

# Stress Testing and Bank Lending

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Stress tests convey information about the strictness of future tests, creating incentives for banks to alter their future lending behavior. Regulators recognize and use this influence: they may conduct softer stress tests to encourage lending or tougher stress tests to reduce risk-taking. This information management can lead to inefficiencies when (a) the test loses credibility or (b) the test becomes self-fulfilling. In addition, banks may distort their lending behavior in anticipation of the stress test design, leading to further surplus losses. The analysis applies to banking supervision and regulation more broadly. (*JEL* G21, G28)

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Stress tests, a new policy tool for bank regulators, were first used in the recent financial crisis and have become regular exercises since then. They assess a bank's ability to withstand adverse shocks and are generally accompanied by measures to encourage banks found to be at risk to boost their capital.

Naturally, bank behavior changes in response to stress tests. [Acharya, Berger, and Roman \(2018\)](#) find that all banks that underwent the U.S. SCAP and CCAR tests reduced their risk by raising loan spreads and decreasing their commercial real estate credit and credit card loan activity.<sup>1</sup>

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<sup>1</sup> [Brauning and Fillat \(2020\)](#), [Berrospide and Edge \(2019\)](#), [Connolly \(2017\)](#), [Cortés et al. 2020](#); [Calem, Correa, and Lee \(2020\)](#), and [Konietschke, Ongena, and Marques \(2022\)](#) have similar findings.

Regulators must take banks' reactions into account when conducting the tests. One might posit that if regulators want to boost lending, they might make stress tests softer. Indeed, in the case of bank ratings, Agarwal et al. (2014) show that state-level banking regulators gave banks higher ratings than federal regulators (because of concerns over the local economy), which led to more bank failures. Baudino (2020) documents the postponement of stress test exercises by the ECB and the Bank of England with the onset of the COVID-19 pandemic in 2020; she notes that an *ad hoc* replacement stress test by the Bank of England, "encourage[d] banks to support lending."

In this paper, we study the interaction between stress testing and bank lending. Banks may take too much risk or not lend enough. Regulators anticipate this by designing a stress test that is either tough or soft. This is a form of reputation management that may be efficient if it incentivizes banks to take actions more in line with social objectives. However, these incentives may lead to surplus losses when (a) the test loses credibility or (b) the regulator gets trapped in a self-fulfilling reputation-building exercise. Further losses may arise due to the bank manipulating its lending choice in anticipation of the stress test design. Losses result from excess default, excess recapitalization, or inefficiently low levels of lending to the real economy.

In the model, there are two sequential stress test exercises. For simplicity, one bank is tested in both exercises. Each period, the bank decides whether to originate a risky loan or to invest in a risk-free asset. The regulator can learn the quality of the risky loan and may set up a lending facility (which we refer to as "failing" the stress test). This action is observable. Depositors will draw inferences from it and decide whether to run. The possibility of a run provides discipline for the bank, as the bank would prefer to draw on the lending facility and recapitalize rather than face a run.

The regulator may be one of two types: high cost or low cost (of supporting banks in distress). The regulator knows its type, while all other agents are uncertain about it. Both types are strategic and maximize social surplus. Failing a bank incurs the cost of recapitalization but generates the benefit of reducing costly default. After the first stress test result, the bank updates its beliefs about the regulator's type, decides whether to make a risky loan, and undergoes a second stress test. Thus, the regulator's first stress test serves two purposes: to possibly induce the bank to boost its capital in the first period *and* to signal the regulator's willingness to induce the bank to raise capital in the second period.

Banks may take too much or too little risk from the regulator's point of view. On the one hand, the bank may take too much risk, as it doesn't internalize the externalities that a default would impose. On the other hand, the bank's owners may take too little risk to avoid the risk of a run/recapitalization.<sup>2</sup>

<sup>2</sup> Thakor (1996) provides evidence that the adoption of risk-based capital requirements under Basel I and the passage of FDICIA in 1991 led banks to shift their portfolios away from risky lending and toward Treasury investments, potentially prolonging the economic downturn.

The regulator faces a natural trade-off in conducting the first stress test:

First, the regulator may want to build a reputation for being soft in order to increase the bank's lending in the second period. There are "soft" equilibria in which the regulator attempts to create the perception of being high-cost (and therefore being soft in the second stress test) by passing banks that would normally fail. This is reminiscent of the EU's 2016 stress test, which eliminated the pass/fail grading scheme, found only one bank to be undercapitalized, and allowed one bank to circumvent one of the rules of the test.<sup>3</sup>

Second, the regulator may instead want to build a reputation for being tough in order to prevent future excessive risk-taking. The regulator may build the reputation of being low-cost (and therefore being tough in the second stress test) by failing banks that would normally pass. The United States has routinely been criticized for being too tough: imposing very adverse scenarios; not providing the stress test model to banks; accompanying the test with asset quality reviews; and conducting qualitative reviews, all of which combine to create a stringent test.<sup>4,5</sup>

Finally, there are other types of equilibria. There is an equilibrium in which the regulator doesn't engage in reputation building. In this equilibrium, the regulator grades the bank in accordance with the bank's quality, as in a one-period model. This occurs when the prior of the regulator's type is extreme; agents are convinced that the regulator is of one type so that there is no scope for reputation management. There is also an equilibrium in which the regulator always passes the bank, providing no information about the bank or the regulator's type.<sup>6</sup> This is an equilibrium in which the regulator is unable to credibly transmit information.

While reputation building may increase surplus by allowing the regulator to influence bank lending, it may also lead to surplus losses from the strategic interactions it engenders.

The first surplus loss is due to the presence of the uninformative equilibrium in which the regulator passes the bank with certainty. This exists when reputation building destroys the stress test's credibility; depositors don't respond to a failing grade by withdrawing, thus removing the discipline that enables a fail to result in a recapitalization of the bank. This leads to a strict loss of surplus, as without the possibility of reputation building, the equilibrium

<sup>3</sup> The bank found to be undercapitalized, Monte dei Paschi di Siena, had already failed the 2014 stress test and was well known by the market to be in distress. It was also revealed that Deutsche Bank was given an exception to the stress test rules, resulting in it appearing to have more capital (Noonan, Binham, and Shoter 2016). We note that the 2018 test also had no pass/fail requirement, and all banks tested were judged to be well capitalized.

<sup>4</sup> A discussion of this and the recent tilt toward leniency is in Schroeder and Price (2017).

<sup>5</sup> There are also equilibria where the high-cost regulator is tough and the low cost regulator is soft, as their incentives differ toward what they prefer to happen in the second period.

<sup>6</sup> If the social benefit of lending is very low, we also demonstrate the possible existence of a money-burning equilibrium, in which the regulator does not influence withdrawals/recapitalization but may still offer a costly stress test with a positive probability of failing in order to signal the regulator's type.

would be informative, and the regulator would be able to induce very risky banks to recapitalize.

The second surplus loss arises when the reputation-building equilibrium is self-fulfilling. The equilibrium features wasteful attempts at signaling by the regulator. This results in losses in the first period due to inefficient recapitalization of the bank and in the second period due to the bank's inefficient lending decision, which reflects the bank's belief about how the regulator will conduct the second stress test. Moreover, this equilibrium always coexists with another equilibrium with higher surplus: an equilibrium in which the regulator does not build reputation and chooses its static optimum in both periods. This means that coordination failure can lead to a loss of surplus.

The third surplus loss comes from the bank manipulating its lending choice in anticipation of the stress test design. While the regulator may be managing its reputation so as to influence the bank's subsequent choice and increase surplus, the bank understands these incentives *ex ante* and may distort its lending before the first stress test.

The first and second surplus losses are “policy traps”<sup>7</sup>: situations in which a regulator with an objective to maximize social welfare becomes trapped into inefficient actions by the interaction of signaling and beliefs (e.g., [Morris 2001](#)).

All of the surplus losses point to possible gains from limiting the regulator's reputation concerns when setting stress testing criteria. In practice, this may be difficult to achieve, as pressures on regulators may change. Banking regulators do attempt to restrict their choices by announcing stress test scenarios in advance and undertaking costly audits of bank data (e.g., asset quality reviews).

We also look at three extensions. First, when the social benefit of lending is low, stress tests may be money-burning exercises that reveal no information about the bank. Second, we add the possibility that the regulator's lending facility may not be set up in time, which allows bank runs to happen on the equilibrium path. The key results of the paper remain, though the regulator may display more caution in failing banks. Third, we allow the regulator to force the bank to recapitalize. In this situation, the credibility issue the regulator faces in the main model disappears.

The model applies to banking supervision and regulation more broadly. The key element of the model is that the regulator has more information than the other agents. There are two types of informational advantage in the model:

- Information about regulatory costs: Given that (a) increased lending may come with risk to the economy, and (b) bank distress may have systemic consequences, there is ample motivation to keep this information/intention private. This uncertainty may also arise from

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<sup>7</sup> [Angeletos, Hellwig, and Pavan \(2006\)](#) coined this term. Their environment (and the “traps”) differ as they look at policies that convey information that is relevant for a coordination game.

the political process. Decision-making may be opaque, bureaucratic, or tied up in legislative bargaining. [Morrison and White \(2013\)](#), [Boot and Thakor \(1993\)](#), and [Shapiro and Skeie \(2015\)](#) demonstrate that superior information by a banking regulator may affect forbearance, bank closures, and bailouts.

- Information about bank risk: The regulator could have generated private information from close supervision of banks. Banking supervisors in the United States have private rating systems and their actions have been found to affect bank risk-taking, much along the lines of the stress tests we discuss in the paper.<sup>8</sup> Another possible source of private information comes from having done a stress test on many banks. In this case, the regulator may have gathered more information on asset values and liquidity than any individual bank has. Given this, the regulator may understand more about systemic risk and tail risk (not modeled here). This is an element of the macroprudential role of stress tests.<sup>9,10</sup>

There is little direct evidence, but much indirect evidence, of banking regulators behaving strategically. The variance in stress test results to date seem to support the idea of regulatory discretion.<sup>11</sup> Beyond [Agarwal et al. \(2014\)](#), cited above, [Bird, Karolyi, and Ruchti \(2023\)](#) show that U.S. stress tests were soft toward large banks and tough on poorly capitalized banks, affecting bank equity issuance and payout policy. The Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was interpreted as a suggestion that the bank lower its Libor submissions.<sup>12</sup> [Hoshi and Kashyap \(2010\)](#) and [Skinner \(2008\)](#) discuss accounting rule changes that the government of Japan used to improve the appearance of its financial institutions during the country's crisis.<sup>13</sup>

<sup>8</sup> [Hirtle and Kovner \(2022\)](#) survey the literature on bank supervision in the United States, while [Eisenbach, Lucca, and Townsend \(2022\)](#) and [Hirtle, Kovner, and Plosser \(2020\)](#) are recent contributions that highlight these issues. They note connections between supervision and risk-taking, though do not find much of a connection between supervision and lending.

<sup>9</sup> [Leitner and Yilmaz \(2019\)](#) examine the possibility that banks may distort the information the regulator gathers; for simplicity, we do not model this effect.

<sup>10</sup> In an earlier version of this paper, [Shapiro and Zeng \(2021\)](#), using a somewhat different model, we demonstrate that removing the information asymmetry about bank risk does not affect the results qualitatively.

<sup>11</sup> The 2009 U.S. SCAP was widely perceived as a success ([Goldstein and Sapra 2014](#)), with subsequent U.S. tests retaining credibility. European stress tests have varied in perceived quality ([Schuermann 2014](#)), with the early versions so unsuccessful that Ireland and Spain hired independent private firms to conduct stress tests on their banks.

<sup>12</sup> The CEO of Barclays wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appeared in the *Financial Times* ([Masters, Parker, and Burgess 2012](#)).

<sup>13</sup> Nevertheless, stress tests do contain significant information that is valued by markets. ([Flannery, Hirtle, and Kovner \(2017\)](#) demonstrate this and survey recent evidence.).

Several recent theoretical papers tackle stress tests.<sup>14</sup> Quigley and Walther (2022) show that more disclosure by a bank regulator decreases the amount of information that banks provide to the public, and that the regulator may take advantage of this to stop runs. Bouvard, Chaigneau, and de Motta (2015) show that transparency is better in bad times and opacity is better in good times. Goldstein and Leitner (2018) find a similar result in a very different model, in which the regulator is concerned about risk sharing (the Hirshleifer effect) between banks. Williams (2017) looks at bank portfolio choice and liquidity in this context. Orlov, Zryumov, and Skrzypacz (2023) show that the optimal stress test will test banks sequentially. Faria-e-Castro, Martinez, Philippon (2017) demonstrate that stress tests will be more informative when the regulator has a strong fiscal position (to stop runs). In contrast to these papers, in our model, the stress test works through costly signaling (setting up a lending facility) and reputational incentives rather than communication through commitment to a disclosure rule.<sup>15</sup> In addition, we allow banks' endogenous choice of risk to be a key element of stress testing.<sup>16</sup>

Our paper is related to the literature on bad reputation (Morris 2001; Ely and Valimaki 2003; Ely, Fudenberg, and Levine 2008) – where the reputation concerns of a long-run player distort its actions and can lead to inefficiencies in the form of exit by a short-run player. Our paper is similar in the sense that the long-run player (the regulator) has private information on two dimensions (here, the bank's riskiness and the regulator's own cost of setting up a lending facility). However, in our model, the short-run player's (the bank's) choice is not about exit (it is an investment choice), and there is no "bad" type of regulator as neither regulator type has a conflict of interest; both regulators want to maximize social welfare. In the aforementioned papers, inefficiency results from the good type trying to separate from the bad type, distorting the short-run player's action in anticipation. In our model, separating incentives do not result in distortions in the bank's first investment decision. When the benefit of risky lending is high (low), both regulator types run a soft (tough) test, which (a) implies that one regulator type is separating and one is mimicking, and (b) results in the bank choosing the risky asset (safe asset), which can be surplus enhancing.<sup>17</sup> In contrast, pooling incentives for the regulator types (which is

<sup>14</sup> A few papers investigate regulatory disclosure. Goldstein and Sapra (2014) survey the disclosure literature to describe the costs and benefits of information provision for stress testing. Prescott (2008) argues that more information disclosure by a bank regulator decreases the amount of information that banks provide to the regulator. Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2015) analyze government interventions that rely on and endogenously determine market information.

<sup>15</sup> Quigley and Walther (2022) and Bouvard, Chaigneau, and de Motta (2015) do not have commitment or reputation.

<sup>16</sup> In Leitner and Williams (2023) and Dogra and Rhee (2021), the regulator chooses a disclosure rule, and banks react by altering their risk profile.

<sup>17</sup> Note that when reputation building/separation in our model is self-fulfilling, there may be distortions in the bank's investment. However, the self-fulfilling nature of the reputation building is not present in the "bad reputation"

often present in “good reputation” models) can lead to distortion in the bank’s investment decision.<sup>18</sup>

## 1. The Model

We consider a model with three risk-neutral agents: the regulator, the bank and a depositor. The model has two periods  $t \in \{1, 2\}$ , and the regulator conducts a stress test for the bank in each period.

We now provide a very basic timeline of each period. In each period  $t$ , where  $t = \{1, 2\}$ , there are four stages:

1. The bank raises deposits and makes an investment choice.
2. The regulator conducts the stress test and recapitalization may occur.
3. The depositor makes the withdrawal decision.
4. Payoffs realize.

In the following subsections, we discuss each aspect in detail: the bank, the stress test, recapitalization, the preferences of the regulator, and the regulator’s reputation.

### 1.1 The bank

We assume that, in each period, the bank is managed by a myopic banker, whose objective is to maximize the expected profits of the bank in that period. The myopia may be due to compensation oriented toward short-term results or pressure from short-term-oriented shareholders, such as hedge funds. In any case, this (a) simplifies the banker’s problem in period 1 and (b) allows for a welfare comparison between periods 1 and 2.<sup>19</sup>

We now examine the bank’s choice in each period.

At stage 1, the bank raises one unit of funds from the depositor and makes an investment, which is observable.

To raise the funds for investment, the bank issues one unit of demandable deposits, which promises to repay an endogenously determined interest rate  $r_t \geq 1$  and can be withdrawn early (a run) at stage 3 or withdrawn after payoffs are realized at stage 4. To abstract away from coordination problems, we assume that there is only one depositor. This implies that any run is based

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literature, meaning this distortion arises through a different mechanism (see the discussion of the literature on policy traps above).

<sup>18</sup> For example, suppose the high-cost regulator prefers that the bank chooses the safe asset but the low-cost regulator prefers that the bank chooses the risky asset. In this case, the high-cost regulator tries to pool by failing the bank excessively, which can cause the bank to choose the safe asset and destroy surplus for the low-cost regulator (but increase surplus for itself).

<sup>19</sup> Note that in standard models of reputation, those players not building their reputation are generally short-lived (e.g., Mailath and Samuelson 2006).



on fundamentals. Note that there is no deposit insurance in the model, so the deposits may represent large deposits or wholesale debt.

The bank has market power vis-à-vis the depositor and makes a take-it-or-leave-it offer of  $r_t$  to the depositor at stage 1 to raise funds. The depositor's outside option is safe storage with a return of 1 at each stage.

The bank can choose between two possible investments. The first is a safe asset that returns  $R_0 > 1$  at stage 4. The second is a risky loan that returns  $R > R_0$  with probability  $1 - \theta_t$ , or defaults and returns 0 with probability  $\theta_t$ . The loan's default probability  $\theta_t \in [0, \bar{\theta}]$  is distributed with a p.d.f.  $h(\cdot)$  and c.d.f.  $H(\cdot)$ . We assume that  $\theta_t$  is uniformly distributed, that is,  $h(\theta) = \frac{1}{\bar{\theta}}$ . The risky loan's expected default probability is, thus,  $\mathbb{E}[\theta] = \frac{\bar{\theta}}{2}$ . We assume that the expected payoff of the risky loan is higher than that of the safe investment, representing a risk/return trade-off:

**Assumption 1.**  $(1 - \mathbb{E}[\theta])R > R_0$ .

If the depositor withdraws at stage 3, the bank may liquidate its assets and use the proceeds to repay the depositor. The risky loan has a liquidation value of  $L$  per unit. We assume that the liquidation value of the bank's risky loan is sufficiently low that the depositor would earn a higher expected payoff from not withdrawing (even with the lowest promised repayment of  $r_t = 1$ ) rather than liquidating:

**Assumption 2.**  $L < 1 - \mathbb{E}[\theta]$ .

Nevertheless, there is the possibility that the bank may be liquidated if the depositor learns that the risky loan is likely to default.

If the asset is not liquidated at stage 3, the bank uses the payoff of its investment to repay the depositor at stage 4 and pays out the residual payoff (if there is any) to its owners as dividends.

For simplicity, we assume that if the bank defaults in the first period, it is restructured and can operate again in the second period.<sup>20</sup>

## 1.2 Recapitalization

A bank that originates a risky loan faces the risk of having to liquidate it at stage 3 if the depositor withdraws early. The bank may prevent this by raising funds to repay the depositor, which we call recapitalization. Specifically, we assume that the regulator may set up a lending facility at a cost (which we detail in Subsection 1.5). If the facility has been set up, the bank may recapitalize by accessing it.

<sup>20</sup> This simplifies the analysis; without this assumption, the analysis would have to account for the state in which the bank fails and disappears. Alternatively, this scenario can be interpreted as a new bank entering if the first bank defaults (with the stress test outcomes from the first period observable).



There is a fixed cost  $C$  to the regulator of providing the funds.<sup>21</sup> This may represent the cost of providing real-time access to funds, the cost of the regulator diverting funds to its lending facility, or the premium the regulator requires in order to justify such an operation. The bank chooses to borrow  $\kappa$  units of capital via the lending facility with an expected repayment that is equal to  $\kappa + C$ . We demonstrate in Lemma 1 that the bank fully recapitalizes (obtains  $\kappa = r_t$ ).

Recapitalization then ensures that the bank does not default at stage 4. We assume that the return on the risky loan is sufficiently high (even at the highest default probability  $\bar{\theta}$ ), so that recapitalization is always feasible (even with the highest possible amount of capital required; this occurs when  $r_t$  is equal to  $\bar{r} = 1/(1 - \mathbb{E}[\theta])$ ):

**Assumption 3.**  $(1 - \bar{\theta})R > \bar{r} + C$ .

### 1.3 Stress testing

At stage 2, the regulator conducts the stress test. We model the stress test as the acquisition and signalling of information about the bank's risks.

