Supersymmetry and Electroweak Fine Tuning

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Abstract

Low scale supersymmetry (SUSY) is a compelling solution to the electroweak hierarchy problem. However, increasingly strong limits on the masses of superpartners, first from LEP and now the LHC, mean that the simplest models require significant fine tuning. This thesis is dedicated to the study of a possible alternative low energy superpartner spectrum, natural SUSY, in which only superparticles directly involved in stabilising the electroweak scale are light, alleviating collider limits and potentially reducing tuning.

After reviewing how low scale SUSY is motivated by the hierarchy problem, we build a model of SUSY breaking and mediation that successfully generates a natural SUSY spectrum. This also suppresses the first two generation fermion Yukawas, and leads to small parameters in the hidden sector, which are required for successful SUSY breaking. A challenge in models of natural SUSY is raising the physical Higgs mass to 125 GeV, and we study the possibility that this could occur through the addition of a singlet to the theory. If stops are very light, the coupling of the singlet to the Higgs needs to be so large that it becomes non-perturbative before the scale of grand unification, raising the concern that precision gauge coupling unification may be upset. However, we find that this is not necessarily the case. Rather it is possible this could correct for the present $\sim 3\%$ discrepancy in the two-loop minimal supersymmetric model's unification prediction.

We then turn to the fine tuning in models of natural SUSY, emphasising that this should be measured with respect to the theory's ultraviolet (UV) parameters. We show that the first two generation sfermions can be made relatively heavy, beyond LHC reach, without introducing tuning. However, the gluino generates a significant tuning through the stops during the renormalisation group flow. As a result, there is no fine tuning benefit in reducing the stop masses below $(50 - 75)\,\%$ of the weak scale gluino mass, and we obtain strong lower bounds on the tuning of theories compatible with collider limits. We also study theories with Dirac gauginos, which have relatively low fine tuning even if the scale of mediation is high. Finally, we consider the effect of relaxing a common assumption and allowing the hidden SUSY breaking sector to modify the running of the visible sector soft masses. This may plausibly occur in realistic models and could dramatically reduce the fine tuning of SUSY theories.

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Statement of Originality

This thesis is based on original research and contains no material that has already been accepted, or is concurrently being submitted, for any degree or diploma or certificate or other qualification in this university or elsewhere. To the best of my knowledge and belief this thesis contains no material previously published or written by another person, except where due reference is made in the text.

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Chapter 1: Introduction and Motivation

1.1 The Standard Model, Electroweak Symmetry Breaking and the Hierarchy Problem

The Standard Model (SM) of particle physics is a spectacularly successful theory of the varieties and interactions of subatomic particles that is perfectly consistent with almost every laboratory and collider experiment carried out to date, with the exception of neutrino oscillations [1].[1] The electromagnetic, weak, and strong nuclear forces arise from the gauge bosons of the gauge symmetry group of the theory, $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$, under which the matter content is charged. Spontaneous symmetry breaking via a non-zero Higgs vacuum expectation value (VEV) breaks $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \to \mathrm{U}\,(1)_\mathrm{EM}$ giving masses to three of the gauge bosons in the electroweak (EW) sector. This also generates Dirac masses for the (chiral) fermion matter apart from neutrinos. At low energy scales the strong nuclear interaction runs into strong coupling, and quarks and gluons confine leading to hadrons which gain their dominant masses from strong coupling effects.[2]

However, despite all of its triumphs, the SM has a number of deficiencies. It cannot be a valid description of Nature at energies above the Planck mass (M_Pl) [4]. It also has a large number of free parameters, including a complicated array of fermion masses and mixings that is suggestive of some new structure [5], and there must exist mechanisms to generate the required baryon asymmetry in the Universe and neutrino masses [6]. Further, the fermion matter suggestively falls into SU(5) multiplets, which together with the apparent success of gauge unification in minimal supersymmetric extensions suggests the gauge groups may unify at some high scale [7]. Cosmological inflation is a very attractive solution to the horizon, flatness and monopole problems of the Standard Cosmology and also requires the addition of new states and scales [8, 9]. Finally, the SM gives no explanation for the extreme smallness of the CP violating coupling in the QCD sector, $\overline{\theta} \lesssim 10^{-9}$ [10].

The classic example of how these shortcomings necessitate an extended theory with new high scales arises from gravity. Gravity is perturbatively non-renormalisible, a perturbative expansion of the Einstein-Hilbert interaction includes arbitrarily high powers of $\frac{1}{M_\mathrm{Pl}}$. While

[1]The SM does requires extension to accommodate astrophysical and cosmological observations, most notably dark matter and possibly dark energy too [2]. However, the state that constitutes dark matter does not necessarily require any significant (or indeed any non-gravitational) couplings to the visible sector, and the source and dynamics of dark energy is still very poorly understood.

[2]Although pions only receive masses from QCD effects because of explicit chiral symmetry breaking from non-zero explicit quark masses and the $\mathrm{U}\,(1)_\mathrm{EM}$ charges [3].

such an expansion is predictive at low energies where all higher dimensional operators are unimportant, at higher energies, $E \gtrsim M_{\rm Pl}$, the theory has infinitely many important parameters, and hence has no predictive power [4]. Consequently, the gravitational sector must either run into some as yet unknown strongly coupled fixed point (a proposed scenario known as *asymptotic safety*) [11], or the theory is UV completed before $M_{\rm Pl}$ is reached. Given a dearth of evidence for the existence of a strongly coupled fixed point, the standard viewpoint is to regard the Einstein-Hilbert action as only a low energy effective field theory arising out of UV dynamics.[3]

In light of the required additions to it, the modern perspective is to view the SM as an effective field theory, valid up to some cutoff [14]. At the cutoff either new degrees of freedom appear in addition to the SM states, or the SM states themselves may turn out to be composite and the underlying degrees of freedom are revealed. In the later case, the underlying degrees of freedom may be standard quantum field theory (QFT) states, as in models of technicolour or extended objects in string theory models. In reality it is likely that there are multiple such cutoffs, for example it may be that new states appear not far from the weak scale, followed by additional states at the scale of a grand unified theory (GUT), and a transition into a string theory at the string scale.

Since the SM is now simply an effective field theory, in addition to the usual renormalisible operators, the low energy effective theory is expected to contain non-renormalisible operators suppressed, parametrically, by powers of the cutoffs. For example, integrating out heavy right handed neutrinos (or other appropriate matter content) leads to neutrino masses through the operator [15]

$$\mathcal{L} \supset \frac{y^2}{\Lambda} LLHH \ , \tag{1.1}$$

where y is a coupling constant, L is the left handed lepton doublet, H is the Higgs scalar doublet, and Λ is the mass scale of the right handed neutrinos. For models where there is a cutoff near the weak scale, there are very strong constraints on the form of the higher dimensional operators, and consequently on the underlying physics, from precision collider observations. For example, flavour constraints require that, if new states and interactions are introduced near the EW scale, the global flavour symmetries of the SM must remain only broken by the SM Yukawa couplings [16].[4] Additionally, precision EW constraints

[3] It appears challenging to find a UV completion that is a normal quantum field theory [12], however it is believed that a string theory can provide a suitable completion [13].

[4] In fact, even this assumption is not sufficiently stringent if the new physics appears at sufficiently low

are very hard to accommodate in models where EW symmetry breaking arises from strong dynamics [17, 18].

However, once the SM is viewed as an effective field theory there is a 'hierarchy problem' of why the EW scale is so small compared to the other, higher, scales in the theory [19]. As we will discuss shortly, in the SM the EW VEV is set by, and parametrically the same scale as, the Higgs mass squared parameter. So, more precisely, the hierarchy problem boils down to the question of why is the renormalised Higgs soft mass squared parameter, evaluated at the scale $\mu \sim$ TeV, so much smaller than the high scales in the theory?

This is a problem because the running Higgs mass squared parameter $m_h^2(\mu)$ receives contributions from any extension of the SM that is not very weakly coupled to the Higgs sector (in a way that is not invariant under a shift of the Higgs field). For example, consider the addition of a new heavy scalar to the theory, which couples to the Higgs through a term in the Lagrangian at the UV cutoff of the theory

$$\mathcal{L} \supset -m_\phi^2 |\phi|^2 - m_h^2 |h|^2 - \lambda |\phi|^2 |h|^2 \ , \tag{1.2}$$

where ϕ is the new scalar, and h is the physical Higgs (no symmetry can forbid such a coupling assuming the mass terms are present). As a result of running, a mass shift for the Higgs is generated at one loop

$$\delta m_h^2 = \frac{\lambda m_\phi^2}{16\pi^2} \log\left(\frac{\Lambda_{\rm UV}^2}{m_\phi^2}\right) \ , \tag{1.3}$$

where $\Lambda_{\rm UV}$ is the UV cutoff of the theory [20]. There is also a contribution that is not logarithmically enhanced arising as a threshold correction when the heavy scalar is integrated out of the theory. Contributions of this form arise in GUT theories, giving an explicit realisation of the hierarchy problem [19].

Similarly, some realisations of axion models, which provide a solution to the strong CP problem, include an additional Dirac fermion that is vector-like under the SM gauge groups and typically has a mass $\sim 10^{11}\,$GeV [21, 22]. This gives a three-loop contribution to the Higgs mass through gauge interactions that is numerically large ($\delta m_h^2 \sim \left(10^7\,\text{GeV}\right)^2$ if running begins at the Planck Scale). High scale UV completions where SM states are composite (including string theories) also give large contributions, modified by a form factor that depends on the composite dynamics [20].

scale.

The Higgs part of the SM effective Lagrangian is

$$\mathcal{L} \supset -m_h^2 \left|h\right|^2 - \lambda \left|h\right|^4 \ , \tag{1.4}$$

where all parameters are evaluated at the weak scale. Consequently the Higgs obtains a VEV, $\langle h \rangle = \sqrt{\frac{-m_h^2}{2\lambda}}$ and expanding fluctuations around this vacuum the physical Higgs mass is $m_h = \sqrt{-m_h^2}$. For correct EW symmetry breaking the VEV must be 246 GeV, and the physical Higgs mass is observed to be $\sim 125\,\text{GeV}$. The Higgs mass squared parameter at a low scale, which appears in Eq. (1.4), can be written as

$$m_h^2\left(\Lambda_{\text{EW}}\right) = m_h^2\left(\Lambda_{\text{UV}}\right) + \delta m_h^2 \ , \tag{1.5}$$

where δm_h^2 contains all of the contributions from running. Unless the SM is completed to a theory without a hierarchy problem close to the EW scale, the large contributions to δm_h^2 from the high scales in the theory mean that a very high degree of cancellation between the two terms in Eq. (1.5) is required to obtain a low EW scale. For this to occur the parameters that contribute to $\delta m_h^2\left(\Lambda_{\text{EW}}\right)$ (and $m_h^2\left(\Lambda_{\text{UV}}\right)$ itself) must have precisely related values at the UV boundary of the renormalisation group (RG) flow, referred to as a *tuning* of these parameters.

The degree of tuning Δ_i with respect to a parameter p can be quantified as the fractional change in the EW scale (parametrised by the mass of the Z boson m_Z) in response to a fractional change in the parameter at the high scale [23, 24]

$$\Delta_p = \frac{p\left(\Lambda_{\text{UV}}\right)}{m_Z^2}\frac{\partial m_Z^2}{\partial p\left(\Lambda_{\text{UV}}\right)} = \frac{\partial \log m_Z^2}{\partial \log p\left(\Lambda_{\text{UV}}\right)} \ . \tag{1.6}$$

For example, in the SM the tuning with respect to the UV value of the Higgs mass squared is

$$\Delta_{m_h^2} \sim \frac{1}{\lambda}\frac{m_h^2\left(\Lambda_{\text{UV}}\right)}{m_Z^2} \sim \frac{1}{\lambda}\frac{\delta m_h^2}{m_Z^2} \ . \tag{1.7}$$

Taking $\delta m^2 \sim \left(10^{18}\,\text{GeV}\right)^2$, as a reasonable estimate of the correction from new physics at the Planck Scale, leads to a tuning of $\Delta_{m_h^2} \sim 10^{32}$.

Use of the measure Eq. (1.6) implicitly assumes that the parameter is being varied around a value which is not specially preferred in the underlying UV theory [25] (that is, the prior for the parameter is not too far from flat under $\mathcal{O}\left(1\right)$ changes in its value). In this case $\frac{1}{\Delta_i}$ gives an estimate of the probability that the required cancellation takes place for a randomly chosen value of the parameter p.

In extensions of the SM with more complex Higgs sectors, such as supersymmetry, even though the details of the Higgs sector are more involved the EW scale dependence on the Higgs mass squared parameter(s) remains strong

$$\frac{\partial m_Z^2}{\partial m_h^2\left(m_Z\right)} \sim 1 \ , \tag{1.8}$$

and consequently large contributions to the mass squared parameter again lead to tuning.

It should be noted that fine tuning is essentially an aesthetic problem. The contributions to δm^2 arise from a variety of sources, all disconnected from each other, at vastly different energy scales and, naively, with nothing to do with the physics that sets the UV parameters. Consequently, there is no reason to expect any cancellation, and certainly not to the degree that appears to be present in the SM. While, clearly, theories with greater fine tuning are less appealing, there is no sharp upper limit on the tuning that may be regarded as acceptable, and even the measure itself is only defined up to $\mathcal{O}\left(1\right)$ factors.[5]

Importantly, the reason the Higgs mass squared operator receives large corrections is that the SM has no additional symmetry in the limit that $m_h^2 \to 0$.[6] As a result, the corrections *do not* have to be proportional to the UV value of the Higgs mass squared. This is in contrast to what would occur for a new Dirac fermion, with mass term $m_D\overline{\psi}\psi$. In the limit $m_D \to 0$ there is an additional chiral symmetry acting to rotate the two Weyl components of ψ independently. As a result, if m_D was zero, the symmetry would result in no mass term being generated during running. Further, if the mass is non-zero, the corrections to it necessarily take the form $\delta m_D \sim m_D \log\left(\frac{\Lambda_{\rm UV}}{m_D}\right)$. Provided m_D is small at the assumed UV cutoff of the theory it therefore remains small during a finite period of running. A small parameter that leads to an enhanced symmetry in the limit that it vanishes is known as *technically natural* [26].

Apart from the Higgs mass squared (and the cosmological constant [27], which we do not consider), all parameters in the SM are only logarithmically sensitive to higher scales in the theory. As a result the corrections to these are comparatively small, and there is only a hierarchy problem for the Higgs mass squared parameter.

Solutions to the hierarchy problem extend the SM so that no significant cancellations are required to obtain a light EW scale (the EW scale is then said to be *stabilised*). Once this

[5]There is additional uncertainty surrounding the correct choice of the underlying UV parameters as we discuss in Section 2.10.

[6]In the presence of high scales there is no conformal symmetry, and moreover conformal symmetry is broken by loop effects, for example through the running of the coupling constants.

has been achieved, there remains a second question of why the EW scale is exponentially separated from high scales at all (even though no tuning is required to keep it there during the RG flow). However, it turns out that solutions to the tuning problem can usually accommodate solutions to this fairly straightforwardly.

There are three proposed solutions to the hierarchy problem that are plausible; weak scale supersymmetry, some form of strong dynamics, and theories in which gravity becomes strongly coupled close to the weak scale. In this thesis we focus on supersymmetry, which is a perturbative solution to the hierarchy problem, and delay a detailed description of this until the next chapter. Here we simply note that the minimal versions of supersymmetry that solve the hierarchy problem predict new states that would have already been observed by particle colliders (first LEP and now the LHC), motivating more complex model building and assessment of the fine tuning of theories.

The main alternative solution is some form of strong dynamics near the weak scale. In the original form of such models, technicolour, an extra gauge group runs into strong coupling and new fermions F charged under this gauge group are postulated to form a condensate $\left\langle \overline{F}F \right\rangle \neq 0$, generating masses for the W and Z bosons [28–30]. These models resemble QCD in the SM, which itself breaks EW symmetry, albeit at a scale that is much too low for the observed phenomenology. SM fermion masses can arise through additional gauge dynamics however this requires more model building and it is complicated to obtain sufficiently large masses, especially for the third generation fermions [31].

In technicolour models there is no fundamental scalar Higgs, and consequently, provided the separation between the compositeness scale and the EW scale is natural, there is no hierarchy problem. The scale of strong coupling is exponentially separated from high scales in the theory by $\Lambda_{\rm TC} \sim e^{-\frac{1}{b_0 \alpha(\Lambda_{\rm UV})}} \Lambda_{\rm UV}$ where $\alpha\left(\Lambda_{\rm UV}\right)$ is the gauge coupling at a high scale and the beta function coefficient is defined as $\frac{\partial \alpha}{\partial \log \mu} = -b_0 \alpha^2$. This explains the small size of the EW scale relative to $M_{\rm Pl}$, as well as stabilising it against radiative corrections. However, despite being theoretically well motivated, the simplest examples of such theories are strongly disfavoured since they give much too large contributions to EW precision observables that are observed to agree to high accuracy with the SM predictions [32]. Further, additional assumptions and model building are required to avoid excessively large rates of highly constrained processes such as flavour changing neutral currents [33]. Like supersymmetry, LHC limits on new states forces the reintroduction of some tuning in the simplest models, and (unlike supersymmetry) the observation of a boson with couplings close to those of a light SM Higgs is very challenging to accommodate in technicolour models (attempts to accommodate such a Higgs include, for example, [34]). While it may be possible to build more complex models that simultaneously evade all these problems, such model building is complicated since it involves strong dynamics that is very hard to calculate.

A similar class of theories, that are especially interesting due to the observation of a light Higgs, are composite Higgs models [35–37]. In these, a Higgs like state arises as a pseudo-Goldstone boson from some strong dynamics and appears in the effective field theory. SM fermions are typically assumed to be partially composite allowing for their masses to be generated. Similarly to technicolour, it is challenging to build explicit UV models with calculable dynamics, and the absence of any observed new states at the LHC also provides strong constraints and may necessitate the reintroduction of some tuning. Constraints from flavour observables are typically weaker than in technicolour models but not entirely safe [38, 39].

Another, dramatic, possibility is that gravitational interactions become strong near the weak scale, due to the presence of an extra dimension with size not far from $\frac{1}{m_Z}$. As a result, the cutoff of the SM is lowered to close to the weak scale and the corrections to the Higgs mass parameter are relatively small. In the original versions of these models the fundamental Planck scale is low, and gravity only appears weak at large distances due to gravitational flux 'leaking' out into the extra dimensions while the SM fields are confined to a brane [40]. Later versions employ a warped extra dimension that scales the effective Higgs VEV to close to the EW scale [41].[7] Again, these models are under pressure due to the lack of observation of the new states close to the weak scale, and most realisations now include significant tuning, and flavour observations are hard to accommodate.

A postulated alternative resolution to the hierarchy problem is that there are no new scales with significant couplings to the SM that are not near the weak scale [43, 44]. If this were the case there would be no large corrections to the Higgs mass squared parameter and consequently no tuning. This not only requires all the shortcomings of the SM to be resolved close to the weak scale (or in a way that is only very weakly coupled to the Higgs) but also a smooth transition into quantum gravity without it counting as a high scale. Additionally, in the absence of new dynamics the U(1) hypercharge runs into a Landau pole. The pole is

[7] It is thought that the later class of models are actually equivalent to some strongly coupled models through the AdS/CFT duality [42].

far above the Planck scale and therefore not usually regarded as significant, however it does mean that, even if quantum gravity resolves itself, the SM cannot be a UV complete theory with no new scales. It is straightforward to show that any field theory effect that changes the running sufficiently to avoid this counts as a new scale in the sense of the hierarchy problem [45]. Consequently, this proposal still requires new matter charged under the SM gauge groups close to the weak scale (or the transition to quantum gravity to effect the gauge couplings in a very surprising way).

It is also possible that there is an anthropic solution to the hierarchy problem, if separate universes scan over different UV parameters, and only those with a light EW scale are suitable for developing life. However, there is no compelling reason to believe a light EW scale is required for life [46], and there is no known mechanism by which an enormous number of universes with differing parameters can be generated.[8] A final problem with an anthropic solution is that, while there are reasons to believe a landscape may prefer a high supersymmetry breaking scale so that weak scale supersymmetry would not appear [48, 49], there is no convincing reason that technicolour would be disfavoured. All current prejudice against a technicolour solution to the hierarchy problem is based on observations. *A priori* there is no reason to think the 'landscape' of theories should prefer a very highly tuned Higgs to the simple addition of an extra asymptotically free gauge group.

In the remainder of this thesis we study the viability of low scale supersymmetry as a solution to the hierarchy problem, and the extent to which it is possible to realise models with low fine tuning, in light of increasingly stringent limits from the LHC.

[8]While string theory certainly allows for the parameters of the theory to be altered by differing compactifications of the extra dimensions, there is as yet no convincing mechanism by which many disconnected regions with differing compactificatons can actually, dynamically, occur (although there are proposals [47]).

Chapter 2: Supersymmetry

Supersymmetry is an extension of space-time symmetry that relates fermions and bosons, and consequently to be realised in the real world requires an (approximate) doubling of the particle content of the SM. To be compatible with the non-observation of superpartners it must be a broken symmetry, and if (softly) broken in the visible sector close to the weak scale can provide a solution to the hierarchy problem [19, 50].[1] Additionally minimal models lead to gauge unification [51, 52] and can contain viable dark matter (DM) candidates [53]. It has also been suggested that supersymmetry is a requirement of a consistent quantum theory of gravity (see for example [54]). While this only necessitates supersymmetry at a high scale, and certainly does not require it to have any connection to the weak scale or the hierarchy problem, it is interesting that it is separately motivated. Even if not realised in the real world, theories with supersymmetry are highly mathematically interesting and lead to additional calculational power that allows for insights into phenomena such as confinement that may be relevant to ordinary gauge theories [55]. In this chapter we briefly review the theory and models relevant to this thesis (many more details, important results, and original references are given, for example, in [56–58]).

2.1 The Supersymmetry Algebra

The symmetry structure of flat space-time is given by the Poincaré group, which contains rotations, boosts, and translations. There are ten independent generators with commutation relations given by

$$\begin{aligned} [P_\mu, P_\nu] &= 0, \\ [M_{\mu\nu}, P_\lambda] &= i\left(\eta_{\nu\lambda} P_\mu - \eta_{\mu\lambda} P_\nu\right), \\ [M_{\mu\nu}, M_{\rho\sigma}] &= -i\left(\eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho}\right), \end{aligned} \tag{2.1}$$

where η is the Minkowski metric (with signature $(+---)$), $M_{\mu\nu} = -M_{\nu\mu}$ generates Lorentz transformations, and P_μ generates translations. The Coleman-Mandula theorem [59] severely constrains possible extensions to the Poincaré symmetry of space-time. Subject to mild assumptions, it shows that any symmetry group of an S-matrix that contains the Poincaré

[1] Superficially it may seem highly artificial that so many ordinary particles have been discovered before any superpartners to already discovered particles have been found, however the particles not yet observed are exactly those that can gain large soft SUSY breaking masses in the limit where all symmetries of the SM are unbroken. Therefore this is not a particularly surprising scenario.

group can be locally decomposed into a direct product of a symmetry group and the Poincaré group, that is, the symmetries 'factorise' into internal and spacetime symmetries without mixing.[2] In the SM, the internal group consists of the gauge symmetries of the theorem.

However, the Coleman-Mandula theorem implicitly assumes commuting charges. Supersymmetry evades it by introducing anticommuting, spinorial, charges (that is, generators in the representation $\left(\frac{1}{2},0\right)$ or $\left(0,\frac{1}{2}\right)$ of the Lorentz group). Since the generators are not Lorentz scalars, this is a non-trivial extension of space-time symmetry. Shortly afterwards, the Haag-Lopuszanski-Sohnius theorem showed that spinorial charges are the maximal extension of the space-time symmetry under a weaker set of assumptions than the Coleman-Mandula theorem [60].[3]

For the purposes of this work, we primarily consider $\mathcal{N}=1$ supersymmetry, which contains one set of supersymmetry generators. Higher $\mathcal{N}$ theories are non-chiral in four dimensions and therefore cannot describe the visible sector in models that are purely field theoretic and contain only four dimensions. However, it is very plausible that they may be important in models with additional extra dimensions where a chiral low-energy theory can be obtained by the compactification of additional dimensions [61].

Assuming a generalised Jacobi identity, the supersymmetry algebra is almost entirely fixed. The $\mathcal{N}=1$ version has one Weyl conserved charge Q_α along with its conjugate $Q^\dagger_{\dot\alpha}$, and is given by Eq. (2.1) supplemented by

$$\begin{aligned} [P_\mu, Q_\alpha] &= 0 \ , & [M^{\mu\nu}, Q_\alpha] &= i\left(\sigma_{\mu\nu}\right)_\alpha^\beta Q_\beta \ , \\ \left\{Q_\alpha, Q^\dagger_{\dot\beta}\right\} &= 2\left(\sigma^\mu\right)_{\alpha\dot\beta} P_\mu \ , & \{Q_\alpha, Q_\beta\} &= 0 \ , \end{aligned} \tag{2.2}$$

where α is a spinor index, $\sigma_{\mu\nu}=\frac{i}{4}\left(\sigma_\mu\overline{\sigma_\nu}-\sigma_\nu\overline{\sigma_\mu}\right)$ with σ_μ the sigma matrices, and we use two component notation. Mathematically, this is a *graded Lie algebra.* The commutator of the SUSY generators with internal symmetry generators vanishes, with exception of the generator of the *R-symmetry* that acts as $Q_\alpha\mapsto e^{i\lambda}Q_\alpha$ and $Q^\dagger_{\dot\alpha}\mapsto e^{-i\lambda}Q^\dagger_{\dot\alpha}$ and satisfies

$$\begin{aligned} [Q_\alpha, R] &= Q_\alpha \ , \\ \left[Q^\dagger_{\dot\alpha}, R\right] &= -Q^\dagger_{\dot\alpha} \ . \end{aligned} \tag{2.3}$$

The irreducible representations of the SUSY algebra consist of collections of particles

[2]A notable exception are conformal theories which have an enlarged symmetry group, and evade the Coleman-Mandula theorem by not having a well defined S-matrix.

[3]Supersymmetry remains the maximum possible extension in the case of extended objects that arise in string theories.

(*multiplets*), which are related by the action of the generators Q_α, $Q^\dagger_{\dot\alpha}$ and consequently have spins differing by (multiples of) $\frac{1}{2}$. It can be shown that representations contain the same number of (on shell) bosonic and fermionic degrees of freedom, and since the operator P^2 commutes with Q_α all states in the same multiplet have the same mass while supersymmetry is unbroken (in flat space-time). Although it is possible to construct supersymmetric theories directly by writing down a Lagrangian with suitable matter content and couplings, a particularly convenient formulation is described in Section 2.2.

We work mostly in the framework of *global* (also known as *rigid*) supersymmetry, where the parameter of the supersymmetry transformation does not depend on the space-time coordinate. The extension to local transformations, *supergravity* (SUGRA), automatically includes general relativity. At energy scales much lower than the Planck mass, it may be hoped that all effects of SUGRA are unimportant and rigid supersymmetry is a good description. Such an assumption cannot be entirely accurate when studying supersymmetry breaking; the mass of the gravitino (the spin $\frac{3}{2}$ superpartner of the graviton) arises from eating the massless goldstone fermion that appears when supersymmetry is broken and this is explicitly a SUGRA effect [57]. Also, in some calculable string theory completions achieving moduli heavy enough for acceptable cosmology requires significant SUSY breaking in the gravitational sector of the theory, which typically has a large effect on the visible sector [62–64]. However, there may well be large regions of the string landscape where such behaviour is not typical, and we usually adopt the common approach of assuming this high-scale physics does not have a significant effect on physics at lower scales.

2.2 Superspace

Superspace is a construction that allows the particles that make up a representation of the superalgebra to be assembled into a single object, a *superfield*. In order to combine bosons and fermions into an object with consistent transformations under the Lorentz group, a new two component spinor θ^α and its conjugate $\theta^\dagger_{\dot\alpha}$ are introduced with components satisfying the anticommutation relations $\{\theta_\alpha, \theta_\beta\} = 0$ (consequently, $\theta^2_\alpha = 0 = \theta^{\dagger 2}_{\dot\beta}$ and the components of these spinors are Grassmann variables).

A superfield is defined as a function of the space-time coordinates and the variables θ and $\theta^\dagger$. The power series expansion in a Grassmann variable necessarily terminates, and so a generic scalar superfield can be expanded

$$\begin{aligned}\Phi(x,\theta) = {} & \phi(x) + \theta\psi(x) + \theta^\dagger\chi^\dagger(x) + \theta\theta M(x) + \theta^\dagger\theta^\dagger N(x) + \theta\sigma^\mu\theta^\dagger V_\mu(x) \\ & + \theta\theta\theta^\dagger\lambda^\dagger + \theta^\dagger\theta^\dagger\theta\rho(x) + \theta\theta\theta^\dagger\theta^\dagger D(x) \ ,\end{aligned} \tag{2.4}$$

where ψ, χ, λ, ρ are fermions, V is a vector and the other fields are scalars. Superfields with Lorentz indices can also be constructed, but are not important for our purposes.

There is an explicit representation of the supersymmetry generators as differential operators on superspace, given by

$$\begin{aligned} Q_\alpha &= -i\partial_\alpha - i\sigma^\mu_{\alpha\dot\beta}\theta^{\dagger\dot\beta}\partial_\mu \ , \\ Q^\dagger_{\dot\alpha} &= i\partial^\dagger_{\dot\alpha} + \theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \ . \end{aligned} \tag{2.5}$$

It is also very useful to define covariant derivatives

$$\begin{aligned} D_\alpha &= \partial_\alpha + i\sigma^\mu_{\alpha\dot\beta}\theta^{\dagger\dot\beta}\partial_\mu \ , \\ D^\dagger_{\dot\alpha} &= -\partial^\dagger_{\dot\alpha} - i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \ , \end{aligned} \tag{2.6}$$

which are constructed to anticommute with the SUSY generators

$$\{D_\alpha, Q_\beta\} = \left\{D_\alpha, Q^\dagger_{\dot\beta}\right\} = \left\{D^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\right\} = \left\{D^\dagger_{\dot\alpha}, Q_\beta\right\} = 0 \ . \tag{2.7}$$

The action of an infinitesimal SUSY transformation on the general scalar multiplet can be obtained from Eq. (2.5), and shows Φ is a basis for a (reducible) representation of the SUSY algebra.[4] The transformation of the $\theta^2\theta^{\dagger 2}$ component, $D(x)$, is a total space-time derivative $\delta D = \frac{i}{2}\partial_\mu\left(\xi\sigma^\mu\lambda^\dagger - \rho\sigma^\mu\xi^\dagger\right)$, where ξ is the transformation parameter.

A phenomenologically important superfield, the *chiral superfield*, is obtained by imposing the restriction $D^\dagger_{\dot\alpha}\Phi = 0$ (similarly, antichiral superfields are defined by $D_\alpha\Phi = 0$). The covariant nature of the derivative ensures that the resulting superfield furnishes a representation of SUSY. The expansion of the chiral superfield is most simply written as

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \ , \tag{2.8}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\theta^\dagger$, ψ is a chiral Weyl fermion, and ϕ is a scalar. From this it is straightforward to obtain an expression for the components of the chiral superfield in $\left(x,\theta,\theta^\dagger\right)$ space. Like $D(x)$ in the general scalar multiplet, the SUSY transformation of the scalar field $F(x)$ is a total derivative, $\delta F(x) = -\sqrt{2}i\partial_\mu\psi(x)\sigma^\mu\xi^\dagger$. Due to the chain rule, any product

[4]Non-linear realisations of supersymmetry also exist, but are not important for us.

or sum of only chiral superfields is also a chiral superfield (likewise anti-chiral superfields, but not combinations of chiral and anti-chiral superfields).

The second important multiplet is obtained by imposing the condition $V\left(x,\theta,\theta^\dagger\right) = V^\dagger\left(x,\theta,\theta^\dagger\right)$, on the (now renamed) general scalar multiplet, which is again a covariant constraint. The resulting multiplet contains a vector field and allows for supersymmetric gauge theories to be constructed. After fixing part of the supergauge symmetry (to be defined below) leaving ordinary gauge transformations unfixed, the vector supermultiplet can be expanded as

$$V\left(x,\theta,\theta^\dagger\right) = \theta\sigma^\mu\theta^\dagger A_\mu\left(x\right) + \theta^2\theta^\dagger\lambda^\dagger\left(x\right) + \theta^{\dagger 2}\theta\lambda\left(x\right) + \frac{1}{2}\theta^2\theta^{\dagger 2}D\left(x\right) \ , \tag{2.9}$$

which contains a vector field A_μ, a gaugino λ, and an auxiliary field D. This is known as the Wess-Zumino gauge [65]. In this gauge, the definition of the vector supermultiplet's infinitesimal transformation under the supersymmetric version of a gauge transformations takes the simple form

$$\delta V = i\left(\Lambda - \Lambda^\dagger\right) - \frac{i}{2}\left[\left(\Lambda + \Lambda^\dagger\right), V\right] \ , \tag{2.10}$$

where, in the non-abelian case, the objects V and Λ are implicitly matrices, $V = T^a_{ij}V_a$ and $\Lambda = T^a_{ij}\Lambda_a$ with T^a the generator of a representation of the gauge group's Lie algebra, and Λ_a an arbitrary chiral superfield. It can be shown that the vector component transforms appropriately to be a gauge field, and λ_α and D transform in the adjoint. It is also useful to define a chiral superfield

$$W_\alpha = T^a W^a_\alpha = -\frac{1}{8}D^\dagger D^\dagger e^{-2V} D_\alpha e^{2V} \ . \tag{2.11}$$

This transforms covariantly under supergauge transformations

$$W_\alpha \mapsto e^{-2i\Lambda} W_\alpha e^{2i\Lambda} \ , \tag{2.12}$$

and can be evaluated in Wess-Zumino gauge as

$$W^a_\alpha|_{\mathrm{WZ}} = \lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2}\left(\sigma^\mu\overline{\sigma}^\nu\theta\right)_\alpha F^a_{\mu\nu} + i\theta\theta\left(\sigma^\mu\left(\partial_\mu\lambda + gf^{abc}A^b_\mu\lambda^c\right)^{\dagger a}\right)_\alpha \ , \tag{2.13}$$

where $F^a_{\mu\nu}$ is the usual gauge field strength.

Under a gauge transformation, a chiral multiplet is defined to transform as

$$\Phi \mapsto e^{-2i\Lambda'}\Phi \ , \tag{2.14}$$

where $\Lambda' = t^a_{ij}\Lambda_a$, with t_a the generators of the representation of the gauge algebra that Φ is in, reproducing the correct transformations for the component fields under ordinary gauge transformations.

These objects allow the construction of a generic $\mathcal{N} = 1$ Lagrangian containing chiral matter and gauge fields. In order that the action is invariant under the super-Poincaré symmetry we require that the Lagrangian transforms as a Lorentz scalar density, and its variation under the supersymmetry transformations is a space-time total derivative. Consider an object

$$\int \mathrm{d}^2\theta\, \mathrm{d}^2\theta^\dagger\, K\left(\Phi_i, \Phi_i^\dagger, W_\alpha, W_{\dot\alpha}^\dagger V, ...\right) \ , \tag{2.15}$$

where K is an arbitrary real (to make the Lagrangian real) function of any of the multiplets in the theory, known as the *Kähler potential*. The integral over superspace picks out the D component of the superfield $K\left(\Phi_i, \Phi_i^\dagger, ...\right)$ (any product of superfields is also a superfield), which is suitable for inclusion in a Lagrangian.[5]

Similarly the θ^2 component of a chiral superfield $F(x)$ also transforms as a total derivative, therefore the object

$$\int \mathrm{d}^2\theta\, W\left(\Phi_i, \Phi_j, ...\right) + \text{h.c.} \ , \tag{2.16}$$

is suitable for inclusion in the Lagrangian. Importantly W, the *superpotential*, only depends on chiral superfields (including the hermitian conjugate of any anti-chiral superfields) in order that it is a chiral superfield itself. When the theory is written in terms of chiral fields it is a holomorphic function of these.

Of course, the action must also be (super)gauge invariant, and contain the standard kinetic terms for the matter and gauge sectors. The superpotential can be made gauge invariant provided that, for each term, the product of the gauge representations of the superfields contains a singlet of the representation. It can be seen that a dimension two operator in the Kähler potential can be made invariant by the inclusion of the vector superfield in the form

$$\int \mathrm{d}^2\,\theta \mathrm{d}^2\theta^\dagger\, \Phi^\dagger e^{2g_a t^a V_a} \Phi \ , \tag{2.17}$$

which is invariant due to the exponentiated version of Eq. (2.10). As well as the kinetic terms for the fermion and scalar in the chiral multiplet, this contains the couplings of the gauge bosons and gauginos to the matter fields. More generally, the Kähler potential can be

[5] Integration over superspace is reviewed in many places for example [56].

made gauge invariant if it is defined as a function

$$K\left(\Phi^{\dagger}, e^{2g_a T^a V^a}\Phi, ...\right) \ , \tag{2.18}$$

and the product of the representations of the chiral multiplets in each term contains a singlet.

Kinetic terms for gauge fields and gauginos arise from the inclusion of the gauge invariant term

$$\int \mathrm{d}^2\theta \left(\frac{1}{4} - \frac{i g_a^2 \theta_{YM}}{32\pi^2}\right) \mathrm{Tr}\left(W^a_\alpha W_\alpha\right) + \text{h.c.} \ , \tag{2.19}$$

where θ_{YM} is a CP violating phase. Commonly, the vector superfield is rescaled so that the coefficient of the $\mathrm{Tr}\,(WW)$ term is $\frac{\tau}{16\pi i}$ where $\tau = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2}$ is the *holomorphic gauge coupling.* There is one more term that can be added to the Lagrangian for U(1) gauge groups,

$$\xi \int \mathrm{d}^2\theta \, \mathrm{d}^2\theta^{\dagger} \, V = \frac{1}{2}\xi D \ , \tag{2.20}$$

known as the Fayet-Iliopoulos (FI) term, which is gauge and supersymmetry invariant (since the D component is a gauge singlet for a U(1) group) [66]. Other objects that may be thought to give further interactions, such as by allowing covariant derivatives in the superpotential (for example, the object $DD^{\dagger}\Phi$ is automatically a chiral multiplet) can be shown to give no new couplings. The expressions given so far lead to the most general interaction of chiral and vector supermutiplets.[6]

In a renormalisible theory, the superpotential has dimensions $[\text{mass}]^3$, the Kähler potential $[\text{mass}]^2$, and the coefficient of $\mathrm{Tr}\,(WW)$ is just the numerical factor defined above (the multiplet V has mass dimension 0). In non-renormalisable theories, the superpotential and Kähler potential can include higher dimensional terms, and the gauge kinetic function can be a function of the chiral multiplets.

Once the Lagrangian has been written down in superspace, it is straightforward to expand the components to obtain the full set of interactions. Importantly, the highest component of the chiral multiplets $F\,(x)$ and of the vector multiplets $D\,(x)$ do not have kinetic terms. Consequently they are non-dynamical and, since their equations of motion are purely algebraic, can be integrated out of the theory.[7]

[6]It is interesting that not only does supersymmetry demand relations between the coupling constants of scalars and fermions, but actually *forbids* the existence of certain terms from the Lagrangian, for example a theory with unbroken supersymmetry can never include a dipole operator $\overline{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$ [67].

[7]Though non-dynamical these fields play an important role in allowing supersymmetry to be realised off-shell. For example, they lead to the theory having the same number of bosonic and fermionic degrees of freedom off shell.

Some Lagrangians are also invariant under an R-symmetry, which does not commute with the SUSY generators, and instead satisfies Eq. (2.3). If this is a symmetry, terms in the superpotential have R-charge +2 and in the Kähler potential R-charge 0. The components of a chiral multiplet, ϕ, ψ and F have R-charges r_Φ, $r_\Phi - 1$, and $r_\Phi - 2$ respectively. From Eq. (2.19), gauge vectors necessarily have R-charge 0, gauginos +1, and D 0.

2.3 Non-renormalisation Theorems

A major advantage of supersymmetric theories, compared to general QFTs, is that their quantum corrections are highly constrained. In particular, this allows for strong statements to be made about the behaviour of supersymmetric theories under renormalisation, and during strong coupling.

The first important result is that the superpotential is not perturbatively renormalised. More precisely, the most general term that can be generated by loops can be written in the form of an integral over $\mathrm{d}^2\,\theta \mathrm{d}^2\theta^\dagger$, and so interpreted as a correction to the Kähler potential. This was first proved using supergraph techniques [68], and later through a holomorphy argument that we very briefly review following [69].

First, all coupling constants in the superpotential are promoted to chiral superfields with scalar component expectation values equal to the coupling constants (known as *spurions*). By integrating out heavy modes a Wilsonian effective action is obtained that describes the interactions of the degrees of freedom in the theory with energies less than the cutoff. Provided SUSY is not spontaneously broken, this effective action is also supersymmetric, and consequently can be written as a combination of an effective Kähler potential and an effective superpotential.[8]

The effective superpotential is holomorphic in not only the dynamical chiral multiplets, but also the spurion coupling multiplets. Additionally, in the limit that the expectation values of the spurions (that is, the coupling constants) go to zero, the theory respects an enlarged symmetry group. Regarding the coupling constants as spurions, this symmetry is only *spontaneously* broken, and consequently constrains how the spurions and normal multiplets may appear in the effective superpotential. Finally, the behaviour of the theory in the limit that the coupling constants go to zero must match up to the tree-level action.

[8] The non-renormalisation theories apply only to the *Wilsonian* effective action, not the 1PI effective action, which contains the effects of massless states that can introduce non-holomorphic interactions.

Combined, these restrictions turn out to be severe enough to forbid any perturbative terms in the effective superpotential beyond those that appear at tree level.

The Kähler potential can receive perturbative corrections, both in the form of wavefunction renormalisation and the appearance of new terms. However, due to gauge invariance, the combination $\Phi^\dagger e^{2gV}\Phi$ has to renormalise to $Z_\phi \Phi^\dagger e^{2gV}\Phi$. After canonically normalising the kinetic terms for the chiral superfields, superpotential parameters are renormalised. However, this renormalisation is only logarithmic and is proportional to the parameters themselves. Additionally, non-perturbative effects can lead to new terms in the superpotential proportional to (positive) powers of the dynamically generated scales of any gauge groups in the theory. These are highly important in theories that run into strong coupling [70].

