	$\delta f^{sim}(\%)$
1.6	1.595	144.8	138.4	144.0	-0.6	3.9
1.8	1.785	142.3	137.6	142.9	0.4	3.7
2.0	1.995	140.8	137.7	142.6	1.3	3.4
2.2	2.190	140.0	138.8	142.4	1.7	2.5
2.4	2.390	139.9	140.6	144.2	3.0	2.5

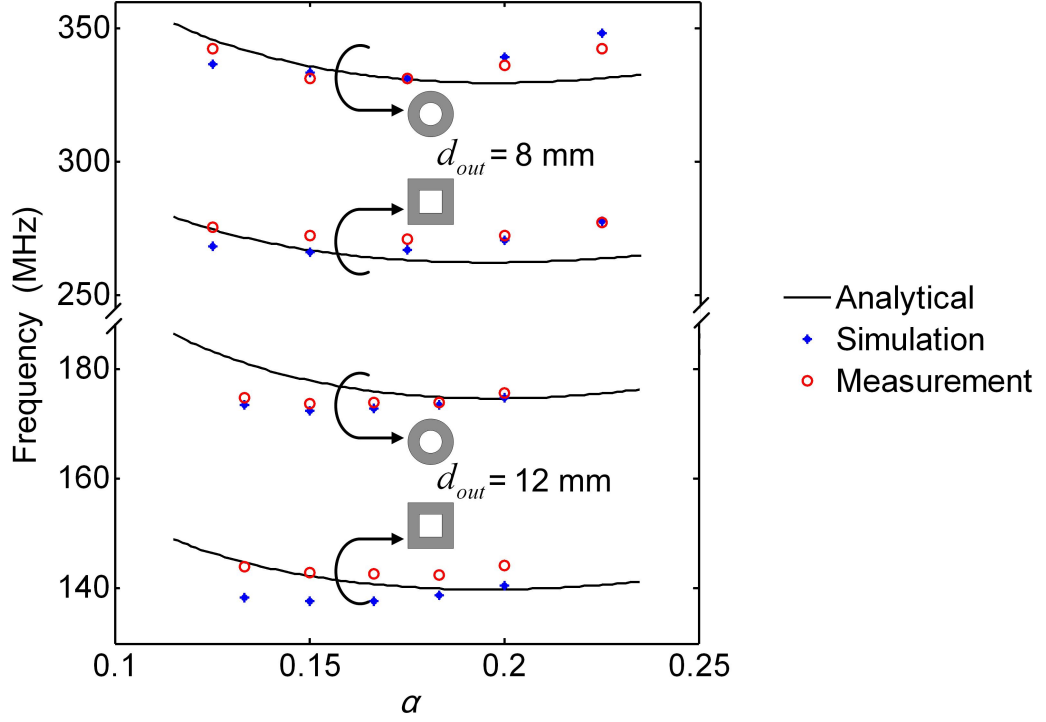


Figure 2-8: Variation of the resonant frequency of helical resonators with metallic fill ratio α for both circular and square shapes with two different outmost dimension $d_{out} = 8$ mm and 12 mm.

respectively, and the relative error of the quasi-static model δf^{ana} is within the range of $\pm 5\%$ (f^{ana} and δf^{ana} in the tables for circular elements denote the results of the modified analytical model, which will be illustrated in § 2.3.3). However, the minimum resonant frequency and the corresponding maximum achievable miniaturisation level are in good agreements with the measurement and simulation.

For identical physical dimensions, the square helical resonators achieve 20% smaller electrical sizes than their circular alternative. The bigger discrepancies between the measured data and simulation results for square helical resonators can be largely attributed to a small shift in fabrication, which results in imperfect alignment of the upper and lower

copper patterns by 0.13 mm. A similar shift is also present in the circular resonators but in this case has negligible impact. The mean error in dimensions of all fabricated helical resonators is -0.012 mm, while the standard deviation is 0.011 mm.

The coincidence of the analytical, numerical and experimental results illustrates that the simple circuit model provides useful insight into the potential limitations for miniaturisation of helical resonators, although further refinements to improve the correspondence are possible. Fabrication errors, in our case a horizontal shift of the metal patterns, may cause significant error for elements with very thin substrates such as the 30 μm layer used in our experiments.

In order to investigate the limits of the achievable miniaturisation level, the variation of the normalised electrical size Λ of an optimised helical resonator with d_{out} , calculated from (2.13), is plotted in Fig. 2-9 for the two topologies with four different substrate thicknesses, together with the numerical simulation data. The agreements between the two results are quite good. It is clear that a normalised electrical size of below 1% can be easily realised using thin-film substrates, which is an order smaller than that of an ordinary planar SRR.

Since the quasi-static circuit models for BC-SRR and helical resonator only differ by a constant coefficient in the global capacitance, a similar behaviour for the BC-SRR with an equal optimal fill ratio should also exist accordingly.

In conclusion, an optimal metallic fill ratio of around 0.2 is demonstrated to exist for both circular and square helical resonators, which can achieve minimum resonant frequency and normalised electrical size for any given outmost dimension of element. The minimum achievable electrical size can be reduced by increasing the radial size and decreasing the substrate thickness, thus thin films with a thickness much smaller than the

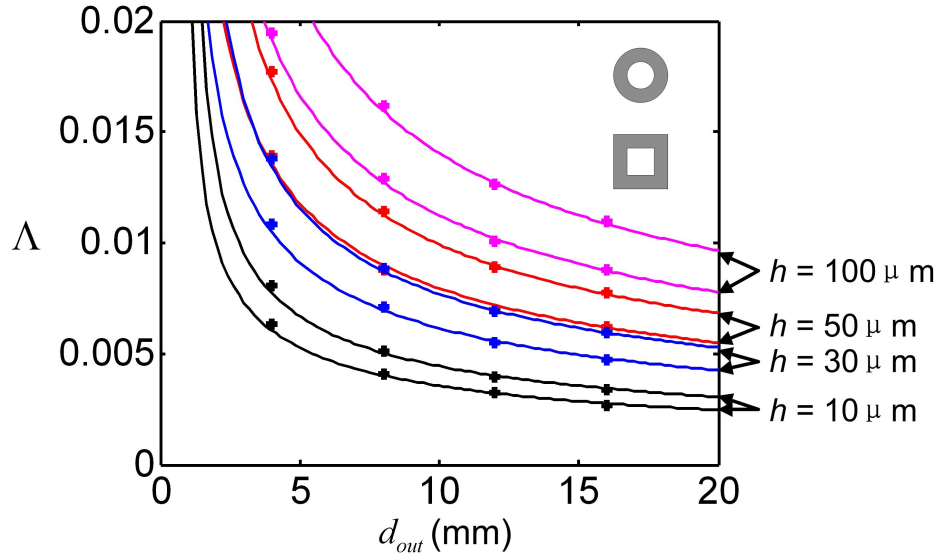


Figure 2-9: Variation of the normalised electrical size Λ of an optimised helical resonator with its outmost dimension d_{out} for the two topologies with four different thicknesses. The lines represent results from analytical model while the dots represent the numerical simulation data. The upper and lower curves correspond to the circular and square helical resonator separately for a given substrate thickness h .

helical strip width should be utilised. Since the size of thin-film helical resonators can easily fall below 1% of the free space wavelength at resonance, a metamaterial composed from them can closely approximate a continuous medium, which is required for implementation of high quality imaging devices and other metamaterial components. The helical resonators are scalable and can be used for frequency up to IR or optics, but the analysis should be adjusted to include the additional high frequency effects (e.g. the kinetic inductance for frequency above 10 THz).

2.3 Modelling the Quality Factor of Helical Resonators

Miniaturised elements with high quality factor Q are required for assembling metamaterials with good homogeneity and low loss, which can better approximate ordinary continuous media. In addition, it has been illustrated in [79] that the resonant effective permeability of an artificial material comprising of resonating particles can be estimated from the Q of individual elements. This discovery greatly facilitates the design process, therefore the element's Q is of key importance in metamaterial design.

An analytical model of the quality factor of individual resonators can provide a clear physical insight into the effect of the various existing loss mechanisms arising from conductor resistance, dielectric absorption and radiation, so that the design of elements with engineerable loss properties can be realised. However, so far, only a few references specifically address the Q of metamaterial elements, predominantly based on the results from numerical simulation rather than analytical approach [76]-[82]. Bilotti et al. developed a comprehensive analytical model that is applicable to various planar magnetic resonators employing edge-coupled self-capacitance [80],[81]. However, radiation loss was not incorporated into the analysis, and there was no experimental verification of the model proposed.

This section focuses on the accurate analytical modelling of quality factor of helical resonators. In § 2.3.1, analytical expressions for Q associated with different loss mechanisms are derived. Their accuracies are verified by comparing with numerical simulation and experimental results in § 2.3.2. The loss incurred from the dielectric absorption appears to dominate the quality factor of helical resonators in this configuration. After scrutinising the discrepancies in the model predictions, the accuracy of quasi-static equivalent RLC circuit model is greatly improved by using geometric modification as

introduced in § 2.3.3, which reflects the effect of actual spatially nonuniform conduction current flow at element resonance.

From the previous sections, it can be known that the circular and square helical resonators have quite similar properties. Therefore only circular elements are investigated in this section, while the obtained conclusions can also be extended to the square counterparts with simple modifications.

2.3.1 Analytical Expressions for Quality Factors of Helical Resonators

As has been described, the Q of resonator elements determines the equivalent permeability function of the developed artificial material, which on the other hand is limited by the loss inherent in the element. Generally speaking, there are three different loss mechanisms associated with a dielectric loaded resonating element, including conductor resistance, dielectric absorption and radiation. The quasi-static equivalent RLC circuit approach is employed again to estimate the quality factor of helical resonators by calculating the energy stored and loss dissipated in the element at resonance, whose validity has been demonstrated as in [83],[84]. The whole equivalent circuit as depicted in Fig. 2-4 will be utilised, with the analytical expressions for various resistances be derived. Therefore this equivalent circuit model incorporates all the loss mechanisms, and the overall Q of such a resonator can then be calculated as [86]

$$\frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r} \quad (2.14)$$

where Q_c , Q_d and Q_r are the quality factors related to the losses induced from the metallic conductor, dielectric substrate and far-field radiation respectively.

Q_c can be regarded as the quality factor of the equivalent inductor, thus has the following definition

$$Q_c = \frac{\omega_0 L}{R_c} \quad (2.15)$$

where ω_0 is the angular frequency at resonance, L is the quasi-static inductance as in (2.3), and R_c is the equivalent series resistance reflecting conductor loss, which can be calculated as follows by neglecting the curvature effects

$$R_c = \frac{\rho_c}{wt'} 2\pi r \quad (2.16)$$

$\rho_c / (wt')$ is the per-unit-length resistance along the current path, where ρ_c is the electrical resistivity of the metal, r and w are the mean radius and strip width of elements, and t' is the effective metallisation thickness which will be explained as follows.

This estimation is based on assuming uniform current distribution among the cross-section of current path, which is a narrow rectangle for the equivalent disk employed. However, the conduction current flow tends to concentrate near the metal surface at high frequency as the EM fields attenuate when penetrating through the conductor, which is well known as the "skin effect". Due to the much smaller conductor thickness t comparing with its width w , only the concentration close to the broad surfaces of metal disk need to be considered. The skin depth δ at frequency ω_0 is given as

$$\delta = \sqrt{\frac{2\rho_c}{\mu_0\omega_0}} \quad (2.17)$$

When the concentration effect is not significant, the current density can still be regarded as constantly distributed across the rectangular cross-section. On the other hand, when the skin depth is smaller than a half of the metallisation thickness t , the current flow

can be approximated to have "sandwich" shaped distribution among the cross-section: uniform flow within a layer of thickness δ below upper and lower broad surfaces of disk, while no current exists beyond these two thin layers. Thus the resistance can be calculated by (2.16) with an effective thickness of metallisation $t' = 2\delta$. In summary, the effective thickness of equivalent conducting disk t' in (2.16) is introduced as

$$t' = \min(t, 2\delta) \quad (2.18)$$

Similarly, Q_d can be considered as the quality factor of the equivalent parallel plate capacitor. The inherent shunt resistance R_d reflecting dissipation loss in the dielectric substrate thus has the following form

$$R_d = \frac{h}{\sigma_d S} \quad (2.19)$$

where

$$\sigma_d = \omega \epsilon_0 \epsilon_r \tan \theta \quad (2.20)$$

is the effective conductivity of dielectric substrate, ϵ_r and $\tan \theta$ denote the relative permittivity and the loss tangent of dielectric respectively. S is the area of metallisation in an element. Therefore Q_d can be deduced as follows

$$\begin{aligned} Q_d &= \omega_0 R_d C \\ &= \omega_0 \frac{h}{\sigma_d S} \frac{\epsilon_0 \epsilon_r S}{h} \\ &= \frac{\omega_0 \epsilon_0 \epsilon_r}{\sigma_d} \\ &= \frac{1}{\tan \theta} \end{aligned} \quad (2.21)$$

which is only relevant with the inherent loss property of substrate. Such a concise result has also been found in the analysis of dielectric loaded metallic cavity resonators [86].

If the physical size of resonator is in a scale comparable with the free space wavelength at resonance (as the case of SRRs), the loss due to far-field radiation cannot be neglected [38]. Generally speaking, the accurate estimation of equivalent radiation resistance would be quite cumbersome due to the complex current distribution on the resonator. However, the radiation resistance of a circular helical resonator can be reasonably approximated by that of a loop antenna [87]

$$R_r = 320\pi^6 \left(\frac{r}{\lambda}\right)^4 \quad (2.22)$$

with a radius equal to the mean radius r of the helical structure, due to the uniform current distribution and relatively small strip width. Q_r has the same form as Q_c

$$Q_r = \frac{\omega_0 L}{R_r} \quad (2.23)$$

Then the effect of far-field radiation loss is incorporated into the quality factor analysis.

2.3.2 Validation of the Model's Accuracy

In order to validate the accuracy of the above quasi-static model, the Q of the fabricated circular helical resonators as in § 2.2 are extracted from data of numerical simulation (denoted by Q^{sim}) and measurement (Q^{mea}), and then compared with the estimation from the analytical model (Q^{ana}) as shown in Fig. 2-10 (dashed lines) and Table 2.9 - 2.10. $Q^{ana'}$ and $\delta Q^{ana'}$ represent the results of the modified model which will be explained

in § 2.3.3. The relative errors are still defined as

$$\delta Q^{ana/sim} = \frac{Q^{mea} - Q^{ana/sim}}{Q^{mea}} \quad (2.24)$$

CST Micro-StripesTM can provide results of the total stored energy and losses incurred from different mechanisms, thus can determine Q_c , Q_d and Q_r separately and hence be used to verify the analytical predictions in detail. The comparison of the three components of Q are exhibited in Fig. 2-11, 2-12 and Table 2.11 - 2.16, respectively. The plot of Q_d is omitted since its theoretical value is a constant independent with element dimensions. The experimental Q are extracted from measured S_{21} transmission spectrum using the nonlinear least-squares fit to the phase of S_{21} versus frequency curve, which is the most accurate and precise method [88]. Those details of simulation and experiment will be further specified in Chapter 3.

In Fig. 2-10 and Table 2.9, 2.10, the overall quality factor monotonically increases with the metallic fill ratio α . Good consistency exists between the results from simulation and measurement, while an overestimation of more than 10% from analytical model can be observed. This discrepancy is attributed to the significant underestimation of conduction loss R_c (almost one-third smaller) deduced from the uniform current distribution assumption, because the concentration of current leads to higher conductor loss and hence lower Q_c as illustrated in Fig. 2-11 and Table 2.11, 2.12. This will be further analysed in the next section.

As can be seen from Table 2.13, 2.14, the simulated Q_d of all helical resonators are quite close to the theoretically predicted value of 167, which dominates the total losses in the current resonator configuration. Therefore thin-film substrate with low loss tangent should be utilised to fabricate miniaturised helical resonators with high quality factors.

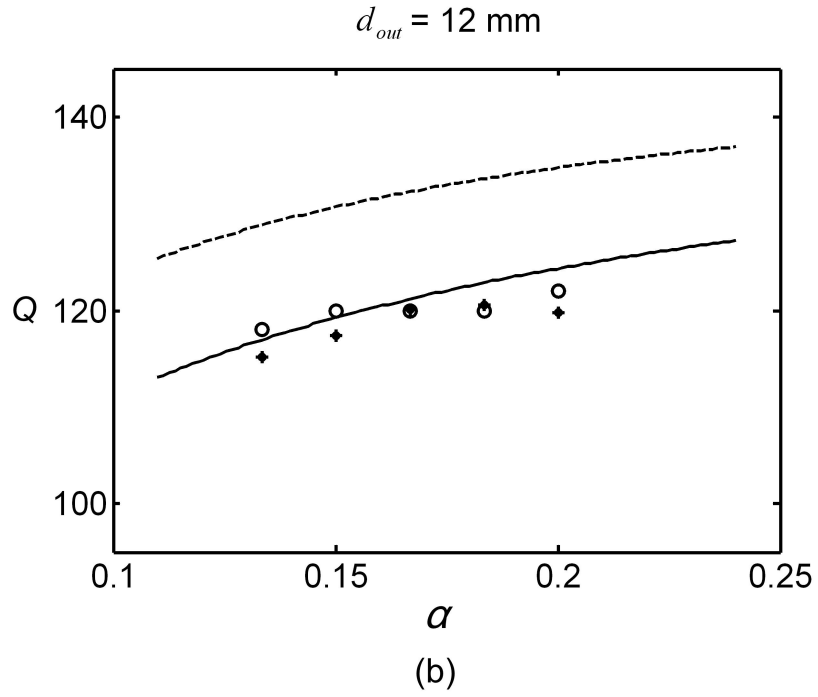
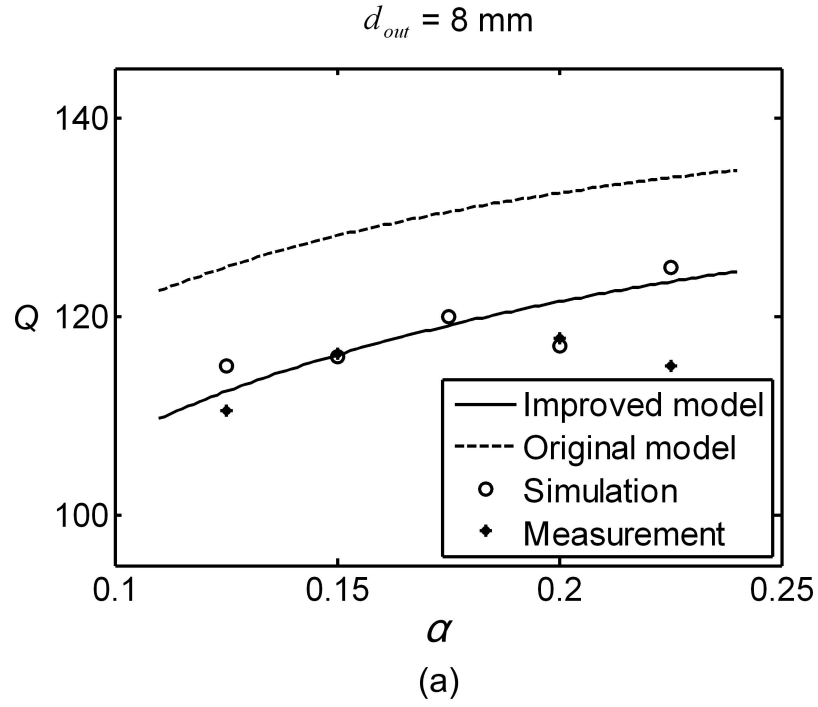


Figure 2-10: Variation of the overall quality factor with metallic fill ratio α for circular helical resonators with two different outer diameters d_{out} (a) 8 mm. (b) 12 mm. The solid lines represent results from quasi-static circuit model with modified element size, while the dashed lines represent results from model with original size.

Table 2.9: Comparison of the overall quality factors of circular helical resonators from quasi-static circuit model with original dimensions, modified dimensions, numerical simulation and measurement. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 8 \text{ mm}$.

$w(\text{mm})$	Q^{ana}	Q^{anal}	Q^{sim}	Q^{mea}	$\delta Q^{ana}(\%)$	$\delta Q^{anal}(\%)$	$\delta Q^{sim}(\%)$
1.0	125	112	115	111	-12.6	-0.9	-3.6
1.2	128	116	116	116	-10.3	0.0	0.0
1.4	131	119	120	113	-15.9	-5.3	-6.2
1.6	132	122	117	118	-11.9	-3.4	0.8
1.8	134	124	125	115	-16.5	-7.8	-8.7

Table 2.10: Comparison of the overall quality factors of circular helical resonators from quasi-static circuit model with original dimensions, modified dimensions, numerical simulation and measurement. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 12 \text{ mm}$.

$w(\text{mm})$	Q^{ana}	Q^{anal}	Q^{sim}	Q^{mea}	$\delta Q^{ana}(\%)$	$\delta Q^{anal}(\%)$	$\delta Q^{sim}(\%)$
1.6	129	117	118	115	-12.2	-1.7	-2.6
1.8	131	120	120	117	-12.0	-2.6	-2.6
2.0	132	121	120	120	-10.0	-0.8	0.0
2.2	134	123	120	120	-11.7	-2.5	0.0
2.4	135	124	122	120	-12.5	-3.3	-1.7

Since the Q_c for the fabricated resonators with optimal metallic fill ratio are around 400, then the conductor loss will dominate if $\tan\theta < 1/400 = 0.0025$, which can be regarded as a limit when looking for better dielectric materials.

From Fig. 2-12 and Table 2.15, 2.16, the analytical predictions on Q_r deduced from loop model also have a monotonic relation with the metallic fill ratio, which are roughly 10% lower than those estimated from simulation. The agreements are reasonably good given the coarse approximation adopted. These values of Q_r are several orders higher than the other two components due to the miniaturised physical sizes of helical resonators, thus can be safely neglected in the current element configuration. However, when the resonator size is comparable with the wavelength at resonance, such as the case of SRRs, radiation loss will become significant and must be incorporated into the modelling process [38].

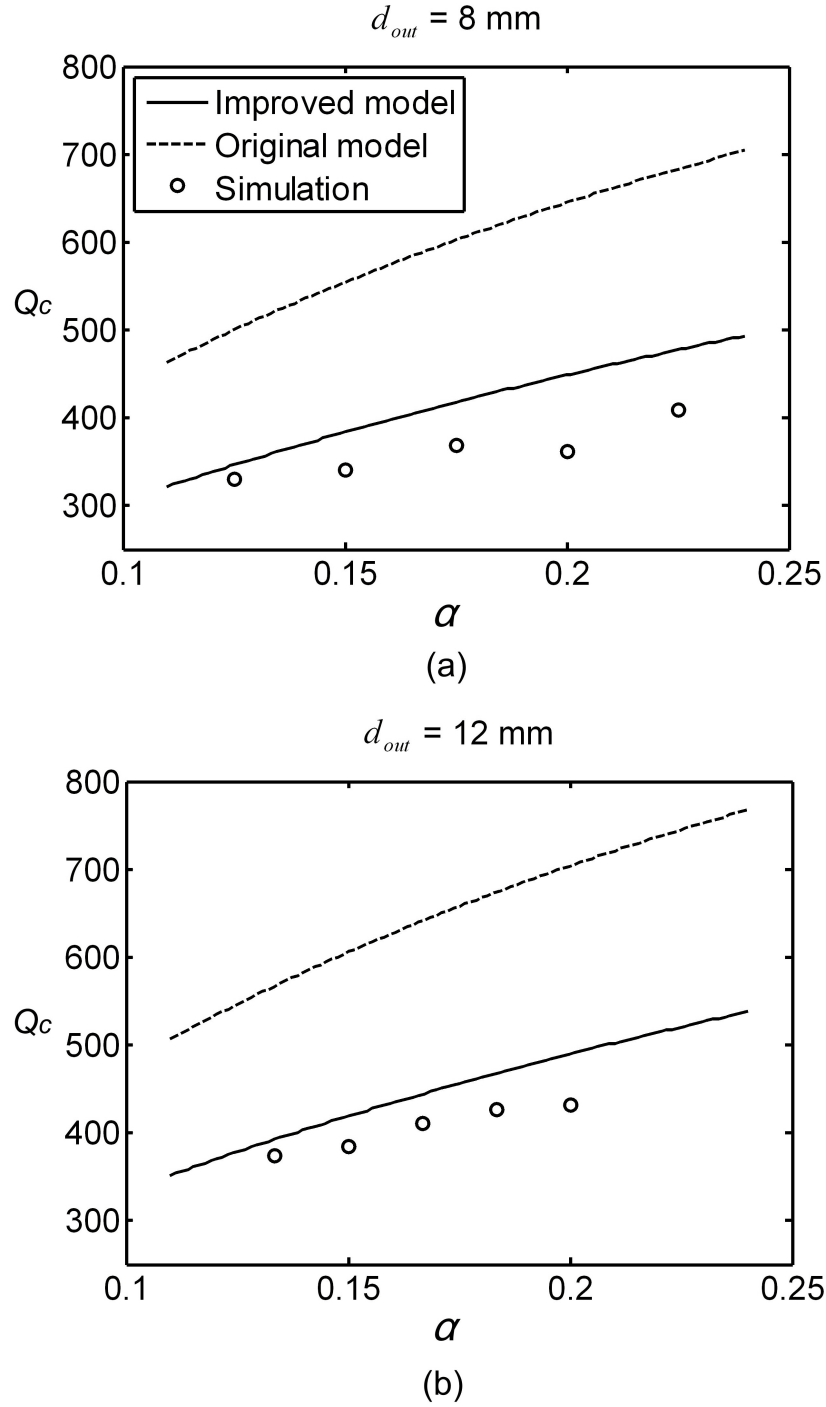


Figure 2-11: Variation of quality factor reflecting conduction loss with metallic fill ratio α for circular helical resonators with two different outer diameters d_{out} (a) 8 mm. (b) 12 mm. The solid lines represent results from quasi-static circuit model with modified element size, while the dashed lines represent results from model with original size.

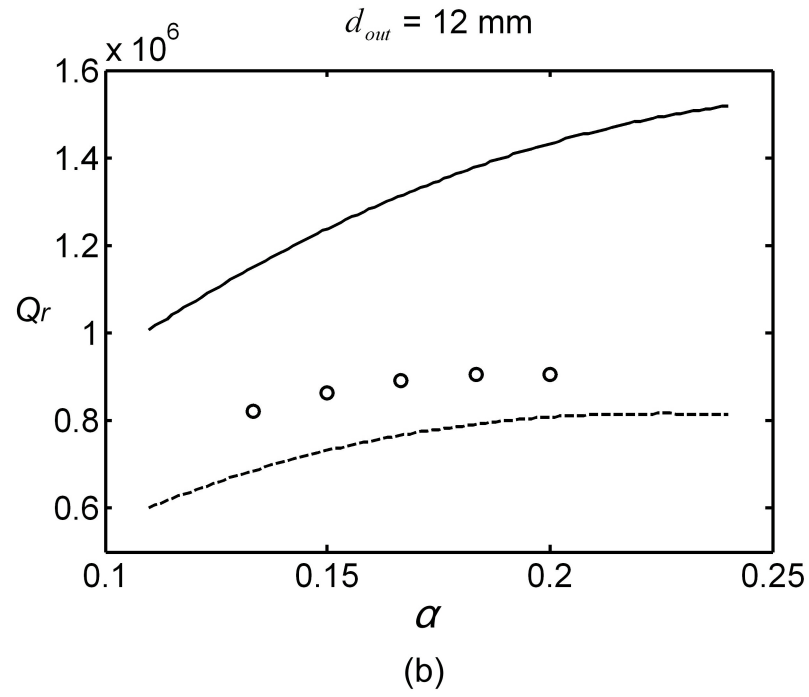
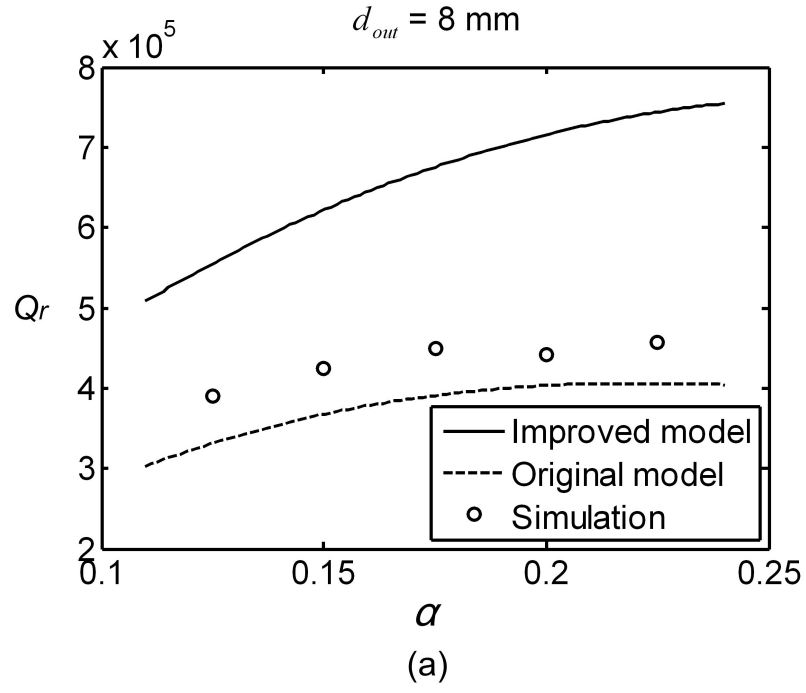


Figure 2-12: Variation of quality factor reflecting radiation loss with metallic fill ratio α for circular helical resonators with two different outer diameters d_{out} (a) 8 mm. (b) 12 mm. The solid lines represent results from quasi-static circuit model with modified element size, while the dashed lines represent results from model with original size.