The regulator perfectly observes the default probability of the bank's risky loan  $\theta_t$ , and no other party (including the bank itself) knows this information. The regulator then decides whether to set up a lending facility. We will henceforth refer to the regulatory actions of setting up a lending facility as "failing" and of not setting up a lending facility as "passing."<sup>22</sup>

A bank that fails the stress test may face depositor withdrawals unless it recapitalizes. The stress test results themselves are cheap talk, but setting up a lending facility incurs costs for the regulator,<sup>23</sup> enabling informative communication. By revealing negative information about the bank to the depositor, the regulator can create a credible liquidation threat from depositor withdrawals that prompts voluntary recapitalization by the bank.

Nevertheless, the stress test has a dual role, as it also conveys information about the regulator's type, which is private information (detailed below). In the second period, the bank reacts to the information inferred from the first-period stress test, forming the basis of the reputation mechanism.

### 1.4 Regulatory preferences

The regulator's objective is to maximize social surplus. This includes the net payoff of the asset chosen by the bank and the externalities from the bank's risky lending. We now detail these externalities.

<sup>21</sup> This could also include a marginal cost without changing the results; we write it as only a fixed cost for simplicity.

<sup>22</sup> To be precise, a "fail" is a costly announcement about the creation of a lending facility for the bank. A bank that fails the stress test, however, may choose not to use the lending facility.

<sup>23</sup> We will specify the costs below.

The social cost of risky lending is the cost to society of a bank default,  $D$ . A bank default can occur at stage 3 if the depositor withdraws early, or at stage 4 when the bank's risky loan fails and the bank does not have sufficient capital to meet the deposit repayment. The cost of bank default may represent the cost to resolve the bank, the cost of contagion, and/or the asset market disruption from the fire sale of bank assets.

Two social costs are associated with a bank recapitalization at stage 2. They are the social cost  $\xi$  (discussed in detail in the next section) of setting up a lending facility and the cost  $C$  of utilizing the lending facility.

Finally, there is one more potential externality, which we call the social benefit of risky lending: if the bank originates a risky loan at stage 1, it generates a positive externality equal to  $B$ . Broadly, increased credit is positively associated with economic growth and income for the poor (both across countries and across U.S. states; see [Demirgüç-Kunt and Levine 2018](#)).<sup>24</sup>

### 1.5 Regulator reputation

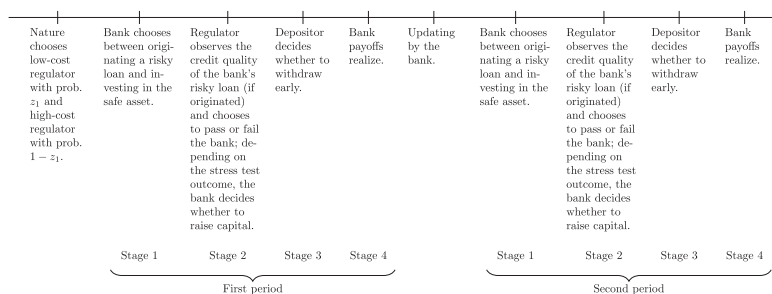
We assume that the regulator is long-lived and has a discount factor  $\delta \geq 0$  for the payoffs from the second period, in which  $\delta$  may be larger than 1 (as in, e.g., [Laffont and Tirole 1993](#)). The discount factor captures the relative importance of the future of the banking sector for the regulator. For simplicity, we do not allow for discounting within a period.

The regulator faces different possible costs of setting up a lending facility/failing a bank in the stress test  $\xi \in \{\xi_h, \xi_\ell\}$ , where  $\xi_h > \xi_\ell$ . We henceforth refer to this cost as the regulator's type  $\tau \in \{h, \ell\}$ . These costs may vary due to the amount of political support for failing banks, the beliefs of top regulators regarding the ability to save banks, or the mechanics of supporting banks.<sup>25</sup> As will become clear, the high-cost regulator is relatively softer toward the bank during a stress test, whereas the low-cost regulator is relatively tougher.

The regulator knows its own type, but during the stress test in period  $t$  (where  $t = \{1, 2\}$ ), the banker and the depositor are uncertain about the regulator's type. In each period  $t \in \{1, 2\}$ , these agents believe that the regulator is of the low-cost type with probability  $z_t$ . In our model,  $z_1$  is the probability with which nature chooses the regulator to be a low-cost type. Consequently,  $z_2$  is the updated belief that the regulator is a low-cost type after the first-period stress test. Our model, thus, allows us to endogenize the regulator's incentives to build a reputation of softness or toughness.

<sup>24</sup> [Garmaise and Moskowitz \(2006\)](#) provide causal evidence of the social effects of credit allocation, such as reduced crime.

<sup>25</sup> [Shapiro and Skeie \(2015\)](#) summarize the uncertainty regarding capital injections in the United States and Europe during the 2008 financial crisis. In a more recent example of regulatory uncertainty, Treasury head Janet Yellen and Fed chair Jerome Powell waffled on whether deposit insurance would be increased for all banks in response to the SVB failure; meanwhile the legality of doing so was also dubious ([Rampell 2023](#)).



**Figure 1**  
Timeline of events

We assume that the cost of failing the bank,  $\xi_\tau$ , is less than the gain from not liquidating the bank (even when the bank's risky loan has the highest default probability):

**Assumption 4.**  $\xi_h < (1 - \bar{\theta})R - L$ .

This assumption rules out the unintuitive scenario in which the regulator would pass a bank only for it to be liquidated. This cannot be an equilibrium because the regulator would then prefer to fail the bank to avoid its liquidation.<sup>26</sup>

## 1.6 Summary of timing

The regulator performs a stress test of the bank in each period. At the beginning of the second period, the beliefs about the regulator's type are updated depending on the result of the bank's stress test in the first period. In principle, the beliefs also could be conditioned on the payoffs of the bank's first period investment. For simplicity of presentation, we don't allow this; however, in an extension in Section 4.2, we demonstrate that the results are similar. Figure 1 illustrates the timing.

We assume that the probability that the risky loan defaults in the second period is independent of whether the risky loan defaults in the first period, and that the regulator's type is independent of the quality of the bank's risky loans. Furthermore, the regulator's type remains the same in both periods.

We use the equilibrium concept of the Perfect Bayesian equilibrium to solve the game. We use the concept of perfect sequential equilibrium (PSE) of Grossman and Perry (1986) to refine off-equilibrium-path beliefs where possible.<sup>27</sup>

<sup>26</sup> Note that if the depositor finds it optimal to withdraw when the bank passes, it must not find it optimal to withdraw if the bank fails, given that the average probability of default is not too high by Assumption 2. We formally show this in the proof of Proposition 2.

<sup>27</sup> This is a commonly used refinement (see, e.g., Diamond 1989; Maskin and Tirole 1992; Fishman 1989). We summarize how to apply this refinement in the beginning of the appendix.

## 2. One-Period Benchmarks

We present two one-period benchmarks in this section in order to highlight the role of stress tests in the model.

### 2.1 First-best benchmark

The first best consists of the choices that a social planner would make for all agents with full information. The social planner's preferences coincide with the regulator's. The solution is characterized by an efficient recapitalization policy at stage 2 and an efficient investment decision at stage 1, and is summarized as follows.

**Proposition 1.** The first-best benchmark is as follows.

- At stage 1, given the regulator type  $\tau \in \{\ell, h\}$ , there exists  $B_\tau$ , where  $B_\ell < B_h$ , such that the bank originates a risky loan if and only if  $B \geq B_\tau$ .
- At stage 2, the bank recapitalizes if and only if it has originated a risky loan and  $\theta \geq \theta_\tau^{FB} \equiv \frac{\xi_\tau + C}{D}$ . At stage 3, the depositor does not withdraw.

To understand this proposition, consider, first, the recapitalization policy at stage 2. If the bank invests in the safe asset, it never defaults and, therefore, does not recapitalize. If the bank originates a risky loan, it should recapitalize if and only if the quality of its risky loan  $\theta$  satisfies:

$$\xi_\tau + C - \theta D \leq 0. \quad (1)$$

That is, the banks should recapitalize if the expected social cost of recapitalization,  $\xi_\tau + C$ , stemming from both setting up and lending through the facility for a type- $\tau$  regulator, is less than the expected social cost of bank default,  $\theta D$ . This is equivalent to  $\theta \geq \theta_\tau^{FB}$ , where  $\theta_\tau^{FB} \equiv \frac{\xi_\tau + C}{D}$ .

We now turn to the efficient investment decision at stage 1. When the bank invests in the safe asset, we write the expected social surplus by  $U^0$ :

$$U^0 \equiv R_0 - 1. \quad (2)$$

When the bank originates a risky loan, we write the expected social surplus for a  $\tau$ -type regulator by  $U_\tau^R(\theta_\tau^{FB})$ , which takes into account the efficient recapitalization policy characterized above and consists of the social benefit of bank lending and the expected NPV of the risky loan, less the expected cost of bank default and the expected cost of recapitalization:

$$U_\tau^R(\theta_\tau^{FB}) = B + (1 - \mathbb{E}[\theta])R - 1 - \int_0^{\theta_\tau^{FB}} \theta dH(\theta)D - \int_{\theta_\tau^{FB}}^{\bar{\theta}} dH(\theta)(\xi_\tau + C). \quad (3)$$

It then follows that the bank originates a risky loan in the first best if and only if it leads to a higher expected social surplus, that is, if and only if  $B \geq B_\tau$ , where  $B_\tau$  is defined such that  $U^0 = U_\tau^R(\theta_\tau^{FB})$ .

Finally, since liquidating the bank is never efficient (Assumption 2), the depositor does not withdraw at stage 3.

## 2.2 One-period equilibrium

We now consider the equilibrium in a one-period model. We proceed by backward induction and analyze first the bank's recapitalization decision at stage 2 and the depositor's withdrawal decision at stage 3, given the bank's investment decision at stage 1. If the bank invests in the safe asset at stage 1, it faces no depositor withdrawal and no need for recapitalization.

Suppose that the bank originates a risky loan at stage 1.  $\hat{\theta}$  denotes the belief held by the bank and the depositor about the default probability of the bank's risky loan, given the information revealed by the regulator (the stress test) in stage 2. Consider the depositor's withdrawal decision at stage 3 if the bank does not recapitalize at stage 2. In this case, the depositor withdraws if and only if his expected payoff from holding the deposit until maturity is less than the payoff from early withdrawal and liquidating the bank:

$$(1 - \hat{\theta})r \leq L \Leftrightarrow \hat{\theta} \geq \underline{\theta}(r) \equiv 1 - \frac{L}{r}. \quad (4)$$

We can now consider the bank's decision to recapitalize at stage 2.

**Lemma 1.** Suppose that the bank originates a risky loan at stage 1.

- If  $\hat{\theta} \geq \underline{\theta}(r)$ , the bank raises  $r$  units of capital at stage 2 by accessing the lending facility (if available), and the depositor does not withdraw at stage 3 if and only if the bank recapitalizes.
- If  $\hat{\theta} < \underline{\theta}(r)$ , the bank does not recapitalize at stage 2 and the depositor does not withdraw at stage 3.

Lemma 1 highlights the disciplining effect of revealing negative information during stress testing. If the stress test reveals negative information about the bank (such that  $\hat{\theta} \geq \underline{\theta}(r)$ ), the bank faces the threat of a depositor withdrawal—which leads to voluntary recapitalization.

This result then allows us to characterize the equilibrium.  $\hat{\theta}^p$  and  $\hat{\theta}^f$  denotes the beliefs about the default probability held by the bank and the depositor at stage 2, given that the bank passes and fails the stress test, respectively. Consider an equilibrium in which the stress test is sufficiently informative, such that  $\hat{\theta}^f \geq \underline{\theta}(r) \geq \hat{\theta}^p$ . That is, by Lemma 1, the depositor prefers to withdraw early, at stage 3, if and only if the bank fails the stress test and does not recapitalize at stage 2.

In such an equilibrium, the regulator's stress test strategy compares the social cost of passing and failing the bank. The trade-off is exactly the same as in Proposition 1 and the regulator's stress-testing strategy is characterized by the cutoff  $\theta_{\tau}^{FB}$ , which induces the efficient recapitalization decision by the bank.

Anticipating the stress test and, thus, the bank's recapitalization behavior at stage 2, the promised repayment  $r$  is set at stage 1 such that the depositor breaks even:

$$\left(1 - z \int_0^{\theta_{\tau}^{FB}} \theta dH(\theta) - (1 - z) \int_0^{\theta_h^{FB}} \theta dH(\theta)\right) r = 1. \quad (5)$$

This expression reflects the fact that the depositor receives the promised repayment in full unless the bank does not recapitalize (if  $\theta \leq \theta_\tau^{FB}$ , in which case it passes the stress test) and subsequently defaults (with probability  $\theta$ ).

This is indeed an equilibrium if the posterior beliefs about the bank's risky loan's default probability satisfy  $\hat{\theta}^f \geq \underline{\theta}(r) \geq \hat{\theta}^p$ , given the regulator's stress test strategy described above. Assumption 5 provides a sufficient condition for this equilibrium to exist and be the unique equilibrium:

**Assumption 5.**  $1 - \bar{\theta} < 1 - \frac{\xi_\tau + C}{D} < (1 - \mathbb{E}[\theta])L$  for all  $\tau \in \{h, \ell\}$ .

The first inequality states that the social cost of a bank default is large relative to the cost of failing and recapitalizing the bank, so that both regulator types find it optimal to fail a bank if the bank's risky loan has the lowest quality ( $\bar{\theta}$ ). The second inequality states that the marginal quality of a bank that just fails the stress test is sufficiently low that the depositor prefers to withdraw early and liquidate the bank if it has not been recapitalized:  $\theta_\tau^{FB} = \frac{\xi_\tau + C}{D} > \underline{\theta}(r)$ , even with the highest possible promised repayment (when  $r$  is equal to  $\bar{r} = 1/(1 - \mathbb{E}[\theta])$ ). This ensures that the liquidation threat is credible in equilibrium if the bank fails the stress test. The equilibrium is summarized in the following proposition.

**Proposition 2.** There exists a unique equilibrium in the one-period benchmark as follows.

- If the bank originates a risky loan at stage 1, the type- $\tau$  regulator fails the bank at stage 2 if and only if  $\theta \geq \theta_\tau^{FB} \equiv \frac{\xi_\tau + C}{D}$ , and the bank recapitalizes if and only if it fails the stress test.
- The bank originates a risky loan at stage 1 if and only if

$$(1 - \mathbb{E}[\theta])R - R_0 \geq \left( z \int_{\theta_\ell^{FB}}^{\bar{\theta}} dH(\theta) - (1 - z) \int_{\theta_h^{FB}}^{\bar{\theta}} dH(\theta) \right) C. \quad (6)$$

This result shows that if the bank originates a risky loan, then information revealed by the stress test improves the expected social surplus by creating a credible liquidation threat that leads to efficient recapitalization by the bank. This is in stark contrast to the results in Goldstein and Leitner (2018) and Bouvard, Chaigneau, and de Motta (2015). In those papers, in good times (defined as the expected quality of the bank being above a threshold), the regulator prefers to reveal no information since there will be no ex post runs. Our setup also assumes that there are “good times” in the model: without information, there is no reason to run (Assumption 2). However, the regulator can find it advantageous to reveal information to discipline the banks, because their riskiness may be ameliorated through recapitalization.

This presents its own dilemma, however, as the possibility of discipline may result in an ex ante reaction by the bank to change the riskiness of its portfolio.

Proposition 2 shows that the bank originates a risky loan if and only if (6) is satisfied: that is, if the difference between the expected return on the risky loan and the safe investment is greater than the expected recapitalization cost. This can differ from the efficient investment decision described in Proposition 1, resulting in welfare losses.

In the dynamic game, the regulator will have scope for influencing bank lending and potentially reducing this surplus distortion through reputation management. We now illustrate the value of reputation management through a formal comparison of this benchmark with the first-best.

Since the regulator's type is unknown, the bank's investment decision at stage 1 anticipates the stress test outcomes at stage 2 and takes into account the probability  $z$  that the regulator is of the low-cost type. Since  $\theta_h^{FB} > \theta_\ell^{FB}$ , the high-cost regulator is more likely than the low-cost regulator to pass a bank, that is, it is softer (and the low-cost regulator is tougher). To focus on the interesting parameter space, we make the following assumption to ensure that the bank's investment decision at stage 1 is sensitive to the regulator's reputation in equilibrium:

**Assumption 6.**  $(1 - \frac{\xi_h + C}{\theta D})C < (1 - \mathbb{E}[\theta])R - R_0 < (1 - \frac{\xi_\ell + C}{\theta D})C$ .

Proposition 2 and this assumption imply that if the regulator were known to be high-cost, the bank would prefer the risky loan, whereas if the regulator were known to be low-cost, the bank would prefer the safe asset. Using this assumption, the bank's investment decision can be summarized as follows.

**Corollary 1.** In the one-period equilibrium, the bank originates a risky loan if and only if  $z \leq z^*$ , where  $z^* \in (0, 1)$  is defined such that (6) holds with equality.

We can now compare the expected social surplus in the one-period equilibrium to the first-best benchmark.

**Proposition 3.** In the one-period equilibrium,

- if  $B \geq B_\tau$ , the expected surplus for the type- $\tau$  regulator is decreasing in its reputation for having a low-cost,  $z$ , and equal to the first-best if and only if  $z \leq z^*$ ;
- if  $B \leq B_\tau$ , the expected surplus for the type- $\tau$  regulator is increasing in its reputation for having a low-cost,  $z$ , and equal to the first-best if and only if  $z \geq z^*$ .

This result implies that the regulator may benefit from either “signaling” or “hiding” its type, in order to induce a more efficient investment choice. For  $B \geq B_h$ , both types of the regulator benefit from a reputation for having a high



cost ( $z \leq z^*$ ); the high-cost regulator, thus, wishes to separate from the low-cost type, while the low-cost regulator wishes to pool with the high-cost type. Symmetrically, for  $B \leq B_\ell$ , both types of the regulator benefit from a reputation for having a low-cost ( $z \geq z^*$ ); the low-cost regulator wishes to separate from the high-cost type, while the high-cost regulator wishes to pool with the low-cost type. Finally, for an intermediate level  $B \in (B_\ell, B_h)$ , both types want to pretend to be the other type: the high-cost regulator benefits from a reputation for toughness (higher  $z$ ), while the low-cost regulator benefits from a reputation for softness (lower  $z$ ).

### 3. Stress Testing and Reputation

In this section, we analyze how the regulator's reputation-building incentives affect the equilibrium outcomes. We proceed by backward induction: characterizing the equilibrium in the second period and then going back to detail the equilibrium in the first period.

#### 3.1 The second period

The equilibrium outcome in the second period is identical to that in the one-period benchmark described in Proposition 2 and Corollary 1. That is, in each period  $t \in \{1, 2\}$ , if the bank originates a risky loan at stage 1, the type- $\tau$  regulator fails the bank at stage 2 if and only if  $\theta_2 \geq \theta_{2,\tau}^* \equiv \theta_\tau^{FB} = \frac{\xi_\tau + C}{D}$ , and the bank recapitalizes if and only if it fails the stress test. Anticipating the stress test outcomes, the bank originates a risky loan at stage 1 if and only if  $z_2 \leq z^*$ . That is,  $\lambda^*(z)$  denote the probability that the bank originates a risky loan in the second period. Thus we have  $\lambda^*(z) = 1$  for all  $z_2 < z^*$ ,  $\lambda^*(z) = 0$  for all  $z_2 > z^*$ , and  $\lambda^*(z) \in [0, 1]$  for  $z_2 = z^*$ . The regulator's expected surplus in the second period is, then, given by

$$U_{2,\tau}^*(z_2) = \lambda^*(z_2)U_\tau^R(\theta_{2,\tau}^*) + (1 - \lambda^*(z_2))U^0, \quad (7)$$

where  $U^0$  and  $U_\tau^R(\cdot)$  are given in (2) and (3), respectively.

