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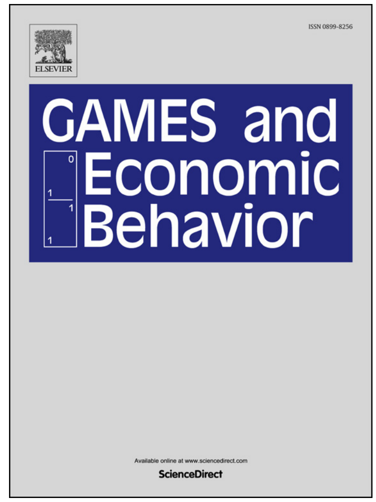

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PII: S0899-8256(24)00133-7
DOI: https://doi.org/10.1016/j.geb.2024.09.001
Reference: YGAME 3754

To appear in: *Games and Economic Behavior*

Received date: 30 April 2024

Please cite this article as: I. Moreno de Barreda, Cheap Talk With Two-Sided Private Information, *Games and Economic Behavior*, doi: https://doi.org/10.1016/j.geb.2024.09.001.

Cheap Talk With Two-Sided Private Information*

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Abstract

This paper studies how the transmission of information from a biased expert to a decision maker is affected when the latter has access to an unbiased symmetric private signal. The extra information has two distinct effects on the expert's incentives to communicate. First, there is an *information effect* that allows the decision maker to choose a better action on expectation. This reduces the implicit cost of transmitting coarse messages and hence hampers communication. Second, there is a *risk effect* that arises because the extra information introduces uncertainty to the expert. For risk averse experts, this effect increases the cost of sending coarse messages and hence favours communication. I show that the information effect dominates the risk effect, and for any symmetric signal structure there are always sufficiently biased experts for which communication is no longer possible in equilibrium. Moreover, for any bias of the expert, no communication is possible if the signal structure is sufficiently precise. For the uniform signal structure I show that communication decreases with the precision of the signal. Finally, I provide non degenerate examples for which the decision maker's private information cannot make up for the loss of communication implying that the welfare of both agents decreases.

Keywords: Communication, two-sided private information, cheap talk, information acquisition
JEL: C72, D82, D83.

1. Introduction

Decision makers often seek advice from better informed experts. Examples range from management consulting to political, financial and medical advice[1]. In most of the cases, the interests of the expert are not perfectly aligned with those of the decision maker and as a result the expert has an incentive to manipulate his information. Crawford and Sobel (1982)[2] (CS henceforth) study the strategic information transmission between a biased expert (he) and an uninformed decision maker (she) when contracts or other commitment devices are not available[3]. They show that only coarse information can be transmitted in equilibrium, even though, when the divergence of preferences is small, the expert would be better off if he could commit to reveal his information truthfully.

In practice though, given this poor information transmission, it is likely that the decision maker will seek other sources of information, either through further consultation or through private research. The aim of this paper is to analyse how the access to a private source of information affects the expert's incentives to transmit information in the first place. Will an expert knowing he is not the only source

*I am indebted to Gilat Levy for her advice, support and constant encouragement. I also thank Can Celiktemur, Tom Cunningham, Erik Eyster, Nathan Foley-Fisher, Kohei Kawamura, Hande Kucuk-Tuger, Kristof Madarazs, Meg Meyer, Francesco Nava, Ronny Razin, Francesco Squintani, Balazs Szentes and the participants of seminars at LSE, UAB, ESSET 2012, Essex, Oxford and Paris School of Economics, for helpful comments and discussions. All remaining errors are my own.

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[1]Cheap talk games have been applied to study communication in a wide variety of areas. See Morgan and Stocken (2003) for an application to finance, Gilligan and Krehbiel (1989); Stein (1989); Austen-Smith (1993); Krishna and Morgan (2001b) and Morgan and Stocken (2008) for applications to political science and Galeotti et al. (2013) for an application to organisation design and sociology.

[2]Green and Stokey (2007), which circulated in 1981, also study the information transmission between two agents. They analyse the welfare implications of improving the information available to the expert. Along the same lines, Fischer and Stocken (2001) and Ivanov (2010) study the relation between the precision of the expert's information and the communication between the two players.

[3]Dessein (2002) and Alonso and Matouschek (2008) analyse the case in which the decision maker can commit to delegate her decision to the expert. Goltsman et al. (2009) and Kovac and Mylovanov (2009) study cases in which the decision maker can commit to a mechanism. Kamenica and Gentzkow (2011) and Kolotilin (2018) study the case in which the sender can commit to a message structure ex-ante. Finally Krishna and Morgan (2008) analyse optimal contracts with full and imperfect commitment.

of information be willing to provide more precise reports, or will he be deterred to provide information? If that was the case, how much would be lost by the loss of communication? Would it be worth for the decision maker to acquire extra information?

In order to examine these issues I extend the quadratic-uniform canonical model of CS by allowing the decision maker to access a symmetric and unbiased signal about the relevant variable prior to making her decision. The introduction of this extra source of information has two distinct effects on the incentives of the expert to communicate. On the one hand, there is an *information effect* that arises because more information allows the decision maker to choose better actions in expectation, reducing the implicit cost to the expert of sending coarse messages. This effect is stronger for coarser messages because the decision maker relies more on her private information for those messages. Therefore the information effect favours coarser messages over more precise ones, and hence hurts communication. On the other hand, there is a *risk effect* that occurs because the expert is no longer certain of the decision maker's reactions to his messages. Once more, this effect is stronger the coarser the message is and since the expert is risk averse, he has an incentive to report more precise messages and thus reduce the variance of the decision maker's actions. This effect helps communication.

I show that the information effect dominates the risk effect and as a result, if the expert is sufficiently biased all communication is lost in equilibrium. Moreover, fixing the expert's bias, it is possible to find a symmetric signal structure such that there is no communication in equilibrium. In general, for moderate biases, some information might be transmitted in equilibrium. However, for the Normal and Uniform families of signals, the information transmission decreases with the accuracy of the signal. Most strikingly, I show that sometimes the decision maker's private information cannot make up for the loss of communication and as a result the welfare of both agents declines. In those cases the decision maker would be better off if she could commit not to acquire private information.[4]

To gain some intuition of the results, consider a decision maker who wants to choose an action $y \in \mathbb{R}$ to minimise the distance to an unknown state of the world. For simplicity, suppose that the state of the world, θ, takes one of the values $\{0, \frac{1}{2}, 1\}$ with equal probability. The decision maker consults an expert who perfectly knows the true state of the world, but who would like a higher action to be implemented. For instance, suppose that the expert wants the decision maker to choose the action $y = \theta + \frac{1}{3}$, where $\frac{1}{3}$ represents the bias of the expert[5]. Along the lines of CS, full revelation is not possible in equilibrium because the expert observing the lowest state of the world has an incentive to deviate and pretend that he observed $\theta = \frac{1}{2}$. In the most informative equilibrium[6], the expert reveals the lowest state of the world and pools the two higher states. Observe that in this equilibrium when the expert observes the lowest state of the world he does not have an incentive to deviate, because doing so would lead to an action $y = \frac{3}{4}$, further away from his optimal action. Given this equilibrium, the ex-ante expected utility of an uninformed decision maker is $EU^D = -\frac{1}{24}$.