Table 2.11: Comparison of the quality factor reflecting conduction loss of circular helical resonators from quasi-static circuit model with original dimensions, modified dimensions, and numerical simulation. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 8 \text{ mm}$.

$w(\text{mm})$	Q_c^{ana}	$Q_c^{ana'}$	Q_c^{sim}	$\frac{Q_c^{sim}-Q_c^{ana}}{Q_c^{sim}}(\%)$	$\frac{Q_c^{sim}-Q_c^{ana'}}{Q_c^{sim}}(\%)$
1.0	500	346	329	-52.0	-5.2
1.2	554	383	340	-62.9	-12.6
1.4	602	417	369	-63.1	-13.0
1.6	645	449	361	-78.7	-24.4
1.8	683	477	409	-67.0	-16.6

Table 2.12: Comparison of the quality factor reflecting conduction loss of circular helical resonators from quasi-static circuit model with original dimensions, modified dimensions, and numerical simulation. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 12 \text{ mm}$.

$w(\text{mm})$	Q_c^{ana}	$Q_c^{ana'}$	Q_c^{sim}	$\frac{Q_c^{sim}-Q_c^{ana}}{Q_c^{sim}}(\%)$	$\frac{Q_c^{sim}-Q_c^{ana'}}{Q_c^{sim}}(\%)$
1.6	566	392	373	-51.7	-5.1
1.8	605	419	384	-57.6	-9.1
2.0	641	444	411	-56.0	-8.0
2.2	673	467	426	-58.0	-9.6
2.4	704	489	432	-63.0	-13.2

Table 2.13: Comparison of the quality factor reflecting dielectric absorption of circular helical resonators from quasi-static circuit model and numerical simulation. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 8 \text{ mm}$.

$w(\text{mm})$	Q_d^{ana}	Q_d^{sim}	$\frac{Q_d^{sim}-Q_d^{ana}}{Q_d^{sim}}(\%)$
1.0	167	177	5.6
1.2	167	176	5.1
1.4	167	178	6.2
1.6	167	173	3.5
1.8	167	180	7.2

Table 2.14: Comparison of the quality factor reflecting dielectric absorption of circular helical resonators from quasi-static circuit model and numerical simulation. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 12 \text{ mm}$.

$w(\text{mm})$	Q_d^{ana}	Q_d^{sim}	$\frac{Q_d^{sim}-Q_d^{ana}}{Q_d^{sim}}(\%)$
1.6	167	173	3.5
1.8	167	175	4.6
2.0	167	169	1.2
2.2	167	167	0.0
2.4	167	170	1.8

Table 2.15: Comparison of the quality factor reflecting far-field radiation of circular helical resonators from quasi-static circuit model with original dimensions, modified dimensions, and numerical simulation. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 8 \text{ mm}$.

$w(\text{mm})$	$Q_r^{ana}(\times 10^5)$	$Q_r^{ana'}(\times 10^5)$	$Q_r^{sim}(\times 10^5)$	$\frac{Q_r^{sim}-Q_r^{ana}}{Q_r^{sim}}(\%)$	$\frac{Q_r^{sim}-Q_r^{ana'}}{Q_r^{sim}}(\%)$
1.0	3.3	5.6	3.9	15.4	-43.6
1.2	3.7	6.2	4.3	14.0	-44.2
1.4	3.9	6.8	4.5	13.3	-51.1
1.6	4.0	7.2	4.4	9.1	-63.6
1.8	4.1	7.4	4.6	10.9	-60.9

Table 2.16: Comparison of the quality factor reflecting far-field radiation of circular helical resonators from quasi-static circuit model with original dimensions, modified dimensions, and numerical simulation. $\epsilon_r = 3.1$, $\tan\theta = 0.006$, $h = 30 \mu\text{m}$, $d_{out} = 12 \text{ mm}$.

$w(\text{mm})$	$Q_r^{ana}(\times 10^5)$	$Q_r^{ana'}(\times 10^5)$	$Q_r^{sim}(\times 10^5)$	$\frac{Q_r^{sim}-Q_r^{ana}}{Q_r^{sim}}(\%)$	$\frac{Q_r^{sim}-Q_r^{ana'}}{Q_r^{sim}}(\%)$
1.6	6.8	11.5	8.2	17.1	-40.2
1.8	7.3	12.4	8.6	15.1	-44.2
2.0	7.7	13.1	8.9	13.5	-47.2
2.2	7.9	13.8	9.0	12.2	-53.3
2.4	8.1	14.3	9.1	11.0	-57.1

2.3.3 Improved RLC Circuit Model for Helical Resonators

It has been demonstrated in previous sections that the established quasi-static circuit model can provide reasonable estimations on the resonant frequency and quality factors reflecting dielectric absorption and radiation loss of helical resonators. However, the deviation in resonant frequencies tends to increase when the element strip width w is either small or large comparing with its diameter d_{out} , and the conduction loss is greatly underestimated. The former can be solely attributed to the inaccuracy of the analytical expression in estimating inductance L (2.3) since the parallel plate model for capacitance C (2.6) is believed to be precise for the adopted thin-film structure, while the latter is due to the concentration of conduction current on the element at resonance.

The actual conduction current distribution on an optimised helical resonator ($\alpha =$

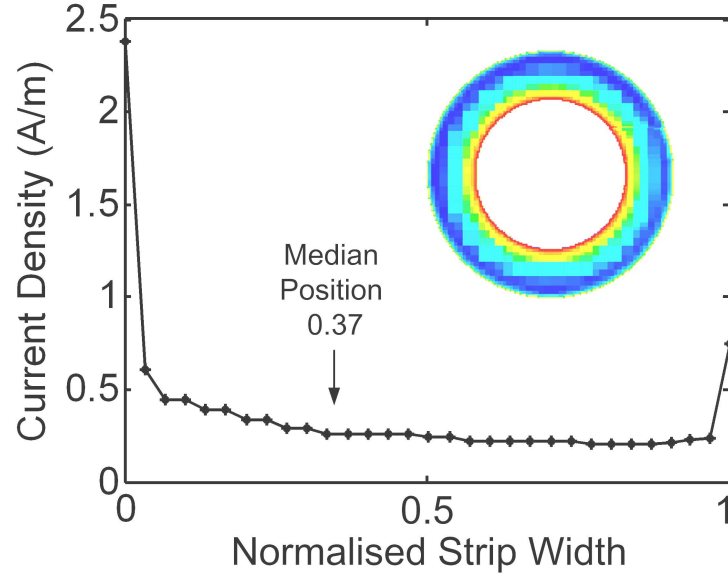


Figure 2-13: Radial distribution of conduction current density from the inner to the outer edge for a sample optimised circular helical resonator at resonance. The dots represent simulation results using CST Micro-StripesTM, and the line is for visual guidance. The inset is a colour plot of surface current distribution with red/blue denote higher/lower current density respectively.

$w/d_{out} = 0.2$) at resonance simulated by CST Micro-StripesTM is plotted in Fig. 2-13. It's clear that the current flow is concentrated towards the inner edge of the ring, and peak density appears near both edges. The median position of the current density calculated using the adopted discretization across the strip width is $0.37w$ from the inner edge. This distribution is qualitatively consistent with the results reported in [85], which leads to a reduction in the inductance L and an increase in the resistance R_c of helical resonators in the quasi-static circuit model, comparing with the corresponding ones estimated from uniform current distribution assumption.

From analysis in terms of basic physics, such a current distribution can be qualitatively understood. If the equivalent thin planar conductive disk employed for calculating inductance and conduction loss is hypothetically divided into a series of circular fila-

ments with different radius, these can be regarded as parallel connected inductors which are excited by the same electromotive force. Since the resistance and inductance of a single filament ring is roughly proportional to its radius, the current flowing on ring with smaller radius is larger, therefore its distribution tends to concentrate near the inner edge. Such an effect would be more pronounced at helical element with larger metallic fill ratio. Although this coarse analysis neglects the mutual interaction among thin rings, it can still be used as an intuitive explanation. In addition, the concentration of current is further enhanced by the skin effect at the inner edge.

Theoretically, the quasi-static inductance and resistance of a single conductive disk can be accurately calculated if the current density distribution across its width is known. However, it has been illustrated that the estimation of such actual distributions would be quite cumbersome, if not impossible, by Grover [89] who has engaged in the calculation of static self and mutual inductances of various structures. Most of his developed formulas and tables have been adopted by the National Bureau of Standards in the United States because of their very good accuracies. In addition, he has pointed out that the concentration of current near the inner edge of ring at high frequency effectively reduces the area of cross-section of current path, which moderately changes the value of inductance L but greatly increases the resistance R_c . These well echo the conclusions observed from the analysis in previous subsection.

In order to accurately estimate the impact of nonuniform current distribution at resonance on quasi-static circuit parameters of helical resonators, firstly another analytical expression for self inductance in [77] is employed, which again considers a conductive disk as a one-turn spiral,

$$L = \mu_0 r \left[\ln \left(4.92 \frac{r}{w} \right) + 0.2 \left(\frac{w}{2r} \right)^2 \right] \quad (2.25)$$

This expression has much simpler form than the accurate formulas for circular coils with rectangular cross-sections as given in [89],[90], but very good coincidence has been achieved. The relative errors are within 2% for all the element sizes that have been discussed so far, which verifies its accuracy. In comparison, the expression (2.3) gives smaller values of inductance L when the strip width w is small comparing with outer diameter d_{out} . It has to be emphasised that the accurate formulas in references were all derived in the static regime, i.e., the current distributions are again assumed constant.

Then, being inspired by Grover's insight, the author proposes an assumption to make modifications to the geometric parameters of the equivalent planar disk as follows when calculating inductance L and resistance R_c

$$w' = \frac{w}{\beta_1} \quad (2.26)$$

$$r' = r - \frac{w}{2} + \frac{w'}{\beta_2} \quad (2.27)$$

where the parameters with and without prime denote those of the equivalent disk and actual element, respectively. The starting point for the modification is that the equivalent disk, still with uniform current distribution, should have a smaller radius and width so as to approximate the effect of current concentration near the inner edge. In addition, the actual inner edge, where the maximum of current circulates around, should be roughly retained. By adopting these adaptations in element dimensions, the effects of current concentration are aimed to be fully incorporated, so that both inductance L and conduction resistance R_c should be manipulated to have better correspondence with the actual quasi-static values simultaneously. The calculations of other circuit parameters remain unchanged.

The coefficients $\beta_{1,2}$ can be extracted, for example, by fitting the analytically cal-

culated resonant frequencies to the measured results for the fabricated circular helical resonators with two different outer diameters as in Table 2.5 and 2.7. In this way, the equivalent size modification are obtained via fitting the results of inductance L , which in turn determines the correct level of conduction loss. The obtained coefficients are

$$\beta_1 = 1.66, \beta_2 = 2.57 \quad (2.28)$$

The above process is in a way similar to Wheeler's work of producing accurate empirical formulas for coil inductances [91],[92]. The obtained coefficients are applicable for accurately estimating the inductance and resistance of metal disks with similar sizes, while the range of validity is left to be examined. Strictly speaking, a lookup table of those two coefficients should be produced for conducting metal disks with a series of dimensions, in order to provide more accurate and reliable data for actual element design. The analytically estimated resonant frequencies using the adapted and the original element size are compared in Fig. 2-14. Those obtained from full-wave numerical simulation and measurement are also provided as reference. The abscissa is the metallic fill ratio α of a helical resonator. The model with geometric modification has been optimised to have more accurate correspondence to the simulation and experimental results than the origin model. The relative errors between analytical model and measured results are within 1% for almost all resonators, as listed in Table 2.5 and 2.7.

In § 2.2, it was pointed out that maximum miniaturisation of helical resonators could be achieved at an optimal metallic fill ratio of around 0.2 independent of diameter. However, it can be seen from Fig. 2-14 that the optimal ratio deduced from the analytical model with geometric correction is around 0.16, which is also of better correspondence with the experimental results. The reduction in the value of optimal ratio is because the

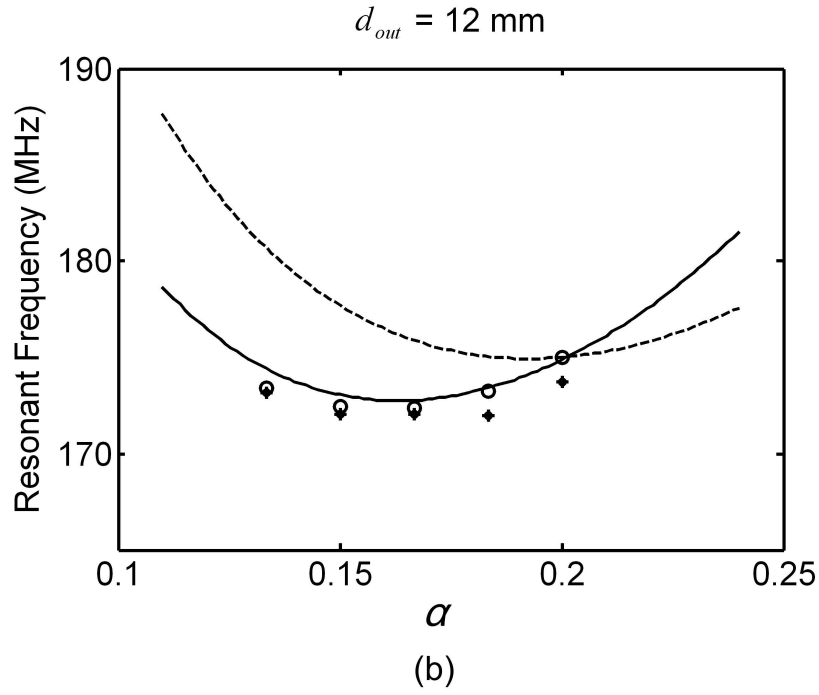
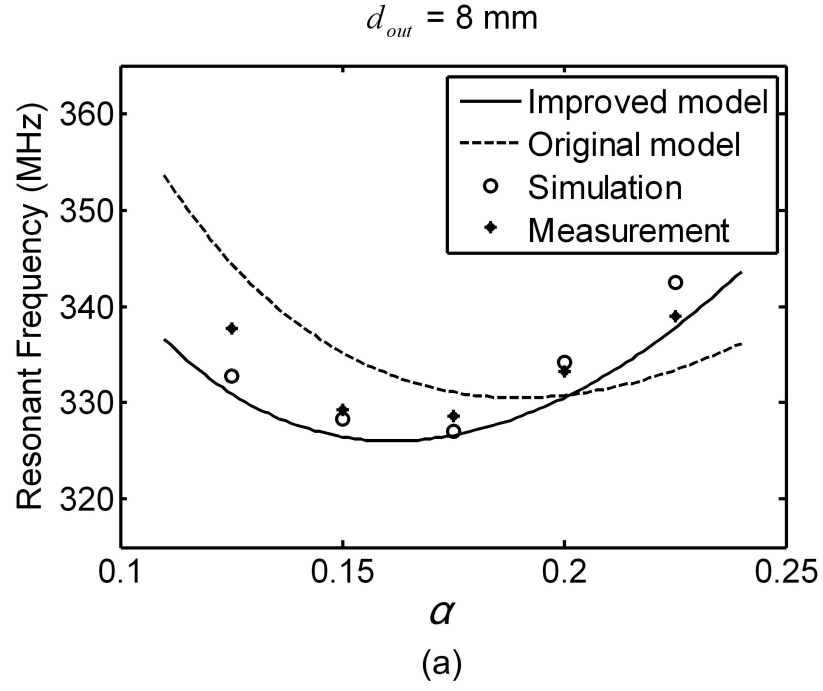


Figure 2-14: Comparison of calculation results of resonant frequency for a series of circular helical resonators with two different outer diameters d_{out} (a) 8 mm and (b) 12 mm and a range of strip widths w . Substrate specification: $h = 30 \text{ } \mu\text{m}$, $\epsilon_r = 3.1$, $\tan \theta = 0.006$. The solid lines represent results from quasi-static circuit model with modified element size, while the dashed lines represent results from model with original size.

quasi-static inductance L estimated by the revised model decreases more rapidly than the original model when the strip width w increases while the outer diameter d_{out} remains fixed. In addition, the maximum miniaturisation level realised at the optimal ratio is also greater than the one using the original model, which further illustrates that the helical resonator is an excellent candidate for developing artificial materials with better homogeneity.

Next the impact of geometry modification on the calculated quality factors will be explored. As shown in Fig. 2-10, 2-11 and Table 2.9 - 2.12, if the element dimensions are modified by the factors β (2.28) retrieved from the correspondence of inductance, the analytically calculated Q and Q_c are in quite good agreements with both simulation and experimental results, which demonstrate precise estimations of the conduction resistance R_c . Good coincidences also exist for other fabricated elements, which are not incorporated when fitting the correction factors β . Therefore by employing the above geometric modification method, the quasi-static circuit model with improved parameters can give very accurate results on resonant frequencies and quality factors of helical resonators than the model with original element size, which paves an easy way of circuit parameter extraction at high frequency involving complex current distributions.

It should be pointed out from Fig. 2-12 and Table 2.15, 2.16 that such adaptations in element dimensions should not be applied when calculating the radiation loss, which leads to greater discrepancies in the estimation of Q_r .

2.4 Effect of Miniaturisation on Quality Factors

It is well known that simply scaling down the size of resonators will lead to degradation of their quality factors due to the difficulty in confining the resonant EM fields within subwavelength volumes. Consequently, controlling losses may be a challenge in magnetic metamaterials when miniaturised resonators are employed in order to better approximate a continuous medium [79]. In this section, the influence of miniaturisation level on the Q of helical resonators will be investigated using the developed analytical circuit model.

In order to give a fair comparison, several circular helical resonator elements with different outer diameters d_{out} but the same resonant frequency at $f_0 = 300$ MHz are designed. The corresponding free space wavelength at resonance is $\lambda = 1$ m. The substrate has the same parameters as the thin-film polyimide used in previous sections ($\epsilon_r = 3.1$, $\tan \theta = 0.006$, $t = 18 \mu\text{m}$), except that the substrate thickness h will be estimated using the analytical model to maintain the resonant frequency. All the elements have optimal metallic fill ratio of $\alpha_{opt} = 0.16$, when the resonant frequency is minimum for a given set of d_{out} and h . After obtaining the detailed dimensions of each element, the corresponding quality factors are calculated using the model with geometric modification as developed in § 2.3.3.

Fig. 2-15 shows how the Q varies with the resonator's normalised electrical size Λ , accompanied by simulation results using CST Micro-StripesTM. Fig. 2-15(a) depicts the calculated behaviour of the two dominant components of the quality factor (Q_c and Q_d) as a function of Λ . Q_d are predicted to be independent with the element scale, and the computed points from simulation confirm this. Q_c on the other hand have a linear dependence on Λ , with possible high value for relatively large devices but dropping below the dielectric value for very small resonators. Between these limits there is a crossover

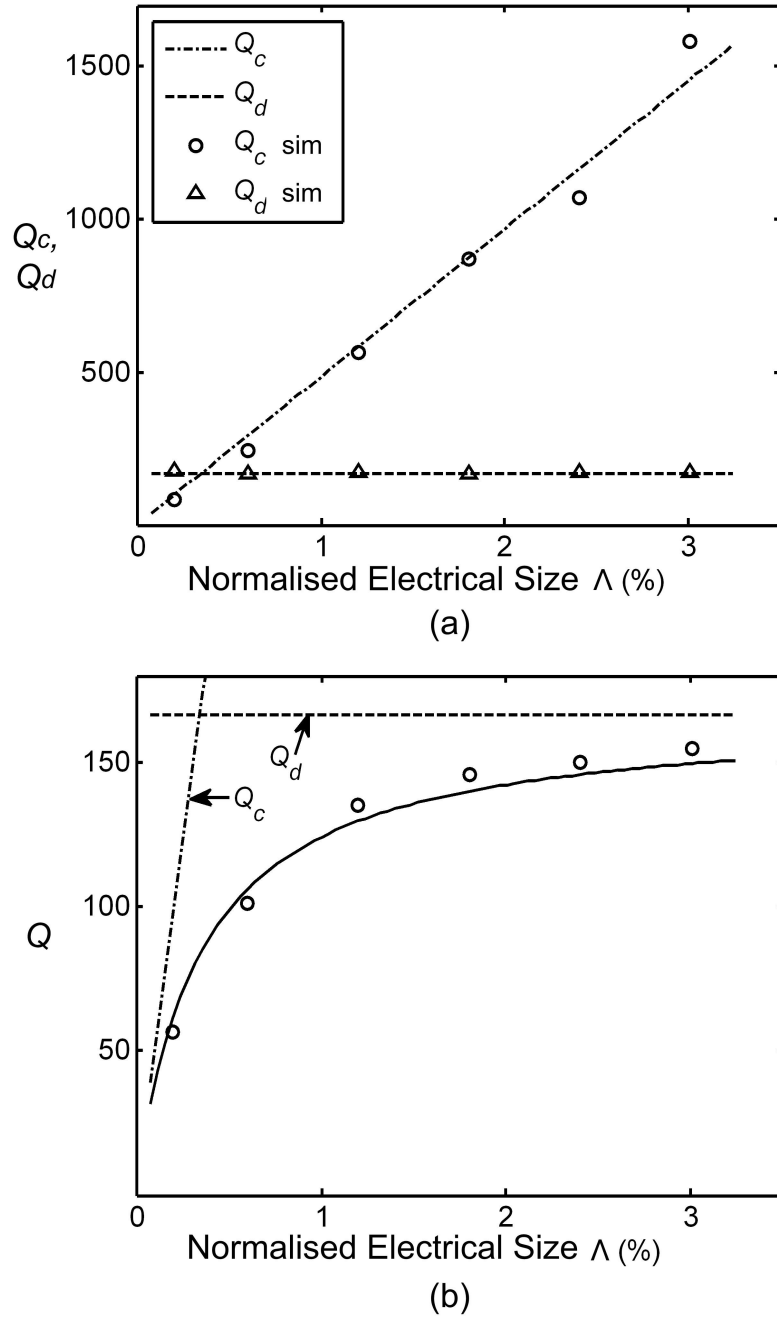


Figure 2-15: Dependence of (a) Q_c and Q_d ; (b) overall quality factor Q on the normalised electrical size Λ of optimised circular helical resonators with fixed resonant frequency at 300 MHz. The lines represent results from quasi-static circuit model and the dots and triangles represent CST Micro-StripesTM simulation results.

from dielectric dominated losses to conduction dominated behaviour. In addition, the contribution from radiation (Q_r) is a very large value and cannot be plotted on the same scale effectively.

The overall Q is shown in Fig. 2-15(b) which is asymptotic at larger values of Λ to the constant component Q_d . Thus, via the losses inherent in the dielectric, the overall Q of a resonator has a scale independent upper maximum limit. This upper limit is a crucial parameter for determining the maximum positive/negative values of the resonant effective medium permeability of assembled magnetic material that can be achieved [79]. On the other hand, for greater scaling when Λ is much smaller Q_c is dominant. At the crossover point the effective Q is a half that of the Q_d limit. Below this point, Q very rapidly decays which shows that attempting to develop high Q resonator below this scaling is effectively impossible due to the limits imposed by the conduction losses. Unless cryogenic cooling and superconducting materials (e.g. high temperature YBCO) are used, the current fabrication using copper is quite close to the best possible limit, although further improvement may be available by using silver.

It will then be of interest to investigate the crossover point of normalised electrical size Λ_0 when the Q_c and Q_d are equal to each other for optimised helical resonators with fixed resonant frequency. After some algebraic deduction the following equation can be obtained

$$\Lambda_0 = \frac{\beta_1 \rho_c}{c_0 \mu_0 \alpha_{opt} t' \tan \theta [\ln(4.92\gamma) + 0.05\gamma^{-2}]} \quad (2.29)$$

where $\beta_{1,2}$ are the geometric correction factors, ρ_c is the electrical resistivity of the metal, c_0 is the speed of EM wave in free space, $\alpha_{opt} = 0.16$ is the optimal ratio, t' is the effective thickness of metal strip as in (2.18) and $\tan \theta$ is the loss tangent of substrate. γ is the

ratio of modified mean radius and width of the element at optimal ratio

$$\gamma = \frac{r'}{w'} = \frac{\frac{1}{2} - \alpha_{opt} + \frac{\alpha_{opt}}{\beta_1\beta_2}}{\frac{\alpha_{opt}}{\beta_1}} \quad (2.30)$$

which is a constant. It can then be inferred from (2.29) that Λ_0 is inversely proportional to $\tan \theta$ and independent of other parameters of the dielectric substrate, provided that the resonant frequency remains unchanged. For example, for the substrate used here with $\tan \theta = 0.006$, $\Lambda_0 = 0.35\%$. However if a better substrate is employed, such as Rogers 5880 with $\tan \theta = 0.0009$, then Λ_0 will increase to 2.33% . Therefore by using substrates with smaller loss tangent, resonators with larger Q at large sizes can be realised. On the other hand, Λ_0 increases with the use of better dielectrics, making the Q of the fabricated resonating elements more sensitive to the miniaturisation.

2.5 Summary and Conclusions

This chapter develops a quasi-static equivalent RLC circuit model for analysing the resonance properties of thin-film helical resonators, which is a type of magnetic metamaterial elements with miniaturised physical dimensions comparing with the wavelength of interest. The analytical expressions for circuit parameters have been derived, including resistance, inductance and capacitance, so that the resonant frequency and quality factor of helical element have explicit forms.

The developed analytical model provides a convenient while accurate way to investigate the impact of various geometric parameters on resonance properties of elements. By utilising thin-film substrate with a thickness much smaller than the metallic strip

width, helical resonators with normalised electrical size of only a several percent have been fabricated. A metamaterial composed from them can closely approximate a continuous medium. In addition, an optimal metallic fill ratio of around 0.16 is demonstrated to exist for both circular and square helical elements, which can achieve minimum resonant frequency and normalised electrical size for any given outmost dimension of element.

The nonuniform conduction current distribution at resonance effectively reduces the cross-section of current path, which introduces a moderate change in the equivalent inductance but huge impact on the conduction resistance and associated quality factors of resonators, especially for those whose widths are comparable with the radius. The difficulty in accurate estimation of current density distribution has been circumvented by simple modifications on the element dimensions, and precise results for both resonant frequency and quality factor have been obtained with good correspondence with both numerical simulation and measurement.

The three loss mechanisms present in helical resonators have been analysed. For our fabricated elements, the dielectric loss is dominant while the radiation loss can be safely neglected. Based on electrical size, very small resonators have a low Q dominated by conduction losses, medium sized resonators have Q that have a constant upper limit determined by the loss tangent of substrate, while radiation loss will influence the Q of large resonators with sizes comparable with the wavelength at resonance, such as SRRs.

The impact of miniaturisation on element's Q is significant, and a linear dependence of Q_c on the normalised electrical size has been discovered for resonators with optimal ratio and fixed resonant frequency. Consequently, the overall Q of a resonator will remain relatively constant until its electrical size is reduced below a certain critical point, which is inversely proportional to the loss tangent of substrate. After this point, Q degrades

rapidly following the limit imposed by conduction losses.

The results in this chapter illustrate that the helical resonators can be made at extremely small sizes with usefully high quality factors, and therefore can be employed to construct metamaterials with better homogeneity and lower loss. In addition, the fabrication of helical elements only requires normal lithography technique as for making ordinary printed circuit boards, which enables mass production. Furthermore, the symmetrical layout of helical resonator element offers great facilitation in actual fabrication, as only one lithography mask is actually needed for patterns on both sides of substrate.

Chapter 3

Extraction of Resonance Parameters of Helical Resonators from the Simulation and Measurement

In Chapter 2, the resonance parameters (resonant frequency and quality factor) of the helical resonators deduced from the analytical RLC circuit model have been compared with the results from numerical simulation using CST Micro-StripesTM and experimental measurement. In this chapter, the details of retrieving these parameters from the recorded responses of simulation and measurement will be illustrated. Especially, measurement of the complex transmission coefficient S_{21} as a function of frequency has been conducted using two small non-resonant loop dipoles as the magnetic probes weakly coupled to the resonator under test, and the parameters are extracted using the nonlinear least-squares fit to the phase of S_{21} data versus frequency. This method has been demonstrated to have the best performance in terms of both precision and robustness in a reference, and will be

re-examined for accuracy when being applied to the measured data of helical resonators using dipole probes. In addition, the impact of the coupling strength of probes to the resonator under test on the accuracy of parameter extraction will be investigated.

3.1 Simulation Results of Helical Resonators using CST Micro-StripesTM

The commercial software CST Micro-StripesTM is used throughout this thesis to simulate the responses of helical resonator elements and arrays to the external electromagnetic (EM) field excitations, which employs the time-domain three dimensional transmission-line method [78]. The simulated model is spatially discretized into a large amount of small cuboids, so that the electric and magnetic fields within each cuboid, which are linearly represented by the voltage and current along the equivalent transmission line, are assumed constant. A pulse excitation in the time-domain is fed into the system when the simulation starts, and the transient responses of EM fields at certain prescribed positions are recorded as the simulation proceeds, which finally terminates after sufficient attenuation. The associated frequency responses can then be calculated as the Fourier transform of the corresponding truncated time-domain ones, thus broadband frequency response can be obtained after a single run. In addition, the distribution of conduction current flow, as well as the integrals of energy and loss within a finite region around the model at specified frequencies can also be calculated, which can thus be used to estimate the quality factors.

