

## Euclid Quick Data Release (Q1)

### XXIX. The strong-lensing discovery engine D: Double-source-plane lens candidates

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## ABSTRACT

Strong gravitational-lensing systems with multiple source planes are powerful tools for probing the density profiles and dark matter substructure of galaxies. The ratio of the Einstein radii is related to the dark energy equation of state through the cosmological scaling factor  $\beta$ . Galaxy-scale double-source-plane lenses (DSPLs) are extremely rare, however. We report the discovery of four new galaxy-scale DSPL candidates in the Euclid Quick Release 1 (Q1) data. These systems were initially identified through a combination of machine-learning lens-finding models and subsequent visual inspection from citizens and experts. We applied the widely-used LensPop lens-forecasting model to predict that the full *Euclid* survey will discover 1700 DSPLs. This scales to  $6 \pm 3$  DSPLs in  $63 \text{ deg}^2$ , which is the area of Q1. The number of discoveries in this work is broadly consistent with this forecast. We present lens models for each DSPL and infer their  $\beta$  values. Our initial Q1 sample demonstrates that *Euclid* promises to discover these rare objects.

**Key words.** gravitational lensing: strong – galaxies: structure – cosmological parameters

## 1. Introduction

Strong gravitational lensing occurs when a background source and a foreground massive object align along our line of sight, which causes the light from the source to be split into multiple images or to form an Einstein ring (Einstein 1936; Zwicky 1937a,b). There can be multiple sources at different redshifts behind the same lens galaxy at times that form Einstein rings at different radii. These systems are referred to as compound lenses or double-source-plane lenses (DSPLs).

In strong-lensing systems, the radius of the Einstein ring is known as the Einstein radius (parameterised by  $\theta_E$ ) and depends on both the mass profile of the lens and on the cosmological distances that are involved. Lens modelling on systems with a single Einstein ring can constrain the local logarithmic density slope in the region in which the strongly lensed images are observed when a specific parametric form is assumed for the total mass profile of the lens galaxy (e.g., Suyu & Halkola 2010; Birrer & Amara 2018; Nightingale et al. 2021; Galan et al. 2022). Systematics affect the results, however, such as the mass-sheet degeneracy (Falco et al. 1985; Schneider & Sluse 2013): When a constant mass sheet is added and the convergence is rescaled, mass-sheet transformations (MST) can alter the shape of the galaxy mass profile and rescale the size of the source while keeping all lensing observables unchanged. In DSPL systems, an additional source plane can break the mass-sheet degeneracy of the lens if the mass contribution of the first source is neglected (Bradač et al. 2004) and the cosmology is known.

The presence of two rings at different radii makes DSPLs powerful tools for probing the mass distribution of galaxies, based on which, the properties of dark matter can be constrained. The second ring provides an additional aperture within which the total mass is well constrained, so that we can disentangle the distribution of dark and luminous matter in the lens without additional kinematic data (Sonnenfeld et al. 2012). When a dark matter subhalo exists in the lens plane, the inclusion of a second source probes the mass distribution on a larger radius. This can break degeneracies in the lens model and improve the constraints on its mass properties (see Figure 6 in Enzi et al. 2025, where the posteriors of the dark matter subhalo when modelled with two arcs are tighter). For example, several observations and analyses of the so-called Jackpot lens SDSS J0946+1006 have suggested the existence of an over-concentrated subhalo (see Vegetti et al. 2010; Minor et al. 2021; Ballard et al. 2024; Despali et al. 2025; Enzi et al. 2025).

The DSPLs have also been used to constrain the equation of state of dark energy. For a system with multiple sources at different redshifts, the ratio of their Einstein radii is related to cosmological parameters such as the dark energy equation of state (parametrised by  $w$ ) through the cosmology scaling factor  $\beta$ . The cosmological measurements derived from DSPLs are independent of the Hubble constant and complement other cosmological measurements, as demonstrated by previous studies (see e.g., Gavazzi et al. 2008; Collett et al. 2012; Johnson et al. 2025). Collett & Auger (2014) showed that a single galaxy-scale DSPL system can provide competitive cosmological constraints assuming a power-law mass profile of the lens galaxy. Although

**Table 1.** Sky locations are RA and Dec of the four lens systems.

| Name             | RA      | Dec     |
|------------------|---------|---------|
| Teapot Lens      | 273°595 | 67°138  |
| Cosmic Dartboard | 59°565  | −50°948 |
| Galileo’s Lens   | 66°685  | −48°111 |
| Cosmic Ammonite  | 61°239  | −49°373 |

cluster-scale lenses typically have multiple source planes and are also capable of measuring cosmological parameters (e.g., Soucail et al. 2004; Jullo et al. 2010; Caminha et al. 2016, 2022; Acebron et al. 2017; Magana et al. 2018), their complex mass distributions limit the precision. While selecting strong-lensing clusters or galaxy groups with simpler mass distributions is a possible alternative (Bolamperti et al. 2024), galaxy-scale lenses generally exhibit a lower systematic uncertainty due to their inherently simpler mass profiles. Schneider (2014) argued, however, that analogue of the MST exists for DSPL systems that can render the cosmological constraints significantly less restrictive. A 1% bias in  $\beta$  can give rise to a bias in  $w$  of approximately 0.3. Additional data from stellar kinematics, for example, can help to break this degeneracy, but competitive results require unbiased measurements of the kinematics and correct assumptions about the mass profiles. These are both challenging tasks.

Unfortunately, galaxy-scale strong lenses with multiple background sources are exceedingly rare, and only a handful of examples are known in the literature: the Jackpot lens (Gavazzi et al. 2008), the Eye of Horus (Tanaka et al. 2016), and J1721+8842 (Dux et al. 2025). Other DSPLs whose sources have similar Einstein radii are DES0408−5354 (Lin et al. 2017) and the Cosmic Horseshoe (Belokurov et al. 2007). This rarity arises from the fact that whilst the probability of being a single-plane lens scales as  $\theta_E^2$ , the probability of lensing two sources scales as  $\theta_E^4$ . Even with the depth and resolution of the *Hubble* Space Telescope, only about one in a hundred lenses is expected to be a DSPL (Gavazzi et al. 2008), although the exact rate depends on the depth, resolution, and wavelength of the observations (Collett et al. 2012). The *Euclid* space telescope provides an unprecedented inspection of the sky, and forecasts show that about 1700 DSPLs will be discovered during the survey (rescaling from the 170 000 single-source-plane lenses forecast in Collett 2015).

We report the discovery of four new galaxy-scale DSPL candidates in the 63 deg<sup>2</sup> of the Euclid Quick Release 1 (Q1 Euclid Quick Release Q1 2025). The  $I_E$ -band images and coloured images of the four systems are presented in Fig. 1, and their coordinates are shown in Table 1. Based on the morphology of the four discovered lens systems, they were named Teapot Lens, Cosmic Dartboard, Galileo’s Lens (this lensing configuration somewhat resembles Galileo’s hand-drawn depiction of Saturn), and Cosmic Ammonite.

These four new DSPLs were discovered using the strong-lensing discovery engine, which is a structured workflow that integrates machine learning with citizen-science classifications, followed by expert grading. This paper is part of a series of papers, with Euclid Collaboration: Walmsley et al. (2026) describing the *Euclid* lens-finding engine and new discoveries. Euclid Collaboration: Rojas et al. (2026) describes our effort to build a suitable training sample for Q1 and the lens search with high-velocity dispersion galaxies.