Suppose now that the decision maker has access to an informative signal, s, which takes values in $\{0, \frac{1}{2}, 1\}$ with the following conditional probability matrix:

$$P = \begin{array}{c} s \setminus \theta \\ 0 \\ \frac{1}{2} \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & \frac{1}{2} & 1 \end{array} \\ \begin{pmatrix} 0.7 & 0.15 & 0 \\ 0.3 & 0.7 & 0.3 \\ 0 & 0.15 & 0.7 \end{pmatrix} \end{array}$$

where $P_{s\theta} = Prob(s|\theta)$ is the conditional probability of observing signal s given that the state of the world is θ.[7] Given this signal structure, the expert can no longer credibly separate the lowest state from the other two. The reason is that when he observes that $\theta = 0$, he knows that the decision maker will receive the signal $s = 0$ with high probability. If he lies and reports that $\theta \in \{\frac{1}{2}, 1\}$, with probability 0.7 the decision maker will choose $y = \frac{1}{2}$ and with probability 0.3 she will choose $y = \frac{13}{20}$, leading to an expected utility to the expert of $-\frac{1783}{36000} \simeq -0.0495$, which is higher than the expected utility he would

[4] Of course there is a commitment problem because ex-post (once the expert has sent his message) the decision maker is always better off by acquiring information.

[5] To be more precise I am assuming in this example that both agents have quadratic loss utilities given by $u(y, p) = -(y - p)^2$ where p represents the peak of the preferences which is θ for the decision maker and $\theta + \frac{1}{3}$ for the expert.

[6] There is an issue of multiplicity of equilibria in cheap talk games. In particular there is always a babbling equilibrium. CS show that for the case of quadratic-loss utilities, the most informative equilibrium is preferred by both agents to any other equilibrium and hence I focus on this one. For refinements of equilibria in cheap talk games, see Matthews et al. (1991); Farrell (1993); Rabin (1990) and Chen et al. (2008) among others.

[7] Observe that the signal is not only informative, but it is *affiliated* with the state of the world, meaning that higher realisations of the signal lead to higher posterior beliefs about θ in the first order stochastic dominance.

have if he truthfully revealed that $\theta = 0$ (in that case the utility for the expert would be $-\frac{1}{9} \simeq -0.1111$). Therefore, the introduction of the private information prevents the expert from revealing any information at all.[8] Moreover, the ex-ante utility of the decision maker when she has access to the signal (and hence does not receive informative messages from the expert) is $EU^D = -\frac{6}{85}$, which is lower than what she had in the uninformed case.

This example shows that allowing the decision maker to have access to a private signal lowers the incentives of the expert to reveal information because he knows that the signal will shift the decision maker's action towards the true state of the world, making exaggeration less costly. Moreover, given the loss of communication, the decision maker is better off if she has not access to extra information.

Related Literature. A few papers have studied information transmission when the decision maker is privately informed. Some early contributions by Seidmann (1990), Watson (1996) and Olszewski (2004) explore ways in which private information might facilitate communication. In Seidmann (1990), the private information allows the decision maker to distinguish between experts who have the same preferences over choices but different preferences over lotteries. In Watson (1996), the two parties have complementary information and the expert, unsure of which one is the best action to choose, prefers to truthfully reveal his information. In Olszewski (2004), the expert is concerned about being perceived as honest and the decision maker uses her information to cross-check the expert's statements and hence disciplines the expert. All these papers show different model assumptions under which the private information could be used to extract further information from the expert. I see them as complementary to this study. In this paper I highlight that whenever the decision maker's information acts as a substitute for the expert's information, a new effect arises that hinders communication and needs to be taken into account.

My paper is most closely related to Chen (2012); Lai (2014) and Ishida and Shimizu (2016). These papers introduce information to the decision maker within the standard framework of CS. Chen (2012) considers a binary state version of CS and studies the optimal timing of the sender's report when the players have access to a public signal, i.e., it compares the receiver's equilibrium payoff when the public information is available before the communication stage to the receiver's equilibrium payoff if the public information comes afterwards. In this paper I compare the equilibrium of the model with private information (analogous to the case where the public signal arrives after the sender's report) to the equilibrium in CS. In other words, the question of this paper is whether it is worth acquiring information at all. Lai (2014) studies communication between an expert and an *amateur* who knows whether the state of the world lies below or above a cutoff point which is her private information. As in the present paper, Lai finds that the expert in the *amateur* model is less willing to provide information. However, the decision maker always ex-ante benefits from having access to the extra information. The setup of this paper allows for more flexible signal structures. In particular, I am able to explore the communication as the signals become smoothly more precise and I find that having access to information might reduce the ex-ante welfare of the decision maker. Ishida and Shimizu (2016) analyse the case when both the expert and the decision maker have discrete imperfect signals about a binary state of the world. In their paper the preferences of the two agents differ only when there is enough uncertainty (to both players) about the state of the world. They provide conditions for the non existence of communication in equilibrium. In particular, they show that when the two agents are equally (well) informed no information can be revealed in equilibrium because the sender's message becomes pivotal only when it disagrees with the receiver's information bringing back the uncertainty and hence the divergence of preferences.

Also related to this paper are models which introduce multiple experts because each expert represents a different source of information to the decision maker. Austen-Smith (1993) analyses the case of an uninformed House that refers legislation to two expert committees (which are imperfectly informed) under open rule. In a binary set up, he finds that any single committee is willing to provide more information under single referral than multiple referral. However, the information content of multiple referral is superior to that of single referral. In Krishna and Morgan (2001a) a decision maker can sequentially consult experts with different biases. They find that if the experts have similar biases the decision maker cannot do better than to ignore the messages of the most biased expert. Galeotti et al. (2013) study communication across a network where all the agents are at the same time senders and receivers with binary information. They find that the willingness of a player to communicate with a neighbour declines with the number of opponents who communicates to this neighbour. In all these papers there is an equilibrium in which the decision maker ignores the report of all except one expert, who then babbles, and as a result, consulting multiple experts cannot be detrimental. By contrast, in

[8]These results are not driven by the presence of zeros in the conditional probability matrix.

the setup of the present paper, it is never rational for the decision maker to ex-post ignore her signal and hence the welfare implications can be negative.

Finally, there are some papers which study the effect of uncertainty (to both agents[9]) on the incentives to communicate. Krishna and Morgan (2004) introduce a jointly controlled lottery together with multiple rounds of communication in the CS framework and show that the resulting equilibria Pareto dominate those of the original model. Blume et al. (2007) introduce error in the message transmission. They show that adding noise to the model almost always leads to a Pareto improvement. Goltsman et al. (2009) study optimal mediation in communication games. They find that mediators should optimally introduce noise in their reports because this eases the incentive compatibility constraints of the expert. In all these papers the uncertainty is independent of the state of the world. By contrast, I show that if instead of pure noise the decision maker receives an informative noisy signal, the results can be reversed.

The rest of the paper is organised as follows. Section 2 states the model and shows the existence of the partition equilibria. Section 3 exposes the main results of the paper. I analyse how the access to information affects the incentives of the expert to communicate and I illustrate the welfare implications for two families of signal structures, the Normal and the Uniform case. Section 4 discusses some of the assumptions of the model and Section 5 concludes.