The running of gauge couplings in SUSY theories is also very constrained by holomorphy. Consider super-QCD with gauge group $\mathrm{SU}(N)$ and F flavours of chiral multiplets in the fundamental and anti-fundamental representations, and an action normalised as

$$\mathcal{L} \supset \frac{1}{16\pi i}\int \mathrm{d}^2\theta\, \tau\, \mathrm{Tr}\left(W^\alpha W_\alpha\right) \ , \tag{2.21}$$

where τ is the holomorphic gauge coupling. The real part of τ contains θ_{YM} which couples to a total derivative, and is therefore not perturbatively renormalised. The Wilsonian effective gauge coupling must remain a holomorphic function of τ (since it can be promoted to the expectation of a chiral field), and consequently the beta function of the Wilsonian gauge coupling must be a holomorphic function of τ. As a result the perturbative beta function must simply be an imaginary constant, independent of τ. This arises from the one-loop diagrams, and no higher terms contribute in perturbation theory, although there are corrections from non-perturbative effects. In particular, for an asymptotically free gauge theory, the holomorphic coupling at a scale μ is given by

$$\tau\left(\mu\right) = \frac{b}{2\pi i}\log\left(\frac{\Lambda}{\mu}\right) + \sum_{c=1}^{\infty} a_n\left(\Phi_i, \lambda, \mu\right)\Lambda^{bc} \ , \tag{2.22}$$

where b is the one-loop beta function coefficient, and the holomorphic scale $\Lambda = |\Lambda|\, e^{\frac{i\theta_{YM}}{b}}$ with $|\Lambda|$ the dimensionful scale associated to the gauge theory (the scale where the perturbative gauge coupling prediction is infinite), Φ_i are the chiral superfields, and λ_i are Yukawa couplings [71]. The beta function coefficient is given by

$$b = 3T\left(Ad\right) - \sum_i T\left(r_i\right) \ , \tag{2.23}$$

where i labels the chiral matter content of the theory, $T(r_i)$ is the Dynkin index of the representation r_i of the matter under the gauge group, and Ad denotes the adjoint representation.

The discussion so far however only applies to the holomorphic gauge coupling. To obtain the physical gauge coupling, the kinetic terms of the vector and chiral multiplets must be rescaled as $V' = gV$ and $\Phi_i' = Z_{\Phi i}^{1/2}\Phi_i$ respectively. However, rescaling the fermionic components of these multiplets is anomalous and the physical gauge coupling g_p is related to the holomorphic coupling by

$$\frac{1}{g_p^2} = \mathrm{Im}\left(\frac{\tau}{4\pi}\right) - \frac{2T(Ad)}{8\pi^2}\log(g_p) - \sum_i \frac{T(r_i)}{8\pi^2}\log(Z_i) \ . \tag{2.24}$$

Consequently, the running of the physical gauge coupling is given by

$$\beta(g_p) = -\frac{g^3}{16\pi^2}\frac{3T(Ad) - \sum_i T(r_i)(1-\gamma_i)}{1 - T(Ad)\frac{g^2}{8\pi^2}} \ . \tag{2.25}$$

This is the famous exact Novikov-Shifman-Vainshtein-Zakharov (NSVZ) beta function [72]. It disagrees with the holomorphic beta function at two loops, in contrast to the usual results in gauge theories, since there is no analytic map between the two beta function (due to the logarithms in Eq. (2.24)).

Finally, we briefly note that though we have focused on $\mathcal{N} = 1$ theories, theories with more supersymmetry have even more constrained renormalisation properties. In $\mathcal{N} = 2$ theories the beta function is only corrected perturbatively at one loop and the superpotential is entirely fixed by the matter content of the theory [73]. Furthermore, $\mathcal{N} = 4$ super-Yang-Mills is actually conformal [74].

2.4 The MSSM and Soft Breaking

The Minimal Supersymmetric extension of the SM (the MSSM), is attractive in its simplicity [56]. The field content is given in Table 2.1, and with the exception of the Higgs sector it is obtained by simply promoting the chiral fermion content to chiral superfields and the gauge bosons to vector superfields. Two Higgs doublets with conjugate gauge charges are needed to cancel anomalies in the hypercharge and SU(2) sector as well as the Witten anomaly (which demands an even number of fermion SU(2) doublets). Also, since $H^\dagger$ can not appear in the superpotential, a second doublet is required if both up- and down-type fermion mass terms are to be generated through superpotential interactions.[9]

[9] In extensions it is also possible to generate down-type fermion masses through Kähler interactions, if the cutoff of the theory is very low, however a second Higgs like doublet (or other EW charged matter content)

	bosons	fermions	SU(3)	SU(2)	U(1)
g	g_μ^a	$\tilde{g}^a$	Ad	1	0
W	$W_\mu^{\pm,3}$	$\tilde{W}^{\pm,3}$	1	Ad	0
B	B_μ	$\tilde{B}$	1	1	0
Q_i	$\left(\tilde{u}_L, \tilde{d}_L\right)_i$	$(u_L, d_L)_i$	$\square$	$\square$	$\frac{1}{6}$
$\overline{u}_i$	$\tilde{u}_{Ri}^*$	$u_{Ri}^\dagger$	$\overline{\square}$	1	$-\frac{2}{3}$
$\overline{d}_i$	$\tilde{d}_{Ri}^*$	$d_{Ri}^\dagger$	$\overline{\square}$	1	$\frac{1}{3}$
L_i	$(\tilde{\nu}_L, \tilde{e}_L)_i$	$(\nu, e_L)_i$	1	$\square$	$-\frac{1}{2}$
$\overline{e}_i$	$\tilde{e}_{Ri}^*$	$e_{Ri}^\dagger$	1	1	1
H_u	(H_u^+, H_u^0)	$\left(\tilde{H}_u^+, \tilde{H}_u^0\right)$	1	$\square$	$\frac{1}{2}$
H_d	$\left(H_d^0, H_d^-\right)$	$\left(\tilde{H}_d^0, \tilde{H}_d^-\right)$	1	$\square$	$-\frac{1}{2}$

Table 2.1: The MSSM field content, multiplets above the double line are vector multiplets, and below are chiral multiplets. Here the U(1) charge assignments are in the SM normalisation rather than SU(5). $\square$ ($\overline{\square}$) denotes the fundamental (antifundamental) representation, Ad the adjoint, and $i = 1, 2, 3$ labels the generations.

The MSSM superpotential is given by

$$W_{\text{MSSM}} = H_u \overline{u} Y_u Q - H_d \overline{d} Y_d Q - H_d \overline{e} Y_e L + \mu H_u H_d \ , \tag{2.26}$$

where Y_u, Y_d, and Y_e are the 3×3 Yukawa matrices, and the relative minus sign is a convention. The μ parameter is unique in that it is dimensionful, but is still required to be close to m_Z for EW symmetry breaking without fine tuning.[10]

The superpotential Eq. (2.26) leads to the interactions between fermions and the Higgs (and consequently fermion mass terms), as well as interactions between a fermion, a sfermion, and a Higgsino as required by supersymmetry. Integrating out the auxiliary fields F gives 4-scalar interactions proportional to Yukawa couplings squared, and also 3-scalar interactions once the Higgses obtain VEVs. The μ term leads to Higgs and Higgsino masses. The kinetic terms take the form of Eq. (2.17), which include the couplings of the gauge bosons to fermions, and also couplings of gauginos to a fermion and sfermion. D-terms from the vector multiplets lead to 4-scalar interactions proportional to gauge couplings squared.

The W and Z gauge bosons get masses by 'eating' the Goldstone bosons once the two Higgs doublets gain a VEV. This is a supersymmetric process so the associated gauginos also acquire supersymmetry preserving masses through interactions with the Higgsinos, known as the *superhiggs* effect. Neutrino masses are not included in the MSSM, however can be generated through a variety of additional field content as in the SM [77].

is still required to cancel the anomalies [75, 76].

[10] Requiring this is one aspect of the second part of the hierarchy problem discussed in the introduction: why the EW VEV happens to be small, even if this smallness is stable against radiative corrections.

However, Eq. (2.26) is not the most general superpotential consistent with gauge symmetries. Extra superpotential terms of the form

$$W = \alpha^{ijk} L_i L_j \overline{e}_k + \beta^{ijk} L_i Q_j \overline{d}_k + \mu'^i L_i H_u + \delta^{ijk} \overline{u}_i \overline{d}_j \overline{d}_k \ , \tag{2.27}$$

are also allowed. These violate baryon and lepton number, which are accidental symmetries of the SM. If present, such terms lead to proton decay that is much too fast compared to current experimental limits even if the superpartners have masses close to the Planck scale (the proton lifetime in the presence of such terms is $t_d \sim \left(\frac{m_{\tilde{q}}}{\text{TeV}}\right)^4 10^{-11}\,\text{s}$, where $m_{\tilde{q}}$ is the typical squark mass, while the observed limit is $t_d \gtrsim 10^{39}\,\text{s}$) [56].[11] The simplest way to protect against proton decay is by imposing an additional Z_2 symmetry, R-parity, under which SM states have charge $+1$, and superpartners have charge -1, which forbids all of the terms in Eq. (2.27).[12] The consequences of R-parity are phenomenologically significant: the lightest superpartner (LSP) is stable and so is a DM candidate; at colliders superparticles are produced in pairs; and any superpartner produced at a collider decays to an odd number of the LSP. There are also other symmetry structures that can prevent proton decay without forbidding all of the terms in Eq. (2.27), which have differing phenomenology to R-parity conserving models [79].

Of course, the MSSM as so far described is not an accurate description of Nature; there have been no observations of superpartners, and consequently SUSY must be spontaneously broken. As we discuss in Section 2.7, this necessarily occurs in a new sector of the theory, and is then mediated to the visible sector. The dominant SUSY breaking induced in the MSSM is in the form of parameters with positive mass dimension (since SUSY breaking is assumed to occur in a sector at a higher energy scale, and communicated via suppressed operators). Such breaking is known as *soft* breaking, and the most general expression for the MSSM soft terms is

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{2}\left(M_i \tilde{g}_i \tilde{g}_i\right) + \text{h.c.} - \left(H_u \tilde{\overline{u}} A_u \tilde{Q} - H_d \tilde{\overline{d}} A_d \tilde{Q} - H_d \tilde{\overline{e}} A_e \tilde{L}\right) + \text{h.c.} \\ & - 2\tilde{Q}^* m_Q^2 \tilde{Q} - \tilde{L}^* m_L^2 \tilde{L} - \tilde{\overline{u}}^* m_{\overline{u}}^2 \tilde{\overline{u}} - \tilde{\overline{d}}^* m_{\overline{d}}^2 \tilde{\overline{d}} - \tilde{\overline{e}}^* m_{\overline{e}}^2 \tilde{\overline{e}} \\ & - m_{Hu}^2 H_u^* H_u - m_{Hd} H_d^* H_d - \left(b H_u H_d + \text{h.c.}\right)\ , \end{aligned} \tag{2.28}$$

[11] Dimension-5 operators that are typically expected to arise suppressed by M_{Pl} can also lead to too fast proton decay, unless forbidden by a flavour symmetry in the UV theory [78].

[12] Since all vertices have an even number of fermions, R-parity is equivalent to a matter parity symmetry where states have charge $(-1)^{3(B-L)}$, and consequently is not truly an R-symmetry, though it can arise as the remnant of one.

where i represents any of the gauge groups, and the A and m_f^2 parameters are 3×3 matrices in flavour space. Notably, the (Majorana) gaugino soft masses necessarily break any R-symmetry in the theory, which has important implications for the mediation of SUSY breaking.

Although the MSSM has an enormous number of physical parameters, in total 105 more than the SM, they are highly constrained by flavour and CP observations [80, 81]. For example, there are very strong bounds from measurements of $\mu \to e\gamma$, K^0-$\overline{K}^0$ mixing, and $b \to s\gamma$ decays, as well as from D and B systems [82–85]. Further constraints on the CP-violating parameters arise from limits on the neutron and electron dipole moments [86, 87]. These constraints can be evaded if the soft masses are nearly universal and there are no new phases in the gaugino sector, and if the A-terms are either small or close to proportional to the associated Yukawa matrices. Alternatively, the sfermion masses may be close to being aligned with the fermion Yukawa matrices, or (some of) the superpartners could have relatively large masses suppressing the dangerous processes (or some combination of these possibilities) [56].

The RG equations for the SUSY preserving parameters in the MSSM can be calculated perturbatively. Above the scale of the soft masses, the superpotential parameters renormalise only due to the anomalous dimensions of the appropriate fields. For example

$$\frac{\mathrm{d}y_t}{\mathrm{d}t} = y_t \left(\gamma_{Hu} + \gamma_{Q3} + \gamma_{\overline{u}3}\right) = \frac{y_t}{16\pi^2}\left(6y_t^* y_t + y_b^* y_b - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right) , \tag{2.29}$$

using the one-loop expression for the anomalous dimensions, which in general is a matrix given by

$$\gamma_j^i = \frac{1}{16\pi^2}\left(\frac{1}{2}y^{imn}y_{jmn}^* - 2g_a^2 C_a\left(i\right)\delta_j^i\right) , \tag{2.30}$$

where y^{imn} is the Yukawa coupling between the states labelled i, m, and n and C is the quadratic Casimir.

At one loop, the Majorana gaugino soft masses renormalise as

$$\frac{\mathrm{d}M_i}{\mathrm{d}t} = \frac{1}{8\pi^2}b_i g_i^2 M_i , \tag{2.31}$$

where $b_i = \{33/5, 1, -3\}$. Since the coefficients b_i are exactly the beta function coefficients of the gauge coupling, the combination $\frac{M_i}{g_i^2}$ is a renormalisation group invariant. This is not surprising, the gaugino mass can be included as a θ^2 expectation value in the gauge coupling spurion, and the two components renormalise together.[13]

[13]More formally, an RG invariant can be constructed from the spurion that includes the physical gauge

The one loop renormalisation of the third generation sfermions and Higgs are given by expressions that take the form

$$\begin{aligned}\frac{\mathrm{d}m_Q^2}{\mathrm{d}t} &= \frac{1}{16\pi^2}\left(X_t + X_b - \frac{32}{3}g_3^2\left|M_3\right|^2 - 6g_2^2\left|M_2\right|^2 - \frac{2}{15}g_1^2\left|M_1\right|^2 + \frac{1}{5}g_1^2 S\right) ,\\ \frac{\mathrm{d}m_{Hu}^2}{\mathrm{d}t} &= \frac{1}{16\pi^2}\left(3X_t - 6g_2^2\left|M_2\right|^2 - \frac{6}{5}g_1^2\left|M_1\right|^2 - \frac{3}{5}g_1^2 S\right) \ ,\end{aligned} \tag{2.32}$$

where $X_t = 2\left|y_t\right|^2\left(m_{Hu}^2 + m_{Q3}^2 + m_{\overline{u}3}^2\right) + 2\left|A_t\right|^2$, $X_b = 2\left|y_b\right|^2\left(m_{Hd}^2 + m_{Q3}^2 + m_{\overline{d}3}^2\right) + 2\left|A_b\right|^2$, and $S = \mathrm{Tr}\left(m_{\Phi i}^2 Y_i\right)$. Terms including the, negligibly small, first two generation Yukawa couplings have been dropped. At two loops the first two generation sfermion masses feed into the stop and Higgs soft masses squared through gauge couplings, which can be a significant effect as we study in Section 5.

Since in the limit that the soft masses go to zero SUSY is restored, the RG corrections to the Higgs soft mass squared parameter are proportional to the soft masses and consequently small provided the soft masses are close to the EW scale. As long as the Higgs soft masses do not receive any other large contributions by coupling strongly to sectors with broken SUSY, the hierarchy problem is solved. This is in stark contrast to if the visible sector included hard SUSY breaking interactions, for example by a shift in the quartic Higgs-Higgs-Stop-Stop coupling Δy_t^2, which would lead to corrections to the Higgs mass squared of typical size $\delta m_h^2 \sim \Delta y_t^2 \Lambda_{\mathrm{UV}}^2$ where Λ_{UV} is the high scale that altered the quartic coupling.

Of course, there any many proposed extensions to the MSSM. Of primary interest to us is the *next-to-minimal supersymmetric SM* (NMSSM), which involves extending the MSSM with an additional singlet [90]. This has a modified Higgs sector compared to the MSSM, described in Section 2.6.

Another interesting extension are Dirac gauginos [76, 91–96]. In these models, there are R-symmetry preserving soft gaugino masses that arise through couplings to new chiral multiplets in the adjoint of the gauge group. If the chiral superfields are labelled A_1, A_2, A_3 (in the adjoints of U(1), SU(2), and SU(3), respectively), the mass terms can be written in terms of spurions M_i^α with non-zero θ components $\langle M_i^\alpha\rangle = \theta^\alpha m_i$ as

$$\int \mathrm{d}^2\theta \sum_{i=1}^{3} \sqrt{2} M_i^\alpha \mathrm{Tr}\left(W_{\alpha i} A_i\right) \supset \sum_{i=1}^{3} -m_i \lambda_{ia}\psi_{ia} \ , \tag{2.33}$$

where ψ_i is the fermion component of A_i, and the index a labels the generators of the group.

coupling and gaugino mass, so that this relation is maintained at all orders in perturbation theory and during strong coupling [88, 89].

The spurions M_i could arise from a D-component of a new hidden sector which couples via a non-renormalisible operator. The operator Eq. (2.33) also leads to masses for the real scalar components of A_i, and modifies the D-term potentials for the visible sector gauge groups.

A particularly interesting property of theories where the dominant SUSY breaking gaugino masses arise from terms of the form Eq. (2.33) is that, unlike the MSSM, there is no logarithmically divergent contribution to the sfermion masses from the gaugino masses [91]. Such a contribution is usually expected to turn on at the mediation scale, leading to a correction to the sfermion soft mass containing a large logarithm. However, in Dirac gaugino models the only possible sfermion mass counterterm is

$$\int \mathrm{d}^4\theta\, \theta^2\theta^{\dagger 2}\, \frac{m_i^4}{\Lambda^2} Q^\dagger Q \ , \tag{2.34}$$

which vanishes in the limit that the cutoff Λ is taken to infinity. Consequently, there can be no corrections to the sfermion masses sensitive to the UV cutoff of the theory and enhanced by a large logarithm $\log\left(\frac{\Lambda_{UV}}{m_Z}\right)$. Instead, there are only finite contributions to the sfermion masses, a point which will be important when we discuss the tuning of these theories in Section 5.3. This finiteness is due to an effective $\mathcal{N}=2$ supersymmetry in the gauge sector of the theory (an $\mathcal{N}=2$ vector multiplet consists of $\mathcal{N}=1$ vector and chiral multiplet, which is exactly the matter content in the gauge sector of these models) which constrains the running of the theory even more than in $\mathcal{N}=1$ theories. Model building is however not completely straightforward: generating scalar soft masses for the adjoint chiral multiplets is challenging, and UV completions of SUSY breaking and mediation are complex [91]. Also, gauge unification does not occur without additional states [94].

2.5 Unification

An encouraging feature of the MSSM is that, if the superpartners are near the weak scale, it leads to successful SU(5) gauge unification at a scale $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$ [51, 52]. This is shown in Fig. 2.1, where the gauge couplings in the SM and the MSSM are plotted (assuming no additional matter charged under the SM gauge group). The calculation of gauge running has been performed to two loops (including one-loop threshold effects) [97], although there is significant model dependence from threshold corrections at the GUT scale [98].[14] Agreement

[14] Threshold corrections are the non-logarithmically enhanced corrections to the parameters of the theory that appear in an effective field theory as a result of integrating out heavy states from the full theory, see for example [99].

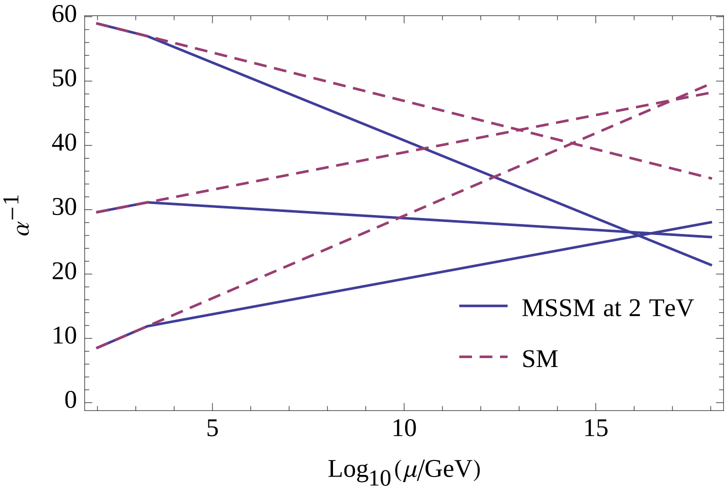

Figure 2.1: Running of gauge couplings in the SM and the MSSM with superpartners at 2 TeV, assuming SU(5) normalisation of the hypercharge.

with precision unification is good, under the assumption that there is a relatively small threshold correction at the GUT scale, $\Delta\alpha_3/\alpha_3\,(M_G) \sim 3\%$.[15]

In the SM the normalisation of the hypercharge is unfixed, and statements about unification are meaningful only once a specific GUT group is chosen. Consequently, achieving unification by adding extra matter, at an energy scale that is free to be fixed, for a particular U(1) normalisation is not difficult; three straight lines crossing at a point requires one parameter to be adjusted. The reason that unification in the MSSM is interesting is that it is automatically achieved in the minimal SUSY model, with the hypercharge normalisation that arises from simple GUT models, and superparticles in the correct mass range to also be relevant to the hierarchy problem, all of which are independently motivated.

While GUTs are an attractive possibility, they are not without problems. Since quarks and leptons are now in the same gauge multiplets nucleon decay is automatically a possibility. In SM GUTs any decays mediated through dimension-5 operators (generated after additional heavy gauge bosons that appear in these models are integrated out) are too rapid to be consistent with observation, and dimension-6 operators are also severely constrained by proton decay. The situation is mildly improved in supersymmetry, since the GUT scale is raised, reducing the decay rate. However, even if symmetries forbid these dangerous operators at tree level, they are typically regenerated by exchange of the additional, coloured, Higgsinos that appear in SUSY GUT models. Consequently, the limits are quite severe, and

[15] In simple 4D SUSY GUTs there a threshold correction at the GUT scale from the additional Higgs triplets, which goes in the wrong direction, requiring a larger opposite sign contribution from the GUT breaking sector of the theory [100].

(barring tuning in the flavour structure) typically rule out minimal SU(5) models [1]. It is possible to build models with non-minimal Higgs sectors (either in SU(5) or SO (10)) that evade current limits, or alternative constructions such as orbifold GUTS can also lead to viable theories [101]. Obtaining sufficient mass splitting of the doublet and triplet Higgs states is also problematic [102], and may be suggestive of some structure beyond pure 4D field theory.

In GUT models the Yukawa couplings of the tau lepton and bottom quark are typically expected to unify at the GUT scale, since both arise (in an SU(5) like model) from terms of the form $\lambda \mathbf{10}\, \overline{\mathbf{5}}\, \overline{\mathbf{5}}_{\mathrm{H}}$ (where H labels the multiplet containing the Higgs). In minimal SO(10) models the top Yukawa is also expected to unify with these two couplings at the high scale since there is only one Yukawa term $\lambda \mathbf{16}\, \mathbf{16}\, \mathbf{10_H}$ [103]. These relations can be satisfied in some regions of SUSY parameter space [104–107].[16] The first two generation Yukawas typically cannot unify, which naively is in contradiction with the requirement of a GUT, however this requirement can be evaded by the introduction of flavour symmetries [108].

2.6 The (N)MSSM Higgs Sector

The Higgs sector in the MSSM is more complicated than the SM due to the two Higgs doublets, $H_u = (H_u^0, H_u^+)$ and $H_d = \left(H_d^-, H_d^0\right)$. The MSSM superpotential leads to terms quadratic in the Higgs fields and the gauge D-terms lead to quartic scalar interactions. Using gauge invariance, H_u^+ can be taken to have zero VEV without loss of generality. From the full form of the potential it can be shown that this leads to $\left\langle H_d^- \right\rangle = 0$ [56]. The scalar potential, in terms of parameters evaluated at a low scale, then reduces to

$$\begin{aligned} V =& \frac{1}{8}\left(g^2 + g'^2\right)\left(\left|H_u^0\right|^2 + \left|H_u^+\right|^2 - \left|H_d^0\right|^2\right) \\ &+ \left(|\mu|^2 + m_{Hu}^2\right)\left|H_u^0\right|^2 + \left(|\mu|^2 + m_{Hd}^2\right)\left|H_d^0\right|^2 - \left(b H_u^0 H_d^0 + \text{c.c.}\right) \ . \end{aligned} \tag{2.35}$$

The conditions for Eq. (2.35) to give a stable EW symmetry breaking vacuum are

$$\begin{aligned} &2b < 2\,|\mu|^2 + m_{Hu}^2 + m_{Hd}^2 \ , \\ &b^2 > \left(|\mu|^2 + m_{Hu}^2\right)\left(|\mu|^2 + m_{Hd}^2\right) \ . \end{aligned} \tag{2.36}$$

If these inequalities are satisfied, both H_u^0 and H_d^0 gain VEVs, which for correct EW symmetry breaking must satisfy $\langle H_u^0 \rangle^2 + \langle H_d^0 \rangle^2 = (174\,\mathrm{GeV})^2$, and the ratio of VEVs is defined as

[16] As we discuss in Section 4 there are other effects that could modify these properties.

$\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$. The minimum of the potential Eq. (2.35) satisfies

$$\begin{aligned} \sin(2\beta) &= \frac{2b}{m_{Hu}^2 + m_{Hd}^2 + 2|\mu|^2} \ , \\ m_Z^2 &= \frac{|m_{Hd}^2 - m_{Hu}^2|}{\sqrt{1-\sin^2(2\beta)}} - m_{Hu}^2 - m_{Hd}^2 - 2|\mu|^2 \ , \end{aligned} \tag{2.37}$$

which in the limit of large $\tan\beta$ reduces to

$$m_Z^2 = -2\left(m_{Hu}^2 + |\mu|^2\right) + \frac{2}{\tan^2\beta}\left(m_{Hd}^2 - m_{Hu}^2\right) + \mathcal{O}\left(1/\tan^4\beta\right) \ . \tag{2.38}$$

For the conditions Eq. (2.36) to be satisfied requires $m_{Hu}^2 \neq m_{Hd}^2$ at the weak scale. This can readily be achieved through radiative EW symmetry breaking, even if the two Higgs have the same soft mass at the mediation scale. In this, the up-type Higgs mass squared parameter is driven to negative values during running due to the large Yukawa coupling to the stops [109].[17]

After the Goldstone bosons have been eaten, the remaining Higgs sector matter content consists of two neutral CP-even scalars, a CP-odd neutral scalar, a charge $+1$ scalar and a charge -1 scalar. In the limit of very heavy superpartners, the masses of all of these states, except the lightest CP even scalar h^0, can be arbitrarily large at tree level and the properties of h^0 converge to those of the SM Higgs. However, the mass of h^0 is bounded by m_Z at tree level, and including the leading one-loop corrections is given by

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2}\left[\log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{X_t^2}{12 m_{\tilde{t}}^2}\right)\right] \ , \tag{2.39}$$

where $X_t = A_t - \mu/\tan\beta$ is the stop mixing parameter and $m_{\tilde{t}}^2 = m_{\tilde{t}1} m_{\tilde{t}2}$ (the next corrections can also be important, and are given in [110]). Generating the observed $\sim 125\,\text{GeV}$ Higgs mass in the MSSM is challenging without reasonably large stop masses (that lead to fine-tuning), or very large A-terms [111]. This is one of the motivations for extended Higgs sectors that we review shortly and study in Section 4.

For correct EW symmetry breaking (without fine tuning in Eq. (2.37)) the MSSM requires the supersymmetry preserving mass μ to be close to the SUSY breaking mass $\sqrt{b}$ and the Higgs soft masses, this is known as the μ problem. Solutions to this often involve forbidding a bare μ term, and allowing it to arise as a SUSY breaking effect. For example the Giudice-Masiero mechanism [112] postulates the existence of a Kähler potential term $\frac{X^\dagger}{M_{\text{Pl}}} H_u H_d$, which leads to a μ term if X acquires a SUSY breaking F-term VEV. Assuming

[17] This can be made more precise by analysing the eigenvalues of the RG equations.

the dominant mediation to the SM is through Planck-suppressed operators the μ term generated is parametrically the same size as the other soft masses. The μ problem is typically rather more severe in models of gauge mediation, and a further problem arises that even if the correct size μ parameter is generated, the b parameter then often has size $\sim 16\pi^2\mu^2$ and is unacceptably large. However, several solutions have been proposed, see for example [113].

An alternative, attractive, solution is to forbid the μ parameter by a new symmetry (often taken to be a Z_3), but introduce a new singlet, S, that has a superpotential coupling $\lambda S H_u H_d + \kappa S^3$, and gains a weak scale VEV. This is the previously mentioned NMSSM [90]. The Higgs sector contains additional CP-even and CP-odd scalar and singlet fermion degrees of freedom that mix with the neutral Higgs. An important effect is that there is an additional contribution to the tree-level physical Higgs mass that arises from the F-term of the superfield S. This modifies the tree level expression into

$$m_{h0}^2 \simeq m_Z^2 \cos^2 2\beta + \lambda^2 \sin^2 2\beta \ , \tag{2.40}$$

while the leading stop loop correction Eq. (2.39) remains unchanged. There are also further corrections proportional to $g^4, g^2\lambda^2, g^2\kappa^2, \lambda^4, \kappa^4$ that are typically less important [90]. As a result of the additional contribution in Eq. (2.40) it is possible to obtain a Higgs with a mass of 125 GeV without heavy stops (this is studied in Section 4).

There are a number of modifications to collider phenomenology in the NMSSM compared to the MSSM as a result of the additional states. The NMSSM Higgs sector can allow new Higgs to Higgs decay, both for charged and uncharged Higgses, that are potentially observable at the LHC [114]. The neutralino sector is complicated by the fermionic component of S, modifying production and decay channels, and potentially leading to displaced vertices if there are small couplings between a bino like next to lightest supersymmetric partner (NLSP) and a singlino like LSP [115, 116]. Additionally, there are NMSSM specific effects on B physics [117] and precision observables such as the anomalous magnetic moment of the muon [118]. The phenomenology of (DM) candidates in the NMSSM can also differ from the MSSM, especially if the LSP is mostly singlino like [118].

However, there are problems with the simplest implementations of these models. Often couplings to heavy fields in the theory (for example during the mediation of SUSY breaking) induce linear terms in the superpotential $\sim \xi S$ or soft Lagrangian $\sim \xi_S \tilde{S}$ (where $\tilde{S}$ is the scalar component of S). If these dimensionful parameters are large compared to the weak scale there is either no EW symmetry breaking or a fine-tuning problem. This is known as the tadpole problem [119], and can be evaded if there are discrete symmetries in the theory [120]. Such symmetries can forbid all operators couplings heavy states to S to a sufficiently high order in perturbation theory that the induced tadpole terms are small compared to the EW scale.

A second problem arises from the spontaneous breaking of the discrete symmetry (that forbids a tree-level μ term) in the early Universe. In particular, during EW symmetry breaking different regions in the Universe may have the same vacuum energy but different phases of $\langle H_u \rangle$, $\langle H_d \rangle$, and $\langle S \rangle$. Such regions are separated by domain walls, which often dominate the Universe's energy density ruining the successful predictions of big bang nucleosynthesis and leading to much too large anisotropies in the cosmic microwave background [121]. This is known as the domain wall problem, and may be evaded by allowing small violations of the discrete symmetry, for example by Planck-suppressed operators. The symmetry violating operators shift the relative energy of the vacua slightly, avoiding domain walls. However, care is required to avoid reintroducing a tadpole problem, and more complex models (for example, involving gauged R-symmetries) may be necessary [122].

2.7 Supersymmetry Breaking

We now turn to the question of how to break SUSY in a hidden sector. This is a requirement of models of gauge mediation, and some models of gravity mediation (for example, this could be required in heterotic string theory completions). In supersymmetric theories the energy operator can be written as $H = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right)$. Since a SUSY preserving vacuum state is annihilated by Q_α, a vacuum is SUSY breaking if and only if it has positive energy. The scalar part of the potential is $V \sim \frac{1}{2} F^{i*} F_i + \frac{g^2}{2} D^{a*} D^a$, and so if $F_i = 0$ and $D^a = 0$ cannot be simultaneously satisfied for all i and a the vacuum breaks SUSY.[18] Despite SUSY being broken, the mass sum rule

$$\mathrm{Tr}\left(m^2_{scalars}\right) - 2\mathrm{Tr}\left(m^2_{fermions}\right) = 2\sum_a D_a \mathrm{Tr}\left(g t_a\right) \ , \tag{2.41}$$

is still satisfied at tree level in theories with only renormalisible operators (it can be modified by loop corrections). The right-hand side is zero for any compact simple non-abelian group (due to the trace over the generator), and also vanishes for U(1) theories without a mixed

[18]It can be shown that, in the absence of a Fayet-Iliopoulos term, if a theory has a solution to all $F_i = 0$ there is necessarily a simultaneous solution to all $D^a = 0$.

gauge-gravity-gravity anomaly since this leads to the sum of charges vanishing (as is the case for U(1) hypercharge). This is the reason SUSY cannot be broken by the visible sector in simple models; some bosons would be too light for acceptable phenomenology.[19]

An interesting quantity in supersymmetric theories is the Witten index [123]

$$\mathrm{Tr}\,(-1)^F = \sum_E n_B\,(E) - n_F\,(E) \ , \tag{2.42}$$

where $n_B\,(E)$ ($n_F\,(E)$) are the number of bosonic (fermionic) states with energy E. Even if SUSY is spontaneously broken, the action of the SUSY generators remains well defined and therefore finite energy bosonic and fermionic states are paired up. However, zero-energy states are annihilated by the SUSY generators and so can have a mismatch between bosonic and fermionic degrees of freedom. Therefore the Witten index reduces to $\mathrm{Tr}\,(-1)^F = n_B\,(0) - n_F\,(0)$. Since a vacuum is supersymmetric if it has zero energy, a theory with a non-zero index has at least one SUSY preserving vacuum. However, a theory having zero index does not guarantee that there are no SUSY preserving vacua.

The Witten index is a topological quantity, independent of the values the parameters of the theory take. This is because the only way it could change as parameters are adjusted is if finite energy states moved to zero energy or vice versa. However, since finite energy states are paired, this cannot actually change the value of the index.[20] It can be shown that super-Yang-Mills (SYM) theories with massive vector-like matter have non-zero index and consequently do not have a stable SUSY breaking vacua [123].

A simple superpotential that leads to F-term breaking is the O'Raifeartaigh model [124]. This is a theory with three chiral superfields and a superpotential given by

$$W = -k^2\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2 \ . \tag{2.43}$$

The conditions $F_1 = 0$ and $F_2 = 0$ cannot be simultaneously satisfied, and if $m^2 > yk$ the minimum of the potential is at $\phi_2 = \phi_3 = 0$ (where ϕ_i is the scalar component of Φ_i). ϕ_1 is not fixed at tree level, however as supersymmetry is broken there are now loop corrections to its mass that lead to a stable SUSY breaking minimum at $\langle\phi_1\rangle = 0$. There is a massless fermion in the broken theory, the goldstino. The appearance of a massless goldstino is a

[19] Models of (semi)-direct gauge mediation effectively break SUSY in a much more complex visible sector, by adding a large number of additional heavier fields that allow the MSSM superpartners to all be heavier than their partners.

[20] We neglect a number of subtleties around the calculation of the index including regularisation and behaviour of fields running away to infinity.

general feature of spontaneous SUSY breaking, even if this occurs during strong coupling. As mentioned previously, in SUGRA the goldstino is eaten by the gravitino and as a result, the gravitino gains a mass $m_{3/2} = \frac{F}{\sqrt{3}M_{\rm Pl}}$.

Alternatively, SUSY can be broken through a D-term. For example, a theory with U(1) gauge group and FI term ξ has a scalar potential

$$\begin{aligned} V &= \frac{1}{2}D^2 - \xi D + gD\sum_i q_i \phi^{i*}\phi_i \ , \\ \implies D &= \xi - g\sum_i q_i \phi^{i*}\phi_i \ . \end{aligned} \tag{2.44}$$

If the chiral superfields all have large positive masses, this leads to a D-term expectation value $\langle D\rangle = \xi$.

There are several conditions on whether a theory is expected to break supersymmetry. Firstly, if a theory spontaneously breaks a global symmetry but has no (non-compact) flat directions it breaks SUSY. This is because a chiral multiplet includes two scalars so if SUSY was preserved there would be a second massless scalar in the same multiplet as the Goldstone boson [125]. In practise this condition is not straightforward to use since finding if a global symmetry is broken is typically as hard as directly calculating if SUSY is broken. However, it can give an indication since t'Hooft's anomaly matching criteria [26] (which constrains the properties of theories as they pass through strong coupling) is only satisfied if global symmetries are unbroken. If anomaly matching cannot be straightforwardly satisfied and the theory has no run away directions it is plausible SUSY is broken. A second theory is that (assuming the low-energy effective theory contains no gauge fields) a spontaneously broken R symmetry generically leads to supersymmetry breaking [126]. If the superpotential has either no symmetries or a global symmetry, the F-term equations have the same number of unknowns as equations, so a solution generically exists. However, if the theory has an R-symmetry that is spontaneously broken by a field ϕ_1, with R-charge q_1, getting a VEV, the superpotential can be written in terms of a new set of variables as

$$\begin{aligned} W &= \phi_1^{2/q_n} f\left(X_i\right) \ , \\ X_i &= \frac{\phi_i}{\phi_1^{q_i/g_n}} \ , \end{aligned} \tag{2.45}$$

where $i = 2, ..., n$, and n is the number of chiral multiplets in the theory. As a result, the F-term equations take the form of n conditions on the function $f\left(X_i\right)$, which is a function of $n-1$ variables, and so generically cannot be simultaneously solved. Majorana gaugino masses are forbidden by an R-symmetry, and even though it is spontaneously broken this can result in models having gaugino masses that are much smaller than the sfermion masses leading to fine-tuning problems [127] (alternatively, if the mediation mechanism does not respect the R-symmetry there is a danger it will lead to new SUSY preserving vacua [128]).

In a supergravity completion, the scalar potential is given by

$$V = e^{K/M_{\rm Pl}^2}\left(K^{ij*}F_iF_{j*} - 3\frac{|W|^2}{M_{\rm Pl}^2}\right) , \tag{2.46}$$

where K is the Kähler potential, $K^{ij*} = \partial_i\partial_{j*}K$, and the F-terms have been generalised to $F_i = D_iW \equiv \partial_iW + \frac{(\partial_iK)}{M_{\rm Pl}}W$ [57]. After supersymmetry breaking the first term gives a positive contribution, and the superpotential must gain a vacuum expectation value $\langle W\rangle \sim M_s^2M_{\rm Pl}$ to obtain an (almost) vanishing cosmological constant, where M_s is the SUSY breaking scale.[21] This superpotential expectation breaks any R-symmetry in the theory, and even if it arises in a different sector to the SUSY breaking can feed into it through higher-dimensional operators potentially modifying the SUSY-breaking dynamics.

In light of these considerations, the requirement that the SUSY-breaking vacuum is the absolute minimum of the potential is often relaxed [129], and models with metastable SUSY breaking vacua are phenomenologically acceptable provided they are sufficiently long lived. These only require an accidental R-symmetry at some points in field space, and can evade the problems above.

Although technically natural, the models so far require small mass scales and parameters to be put in by hand. A better alternative is for spontaneous supersymmetry breaking to be triggered by a hidden sector running into strong coupling at a scale exponentially separated from other scales in the theory [130]. To study such models is challenging since it requires an understanding of strong coupling. However, supersymmetry allows for new insights into such regions, and some examples of SUSY-breaking sectors have been found, one of which is reviewed in the next section.

2.8 Seiberg Duality and the ISS Model

As well as constraining the renormalisation properties of theories, supersymmetry also allows for an increased understanding of the behaviour of theories during regions of strong coupling. A famous example of this is the Seiberg-Witten theory [131], which gives an exact description

[21] Of course, this requires an extraordinary level of tuning, which we do not even attempt to address.

of the massless low-energy degrees of freedom in $\mathcal{N} = 2$ theories. Here we review a duality in $\mathcal{N} = 1$ theories, important for our later work, known as Seiberg duality [132]. This gives detailed information on the low-energy behaviour of gauge theories, especially super-QCD models.

Consider a SU (N) gauge theory with F flavours of vector-like fermions in the fundamental and anti-fundamental representations. This has a large moduli space, and if the theory has $F > N$ a typical point in the moduli space breaks the gauge symmetry completely through scalar VEVs. Generically, the low-energy theory contains $2NF - (N^2 - 1)$ light degrees of freedom described by the gauge invariant baryons and meson superfields

$$\begin{aligned} M_i^j &= \overline{\Phi}^{jn}\Phi_{ni} \ , \\ B_{ij\ldots} &= \Phi_{ni}\Phi_{mj}\ldots\epsilon^{nm\ldots} \ , \\ \overline{B}^{ij\ldots} &= \overline{\Phi}^{ni}\overline{\Phi}^{mj}\ldots\epsilon_{nm\ldots} \ , \end{aligned} \tag{2.47}$$

where additional constraint equations between these states ensure the correct number of degrees of freedom. For $F \geq 3N$ it can be shown that the theory is not asymptotically free, so is simply a low-energy effective theory [58]. For $\frac{3}{2}N < F < 3N$ there is an interacting conformal infra-red (IR) fixed point, while for $N + 1 < F \leq \frac{3}{2}N$ the theory runs into strong coupling.[22] For $F < N$ there is a run away direction in the low-energy theory. This is because the dynamically generated superpotential is

$$\begin{aligned} W &\sim \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} \ , \\ V &\sim \left|\frac{\partial W}{\partial M}\right|^2 \sim |M|^{\frac{-2N}{N-F}} \ , \end{aligned} \tag{2.48}$$

which drives $M \to \infty$ if $F < N$ [125]. The cases $F = N$ and $F = N + 1$ are special, in the former the theory is confining and quantum corrections to the classical constraint equations push it away from the origin in field space ($M = B = \overline{B} = 0$) so at least some of the global symmetries are broken. In the later, the theory is confining, but the moduli space includes the origin where all the global symmetries are unbroken [58].

