Between-desk-teaching as a deliberate act of making content available: The case of Bernie teaching ratio

Nick Andrews

University of Oxford, UK; nicholas.andrews@education.ox.ac.uk

Between-desks-teaching is recognised as a feature of mathematics classrooms across the world. Previous studies have sought to describe the range of purposes of this lesson event within a lesson and have been concerned with the orchestration of the classroom. Yet there has been little attention given to the role of between-desks-teaching beyond a single lesson, such as its contribution to how mathematical content is made available to learners over time. The current study explores what is happening mathematically when teachers and learners engage in between-desks-teaching across a topic. There is a particular focus on one case in which this lesson event features extensively. This case reveals how between-desks-teaching can have a strategic purpose and has potential to be used deliberately when teaching a topic.

Keywords: Mathematics instruction, Instructional design, Student centred learning, Research methodology, Teacher education.

Introduction

In this paper I focus on between-desk-teaching and how it can be distinctive in terms of its purpose when teaching a topic over a series of lessons. To do so, I draw on data from a wider exploratory study of four cases of classroom practice over a series of lessons. I then focus on one particular case in order to understand better the distinctive contribution of between-desks-teaching to how subject matter is transformed and connected over time in this context. This study therefore, in common with previous CERME papers (Skott, Mosvold, & Sakonidis, 2018), brings to the surface a specific act within mathematics teaching and discusses how it provides opportunities for learning.

Between-desk-teaching has been identified as a feature or "lesson event" of mathematics classrooms across the world (O'Keefe, Xu, & Clarke, 2006, p.73). It is a distinct event from 'classwork', which might include teacher exposition and whole-class discussion, but is a sub-set of 'seatwork', which is when "students work individually or in small groups on assigned tasks" and "talk is mostly private — teacher-student or student-student" (Stigler & Hiebert, 1999, p.67). It is this mostly private teacher-student talk, while students work on assigned tasks, which defines between-desk-teaching, although O'Keefe et al. (2006) also point to instances where this talk becomes more public. Inoue, Asada, Maeda, & Nakamura (2019) describe how between-desk-teaching offers learners individual support when problem solving, while Jakonen (2018) states that teacher actions during this lesson event are 'occasioned' in that they are sensitive to the learner's present situation. Yet between-desk-teaching is often 'repetitive' in that the teacher addresses the same issue both with multiple learners and also potentially multiple times with the same learner (Jakonen, 2018).

O'Keefe et al. (2006) discuss both the monitoring and guiding functions of between-desk-teaching. Providing feedback, including references to learning goals (Svanes & Klette, 2018), or observing

learners working serve a monitoring function. The guiding function can be associated with either 'extending' or 'enabling' teacher prompts (Roche & Clarke, 2015). Extending prompts are typically discussed in the literature with reference to challenge and questioning (e.g. Svanes & Klette, 2018), while enabling prompts may involve telling, modelling or offering hints (Svanes & Klette, 2018), or referring to prior learning or suggesting alternative approaches (Inoue et al., 2019).

While between-desk-teaching can be analysed as a distinct lesson event, or as an event within a lesson (Inoue et al., 2019), O'Keefe et al.'s (2006) study emphasises that in order to understand its purpose consideration needs to be given to the events that surround it and whether these events are classwork or seatwork. In terms of how content is made available to learners, between-desk-teaching can be seen as a bridge between independent-of-the-teacher seatwork and classwork, or vice versa. When the flow is from seatwork to classwork, the purpose of between-desk-teaching is more likely to be 'monitoring student activity' (O'Keefe et al., 2006) in order to inform the precise nature and focus of the classwork to follow, whereas when the flow is from classwork to seatwork the purpose is more likely to be 'guiding student activity' and making connections between the assigned task and the whole-class teaching the preceded it (Inoue et al., 2019). This emphasises how these lesson events mutually influence each other, but that there is a pedagogical choice regarding their sequencing.

The previous studies of between-desks-teaching discussed above focus particularly on the nature of teacher-learner participation during this lesson event rather than how the subject matter is being transformed and connected through these interactions. My own work however has focused on how subject matter is made available to learners over the course of teaching a topic (Andrews, 2017), through the actions of the teacher. I use the term 'manifestation' of a mathematical concept to refer to the form of content I observe the teacher offering learners at any moment in time, and a manifestation may be categorised as visual, technical, functional, or a combination of these. Visual manifestations often include diagrams or forms of imagery, technical manifestations often relate to methods or symbolic manipulation, and functional manifestations include worded problems and applications of methods.