2. The Model

2.1. Setup

There are two players, an expert (E or he) and a decision maker (D or she). The expert privately and perfectly observes the state of the world θ,[10] a random variable uniformly distributed on $[0, 1]$. The decision maker, by contrast, only receives a noisy signal s affiliated with θ.[11] The conditional distribution of the signal is common knowledge, but the realisation s is privately observed by the decision maker. I assume that given θ the signal is symmetrically distributed around θ with density $f(s - \theta)$, where $f(\cdot)$ is symmetric around 0, positive on the whole real line and log-concave. I refer to θ and s as the *types* of the expert and the decision maker respectively and I identify the signal structure with f, the density of the signal.

After learning θ, the expert sends to the decision maker a costless message m from an arbitrarily large topological space $\mathcal{M}$. The decision maker, taking into account her private signal and the expert's message, chooses an action $y \in \mathbb{R}$ that affects both agents' payoffs.

The players' payoffs are given by the following utility functions:
$u^D(y, \theta) = u(y - \theta)$
$u^E(y, \theta, b) = u(y - (\theta + b))$
where $u(\cdot)$ is a strictly concave, twice differentiable and symmetric function around 0. The parameter b represents the bias of the expert; given a realisation of the state of the world θ, the expert would like the decision maker to choose action $\theta + b$, whereas the optimal action for the decision maker is to match the state of the world. I consider the case $b > 0$ although all the results can be replicated for negative biases.

Before moving to the equilibrium analysis of the model it is worth explaining the motive of the symmetric assumptions made both for the signal and state distributions and for the payoff functions. As long as the signal received by the decision maker and the state of the world are affiliated, the conditional density of the signal is log-concave and the payoff functions are concave in y and supermodular in (y, θ), the existence of the monotone partition equilibrium (Theorem 1) can be shown. The symmetric assumption facilitates the comparison of the different equilibria within and between different signal structures because it implies that the preferences of the expert over messages in equilibrium depend only on the distance between the state of the world and the states induced by the message and not on their actual values. As a result, any property established for one message can be easily translated and compared to properties for other messages.

[9]Another branch of the literature introduces uncertainty on the preferences of the expert. See for example Li and Madarasz (2007); Morgan and Stocken (2003); Wolinsky (2003) and Dimitrakas and Sarafidis (2005). They find that more information can be transmitted because the decision maker is less sensitive to the message of the expert.

[10]With some abuse of notation I use the same character to denote a random variable and its realisation.

[11]The fact that θ and s are affiliated implies that higher realisations of s lead to higher posterior beliefs about θ in the first-order stochastic dominance. For unidimensional random variables, being affiliated is equivalent to the joint density distribution being log-supermodular in (s, θ) or that the conditional density distribution of s given θ satisfies the *Monotone Likelihood Ratio Property* (MLRP): $f(s|\theta)f(s'|\theta') > f(s|\theta')f(s'|\theta)$ for all $s > s'$, $\theta > \theta'$ (Milgrom and Weber, 1982).

Finally, I refer to this model as the *private information* model in contrast to the CS model, where the decision maker is uninformed.

2.2. Equilibrium Analysis

The equilibrium concept I consider is the *Bayesian Nash Equilibrium* (BNE). A message strategy for the expert is a probability distribution q over the Borel measurable subsets of $[0,1] \times \mathcal{M}$. Given θ, I denote by $q(\cdot \mid \theta)$ the conditional distribution induced by q. A *pure*[12] action strategy for the decision maker is a function $y : \mathcal{M} \times \mathbb{R} :\to [0,1]$. The strategies $(q(\cdot), y(\cdot))$ constitute a BNE if:

(E1) for each m and s, such that $\int_0^1 q(m|t)dt > 0$,

$$y(m,s) = \arg\max_y \int_0^1 u(y-\theta)g(\theta|m,s)d\theta$$

where $g(\theta|m,s) = q(m|\theta)f(s-\theta)/\int_0^1 q(m|t)f(s-t)dt$

(E2) whenever $q(m|\theta) > 0$ and $m' \in \mathcal{M}$,

$$\int_{\mathbb{R}} u\left(y(m,s) - (\theta+b)\right) f(s-\theta)ds \geq \int_{\mathbb{R}} u\left(y(m',s) - (\theta+b)\right) f(s-\theta)ds$$

That is, after receiving her signal and the message from the sender, the decision maker correctly updates the distribution over the possible states of the world and chooses her optimal action. On the other hand, the expert sends the message that maximises his payoff given the decision maker's strategy.[13]

If the signal s were independent of θ, the setup would correspond to the canonical model of CS. However, when the signal is informative two main differences arise. First, the expert is no longer able to perfectly forecast the reaction of the decision maker to his message. Although there is a unique best action for the decision maker, this action depends on the signal she receives and hence from the point of view of the expert a message induces a lottery over actions rather than a single action. Second, since the signal and the state of the world are affiliated, the distribution of the actions induced by a message depends on the expert's type. In other words, two experts sending the same message face different lotteries. As a consequence, the set of experts who prefer one message to another does not need to form an interval as in CS. This fact makes it difficult to provide a complete characterisation of the equilibria. However, as shown below, it is possible to characterise the set of monotone partition equilibria that share the same structure as the ones characterised by CS. In particular, as the signal becomes less informative, they converge to the equilibria in CS. In the remainder of the paper I focus exclusively on these equilibria.

2.3. Monotone Partition Equilibria

An equilibrium is said to be a monotone partition equilibrium if the state space $[0,1]$ can be partitioned into intervals such that the expert strategy induces the same conditional distribution for all types in a given interval, which has disjoint support from the conditional distributions induced when the type lies in another interval. Formally:

Definition 1. *An BNE equilibrium* $(q(\cdot), y(\cdot))$ *is a* monotone partition equilibrium of size N*, if there exists a partition* $0 = a_0 < a_1 < ... < a_N = 1$ *such that* $q(m|\theta) = q(m|\theta')$ *if* $\theta, \theta' \in (a_i, a_{i+1})$*, and if* $q(m|\theta) > 0$ *for* $\theta \in (a_i, a_{i+1})$ *then* $q(m|\theta') = 0$ *for all* $\theta' \in (a_j, a_{j+1})$ *with* $j \neq i$*.*

I consider any two equilibria with strategies that agree almost everywhere to be identical. Hence, the definition above does not determine the strategy of boundary types $\theta = a_i$.

To simplify the exposition of the argument and given that the only information that the decision maker induces from a message is the interval in which the actual state of the world lies, I identify a

[12] This is without of generality given the concavity of preferences of the decision maker.

[13] Note that perfection cannot refine the set of equilibria in this set up. We can always specify beliefs for unsent messages that replicate equilibrium beliefs. The assumption of full support in the signal structure is important though. If the support of the signal varies with θ, it might be the case that an expert deviating by sending a message which corresponded to another type would be discovered. In this case, perfection might impose an extra restriction since some out of equilibrium beliefs need to be specified and the beliefs should be consistent with the information revealed by the signal, i.e, it has to satisfy that if $g(\theta|m,s) > 0$, then $f(s-\theta) > 0$.

message with the interval of types that send it given the strategy $q(\cdot)$ in consideration, i.e. $m \equiv [\underline{a}, \overline{a}]$ if $[\underline{a}, \overline{a}] = cl(\{\theta \in [0,1] \mid q(m|\theta) > 0\})$, where $cl(A)$ denotes the closure of the set A.