Sieberg duality proposes that the deep IR behaviour of (certain classes of) $\mathcal{N} = 1$ gauge theories are completely equivalent to the IR behaviour of different gauge theories (in condensed matter terminology, they are in the same universality class). In analogy with elec-

[22] It can be shown that in this region the theory cannot flow into an IR conformal fixed point, since such a point would violate unitarity conditions.

tromagnetic duality the two theories are referred to as the *electric* and *magnetic* theories, respectively. In particular for sQCD, if $F > N+1$, there is a dual theory that is a $\mathrm{SU}(F-N)$ gauge theory with matter content consisting of F flavours of 'dual' quarks q and antiquark $\tilde{q}$ superfields transforming in the fundamental and antifundamental of $\mathrm{SU}(F-N)$, and a gauge singlet meson superfield (that can be thought of as the meson of the electric theory) M_e with mass dimension 2. The dual theory has a superpotential

$$W_{\mathrm{mag}} = \frac{1}{\hat{\Lambda}} M_e q \tilde{q} \ , \tag{2.49}$$

where $\hat{\Lambda}$ is parametrically related to the dimensionful scales of the UV and IR theories (but not entirely fixed by holomorphy). This superpotential is essential for ensuring the degrees of freedom in the two theories match up. If the electric theory has quark mass terms in the form of a superpotential

$$W_{\mathrm{el}} = \sum_{i=1}^{F} m_i Q^i Q_i \ , \tag{2.50}$$

where the mass matrices have already been diagonalised, this maps into the dual theory as a superpotential term

$$W_{\mathrm{mag}} \supset \sum_i m_i {M_e}_i^i \ . \tag{2.51}$$

Rescaling the meson to mass dimension 1, and defining $\mu^2 = -m\hat{\Lambda}$, the superpotential of the theory dual to an electric theory with universal quark masses is

$$W_{\mathrm{mag}} = h\tilde{q}Mq - h\mu^2 \Lambda \mathrm{Tr}\,(M) \ , \tag{2.52}$$

where h is a coupling constant unfixed by holomorphy.

The behaviour of the magnetic theory is interesting. Assuming vanishing (or small compared to Λ) quark masses in the electric theory, the beta function of the dual theory is

$$\beta(\tilde{g}) \sim -\tilde{g}^3 \left(3(F-N) - F\right) \sim -\tilde{g}^3 (2F - 3N) \ , \tag{2.53}$$

where $\tilde{g}$ is the gauge coupling of the dual theory. Consequently, in the region $N+1 > F \geq \frac{3}{2}N$ the dual theory loses asymptotic freedom and has a trivial IR fixed point $\tilde{g}^2 = h^2 = 0$, so in the IR is a free theory of composite states. This means the dual theory is weakly coupled in a regime where the original theory is strongly coupled and vice versa. For $\frac{3}{2}N < F < 3N$ the dual theory has an interacting IR fixed point at finite $\tilde{g}^2$ and h^2. In this parameter range the duality is between two different theories with IR fixed points that describe the same physics.

The case $F = N + 1$ is subtle since there is no dual gauge group, however a careful analysis shows that the duality still applies near the origin in field space.

Although the duality is a conjecture (even the existence of the fixed points is only certain for $F = N(3 - \epsilon)$ where ϵ is infinitesimal) it is supported by a large amount of evidence. One consistency check is that the global anomalies match in the original and dual theories. Also, the moduli spaces are the same dimension in both theories and the gauge invariant operators match. Giving one flavour of quark a large mass in the original theory (reducing F to $F - 1$ in the effective theory) has the correct effect on the dual theory. Finally, taking the dual of the dual theory maps the original theory onto itself as required [58].

Seiberg duality can be used to study a model of dynamical SUSY breaking, the Intriligator-Seiberg-Shih (ISS) model [129]. This consists of SQCD in the region $N + 1 < F < \frac{3}{2}N$, with quark masses m much smaller than the dynamical scale of the gauge group Λ.[23] Below the strong coupling, the theory is best described by its magnetic dual, which has superpotential given by Eq. (2.52). The meson F-terms, $F_{M_i^j} = h\tilde{q}_i^a q_a^j - h\mu^2\delta_i^j$ (i, j are flavour indices and a is a colour index), cannot simultaneously vanish since the rank of $\tilde{q}_i^a q_a^j$ is $F - N$ while the rank of δ_i^j is N, and consequently SUSY is broken in this effective theory.

Actually, the theory has a SUSY preserving vacuum (as must be the case from the Witten index) at large field values where the quark mass term is not a small perturbation of the theory, and Seiberg duality is not accurate. However, it can be shown that a Coleman-Weinberg potential lifts the tree-level flat directions near the origin in field space, resulting in a metastable SUSY breaking vacua at

$$M = 0, \qquad q = \tilde{q} = \begin{pmatrix} \mu \mathbf{1_{F-N}} \\ 0 \end{pmatrix}. \tag{2.54}$$

The induced SUSY breaking is given by $V = \sum_i F_i^* F_i = (F - N)\, h^2\mu^4$, and can be understood in terms of an approximate R-symmetry in the magnetic theory. The superpotential Eq. (2.52) is generic for a theory in which the superfields are charged under an R-symmetry as $[\Phi] = 2$ and $[q] = [\tilde{q}] = 0$.

The decay rate of the metastable SUSY breaking vacua is parametrically $\Gamma \sim e^{-S_b}$, where $S_b \sim \left(\frac{\Lambda}{m}\right)^{\frac{6N-4F}{N}}$, so the typical lifetime can be much longer than the age of the Universe for suitable parameter choices. It has also been suggested that the early Universe may drive the

[23] Some masses much larger than Λ are also allowed since the corresponding states are simply integrated out of the theory.

system into the metastable vacuum [133]. Actually, the ISS model so far described is not entirely satisfactory; although Λ is exponentially separated from other scales in the theory, the small mass scale m is still put in by hand, defeating the object of dynamical SUSY breaking. In Section 3, we study a refinement that generates a small mass scale automatically.

2.9 Supersymmetry Mediation

The SUSY-breaking sector must be connected to the visible sector to induce soft masses for the visible sector superpartners. A simple way for this to occur is through gauge mediation. In this, additional (relatively heavy) messenger fields are introduced that are charged under the gauge groups of the SM and also coupled to the SUSY-breaking sector. The messenger fields obtain SUSY-breaking mass splittings from their couplings to the SUSY-breaking sector, and these induce visible sector gaugino masses at one loop and sfermion masses squared at two loops. Typically the messenger fields are taken to be complete representations of SU(5) so that gauge unification is preserved.

For example, suppose the theory contains n_m copies of chiral and anti-chiral messenger superfields, Ψ and Ψ^c, in the 5 and $\overline{5}$ of SU(5). These can couple to a gauge singlet in the SUSY-breaking sector X (which could be a composite operator) that gains an F-term expectation value and a scalar expectation value $\langle X \rangle = X_0 + F_X \theta^2$, through a superpotential term

$$W \supset X \Psi \Psi^c \ . \tag{2.55}$$

If the messengers receive no other mass contributions, this leads to SUSY-preserving mass terms for the messenger fermions, $m_{\text{mess}} = X_0 \psi \psi^c + \text{h.c.}$, and scalars $X_0^2 \left(|\phi|^2 + |\phi^c|^2 \right)$. There are also SUSY-breaking scalar masses squared from the term in the Lagrangian $F_X \phi \phi^c + \text{h.c.}$, and so the scalar mass eigenstates have masses squared of $X_0^2 \pm F_X$.[24]

The leading soft masses induced in the visible sector (in an expansion in the parameter $\frac{F_X}{m_{\text{mess}}}$) can be obtained through holomorphy [88, 89]. This is a good approximation if the SUSY-breaking sector is not too close to the weak scale. Once the messengers have been integrated out of the theory the gauge kinetic term can be written

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \, \tau \left(X, \mu \right) W^{\alpha} W_{\alpha} \ . \tag{2.56}$$

[24]If the messengers receive a SUSY preserving mass from a different source, this can just be written in combination with the SUSY-breaking masses from X through a spurion chiral superfield which couples as in Eq. (2.55) and the analysis is unchanged.

Gaugino masses arise through a θ^2 term in τ, which is generated by τ's dependence on X. The leading dependence is given by

$$M_\lambda = \frac{i}{2\tau}\frac{\partial \tau}{\partial X}\bigg|_{X=M} F_X \ . \tag{2.57}$$

Since the Wilsonian holomorphic gauge coupling is

$$\tau\left(X,\mu\right) = \tau\left(\Lambda_{\mathrm{UV}}\right) + \frac{ib_0}{2\pi}\log\left(\frac{X}{\Lambda_{\mathrm{UV}}}\right) + \frac{ib_1}{2\pi}\log\left(\frac{\mu}{X}\right) \ , \tag{2.58}$$

the gaugino soft masses are

$$M_\lambda = \frac{\alpha\left(\mu\right)}{4\pi} n_m \frac{F}{m_{\mathrm{mess}}} \ . \tag{2.59}$$

Similarly, the leading sfermion soft masses can be obtained from the dependence of the chiral multiplets' wavefunction renormalisation on the spurion X, giving

$$m^2_{\phi_i} = 2\left(\frac{F_X}{m_{\mathrm{mess}}}\right)^2 n_m \sum_a C_a\left(i\right)\left(\frac{\alpha_a}{4\pi}\right)^2 \ , \tag{2.60}$$

where $C_a\left(i\right)$ is the quadratic Casimir of the representation of the sfermion ϕ_i for the group labelled by a. Parametrically, the sfermion and gaugino soft masses are at the same scale, however A-terms are very suppressed. This derivation shows that the the leading contribution to the soft masses arises as a threshold effect when the messengers are integrated out at an energy scale m_{mess}.

Separately to the fact that Eq. (2.59) is to leading order in $\frac{F_X}{m_{\mathrm{mess}}}$, it also misses higher-order effects due to the fields in the theory not being canonically normalised. The physical gaugino mass arises from the θ^2 component of the real gauge coupling defined by

$$R = S + S^\dagger + \frac{T\left(Ad\right)}{8\pi^2}\log\left(S+S^\dagger\right) - \sum_i \frac{T\left(r_i\right)}{8\pi^2}\log\left(Z_i\right) + \ldots \ , \tag{2.61}$$

where S is defined as the chiral superfield appearing in the Lagrangian as

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \, \frac{1}{2} S W^\alpha W_\alpha \ , \tag{2.62}$$

and the ellipses represent two-loop corrections. This is exactly the extension of Eq. (2.24) to the case of superfields.

If required, the full expression for the soft masses to all orders in $\frac{F_X}{m_{\mathrm{mess}}}$ can be obtained by evaluating the loop diagrams that lead to the soft masses [134]. Again, it can be seen that the masses are generated by momenta near the scale m_{mess}. It is also possible to calculate the visible sector soft masses induced from a generic messenger mass matrix (expressions for this are given in Appendix A.1). For example, the messenger scalars can obtain SUSY-breaking mass terms of the form $\left|\frac{F_X}{M_*}\right|^2 \phi^\dagger\phi$ (where M_* is some higher scale) from a Kähler potential term $\frac{X^\dagger X}{M_*^2}\Psi^\dagger\Psi$. If the soft masses are purely of this form there is an effective R-symmetry in the messenger sector and consequently no gaugino masses are induced (in contrast any possible R-symmetry is violated by a coupling of the form Eq. (2.55) when X has an F-term expectation value, and the messengers have SUSY preserving masses). If the messenger mass matrix is such that its supertrace does not vanish, there is an additional contribution to the sfermion masses from the so-called ϵ scalars, which arise from evaluating loop integrals of vectors in $4-2\epsilon$ dimensions (when regularising loop integrals by dimensional reduction). Unlike the other contributions these corrections are logarithmically divergent $\sim \log(\Lambda_{\text{UV}})$, and occurs because (if SUSY is only spontaneously broken) the theory is completed to a theory with vanishing mass supertrace at some higher scale [134].

A significant advantage of gauge mediation is that minimal models are entirely flavour blind [56], and consequently dangerous flavour-violating operators in the visible sector are avoided. Notably, models with messengers in complete GUT multiplets lead to gaugino masses in the GUT unified patten $\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}$. The maximum number of SU(5) messenger multiplets is constrained to $n_m < \frac{150}{\log\left(\frac{M_{\text{GUT}}}{m_{\text{mess}}}\right)}$ if unification is required to be perturbative. Alternatively, if mediation occurs through a new gauge group that has been Higgsed this can be included in the analysis [135]. More generally, gauge mediation from a generic, possibly strongly coupled breaking and mediating sector can be studied through the formalism introduced in [136], although this is not necessary for our purposes. An interesting alternative to the set up so far described is direct gauge mediation. In this the SUSY-breaking fields have SM gauge charges, and mediation proceeds without requiring an additional messenger sector [137].

Of course there are other mediation possibilities. In gravity mediation, the SUSY breaking sector is coupled to the MSSM through Planck scale physics (see for example [57, 61]). Gaugino masses arise if the gauge kinetic function includes a non-renormalisible term

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \left(\frac{1}{g_a^2} - \frac{f_a}{M_{\text{Pl}}}X + ...\right) W_a^\alpha W_{a\alpha} \,, \tag{2.63}$$

where f_a is a constant (with mass dimension zero). Similarly, sfermion masses are generated through non-renormalisible operators in the Kähler potential, and A-terms from the superpotential. If the suppression of these operators is the Planck scale, all soft masses are comparable to the gravitino mass. However, in completions, the actual suppression of these operators could be exponentially separated from the Planck scale, for example if there is an exponentially large volume arising from a string compactification [64]. While gravity mediation is very well motivated, the details of the breaking is explicitly dependent on the UV completion. One problem that arises in many completions is obtaining soft terms that do not lead to excessively large flavour changing currents and CP violation.[25]

Another mediation mechanism is anomaly mediation, which is the combination of several effects [138–142]. One source is that SUSY breaking leads to a positive vacuum energy contribution, and so de Sitter spacetime. Therefore prior to SUSY breaking the theory must begin in Anti-de Sitter (AdS) space [143]. However, in AdS the SUSY preserving masses of multiplets are split. Once space-time is uplifted to flat space, these mass splittings are generically preserved, but now break supersymmetry (the splittings depend on how far in AdS the theory begins, which is fixed by how far SUSY breaking uplifts it). Another source of anomaly mediation is linear couplings of fields charged under the SM gauge group to a SUSY breaking spurion in the Kähler potential. These feed into the real gauge coupling Eq. (2.61), or equivalently can be shifted into the gauge coupling by an anomalous rotation in field space. In string theories there are also further sources [144]. While anomaly mediation is flavour blind and theoretically well motivated, minimal models lead to tachyonic sleptons, and evading this requires extra model building.

Regardless of the mediation mechanism, having calculated the soft masses at the mediation scale (that is the messenger mass in gauge mediation, or the scale of the higher dimension operators in gravity or anomaly mediation), in order to find the soft terms near the weak scale the theory must be run down using the RG equations. Typically, these are taken to be the MSSM equations given in Section 2.4 (although we discuss an alternative in Section 6).

The RG flow leads to another mediation possibility. If the gaugino soft masses are generated at a high scale, then during RG flow they can induce soft masses in the sfermions that are of comparable size if the running is from a relatively high scale [145]. The initial condition of only gaugino soft masses most naturally arises from models with extra dimensions, or models with multiple sets of gauge groups (so called deconstructed models that are closely related to extra dimensional models).

[25] This arises because, although gravity couples universally at low scales, gravity mediation is sensitive to the details of the UV theory, which also needs to explain the fermion mass structure and therefore is manifestly not flavour universal.

A significant difference between gauge and gravity mediation is the mass of the gravitino, in gravity mediation this is parametrically the same size as the other soft masses in the theory, while in models of gauge mediation the gravitino mass is usually much smaller and it is the LSP. Consequently, in models of gauge mediation the gravitino is a (DM) candidate, and requiring that it does not overclose the Universe can give stringent constraints on the reheating temperature after inflation. More generally, even if it is not the DM, since the gravitino couplings are suppressed by the Planck scale, it can have a significant effect on cosmology [146–148], for example if it decays during big bang nucleosynthesis.

2.10 Fine Tuning

We now return to the details of fine tuning, specifically in supersymmetric theories. In principle, once a definition of fine tuning has been decided upon, the procedure of calculating it is straightforward. Once a UV complete model (that leads to correct EW symmetry breaking) is specified, the sensitivity of the EW scale is obtained by varying the fundamental underlying parameters, and evaluating Eq. (1.6). This requires the UV complete theory to be run down to the weak scale, so that radiative corrections to the parameters that appear in the low-energy potential are included [24, 149, 150]. The overall tuning of the theory can either be defined as $\max(\Delta_p)$, where Δ_p is the tuning with respect to the parameter p, or as $\sqrt{\sum_p \Delta_p^2}$.[26]

In practice however this procedure is not straightforward. The running from the UV cutoff depends on the complete underlying theory, including all the higher-dimensional operators, which is usually unknown. This is because, although the effective theory at energies far below the UV cutoff is insensitive to higher dimensional operators, it is not obvious how fast the effects of these turn off and if they give any significant contributions before this happens. Instead, what has to be done is take some particular boundary conditions at an assumed UV cutoff of the RG flow. These can then be run down to the weak scale (using perturbative RG equations), and it is hoped that the tuning obtained is a good approximation to the true tuning of the theory.

The collider limits on Higgsinos are comparatively weak, and consequently μ can be close to the weak scale [56]. Therefore, provided the theory does not require a large value of μ to

[26]The former choice runs the risk of allowing a more complex theory to 'hide' the true tuning of the theory by introducing multiple new parameters, each of which are individually tuned to some extent. However, apart from this caution, the two measures are usually comparable, and give similar results.

obtain the correct EW VEV, the tuning from this parameter can be relatively small (from Eq. (2.38)). The dominant tuning typically arises through the up-type Higgs soft mass and the radiative corrections to it. For typical spectra compatible with collider limits, the most important contributions arise from the stop masses, and the gluino which feeds into the Higgs soft mass squared through the stop at two loops. We study this in detail in Section 5.

The size of the radiative corrections depends on how long the theory is running, so different theories with the same EW spectrum can have vastly different fine tuning if they have different mediation scales. Consider the tuning induced by a stop with mass 1 TeV. This can be estimated as

$$\Delta_{\tilde{t}} \sim 2\frac{m_{\tilde{t}}^2}{m_Z^2}\frac{6y_t^2}{16\pi^2}\log\left(\frac{m_{\text{med}}}{m_Z}\right) \ , \tag{2.64}$$

(where the running of the stop soft mass has been neglected) [151]. If the mediation scale is $\sim 10\,\text{TeV}$ the tuning is $\Delta_{\tilde{t}} \sim 20$, while if the mediation scale is $10^{18}\,\text{GeV}$ it is much larger $\Delta_{\tilde{t}} \sim 170$.

Even theories with the same Lagrangian at the assumed UV cutoff of the RG flow can have differing tuning if they arise out of different underlying models, with different fundamental parameters that can be varied.[27] Equivalently, the fine tuning measure is not invariant under redefinitions of the fundamental parameters of the theory. For example, in GUT models gaugino masses are unified and the only parameter that can be changed is the unified gaugino mass, while in other completions the gaugino masses can be varied independently, giving a different tuning. Another example arises in a model where the first two generation sfermion masses are all fixed equal at the UV boundary by the underlying theory. This can lead to cancellations that would not be observed if they were independent. Assuming running from the GUT scale at $10^{16}\,\text{GeV}$, and $\tan\beta = 10$, the dependence of the weak scale Higgs mass on the first two generation sfermion masses (at the UV boundary) is [152]

$$-2\delta m_{Hu}^2\left(m_Z\right) \supset 0.051 m_{Q2}^2 - 0.11 m_{\overline{u}2}^2 + 0.051 m_{\overline{d}2}^2 - 0.052 m_{L2}^2 + 0.053 m_{E2}^2 + [2 \mapsto 1] \ . \tag{2.65}$$

If the first two generation sfermion masses squared are all fixed equal to a common UV parameter m_s^2, there is a cancellation in Eq. (2.65), and $-2\delta m_{Hu^2} \sim -0.014 m_s^2$, less than the tuning if all the soft masses were independent.[28] UV models that lead to such cancella-

[27] It is also possible to define another measure of fine tuning that does not allow for cancellations, and typically gives an upper bound on the tuning of a model [150].

[28] The sensitivity of the weak scale to whatever effects set $\tan\beta = 10$ also needs to be considered in any complete model relying on this cancellation.

tions are good candidates for theories for low fine tuning, provided they are independently motivated and the relations between soft parameters have not simply been put in 'by hand'. Further, fine tuning calculations usually simply assume the running arises from the visible sector beta functions, however in Section 6, we study the effects when this assumption is relaxed (as may plausibly occur in realistic models).

Notably, the fine tuning only measures the *sensitivity* of the EW scale to the underlying parameters. We make no attempt to quantify the likelihood that a particular type of theory is actually realised, which would require a measure on 'theory space'. For example, it is very difficult to know the probability of a new gauge group, under which only the first two generations are charged, actually existing. However, a highly convoluted theory, which is exceptionally hard to realise from a sensible UV completion, is not compelling simply because it has low fine tuning.[29]

Another possible measure of the fine tuning of a theory is the tuning evaluated purely at the EW scale [150, 152]. This starts from the (radiatively-corrected) EW potential

$$\frac{m_Z^2}{2} = \frac{-\left(m_{Hu}^2 + \Sigma_u\right)\tan^2\beta + m_{Hd}^2 + \Sigma_d}{\tan^2\beta - 1} - \mu^2 \ , \tag{2.66}$$

where Σ_u (Σ_d) are the loop corrections from to states that couple to the up-type (down-type) Higgs, and all quantities are evaluated at the weak scale. The low scale tuning is then defined as

$$\Delta_{\rm EW} = \max\left(\left|\frac{m_{Hu}^2\tan^2\beta}{\tan^2\beta-1}\right|, \left|\frac{\Sigma_u\tan^2\beta}{\tan^2\beta-1}\right|, \left|\frac{m_{Hd}^2}{\tan^2\beta-1}\right|, \left|\frac{\Sigma_d}{\tan^2\beta-1}\right|, \left|-\mu^2\right|\right) \times \frac{2}{m_Z^2} \ . \tag{2.67}$$

Numerically, Σ_u is usually dominated by the top squark loops, and is given by $\Sigma_u \sim \frac{3f_t^2}{16\pi^2} m_{\tilde{t}}^2 \log\left(\frac{m_{\tilde{t}}^2}{Q^2}\right)$ (there are also extra terms due to stop mixing [153]), where Q is the scale choice (usually optimised to $Q^2 = m_{\tilde{t}1} m_{\tilde{t}2}$). This quantity does not take into account the effects of running from a high scale and consequently misses the large logarithmically enhanced terms that can arise in the true tuning of the theory.[30] However, it does give a lower bound on the tuning that a theory with a particular EW spectrum can have.

[29] The distributions of vacua in SUGRA compactified from 10 to 4 dimensions actually seem to suggest low energy SUSY itself may be disfavoured (by a stronger amount than a tuned Higgs is disfavoured in stable compactifications with zero cosmological constant, under some assumed distribution of potentials [48, 49]. However, at present, the understanding of moduli stabilisation and the string landscape is not developed enough to understand if this is representative of real string theory dynamics.

[30] To see the effect of this running the potential has to be written in terms of the high scale quantities $m_{Hu}^2(Q) = m_{Hu}^2(\Lambda_{\rm UV}) + \delta_{m_{Hu}^2}$

2.11 Low-Energy Spectra, Collider Detection, and Natural SUSY

While the (weak scale) mass spectrum of a theory depends on the details of the model and parameters, there are some features that often appear in minimal models of both gauge and gravity mediation [56]. Many conventional models of both gauge and gravity mediation predict gaugino masses in the GUT unification pattern, although there are string theory completions that do not have this property [63, 64]. The sfermion masses are often assumed to be universal at the mediation scale in gravity mediated models, while in minimal gauge mediation models these depend on the representation of the gauge group that the sfermion is in. Additionally, all generations of sfermions are often taken to have the same soft masses (which, as discussed, helps to evade flavour constraints) [56].

The lightest states tend to be the neutralinos and right-handed sleptons (apart from possibly the gravitino), while SU(2) charged states are somewhat heavier. The gluino is typically relatively heavy, since α_3 is large at low energies. Coloured sfermions are usually significantly heavier than other sfermions, in gauge mediation because they obtain larger masses at the mediation scale and in minimal gravity mediation because they get a large positive mass contribution from the gluino during running.[31]

The possible signatures of supersymmetry at colliders, and the limits that arise from negative search results at LEP, the Tevatron, and now the LHC are the topic of an enormous body of work. The classic signatures of SUSY at hadron colliders are events with some number of jets, some number of leptons and missing transverse energy. The missing energy arises from the LSP escaping the detector (assuming R-parity, however the limits are not actually relaxed significantly if the theory is R-parity violating). Of course, understanding the SM backgrounds, which mostly arise from neutrino production and jet mismeasurement, is vital to obtain limits (amongst many other studies, see for example [154–156]).

The strongest limits on superpartners are on gluinos and the first two generation sfermions. Both are efficiently produced from the parton contribution of the protons by strong interactions (unless sufficiently heavy that their production is kinematically suppressed). The limits on the third generation sfermion masses are much weaker than those on the first two generation sfermion masses, due to the small mixing between the first two generations and the third generation [157].

[31] An exception to this is if the universal scalar soft mass is much larger than the gaugino mass, in which case the coloured sfermions can be lighter than other sfermions due to to a negative contribution to their masses from each other during running.

In a simplified phenomenological model containing only the first two generation squarks (with a universal mass), neutralinos and gluinos the limit on the squark masses (combined from a variety of searches) is in the region of 1.7 TeV even if the gluino is in the region of 2 TeV (the mass of the gluino affects the production rate, and consequently the mass limits on sfermions). Similarly, no matter how heavy the squarks are, the limits on the gluino are around ~ 1.4 TeV [158]. The limits can also be cast into the mSUGRA plane (mSUGRA is a theory with unified gaugino masses and a common sfermion mass at a high scale). In this case the limits on the squark masses are at least 2 TeV (demanding that the gauginos are not so heavy that the LSP is a phenomenologically unacceptable stau).

Motivated by experimental constraints, it is interesting to consider more complicated theories that lead to a low energy mass spectrum with the first two generation sfermions somewhat heavy, while keeping third-generation squarks, especially stops, and electroweak gauginos and Higgsinos light, the so called *natural SUSY* scenario [159, 160]. This allows for experimental constraints to be weakened and since the first two generations couple only relatively weakly to the Higgs, it may be hoped that such spectra do not lead to large fine tuning. In this thesis we study the model building possibilities of natural SUSY spectra, and the possibility that these reduce fine tuning.

In models of natural SUSY, the limits on gluinos are typically in the region of $1.3-1.4$ TeV but this depends strongly on the simplified model being considered [161, 162]. This typically arise from the decays $\tilde{g} \to t\bar{t}\tilde{\chi}_0$, $\tilde{g} \to b\bar{t}\tilde{\chi}_+$ and $\tilde{g} \to b\bar{b}\tilde{\chi}_0$, which cannot be evaded unless μ is large (introducing significant tuning) [157].

Stops masses are typically limited to $\gtrsim 400-700$ GeV, however this is very dependent on the model and the mass splittings in the spectrum [163–165]. Production is either through gluinos or other squarks, or directly if all other strongly coupled superparticles are much heavier. For sufficiently heavy stops a typical decay channel is $\tilde{t} \to t\tilde{\chi}_0$, where the final states are on-shell and $\tilde{\chi}_0$ is the lightest neutralino. In this case the neutralino can lead to large missing transverse energy signatures. If the stops are lighter the top has to be off-shell, and the decay is directly $\tilde{t} \to bW^+\tilde{\chi}_0$, possibly leading to observable deviations in kinematic variables. If the neutralino is very light and the stop is only slightly heavier than the top, the neutralino carries off very little energy and the event is hard to distinguish from pair production. Of course more sophisticated searches for light stops covering a range of spectra, see for example [166], have been considered, and are being carried out.

A wide range of theories of SUSY breaking and mediation that can lead to a natural Spectrum have been proposed. An appealing feature of many models is the ability to link the large third generation Yukawas in the SM to the smaller soft mass of the stop relative to the other sfermions. In the original models, the first two generation sfermions are taken to be charged under some new gauge symmetry that mediates additional SUSY breaking masses to these states, but not the third generation sfermions, which gain soft masses either through some other gauge mediation or gravity mediation [167, 168]. Similarly, there could also be only one set of messenger states, but with additional flavour structure [135]. In the next chapter we build a supersymmetric model that generates a natural SUSY spectrum, while simultaneously explaining the suppressed first two generation sfermions in the SM, and the appearance of mass parameters in a dynamical SUSY breaking sector.

Other possibilities for generating a natural SUSY spectrum include deconstructed models, in which the SM gauge group is extended to $G^1_{\mathrm{SM}} \times G^2_{\mathrm{SM}}$, where each of $G^{1,2}_{\mathrm{SM}}$ are the full SM gauge group, and the third generation states are taken to be charged under a different group to the other matter [169]. These models are closely related to other theories where the third generation states are localised differently to the first two generations in an extra dimension, also generating a natural spectrum [170]. It is also possible that a natural SUSY spectrum could arise out of a combination of strong dynamics (in the form of a composite Higgs model), and supersymmetry [171].

Chapter 3: Building a Model of Natural SUSY

This chapter is based on [172], work done in collaboration with John March-Russell.

As discussed in the previous chapter, if softly-broken SUSY is to be a successful theory of the weak-scale, solving the hierarchy problem, then it must meet a number of serious challenges. First, on the theoretical side, there is still significant uncertainty over the mechanism of SUSY breaking and its mediation to the visible sector. From the perspective of the hierarchy problem the most attractive possibility, as first argued by Witten [130], is the dynamical breaking of SUSY via dimensional transmutation and non-perturbative effects. Despite this attractive feature, many models of dynamical supersymmetry breaking (DSB) still require small parameters, or masses to be parametrically suppressed relative to other scales in the theory. A particularly well known example is the, previously mentioned, ISS model [129] where small parameters are required to ensure that (in the presence of the phenomenologically required R-symmetry breaking) a metastable vacuum is sufficiently long-lived to be viable.

An appealing approach to deal with this is through so-called *retrofitting* [128], where IR irrelevant operators generate small parameters which would otherwise be forbidden by symmetries of the theory. In the case that the operators introduce a small amount of R-symmetry breaking, this is not a surprising scenario: vanishing of the vacuum energy post SUSY breaking requires the superpotential to have a R-symmetry violating expectation value, which is transmitted through supergravity to produce the required operators.

Second, on the phenomenological side, there is increasing tension between the requirement that superpartners should be close to the EW scale to prevent the reintroduction of a little hierarchy problem and negative results of collider searches first at LEP and now at the LHC [1]. One possibility to weaken these experimental constraints is to have, as previously mentioned, a natural SUSY spectra. Although there has been much phenomenological study of this case (for example [151]) it is unclear how such spectra may be realised from a UV theory in a way that maintains the successes of low-energy SUSY such as the gauge unification prediction of $\sin^2\theta_w(m_Z)$ and radiative EW symmetry breaking. The issue is again the appearance of small parameters, in this case the ratio of third to first and second generation sfermion masses. Of course, there is another well-known problem involving unexpectedly small parameters: the SM fermion masses themselves which exhibit a large range of values.

A very popular way of dealing with this, which again involves irrelevant operators generating terms that are forbidden at leading order by a new symmetry, is the Froggatt-Nielsen mechanism [5].

Our aim in this chapter is to argue that these disparate cases may in fact be directly related, with the same broken symmetry leading to small parameters in both the SUSY-breaking and visible fermion/sfermion flavour sectors. In particular, we consider theories where there is an additional underlying U(1) gauge symmetry broken at high scale. While such symmetries may simply be regarded as a feature of an effective theory, they often automatically appear in an underlying string theory model. This case is highly attractive since these symmetries are naturally anomalous in the field-theory limit before a generalised Green-Schwarz mechanism is included which typically leads the associated gauge bosons to gain a large SUSY preserving mass [173]. Additionally, they can be linked to generation of the visible sector fermion masses in brane stack models, whereby different generation fermions are charged differently, as recently discussed in [174, 175].

The possible role of U(1) gauge symmetries in breaking and mediating SUSY has been studied extensively and it has previously been proposed to generate flavour structure in sfermion masses, see e.g., [167, 168, 176–187].[1] Many previous models have proposed the fields that break the U(1) can be directly involved in the SUSY breaking sector. While this is an attractive prospect it leads to issues such as the dilaton necessarily gaining an F-term that may dominate the mediation [190]. An additional problem is if gaugino and third generation soft masses are generated through gravity mediation it is very hard to avoid dangerous flavour changing processes without making the first two squarks generations so heavy as to drive the stops tachyonic during running [191] (this is discussed further in Section 5).

In contrast, in the models we consider, the U(1) vector multiplet receives a mass at a high scale and only acts as an additional messenger interaction without being directly involved with the SUSY-breaking. Importantly, since the SUSY breaking sector is charged under this gauge symmetry, there is an additional contribution to the MSSM soft masses from a contact interaction after integrating out the heavy vector multiplet.[2] Then, as we will argue, if only the first two generation sfermions are charged under the broken U(1), this can lead to first two

[1] Alternative UV models that could realise natural SUSY spectra have also been proposed [135, 188, 189].

[2] Operators generated by integrating out heavy gauge fields have previously been proposed as a viable mechanism of mediating supersymmetry breaking [192,193], and have been studied in the context of dynamical SUSY breaking and gauge mediation with universally charged MSSM fields [194].

generation sfermion soft masses a factor of a few larger than the gauge mediated soft masses, and therefore the first two generation sfermions can be heavy enough to evade detection and realise natural SUSY, but not so heavy as to drive the stop tachyonic through RG running. However, with first two generation sfermions in the mass range of a few TeV, flavour violation is not adequately suppressed unless there is a high degree of degeneracy between these sfermions. Because of this we take the first two generations to be charged equally under the U(1), so both broken U(1) mediation and the competing SM gauge mediation are flavour universal, leading to flavour observables within current limits. Of course, a consequence of this is that the observed hierarchies in first and second generation fermion masses and mixing cannot be 'explained' by selection rules following from the breaking of U(1), and only the hierarchy and mixing between the third generation and the lower generations is due to the Froggatt-Nielsen mechanism. Our attitude here is that the flavour structure of the first two generations is set by high-scale physics which is independent of SUSY-breaking dynamics. As we will show this is allowed since, in our model there can be $\mathcal{O}(1)$ breaking of the flavour symmetry of the lower generation fermions consistent with the fact that the sfermion partners simultaneously possess an effective flavour symmetry that is only very weakly broken at loop order by the tiny first and second generation Yukawas.

Turning to the organisation of this Section, in Section 3.1 we introduce the overall structure of our models and the basic mechanisms of SUSY breaking and mediation in a field theory setting, illustrating the ideas first using a Polonyi model, and then a fully dynamical ISS model. Following this in Section 3.2, we examine how such models may naturally appear from an underlying string theory possessing anomalous U(1) gauge symmetries. In Section 3.3 we consider the low-energy spectrum of soft terms obtained, while in Section 3.4 we note some additional interesting phenomenological possibilities and discuss signatures.

3.1 Structure of Field Theory Implementation

We begin by discussing the implementation of our models in a low-energy field theory setting using a Polonyi model as a straightforward example of the SUSY breaking sector. Following this we implement a fully dynamical example, an ISS model.

3.1.1 Low-Energy Polonyi Model

The underlying theory is specified by four sectors. At the highest scale a sector that breaks the U(1), a DSB sector, a messenger sector, and the visible sector. All sectors involve fields charged under the U(1) symmetry, and the superpotential takes the form

$$W = W_{\mathrm{U(1)}} + W_{\mathrm{DS}} + W_{\mathrm{mess}} + W_{\mathrm{visible}} \ , \tag{3.1}$$

with a UV cutoff at a scale M_* and canonical Kähler potential up to irrelevant operators suppressed by powers of M_*. The sector $W_{U(1)}$ involves only fields S_i, with U(1) charge i, and spontaneously breaks the U(1) gauge symmetry through fields S_{+1} and S_{-1} getting vacuum expectation values (VEVs) v leading to a gauge boson mass $m_{Z'} = g'v$. Here g' is the U(1) gauge coupling, and we assume that the scale of $W_{U(1)}$ is sufficiently above the other sectors that the VEVs of fields S are rigidly fixed. Hence, once this symmetry breaking occurs, the fields $S_{\pm 1}$ in the other sectors may be replaced by their expectation values v. This leads to a small ratio in the theory we denote by $\epsilon = \frac{v}{M_*}$.[3]

The DSB sector has fields charged under the U(1) symmetry and the superpotential includes irrelevant operators generated at the cutoff of the theory with the form

$$\Delta W_{\mathrm{DS}} = \frac{S^n}{M_*^m} \mathcal{O}_{\mathrm{DS}} \ , \tag{3.2}$$

where n and m are integers and $\mathcal{O}_{\mathrm{DS}}$ are operators involving the fields in this sector. Once some of the S_i obtain a VEV these couplings lead to small mass terms and parameters. In particular, consider a very simple sector of Polonyi form with one field Φ with charge +6 under the U(1) symmetry. After $S_{\pm 1}$ gain their common VEV, the superpotential is

$$W \supset \frac{S_{-1}^6}{M_*^4} \Phi = \epsilon^4 v^2 \Phi \ , \tag{3.3}$$

leading to a SUSY-breaking F-term $F_\Phi = \epsilon^4 v^2$.

In the messenger sector there are fields, $\{\psi, \psi^c\}$, that form a vector-like pair under the SM gauge groups which act as messengers of gauge mediation. They are charged under the U(1) with couplings to the fields S and also to the DSB sector of the form

$$W_{\mathrm{mess}} = \frac{S^{p+1}}{M_*^p} \psi\psi^c + \frac{S^{n'}}{M_*^{m'}} \mathcal{O}'_{\mathrm{DS}} \psi\psi^c \ , \tag{3.4}$$

[3] In a complete theory it is also necessary to explain the suppression of v relative to M_*. However, since the required suppression is only a factor of $\sim 20 - 100$ it is plausible it can arise without significant tuning (for example due to a small coupling constant or loop factor).

where $\mathcal{O}'_{\rm DS}$ is an operator in the SUSY-breaking sector that gains an F-term expectation value (and is not typically the same operator as $\mathcal{O}_{\rm DS}$ in Eq. (3.2)), and n' and m' are integers. We further assume they have a potential (either at tree or loop level) such that the SUSY-breaking minimum remains either a stable or metastable state. Taking the combination $\psi\psi^c$ to have charge -4 this sector includes a mass term and interactions with Φ given by

$$\begin{aligned} W &\supset \frac{S_{-1}^2}{M_*^2}\Phi\psi\psi^c + \frac{S_1^3}{M_*^4}\psi\psi^c \\ &= \epsilon^2\Phi\psi\psi^c + \epsilon^3 v\psi\psi^c \ . \end{aligned} \tag{3.5}$$

Due to the coupling between the field Φ and the messengers, there will be gauge-mediated soft masses roughly of size

$$m_{\rm gauge} \sim \left(\frac{\alpha}{4\pi}\right)\frac{F_{\rm eff}}{m_{\rm mess}} \sim \left(\frac{\alpha}{4\pi}\right)\epsilon^3 v \ , \tag{3.6}$$

where $F_{\rm eff} = \epsilon^6 v^2$ is the effective F-term felt by the messenger fields due to its coupling to Φ. In order that these soft terms are close to the EW scale, for values of ϵ appropriate to fermion masses, v and M_* must be relatively close to the weak scale, hence this is very low scale gauge mediation with messenger masses an inverse loop factor above the weak scale. This is phenomenologically beneficial as it results in relatively little running and the first two generations can be pushed heavier without leading to a tachyonic stop.

Finally the visible sector superpotential takes the form

$$W_{\rm visible} = c_{ij}\left(\frac{S_{-1}}{M_*}\right)^{q_{ij}}\mathcal{O}^{ij}_{\rm visible} \ , \tag{3.7}$$

where i, j are generation indices. The parameters c_{ij}, which are not constrained by the U(1) symmetry, are set by UV physics at (or above) the scale M_* where the irrelevant operators are generated, and may or may not satisfy other symmetry relations. After U(1) symmetry breaking the effective Yukawa couplings relevant to IR physics which set the observed fermion mass ratios and CKM mixings are

$$\lambda_{ij} = \epsilon^{q_{ij}} c_{ij} \ . \tag{3.8}$$

As is well known, the observed third-generation fermion masses and mixings have properties which set them apart from the lower generations: not only is the top Yukawa coupling $\mathcal{O}(1)$ (as can be those of the bottom and tau if $\tan\beta$ is large) in distinction to the suppressed first and second generation couplings, but SU(5) SUSY unification predictions work well for m_b/m_τ, while the remaining predictions fail badly. In addition, if the experimentally observed ratios of second generation to third generation fermion masses at a low scale are run to the GUT scale, assuming weak scale SUSY, the resulting ratios and mixings $m_c/m_t \approx 1/300$, $m_s/m_b \approx 1/40$, $m_\mu/m_\tau \approx 1/17$ and $V_{cb} \approx 1/25$ are well-described by a structure of Yukawa couplings for the up and down quarks and leptons depending on a single small parameter $\epsilon \sim 1/20$ of the form

$$U \simeq \begin{pmatrix} \epsilon^2 & \epsilon \\ \epsilon & 1 \end{pmatrix}, \qquad D \simeq \begin{pmatrix} \epsilon & * \\ ** & 1 \end{pmatrix}, \qquad E \simeq \begin{pmatrix} \epsilon & * \\ ** & 1 \end{pmatrix}, \tag{3.9}$$

where here "$*$" and "$**$" denote entries that are $\mathcal{O}(1)$ (respectively $\mathcal{O}(\epsilon)$) or smaller, see e.g. [195, 196]. This structure strongly suggests that some dynamics sets this pattern, such as that following from a Froggatt-Nielsen mechanism [5], or from extra-dimensional orbifold-GUT constructions [195–198]. This is particularly the case since, as far as we are aware, there is no anthropic reason for the second-generation masses and V_{cb} to take their observed values. On the other hand the masses of the first generation quarks, as well as the mass of the electron, do not fit so nicely with any simply dynamical mechanism depending on only one small parameter, and are, in addition, (remarkably) in accord with the anthropic "catastrophic boundaries" linking m_u, m_d, m_e, with λ_{QCD} and α_{EM} [199, 200]. In particular, this is the claim that small variations in the values of these parameters would lead to dramatic changes in the physics of the Universe at much larger scales.