In order to connect the sequencing of manifestations with opportunities for learning, I draw on a theory of growth in mathematical understanding (Pirie & Kieren, 1989; Pirie & Kieren, 1994). This constructivist theory proposes a progression of levels, each of which represents a reorganisation of knowledge structures, from 'primitive knowing', through 'image having' and 'formalising', to 'structuring' and beyond (Pirie & Kieren, 1989). I associate visual, technical and functional manifestations with opportunities to focus learners' attention on image having, formalising and structuring respectively. I associate visual-functional, visual-technical and functional-technical manifestations with opportunities to shift the focus of learners' attention, captured by Pirie & Kieren's (1989) terms 'image forming', 'property noticing' and 'observing' respectively.

In a recent paper I highlight mathematical differences between what is addressed during classwork and seatwork in each of the four cases studied (Andrews, 2018). One pattern in practice revealed in this paper is a flow from classwork to seatwork within lessons and also across lesson series, with greater foregrounding of visual manifestations during classwork and functional manifestations

during seatwork. This analysis suggests a degree of alignment in these cases between how content is made available to learners over time and the underlying progression proposed in Pirie & Kieren's theory. The current study builds on this recent paper and explores ways in which between-desk-teaching might support this progression.

From the range of purposes of between-desk-teaching identified above, I focus particularly in this paper on the guiding function and references back to prior learning (Inoue et al., 2019). Pirie and Kieren describe growth as "levelled but non-linear" (Pirie & Kieren, 1994, p.166) and use the term 'folding back' to describe returning to a previous level "in order to extend one's current, inadequate understanding" (Pirie & Kieren, 1994, p.173). I associate teacher references during between-desk-teaching to prior learning with offering explicit opportunities for learners to 'fold back' to a previous level of understanding in order to come to understand the assigned task at hand. In such a situation the between-desk-teaching will not, in terms of manifestation, align to the assigned task but rather more closely to what has been offered previously (for example through classwork). The current study explores therefore whether during seatwork between-desk-teaching aligns with the assigned task, or is more associated with opportunities for folding back. Addressing this question over a series of lessons on a given topic rather than an individual lesson affords exploring patterns in practice, which in turn reveals if between-desk-teaching is being deployed as a strategic act for transforming and connecting the subject matter of a topic.

Methodology

A summary of the approach taken to address the current research question will be offered here. A methodological paper detailing the range of approaches taken in the wider study is being worked on separately, and this will in due course provide fuller details and justifications of methods employed. The approach is an analysis of the text of what the teacher is seen to offer learners over the course of a series of lessons on a single mathematical topic. By text here I mean the literal text of teacher-talk during lessons, whether that be public or privately to individual learners, and also the tasks, video clips, simulations, physical resources and other teaching media that the teacher makes available.

A quantitative approach is taken in order to detect differences in manifestation over periods of time through applying what I call a *tri-polar analysis*. The relative stressing of the three elements of the triad {visual, functional, technical} in the ratio $v : f : t$ and satisfying the condition that $v + f + t = 1$ can be succinctly represented by the Barycentric co-ordinate (v, f, t).

How the teacher makes content available to learners is conceptualised as having two elements: teacher-talk and given-tasks. Teacher-talk may be public (for example, during periods of classwork) or private (during seatwork). Given-tasks are only present during seatwork and are the prompts that initiate subsequent activity in a lesson rather than that activity itself. The combination of teacher-talk and given-task during an individual between-desk-teaching event may serve to alter what is offered to learners when compared to what would have been available through the given-task alone. Therefore the effect of between-desk-teaching is revealed in any differences between what is offered during seatwork through considering both teacher-talk and given-task in unison to what would have been offered over the same time period through the given-task alone.