I denote by $U_f^E(\underline{a}, \overline{a}, \theta)$, or with some abuse of notation $U_f^E(m, \theta)$, the expected utility of an expert with type θ who sends message $m = [\underline{a}, \overline{a}]$, when the conditional density of the signal is given by f. To simplify the notation, whenever there is no ambiguity I omit the reference to the bias b.

Finally, to be able to make welfare comparisons between equilibria, I make the following assumption:

Assumption A: *$U_f^E(\underline{a}, a, \underline{a})$ is quasi-concave in a for $a \geq \underline{a}$.*[14]

Assumption A guarantees that given $a_{i-1} \leq a_i$ there is at most a unique a_{i+1} which satisfies the Condition (A_f) below. It allows a stronger version of condition (M) in CS[15] to be proved, which in particular ensures that there is at most one partition of size N satisfying (A_f) and this allows comparative statics of the equilibria.

The following results mimics Theorem 1 in CS to characterise the set of monotone partition equilibria:

Theorem 1. *For any $b > 0$, there exists a positive integer $N_f(b)$ such that, for every $1 \leq N \leq N_f(b)$:*

1. *there exists a monotone partition equilibrium of size N, characterised by a partition $0 = a_0 < a_1 < ... < a_N = 1$ satisfying*

$$U_f^E(a_{i-1}, a_i, a_i) = U_f^E(a_i, a_{i+1}, a_i) \qquad (A_f)$$

2. *If Assumption A is satisfied, then there is a unique monotone partition equilibrium for each N, and $a_{i+1} - a_i > a_i - a_{i-1}$ for all $i = 1, ..., N-1$. Moreover, both the decision maker and the expert ex-ante prefer equilibrium partitions with more intervals.*

These are no other monotone partition equilibria. [16]

All proofs are found in the Appendix.

Theorem 1 establishes that for each positive integer up to a finite number $N_f(b)$, there exists a unique monotone partition equilibrium of that size, for which the intervals are increasing in length. In particular, this implies that the messages sent by the expert in equilibrium are less precise as the state of the world increases. Finally, Theorem 1 also states that the equilibria can be Pareto ranked, and that the equilibrium with size $N_f(b)$ ex-ante Pareto dominates all the others. On the basis of this last statement, for the welfare analysis in Section 3, I focus on the equilibrium partition with the highest number of intervals.

3. Communication

The access to information has a direct and an indirect effect on the decision maker's welfare. On the one hand, fixing the information transmission, a better informed decision maker is able to choose more accurate actions and hence this direct effect of information enhances her welfare. On the other hand, the access to information alters the incentives of the expert transmit information. The aim of this section is to study this second indirect effect of the access to information. In order to do so I start by providing a definition of when a partition is more informative than another, along with a useful preliminary result that allows me to compare equilibrium partitions across different signal structures in terms of communication.

3.1. Definition and Preliminary Result

Definition 2. *A partition is more informative than another if* ex-ante *the decision maker prefers the former over the latter.*

Two observations are in order here. First, given the symmetric setup, the decision maker in the private information model has ex-ante the same preferences over partitions as the uninformed decision maker (CS). That is, if the decision maker had to rank the partitions before receiving the expert's message and before learning her signal, she would rank them in exactly the same order as a decision maker that doesn't have access to private information. Hence, this definition is independent of the signal structure.

[14] Assumption A is satisfied by all the examples given in the paper.
[15] See Proposition 6 in Appendix Appendix A.2.
[16] All the comparative statics with respect to the divergence of preferences b established in CS can also be transferred to the private information model. Since this is not the focus of the paper I do not state them here.

Without loss of generality in what follows I measure the information of a partition by the ex-ante expected payoff of a decision maker that doesn't have access to private information.

Second, note that I use this definition to compare partition equilibria that arise from two different signal structures. However, to state whether one equilibrium is more informative than the other I evaluate the partitions from the point of view of a decision maker that does not have access to any further information (or analogously, fixing the signal structure). The reason is that I want to isolate the indirect effect that the private information has on welfare through the loss of communication from the direct effect that an extra piece of information has on welfare through better decision making. In Section 3.4, I consider the total welfare effect.

To gain some intuition about which partitions are preferred to others, consider the decision maker that has no access to information. Fixing the number of intervals in the partition, and given symmetric concave preferences, the optimal partition is the one with intervals of equal size, because it is the one that minimises the riskiness of the decision. Analogously, dividing one interval into two or more subintervals improves the partition because it reduces the riskiness that the decision maker faces.

The following result simplifies the comparison of different information structures in terms of communication. It states that it is enough to look at how the indifference condition is affected when the signal structure changes to conclude whether there is more or less communication in equilibrium.

Proposition 1. *Suppose that f and f' are two signal structures satisfying the following condition:*

$$\text{(C):}\quad \text{For any } a_{i-1} \leq a_i \leq a_{i+1} \text{ such that } U_f^E(a_{i-1}, a_i, a_i) = U_f^E(a_i, a_{i+1}, a_i),$$
$$U_{f'}^E(a_{i-1}, a_i, a_i) < U_{f'}^E(a_i, a_{i+1}, a_i).$$

Then $N_f(b) \geq N_{f'}(b)$. Moreover, if a and a' are two equilibrium partitions of size N of the private information model with signal structures f and f' respectively, then $a_i > a'_i$ for all $1 \leq i \leq N-1$. Namely, there is less communication in equilibrium in the private information model with signal structure f' than when the signal structure is f.

In particular, to compare the CS model with the private information model, it is enough to consider any three points that satisfy the indifference condition in the CS model, that is, such that $a_{i+1} - a_i = a_i - a_{i-1} + 4b$,[17] and see which interval, $[a_{i-1}, a_i]$ or $[a_i, a_{i+1}]$ the expert with type a_i prefers when the decision maker has access to private information.

The intuition behind this result is as follows. Consider an equilibrium with a particular signal structure f that has just two intervals. The partition equilibrium is then given by $\{0, a, 1\}$. Suppose now that there is a signal structure f' that satisfies Condition (C). This means that the expert with type a, facing a decision maker that has access to a signal structure f' strictly prefers the upper interval $[a, 1]$ to the interval $[0, a]$. The new equilibrium would then need to make the upper interval less attractive and the lower one more attractive, which implies the upper interval has to be larger. Therefore the new indifferent point a' would lie to the left of a. Recall that by Theorem 1, the indifferent expert had type $a < \frac{1}{2}$ and hence the new partition $\{0, a', 1\}$ is more uneven or provides less information ex-ante than the previous one. This intuition transfers to equilibria of more than two intervals.

3.2. Analysis

For the rest of the paper I will restrict my attention to the case of quadratic-loss utilities[18] given by:

$u^D(y, \theta) = -(y - \theta)^2$

$u^E(y, \theta, b) = -(y - (\theta + b))^2$.