Because of this we now make the crucial assumption, different from many previous studies, that the physics that sets the 2-3 inter-generational mass ratios and mixings is different than that which sets the 1-2 ratios and mixings. Specifically our starting place is that second-third generational physics is set by the U(1)-dependent factors $\epsilon^{q_{ij}}$ while the first-second generation physics is set by the c_{ij}'s which are *not* determined by our broken gauged U(1).[4]

In detail, the up-like-Higgs and top-quark multiplets are uncharged under the U(1), such that a superpotential term $W \supset H_u q_{L3} u^c$ is allowed, with an order 1 coefficient to match observation. In contrast the first two generation fields of the same SM quantum numbers are taken to be charged *equally* under the U(1), leading to mass terms that are suppressed by equal powers of ϵ. Since the U(1) symmetry is abelian the $\mathcal{O}(1)$ coefficients that dress

[4]The mixings and hierarchies between the lighter two generations may result from another broken Froggatt-Nielsen flavour symmetry such as U(1) or SU(2) which is either not gauged, or does not interact with the SUSY breaking sector, or, alternatively may instead be the result of landscape scanning of the coefficients c_{ij} subject to the strong anthropic constraints that they must obey. The important point for our work is that we do not need to specify this physics as long as it is independent of (commutes with) our U(1) that interacts with the DSB and similarly retrofits some if its couplings.

these couplings possess no symmetry properties, and can lead to the observed mass splitting of the first two generation visible sector fermions and the Cabbibo mixing structure, as we discuss in Section 3.3. (When investigating particular models, we will give explicit charge assignments and show the textures generated in the visible fermion masses.)

For phenomenologically viable charge assignments, including only MSSM matter, the U(1) symmetry would appear to have anomalies of the form $\mathrm{U}(1) \times G^2_{\mathrm{visible}}$ and $\mathrm{U}(1)^2 \times \mathrm{U}(1)_{\mathrm{Y}}$, however these can be cancelled by the messenger fields (or other matter which is chiral under the U(1) and vector-like under the visible sector groups). Choosing a GUT compatible U(1) charge assignment for the visible sector allows these anomalies to be cancelled by matter in complete GUT multiplets hence gauge unification is preserved.[5]

After U(1) symmetry breaking, as a result of integrating out the heavy U(1) gauge boson there will be a Kähler contact operator [190], derived in Appendix A.1, between any two fields charged under the U(1) symmetry. This is important for our phenomenology as it leads to an extra coupling between the field which obtain an F-term, Φ, and the first two generation MSSM fields (and third generation down type quarks and leptons), $Q_{1,2}$,

$$\int \mathrm{d}^4\theta \, c_i g^2 \left(\frac{\Phi^\dagger \Phi Q^\dagger_{1,2} Q_{1,2}}{m^2_{Z'}} \right) \, . \tag{3.10}$$

Here $m_{Z'}$ is the mass of the heavy U(1) gauge boson, while $c_i \sim q_\Phi q_{1,2}$ depends on the U(1) charges of the fields. Since $m_{Z'} = g' v$ the dependence on g' drops out leading to soft masses for the first two generations

$$m^2_K = -c_i \frac{|F_\Phi|^2}{v^2} \, . \tag{3.11}$$

At the scale these interactions are generated, the coefficients c_i depend only on the U(1) gauge charges of the fields, and therefore can naturally be equal for the first two generations by a discrete choice of the charges. During RG evolution down to the weak scale the dominant running effects will be due to SM gauge interactions which are still universal. The only deviations from universality are due to the first and second generation Yukawas and have a negligible effect. Therefore, flavour changing currents are not generated in the visible sector, which is crucial for acceptable spectra. In order that sfermions receive a positive mass contribution, it must be assumed that $c_{1,2} < 0$. In Appendix A.1 it is seen that this

[5] It is possible to arrange, either by choice of U(1) charges or by the geometric localisation of the messenger fields, that less suppressed interactions between S and the visible sector are not generated upon integrating out these states.

can be easily realised in UV completions, simply if the fields Φ and $Q_{1,2}$ have the same sign charge. Since the exact properties of the underlying theory are unknown, we fix and overall normalisation by setting $c_i = -q_i$ for all states in the visible sector with U(1) charge q_i. The qualitative properties of the spectra obtained are not especially sensitive to this assumption.

Since the third generation up-type quarks are uncharged under the U(1) there are no such terms generated for the stops through this type of interaction. Further, integrating out the gauge multiplet will not generate terms of the form $\int d^4\theta\, f\left(\Phi^\dagger, \Phi\right) Q_{1,2}^\dagger Q_3$ hence these are suppressed relative to the Kähler mass terms Eq. (3.11). Since the Higgs fields are uncharged under this symmetry, the soft masses $m_{H_d}^2$ and $m_{H_u}^2$ are not large which is beneficial in avoiding large fine tuning of the EW scale. An important assumption we are making is that there is no additional field content in the UV theory that generates significant Kähler couplings between Φ and the top multiplet. In a realistic UV completion these will appear at some level, however may naturally be expected to be suppressed by either M_* or M_{Pl} and therefore be negligible compared to the other contributions.

In the model considered here the interaction Eq. (3.11) generates masses

$$m_K^2 = -c_i \frac{|F_\Phi|^2}{v^2} = \epsilon^8 v^2 \ . \tag{3.12}$$

There will be a similar coupling to the messenger fields,

$$\int \mathrm{d}^4\theta\, c_i \frac{\Phi^\dagger \Phi}{v^2} \psi^\dagger \psi \ . \tag{3.13}$$

Since the messenger mass is close to $\sqrt{F}$, the SUSY breaking from this term can lead to slight corrections to the gauge mediated masses induced in the visible sector compared to the normal formula derived assuming analytic continuation. In Appendix A.2 we give the general formula which we use in our later phenomenological analysis. While the exact form of these corrections is complicated their effect is straightforward: both the next corrections in $\frac{F}{m}$ and those from SUSY breaking diagonal masses tend to increase the sfermion masses relative to gaugino masses as they do not break R-symmetry.

The key phenomenological feature of our models is that the ordinary gauge mediated contribution from messenger fields will compete with this Kähler contribution to the first two generations leading to a scenario where the first two generation sfermions become relatively heavy, while the stop quarks stay light, realising a natural SUSY spectrum. These contributions can give phenomenologically reasonable soft terms and natural spectra with appropriate choices of M_* for ϵ in the range suggested by the fermion mass hierarchy. In Section 3.3 we study the MSSM spectra for reasonable choices, however first we examine a more sophisticated and UV complete model of SUSY breaking.

3.1.2 ISS Model of Supersymmetry Breaking and Mediation

While general features of these models can be realised in many examples we now consider the ISS model [129] as an example of a fully dynamical SUSY breaking sector, which, once including the suppression from retrofitted couplings, needs no small scales or couplings. In particular this allows the very natural possibility of associating M_* with the GUT scale. A retrofitted model has previously been studied in [201], and here we consider a mediation to the visible sector through the addition of messenger fields.

The theory is a simple modification of that described in Section 2.8. Consider supersymmetric QCD with gauge group $\mathrm{SU}(N_c)$ and N_f quarks, Q_i, $\tilde{Q}_i$, in the range $N_c + 1 \leq N_f < \frac{3}{2}N_c$. The quarks have charge $+n/2$ and the messenger fields $-n/2$ under the U(1) which is broken in a separate sector by two fields with charge ± 1, $\langle S_{\pm 1}\rangle = v$. The $\mathrm{SU}(N_c)$ gauge coupling is asymptotically free and the theory has a dynamical scale, Λ, above which the superpotential is given by

$$W = \frac{1}{M_*} Q_i \tilde{Q}^i \psi \psi^c + \frac{S_{-1}^n}{M_*^{n-1}} Q_i \tilde{Q}^i + \frac{S_1^n}{M_*^{n-1}} \psi \psi^c \ . \tag{3.14}$$

Below Λ the theory is given by the Seiberg dual which consists of magnetic degrees of freedom: dual quarks q, $\tilde{q}$ and the (canonically normalised) meson of the electric theory $\Phi_i^j = \frac{Q_i \tilde{Q}^j}{\Lambda}$ with superpotential

$$W = \Phi_i^j q^i \tilde{q}_j + \frac{v^n}{M_*^{n-1}} \Lambda \mathrm{Tr}(\Phi) + \left(\frac{v^n}{M_*^{n-1}} + \frac{\Lambda}{M_*} \mathrm{Tr}(\Phi) \right) \psi \psi^c \ . \tag{3.15}$$

With this superpotential, neglecting the small coupling to the messenger fields, the F-terms of the meson field are given by $F_{\Phi_j^i} = \tilde{q}_j q^i - m\Lambda \delta_j^i$, where $m = \frac{v^n}{M_*^{n-1}}$.

As usual the differing ranks of the two contributions to F_Φ imply that not all F-terms can vanish and therefore SUSY is broken in a metastable vacuum with Φ gaining an F-term of order $F_\Phi \sim \frac{v^n \Lambda}{M_*^{n-1}}$.[6] Since the mass term in the electric theory, $\sim \frac{v^n}{M_*^{n-1}}$, can naturally be much smaller than Λ, the F-term can be suppressed away from other scales in the theory allowing

[6]This F-term depends on an $\mathcal{O}(1)$ coefficient, which is undetermined by holomorphy and therefore unknown. However all the soft mass contributions will be seen to depend on F_Φ in the same way therefore this leads to no alteration in the phenomenology.

for small SUSY breaking soft terms to be generated in the visible sector after mediation.

Including the non-renormalisable couplings to messengers in the electric superpotential explicitly breaks the R-symmetry of the magnetic theory (discussed in Section 2). Consequently, this also potentially creates new SUSY preserving vacua. However, the R-breaking is small so the metastable SUSY-breaking vacua can be long lived. This can be connected to the requirement that the cosmological constant vanishes by making the sector that gives the VEV $\langle S\rangle \neq 0$ the same sector that gives a constant contribution to the supergravity scalar potential.

Regarding the visible sector soft masses gauge mediation will give a contribution

$$m_{\rm gauge} \sim \left(\frac{\alpha}{4\pi}\right)\frac{M_*^{n-2}\Lambda F_\Phi}{v^n} \sim \left(\frac{\alpha}{4\pi}\right)\frac{\Lambda^2}{M_*} \ , \tag{3.16}$$

which can be close to the EW scale without fine tuning, as a large hierarchy between Λ and M_* is natural. In addition as in the simple Polonyi model, integrating out the heavy U(1) gauge boson leads to a Kähler contact operator between Φ and other U(1) charged fields. In the electric theory this is given by

$$\int {\rm d}^4\theta\, c_i g^2 \left(\frac{\tilde{Q}^\dagger\tilde{Q} + Q^\dagger Q}{m_{Z'}^2}\right) Q^\dagger_{\rm MSSM} Q_{\rm MSSM} \ . \tag{3.17}$$

In the magnetic regime the Kähler potential is given by

$$\int {\rm d}^4\theta\, c_i g^2 \frac{\Phi^\dagger\Phi}{m_{Z'}^2} Q^\dagger_{\rm MSSM} Q_{\rm MSSM} \ . \tag{3.18}$$

This induces masses for the first two generations

$$m_K^2 = -c_i\frac{|F_\Phi|^2}{v^2} = -c_i\epsilon^{2n-2}\Lambda^2 \ . \tag{3.19}$$

As in the previous model, there will also be a coupling to the messenger fields:

$$\int {\rm d}^4\theta\, c_i \frac{\Phi^\dagger\Phi}{v^2}\psi^\dagger\psi \ . \tag{3.20}$$

The qualitative features of such a model are rather similar to that of the simple Polonyi case. Some details differ, however. In particular since the soft terms are set by the dynamical scale Λ which can be exponentially separated from M_* (and in fact must be for reasonable gauge mediated soft masses), the two scales v and M_* can now be large.

3.2 String Theory Implementation

Since with phenomenologically viable charge assignments the U(1) symmetry naturally possesses mixed anomalies with the SM gauge groups (at least at the level of triangle diagrams involving chiral fermions, and before including the contribution from messenger fields), it is tempting to associate it with the "anomalous" symmetries necessarily found in realistic compactifications of string theories which are rendered consistent by the generalised Green-Schwarz mechanism. While there are various possible stringy UV completions of our models we focus on IIB theories as we now explain.[7]

In traditional heterotic string theory a U(1) with anomalies cancelled by the Green-Schwarz mechanism necessarily obtains a large Fayet-Iliopoulos term $\xi = g^2 M_{\rm Pl}^2 \delta_{GS}/16\pi^2$ where δ_{GS} is the mixed U(1)-gravity^2 anomaly coefficient which must be non-zero (however see [202]). Then the D-term contribution to the action is given by $\frac{g^2}{2}(\xi + \sum_{S_j} j S_j K_j)^2$ where S_j are all fields charged under the U(1) (with charge j), and K_j is the derivative of the Kähler potential with respect to S_j. In order that this does not lead to excessively large SUSY breaking at least one of the fields must gain a VEV, and this VEV is automatically as large as the mass of the U(1) gauge boson. A theory of this type could in principle be used to generate retrofitted models of the form discussed in the previous section if the irrelevant operators appear in the effective field theory by integrating out matter of typical mass $M_{\rm Pl}$.[8] However for our particular case there are some problems with using this traditional heterotic construction. In particular, the requirement of universal mixed anomalies (up to Kac-Moody level factors) too-severely restricts our model-building freedom, while the form of the D-term with non-zero FI term implies that only fields of either positive or negative charge will gain VEVs, not both. Hence, we consider a slightly different scenario using an underlying IIB string theory (such a IIB construction was recently used to implement a Froggatt-Nielsen mechanism in [174, 175]), which leads to a similar but not identical structure to the models of the previous section.

In Type IIB string theory, unlike in traditional heterotic theories, *non-universal* mixed anomalies can be cancelled by massless twisted closed string modes which shift under an anomalous transformation. In the process the U(1) gauge boson gains a mass through the

[7] Our summary of the appearance of such symmetries in string theory follows the discussion in [173] which contains further details.

[8] In this case there is the beneficial feature that the ratio $\frac{\langle S_i \rangle}{M_{\rm Pl}} \sim 0.01$ is automatically appropriate for the fermion mass hierarchies as has been noted by many authors.

Stueckelberg mechanism. An important difference with the heterotic case is that, depending on the underlying geometry, the Fayet-Illiopoulos term can be zero, allowing in the IIB case the situation where no fields charged under the U(1) symmetry necessarily gain VEVs. Hence, at the perturbative level, a global U(1) symmetry can survive in the low-energy theory below the mass of the vector boson, this symmetry only being explicitly broken by non-perturbative effects which can naturally be very small [173]. The charges of fields under the global U(1) are identical to their charges under the gauged U(1). One further advantage of IIB models is that by utilising intersecting brane stack constructions it is straightforward to build theories such that only some generations are charged under the anomalous U(1).

With this UV completion, the structure of our models is as follows. At the string scale $M_* = M_{string}$ there is an anomalous U(1) gauge symmetry. Through the Stueckelberg mechanism the associated gauge boson gains a mass $m_{Z'}$ leaving an (approximate, anomalous) global symmetry. Integrating out this heavy state leads to Kähler contact operators with coefficients determined by the charges of the fields involved and the gauge boson mass. Often it is assumed that the vector boson mass is given by gM_*. However as shown in [203] this relation can be modified in the case of asymmetric compactifications by ratios of volume factors, which can be parametrically less than 1. We include these effects though a parameter λ and write $m_{Z'} = \lambda g M_*$.

At a lower energy scale the approximate global symmetry is broken by fields S_1 and S_{-1} gaining common VEVs v with $\epsilon = \frac{v}{M_*} \ll 1$ (these VEVs slightly correct the vector boson mass). As in the previous section, fields in the DSB sector and the visible sector have U(1) charges such that global symmetry forbids some mass terms and parameters at leading order, these terms being generated from irrelevant operators of the form $W_{\rm DS} \supset \frac{S^n}{M_*^m}\mathcal{O}_{\rm DS}$ and $W_{\rm visible} \supset \frac{S^p}{M_*^p}\mathcal{O}_{\rm visible}$, so suppressing couplings by powers of ϵ.

The resulting soft term structure at the scale of SUSY breaking is similar to the field theory case. There will be a universal gauge mediated contribution and also masses from contact terms generated between fields in the SUSY breaking and visible sectors as a result of integrating out the heavy gauge boson. In the present models $m_{Z'} = g\lambda M_*$, which is of slightly different parametric form compared to the field theory implementation, resulting in a small shift in the relative size of the Kähler contribution. For example, in the ISS model, the Kähler mass contribution to the first two generation sfermions Eq. (3.18) is

$$m_K^2 = -c_i \frac{|F_\Phi|^2}{(\lambda M_*)^2} = -c_i \left(\frac{\epsilon^n \Lambda}{\lambda}\right)^2 \ . \tag{3.21}$$

One notable change in the phenomenology is that the scale of mediation is typically high. As a consequence, there will be large logarithms when the soft masses are run to the weak scale. In Section 3.3 we will see that this can make it harder to obtain viable spectra with large splitting between different generation sfermion masses. Additionally, one might be legitimately concerned about whether the Kähler contribution will dominate over other generic contributions that may be expected to also couple the SUSY-breaking and visible sectors with suppression by the string scale. If the two sectors are approximately sequestered, with communication only occurring through the U(1) gauge multiplet and messenger fields, the only extra contribution will be a small, generation universal, anomaly mediated soft mass. This is the scenario we study in detail in Section 3.3, by taking the parameter $\lambda = 1$. However, the extent to which two sectors may be completely sequestered is still unclear (see for example [204–206]). Alternatively λ can be fairly small of order 0.01, slightly lowering the scale of mediation. This will enhance the Kähler and gauge mediated contributions sufficiently that they can dominate over couplings suppressed by the string or Planck scale.[9]

3.3 MSSM Spectra

Having discussed the main features of our models, in this section we study in some detail the pattern of soft terms obtained in the MSSM sector. The spectra of soft masses in the previous sections are valid at the energy scale where SUSY-breaking is mediated to the visible sector. For the gauge and Kähler contributions this is the mass of the messenger fields and the SUSY breaking sector respectively.

To make any phenomenological predictions it is necessary to run the soft masses to the weak scale. While doing this there will be two dominant and competing effects on the stop masses [191]: 1) the non-zero gaugino masses will tend to pull the third generation soft masses squared to larger values, as in gaugino mediated scenarios, and 2) the large first and second generation masses from Kähler mediation will push the third generation soft masses squared towards negative values. In cases of very low scale mediation these effects have a

[9] In full string constructions it can sometimes be the case that the Kähler contribution is only one of a number of similar sized universal contributions (at least between the first two generations) [186, 187]. We do not consider such modifications here.

reasonably small impact due to the small size of the logarithms, while in models with higher scale mediation these effects can be significant and limiting of the low-energy spectra that can be obtained. First we consider the field theory case, with low-scale SUSY breaking, using the particular example of the Polonyi Model discussed in Section 3.1.1, then we examine the sting motivated case with the ISS model of Section 3.1.2.

3.3.1 Polonyi Model

Recall, $F_{\text{eff}} = \epsilon^6 v^2$, $m_{\text{mess}} = \epsilon^3 v$, and a Kähler mass contribution $m_K = \epsilon^4 v$. The charge assignment to MSSM fields is given by Table 3.1, therefore the SM fermion masses dictate $\epsilon \sim 0.1$, and hence to obtain a reasonable spectrum of soft terms requires $M_* \sim 10^8$ GeV and $v \sim 10^7$ GeV.

	q_L	u^c	e^c	L	d^c
generation 1	1	1	1	1	1
generation 2	1	1	1	1	1
generation 3	0	0	0	0	0

H_u	H_d
0	0

Table 3.1: Charge assignments for low scale breaking

As discussed, the third generation superfields are uncharged and hence obtain Yukawas of $\mathcal{O}(1)$, while mass terms for the first two generations have non-zero net U(1) charge therefore are generated only once S_{-1} gains a VEV. Due to the GUT-consistent structure of charges, the lepton mass hierachy is parametrically the same as that of the down-type quarks, although the two sets of coefficients are not equal. The resulting up- and down-like Yukawas are given by

$$U = \begin{pmatrix} c_{11}\epsilon^2 & c_{12}\epsilon^2 & c_{13}\epsilon \\ c_{21}\epsilon^2 & c_{22}\epsilon^2 & c_{23}\epsilon \\ c_{31}\epsilon & c_{32}\epsilon & c_{33} \end{pmatrix} , \qquad D = \begin{pmatrix} c'_{11}\epsilon^2 & c'_{12}\epsilon^2 & c'_{13}\epsilon \\ c'_{21}\epsilon^2 & c'_{22}\epsilon^2 & c'_{23}\epsilon \\ c'_{31}\epsilon & c'_{32}\epsilon & c'_{33} \end{pmatrix} , \tag{3.22}$$

where c_{ij} and c'_{ij} are coefficients which, as discussed, are not subject to any symmetry structure. Before inclusion of these coefficients the U(1) charges lead to a mass spectrum of SM fermions parametrically of the form

$$m_{\text{up}} \sim \langle H_u \rangle \begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \end{pmatrix} , \qquad m_{\text{down}} \sim \langle H_d \rangle \begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \end{pmatrix} \sim m_{lepton} , \tag{3.23}$$

while the 2-3 block of the CKM matrix is of the correct form

$$V_{\rm CKM} \sim \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} . \tag{3.24}$$

As discussed in Section 3.1 the mixings and mass-hierarchies involving the first generation are not set by the broken U(1) but depend on the coefficients c_{ij} and c'_{ij} for i or $j \in \{1,2\}$ which depend upon independent physics. This physics might be an additional UV flavour symmetry that is independent of SUSY-breaking dynamics, or it might be the result of a random anarchic structure. For instance, if the $\mathcal{O}(1)$ coefficients c_{ij} and c'_{ij} take random values over a finite range, for example a flat distribution in $[0,1]$, the total 3×3 CKM structure can easily be close to that observed.[10] Additionally, these coefficients and level repulsion in the eigenvalues of the mass matrices can account for the fairly large splitting observed between the first two generation fermions. In any case, in our model, there is strong alignment between the third generation sfermion and fermion mass eigenstates. Typically, the first two generation fermion mass eigenstates contain at most a component of size ϵ of the third generation U(1) eigenstate, while the first two generation sfermion masses are equal to high precision.

In order to study the spectrum of sfermion masses that may occur in such a theory it is most interesting to fix the gauge mediated contribution to these masses so that the gluino is around 1.5 TeV close to current limits. This fixes the combination

$$m_{\rm gauge} \sim \left(\frac{\alpha_3}{4\pi}\right) \epsilon^3 v \sim 10^3 \, {\rm GeV} . \tag{3.25}$$

Therefore the Kähler contribution is given by

$$m_K \sim \epsilon^4 v \sim \left(\frac{4\pi}{\alpha_3}\right) \epsilon m_{\rm gauge} , \tag{3.26}$$

which depends only on the value of the parameter ϵ. In addition, we choose the number of pairs of messenger fields $n_m = 5$. This increases the gauge mediated gluino mass, which is proportional to n_m, relative to the stop mass which is proportional to $\sqrt{n_m}$, but is not so large as to lead to a Landau pole for the SM gauge couplings below the GUT scale.[11]

[10]The required CP phase of the CKM matrix can arise from the values of the order one coefficients that appear in the superpotential terms which generate the yukawa couplings.

[11]In this model, anomaly cancellation requires additional matter charged under the U(1) and MSSM gauge groups. We assume these fields have charges such that they do not couple strongly to the SUSY breaking sector, and are not sufficiently numerous that they lead to a Landau pole. Alternatively, anomaly cancellation with no extra matter is possible if there are fewer messenger fields present. The only effect of such a modification is the gluino mass will be lowered towards that of the stop.

In Fig. 3.1 (*top*) we plot the soft masses obtained at the scale $\sqrt{F}$ by allowing ϵ to vary while keeping the gauge mediated contribution fixed. As ϵ increases the first two generations obtain increasing masses from the Kähler operator resulting in a natural SUSY spectrum. As discussed we need to run the spectrum to the weak scale. The Kähler contribution to the first two generation soft masses turns on at a scale $\sqrt{F} \sim \epsilon^2 v$ while gauge mediated contributions to these and the gaugino and third generation soft masses begins at $m_{\text{mess}} \sim \epsilon^3 v$. Depending on the charge assignments, and the particular value of ϵ, it is possible that the sbottom or stop may be driven tachyonic at some point in this energy regime. Such an event is not necessarily problematic if these states run back to positive mass squared before the weak scale. Provided $m_{\tilde{t}}\,(m_Z) > \frac{1}{10} M_3\,(m_Z)$ the EW breaking vacuum is sufficiently meta-stable against decays to a colour breaking vacuum compared to the lifetime of the Universe [207–209]. This relation is typically satisfied for our models.[12]

Below a scale $m_{1,2}$ the first two generation sfermions are integrated out of the theory and have no further effect on the third generation running, while the positive contribution from the gluino persists until the gluino mass is reached. Additionally the gauginos and first two generation sfermion masses also flow. We solve the RG equations numerically and plot the mass spectrum at the weak scale in Fig. 3.1 (*bottom*). As ϵ increases the Kähler mass contribution increases and during running the stop and stau masses are driven smaller, until at $\epsilon \sim 0.2$ the right-handed stau becomes tachyonic at the weak scale and the spectrum is not phenomenologically viable.

The key point of our models is that for values of ϵ motivated by the fermion mass hierarchy the split between the first two generation soft masses and the third is sufficiently large to realise natural SUSY, but not so large as so lead to tachyonic third generation states. The NLSP (after the gravitino) is typically a stau, which is fairly light, and can modify cosmology and certain collier signals as we will discus later. As a representative example of the full spectra that may typically be obtained, we show the field content for $\epsilon = 0.10$ under the current assumptions in Fig. 3.2. This is a reasonable value in the middle of the plausible range without fine tuning to the edge of the allowed region.

[12]The energy region where such states are tachyonic is fairly small hence there is little danger of reheating after inflation into a colour breaking vacua, and even in this case it has been suggested that the EW vacua may be favoured [210].

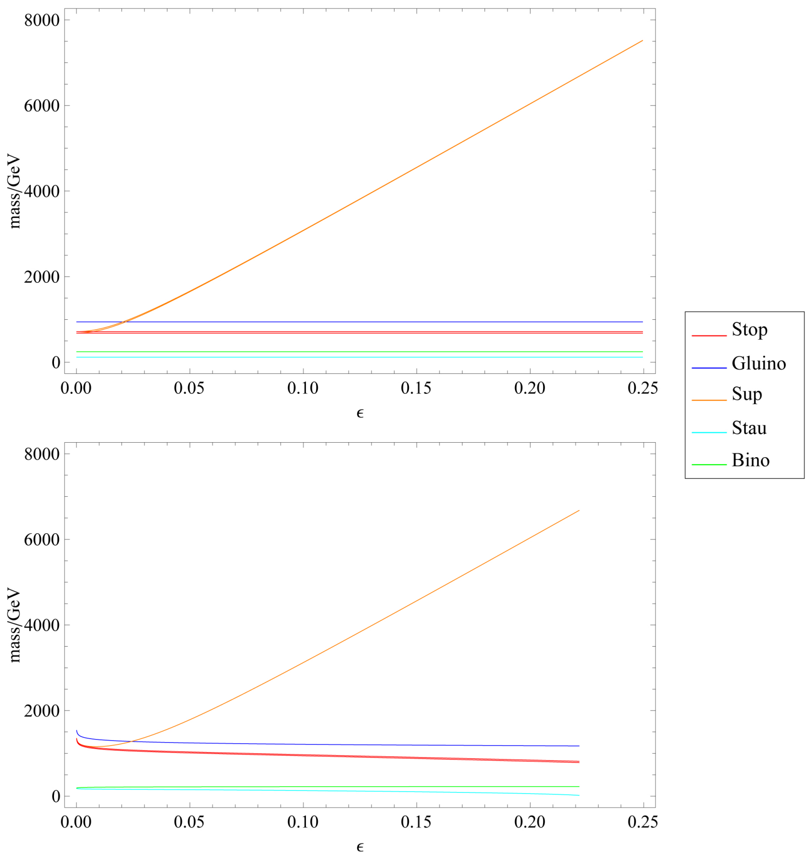

Figure 3.1: The spectrum of superparticles before (top) and after (bottom) running to the weak scale in the Polonyi model. F and v have been fixed to give a gluino in the region of 1.3 TeV after running, close to current LHC limits, while M_* is varied changing ϵ and therefore the relative importance of the Kähler interactions. For $\epsilon > 0.22$ the first two generation sfermions are so heavy that a stau is driven tachyonic during running and the weak scale spectrum is not phenomenologically viable.

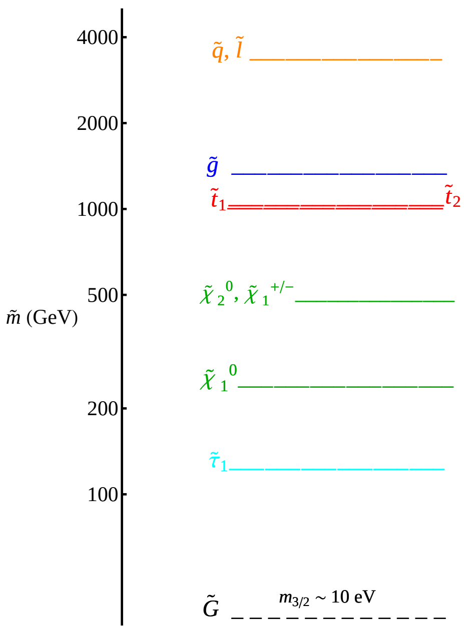

Figure 3.2: A typical spectrum of superparticles after running to the weak scale in the Polonyi model. F and v are fixed to give a gluino in the region of 1.3 TeV and as a representative example $\epsilon = 0.1$. There is a very light gravitino LSP and the right-handed stau is the NLSP.

3.3.2 ISS Model

It is also interesting to see to what extent we can realise natural SUSY in the ISS model at a relatively high scale. In this case we take the string motivated Kähler contribution, $m_K \sim \frac{F}{M_*}$. To simplify the analysis we assume the parameter $\lambda = 1$, and take the U(1) charge assignment of Table 3.2.[13] This has the phenomenological benefit that it gives the right-handed stau a large mass, preventing it running tachyonic, which would otherwise place the strongest limit on the allowed values of ϵ. Before inclusion of the c_{ij} and c'_{ij} coefficients

	q_L	u^c	e^c	L	d^c
generation 1	1	1	1	1	1
generation 2	1	1	1	1	1
generation 3	0	0	1/2	1/2	1

H_u	H_d
0	0

Table 3.2: Charge assignments for high scale breaking

these give a mass pattern

$$m_{\rm up} \sim \langle H_u \rangle \begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \end{pmatrix} , \qquad m_{\rm down} \sim \langle H_d \rangle \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \end{pmatrix} \sim m_{lepton} , \tag{3.27}$$

while the third-second generation sub-block of the CKM matrix is again of the form (3.24).

Reasonable splitting of the third generation leads to $0.007 \lesssim \epsilon \lesssim 0.05$. This also gives a CKM matrix of the correct form to leading order. Since the third generation down sector masses are suppressed by a factor of ϵ, $\tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle} \sim \frac{m_t}{m_b}\epsilon \sim 1$ assuming Yukawa coefficients in the third generation are $\mathcal{O}(1)$. This may be phenomenologically favoured over the alternative of large $\tan\beta \sim \frac{m_t}{m_b} \sim 40$ in enhancing the Higgs mass to 125 GeV in an NMSSM like model (see [90] for a review). As the bottom Yukawas are small, even though the Kähler mass contribution will lead to multi-TeV scale bottom squarks these do not lead to fine tuning of the EW scale.

Again we take there to be five pairs of messenger fields, and as well motivated by string compactifications, $M_* = 10^{16}$ GeV. In order to obtain a gauge mediated contribution to soft masses (and in particular the gaugino masses) of order TeV such that these are close to current limits but not excluded requires $\Lambda \sim 10^{10}$GeV. To obtain Kähler contributions to the first two generation masses that are also a few TeV for reasonable values of ϵ, we take the charge, introduced in Section 3.1.2, $n = 4$. Of course, this is a particular choice which leads

[13] In this case we are choosing a U(1) charge structure that is not compatible with a traditional 4D GUT. However it is compatible with an orbifold GUT structure, which can result from an underlying IIB D-brane model, with split matter multiplets [197, 198]. Thus precision SUSY gauge-coupling unification can be maintained.

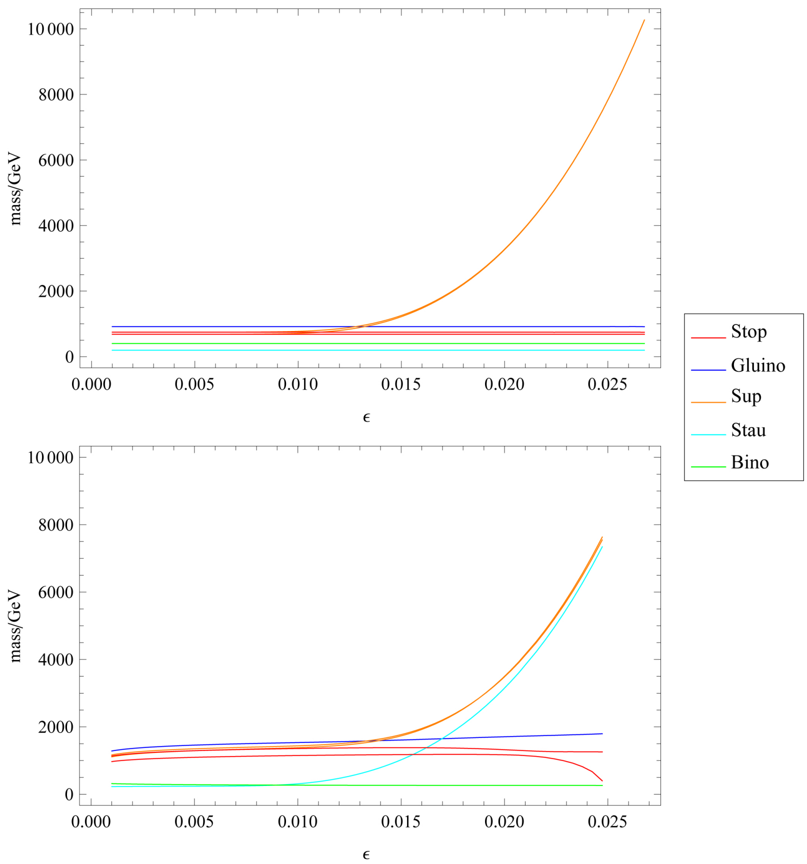

Figure 3.3: The spectrum of superparticles before (top) and after (bottom) running to the weak scale in the string motivated case. Λ and M_* have been fixed to give a gluino in the region of 1TeV after running, and ϵ is allowed to vary.

to viable natural spectra, however as a discrete value it is plausible and not a fine tuning in the sense of a continuous parameter. Such a choice also has the benefit of setting the mass of the messengers and also the coefficient of $Q_i\tilde{Q}^i$ in the electric ISS superpotential equal to $\epsilon^3 v \sim 10^8\,\text{GeV}$. Since this is much less than the strong coupling scale of the ISS theory it is valid to use the Seiberg dual of this theory, and the SUSY breaking vacua obtained is sufficiently long lived.

Unlike the field theory case, the gauge mediated contribution, $m_{\text{gauge}} \sim \left(\frac{\alpha_3}{4\pi}\right)\frac{\Lambda^2}{M_*}$, is independent of ϵ. Therefore in studying the spectra we simply fix Λ and M_* and allow ϵ to vary, which changes the Kähler mediated contribution $m_K \sim \epsilon^4\Lambda$. Since the running occurs over a long period it is important to use the full RG equations and our analysis is done using `SOFTSUSY` [211]. The spectrum obtained before and after running is shown in Fig. 3.3.

For choices of $\epsilon \simeq 0.02$ a natural SUSY spectrum with light stops and heavy first two generations is obtained, however the range of ϵ that generates such a spectrum is smaller than in the field theory case. There are two reasons for this, firstly, since SUSY is broken at a higher scale there is more running, therefore for a given stop mass the first two generations cannot be as heavy as previously. This effect is unavoidable in any model of natural SUSY that uses high scales as may be natural in string completions. Secondly, in this scenario the Kähler masses have a power law dependence on ϵ while the gauge mediated contribution has no such dependence, therefore relatively small changes in ϵ change the Kähler masses significantly. This may be regarded as a defect of the model, however our purpose is only to demonstrate that natural SUSY spectra are possible in realistic completions.

As previously discussed, in order to obtain a vanishing cosmological constant, there must be an R-symmetry breaking constant superpotential term generated by the theory. Depending on the particular dynamics, the sector that generates the VEV for the S fields may play such a role.

3.4 Variations and Signatures

3.4.1 Variant Spectra

So far we have not addressed the μ/B_μ problem that is very commonly found in models of gauge mediated SUSY breaking [212]. This may be solved using a mechanism completely separate from the U(1) and generation of a natural SUSY spectrum, or alternatively, with minor alternations to the charge structure could be solved automatically in our models. Suppose the down-type Higgs has charge $\frac{1}{2}$ under the U(1) symmetry. Then suitable choices of the charges of the lepton and down-type superfields can still lead to viable fermion mass patterns and natural soft mass spectra (for example one may shift the charge assignments of all three generations of d^c and L fields by $-\frac{1}{2}$ from their values in Table 3.2). As a consequence both the μ and B_μ terms are forbidden at tree level, while the down type Higgs obtains a significant Kähler soft mass $m_{H_d}^2 \sim (4\,\mathrm{TeV})^2$ while $m_{H_u}^2 \sim (200\,\mathrm{GeV})^2$. If the DSB and messenger sectors additionally involve fields with charge $\pm\frac{1}{2}$ gaining VEVs or F-terms, it is possible to generate μ and B_μ, and depending on the explicit model, these satisfy the standard relation $B_\mu \sim 16\pi^2\mu^2$. However, this pattern of soft masses and parameters with $m_{H_d}^2$ large now realises lopsided gauge mediation [213], at least as far as the Higgs sector of the theory is concerned, and which leads to viable EW symmetry breaking without excessive fine tuning.

While we have studied charge assignments such that the first two generation sfermions gain large soft masses, there is an alternative option that can lead to natural SUSY spectra.

If the top superfields are charged appropriately, it is possible the stops gain a *negative* mass contribution from the Kähler couplings pushing them to a lower mass than the first two generation sfermion masses. Such charge assignments can also generate the fermion mass hierarchy if the Higgs multiplets are charged under the U(1). In fact, even if the theory is such that the stops have negative mass squared at the mediation scale it is possible, after running, to obtain viable spectrum with non-tachyonic stops at the weak scale if the gluino is sufficiently heavy. This was first raised as a possibility in [214] where it was suggested as a mechanism for obtaining a spectrum with low fine tuning. In such a model the stop may be expected to be tachyonic for a relatively large range of energies. Hence, even if the lifetime of the EW breaking and colour-preserving vacuum is sufficiently long, there is a concern about whether the Universe is likely to find itself in this metastable state after reheating. While we have not investigated this scenario in detail, preliminary investigation demonstrates that it is possible to obtain reasonably natural spectra consistent with LHC constraints. However, as in the models we have focussed on, very light stops, possibly down to about 400GeV, require some fine tuning of the parameters of the theory.

As an additional possibility, if we reject the requirement of naturalness, it is also easily possible to generate a split [215, 216] or mini-split spectrum within these models [217]. This could occur if all the quark superfields have the same sign charge under the U(1), hence all sfermions gain a large positive mass contribution from Kähler interactions. In order to obtain a viable fermion mass spectrum this would require the Higgs fields to have the opposite charge. In this case since the SUSY flavour problem is solved by decoupling, the U(1) could also generate the texture in the first two generation fermion masses.

3.4.2 Collider Signals, Flavour and Higgs

The collider signals of the natural SUSY spectra typical of our models have been studied extensively. Depending on the charge assignments, a bino or stau is generically the NLSP, which for very low gravitino mass may decay in a typical detector distance while for larger gravitino mass will escape the detector. Both cases lead to clear signals that can be studied at the LHC. However, the relatively heavy gluino masses ($m_{\tilde{g}} > 1.5\,\text{TeV}$) and especially the almost decoupled first two generations, reduce production cross sections dramatically, and spectra are typically well within current LHC limits such that a light stop is not ruled out. As we have seen, a very light stop is hard to achieve, a more realistic model has been seen in Fig. 3.2. Since in this case the stop is not especially light (though still far below current bounds for squark masses in generation universal models) such a spectrum will be challenging to discover at the LHC until a large integrated luminosity has been accumulated. Additionally, if R-parity is broken, spectra with a lighter gluino may be compatible with LHC constraints, allowing lighter stops in our models [162]. More detailed analysis of the expected signals and phenomenology can be found in for example [166, 218–224].

A common concern in SUSY theories is suppressing flavour changing effects to safe levels. In our model these effects can be well within current limits. The ordinary gauge mediated contribution is automatically flavour blind as normal. Additionally the Kähler contribution to the first two generation sfermions is universal. Therefore flavour changing effects occur only due to the small mixing in the CKM matrix between the first two generations and the third generation. More precisely, the first two fermion mass eigenstates include only a component of the third generation U(1) eigenstate of size ϵ. In order to produce a realistic CKM matrix ϵ must satisfy $\epsilon \sim V_{cb} \sim 0.04$. Hence, the sfermion mass squared matrix differs from diagonal in the first two generation sector at most by by elements like $V_{cb}^2 m_{\tilde{t}}^2$. There is also additional suppression of flavour changing effects due to the relatively large masses of the first two generation sfermions. Utilising the expressions in [81] we find that CP-conserving flavour changing effects are typically well within experimental limits. CP-violating processes generally give stronger constraints; if the first two generation sfermions are near their maximum allowed mass and ϵ fairly small these can be within current limits for $\mathcal{O}(1)$ phases in the soft terms. Alternatively, we can assume the UV theory is such that these phases are small or zero.[14]

While it may be hoped that light stops allow a theory without excessive fine tuning (we discuss this further in Section 5), it is not immediately obvious how to combine such a spectrum with a lightest Higgs mass of 126 GeV as recently discovered by ATLAS and CMS, since as discussed the tree level Higgs mass is bounded by m_Z at tree level. We discuss the resolution to such a problem in an NMSSM like model in the next section.

3.4.3 Axions and Cosmology

In the string motivated case an approximate global symmetry is spontaneously broken and there will be an axion present in the low energy theory, which will, because of the $U(1) \times$

[14]The counting of physical phases also depends on the mechanism that generates μ and $B\mu$, hence is model dependent.

$G^2_{\rm MSSM}$ anomaly, have couplings to the MSSM gauge multiplets. Therefore in the case of relatively high scale mediation this state could even play the role of the QCD axion. This is by no means necessary, however. For example, there may be couplings between the axion and any hidden gauge groups, for instance in the DSB sector, depending on the particular anomaly coefficients of the theory. Such anomalous couplings to a hidden gauge sector typically imply that the axion-like states gains a large mass of order $\frac{\Lambda^2_{\rm hidden}}{f_a}$, and therefore cannot be the QCD axion. On the other hand this allows current astrophysical and direct search bounds to be easily evaded even if $f_a \sim v \ll 10^9\,{\rm GeV}$.