The layout for simulating a circular helical resonator is depicted in Fig. 3-1. The resonator element is spatially discretized in all three dimensions, and the lengths of mesh

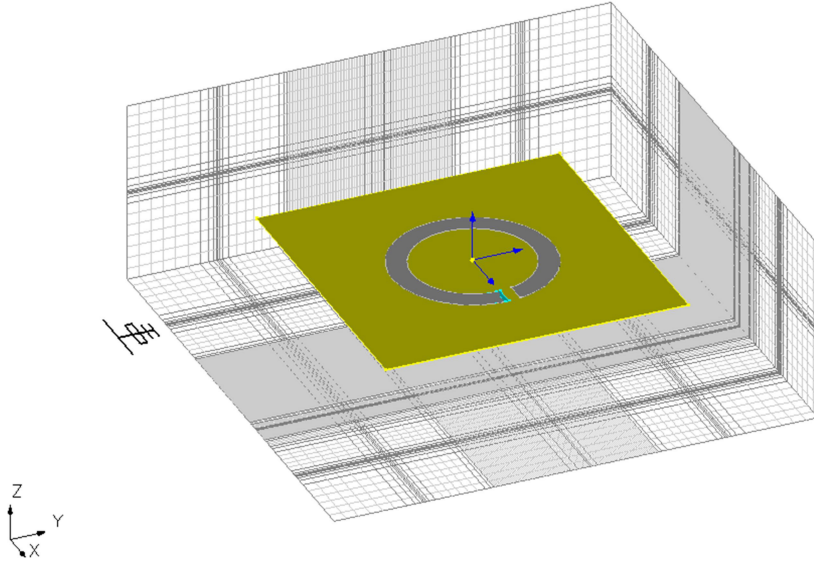


Figure 3-1: The diagram of simulating a circular helical resonator using CST MicroStripesTM. The element is placed within $x - y$ plane, and the plane wave excitation indicated by the antenna propagates along $\hat{y}$ axis, with the electric and magnetic fields along $\hat{x}$ and $\hat{z}$ axes, respectively.

grid along $\hat{z}$ axis are comparatively small to incorporate the small thickness of the metallic pattern and thin-film substrate. The element is being excited by a pulsed plane wave with the electric field pointing in $\hat{x}$ direction as shown by the antenna outside the mesh, and the magnetic field is along the element's axis $\hat{z}$ which induces resonance. A finite cuboid "absorbing boundary" is placed around the element to minimise the reflections at the boundary. It should be pointed out that other boundary types are also available. For example, the "wrap around boundary" will be employed in Chapter 4 to simulate the EM properties of periodic structures comprising of helical resonators.

The transient responses at the centre of the resonator element are recorded, and the corresponding frequency responses can be obtained as illustrated above. Sample time- and frequency-domain responses of the amplitude of magnetic field are depicted in Fig. 3-2. As can be seen, the pulse excitation of the resonator element induces field oscillation

with damping amplitude. The damping is not significant, which demonstrates the low loss and high quality factor attributes of the helical resonator. The corresponding frequency response exhibits a maximum at certain frequency, while the ripple is caused by the truncation in the time-domain.

Strictly speaking, the simulation should proceed until the oscillation in the time domain has sufficiently decayed. However, it has been demonstrated that the resonant frequency can be determined by Fourier transforming just the initial part of the time-domain response, as the further advancement of simulation will mainly affect the responses away from the resonance [93]. Actually, when the same simulation as in Fig. 3-2 is repeated with 10 times the duration time, the ripples exhibited in the frequency response are greatly suppressed, but the value of the resonant frequency remains almost unchanged.

CST Micro-StripesTM can get both the stored energy and power loss in a cuboid region enclosing the model under simulation at given frequency points by calculating the distribution of EM fields and surface currents, respectively. Both can be determined for each separate material body, which enables estimating partial quality factors reflecting different loss mechanisms according to the following equation

$$Q = \omega \frac{E_{total}}{P_l} \quad (3.1)$$

where ω is the angular frequency, E_{total} denotes the total stored energy, and P_l is the loss incurred. It has also been instructed that the time-domain truncation will reduce the calculated energy and loss by the same factor [93]. Since the quality factor is the ratio between the two, its value should not be greatly influenced by the truncation when being requested at resonance. This conclusion has been confirmed by actual simulations

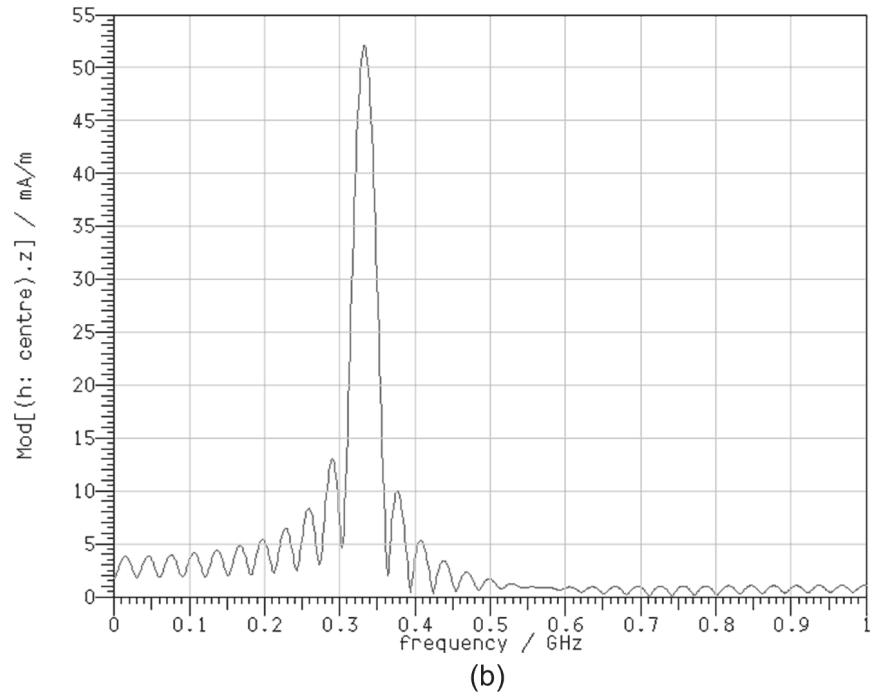
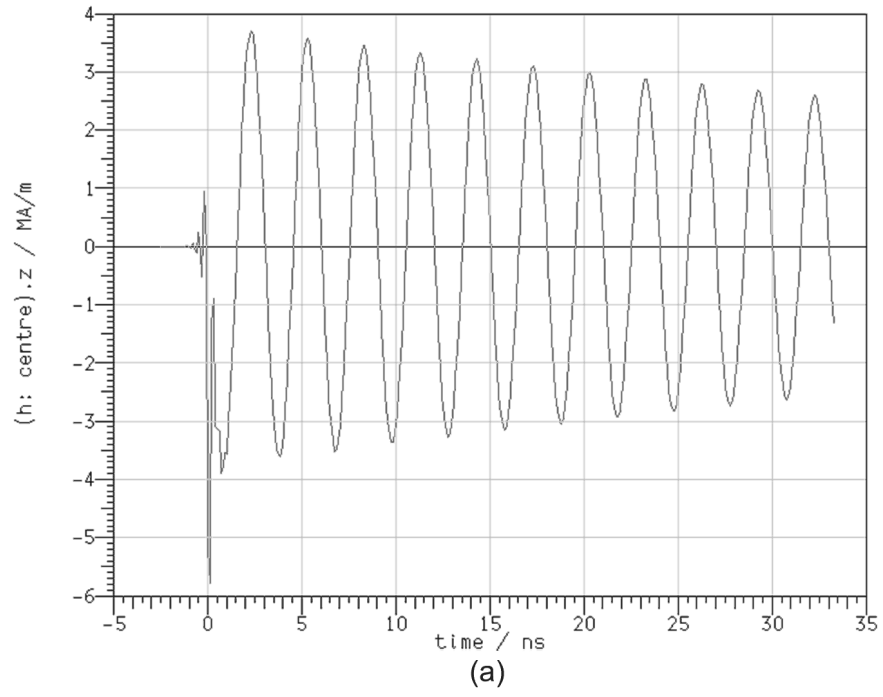


Figure 3-2: Examples of the simulated magnetic field at the centre of a helical resonator to pulse plane wave excitation using CST Micro-StripesTM. (a) time-domain response; (b) frequency-domain response.

that the increase in the duration time will enlarge both the calculated energy and loss by almost the same factor at the resonant frequency.

Since the ripple is significant in the obtained frequency response, the resonant frequency from the simulation is determined as the frequency point where the calculated energy and loss reach maximum, which is slightly bigger than the frequency corresponding to the highest field amplitude.

It should be pointed out that the radiation loss incurred in a helical resonator element is calculated as the power emitted from the equivalent surface enclosing the model, whose value is stored in a file different from the one for energy and loss.

3.2 Parameter Extraction from Measured Transmission Frequency Responses

Research on parameter extraction of microwave resonators from their measured responses has a long history [94]-[105]. It has been demonstrated that the retrieved results for resonant frequency are quite robust, while the accurate estimation of the quality factor proves to be more difficult. In recent years, network analysers have been widely employed which inject energy to the resonator under test and record the resonating frequency response. The parameters are then extracted from the recorded data using different algorithms [97]-[103].

There are several ways of characterising the frequency responses of resonators, among which the transmission-type measurement has been widely adopted due to its better accuracy [100],[102]. This method is employed in this thesis as well, which is implemented

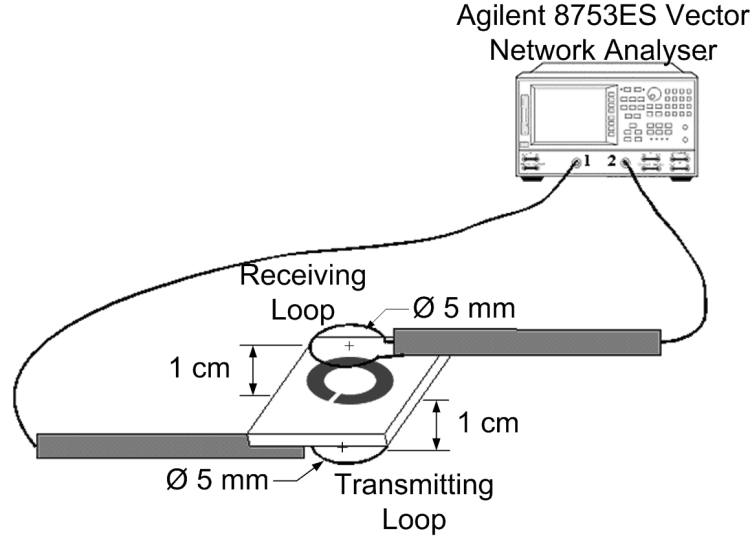


Figure 3-3: Diagram of transmission-type measurement of helical resonator elements. Two loop probes are placed concentrically and in proximity with the resonator under test, and the frequency response of complex transmission coefficient S_{21} is recorded by the network analyser.

by placing a transmitting and a receiving probe both in proximity to the resonator under test placed on a thin piece of balsawood, and recording the complex transmission coefficient S_{21} as a function of frequency. The layout of the experiment is plotted in Fig. 3-3. The probes are in the form of small non-resonant circular loop, which were made by bending the protruding end of the inner conductor of a coaxial cable into a circle and then connecting it to the outer conductor. The two probes have the same size with a diameter of 5 mm, which are much smaller comparing with the free space wavelength at the resonance of the helical resonator element.

3.2.1 Analytical Model for Characterising the Transmission Measurement

The equivalent circuit model widely employed for illustrating the transmission measurement of microwave resonators is depicted in Fig. 3-4. The resonator element under test

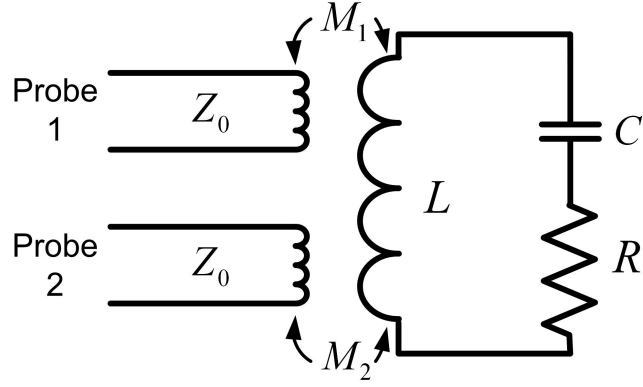


Figure 3-4: The "series-coupled" equivalent circuit characterising the transmission-type measurement of microwave resonators. The two probes with characteristic impedance Z_0 are coupled to the RLC resonator by mutual inductances M_1 and M_2 . There is no direct coupling between the two probes.

is characterised by a lumped series RLC resonant circuit, and the two probes with characteristic impedance Z_0 are coupled to the resonator via mutual inductances $M_{1,2}$. It needs to be emphasised that there is no direct signal path between the two probes without passing through the resonator in this model. Such a circuit topology will be termed "series-coupled" in the following text. In the limit of weak coupling, the expression for the complex transmission coefficient can be derived as

$$S_{21}(f) = \frac{|S_{21}|_{\max}}{1 + jQ \left(\frac{f}{f_0} - \frac{f_0}{f} \right)} \quad (3.2)$$

where f_0 and Q are the resonant frequency and unloaded quality factor of the element, $|S_{21}|_{\max}$ is the maximum amplitude of the transmission coefficient which occurs at the resonance:

$$|S_{21}|_{\max} = \frac{8\pi^2 f^2 M_1 M_2}{Z_0 R} = 2\sqrt{\beta_1 \beta_2} \quad (3.3)$$

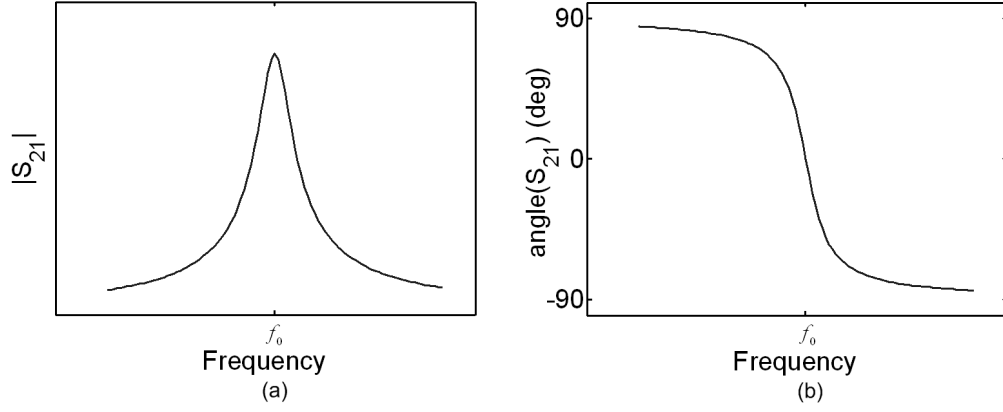


Figure 3-5: Typical frequency responses of the (a) amplitude and (b) phase of the complex transmission coefficient S_{21} derived from the weakly series-coupled RLC circuit model.

this expression again is valid in the weak coupling limit when the coupling coefficients $\beta_{1,2} \ll 1$, where

$$\beta_i = \frac{(2\pi f M_i)^2}{Z_0 R}, i = 1, 2 \quad (3.4)$$

Typical frequency responses of the amplitude and phase of S_{21} as given in (3.2) have the shape as plotted in Fig. 3-5. Its amplitude reaches maximum at resonance and monotonically decreases when the frequency deviates from resonance. Its phase is positive before reaching the resonant frequency, and monotonically turns negative beyond resonance.

The measured S_{21} as a function of frequency from the experiment configuration as illustrated in Fig. 3-3, however, is not exactly the same as the one given by the series-coupled RLC circuit model. Since the circuit analysis is just a simplified approach for characterising the responses of helical element to external EM excitations, a model derived from the quasi-static field analysis should provide more comprehensive descriptions.

From the quasi-static analysis as in Chapter 2, the resonator is excited by the magnetic field which is induced by the current circulating around the transmitting loop. In addition, the receiving loop essentially detects the overall magnetic field near the resonator, since its electrical size is too small to behave as an efficient receiving antenna and have significant impact on the overall EM field distribution. In terms of circuit analysis, it can also be claimed that the two dipole probes are weakly coupled to the resonator under test, but the measured S_{21} is linearly related to the intensity of the overall magnetic field.

Assuming that the transmitting loop generates a magnetic field of B_i along the element's axis which excites the resonator element. Then the expressions of the induced quasi-static electromotive force and current circulating around the resonator are reproduced here from the results in Chapter 2:

$$\varepsilon = -j\omega B_i S \quad (3.5)$$

$$I = \frac{\varepsilon}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{S}{L\left(\frac{1}{\omega^2 LC} - 1 + j\frac{R}{\omega L}\right)} B_i \quad (3.6)$$

where S is the area enclosed by the resonator and R, L and C denote the equivalent circuit parameters. Such a loop current will introduce a secondary scattering magnetic field, which has the following expression at the element centre

$$B_s = \frac{\mu_0 I}{2r} \quad (3.7)$$

r is the radius of the circular resonator. Thus the total magnetic field can be written as

$$\begin{aligned} B &= B_i + B_s \\ &= \left[1 - j \frac{\mu_0 \pi^2 r f / R}{1 + jQ \left(\frac{f}{f_0} - \frac{f_0}{f} \right)} \right] B_i \end{aligned} \quad (3.8)$$

$$= \left[1 + \frac{(\mu_0 \pi r / 2L) \omega^2}{\omega_0^2 - \omega^2 + j\omega\omega_0/Q} \right] B_i \quad (3.9)$$

As can be seen, the expression for the magnetic field (3.8) is different from the transmission response derived from the series-coupled circuit model (3.2). The numerator is frequency dependent, and an additional constant 1 reflecting the incident field presents in the parenthesis. The introduction of this constant can be regarded as a direct signal path from the transmitting to the receiving probe. From the analysis for microwave bandpass filters with cross-coupled circuit topologies [106], such an additional cross-coupling path may lead to a "transmission null" in the frequency response when the signal from the two paths cancel each other, which agrees with the attribute of the developed function for magnetic field (3.8) as will be illustrated later.

Due to the difference in the transmission frequency response, the parameter extraction methods which have been developed for measurement with the series-coupled circuit model should be re-examined for accuracy when being applied to the current experiment configuration using concentric dipoles, which is the content of the next section.

In fact, the equivalent expression for the overall magnetic field (3.9) is in the same Lorentzian form as the one that has been commonly accepted for describing the equivalent permeability μ of metamaterials comprising of dense (in terms of the wavelength) arrays

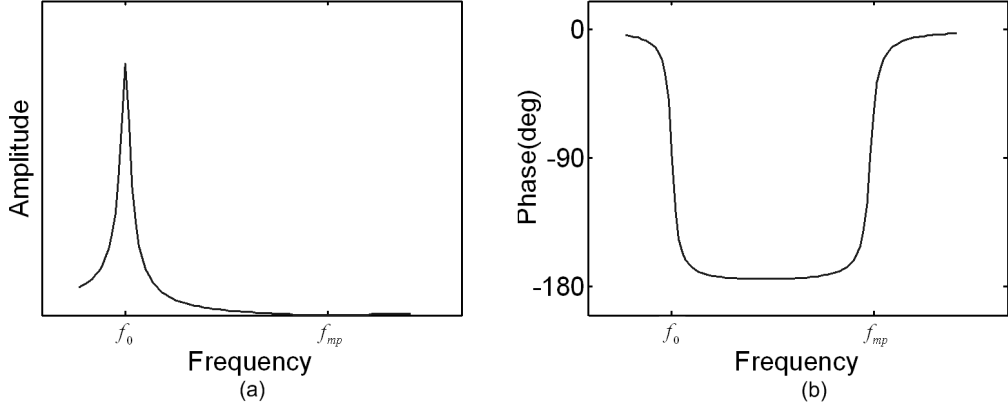


Figure 3-6: Typical frequency responses of the (a) amplitude and (b) phase of the Lorentzian coefficient derived from the quasi-static local field approach.

of magnetic resonating elements [84]

$$\mu = \mu_0 \left(1 + \frac{A\omega^2}{\omega_0^2 - \omega^2 + j\omega\Gamma} \right) \quad (3.10)$$

It is then of interest to briefly introduce the homogenisation approach that has been widely employed in the research on characterising metamaterials using effective medium theory. It was argued in Pendry's pioneering paper [13] that the equivalent permeability μ of metamaterials comprising of regular array of elements can be defined as the ratio of the total magnetic field averaged on the individual unit to the incident magnetic field. This is also in line with the Lorentz local field homogenisation theory [107]-[108] for characterising artificial materials. Thus the coincidence between the equivalent permeability in (3.10) and the total excited magnetic field in (3.9) can be explained. Typical frequency responses of the amplitude and phase of the Lorentzian coefficient as given in (3.9), (3.10) are plotted in Fig. 3-6.

The responses around the resonant frequency f_0 are similar to those developed from

weakly coupled series circuit model. However, there exists a frequency point f_{mp} where the amplitude vanishes ("transmission null" in circuit analysis). It is the so-called "magnetic plasma frequency" where the scattered magnetic field essentially cancels out the incident field, making the equivalent permeability $\mu = 0$. As can be seen, if the artificial material composed from magnetic resonators is characterised by equivalent material parameters, it exhibits both paramagnetic and diamagnetic behaviour before and after resonance separately. Strong diamagnetism appears at the frequency range from f_0 to f_{mp} where the real part of μ is negative. The extent of above range increases with the positive constant A in (3.10) which reflects the density of the artificial material [79].

3.2.2 Comparison of Parameter Extraction Methods

The 3 dB method is the simplest way of retrieving the resonance parameters. If the transmission coefficient S_{21} has the form as deduced from the series-coupled circuit model (3.2), the 3 dB method determines the frequency corresponding to the maximum magnitude $|S_{21}|_{\max}$ as the resonant frequency f_0 . The half power points with an amplitude of $|S_{21}|_{\max}/\sqrt{2}$ are found on either side of the resonance and the difference of those frequency positions is the bandwidth $\Delta f_{3\text{ dB}}$. The quality factor is then given by

$$Q_{3\text{ dB}} = \frac{f_0}{\Delta f_{3\text{ dB}}} \quad (3.11)$$

Although quite straightforward and physically meaningful, the 3 dB method may give poor results because it relies solely on the discrete data rather than a fit. If the frequency response is extremely narrowband for resonator with high quality factor, the resolution in frequency domain can be too low to capture the whole sharp response which leads to significant errors. In addition, other corruptions are introduced to the measured data

under actual experimental conditions. The possible reasons include crosstalk between the probes and cables, the phase changes in S_{21} due to the finite length of cables, and unavoidable noise. Thus methods incorporate curve fitting should be employed in order to improve the estimation accuracy.

Petersan [88] compared the precision and robustness of several methods for estimating resonance parameters from the measured transmission spectrum of a microwave resonator. He has concluded that two methods have the best performances, namely the nonlinear least-squares fit of the phase of S_{21} versus frequency method and the nonlinear least-squares fit of $|S_{21}|$ to the Lorentzian curve method separately. The former has a major advantage by using vector data so that it allows one to perform corrections to remove cross talk, delay, and nearby resonances. The latter is preferable for very noisy data. Their basic principles will be explained in detail as follows.

For the Lorentzian magnitude fit method, the amplitude of S_{21} versus frequency data are fit to a Lorentzian curve with the following expression

$$|S_{21}(f)| = A_1 + A_2 f + \frac{|S_{\max}| + A_3 f}{\sqrt{1 + 4 \left(\frac{f - f_0}{\Delta f_L} \right)^2}} \quad (3.12)$$

using a nonlinear least-squares fit. The resonant frequency f_0 , bandwidth Δf_L , maximum magnitude $|S_{\max}|$, and three constant reflecting unrealistic data corruption including constant background A_1 , slope on the background A_2 , and skew A_3 are used as fitting parameters. The Q is then calculated using the values of f_0 and Δf_L from the final fit results

$$Q_L = \frac{f_0}{\Delta f_L} \quad (3.13)$$

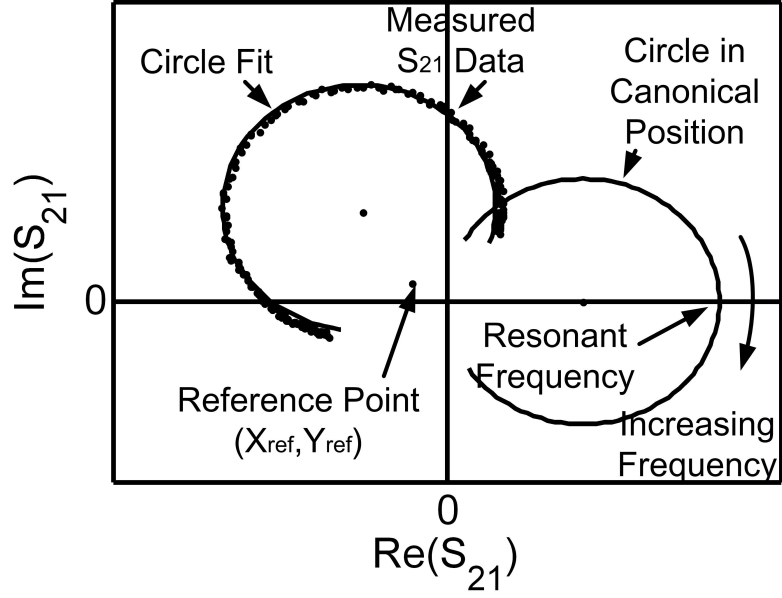


Figure 3-7: Measured data of complex transmission coefficient S_{21} , their circle fit as well as the ideal circle of S_{21} in the canonical position. Large dots indicate circle centres and reference point.

Although having the same mathematical form, this method is substantially more robust and accurate than the 3 dB method.

The phase frequency fit method, on the other hand, relies on a circle fit to the complex S_{21} data as a step to finding f_0 and Q . It can be verified that the plot of the imaginary part of S_{21} given in (3.2) versus its real part (with frequency as a parameter) forms a circle in the canonical position with its centre on the real axis. The frequency point corresponding to the intersection of the circle to the real axis is the resonant frequency. The actual measured data, however, can be significantly altered due to the similar undesired aspects in the real measurement situation as explained above. A typical set of measured complex S_{21} is given in Fig. 3-7, which can be fit to a circle with the following general expression

$$(x - x_c)^2 + (y - y_c)^2 = r^2 \quad (3.14)$$

where (x_c, y_c) is the coordinate of the circle centre, and r is the radius of the circle. These three parameters are aimed to be regressed from fitting the data to the circle. (3.14) can be rearranged as

$$2xx_c + 2yy_c + (r^2 - x_c^2 - y_c^2) = x^2 + y^2 \quad (3.15)$$

Let

$$z_c = r^2 - x_c^2 - y_c^2 \quad (3.16)$$

then (3.15) becomes a linear relation among x_c, y_c and z_c . The real and imaginary parts of the measured data of complex S_{21} are substituted into the equation as the coefficients

$$x = \text{Re}(S_{21}), y = \text{Im}(S_{21}) \quad (3.17)$$

Then the best linear fit results of x_c, y_c and z_c can be estimated using standard linear least-squares algorithm. The radius of the circle

$$r = \sqrt{x_c^2 + y_c^2 + z_c} \quad (3.18)$$

In the circle fit, a nonuniform weighting scheme is adopted which assigns the data points closer to the resonant frequency heavier weights than those further away. Since the transmission near resonance has greater amplitude, such a scheme can further suppress the interference of noise.

The specific weights are calculated by first locating the reference point (x_{ref}, y_{ref}) midway between the first and last data point in the complex plane, as shown in Fig.

3-7. Next, the distance from the reference point to each data point (x_i, y_i) is calculated, whose square is the weight assigned to that data point

$$w(i) = (x_{ref} - x_i)^2 + (y_{ref} - y_i)^2 \quad (3.19)$$

After obtaining the centre and radius of the best fit circle, the measured data are rotated around the origin altogether so that the centre of the best fit circle falls on the real axis. The phase angles of every data points after rotation with respect to the positive real axis are then calculated, and fit as a function of frequency of the following form using a nonlinear least-squares fit method.

$$\theta(f) = \theta_0 + \tan^{-1} \left[Q \left(\frac{f}{f_0} - \frac{f_0}{f} \right) \right] \quad (3.20)$$

In this equation θ_0 , the phase at which the resonance occurs (close to zero), f_0 , and Q are determined from the fit. A similar weighting scheme is again employed to emphasise data near resonance. A typical set of fit results to the phase of the calibrated measured S_{21} data is plotted in Fig. 3-8. As can be seen, the agreement is quite good around the resonance where the phase approaches zero, while degraded away from resonance.