Euclid Collaboration: Lines et al. (2026) explains the machine-learning models we used to find the strong lenses. Finally, Euclid Collaboration: Holloway et al. (2026) introduces an ensemble classifier that combines the citizen-science and different machine-learning classifiers.

The paper is organised as follows. In Sect. 2 we briefly summarise the *Euclid* survey and the lens-finding methods we employed. Section 3 introduces the lens-modelling algorithm, followed by the lens models and spectra, which are presented in Sect. 4. In Sect. 5 we present the forecast of the number of DSPLs that *Euclid* will discover in its planned survey area. Finally, Sect. 6 presents our conclusions. We discuss the limitations of our lens-finding approach and provide examples of typical false DSPL systems in Appendix A.

## 2. Search for a *Euclid* double-source-plane lens

The search for DSPLs is a by-product of the strong-lensing discovery engine, which had the primary goal of finding single-source-plane galaxy-scale strong lenses in *Euclid* data. In the lens-finding pipeline described in Euclid Collaboration: Walmsley et al. (2026), the lens candidates selected by machine-learning and citizen scientists (from objects that are brighter than 22.5 in  $I_E$  in the 63 deg<sup>2</sup> of Q1) are sent to the Galaxy Judges project, where a group of strong-lensing experts provides a final grade for each lens candidate. Following this semi-automated search for all galaxy-galaxy lenses, a group (approximately ten) of experienced experts re-inspected the top 10 000 lenses and identified four DSPLs by eye. All four of our DSPL candidates are ranked in the top 250 lenses by the expert visual inspection.

The primary criterion for identifying a DSPL system are two pairs of arcs at different Einstein radii. The colours of the arcs serve as a key indicator to determine whether they originate from the same source plane. When a lens is identified as a potential DSPL candidate, preliminary lens modelling (described in Sect. 3) is performed to further evaluate the plausibility of its configuration.

*Euclid* offers significant advantages for DSPL searches due to its high-resolution imaging over a large area. Its point-spread function (PSF) has a full width at half maximum of approximately 0′.16 in the  $I_E$  band (Euclid Collaboration: Mellier et al. 2025; Euclid Collaboration: McCracken et al. 2026). This high resolution enables the separation of lens light from the arcs and allows us to resolve multiple arcs. Although the infrared channels have a much lower resolution, their colour information is helpful for associating different arcs.

## 3. Modelling the double-source-plane lens

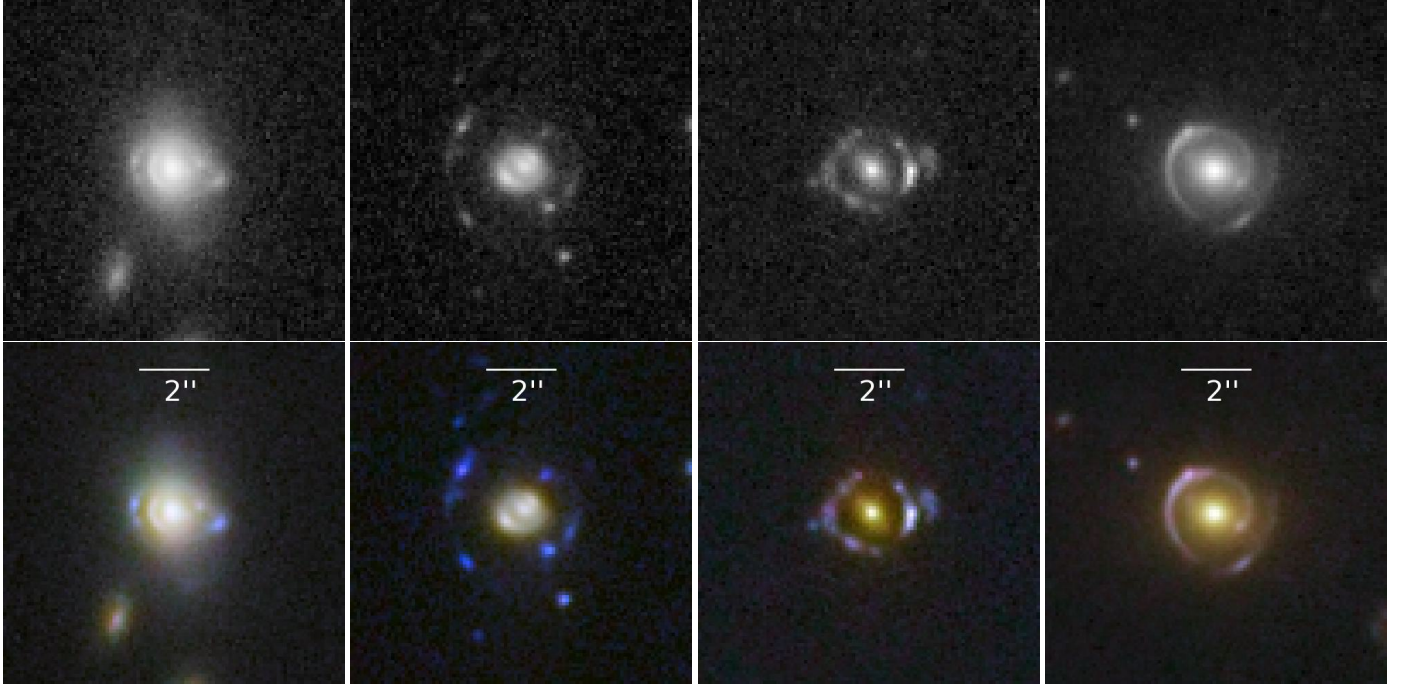
### 3.1. Background theory

For a lensed image at position  $\theta$ , the scaled deflection angle of a galaxy  $\alpha(\theta)$  is related to its lensing potential  $\psi$  via

$$\alpha(\theta) = \nabla\psi(\theta), \quad (1)$$

and the relation between lensing potential and lensing convergence is

$$\kappa(\theta) = \frac{1}{2} \nabla^2 \psi(\theta), \quad (2)$$



**Fig. 1.**  $I_E$  (top panels) and  $I_E$ - $Y_E$ - $J_E$  coloured images (bottom panels) of the DSPL candidates. From left to right, we show Teapot Lens, Cosmic Dartboard, Galileo's Lens, and Cosmic Ammonite. North is up, and east is to the left.

where convergence is defined as

$$\kappa(\theta) \equiv \frac{\Sigma(\theta)}{\Sigma_{\text{cr}}} \quad (3)$$

The convergence is the surface mass density normalised by the critical lensing surface density,

$$\Sigma_{\text{cr}} \equiv \frac{c^2 D_s}{4\pi G D_l D_{ls}} \quad (4)$$

where  $D$  is the angular diameter distance between two objects, and the subscripts  $l$  and  $s$  denote the lens galaxy and the source galaxy, respectively.