Given these utilities, the decision maker's optimal action when she receives message m and signal s is to match her expectation about the state of the world: $y_f(m, s) = E_f[\theta|m, s]$. Moreover, the expected utility of an expert with type θ who sends message m can be written as:

$$U_f^E(m, \theta) = -\hat{\sigma}_f^2(m, \theta) - (\hat{y}_f(m, \theta) - (\theta + b))^2$$

where $\hat{y}_f(m, \theta)$ and $\hat{\sigma}_f^2(m, \theta)$ are the expectation and the variance of the actions chosen by the decision maker when the expert sends message m and has type θ.[19] Hence, the expert's expected utility depends

[17] In the CS model, the decision maker's best response upon receiving an interval is to choose the middle point. Hence the action induced by $[a_{i-1}, a_i]$ is $\frac{a_{i-1}+a_i}{2}$ and the action induced by $[a_i, a_{i+1}]$ is $\frac{a_i+a_{i+1}}{2}$. An expert with type a_i is indifferent between these two actions if $(a_i + b) - \frac{a_{i-1}+a_i}{2} = \frac{a_i+a_{i+1}}{2} - (a_i + b)$ which simplifies to $a_{i+1} - a_i = a_i - a_{i-1} + 4b$

[18] Section 4.2 provides a discussion of the results for other symmetric preferences.

[19] Namely, $\hat{y}_f(m, \theta) = \int_{\mathbb{R}} y_f(m, s) f(s - \theta) ds$ and $\hat{\sigma}_f^2(m, \theta) = \int_{\mathbb{R}} (y_f(m, s) - \hat{y}_f(m, \theta))^2 f(s - \theta) ds$.

only on the variance of the actions and the distance between the expert's peak and the expected action of the decision maker.

We are now ready to analyse the effect of the access to information on the communication in equilibrium. Consider three points $\{a_{i-1}, a_i, a_{i+1}\}$ that satisfy the indifference condition in the CS model (I denote the indifference condition in the CS model by (A_{CS}) by analogy to (A_f)). By Proposition 1, we need to compare the expected utility in the private information model of an expert with type a_i sending the message $m_i = [a_{i-1}, a_i]$ versus sending message $m_{i+1} = [a_i, a_{i+1}]$. Since $\{a_{i-1}, a_i, a_{i+1}\}$ satisfy the indifference condition in the CS model, this is equivalent to compare the change in the expert's expected utility due to the introduction of the privation information between the two messages, i.e. to compare $U_f^E(m_{i+1}, a_i) - U_{CS}^E(m_{i+1}, a_i)$ to $U_f^E(m_i, a_i) - U_{CS}^E(m_i, a_i)$.

Denote by $y_{CS}(m)$ the action chosen by an uninformed decision maker upon receiving message m.[20] Consider message m_i (the same decomposition applies to m_{i+1}), the change in the expert's expected utility due to the introduction of private information is:

$$U_f^E(m_i, a_i) - U_{CS}^E(m_i, a_i) = \underbrace{-\hat{\sigma}_f^2(m_i, a_i)}_{Risk\ \ Effect} + \underbrace{(y_{CS}(m_i) - (a_i + b))^2 - (\hat{y}_f(m_i, a_i) - (a_i + b))^2}_{Information\ \ Effect}$$

The introduction of private information has two effects on the expert's expected utility: an *information effect* and a *risk effect*. The information effect arises because the signal allows the decision maker to choose better actions on average. In expectation, her action will be closer to a_i than it was before, and hence closer to the expert's best action $a_i + b$.[21] Therefore, the information effect has a positive impact on the expected utility of the expert with type a_i. The risk effect occurs because the expert is no longer certain of the response of the decision maker to his message. Since the expert is risk averse, he dislikes this uncertainty and the risk effect has always a negative impact in the expert's expected utility.

The next result states that the information effect dominates the risk effect for the two intervals.

Proposition 2. *Consider* $0 \leq a_{i-1} \leq a_i \leq a_{i+1} \leq 1$ *satisfying* (A_{CS})*, and denote* $m_i = [a_{i-1}, a_i]$ *and* $m_{i+1} = [a_i, a_{i+1}]$*, then:*

1. $U_f^E(m_i, a_i) - U_{CS}^E(m_i, a_i) \geq 0$
2. $U_f^E(m_{i+1}, a_i) - U_{CS}^E(m_{i+1}, a_i) \geq 0$

In other words, an indifferent expert in the CS model would benefit if (keeping the CS message strategy fixed) the decision maker acquired information.[22] Proposition 2 allows us to conclude that in some cases the acquisition of information leads to the complete loss of communication.

Theorem 2. 1. *For any signal structure* f*, there exist* $\bar{b} < \frac{1}{4}$ *such that if* $b > \bar{b}$*, there is no communication in the private information model.*

2. *For any* $b > 0$*, there exists a signal structure* f *such that there is no communication in the private information model.*

Observe that for $b < \frac{1}{4}$ there exists an informative equilibrium in the CS model, and hence part 1 of Theorem 2 refers to situations where there was some information transmitted in equilibrium, but once the expert is aware that the decision maker has access to a private signal, no information is revealed in equilibrium.

The intuition behind the proof of Theorem 2 is very simple. Consider an expert with bias $b = \frac{1}{4}$ in the CS model. In this case the expert with type $\theta = 0$ is indifferent between perfectly revealing his type or pooling with the rest of the interval. If the expert perfectly reveals his type, the introduction of private information is irrelevant since the decision maker perfectly learns from the message the type of the expert (the information and the risk effect are both equal to zero). However, if the expert pools with the rest of the interval, Proposition 2 says that the introduction of information has a positive impact on the expert's

[20] $y_{CS}(m)$ is the midpoint of the interval m.

[21] This is also true, although less straight forward, in the case of message m_{i+1}. The reason is that the length of the interval m_{i+1} is at least $4b$, and hence $y_{CS}(m_{i+1}) - (a_i + b) > b$. Thus any action closer to a_i is also closer to $a_i + b$ than $y_{CS}(m_{i+1})$ was.

[22] Observe that this is not true for any expert. For instance consider $\theta = \frac{a_i + a_{i+1}}{2} - b$, it is clear that this expert sending message $[a_i, a_{i+1}]$ prefers to face an uninformed decision maker. Also, although it is always true that an expert with type a_i sending message $[\underline{a}, a_i]$ always benefits from the decision maker's information, the same expert sending message $[a_i, \overline{a}]$ only benefits if $\overline{a} - a_i \geq 4b$ (which in this case is guaranteed by (A_{CS})).

expected utility. Hence in the private information model, an expert with bias $b = \frac{1}{4}$ strictly prefers to send message $[0, 1]$. By continuity, there exists a $\bar{b} < \frac{1}{4}$ such that these preferences are preserved for any bias $\bar{b} < b < \frac{1}{4}$, and by Proposition 1 no communication is possible in equilibrium.

For the second statement of Theorem 2, observe that for any $b > 0$, we can find a sufficiently precise signal structure such that the lottery over actions induced by message $[0, 1]$ is preferred by an expert with type $\theta = 0$ to the constant action $y = 0$. This being the case, no information can ever be transmitted in equilibrium.

Theorem 2 relates to cases where the access to information prevents any communication in equilibrium. In general though, the introduction of private information affects the partition equilibria but does not need to eliminate all the communication between the agents. In what follows I compare the information and risk effect across the messages m_i and m_{i+1} for the expert with type $\theta = a_i$.