The above two curve fitting methods have been demonstrated to be quite precise and robust for data extracted from experiment with configuration represented by the series-coupled circuit as in Fig. 3-5. In order to thoroughly check their accuracies when being applied to the current configuration, artificial complex S_{21} data were created in Lorentzian form as in (3.9). For simplicity, helical resonators with optimal dimension ratio $\alpha = 0.16$ were assumed, so that the coefficient of ω^2 in the numerator is a constant 0.61. The resonant frequency was given as $f_0 = 1$ GHz, and two values of quality factor

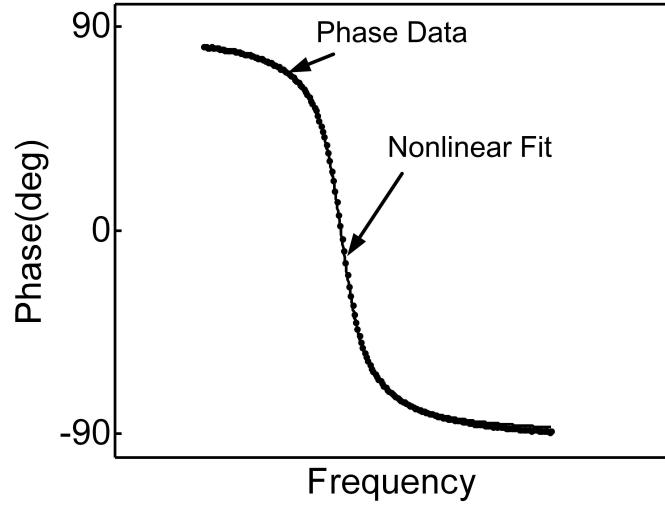


Figure 3-8: Phase of the calibrated measured S_{21} data as a function of frequency as well as their nonlinear fit. The complex S_{21} data are rotated around the origin so that the centre of their best fit circle locates on the positive real axis, and the phase to each point is calculated from the positive real axis.

$Q = 30$ and 300 were selected which roughly represent the lower and upper limits of the achievable values for the actual fabricated resonators. The data were created within a frequency range from 0.9 to 1.1 GHz with 1 MHz spacing.

The noise was introduced to both the real and imaginary parts of the data as Gaussian distributed random numbers, with an amplitude scaled to the maximum amplitude of S_{21} according to the signal-to-noise ratio (SNR) specified. The resonance parameters f_0 and Q were extracted from the artificial data using the above two methods and 3 dB method, with a SNR ranges from 5 to 500 . The nonlinear least-square fit were conducted using the relevant curve fitting function in Matlab. At each SNR level, the process was repeated 100 times, and the mean values of retrieved f_0 and Q as well as their standard deviation σ_{f_0} and σ_Q were calculated for each method so as to characterise both their accuracy and robustness.

The relation between the mean values and normalised standard deviations (standard deviation σ_{f_0} and σ_Q normalised by their mean values separately) of retrieved f_0 and Q versus SNR for helical resonators with $Q = 30$ and 300 are illustrated in Fig. 3-9 and Fig. 3-10 separately. The phase frequency fit method outperforms the other two in terms of both accuracy and robustness. Its mean values consistently equal the given theoretical values in the two graphs. For resonator with $Q = 30$, the normalised standard deviation are within 0.03% and 2% for f_0 and Q separately. For resonator with $Q = 300$, these are within 0.01% and 6% for f_0 and Q separately. Therefore the increase in quality factor improves the estimation accuracy for f_0 while degrades the accuracy for Q itself.

The 3 dB method exhibits much larger deviation for resonator with $Q = 30$, which can be attributed to its poor resilience to the influence of noise. It can accurately estimate the resonant frequency when $Q = 300$, but the estimated quality factor differs from the theoretical value. This is because it fails to capture the sharp resonant response due to the limited frequency resolution of the discrete data points. On the other hand, the Lorentzian amplitude fit method consistently underestimates both f_0 and Q when the SNR is low. Generally speaking, all the three methods exhibit improved accuracy as the SNR increases, which agrees with the intuition as the influence of noise declines.

From the above analysis, it can be seen that the phase frequency fit method can give very accurate estimation on both the resonant frequency and quality factor of helical resonators from the complex transmission coefficient S_{21} with the form of (3.9). The standard deviation of the results for quality factor is within several percent of the estimated value, which fully demonstrates its validity when being applied to the measured S_{21} data from the experiment configuration using magnetic dipole probes. This method has been employed for parameter extraction from the actual measured S_{21} data.

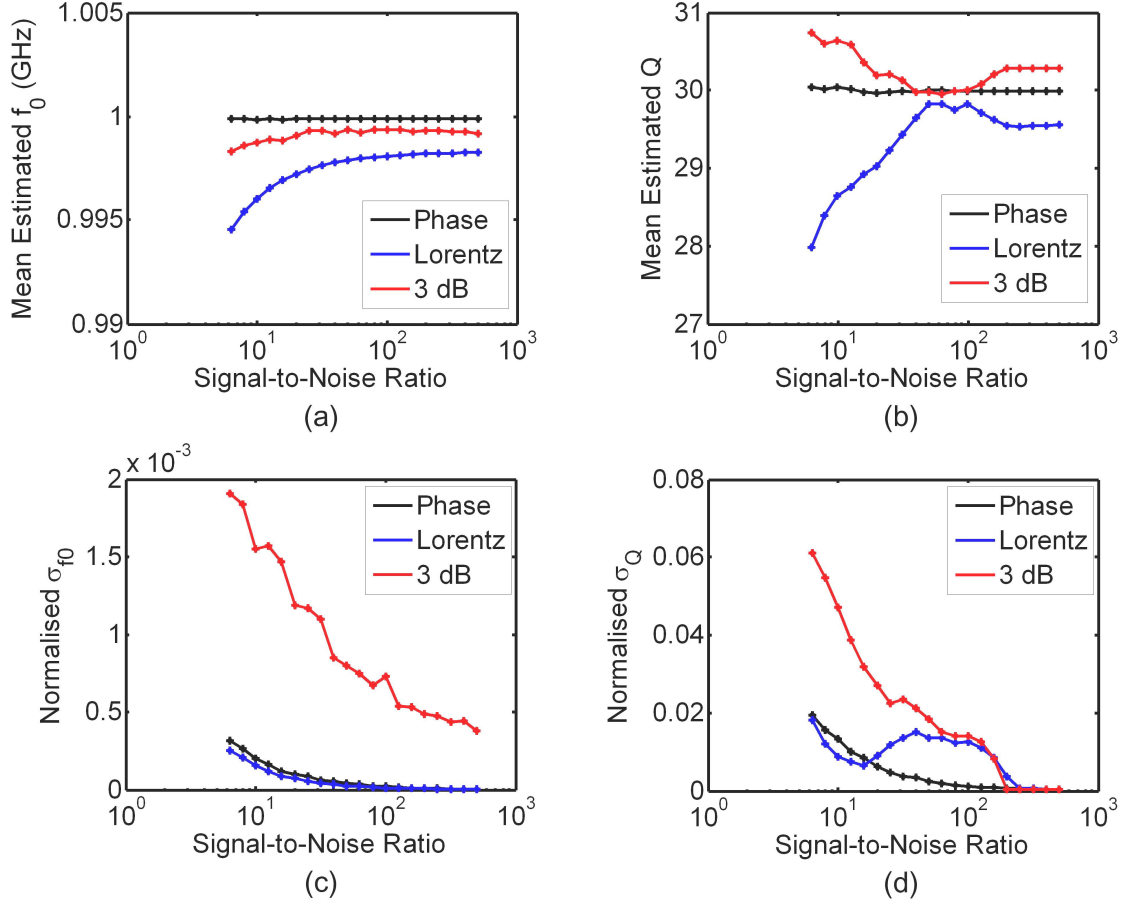


Figure 3-9: Mean values of (a) resonant frequency and (b) quality factor as well as their standard deviations (c)(d) normalised by corresponding mean values estimated from created complex S_{21} data with different signal-to-noise ratio (SNR) levels using phase frequency fit method (black), Lorentzian magnitude fit method (blue) and 3 dB method (red). The dots represent the discrete calculation results for different SNR ratios while the lines connecting these dots are for visual guidance. The helical resonator with optimal dimension ratio has a resonant frequency of 1 GHz and a quality factor of 30.

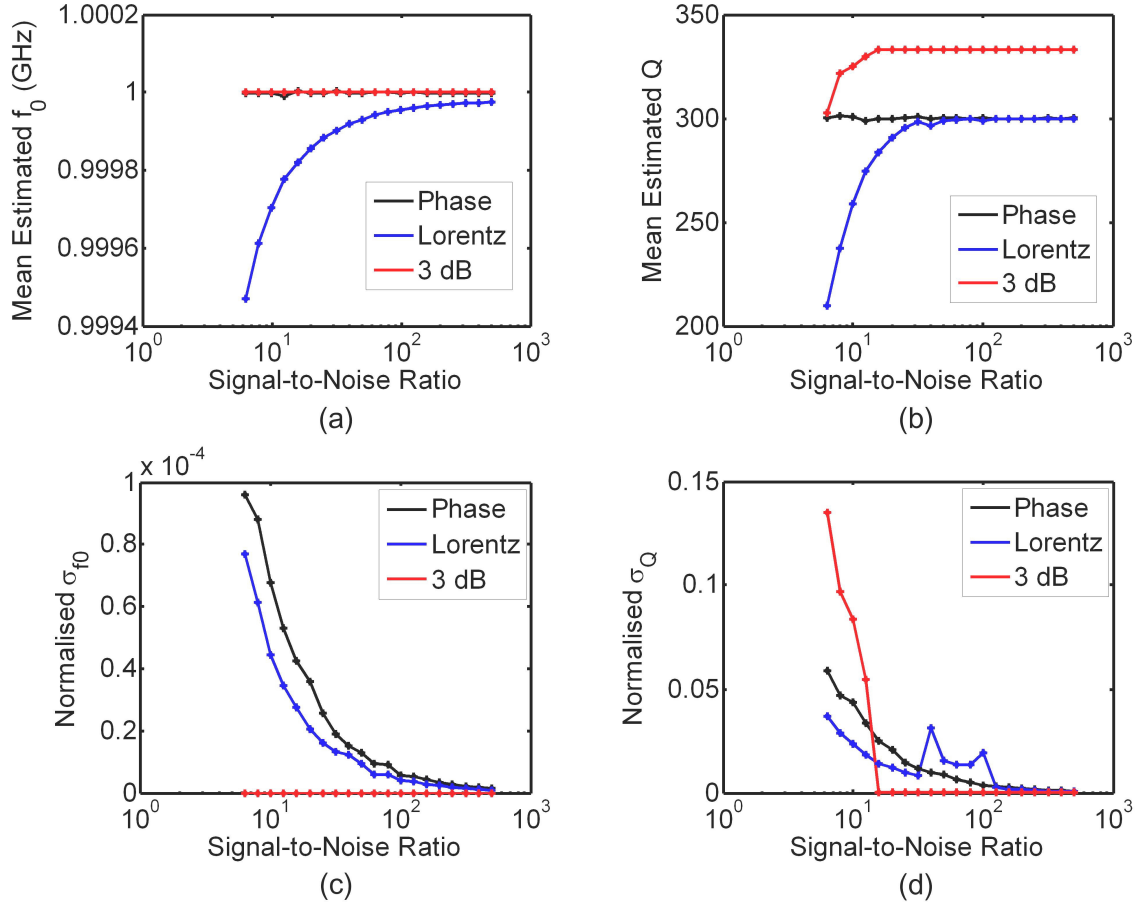


Figure 3-10: Mean values of (a) resonant frequency and (b) quality factor as well as their standard deviations (c)(d) normalised by corresponding mean values estimated from created complex S_{21} data with different signal-to-noise ratio (SNR) levels using phase frequency fit method (black), Lorentzian magnitude fit method (blue) and 3 dB method (red). The dots represent the discrete calculation results for different SNR ratios while the lines connecting these dots are for visual guidance. The helical resonator with optimal dimension ratio has a resonant frequency of 1 GHz and a quality factor of 300.

3.2.3 Impact of Coupling Strength on the Extracted Resonance Parameters

When measuring the quality factor of resonators, it is well known that the probes should be weakly coupled to the resonator under test, so that only a negligible part of energy oscillating within the resonator would be exported, and the measured Q can approximate the unloaded Q of a resonator. On the other hand, the signal around resonance recorded by the network analyser should be significantly greater than the noise threshold. Therefore the appropriate level of external coupling in the experiment needs to be quantitatively clarified.

In order to investigate the impact of coupling strength on the retrieved resonance parameters, the measurement of transmission spectrum S_{21} were repeated using magnetic dipoles with different sizes and spacing with the resonator under test to provide various level of external coupling. The resonance parameters were then extracted using the phase frequency fit method. It was discovered that the results of resonant frequency were quite consistent and irrelevant with the strength of coupling. However, the quality factor Q could be greatly affected.

When the coupling was too small (measured $|S_{21}|_{\max} < -40$ dB), the extracted Q s were inconsistent, but systematically higher than the results given by the accurate analytical model developed in Chapter 2 due to the significant influence of noise. On the other hand, when the coupling was too strong (measured $|S_{21}|_{\max} > -20$ dB), the extracted Q s were smaller than the analytical predictions because the energy exported by the probes accounts for a significant part of the total energy which degrades the loaded Q s. When the coupling level is moderate (-40 dB $< |S_{21}|_{\max} < -20$ dB), the retrieved results were consistent and coincided well with analytical predictions. Therefore an appropriate coupling level is required for the accurate extraction of resonance parameters even when

the most precise fitting method has been employed.

In addition, another set of measurements were conducted following the configuration given in other references [81],[109], in which two monopoles were used as probes and placed in parallel with the resonator in between. It was discovered that although the resonator could be excited by the penetrating magnetic field of the monopole, its resonance could only slightly affect the measured transmission frequency response because the two monopoles were mainly coupled to each other via electric rather than magnetic field. On the other hand, the loop probes employed in our experiment can provide the appropriate level of magnetic coupling. Therefore it can be safely concluded that the monopoles are not suitable for measuring magnetic metamaterial elements.

3.3 Conclusions

This chapter gives a detailed explanation on the process of simulating the response of individual helical resonator using CST Micro-StripesTM, as well as the method for extracting resonant frequency and quality factor from the measured frequency response of complex transmission coefficient S_{21} . CST Micro-StripesTM is a time-domain simulation software using transmission line method, thus the transient response is recorded as the simulation proceeds and the frequency response is obtained as the Fourier transform of the truncated time response. The energy and loss (including radiation loss) incurred at specified frequencies can also be calculated from the distribution of EM fields as well as current flow, which can thus be used to estimate the quality factors. Only some initial part of transient response is needed to estimate the resonant frequency and quality factor.

The measurement of the complex transmission coefficient S_{21} as a function of fre-

quency has been conducted using two small loop dipoles as the magnetic probes weakly coupled to the resonator under test. The frequency response is in different form as the one developed from series-coupled circuit model, thus several parameter extraction methods have been re-examined for precision and robustness using artificially constructed S_{21} data incorporating random noise. The nonlinear least-squares fit to the phase of S_{21} versus frequency method proves to have the best accuracy and robustness, and has been employed to analyse the actual measured data using dipole probes.

In addition, the impact of the coupling strength of probes to the resonator under test on the accuracy of parameter extraction has been investigated. When the maximum amplitude of measured S_{21} were within a range of -40 dB to -20 dB, the retrieved results were consistent and had good agreements with the predictions from accurate analytical model developed in Chapter 2. Therefore both an appropriate coupling level and a precise parameter extraction method are required for the accurate extraction of resonance parameters for helical resonators.

Chapter 4

The Dichroism Properties of Two-Dimensional Interlocking Square Arrays of Chiral Resonators

After thoroughly investigating the characteristics of individual helical resonators, the properties of assembled arrays are consequently aimed to be explored. The helical resonator is inherently chiral, i.e., its image cannot be superimposed on itself, which enables the assembled metamaterial to have unusual characteristics [110]-[116]. Especially, it has been demonstrated that negative refraction of one of the left-/right-handed circularly polarised electromagnetic (EM) waves, both of which are eigensolutions of the EM wave propagation in chiral media [117], can be realised via strong chirality with no requirements on negative values of permittivity and/or permeability [110]-[112]. Such an attribute leads to much more flexible design requirements of metamaterials than the traditional double negative prototype.

As been illustrated in § 1.3, most of the actual fabricated metamaterial samples based on resonating elements are comprised of only a few layers of two-dimensional arrays of elements. Their properties can be more accurately characterised by analysing the transmission of EM waves through the planar array, compared with employing the effective medium theory. This can be done by treating the array as an equivalent frequency selective surface, and deducing the relations between macroscopic transmission properties and microscopic characteristics of elements [118]-[124].

In this chapter an interlocking square lattice of helical resonators will be investigated, which can be regarded as a two-dimensional (2D) version of metamaterials. Due to the inherent chirality of the elements, the assembled array exhibits dichroism, which is characterised by the difference in the macroscopic transmissions of the two circularly polarised EM waves with the same wave vector through the array. The dichroism is most significant around the element resonance.

The aim of this chapter is to develop an analytical model, which can provide a physical insight into the interaction of EM wave with the infinite array of elements and enable the design of 2D metamaterial according to the required transmission properties. In the model the macroscopic EM fields (on a scale comparable with the free space wavelength of interest) on both sides of the array are expressed as a function of the electric and magnetic polarisabilities of the individual chiral elements and their density of distribution. Therefore the impact of the characteristics of individual elements on the macroscopic array transmission performance can be revealed. It will be illustrated that the quality factor of elements plays a crucial role on the dichroism of such a 2D interlocking array, while an optimal lattice spacing exists for given elements which can achieve the greatest difference in the absorption of the two circularly polarised EM waves around resonance.

It has to be emphasised that the definitions of handedness of circularly polarised EM waves in this thesis are in accordance with the IEEE standard (i.e., left/right handedness is determined by pointing one's left/right thumb in the same direction that the wave is propagating, and then matching the curling of one's fingers to the temporal rotation of the field) [125],[126], and time harmonic term of $e^{j\omega t}$ is assumed unless specified otherwise. These are in opposite to those widely used by physicists, therefore great attention have to be paid when making comparisons with other published results, as will be illustrated later.

4.1 Modelling the Macroscopic Transmission Properties of Two-Dimensional Array

The element used in this chapter is a modified version of helical resonator - the "double" helical resonator. As shown in Fig. 4-1(a), its topology has better symmetry than the original one, and can in theory be applied for the design of three-dimensional isotropic artificial media [127].

By conducting similar quasi-static circuit analysis as in Chapter 2, it can be derived that a double helical resonator has the same self-inductance L as that of a helical resonator with the same size. The self-capacitance C , however, is the series connection of the capacitances of two half helices, while that of a helical resonator is the parallel connection of the two. Therefore a double helical resonator has a quarter of the self-capacitance as that of a helical resonator with the same physical dimensions, and two times the resonating frequency of the latter as a consequence. In the following text the "helical resonator" refers to double helical resonator unless specified otherwise.

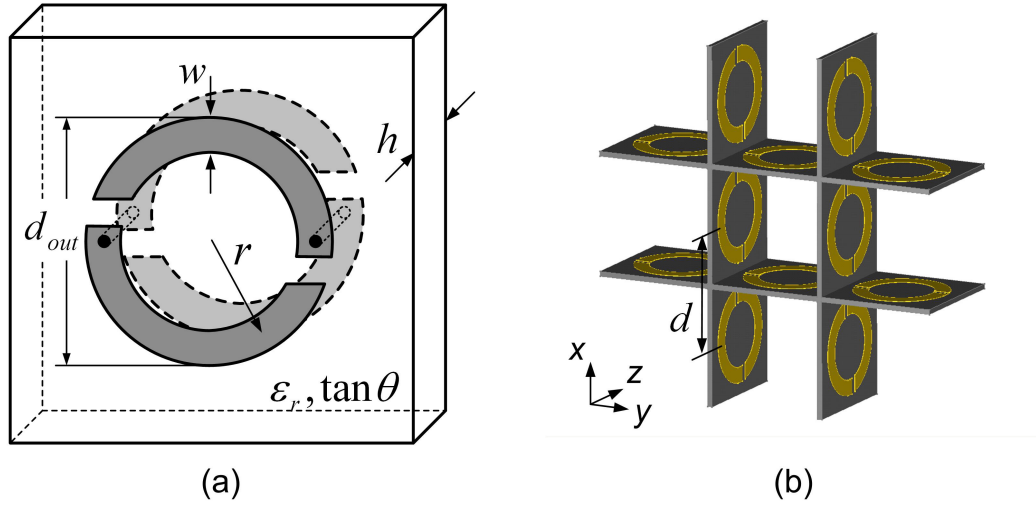


Figure 4-1: (a) The layout of a double helical resonator. Grey zones with solid boundaries correspond to conductor on top of the dielectric substrate while those with dashed boundaries correspond to conductor on the bottom level. The metal patterns on the two sides are connected by two metallic vias, forming a double helix pattern. (b) Part of an infinite two-dimensional (2D) interlocking square array of double helical resonators. The element orientations are in line with the principal axes $\hat{x}$ and $\hat{y}$.

It has been experimentally verified that a 2D interlocking array of double helical resonators offers both significant dichroism around element resonance and symmetry in the transmissions along both propagation directions [128],[129], which lead to great application potential in building isotropic three-dimensional metamaterials. Thus the development of an analytical model for characterising the array properties is of significant importance so as to engineer the dichroism properties of such a 2D slab.

In this chapter an analytical model will be developed which associates the macroscopic EM fields to the small scale properties of individual helical resonator elements as well as their interactions. The 2D slab will be regarded as continuous distribution of electric and magnetic currents (and polarisation densities) across an equivalent surface, provided that the slab thickness is much smaller than the operating wavelength, which contributes

to the dichroism in the macroscopic transmission. These continuous surface polarisation densities are then related to the discrete dipole moments induced at the centre of elements, which are assumed to be characterised completely by their electric and magnetic polarisabilities and density of distribution. Such relations have been named "generalised sheet transition conditions (GSTCs)" in previously published work [119],[120].

It is assumed throughout the modelling procedure that the array is a two-dimensional infinite square lattice with rigid period, under normal incidence of planar EM wave. Therefore the elements with the same orientation are uniformly excited.

4.1.1 Quasi-Static Analysis of Excitation and Interaction of Chiral Elements

Due to the inherent chirality of the helical resonators, both the electric and magnetic dipole moments will be induced when the elements are subjected to external electric and/or magnetic field excitation [107]. The electric dipole comes from the strong electric field between the broadside coupled upper and lower rings, while the magnetic dipole arises from the resonating current flowing along the metal path. The cross-coupled expressions linking dipole moments and excitation fields are as follows

$$p_n = \alpha^{ee} E_n + \alpha^{em} B_n \quad (4.1)$$

$$m_n = -\alpha^{em} E_n + \alpha^{mm} B_n \quad (4.2)$$

where E_n/B_n are the local excitation fields and p_n/m_n are the induced electric/magnetic dipole moments separately, both of which are oriented parallel to the normal axis of the

element. The quasi-static polarisabilities α of double helical resonators are as follows:

$$\alpha^{mm} = \frac{\pi^2 r^4}{L} \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)^{-1} \quad (4.3)$$

$$\alpha^{em} = \pm j \frac{2\pi r^2 h}{\epsilon_r \omega L} \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)^{-1} \quad (4.4)$$

$$\alpha^{ee} = \frac{4h^2}{(\epsilon_r \omega)^2 L} \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)^{-1} \quad (4.5)$$

where r and h are the geometric parameters as shown in Fig. 4-1(a), L , R , and ω_0 are the self-inductance, resistance, and frequency of resonance, respectively. The quasi-static resistance incorporates the loss incurred from both conductor and dielectric. ϵ_r is the relative dielectric constant of the substrate. The sign of α^{em} depends on the element's handedness: positive sign corresponds to left-handed helix while minus sign applies to right-handed one. The detailed derivation process of these polarisabilities are listed in Appendix I. For single helical resonators, the coefficients of 2 and 4 in the expressions for α^{em} (4.4) and α^{ee} (4.5) are absent.

It should be pointed out that the radiation effects have been excluded from these expressions due to their quasi-static nature, which will be incorporated in § 4.1.3 so as to provide more accurate estimations on the macroscopic transmission properties as well as fulfill energy conservation requirements. In addition, nonresonant electric polarisabilities are also present in the plane of the elements, with constant value of $16\epsilon_0 r_{ext}^3/3$. This value is generally much smaller than the electric polarisability α^{ee} around resonance, therefore can be safely neglected in the modelling process.

From (4.3) – (4.5), it is known that the following general relation holds for the three polarisabilities

$$\alpha^{mm} \cdot \alpha^{ee} = -(\alpha^{em})^2 \quad (4.6)$$

The excited electric and magnetic dipoles will then induce secondary scattering fields. The full-wave fields generated by an electric dipole $\vec{p}$ have the following form [9]:

$$\vec{E}^p(\vec{r}) = [\vec{p} - \hat{r}(\hat{r} \cdot \vec{p})] \frac{k^2 e^{-jkr}}{4\pi\epsilon_0 r} + [3\hat{r}(\hat{r} \cdot \vec{p}) - \vec{p}] \left(\frac{1}{r^3} + \frac{jk}{r^2} \right) \frac{e^{-jkr}}{4\pi\epsilon_0} \quad (4.7)$$

$$\vec{H}^p(\vec{r}) = (\hat{r} \times \vec{p}) \left(1 + \frac{1}{jkr} \right) \frac{k^2 c_0 e^{-jkr}}{4\pi r} \quad (4.8)$$

where $\vec{r}$ is the vector pointing from the dipole to the observation point, while $\hat{r} = \vec{r}/|\vec{r}|$ is the unit vector in the direction of $\vec{r}$. k and c_0 are the phase constant and speed of EM wave in free space, respectively.

The corresponding results for full-wave EM fields generated by a magnetic dipole $\vec{m}$ follow from the duality principle

$$\vec{E}^m(\vec{r}) = (\vec{m} \times \hat{r}) \left(1 + \frac{1}{jkr} \right) \frac{k^2 Z_0 e^{-jkr}}{4\pi r} \quad (4.9)$$

$$\vec{H}^m(\vec{r}) = [\vec{m} - \hat{r}(\hat{r} \cdot \vec{m})] \frac{k^2 e^{-jkr}}{4\pi r} + [3\hat{r}(\hat{r} \cdot \vec{m}) - \vec{m}] \left(\frac{1}{r^3} + \frac{jk}{r^2} \right) \frac{e^{-jkr}}{4\pi} \quad (4.10)$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi\Omega$ is the free space wave impedance.

From the above, it can be seen that the effective fields acting to polarise an element inside such a 2D lattice are the superposition of two parts: the contribution from the external incident fields as well as those from all the other dipoles except the one under consideration. The latter will be termed as "interaction field" hereafter.

Due to the infinite periodicity of the array, and from considerations of symmetry, the origin of the three-dimensional Cartesian coordinate system for the array can be set at the centre of an individual element to be investigated. The other element centres all

locate within the $x - y$ plane, and all the elements are set to be oriented in line with either $\hat{x}$ or $\hat{y}$, as indicated in Fig. 4-1(b). The normally incident EM wave propagates along $+\hat{z}$ direction through the array.

The direct infinite summation involved in calculating the electric and magnetic interaction fields may raise concerns in terms of convergence. Because the elements have subwavelength dimensions, the detailed fields associated with individual elements are generally unimportant with respect to the overall wave propagation properties of the composite; thus a homogenisation procedure can be applied as follows. The infinite array of discrete elements, except the one at the origin, are replaced by a continuous distribution of dipole moments on an equivalent planar sheet coinciding with the $x - y$ plane, with a punctured circular hole of radius R_a around the origin. This procedure is illustrated in Fig. 4-2, which is in a way similar to the Clausius-Mossotti theory for determining the effective dielectric constant of a volume distribution of small scatterers [108]. The discrete and continuous dipole moments are related as

$$\vec{p}_s = \frac{\vec{p}}{S_0} \quad (4.11)$$

$$\vec{m}_s = \frac{\vec{m}}{S_0} \quad (4.12)$$

where $\vec{p}_s$ and $\vec{m}_s$ are the electric and magnetic surface dipole moments density (polarisation density) respectively. S_0 is the area of unit cell. The interaction fields generated by the two configurations should be equal, therefore can be written as follows

$$\vec{E}^{int} = \sum \vec{E}^{p,m}(\vec{r}) = \frac{1}{S_0} \int_{r>R_a} \vec{E}^{p_s,m_s}(\vec{r}) dS \quad (4.13)$$

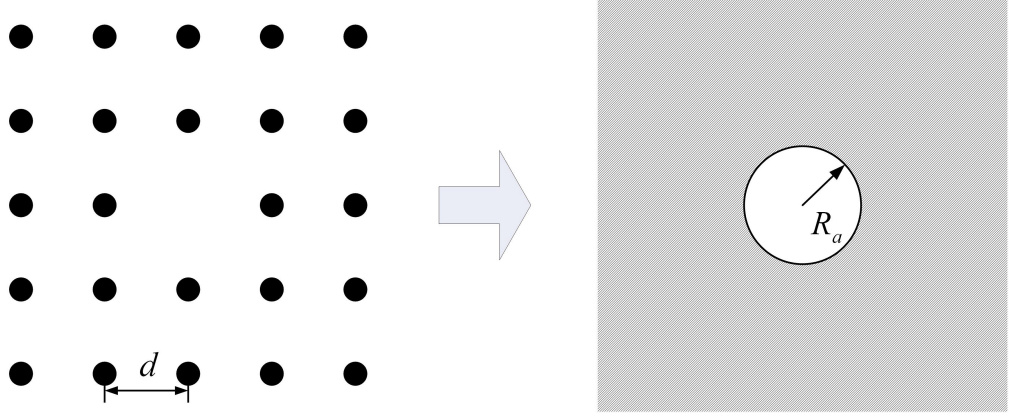


Figure 4-2: The approximation of a layer of infinite regular array of discrete elements, except the one at the origin, by a continuous current distribution on an equivalent planar surface with a punctured circular hole around the origin.

$$\vec{H}^{int} = \sum \vec{H}^{p,m}(\vec{r}) = \frac{1}{S_0} \int_{r>R_a} \vec{H}^{ps,m_s}(\vec{r}) dS \quad (4.14)$$

Substituting the expressions for full-wave electric and magnetic fields (4.7) – (4.10) into the above equations, these integrals can be deduced into very simple forms after some algebra as follows

$$\vec{E}^{int,p} = \frac{-jk\vec{p}}{4\epsilon_0 S_0} \left(1 - \frac{1}{jkR_a}\right) e^{-jkR_a} \quad (4.15)$$

$$\vec{H}^{int,m} = \frac{-jk\vec{m}}{4S_0} \left(1 - \frac{1}{jkR_a}\right) e^{-jkR_a} \quad (4.16)$$

$$\vec{E}^{int,m} = \vec{H}^{int,p} = 0 \quad (4.17)$$

In Appendix II the details are given on how these interaction fields created by continuous surface polarisation densities are evaluated.