In a DSPL system, the lens equation of the first source plane can be written as

$$\theta_1 = \theta - \beta \alpha_1(\theta), \quad (5)$$

where  $\alpha_1$  is the reduced deflection angle of the first source at the image position  $\theta$ , and  $\theta_1$  is the position of the first source on its source plane. Here,  $\beta$  is the cosmological scaling factor, which is the ratio of the angular diameter distances between the different redshift planes,

$$\beta = \frac{D_{ls1} D_{s2}}{D_{s1} D_{ls2}}, \quad (6)$$

where the subscripts  $s1$  and  $s2$  denote source 1 and source 2, respectively. The lens equation of the second source plane is then

$$\theta_2 = \theta - \alpha_1(\theta) - \alpha_{s1}[\theta - \beta \alpha_1(\theta)], \quad (7)$$

where  $\theta_2$  is the position of the second source on its source plane, and  $\alpha_{s1}$  is the deflection angle of the mass on the first source plane. In a simple case for which we assume that the lens galaxy has a singular isothermal sphere (SIS) profile, the parameter  $\beta$  is

just the ratio of the two Einstein radii (the geometry of the DSPL is in Figure 1 of Collett & Auger 2014).

The lenses are modelled using an elliptical power-law (EPL) mass model with an external shear component. The convergence ( $\kappa_{\text{EPL}}$ ) of the EPL model can be parametrised as

$$\kappa_{\text{EPL}}(\rho, \gamma, q) = \frac{3 - \gamma}{2} \left( \frac{\theta_E}{\rho} \right)^{\gamma-1}, \quad (8)$$

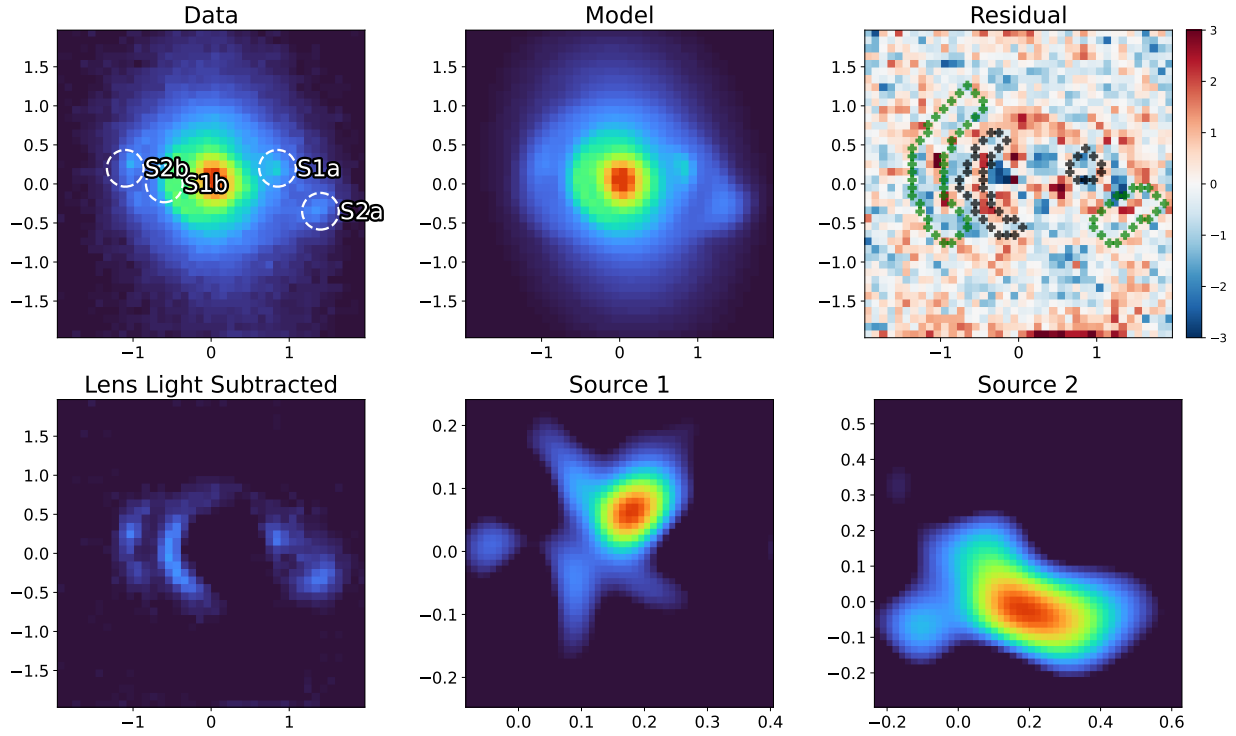
where  $\rho^2 = x^2 q + y^2 / q$  is the squared elliptical radius, and  $q$  is the axis ratio,  $\gamma$  is the power-law index of the density slope, and  $\theta_E$  is the Einstein radius. We used  $\theta_{E1}$  to represent the Einstein radius of the primary lens with respect to source 1,  $\theta_{E2}$  for the Einstein radius of the same lens with respect to source 2 (with source 1 absent), and  $\theta_{\text{sis1}}$  for the Einstein radius when source 1 itself lensed source 2. The cosmological parameters can therefore be measured by relating  $\beta$  from the lens modelling and the redshifts of the system.

### 3.2. Lens modelling strategy

We used the open-source lens modelling code *Herculens*<sup>1</sup> (Galan et al. 2022), which now includes multiplane lensing capabilities (see, e.g. Enzi et al. 2025). *Herculens* is based on the automatic differentiation and compilation features of JAX<sup>2</sup> and can run on graphics processing units (GPUs). We note here that our lens modelling primarily aims to validate that these systems are plausible DSPLs, and we do not to infer the parameters of these systems precisely. The lack of redshift information for either the lens or the sources prevents us from building a robust mass distribution and leverage the full statistical analyses from Markov chain Monte Carlo (MCMC) methods. Our lens modelling is fully capable of recovering the source-plane morphology of the galaxies in order to confirm the double-source-

<sup>1</sup> <https://github.com/Herculens/herculens>

<sup>2</sup> <https://docs.jax.dev/en/latest/>



**Fig. 2.** Lens model of the Teapot Lens.  $I_E$ -band image, lens model, and the residual. The bottom panels display the lens light-subtracted image and two source models. The inner pair of arcs is labelled S1a and S1b, and the outer pair of arcs is labelled S2a and S2b. In the residual plot, the black and green scatter points represent the edges of the masks for the inner and outer arcs, respectively (we did not use the arc mask in our modelling). The axis tick marks are in arcseconds.

plane nature of the systems, however. We briefly summarise the modelling strategy.

The data we used for the modelling came from the  $I_E$  band because it has the highest pixel resolution ( $0''.1 \text{ pixel}^{-1}$ ). We measured the root mean square of the background pixels and inferred the shot noise from the modelled lens light. The PSF was derived from the *Euclid* pipeline. Since the *Euclid*  $I_E$  band spans a wide wavelength range (550–900 nm), the PSF profile will be influenced by the spectral energy distribution of the target. Consequently, lensed arcs with markedly different colours might exhibit different PSFs, which might introduce additional uncertainties into the lens model.

We used the Numpyro implementation of stochastic variational inference (SVI; see Wingate & Weber 2013). We employed the AdaBelief optimiser (Zhuang et al. 2020) with a low-rank multivariate normal distribution as the guiding probability distribution. This guiding distribution is not complex enough to accurately recover the full posterior distribution, but it is computationally much cheaper than MCMC methods because it can use the auto-differentiable nature of JAX (Bradbury et al. 2018). The inference was made through optimising the guiding distribution to minimise the Kullback–Leibler divergence to the true posterior rather than sampling over the posterior.