Consider first the information effect. Observe that the message sent by the expert determines the prior of the decision maker before hearing her signal. Since message m_{i+1} is larger than message m_i, sending m_{i+1} instead of m_i implies that the decision maker will have a less precise prior about the state of the world. But a less precise prior implies that the decision maker will rely more on her signal when updating her posterior. In other words, the actions of the decision maker are more sensitive to her private information the larger the message sent. From the point of view of the expert with type a_i, it means that the expected action of the decision maker will shift towards him by more when he sends m_{i+1} than when he sends m_i. Hence the expert with type a_i, strictly prefers $\hat{y}(m_{i+1}, a_i)$ to $\hat{y}(m_i, a_i)$. Abstracting from risk aversion, this result implies that the message m_{i+1} becomes more attractive to the expert than the message m_i. Hence, the information effect worsens the incentives of the expert to communicate.

Proposition 3. *The information effect hampers communication. Namely, if* $0 \leq a_{i-1} \leq a_i < a_{i+1} \leq 1$ *satisfy* (A_{CS})*, then* $(\hat{y}_f(m_{i+1}, a_i) - (a_i + b))^2 < (\hat{y}_f(m_i, a_i) - (a_i + b))^2$.

Consider now the risk effect. Intuitively, sending a larger message spreads the decision maker's actions across the interval, thereby increasing the variance of the lottery. Hence the risk effect is stronger in m_{i+1} than in m_i. Abstracting from the information effect, a risk averse expert prefers the lower and narrower interval, easing the communication between the agents.

Proposition 4. *The risk effect facilitates communication. Namely, if* $0 \leq a_{i-1} \leq a_i < a_{i+1} \leq 1$ *satisfy* (A_{CS})*, then* $\hat{\sigma}^2_f(m_{i+1}, a_i) > \hat{\sigma}^2_f(m_i, a_i)$.

Proposition 3 and 4 highlight two counterintuitive effects of the acquisition of private information. Better decision making harms communication whereas adding risk helps it. To understand these results, observe that in CS, low type experts were deterred from sending high messages because they led to actions too far away from their preferred actions. When private information is introduced, low type experts know that, even if they send a high message, the action chosen by the decision maker will be pulled towards their preferred action, because the signal received is affiliated with the state of the world. This pulling force makes high messages more attractive and hence, in equilibrium, high intervals are even less precise than before, leading to a loss in communication. The opposite effect arises with the riskiness of the signal. A risk averse expert will try to balance the risk associated with the actions of the decision maker by making the intervals more even, hence favouring communication.

In what follows I provide two examples of how these effects interact with each other considering two particular families of distribution of signals, the normal signals with variance σ^2 and the uniform signals with support $[\theta - \delta, \theta + \delta]$.

3.3. Examples

3.3.1. The Normal Private Information Model

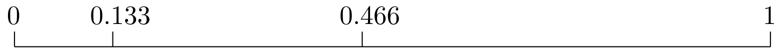
Consider the case in which the signal conditional on θ is distributed normally around θ with variance σ^2.

To be specific, suppose that the bias of the expert is $b = \frac{1}{20}$.[23] For this bias, the most informative equilibrium in the standard CS model is determined by the following partition: $\{0, \frac{2}{15}, \frac{7}{15}, 1\}$. The expert reveals whether the state of the world lies in $[0, \frac{2}{15}]$, in $[\frac{2}{15}, \frac{7}{15}]$ or in $[\frac{7}{15}, 1]$, and the decision maker reacts by choosing the midpoint in each interval.[24]

[23] This bias corresponds to the example illustrated in Crawford and Sobel (1982).

[24] It is easy to check that this in fact constitutes an equilibrium. For instance, when the expert observes $\theta = \frac{2}{15}$ he is indifferent between reporting the first interval and the second, because they lead respectively to actions $a_1 = \frac{1}{15}$ and $a_2 = \frac{9}{30}$, which are equidistant to his preferred action $\frac{2}{15} + \frac{1}{20}$.

In the private information model with $\sigma = 0.3$, the most informative equilibrium is determined by the partition $\{0, 0.0863, 0.59, 1\}$. Figure 1 provides a graphical illustration of the two equilibria.



(a) CS Equilibrium ($b = \frac{1}{20}$)

(b) PI Equilibrium ($b = \frac{1}{20}$, $\sigma = 0.3$)

Figure 1: Monotone Partition Equilibrium in CS model and in the Normal PI model, with $b = \frac{1}{20}$ and $\sigma = 0.3$

To compute the loss of communication due to the introduction of the signal, I compute the ex-ante utility of an uninformed decision maker under both partitions. The loss of communication is $EU^D_{CS,\{0,\frac{2}{15},\frac{7}{15},1\}} - EU^D_{CS,\{0,0.0863,0.39,1\}} = (-0.0159) - (-0.0213) = 0.0054$.

Figure 2 shows the partition equilibria as a function of the variance of the signal. For every σ the partition can be read by tracing the horizontal line at this level. The intersections of the horizontal line with the solid lines correspond to the points of the partition equilibrium. The case of $\sigma = 0.3$ is depicted as an example. The horizontal line cuts the solid lines at $a_1 = 0.0863$ and $a_2 = 0.39$, indicating that the partition equilibrium is $\{0, 0.0863, 0.39, 1\}$.

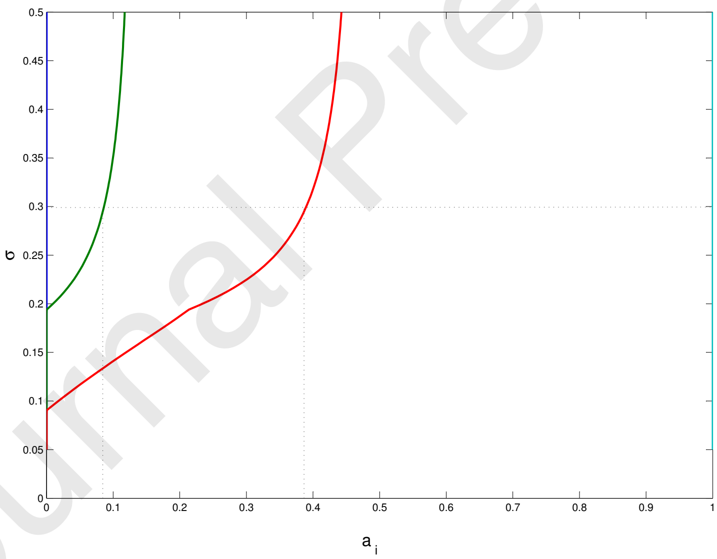


Figure 2: Monotone partition equilibria in the Normal PI model for different variances of the signal. ($b = \frac{1}{20}$)

From Figure 2 we can see that for $\sigma < \sigma_0 \simeq 0.09$ no information is revealed in equilibrium. For $\sigma_0 < \sigma < \sigma_1 \simeq 0.193$ the partition equilibrium contains only two intervals and for $\sigma > \sigma_1$ the partition equilibrium is formed by three intervals. Finally, as σ increases, the equilibrium partition converges to the CS equilibrium.

Figure 2 shows that the communication declines with the precision of the signal. This comparative statics is proven in Theorem 3 for the case of the family of uniform signals.