Define the interaction constant of the array

$$\beta = \frac{-jk}{4\epsilon_0 S_0} \left(1 - \frac{1}{jkR_a}\right) e^{-jkR_a} = \frac{1 - jkR_a}{4\epsilon_0 R_a S_0} e^{-jkR_a} \quad (4.18)$$

then

$$\vec{E}^{int,p} = \beta \vec{p} \quad (4.19)$$

$$\vec{H}^{int,m} = \epsilon_0 \beta \vec{m} \quad (4.20)$$

The deduced explicit results (4.19), (4.20) reveal some important conclusions about the interaction among helical resonator elements. First, it can be seen that sheet electric polarisation distribution contributes to electric field at the hole centre with the same orientation only, and so does the magnetic counterpart. In addition, since the helical resonator element only responds to excitation fields along its normal axis, it can be deduced that one element can only be polarised by the interaction fields induced by all other elements with the same orientation (e.g., $\hat{x}$), while the overall contribution by elements with the other orthogonal orientation (e.g., $\hat{y}$) is ineffective.

Since there is no overall cross-coupling among elements with orthogonal orientations, the array can be divided into two groups of elements according to their orientations and be analysed separately. Due to the rotational symmetry of the array, analysis on only one square lattice of elements with the same orientation is needed. This conclusion greatly facilitates the modelling procedure. In particular, it should be emphasised that the lattice period d in the analytical model denotes the spacing between adjacent elements with the same orientation, and the continuous polarisation densities are the average of dipole moments on a unit cell with an area of $S_0 = d^2$.

The value of the hole radius R_a determines the accuracy of the continuous sheet

approximation. In the static limit ($k \rightarrow 0$), it was given that [118]

$$R_a \approx 0.6956d \quad (4.21)$$

for a distribution of scatterers in a square lattice of period d . More discussions on the accuracy of R_a will be proposed in § 4.1.3 when incorporating the radiation effects.

4.1.2 Macroscopic Transmission of 2D Interlocking Array of Helical Resonators

In order to model the macroscopic transmission properties of the 2D interlocking slab, it is assumed that the discontinuities in the EM fields on both sides of array are attributed to the continuous distribution of surface electric and magnetic currents (polarisation densities) across the unpunctured equivalent sheet. By doing this the microscopic fields near the equivalent sheet or discrete elements are forgone. Then the relations between the external incident fields and the induced surface polarisation distributions are aimed to be developed.

On the other hand, it is known from the analysis in § 4.1.1 that the surface polarisation densities are proportional to the local fields acting on each element, which are the superposition of the external incident fields and the interaction fields. Substituting field expressions (4.15) – (4.17) into dipole moments (4.1), (4.2) lead to

$$\begin{aligned} p_{x,y} &= \alpha^{ee} (E_{x,y}^{ext} + E_{x,y}^{int,p}) + \alpha^{em} \mu_0 (H_{x,y}^{ext} + H_{x,y}^{int,m}) \\ &= \alpha^{ee} (E_{x,y}^{ext} + \beta p_{x,y}) + \alpha^{em} \mu_0 (H_{x,y}^{ext} + \epsilon_0 \beta m_{x,y}) \end{aligned} \quad (4.22)$$

$$\begin{aligned}
m_{x,y} &= -\alpha^{em} (E_{x,y}^{ext} + E_{x,y}^{int,p}) + \alpha^{mm} \mu_0 (H_{x,y}^{ext} + H_{x,y}^{int,m}) \\
&= -\alpha^{em} (E_{x,y}^{ext} + \beta p_{x,y}) + \alpha^{mm} \mu_0 (H_{x,y}^{ext} + \epsilon_0 \beta m_{x,y})
\end{aligned} \tag{4.23}$$

The diochroic nature of a material is characterised by the difference in the macroscopic transmissions of the two circularly polarised EM waves with the same wave vector but different handedness within such a material, which are both eigensolutions for propagation in a chiral medium [117]. Firstly it is assumed that the incident fields are right-handed polarised with the following form

$$\vec{E}^{ext} = (\hat{x} - j\hat{y}) E_0 e^{j(\omega t - kz)} \tag{4.24}$$

$$\vec{H}^{ext} = (j\hat{x} + \hat{y}) \frac{E_0}{Z_0} e^{j(\omega t - kz)} \tag{4.25}$$

It should be highlighted again that the IEEE standard definitions are used, which are in opposite to those widely employed among physicists [125],[126]. Since all elements are oriented in either $\hat{x}$ or $\hat{y}$, the discrete electric and magnetic dipole moments in these two orthogonal directions can be written as

$$p_x = \alpha^{ee} (E_0 + \beta p_x) + \alpha^{em} \mu_0 \left(j \frac{E_0}{Z_0} + \epsilon_0 \beta m_x \right) \tag{4.26}$$

$$m_x = -\alpha^{em} (E_0 + \beta p_x) + \alpha^{mm} \mu_0 \left(j \frac{E_0}{Z_0} + \epsilon_0 \beta m_x \right) \tag{4.27}$$

$$p_y = \alpha^{ee} (-jE_0 + \beta p_y) + \alpha^{em} \mu_0 \left(\frac{E_0}{Z_0} + \epsilon_0 \beta m_y \right) \tag{4.28}$$

$$m_y = -\alpha^{em} (-jE_0 + \beta p_y) + \alpha^{mm} \mu_0 \left(\frac{E_0}{Z_0} + \epsilon_0 \beta m_y \right) \tag{4.29}$$

The time harmonic term $e^{j(\omega t - kz)}$ has been omitted.

The polarisabilities have the balance relation as in (4.6), which can be transformed to

$$\frac{\alpha^{mm}}{\alpha^{em}} = -\frac{\alpha^{em}}{\alpha^{ee}} \stackrel{defined}{=} -jK_{em} \quad (4.30)$$

Then the electric and magnetic dipole moments induced on a chiral helical resonator are linearly correlated as

$$\vec{m} = -jK_{em}\vec{p} \quad (4.31)$$

Therefore K_{em} can be termed as "chirality ratio". From the expressions of polarisabilities (4.3) –(4.5), its expression can be derived as

$$K_{em} = \pm \frac{\pi r^2 \epsilon_r \omega}{2h} \quad (4.32)$$

which is a real number. The definition of its sign is the same as that of α^{em} , i.e., plus for left-handed element and minus for right-handed one. Then the following explicit results for the dipole moments can be deduced after some algebra

$$p_x = \frac{\alpha^{ee} \left(1 - \frac{K_{em}}{c_0}\right)}{1 - \left(1 + \frac{K_{em}^2}{c_0^2}\right) \alpha^{ee} \beta} E_0 \quad (4.33)$$

$$p_y = \frac{-j\alpha^{ee} \left(1 - \frac{K_{em}}{c_0}\right)}{1 - \left(1 + \frac{K_{em}^2}{c_0^2}\right) \alpha^{ee} \beta} E_0 \quad (4.34)$$

$$m_{x,y} = -jK_{em}p_{x,y} \quad (4.35)$$

Similar expressions can be derived for left-handed polarised incident circular wave

$$\vec{E}^{ext} = (\hat{x} + j\hat{y}) E_0 e^{j(\omega t - kz)} \quad (4.36)$$

$$\vec{H}^{ext} = (-j\hat{x} + \hat{y}) \frac{E_0}{Z_0} e^{j(\omega t - kz)} \quad (4.37)$$

The induced electric dipole moments are

$$p_x = \frac{\alpha^{ee} \left(1 + \frac{K_{em}}{c_0}\right)}{1 - \left(1 + \frac{K_{em}^2}{c_0^2}\right) \alpha^{ee} \beta} E_0 \quad (4.38)$$

$$p_y = \frac{j\alpha^{ee} \left(1 + \frac{K_{em}}{c_0}\right)}{1 - \left(1 + \frac{K_{em}^2}{c_0^2}\right) \alpha^{ee} \beta} E_0 \quad (4.39)$$

After obtaining the expressions of dipole moments and associated surface polarisation densities, the macroscopic transmission properties of unpunctured continuous sheet are aimed to be modelled with the aid of classical boundary conditions [108]. It is known that the surface electric and magnetic polarisation densities $\vec{p}_s, \vec{m}_s$ are related to relevant surface current densities $\vec{J}, \vec{J}_m$ as follows

$$\vec{J} = \frac{\partial}{\partial t} \vec{p}_s = j\omega \vec{p}_s \quad (4.40)$$

$$\vec{J}_m = \frac{\partial}{\partial t} \mu_0 \vec{m}_s = j\omega \mu_0 \vec{m}_s \quad (4.41)$$

while the classical boundary conditions give

$$\hat{z} \times (\vec{H}_2 - \vec{H}_1) = \vec{J} \quad (4.42)$$

$$(\vec{E}_2 - \vec{E}_1) \times \hat{z} = \vec{J}_m \quad (4.43)$$

where $\hat{z}$ is the normal direction of the continuous sheet (and direction of incidence). Subscripts 1,2 represent the illuminated and transmitted sides of the sheet separately. In the current configuration both sides of the slab are free space, therefore surface current densities $\vec{J}/\vec{J}_m$ create magnetic/electric fields with the same amplitude but opposite directions on the two sides of the sheet, respectively. By summing up the contributions from both surface electric and magnetic polarisation densities $\vec{p}_s, \vec{m}_s$ we have

$$\vec{E}^{trans} = \vec{E}^{ext} - \frac{j\omega Z_0 \vec{p}_s}{2} + \hat{z} \times \frac{j\omega \mu_0 \vec{m}_s}{2} \quad (4.44)$$

$$\vec{E}^{refl} = -\frac{j\omega Z_0 \vec{p}_s}{2} - \hat{z} \times \frac{j\omega \mu_0 \vec{m}_s}{2} \quad (4.45)$$

where $\vec{E}^{trans}$ and $\vec{E}^{refl}$ denote the electric fields of EM wave transmitted through and reflected by the slab at the position of the equivalent planar sheet separately. $\vec{E}^{trans}$ corresponds to $\vec{E}_2$ in the boundary condition (4.43), while $\vec{E}^{ext} + \vec{E}^{refl}$ equals $\vec{E}_1$.

Define the transmission and reflection coefficients for circularly polarised EM waves

$$T_{ab} \stackrel{defined}{=} \frac{E_a^{trans}}{E_b^{ext}} \quad (4.46)$$

$$R_{ab} \stackrel{defined}{=} \frac{E_a^{refl}}{E_b^{ext}} \quad (4.47)$$

where subscripts a and b are either r or l , representing right-/left-handed polarisations separately. a and b indicate the polarisation states of transmitted/reflected and incident waves, respectively. Substituting the expressions for dipole moments (4.33), (4.34), (4.38), (4.39) into the field equations (4.44), (4.45), the transmission and reflection coefficients of a cir-

cular wave can be deduced as follows:

$$T_{rr} = 1 - \frac{\frac{j\omega Z_0 \alpha^{ee}}{2S_0} \left(1 - \frac{K_{em}}{c_0}\right)^2}{1 - \left(1 + \frac{K_{em}^2}{c_0^2}\right) \alpha^{ee} \beta} \quad (4.48)$$

$$T_{ll} = 1 - \frac{\frac{j\omega Z_0 \alpha^{ee}}{2S_0} \left(1 + \frac{K_{em}}{c_0}\right)^2}{1 - \left(1 + \frac{K_{em}^2}{c_0^2}\right) \alpha^{ee} \beta} \quad (4.49)$$

$$T_{rl} = T_{lr} = 0 \quad (4.50)$$

$$R_{rl} = R_{lr} = -\frac{\frac{j\omega Z_0 \alpha^{ee}}{2S_0} \left(1 - \frac{K_{em}}{c_0}\right)^2}{1 - \left(1 + \frac{K_{em}^2}{c_0^2}\right) \alpha^{ee} \beta} \quad (4.51)$$

$$R_{rr} = R_{ll} = 0 \quad (4.52)$$

These analytical results provide explicit characterisation on the dichroism properties of the infinite 2D array of chiral elements. First, as can be seen in the expressions of T_{rr} (4.48) and T_{ll} (4.49), the difference in these two transmissions, which contributes to dichroism, arises from the chirality of the elements (magnetoelectric polarisability α^{em}) which is in line with intuition. Second, several other factors influence the dichroism, including element properties such as the polarisability α^{ee} and the chirality ratio K_{em} , as well as the interaction constant β which reflects attributes of the array (elements densities depicted by S_0 and the hole radius R_a). The impact of these factors on the macroscopic dichroism of array will be assessed in § 4.3. Finally, there is no polarisation conversion effect between the two circular EM waves, as stated in (4.50). It should be emphasised again that the lattice period d denotes the spacing between adjacent elements with the same orientation.

Unlike the transmission coefficients, the reflection coefficients are the same for the two polarisation states, as illustrated in (4.51) and (4.52). Therefore it can be inferred that the part of energy represented by the difference in the transmission coefficients of circularly polarised plane waves is dissipated within the lossy chiral array rather than reflected back. It needs to be pointed out that the handedness of reflected circular wave is in opposite to those of incident and transmitted waves since the propagation direction is reversed.

The model developed has established a generalised explicit relation between the microscopic characteristics of individual elements to the macroscopic transmission properties of the whole array, describing the physical nature of excitation of elements by external incidence and interaction among elements in the array, therefore can find great application potential for analysing frequency selective surfaces comprising of regular array of elements. The procedure is also straightforward, without calculating complicated double Fourier transformations and Bessel function integrals as in other references [119],[120] which also introduces a great deal of approximations as a consequence. Thus the accuracy of the model can be guaranteed. In following sections the model will be employed for predicting the transmission properties of 2D chiral arrays, as well as analysing their dependence on various microscopic parameters.

The approximations and assumptions involved in the modelling procedure include the following aspects.

- Firstly, the elements are characterised as discrete dipole moments via their polarisabilities and the local fields acting on them, therefore the physical dimensions of elements should be small compared with the free space wavelength around resonance.

- Secondly, only fundamental mode dipole interactions among elements have been taken into account, while higher order interactions are neglected. This is valid when the size of elements are not significant comparing with their spacing. An upper limit ratio of 0.7 has been suggested in [120] for valid approximation of dipole interaction.
- Thirdly, the infinite array of elements is replaced by a continuous sheet when modelling the macroscopic transmission properties, with the surface polarisation densities equal the average of the discrete dipole moments over a unit area of the square lattice. Such approximation holds true when the lattice spacing is also much smaller than the wavelength of interest (an upper limit ratio of 0.5 has also been suggested in [120]).
- In addition, the deduced macroscopic fields vary only on a scale comparable with the free space wavelength at the frequency range of interest, therefore the precise knowledge of field distribution near individual elements are excluded.
- Finally, the expressions for interaction constant β (4.18) and the hole radius R_a (4.21) are developed in static limit, which have to be re-examined for accuracy when being applied to the current situation. In § 4.1.3 improved expressions for β and polarisabilities will be derived which incorporate energy conservation, thus can provide results that are more physically meaningful.

In summary, the elements should be "densely" distributed in terms of the free space wavelength of interest, but "sparse" in terms of the element size compared with the lattice spacing. The valid range of the approximations are aimed to be quantitatively assessed in the following by comparison with results from numerical simulations and experimental measurements.

4.1.3 Improved Model Incorporating Energy Conservation

In this subsection modified expressions for the interaction constant β as well as polarisability α^{ee} will be derived. Specifically, their imaginary parts are revised from energy conservation point of view, thus can provide results which better comply with physical laws than the original ones deduced from quasi-static analysis.

The modification of β can be derived with the aid of lossless non-chiral electric dipoles. First we analyse an individual lossless electric dipole scatterer under external polarisation E^{ext} in free space. The induced dipole moment is

$$p = \alpha^{ee} E^{ext} \quad (4.53)$$

Thus the time-averaged power absorbed from the incident field to excite the dipole is

$$\begin{aligned} P^{abs} &= \frac{1}{2} \operatorname{Re} (E^{ext} \cdot J^*) = \frac{1}{2} \operatorname{Re} \left(E^{ext} \cdot \frac{\partial}{\partial t} p^* \right) \\ &= \frac{1}{2} \operatorname{Re} (-j\omega E^{ext} p^*) = \frac{1}{2} \operatorname{Re} (-j\omega (\alpha^{ee})^*) |E^{ext}|^2 \\ &= -\frac{\omega}{2} \operatorname{Im} (\alpha^{ee}) |E^{ext}|^2 \end{aligned} \quad (4.54)$$

where the asterisk represents complex conjugate. J denotes the electric current induced. Comparing with quasi-static analysis, an imaginary part reflecting the scattering effect is introduced into the polarisability α^{ee} of element, which is a complex number even for lossless case.

On the other hand, an electric dipole radiates power

$$P^{rad} = \frac{\mu_0 \omega^4 |p|^2}{12\pi c_0} = \frac{\mu_0 \omega^4 |a^{ee}|^2}{12\pi c_0} |E^{ext}|^2 \quad (4.55)$$

No energy is dissipated inside a lossless element, therefore the above two powers in (4.54) and (4.55) should be equal, which leads to

$$\text{Im}(\alpha^{ee}) = -\frac{\mu_0 \omega^3 |a^{ee}|^2}{6\pi c_0} \quad (4.56)$$

and

$$\text{Im}\left(\frac{1}{\alpha^{ee}}\right) = \frac{\mu_0 \omega^3}{6\pi c_0} \quad (4.57)$$

As can be seen, the introduction of such an imaginary part reflects the radiation from an individual dipole.

Next, an infinite regular 2D square array based on these lossless electric dipole scatterers will be analysed from energy conservation point of view. From the definition of β (4.18), it is known that the local electric field exciting each individual element in the array is the sum of external incidence and β times the dipole moment, i.e.,

$$p = \alpha^{ee} (E^{ext} + \beta p) \quad (4.58)$$

which gives

$$p = \frac{\alpha^{ee}}{1 - \alpha^{ee}\beta} E^{ext} \quad (4.59)$$

Adopting the same approximation as in § 4.1.1 which replaces the infinite array of

electric dipoles by a planar sheet with equivalent electric surface current flow

$$\vec{J} = \frac{j\omega\vec{p}}{S_0} \quad (4.60)$$

Transmission of EM wave through a lossless sheet fulfills the following energy conservation condition (with no contribution from the magnetic counterpart)

$$\text{Re} \left(\vec{E}^{total} \cdot \vec{J}^* \right) = 0 \quad (4.61)$$

where $\vec{E}^{total}$ represents the total average electric field at the sheet position. From transmission conditions (4.44) we have

$$\vec{E}^{total} = \vec{E}^{ext} - \frac{j\omega Z_0}{2S_0} \vec{p} \quad (4.62)$$

where S_0 denotes the unit cell area that is occupied by a single dipole in the array. Substituting (4.59), (4.60) and (4.62) into (4.61), the following results can be deduced

$$\begin{aligned} \text{Re} \left(\vec{E}^{total} \cdot \vec{J}^* \right) &= \text{Re} \left[\left(\frac{1 - \alpha^{ee}\beta}{\alpha^{ee}} p - \frac{j\omega Z_0}{2S_0} p \right) \cdot \left(\frac{j\omega p}{S_0} \right)^* \right] \\ &= \text{Re} \left[\left(-\frac{j\omega}{S_0} \frac{1 - \alpha^{ee}\beta}{\alpha^{ee}} - \frac{\omega^2 Z_0}{2S_0^2} \right) |p|^2 \right] = 0 \end{aligned} \quad (4.63)$$

which leads to

$$\text{Im} \left(\frac{1}{\alpha^{ee}} - \beta \right) = \frac{Z_0 \omega}{2S_0} \quad (4.64)$$

and

$$\text{Im}(\beta) = \frac{\mu_0 \omega^3}{6\pi c_0} - \frac{Z_0 \omega}{2S_0} \quad (4.65)$$

This imaginary part, which is physically responsible for the energy transmission, only depends on the frequency and the array period specified. It is interesting to notice that by incorporating both the power scattered by individual dipole (ω^3 term) and that by plane-wave propagation (ω term), such an imaginary part guarantees that the overall energy of EM wave in a regular array is transported by plane wave propagation only, with no individual scattering occurring. Thus the energy conservation law can be fulfilled.

Since β characterises the interaction of a single element with the fields generated by all other elements, it should be independent with the intrinsic properties of particular elements. Therefore the modified expression for the interaction constant β incorporating energy conservation has the following form

$$\begin{aligned}\beta &= \operatorname{Re} \left[\frac{1}{4\epsilon_0 R_a S_0} (1 - jkR_a) e^{-jkR_a} \right] + j \left(\frac{\mu_0 \omega^3}{6\pi c_0} - \frac{Z_0 \omega}{2S_0} \right) \\ &= \frac{\cos(kR_a) - kR_a \sin(kR_a)}{4\epsilon_0 R_a S_0} + j \left(\frac{\mu_0 \omega^3}{6\pi c_0} - \frac{Z_0 \omega}{2S_0} \right)\end{aligned}\quad (4.66)$$

which should hold true for both lossless and lossy elements. The above process is also reported in [121].

The level of modification on the imaginary part of β can be assessed as follows. Let

$$\begin{aligned}\operatorname{Im} \left[\frac{1}{4\epsilon_0 R_a S_0} (1 - jkR_a) e^{-jkR_a} \right] &= -\frac{\sin(kR_a) + kR_a \cos(kR_a)}{4\epsilon_0 R_a S_0} \\ &= \frac{\mu_0 \omega^3}{6\pi c_0} - \frac{Z_0 \omega}{2S_0}\end{aligned}\quad (4.67)$$

By employing Taylor series expansion of the trigonometric functions, it can be deduced that

$$\sin(kR_a) + kR_a \cos(kR_a) = 2kR_a - \frac{2}{3}(kR_a)^3 + O[(kR_a)^5] \quad (4.68)$$

therefore

$$-\frac{2kR_a - \frac{2}{3}(kR_a)^3}{4\epsilon_0 R_a S_0} = -\frac{Z_0\omega}{2S_0} + \frac{\omega^3 R_a^2}{6\epsilon_0 S_0 c_0^3} = \frac{\mu_0\omega^3}{6\pi c_0} - \frac{Z_0\omega}{2S_0} \quad (4.69)$$

which finally leads to

$$R_a^2 = \frac{S_0}{\pi} = \frac{d^2}{\pi} \quad (4.70)$$

$$R_a = \frac{d}{\sqrt{\pi}} \approx 0.5642d \quad (4.71)$$

Comparing with the result (4.21) developed from quasi-static analysis, it can be seen that the hole radius R_a is reduced in order to obtain better accuracy under the current situation. Actually, it has been discovered that the developed array transmission properties are much more resilient to the change in the real part of β than that in the imaginary part. Since its imaginary part has been accurately estimated according to energy conservation law, in the following text β will have the form as in (4.66) with the static value of R_a (4.21) still being employed.

After obtaining the improved expression for the interaction constant, the polarisabilities of double helical resonator elements deduced in § 4.1.1 are subsequently aimed to be revised. The chiral elements are again assumed to be lossless in the first instance.

Now the energy conservation condition for transmission of planar EM waves with arbitrary polarisation through a lossless sheet is as follows, incorporating the contribution from the magnetic part

$$\begin{aligned} \text{Re} \left[\vec{E}^{total} \cdot \vec{J}^* + \left(\vec{H}^{total} \right)^* \cdot \vec{J}_m \right] &= 0 \\ \Rightarrow \text{Re} \left[\left(\vec{E}^{ext} - \frac{j\omega Z_0}{2S_0} \vec{p} \right) \cdot \left(\frac{j\omega}{S_0} \vec{p} \right)^* + \left(\vec{H}^{ext} - \frac{j\omega\mu_0}{2Z_0 S_0} \vec{m} \right)^* \cdot \left(\frac{j\omega\mu_0}{S_0} \vec{m} \right) \right] &= 0 \end{aligned} \quad (4.72)$$

Assuming that the linear relations among the three polarisabilities (4.6) , (4.30) , (4.31) developed in the quasi-static situation still hold, with a real number chirality ratio K_{em} , the above energy conservation condition can be rewritten as

$$\text{Re} \left[-j\vec{E}^{ext} \cdot \vec{p}^* + K_{em}\mu_0 \left(\vec{H}^{ext} \right)^* \cdot \vec{p} \right] = \frac{\omega Z_0}{2S_0} \left(1 + \frac{K_{em}^2}{c_0^2} \right) |\vec{p}|^2 \quad (4.73)$$

On the other hand, the excited vector electric dipole moment on individual chiral element has the following form

$$\begin{aligned} \vec{p} &= \alpha^{ee} \left(\vec{E}^{ext} + \beta \vec{p} \right) + \alpha^{em} \mu_0 \left(\vec{H}^{ext} + \epsilon_0 \beta \vec{m} \right) \\ &= \alpha^{ee} \left(\vec{E}^{ext} + \beta \vec{p} \right) + jK_{em} \alpha^{ee} \mu_0 \left(\vec{H}^{ext} - jK_{em} \epsilon_0 \beta \vec{p} \right) \\ &\Rightarrow \left[\frac{1}{\alpha^{ee}} - \left(1 + \frac{K_{em}^2}{c_0^2} \right) \beta \right] \vec{p} = \vec{E}^{ext} + jK_{em} \mu_0 \vec{H}^{ext} \end{aligned} \quad (4.74)$$

By calculating the inner product with $-j\vec{p}^*$ on both sides, we have

$$\begin{aligned} -j \left[\frac{1}{\alpha^{ee}} - \left(1 + \frac{K_{em}^2}{c_0^2} \right) \beta \right] |\vec{p}|^2 &= -j\vec{E}^{ext} \cdot \vec{p}^* + K_{em}\mu_0 \vec{H}^{ext} \cdot \vec{p}^* \\ &\Rightarrow \text{Im} \left[\frac{1}{\alpha^{ee}} - \left(1 + \frac{K_{em}^2}{c_0^2} \right) \beta \right] |\vec{p}|^2 \\ &= \text{Re} \left[-j\vec{E}^{ext} \cdot \vec{p}^* + K_{em}\mu_0 \vec{H}^{ext} \cdot \vec{p}^* \right] \\ &= \text{Re} \left[-j\vec{E}^{ext} \cdot \vec{p}^* + K_{em}\mu_0 \left(\vec{H}^{ext} \right)^* \cdot \vec{p} \right] \\ &= \frac{\omega Z_0}{2S_0} \left(1 + \frac{K_{em}^2}{c_0^2} \right) |\vec{p}|^2 \\ &\Rightarrow \text{Im} \left(\frac{1}{\alpha^{ee}} \right) = \left(1 + \frac{K_{em}^2}{c_0^2} \right) \left(\text{Im}(\beta) + \frac{\omega Z_0}{2S_0} \right) \end{aligned} \quad (4.75)$$

Substituting the modified expression of imaginary part of the interaction constant β

(4.65) leads to the following result

$$\text{Im} \left(\frac{1}{\alpha^{ee}} \right) = \left(1 + \frac{K_{em}^2}{c_0^2} \right) \frac{\mu_0 \omega^3}{6\pi c_0} \quad (4.76)$$

Comparing with that for the electric polarisability of non-chiral element (4.57), the imaginary part increases by $(K_{em}/c_0)^2$ which reflects the impact of chirality, as the excitation of both electric and magnetic dipole moments imply greater power absorption and radiation from a chiral element than its non-chiral counterpart. The increased amount is directly relevant with the chirality ratio, which coincides with intuition.

The above analysis is based on lossless assumption. When dissipation inside individual element is introduced, the full expression for the electric polarisability of double helical resonator can be written as

$$\alpha^{ee} = \left\{ \frac{(\epsilon_r \omega)^2 L}{4h^2} \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right) + j \left[1 + \left(\frac{\epsilon_r \omega \pi r^2}{2hc_0} \right)^2 \right] \frac{\mu_0 \omega^3}{6\pi c_0} \right\}^{-1} \quad (4.77)$$

Comparing with the quasi-static result (4.5), an additional imaginary part is introduced into the denominator to reflect the scattering effect.