Further, depending on the mass and decay constant of the axion, as well as the initial misalignment angle and thermal history of the Universe, this can provide a significant component of the DM. In fact it may be highly beneficial to couple the QCD axion to the DSB sector: typically overproduction of the axino and saxion, combined with gravitino limits, strongly constrains the reheat temperature over a large parameter space [225]. However if there is a significant coupling between the axion multiplet and the SUSY-breaking sector the axino and saxion can gain large masses greatly relaxing these limits [226]. The presence of a light axion degree of freedom coupling both to the DSB and visible sectors is similar to a scenario we recently studied where the axion was the primary mediator of SUSY breaking [227], although in the present models the axion multiplet does not typically gain a significant F-term.

Apart form the possibility of such an axion, the cosmology of the models are fairly similar to that of normal gauge mediated models. One exception is when the U(1) charge assignments are such that there is a light stau in the theory, in which case it is typically the NSLP after running (see for example Fig. 3.2). This may be beneficial for cosmology; since a stau NLSP leads to decay hadronically it can decay into the gravitino later than other NLSP candidates without disrupting big bang nucleosynthesis. As a result a heavier gravitino is compatible with observations, permitting a higher reheat temperature without gravitino overproduction aiding inflation model building. More precisely, it has been suggested that F in the region $\sqrt{F} \sim 10^{8.5-10}\,{\rm GeV}$ and thus $m_{3/2} \sim 0.1 - 100\,{\rm GeV}$ (which is compatible with a GUT scale value of M_*) may permit reheat temperatures up to roughly $10^9\,{\rm GeV}$ [228].

Chapter 4: Running Through Strong Coupling in λSUSY

This chapter is based on [229], work done in collaboration with John March-Russell and James Unwin .

As discussed in Section 2, the Higgs recently discovered by ATLAS and CMS [230, 231] with a mass around 125 GeV is potentially problematic for models of SUSY. Common approaches to raising the mass of the lightest Higgs state are through large loop corrections, new contributions to the quartic Higgs coupling, or via level repulsion due to mixing between the Higgs and a SM singlet state.

Probably the most studied possibility involves stop squarks significantly heavier than the top quark leading to large contributions from the stop loops. In models with universal sfermion masses, collider limits typically force the stop masses to be sufficiently heavy that the requirement that $m_h \approx 125$ GeV can be achieved in the MSSM provided $\tan\beta$ is small. However, in models of natural SUSY, the weak scale stop masses are often significantly lighter than the universal limits in an attempt to reduce fine tuning. Specifically, it is difficult to obtain $m_h \approx 125$ GeV with stop masses ($m_{\widetilde{t}} \lesssim 1.5$ TeV) in the MSSM unless there is near-maximal mixing between $\widetilde{t}_L$ and $\widetilde{t}_R$ [111, 232, 233], requiring very large A-terms. However, these are difficult to generate in models of gauge mediation, which are attractive for minimising fine tuning since they can have a low mediation scale.

In this section we study the well-motivated approach of introducing a new source for the quartic Higgs interaction via the superpotential term $\lambda S H_u H_d$, which involves a new SM singlet state S, as found the Next-to-Minimal Supersymmetric Standard Model (NMSSM). Including this as well as leading loop corrections leads to contributions to the mass of the lightest SM-like Higgs state of the form Eq. (2.40).[1]

For sizeable $\lambda \gtrsim 0.6$ the new NMSSM contribution provides the dominant correction to the Higgs mass and one can obtain $m_h \approx 125$ GeV whilst maintaining natural stop masses and small stop mixing. Moreover, the NMSSM is far from an *ad hoc* solution, since it also provides a solution to the μ-problem of the MSSM [56]. It is notable that if the coupling

[1]For simplicity, and motivated by minimal constructions, we shall assume that $\widetilde{t}_1$ and $\widetilde{t}_2$ are approximately degenerate; our conclusions are not substantially altered upon relaxation of this assumption. Throughout we shall consider only models in which A-term contributions are negligible.

$\lambda \gtrsim 0.7$ at the weak scale then it will run non-perturbative before the unification scale. It is then natural to be concerned that such large values may result in undesirable side-effects on precision gauge coupling unification. The aim of this chapter is to quantify the impact on unification of λ running through a period of strong coupling.

Experience with the running of the QED coupling through the QCD strong coupling regime is indicative that non-perturbative dynamics in some sector of a theory is not necessarily disastrous for the evolution of an independent gauge coupling, despite naive expectations based upon cursory examination of the RGEs. In fact, the corrections to the QED coupling α_{EM} generated during the region in which QCD is strongly coupled can be measured experimentally, as well as estimated from semi-rigorous theoretical calculations, and is of order a few percent [234]. Furthermore, arguments based on holomorphy [72, 235–237] lead us to believe that the strong coupling in λSUSY should not damage gauge unification. In this section we demonstrate that provided the coupling λ remains non-perturbative for roughly less than an order of magnitude in energy then this in fact can likely increase the precision of gauge coupling unification, correcting the present 3% discrepancy in MSSM gauge unification [1] due to the strong coupling constant running too fast.[2] While it is entirely possible that this present deviation between the predicted $\alpha_s(m_Z)$ and the measured value may be resolved by threshold corrections near the weak or GUT scale [245, 246], there are well motivated cases where these are naturally small [98, 243]. We thus find it intriguing that λSUSY models may not disturb, but even improve, unification.

This section is ordered as follows: in Section 4.1 we study how the Higgs mass depends on the parameters $\tan\beta$, $m_{\tilde{t}}$ and λ_0 (the weak scale value of λ) and determine the values of these which result in a lightest SM-like Higgs boson at $m_h \approx 125$ GeV. Further we identify the parameter regions which result in λ running non-perturbative before the unification scale and discuss how the scale of strong coupling depends on these parameters. In Sections 4.2 and 4.3 we demonstrate that running through a region in which λ becomes non-perturbative can improve the precision of unification. We also consider a possible link with the observed hierarchy in up-type to down-type quark masses, especially, m_t/m_b.

[2] Alternative suggestions to improved the precision of gauge coupling unification include [98, 238–244].

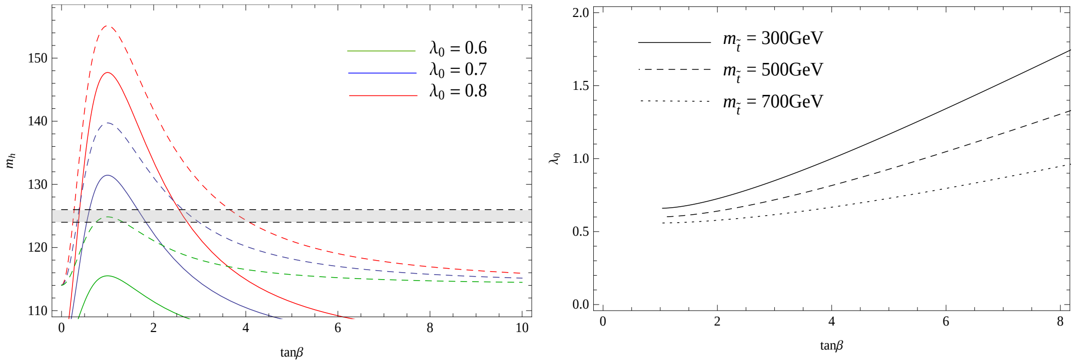

Figure 4.1: **Left.** The variation of the SM-like Higgs mass as a function of $\tan\beta$ for $m_{\tilde{t}} = 300$ GeV (solid curves) and $m_{\tilde{t}} = 500$ GeV (dashed curves) and different values of the starting (weak scale) coupling λ_0 as indicated. The shaded region corresponds to the possible Higgs signal at 124-126 GeV. **Right.** The relationship between $\tan\beta$ and λ_0 which gives $m_h = 125$ GeV for different stop masses. For $m_{\tilde{t}} \lesssim 500$ GeV this requires $\lambda_0 \gtrsim 0.65$ and λ may run non-perturbative before $M_{\rm GUT}$.

4.1 The 125 GeV Higgs in the NMSSM and λSUSY

To solve the μ-problem of the MSSM the superpotential term $\mu H_u H_d$ is replaced[3] in the NMSSM by a trilinear interaction $\lambda S H_u H_d$ involving a dynamical SM singlet chiral superfield, S, and the μ-term is reintroduced upon S acquiring a VEV. Possible mechanisms for generating a VEV for S in the context of λSUSY are discussed in [247]. The introduction of S leads to possible new terms in the superpotential

$$\mathcal{W} = \mathcal{W}_{\rm MSSM} + \lambda S H_u H_d + \xi S + \mu' S^2 + \kappa S^3 \ . \tag{4.1}$$

Note that some additional symmetries must be imposed in order to remove the dangerous tadpole term ξS (unless the field S is composite with suitably low compositeness scale) and in simplified scenarios it is often assumed that the cubic term κS^3 is also forbidden. Note that if the trilinear term is allowed in the superpotential the RGEs imply that κ quickly evolves to small values at lower energies [247] and thus we shall neglect the cubic term henceforth.[4]

The leading corrections to the tree-level Higgs mass come from the F-term associated with $\lambda S H_u H_d$ and the stop loops, as given in Eq. (2.40). Thus the physical mass of the lightest SM-like Higgs scalar depends on $\tan\beta$, the couplings λ and κ, and the stop mass $m_{\tilde{t}}$. To give an idea of the dependence we use Eq. (2.40)) to calculate the mass of the lightest SM-like Higgs, following [248], as a function of $\tan\beta$ for differing values of $m_{\tilde{t}}$ and λ_0, defined as the value of the coupling λ at the weak scale,[5] this is shown in Fig. 4.1 (*left*) (see also [250]).

[3]In λSUSY an explicit $\hat{\mu} H_u H_d$ term is often added, which is taken to be a small PQ-breaking term.
[4]Sizeable κ at the weak scale would result in λ running faster, becoming non-perturbative at a lower scale.
[5]We have neglected two-loop contributions which generally change the Higgs mass by a few GeV. The results

Observe that $m_h = 125$ GeV cannot be obtained for $\lambda_0 = 0.6$ in the case that $m_{\tilde{t}} \lesssim 500$ GeV.

At low $\tan\beta$ increasing λ_0 in the NMSSM allows for smaller stop masses [248, 251]. In λSUSY models, where $\lambda_0 \sim 2$ is very large it may also reduce the EW tuning of the theory by reducing the sensitivity of the Higgs potential to corrections to the Higgs soft masses [248]. In this case mixing between the singlet and the Higgs is actually used to lower m_h, due to level repulsion [248, 252], allowing a larger value of $\lambda_0 \sim 2$ whilst obtaining the desired Higgs mass (experimental constraints on models with large λ_0 have been discussed in [253]). Alternatively, if the Higgs-singlet mixing is small then $m_h \approx 125$ GeV can be obtained with low stop masses and without stop mixing for somewhat smaller values of λ_0. However, with light stops and small mixing one requires $\lambda_0 \gtrsim 0.7$ and the coupling will generally run non-perturbative before the GUT scale.[6]

In Fig. 4.1 (*left*) the curves with $\lambda_0 = 0.7, 0.8$ have two values of $\tan\beta$ which satisfy $m_h = 125$ GeV, the lower solution, however, requires $\tan\beta < 1$ and such low values are theoretically disfavoured as they result in the top Yukawa running non-perturbative before the unification scale - in the NMSSM $\tan\beta \gtrsim 1.5$ is required in order to preserve perturbative *SM couplings* up to the unification scale (by adding additional matter in $5 + \overline{5}$ pairs one can allow $\tan\beta \gtrsim 1$ [247, 254]).[7] Consequently, there is a definite relation between λ_0 and $\tan\beta$ depending only on $m_{\tilde{t}}$ which we display in Fig. 4.1 (*right*). We observe that a Higgs in the signal region can be obtained for a range of parameters, with, in many cases, λ becoming strongly coupled before the unification scale.

In Fig. 4.2 we use the one-loop RGE evolution of λ (see e.g. [90]) to study the parameter dependence of the scale μ at which λ becomes strongly coupled, which we define as $\lambda(\mu) \sim \sqrt{4\pi}$ (the results are insensitive to the exact definition). Judicious parameter choices, with the inclusion of some mixing, can result in perturbativity of λ up to the unification scale for models with $m_{\tilde{t}} \lesssim 500$ GeV. With small mixing, it can be seen from Fig. 4.2 that for $m_{\tilde{t}} \lesssim 500$ GeV (with our previously stated assumptions), the coupling λ always runs non-

obtained agree well with calculations performed using the numerical code `NMHDECAY` [249], which include further radiative corrections beyond Eq. (2.40).

[6]Following Hall *et al.* [248], we conservatively neglect singlet-Higgs mixing which would reduce the mass of the lightest SM-like Higgs. As we are concerned here with the scenario in which the coupling λ is large and the stops are light, higher-order corrections to the Higgs mass involving stop loops are small. We consider only models in which A-term contributions are negligible, corrections to the Higgs mass due to moderate stop mixing δ_X compared to the correction δ_λ due to $\lambda S H_u H_d$ is $\frac{\delta_X}{\delta_\lambda} \sim \frac{0.068}{\lambda^2 \sin^2 2\beta}$ (taking $X_t \simeq m_{\tilde{t}}$). In models of interest to us here $\lambda \gtrsim 0.7$ and $\tan\beta$ is small, giving $\delta_X/\delta_\lambda \lesssim 0.2$ and for larger values of λ ($\simeq 2$) the δ_X correction is further suppressed.

[7]Although models in which non-SM-singlet states such as the Higgs doublets or $\overline{u}_3$ are composite states are of interest (see e.g. [255], in the non-SUSY case), in this work we consider the simplest case in which only SM-singlet states are composite and have large interactions at some scale.

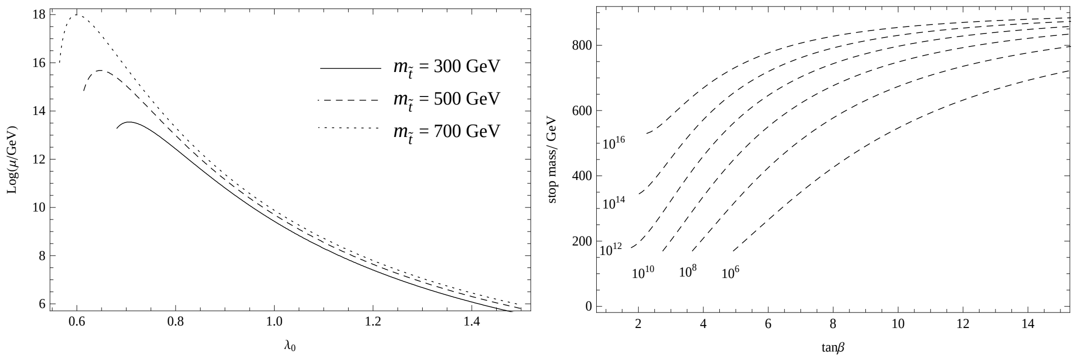

Figure 4.2: **Left.** The strong coupling scale for λ against λ_0 for various stop masses. Note (with our assumptions) for $m_{\tilde{t}} \lesssim 500$ GeV the coupling λ runs non-perturbative before $M_{\rm GUT}$. We fix $\tan\beta$ such that $m_h = 125$ GeV and $\tan\beta > 1.5$. **Right.** Contour plot showing the dependence on $\tan\beta$ and $m_{\tilde{t}}$ of the strong coupling scale for λ, displaying contours for scales larger than 10^6 GeV only. We fix λ_0, the weak scale value of λ, such that $m_h = 125$ GeV.

perturbative before the unification scale. Depending on the parameter choices this can occur anywhere from 10^5 GeV to just below the unification scale. As noted previously, large λ_0 may reduce the fine-tuning, hence λSUSY provides a well-motivated scenario in which we expect either new physics to appear before the non-perturbative scale, or the theory to run through a strong coupling regime.

4.2 Running Through Strong Coupling

If λ runs to strong coupling then there are two conceivable scenarios. The theory may remain in a quasi-conformal strong coupling regime all the way to the GUT scale (which need only be an order of magnitude higher in energy scale in some cases). Alternatively, after a brief period of strong coupling the degrees of freedom may recombine such that the theory reverts back to a weakly-coupled system with the IR fields composites of the UV degrees of freedom. Examples of the first case occur in Randall-Sundrum-like models where the IR brane scale is the strong coupling scale, while explicit realisations of the second scenario can arise, for example, in [256] and the Fat Higgs models [237, 257–261]. In both cases the period of strong coupling will modify gauge coupling unification. As we shall see, however, it will not necessarily destroy successful unification and in some cases can enhance the precision. From the perspective of unification we are most interested in the case where the SM gauge coupling β-function coefficients below and above the strong coupling regime are such as that the ratios of differences $\frac{b_2-b_3}{b_1-b_2}$ are unaltered, thus maintaining the success of SUSY unification at the leading one-loop logarithmic level. An example of this case occurs when the singlet field S is composite but the Higgs fields are fundamental; such a model was constructed in [237]. We will argue, self-consistently, that even though λ becomes non-perturbative and S is replaced by some more elementary degrees of freedom, SM gauge couplings remain perturbatively small throughout the strong coupling region and the effect of this regime is of the form of a threshold correction whose size can be estimated with not unreasonable assumptions.

To quantify the effect of the strong coupling period on gauge unification, consider a theory where λ becomes strongly coupled at a scale μ_- and remains so until some higher scale μ_+ at which the theory UV completes to a more fundamental weakly-coupled theory. The scenario in which the theory remains strongly coupled up to the GUT scale is simply a special case for which μ_+ is identified with $M_{\rm GUT}$. Recalling that the holomorphic Wilsonian gauge kinetic function is renormalised only at one loop, the strongly-coupled sector modifies the MSSM β-functions solely through the anomalous rescaling of matter fields needed to canonically normalise the Kähler potential. The effect on the running is encapsulated in the NSVZ β-function for the gauge-coupling evolution in a supersymmetric Yang-Mills-matter theory [72, 235]:

$$\beta_{g_a} \equiv \frac{\mathrm{d}g_a}{\mathrm{d}t} = \frac{g_a^3}{16\pi^2} b_a \ , \tag{4.2}$$

with $t = \ln\left(Q/M_{\rm GUT}\right)$ and

$$b_a = -\frac{3C_2(G_a) - \sum_R T_a(R)\left[1-\gamma_R\right]}{1 - \frac{g_a^2}{8\pi^2}C_2(G_a)} \ , \tag{4.3}$$

where the index R labels all matter representations, $T_a(R)$ is the quadratic index of R, $C_2(G_a)$ is the quadratic Casimir of the group G_a (normalised so that $C_2(\mathrm{SU}(N)) = N$ and $T_2(\Box) = \frac{1}{2}$), and γ_R are the matter field anomalous dimensions. The use of the supersymmetric β-function is justified as the non-perturbative scales we consider are much larger than the scale of soft supersymmetry breaking of order TeV. In Eq. (4.2) g_a is the canonically normalised 'physical' gauge coupling of the one particle irreducible (1PI) effective action, and not the holomorphic coupling, a change which leads to the non-trivial denominator (see [236] for details). In the cases of interest the factor $\frac{g_a^2}{8\pi^2}C_2(G_a)$ is small as the SM gauge couplings g_a will remain perturbative, hence the denominator may be approximated by 1 if we work to one-loop order in SM gauge couplings in the mixed gauge coupling-γ_R terms (but non-perturbative in λ).

Outside of the strong coupling region the anomalous dimensions, γ_R, are loop suppressed and small for all fields, and the one-loop β-functions are those of the MSSM

$$b_a^{(0)} \simeq -\left(3C_2(G_a) - \sum_R T_a(R)\right) \ , \tag{4.4}$$

while in the region of strong coupling b_a picks up a new contribution due to non-SM-singlet fields with large anomalous dimensions

$$\Delta b_a^{(\mathrm{SC})} \simeq -\sum_R T_a(R)\gamma_R \ . \tag{4.5}$$

In the NMSSM the only fields with SM gauge charges that are coupled directly to the strongly interacting sector are H_u and H_d, and therefore these fields alone pick-up significant anomalous dimensions (of order one) at the point that the coupling λ becomes large. However, the large anomalous dimensions for the Higgs fields will feed into the Yukawa interactions and, as a result, the top Yukawa may subsequently also develop a large anomalous dimension depending on the size of the strong coupling region and the magnitude of γ_{H_u}; we shall discuss this in detail shortly.

We make the reasonable assumption that during the period of strong coupling, $\mu_- < \mu < \mu_+$, the anomalous dimensions of H_u and H_d are not $\gg 1$ (this assumption will be quantified shortly). Hence, calling g_a the 'unperturbed' RGE gauge coupling trajectory, i.e. neglecting corrections due to $\Delta b_a^{(\mathrm{SC})}$, the RGEs for the gauge couplings can be approximated as

$$\beta_{g_a} = \frac{g_a^3}{16\pi^2}(b_a^{(0)} + \Delta b_a^{(0)} + \Delta b_a^{(\mathrm{SC})}) \ , \tag{4.6}$$

where $g_a = g_a^{(0)} + \Delta g_a$ is the modified coupling trajectory and the effects of MSSM two-loop diagrams, corrections due to Yukawa interactions and scheme conversion effects are included as an additional perturbation $\Delta b_a^{(0)}$ (which from numerical studies is known to be small in practice, and which we later include). Writing the formal solution to Eq. (4.2) as an integral from the IR weak scale to the UV GUT scale we get

$$\int_g^{g_a(m_Z)} \frac{\mathrm{d}g_a}{g_a^3} = \int_0^{t_{\mathrm{IR}}} \frac{b_a \mathrm{d}t}{16\pi^2} \ , \tag{4.7}$$

where g is the (normalised) unified coupling at the GUT scale and $t_{\mathrm{IR}} = m_Z/M_{\mathrm{GUT}}$. The two-loop MSSM and scheme conversion corrections, $\Delta b_a^{(0)}$, are small and therefore induce small finite corrections $\Delta_a^{(0)}$ to the final value of the gauge couplings at the UV scale. The corrections $\Delta_a^{(0)}$ are independent of γ_R to leading order, and thus can be well-approximated by constant numerical shifts derived from numerical solution of the usual two-loop MSSM

RGEs. As the behaviour of b_a is different in the region of strong coupling, the integration should be partitioned thus

$$\int_0^{t_{\rm IR}} \frac{b_a {\rm d}t}{16\pi^2} = \int_0^{\ln\left(\frac{\mu_+}{M_{\rm GUT}}\right)} \frac{b_a^{(0)}{\rm d}t}{16\pi^2} + \int_{\ln\left(\frac{\mu_+}{M_{\rm GUT}}\right)}^{\ln\left(\frac{\mu_-}{M_{\rm GUT}}\right)} \frac{(b_a^{(0)}+\Delta b_a){\rm d}t}{16\pi^2} + \int_{\ln\left(\frac{\mu_-}{M_{\rm GUT}}\right)}^{\ln\left(\frac{m_Z}{M_{\rm GUT}}\right)} \frac{b_a^{(0)}{\rm d}t}{16\pi^2} + \frac{1}{2}\Delta_a^{(0)} \ . \tag{4.8}$$

To parameterise the effects of the strong coupling, we approximate γ_R by a constant over the entire region $\mu_- < \mu < \mu_+$ and their usual perturbative value everywhere else. This, of course, is not meant to be a realistic description of the behaviour of γ_R in the strong coupling regime. Nevertheless, in a self consistent perturbative expansion in the SM gauge couplings, the leading effect of the large anomalous dimensions is expressible purely as an integral of $\sum_R T_a(R)\gamma_R$ over the strong coupling regime, the sign and size of which we can parameterise in terms of a constant over $\mu_- < \mu < \mu_+$. Specifically, from Eq. (4.7) we then obtain

$$\frac{16\pi^2}{g_a^2(m_Z)} = \frac{16\pi^2}{g^2} + \left[L_a + \Delta_a^{\rm SC} + \Delta_a^{(0)}\right] \ , \tag{4.9}$$

$$L_a = b_a^{(0)} \ln\left(\frac{M_{\rm GUT}^2}{m_Z^2}\right) \ , \tag{4.10}$$

and we have used Eq. (4.5) in defining

$$\Delta_a^{\rm SC} \equiv -\sum_R T_a(R)\gamma_R \ln\left(\frac{\mu_+^2}{\mu_-^2}\right) \ . \tag{4.11}$$

Only the Higgs sector is directly sensitive to the coupling λ, thus we expect only $\Delta_{1,2}^{(\rm SC)} \neq 0$ and $\Delta_3^{(\rm SC)} = 0$, up to small corrections. The sign of the corrections $\Delta_{1,2}^{\rm SC}$ is important to us. In the perturbative λ regime the Higgs anomalous dimensions are given at one-loop by

$$\begin{aligned} \gamma(H_u) &= \frac{1}{32\pi^2}\left(2\lambda^2 + 6h_t^2 - g_1^2 - 3g_2^2\right) \ , \\ \gamma(H_d) &= \frac{1}{32\pi^2}\left(2\lambda^2 + 6h_b^2 + 2h_\tau^2 - g_1^2 - 3g_2^2\right) \ , \end{aligned} \tag{4.12}$$

where h_i, for $i = t, b, \tau$, are the SM Yukawa couplings. Then from definition Eq. (4.11) and since $T_{1,2}(H_u, H_d) > 0$, both $\Delta_1^{\rm SC}(H_u, H_d) \leq 0$ and $\Delta_2^{\rm SC}(H_u, H_d) \leq 0$. Outside of the perturbative regime we cannot make a rigorous statement as the usual unitarity constraint on the wavefunction renormalisation coefficient, $0 \leq Z \leq 1$, implies only that (the λ-dependent pieces of) $\gamma(H_u, H_d) \geq 0$ in perturbation theory. Nevertheless, a possibility, in the cases of most interest to us, where the theory doesn't UV complete to a quasi-superconformal model, is that $\Delta_1^{\rm SC} = \Delta_2^{\rm SC} \leq 0$ remains true. If the theory remains strongly coupled for roughly an order of magnitude, the typical size of the deviation due to strong coupling is $\Delta_a^{\rm SC} \sim -5$

from Eq. (4.11), which is parametrically smaller than the standard size RGE-resummed loop corrections $L_2 \approx 66$ and $L_1 \approx 198$. This allows us to perform expansions in the small quantities $\Delta_a^{(\mathrm{SC})}/L_a$ to solve for the modified gauge coupling RG trajectories.

The Higgs anomalous dimensions γ_{H_u} and γ_{H_d} feed directly into the RGE evolution of the top and bottom Yukawas, respectively, which in the strongly coupled region, to leading order, evolve according to

$$\frac{\mathrm{d}h_t}{\mathrm{d}t} \simeq \gamma_{H_u} h_t \ , \quad \text{and} \quad \frac{\mathrm{d}h_b}{\mathrm{d}t} \simeq \gamma_{H_d} h_b \ . \tag{4.13}$$

So far our results have only depended upon the sum of the Higgs anomalous dimensions $(\gamma_{H_u} + \gamma_{H_d})$, since $T_a(H_u) = T_a(H_d)$. Whilst an extrapolation of Eq. (4.12), which gives the perturbative forms of γ_{H_u} and γ_{H_d}, would suggest that $\gamma_{H_u} \simeq \gamma_{H_d}$ for large λ, in the non-perturbative regime these expressions are no longer reliable and this need not necessarily be the case. From a top-down perspective it is natural that no two operators of the strongly interacting theory not appearing in a single irreducible multiplet of the symmetry group of the UV theory should have the same operator dimension, thus implying that $\gamma_{H_u} \neq \gamma_{H_d}$ in general. In fact any dynamical explanation of the MSSM flavour structure must violate a naïve extrapolation of the perturbative expression so that the anomalous dimension of the bottom quark mass term (and first two generation fermion mass terms) is large while that of the top remains small, for example as discussed in [262].

If h_t is not to become non-perturbatively large itself (likely implying that $\overline{u}_3$ and/or Q_3 are also composite states), we require that $\gamma_{H_u} < \gamma_{H_d}$, with γ_{H_u} bounded above by

$$\gamma_{H_u} \lesssim \frac{0.5}{\ln\left(\frac{\mu_+}{\mu_-}\right) / \ln(10)} \ . \tag{4.14}$$

The difference $(\gamma_{H_u} - \gamma_{H_d})$ allows an interesting possibility, providing an explanation for the hierarchy between up-like and down-like quark masses which does not rely on large $\tan\beta$, as is usually assumed, but instead is due to the greater running of h_b compared to h_t, starting from a common value $h_b \simeq h_t \simeq \mathcal{O}(1)$ at the GUT scale. Specifically, if

$$(\gamma_{H_u} - \gamma_{H_d})\ln\left(\frac{\mu_-}{\mu_+}\right) \sim 4 \ , \tag{4.15}$$

then the observed small ratio m_b/m_t is obtained without resort to $\tan\beta \gg 1$. In fact if the Higgs contribution due to $\lambda S H_u H_d$ is to raise the Higgs mass to 125 GeV, then $\tan\beta \lesssim 10$ is required, as illustrated in Fig. 4.1, so an independent explanation of the top to bottom mass hierarchy is necessary.

Alternatively, if $\gamma_{H_u} \gtrsim 0.5$, then the top will also generally develop a sizeable anomalous dimension shortly after the period of strong coupling begins. This provides an additional contribution to $\Delta_a^{\rm SC}$:

$$\Delta_a^{\rm SC} \equiv -\left(\sum_{R=H_u,H_d} T_a(R)\gamma_R\right)\ln\left(\frac{\mu_+^2}{\mu_-^2}\right) - \theta(\mu_+ - \mu_t)\left(\sum_{R=t,Q} T_a(R)\gamma_R\right)\ln\left(\frac{\mu_+^2}{\mu_t^2}\right) \ , \quad (4.16)$$

where μ_t is the scale at which the top Yukawa becomes non-perturbative. Note that in the case that $\gamma_{H_u} \simeq \gamma_{H_d}$ we expect that the top Yukawa runs non-perturbative shortly after λ, and therefore $\mu_t \simeq \mu_-$. Importantly, since $T_a(t,Q) > 0$, and $\gamma_{\overline{u}_3}$ and γ_{Q_3} inherit the same sign as γ_{H_u} (at least if the leading perturbative results for the sign of $\gamma_{\overline{u}_3}$ and γ_{Q_3} hold), these corrections have the same sign as those due to $\gamma_{H_{u,d}}$, and as we shall see shortly, this only results in a slight deflection in the RGE trajectories of the gauge couplings.

4.3 Effects of Strong Coupling on SM Gauge Couplings at m_Z

Taking the measured low-energy gauge parameters $\alpha_{\rm em}|_{\overline{\rm MS}}$, m_Z and $\sin^2\theta_w|_{\overline{\rm MS}}$ as inputs allows a prediction for $\alpha_S(m_Z)|_{\overline{\rm MS}}$. From Eq. (4.9) these quantities can be expressed as

$$\sin^2\theta_W = \frac{3}{8}\left[1 - \left(b_1^{(0)} - \frac{5}{3}b_2^{(0)}\right)\frac{\alpha_{\rm em}}{2\pi}\ln\left(\frac{M_{\rm GUT}}{m_Z}\right)\right] + \Delta^{s_w} \ , \quad (4.17)$$

$$\alpha_s^{-1}(m_Z) = \frac{3}{8\alpha_{\rm em}}\left[1 - \left(b_1^{(0)} + b_2^{(0)} - \frac{8}{3}b_3^{(0)}\right)\frac{\alpha_{\rm em}}{2\pi}\ln\left(\frac{M_{\rm GUT}}{m_Z}\right)\right] + \Delta^{\alpha_s} \ , \quad (4.18)$$

where Δ^{s_w} and Δ^{α_s} are corrections to the one-loop form due to two-loop SM corrections, Yukawa interactions, scheme dependent effects and, now, also the effects of running through a regime of strong coupling. To study the effect of the period of strong coupling on the SM gauge couplings we write $\Delta_a = \Delta_a^{(0)} + \Delta_a^{\rm SC}$ where $\Delta_a^{(0)}$ are the standard MSSM values which are known (see e.g. [263, 264]) to be $(\Delta_1^{(0)}, \Delta_2^{(0)}, \Delta_3^{(0)}) \simeq (11.6, 13.0, 7.0)$ and $\Delta_a^{\rm SC}$ is the additional correction due to running through a period of strong coupling. The form of the corrections is given by

$$\begin{aligned}\Delta^{s_w} &= -\frac{\alpha_{\rm em}}{4\pi}\left(\frac{1}{1+\frac{5}{3}}\right)\left[\Delta_1 - \frac{5}{3}\Delta_2\right], \\ \Delta^{\alpha_s} &= -\frac{1}{4\pi}\left(\frac{1}{1+\frac{5}{3}}\right)\left[\Delta_1 + \Delta_2 - \left(1+\frac{5}{3}\right)\Delta_3\right].\end{aligned} \quad (4.19)$$

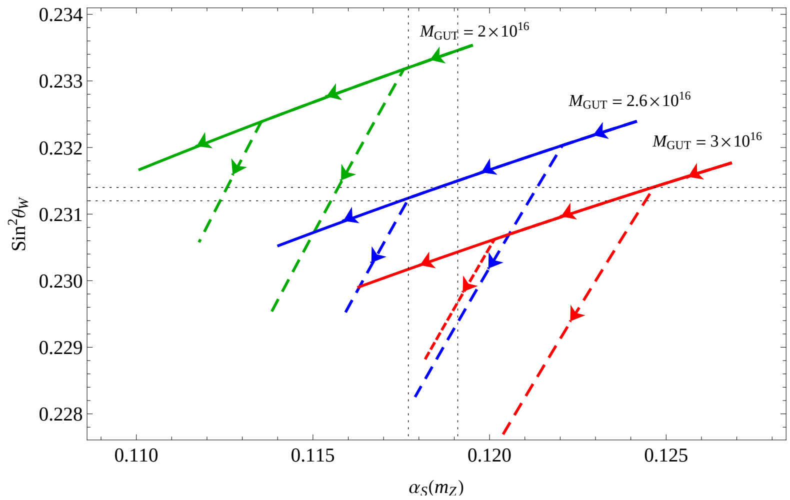

Figure 4.3: The plot shows the effect of $\Delta_a^{\rm SC}$ on the predicted values of $\alpha_s(m_Z)$ and $\sin^2\theta_W$ for a range of unification scales $M_{\rm GUT}$. The start point of each curve indicates the MSSM value (i.e. $\Delta_a^{\rm SC}=0$) and the arrows indicate the trajectories for increasing values of the quantity $t \equiv (\gamma_{H_u}+\gamma_{H_d})\ln(\mu_+/\mu_-)$, showing $0\le t\le 6$. The default preferred scenario is shown by the solid lines which assume negligible anomalous dimensions for the top states, a self-consistent assumption if γ_{H_u} is not too large ($\lesssim 1$). In the case of large γ_{H_u} the top Yukawa coupling runs to non-perturbative values leading to large anomalous dimensions for the 3rd-generation states Q_3 and $\overline{u}_3$. Assuming $\gamma_{\overline{u}_3}\simeq\gamma_{Q_3}\neq 0$, the trajectory will be deflected depending on the scale at which h_t becomes non-perturbative, as shown schematically by the heavy dashed lines as this scale is varied over the allowed range. The black dotted lines show the preferred region as indicated by current experimental measurements [1] (including errors): $\sin^2\theta_w|_{\overline{\rm MS}}=0.2313\pm0.001$ and $\alpha_s(m_Z)=0.1184\pm0.007$.

Expanding the Δ_a and using the numerical values for the MSSM corrections in order to assess the impact of the corrections due to strong coupling gives

$$\begin{aligned}\Delta^{s_w} &\simeq \frac{\alpha_{\rm em}}{32\pi}\left[5\Delta_2^{\rm SC}-3\Delta_1^{\rm SC}+30.2\right],\\ \Delta^{\alpha_s} &\simeq \frac{1}{32\pi}\left[8\Delta_3^{\rm SC}-3\Delta_1^{\rm SC}-3\Delta_2^{\rm SC}-17.8\right].\end{aligned}\tag{4.20}$$

The low-energy gauge parameters are well measured and there is a reasonable level of agreement with the predictions of gauge coupling unification assuming the MSSM spectrum. However, as stated previously there is a $\sim 3\%$ deviation between the predictions for $\alpha_s(m_Z)$ from MSSM unification and the measured values [1] of $\sin^2\theta_w|_{\overline{\rm MS}} = 0.2313\pm0.001$ and $\alpha_s(m_Z)=0.1184\pm0.007$. In Fig. 4.3 we plot the low-energy observables as a function of $M_{\rm GUT}$ and the quantity $(\gamma_{H_u}+\gamma_{H_d})\ln(\mu_+/\mu_-)$. The new corrections entering due to the region of strong coupling have the right sign if, as expected, $\Delta_a^{\rm SC}\le 0$, and possibly even the correct magnitude, to correct for the discrepancy in MSSM unification.

First we shall consider the scenario in which the anomalous dimension of the top is negligible, as is the case if the anomalous dimension for γ_{H_u} is small and it is primarily γ_{H_d} which is responsible for deviations in the evolution of the gauge couplings. In this situation it is straightforward to determine the parameter values of a strong coupling regime that gives precision unification; using Eq. (4.19) and Eq. (4.17) yields

$$\sin^2\theta_W \simeq \frac{3}{8} + \frac{\alpha_{\rm em}}{16\pi}\left[\frac{1}{2}\left[5\Delta_2^{\rm SC} - 3\Delta_1^{\rm SC} + 30.2\right] - \left(3b_1^{(0)} - 5b_2^{(0)}\right)\ln\left(\frac{M_{\rm GUT}}{m_Z}\right)\right] \ . \tag{4.21}$$

Recall, in the (N)MSSM the one-loop β-function coefficients are $b_1^{(0)} = 11$, $b_2^{(0)} = 1$ and $b_3^{(0)} = -3$ and that $\alpha_{\rm em} = 1/127.9$. Substituting $\sin^2\theta_W \simeq 0.2313$ leads to

$$\ln\left(\frac{M_{\rm GUT}}{m_Z}\right) \simeq 33.53 + \frac{5}{56}\Delta_2^{\rm SC} - \frac{3}{56}\Delta_1^{\rm SC} \ . \tag{4.22}$$

Similarly, from Eq. (4.20) and Eq. (4.18) we obtain

$$\alpha_s^{-1}(m_Z) \simeq \frac{3}{8\alpha_{\rm em}} + \frac{1}{16\pi}\left[\frac{1}{2}\left[8\Delta_3^{\rm SC} - 3\Delta_1^{\rm SC} - 3\Delta_2^{\rm SC} - 17.8\right] - \left(3b_1^{(0)} + 3b_2^{(0)} - 8b_3^{(0)}\right)\ln\left(\frac{M_{\rm GUT}}{m_Z}\right)\right],$$

and by comparison with Eq. (4.22) we have

$$\alpha_s(m_Z) \approx 0.129 + 5.3\times 10^{-3}\times\left[\frac{3}{7}\Delta_2^{\rm SC} - \frac{3}{28}\Delta_1^{\rm SC} - \frac{1}{4}\Delta_3^{\rm SC}\right] \ . \tag{4.23}$$

Thus in order to obtain the observed value $\alpha_s(m_Z) \approx 0.118$ it is required that

$$\Delta_3^{\rm SC} \approx 8.3 - 0.43\Delta_1^{\rm SC} + 1.71\Delta_2^{\rm SC} \ . \tag{4.24}$$

In the case that only the Higgs acquire large anomalous dimensions, we have $\Delta_1^{\rm SC} = \Delta_2^{\rm SC}$ and $\Delta_3^{\rm SC} = 0$ and hence the GUT scale can be expressed as a function of a single argument

$$M_{\rm GUT} \sim m_Z \exp\left(33.5 - \frac{\Delta_1^{\rm SC}}{28}\right) \ . \tag{4.25}$$

The observed value of $\alpha_s(m_Z) \approx 0.118$, given in Eq. (4.24), is obtained for $\Delta_1^{\rm SC} = \Delta_2^{\rm SC} = -6.5$, which corresponds to a unification scale of $M_{\rm GUT} \approx 2.6\times 10^{16}$ GeV. Note that the unification scale is slightly raised compared to the standard MSSM prediction, slightly lengthening the predicted proton lifetime arising from dimension six X and Y gauge boson exchange (see e.g. [265–267]), as $\tau_p \propto M_X^4/\alpha_{\rm GUT}^2$ (in addition, $1/\alpha_{\rm GUT}$ increases slightly in our scenario from ~ 23.6 to ~ 23.8 for $M_{\rm GUT} = 2.6\times 10^{16}$, further increasing the proton lifetime, though this is a subdominant effect). Furthermore, since $T(H_{u,d})|_{\rm U(1)} = 1$ we may write

$$\Delta_1^{\rm SC} \sim -2(\gamma_{H_u} + \gamma_{H_d})\ln\left(\frac{\mu_+}{\mu_-}\right) \ . \tag{4.26}$$

For example, in the case that $\mu_+/\mu_- \simeq 10$ to obtain $\Delta_1^{\rm SC} = -6.5$ we require an anomalous dimension of $(\gamma_{H_u} + \gamma_{H_d}) \sim 1.4$, in accord with our expectation for the effective magnitude of the anomalous dimensions during a regime of strong coupling. If $\mu_+/\mu_- \sim 2$ then the required anomalous dimension increases to $(\gamma_{H_u} + \gamma_{H_d}) \sim 4.6$, still within reasonable values.

It is likely, however, that if $\gamma_{H_u} \log\left(\frac{\mu_+}{\mu_-}\right) \gtrsim 0.5$ then non-perturbative effects due to the top also affect the evolution of the gauge couplings. The case where these effects turn on quickly is shown as dashed curves in Fig. 4.3. However, for an appropriate choice of $M_{\rm GUT}$ it is clear that precision unification can be achieved regardless of how quickly the non-perturbative effects due to the top enter, provided the period of strong coupling is not too long. Of course it would be false to claim that a period of strong coupling fixes the discrepancy between the MSSM two-loop prediction of $\alpha_s(m_Z) \sim 0.129$ and the measured value, rather, our point is that an epoch of strong coupling (with the theory UV completing in such a way that $\frac{b_3-b_2}{b_2-b_1}$ remains unchanged) is not disastrous for precision unification and may even be advantageous.

Another interesting scenario which realises precision unification via running through strong coupling is the case where the strong coupling region immediately precedes the GUT scale and μ_+ is identified with this unification scale. In this scenario one need not be concerned if the top Yukawa runs non-perturbative. Such strong coupling unification has been previously argued to have advantages for stabilising the string dilaton and may also have interesting consequences for the SUSY spectrum [268]. Note that, in Section 4.1 we identified the parameter regions in which this situation is realised, for example, from inspection of the right panel of Fig. 4.2 we observe that for 500 GeV stops and $\tan\beta \simeq 3$, then the strong coupling window starts at $\mu_- \sim 10^{15}$ GeV, only an order of magnitude below the GUT scale.