Since the inner arcs of our DSPL candidates are mixed with the lens light, we modelled the lens light simultaneously with the arcs. In our model, the lens light was modelled with the sum of five elliptical Gaussian luminosity profiles with flexible centres (Shajib 2019; He et al. 2024; Enzi et al. 2025). The sources were first modelled with a single elliptical Gaussian luminosity profile to provide an approximate answer. Then, we used the parametrised lens model as the starting point and described the sources as fields that vary on regularly pixelated grids. The pixelated source was regularised by multiplying the square root of

the Matérn power spectrum (see Stein 2012) with white noise (sampled from a standard normal distribution) and then applying the inverse-Fourier transform to the result. A flat prior was applied to every mass model parameter.

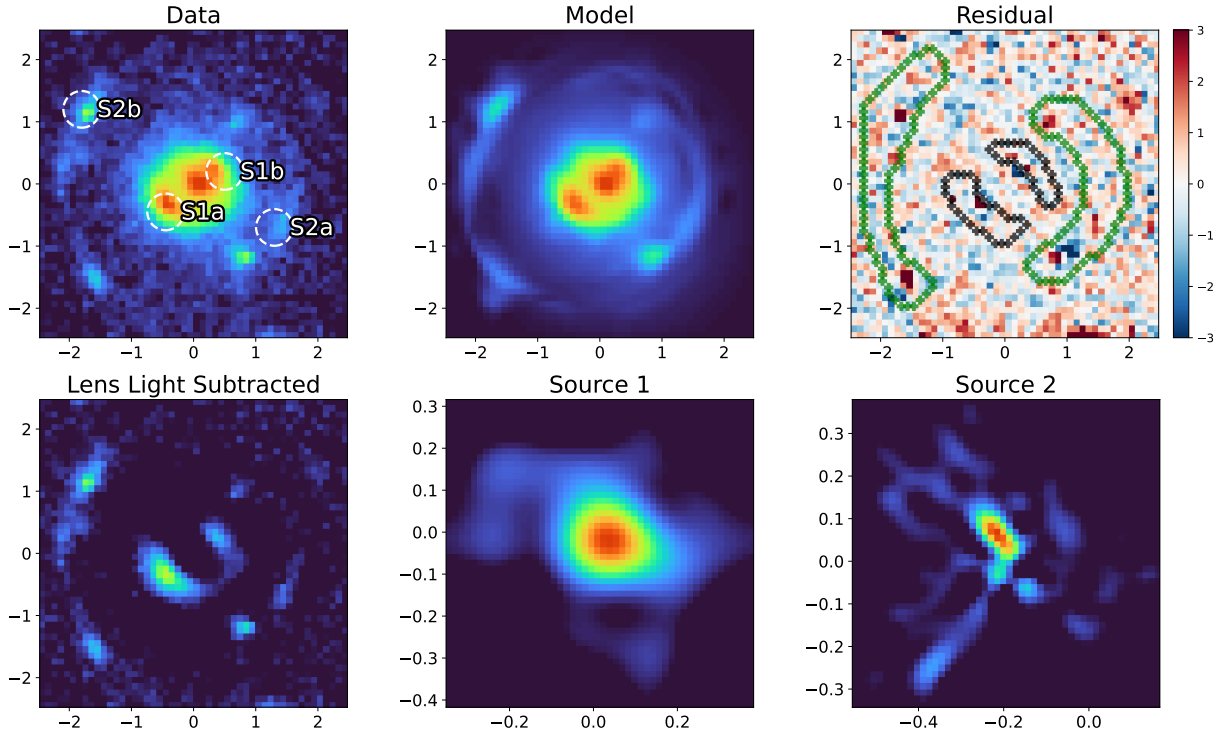
For each of our targets, we ran 20 000 iterations in 10 SVIs. Each SVI realisation had different random initialisations, and we chose the SVI result with the lowest average loss value to be our lens model. The lens model results are shown in Table 2. We do not show the uncertainty of the lens model because SVI tends to underestimate the lens model uncertainty. The expected uncertainties from the subsequent MCMCs are typically about 0.05 for each parameter.

#### 4. Lens model of DSPL candidates

In this section, we present and analyse the lens models of our DSPL candidates (Teapot Lens, Cosmic Dartboard, Galileo’s Lens, and Cosmic Ammonite). We also present one system that we initially thought was a DSPL candidate, the Crackpot, which we found was better modelled with two sources on a single plane (Sect 4.5). Since the Crackpot is not a DSPL lens, it is not shown in Table 2.

##### 4.1. Teapot Lens

This lens was discovered during the visual inspection of the top 20 000 ranked lenses according to the lens-finding version of the machine-learning model Zoobot (see Euclid Collaboration: Lines et al. 2026). It was ranked 2349 by Zoobot and 43 in Galaxy Judges. It was classified as a grade A lens. The system features two concentric rings at distinct Einstein radii, each with a different colour. This means that it is a highly promising DSPL candidate.



**Fig. 3.** Same as Fig. 2, but for Cosmic Dartboard.

The lens model of Teapot Lens is presented in Fig. 2. The lens light-subtracted image reveals a distinct pair of inner rings. The results of the lens model indicate an inner ring at  $0''.62$  and a  $\beta$  of 0.74, leaving a clean residual within the arc position. The positive residual near the centre might be due to multiple reasons, such as dust lanes or an AGN in the lens galaxy; a PSF mismatch that affects the lens light subtraction; a satellite galaxy located close to the primary lens; or an inner zig-zag image of the second source (Collett & Bacon 2016).

#### 4.2. Cosmic Dartboard

This lens was discovered during the visual inspection of the top 20 000 ranked lenses. It ranked 1902 by Zoobot and 94 in Galaxy Judges. It was classified as a grade A lens. The lens model of the Cosmic Dartboard is presented in Fig. 3. Our lens model successfully recovered all the details in the image. Because the resolution is limited, however, it is challenging to perfectly distinguish the lens light from the inner arc. The reconstruction of the outer arc might be significantly improved with data with a higher signal-to-noise ratio.

The inner ring of this lens was inferred to be at  $0''.45$ , while the  $\beta$  value is 0.51. The first source is massive according to the result from the lens model ( $\theta_{\text{sis1}} = 0''.48$ ) because it appears to be an elliptical galaxy. If spectroscopic redshifts are obtained, the large Einstein radius ratio should provide valuable insights for cosmological measurements as the image configuration suggests  $z_{s1} \ll z_{s2}$ , which would provide the optical lever arm for constraining dark energy (Collett et al. 2012).