The other polarisabilities can be deduced as follows by employing the linear dependence (4.31) – (4.32)

$$\alpha^{em} = \pm \left[\left(\frac{2h}{\epsilon_r \omega \pi r^2} + \frac{\epsilon_r \omega \pi r^2}{2hc_0^2} \right) \frac{\mu_0 \omega^3}{6\pi c_0} - j \frac{\epsilon_r \omega L}{2\pi r^2 h} \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right) \right]^{-1} \quad (4.78)$$

$$\alpha^{mm} = \left\{ \frac{L}{(\pi r^2)^2} \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right) + j \left[\left(\frac{2h}{\epsilon_r \omega \pi r^2} \right)^2 + \frac{1}{c_0^2} \right] \frac{\mu_0 \omega^3}{6\pi c_0} \right\}^{-1} \quad (4.79)$$

It can be seen that additional real and imaginary part are incorporated into the denominator in the expressions for cross- and co-polarisability, respectively.

In this subsection, the expressions for the array interaction constant β and element polarisabilities are revised, so that the scattering effects of individual elements are eliminated, and the energy of EM wave through the array are transmitted in plane wave form only which fulfills the energy conservation law. Therefore these expressions (4.66), (4.77) – (4.79) should be employed instead of the original ones (4.18), (4.3) – (4.5) when estimating the transmission properties of 2D chiral array so as to obtain physically meaningful results.

4.2 Validation of the Analytical Model

4.2.1 Validation Using Reported Results

In order to validate the model developed, in this section the numerical simulation and experimental results reported in previously published work are employed [128],[129]. In these references a 2D interlocking square lattice of double helical resonators on both sides of FR-4 printed circuit boards were fabricated and assembled, whose parameters are listed in Table 4.1.

Table 4.1: The parameters of the 2D interlocking array of double helical resonators as in [128].

Substrate thickness (mm)	1.6	Outer radius (mm)	2.25
Relative dielectric constant	3.76	Metal strip width (mm)	1
Substrate loss tangent	0.0186	Gap width (mm)	0.3
Metal thickness (μm)	36	Via diameter (mm)	0.5
Lattice spacing (mm)	8	Number of unit cells in one	26
Slab size (mm^2)	208×208	dimension	

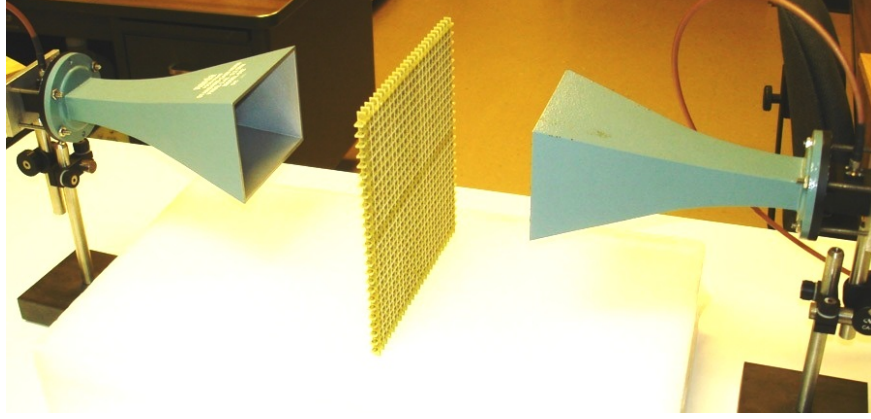


Figure 4-3: Measurement of transmission responses through the 2D chiral array by a pair of horn antennas, which were connected to a vector network analyser (not shown) [129].

The macroscopic transmissions of left- and right-handed circularly polarised EM waves through the slab were simulated and measured. Two linearly polarised horn antennas were employed as transmitter and receiver, as shown in Fig. 4-3, and the frequency responses for transmission of linear waves with two different orthogonal polarisations were acquired by a network analyser. The desired transmission coefficients for circularly polarised EM waves were deduced from linear combination of these responses for linearly polarised waves. The details of the measurement and data analysis will be illustrated in § 4.2.3.

The two sets of results derived from simulation and measurement separately are reproduced in Fig. 4-4, which are in good agreements. The resonant frequency of individual elements is around 7.8 GHz, and significant difference in the transmission responses appears around the element resonance. The "left-handed" circular EM wave experiences negligible absorption, while the other shows attenuation of around 6 dB. The notations T_{++} and T_{--} in the figure correspond to T_{rr} and T_{ll} in this thesis, respectively, while R denotes the identical reflection coefficient of the two polarisations. The polarisation

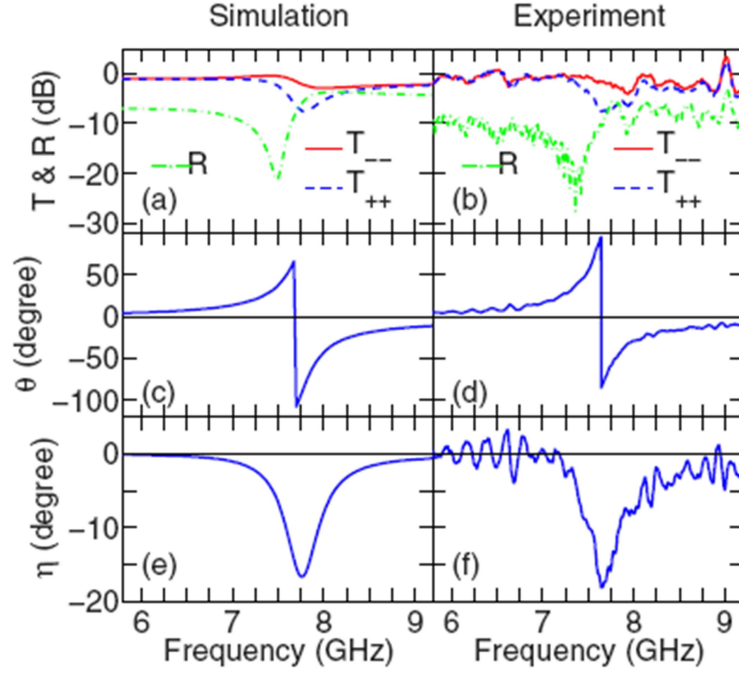


Figure 4-4: Simulation and measured results for the dichroism properties of 2D array of double helical resonators [128],[129]. The transmission and reflection amplitudes for the two circularly polarised EM waves (top panel), the azimuth rotation angle θ (middle panel) and the ellipticity η (bottom panel).

azimuth rotation,

$$\theta = \frac{1}{2} (\arg(T_{rr}) - \arg(T_{ll})) \quad (4.80)$$

and the ellipticity

$$\eta = \frac{1}{2} \sin^{-1} \left(\frac{|T_{rr}|^2 - |T_{ll}|^2}{|T_{rr}|^2 + |T_{ll}|^2} \right) \quad (4.81)$$

were also calculated, which gauge the differences in phase and amplitude of the transmissions of the two circular waves and thus characterise the dichroism. The peak values of the azimuth rotation θ (-100°) and ellipticity η (-17°) all appear at resonance.

In the following the developed analytical model is employed to estimate the dichroism properties of such an array. The elements are left-handed helices, therefore the sign of α^{em}

is positive. After substituting the parameters in Table 4.1 into the model, the frequency responses of various dichroism properties are obtained as exhibited in Fig. 4-5.

It can be noticed that the results from the analytical model have good correspondence with those from reference [128],[129] in absolute values, but some have reversed signs. For example, it is the right-handed circular wave that experiences negligible attenuation, and the ellipticity η is always positive. However, the results for azimuth rotation angle θ have good correspondence. The maximum attenuation of T_l is 8.0 dB, while the peak values of the azimuth rotation θ and ellipticity η are 86° and 22° respectively, all of which exhibit slightly larger dichroism than the reported results. The 3 dB bandwidth is 0.26 GHz. An upward frequency shift of around 0.2 GHz has been observed for the analytical results comparing with those from reference. This can be attributed to the inaccuracy in the prediction of element resonant frequency using quasi-static *RLC* model for the current resonator, whose thickness is already comparable with its radius. The relative error is $0.2 / 7.8 \approx 2.6\%$ which is still quite acceptable. The key parameters are summarised and compared in Table 4.2.

The reversed responses arise from the difference in both the definition on handedness of circular polarisation and the time harmonic assumption. In references [128],[129] the definitions of circular polarisation widely adopted by physicists, and $e^{-j\omega t}$ assumption, were employed; while IEEE standard definitions and $e^{j\omega t}$ assumption are used in the

Table 4.2: Comparison of parameters characterising dichroism of the 2D interlocking array of double helical resonators from the analytical model and reference [128].

Parameters	Analytical Model	Reference
Resonant Frequency (GHz)	8.0	7.8
$\min(T_{rr})$ (dB)	0	-6
$\min(T_l)$ (dB)	-8.0	0
Azimuth Rotation θ (deg)	86	-100
Ellipticity η (deg)	22	-17

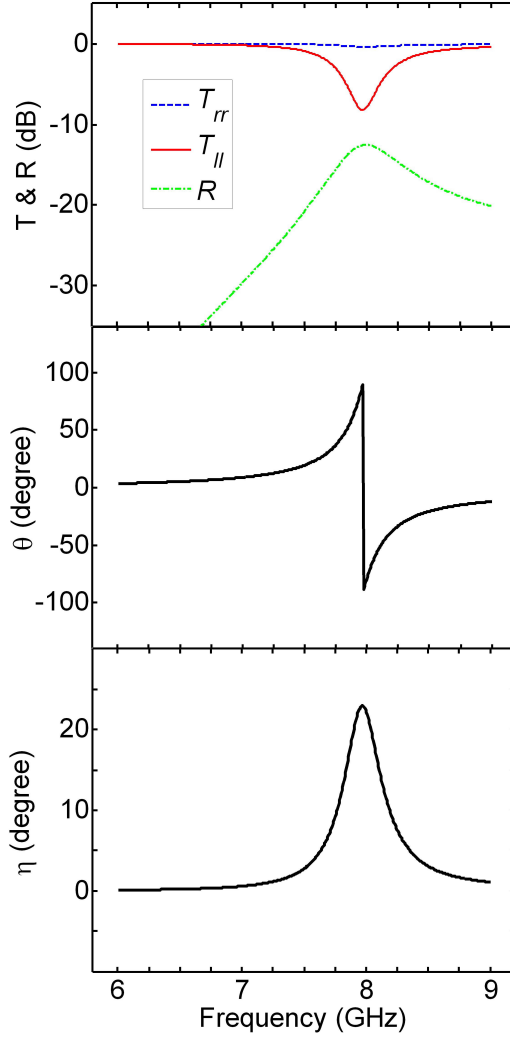


Figure 4-5: The transmission and reflection amplitudes for the two circularly polarised EM waves (top panel), the polarisation rotation angle θ (middle panel) and the ellipticity η (bottom panel) calculated from the analytical model.

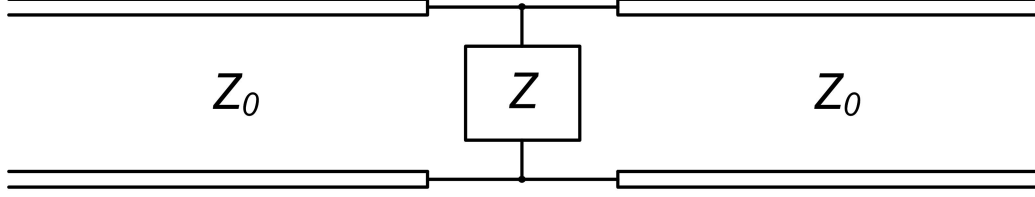


Figure 4-6: Transmission line model loaded with a shunt resonator.

model developed [125],[126]. These two definitions on handedness of circular polarisation are in contrary with each other, which leads to the reversed responses in transmission and ellipticity η , as can be comprehended. The similarity in azimuth rotation angle θ can be further illustrated using the conventional transmission line theory as follows.

Consider a shunt impedance Z loaded between a pair of transmission lines with characteristic impedance Z_0 , as depicted in Fig. 4-6. $e^{j\omega t}$ assumption is adopted. Z is in series resonant form, i.e.,

$$Z = R + j \left(\omega L - \frac{1}{\omega C} \right) \quad (4.82)$$

Then its imaginary part will have opposite sign if $e^{-j\omega t}$ is used. The traditional transmission line theory gives a transmission coefficient of

$$T = \frac{1}{1 + \frac{Z_0}{2Z}} = \left[1 + j \frac{Z_0}{2\omega L \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)} \right]^{-1} \quad (4.83)$$

where $\omega_0 = 1/\sqrt{LC}$ is the resonant frequency. For frequencies far away from resonance, the shunt impedance Z is much larger than the characteristic impedance Z_0 , and the transmission coefficient $T \approx 1$. At resonance, $T = (1 + Z_0/2R)^{-1} < 1$. Therefore the transmission line model imitates the absorption of normally incident circularly polarised

plane wave through the 2D array of chiral resonators.

The phase response will be investigated further. If the influence of resistance R is neglected, when $\omega < \omega_0$, the denominator has a positive imaginary part, therefore T has a negative imaginary part which corresponds to phase lag. When $\omega > \omega_0$, T has a positive imaginary part which corresponds to phase advance. On the other hand, if $e^{-j\omega t}$ is used, the reverse holds true.

These conclusions can be applied to verify the analytically derived phase response reflecting dichroism of the chiral array. For the current 2D array of left-handed helix elements, the propagation of left-handed circularly polarised EM wave (in IEEE standard) can be characterised by the loaded transmission line model, while the right-handed counterpart experiences negligible absorption that also incorporates little variation in the phase of transmitted signal. From the definition (4.80) it can be known that the polarisation rotation angle θ is in opposite to the phase response of resonator-loaded transmission line as derived above, i.e., positive before resonance and negative after resonance. The reverse holds true for arrays of right-handed chiral elements. Due to both the reversed definition on handedness and time harmonic assumption adopted, the phase response from the analytical model corresponds to that from the reference [128],[129].

In conclusion, the analytical model provides results that are in good coincidence with those reported from reference when being applied to predict the transmission properties of 2D interlocking array of left-handed double helical resonators. It should be emphasised that the diameter of elements and the lattice spacing are 0.12 and 0.21 times the free space wavelength at resonance separately, while their ratio is $2 \times 2.25 / 8 = 0.57$. The former two numbers demonstrate the validity of the analytical model derived in terms of element size and the "dense" level of array, while the latter ratio reflects the range of

effectiveness of dipole interactions among the elements.

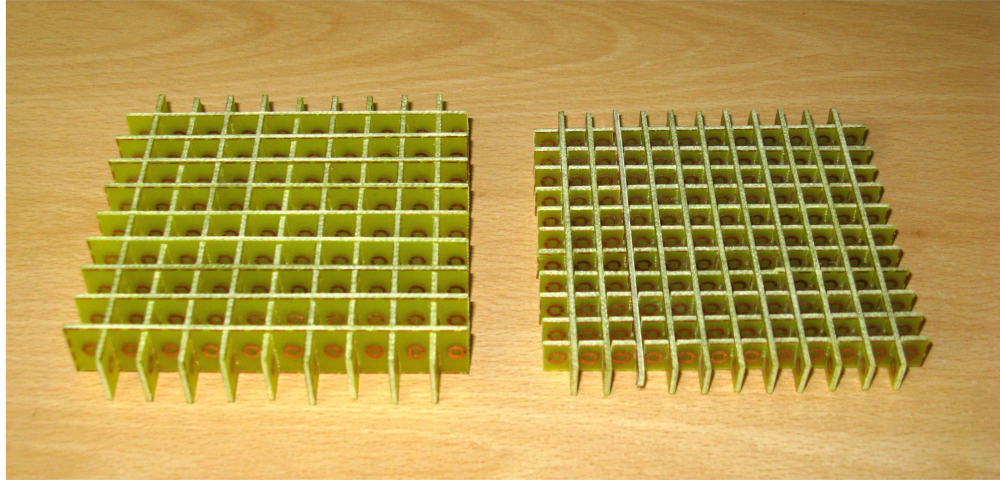
4.2.2 Validation by Measurement of Fabricated Sample Arrays

In order to further validate the accuracy of the derived analytical model, as well as investigate the influence of physical parameters of the array on the transmission properties, two interlocking square arrays of double helical resonators with different lattice spacings were fabricated and assembled, as shown in Fig. 4-7. Their parameters are listed in Table 4.3. One array has the same period (8 mm) as the one reported before in reference [128],[129], and the other has a larger period (10 mm). The number of elements are different in the two arrays in order to roughly maintain the overall slab size, and eliminate the impact of possible edge diffraction when making comparisons between the transmissions of EM wave through the two arrays. It has to be emphasised that the elements are right-handed helices whose physical dimensions are almost the same as the left-handed ones in § 4.2.1, which were fabricated on a FR-4 PCB with a different dielectric constant.

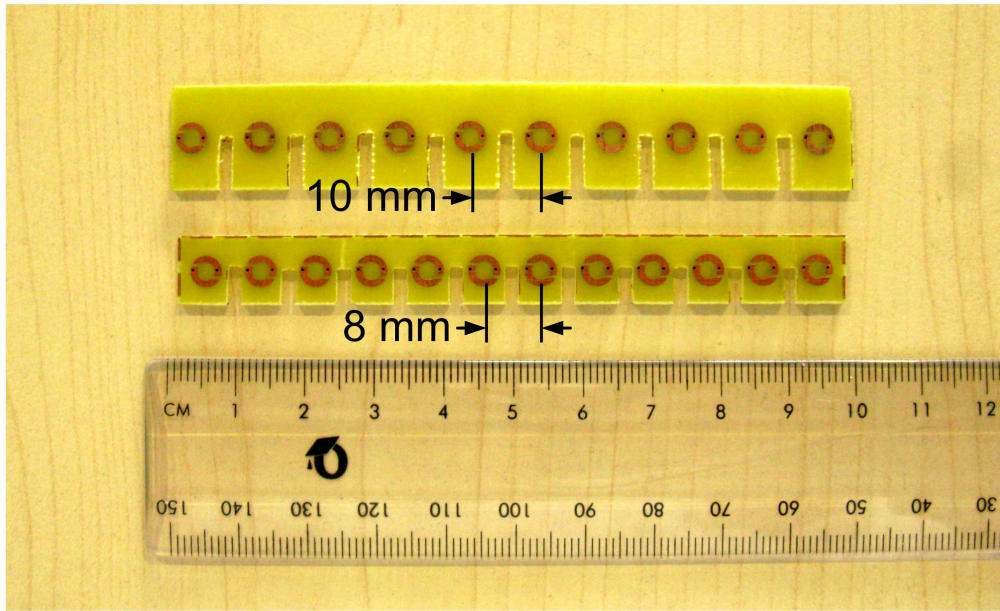
The frequency responses of dichroism properties of the fabricated chiral arrays were deduce from the measured transmission spectrums, which are plotted in Fig. 4-8 together with the results from analytical model as well as simulation using CST Micro-StripesTM [78]. The details of numerical simulation and measurement will be summarised in §

Table 4.3: The parameters of the fabricated 2D interlocking arrays of double helical resonators.

Substrate thickness (mm)	1.6	Outer radius (mm)	2.3
Relative dielectric constant	4.55	Metal strip width (mm)	1
Substrate loss tangent	0.0175	Gap width (mm)	0.5
Metal thickness (μm)	36	Via diameter (mm)	1
Lattice spacing (mm)	8	No. of unit cells in one dimension	12
	10		10
Slab size (mm^2)	100×100		



(a)



(b)

Figure 4-7: Pictures of (a) the two fabricated 2D interlocking square arrays of double helical resonators and (b) the comprising 1D array units on comb-shape PCB board.

Table 4.4: Comparison of parameters characterising dichroism of the fabricated 2D interlocking array with $d = 10$ mm from the analytical model, numerical simulation and measurement.

Parameters	Analytical	Simulation	Measurement
Resonant Frequency (GHz)	7.8	7.8	7.8
$\min(T_{rr})$ (dB)	-17	-13	-23
Azimuth Rotation θ (deg)	90	90	115
Ellipticity η (deg)	-37	-30	-41

Table 4.5: Comparison of parameters characterising dichroism of the fabricated 2D interlocking array with $d = 8$ mm from the analytical model, numerical simulation and measurement.

Parameters	Analytical	Simulation	Measurement
Resonant Frequency (GHz)	7.8	7.9	7.7
$\min(T_{rr})$ (dB)	-10	-7	-13
Azimuth Rotation θ (deg)	90	90	120
Ellipticity η (deg)	-25	-17	-31

4.2.3. The comparison of key parameters are listed in Table 4.4 and 4.5. Now it is the right-handed circular wave that experiences significant attenuation, and the corresponding ellipticity is negative. Good coincidences exist between analytical and experimental results, in which the fluctuation beyond 8 GHz can be attributed to the ineffective operation of the transmitter/receiver horn antennas, while the numerical data exhibit smaller chirality. Since the frequency responses of transmission from numerical simulation are the Fourier transformation of the corresponding truncated responses in the time-domain under pulse excitation, the truncation introduces unavoidable expansion of the bandwidth and decrease in the level of absorption in the frequency responses. More accurate results can be obtained by significantly increasing the simulation time, which can only provide marginal improvements.

All the results illustrate that the array with larger spacing ($d = 10$ mm) exhibits bigger difference in the transmission of the two circular waves around the element resonance, and a consequent greater ellipticity, while its bandwidth of absorption is smaller than that of

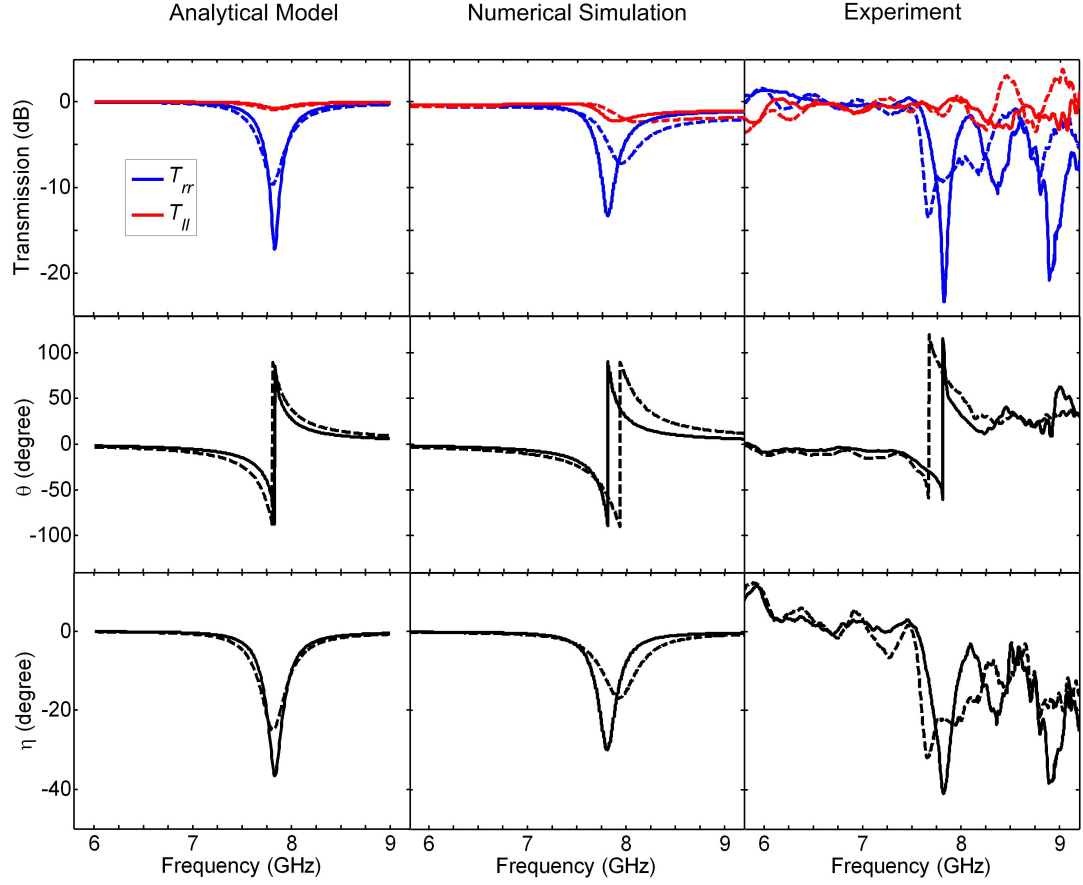


Figure 4-8: Comparison of circular transmission coefficients (top panel), linear polarisation rotation angle (middle panel) and ellipticity (bottom panel) obtained from analytical model (left column), numerical simulation (middle column) and experiment (right column). The solid lines represent results of array with $d = 10$ mm spacing while dashed lines represent those of the $d = 8$ mm array.

the array with smaller spacing ($d = 8$ mm). This conclusion is in line with intuition, as array with smaller spacing has stronger coupling among elements, which tends to result in wider bandwidth. The impact of lattice spacing on the dichroism properties of the chiral array will be further investigated in § 4.3.

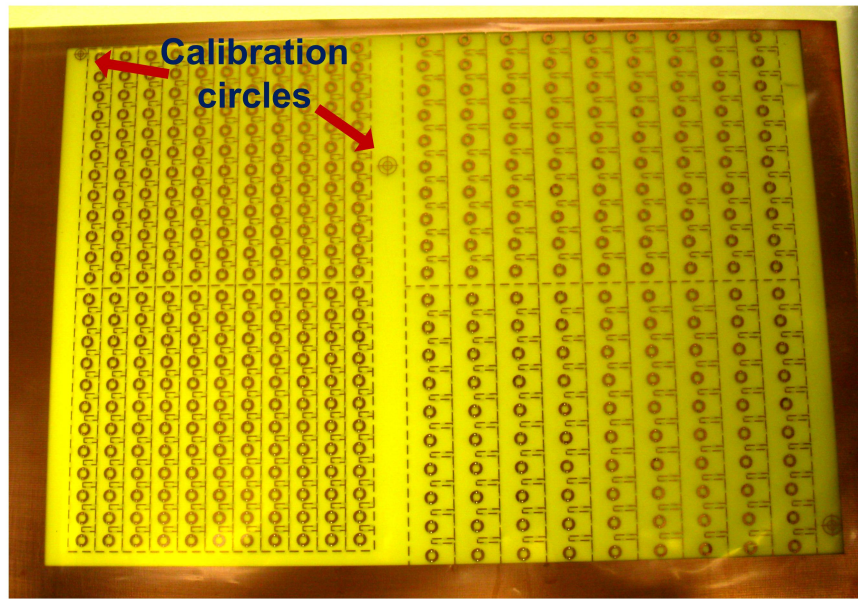
In conclusion, good correspondence in the dichroism properties have been obtained from results derived from analytical model, numerical simulation and experimental measurement for the fabricated arrays of right-handed double helical resonators resonating around 7.8 GHz. The element size (diameter) is roughly the same as that in § 4.2.1, which is 0.12 times the free space wavelength at element resonance. For the array with larger spacing ($d = 10$ mm), the ratio between spacing and the free space wavelength at resonance is 0.26. This value states the valid range of the derived analytical model in terms of the array density.

4.2.3 Details of Measurement and Simulation of 2D Chiral Arrays

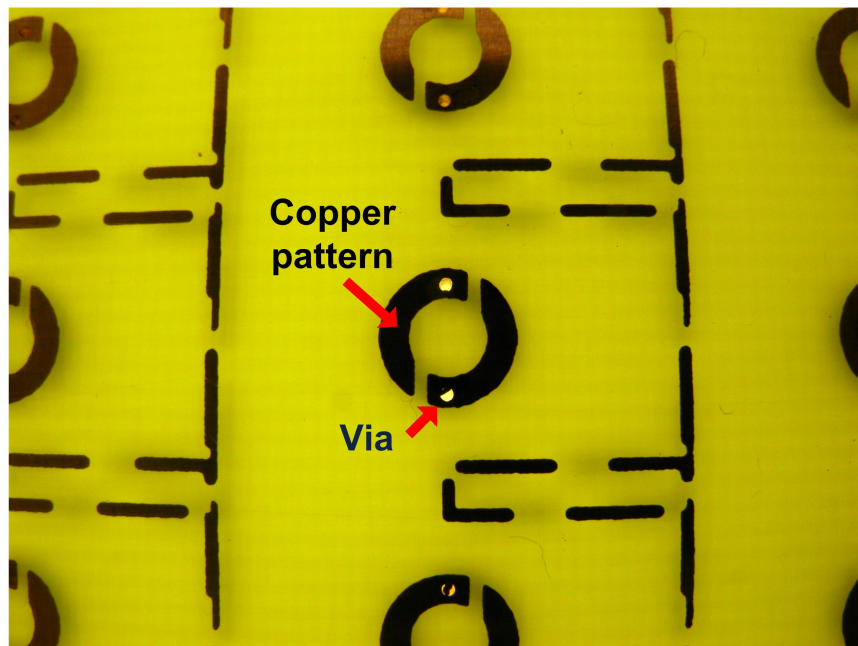
Measurement of Fabricated Finite 2D Interlocking Chiral Arrays

As mentioned in § 4.2.2, two interlocking square arrays of double helical resonators with roughly the same size but different periods were fabricated and assembled, with parameters listed in Table 4.3. The detailed process are explained below.