#### 3.2 The first period

We analyze the equilibrium in the first period by working our way backward: characterizing first the stress test equilibrium at stage 2 given the bank's investment, and then the bank's investment decision at stage 1.

Suppose that the bank invests in the safe asset at stage 1. Since there is no risk of a bank default, the depositor does not withdraw at stage 3, and the bank does not recapitalize at stage 2 regardless of the stress test outcome. Nevertheless, anticipating that the bank's investment strategy in the second period depends on the regulator's reputation, the regulator may have incentives to fail the bank in the first period if it wishes to manipulate its reputation. This would be a form of money burning. The following lemma shows that such incentives never arise as long as the net social benefit of bank lending is not too low:

**Lemma 2.** There exists a  $\underline{B}$ , where  $\underline{B} < B_\ell$ , such that if  $B > \underline{B}$ , then in any equilibrium in which the bank invests in the safe asset at stage 1 in the first period, both types of the regulator pass the bank at stage 2 with certainty.

In the remainder of the analysis in this section, we restrict attention to the parameter space in which such money-burning incentives do not arise; in Section 4.1, we examine the complementary parameter space and examine the robustness of our results.

**Assumption 7.**  $B > \underline{B}$ .

Suppose that the bank originates a risky loan at stage 1. Consider, again, an equilibrium in which the stress test is sufficiently informative that the beliefs held by the bank and the depositor at stage 2, given the stress test result, satisfy  $\hat{\theta}_1^f \geq \underline{\theta}(r_1) \geq \hat{\theta}_1^p$ . That is, by Lemma 1, the depositor prefers to withdraw early at stage 3 if and only if the bank fails the stress test and does not recapitalize at stage 2.

The stress test result of the bank in the first period conveys not only information about the quality of the bank's risky loan but also information about the regulator's type. Since the bank's investment decision in the second period depends on the regulator's updated reputation  $z_2$ , the regulator in the first period may face reputation-building incentives during the stress test. Specifically, let  $\lambda^p$  and  $\lambda^f$  denote the probability that the bank originates a risky loan in period 2, given that the bank passes or fails the stress test in period 1, respectively. Anticipating the bank's investment strategy in the second period, the type- $\tau$  regulator's stress test strategy in the first period is characterized by a cutoff  $\theta_{1,\tau}^*$  such that the regulator is indifferent between passing and failing the bank:

$$\underbrace{(\xi_\tau + C - \theta_{1,\tau}^* D)}_{\text{First period surplus effect}} + \underbrace{\delta(\lambda^p - \lambda^f)[U_\tau^R(\theta_{2,\tau}^*) - U^0]}_{\text{Reputation effect}} = 0. \quad (8)$$

Analogous to the regulator's incentives in the second period given in (1), the first term in (8) captures the regulator's trade-off between the cost of recapitalizing the bank and the expected cost of a bank default. The second term reflects the regulator's reputation-building incentives since passing the bank in the first period changes the probability that the bank will originate a risky loan in the second period from  $\lambda^f$  to  $\lambda^p$ , while the regulator derives an expected surplus of  $U_\tau^R(\theta_{2,\tau}^*)$  if the bank originates a risky loan in the second period and an expected surplus of  $U^0$  if the bank invests in the safe asset.

Given the regulator's stress-testing strategy,  $\theta_{1,\tau}^*$ , the regulator's reputation is updated following Bayes' rule.  $z_2^p$  and  $z_2^f$  denote the updated belief at the beginning of the second period about the probability that the regulator is of the low-cost type, given that the bank passes and fails the stress test, respectively, in

the first period.<sup>28</sup> Given the regulator's stress-testing strategy,  $\theta_{1,\tau}^*$ , the updated reputation of the regulator conditional on the bank passing and failing the stress test, respectively, is

$$z_2^p(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) = \frac{\theta_{1,\ell}^* z_1}{\theta_{1,\ell}^* z_1 + \theta_{1,h}^* (1 - z_1)}, \quad (9)$$

$$z_2^f(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) = \frac{(\bar{\theta} - \theta_{1,\ell}^*) z_1}{\bar{\theta} - \theta_{1,\ell}^* z_1 - \theta_{1,h}^* (1 - z_1)}. \quad (10)$$

Since the low-cost regulator is tougher than the high-cost regulator, the posterior reputation is softer if the bank passes the stress test than if the bank fails the stress test, that is,  $z_2^p(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) < z_2^f(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ .

To characterize the bank's investment in the second period after originating a risky loan in the first period, it is useful to define cutoffs  $\underline{z}_1$  and  $\bar{z}_1$  such that:

$$z_2^p(\theta_\ell^{FB}, \theta_h^{FB}; \underline{z}_1) = z^*, \text{ and} \quad (11)$$

$$z_2^f(\theta_\ell^{FB}, \theta_h^{FB}; \bar{z}_1) = z^*. \quad (12)$$

Recall that  $\theta_\tau^{FB}$  characterizes the regulator's optimal stress-testing strategy in the one-period equilibrium (Proposition 2), in which the regulator faces no reputation concerns. It follows naturally that  $0 < \underline{z}_1 < z^* < \bar{z}_1 < 1$ .

The terms  $z_2^p(\theta_\ell^{FB}, \theta_h^{FB}; z_1)$  and  $z_2^f(\theta_\ell^{FB}, \theta_h^{FB}; z_1)$ , then, characterize the regulator's posterior reputation at the end of the first period if the type- $\tau$  regulator follows the strategy  $\theta_\tau^{FB}$ . When  $z_1 \leq \underline{z}_1$ , the prior reputation of the regulator is so soft that the bank originates a risky loan in the second period both when it passes and when it fails the stress test in the first period. When  $z_1 \geq \bar{z}_1$ , the prior reputation of the regulator is so tough that the bank invests in the safe asset in the second period both when it fails and when it passes the stress test in the first period. For intermediate levels of  $z_1 \in (\underline{z}_1, \bar{z}_1)$ , the bank is responsive to the stress test result; it originates a risky loan in the second period if it passes the stress test in the first period but invests in the safe asset in the second if it fails the stress test in the first period. Thus, the regulator may be able to manage its reputation in this region and achieve higher surplus.

Next, we characterize the equilibria by dividing the parameter space based on the regulator's prior reputation  $z_1$ .

**3.2.1 Intermediate  $z_1$ : Reputation building** We now use the cutoffs  $\underline{z}_1$  and  $\bar{z}_1$ , which are given by (11) and (12), respectively. For intermediate levels of the regulator's initial reputation  $z_1 \in (\underline{z}_1, \bar{z}_1)$ , the regulator's stress-testing strategy

<sup>28</sup> Recall that the bank's investment payoff in the first period is not observable. Therefore, the updating of the regulator's reputation depends only on the stress test result in the first period. In Section 4.2, we analyze the robustness of this simplifying assumption.

in the first period (given that the bank originates a risky loan) diverges from its strategy in the second period (given that the bank originates a risky loan) due to reputation-building incentives. To see this, suppose that the regulator adopts the same strategy in the first period as in the second period:  $\theta_{1,\tau} = \theta_{2,\tau}^* = \theta_{\tau}^{FB}$ . From the definitions of  $\underline{z}_1$  and  $\bar{z}_1$  above, this results in posterior reputation such that  $z_2^p(\theta_{1,\ell}, \theta_{1,h}; z_1) < z^* < z_2^f(\theta_{1,\ell}, \theta_{1,h}; z_1)$ . That is, the bank subsequently originates a risky loan in the second period if it passes the stress test in the first period, but invests in the safe asset if it fails the stress test in the first period. Since the bank's investment decision in the second period depends on the stress test result in the first period, the regulator may have incentives to excessively pass or fail. The regulator's incentives will be driven by its benefit from risky lending  $B$ : when  $B$  is large, the regulator might lower its threshold for passing in order to induce the bank to originate a risky loan in the second period; when  $B$  is small, the regulator might increase its threshold for passing in order to induce the bank to invest in the safe asset in the second period. The following proposition summarizes the regulator's equilibrium stress-test strategy in the first period, conditional on the bank originating a risky loan at stage 1 (we look at the bank's optimal decision subsequently).

**Proposition 4.** For  $z_1 \in (\underline{z}_1, \bar{z}_1)$ , in the first period, if the bank originates a risky loan at stage 1, there exists a unique equilibrium at stage 2. The unique equilibrium is one of the following two types.

- **Reputation-building equilibrium:** The type- $\tau$  regulator fails the bank at stage 2 if and only if  $\theta_1 \geq \theta_{1,\tau}^*$  and the bank recapitalizes if and only if it fails the stress test.
  - If  $B > B_h$ , then  $\theta_{1,\tau}^* > \theta_{2,\tau}^*$  for both types  $\tau$ ; that is, both regulator types are softer in the first period than in the second period in order to incentivize lending.
  - If  $B \in [B_\ell, B_h]$ , then  $\theta_{1,h}^* \leq \theta_{2,h}^*$  and  $\theta_{1,\ell}^* \geq \theta_{2,\ell}^*$ ; that is, the high-cost regulator is tougher in the first period in order to discourage risk-taking, while the low-cost regulator is softer in the first period in order to incentivize lending.
  - If  $B < B_\ell$ , then  $\theta_{1,\tau}^* < \theta_{2,\tau}^*$  for both types  $\tau$ ; that is, both regulator types are tougher in the first period than in the second period in order to discourage risk taking.
- **Always-pass equilibrium:** Both types of regulator pass the bank with certainty, and the bank does not recapitalize regardless of the stress test outcome.

There exists  $\tilde{\delta}(B, z_1)$ , such that the reputation-building equilibrium does not exist if  $B < B_\ell$  and  $\delta > \tilde{\delta}(B, z_1)$ .

This result highlights that the regulator may face incentives to build a reputation for softness or toughness depending on the benefit from risky lending,  $B$ .

U.S. stress tests have generally been regarded as much tougher than European ones. First, the Federal Reserve performs the stress test itself on data provided by the banks (and does not provide the model to the banks), whereas in Europe, it has been the case that the banks themselves perform the test. Second, the U.S. stress tests have regularly been accompanied by Asset Quality Reviews, whereas this has been infrequent for European stress tests. Third, one of the most feared elements of the U.S. stress tests has been the fact that there is a qualitative element that can be (and has been) used to fail banks.<sup>29</sup> A possible reason that European authorities did not include such a qualitative element is that they prioritized stimulating lending, given the slow recovery after the crisis.

While the initial European stress tests performed poorly (e.g., passing Irish banks and Dexia), one might argue that during crisis times, the main focus was on preventing runs, and without a fiscal backstop, it was difficult to maintain credibility (Faria-e-Castro, Martinez, Philippon 2017). We argue that in normal times, a stress test may be soft in order to incentivize banks to lend to the real economy. This may explain the 2016 EU stress test, which eliminated the pass/fail criteria, reduced the number of banks stress-tested by about half, used less-adverse scenarios than did the United States and United Kingdom, and singled out only one bank as undercapitalized—Monti dei Paschi di Siena, which had failed the previous (2014) stress test and was well known to be in distress. The press also revealed that Deutsche Bank was given a special exception to one of the stress test rules, boosting its recorded capitalization.<sup>30</sup>

Interestingly, Proposition 4 also shows that the regulator's reputation-building incentives can destroy the stress test's credibility and result in an "always-pass" equilibrium in which both types of regulator pool at passing the bank with certainty, an equilibrium in which the stress test is uninformative about the bank's asset risk and, thus, does not induce any recapitalization.

This is because, since failing the bank is costly, the regulator finds it optimal to do so only if it can induce recapitalization by creating the threat of a depositor withdrawal. Such a threat is credible if the perceived default probability is large enough when the bank fails the stress test, that is,  $\hat{\theta}_1^f \geq \underline{\theta}(r_1)$ .

Proposition 4 shows that this condition no longer holds when the social benefit of risky lending,  $B$ , is sufficiently low, and the regulator's reputation concern,  $\delta$ , is high. This is because, in this case, the stress test would be tough in the first period, as the regulator attempts to build reputation to deter risk-taking by the bank in the second period. Given a tough stress test, the depositor

<sup>29</sup> The qualitative element for domestic banks was removed in March 2019 (see Stacey and Fleming 2019).

<sup>30</sup> See Noonan, Binham, and Shotton (2016). We also note that in the 2018 test, there was no pass/fail requirement, only 48 banks were included, and all banks tested were judged to be well capitalized.

rationally expects that a bank may fail the stress test while having a relatively low probability of default. Therefore, when  $B$  is sufficiently low and  $\delta$  is sufficiently high, the reputation-building behavior of the regulator leads to a sufficiently tough stress test, where the posterior belief is  $\hat{\theta}_1^f \leq \underline{\theta}(r_1)$  and the depositor would not withdraw.<sup>31</sup>

When an informative stress test does not exist, the always-pass equilibrium is the unique equilibrium. This is sustained by the beliefs such that  $\hat{\theta}_1^p, \hat{\theta}_1^f < \underline{\theta}(r_1)$ ; that is, the bank does not recapitalize whether or not it passes the stress test. Notice that the always-pass equilibrium does not survive our refinement of perfect sequential equilibrium (PSE) when the reputation-building equilibrium exists. This gives us uniqueness.

Having characterized the regulator's stress test in equilibrium at stage 2, we finally consider the bank's investment decision at stage 1.

**Proposition 5.** For  $z_1 \in (\underline{z}_1, \bar{z}_1)$ , the bank's investment choice in the unique equilibrium is as follows.

- In the **reputation-building equilibrium**, there exists  $B^*(z_1, \delta) \in (B_\ell, B_h)$ , such that
  - if  $B \geq B^*(z_1, \delta)$ , the bank originates a risky loan in the first period; it subsequently originates a risky loan in the second period with strictly higher probability if it passes the stress test in the first period than if it fails the stress test;
  - if  $B \leq B^*(z_1, \delta)$ , the bank invests in the safe asset in the first period; it subsequently originates a risky loan in the second period if and only if  $z_1 \leq z^*$ , where  $z^*$  is defined in Corollary 1.
- In the **always-pass equilibrium**, the bank originates a risky loan in the first period; it subsequently originates a risky loan in the second period if and only if  $z_1 \leq z^*$ .

This result follows from Proposition 4. In the reputation-building equilibrium, the regulator is softer (tougher) when the social benefit of risky lending  $B$  is higher (lower). Therefore, the bank originates a risky loan in the first period if  $B$  is high and invests in the safe asset if  $B$  is low. Subsequently, in the second period, if the bank originated a risky loan in the first period, then its second-period investment choice reflects the information learned: a pass (fail) was more likely to come from a regulator of type  $h$  ( $l$ ), and, therefore, the banks

<sup>31</sup> Symmetrically, one might be concerned that an excessively soft stress test could also destroy the test's credibility: Given a soft stress test, the depositor rationally expects that a bank may pass the stress test while having a relatively high probability of default. In this case, the reputation-building behavior of the regulator can lead to a posterior belief  $\hat{\theta}_1^p \geq \underline{\theta}(r_1)$ . Nevertheless, this never happens, as even with the extreme version of a soft test in which the bank almost always passes, the posterior belief  $\hat{\theta}_1^p$  is equal to the average quality of the risky loan, which is good enough to avoid withdrawal given Assumption 2.

take more (less) risk. If the bank invested in the safe asset in the first period, then no information about the regulator is revealed, and the bank's investment decision in the second period depends only on the prior belief  $z_1$  about the regulator's type.

In the always-pass equilibrium, the stress test in the first period is uninformative about the bank's asset risk and, thus, does not lead to any recapitalization. Therefore, the bank originates a risky loan in the first period. Since the stress test in the first period also provides no information about the regulator's type, the bank's investment decision in the second period depends only on the prior belief  $z_1$  about the regulator's type.

**3.2.2 Extreme  $z_1$ : No-reputation building and self-fulfilling reputation building** When the prior reputation of the regulator is either very low or very high, the regulator may not face the reputation-building incentives present in the last section when conducting the stress test in the first period.

To see this, suppose, again, that the regulator adopts the same stress-testing strategy in the first period as in the second period. Consider, first, the region where the prior reputation of the regulator is sufficiently soft (high-cost), that is,  $z_1 < \underline{z}_1$ . By the definition of  $\underline{z}_1$ , we have that  $z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) < z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) < z^*$  for all  $z_1 < \underline{z}_1$ . That is, the reputation of the regulator is so soft that even if the bank fails the stress test in the first period, the bank still prefers to originate a risky loan in the second period. As a result, since the bank's investment decision in the second period does not depend on the stress test result in the first period, the regulator faces no reputation-building incentives in the first period. That is,  $\lambda^p = \lambda^f = 1$  and the second term in (8) is zero. It is, therefore, indeed an equilibrium for the regulator to have a stress-testing strategy in the first period identical to that in the second period. Analogously, if the regulator's reputation is sufficiently tough (low-cost), that is,  $z_1 > \bar{z}_1$ , the bank invests in the safe asset in the second period for any stress test result, even if the bank passes the stress test in the first period, that is,  $z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > z^*$  and  $\lambda^p = \lambda^f = 0$ . The regulator again faces no reputation-building incentives and its equilibrium stress test strategy in the first period is identical to that in the second period. This result is formalized in the following proposition.

**Proposition 6.** For  $z_1 \notin (\underline{z}_1, \bar{z}_1)$ , in the first period, if the bank originates a risky loan at stage 1, the equilibrium at stage 2 is as follows:

- **No-reputation-building equilibrium:** There always exists an equilibrium in which the type- $\tau$  regulator fails the bank at stage 2 if and only if  $\theta_1 \geq \theta_{1,\tau}^* = \theta_{2,\tau}^* = \frac{\xi_\tau + C}{D}$ , and the bank recapitalizes if and only if it fails the stress test.
- **Self-fulfilling reputation-building equilibrium:** Recall  $\tilde{\delta}(B, z_1)$  defined in Proposition 4. There exists  $B'_\tau$  for  $\tau \in \{h, \ell\}$ , where  $B'_h > B_h > B_\ell > B'_\ell$ , and  $\underline{\delta}(B, z_1)$ , such that



- for  $z_1 < \underline{z}_1$ ,  $B > B'_h$ , and  $\delta \geq \underline{\delta}(B, z_1)$ , there exists a reputation-building equilibrium in which the type- $\tau$  regulator fails the bank at stage 2 if and only if  $\theta_1 \geq \theta_{1,\tau}^*$ , where  $\theta_{1,\tau}^* > \theta_{2,\tau}^*$ , and the bank recapitalizes if and only if it fails the stress test;
- for  $z_1 > \bar{z}_1$ ,  $B < B'_\ell$ , and  $\delta \in [\underline{\delta}(B, z_1), \bar{\delta}(B, z_1)]$ , there exists a reputation-building equilibrium in which the type- $\tau$  regulator fails the bank at stage 2 if and only if  $\theta_1 \geq \theta_{1,\tau}^*$ , where  $\theta_{1,\tau}^* < \theta_{2,\tau}^*$  and the bank recapitalizes if and only if it fails the stress test.

Proposition 6 demonstrates that, while there always exists a no-reputation-building equilibrium for  $z_1 \notin (\underline{z}_1, \bar{z}_1)$ , there is also a self-fulfilling reputation-building equilibrium that coexists with the no-reputation-building equilibrium for some parameters.<sup>32</sup>

The equilibrium multiplicity is driven by a strategic interaction between the regulator's stress test in the first period and the bank's lending decision in the second period. Consider the case in which the regulator's prior reputation is soft ( $z_1 < \underline{z}_1$ ). In this case, the bank originates a risky loan in the second period after passing the stress test in the first period as  $z_2^p \leq z_1$ . However, after failing the stress test in the first period, the bank may originate a risky loan or invest in the safe asset in the second period, depending on the regulator's reputation  $z_2^f$ .