#### 4.3. Galileo's Lens

This lens was first discovered in Space Warps and Galaxy Judges. It ranked 3 by Zoobot (although it was missed as a DSPL candidate in the first pass of Zoobot by TC, TL, and NL)

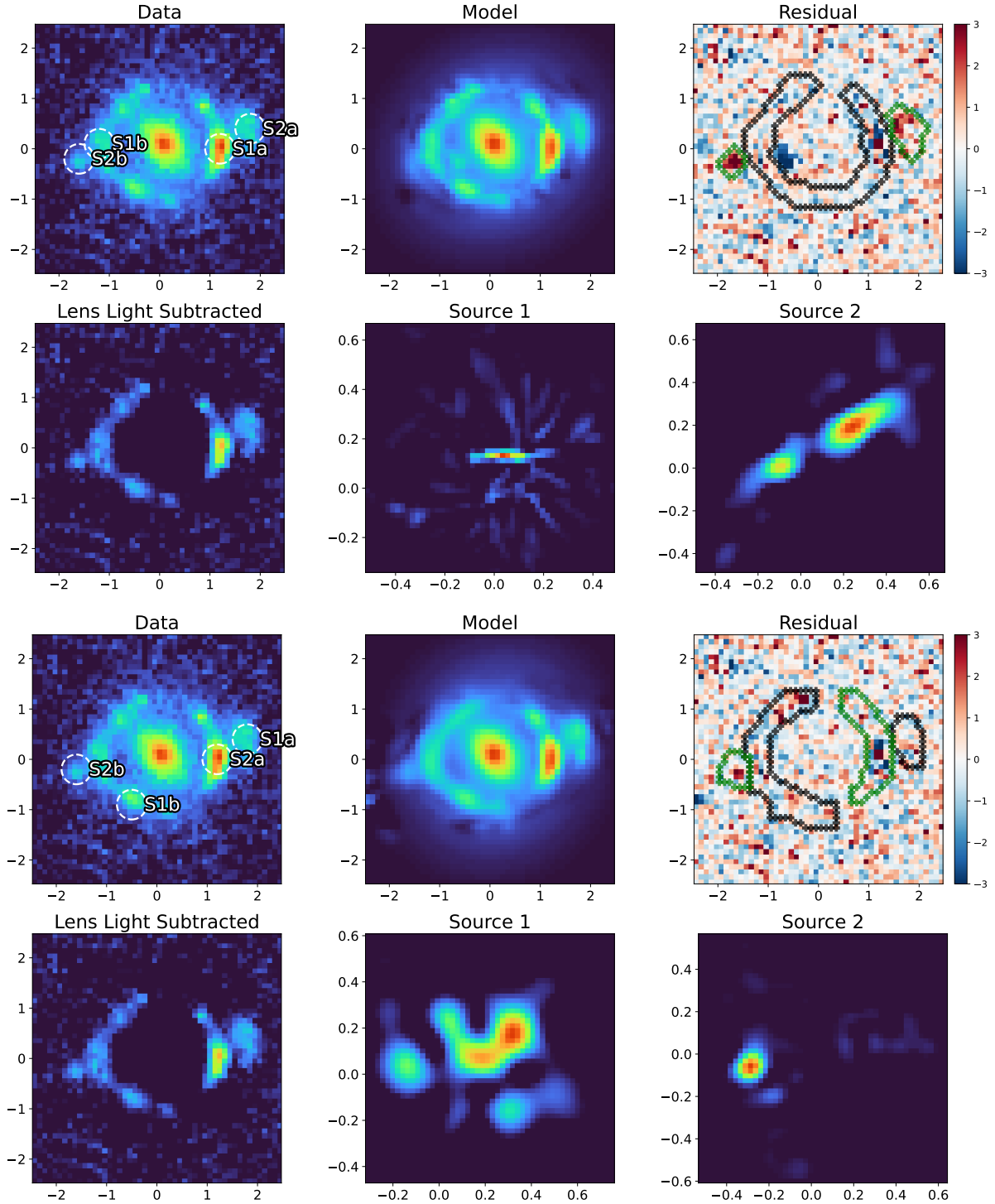
and 19 in Galaxy Judges. It was classified as a grade A lens. This system was proposed as a DSPL system because it features a central ring structure and two faint arcs in the outskirts. The apparent inner ring has an unusual configuration, however, and the colours of all the arcs are similar. This makes it difficult to classify the source confidently. It is also difficult to explain the highly elliptical central lens light and the nearly circular ring by merely classifying it as a ring galaxy.

In Fig. 4, we present two lens models. The first model assumed a DSPL configuration, and the second model considered a lens with two sources situated on the same redshift plane. Neither model can explain the lensing configuration really well. This system might be a single ring system in which the two blue objects nearby are just field galaxies. Spectroscopic data or imaging with a higher resolution should provide definitive evidence to decide whether this system is indeed a DSPL.

#### 4.4. Cosmic Ammonite

This lens was discovered during the visual inspection of the top 1000 Galaxy Judges candidates. It ranked 7 in Zoobot (but was initially assumed to be a multi-component single-source plane system) and 207 in Galaxy Judges. It was classified as a grade B lens, but the expert scores were not unanimous, with four A\* votes (lens with special interest), one A vote, three B votes, and two X votes (non-lens).

The lens model of Cosmic Ammonite is presented in Fig. 5. Our lens model successfully recovered all the details in the image. The  $\beta$  of this system is constrained to be around 0.85. Although this system was modelled with excellent fidelity and is almost certainly a DSPL, it received a surprisingly low score in Galaxy Judges. The best explanation is that the lensing configuration of this system is uncommon, so that human experts are less confident about this object. We initially modelled this system with the expectation that it would be ruled out as a DSPL, with



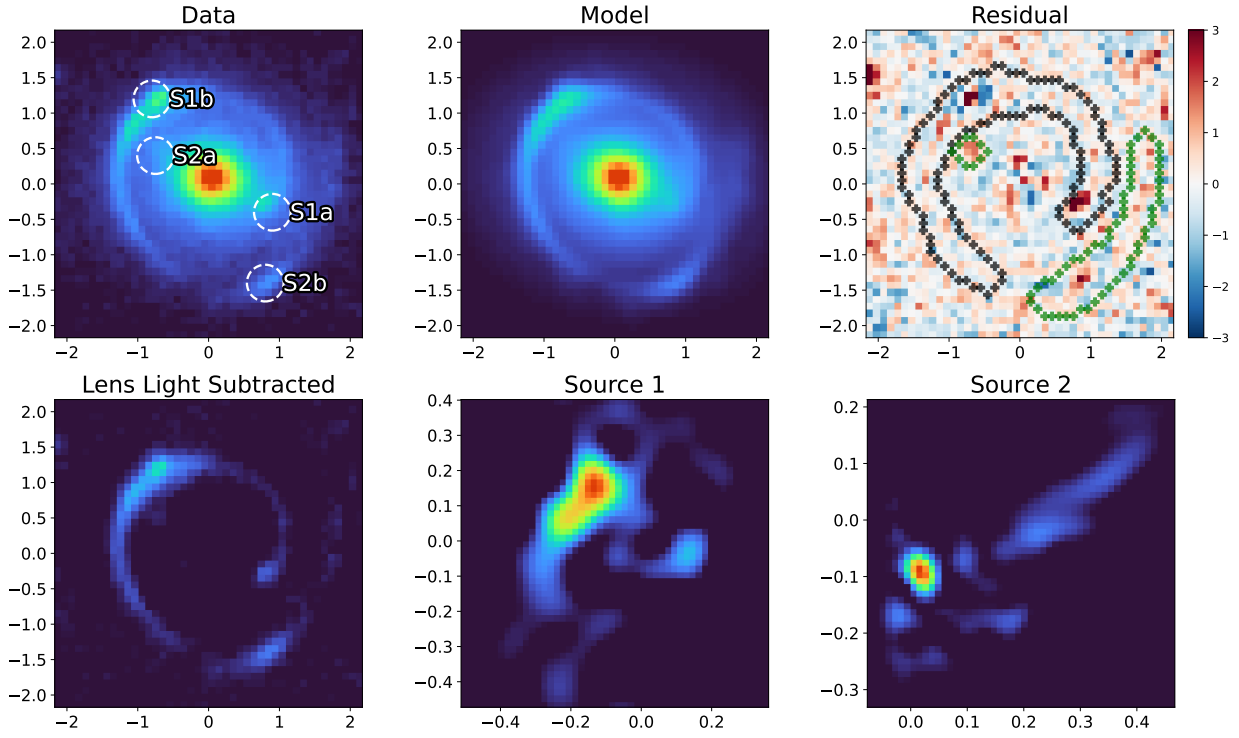
**Fig. 4.** Same as Fig. 2, but for Galileo’s Lens. The top six panels show the lens modelled with a single-source plane. The bottom six panels show the lens modelled with double-source-planes.