For the current study, the text of each lesson series is parsed into 30-second intervals, each of which is then qualitatively coded for manifestation. Each manifestation category is associated with a Barycentric co-ordinate. The quantitative code for teacher-talk for the interval is the mean of the Barycentric co-ordinates of all of the manifestation categories present in the teacher-talk in that interval (or (0, 0, 0) if there is no teacher-talk), and the quantitative code for given-task for the interval is determined similarly. The overall quantitative code for the interval (referred to as the interval centre) is either the co-ordinate for teacher-talk if there is no given task, the co-ordinate for given-task if there is no teacher-talk, or the mean of the teacher-talk and given-task co-ordinates. Each 30-second interval is also coded as either seatwork or classwork. The seatwork centre for the lesson series is then the mean of the Barycentric co-ordinates of all the intervals coded as seatwork, and the classwork centre is similarly defined. The given-task centre is the mean of the quantitative codes for given-task of all the intervals coded as seatwork. A statistical analysis of the visual co-ordinates for each interval using the Wilcoxon rank-sum test (Wilcoxon, 1945) allows the stressing of visual manifestation between classwork and seatwork, and between seatwork and given-task to be compared quantitatively (reported here using an effect size), and a similar analysis is conducted on the functional and then the technical co-ordinates. The extent of differences is represented by the effect size, and an effect size greater than .3 is treated as a material difference. By this I mean that the difference is large enough to infer systematic differences in how content is made available associated with strategic differences in teacher action.

The current paper initially reports on four lesson series taught in local secondary schools in England with students aged between 13 and 16. One of the teachers, Ashley taught a series of five lessons on geometrical constructions. Bernie was at the same school as Ashley and taught a series of four lessons on ratio with one class and three lessons on linear equations with another. Courtney was at a different school and taught a series of four lessons on linear equations. In each case, the teacher determined the topic and the teaching approach. The four cases are used to address the question of whether there are mathematical differences in what the teacher makes available through between-desk-teaching when compared to the underlying given-task alone. A closer analysis is then conducted of Bernie teaching ratio in order to understand better between-desk-teaching in this case.

Cross-case findings

Table 1 sets out the seatwork and given-task centres for the four lesson series studied. For each case the statistical significance of any differences between seatwork and given-task are given. The number (n) of 30-second intervals given over to seatwork is also given in each case.

The seatwork and given-task centres are descriptive statistics, and so comparisons between them only indicate where there are differences. It is the statistical analysis that quantifies these differences. The Barycentric co-ordinates highlight differences in each case between the seatwork and given-task centres, which indicate that during between-desk-teaching teacher-talk did not always align mathematically with the given-task. Instead this form of teacher-learner participation changed the mathematical nature of the activity. The greatest misalignment between seatwork and given-task was in the case of Bernie teaching ratio, and as this is an extreme case it has been selected for further analysis below. As this difference was material, the inference is that between-

desk-teaching was being deployed as a strategic act for transforming and connecting the subject matter of a topic in this case.

Case		Seatwork	Given-task
Ashley teaching geometric constructions	*n*	410	
	Centre	(0.24, 0.27, 0.49)	(0.24, 0.32, 0.44)
	Difference	**	
Bernie teaching ratio	*n*	301	
	Centre	(0.06, 0.29, 0.65)	(0.00, 0.39, 0.61)
	Difference	MAT	
Bernie teaching linear equations	*n*	208	
	Centre	(0.20, 0.02, 0.78)	(0.18, 0.02, 0.80)
	Difference	NS	
Courtney teaching linear equations	*n*	270	
	Centre	(0.17, 0.09, 0.74)	(0.15, 0.12, 0.73)
	Difference	NS	

N.B. Difference indicators show whether differences between the centres were material (MAT), statistically significant at the 1% level (**), or not statistically significant at the 1% level (NS)

Table 1: The seatwork and given-task centres of the four lesson series

The particular case of Bernie teaching ratio

The lesson series was concerned with translating part-to-part relationships into part-to-whole relationships, such as dividing quantities in a given ratio. In addition to the seatwork and given-task centres given in table 1, the classwork centre is also given in table 2 to allow for further comparison.