If the bank conjectures that the regulator faces no reputation-building incentives and adopts the stress-testing strategy in the first period of  $\theta_{1,\tau}^* = \theta_{2,\tau}^*$ , then  $z_2^f \leq z_1$  for all  $z_1 < \underline{z}_1$  by the definition of  $\underline{z}_1$  given in (11). In this case, the bank originates a risky loan in the second period after failing the stress test in the first period, as well as after passing the stress test, indeed resulting in no reputation-building incentives (the no-reputation-building equilibrium). Suppose, instead, that the bank conjectures that the regulator adopts a softer stress-testing strategy in the first period due to the concern that the bank would invest in the safe asset in the second period after failing the stress test in the first period. In this case, when the social benefit of risky lending  $B$  is sufficiently high ( $B > B'_h$ ), both types of the regulator avoid a reputation of toughness by passing the bank in the first period with higher probabilities. Yet, since a low-cost regulator is more likely than the high-cost regulator to fail the bank, the more the regulator types avoid failing the bank, the more likely it is that a fail indicates that the regulator is of the low-cost type, that is, the higher is  $z_2^f$ .<sup>33</sup> In turn, the bank finds it optimal to invest in the safe asset in the second period

<sup>32</sup> The self-fulfilling reputation-building equilibrium survives the PSE refinement. Since both passing and failing the stress test occur with strictly positive probability, there are no off-equilibrium beliefs, and, thus, belief-based equilibrium refinements, such as the PSE have no bite.

<sup>33</sup> For all  $B \geq B_h$ , reputation concerns compel both types of the regulator to avoid a reputation of toughness by passing the bank in the first period with higher probabilities. Since the expected social surplus from risky lending is higher for the low-cost regulator than for the high-cost regulator, the reputation effects are stronger for the

after failing the stress test in the first period, justifying a softer testing strategy. It is, indeed, the self-fulfilling beliefs about the regulator's reputation-building that leads to equilibrium multiplicity.<sup>34</sup>

Symmetrically, consider the case in which the regulator's prior reputation is tough ( $z_1 > \bar{z}_1$ ). In this case, the bank invests in the safe asset in the second period after failing the stress test in the first period, as  $z_2^f \geq z_1$ . However, after passing the stress test in the first period, the bank may originate a risky loan or invest in the safe asset in the second period, depending on the regulator's reputation  $z_2^p$ . Analogously, if the bank conjectures that the regulator faces no reputation-building incentives and adopts the stress-testing strategy in the first period of  $\theta_{1,\tau}^* = \theta_{2,\tau}^*$ , then, indeed,  $z_2^p \geq z_1$ , and the bank finds it optimal to invest in the safe asset in the second period even after passing the stress test in the first period. Suppose, instead, that the bank conjectures that the regulator adopts a tougher stress-testing strategy in the first period due to the concern that the bank would originate a risky loan in the second period after passing the stress test in the first period. Then, when the social benefit of risky lending  $B$  is sufficiently low ( $B < B'_l$ ), the reputation-building behavior of the regulators makes it more likely that a pass indicates a high-cost regulator. In turn, the bank finds it optimal to originate a risky loan in the second period after passing the stress test in the first period, justifying a tougher stress-testing strategy. Notice that, in this case, analogously to the reputation-building equilibrium described in Proposition 4, this stress-testing strategy can be sustained as an equilibrium only if it is not too tough, so that it can induce recapitalization by creating the threat of a depositor withdrawal. That is, the self-fulfilling stress test exists only if the reputation concern is not too large, that is,  $\delta \leq \bar{\delta}(B, z_1)$ .

Having characterized the regulator's stress test in equilibrium at stage 2, we finally consider the bank's investment decision at stage 1 and summarize it in the following proposition.

**Proposition 7.** For  $z_1 \notin (z_1, \bar{z}_1)$ , the bank's investment in equilibrium is as follows.

low-cost regulator  $\frac{U_\ell^R(\theta_{2,\ell}^*) - U^0}{U_h^R(\theta_{2,h}^*) - U^0} > 1$ . However, the relative difference is diminishing as  $B$  becomes larger, that is,  $\frac{U_\ell^R(\theta_{2,\ell}^*) - U^0}{U_h^R(\theta_{2,h}^*) - U^0}$  decreases and approaches 1 as  $B$  increases. For  $B$  not too large ( $B \in [B_h, B'_h]$ ), the low-cost regulator has much stronger reputation concerns to pass the bank (act softly) than the high-cost regulator has. In this case, reputation concerns that make both types of the regulator avoid failing the bank also reduce the likelihood that a fail indicates that the regulator is of the low-cost type,  $z_2^f$ . For  $B$  sufficiently large ( $B > B'_h$ ), the difference between the reputation-building incentives for the low-cost and high-cost regulators becomes sufficiently small. In this case, as both types of the regulator become similarly softer, it becomes proportionally more likely that a fail indicates a low-cost regulator since the low-cost regulator has a higher probability of failing the bank. We detail this mathematically in the proof of Proposition 6.

<sup>34</sup> In a different context, Ordoñez (2013, 2018) shows that banks' reputation concerns, which provide discipline to keep banks from taking excessive risk, can lead to fragility and a crisis of confidence in the market. Other theories have predicted self-fulfilling bank lending freezes due to interdependence of banks' lending opportunities (Bebchuk and Goldstein 2011) and the fear of future fire sales (Diamond and Rajan 2011).

- In the no-reputation-building equilibrium,
  - for  $z_1 < \underline{z}_1$ , the bank originates a risky loan in both periods;
  - for  $z_1 > \bar{z}_1$ , the bank invests in the safe asset in both periods.
- In the self-fulfilling reputation-building equilibrium,
  - for  $z_1 < \underline{z}_1$ ,  $B > B'_h$ , and  $\delta > \underline{\delta}(z_1, B)$ , the bank originates a risky loan in the first period; it subsequently originates a risky loan in the second period if and only if it passes the stress test in the first period;
  - for  $z_1 > \bar{z}_1$ ,  $B < B'_\ell$ , and  $\delta \in [\underline{\delta}(B, z_1), \bar{\delta}(B, z_1)]$ , the bank invests in the safe asset in both periods.

This result follows from Proposition 6. In the no-reputation-building equilibrium, the regulator's stress-testing strategy is identical in both periods. As a result, the bank's investment decision is also the same in both periods. It originates a risky loan if the prior reputation of the regulator is soft and invests in the safe asset if the prior reputation of the regulator is tough.

In the self-fulfilling reputation-building equilibrium, the regulator's stress-testing strategy in the first period diverges from that in the second period. When the regulator's prior reputation is soft ( $z_1 < \underline{z}_1$ ), the regulator faces self-fulfilling incentives to avoid a reputation of toughness and, therefore, passes the bank with a risky loan with higher probability in the first period. The bank thereby enjoys even higher payoffs from originating a risky loan in the first period. However, if the bank fails the stress test in the first period, the equilibrium posterior reputation becomes sufficiently tough, so that the bank invests in the safe asset in the second period if it fails the stress test in the first period. When the regulator's prior reputation is tough ( $z_1 > \bar{z}_1$ ), the regulator faces self-fulfilling incentives to avoid a reputation of softness and, therefore, fails the bank with a risky loan with higher probability in the first period. Thus, the bank finds it optimal to invest in the safe asset in the first period.<sup>35</sup> Subsequently, since no information about the regulator's type is revealed, the bank again finds it optimal to invest in the safe asset in the second period.<sup>36</sup>

### 3.3 Surplus effects of reputation management in stress testing

In this section, we analyze how the regulator's reputation concerns affect the expected social surplus. To do so, we first establish a benchmark in which the regulator is myopic and takes actions to maximize the expected social surplus within each period. This is a regulator who has no reputation concerns. We then compare the equilibrium outcome to the benchmark with a myopic regulator.

<sup>35</sup> Notice that reputation-building occurs off-equilibrium, in the subgame after the bank originates a risky loan.

<sup>36</sup> In this case, the equilibrium outcome in the self-fulfilling equilibrium coincides with the outcome in the no-reputation-building equilibrium.

**3.3.1 Benchmark with a myopic regulator** Since the regulator is myopic, the equilibrium outcome in each period is identical to that in the one-period benchmark described in Proposition 2 and Corollary 1 in the previous section. It is also equivalent to the no-reputation-building equilibrium summarized in Propositions 6 and 7, with the caveat that, when the regulator is myopic, it exists for all  $z_1$ . For clarity, we summarize the myopic benchmark:

**Corollary 2.** There exists a unique equilibrium in the benchmark with a myopic regulator:

- In each period  $t \in \{1, 2\}$ , if the bank originates a risky loan at stage 1, the regulator fails the bank at stage 2 if and only if  $\theta_t \geq \theta_\tau^{FB} = \frac{\xi_\tau + C}{D}$ , and the bank recapitalizes if and only if it fails the stress test.
- The bank's investment decision depends on the regulator's prior reputation  $z_1$  as follows.
  - For  $z_1 \leq \underline{z}_1$ , the bank originates a risky loan in both periods.
  - For  $z_1 \in (\underline{z}_1, z^*]$ , the bank originates a risky loan in the first period; the bank originates a risky loan in the second period if it passes the stress test in the first period and invests in the safe asset in the second period if it fails the stress test in the first period.
  - For  $z_1 \geq z^*$ , the bank invests in the safe asset in both periods.

**3.3.2 Surplus effects of reputation concerns** The following proposition summarizes the surplus effects of reputation concerns for intermediate values of the initial reputation of the regulator  $z_1 \in (\underline{z}_1, \bar{z}_1)$ .

**Proposition 8.** Consider  $z_1 \in (\underline{z}_1, \bar{z}_1)$ .

- If the unique equilibrium is a reputation-building equilibrium, there are two cases.
  - If the bank's first-period investment choice in the reputation-building equilibrium is the same as that in the benchmark with a myopic regulator, the expected social surplus is higher in the reputation-building equilibrium than in the benchmark with a myopic regulator.
  - If the bank's first-period investment choice in the reputation-building equilibrium is different from that in the benchmark with a myopic regulator, the expected social surplus can be higher or lower than in the benchmark with a myopic regulator.
- If the unique equilibrium is an always-pass equilibrium, the expected social surplus is strictly lower in the always-pass equilibrium than in the benchmark with a myopic regulator.

These results demonstrate that whenever the bank's first-period investment decision remains the same in both the reputation-building equilibrium and the myopic benchmark, the expected social surplus is larger in the reputation-building equilibrium. This is because reputation-building by the regulator trades off the cost of the distorted recapitalization decision for the bank in the first period against the benefit of inducing a more desirable investment choice by the bank in the second period.

However, reputation-building can also reduce the expected social surplus due to two distortions. First, it might result in the bank making a less desirable investment choice in the first period, anticipating the regulator's stress-testing strategy. In this case, the regulator's modification of its static/myopic stress test in order to influence future bank lending backfires as the bank changes its behavior in the first period. Second, reputation-building can destroy the credibility of the stress test and result in an always-pass equilibrium. In this case, the expected social surplus is strictly lower as the bank takes excessive risk and does not recapitalize in the first period.

Next, we consider the surplus for extreme values of the initial reputation of the regulator  $z_1 \notin (\underline{z}_1, \bar{z}_1)$ .

**Proposition 9.** Consider  $z_1 \notin (\underline{z}_1, \bar{z}_1)$ .

- In the no-reputation-building equilibrium (which always exists), the expected social surplus is identical to that in the benchmark with a myopic regulator.
- In the self-fulfilling reputation-building equilibrium (whenever it exists), the expected social surplus is lower (and strictly lower if it leads to a different outcome) than in the benchmark with a myopic regulator.

The no-reputation-building equilibrium leads to the same outcome as in the benchmark with a myopic regulator and, thus, has the same expected social surplus.

However, the self-fulfilling reputation-building equilibrium, when it leads to a different outcome than the myopic benchmark does, has a strictly lower expected social surplus than the myopic benchmark. Recall that this occurs when the regulator's prior reputation is sufficiently soft (Proposition 7).<sup>37</sup> Reputation-building in this case features wasteful attempts at signaling by the regulator that lead to two inefficiencies. First, the regulator's stress-testing strategy in the first period is too soft compared to the no-reputation-building equilibrium, resulting in insufficient recapitalization. Second, the

<sup>37</sup> In contrast, when the regulator's prior reputation is sufficiently tough, although self-fulfilling reputation-building distorts the regulator's stress testing strategy when the bank originates a risky loan (Proposition 6), the bank chooses the safe investment in equilibrium. In this case, the equilibrium outcome and, thus, the expected social surplus coincide with that in the benchmark with a myopic regulator.

bank interprets failing the stress test in the first period as indicative of a low-cost regulator and switches to the safe asset in the second period, resulting in insufficient lending.

Finally, this result implies that, whenever multiple equilibria exist, the no-reputation-building equilibrium dominates the self-fulfilling reputation-building equilibrium.

## 4. Discussion and Extensions

Having characterized the model's equilibria, we now extend the model to demonstrate the robustness of the baseline results. See the [Internet Appendix](#) for all proofs for this section.

### 4.1 Reputation-building through money burning

Recall that we have restricted attention to a social benefit of risky lending  $B$  that is not too low by Assumption 7. In this section, we consider the case of the complementary parameter space and show that there can exist a money-burning equilibrium when  $B$  is sufficiently low and  $\delta$  is sufficiently high.

**Proposition 10.** For  $B \leq \underline{B}$  and  $\delta \geq \frac{\xi_\ell}{U^0 - U_\ell^R(\theta_{2,\ell}^*)}$ , there exist money-burning equilibria as follows:

- In the first period, the bank originates a risky loan at stage 1; the high-cost regulator passes the bank with strictly higher probability than the low-cost regulator; the bank does not recapitalize regardless of the stress test result.
- In the second period, the bank originates a risky loan in the second period with strictly higher probability if it passes the stress test than if it fails the stress test in the first period.

In a money-burning equilibrium, failing the stress test in the first period does not induce any recapitalization. Yet the low-cost regulator fails the bank and incurs the cost  $\xi_\ell$  to signal its type. The signaling benefit stems from influencing the bank's investment decision in the second period. That is, since the social benefit of lending  $B$  is low, the low-cost regulator is willing to signal its toughness in order to incentivize the bank to invest in the safe asset in the second period with higher probability. However, since the high-cost regulator derives even lower social surplus from risky lending than the low-cost regulator does (because of the former's higher cost of recapitalizing the bank), there are strong mimicking incentives for the high-cost regulator. Therefore, a credible signaling equilibrium can emerge only when  $\delta$  is sufficiently high, so that signaling incentives are strong, and when  $B$  is sufficiently low, so that the signaling incentives of the low-cost regulator are sufficiently strong compared to the mimicking incentives of the high-cost regulator.

The emergence of a money-burning equilibrium in this case also influences the bank's investment decision in the first period. In a money-burning equilibrium, the stress test result is not informative about the bank's riskiness. The bank, therefore, faces no threat of depositor withdrawal, and would not recapitalize when it originates a risky loan in the first period. Given this, the bank prefers to originate a risky loan in the money-burning equilibrium.

Notice that, given  $B < \underline{B}$  and  $\delta \geq \frac{\xi_\ell}{U^0 - U_\ell^R(\theta_{2,\ell}^*)}$ , the money-burning equilibrium can coexist with the reputation-building equilibrium characterized in Propositions 4–5 and with the no-reputation-building equilibrium and the self-fulfilling reputation-building equilibrium characterized in Propositions 6–7. However, it does not coexist with the always-pass equilibrium characterized in Propositions 4–5.

## 4.2 Reputation updating and the bank's investment payoffs

In the baseline analysis in Section 3, we assumed that the updated beliefs about the regulator's type at the beginning of the second period depend only on the result of the bank's stress test, and not on the payoffs of the bank's investment.

Suppose, instead, that the belief updating depends also on the payoffs of the bank's investment.  $z_2^{p,R}$  and  $z_2^{p,0}$  denote the updated beliefs, given that the bank passes the stress test in the first period and realizes payoffs of  $R$  and  $0$ , respectively. Similarly,  $z_2^{f,R}$  and  $z_2^{f,0}$  denote the updated beliefs given that the bank fails the stress test in the first period.

Let  $\lambda^{p,R}$  and  $\lambda^{p,0}$  denote the probabilities that the bank originates a risky loan in the second period, given that the bank passes the stress test in the first period and realizes payoffs of  $R$  and  $0$ , respectively. Let  $\lambda^{f,R}$  and  $\lambda^{f,0}$  be defined analogously when the bank fails the stress test in the first period.

Analogous to Equation (8), the regulator's stress-testing strategy in the first period is characterized by a cutoff  $\theta'_{1,\tau}$  such that the regulator is indifferent between passing and failing the bank:

$$(\xi_\tau + C - \theta'_{1,\tau} D) + \delta [(1 - \theta'_{1,\tau})(\lambda^{p,R} - \lambda^{f,R}) + \theta'_{1,\tau}(\lambda^{p,0} - \lambda^{f,0})] [U_\tau^R(\theta_{2,\tau}^*) - U^0] = 0. \quad (13)$$

Under Assumption 7, in Proposition IA.1 in the Internet Appendix, we show that the equilibria are qualitatively the same as the equilibria characterized in the main model in Propositions 4–7.

## 4.3 Stress testing and bank runs

In the baseline model, we define the regulatory action of setting up a lending facility as “failing” the bank in the stress test. As a result, although failing the stress test reveals negative information about the bank and creates the *threat* of a depositor withdrawal (Lemma 1), there is never a bank run in equilibrium, because the bank always recapitalizes by accessing the lending facility after failing the stress test.



As highlighted by [Bouvard, Chaigneau, and de Motta \(2015\)](#) and [Goldstein, and Leitner \(2018\)](#), a key concern about stress tests is that revealing negative information can trigger welfare-destroying bank runs. We now extend the model to allow for bank runs to occur in equilibrium by assuming that, upon failing the stress test, the lending facility is available only with some probability. This risk may stem from the political uncertainty or administrative challenges in providing timely intervention.<sup>38</sup> As a result, the regulator's decision to fail a bank must take into account the possibility of triggering a bank run.

Specifically, we modify the baseline setup by assuming that “fail” is a costly (and observable) effort to set up a lending facility. The lending facility, however, only becomes available with probability  $\gamma$ , and is not available with probability  $1 - \gamma$ . The bank can recapitalize only if the lending facility is available. In the first-best benchmark, the social planner prefers to fail if and only if the surplus from failing is larger than the surplus from passing:

$$-\xi_\tau - \gamma C + \gamma[(1 - \theta)R] + (1 - \gamma)[(1 - \theta)R - \theta D] \geq (1 - \theta)R - \theta D. \quad (14)$$

Failing incurs the costs of recapitalizing ( $\xi_\tau$  and, with probability  $\gamma$ , the cost  $C$ ) but has benefits if the lending facility is available, as it avoids the cost of default. Let us define  $\hat{\xi}_\tau \equiv \frac{\xi_\tau}{\gamma}$  as the effective cost of failing the bank. Correspondingly, in the first-best benchmark, the regulator fails the bank if and only if  $\theta \geq \hat{\theta}_\tau^{FB} \equiv \frac{\hat{\xi}_\tau + C}{D}$ . Notice that in the first best, the social planner would not allow a bank to be liquidated.

Now consider the full model. The regulator's stress-testing strategy in the second period is characterized by a passing threshold  $\hat{\theta}_\tau^*$  such that  $\theta \geq \hat{\theta}_\tau^*$ , where the regulator prefers to fail if and only if the surplus from failing is larger than the surplus from passing:

$$-\xi_\tau - \gamma C + \gamma[(1 - \theta)R] + (1 - \gamma)(L - D) \geq (1 - \theta)R - \theta D. \quad (15)$$

Here, there is a sharp contrast with the first best.<sup>39</sup> When the regulator chooses to fail the bank, if there is no lending facility, there will be a run. This means the bank will be liquidated, yielding a payoff of  $L$  and incur the default cost with probability one.

Interestingly, the threshold probability for passing the bank derived from (15) implies that the regulator is more cautious in the second period compared to the first best, that is,  $\hat{\theta}_\tau^* > \hat{\theta}_\tau^{FB}$ . In the baseline model, the regulator follows the first-best strategy in the second period (characterized in Proposition 2). This difference arises because the regulator is concerned with the risk of triggering a run that leads to inefficient liquidation when the lending facility is unavailable.

<sup>38</sup> The recent run on Silicon Valley Bank highlights the difficulty for supervisors to step in and intervene in a timely manner (see, e.g., [Tett 2023](#)).

<sup>39</sup> Recall that, in the baseline model, the regulator follows the first-best strategy in the second period (characterized in Proposition 2).