the two sources on a single plane. The modelling result shows, however, that this is a confident DSPL with  $\beta \approx 0.85$ . This demonstrates the power of lens modelling over human experts in identifying DSPLs with  $\beta$  not much smaller than one.

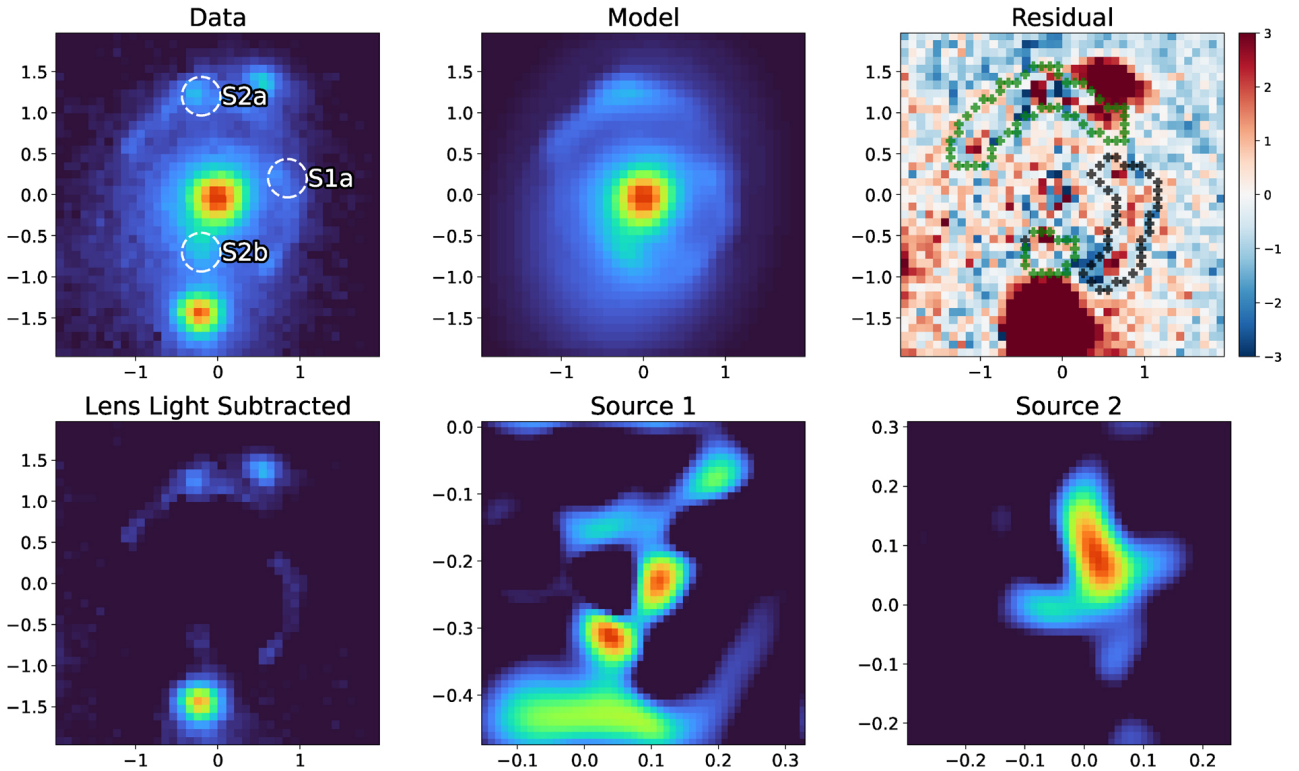
#### 4.5. The Crackpot: A single-plane system with two sources

This lens was first discovered during the visual inspection of high-velocity dispersion galaxies from the Dark Energy Spectro-

scopic Instrument (Euclid Collaboration: Rojas et al. 2026) as part of the effort to build a training set for machine learning. It was later rediscovered in Space Warps and Galaxy Judges, ranking 622 in the Zoobot and 237 in Galaxy Judges. It was classified as a grade B lens. As shown in Fig. 7, the system features two blue arcs at different radii. It is a lens with two bright lensing elliptical galaxies. While the counter-image of the top arc is clearly visible in the bottom left corner of the lens, the counter-image of the inner arc is only faintly visible in the



**Fig. 5.** Same as Fig. 2, but for Cosmic Ammonite.



**Fig. 6.** Same as Fig. 2, but for the Crackpot lens.

lens light-subtracted image. Its identification is therefore less convincing.

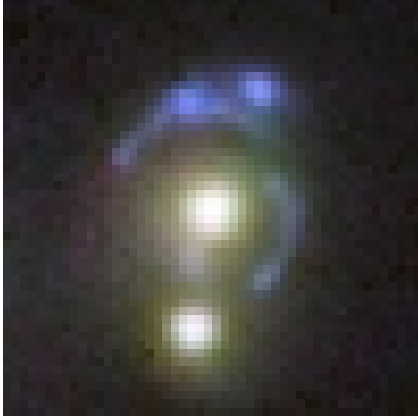
Figure 6 shows the lens model of the Crackpot lens. If the faint inner arc is real, the double-source-plane lens model suggests that the two images have similar Einstein radii. The nearby galaxy at the bottom of the image complicates the mass pro-

file, however. For simplicity, we did not include the mass of the southern galaxy, and we masked the blue blob in the top right corner of the northern source. Based on these factors, this lens is probably not a DSPL, but rather a lens with two sources at the same redshift. We named this system the Crackpot as a tongue-in-cheek reference to the fact it is not a Jackpot lens after all. A

**Table 2.** Properties and lens model parameters of the four systems.

| Name             | $I_E$ (lens) | $I_E$ (s1) | $I_E$ (s2) | $\beta$ | $\theta_{E1}$ | $\theta_{\text{sis1}}$ | $\theta_{E2}$ | $\gamma$ | $\phi$ | $q$  |
|------------------|--------------|------------|------------|---------|---------------|------------------------|---------------|----------|--------|------|
| Teapot Lens      | 20.0         | 23.3       | 24.0       | 0.74    | 0′′62         | 0′′13                  | 0′′89         | 1.83     | −0.44  | 0.63 |
| Cosmic Dartboard | 21.2         | 22.9       | 23.2       | 0.51    | 0′′45         | 0′′48                  | 1′′31         | 1.63     | −1.32  | 0.64 |
| Galileo’s Lens   | 22.3         | 22.7       | 24.4       | 0.69    | 1′′09         | 0′′28                  | 1′′52         | 2.11     | 0.39   | 0.76 |
| Cosmic Ammonite  | 20.0         | 22.3       | 24.2       | 0.85    | 1′′18         | 0′′15                  | 1′′35         | 2.22     | 0.17   | 0.72 |

**Notes.** We show the lensed  $I_E$ -band magnitudes of the lens, source 1 (s1), and source 2 (s2). The lens model parameters are the best solutions after 10 SVI runs per lens. The uncertainties are typically around 0.05 for each parameter from subsequent MCMCs.