3.3.2. The Uniform Private Information Model

Consider now that the signals are distributed uniformly on $[\theta - \delta, \theta + \delta]$.[25] The parameter δ plays the same role as σ in the Normal example. Upon receiving a signal s and a message $m = [\underline{a}, \overline{a}]$ the decision maker's posterior distribution of θ is uniform on the interval $[\max\{\underline{a}, s-\delta\}, \min\{\overline{a}, s+\delta\}]$[26]. Given these beliefs, the optimal action for the decision maker is[27]:

$$y(\underline{a}, \overline{a}, s, \delta) = \frac{\max\{\underline{a}, s-\delta\} + \min\{\overline{a}, s+\delta\}}{2}$$

Theorem 3. *In the Uniform Private Information model, an increase in the precision of the signal (a decrease in δ) leads to less communication in equilibrium.*

Theorem 3 states that the communication from the expert declines with the accuracy of the decision maker's information. Figure 3 illustrates the comparative static results for the family of uniform signals. As in the Normal Private Information case, no communication is possible for sufficiently precise signals. As the precision of the signal decreases (i.e. δ increases) more and more communication arises in equilibrium, converging to the CS most informative equilibrium as the signal becomes uninformative.

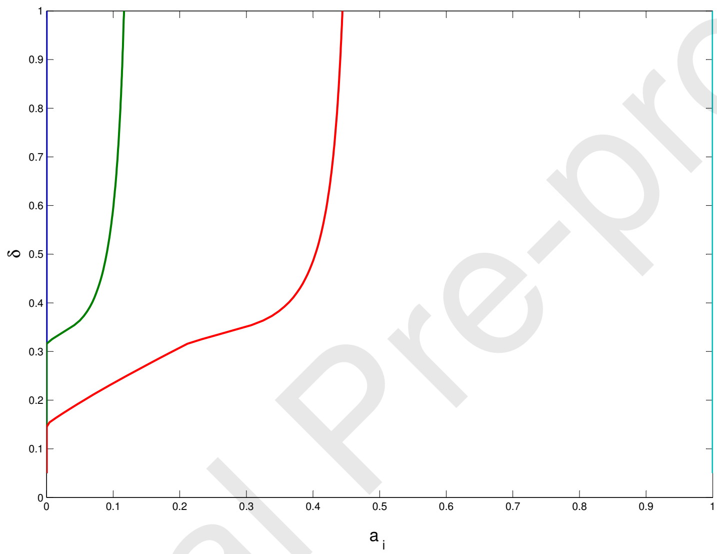


Figure 3: Monotone Partition equilibria in the Uniform PI model for different values of δ. ($b = \frac{1}{20}$)

Even if communication is hampered due to the introduction of the signal, the signal itself may provide enough information to make up for the loss in communication. I now study the total welfare implications of the access to private information.

[25] All the derivations related to this example, including the proof of Theorem 3, can be found in Appendix Appendix B

[26] Note that this signal structure does not satisfy the full support assumption. As it is clear in the example, this assumption is not necessary for the existence of the monotone partition equilibria or for establishing its properties. However, the fact that the support of the signal varies with θ implies that an expert pretending to have another type might be discovered and this gives rise to more equilibria constructed by using out of equilibrium threats. For instance full revelation could be supported in an equilibrium using the following strategies:

$$q(m|\theta) = \begin{cases} 1 & if \quad m = \theta \\ 0 & otherwise \end{cases} \qquad y(m, s) = \begin{cases} m & if \quad m \in [s-\delta, s+\delta] \cap [0,1] \\ -b^2 - 4\delta b & otherwise \end{cases}$$

Here I take a mild approach because I am interested in understanding how similar signals with full support (in which threatening with off equilibrium actions is not possible) affect the incentives to communicate. Note as well that full revelation cannot be supported if we impose perfection and require that out of equilibrium beliefs should be consistent with the information provided by the signal, that is, the support of the beliefs after signal s should be a subset of $[s-\delta, s+\delta] \cap [0,1]$.

[27] All the functions previously defined will be indexed by δ to indicate the signal structure in consideration.

3.4. Welfare

The welfare implications of the introduction of private information to the decision maker are not straight forward. As we have seen, the introduction of private information can have a negative effect on the incentives of the expert to communicate in equilibrium. On the other hand, given a message, a better informed decision maker is able to choose more accurate actions.

Clearly, if the divergence of preferences is such that there is no communication in the CS model ($b \geq \frac{1}{4}$), private information is always welfare improving.[28] Similarly, if the information is very precise, the decision maker is better off even if no information is ever transmitted from the expert.

However, access to private information is not always welfare improving. In what follows I explore the examples above and show that for each family of signals there is a range of parameters for which increasing the accuracy of the signal reduces welfare and, strikingly, the welfare falls below the welfare level in the CS model.

3.4.1. The Normal Private Information Model (Continues)

Coming back to the example depicted in Figure 1, the welfare gain due to better decision making can be computed as the difference in the ex-ante utility of the informed and the uninformed decision maker under the new partition: $EU^D_{PI,\{0,0.0863,0.39,1\}} - EU^D_{CS,\{0,0.0863,0.39,1\}} = (-0.0162) - (-0.0213) = 0.0051$.

The overall welfare effect of the addition of information can be computed by combining the loss in communication with the gain in accuracy. In this particular case the overall effect is negative: $EU^D_{PI,\{0,0.0863,0.39,1\}} - EU^D_{CS,\{0,\frac{2}{15},\frac{7}{15},1\}} = (-0.0162) - (-0.0159) = -0.0003$; the decision maker would be better off if she could commit to have no access to this additional information.

Figure 4 shows the ex-ante expected utility of the decision maker for different variances of the signal. The horizontal dashed line corresponds to the ex-ante utility in the CS model.

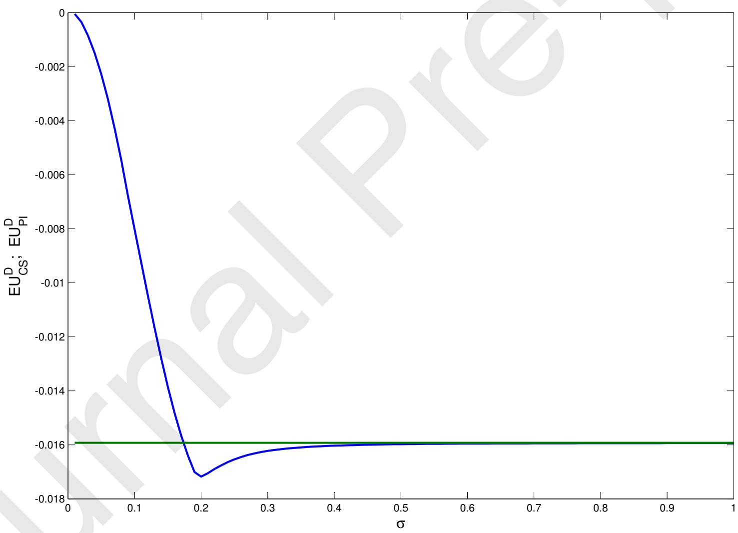


Figure 4: Ex-ante expected utility of the decision maker in the Normal PI model, for different variances of the signal. The horizontal line represents the ex-ante expected utility of the decision maker in the CS model. ($b = \frac{1}{20}$)

As can be seen from Figure 4 unless the precision of the signal is sufficiently high ($\sigma < 0.1735$), the decision maker is better off not seeking external information. The minimum ex-ante utility is reached at $\sigma = 0.1930$ which corresponds to the case where the partition equilibrium of the model passes from being of size 3 to size 2.

[28]See Persico (2000) and Athey and Levin (2001). They show that for decision problems where the signals are affiliated to the state of the world and the payoff of the decision maker satisfies the single crossing condition in (θ, y) (see Milgrom and Shannon (1994)), the ex-ante utility of the decision maker increases with the accuracy of the signals.