Finally, since the motivation for λSUSY is predicated on the 125 GeV Higgs signal, it is worth investigating if other aspects of Higgs phenomenology, particularly the production cross-section and branching ratios, favour the λSUSY scenario. Currently, the branching ratios seem roughly SM-like, however there is still significant room for deviations to be observed in the future [269–272]. As λSUSY is a leading mechanism for raising the Higgs mass in models with light stops, and strong coupling need not adversely affect precision gauge coupling unification, new anomalies arising in the data certainly warrant dedicated studies in the context of λSUSY.

Chapter 5: Fine Tuning in Models of Natural SUSY

This chapter is based on [273].

While natural SUSY spectra provide hope for an EW sector without significant fine tuning [162, 166, 218–220, 222, 274–278], as was quickly realised after their initial proposal (and seen in chapter 3) it is difficult to preserve such a spectrum during the RG flow to the EW scale [177, 191]. On one hand, the heavy first two generation sfermions tend to drive the stops tachyonic, while on the other, a gluino above the current experimental limit will tend to pull the stops to high masses.

Quantifying the fine tuning of a model is a useful tool to study the viability of particular low-energy spectra [279]. This has been applied in a large number of studies of supersymmetric models, for example to strongly constrain spectra with universal sfermion masses [280–284], and has also been studied in the context of natural spectra [149, 151, 285–287]. In this section, we study the tuning associated with natural SUSY spectra in detail. First, expressions for the fine tuning required to obtain stops significantly lighter than gluinos and the first two generation sfermions are derived. We then extend previous approximate results for the fine tuning of the EW scale introduced due to heavy gluinos and sfermions.

It is found that if there is a Majorana gluino with soft mass above 1.5 TeV there is no fine tuning benefit to decreasing the stop masses below 1.5 TeV, if mediation is from close to the GUT scale, due to the tuning from the gluino dominating. However, while there is no benefit to reducing the stop mass, provided the stop is not too light ($\gtrsim 500\,\mathrm{GeV}$) doing so does not make the tuning of the theory worse and is not actively disfavoured. Similarly, for low-scale mediation (from 10^6 GeV), and a Majorana gluino mass of 1.5 TeV, the stop can typically be as heavy as 1 TeV. Consequently, applying current experimental constraints, barring surprising cancellations , there are strong lower bounds on the fine tuning of natural SUSY theories, even though there are regions of parameter space where the LHC has not excluded light stops. As a result, in the regions of lowest fine tuning in these theories, a physical Higgs mass of 125 GeV can arise directly from stop-loop corrections if the theory has large A-terms, or from an NMSSM structure without couplings running non-perturbative (as studied in Section 4).

As discussed in Section 2.10, a theory's fine tuning is measured with respect to the parameters at an assumed UV boundary of the RG flow.[1] In contrast, the weak scale parameters have values which are strongly coupled together by the RG equations, and typically cannot accurately quantify the tuning [150, 288]. Of course, choosing the independent variables at the UV boundary requires some assumptions about the mediation of SUSY-breaking, and possible correlations between soft terms at this scale. We further assume there is no new physics between the UV boundary and the weak scale that modifies the running (in Section 6 we study the possibility that interactions with the SUSY breaking sector can violate this assumption).

The independent variables are typically taken to be the gluino mass squared (the other gauginos are less important and we do not assume a GUT structure), the stop mass squared and the mass squared of the first two generation sfermions, which are assumed to be universal based on strong flavour constraints [289].[2] This choice is reasonable; a natural Spectrum is often obtained by including several sources of SUSY breaking, and hence these masses may be adjusted independently [167, 168, 295–297]. Also, in both gravity mediation [298] and the most general models of gauge mediation [136], the gauge fermion and sfermion masses generated are independent.

Alternatively, both the gluino and stop masses at the UV renormalisation boundary may both be generated through a single F-term, for example as in the model described in Section 3.[3] In this case, varying the gluino mass will be correlated to varying the UV stop mass, and so the F-term is the fundamental parameter. As we discuss later, this scenario makes the tuning of natural SUSY spectra worse since increasing the F-term increases the weak scale stop mass both directly though the UV stop mass, and through the increased running from a more massive gluino. Another issue is whether the left- and right-handed stop masses should be regarded as a single parameter, as occurs if both gain their soft masses through the same mediation mechanism. This is the case in many models of natural SUSY, but is not required in generic mediation models. We give results for both the case where these are independent, and when they are not.

There are possible ways our arguments may be evaded. It might be that the mediation mechanism gives a pattern of soft masses that happens to lead to cancellations in the RG flow, so that the shift in the Higgs soft mass is smaller than expected (this is the case

[1]However the location of this boundary, and the set of independent parameters there, is only physically meaningful once a complete UV theory, including all higher-dimensional operators, is specified.

[2]Though this assumption can be relaxed [290–294].

[3]In the model of Section 3, there is also a potential tuning from the parameter ϵ, which we do not consider.

for focus point spectra [299, 300] and as discussed around Eq. (2.65)). However, such a mechanism would need to couple the stop, gluino, and first two generation sfermions in a highly non-trivial way despite their soft masses coming from very different sources (typically R-symmetry preserving SUSY breaking, R-symmetry breaking SUSY breaking and another mediation mechanism). Therefore, this does not seem a strong assumption.[4] We also assume the Higgs potential is either that of the MSSM or the NMSSM with the parameter λ_0 not very large at the weak scale. As mentioned in Section 4, if $\lambda_0 \gtrsim 1$ the Higgs potential can be significantly modified, and the sensitivity to the Higgs soft masses (and contributions to these) could be reduced.

As discussed in Section 2.10, we make no attempt to quantify the probability, over the 'theory space' of SUSY breaking and mediation mechanisms, that the initial UV parameters begin in the correct region to allow for a natural SUSY spectrum at the weak scale. Such a starting point requires multiple forms of mediation which, *a priori*, could lead to a separation between the gluino and sfermion masses that is far too large to give a viable natural SUSY spectrum at the weak scale. Consequently, it is unclear how likely it is that a natural SUSY spectra is actually realised (although models such as that studied in Section 3 have other benefits, so may perhaps be relatively more common). However, it is unknown if the concept of a 'theory space', let alone a measure on it, is well defined so we do not consider this issue any further.

While the main focus of our work is on conventional Majorana gauginos, we also study the EW fine tuning in a simple model of Dirac gauginos (described in Section 2.4). It is found that in this model the tuning is independent of the mediation scale and comparable to an MSSM theory with very low mediation scale. Consequently, this is a good option for reducing fine tuning in models where the mediation scale is required to be high, for example in string theory completions.

In Section 5.1 we discus the fine tuning of the UV parameters required to obtain a light stop after running. Section 5.2 contains the main results on the tuning of the EW VEV, while Section 5.3 contains our discussion of Dirac gauginos.

[4] In contrast focus point scenarios typically only involve one, simple, form of mediation to all MSSM fields, hence can occur as a result of single numerical coincidence.

5.1 Fine Tuning to Obtain a Light Stop

First, we consider the fine tuning of the gluino and first two generation soft masses required to obtain a light stop at the weak scale. In analogy to the EW tuning, this is defined as

$$Y_p = \left| \frac{\partial \log m_{\tilde{t}}^2 \left(m_Z\right)}{\partial \log p} \right| \ , \tag{5.1}$$

where p is one of M_3^2, $\tilde{m}_{1,2}^2$ or $m_{\tilde{t}}^2$ evaluated at the UV boundary, and $\tilde{t}$ is the stop state which receives the greatest fine tuning. In this section we use the convention that soft terms without their scale specified are evaluated at the UV boundary of the RG flow of the theory, Λ_{UV}, which is parametrically the scale at which SUSY breaking is mediated.

The RG equations for the stops in the presence of heavy sfermions are well known [56,191]. Since we are interested in the effect of the gluino and sfermion masses and these dominate the RG equations, it is sufficient to include only these leading terms. The RG equation of the stop soft mass is then given by

$$\frac{\mathrm{d}}{\mathrm{d}t} m_{\tilde{t}}^2 = -\frac{8}{4\pi} \sum_i \alpha_i \left(t\right) C_i M_i^2 + \frac{2}{\pi^2} \left(\sum_i \alpha_i^2 \left(t\right) C_i \right) \tilde{m}_{1,2}^2 \ , \tag{5.2}$$

where C_i is the Casimir of the stop state with respect to the gauge group labelled by i (and α_1 is GUT normalised). We further assume the right-handed bottom sfermion and the staus remain relatively light such that they do not have a significant effect on the running of the stops, but not so light as to be driven tachyonic (giving these states relatively large masses does not change the results dramatically). We take the heavy first two generations to have a constant mass which is a reasonable approximation if they begin fairly heavy as in natural spectra.[5] Following [191], at this level of approximation the RG flow can be solved exactly to give

$$\begin{aligned} m_{\tilde{t}}^2 \left(m_Z\right) = & m_{\tilde{t}}^2 \left(\Lambda_{\mathrm{UV}}\right) - \sum_i \frac{2}{b_i} C_i \left(\frac{1}{\left(1 + \frac{b_i}{2\pi} \log\left(\frac{\Lambda_{\mathrm{UV}}}{M_i(m_Z)}\right) \alpha_i \right)^2} - 1 \right) M_i^2 \\ & + \sum_i \frac{4}{\pi b_i} \alpha_i \left(\Lambda_{\mathrm{UV}}\right) \left(\frac{1}{1 + \frac{b_i}{2\pi} \log\left(\frac{\Lambda_{\mathrm{UV}}}{\tilde{m}_{1,2}(m_Z)}\right) \alpha_i} - 1 \right) C_i \tilde{m}_{1,2}^2 \ , \end{aligned} \tag{5.3}$$

where the gauge beta-function coefficients are defined as $\frac{d}{dt}\left(\frac{1}{\alpha_i}\right) = -\frac{b_i}{2\pi}$ (and Eq. (5.3) is

[5] We are interested in spectra where the stops are fairly light at the UV scale and remain relatively light during running. Hence, the overall shift in their mass during running is $\lesssim 500\,\mathrm{GeV}$. The first two generation's dominant running is the same as the stops hence these run by a similar amount, which is negligible if they start at $\mathcal{O}\left(10\,\mathrm{TeV}\right)$.

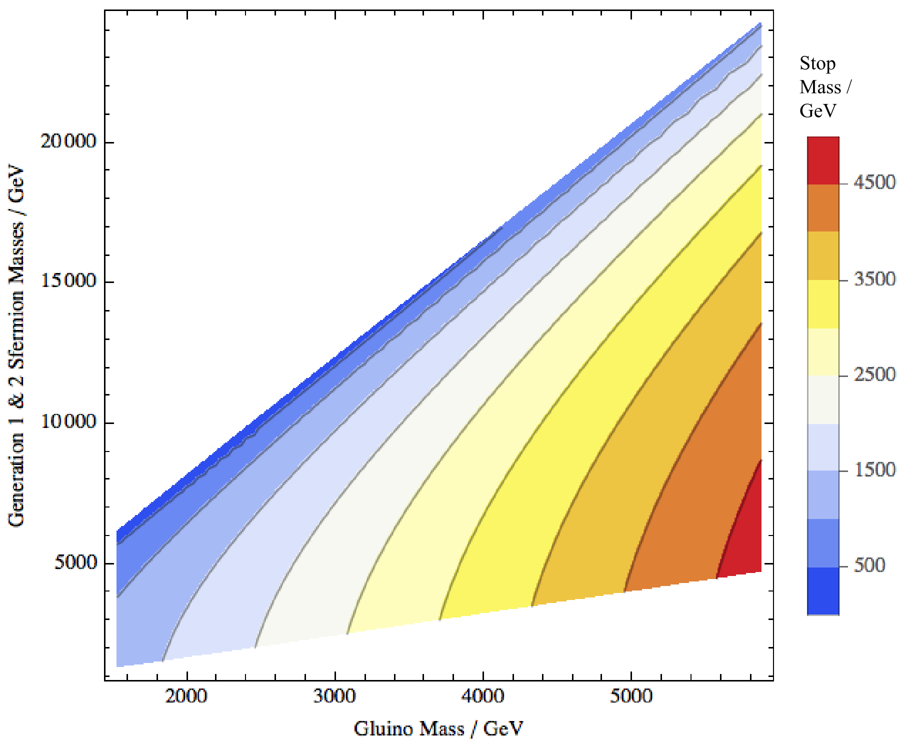

Figure 5.1: The stop mass obtained at the weak scale as a function of the weak scale gluino and first two generation sfermion masses, after running from the GUT scale at 10^{16} GeV assuming an initial mass of 200 GeV. The lower cutoff is due to the gluino increasing the the first two generation sfermions masses during running, while the upper cutoff is due to the stop running tachyonic above this line.

written in terms of the UV values of the gauge couplings).[6] The contribution from the first two generation sfermion turns off at an energy scale $\tilde{m}_{1,2}$ and the gaugino contribution is present until the scale M_i.

In Fig. 5.1 we plot the weak scale lightest stop mass as a function of the weak scale gluino and first two generation sfermion masses, after numerically running from 10^{16} GeV with a UV stop mass of 200 GeV (using the full two loop RG equations). For a given gluino mass, above a certain sfermion mass the stops run tachyonic and the theory is not viable. To obtain the light stops needed for a natural SUSY spectrum requires M_3 and $\tilde{m}_{1,2}$ to be such that the stop is in the thin strip close to this boundary. The relatively small effect of the gluino increasing the mass of the first two generation sfermions during running leads to the lower cutoff in this plot.

Now it is straightforward to write down the fine tuning with respect to the UV gaugino and first two generation masses. There will be two contributions to the fine tuning, one

[6] Throughout this section we calculate weak scale parameters by solving one and two loop RG equations, this is equivalent to an all order summation of the leading logarithms [301].

directly from the dependence on $\tilde{m}_{1,2}^2$, and the other from dependence inside the logarithm,

$$
\begin{aligned}
Y_{\tilde{m}_{1,2}^2} &= \frac{\tilde{m}_{1,2}^2}{m_{\tilde{t}}^2}\frac{\partial m_{\tilde{t}}^2}{\partial \tilde{m}_{1,2}^2} \\
&= \frac{\tilde{m}_{1,2}^2}{m_{\tilde{t}}^2}\sum_i \frac{4C_i}{\pi b_i}\alpha_i\left(\Lambda_{\rm UV}\right)\left(\frac{1}{1+\frac{b_i}{2\pi}\log\left(\frac{\Lambda_{\rm UV}}{\tilde{m}_{1,2}}\right)\alpha_i}-1\right) \\
&\qquad\qquad\qquad + \frac{\tilde{m}_{1,2}^2}{m_{\tilde{t}}^2}\sum_i \frac{C_i\alpha_i^2\left(\Lambda_{\rm UV}\right)}{\pi^2}\left(\frac{1}{1+\frac{b_i}{2\pi}\log\left(\frac{\Lambda_{\rm UV}}{\tilde{m}_{1,2}}\right)\alpha_i}\right)^2 .
\end{aligned}
\tag{5.4}
$$

The second term from the variation of the logarithm is typically significantly smaller than the first and slightly reduces the fine tuning. It appears because if the mass of the first two generation sfermions increases then there will be slightly less running. Actually, to the accuracy required we do not need to include this effect (but we retain the full dependence for completeness). Similarly,

$$
\begin{aligned}
Y_{M_i^2(m_Z)} &= -\frac{M_i^2}{m_{\tilde{t}}^2}\frac{2}{b_i}C_i\left(\frac{1}{\left(1+\frac{b_i}{2\pi}\log\left(\frac{\Lambda_{\rm UV}}{M_i(m_Z)}\right)\alpha_i\right)^2}-1\right) \\
&\qquad\qquad - \frac{M_i^2}{m_{\tilde{t}}^2}\frac{C_i}{\pi}\alpha_i\frac{1}{\left(1+\frac{b_i}{2\pi}\log\left(\frac{\Lambda_{\rm UV}}{M_i(m_Z)}\right)\alpha_i\right)^3} .
\end{aligned}
\tag{5.5}
$$

The greatest fine tuning from the heavy sfermions will occur on the left-handed stop. Even though the beta function coefficients b_2 and b_3 have opposite signs, their overall contributions to Eq. (5.4) go in the same direction. The gluino couples equally to the left- and right-handed stops, so the tuning with respect to its mass is equal for both.

Finally, there is also a tuning with respect to the initial stop masses. This can be evaluated as a perturbation to the RG trajectory obtained already. If the stop soft masses are independent, a perturbation to the initial left handed soft mass, $\Delta m_{\tilde{Q}3}^2$, will satisfy

$$
\frac{\mathrm{d}}{\mathrm{d}t}\left(\Delta m_{\tilde{Q}3}^2\right) \supset \frac{2y_t^2}{16\pi^2}\Delta m_{\tilde{Q}3}^2 , \tag{5.6}
$$

and will also feed into the right-handed stop and up-type Higgs mass since the RG includes

$$
\frac{\mathrm{d}}{\mathrm{d}t}\left(\Delta m_{\tilde{u}3}^2\right) \supset \frac{4y_t^2}{16\pi^2}\Delta m_{\tilde{Q}3}^2, \tag{5.7}
$$

$$
\frac{\mathrm{d}}{\mathrm{d}t}\left(\Delta m_{Hu}^2\right) \supset \frac{6y_t^2}{16\pi^2}\Delta m_{\tilde{Q}3}^2 . \tag{5.8}
$$

At this level of approximation, the beta functions are linear in $m_{\tilde{t}}^2$, so the evolution of the perturbation during running may be obtained by integrating the system of RG equations Eq.

(5.6), Eq. (5.7), and Eq. (5.8) giving

$$\Delta m^2_{\tilde{Q}3}(m_Z) = \frac{1}{6}\left(5 + \left(\frac{m_{\tilde{Q}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)}\right)\Delta m^2_{\tilde{Q}3}(\Lambda_{\text{UV}}), \tag{5.9}$$

$$\Delta m^2_{\tilde{u}3}(m_Z) = \frac{1}{3}\left(-1 + \left(\frac{m_{\tilde{Q}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)}\right)\Delta m^2_{\tilde{Q}3}(\Lambda_{\text{UV}}) \ . \tag{5.10}$$

Similarly, a perturbation to the right-handed stop leads to

$$\Delta m^2_{\tilde{u}3}(m_Z) = \frac{1}{3}\left(2 + \left(\frac{m_{\tilde{u}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)}\right)\Delta m^2_{\tilde{u}3}(\Lambda_{\text{UV}}), \tag{5.11}$$

$$\Delta m^2_{\tilde{Q}3}(m_Z) = \frac{1}{6}\left(-1 + \left(\frac{m_{\tilde{u}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)}\right)\Delta m^2_{\tilde{u}3}(\Lambda_{\text{UV}}) \ . \tag{5.12}$$

The expressions Eq. (5.9) and Eq. (5.11) are numerically largest, therefore the fine tunings are approximately

$$Y_{m^2_{\tilde{Q}3}} = \frac{m^2_{\tilde{Q}3}(\Lambda_{\text{UV}})}{m^2_{\tilde{Q}3}(m_Z)}\left(\frac{5}{6} + \frac{1}{6}\left(\frac{m_{\tilde{Q}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)}\right), \tag{5.13}$$

$$Y_{m^2_{\tilde{u}3}} = \frac{m^2_{\tilde{u}3}(\Lambda_{\text{UV}})}{m^2_{\tilde{u}3}(m_Z)}\left(\frac{1}{3}\left(\frac{m_{\tilde{u}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} + \frac{2}{3}\right) \ . \tag{5.14}$$

If there is a small separation between the mediation scale and the weak scale $\left(\frac{m_{\tilde{Q}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} \sim 1$ and the fine tuning $Y_{m^2_{\tilde{Q}3}} \sim \frac{m^2_{\tilde{Q}3}(\Lambda_{\text{UV}})}{m^2_{\tilde{Q}3}(m_Z)}$ as is the leading-order expectation. However if there is a large separation between these scales then running proceeds for sufficiently long that the back-reaction from a perturbation suppresses itself, reducing the tuning. For a mediation scale of 10^{16} GeV,

$$\left(\frac{m_{\tilde{Q}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} \sim 0.1 \ , \tag{5.15}$$

so this can be a non-negligible effect. The tuning of the left-handed stop is greater since it is less strongly damped by the RG flow.

In the case where these two stop masses are linked, the RG equations for the perturbation are modified since the left-handed stop perturbation feeds into the right-handed stop perturbation and vice versa. These are easily integrated to obtain

$$\Delta m^2_{\tilde{Q}3}(m_Z) = \frac{1}{3}\left(\left(\frac{m_{\tilde{t}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} + 2\right)\Delta m^2_{\tilde{t}3}(\Lambda_{\text{UV}}) \ , \tag{5.16}$$

$$\Delta m^2_{\tilde{u}3}(m_Z) = \frac{1}{3}\left(2\left(\frac{m_{\tilde{t}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} + 1\right)\Delta m^2_{\tilde{t}3}(\Lambda_{\text{UV}}) \ . \tag{5.17}$$

Therefore,

$$Y_{m^2_{\tilde{t}3}} = \frac{m^2_{\tilde{t}3}(\Lambda_{\text{UV}})}{m^2_{\tilde{Q}3}(m_Z)}\left(\left(\frac{m_{\tilde{t}3}}{\Lambda_{\text{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} + 1\right) \ . \tag{5.18}$$

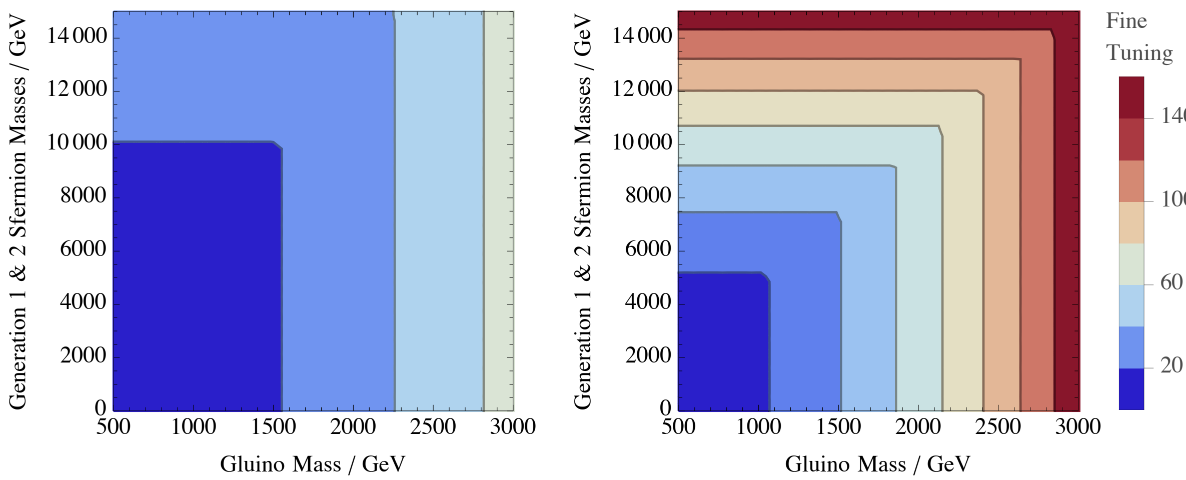

Figure 5.2: The fine tuning required to obtain a stop mass of 200 GeV at the weak scale **Left:**With a UV boundary of 10^{16} GeV. **Right:** With a UV boundary of 10^{6} GeV, as a function of the *weak scale* gluino mass and the UV value of the first two generation sfermion masses.

As before, if $\Lambda_{\rm UV}$ is not too large, the damping is not significant and these expressions reduce to the leading order expectation $Y_{m^2_{\tilde{t}3}} \sim \frac{m^2_{\tilde{t}3}(\Lambda_{\rm UV})}{m^2_{\tilde{Q}3}(m_Z)}$. However, if $\Lambda_{\rm UV}$ is close to the GUT scale the difference can be significant.

To gauge the severity of these fine tunings, recall the approximate expression for the tuning of the EW scale with respect to the stop soft mass Eq. (2.64). An mSUGRA spectrum with sfermions and gluinos at 2500 GeV would have an EW tuning with respect to the stop soft mass of $\Delta_{\tilde{Q}3} \sim 350$ for $\Lambda_{\rm UV} = 10^6$ GeV and $\Delta_{\tilde{Q}3} \sim 1500$ for $\Lambda_{\rm UV} = 10^{16}$ GeV. In contrast, a natural spectra with $m_{\tilde{t}} = 200$ GeV, $\tilde{m}_{1,2} \sim 10^4$ GeV and $M_3 \sim 2500$ GeV at the weak scale, and $\Lambda_{\rm UV} = 10^{16}$ GeV, has a tuning of the stop mass of

$$Y_{\tilde{m}^2_{1,2}} \sim 80 \ , \quad Y_{M^2_3} \sim 115 \ , \quad Y_{m^2_{\tilde{t}3}} \sim 15 \ , \quad Y_{m^2_{\tilde{Q}3}} \sim 20 \ . \tag{5.19}$$

Meanwhile a theory with low-scale mediation, with $\Lambda_{\rm UV} = 10^6$ GeV, has tuning

$$Y_{\tilde{m}^2_{1,2}} \sim 20 \ , \quad Y_{M^2_3} \sim 50 \ , \quad Y_{m^2_{\tilde{t}3}} \sim 20 \ , \quad Y_{m^2_{\tilde{Q}3}} \sim 25 \ . \tag{5.20}$$

These agree with the variations evaluated numerically using the code `SOFTSUSY` [211] to around 10% . The leading error is due to the back-reaction of a perturbation to $m_{\tilde{t}}$ on its own RG group equation, however this level of accuracy is not required for our purposes.

Defining the overall fine tuning as $Y = \max\left(\{Y_{M^2_3}, Y_{\tilde{m}^2_{1,2}}, Y_{m^2_{\tilde{t}3}}\}\right)$, in Fig. 5.2 we plot the fine tuning required to obtain a weak scale stop of mass 200 GeV in the plane $M_3\,(m_Z)$, $\tilde{m}_{1,2}$ for low and high scale mediation, assuming the two stop masses are not independent in the

UV. This shows that it is possible to obtain a fairly light stop in the presence of a gluino mass of around 2 TeV and first two generation sfermion masses at about 5 TeV with a tuning of order $5 - 100$ depending on the scale of mediation. While significant, this is is not large compared to that found in the EW sector of typical SUSY models. Therefore theories with a light stop are not automatically disfavoured by tuning arguments.

5.2 Electroweak Fine Tuning in Models of Natural SUSY

We now study whether a natural SUSY scenario, compatible with current limits, can lead to an EW sector with low fine tuning. For simplicity, we assume the MSSM RG flow and that $\tan\beta$ is fairly large, in which case the EW scale is given by Eq. (2.38).

Consider the dependence on the first two generation sfermion masses. Just integrating the two-loop expression for the beta function of the up type Higgs mass, from SU(2) and U(1) gauge interactions,

$$\frac{\mathrm{d}m_{Hu}^2}{\mathrm{d}t} \supset \frac{2}{\pi^2}\left(\sum_i \alpha_i^2\left(t\right) C_i\left(H_u\right)\right)\tilde{m}_{1,2}^2 \ , \tag{5.21}$$

without considering the RG flow of the other parameters, gives a contribution enhanced by a single logarithm. This leads to a relatively small tuning if $\tilde{m}_{1,2}$ is of order a few TeV. However, the EW VEV has a strong dependence on the stop mass, which itself has a significant dependence on $\tilde{m}_{1,2}^2$, and can lead to a significant tuning even though it is a higher order effect.

To calculate the EW tuning with respect to the sfermions we study only the dominant terms in the RG flow (rather than solving the full two-loop RG equations, which can only be done numerically). The important terms are the coupling of the Higgs to the stop in Eq. (2.32), and the dynamics of the stop, sfermion, gluino system solved in Eq. (5.3). This neglects effects such as the Higgs back-reaction on its own mass and the stop and the RG flow of the first two generation sfermions.[7]

Under these assumptions, the shift in the Higgs soft mass due to a perturbation in the UV value of the first two generation sfermion masses squared, including the direct 2-loop contribution and the resummed contribution through the stops, can be calculated analytically. The later is obtained from the change in the contribution from the stop to the Higgs soft

[7] We are also neglecting the running of λ_t (but the running of α_s is included). The results obtained agree with numerical solutions of the RG with all significant two-loop contributions equations to within approximately 10%.

mass during RG flow when the stop-gluino-sfermion system (solved in Eq. (5.3)) is perturbed. Integrating the RG equation leads to

$$\Delta m_{Hu}^2 \left(m_Z\right)|_{M_3} = \int_{t_\Lambda}^{t_{M3}} \Delta\beta_{m_{Hu}^2}\left(t\right)|_{M3}\,\mathrm{d}t \tag{5.22}$$

$$= \int_{t_\Lambda}^{t_{M3}} \frac{\partial\beta_{m_{Hu}^2}\left(t\right)}{\partial m_{\tilde{t}}^2\left(t\right)}\Delta m_{\tilde{t}}^2\left(t\right) + \frac{\partial\beta_{m_{Hu}^2}\left(t\right)}{\partial\tilde{m}_{1,2}^2\left(t\right)}\Delta\tilde{m}_{1,2}^2\left(t\right)\,\mathrm{d}t\ , \tag{5.23}$$

where the first term is the contribution through the stop, and the second is the direct two-loop contribution. The fine tuning is then given by (using Eq. (2.38))

$$\frac{\mathrm{d}t\left(\log m_Z^2\right)}{\mathrm{d}t\left(\log\tilde{m}_{1,2}^2\right)} = \frac{2\tilde{m}_{1,2}^2}{m_Z^2}\frac{\partial}{\partial\left(\tilde{m}_{1,2}^2\right)}\int_{t_\Lambda}^{t_{m1,2}} \frac{\partial\left(\frac{d}{dt}m_{Hu}^2\right)}{\partial m_{\tilde{t}}^2\left(t\right)}m_{\tilde{t}}^2\left(t\right) + \frac{\partial\left(\frac{d}{dt}m_{Hu}^2\right)}{\partial\tilde{m}_{1,2}^2\left(t\right)}\tilde{m}_{1,2}^2\left(t\right)\,\mathrm{d}t\ . \tag{5.24}$$

Using Eqs.(5.3) and (5.21), with a factor of two since the coupling occurs through both the left- and right-handed stops, leads to

$$\begin{aligned}\Delta_{\tilde{m}_{1,2}^2} &= \frac{\tilde{m}_{1,2}^2}{m_Z^2}\frac{\partial}{\partial\left(\tilde{m}_{1,2}^2\right)}\int_{t_\Lambda}^{t_{m12}} \frac{3m_t^2}{4\pi^2 v^2\cos\left(2\beta\right)}\sum_i \frac{8C_i}{\pi b_i}\alpha_i\left(\frac{1}{1+\frac{b_i\alpha_i}{2\pi}\left(t_\Lambda - t\right)}-1\right)\tilde{m}_{1,2}^2 \\ &\qquad + \frac{2}{\pi^2}\left(\sum\frac{\alpha_i^2 C_i\left(H_u\right)}{1+\frac{b_i\alpha_i}{2\pi}\log\left(\frac{\Lambda_{\mathrm{UV}}}{\tilde{m}_{1,2}}\right)}\right)\tilde{m}_{1,2}^2\,\mathrm{d}t \\ &= \frac{\tilde{m}_{1,2}^2}{m_Z^2}\frac{\partial}{\partial\left(\tilde{m}_{1,2}^2\right)}\sum_i\left[A\frac{8C_i}{\pi b_i}\alpha_i\left(\log\left(\frac{\Lambda}{\tilde{m}_{1,2}}\right)-\frac{2\pi}{\alpha_i b_i}\log\left(1+\frac{b_i\alpha_i}{2\pi}\log\left(\frac{\Lambda}{\tilde{m}_{1,2}}\right)\right)\right)\right. \\ &\qquad \left. +\frac{4C_i}{\pi b_i}\alpha_i\left(\frac{1}{1+\frac{b_i}{2\pi}\log\left(\frac{\Lambda_{\mathrm{UV}}}{\tilde{m}_{1,2}(m_Z)}\right)\alpha_i}-1\right)\right]\tilde{m}_{1,2}^2\ ,\end{aligned} \tag{5.25}$$

where $A = \frac{3m_t^2}{4\pi^2 v^2\cos(2\beta)}$. As before, each term gives two contributions to the fine tuning. The largest is from the direct variation of the initial sfermion masses, while the second contribution comes from varying the end point of the logarithm, and is smaller.

Intuitively, the first term consists of two tunings at different levels in the theory, the EW VEV is tuned by the mass of the stop, which is itself tuned by the first two generations. The overall tuning is effectively obtained by multiplying these together, and weighting by a factor less than 1 to account for the gluino only generating a change in the stop mass after some running has occurred. The second term (the two loop direct contribution) typically gives a shift in the mass squared of around $(10-50)\,\%$ of the first term, and acts in the opposite direction reducing the total fine tuning. This is because the direct contribution decreases the Higgs mass squared, while the indirect contribution decreases the stop mass squared resulting in a less negative Higgs mass squared. Since it is a higher loop effect, the indirect contribution is greatest when the mediation scale is highest.

For natural SUSY spectra, the shift in the Higgs mass directly from the gluino is completely negligible compared to the logarithm squared contribution that occurs through the stop mass.[8] Similarly to the previous calculation, this gives

$$\begin{aligned}\Delta_{M_3^2} &= \frac{M_3^2}{m_Z^2} A \frac{\partial}{\partial\left(M_3^2\right)} \int_{t_\Lambda}^{t_{M_3}} \frac{4}{b_3} C_3 \left(\frac{1}{\left(1+\frac{b_3\alpha_3}{2\pi}\left(t_\Lambda - t\right)\right)^2} - 1 \right) M_3^2 \, \mathrm{d}t \\ &= \frac{M_3^2}{m_Z^2} A \frac{\partial}{\partial\left(M_3^2\right)} \frac{4}{b_3} C_3 \frac{\frac{b_3\alpha_3}{2\pi}\log^2\left(\frac{\Lambda}{M_3}\right)}{1+\frac{b_3\alpha_3}{2\pi}\log\left(\frac{\Lambda}{M_3}\right)} M_3^2 \ .\end{aligned} \tag{5.26}$$

Next, we turn to the tuning with respect to the initial stop mass. Since the RG equation governing the behaviour of a perturbation at the UV boundary of the stop mass may be solved exactly (at one-loop order), as in Eq. (5.10), we can evaluate the shift in the low energy Higgs soft mass directly. This leads to

$$\Delta m_{Hu}^2\left(m_Z\right) = \frac{1}{2}\left(\left(\frac{m_{\tilde{Q}3}}{\Lambda_{\mathrm{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} - 1\right) \Delta m_{\tilde{Q}3}^2\left(\Lambda_{\mathrm{UV}}\right) \ , \tag{5.27}$$

$$\Delta_{m_{\tilde{Q}3}^2} = \frac{m_{\tilde{Q}3}^2}{m_Z^2} \frac{\partial}{\partial\left(m_{\tilde{Q}3}^2\right)} \left(\left(\frac{m_{\tilde{Q}3}}{\Lambda_{\mathrm{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} - 1\right) \Delta m_{\tilde{Q}3}^2\left(\Lambda_{\mathrm{UV}}\right) \ , \tag{5.28}$$

for the left-handed stop. The expression for the right-handed stop is given by

$$\Delta_{m_{\tilde{u}3}^2} = \frac{m_{\tilde{u}3}^2}{m_Z^2} \frac{\partial}{\partial\left(m_{\tilde{u}3}^2\right)} \left(\left(\frac{m_{\tilde{u}3}}{\Lambda_{\mathrm{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} - 1\right) \Delta m_{\tilde{u}3}^2\left(\Lambda_{\mathrm{UV}}\right) \ . \tag{5.29}$$

Alternatively, if we regard the UV masses of the left and right handed stops as one variable a similar computation easily gives

$$\Delta_{m_{\tilde{t}3}^2} = 2\frac{m_{\tilde{t}3}^2}{m_Z^2} \frac{\partial}{\partial\left(m_{\tilde{t}3}^2\right)} \left(\left(\frac{m_{\tilde{t}3}}{\Lambda_{\mathrm{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} - 1\right) \Delta m_{\tilde{t}3}^2\left(\Lambda_{\mathrm{UV}}\right) \ . \tag{5.30}$$

If an MSSM Higgs sector is assumed, solving the same set of RG equations gives the tuning from the Higgs soft mass at the UV boundary of the RG flow

$$\Delta_{m_{Hu}^2} = \frac{m_{Hu}^2}{m_Z^2} \frac{\partial}{\partial\left(m_{Hu}^2\right)} \left(\left(\frac{m_{Hu}}{\Lambda_{\mathrm{UV}}}\right)^{\left(3y_t^2/4\pi^2\right)} + 1\right) \Delta m_{Hu}^2\left(\Lambda_{\mathrm{UV}}\right) \ . \tag{5.31}$$

Assuming $\tan\beta$ is moderately sized the tuning with respect to m_{Hd}^2 is negligible compared to that from m_{Hu}^2. If the Higgs sector is more complicated, for example in the NMSSM, the exact expression here will be modified however it is still expected to still take the form

[8]The NLL direct gluino contribution is two-loop order but only enhanced by a single logarithm compared to the two-loop, contribution enhanced by two logarithms.

$\Delta_{m^2_{Hu}} \lesssim 2\frac{m^2_{Hu}}{m^2_Z}$, with the equality satisfied if $\Lambda_{\rm UV} \sim m_{Hu}$ so there is very little running.

Finally, we consider the μ and $B\mu$ parameters (again assuming the MSSM Higgs sector). These do not feed strongly into other soft masses during RG flow, and the tuning with respect to them is given by

$$\Delta_{\mu^2} = 2\frac{\mu^2\left(\Lambda_{\rm UV}\right)}{m_Z^2}\frac{\partial\mu^2\left(m_Z\right)}{\partial\mu^2\left(\Lambda_{\rm UV}\right)}, \tag{5.32}$$

$$\Delta_{B\mu} = 2\frac{B\mu\left(\Lambda_{\rm UV}\right)}{m_Z^2}\frac{\partial B\mu\left(m_Z\right)}{\partial B\mu\left(\Lambda_{\rm UV}\right)}\ . \tag{5.33}$$

For $\mu = 400\,\text{GeV}$ and $B\mu = 200\,\text{GeV}$ at the weak scale, solving the RG equations for these terms numerically gives

$$\Delta_{\mu^2} \sim 40\ , \tag{5.34}$$

$$\Delta_{B\mu} \sim 10\ , \tag{5.35}$$

for both high- and low-scale mediation. Since these values of μ and $B\mu$ are allowed by collider constraints, and it will turn out that the tunings are less than those from the stops, gluinos, and sfermions, the tunings from these parameters may be neglected. Once these parameters are fixed, the Higgs soft masses in the IR (and therefore after the RG flow at the UV boundary) are fixed by Eq. (2.38).[9]

The overall EW fine tuning is defined as $\Delta = \max\left(\{\Delta_{\rm p}\}\right)$. Initially, we focus on the tuning introduced by the gluino mass, stop mass, and sfermion masses which are fairly independent of the details of the Higgs sector. In contrast, the fine tuning from the Higgs soft mass is dependent on both the $\mu/B\mu$ parameters, and whether the theory is the MSSM, the NMSSM, or another extension (which may be needed to obtain the correct physical Higgs mass in some regions of parameter space). Because of this, the fine tuning from m^2_{Hu} in a typical MSSM Higgs sector is studied separately at the end of this section. There it is seen that the conclusions we draw about the overall tuning of the theory in this section are valid.

Considering the stop, gluino and sfermion soft masses, expanding the fine tuning expressions Eqs.(5.25), and (5.26) in the parameter $\frac{b_3\alpha_3}{2\pi}\log\left(\frac{\Lambda_{\rm UV}}{m_Z}\right)$ and retaining only the leading dependence recovers the expressions in previous papers [151]. However, since α_3 is fairly large over all energy scales, and we are potentially interested in high-scale models which can have large logarithms, we retain the full dependence (this can lead to a factor of two difference in

[9] An alternative but equivalent approach would be to fix the UV boundary stop soft masses at a relatively small value in which case μ and $B\mu$ would be determined by the same relation.

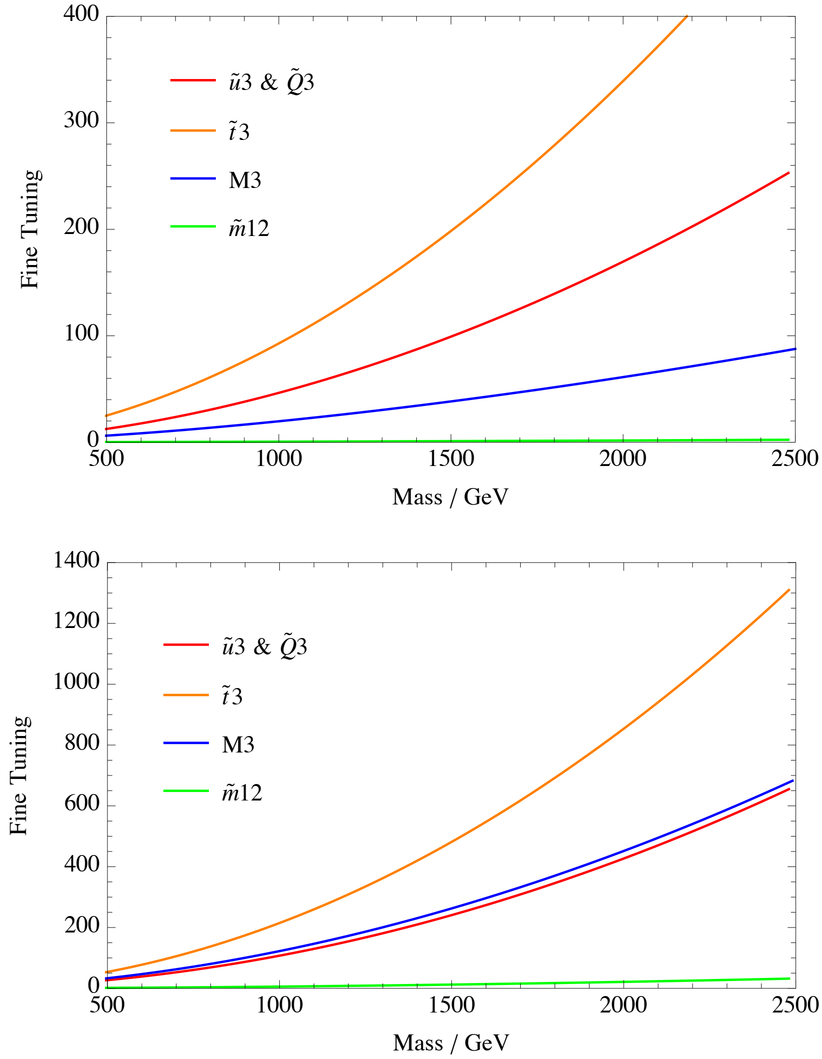

Figure 5.3: The fine tuning in the EW sector as a function of the soft parameters, for low-scale mediation with $\Lambda_{\rm UV} = 10^6\,$GeV (top) and high-scale mediation $\Lambda_{\rm UV} = 10^{16}\,$GeV (bottom). The plots are a function of the *weak scale* gluino mass since its running is fairly independent of the other parameters in the theory. The other masses are the values at the mediation scale, which may run to smaller or larger values when evolved to the weak scale.

some expressions). In Fig. 5.3 the fine tuning is plotted as a result of the UV soft parameters for low- and high-scale mediation, both for the cases where the stop masses are independent in the UV and when they are not. When they are both set by one parameter the fine tuning is worse since both feed into the up type Higgs mass simultaneously.