**Fig. 7.** Colour image of the Crackpot lens.

more definitive judgement will require higher-resolution imaging observations and spectral information.

## 5. Expected rate and population of DSPLs in *Euclid*

The *Euclid* survey is expected to enable the construction of a sample of approximately 1700 galaxy-scale DSPL systems. This forecast is derived from the LensPop package (Collett 2015), modified to include multiple background sources (we neglected the mass of the first source). We also imposed more stringent constraints than the LensPop defaults. Specifically, we required that both sources have one or more arcs with a length of 0′′3, which ensured that the density slope of the lens can be recovered from *Euclid* imaging alone with a reasonably high possibility. The redshift distribution for lenses and sources is shown in Fig. 8. Although most compound lenses are at  $z \approx 0.5$ , a significant fraction of the lenses are at  $z < 0.2$  and at  $z > 1$ , which is promising for a precise constraint on the mass distribution in lenses. Of the 1700 expected *Euclid* DSPLs, approximately  $6 \pm 3$  should fall inside the Q1 footprint (assuming that the DSPLs are distributed randomly). The entire *Euclid* survey will cover about  $14\,000 \text{ deg}^2$ , whereas the Q1 footprint we considered spans only  $63 \text{ deg}^2$ . The forecast number of systems in this small area is expected to be modest, and even a difference of a few lenses (e.g. finding four instead of the predicted six) can therefore be attributed to small-number statistics. Poisson fluctuations and cosmic variance can easily shift the observed number by a few systems relative to theoretical forecasts.

Our four DSPLs already more than double the galaxy-galaxy DSPL population. This is expected to be useful for cosmography, but without any redshift information, it is impossible to map  $\beta$  values onto cosmological parameter constraints. Notably, Cosmic Dartboard has  $\beta \approx 0.51$ , which is quite rare based on the  $\beta$  distribution of the mock catalogue, as shown in Fig. 9 (only 6%

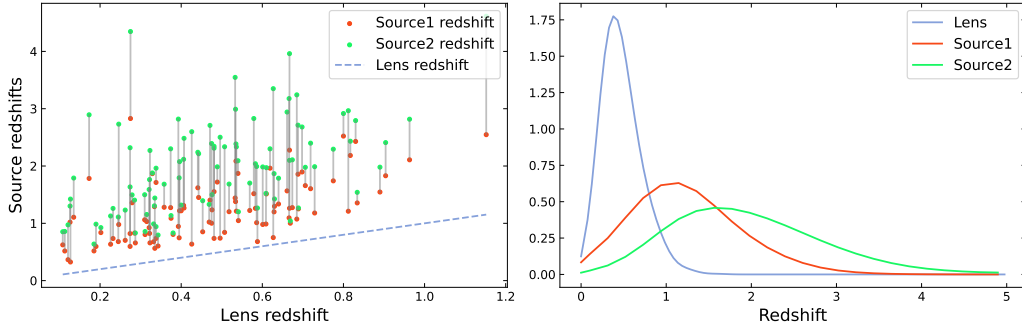
of the DSPLs have  $\beta < 0.5$ , and the probability of having at least one DSPL with  $\beta < 0.5$  in four systems therefore is 0.2). This indicates that our forecast might underestimate the DSPL population observed by *Euclid*. Theoretically, lower  $\beta$  values (two source galaxies at very different redshift) are more sensitive to the cosmology parameters because a DSPL with a decreased  $\beta$  provides a stronger geometric leverage and mitigates degeneracies among the cosmological parameters. For instance, when  $\beta \approx 1$  ( $z_{s1} \approx z_{s2}$ ), variations in cosmological parameters may cause  $\beta$  to remain close to one.

## 6. Conclusions

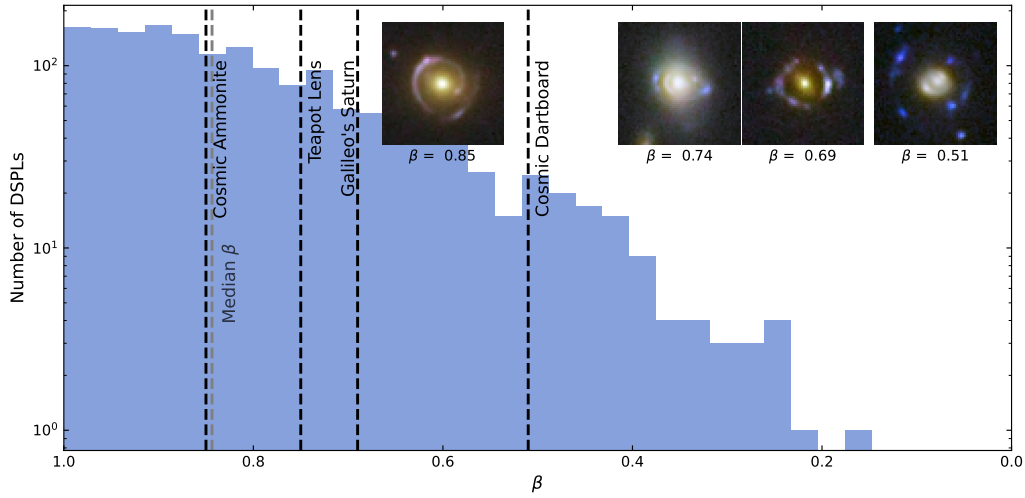
We have identified four new galaxy-scale DSPL candidates in Q1. This significantly extends the limited sample of known DSPL systems. Our detailed lens modelling again showed that they probably are DSPLs, and we derived the  $\beta$  parameters (the cosmological scaling factor that represents the distance ratio) for each candidate, which demonstrates that they can be used in cosmological studies. For a handful of DSPL candidates, higher-resolution imaging and spectroscopic redshifts are needed for a definitive conclusion about their nature.

We employed the single-source plane lens search to find interesting DSPL candidates. We did not train any machine or human classifiers to explicitly search for DSPLs. One consequence of not creating machine-learning lens-finders trained on DSPLs is that they do not learn to value DSPL systems higher than regular lenses. This means that the DSPLs will not necessarily be ranked exceptionally highly among the lenses; this was found in Q1 (Euclid Collaboration: Lines et al. 2026). Future *Euclid* single-plane lens searches will be made over larger areas, which means that we will be unable to visually inspect the same proportion of the total sample as in Q1: The proportion of the total number of images that were visually inspected in Q1 scaled up to the full *Euclid* survey would correspond to visually inspecting six million images, which is likely intractable. The DSPLs and the single-source-plane lenses found in Q1 will likely enable a significant improvement in the machine-learning performance, and the training of machine-learning models on DSPLs will increase the probability that they are recovered in future *Euclid* lens searches. A dedicated DSPL search might be needed for *Euclid* DR1 and beyond, however.