In the first period, analogous to (8), the regulator's decision to pass a bank trades off the first period surplus effect, which now takes into account the risk of triggering a bank run by failing the bank and is captured by the left-hand side of (15), against a reputation effect that is defined similarly to the second term of (8). In the [Internet Appendix](#), we show that the equilibria are qualitatively the same as those characterized in the baseline model in Propositions 4–7.

#### 4.4 Forced recapitalization

In the baseline model described in Section 1, we assumed that the regulator can set up a lending facility, but the bank can voluntarily choose whether to recapitalize by accessing the lending facility. As demonstrated, information revealed during stress tests can instill market discipline that compels weaker banks to recapitalize in equilibrium.

In practice, regulators could have the authority to force recapitalization. For instance, stress tests in the United States can trigger a temporary restriction on dividend payments, effectively recapitalizing the bank.<sup>40</sup>

The reputation building incentives of the regulator analyzed in the baseline model remain even if failing the bank implies forced recapitalization.

The only difference from the baseline model is that the reputation-building equilibrium described in Propositions 4 and 5 exists and is the unique equilibrium for all  $z_1 \in (\underline{z}_1, \bar{z}_1)$ , whereas the always-pass equilibrium no longer exists.<sup>41</sup> Recall that the always-pass equilibrium arises in the baseline model instead of the reputation-building equilibrium because of the interaction between the regulator's reputation building incentives and the depositor's belief updating process: as the stress test becomes tougher, the depositor no longer finds it optimal to withdraw from a bank that fails the stress test even if it does not recapitalize; this leads to a loss of market discipline to compel the bank to recapitalize. If the regulator is able to force recapitalization directly, market discipline is no longer needed. Therefore the reputation-building equilibrium always exists under forced recapitalization.

## 5. Conclusion

A recent addition to the regulatory toolkit, stress tests provide assessments of bank risk in adverse scenarios. Revealing negative information can force banks to raise capital. However, regulators have incentives to be tough, by failing even some safe banks, or to be soft, by passing some risky banks. These incentives are driven by the weight that the regulator places on lending in the economy in

<sup>40</sup> For example, following the stress test of 2020, the Federal Reserve Board required large banks to preserve capital by suspending share repurchases and capping dividend payments.

<sup>41</sup> Specifically, for  $B < B_\ell$  and  $\delta > \bar{\delta}(B, z_1)$ , the always-pass equilibrium is the unique equilibrium in the baseline model, whereas the reputation-building equilibrium is the unique equilibrium for this parameter range when the regulator can force recapitalization.

relation to financial stability. Banks respond to the softness of the stress test by altering their lending policies. This may lead to a loss of surplus due to the test losing its credibility, self-fulfilling reputation management by the regulator, and ex ante inefficient choices by the bank.

It would be interesting to extend the model to study stress testing with multiple banks in a macroprudential setting.

## A. Appendix

### A.1 Perfect Sequential Equilibrium

We use perfect sequential equilibrium (PSE), as defined by Grossman and Perry (1986), to refine the set of sequential equilibria. A brief summary of how to apply this concept is as follows: the first requirement of perfect sequential equilibria is to define best responses to off-equilibrium-path actions. Then, one can search for an off-equilibrium-path updating rule. If the types given positive weight in the updating rule would deviate, and the types given zero weight would not deviate, then this updating rule is called credible and the equilibrium is not a PSE.

### A.2 Proof of Proposition 1

This result follows from the analysis following the proposition.

### A.3 Proof of Lemma 1

To prove this lemma, we first consider the depositor's withdrawal decision at stage 3, given the bank's recapitalization decision at stage 2, and then characterize the bank's optimal recapitalization decision at stage 2.

If the bank raises  $\kappa$  unit of capital at stage 2, the depositor does not withdraw at stage 3 if and only if his expected payoff from waiting until stage 4 is higher than from withdrawing at stage 3:

$$(1 - \hat{\theta})r + \hat{\theta} \min\{\kappa, r\} \geq \min\{\kappa + L, r\}. \quad (\text{A.1})$$

The min operators capture the fact that, if the bank is unable to meet the promised repayment  $r$  at either stage 3 or stage 4, the depositor receives the  $\kappa$  unit of capital if the bank's risky loan fails at stage 4 (second term on the left-hand side), or receives  $\kappa$  and the liquidation proceeds  $L$  at stage 3 (the term on the right-hand side).

We now turn to the bank's recapitalization decision (if a lending facility is available). Recapitalization of amount  $\kappa$  is feasible only if the depositor does not withdraw at stage 3, that is, if (A.1) holds. In that case, the bank's expected payoff is

$$(1 - \hat{\theta}) \max\{R + \kappa - r, 0\} + \hat{\theta} \max\{\kappa - r, 0\} - (C + \kappa). \quad (\text{A.2})$$

The last term in this expression captures the cost of capital. Notice that the expression is weakly decreasing in  $\kappa$ . Therefore, if the bank recapitalizes, it optimally raises the minimum amount of capital that satisfies (A.1).

We can now characterize the bank's optimal recapitalization decision for the two cases stated in this lemma.

- If  $\hat{\theta} \geq \underline{\theta}(r)$ , that is, (4) holds—the depositor withdraws if the bank does not recapitalize, and the bank's expected payoff without recapitalization is 0. Moreover, in this case, we can show that (A.1) holds if and only if  $\kappa \geq r$ . This is because (a) (4) implies that (A.1) does not hold for  $\kappa = 0$ ; (b) for all  $\kappa \in [0, r - L]$ , the left-hand side of (A.1) is increasing at rate  $\hat{\theta}$  whereas the right-hand side is increasing at rate  $1 > \hat{\theta}$ , implying that (A.1) does not hold for all  $\kappa \in [0, r - L]$ ; (c) for all  $\kappa \in [r - L, r)$ , (A.1) is equivalent to  $(1 - \hat{\theta})r + \hat{\theta}\kappa \geq r$ , which does not hold; and (d) for all  $\kappa \geq r$ , both the left- and the right-hand sides of (A.1) are equal to  $r$ , implying that (A.1) holds with equality. Therefore, the bank optimally raises the minimum amount of capital that ensures no depositor withdrawal,  $\kappa = r$ , at stage 2. This results in an expected payoff to the bank equal to  $(1 - \hat{\theta})R - C - r$ .

- If  $\hat{\theta} < \underline{\theta}(r)$ , that is, (4) does not hold, the depositor does not withdraw even if the bank does not recapitalize. Therefore, the bank optimally does not recapitalize and obtains an expected payoff equal to  $(1 - \hat{\theta})(R - r)$ .

#### A.4 Proof of Proposition 2

The proof proceeds in two steps. First, notice that, given the stress-testing strategy described in the first bullet point of Proposition 2, the bank's equilibrium investment decision is as described in the second bullet point of Proposition 2.

Next, it remains to show that the stress-testing strategy described in the first bullet point of Proposition 2 is the unique equilibrium strategy at stage 2 if the bank originates a risky loan at stage 1.

Before we proceed, let us first characterize the equilibrium fully at stage 2 if the bank originates a risky loan at stage 1. Suppose that the bank originates a risky loan at stage 1. At stage 2, the regulator's stress-testing strategy is to fail the bank if and only if  $\theta \geq \theta_\tau^{FB} = \frac{\xi_\ell + C}{D}$ . We now derive the posterior beliefs  $\hat{\theta}^p$  and  $\hat{\theta}^f$  given the regulator's stress-test strategy. Notice that the first inequality in Assumption 5 ( $1 - \hat{\theta} < 1 - \frac{\xi_\ell + C}{D}$ ) implies that  $\theta_\tau^{FB} = \frac{\xi_\ell + C}{D} \in (0, \bar{\theta})$ . This then implies that both passing and failing the stress test occur on the equilibrium path, and  $\hat{\theta}^p, \hat{\theta}^f$  must satisfy Bayes' rule:

$$\hat{\theta}^p = \mathbb{E}[\theta | \theta \leq \theta_\tau^{FB}], \quad \hat{\theta}^f = \mathbb{E}[\theta | \theta \geq \theta_\tau^{FB}]. \quad (\text{A.3})$$

At stage 1, to raise funds to originate a risky loan, the interest rate  $r$  satisfies

$$\left(1 - Pr[\theta \leq \theta_\tau^{FB}] \hat{\theta}^p\right) r = 1. \quad (\text{A.4})$$

The left-hand side of this expression takes into account that, by Lemma 1, the bank raises  $r$  unit of capital at stage 2 if it fails the stress test. Therefore, the depositor expects to be repaid in full unless the bank passes the stress test, which occurs with probability  $Pr[\theta \leq \theta_\tau^{FB}]$ . From (A.4), we have

$$r = \frac{1}{1 - \left(z \int_0^{\theta_\tau^{FB}} \theta dH(\theta) + (1 - z) \int_0^{\theta_\tau^{FB}} \theta dH(\theta)\right)} \leq \frac{1}{1 - \int_0^{\bar{\theta}} \theta dH(\theta)} = \frac{1}{1 - \mathbb{E}[\theta]}. \quad (\text{A.5})$$

We now show that the above equilibrium exists. The equilibrium exists if and only if  $\hat{\theta}^f \geq \underline{\theta}(r) \geq \hat{\theta}^p$ . First, we have

$$\hat{\theta}^p = \mathbb{E}[\theta | \theta \leq \theta_\tau^{FB}] < \mathbb{E}[\theta_2]. \quad (\text{A.6})$$

Using Assumption 2, we then have, for all  $r \geq 1$ ,

$$\hat{\theta}^p < \mathbb{E}[\theta_2] < 1 - L \leq \frac{r - L}{r} = \underline{\theta}(r). \quad (\text{A.7})$$

Second, we have

$$\hat{\theta}^f = \mathbb{E}[\theta | \theta \geq \theta_\tau^{FB}] > \theta_\tau^{FB} = \frac{\xi_\ell + C}{D}. \quad (\text{A.8})$$

Using the second inequality in Assumption 5 and  $r \leq \frac{1}{1 - \mathbb{E}[\theta_2]}$  derived above, we then have

$$\hat{\theta}^f > \frac{\xi_\ell + C}{D} > 1 - (1 - \mathbb{E}[\theta_2])L \geq \frac{r}{r - L} = \underline{\theta}(r). \quad (\text{A.9})$$

In the last part of this proof, we show that this equilibrium is unique. Notice that the discussion preceding Proposition 2 implies that this is the unique equilibrium in which  $\hat{\theta}^f \geq \underline{\theta}(r) \geq \hat{\theta}^p$ . It then remains to show that any equilibrium must have  $\hat{\theta}^f \geq \underline{\theta}(r) \geq \hat{\theta}^p$ . We do so by establishing a sequence of results below. We first show that any equilibrium in which the bank passes and fails the stress test with strictly positive probabilities has  $\hat{\theta}^f \geq \underline{\theta}(r) \geq \hat{\theta}^p$ . We then show that any equilibrium in which the regulator either passes or fails the bank with certainty is not a PSE.

First, any equilibrium in which the bank passes and fails the stress test with strictly positive probabilities has  $\hat{\theta}^f \geq \underline{\theta}(r) \geq \hat{\theta}^p$ . We prove this by contradiction.

- Conjecture an equilibrium in which the bank passes and fails the stress test with strictly positive probabilities and  $\hat{\theta}^f, \hat{\theta}^p \leq \underline{\theta}(r)$ . Lemma 1 implies that the bank does not recapitalize both after passing and after failing the stress test at stage 2. It then follows that the regulator strictly prefers to pass the bank at stage 2 to save on the cost  $\xi_\tau$ . This contradicts the supposition that the bank passes and fails the stress test with strictly positive probabilities.
- Conjecture an equilibrium in which the bank passes and fails the stress test with strictly positive probabilities and  $\hat{\theta}^f, \hat{\theta}^p \geq \underline{\theta}(r)$ . Since  $\hat{\theta}^f, \hat{\theta}^p$  must satisfy Bayes' Rule, this implies that  $\mathbb{E}[\theta] \geq \underline{\theta}(r)$ , contradicting Assumption 2.
- Conjecture an equilibrium in which the bank passes and fails the stress test with strictly positive probabilities and  $\hat{\theta}^p \geq \underline{\theta}(r) \geq \hat{\theta}^f$ . Lemma 1 implies that the bank cannot recapitalize after passing the stress test at stage 2, and the depositor withdraws at stage 3, whereas the bank does not recapitalize after failing the stress test at stage 2, and the depositor does not withdraw at stage 3. It then follows that the regulator strictly prefers to fail the bank at stage 2 in order to avoid liquidating the bank for all  $\tau$  and for all  $\theta$ :

$$(1 - \theta)R - \theta D - \xi_\tau > L - D, \quad (\text{A.10})$$

which is implied by Assumption 4.

Next, we show that an equilibrium in which the regulator fails the bank with certainty is not a PSE. In such an equilibrium,  $\hat{\theta}^f = \mathbb{E}[\theta] < \underline{\theta}(r)$ , where the inequality follows from Assumption 2. We now demonstrate a credible updating rule following the bank passing the stress test that is consistent with both regulator types passing the bank with certainty. This updating rule implies that  $\hat{\theta}^p = \hat{\theta}^f = \mathbb{E}[\theta] < \underline{\theta}(r)$ . By Lemma 1, both after passing and after failing the stress test, the bank does not recapitalize at stage 2, and the depositor does not withdraw at stage 3. It then follows that the regulator strictly prefers to pass the bank at stage 2 with certainty to save on the cost  $\xi_\tau$ .

Lastly, we show that an equilibrium in which the regulator passes the bank with certainty is not a PSE. In such an equilibrium, we have  $\hat{\theta}^p = \mathbb{E}[\theta] < \underline{\theta}(r)$  by Assumption 2. We now demonstrate a credible updating rule following the bank failing the stress test that is consistent with the  $\tau$ -type regulator failing the bank if and only if  $\theta \geq \theta_\tau^*$ . This updating rule coincides with that in the equilibrium described in this proposition, and we showed above that in such an equilibrium, the posterior belief satisfies  $\hat{\theta}^f > \underline{\theta}(r)$  under Assumption 5. It follows that, under this updating rule, the bank recapitalizes at stage 2 if it fails the stress test by Lemma 1. Therefore, the  $\tau$ -type regulator would deviate and fail the bank if and only if  $\theta \geq \theta_\tau^{FB}$ .

This concludes the proof that the equilibrium described in Proposition 2 exists and is unique.

### A.5 Proof of Corollary 1

Corollary 1 immediately follows from Proposition 2 and Assumption 6.

### A.6 Proof of Proposition 3

This result immediately follows from Proposition 1 and the definition of  $B_\tau$  given in Section 2.1.

### A.7 Proof of Lemma 2

Let us define  $\underline{B}$  such that :

$$(\xi_h - \xi_\ell) \left[ U^0 - U_\ell^R(\theta_{2,h}^*) \right] + \xi_\ell \left[ U_h^R(\theta_{2,h}^*) - U_\ell^R(\theta_{2,\ell}^*) \right] = 0, \quad (\text{A.11})$$

where  $U_\tau^R(\theta_{2,\tau}^*)$  is given by (3). Notice that the first term of (A.11) is decreasing in  $B$  and is equal to 0 at  $B = B_\ell$ , while the second term of (A.11) is independent of  $B$  (which enters both  $U_h^R(\cdot)$  and  $U_\ell^R(\cdot)$ ) and is negative. We can now establish that  $\underline{B} < B_\ell$  exists and is unique. This follows because the left-hand side of (A.11) is (a) strictly decreasing in  $B$  and (b) strictly negative for  $B = B_\ell$ .

Suppose the bank invests in the safe asset at stage 1 in the first period. An equilibrium clearly exists in which both types of the regulator pass the bank at stage 2 with certainty. This can be sustained as an equilibrium with the off-equilibrium beliefs about the regulator's reputation  $z_2^f = z_2^p = z_1$ .

We prove that this equilibrium is unique in three steps: First, we consider an equilibrium in which the type- $h$  regulator passes the bank at stage 2 with strictly higher probability than the type- $\ell$  regulator and show that  $B > \underline{B}$  is a sufficient condition for this equilibrium not to exist. Second, we show that there exists no equilibrium in which the type- $\ell$  regulator passes the bank at stage 2 with strictly higher probability than the type- $h$  regulator. Third, we show that there exists no equilibrium in which both regulator types fail the bank at stage 2 with certainty.

Consider first an equilibrium in which the type- $h$  regulator passes the bank at stage 2 with strictly higher probability than the type- $\ell$  regulator. In this case, the posterior belief about the regulator's reputation satisfies  $z_2^p < z_2^f$ , and the probability that the bank originates a risky loan in the second period satisfies  $\lambda^p \geq \lambda^f$ . Given the bank's investment decision in the second period, the regulator's stress-testing strategy in the first period is optimal if and only if

$$\xi_h + \delta(\lambda^p - \lambda^f) \left[ U_h^R(\theta_{2,h}^*) - U^0 \right] \geq 0 \geq \xi_\ell + \delta(\lambda^p - \lambda^f) \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right]. \quad (\text{A.12})$$

Notice that  $B \geq B_\ell$  implies that the second inequality in (A.12) is not satisfied. For  $B < B_\ell$ , (A.12) is equivalent to

$$\frac{\xi_h}{U^0 - U_h^R(\theta_{2,h}^*)} \geq \delta(\lambda^p - \lambda^f) \geq \frac{\xi_\ell}{U^0 - U_\ell^R(\theta_{2,\ell}^*)}. \quad (\text{A.13})$$

In this case, there exists no  $\delta$ ,  $\lambda^p$  and  $\lambda^f$  such that (A.13) holds if

$$\begin{aligned} \frac{\xi_h}{U^0 - U_h^R(\theta_{2,h}^*)} &< \frac{\xi_\ell}{U^0 - U_\ell^R(\theta_{2,\ell}^*)} \\ \Leftrightarrow (\xi_h - \xi_\ell) \left[ U^0 - U_\ell^R(\theta_{2,\ell}^*) \right] &+ \xi_\ell \left[ U_h^R(\theta_{2,h}^*) - U_\ell^R(\theta_{2,\ell}^*) \right] < 0, \end{aligned} \quad (\text{A.14})$$

which is equivalent to  $B > \underline{B}$ . Therefore,  $B > \underline{B}$  is a sufficient condition that an equilibrium does not exist in which the type- $h$  regulator passes the bank with strictly higher probability than the type- $\ell$  regulator.

Next, consider an equilibrium in which the type- $\ell$  regulator passes the bank at stage 2 with strictly higher probability than the type- $h$  regulator. In this case, the posterior belief about the regulator's type satisfies  $z_2^p > z_2^f$ , and the probability that the bank originates a risky loan in the second period satisfies  $\lambda^p \leq \lambda^f$ . Given the bank's investment decision in the second period, the regulator's stress testing strategy in the first period is optimal if and only if

$$\xi_h + \delta(\lambda^p - \lambda^f) \left[ U_h^R(\theta_{2,h}^*) - U^0 \right] \leq 0 \leq \xi_\ell + \delta(\lambda^p - \lambda^f) \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right]. \quad (\text{A.15})$$

Notice that the first inequality and  $\lambda^p \leq \lambda^f$  imply  $B > B_h > B_\ell$ . However, this implies that the second inequality does not hold. Therefore, such an equilibrium does not exist.

Finally, consider an equilibrium in which both regulator types fail the bank at stage 2 with certainty. In such an equilibrium, the posterior belief about the regulator's type satisfies  $z_2^f = z_1$ . We now show that this is not a PSE by demonstrating a credible updating rule following the bank passing the stress test that is consistent with both regulator types passing the bank with certainty, that is,  $z_2^p = z_1$ . Under this updating rule, the bank's investment decision in the second period is unaffected by whether it is passes or fails the stress test in the first period. Therefore, both regulator types would, indeed, deviate and pass the bank with certainty to avoid the cost  $\xi_\tau$  of failing the bank.

This concludes the proof that, for  $B > \underline{B}$ , the equilibrium described in Lemma 2 exists and is unique.

### A.8 Lemma 3 and its proof (Lemma 3 will be used in Propositions 4 and 6)

Lemma 3 below is useful for the proofs of Propositions 4 and 6.