Firstly, all the identical double helical resonator elements were fabricated altogether as two 2D planar arrays on a plate PCB using standard optical lithography technique at the Photofabrication Unit, Department of Physics, University of Oxford, as depicted in Fig. 4-9. Two negative masks were designed each consisting of two 2D rectangular arrays with 9×20 and 11×24 unit cells of double-split-single-ring pattern, respectively.



(a)



(b)

Figure 4-9: Photos of the fabricated (a) square arrays of elements on a plate PCB; (b) the feature of individual double helical resonator. The dashed lines were used for guiding further processing.

The array layouts on the two masks were mirror images of each other, but the element patterns were the same because the metallic patterns of a double helical resonator on both sides of substrate are identical. The masks were printed on two transparent films separately, having a resolution of 600 dpi.

All the through holes were drilled on a bare photosensitised half-ounce copper-clad FR-4 circuit board according to the coordinates provided, which would become the metallic vias later connecting the metallic patterns of double helical resonator elements on both sides of substrate. Three additional holes were made as reference points for aligning patterns, as shown in Fig. 4-9(a) (calibration circles).

The circuit board was then covered by the negative masks on both sides, with the printed sides of films adhering to the board. The accuracy of the pattern alignment was guaranteed by collimating the centres of three target-shaped circles to the reference through holes separately. The masked board was then exposed to ultraviolet light for 5 minutes in a Kinsten three-bulb UV exposure unit. The photoresist was developed in a sodium carbonate solution and then placed in an ferric chloride etching solution for 15 minutes. All the through holes were covered by copper after this procedure forming effective metallic vias. Finally, the board was placed in a stripper solution (potassium hydroxide) for 10 minutes to remove the remaining photoresist.

After the fabrication of planar PCB, the long strip units (one-dimensional array of elements) were cut using a guillotine according to the dashed lines in the fabricated pattern, as shown in Fig. 4-9. The width of strips equals the array spacing. Next, the slots between elements, which were made for the purpose of joining the interlocking arrays of strips, were cut using a bandsaw. All the strip units were stacked together so as to guarantee the consistency in fabrication. The width of slots equals the substrate

thickness while their length equals a half the strip width, thus the cross joints between 1D strip units should be well fit.

The schematic and photo of the experiment for measuring the transmission properties of the 2D chiral arrays are provided in Fig. 4-10. The experiment configuration is essentially the same as that reported in [128],[129] (see Fig. 4-3) and [130], but with improvements as follows. The experiment was conducted in an anechoic chamber, and a 3 mm thick aluminium plate with a 15 cm $\times$ 15 cm square window was placed around the array sample under test. The aim of the latter is to reduce unwanted transmission around the sample edges, as well as provide a physical support. A vector network analyser (Agilent 8722ES) and a pair of identical horn antennas were employed to make measurements, and the measured data were collected by a computer via a Labview GPIB interface.

The antennas operate at a frequency range of 6.0 - 8.2 GHz, during which the reflection coefficient $S_{11} < -10\text{dB}$. This range covers the resonance of individual elements, which is estimated to be 7.9 GHz by the quasi-static circuit model. The 3 dB beamwidth of the horn antenna is 20° in the E plane ($x - z$ plane as in Fig. 4-10(a)) and 30° in the H plane ($y - z$ plane in Fig. 4-10(a)). Both antennas were carefully adjusted to align with the centre of sample throughout the experiment. In order to approximate normal incidence of plane waves on the array, the transmitter horn antenna was placed 55 cm away from the aluminium screen, so that the spread angle of the sample under test seen at the aperture plane of horn antenna is $2\tan^{-1}(5/55) = 10.4^\circ$ which is smaller than the 3 dB beamwidth. Therefore the EM wave incident on the array can be regarded as uniformly distributed. The receiver horn was placed close to the sample on the other side of screen so as to receive signal with larger amplitude.

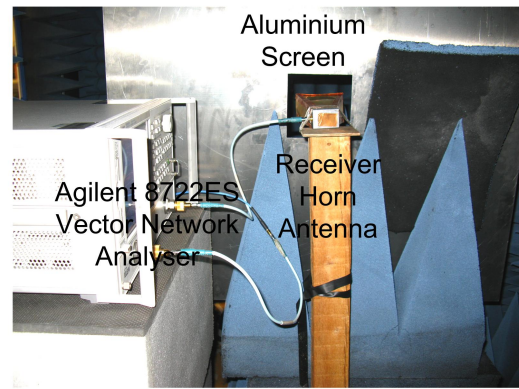
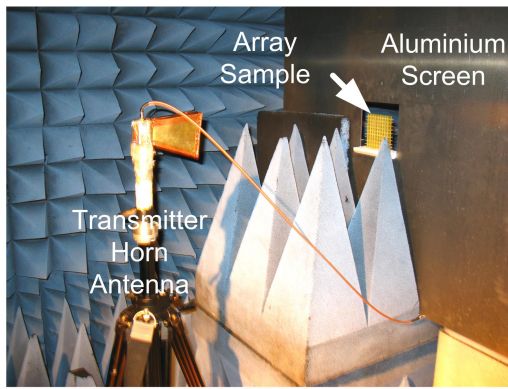
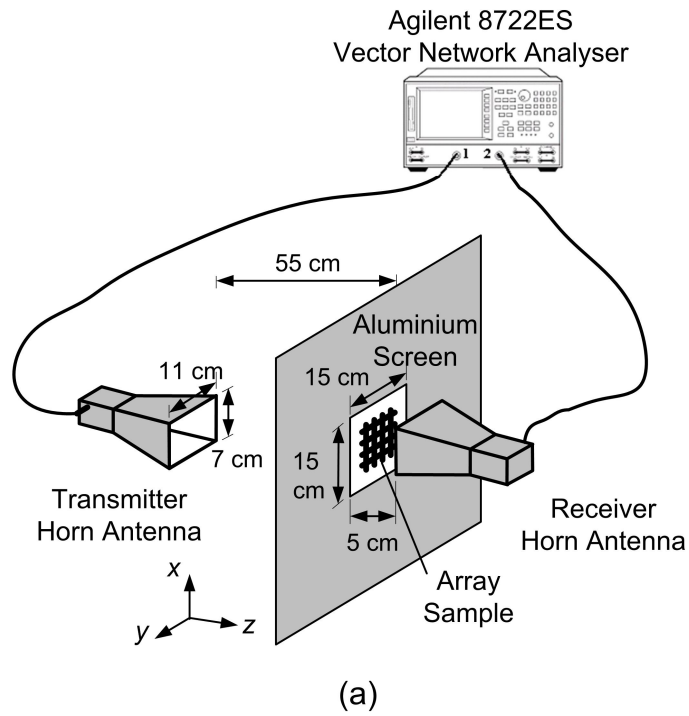


Figure 4-10: Schematic drawing of the measurement of transmissions through square arrays of double helical resonators. The array samples were placed inside the square window within an aluminium screen. A pair of horn antennas and Agilent 8722ES network analyser were employed for measurement.

Since the far-field radiation generated from a horn antenna is linearly polarised, four transmission responses were measured when the transmitter and receiver horns were either horizontally or vertically placed. The co- and cross-polarisation linear transmission coefficients were derived via normalising the measured response through the chiral array by the corresponding reference response without array samples in between. This will be discussed in explicit mathematical formula as follows.

First the reference transmission responses T_{ab}^{ref} between the two antennas through the punctured screen without samples were measured. The definition of the subscripts are the same as before, i.e., a (x or y) denotes the orientation of the E plane of the receiver horn antenna, thus the polarisation of the electric field of the received EM wave, while b (x or y) represents that of the transmitter antenna. The measured frequency response of transmission when the two antennas are placed as depicted in Fig. 4-10(a), for example, is defined as T_{xx}^{ref} . Due to the symmetry in experimental layout, T_{xx}^{ref} and T_{yy}^{ref} should be roughly equal if the phase centres remain steady when rotating both antennas during the experiment. In addition, no linear polarisation conversion effect should exist, i.e., $T_{xy}^{ref} = T_{yx}^{ref} = 0$.

Then if a chiral array is placed inside the square window, the received signal is the transmission of the unloaded screen itself modulated by the co-polarisation linear transmission coefficient of the array. Therefore the measured co-polarisation transmission responses of the array write

$$T_{xx}^{array} = T_{xx} \cdot T_{xx}^{ref} \quad (4.84)$$

$$T_{yy}^{array} = T_{yy} \cdot T_{yy}^{ref} \quad (4.85)$$

The polarisation conversion effect of linear plane wave now exists due to the presence

of the chiral array. If the receiver antenna is rotated 90° clockwise with respect to $\hat{z}$ axis, it is orthogonal to the transmitter antenna (with E plane in $y - z$ plane as in Fig. 4-10(a)), and the received signal is the transmission of the unloaded screen modulated by the cross-polarisation linear transmission coefficient of the array. Thus the measured cross-polarisation transmission responses of the array write

$$T_{yx}^{array} = T_{yx} \cdot T_{xx}^{ref} \quad (4.86)$$

$$T_{xy}^{array} = T_{xy} \cdot T_{yy}^{ref} \quad (4.87)$$

In a word, the linear coefficients of array are the measured transmission responses normalised by the reference responses without loading samples. The transmission coefficients for left- and right-handed circularly polarised EM waves can then be obtained by linear combination of the linear transmission coefficients, which will be deduced next.

Consider a chiral array under the normal incidence of linear planar EM wave

$$\vec{E}_{ext} = \hat{x} E_0 e^{j(\omega t - kz)} \quad (4.88)$$

The transmitted signal have the form

$$\vec{E}_{trans} = (\hat{x} T_{xx} + \hat{y} T_{yx}) E_0 e^{j(\omega t - kz)} \quad (4.89)$$

Similarly, the corresponding signal under the normal incidence of linear planar EM

wave with $\hat{y}$ -polarised electric field have the form

$$\vec{E}_{trans} = (\hat{x}T_{xy} + \hat{y}T_{yy}) E_0 e^{j(\omega t - kz)} \quad (4.90)$$

Therefore, the overall array transmission under the normal incidence of right-handed circularly polarised EM wave

$$\vec{E}_{ext} = (\hat{x} - j\hat{y}) E_0 e^{j(\omega t - kz)} \quad (4.91)$$

are the linear combination of the corresponding transmission responses of linear waves

$$\begin{aligned} \vec{E}_{trans} &= [(\hat{x}T_{xx} + \hat{y}T_{yx}) - j(\hat{x}T_{xy} + \hat{y}T_{yy})] E_0 e^{j(\omega t - kz)} \\ &= [\hat{x}(T_{xx} - jT_{xy}) + \hat{y}(T_{yx} - jT_{yy})] E_0 e^{j(\omega t - kz)} \\ &= T_{rr}(\hat{x} - j\hat{y}) E_0 e^{j(\omega t - kz)} + T_{lr}(\hat{x} + j\hat{y}) E_0 e^{j(\omega t - kz)} \end{aligned} \quad (4.92)$$

where the circular transmission coefficients write

$$T_{rr} = \frac{(T_{xx} + T_{yy}) - j(T_{xy} - T_{yx})}{2} \quad (4.93)$$

$$T_{lr} = \frac{(T_{xx} - T_{yy}) - j(T_{xy} + T_{yx})}{2} \quad (4.94)$$

The other two coefficients can be similarly derived as follows

$$T_{ll} = \frac{(T_{xx} + T_{yy}) + j(T_{xy} - T_{yx})}{2} \quad (4.95)$$

$$T_{rl} = \frac{(T_{xx} - T_{yy}) + j(T_{xy} + T_{yx})}{2} \quad (4.96)$$

From the symmetry point of view, it can be known that the linear transmission coefficients of the fabricated 2D square arrays should in theory satisfy the following relationships

$$T_{xx} = T_{yy}, T_{xy} = -T_{yx} \quad (4.97)$$

which lead to

$$T_{lr} = T_{rl} = 0 \quad (4.98)$$

i.e., no polarisation conversion between the two circular EM waves would occur. However the actual transmission coefficients may not fulfill these relations due to imperfections in the experimental conditions, thus all four linear coefficients were measured and derived in the experiment.

The measured frequency responses of transmission as well as the deduced linear coefficients are depicted in Fig. 4-11. As can be seen from the left plot in Fig. 4-11(a), the amplitude of the two orthogonal co-polarisation transmissions through the unloaded screen (T_{xx}^{ref} and T_{yy}^{ref}) are essentially the same, while the cross-polarisation transmissions (T_{xy}^{ref} and T_{yx}^{ref}) are roughly lower by more than an order of magnitude, thus can be safely neglected in the data analysis procedure. The frequency range of effective transmission between the two antennas matches the result from the reflection measurement of a single antenna, which demonstrates effective normal plane wave excitation of the investigated array of helical elements at resonance.

As for the responses of the screen loaded with chiral samples, the amplitude of the two orthogonal co-polarisation transmissions (T_{xx}^{array} and T_{yy}^{array}) agree well with each other for both of the two arrays, which exhibit sharp decrease around the element resonance comparing with the reference responses without samples (T_{xx}^{ref} and T_{yy}^{ref}). The agreements between those of the cross-polarisation transmissions (T_{xy}^{array} and T_{yx}^{array}) are not as good

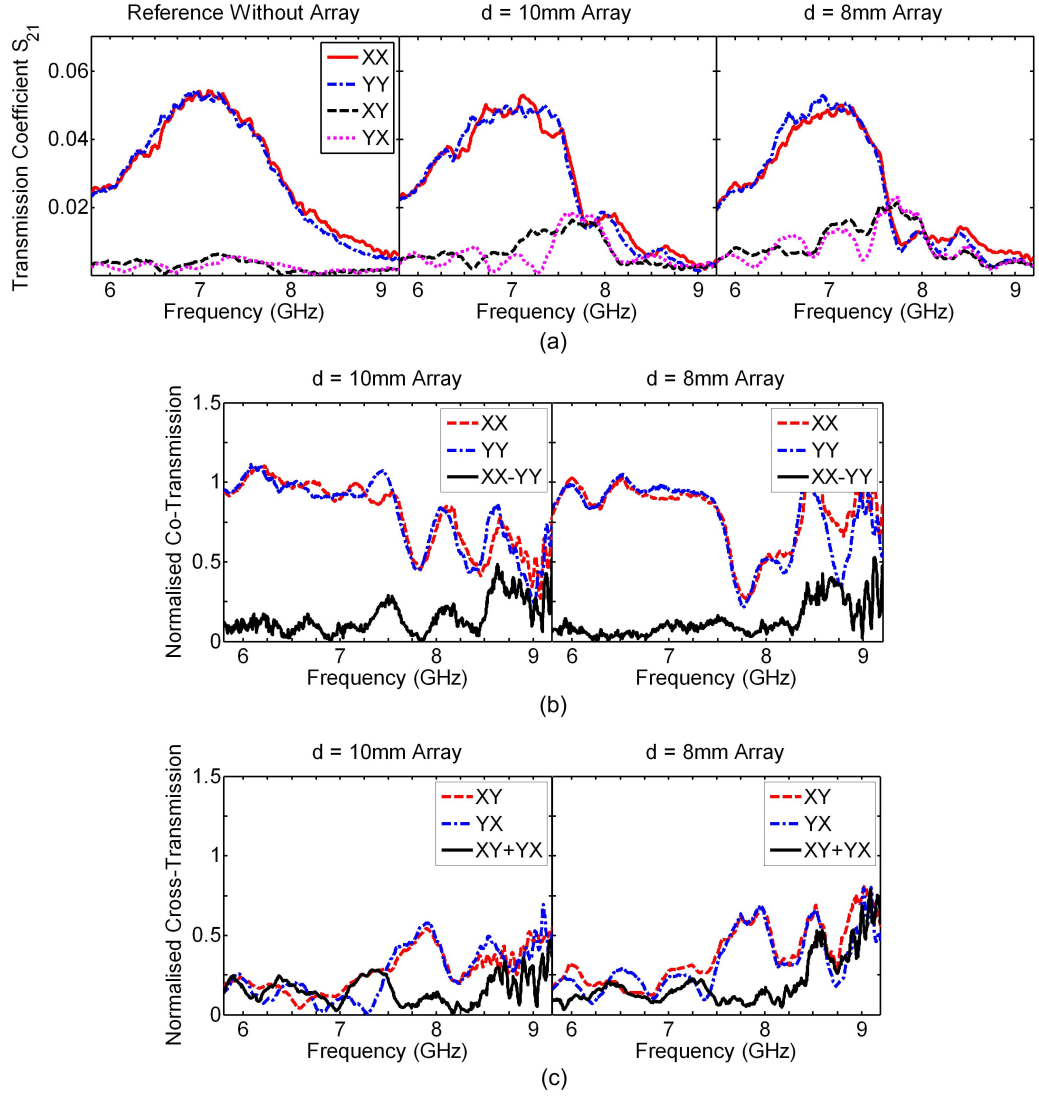


Figure 4-11: (a) Amplitude of the measured transmission responses of punctured aluminium screen itself and when loaded with the two chiral arrays. (b) Amplitude of the derived linear co-polarisation transmission coefficients of the two arrays and their difference. (c) Amplitude of the derived linear cross-polarisation transmission coefficients of the two arrays and their sum.

as the co-polarisation ones, but are still reasonable around the element resonance where their amplitude are most significant. The polarisation conversion for linear EM waves are much greater than those in the reference responses, which illustrates strong chirality presented in the array.

The amplitude of the normalised co-polarisation linear transmission coefficients (T_{xx} and T_{yy}) of the two arrays are exhibited in Fig. 4-11(b). Their differences ($T_{xx} - T_{yy}$) are also plotted so as to verify their coincidence in both amplitude and phase responses. As can be seen, good agreements have been obtained among these coefficients, which are close to 1 except around the element resonance.

The amplitude of the normalised cross-polarisation linear transmission coefficients (T_{xy} and T_{yx}) of the two arrays are shown in Fig. 4-11(c). Their sums ($T_{xy} + T_{yx}$) are also plotted so as to verify the coincidences as above. Again the agreements are not so good beyond the element resonance when the cross-polarisation transmission amplitudes are much smaller than the co-polarisation ones, which can be greatly influenced by unwanted interference signals in the experiments. However, the correspondence around element resonance are pretty good, where the maximum amplitude of cross-polarisation transmissions appear, which demonstrates that the polarisation conversion of linear EM waves in this region can be genuinely attributed to the chirality of the array sample.

The normalised transmission coefficients, both for co- and cross-polarisation, show significant fluctuation above 8.2 GHz. The amplitude of cross-polarisation transmissions are comparable with those of co-polarisation, which can be attributed to the significant reduction in the co-polarisation radiation, as well as the spurious mode excited for both transmitter and receiver horn antennas. In addition, the agreements in the phase of the corresponding orthogonal transmission coefficients (e.g. T_{xx} and T_{yy}) also deteriorate,

which may be due to the crosstalk in the cables connecting the antennas with the network analyser.

After obtaining the normalised linear transmission coefficients, the experimentally derived frequency responses of transmission of circular EM waves with the two polarisation states, together with the polarisation azimuth rotation angle θ and ellipticity η , are calculated for each of the arrays.

There were many challenges when measuring the co- and cross-polarisation transmission responses of the above chiral array samples, as explained below.

- Firstly, the transmitter and receiver antennas should be able to radiate effectively at the operating frequency range of the chiral samples, which is around the resonance of individual elements.
- Secondly, the responses from array samples with finite size were measured to approximate those from infinite array, thus the antennas were carefully aligned to be in line with the centre of sample during the experiment process; 3 dB beamwidths were measured to assure quasi-planar wave incidence, and undesired diffraction around the finite array were roughly eliminated by the aluminium screen placed around the samples.
- Thirdly, the measured dichroism properties depend on four linear transmission responses for each array, and both transmitter and receiver antennas should be rotated in two orthogonal positions during the experiment as a consequence. The free space wavelength at element resonance is about 40 mm, which put stringent requirements on the allowed shift of antenna phase centre and the orthogonality of the two positions.

- Finally, the cross-polarisation transmission is comparatively small, and thus can be easily influenced by the interference signals. In order to receive signal with significant amplitude and suppress the interference, the horn antennas were placed close to each other. In addition, the resulted reduction in the length of cables for connecting the antennas with the vector network analyser further improved the strength of the received signal.

Based on the reported experiment process, further improvements can be made if better standard horn antennas operating at the range where the chirality is significant are available, as the antennas currently employed do not provide maximum radiation at the element resonance. In addition, the utilisation of an antenna support structure with mechanically controllable rotation can offer better stability of phase centres and rotation angles for both transmitter and receiver antennas, which can lead to better accuracy in the measured phase responses of transmissions.

Numerical Simulation of Infinite 2D Interlocking Array

In terms of numerical simulation, CST Micro-StripesTM offers a "wrap around" boundary type that aids the simulation of periodic structures, in which the phase shift between any opposite boundaries of the modelling region is zero. In order to utilise this boundary to simulate the transmission properties of infinite 2D interlocking arrays, a set of four identical isolated double helical resonator elements are employed as a unit composing the array, which are orthogonal to each other in turn and are placed diagonally with respect to $\hat{x}$ and $\hat{y}$ axis (as exhibited in Fig. 4-12). The "wrap around" periodic boundaries in the $x - z$ and $y - z$ planes are then placed symmetrically according to the array spacing required. Plane wave excitations polarised in $\hat{x}$ and $\hat{y}$ are then added, and the

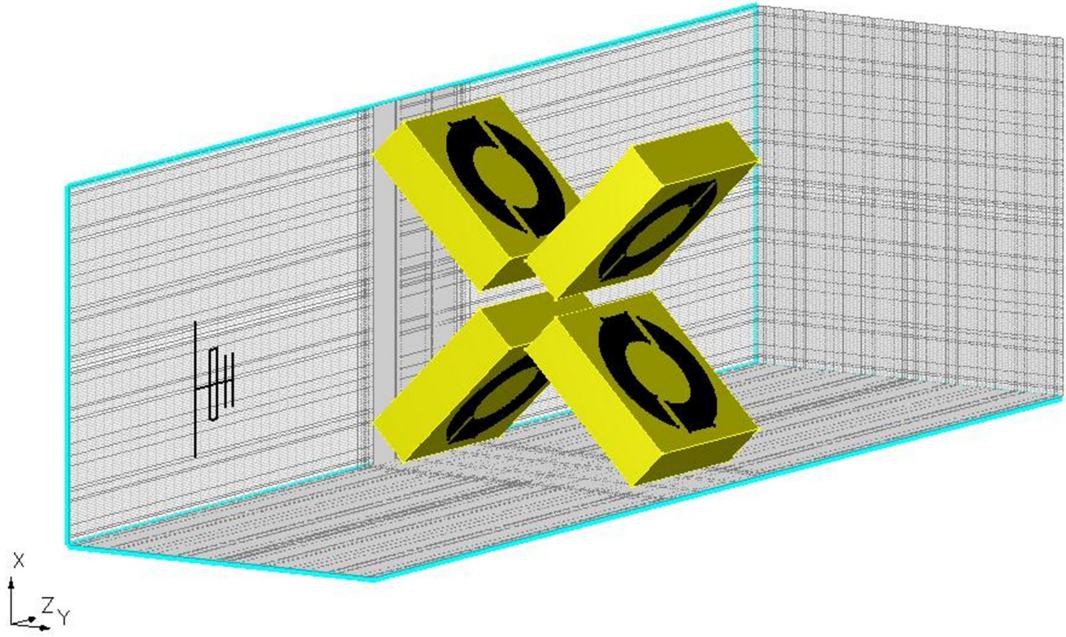


Figure 4-12: Layout of simulation of infinite 2D array of double helical resonators using CST Micro-StripesTM. The four orthogonal elements are utilised as a unit comprising the infinite interlocking array, and periodic boundaries are placed in $x - z$ and $y - z$ planes. The exhibited array is being excited by a linear planar EM wave with $\hat{x}$ -polarised electric field. The transmitted field on the other side of the array is recorded.

amplitude and phase of the transmitted EM wave on the other side of array are recorded. This gives the frequency responses of the normalised transmission coefficients for linearly polarised EM waves, which are different from those in analytical model and experiment as illustrated above. However, the transmission of circular waves, deduced from the linear responses using the same linear algorithm as for measured results, are the same due to the rotational symmetry of the array structure and are thus employed to make comparisons.

4.3 Investigating the Dependence of Dichroism Properties on Element and Array Parameters

So far the analytical model has provided accurate predictions on the transmission properties of two-dimensional chiral arrays. In addition, it has been discovered that arrays with the same elements but different spacings have diverse dichroism characteristics. In this section, the impact of the lattice spacing d and the quality factor Q of elements on the dichroism characteristics of the infinite 2D interlocking square array will be investigated using the analytical model developed.

4.3.1 Impact of the Lattice Spacing of Square Array

In this subsection, the transmission properties are calculated for the infinite arrays comprised of the same right-handed double helical resonators as in § 4.2.2 (their parameters are listed in Table 4.3) with different array spacing d . It has been illustrated that an array with smaller spacing has larger chiral bandwidth due to stronger coupling among elements. In order to make fair comparisons, the greatest achievable ellipticity η by each array is employed as the benchmark. The element quality factor $Q = 55$ is used, which is calculated by the improved quasi-static *RLC* model as in Chapter 2.

The relation between the maximum achievable η and lattice spacing d for the given element is plotted in Fig. 4-13(a). An optimal spacing $d_{opt} = 11.73$ mm exists when the array has the largest ellipticity of -45° . This is the lower limit for ellipticity η as can be

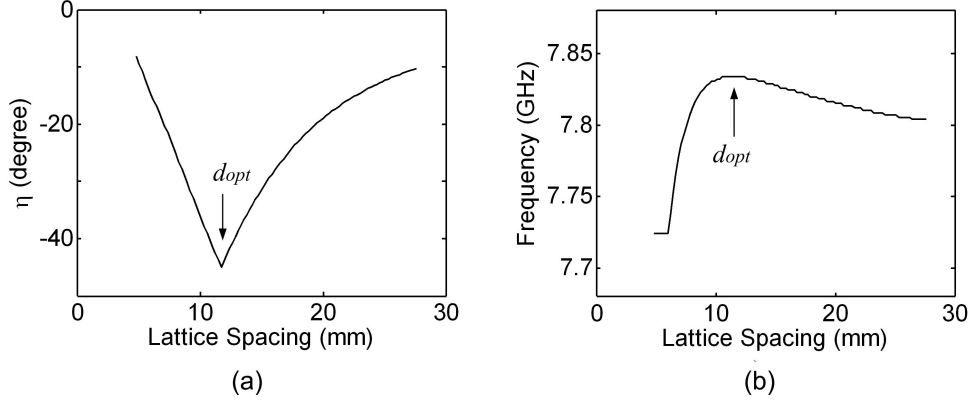


Figure 4-13: The relation between the lattice spacing d of 2D interlocking square array comprising of the double helical resonator elements as given in Table 4.3 and (a) maximum achievable ellipticity η ; (b) the frequency corresponding to maximum η .

seen from its definition (4.81)

$$\eta = \frac{1}{2} \sin^{-1} \left(\frac{|T_{rr}|^2 - |T_{ll}|^2}{|T_{rr}|^2 + |T_{ll}|^2} \right)$$

which implies that transmission of the right-handed circular wave through the chiral slab is essentially suppressed at the resonance.

In addition, the relation between the frequency corresponding to the greatest η and lattice spacing d is plotted in Fig. 4-13(b). When the lattice spacing is smaller than d_{opt} , the frequency increases with the spacing as the coupling among elements attenuate, thus the frequency would approach the individual element resonance. When the lattice spacing expands beyond d_{opt} , however, the frequency gradually drops with the further increase of spacing.

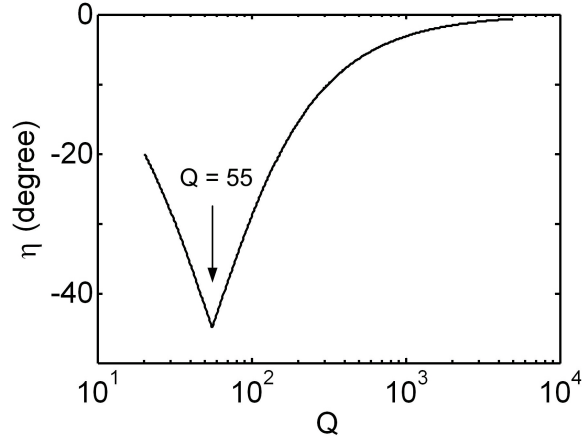


Figure 4-14: The relation between the quality factor Q of double helical resonator elements with the size given in Table 4.3 and the maximum achievable ellipticity η of the comprising infinite 2D array.

4.3.2 Impact of the Quality Factor of Elements

This subsection explores the impact of quality factor Q of individual elements on the array dichroism properties. The infinite 2D array still consists of the same double helical resonator as in Table 4.3, except that Q will be specified rather than calculated from quasi-static model. The array spacing equals the optimal value of $d_{opt} = 11.73$ mm as given in the previous subsection.

The relation between the quality factor Q of element and the maximum achievable η of the assembled chiral array is plotted in Fig. 4-14. The lower limit of ellipticity (-45°) is achieved when $Q = 55$, which is the Q of the original element thus coincides with the result in previous subsection. The ellipticity decreases when Q deviates from this optimal value. In addition, the frequency corresponding to the greatest η is independent with the element's Q , which also accords with intuition.