For a given UV stop mass a larger UV gluino or sfermion mass is never preferred.[10] However, provided $\Delta_{M_3^2}$ and $\Delta_{\tilde{m}^2_{1,2}}$ remain smaller than $\Delta_{m^2_{\tilde{t}}}$, increasing the gluino or sfermion masses does not make the fine tuning worse (at least with the measure of fine tuning adopted here). Consequently, collider bounds can be somewhat alleviated without introducing fine

[10]This does not necessarily hold for the weak scale stop mass though.

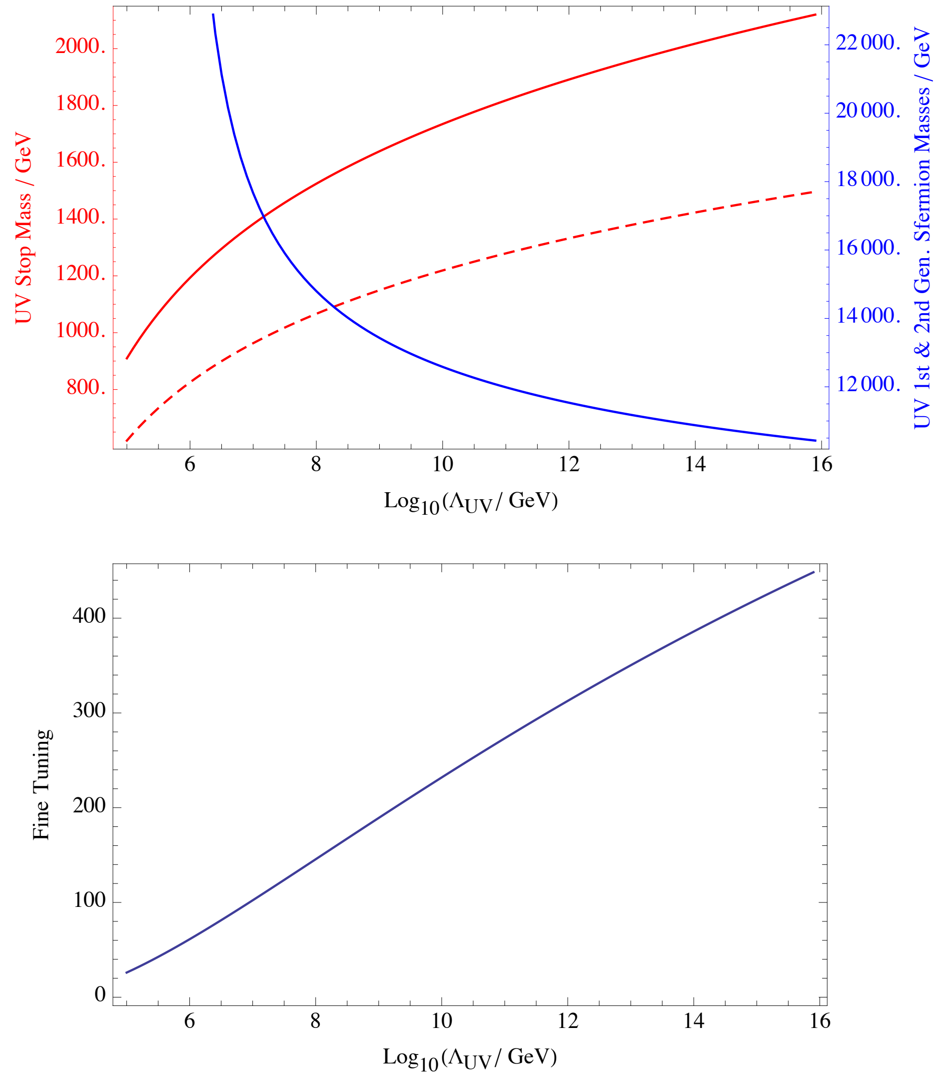

Figure 5.4: **Top:** The UV stop (red) and sfermion masses (blue) that lead to the same fine tuning of the EW scale as a gluino with weak scale mass of 2 TeV as a function of the mediation scale. We show both the case where the left and right handed stops are independent parameters (solid lines) and when they are fixed equal (dashed). Lowering the stop or sfermion masses below these masses does not improve the fine tuning of the theory, and so this graph limits the extent to which a natural SUSY theory is beneficial. **Bottom:** The fine tuning corresponding to a 2 TeV gluino as a function of mediation scale. By construction, this is the same as the fine tuning generated by stops at the masses in the top panel. If fine tuning better than 1% is imposed then the mediation scale is limited to $\Lambda_{\rm UV} < 10^7$ GeV.

tuning. It is interesting to ask what the values of $m_{\tilde{t}}^2$, M_3^2, and $\tilde{m}_{1,2}^2$ that saturates a given fine tuning are. In particular, suppose we fix the gluino mass to be 2 TeV at the weak scale, we wish to know the maximum UV masses the stop and first two generation sfermions may have before they dominate the fine tuning. In Fig. 5.4 we plot the UV masses of the stops and first two generation sfermions for this scenario. If the gluino is at 2 TeV, there is no fine tuning benefit to having UV stop masses below $(1 - 1.5)$ TeV for GUT scale mediation, and $(0.5 - 1)$ TeV for very low scale mediation. From Fig. 5.4 (*bottom*), a gluino of this mass

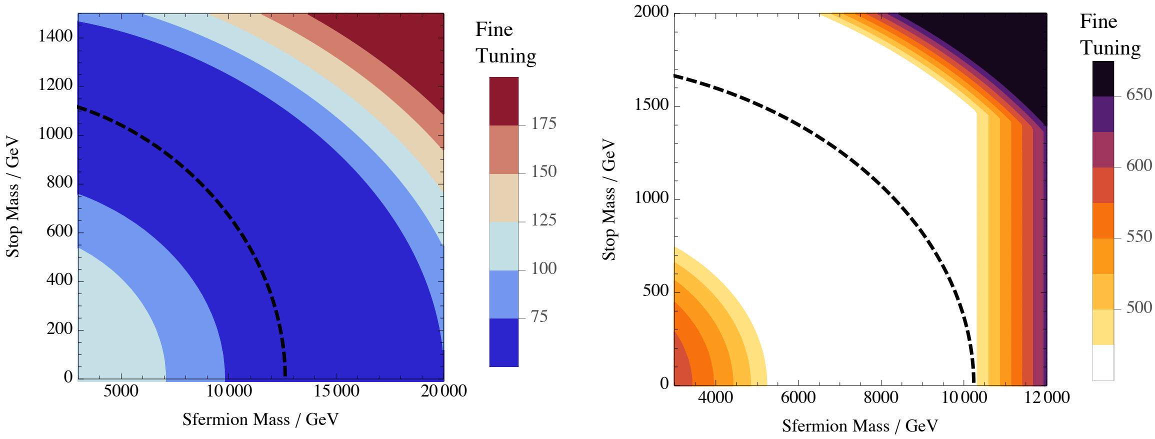

Figure 5.5: The EW fine tuning as a function of the weak scale sfermion and stop masses (assuming the two stops are not independent) with a weak scale gluino mass of 2 TeV for $\Lambda_{\rm UV} = 10^6$ GeV (left), and $\Lambda_{\rm UV} = 10^{16}$ GeV (right). The regions below the dashed black line have a tachyonic stop mass at the UV boundary. Since sfermion masses larger than 3 TeV are not constrained by collider limits, for low-scale mediation there is no improvement in fine tuning through decreasing the stops below 1.4 TeV. For high scale mediation, especially if we demand the stop is not tachyonic at the UV boundary, the majority of the region with the lowest fine tuning actually has a fairly heavy weak scale stop of around 1.5 TeV.

forces the tuning of the EW scale to be at least 400, if running begins at the GUT scale. In contrast, it is easily possible to separate the first two generation sfermions significantly from the gluino and stops without increasing the fine tuning of the theory, which is beneficial for collider limits.

Of course, the relevant quantities for collider physics are the weak scale masses, and (unlike the gluino mass) the running of the stop soft masses depends on the the sfermions and gluino masses. We plot the EW fine tuning as a function of the weak scale stop mass and first two generation sfermion masses with the weak scale gluino mass fixed at 2 TeV, for low- and high-scale mediation, in Fig. 5.5.[11] This is obtained by numerically solving the RG equations between their UV boundary and the weak scale. It is assumed the two stops are not independent, however this does not qualitatively affect the conclusions. In these plots, due to the fixed gluino mass, the smallest possible EW fine tuning is around 60 and 400 for low- and high-scale mediation, respectively (see also Fig. 5.4). The large areas of parameter space with the lowest fine tuning in the centre of both plots have fine tuning dominated by the gluino.

For low-scale mediation, there is no preference for the weak scale stop mass to be lighter than about 1.5 TeV. For high-scale mediation, the largest region of parameter space with low

[11]The weak scale masses here are actually $\overline{\rm MS}$ masses and not pole masses. There is an additional finite correction to convert to the physical stop mass, but this is a small correction.

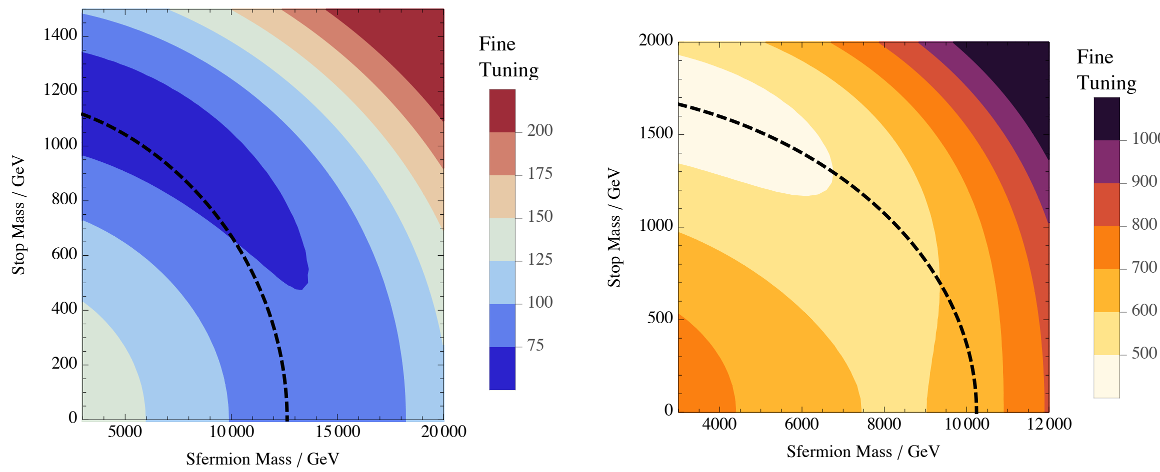

Figure 5.6: The EW fine tuning, using the measure $\delta = \sqrt{\sum \Delta_i^2}$, as a function of the weak scale sfermion and stop masses (assuming the two stops are not independent) with a weak scale gluino mass of 2 TeV for $\Lambda_{\mathrm{UV}} = 10^6$ GeV (left), and $\Lambda_{\mathrm{UV}} = 10^{16}$ GeV (right). The regions below the dashed black line have a tachyonic stop mass at the UV boundary. As a result of the tuning introduced by the sfermions using this measure, lighter sfermions which correspond to heavier weak scale stops are favoured.

fine tuning actually has relatively large stop masses, around 1.5 TeV. In this case, heavy stop masses are even further favoured if we demand the stop is non-tachyonic at the boundary. This is a reasonable restriction since there is a danger such boundary conditions might lead to deep colour breaking vacua in the early universe.[12] As the sfermions tend to decrease the stop mass squared during RG flow down in energy scale, the maximum weak scale stop mass that results in a tachyonic stop in the UV is increased as the first two generation sfermions are made heavier.

In Fig. 5.5 contours of constant UV stop mass are approximately circle arcs concentric with the tachyonic contour. The regions where the tuning contours take the same shape have tuning dominated by the UV stop mass. On the far right side of the plot showing high scale mediation, there is a region where the sfermion soft mass dominates the tuning, indicated by the vertical contours. In the region where the stop is tachyonic at the UV boundary of the RG flow, increasing the weak scale stop mass can actually improve the fine tuning. This occurs since increasing the weak scale stop mass leads to a less tachyonic UV boundary stop mass, and as a result the ratio $\left|\frac{m_{\tilde{t}}^2(\Lambda_{\mathrm{UV}})}{m_{\tilde{t}}^2(m_Z)}\right|$ is smaller.

If instead an alternative definition of fine tuning, $\delta = \sqrt{\sum_i \Delta_i^2}$, is used, increasing the gluino, first two generation sfermion, or stop soft masses at the UV boundary of the RG flow

[12] Although the existence of colour charge breaking vacua is not necessarily problematic if the colour preserving vacua is metastable on timescales longer than the age of the Universe [207, 208, 210].

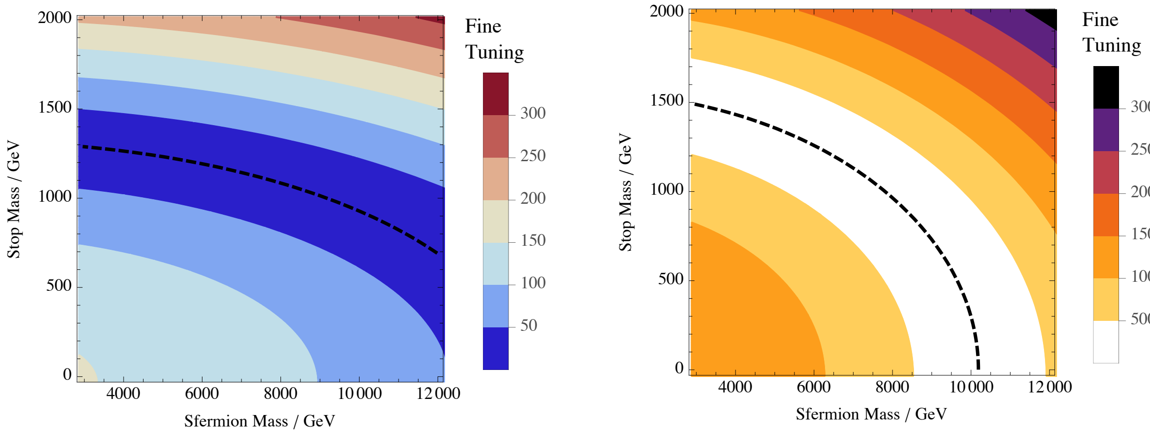

Figure 5.7: The EW fine tuning up type Higgs soft mass at the UV boundary of the RG flow, as a function of the weak scale sfermion and stop masses (assuming the two stops are not independent) with a weak scale gluino mass of 2 TeV for $\Lambda_{\rm UV} = 10^6$ GeV (left), and $\Lambda_{\rm UV} = 10^{16}$ GeV (right). The regions below the dashed black line have a tachyonic Higgs mass at the UV boundary.

always increases the fine tuning. However, since δ is still dominated by whichever tuning is largest, the results are similar to those obtained previously. In Fig. 5.6 we plot the tuning as a function of the weak scale stop and first two generation sfermion masses, with the weak scale gluino fixed at 2 TeV. The fine tuning from the sfermion masses actually results in the regions with the smallest fine tuning having relatively large stop masses.

In the scenario where both the gluino and stop masses depend on the same F-term in the theory, F^2 (or F) is the fundamental parameter that fine tuning should be measured with respect to. Parametrically, the gluino mass is given by $M_3 \sim \frac{F}{M_*}$ and the stop mass also by $m_{\tilde{t}}^2 \sim \frac{F^2}{M_*^2}$ where M_* is the mediation scale. A 1% increase in F^2 generates a 1% increase in both the gluino and stop masses squared. As a result the fine tuning is worse than if the gluino and stop were independent variables.

Finally, we return to the tuning from the UV Higgs soft mass squared. Taking $\mu =$ 400 GeV, in Fig. 5.7 we plot this as a function of the weak scale stop and sfermion masses, with the weak scale gluino fixed at 2 TeV in exact analogy to Fig. 5.5. The fine tuning is calculated by numerically running the IR soft Higgs mass which gives the correct EW scale, to the UV boundary and evaluating Eq. (5.31).[13] Clearly, the tuning from the Higgs soft mass is not especially small. This is to be expected since the Higgs soft mass appears at tree level in the EW VEV. However, in the regions of lowest fine tuning, the tuning from the Higgs mass is typically slightly smaller than that from the other parameters. The plot also shows

[13] We assume vanishing A-terms although these may be important for generating the correct physical Higgs mass in some theories, and if large modify the running slightly.

that in the regions of lowest fine tuning the UV Higgs mass is not far near zero, and there are large parts of parameter space with small fine tuning where the Higgs soft mass squared (at the UV boundary of the RG flow) is positive. For low-scale mediation, the part of parameter space with tuning less than 50 has $|m_{Hu}^2| \lesssim (500\,\mathrm{GeV})^2$ at the UV boundary of the RG flow, and for high-scale mediation the region with tuning less than 500 has $|m_{Hu}^2| \lesssim (1000\,\mathrm{GeV})^2$. The regions with low fine tuning coincide closely with the regions where the other parameters have low fine tuning, therefore the previous estimates of the fine tuning and favoured regions can be valid even when the details of a Higgs sector are included.

5.3 Dirac Gauginos for Natural SUSY

In this section we consider extended theories with Dirac gluinos. As discussed in Section 2.4 these provide an effective way of shielding the stop from large corrections compared to the usual Majorana case. In particular, there are no corrections to the stop mass from the gluino enhanced by a large logarithm of the form $\log\left(\frac{m_{\mathrm{med}}}{m_Z}\right)$. The only contribution is a finite piece generated below the scale where the heaviest part of the effective $\mathcal{N}=2$ multiplet is integrated out, which is typically the sgluon (the new scalar octet partner of the gluon), and above the mass of the gluino [76, 91–96]. As a result these models are a very interesting proposal for a SUSY model without large tuning.

We focus on a simple model, following [91, 95]. There is an additional U(1) gauge group which obtains a D-term expectation value, and has field strength W'. This couples to the visible sector $\mathcal{N}=2$ gauge multiplet, which can be written in $\mathcal{N}=1$ notation as a vector multiplet with field strength W, and a chiral multiplet A in the adjoint of the gauge group, only through a term

$$\int \mathrm{d}^2\theta \, \frac{\sqrt{2}W'_\alpha}{M_*} W_j^\alpha A_j \ , \tag{5.36}$$

where M_* is the mediation scale. It can be shown that this operator also induces a mass for the real component of the sgluon, $\tilde{m}_i^2$, of size $\tilde{m}_3 = 2M_3$, where M_i is the Dirac gaugino mass.[14] In this minimal model there is no direct coupling between the SUSY breaking sector and the sfermions. Instead these are generated only by radiative corrections from the gauge

[14] As discussed in [91], there actually exists another, independent, supersoft term coupling W' and A which gives a mass to the sgluon. For simplicity, we assume this operator is absent from the theory.

sector as discussed in detail in [91]. The induced stop soft mass is given by

$$\begin{aligned}\Delta m_{\tilde{t}}^2 &= \sum_i \frac{C_i \alpha_i}{\pi} M_i^2 \log\left(\frac{\tilde{m}_i^2}{M_i^2}\right) \\ &= \frac{C_3 \alpha_3 M_3^2}{\pi} \log(4) \ ,\end{aligned} \tag{5.37}$$

where we have included only the dominant gluino contribution. The up-type Higgs receives a contribution to its mass from the stop which is only present in the running between the stop soft mass and the scale at which this mass is generated. Since the stop mass is generated only in the small energy range between the sgluon and gluino masses, it is a reasonable approximation to assume it is tuned on instantaneously at the gluino mass.[15] Then the mass shift in the up-type Higgs is given by

$$\Delta\left(\delta m_{Hu}^2\right) = -\frac{3\lambda_t^2}{8\pi^2} m_{\tilde{t}}^2 \log\left(\frac{M_3^2}{m_{\tilde{t}}^2}\right) \ , \tag{5.38}$$

which is clearly very suppressed relative to the MSSM case. Since the sgluon is heavier than the gluino, the energy range during which the gluino mass feeds into the stop mass is separated from that in which the stop mass feeds into the Higgs mass, hence there is no need to carry out an integration over energies. The overall dependence of the Higgs mass on the gluino mass is then given by

$$\begin{aligned}\Delta m_{Hu}^2 &= \frac{-3\lambda_t^2}{8\pi^2} m_{\tilde{t}}^2 \log\left(\frac{M_3^2}{m_{\tilde{t}}^2}\right) \\ &= \frac{3\lambda_t^2}{8\pi^2} \frac{C_3 \alpha_3 M_3^2}{\pi} \log(4) \log\left(\frac{C_3 \alpha_3}{\pi} \log(4)\right) \ .\end{aligned} \tag{5.39}$$

Hence, the fine tuning is

$$\begin{aligned}\tilde{\Delta}_{M_3^2} &= \frac{M_3^2}{m_Z^2} \frac{3\lambda_t^2}{2\pi^2} \frac{C_3 \alpha_3}{\pi} \log(4) \log\left(\frac{C_3 \alpha_3}{\pi} \log(4)\right) \\ &\simeq 0.0282 \frac{M_3^2}{m_Z^2} \ .\end{aligned} \tag{5.40}$$

In these theories the stop masses are not independent variables since both are generated through the gaugino masses, and cannot be adjusted independently. Therefore we take the gluino mass as the only independent variable. While, as previously discussed, using the weak scale value is an approximation, it is sufficient since there is very little running in such a theory. Further, since the running occurs over a very small range of energies the gauge

[15] This assumption leads to an error in the size of the logarithm in Eq. (5.37) of $\frac{1}{2}\log 2 \simeq 0.3$, where the factor of $\frac{1}{2}$ is due to the finite energy range taken for the stop mass to be generated from the gluino mass. Since the typical value of the logarithm is $\log\left(\frac{M_3}{m_{\tilde{t}}}\right) \simeq 2.5$ this is negligible at the accuracy to which we are working.

couplings can be taken to be constant to a good approximation.

The indirect fine tuning of the Higgs by the gluino through the stop mass, which was found to be the dominant contribution in the Majorana case, still appears as a logarithm squared, however now goes as

$$\log\left(\frac{M_3}{m_{\tilde{t}}}\right)\log\left(\frac{\tilde{m}_i}{M_i}\right) \ , \tag{5.41}$$

which of course is much suppressed. Importantly, this is independent of the mediation scale, in effect the scale where a full $\mathcal{N}=2$ spectrum appears is acting as a UV boundary of the RG flow. This is a desirable alternative to a conventional model with a very low cutoff since it is still compatible with a string theory completion [302], and avoids problematic higher dimensional operators from a SUSY breaking and mediation sector which is not far separated in energy scale from the weak scale. Dirac models may also appear naturally out of models with spontaneous supersymmetry breaking [303].

Since we are dealing with logarithms of $\mathcal{O}(1)$, these terms now no longer necessarily dominate over other non-logarithmic corrections. To obtain an accurate measure of fine tuning these should be included. In particular, the non-enhanced terms are the reason that it is not possible to make the fine tuning arbitrarily small for heavy superpartners by taking $\tilde{m}_3 = M_3$ and $M_3 = m_{\tilde{t}}$. The threshold corrections from the gluino can be calculated from [103]. These, along with the other corrections lead to an order 10% difference to the stop masses. Consequently, our results are reasonably accurate.

As the logarithms are small, it is necessary to check that the one-loop contribution of the electroweakinos to the Higgs mass does not dominate the fine tuning. These give a contribution to the Higgs mass

$$\delta m_{Hu}^2 = \delta m_{Hd}^2 = \frac{\alpha_2\left(M_2\right)C_2 M_2^2}{\pi}\log\left(\frac{\tilde{m}_2^2}{M_2^2}\right) \ , \tag{5.42}$$

which leads to a tuning of the EW VEV with respect to the wino mass of approximately

$$\begin{aligned}\Delta_{M2} &= \frac{M_2^2}{m_Z^2}\frac{2\alpha_2\left(M_2\right)C_2}{\pi}\left(\log\left(\frac{\tilde{m}_2^2}{M_2^2}\right)-1\right)\\ &= \frac{M_2^2}{m_Z^2}\frac{2\alpha_2\left(M_2\right)C_2}{\pi}\left(\log\left(4\right)-1\right)\\ &= 0.0062\frac{M_2^2}{m_Z^2} \ .\end{aligned} \tag{5.43}$$

Since the wino is typically significantly less massive than the gluino, this is only a small contribution to the fine tuning.

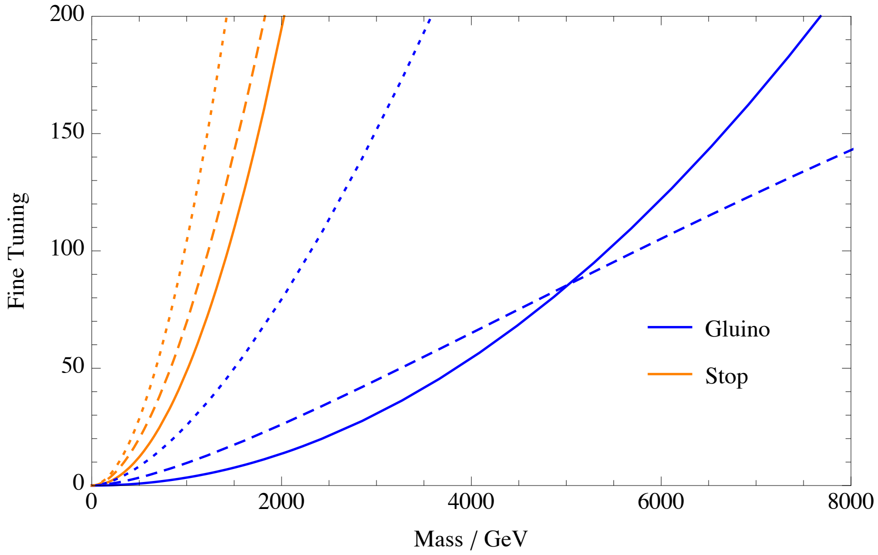

Figure 5.8: The EW fine tuning of the minimal Dirac model as a function of gluino and stop masses (solid lines). Note that, in this model, the stop mass is a function of the gluino mass, hence these are not independent variables. For comparison we also plot the fine tuning for the MSSM, obtained in Section 5.2, for the cases $\Lambda_{\rm UV} = 10^5\,\text{GeV}$, dashed lines, and for $\Lambda_{\rm UV} = 10^6\,\text{GeV}$, dotted lines. It is seen that while the Dirac model gives comparable fine tuning to a very low scale MSSM model, it quickly leads to an improvement in fine tuning as the UV boundary is increased.

In Fig. 5.8 we plot the fine tuning as a function of the gluino mass and also plot the stop mass which is fixed by the gluino mass (solid lines).

By comparison with the expressions found in the previous section, we find the fine tuning as a function of stop mass is comparable to an MSSM model with a very low cutoff of $\Lambda_{\rm UV} = 10^5\,\text{GeV}$ (with both stops masses fixed by one parameter). However, as the cutoff $\Lambda_{\rm UV}$ is raised, Dirac gauginos quickly lead to a benefit in reducing the fine tuning. Hence, for string models, a Dirac gluino provides a very strong option to retain as natural a spectrum as possible, as well as being well motivated theoretically. Of course, a disadvantage of such models is that the $\mathcal{N} = 2$ scalar partners spoil traditional SUSY gauge unification unless other new states are also present, requiring more model building.

Chapter 6: Hidden Sector Renormalisation and Fine Tuning

This chapter is based on [304].

In this chapter we study the effect of relaxing the assumption, made in Section 5 and the vast majority of the literature, that the RG equations are simply those of the (N)MSSM (or more generally of the visible sector matter and couplings). In particular, we show that the EW fine tuning of a SUSY theory can be substantially reduced through the effects of hidden sector renormalisation, first studied in [262,305,306], and later expanded on in [307–315]. This is an effect where the RG flow of MSSM scalar soft masses is modified by the details of the hidden SUSY-breaking sector. If the SUSY-breaking sector runs through a region of strong coupling close to a conformal fixed point, and the operator coupling the Higgs superfields to the SUSY breaking spurion obtains a large anomalous dimension, it can efficiently suppress the Higgs soft mass. This washes out the dependence of the EW scale on the superpartner masses and as a result the fine tuning of phenomenologically viable theories can be significantly reduced.

More precisely, suppose the operator that generates the Higgs soft mass obtains a large anomalous dimension between energy scales Λ_1 and Λ_2. Then, approximating the anomalous dimension, γ, as a constant in this region, after the strong coupling region the Higgs soft mass is given by

$$m_{Hu}^2\left(\Lambda_1\right) \approx m_{Hu}^2\left(\Lambda_2\right)\left(\frac{\Lambda_1}{\Lambda_2}\right)^{\gamma} \ . \tag{6.1}$$

Any feed into the Higgs mass from superparticles above, or during, the strong coupling region is strongly suppressed if the strong coupling regime lasts for a relatively long time. Provided the strong coupling ends not far from the weak scale, there is little time for a dependence on the superparticle masses to reemerge, and the fine tuning is reduced.[1]

Rather than attempting to study particular examples of strong dynamics explicitly, which is both notoriously difficult and may not capture generic features of such sectors, we simply parameterise the effect of the strong coupling region by assuming certain operators get large anomalous dimensions in this region. Of course, the lack of an example of a theory combining all the required elements is a significant deficiency of our present work. However, given that

[1] The possibility that the Higgs mass operator may obtain a large anomalous dimension reducing fine tuning in SUSY theories has previously been considered [316–319]. The main difference in the models we study, is that the strong coupling and large anomalous dimensions arise directly in the SUSY breaking or mediation sector, rather than some additional sector coupled to the Higgs.

only a handful of models of dynamical supersymmetry breaking are known, and even fewer are actually calculable, this is perhaps acceptable. Later we argue that it is possible that sectors with the appropriate dynamics can exist and describe models which exhibit some of the required features.

In the simplest implementations, all chiral multiplets are assumed to couple universally to the hidden sector, and the soft mass operators obtain equal anomalous dimension during the strong coupling period. Due to the universal couplings, the sfermion masses, as well as the Higgs mass, are suppressed during the RG flow. Obtaining weak scale sfermion masses in the region of several TeV requires them to be heavier than normal at the UV boundary of the RG flow, and so the tuning with respect to these states is not reduced. If the hidden sector is approximately supersymmetric during the strong coupling region, the gaugino mass operator is protected from receiving a large anomalous dimension by non-renormalisation theorems and holomorphy. Even if the sector is non-supersymmetric, it is quite plausible that models exist where this operator does not obtain a large anomalous dimension since it is distinguished from the operators that generate scalar soft masses, for example due to its R-symmetry breaking nature. Therefore, the tuning with respect to gaugino masses is substantially reduced. Due to large production cross sections and dramatic decay signals, there are stringent collider limits on the gluino mass. A large gluino mass feeds strongly into the Higgs mass through the stops during RG flow, so this is often the dominant tuning in a theory [273, 320], and even these most basic models of hidden sector renormalisation can be a substantial improvement over traditional theories. Of course, this improvement is only present if the strong coupling region happens to end close to the gluino mass, so that the gluino mass does not regenerate a shift in the stop masses. The reduction of tuning with respect to the wino and bino are similarly dependent on this coincidence, but are typically dominated by the gluino if these masses are assumed to unify at the UV boundary. Later, we quantify how close these two scales must be so that there is an efficient reduction in tuning.

More complex models with extra interactions between the visible and hidden sector can reduce the fine tuning with respect to the sfermion masses as well. For example, this can occur in a theory where the Higgs has extra couplings to the SUSY-breaking sector. These may cause the Higgs soft mass operator to gain a large anomalous dimension, while the sfermion operators do not, reducing the dependence of the Higgs mass on the sfermions. Since the sfermion couplings remain universal, strong constraints from flavour observables that are often challenging to accommodate in SUSY models are satisfied. An even more exotic possibility is that the Higgs soft mass operator obtains an opposite sign anomalous dimension to the sfermion mass operators. This leads to an enhancement of the sfermions masses while the Higgs mass is still suppressed. Potentially, soft masses in the region of 10 TeV can be obtained without making the usual tuning of approximately 1% any worse.

As well as the SUSY particles' soft masses, the fine tuning of the EW VEV depends on the μ and $B\mu$ parameters at tree level. To obtain low fine tuning, these must be relatively small. Since the LHC is fairly insensitive to charginos, this is not an very severe constraint. It does however open up the prospect that a future collider may discover light charginos, with other superpartners potentially much heavier. A further attractive possibility for future work would be to build a model where appropriately sized μ and $B\mu$ terms are generated through hidden sector renormalisation (as has been previously studied) while simultaneously explaining why only the Higgs mass operator gains a large anomalous dimension. For the majority of our study we consider traditional Majorana gauginos which feed into the sfermions at all energy scales. Later we briefly comment on the interesting extension of the MSSM to Dirac gauginos, which can reduce the gaugino fine tuning even further, although the tunings with respect to the μ parameter and initial Higgs soft mass are unchanged.

As discussed previously raising the physical Higgs mass to 125 GeV is not always straightforward in supersymmetric models. In the simplest versions of the models considered in this work, all sfermion masses are close to universal at the weak scale. Consequently, collider limits typically force the stops to be fairly heavy in the region of 1.5 TeV, and the physical Higgs mass can be raised to the required value through the one loop correction Eq. (2.39). Notably, this correction is cut off by the mass of the stops, not the UV boundary of the RG flow, since it is the quartic Higgs coupling which is important. Therefore, the strong coupling region (which is typically above the scale of the stops) does not alter the form of the correction to the physical Higgs mass. In more complicated models, the stops may be significantly lighter than the other sfermions, and the Higgs mass can be raised either through large A-terms increasing the correction in Eq. (2.39), or by introducing an NMSSM structure as studied in Section 4.

Turning to the structure of this section, we begin in Section 6.1 by briefly reviewing hidden sector renormalisation. In Section 6.2 we discuss the mechanism that reduces the fine tuning, and carry out a full numerical study of the fine tuning in models with hidden sector renormalisation. Section 6.3 contains a discussion of the types of theory that can may lead to the required dynamics and other model building possibilities.

6.1 Hidden Sector Renormalisation

The models we study are similar to those introduced in [305], with the crucial difference that the region of strong coupling occurs close to the weak scale. Consider a SUSY breaking sector with a spurion X, which receives an F-term, F_0, at the UV boundary of the RG flow. The visible sector scalars and gauginos get mass through terms in the effective Lagrangian

$$\mathcal{L} \supset \int \mathrm{d}^4\theta\, a_i \frac{X^\dagger X}{M_*^2} \Phi_i^\dagger \Phi_i + \int \mathrm{d}^2\theta\, w_n \frac{X}{M_*} W_{n\alpha} W_n^\alpha + \text{h.c.} \,, \tag{6.2}$$

where Φ_i represents the visible sector chiral superfields, W_n is the gauge field strength (corresponding to the gauge group n), and M_* is some high-energy scale in the theory. These terms may be generated by integrating out messenger fields in models of gauge mediation or interactions with other heavy states.

We consider both models where the hidden sector is approximately supersymmetric during the strong coupling region and also models where SUSY is broken at this scale. If the hidden sector is supersymmetric, the holomorphic coupling w_n is not renormalised perturbatively.[2] As a result, the physical gaugino mass only flows due to the wavefunction renormalisation of X, along with the standard flow of the gauge coupling. Denoting the wavefunction renormalisation of X by Z_X, and normalising such that $Z_X = 1$ at the UV boundary of the RG flow, the physical gaugino mass at a scale μ is

$$M_n\left(\mu\right) = g_n^2\left(\mu\right) w_n \frac{F\left(\mu\right)}{M_*} \,. \tag{6.3}$$

where we have defined $F\left(\mu\right) = \frac{F_0}{Z_X^{1/2}\left(\mu\right)}$. Here, $g\left(\mu\right)$ is the gauge coupling in a basis where all fields are canonically normalised. Its RG evolution is given by the NSVZ beta function,

$$\beta\left(g\right) = -\frac{g^3}{16\pi^2} \frac{3T\left(Ad\right) - \sum_i T\left(R_i\right)\left(1-\gamma_i\right)}{1 - \frac{g^2}{8\pi^2} T\left(Ad\right)} \,, \tag{6.4}$$

where i labels the matter fields in the theory, which are in the representation R_i, and have anomalous dimension γ_i, and $T\left(R_i\right)$ is the Dynkin index of the representation R_i. This arises as a combination of a one loop exact renormalisation of the holomorphic gauge coupling, and

[2] Non-perturbative renormalisation of holomorphic couplings is not forbidden by non-renormalisation theorems, and may be important in the strong coupling region. We assume that this does not lead to new operators that generate soft masses in the visible sector.

a rescaling anomaly from canonically normalising the fields in the theory.

In contrast, the non-holomorphic operator that leads to scalar masses is renormalised. Crucially, this is separate, and in addition to, the wavefunction renormalisation of X. It is this renormalisation that means that the dynamics of the hidden sector do not simply result in a rescaling of all soft masses. Including the wavefunction renormalisation of X through the rescaling of F_0, the scalar masses are

$$m_i^2 = a_i(\mu) \frac{F(\mu)^2}{M_*^2} \; . \tag{6.5}$$

Here $a_i(\mu)$ is the renormalised coupling, which evolves according to

$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = \tilde{\gamma}_i a_i - \frac{1}{16\pi^2} \sum_n 8C_n(R_i)\, g_n^6 w_n^2 + \ldots \; , \tag{6.6}$$

where $t = \log\mu$, and C_n is the quadratic Casimir of the state i with respect to the gauge group n. $\tilde{\gamma}_i$ is the extra contribution to the anomalous dimension of the operator from hidden sector effects beyond wavefunction normalisation of X, and three ellipses represents other visible sector one-loop effects, for example those proportional to the Yukawa couplings, and terms from higher loops.[3]

If the theory is non-supersymmetric during the strong coupling region, consistently packaging the fields of the theory into supermultiplets is no longer possible. Regardless, we will see in Section 6.3 that there are models where the operators that generate the visible sector masses may continue to be renormalised below the scale $\sqrt{F}$. Of course, the argument from holomorphy protecting the gaugino mass does not hold in this case. However, even if SUSY is broken, it seems plausible that there exist theories where the gaugino mass operator gains a far smaller anomalous dimension than the scalar mass operators. This is because the gaugino mass is an R-symmetry breaking operator and the interactions of a vector multiplet are necessarily different to those of a chiral multiplet.

While it is possible to study the effects of a particular model of the hidden sector, similarly to [315], we take an alternative approach and *parameterise* the impact of running through a strong coupling regime. This is done by turning on large anomalous dimensions for some operators in the energy region of strong coupling. Due to unitarity, physical operators have positive total anomalous dimension at one loop [321], however this requirement does not persist at higher orders and so does not apply during strong coupling. For simplicity, we

[3]For simplicity, we assume throughout that the SM states do not couple significantly to any hidden sector operators other than X and $X^\dagger X$. The extension to more general cases is straightforward [306].

also assume that $Z_X(\mu) = 1$ at all energy scales, so that the physical F-term of X does not flow. This does not alter the phenomenology of our models, since it is the coefficients of the operators, a_i and w_n, which feed into each others RG, not the masses. Making this assumption just means the RG flow of the soft masses is not rescaled relative to that of the couplings.

6.2 Fine tuning in the Presence of a Strongly Coupled Hidden Sector

We now study the impact of hidden sector renormalisation on fine tuning, for simplicity assuming $\tan\beta$ is fairly large. It is also convenient to study the low-scale fine tuning $\Delta_{\rm EW}$, defined in Section 2.10. As discussed in Section 2.10 and [150, 152], this gives a lower bound on the fine tuning of a theory, avoiding assumptions about the UV completion of the low-energy effective field theory. For example, correlations between soft parameters at the UV cutoff of the theory could mean that a theory's true fine tuning is much lower than a naive estimate of the tuning based on high-scale parameters would suggest. Similarly, the models we study in this paper are examples of theories where assuming the RG flow is just that of the MSSM would lead to an overestimate of the high-scale tuning. Effectively, hidden sector renormalisation can lower the tuning with respect to the high-scale parameters towards the lower bound set by the tuning with respect to the weak-scale parameters.

Another fine tuning measure, which is interesting for the theories considered here, is the the tuning with respect to the values of those parameters of the theory immediately after exiting the strong coupling region that are assumed independent (as usual, it is necessary to assume some particular boundary conditions since we have no knowledge of the higher dimensional operators arising from the strong coupling region). Although this measure has the potential to miss correlations occurring between parameters and effects from running down to Λ_1, we do not know the complete dynamics of any explicit models and can therefore not properly calculate these effects anyway. Consequently, it gives a sensible estimate of a lower bound on the model's fine tuning. When we study particular spectra and RG flows it will be seen that this measure is typically slightly smaller, but fairly close to the fine tuning with respect to the UV parameters. Similarly to the low-scale tuning, our point is to show that a period of strong coupling that does not last too long and could occur in reasonable models can efficiently lower the high scale tuning to close to this value.

To obtain phenomenologically acceptable models with reduced fine tuning, the strong coupling region has to end not far from the weak scale (typically at a few TeV) so that a perturbation to the Higgs soft mass is not regenerated before the superparticles are integrated out of the theory. Additionally, the strong coupling must extend over at least roughly one order of magnitude in energy scale, so that the Higgs soft mass is sufficiently suppressed. This can occur if the RG flow passes very close to an interacting conformal fixed point. Depending on the details of the models involved, the theory may be either supersymmetric and non-supersymmetric during the strong coupling regime. If the SUSY-breaking sector itself is responsible for the hidden sector renormalisation, the scale of mediation must be low, however if it is the messenger sector that runs to strong coupling near the weak scale the scale of mediation can be high. In Sector 6.3, we consider more model building issues and discuss how the required features could be realised.