Let us define  $\theta_{1,\tau}(\delta) \in [0, \bar{\theta}]$  as

$$\theta_{1,\tau}(\delta) = \min \left\{ 0, \max \left\{ \frac{\xi_\tau + C + \delta \left[ U_\tau^R(\theta_{2,\tau}^*) - U^0 \right]}{D}, \bar{\theta} \right\} \right\}. \quad (\text{A.16})$$

Recall that  $U_\tau^R(\theta_\tau^*)$  is defined in (3) and depends on  $B$ .

Let us define  $\bar{\delta}(B)$  as the smallest  $\delta$  that satisfies

$$\theta_{1,\ell}(\delta) = \theta_{1,h}(\delta). \quad (\text{A.17})$$

Notice that, for  $\delta=0$ , we have  $0 < \theta_{1,\ell}(0) = \theta_{2,\ell}^* < \theta_{2,h}^* = \theta_{1,h}(0) < \bar{\theta}$ . We characterize the properties of  $\bar{\delta}(B)$  and define the thresholds  $B'_h$  and  $B'_\ell$  below.

- Suppose  $B \in [B_\ell, B_h]$ , where  $B_\ell$  and  $B_h$  are defined in Proposition 1,  $\theta_{1,\ell}^*(\delta)$  is increasing in  $\delta$  while  $\theta_{1,h}^*(\delta)$  is decreasing in  $\delta$ . For  $\delta \rightarrow \infty$ , we have  $\theta_{1,\ell}(\delta) = \bar{\theta} > \theta_{1,h}(\delta) = 0$ . Therefore, in this case,

$$\theta_{1,\tau}(\bar{\delta}(B)) = \frac{\xi_\tau + C + \bar{\delta}(B) \left[ U_\tau^R(\theta_{2,\tau}^*) - U^0 \right]}{D} \in (0, \bar{\theta}). \quad (\text{A.18})$$

Using the fact that (A.18) holds for all  $\tau \in \{h, \ell\}$ , we can derive  $\bar{\delta}(B)$ :

$$\bar{\delta}(B) = \bar{\delta} = \frac{\xi_h - \xi_\ell}{U_\ell^R(\theta_{2,\ell}^*) - U_h^R(\theta_{2,h}^*)}. \quad (\text{A.19})$$

Notice that  $\bar{\delta}$  is independent of  $B$  because in the denominator,  $U_\tau^R(\theta_{2,\tau}^*)$ , defined in (3), depends on  $B$  for both  $\tau \in \{\ell, h\}$  and, as a result  $U_\ell^R(\theta_{2,\ell}^*) - U_h^R(\theta_{2,h}^*)$ , does not.

- For  $B > B_h$ ,  $\frac{\partial \theta_{1,\tau}(\delta)}{\partial B} = \frac{\delta}{D}$  for  $\tau \in \{\ell, h\}$  implies that, there exists  $B'_h > B_h$ , such that for all  $B < B'_h$ , (A.18) holds and  $\bar{\delta}(B)$  is given by (A.19); for  $B \geq B'_h$ , (A.18) no longer holds, as  $\theta_{1,h}(\bar{\delta}(B))$  is capped at the upper bound  $\bar{\theta}$ . It then follows that  $B'_h$  is defined such that

$$\theta_{1,h}(\bar{\delta}) = \frac{\xi_h + C + \bar{\delta} \left[ U_h^R(\theta_{2,h}^*) - U^0 \right]}{D} = \bar{\theta}, \quad (\text{A.20})$$

where  $\bar{\delta}$  is defined by (A.19). After some algebraic manipulation, we have that  $B'_h > B_h$  is defined by

$$\frac{\xi_h - \xi_\ell}{U_\ell^R(\theta_{2,\ell}^*) - U_h^R(\theta_{2,h}^*)} \left[ U_h^R(\theta_{2,h}^*) - U^0 \right] = \bar{\theta} D - (\xi_h + C). \quad (\text{A.21})$$

- For  $B < B_\ell$ ,  $\frac{\partial \theta_{1,\tau}(\delta)}{\partial B} = \frac{\delta}{D}$  for  $\tau \in \{\ell, h\}$  implies that, there exists  $B'_\ell < B_\ell$ , such that for all  $B > B'_\ell$ , (A.18) holds and  $\bar{\delta}(B)$  is given by (A.19); for  $B < B'_\ell$ , (A.18) no longer holds, as  $\theta_{1,\ell}(\bar{\delta}(B))$  is bounded at the lower bound of 0. It then follows that  $B'_\ell < B_\ell$  is defined such that

$$\theta_{1,\ell}(\bar{\delta}) = \frac{\xi_\ell + C + \bar{\delta} \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right]}{D} = 0, \quad (\text{A.22})$$

where  $\bar{\delta}$  is defined by (A.19). After some algebraic manipulation, we have that  $B'_\ell$  is defined by

$$\frac{\xi_h - \xi_\ell}{U_\ell^R(\theta_{2,\ell}^*) - U_h^R(\theta_{2,h}^*)} \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right] = -(\xi_\ell + C). \quad (\text{A.23})$$



**Lemma 3.** For all  $\delta < \bar{\delta}(B)$ , where  $\bar{\delta}(B)$  is defined in (A.17), we have

- $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is increasing in  $\delta$  if and only if  $B \geq B'_h$ , where  $B'_h$  is defined by (A.21);
- $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is decreasing in  $\delta$  if and only if  $B \leq B'_\ell$ , where  $B'_\ell$  is defined by (A.23).

Recall that  $z_2^p(\cdot)$  and  $z_2^f(\cdot)$  are defined in (9) and (10), respectively. We now proceed to prove this lemma. We first show that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is increasing in  $\delta$  if and only if  $B \geq B'_h$ . Recall that the threshold  $B_\tau$ ,  $\tau \in \{\ell, h\}$ , is defined in Proposition 1. Consider the following three cases of  $B$ :

- For  $B \in [B_\ell, B_h]$ ,  $\theta_{1,\ell}(\delta)$  is increasing in  $\delta$  while  $\theta_{1,h}(\delta)$  is decreasing in  $\delta$ . Therefore,  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is decreasing in  $\delta$ .
- For  $B < B_\ell$ ,  $\theta_{1,\tau}(\delta) \leq \theta_{2,\tau}^* < \bar{\theta}$  and  $\theta_{1,\tau}(\delta)$  is decreasing in  $\delta$  for all  $\tau \in \{\ell, h\}$ . We can show that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is decreasing in  $\delta$ :

- For  $\delta$  such that  $\theta_{1,h}(\delta) \geq \theta_{1,\ell}(\delta) \geq 0$ , we have

$$\frac{dz_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)}{d\delta} = \frac{1 - z_1 \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right] + z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \left[ U_\ell^R(\theta_{2,\ell}^*) z_1 + U_h^R(\theta_{2,h}^*) (1 - z_1) - U^0 \right]}{D \left( \bar{\theta} - \theta_{1,\ell}(\delta) z_1 - \theta_{1,h}(\delta) (1 - z_1) \right)}. \quad (\text{A.24})$$

We have that (a)  $z_1 < z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  for all  $\delta < \bar{\delta}(B)$  and (b)  $U_h^R(\theta_{2,h}^*) < U_\ell^R(\theta_{2,\ell}^*) < U^0$ , where the first inequality follows because the low-cost regulator derives higher surplus from the risky loan than the high-cost regulator (where  $U_\tau^R(\theta_\tau^*)$  is given by (3)), and the second inequality is satisfied for all  $B < B_\ell$ . These two properties imply that the term in the first square bracket in the numerator of (A.24) is negative but strictly less negative than the term in the second square bracket. Therefore, (A.24) is strictly negative.

- For  $\delta$  such that  $\theta_{1,h}(\delta) > \theta_{1,\ell}(\delta) = 0$ ,  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is decreasing in  $\delta$  because  $\theta_{1,h}(\delta)$  is decreasing while  $\theta_{1,\ell}(\delta)$  remains constant.
- For  $B > B_h$ ,  $\theta_{1,\tau}(\delta) \geq \theta_{2,\tau}^* > 0$  and  $\theta_{1,\tau}(\delta)$  is increasing in  $\delta$  for all  $\tau \in \{\ell, h\}$ . We can show that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is increasing in  $\delta$  if and only if  $B < B'_h$ :

- For  $\delta$  such that  $\theta_{1,\ell}(\delta) \leq \theta_{1,h}(\delta) \leq \bar{\theta}$ ,  $\frac{dz_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)}{d\delta}$  is given by (A.24). Moreover, notice that the first derivative of the numerator of (A.24) with regard to  $\delta$  has the same sign as  $\frac{dz_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)}{d\delta}$ . This implies that  $\frac{dz_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)}{d\delta} > 0$  for all  $\delta$  if and only if it is positive at  $\delta = 0$ , that is,

$$-z_1 \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right] + z_2^f(\theta_{2,\ell}^*, \theta_h^*; z_1) \left[ U_\ell^R(\theta_{2,\ell}^*) z_1 + U_h^R(\theta_{2,h}^*) (1 - z_1) - U^0 \right] > 0. \quad (\text{A.25})$$

After some algebraic manipulation, this expression is independent of  $z_1$  and is equivalent to  $B > B'_h$ , where  $B'_h$  is defined in (A.21).

- For  $\delta$  such that  $\theta_{1,\ell}(\delta) < \theta_{1,h}(\delta) = \bar{\theta}$ ,  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) = 1$ .

Analogously, we can show that  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is decreasing in  $\delta$  if and only if  $B \leq B'_\ell$ . We omit the details of the derivation.

### A.9 Proof of Proposition 4

In this proof, we first characterize an equilibrium in which the bank recapitalizes in the first period if and only if it fails the stress test. We show that, in this equilibrium, the bank originates a risky loan with strictly higher probabilities after passing the stress test in the first period than after failing the stress test, that is,  $\lambda^p > \lambda^f$ . This is the reputation-building equilibrium described in the first bullet point of this proposition. We then characterize an equilibrium in which the bank does not recapitalize in the first period, regardless of the stress test result. We show that, in this equilibrium, the regulator passes the bank with certainty. This is the always-pass equilibrium described in the second bullet point of this proposition. Moreover, we show that the always-pass equilibrium exists if and only if the reputation-building equilibrium does not exist. Finally, we show that there exists no equilibrium in which the bank recapitalizes in the first period regardless of the stress test result.

Consider, first, an equilibrium in which the bank recapitalizes in the first period if and only if it fails the stress test. In this equilibrium, the regulator's stress-testing strategy  $\theta_{1,\tau}^*$  satisfies (8). There are four possibilities for the bank's investment decision in the second period.

- $\lambda^p = 1$  and  $\lambda^f = 0$ . That is, the bank originates a risky loan in the second period if and only if it passes the stress test in the first period. In this equilibrium, using (8) and setting  $\lambda^p = 1$  and  $\lambda^f = 0$ , the regulator's stress-testing strategy is given by  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta)$ , which is defined in (A.16). It then follows that  $\theta_{1,\tau}^*$  satisfies the properties described in this proposition:  $\theta_{1,\tau}^* \geq \theta_{2,\tau}^*$  if and only if  $B \geq B_\tau$ .

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^p(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) \leq z^* \leq z_2^f(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ , where  $z_2^p(\cdot)$  and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. We now characterize the conditions on  $B$ ,  $z_1$  and  $\delta$  such that this condition is satisfied.

- For  $z_1 \in (\underline{z}_1, z^*]$ , we have  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) < z_1 \leq z^*$  for all  $\delta \leq \bar{\delta}(B)$ . We now show that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \geq z^*$  if and only if either (a)  $B \leq B_h'$  and  $\delta \leq \underline{\delta}(B, z_1)$  or (b)  $B > B_h'$ .
  - \* For  $B \leq B_h'$ , we have (a)  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is decreasing in  $\delta$  (by Lemma 3); (b)  $z_2^f(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > z^*$  by the definition of  $\underline{z}_1$  in (11); and (c)  $z_2^f(\theta_{1,\ell}(\bar{\delta}(B)), \theta_{1,h}(\bar{\delta}(B)); z_1) = z_1 \leq z^*$  by the definition of  $\bar{\delta}(B)$  in (A.17). These properties imply that there exists  $\underline{\delta}(B, z_1) \in (0, \bar{\delta}(B)]$ , such that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \geq z^*$  if and only if  $\delta \leq \underline{\delta}(B, z_1)$ .
  - \* For  $B > B_h'$ , we have (a)  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is increasing in  $\delta$  (by Lemma 3) and (b)  $z_2^f(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > z^*$  by the definition of  $\underline{z}_1$  in (11). These properties imply that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \geq z^*$  for all  $\delta$ .
- For  $z_1 \in [z^*, \bar{z}_1]$ , we have  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) > z_1 \geq z^*$  for all  $\delta \leq \bar{\delta}(B)$ . We now show that  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \leq z^*$  if and only if either (a)  $B \geq B_\ell'$  and  $\delta \leq \underline{\delta}(B, z_1)$  or (b)  $B \leq B_\ell'$ .
  - \* For  $B \geq B_\ell'$ , we have (a)  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is increasing in  $\delta$  (by Lemma 3); (b)  $z_2^p(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) < z^*$  by the definition of  $\bar{z}_1$  in (12); and (c)  $z_2^p(\theta_{1,\ell}(\bar{\delta}(B)), \theta_{1,h}(\bar{\delta}(B)); z_1) = z_1 \geq z^*$  by the definition of  $\bar{\delta}(B)$  in (A.17). These properties imply that there exists  $\underline{\delta}(B, z_1) \in (0, \bar{\delta}(B)]$ , such that  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \leq z^*$  if and only if  $\delta \leq \underline{\delta}(B, z_1)$ .
  - \* For  $B < B_\ell'$ , we have (a)  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is increasing in  $\delta$  (by Lemma 3) and (b)  $z_2^p(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) < z^*$  by the definition of  $\bar{z}_1$  in (12). These properties imply that  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \leq z^*$  for all  $\delta \leq \bar{\delta}(B)$ . In this case, let us define  $\underline{\delta}(B, z_1) = \bar{\delta}(B)$ .
- $\lambda^p = 1$  and  $\lambda^f \in (0, 1]$ . That is, the bank originates a risky loan in the second period with certainty if it passes the stress test in the first period, and with probability  $\lambda^f \in (0, 1]$  if it

fails the stress test in the first period. In this equilibrium, using (8) and setting  $\lambda^P = 1$ , the regulator's stress testing strategy can be expressed as  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta(1 - \lambda^f))$ , where  $\theta_{1,\tau}(\cdot)$  is defined in (A.16).

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^f(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) = z^* > z_2^P(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ , where  $z_2^P(\cdot)$  and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. We now show that there exists  $\lambda^f \in (0, 1]$  that satisfies this condition if and only if  $z_1 \in (\underline{z}_1, z^*]$ ,  $B \leq B'_h$  and  $\delta > \underline{\delta}(B, z_1)$ , where  $\underline{\delta}(B, z_1)$  is defined above.

- For  $z_1 \in (\underline{z}_1, z^*]$ , we have  $z_2^P(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) < z_1 \leq z^*$  for all  $\delta \leq \bar{\delta}(B)$  and for all  $\lambda^f \in (0, 1]$ . We now show that there exists  $\lambda^f \in (0, 1]$  such that  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) = z^*$  if and only if  $B \leq B'_h$  and  $\delta > \underline{\delta}(B, z_1)$ :
  - \* For  $B \leq B'_h$ , we have (a)  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1)$  is increasing in  $\lambda^f$  (by Lemma 3); (b) for  $\lambda^f = 0$ , it is equal to  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$ , which is less than  $z^*$  if and only if  $\delta > \underline{\delta}(B, z_1)$  by the definition of  $\underline{\delta}(B, z_1)$ ; and (c) for  $\lambda^f = 1$ , it is equal to  $z_2^f(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) \geq z^*$  by the definition of  $\underline{z}_1$  in (11). These properties imply that there exists  $\lambda^f = 1 - \frac{\underline{\delta}(B, z_1)}{\delta} \in (0, 1)$  such that  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) = z^*$  if and only if  $\delta > \underline{\delta}(B, z_1)$ .
  - \* For  $B > B'_h$ , we have shown in the previous case that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \geq z^*$  for all  $\delta \leq \bar{\delta}(B)$ . Therefore, no  $\lambda^f \in (0, 1]$  exists such that  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) = z^*$ .
- For  $z_1 \in [z^*, \bar{z}_1)$ , we have  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) > z_1 \geq z^*$  for all  $\delta \leq \bar{\delta}(B)$ . Therefore, no  $\lambda^f \in (0, 1]$  exists such that  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) = z^*$ .
- $\lambda^P \in [0, 1)$  and  $\lambda^f = 0$ . That is, the bank originates a risky loan in the second period with probability  $\lambda^P \in [0, 1)$  if it passes the stress test in the first period, and invests in the safe asset in the second period with certainty if it fails the stress test in the first period. In this equilibrium, using (8) and setting  $\lambda^f = 0$ , the regulator's stress-testing strategy can be expressed as  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta\lambda^P)$ , where  $\theta_{1,\tau}(\cdot)$  is defined in (A.16).
 

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^f(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) > z^* = z_2^P(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ , where  $z_2^P(\cdot)$  and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. We now show that there exists  $\lambda^P \in [0, 1)$  that satisfies this condition if and only if  $z_1 \in [z^*, \bar{z}_1)$ ,  $B \geq B'_\ell$  and  $\delta > \underline{\delta}(B, z_1)$ , where  $\underline{\delta}(B, z_1)$  is defined above.

  - For  $z_1 \in [z^*, \bar{z}_1)$ , we have  $z_2^f(\theta_{1,\ell}(\delta\lambda^P), \theta_{1,h}(\delta\lambda^P); z_1) > z_1 \geq z^*$  for all  $\delta \leq \bar{\delta}(B)$  and for all  $\lambda^P \in [0, 1)$ . We now show that there exists  $\lambda^P \in [0, 1)$  such that  $z_2^P(\theta_{1,\ell}(\delta\lambda^P), \theta_{1,h}(\delta\lambda^P); z_1) = z^*$  if and only if  $B \geq B'_\ell$  and  $\delta > \underline{\delta}(B, z_1)$ :
    - \* For  $B \geq B'_\ell$ , we have (a)  $z_2^P(\theta_{1,\ell}(\delta\lambda^P), \theta_{1,h}(\delta\lambda^P); z_1)$  is increasing in  $\lambda^P$  (by Lemma 3); (b) for  $\lambda^P = 0$ , it is equal to  $z_2^P(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^P(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) \leq z^*$  by the definition of  $\bar{z}_1$  in (12); and (c) for  $\lambda^P = 1$ , it is equal to  $z_2^P(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$ , which is greater than  $z^*$  if and only if  $\delta > \underline{\delta}(B, z_1)$  by the definition of  $\underline{\delta}(B, z_1)$ . These properties imply that there exists  $\lambda^P = \frac{\underline{\delta}(B, z_1)}{\delta} \in (0, 1)$  such that  $z_2^P(\theta_{1,\ell}(\delta\lambda^P), \theta_{1,h}(\delta\lambda^P); z_1) = z^*$  if and only if  $\delta > \underline{\delta}(B, z_1)$ .
    - \* For  $B > B'_h$ , we showed in the previous case that  $z_2^P(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \leq z^*$  for all  $\delta \leq \bar{\delta}(B)$ . Therefore, no  $\lambda^P \in [0, 1)$  exists such that  $z_2^P(\theta_{1,\ell}(\delta\lambda^P), \theta_{1,h}(\delta\lambda^P); z_1) = z^*$ .
  - For  $z_1 \in (\underline{z}_1, z^*]$ , we have  $z_2^P(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) < z_1 \leq z^*$  for all  $\delta \leq \bar{\delta}(B)$ . Therefore, no  $\lambda^P \in [0, 1)$  exists such that  $z_2^P(\theta_{1,\ell}(\delta\lambda^P), \theta_{1,h}(\delta\lambda^P); z_1) = z^*$ .
- $1 > \lambda^P > \lambda^f > 0$ . That is, the bank adopts a mixed strategy in the second period both after passing and after failing the stress test in the first period. In this equilibrium, using (8), the

regulator's stress-testing strategy can be expressed as  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta(\lambda^p - \lambda^f))$ , where  $\theta_{1,\tau}(\cdot)$  is defined in (A.16).