Case		Classwork	Seatwork	Given-task
Bernie teaching ratio	*n*	82	301	301
	Centre	(0.14, 0.11, 0.75)	(0.06, 0.29, 0.65)	(0.00, 0.39, 0.61)
	Difference	MAT	MAT	

N.B. Difference indicators show whether differences between the centres were material (MAT), statistically significant at the 1% level (**), or not statistically significant at the 1% level (NS)

Table 2: The classwork, seatwork and given-task centres for Bernie's lesson series on ratio

Inspection of the statistical analysis of differences between the classwork, seatwork and given-task centres indicate that the visual and technical components of manifestation were stressed more during classwork. For example, in the classwork event of Lesson 1 Bernie displayed the problem "share £20 in the ratio 2 to 3" on the interactive whiteboard and learners were tasked with solving it

individually. The problem was presented along with twenty circles representing the twenty £1 coins that were to be shared between the two people. After allowing a brief period for individual consideration, Bernie led a whole-class sharing of ways to solve this problem. As part of this, Bernie demonstrated how the 20 coins might be shared in the ratio two to three by counting them out two, then three, then two, then three, and so on. This classwork phase of the lesson was categorised as featuring visual and technical manifestations as Bernie offered a visual representation of an enactive method of sharing the money in the given ratio. Building on Lesson 1, Bernie provided an opportunity for the whole class to recap methods for sharing quantities in a given ratio at the start of Lesson 2. The first example offered was: "share £140 in the ratio 3 to 4." Three methods were suggested by learners, the third of which was a simple integer 'scaling up' method: 4 : 3 ≡ 40 : 30 ≡ 80 : 60 and 80+60=140. The limitations of this method were revealed by the second example: "share £45 in the ratio 2 : 3 : 7." Following this, Bernie endorsed a single method, the first step of which was dividing the quantity by the total of the parts. Thus there was an emphasis placed on formal methods in this introductory classwork phase to the lesson, with technical manifestation foregrounded.

Technical and particularly functional manifestations were emphasised in the given-tasks assigned to learners during seatwork. For example, in Lesson 1 learners were provided with the opportunity to practise sharing quantities of money in given ratios, for example: "divide £20 in the ratio 4 : 1", and "divide £10 in the ratio 3 : 2." The given-task here was categorised as technical. In each of the following lessons students were set problems that required sharing in a given ratio, but increasingly these became worded, contextual problems. Thus functional manifestation was more emphasised over time.

The effect of between-desk-teaching was that overall seatwork was materially different to the assigned given-task, with visual manifestation emphasised more and functional less. This can be largely attributed to Bernie drawing on the image of 'moneybags' when between-desk-teaching in order to represent a unitary share, such as in Lesson 1:

> In effect you've got 3 bags of money and 2 bags of money, which altogether is 5 bags of money. If you share your £10 equally into those 5 bags you'll have £2 in each bag won't you? That person gets 3 bags [and] that person gets 2 bags, yes? (Bernie, Ratio Lesson 1)

Between-desk-teaching in Lesson 2 also referenced moneybags and included four separate conversations with different students over a period of 10 minutes in which Bernie offered essentially the same explanation of the solution to the same problem. It is interesting to note that while Bernie used a particular visual method for dividing in a given ratio during classwork in Lesson 1, the visual method offered when between-desk-teaching was equivalent but not identical. Lesson 3 continued to include between-desk-teaching in which Bernie made reference to moneybags, but not Lesson 4. This suggests a progression from Lesson 3 to Lesson 4, as in Lesson 4 there was less recourse to the imagery of moneybags in Bernie's between-desk-teaching.

My reading of the between-desk-teaching actions across Lessons 2 and 3 was as a return to more familiar-to-the-learner manifestations of ratio, which I associated with offering learners explicit opportunity to 'fold back' (Pirie and Kieren, 1989) to a secure level of understanding. In Lessons 2

and 3, this secure level of understanding was inferred to be a visual method for sharing quantities in a given ratio. Whereas in Lesson 4 Bernie was heard to return to more technical manifestations of ratio when between-desk-teaching, which I associated with offering learners explicit opportunity to fold back from an inadequate contextual understanding to a more secure understanding of a method for sharing in a given ratio.

Discussion

There are indications in this lesson series that between-desk-teaching was a lesson event foregrounded by Bernie in the context of teaching ratio to this class. Firstly, in total 52% of lesson time across the series was given over to between-desk-teaching compared to 27% to independent-of-the-teacher seatwork and 21% to classwork. Secondly, I have offered examples of Bernie repeating the same explanation on multiple occasions through between-desk-teaching when an alternative choice could have been to explain once to the whole class. This was an instance of the repetition effect noted by Jakonen (2018).