For our numerical studies, we assume that the sector that becomes strongly coupled is either supersymmetric, or such that the gaugino mass operators do not obtain large anomalous dimensions, so that the tuning with respect to the gaugino masses is reduced. To parameterise the behaviour of the visible sector in response to the hidden sector renormalisation, we take

$$\tilde{\gamma}_j(\mu) = \begin{cases} 1 & \text{if } \Lambda_1 < \mu < \Lambda_2 \\ 0 & \text{otherwise} \end{cases}, \tag{6.7}$$

where j labels the chiral multiplets whose soft mass operators obtain a large positive anomalous dimension when the hidden sector is strongly coupled, between Λ_1 and Λ_2. We have made the assumption that $\tilde{\gamma}_j > 0$, in order that the RG flow decreases soft masses. This parameterisation is well motivated since at conformal fixed points of supersymmetric theories fields often have large anomalous dimensions, of order 1 [262, 322].

The Higgs potential mass squared parameters receive a direct contribution at one loop from the charginos and the bino, and a two-loop double logarithmically enhanced contribution from the gluino through its effect on the stop. For GUT boundary conditions, the second effect is much larger. As shown in Eq. (6.1), after running through strong coupling this contribution is strongly suppressed. The stops contribute to the Higgs mass squared at one loop through an interaction proportional to a Yukawa coupling, while the other sfermions dominantly feed in through a two-loop coupling. While these contributions are suppressed by the strong coupling, the sfermion soft masses are also suppressed so must be larger initially, and there is no improvement in the fine tuning with respect to these parameters.

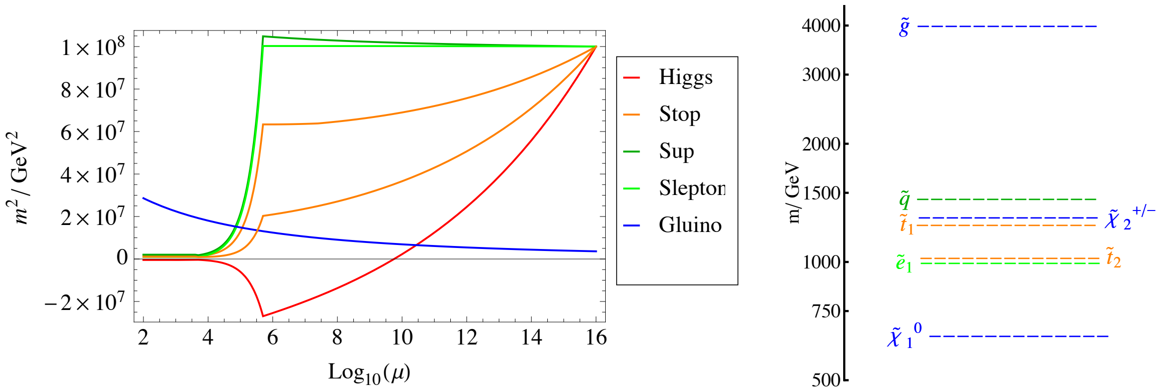

Figure 6.1: **Left:** The RG flow of the soft masses squared in the theory described in the text, assuming universal scalar soft masses at the UV boundary of the flow. The region of strong coupling is clearly visible as an approximately exponential suppression of the scalar soft masses in the region of 10^5 GeV. The Higgs soft mass runs to negative coupling, driven by the terms proportional to the top Yukawa coupling in the RG equations, compatible with radiative EW symmetry breaking. **Right:** The weak scale soft masses in the same theory. The gluino mass is far above LHC reach without being the dominant tuning in the theory, and the sfermion masses are close to current limits.

To study these effects more carefully, we analyse the RG flow of the MSSM in the presence of hidden sector renormalisation numerically. We include the full one-loop equations, and the dominant two-loop effects. Initially, we consider a theory with high-scale mediation. The effect of the hidden sector renormalisation is especially dramatic in this case, and the results are very similar to the low-scale mediation case, except that models with low-scale mediation have slightly less fine tuning. Further, we assume the soft mass operators of all chiral multiplets in the theory get a large anomalous dimension in the strong coupling region. This is completely flavour blind, and attractive in its simplicity. All the dynamics that leads to the large anomalous dimensions can be generated in the hidden sector, without any additions to the visible sector. Also, constraints on flavour changing current are automatically satisfied, even though the strong coupling region is close to the weak scale and higher dimension operators are not strongly suppressed.

Fig. 6.1 (*left*) shows the running in a typical theory, under the assumption of universal soft masses of 10^4 GeV at a mediation scale of 10^{16} GeV, and universal gaugino masses of 1.4 TeV (corresponding to a weak scale gluino mass of 4 TeV), characteristic of a GUT theory. The fine tuning is not substantially altered if the initial soft masses fall into the pattern predicted by minimal gauge mediation. The period of strong coupling is taken to be between 5×10^5 GeV and 5×10^3 GeV and is clearly visible in its effect on the scalar soft masses, while the gluino mass is unaffected. In Fig. 6.1 (*right*), we show the mass spectrum obtained at the weak scale. The sfermions are close to the current experimental bound and the gauginos are far

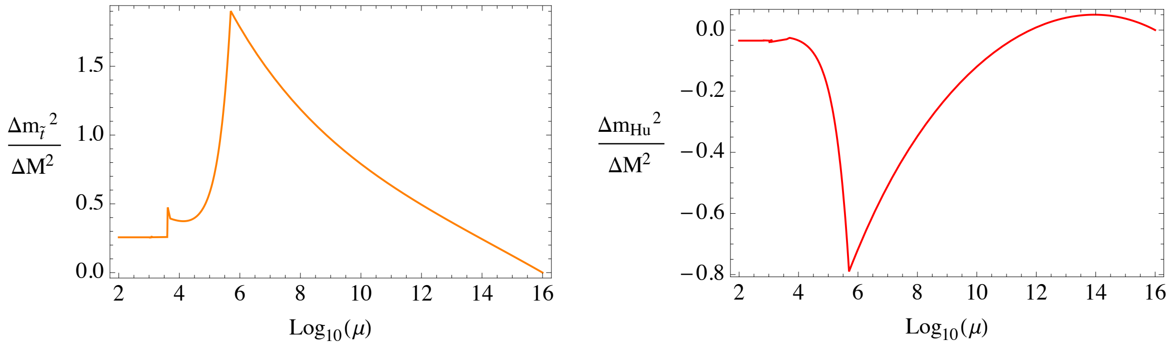

Figure 6.2: **Left:** The perturbation induced in the stop soft mass, $\Delta m_{\tilde{t}}^2$, in response to a perturbation of the universal gaugino mass, ΔM^2, at the UV boundary of the RG flow (taken to be 10^{16} GeV), as a function of the energy scale during the RG flow. **Right:** The perturbation induced in the up type Higgs mass, Δm_{Hu}^2, in response to the same gaugino mass perturbation. The parameters are those of the theory plotted in Fig. 6.1 and described in the text.

above the regions that can be efficiently probed by the LHC. Since the scale of mediation is high, the F-term is large, of order $\sqrt{F} \sim 10^{10}$ GeV.

The fine tuning with respect to the initial universal gaugino mass squared is 15. This a substantial improvement over the typical tuning from a 4 TeV weak scale gluino mass with a mediation scale of 10^{16} GeV which is in the region of 600. Taking a lower mediation scale or the region of strong coupling closer to the weak scale can lower the tuning obtained to 5. The tuning from sfermions is of order 70 which is comparable to that in a model without strong coupling and identical weak scale scalar masses. Assuming just an MSSM Higgs structure, μ is fixed by Eq. (2.38) and induces a substantial tuning of roughly 70. One minor benefit for the fine tuning with respect to sfermions in models with strong coupling is that heavy gluinos slightly reduce production cross sections and consequently alleviate collider bounds on sfermions. However, even with a decoupled gluino, the limits on universal sfermion masses are in the region of 1.4 TeV, which is substantial.

We also plot the perturbation induced in the stop and Higgs soft masses as a result of a perturbation to the universal gaugino mass at the high-scale in Fig. 6.2. Initially a large perturbation in the stop soft mass is induced by the perturbation to the gluino mass (this is identical to the start of the RG flow that would be followed if it was not for the strong coupling). The strong coupling regime is clearly visible and heavily suppresses the perturbation to the stop mass, followed by a short period where the correction is regenerated, before the gluino is integrated out of the theory. The small near vertical drop is because the gluino is now slightly heavier and integrated out of the theory earlier resulting in less running. The Higgs mass (right panel), initially experiences a small positive perturbation as a result of the increased chargino and bino mass. After a short distance in energy scale this is overwhelmed by the two-loop contribution due to the increased mass of the stop. Again, the perturbation is strongly suppressed in the strong coupling region, and the regeneration is negligible.

As discussed, an important assumption is that the strong coupling region must end fairly close to the gluino mass so that a significant shift in the Higgs soft mass squared parameter is not generated below this scale. To quantify this requirement, in Fig. 6.3 we plot the fine tuning with respect to the unified gaugino mass at the UV boundary as a function of the energy scale of the lower end of the strong coupling region, Λ_1. The upper boundary of strong coupling is set by the requirement that $\frac{\Lambda_1}{\Lambda_2} = 100$ is constant, so that the suppression of soft parameters through Eq. (6.1) is the same for all models. The gluino mass is 4 TeV, and all other parameters of the theory as are for Fig. 6.1. For Λ_1 below 4 TeV the gluino is integrated out of the theory within the strong coupling region, and the fine tuning is set by the feed in from the gluino to the Higgs from above the strong coupling region. Of course, this contribution is suppressed by the strong coupling and so gives a much smaller tuning than usual. When Λ_1 is greater than 4 TeV, the fine tuning increases as the gluino regenerates a mass splitting at two-loop order. Efficient reduction of fine tuning requires a fairly close coincidence in scales. However, not surprisingly, there is still a substantial benefit over conventional models with high-scale mediation even if the strong coupling region ends relatively far from the gluino mass.

It is also interesting to consider the low-scale fine tuning, and the tuning if the assumed boundary of the RG flow was simply Λ_1, described in Section 6.2. The gluino tuning assuming the UV boundary is Λ_1 is shown in Fig. 6.3. It can be seen that this is comparable but slightly less than the high-scale tuning of the theory. This shows that a reasonable period of strong coupling, which might be realised in explicit models can efficiently wipe out almost all of the contribution from the gluino above the strong coupling scale. The contribution to the fine tuning evaluated at the EW scale from the gluino is negligible since it does not appear in the one-loop potential. The wino and bino do contribute to this at one loop but assuming GUT unification are sufficiently light that they do not lead to a significant tuning. The tuning from the μ parameter is approximately the same (approximately 70) for all measures of fine tuning, since it does not run by a large amount and is unaffected by the strong coupling region. The high-scale sfermion fine tuning, which is not reduced by the strong coupling,

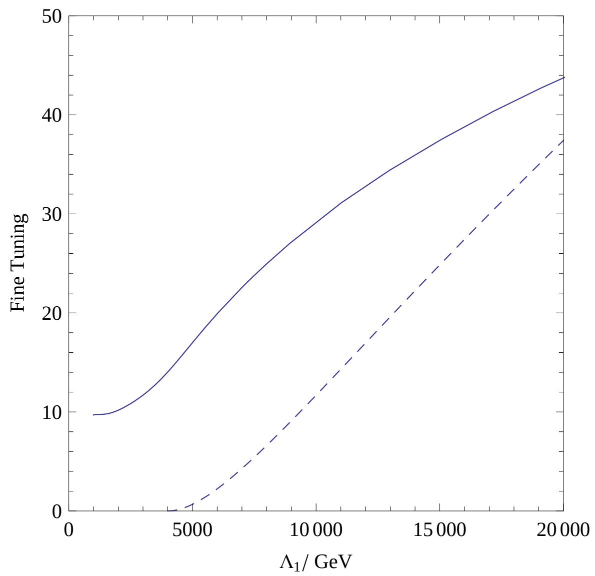

Figure 6.3: The solid line shows the fine tuning with respect to the gluino mass (here assumed to be independent of the other gaugino masses), as a function of the lower end of the strong coupling region, Λ_1, keeping the ratio $\frac{\Lambda_1}{\Lambda_2}$ constant, in a theory with a weak scale gluino mass of 4 TeV and a mediation scale of 10^{16} GeV. As the strong coupling region is separated from the gluino mass the fine tuning increases as a shift in the stop mass is regenerated below the strong coupling region. Below this scale the tuning is solely as a result of the feed-in above the strong coupling scale, which is suppressed but not completely eliminated. For comparison, the dashed line shows the tuning from a 4 TeV weak scale gluino if the assumed UV boundary of the RG flow is Λ_1.

is unsurprisingly much greater than other two measures of tuning, due to the size of the logarithms involved.

There are extensions to the simplest models that reduce the tuning with respect to the sfermion masses. Of course, the pay off for this is that the couplings between the visible sector and hidden sector have to be more complicated. Consider a theory where the sfermions have flavour universal couplings to the hidden sector spurion, through ordinary gauge mediation, but the Higgs fields have additional couplings to the spurion. As a result the initial Higgs soft masses are enhanced compared to normal gauge mediation. Further, assume the extra Higgs couplings result in the Higgs soft mass operator gaining a large anomalous dimension during the strong coupling period, whereas the other operators do not. Such a structure may be obtained, for example, if the Higgs fields are charged under a new gauge symmetry which the other visible sector fields are not.[4] Given that a complete theory must include a mechanism to solve the $\mu/B\mu$ problems it is not implausible that the Higgs fields could have different interactions with the hidden sector to the other chiral multiplets. Flavour observables are still safe since the sfermion couplings are universal. The perturbation to the Higgs mass

[4]Of course, such a symmetry must be strongly broken to allow the SM Yukawa couplings.

from the sfermion masses is suppressed by the strong coupling, but the sfermion masses are not themselves suppressed, reducing the fine tuning from this sector. The initial large Higgs mass is actually useful in finding spectra where the up-type Higgs soft mass is close to zero after running, reducing the required value of μ from Eq. (2.38), and consequently the tuning from this parameter.

In Fig. 6.4 (*left*), we plot the running of a model with these features. The theory has a low mediation scale of $10^6\,\mathrm{GeV}$, and a strong coupling region between $2 \times 10^5\,\mathrm{GeV}$ and $2 \times 10^3\,\mathrm{GeV}$. $\sqrt{F}$ is approximately $10^5\,\mathrm{GeV}$, just inside the strong coupling region. The sfermion soft masses at the UV boundary of the RG flow are taken to fall into the standard gauge mediation pattern. The weak scale masses are shown in Fig. 6.4 (*right*). The gluino is at 2.5 TeV and the squarks are in the region of 1.55 TeV, very close to current limits. For the initial parameters chosen, the Higgs mass squared just runs negative, with a weak scale value of $-(320\,\mathrm{GeV})^2$, and $\mu = 313\,\mathrm{GeV}$. The tuning with respect to the initial gaugino mass squared is approximately 15, with respect to the initial sfermion approximately 20, and that with respect to the initial Higgs mass, and also the initial value of μ is also in the region of 20. This is a substantial improvement over a low-scale gauge mediation model without hidden sector renormalisation, which typically has a tuning of $\mathcal{O}(100)$ in each of these parameters. The dependence of the sfermion fine tuning on the location of the lower cutoff is parametrically similar to that of the gluino plotted in Fig. 6.3, for a large reduction in fine tuning compared to traditional models with low scale mediation these two scale must be rather close.

For comparison, the tuning directly at the EW scale with respect to the 1.5 TeV stops is approximately 10 [153]. Additionally, the tuning if the UV boundary of the RG flow is taken to be Λ_1 is approximately 15. Consequently, the strong coupling region successfully lowers the high-scale tuning towards these lower bounds. This is perhaps not surprising since the strong coupling region ends at $2 \times 10^3\,\mathrm{GeV}$, close to the weak scale, and the fairly low mediation scale means there is little running before the strong coupling region begins. As a result the high-scale tuning is close to the tuning from the scale Λ_1, which itself is close to the tuning evaluated at the EW scale. Again, the tuning of μ is similar for all measures since its running is only a fairly weak effect, and the gluino fine tuning measures have a similar patten to the previous model.

As a entertaining alternative, it is possible that the anomalous dimension of the sfermion

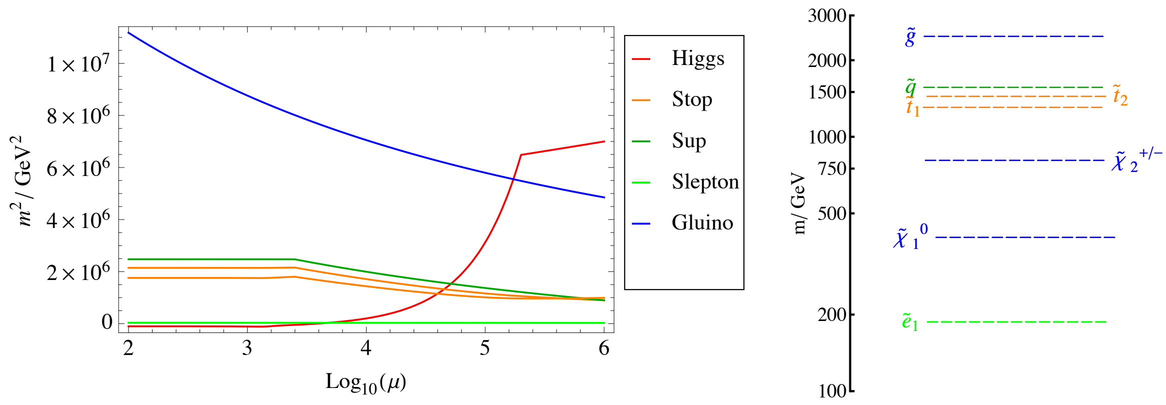

Figure 6.4: **Left:** The RG flow of the soft masses squared in the theory described in the text. Only the Higgs mass squared operator gains a large anomalous dimension in the strong coupling region, and is consequently suppressed. The mediation scale is low, and the sfermions mass ratio at the UV boundary of the RG flow is that of standard gauge mediation. The gauginos masses fall into the GUT pattern, but are assumed to be independent of the sfermion and Higgs masses. **Right:** The weak scale soft masses in the same theory. The UV boundary parameters of the theory are such that the sfermion masses are close to current LHC limits.

mass operators could actually become negative in the strong coupling regime. This would lead to an *enhancement* of the sfermion masses during the strong coupling region, taking them far out of LHC reach without introducing significant fine tuning. It could be that all the sfermions receive enhanced masses, or alternatively just the first two generation sfermions might be enhanced. The later could occur, for example, if these generations are charged under an additional broken gauge group. It is possible such a structure could be linked to the fermion mass hierarchy, in the style of the classic natural SUSY spectra [159, 160].[5] This breaking of a gauged flavour group, could even trigger the supersymmetry breaking sector to run into strong coupling, especially since the SU(2) structures which arise in the model of [297] often appear in interesting candidates for conformal theories.[6]

Assuming all sfermion soft masses are enhanced, it is possible to obtain a model with a weak scale gluino mass of 3 TeV, and sfermion masses in the region of 7 TeV, with an associated tunings of only 25 and 50 with respect to the sfermion and gluino masses respectively. Overall, the tuning of the theory is comparable to the most natural traditional models compatible with collider bounds, even though the scalars are very heavy with a spectrum reminiscent of mini-split supersymmetry [217]. Additionally, as seen from Eq. (2.39), in the models presented here, it is fairly straightforward to obtain a physical Higgs mass of 125 GeV through radiative corrections from the fairly heavy stops.

[5] A similar mechanism for generating a natural SUSY spectrum has been studied in [319].
[6] We are grateful to Matthew McCullough for this observation.

While we have studied hidden sectors with rather dramatic effects on the RG flow, it is also plausible that the fine tuning may be reduced substantially even in a model where the hidden sector does not become strongly coupled, or where the strong coupling does not occur close to the EW scale. For example, a weakly coupled hidden sector could modify the visible sector running in such a way that additional cancellations between the various contributions to the Higgs soft mass appear. This is somewhat analogously to 'focus point' models [323], and relies on the careful analysis of different tuning measures emphasised in [152]. It would be interesting to find examples of theories where this could occur.

6.3 Model Building

We now discuss the possibilities for finding models with the features assumed in the previous section. Finding and studying explicit examples of non-supersymmetric theories which pass close to an interacting conformal fixed point is hard, however several examples are believed to exist. In fact, these types of model have been studied extensively in the context of walking technicolour [324, 325], and it may even be easier to find supersymmetric theories with the appropriate dynamics, for example [326]. Of course it would be highly beneficial to have an example of a complete theory with all the required properties, however in this work we simply argue that such theories may be plausibly realised.[7]

Low-Scale SUSY-Breaking

The simplest implementation of hidden sector renormalisation reducing fine tuning arises in models with low-scale breaking and mediation. In such models, SUSY breaking occurs at approximately the same scale as the strong coupling region, and there is the potential to link these two events, for example running to strong coupling could trigger spontaneous SUSY-breaking as happens in a number of known models.

Typically, due to loop factors that arise in the gauge mediation to the visible sector, $\sqrt{F}$ must be above the weak scale and the lower limit of the strong coupling region. The hidden sector, and SUSY-breaking multiplet, can easily remain dynamical below this scale if some fields in the hidden sector have masses suppressed by loop factors or small coupling constants. For example, the dominant F-term in the theory can arise as an expectation value of a scalar field which receives a mass only at loop order. Additionally SUSY breaking masses

[7]It may be possible to study explicit theories using the techniques of general gauge mediation [136].

that appear in the hidden sector can be somewhat removed from $\sqrt{F}$. If this is the case, the scale of these masses may be close to the end of the strong coupling region. Consequently, at least some of the interactions in the strong coupling region can remain approximately supersymmetric, and results from holomorphy may remain accurate. Alternatively, there may exist mediation mechanisms which do not lead to loop factors, so that $\sqrt{F}$ can be close to the weak scale, and the hidden sector remains supersymmetric during strong coupling. Conversely, the hidden sector may be non-supersymmetric for some or all of the strong coupling region, which is not problematic but does require the extra assumption that the anomalous dimensions of the gaugino mass operators are small.

In models with low-scale mediation, the RG flow of the SM gauge couplings is not necessarily altered; the strong coupling region may only affect matter in the SUSY-breaking sector, which is uncharged under the SM gauge groups. SUSY-breaking messengers, charged under the SM gauge group, might not experience strong coupling depending on the dynamics of their couplings to the hidden sector. Even if the RG flows are modified, gauge unification can be maintained provided the matter content and couplings of the messenger sector are GUT compatible.

To clarify some of these issues we consider the example of the ISS model, previously described in Section 2.8. Although we do not provide a full model (for example, a mediation mechanism) or calculation of anomalous dimensions, this is a good candidate for a theory which might remain strongly coupled for an extended energy range if the theory is close to the edge of the 'conformal window', that is, F is close to $\frac{3}{2}N$ [326]. As usual, the UV description consists of SQCD with massive quarks, of mass m, in the window $N+1 \leq F < \frac{3}{2}N$. The magnetic theory superpotential is of the form given in Eq. (2.52). Typical supersymmetric and non-supersymmetric masses in the theory are $\sim h\mu$, while directions which are pseudo-moduli (including the scalar components of the multiplets that obtain F-terms) only obtain masses at one loop of size $\sim h^2\mu$.

Since it is not fixed by the duality, we suppose the constant h happens to be small in a particular theory. In this case the hierarchy of masses is as shown in Fig. 6.5, and the theory has a significant separation between the onset of the strong coupling region, the scale of the F-terms in the theory and the masses of the fields, as might be expected to occur if the theory is close to the conformal window. Supersymmetry breaking occurs somewhat after the beginning of the strong coupling region. The masses of states in the sector are

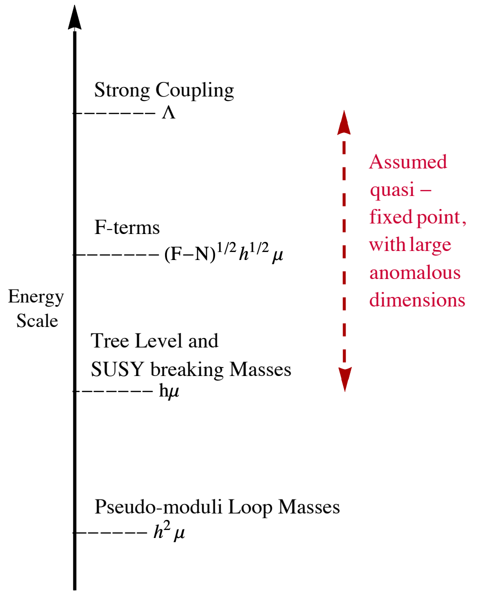

Figure 6.5: The mass scales in the ISS model, which may (once a complete model, including messenger sector, is specified) be a candidate for a theory with low-scale mediation and significant hidden sector renormalisation. The parameters are as defined in the text.

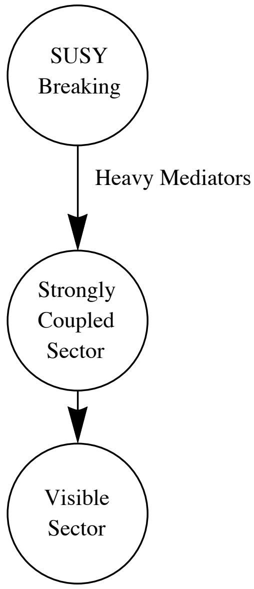

Figure 6.6: A schematic setup which could lead to hidden sector renormalisation, near the weak scale, in a model with high-scale mediation. The strong coupling region may be approximately supersymmetric depending on the interactions between the sectors.

suppressed relative to the SUSY-breaking scale, and therefore the theory remains dynamical until these masses are reached. Additionally, above the scale of the soft masses the theory some interactions of the theory may remain approximately supersymmetric, even though the energy scale is below $\sqrt{F}$. Below the soft masses the theory is non-supersymmetric but remains dynamical until the masses of the lightest states are reached. Consequently, although we have certainly not analysed this theory properly, it is at least a reasonable candidate for giving significant hidden sector renormalisation.

High-Scale SUSY-Breaking

It is also possible to build models with high-scale SUSY breaking and mediation. This case is slightly different, since the SUSY-breaking sector, which is typically dynamical only for a few orders of magnitude below $\sqrt{F}$, cannot be the strong coupling sector. However, hidden sector renormalisation can reduce fine tuning if the messenger sector of the theory is more complicated than usual, and becomes strongly coupled near the weak scale. While the setups involved may seem more contrived than the low-scale case, they are still interesting.

Suppose the theory is as shown in Fig. 6.6. The visible sector is coupled to the supersymmetry breaking sector indirectly, through a sector with light states which themselves couple

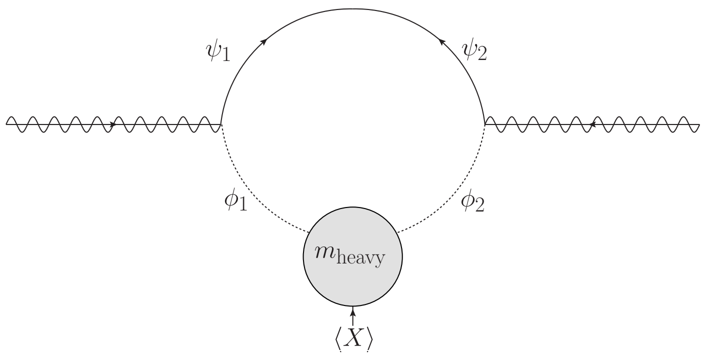

Figure 6.7: A diagram that can lead to visible sector gaugino masses in a model with high scale SUSY breaking. The fields $\psi_{1,2}$ and $\phi_{1,2}$ are the fermions and scalars in the messenger chiral multiplets $\Psi_{1,2}$ and have masses near the TeV scale. Even though the SUSY-breaking scale is high, the messenger multiplets may remain approximately supersymmetric and dynamical until close to the weak scale. Consequently hidden sector renormalisation can reduce the theory's EW fine tuning. Visible sector scalar soft masses squared are be generated through higher loop-order diagrams.

to the SUSY-breaking sector through heavy mediators. If the sector containing light states becomes strongly coupled, the visible sector soft mass operators can still gain large anomalous dimensions. The strong coupling sector is supersymmetric until a scale $\frac{F}{m_{\text{med}}}$, which can be close to the visible sector soft masses and the weak scale. Depending on the model, the strong coupling region may or may not be supersymmetric. A particularly attractive scenario is if the soft masses in the light messenger sector cause the sector to leave the strong coupling regime, so that both are close to the weak scale in a correlated fashion. Depending on the details of the particular model, the scale of mediation can be high (near the mass of the heavier set of messengers), or low, around the mass of the lighter messengers.[8] In these models the RG flow of the SM gauge couplings will be altered, since the messengers, which are charged under the SM gauge groups, gain large anomalous dimensions which affect the beta functions through Eq. (6.4).[9] However, gauge unification can persist if the couplings and matter content respect an underlying GUT structure.

For example, suppose a theory contains messenger chiral superfields, Ψ_1 and Ψ_2, charged under the SM gauge groups, with a supersymmetric Dirac mass, m_{light}, of order 10 TeV,

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \, m_{\text{light}} \Psi_1 \Psi_2 \ . \tag{6.8}$$

The SUSY-breaking sector is parameterised by a spurion X, which obtains an F-term $\langle X \rangle = \theta^2 F$. Further, assume the theory contains heavy fields with typical masses of m_{heavy}, which

[8]The former resembles ordinary gauge mediation, in which visible sector scalar soft masses are generated at the messenger mass scale, despite coupling to the messengers through light gauginos and gauge bosons.

[9]This will also slightly modify the RG flow of the physical gaugino masses, but does not have a significant effect on the fine tuning of the theory.

couple the messengers to the SUSY-breaking sector for example through some new gauge group, although we do not specify the interactions. These induce SUSY-breaking masses of typical size $\frac{F}{m_{\text{heavy}}}$ in the messenger scalars. Soft masses are generated in the visible sector, through diagrams of the form of Fig. 6.7 for gauginos and similar diagrams for the scalar masses. Since the messenger multiplets have masses near the weak scale, they remain dynamical down to low-energy scales. If their interactions are such that they run into a strong coupling regime near the weak scale (for example, if they are charged under some additional gauge group that becomes strongly coupled) the visible sector soft mass operators can obtain a large anomalous dimensions, leading to hidden sector renormalisation.

Other model building possibilities

While we have focused on the benefit of hidden sector renormalisation for fine tuning, there are also potential benefits for building models of SUSY-breaking. A very common issue encountered in theories with dynamical SUSY-breaking is generating weak scale gaugino masses which are not suppressed relative to sfermion masses, leading to unacceptably large tuning if collider constraints are to be satisfied. Suppressed gaugino masses appear because an R-symmetry is a necessary condition for SUSY to be spontaneously broken [126]. Such an R-symmetry however forbids gaugino masses, and even if it is spontaneously broken it is often not broken strongly enough. Alternatively, a metastable SUSY breaking vacua may be obtained in a sector with an approximate R-symmetry. Even here, obtaining both a sufficiently long lived vacua and heavy enough gauginos, makes model building challenging [127]. The theories initially studied, with all chiral multiplets getting a large anomalous dimension in the strong coupling region allows for scalars to start off heavy, yet end up lighter than the gauginos at the weak scale, alleviating this problem without introducing fine tuning.

Finally, if Dirac gauginos are combined with hidden sector renormalisation, in such a way that the theory is in strong coupling for most of the energy region between the masses of the sgauge and gauginos, there is a double suppression of the tuning. It is possible to obtain gluino masses of order 10 TeV without any appreciable fine tuning. While it is remarkable that there may be such little tuning from this sector of the theory, this does not greatly improve the overall fine tuning of the theory. Even with the extended set-ups to reduce sfermion fine tuning discussed in this section, the overall tuning of the theory is still typically

20 due to the initial Higgs soft mass and μ parameter required to obtain the correct EW VEV.

There is an interesting additional feature in models with Dirac gauginos that may be helpful with model building, the visible sector retains an R-symmetry. Suppose the theory has an unusual mediation structure such that $\sqrt{F}$ is below the strong coupling scale. Then the hidden sector passes close to a superconformal fixed point, and necessarily has an almost exact R-symmetry [262, 327]. This can be identified with the R-symmetry in the visible sector, allowing the anomalous dimensions of operators to be evaluated.

Chapter 7: Concluding remarks

In this work we have studied the properties of so called natural SUSY theories, in which only the third generation sfermions, charginos, and neutralinos are light. In Section 3, we built a model of SUSY breaking and mediation that leads to a viable natural SUSY spectrum. While many such models have previously been constructed, the model here has a number of attractive features. In particular, we proposed that a single, spontaneously broken, U(1) gauge symmetry may be responsible for suppressing both the first two generation Yukawa couplings, and also, in a correlated manner, parameters in the dynamical SUSY breaking sector. In the dynamical SUSY-breaking sector, these small parameters are typically required to introduce R-symmetry breaking in a controlled manner and obtain phenomenologically viable meta-stable vacua. The heavy U(1) multiplet mediates a dominant contribution to the first two generation MSSM sfermion soft masses, while gauge mediation provides a parametrically suppressed soft term contribution to the stop and most other states, so realising a natural SUSY spectrum.

However, a Higgs boson as heavy as 125 GeV can be difficult to explain in natural SUSY spectra, due to the relatively light stops. The NMSSM is an attractive framework for raising the Higgs mass. We studied this in Section 4, and found that for very light stop masses, $\lambda_0 \gtrsim 0.7$ is required to obtain the desired Higgs mass and for such large values of λ at the weak scale the coupling will generally become strongly coupled before unification. A coupling becoming strongly coupled before unification raises the concern that successful gauge coupling unification may be adversely affected. However, on the contrary, we argued that gauge coupling unification could actually be improved given a short period of strong coupling. In these advantageous cases, the strong coupling regime corresponds to a threshold effect of sign and size expected to be of the right order to correct the current 3% discrepancy between the two-loop MSSM prediction for $\alpha_s(m_Z)$ and its measured value. Moreover, we argued that in scenarios where $\gamma_{H_u} < \gamma_{H_d}$, a period of strong coupling could also be beneficial for $t-b$ unification.

Given that the motivation for natural spectra is to obtain low EW fine tuning, it is interesting to study the extent to which this can actually be realised. In Section 5, we carried out a careful study of the fine tuning in such theories, improving previous approximation expressions. We obtained lower bounds on the fine tuning of theories for a given gluino mass. For models with high-scale mediation, if there is a Majorana gluino mass of 2 TeV the fine tuning is at least 400, and only constrains the UV stop mass to be below 1.5 TeV. After running to the weak scale, the stop mass can be up to 2 TeV without affected the fine tuning, and the largest regions of parameter space with the lowest fine tuning have fairly heavy IR stop masses of 1.5 TeV. Models with low scale mediation and a 2 TeV Majorana gluino have a fine tuning of at least 50, and the UV stop mass is constrained to be below 500 GeV. After running, the regions with the lowest fine tuning have IR stop masses up to 1.4 TeV.

Consequently, models compatible with LHC bounds can typically raise the Higgs mass to 125 GeV in the regions where they have the lowest fine tuning, either through stop loop corrections (with significant A-terms), or in NMSSM theories without couplings that run non-perturbative. Additionally, in both high- and low-scale mediation models, the masses of the first two generation sfermions may be made very large, far out of reach of the LHC, without introducing additional fine tuning to the theory. We also discussed fine tuning in models of Dirac gluinos. These are found to allow for spectra with moderate fine and significant separation of the gluino and stops, comparable to MSSM theories with very low scale mediation, even if the scale of mediation is high.

Finally in Section 6, we considered the effect of relaxing the usual assumption that the RG flow of the soft masses is fixed by the visible sector matter and interactions. It was found that the EW fine tuning can be significantly reduced by the effect of hidden sector renormalisation in models where the SUSY breaking sector runs through an extended period of strong coupling not far from the weak scale. In the simplest implementation, the fine tuning with respect to the gluino mass may be reduced from order 1000 to order 10, however there is no improvement in the tuning with respect to the sfermion masses. More complicated models, where the Higgs has additional couplings to the SUSY breaking spurion, may reduce the tuning with respect to all parameters of the theory to the region of 20. In this work we simply parameterised the hidden sector through assumed values of anomalous dimensions, and it is clearly an important challenge to build models of the hidden sector with appropriate dynamics.

Appendix A: Appendix A

A.1 Source of the Contact Operator

In this appendix we provide a justification for the interactions Eqs.(3.10) (3.18) that are crucial to our model in Section 3. In the pure field theory case this is obtained by integrating out the heavy gauge multiplet, as discussed in [190] directly leading to an effective term in the Kähler potential

$$\mathcal{L} \supset -\sum_{i,j} \frac{g^2 q_i q_j}{m_{Z'}^2} \int \mathrm{d}^4\theta \; \phi^{\dagger i}\phi_i \phi^{\dagger j}\phi_j \ , \tag{A.1}$$

where ϕ_i and ϕ_j are any fields charged under the gauge symmetry. Alternatively this can be regarded as the vector multiplet gaining a D-term. In the Stueckelberg case, the interaction can also be understood in this way, however it is interesting to also understand it directly from the Lagrangian. For a vector multiplet V, which gains a mass M through interaction with a Stueckelberg field S, and is coupled to hidden sector fields Φ and MSSM fields Q this is given by:

$$\int \mathrm{d}^4\theta \left(\Phi^\dagger e^{q_\Phi g V}\Phi + Q^\dagger e^{q_Q g V} Q + M^2 \left(V + \frac{1}{M}\left(S - S^\dagger\right)\right)^2 \right) \ . \tag{A.2}$$

In the limit that M is much greater than any other mass scale in the theory, V may be integrated out by solving $\frac{\partial K}{\partial V} = 0$ with solution $V = -(g/2M^2)\left(q_\Phi \Phi^\dagger\Phi + q_Q Q^\dagger Q + (S - S^\dagger)/gM\right)$. Inserting this back into the original Kähler potential leads to

$$\int \mathrm{d}^4\theta \left(-\frac{g^2 q_\Phi q_Q}{M^2} \Phi^\dagger\Phi Q^\dagger Q \right) \ . \tag{A.3}$$

The bilinear dependence of these terms on the charges justifies the claim in the text that for a suitable charge assignment a positive mass^2 contribution to sfermion masses arises and also that a sign difference in the charge of Q_3 relative to $Q_{1,2}$ can lead to mass terms of the opposite sign. In the string context we expect $M \sim gM_*$ which results in the dependence on the gauge coupling dropping out.[1] Finally, higher terms in the expansion occur, e.g.,

$$\int \mathrm{d}^4\theta \left(\frac{g^2 q_\Phi^2 q_Q}{M^4} \Phi^\dagger\Phi^\dagger\Phi\Phi Q^\dagger Q \right) \ . \tag{A.4}$$

Such terms, however, are harmless in the relevant parameter range for our discussion.

[1] The limit $g \to 0$ is obscured by these terms as the mass scale $M \sim gM_*$ is no longer large. In the leading and sub-leading terms we have dropped a overall model-dependent coefficients.

A.2 Gauge Mediated Contribution to Soft Masses

In Sections 2 and 3 an approximate expression for the gauge-mediated contribution to soft masses was quoted. Here we give the precise formulae as used in our numerical studies and figures in Section 3.

The two contributions, Eq. (3.5) and Eq. (3.10) in the Polonyi case, and Eq. (3.15) and Eq. (3.18) in the ISS case, result in a messenger scalar mass matrix (assuming Ψ and Ψ^c have the same U(1) charge; the analysis is straightforwardly extended to other cases) of the form

$$\mathcal{L} \supset \begin{pmatrix} \tilde{\Psi}^\dagger & \tilde{\Psi}^{c\dagger} \end{pmatrix} \begin{pmatrix} m_{\text{mess}}^2 + m_K^2 & F_{eff} \\ F_{eff} & m_{\text{mess}}^2 + m_K^2 \end{pmatrix} \begin{pmatrix} \tilde{\Psi} \\ \tilde{\Psi}^c \end{pmatrix} , \tag{A.5}$$

where m_{mess} is the supersymmetric mass of the multiplet, m_K is the mass due to the Kähler interactions, and F_{eff} is the effective F-term felt by messengers through the superpotential.

Hence, the messenger scalar mass eigenstates are given by $\frac{1}{\sqrt{2}}\left(\tilde{\Psi} \pm \tilde{\Psi}^c\right)$ with masses $m_{1,2}^2 = (m_{SUSY}^2 + m_K^2 \pm F_{eff})$, while the messenger fermion masses are simply $m_f = m_{SUSY}$. Gaugino masses are generated through the normal diagrams of gauge mediation, and are given by

$$m_{\lambda i} = \frac{\alpha_i}{4\pi} n_m m_f \left(\left(\frac{m_1^2}{m_1^2 - m_f^2} \right) \log \left(\frac{m_1^2}{m_f^2} \right) - \left(\frac{m_2^2}{m_2^2 - m_f^2} \right) \log \left(\frac{m_2^2}{m_f^2} \right) \right) , \tag{A.6}$$

which reduces to that commonly found through analytic continuation [89] in the limit $F_{eff} \gg m_K^2$ and $F_{eff} \ll m_{SUSY}^2$. However, for the values of parameters we are interested in, these conditions are not satisfied and the full expression (A.6) is required.

The contribution to sfermion masses from gauge mediation with these messenger masses is given by

$$m_{\text{gauge}}^2 = \sum_i \left(\frac{\alpha_i}{4\pi} \right)^2 C_i \, n_m \left[g(m_1, m_2, m_f) + h(m_1, m_2, m_f, \Lambda_{\text{UV}}) \right] . \tag{A.7}$$

Here C_i is the quadratic Casimir of the scalar and

$$h\left(m_1, m_2, m_f, \Lambda_{\text{UV}}\right) = -\left(2m_1^2 + 2m_2^2 - 4m_f^2\right) \log \left(\frac{\Lambda_{\text{UV}}^2}{m_f^2} \right) , \tag{A.8}$$

while $g(m_1, m_2, m_f)$ is the contribution from the normal diagrams of gauge mediation whose lengthy explicit form is given in [134]. As also discussed in [134], the non-vanishing supertrace of the messenger sector results in a negative contribution, described by $h\left(m_1, m_2, m_f, \Lambda_{\text{UV}}\right)$. due to the need to include a counterterm for ϵ-scalar masses in dimensional reduction. The scale Λ_{UV} is the mass at which additional states charged under the MSSM gauge group appear resulting in a vanishing supertrace. For our theories this is naturally $\Lambda_{\rm UV} = M_*$. Due to their large mass these extra states do not contribute significantly to MSSM masses as messengers of gauge mediation.

Unlike gaugino masses, sfermions gain significant masses from gauge mediation even when the messenger masses are dominated by the diagonal Kähler contribution. The reason for this is clear: the Kähler contribution is effectively a D-term mass and does not break an R-symmetry, which protects gaugino masses but not sfermion masses.

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