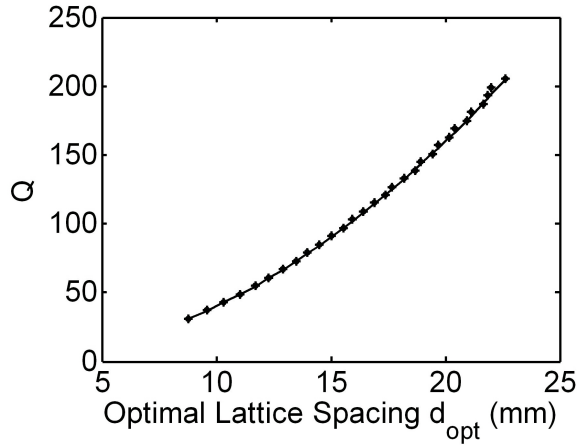


Figure 4-15: The relation between the optimal lattice spacing d_{opt} of the 2D square array of chiral elements when the maximum achievable ellipticity reaches -45° and the corresponding elements' quality factor Q . The dots represent results from the analytical model developed, and the line is a fitting quadratic curve.

It is then of interest to explore the dependence of the optimal spacing d_{opt} on the element's Q , and the results are given in Fig. 4-15. Each data point corresponds to an array configuration where the maximum achievable η is -45° . As can be seen, the optimal array spacing d_{opt} expands monotonically with the increase in Q . Surprisingly, the data points can be well fit to the following simple quadratic relations

$$Q = \kappa d_{opt}^2 \quad (4.99)$$

where the constant $\kappa = 4 \times 10^5$ for the employed double helical elements. The good correspondence in curve fitting can be seen in Fig. 4-15.

It should be emphasised that the developed analytical model is valid when the elements are "densely" distributed in terms of the scale of free space wavelength λ at resonance, thus the results for spacing $d > \lambda/2$ (which is 20 mm in this case) are not reliable.

4.3.3 Summary

In this section, the analytical model has been employed to explore the impact of lattice spacing d and element's quality factor Q on the maximum achievable ellipticity η of the chiral array. The existence of an optimal spacing d_{opt} for given helical elements, as well as an optimal element's Q for given array spacing, have been illustrated which enables the assembled infinite 2D array achieve the extreme ellipticity of $\pm 45^\circ$. This discovery can find potential applications in the design of spatial filters for left-/right-handed circularly polarised EM waves, as $\eta = \pm 45^\circ$ implies zero transmission for one of these polarisation states. Especially, it has been illustrated that the optimal spacing d_{opt} and quality factor Q are related to each other in a simple quadratic manner. Such a behaviour requires further investigation.

4.4 Summary and Conclusions

This chapter includes the development of an analytical model for characterising the dichroism properties of 2D infinite square array of chiral elements, as well as validating its accuracy and the associated applications. By approximating the square array of discrete elements as a continuous sheet with surface currents (dipole moments) distribution, the relations between the external incident fields and the excited dipole moments are established, therefore the transmission of left- and right-handed circularly polarised EM waves can be modelled using classical boundary conditions. The elements are characterised as discrete dipole moments by their polarisabilities and the local fields acting on them, and their fundamental mode interactions are modelled with the aid of the array interaction constant. The model is developed using quasi-static analysis first, then the

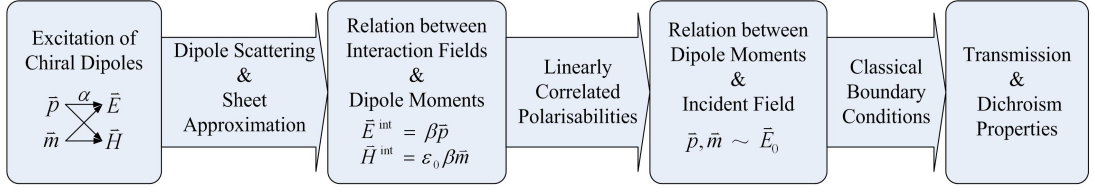


Figure 4-16: Outline of the process of developing the analytical model, which associates the macroscopic array transmission properties to the element characteristics.

expression for polarisabilities and interaction constant are refined to comply with energy conservation law. The outline of the modelling process is illustrated in Fig. 4-16.

The accuracy of the deduced model has been verified by comparing with the results reported in other references, as well as with the experimental measurement conducted by the author. Good correspondence have been achieved for both sets of results. The accurate measurement of cross-polarisation transmissions are much more difficult than that of co-polarisation, as its amplitude is comparatively small and the phase centre of both transmitter and receiver antenna have to be carefully aligned throughout the experiment. It has to be emphasised again that two opposite definitions on circular polarisation and time harmonic assumptions are simultaneously used by researchers, while these employed in this thesis are in accord with the IEEE standards.

The impact of the lattice spacing d and element's quality factor Q on the dichroism properties of the assembled chiral array are investigated using the model, which reveals the existence of a series of optimal array configurations where the ellipticity can reach the extreme value of $\pm 45^\circ$ at resonance. The relation between the spacing d_{opt} and element's Q in the optimal configuration has been illustrated to exhibit an unexpected simple quadratic form, which requires further investigation on its universality and the underlying physical principle.

The model developed fully characterises the physical process of exciting individual elements by both external incidence and interaction among other elements in the array, and the generalised explicit relation between the microscopic characteristics of elements to the macroscopic transmission properties of the whole array has been obtained without involving much approximation. Therefore the model has good accuracy, and can thus find great application potential for analysing frequency selective surfaces comprising of regular array of chiral elements, as well as assist the analysis of isotropic three-dimensional metamaterials comprised of double helical resonators.

The main limitation of the model is that the elements have to be "densely" distributed in terms of the free space wavelength of interest, in order to enable the continuous sheet approximation, but "sparse" when comparing the element size with their spacing so as to validate the dipole approximation. The latter requirement is relatively loose, as it has been demonstrated that the dipole approximation is valid for element diameter as large as 0.7 times the array spacing [120]. In this chapter, the deduced model has been successfully applied to the chiral array with the element diameter and lattice spacing as large as 0.12 and 0.26 times the free space wavelength at element resonance, respectively.

4.5 Appendix I

Following the same quasi-static analysis procedure as in Chapter 2, the electric and magnetic excitations of double helical resonators will be analysed, and their quasi-static polarisabilities will be deduced. It is assumed that the excitation fields are all oriented along the normal axis of resonator.

Firstly, if a single element in free space is excited by a magnetic field B_n along its

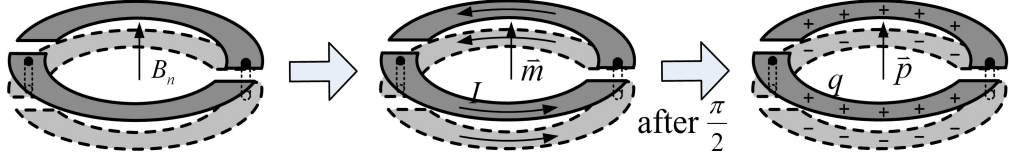


Figure 4-17: Equivalent electric and magnetic dipole moments induced on a right-handed double helical resonator by external magnetic field excitation.

normal axis, a quasi-static electromotive force

$$\varepsilon = -\frac{\partial \psi}{\partial t} = -j\omega B_n S \quad (4.100)$$

will be generated. $S = \pi r^2$ is the area of the inner space enclosed by the helical resonator.

Such a force induces a current

$$I = \frac{\varepsilon}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{\pi r^2}{L\left(\frac{1}{\omega^2 LC} - 1 + j\frac{R}{\omega L}\right)} B_n \quad (4.101)$$

along the circular element, as shown in Fig. 4-17, which contributes to a magnetic dipole of

$$m = IS = \frac{\pi^2 r^4}{L\left(\frac{1}{\omega^2 LC} - 1 + j\frac{R}{\omega L}\right)} B_n \quad (4.102)$$

along its normal axis. Therefore the quasi-static magnetic polarisability

$$\alpha^{mm} = \frac{m}{B_n} = \frac{\pi^2 r^4}{L} \left(\frac{\omega_0^2}{\omega^2} - 1 + j\frac{R}{\omega L}\right)^{-1} \quad (4.103)$$

which is (4.3), where $\omega_0 = 1/\sqrt{LC}$ is the resonant frequency of element.

The current I contributes to accumulation of free charge

$$q = \int I dt = \frac{I}{j\omega} \quad (4.104)$$

on each of the half helix ($+q$ on one end of metal strip at one side of substrate and $-q$ on the other). The total charge is reduced by $1/\epsilon_r$ due to the presence of the dielectric substrate, where ϵ_r is the relative dielectric constant. Therefore the total induced equivalent electric dipole

$$p = \pm \frac{2qh}{\epsilon_r} = \pm \frac{2\pi r^2 h}{j\epsilon_r \omega L \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)} B_n \quad (4.105)$$

and the quasi-static magnetoelectric polarisability

$$\alpha^{em} = \frac{p}{B_n} = \pm j \frac{2\pi r^2 h}{\epsilon_r \omega L} \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)^{-1} \quad (4.106)$$

which is (4.4). The sign depends on the handedness of element. For right-handed helix as shown in Fig. 4-17, the direction of $\vec{p}$ will be along the magnetic field excitation (plus) after a phase delay of $e^{j\frac{\pi}{2}}$, therefore the sign coefficient for α^{em} should be $e^{-j\frac{\pi}{2}} = -j$. On the other hand, the left-handed helix corresponds to $+j$ coefficient.

Next the electric excitation of double helical resonator will be analysed. As exhibited in Fig. 4-18, the element is being excited by an electric field E_n along its normal axis. The induced voltage difference between the broadside coupled upper and lower rings is

$$V = \frac{E_n h}{\epsilon_r} \quad (4.107)$$

The excitation is weakened due to the presence of dielectric. A double helical resonator can be regarded as the series connection of two half helices, therefore the equivalent

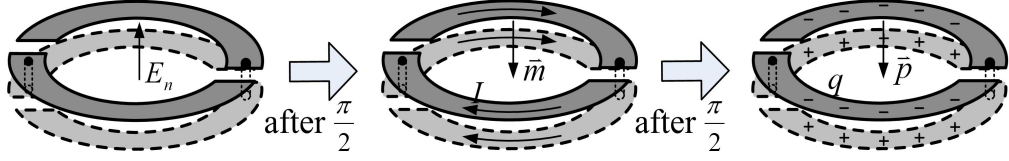


Figure 4-18: Equivalent electric and magnetic dipole moments induced on a right-handed double helical resonator by external electric field excitation.

quasi-static electromotive force $\varepsilon = 2V$, which induces a current

$$I = \frac{\varepsilon}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{2h}{-j\epsilon_r\omega L\left(\frac{\omega_0^2}{\omega^2} - 1 + j\frac{R}{\omega L}\right)} E_n \quad (4.108)$$

along the circular element, as shown in Fig. 4-18. The corresponding magnetic dipole is

$$m = IS = \pm j \frac{2\pi r^2 h}{\epsilon_r\omega L\left(\frac{\omega_0^2}{\omega^2} - 1 + j\frac{R}{\omega L}\right)} E_n \quad (4.109)$$

and the associated polarisability is

$$\alpha^{me} = \frac{m}{E_n} = \pm j \frac{2\pi r^2 h}{\epsilon_r\omega L\left(\frac{\omega_0^2}{\omega^2} - 1 + j\frac{R}{\omega L}\right)}^{-1} \quad (4.110)$$

The amplitude of α^{me} is the same as α^{em} . For right-handed helix as shown in Fig. 4-18, the direction of $\vec{m}$ is in opposite to the electric field excitation (minus) after a phase delay of $e^{j\frac{\pi}{2}}$, as the current tends to flow from higher to lower potential. Therefore the sign coefficient for α^{me} should be $+j$. On the other hand, the left-handed helix corresponds to $-j$ coefficient. In summary, it has been illustrated that

$$\alpha^{me} = -\alpha^{em} \quad (4.111)$$

which fulfills the Onsager symmetries [107].

Similarly, the current I contributes to charge accumulation on both sides of the substrate. The total induced equivalent electric dipole

$$p = \pm \frac{2qh}{\epsilon_r} = \pm \frac{4h^2}{(\epsilon_r \omega)^2 L \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)} E_n \quad (4.112)$$

and the quasi-static electric polarisability

$$\alpha^{ee} = \frac{p}{E_n} = \pm \frac{4h^2}{(\epsilon_r \omega)^2 L \left(\frac{\omega_0^2}{\omega^2} - 1 + j \frac{R}{\omega L} \right)}^{-1} \quad (4.113)$$

According to Fig. 4-18, the direction of $\vec{p}$ will be in opposite to the electric field excitation after a phase delay of $e^{j\pi} = -1$, therefore the sign coefficient for α^{ee} should be positive, which is (4.5). The same conclusion holds for left-handed elements as well.

4.6 Appendix II

This section will derive the relation between the continuous current (polarisation density) distribution on an infinite sheet with a punctured circular hole and the scattered interaction fields at the centre of the hole. The double helical resonator elements in the 2D interlocking slab have two different and orthogonal orientations, and those with the same orientation are identically excited by the normally incident EM wave. Therefore the induced surface polarisation densities are uniform within the equivalent sheet plane.

As an example, the interaction electric field $\vec{E}^{int,p}$ at the centre of the hole scattered

by the uniform surface polarisation density $\vec{p}_s$ on the punctured sheet will be derived. Substituting the full-wave field expression of electric dipole (4.7) into the integral (4.13) we have

$$\begin{aligned}\vec{E}^{int,p} &= \frac{1}{S_0} \int_{r>R_a} \vec{E}^p(\vec{r}) dS \\ &= \frac{1}{4\pi\epsilon_0 S_0} \int_{R_a}^{+\infty} \int_0^{2\pi} \left\{ [\vec{p} - \hat{r}(\hat{r} \cdot \vec{p})] \frac{k^2 e^{-jkr}}{r} + [3\hat{r}(\hat{r} \cdot \vec{p}) - \vec{p}] \left(\frac{1}{r^3} + \frac{jk}{r^2} \right) e^{-jkr} \right\} r d\varphi dr\end{aligned}$$

Since $\vec{p}$ and $\hat{r}$ are all oriented within the integral plane, φ can be regarded as the angle between these two vectors. Therefore

$$\hat{r}(\hat{r} \cdot \vec{p}) = \hat{r} |\vec{p}| \cos \varphi = \vec{p} \cos^2 \varphi + |\vec{p}| \cos \varphi \cdot [\hat{r} - \vec{p}(\hat{r} \cdot \vec{p}) / |\vec{p}|^2] \quad (4.114)$$

The second item can be verified to be perpendicular to $\vec{p}$, while its amplitude is $|\vec{p}| \cos \varphi \sin \varphi$, therefore it will vanish after the loop integration from 0 to 2π . After some transformations it can be deduced that

$$\begin{aligned}\vec{E}^{int,p} &= \frac{\vec{p}}{4\pi\epsilon_0 S_0} \int_{R_a}^{+\infty} \int_0^{2\pi} \left\{ k^2 \sin^2 \varphi + [3 \cos^2 \varphi - 1] \left(\frac{1}{r^2} + \frac{jk}{r} \right) \right\} e^{-jkr} d\varphi dr \\ &= \frac{\vec{p}}{4\epsilon_0 S_0} \int_{R_a}^{+\infty} \left(k^2 + \frac{1 + jkr}{r^2} \right) e^{-jkr} dr\end{aligned} \quad (4.115)$$

Since

$$\frac{\partial}{\partial r} \left(\frac{e^{-jkr}}{r} \right) = -\frac{jkr + 1}{r^2} e^{-jkr} \quad (4.116)$$

then

$$\vec{E}^{int,p} = \frac{-jk\vec{p}}{4\epsilon_0 S_0} \left(1 - \frac{1}{jkR_a} \right) e^{-jkR_a} \quad (4.117)$$

which is (4.15).

When calculating $\vec{H}^{int,p}$, the involved vector direction is $\hat{r} \times \vec{p}$. It can be demonstrated that if the punctured sheet is divided into two halves according to the direction of electric polarisation $\vec{p}$, the contributions from the two halves at the hole centre exactly cancel each other as $\hat{r} \times \vec{p}$ is reversed. Therefore

$$\vec{H}^{int,p} = 0 \tag{4.118}$$

The expressions for $\vec{E}^{int,m}$ and $\vec{H}^{int,m}$ can be similarly derived.

Chapter 5

Conclusions and Future Works

5.1 Summary and Conclusions of the Work in This Thesis

Research on metamaterials has been increasingly booming in recent years, which has developed into a comprehensive subject incorporating contributions from researchers from various disciplines including physics, engineering, chemistry as well as material science. The work in this thesis focuses on the actual composition of metamaterials. In particular, helical resonators with broadside coupled configuration have been demonstrated to have low loss and miniaturised size, which makes them an ideal candidate for assembling metamaterials with better homogeneity, thus their electromagnetic (EM) properties can be better characterised using the effective medium theory. In addition, it has been illustrated that the effective material parameters of magnetic metamaterials can be estimated from the quality factor Q of individual resonators [79], therefore accurate analytical model for the element's Q is aimed to be developed which can greatly facilitates the design process of metamaterials to avoid "blind trials".

The specific research work reported in this thesis has essentially been conducted in the following three areas.

- Analytical modelling of the properties of helical resonator elements;
- Selecting accurate method for retrieving the resonance parameters from the measured data for fabricated elements;
- Analytical modelling of the properties of infinite two-dimensional (2D) interlocking square array composed of helical resonator elements.

In Chapter 2, the properties of individual helical resonator, which is a type of magnetic metamaterial element with miniaturised physical dimension comparing with the free space wavelength at resonance, are investigated. An analytical model has been developed characterising its resonant properties with the aid of quasi-static analysis. The resonant frequency and quality factor of resonator are calculated from its equivalent RLC circuit, where explicit analytical expressions for all the circuit parameters are derived. All the three loss mechanisms present in helical resonators, including conduction loss, dielectric dissipation and radiation, are incorporated in the model.

The analytical model developed provides a convenient while accurate way to investigate the impact of various geometric parameters on the resonance properties of elements. The resonant frequency reduces alongside the increase in the element diameter as well the decrease in the thickness of dielectric substrate. When the outer diameter (side length for square elements) is fixed, however, an optimal metallic fill ratio (ratio of metallic strip width to outmost dimension) of around 0.16 is demonstrated to exist for both circular and square helical elements, which can achieve minimum resonant frequency and normalised electrical size for any given outmost dimension of element. The results of

resonant frequency from the analytical model have been verified using numerical simulation and actual measured data of fabricated elements, which exhibit similar trend with maximum relative error within 10%.

The above error can be partly attributed to the inaccuracy of uniform current distribution assumption adopted in the modelling process. Actually, the distribution of conduction current flow at resonance concentrates towards the inner conductor edge, which effectively reduces the cross section of current path. This introduces a moderate change in the equivalent inductance, but has huge impact on the conduction resistance and associated quality factors of resonators, especially for those whose widths are comparable with the radius. In this thesis, the difficulty in accurate estimation of current density distribution has been circumvented by simple modifications on the elements' dimensions. As a result, the analytical model can give predictions with improved precision for both resonant frequency and quality factor, which are in good correspondence with both numerical simulation and measurement. The relative errors of resonant frequencies are reduced from within 5% to within 2%, while those of quality factors are reduced from 17% to 8%. Especially, the relative errors of quality factor reflecting the conduction loss Q_c are on a scale of 60% for the original model, but are reduced to within 20% for the model with modified elements' dimensions.

The impact of miniaturisation on element Q is theoretically investigated using the model developed, which shows that the dominant loss mechanism in a helical resonator depends on its electrical size. Very small resonators have low Q dominated by conduction losses, medium sized resonators have Q that have a constant upper limit determined by the loss tangent of substrate, while radiation loss influences the Q of large resonators with sizes comparable with the wavelength at resonance, such as SRRs. In addition, a linear dependence of Q_c on the normalised electrical size has been discovered. Conse-

quently, the overall Q of a resonator remains relatively constant approaching the limit by dielectric loss until its electrical size is reduced below a certain critical point, which is inversely proportional to the loss tangent of substrate. After this point, Q degrades rapidly following the limit imposed by conduction losses.

The helical resonator elements operating in the microwave frequency range with different shapes and sizes were fabricated on a thin-film substrate with a thickness of 30 μm . The physical sizes of some elements are below 1% of the free space wavelength at resonance, which contributes to a size reduction of 90% compared with that of a split-ring resonator (SRR) element reported in previously published work. Their quality factors are around 120, which is mainly limited by the dielectric loss. These results have illustrated that the helical resonators can be made at extremely small sizes with usefully high quality factors, and therefore can be employed to construct metamaterials with better homogeneity and lower loss. In addition, the fabrication of helical elements only requires normal lithography technique as for making ordinary printed circuit boards, which enables mass production.

The results from the analytical model are compared with those from numerical simulation using commercial software CST Micro-StripesTM, as well as the parameters retrieved from measurement of the fabricated elements. CST Micro-StripesTM can simulate the time-domain response of individual helical resonator to external pulse EM field excitation. The frequency response is obtained as the Fourier transform of the truncated time response. In addition, both the energy and loss incurred from different mechanism (including radiation loss) at specified frequencies can also be calculated, which can thus be used to estimate the different partial quality factors as in the analytical model.

In the experiment, the frequency responses of complex transmission coefficient S_{21} for

each fabricated helical resonator were measured using a vector network analyser combined with two small loop dipoles as probes. Then the resonant frequency and quality factor were retrieved using nonlinear least-squares fit to the phase of S_{21} versus frequency method. This method has been demonstrated to have the best performance among all the parameter extraction schemes when analysing the response of microwave cavity resonators [88]. In Chapter 3, its performance has been re-examined using artificially constructed S_{21} data with random noise corresponding to the adopted measurement configuration. The results have been proved to be far better than the other two methods in terms of both precision and robustness.

In addition, the impact of the coupling strength of probes to the resonator under test on the accuracy of parameter extraction has been investigated. When the maximum amplitude of measured S_{21} was within the range of -40 dB to -20 dB, the extracted resonant frequencies and quality factors were consistent and had good coincidence with the estimations from the analytical model within a maximum relative error of 7%. Therefore both an appropriate coupling level and a precise parameter extraction method are required for the accurate extraction of resonance parameters for helical resonators.

Based on the research outcome for individual helical resonators, the properties of assembled infinite interlocking square arrays are investigated in Chapter 4, which can be regarded as a two-dimensional metamaterial. A relevant analytical model has been developed, which approximates the array of discrete elements as an equivalent sheet with electric and magnetic surface current (dipole moments) distributions rather than an artificial medium with finite thickness. The relation between the macroscopic transmitted EM fields (on a scale comparable with the free space wavelength of interest) and the small scale properties of individual helical resonators are then established. Especially, the helical resonators are inherently chiral, thus the comprising interlocking array

exhibits dichroism, i.e., circular EM waves with the same wave vector but opposite polarisation states experience different levels of absorption when passing through the array. The transmission of circular waves are related to the equivalent surface current distributions via classical boundary conditions, which are then expressed as functions of the induced discrete dipole moments at the centre of elements and their density of distribution. The helical resonator elements are totally represented by their electric/magnetic polarisabilities, and their coupling are modelled as static dipole interactions with the aid of the array interaction constant. The model is firstly developed using quasi-static analysis, then the expression for polarisabilities and interaction constant are revised to fulfill energy conservation law.

The accuracy of the analytical model derived for the transmission of circular waves has been verified by comparing with the results reported in other references, as well as with the numerical simulation and experimental measurement conducted by the author. The simulation using CST Micro-StripesTM has taken advantage of the "wrap around" boundary type for calculating the responses of periodic structures. In addition, two-dimensional interlocking array samples of helical resonators were fabricated, and their dichroism properties were deduced from the measured transmissions of EM waves with linear polarisations using horn antennas. An algorithm has been illustrated which expresses the transmission coefficients for circularly polarised EM waves as the linear combination of those for linear waves. Good correspondence have been achieved for all sets of results. It should be highlighted that two opposite definitions on circular polarisation and time harmonic assumptions simultaneously exist, while these employed in this thesis are in accord with the IEEE standards [125],[126].

The two approximations employed in developing the analytical model put limitations on the valid range of applications. One is replacing the discrete array of elements by

a continuous sheet, which is justifiable when the elements are "densely" distributed in terms of the free space wavelength of interest. The other is that the interactions among elements are modelled by that of static dipoles, with higher order interactions neglected, which is reasonable when the space between elements is large comparing with the element size. The latter requirement is relatively loose, as it has been demonstrated that the dipole approximation is valid for element diameter as large as 0.7 times the array spacing [120]. In this thesis the analytical model has provided accurate results for characterising the dichroism properties of the fabricated chiral array of double helical resonators, where the element diameter and lattice spacing as large as 0.12 and 0.26 times the free space wavelength at element resonance, respectively.

It has been demonstrated in the experiment process that the accurate measurement of cross-polarisation transmissions is a much more challenging task comparing with that of co-polarisation transmissions. Firstly it's because the amplitude is comparatively small, and the rotation of both transmitter and receiver into two orthogonal positions during the measurement may introduce further interference. Therefore both transmitter and receiver antenna have to be carefully aligned throughout the experiment. Further improvement can be achieved by using an antenna support structure with mechanically controllable rotation.

The analytical model developed has been further applied to analyse the impact of the lattice spacing and element's quality factor on the dichroism properties of the assembled chiral array. It has been revealed that a series of optimal array configurations exist with different combination of lattice spacing and element's quality factor, so that the circular EM waves with one particular polarisation state can be totally attenuated around the element resonance when traversing the array, while the other experiences negligible attenuation. In addition, an unexpected simple quadratic relation between the lattice spacing

and element's quality factor in the optimal configuration has been illustrated. Further investigation is required in order to demonstrate the universality of this conclusion.

The model developed provides a physically meaningful characterisation on excitation of individual elements by both external incidence and interaction among other elements, and the generalised explicit relation between the element attributes to the macroscopic transmission properties of the whole array has been proved to have good accuracy. Therefore the model can find great application potential for analysing frequency selective surfaces comprising of regular array of chiral elements, as well as assist the analysis of isotropic three-dimensional metamaterials comprised by double helical resonators. The chiral metamaterial has become an important constituent in the research on metamaterials [131].

5.2 Possible Future Works

5.2.1 Modelling the Properties of Three-Dimensional Metamaterials

As been illustrated in Chapter 1, most of the actual fabricated metamaterial samples have only a few layers in the lateral dimension due to the finite transmission loss of EM wave through the periodic structure. In addition, it has been illustrated in § 4.1 that the double helical resonator can in theory be employed to construct isotropic artificial media. Therefore it can be envisaged that the assembled bulk artificial material can be regarded as the cascade of layers of two-dimensional arrays as compositing units, and the effective material parameters be derived using the analytical model developed in this thesis for 2D arrays. This method enables accurate theoretical characterisation on the properties of bulk materials, particularly for those with thin layer structures. The miniaturised

physical size and usefully high quality factor Q of helical resonators further contributes to the eligibility of effective medium theory.

5.2.2 Two-Dimensional Chiral Arrays as Spatial Filters

In Chapter 4, it has been theoretically demonstrated that for infinite 2D interlocking array of double helical resonators with certain combination of lattice spacing and element's Q , transmission of circular EM wave with one particular polarisation state is essentially suppressed around the resonance of element, while the other experiences negligible attenuation. An array with this transmission response can be employed as efficient spatial filters, so that circular wave with high purity polarisation state is available after transmitting EM wave with arbitrary polarisation through the array. This conclusion can be further validated by fabricating array samples with prescribed parameters and making measurements on their dichroism properties.

On the other hand, the revealed quadratic relation between the optimal lattice spacing and element's Q needs further theoretical demonstration, in order to verify its universality and reveal the underlying physical principle. The obtained conclusion may find great application in guiding the design of periodic array of chiral elements as efficient frequency selective surfaces.

5.2.3 Investigating the Propagation of Magneto-Inductive Wave in Arrays of Miniaturised Helical Resonators

The magneto-inductive (MI) wave owes its existence to the magnetic couplings among metamaterial elements. Its effect was first realised when researchers including our group

conducted near field imaging experiments using the magnetic conducting "Swiss roll" structure [74],[75],[132]. The distribution of the magnetic field can be transferred through the structure with small distortion just below the resonant frequency of each individual element. However, at higher frequency (above resonance), the excitation and transmission of MI wave lead to strong distortion of the transferred image. The MI wave plays a role similar to surface wave in ordinary materials which cannot be efficiently incorporated into the effective medium theory.

By employing different elements and altering their distance and relative positioning, the coupling among elements can be changed which provides various dispersion characteristics [133]-[137]. In addition, the wavelength of MI wave within the passband is significantly smaller than that of EM wave with the same frequency, thus the MI wave is essentially a slow wave and can be applied to realise various components, e.g., waveguides, couplers, power dividers, transducers, etc [138]-[141]. Specifically, by bringing two planar arrays of metamaterial elements close together, a stopband of MI wave appears in the vicinity of the resonant frequency due to the strong interlayer coupling, which enables good near field subwavelength imaging [142]-[149].

The helical resonator investigated in this thesis has two distinct features: miniaturised size and flexible thin-film structures. It has been demonstrated that the achievable resolution of imaging using MI wave devices is directly related to the physical size of elements. Therefore helical resonator can be employed to fabricate near field imaging devices with superb resolution. In addition, their flexible structures enable the assembled magneto-inductive waveguide to be utilised as adaptable contactless data-transfer channels, which offers the potential of realising "body networking", i.e., connecting a variety of devices at different locations near human body with high bandwidth [150].

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