Furthermore I infer from the statistical analysis that between-desk-teaching was being deployed strategically as a way of making content available to learners. Non-linear opportunities were offered for growth in understanding (Pirie & Kieren, 1994) that were sensitive to the needs of individual learners (Jakonen, 2018). Teacher explanation at the start of the lesson series emphasised visual methods for solving ratio problems, while technical and increasingly functional components of manifestation were emphasised in the given-task that were subsequently assigned. While the methods of problem solving offered through between-desks-teaching in Lessons 2 and 3 were not identical to the methods seen at the start of Lesson 1, there was a clear intention to offer visual representations of problems in order to facilitate their solution, and for these to remain available throughout the lesson series. By Lesson 4 it can be inferred that visual representations were no longer needed, but symbolic representations of contextual problems were being offered through between-desks-teaching by this stage. In summary, whether through conscious deliberation or out of habit Bernie's actions aligned in this context with choosing to use between-desks-teaching in order to offer learners explicit opportunities to fold back to a secure level of understanding from earlier in the lesson series.

Conclusion

Previous studies have considered between-desks-teaching as an isolated event or as an event within a lesson (e.g. Inoue et al., 2019), but the current study has taken the lesson series as the unit of analysis. There is stronger evidence of Bernie's between-desks-teaching being a deliberate act because this phenomenon was explored across a series of lessons. A further affordance of this research design is that through the analysis of teacher-talk and given-task it supports making sense of the contribution of between-desk-teaching to the way content is made available to learners over time. The lesson series is a unit of time over which teachers typically plan, and while the role of between-desks-teaching at this grain size may not be an issue they (and researchers) currently attend to, it could be. The two case studies involving Bernie reported in this paper offer an ideal example of when it might be appropriate for a teacher's attention to be drawn to this. In the case of Bernie teaching ratio, as we have seen, between-desk-teaching was used extensively and strategically. Yet

in the case of teaching linear equations it was not used in this strategic way at all. Such contrasts in the practice provide in my experience powerful prompts for teacher reflection.

References

Andrews, N. (2017). Differences in the forms of content teachers are seen to offer over time: Identifying opportunities for teacher growth. In T. Dooley & G. Gueudet (Eds.). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME10, February 1–5, 2017). Dublin, Ireland: DCU Institute of Education & ERME.

Andrews, N. (2018). Are there mathematical differences in what is addressed through different modes of interaction in classrooms? In F. Curtis (Ed.). *Proceedings of the British Society for Research into Learning Mathematics*, *38*(2), June 2018.

Inoue, N., Asada, T., Maeda, N., & Nakamura, S. (2019). Deconstructing teacher expertise for inquiry-based teaching: Looking into consensus building pedagogy in Japanese classrooms. *Teaching and Teacher Education*, *77*, 366–377.

Jakonen, T. (2018). Professional embodiment: Walking, re-engagement of desk interactions, and provision of instruction during classroom rounds. *Applied Linguistics*. Available from: https://ezproxy-prd.bodleian.ox.ac.uk:5876/applij/advance-article/doi/10.1093/applin/amy034/5123669

O'Keefe, C., Xu, L., & Clarke, D. (2006). Kikan-shido: Between desks instruction. In D. Clarke, J. Emanuelsson, E. Jablonka, & I. Mok (Eds.). *Making connections: Comparing mathematics classrooms around the World*. Rotterdam, Netherlands: Sense Publishers.

Pirie, S. & Kieren, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics*, *9*(3), 7–11.

Pirie, S. & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, *26*(2), 165–190.

Roche, A. & Clarke, D. (2015). Describing the nature and effect of teacher interactions with students during seat work on challenging tasks. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia)*, 532–539. Sunshine Coast: MERGA.

Skott, J., Mosvold, R., & Sakonidis, C. (2018). Classroom practice and teachers' knowledge, beliefs and identity. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.). *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe*. Abingdon, UK: Routledge.

Stigler, J. & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, USA: Free Press.

Svanes, I.K. & Klette, K. (2018). Teachers' instructional practices during pupils' individual seatwork in Norwegian Language arts. *Education Inquiry*, *9*(3), 247–266.

Wilcoxon, F. (1945). Individual comparisons by ranking methods. *Biometrics Bulletin*, *1*(6), 80–83.