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_1^f(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) = z_1^p(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) = z^*$ , implying that  $z_1 = z^*$  and  $\theta_{1,h}^* = \theta_{1,\ell}^*$ . We now show that there exist  $\lambda^p$  and  $\lambda^f$  that satisfy this condition if and only if  $z_1 = z^*$  and  $\delta > \underline{\delta}(B, z^*)$ , where  $\underline{\delta}(B, z_1)$  is defined above and satisfies  $\underline{\delta}(B, z^*) = \bar{\delta}(B)$ . This follows because we have, by the definition of  $\bar{\delta}(B)$ , that  $\theta_{1,h}^*(\delta) > \theta_{1,\ell}^*$  for all  $\delta < \bar{\delta}(B)$ . Therefore, for all  $\delta > \bar{\delta}(B)$ , there exist  $\lambda^p, \lambda^f$  such that  $\delta(\lambda^p - \lambda^f) = \bar{\delta}(B)$ , and the equilibrium satisfies  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta(\lambda^p - \lambda^f)) = \theta_{1,\tau}(\bar{\delta}(B))$  for all  $\tau \in \{h, \ell\}$ .

- $\lambda^p < \lambda^f$ . That is, the bank originates a risky loan in the second period with strictly lower probability if it passes the stress test in the first period than if it fails the stress test in the second period. We show that no such equilibrium exists. This is because  $\lambda^p < \lambda^f$  implies that the incentive to pass the bank, that is, the left-hand side of (8), is strictly higher for the high-cost regulator than for the low-cost regulator. This implies that  $\theta_{1,h}^* > \theta_{1,\ell}^*$ , which implies that  $z_1^f(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) > z_1^p(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ , contradicting the optimality of the bank's investment decision in the second period.

The above analysis shows that, for all  $z_1 \in (\underline{z}_1, \bar{z})$ , there exists a unique set of stress-testing strategies in the first period  $\theta_{1,\tau}^*$  that is optimal for the type- $\tau$  regulator and an investment decision in the second period  $(\lambda^p, \lambda^f)$  that is optimal for the bank. Moreover, the corresponding investment strategy by the bank in the second period is unique for all  $z_1 \neq z^*$  and is given by

$$\theta_{1,\tau}^* = \begin{cases} \frac{\xi_\tau + C + \delta \left[ \frac{U_\tau^R(\theta_{2,\tau}^*) - U^0}{D} \right]}{D}, & \text{if } \delta \leq \underline{\delta}(B, z_1), \\ \frac{\xi_\tau + C + \underline{\delta}(B, z_1) \left[ \frac{U_\tau^R(\theta_{2,\tau}^*) - U^0}{D} \right]}{D}, & \text{if } \delta > \underline{\delta}(B, z_1). \end{cases} \quad (\text{A.26})$$

To conclude the characterization of the reputation-building equilibrium, note that, in order for this to be an equilibrium in which the bank recapitalizes in the first period if and only if it fails the stress test,  $\theta_{1,\tau}^*$  must satisfy  $\hat{\theta}_1^f \geq \underline{\theta}(r_1) \geq \hat{\theta}_1^p$ , where

$$\hat{\theta}_1^f = \frac{z_1 \int_{\theta_{1,\ell}^*}^{\bar{\theta}} \theta_1 dH(\theta_1) + (1 - z_1) \int_{\theta_{1,h}^*}^{\bar{\theta}} \theta_1 dH(\theta_1)}{1 - z_1 H(\theta_{1,\ell}^*) - (1 - z_1) H(\theta_{1,h}^*)}, \quad (\text{A.27})$$

$$\hat{\theta}_1^p = \frac{z_1 \int_0^{\theta_{1,\ell}^*} \theta_1 dH(\theta_1) + (1 - z_1) \int_0^{\theta_{1,h}^*} \theta_1 dH(\theta_1)}{z_1 H(\theta_{1,\ell}^*) + (1 - z_1) H(\theta_{1,h}^*)}, \quad (\text{A.28})$$

and the interest rate  $r_1$  satisfies (5). It follows that  $\hat{\theta}_1^p \leq \mathbb{E}[\theta] < 1 - L \leq \underline{\theta}(r_1)$ , where we have used Assumption 2. Finally,  $\hat{\theta}_1^f \geq \underline{\theta}(r_1)$  is equivalent to

$$g(\theta_{1,\ell}^*, \theta_{1,h}^*) \equiv \frac{1 - \frac{z_1 \int_{\theta_{1,\ell}^*}^{\bar{\theta}} \theta_1 dH(\theta_1) + (1 - z_1) \int_{\theta_{1,h}^*}^{\bar{\theta}} \theta_1 dH(\theta_1)}{1 - z_1 H(\theta_{1,\ell}^*) - (1 - z_1) H(\theta_{1,h}^*)}}{1 - z_1 \int_0^{\theta_{1,\ell}^*} \theta_1 dH(\theta_1) - (1 - z_1) \int_0^{\theta_{1,h}^*} \theta_1 dH(\theta_1)} \leq L, \quad (\text{A.29})$$

which is obtained by evaluating (4) at the equilibrium quantities  $\hat{\theta}_1^f$  and  $r_1$  characterized above. We now show that, first, this condition holds for all  $B \geq B_\ell$ , and, second, for  $B < B_\ell$ , there exists  $\bar{\delta}(B, z_1)$ , such that this condition holds if and only if  $\delta \leq \bar{\delta}(B, z_1)$ .

- First, we show that  $g(\theta_{1,\ell}^*, \theta_{1,h}^*) \leq L$  for all  $B \geq B_\ell$ . To see this, recall that  $g(\theta_{1,\ell}^*, \theta_{1,h}^*)$  is defined as  $(1 - \hat{\theta}_1^f)r_1$  evaluated at the thresholds  $\theta_{1,\tau}^*$  defined for the reputation-building equilibrium. Notice that  $\hat{\theta}_1^f$  is defined in (A.27) and is a weighted average of  $\mathbb{E}[\theta | \theta \geq \theta_{1,\tau}^*]$

for  $\tau \in \{\ell, h\}$ . Thus, we have that  $\hat{\theta}_1^f \geq \theta_{1,\ell}^* \geq \theta_{2,\ell}^*$ , where the first inequality follows because  $\mathbb{E}[\theta | \theta \geq \theta_{1,\tau}^*] \geq \theta_{1,\tau}^*$  and  $\theta_{1,\ell}^* < \theta_{1,h}^*$ , and the last inequality follows because  $B \geq B_\ell$ . This then implies that

$$g(\theta_{1,\ell}^*, \theta_{1,h}^*) \leq (1 - \theta_{2,\ell}^*)\bar{r} \leq L, \quad (\text{A.30})$$

where the last inequality follows from Assumption 5 and  $\bar{r} = 1/(1 - \mathbb{E}[\theta])$ .

- Second, consider  $B < B_\ell$ . We show below that there exists  $\tilde{\delta}(B, z_1) \geq 0$ , such that  $g(\theta_{1,\ell}^*, \theta_{1,h}^*) \leq L$  if and only if  $\delta \leq \tilde{\delta}(B, z_1)$ . This follows because (a) as  $\delta \rightarrow 0$ ,  $g(\theta_{1,\ell}^*, \theta_{1,h}^*) \rightarrow g(\theta_{2,\ell}^*, \theta_{2,h}^*) \leq L$ , which follows from Proposition 2; and (b)  $g(\theta_{1,\ell}^*, \theta_{1,h}^*)$  is quasi-convex in  $\delta$ . This implies that  $\tilde{\delta}(B, z_1) \leq \infty$ ; in the case in which  $\tilde{\delta}(B, z_1) = \infty$ ,  $g(\theta_{1,\ell}^*, \theta_{1,h}^*) \leq L$  for all  $\delta$ .

To see the second property, notice that  $g(\theta_{1,\ell}^*, \theta_{1,h}^*)$  depends on  $\delta$  only through  $\theta_{1,\tau}^*$ , where, using (A.26), we have

$$\frac{d\theta_{1,\tau}^*}{d\delta} = \begin{cases} \frac{U_\tau^R(\theta_{2,\tau}^*) - U^0}{D} & \text{if } \delta \leq \underline{\delta}(B, z_1), \\ 0, & \text{if } \delta > \underline{\delta}(B, z_1). \end{cases} \quad (\text{A.31})$$

This implies that  $\frac{dg(\theta_{1,\ell}^*, \theta_{1,h}^*)}{d\delta} = 0$  for all  $\delta > \underline{\delta}(B, z_1)$ , whereas for  $\delta \leq \underline{\delta}(B, z_1)$ , we have, using  $r_1$  defined in (5):

$$\frac{dg(\theta_{1,\ell}^*, \theta_{1,h}^*)}{d\delta} = r_1 \left( r_1(1 - \hat{\theta}_2^f) \left[ z_1 \frac{\theta_{1,\ell}^*}{\theta} \frac{d\theta_{1,\ell}^*}{d\delta} + (1 - z_1) \frac{\theta_{1,h}^*}{\theta} \frac{d\theta_{1,h}^*}{d\delta} \right] - \frac{d\hat{\theta}_2^f}{d\delta} \right), \quad (\text{A.32})$$

and

$$\begin{aligned} \frac{d^2g(\theta_{1,\ell}^*, \theta_{1,h}^*)}{d\delta^2} &= 2r_1 \frac{dg(\theta_{1,\ell}^*, \theta_{1,h}^*)}{d\delta} \left[ z_1 \frac{\theta_{1,\ell}^*}{\theta} \frac{d\theta_{1,\ell}^*}{d\delta} + (1 - z_1) \frac{\theta_{1,h}^*}{\theta} \frac{d\theta_{1,h}^*}{d\delta} \right] \\ &\quad + r_1^2(1 - \hat{\theta}_2^f) \frac{1}{\theta} \left[ z_1 \left( \frac{d\theta_{1,\ell}^*}{d\delta} \right)^2 + (1 - z_1) \left( \frac{d\theta_{1,h}^*}{d\delta} \right)^2 \right] - r_1 \frac{d^2\hat{\theta}_2^f}{d\delta^2}, \end{aligned} \quad (\text{A.33})$$

where

$$\frac{d^2\hat{\theta}_2^f}{d\delta^2} = - \frac{z_1(1 - z_1) \left[ \left(1 - \frac{\theta_{1,\ell}^*}{\theta}\right) \frac{d\theta_{1,h}^*}{d\delta} + \left(1 - \frac{\theta_{1,h}^*}{\theta}\right) \frac{d\theta_{1,\ell}^*}{d\delta} \right]^2}{\left[1 - z_1 H(\theta_{1,\ell}^*) - (1 - z_1) H(\theta_{1,h}^*)\right]^3}. \quad (\text{A.34})$$

(A.32) and (A.33) imply that, for all  $\delta$  such that  $\frac{dg(\theta_{1,\ell}^*, \theta_{1,h}^*)}{d\delta} = 0$ , we have  $\frac{d^2g(\theta_{1,\ell}^*, \theta_{1,h}^*)}{d\delta^2} > 0$ . Therefore,  $g(B)$  is quasi-convex for all  $\delta$ .

To summarize, the reputation-building equilibrium characterized above exists for all  $B \geq B_\ell$ , and for all  $B < B_\ell$  and  $\delta \leq \tilde{\delta}(B, z_1)$ .

Next, consider an equilibrium in which the bank does not recapitalize in the first period, regardless of the stress test result. In this case, the regulator's stress-testing strategy in the first period does not depend on the quality of the bank's risky loan  $\theta_1$ , and the regulator trades off only the cost of failing the bank  $\xi_1$  against any potential reputation effects. The trade-off facing the regulator in this case is identical to the case if the bank invests in the safe asset in the first period, which is analyzed in Lemma 2. Therefore, Assumption 7 and Lemma 2 imply that, in such an equilibrium, both regulator types pass the bank with certainty. This is the always-pass equilibrium.

This equilibrium always exists and is supported by the off-equilibrium belief that  $\hat{\theta}_1^f = \hat{\theta}_1^p$ ,  $z_2^f = z_2^p$ , and  $\lambda^f = \lambda^p$ .

We now show that this equilibrium is not a PSE whenever a reputation-building equilibrium exists. In the reputation-building equilibrium, the  $\tau$ -type regulator fails the bank if and only if  $\theta \geq \theta_{1,\tau}^*$ . We now prune the always-pass equilibrium by demonstrating a credible updating rule following the bank failing the stress test that is consistent with the  $\tau$ -type regulator failing the bank if and only if  $\theta \geq \theta_{1,\tau}^*$ . Since this updating rule coincides with that in the reputation-building equilibrium, equilibrium condition (A.29) ensures that the posterior belief satisfies  $\hat{\theta}_1^f \geq \underline{\theta}(r_1)$ . It follows that, under this updating rule, the bank recapitalizes at stage 2 if it fails the stress test in the first period. Moreover, the bank follows an equilibrium strategy in the second period,  $(\lambda^p, \lambda^f)$ , that coincides with its strategy in the reputation-building equilibrium. Therefore, the  $\tau$ -type regulator would, indeed, deviate and fail the bank if and only if  $\theta \geq \theta_{1,\tau}^*$ .

Finally, we show that no equilibrium exists in which the bank faces a withdrawal threat, regardless of the stress test result (but is able to recapitalize only if it fails the stress test). If this were the case, we must have  $\hat{\theta}_1^f, \hat{\theta}_1^p > \underline{\theta}(r_1)$ . This, however, contradicts Assumption 2:

- If the bank passes and fails with strictly positive probabilities, then  $\mathbb{E}[\theta]$  must lie between  $\hat{\theta}_1^f$  and  $\hat{\theta}_1^p$  and is, therefore, greater than  $\underline{\theta}(r_1)$ , a contradiction.
- If the bank passes with certainty, then  $\hat{\theta}_1^p = \mathbb{E}[\theta] > \underline{\theta}(r_1)$ , a contradiction.
- If the bank fails with certainty, then  $\hat{\theta}_1^f = \mathbb{E}[\theta] > \underline{\theta}(r_1)$ , a contradiction.

#### A.10 Proof of Proposition 5

This result follows from the stress test outcome if the bank originates a risky loan in the first period as described in Proposition 4 and the bank's investment decision characterized by (6), with the thresholds  $\theta_{\tau}^{FB}$  replaced with  $\theta_{1,\tau}^*$  defined in Proposition 4.

#### A.11 Proof of Proposition 6

In this proof, we first characterize the equilibria in which the bank recapitalizes in the first period if and only if it fails the stress test. We show that this can be either a no-reputation equilibrium or (self-fulfilling) reputation-building equilibrium. Finally, we show that no equilibrium exists in which the bank's recapitalization decision in the first period does not depend on the stress test result.

Consider, first, an equilibrium in which the bank recapitalizes in the first period if and only if it fails the stress test. In this equilibrium, the regulator's stress-testing strategy  $\theta_{1,\tau}^*$  satisfies (8). There are six possibilities for the bank's investment decision in the second period.

- $\lambda^p = \lambda^f = 1$ . That is, the bank originates a risky loan in the second period with certainty. This is a no-reputation-building equilibrium, in which the regulator's stress-testing strategy in the first period is  $\theta_{1,\tau}^* = \theta_{2,\tau}^*$ .

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) < z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) \leq z^*$ , where  $z_2^p(\cdot)$  and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. It then follows from the definition of  $\underline{z}_1$  in (11) that this is the case if and only if  $z_1 \leq \underline{z}_1$ .

- $\lambda^p = \lambda^f = 0$ . That is, the bank invests in the safe asset in the second period with certainty. This is also a no-reputation-building equilibrium, in which the regulator's stress testing strategy in the first period is  $\theta_{1,\tau}^* = \theta_{2,\tau}^*$ .

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) \leq z^*$ , where  $z_2^p(\cdot)$  and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. It then follows from the definition of  $\bar{z}_1$  in (12) that this is the case if and only if  $z_1 \geq \bar{z}_1$ .

- $\lambda^p = 1$  and  $\lambda^f = 0$ . That is, the bank originates a risky loan in the second period if and only if it passes the stress test in the first period. This is a reputation-building equilibrium. In this equilibrium, using (8), the regulator's stress-testing strategy can be expressed as  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta)$ , where  $\theta_{1,\tau}(\cdot)$  is defined in (A.16).

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \leq z^* \leq z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$ , where  $z_2^p(\cdot)$  and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. We now show that this condition is satisfied if and only if either (a)  $z_1 < \underline{z}_1$ ,  $B > B'_h$  and  $\delta > \underline{\delta}(B, z_1)$ , or (b)  $z_1 > \bar{z}_1$ ,  $B < B'_\ell$  and  $\delta > \underline{\delta}(B, z_1)$ .

- For  $z_1 < \underline{z}_1$ , we have  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) < z_1 < \underline{z}_1 < z^*$  for all  $\delta \leq \bar{\delta}(B)$ . Consider  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$ . We have (a)  $z_2^f(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) < z^*$  by the definition of  $\underline{z}_1$  in (11); (b)  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is increasing in  $\delta$  if and only if  $B > B'_h$  (by Lemma 3); and (c) for  $B > B'_h$ ,  $z_2^f(\theta_{1,\ell}(\bar{\delta}(B)), \theta_{1,h}(\bar{\delta}(B)); z_1) = 1$  (by the proof of Lemma 3). These properties imply that there exists  $\underline{\delta}(B, z_1) \in (0, \bar{\delta}(B))$  such that  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \geq z^*$  if and only if  $B > B'_h$  and  $\delta \geq \underline{\delta}(B, z_1)$ .
- For  $z_1 > \bar{z}_1$ , we have  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) > z_1 > \bar{z}_1 > z^*$  for all  $\delta \leq \bar{\theta}(B)$ . Consider  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$ . We have (a)  $z_2^p(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > z^*$  by the definition of  $\bar{z}_1$ ; (b)  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1)$  is decreasing in  $\delta$  if and only if  $B < B'_\ell$  (by Lemma 3); and (c) for  $B < B'_\ell$ ,  $z_2^p(\theta_{1,\ell}(\bar{\delta}(B)), \theta_{1,h}(\bar{\delta}(B)); z_1) = 0$  (by the proof of Lemma 3). These properties imply that there exists  $\underline{\delta}(B, z_1) \in (0, \bar{\delta}(B))$  such that  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) \leq z^*$  if and only if  $B < B'_\ell$  and  $\delta \geq \underline{\delta}(B, z_1)$ .
- $\lambda^p = 1$  and  $\lambda^f \in (0, 1)$ . That is, the bank originates a risky loan in the second period if it passes the stress test in the first period, and with probability  $\lambda^f \in (0, 1)$  if it fails the stress test in the first period. This is a reputation-building equilibrium. In this equilibrium, using (8), the regulator's stress-testing strategy can be expressed as  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta(1 - \lambda^f))$ , where  $\theta_{1,\tau}(\cdot)$  is as defined in (A.16).

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^p(\delta(1 - \lambda^f), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) < z^* \leq z_2^f(\delta(1 - \lambda^f), \theta_{1,h}(\delta(1 - \lambda^f)); z_1)$ , where  $z_2^p(\cdot)$  and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. We now show that there exists  $\lambda^f \in (0, 1)$  that satisfies this condition if and only if  $z_1 < \underline{z}_1$ ,  $B > B'_h$  and  $\delta > \underline{\delta}(B, z_1)$ .

- For  $z_1 < \underline{z}_1$ , we have  $z_2^p(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) \leq z_1 < \underline{z}_1 < z^*$  for all  $\delta \leq \bar{\delta}(B)$  and for all  $\lambda^f \in (0, 1)$ . Consider  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1)$ . We have (a) for  $\lambda^f = 1$ ,  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1)$  is equal to  $z_2^f(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^f(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) < z^*$  by the definition of  $\underline{z}_1$ ; (b)  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1)$  is decreasing in  $\lambda^f$  if and only if  $B > B'_h$  (by Lemma 3); and (c) for  $B > B'_h$ ,  $z_2^f(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) > z^*$  if and only if  $\delta > \underline{\delta}(B, z_1)$  by the definition of  $\underline{\delta}(B, z_1)$  in the previous case. These properties imply that there exists  $\lambda^f = 1 - \frac{\underline{\delta}(B, z_1)}{\delta} \in (0, 1)$  such that  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) = z^*$  if and only if  $B > B'_h$  and  $\delta > \underline{\delta}(B, z_1)$ .
- For  $z_1 > \bar{z}_1$ , we have  $z_2^f(\theta_{1,\ell}(\delta(1 - \lambda^f)), \theta_{1,h}(\delta(1 - \lambda^f)); z_1) > z_1 > \bar{z}_1 > z^*$  for all  $\delta \leq \bar{\delta}(B)$  and for all  $\lambda^f \in (0, 1)$ . Therefore, no such equilibrium exists.
- $\lambda^p \in (0, 1)$  and  $\lambda^f = 0$ . That is, the bank originates a risky loan in the second period with probability  $\lambda^p \in (0, 1)$  if it passes the stress test in the first period and invests in the safe asset in the second period if it fails the stress test in the first period. This is a reputation-building equilibrium. In this equilibrium, using (8), the regulator's stress-testing strategy can be expressed as  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta\lambda^p)$ , where  $\theta_{1,\tau}(\cdot)$  is as defined in (A.16).