Some DSPLs may still remain to be found in Q1, either because they were ranked poorly by the single-source-plane lens machine-learning classifiers or because they were missed in the visual inspection (we do not yet know how our method performs with systems with a small Einstein radius). We also missed any lenses that did not pass our initial lens cuts ( $I_E < 22.5$ ). Some lenses are expected to be faint in  $I_E$  ( $I_E > 24$ , Collett 2015), although these would typically be low-mass or high-redshift lenses (e.g. a lens redshift higher than 1.5). Either scenario is less likely to produce a *Euclid*-detectable DSPL than a galaxy



**Fig. 8.** *Left:* Redshift distribution of 100 example DSPLs from LensPop. *Right:* Histogram of the redshift distribution of 1700 systems drawn from LensPop. The blue line indicates the lens redshift, and the dashed red and green lines mark the redshifts of source 1 and source 2, respectively.



**Fig. 9.** Distribution of  $\beta$  from the forecast. The dashed black lines mark the measured  $\beta$  values for DSPLs discovered in Q1, and the dashed grey line indicates the median  $\beta$  value in the forecast catalogue.

that is more luminous in  $I_E$ . Our forecasts indicate that the full *Euclid* survey at the end of operations is expected to uncover 1700 DSPLs, whilst extrapolating up from our four DSPLs in  $63 \text{ deg}^2$  yields  $1000 (\pm 500)$  DSPLs in the full survey. Regardless of whether we ultimately find 1000 or 1700 lenses in the full *Euclid* data set, it is already clear that *Euclid* will revolutionise the research made using DSPLs.

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## Data availability

The Q1 data is available at the *Euclid* science archive. The forecast population of DSPLs are available at LensPop github repository. The derived data products are available upon request from the corresponding author.

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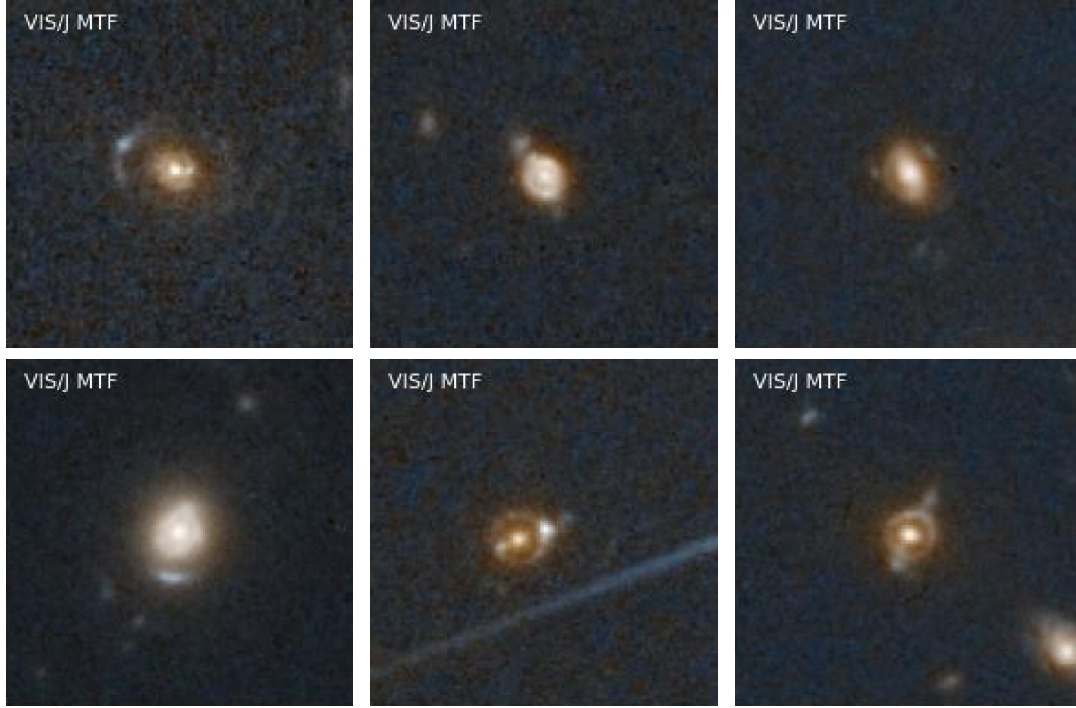
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## Appendix A: Other DSPL lens candidates



**Fig. A.1.** Low probability DSPL candidates selected among the top 1000 ranked galaxies in Galaxy Judges.

Due to the large survey area and high-resolution images provided by *Euclid*, rarer single-plane lensing configurations, and false positives can be discovered which could be misidentified as possible DSPL systems. Figure A.1 shows a sample of lens candidates in Q1 which might be DSPLs but we can not classify them with confidence due to their small Einstein radii. Figure A.2 shows a sample of images that could be confused for DSPLs but we believe are most likely contaminants. Here we outline several candidate configurations that create images that are similar to DSPLs:

- Double cusp configuration: When a source is positioned near a cusp of the tangential caustic, three closely spaced lensed images appear in the lens plane, while a fourth, more isolated and typically less magnified image, forms at a closer distance. If two sources within the same source plane are located near two opposing cusps of the diamond-shaped caustic, the resulting lensing configuration can produce a ring-like structure near the lens, along with two smaller arcs inside the ring. This configuration may resemble a DSPL system and could be mistakenly identified as such.
- Spiral galaxy and lens: In strong-lensing systems where the deflector is a spiral galaxy, features like spiral arms or star-forming rings can sometimes be misidentified as secondary rings at different (smaller or larger) radii. This is because star-forming regions often exhibit a distinct blue colour, similar to that of background lensed sources. Such cases are not common, as face-on spiral galaxies typically have low projected surface mass density, resulting in a small lensing cross-section. However, at least two possible cases were found in our sample although they may not even be lenses. This type of lens has its unique value but it's beyond the scope of this paper.



**Fig. A.2.** Candidate DSPL imposters selected among the top 1000 ranked galaxies in Galaxy Judges.

- Spiral galaxy with multiple arms: As described above, spiral arms can resemble lensed arcs, making certain spiral galaxies appear similar to DSPL systems. This is particularly true when the central bulge of the galaxy is large enough to mimic the appearance of an elliptical galaxy. However, such systems can typically be easily ruled out through lens modelling.
- Elliptical lens with complex shells: In addition to spiral arms, the shell structures of elliptical galaxies can also resemble lensed arcs. These features become more apparent after subtracting the lens light in preliminary lens modelling (Euclid Collaboration: Walmsley et al. 2026). However, such structures are often noisy, as they are typically buried beneath the bright lens light, making them more challenging to identify without careful analysis.
- Lens system with nearby galaxies: galaxies located close in projection to a lens but outside the multiply imaged region can mimic an image from a doubly lensed system. An expert might incorrectly guess that there is an undetected counter-image of this galaxy close to the centre of the primary lens.

The scenarios described above represent the most common cases where DSPL systems can be misidentified. Conversely, a genuine DSPL system can also be misclassified as one of these cases. To make a definitive determination either lens modelling of higher angular resolution images or spectroscopic redshifts for each component, are required.