This is an equilibrium if the bank's investment decision in the second period is optimal, that is,  $z_2^p(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1) = z^* \leq z_2^f(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1)$ , where  $z_2^p(\cdot)$

and  $z_2^f(\cdot)$  are given by (9) and (10), respectively. We now show that there exists  $\lambda^p \in (0, 1)$  that satisfies this condition if and only if  $z_1 > \bar{z}_1$ ,  $B < B'_\ell$  and  $\delta > \underline{\delta}(B, z_1)$ .

- For  $z_1 > \bar{z}_1$ , we have  $z_2^f(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1) \geq z_1 > \bar{z}_1 > z^*$  for all  $\delta \leq \bar{\delta}(B)$  and for all  $\lambda^f \in (0, 1)$ . Consider  $z_2^p(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1)$ . We have (a) for  $\lambda^p = 0$ ,  $z_2^p(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1)$  is equal to  $z_2^p(\theta_{1,\ell}(0), \theta_{1,h}(0); z_1) = z_2^p(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > z^*$  by the definition of  $\bar{z}_1$ ; (b)  $z_2^p(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1)$  is decreasing in  $\lambda^p$  if and only if  $B < B'_\ell$  (by Lemma 3); and (c) for  $B < B'_\ell$ ,  $z_2^p(\theta_{1,\ell}(\delta), \theta_{1,h}(\delta); z_1) < z^*$  if and only if  $\delta > \underline{\delta}(B, z_1)$  by the definition of  $\underline{\delta}(B, z_1)$  above. These properties imply that there exists  $\lambda^p = \frac{\underline{\delta}(B, z_1)}{\delta} \in (0, 1)$  such that  $z_2^p(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1) = z^*$  if and only if  $B < B'_\ell$  and  $\delta > \underline{\delta}(B, z_1)$ .
- For  $z_1 < \bar{z}_1$ , we have  $z_2^p(\theta_{1,\ell}(\delta\lambda^p), \theta_{1,h}(\delta\lambda^p); z_1) < z_1 < \bar{z}_1 < z^*$  for all  $\delta \leq \bar{\delta}(B)$  and for all  $\lambda^p \in (0, 1)$ . Therefore, no such equilibrium exists.
- $\lambda^p < \lambda^f$ . Following similar arguments as in the proof of Proposition 4, we can show that no such equilibrium exists.

To conclude the characterization of equilibria in which the bank recapitalizes in the first period if and only if it fails the stress test, note that, in order for a reputation-building equilibrium to exist, it must satisfy (A.29), as argued in the proof of Proposition 4. This is equivalent to either  $B \geq B_\ell$  or  $B < B_\ell$  and  $\delta \leq \bar{\delta}(B, z_1)$ .

In summary, there are two types of equilibria in which the bank recapitalizes in the first period if and only if it fails the stress test:

- A no-reputation-building equilibrium always exists, in which the regulator's stress-testing strategy in the first period is  $\theta_{1,\tau}^* = \theta_{2,\tau}^*$ .
- A reputation-building equilibrium coexists if and only if either (a)  $z_1 < \bar{z}_1$ ,  $B > B'_h$  and  $\delta \geq \underline{\delta}(B, z_1)$ , or (b)  $z_1 > \bar{z}_1$ ,  $B < B'_\ell$  and  $\delta \in [\underline{\delta}(B, z_1), \bar{\delta}(B, z_1)]$ .

Next, consider an equilibrium in which the bank does not recapitalize in the first period, regardless of the stress test result. Following similar arguments as those in the proof of Proposition 4, we can show that, in such an equilibrium, both regulator types pass the bank with certainty (i.e., it is the always-pass equilibrium), and that this equilibrium is not a PSE, because there is a credible updating rule following the bank failing the stress test that is consistent with the  $\tau$ -type regulator failing the bank if and only if  $\theta \geq \theta_{2,\tau}^*$ .

Finally, consider an equilibrium in which the bank faces a withdrawal threat, regardless of the stress test result (but is able to recapitalize only if it fails the stress test). Again following similar arguments as those in the proof of Proposition 4, we can show that this equilibrium is not a PSE because there is a credible updating rule following the bank passing the stress test that is consistent with both regulator types passing the bank with certainty.

## A.12 Proof of Proposition 7

This result follows from the stress test outcome if the bank originates a risky loan in the first period, described in Proposition 6, and the bank's investment decision characterized by (6), with the thresholds  $\theta_\tau^{FB}$  replaced with  $\theta_{1,\tau}^*$  defined in Proposition 6.

## A.13 Proof of Corollary 2

This result follows from the analysis preceding the proposition.



#### A.14 Proof of Proposition 8

This proposition compares welfare in equilibrium for  $z_1 \in (z_1^*, \bar{z}_1)$  (characterized in Propositions 4–5) with that in the myopic benchmark characterized in Corollary 2.

Consider, first, the parameter space in which the unique equilibrium is a reputation-building equilibrium (i.e., (A.29) holds), and the bank's equilibrium investment choice is the same as in the myopic benchmark. By considering the following two cases, we show that in this case, the expected social surplus is (weakly) higher in the reputation-building equilibrium than in the myopic benchmark:

- For  $z_1 \in [z_1^*, \bar{z}_1]$ , in the myopic benchmark, the bank invests in the safe asset in both periods. If  $B \leq B^*(z_1, \delta)$ , the bank's investment choices in the reputation-building equilibrium are the same. In this case, welfare is identical in the reputation-building equilibrium and the myopic benchmark.
- For  $z_1 \in (z_1^*, z^*]$ , in the myopic benchmark, the bank originates a risky loan in the first period and originates a risky loan in the second period if and only if it passes the stress test in the first period. A reputation-building equilibrium with the same investment choices exists only if  $B \geq B^*(z_1, \delta)$  and either (a)  $B \leq B'_h$  and  $\delta \leq \delta(B, z_1)$  or (b)  $B > B'_h$ : First, from Proposition 5, the bank originates a risky loan in the first period if  $B \geq B^*(z_1, \delta)$ ; second, from the proof of Proposition 4, the bank's investment choice in the second period is to originate a risky loan if and only if it passes the stress test in the first period (i.e.,  $\lambda^p = 1$  and  $\lambda^f = 0$ ) if and only if either (a)  $B \leq B'_h$  and  $\delta \leq \delta(B, z_1)$  or (b)  $B > B'_h$ .

Given the bank's investment choices in both periods described above, the expected social surplus for any stress-testing strategy  $\theta_{1,\tau}$  followed by the type- $\tau$  regulator in the first period can be expressed as:

$$\begin{aligned} W^R(\theta_{1,\ell}, \theta_{1,h}; z_1) = & z_1 \left( U_\ell^R(\theta_{1,\ell}) + \delta \left[ U^0 + Pr(\theta_1 \leq \theta_{1,\ell}) \left( U_\ell^R(\theta_{2,\ell}^*) - U^0 \right) \right] \right) \\ & + (1 - z_1) \left( U_h^R(\theta_{1,h}) + \delta \left[ U^0 + Pr(\theta_1 \leq \theta_{1,h}) \left( U_h^R(\theta_{2,h}^*) - U^0 \right) \right] \right). \end{aligned} \quad (\text{A.35})$$

The first and second lines of this expression represent the expected social surplus when the regulator is of the low-cost and high-cost type, respectively. For each regulator type  $\tau \in \{\ell, h\}$ , the expression captures the expected social surplus of the risky loan in the first period and the discounted expected social surplus of the bank's investment in the second period, which is a risky loan if and only if the bank passes the stress test ( $\theta_1 \leq \theta_{1,\tau}$ ) in the first period.

It then follows that the expected social surpluses in the myopic benchmark and in the reputation-building equilibrium are equal to  $W^R(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1)$  and  $W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ , respectively.

Note that the regulator's equilibrium stress-testing strategy  $\theta_{1,\tau}^*$  is chosen to maximize the expected social surplus given the investment decision of the bank in the second period; that is, the first derivative  $W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$  with respect to  $\theta_{1,\tau}$  given by (8) is equal to zero. Therefore, by the optimality of  $\theta_{1,\tau}^*$ , we have  $W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) > W^R(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1)$ .

Consider, next, the parameter space in which the unique equilibrium is a reputation-building equilibrium ((A.29) holds), and the bank's equilibrium investment choice is different than in the myopic benchmark. In this case, welfare can be higher or lower in the equilibrium than in the myopic benchmark. Consider the following two examples.

- Welfare is strictly lower in the reputation-building equilibrium: Suppose that  $z_1 = z^* - \epsilon_1$ ,  $B \in (B_\ell, B_h)$  such that

$$z_1 \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right] + (1 - z_1) \left[ U_h^R(\theta_{2,h}^*) - U^0 \right] = \epsilon_2 > 0, \quad (\text{A.36})$$

and  $\delta \leq \delta(B, z_1)$ , where  $\epsilon_1, \epsilon_2 > 0$ . In this case, in the myopic benchmark, the bank invests in the safe asset in both periods because  $z_1 < z^*$ . The expected social surplus in the myopic benchmark is, thus, equal to

$$U^0(1+\delta). \quad (\text{A.37})$$

In the reputation-building equilibrium, if the bank originates a risky loan, the type- $\tau$  regulator's stress-testing strategy is given by  $\theta_{1,\tau}^* = \theta_{1,\tau}(\delta)$ , where  $\theta_{1,\tau}(\delta)$  is defined in (A.16). Recall that  $B \in (B_\ell, B_h)$  implies that  $\theta_{1,h}^* < \theta_{2,h}^*$  and  $\theta_{1,\ell}^* > \theta_{2,\ell}^*$ . Now, consider the bank's investment decision in the reputation-building equilibrium, which is given by (6) with the thresholds  $\theta_\tau^{FB}$  replaced with  $\theta_{1,\tau}^*$  as defined in Proposition 4. As  $\delta \rightarrow 0$ ,  $\theta_{1,\tau}^* \rightarrow \theta_{2,\tau}^*$ , which implies that the bank prefers to invest in the safe asset (since  $z_1 < z^*$ ). Moreover, the first derivative of the right-hand side of (6) with  $\theta_{1,\tau}^*$  given by (A.26) with respect to  $\delta$  is equal to

$$-\frac{z_1 \left[ U_\ell^R(\theta_{2,\ell}^*) - U^0 \right] + (1-z_1) \left[ U_h^R(\theta_{2,h}^*) - U^0 \right]}{\bar{\theta}} = -\frac{\epsilon_2}{\bar{\theta}} < 0.$$

Therefore, for  $\epsilon_1 \rightarrow 0$ , there exist  $\epsilon_2$  and  $\delta$  arbitrarily small, such that (6) with  $\theta_{1,\tau}^*$  holds in the reputation-building equilibrium and the bank originates a risky loan in the first period; subsequently, the bank originates a risky loan in the second period if and only if it passes the stress test in the first period. In this case, the expected social surplus in the reputation-building equilibrium is equal to  $W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ .

We can now compare the expected social surplus in the reputation-building equilibrium to that in the myopic benchmark. As  $\epsilon_1, \epsilon_2 \rightarrow 0$  and  $\delta \rightarrow 0$ , we have

$$\begin{aligned} W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) &\rightarrow W^R(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) \\ &= [z_1 U_\ell^R(\theta_{2,\ell}^*) + (1-z_1) U_h^R(\theta_{2,h}^*)] + \delta U^0 \\ &\quad + \delta \left[ z_1 \Pr(\theta_1 \leq \theta_{2,\ell}^*) (U_\ell^R(\theta_{2,\ell}^*) - U^0) + (1-z_1) \Pr(\theta_1 \leq \theta_{2,h}^*) \right. \\ &\quad \left. \times (U_h^R(\theta_{2,h}^*) - U^0) \right] \\ &\rightarrow U^0(1+\delta) + \delta \Pr(\theta_{2,\ell}^* \leq \theta_1 \leq \theta_{2,h}^*) (U_h^R(\theta_{2,h}^*) - U^0) \\ &< U^0(1+\delta), \end{aligned} \quad (\text{A.38})$$

where the last inequality follows from the fact that  $B < B_h$  implies that  $U_h^R(\theta_{2,h}^*) - U^0 < 0$ . Therefore, in this case, the expected social surplus in the reputation-building equilibrium is strictly lower than in the myopic benchmark.

- Welfare is strictly higher in the reputation-building equilibrium: Suppose that  $z_1 = z^* - \epsilon$ ,  $B \geq B_h$ , and  $\delta \leq \delta(B, z_1)$ , where  $\epsilon > 0$ . In this case, in the myopic benchmark, the bank invests in the safe asset in both periods because  $z_1 < z^*$ . The expected social surplus in the myopic benchmark is, thus, given by (A.37). In the reputation-building equilibrium, following similar arguments as in the previous example, we can show that as  $\epsilon \rightarrow 0$ , there exists  $\delta$  arbitrarily small, such that (6) with  $\theta_{1,\tau}^*$  holds and the bank originates a risky loan in the first period; subsequently, the bank originates a risky loan in the second period if and only if it passes the stress test in the first period. In this case, the expected surplus in the reputation-building equilibrium is equal to  $W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ .

It then follows that the expected social surplus in the reputation-building equilibrium is strictly higher than that in the myopic benchmark  $W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1) > W^R(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) > U^0(1+\delta)$ . The first inequality follows from the optimality of  $\theta_{1,\tau}^*$ . The second inequality follows because, in each period, the bank invests in the risky loan with higher probability

in the reputation-building equilibrium than in the myopic benchmark, resulting in strictly higher expected social surplus, as  $B \geq B_h$  implies that  $U_\ell^R(\theta_{2,\ell}^*) > U_h^R(\theta_{2,h}^*) \geq U^0$ .

Consider, finally, the parameter space in which the unique equilibrium is an always-pass equilibrium ((A.29) does not hold). We show that the expected social surplus is strictly lower in the always-pass equilibrium than in the myopic benchmark.

- For  $z_1 \in [z^*, \bar{z}_1]$ , in the myopic benchmark, the bank invests in the safe asset in both periods. The expected social surplus in the myopic benchmark is given by (A.37). In the always-pass equilibrium, since the regulator passes the bank with certainty in the first period, the bank originates a risky loan in the first period; the bank subsequently invests in the safe asset in the second period because it anticipates a tough stress test in the second period ( $z_1 \geq z^*$ ). The expected social surplus in the always-pass equilibrium is, thus, equal to

$$[z_1 U_\ell^R(\bar{\theta}) + (1 - z_1) U_h^R(\bar{\theta})] + \delta U^0 = U_h^R(\bar{\theta}) + \delta U^0. \quad (\text{A.39})$$

This expression reflects that the expected social surplus from a risky loan is the same for both regulator types when the bank passes with certainty.

Recall that an always-pass equilibrium exists if and only if  $B < B_\ell$ , which then implies that  $U_\tau^R(\bar{\theta}) < U_\tau^R(\theta_{2,\tau}^*) < U^0$  for all  $\tau \in \{\ell, h\}$ . Therefore, (A.39) is strictly lower than (A.37), implying that the expected social surplus is strictly lower in the always-pass equilibrium than in the myopic benchmark.

- For  $z_1 \in (z_1^*, z^*)$ , in the myopic benchmark, the bank originates a risky loan in the first period and subsequently originates a risky loan in the second period if and only if it passes the stress test in the first period. The expected social surplus in the myopic benchmark is equal to  $W^R(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1)$ , where  $W^R(\cdot)$  is given in (A.35). In the always-pass equilibrium, the bank originates a risky loan in the first period and does not recapitalize, and it subsequently originates a risky loan in the second period with certainty. The expected social surplus in the myopic benchmark can be expressed as  $W^R(\bar{\theta}, \bar{\theta}; z_1)$ . Recall that we showed above that the existence of an always-pass equilibrium implies that  $B < B_\ell$ . Writing out the difference in expected social surplus:

$$\begin{aligned} & W^R(\bar{\theta}, \bar{\theta}; z_1) - W^R(\theta_{2,\ell}^*, \theta_{2,h}^*; z_1) \\ &= z_1 \int_{\theta_{2,\ell}^*}^{\bar{\theta}} (\xi_\ell + C - \theta D) + \delta [U_\ell^R(\theta_{2,\ell}^*) - U^0] dH(\theta) \\ & \quad + (1 - z_1) \int_{\theta_{2,h}^*}^{\bar{\theta}} (\xi_h + C - \theta D) + \delta [U_h^R(\theta_{2,h}^*) - U^0] dH(\theta) < 0. \end{aligned} \quad (\text{A.40})$$

This expression is strictly negative because, for all  $\tau \in \{\ell, h\}$ , we have  $\xi_\tau + C - \theta D < 0$  for all  $\theta \geq \theta_{2,\tau}^*$  and  $U_\tau^R(\theta_{2,\tau}^*) - U^0 < 0$  for all  $B < B_\ell < B_h$ .

### A.15 Proof of Proposition 9

This proposition compares welfare in equilibrium for  $z_1 \notin (z_1^*, \bar{z}_1)$ , characterized in Propositions 6–7, to that in the myopic benchmark, characterized in Corollary 2.

Consider, first, the no-reputation-building equilibrium, which exists for all  $z_1 \notin (z_1^*, \bar{z}_1)$ . The equilibrium outcome is identical to that in the myopic benchmark. The expected social surplus is, therefore, also the same as in the myopic benchmark.

Consider, next, the self-fulfilling reputation-building equilibrium. We now show that the expected social surplus in the self-fulfilling reputation-building equilibrium is (weakly) lower than in the myopic benchmark.

- For  $z_1 < \bar{z}_1$ ,  $B > B'_h$ , and  $\delta > \underline{\delta}(z_1, B)$ , in the myopic benchmark, the bank originates a risky loan in both periods with certainty. The expected social surplus in the myopic benchmark is, thus, given by

$$[z_1 U_\ell^R(\theta_{2,\ell}^*) + (1 - z_1) U_h^R(\theta_{2,h}^*)](1 + \delta). \quad (\text{A.41})$$

In the self-fulfilling reputation-building equilibrium, the bank originates a risky loan in the first period and subsequently originates a risky loan in the second period if and only if it passes the stress test in the first period. The expected social surplus in equilibrium is given by  $W^R(\theta_{1,\ell}^*, \theta_{1,h}^*; z_1)$ , where  $W^R(\cdot)$  is defined in (A.35). Thus, we have that the expected social surplus is strictly lower in the self-fulfilling reputation-building equilibrium than in the myopic benchmark because (a) the expected social surplus of the bank's risky loan in the first period is lower  $U_\tau^R(\theta_{1,\tau}^*) < U_\tau^R(\theta_{2,\tau}^*)$  due to the optimality of  $\theta_{2,\tau}^*$  and (b) the expected social surplus of the bank's investment in the second period is lower  $U^0 + Pr(\theta_1 \leq \theta_{1,\tau}) (U_\tau^R(\theta_{2,\tau}^*) - U^0) < U_\tau^R(\theta_{2,\tau}^*)$  because  $B > B'_h$  implies  $U_\tau^R(\theta_{2,\tau}^*) > U^0$ .

- For  $z_1 > \bar{z}_1$ ,  $B < B'_\ell$ , and  $\delta > \underline{\delta}(z_1, B)$ , the equilibrium outcome is identical to that in the myopic benchmark. The expected social surplus is, thus, also the same as in the myopic benchmark.

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