To understand why the loss in communication might outweigh the gain in information, it is useful to highlight two facts. First, as the signal is introduced, the largest interval becomes even larger, whereas the smallest ones are reduced (see Figure 2). The addition of information is most useful when θ lies in the big interval, but it is precisely this interval which becomes even larger, losing informativeness. Second, the boundary types (those determining the partition) are the ones that are more sensitive to the introduction of information. The reason is that those types generate signals further away to the midpoint of the interval, and hence lead to a higher impact on the best response of the decision maker. The new partition is determined by the effect of the private information on precisely those types that are most affected, whereas the welfare gain from the extra information is averaged across all types.

In the Normal Information case, the welfare impact of the addition of private information is first decreasing in the precision of the signal and then increasing once a certain threshold of precision is reached (see Figure 4). However, theses dynamics cannot be generalised. Below I illustrate the welfare implications of the access to a Uniform private signal.

3.4.2. The Uniform Private Information Model (Continues)

Figure 5 shows the ex-ante expected utility of the decision maker facing a Uniform signal, for different precisions of the signal.

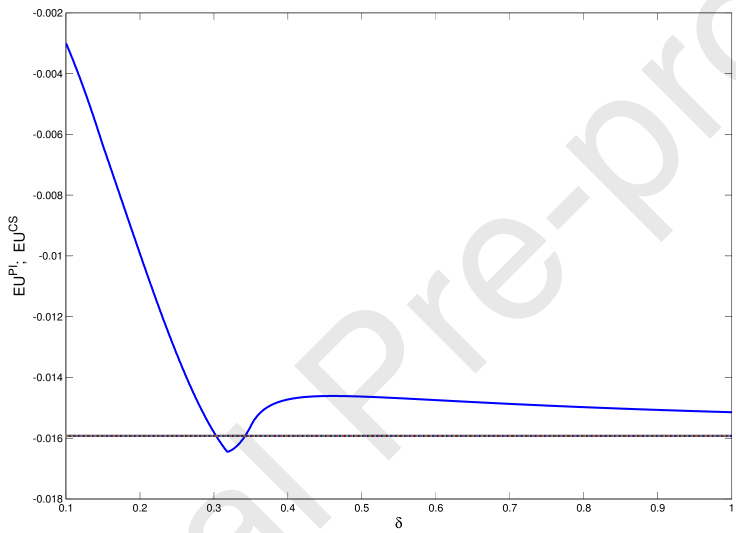


Figure 5: Ex-ante expected utility of the decision maker in the Uniform PI model for different dispersions of the signal. The horizontal line represents the ex-ante expected utility of the CS model. ($b = \frac{1}{20}$)

When $\delta \in [0.302, 0.343]$ the ex-ante welfare of the informed decision maker is lower than the welfare when the decision maker was uninformed. These levels of δ correspond to the cases where the loss of communication is greater (see Figure 3). For these values of δ, a decision maker receiving a low signal is able to reject some high states of the world and hence the information effect in the upper interval is substantially stronger than in the lower interval, leading to a strong decrease in the communication, whereas the increase of welfare given the improvement of the signal is smoother. The minimum ex-ante utility is reached at $\delta = 0.3182$, which corresponds to the case where there are two payoff-equivalent equilibria, one with three intervals and one with two.

These two examples show that a better prepared decision maker does not necessarily lead to a better outcome in equilibrium. Several papers in the literature have suggested that more information might be worse (see, for example, Prendergast (1993) or Aghion and Tirole (1997)). However, the forces driving their results are completely different. Either the expert cares about his reputation and hence wants to pander to what he thinks the decision maker believes, or the decision maker's private information reduces the incentives of the expert to acquire information because she might overrule his proposal. In this setup, however, there are no reputation concerns and the expert already has the information relevant to the decision. The loss of welfare comes from the fact that private information makes it more difficult for the expert to credibly separate low types from high types, because the implicit cost of being vague (i.e. the risk that the decision maker will choose an action too far away from his optimal action) is reduced by the private information.

4. Discussion

4.1. Costly Information Acquisition

I assumed throughout the paper that the access to private information was free. If the information was costly then the decision maker would decide whether or not to acquire extra information upon hearing the expert's message. Suppose that the decision maker has a quasi-linear utility in money,[29] and that the cost of accessing the private signal is given by $c > 0$. In this situation it is worth acquiring information only if the message of the expert is coarse enough (See Persico (2000) and Athey and Levin (2001)). To understand how the incentives to communicate are affected in this new framework, consider a partition equilibria with only two intervals in the CS setup. We know that the upper interval is larger than the lower interval. If the cost of acquiring information is sufficiently low for the decision maker to be willing to acquire information upon receiving any of the messages, then we are back to the private information model studied above. If on the other extreme, the information is so costly that the decision maker never finds profitable to acquire information, then we are in the CS framework. In the most interesting case, in which the cost of information is such that the decision maker is willing to acquire information upon receiving the higher message but not upon receiving the lower one, the incentives to communicate become even weaker than with free private information. The reason is that given Proposition 2, the boundary expert is better off when the decision maker acquires information, which only happens when he sends the upper interval. And hence, the expert strictly prefers the upper interval to the lower one, shifting the indifferent type even further to the left than in the free private information case studied before. This result is summarised bellow:

Proposition 5. *Given a signal structure f, there exists $0 < \underline{c}_f < \overline{c}_f$ such that, for any bias b with $\frac{1}{12} < b < \frac{1}{4}$, if $c \in [\underline{c}_f, \overline{c}_f]$, there is less communication in equilibrium than under the free private information case.*

The restriction on the bias ensures that in the CS most informative equilibrium there are two intervals. If the bias was smaller, then the comparison with the free private information case would not be straightforward because for the range of intervals in which it is not worth to acquire information the CS indifference condition would apply. However, as long it is worth it to acquire information for some interval, the communication would be worsen in comparison to the CS framework.

If instead of a discrete cost of acquiring information, we had a family of signals with different precisions (for example the Normal signals with different variances) and the cost of acquiring information was convex in the precision of the signal, satisfying $Lim_{p\to 0}c(p) = 0$, $Lim_{p\to\infty}c(p) = +\infty$, (so that it is always worth to acquire some information) then intuitively the argument would still go through. In that case, the wider the interval, the more value is attached to extra information and hence the decision maker would buy a more precise signal, making the information effect stronger for the wider intervals than for the narrower ones.[30]

A numerical example of this case is depicted below.

Example 1. *Consider the family of Normal signals where the precision of the signal is given by $p = \frac{1}{\sigma^2}$. Suppose that the cost function is given by $c(p) = 25(\frac{p}{10})^4$. Then for $b = \frac{1}{20}$ the new equilibrium becomes:* $\{0, 0.1065, 0.4180, 1\}$ *and the precisions of the signals bought by the decision maker are* $p(m_1) = 0.16$, $p(m_2) = 2.7278$ *and* $p(m_3) = 6.9051$. *In this case the ex-ante welfare of the decision maker is* -0.0163 *which is smaller than the welfare in the uninformed case, so again, the access to private information is welfare decreasing to the decision maker.*

[29] The decision maker's utility function can be written as $\tilde{u}^D(y, \theta, t) = t + u(y - \theta)$, where t represents money.

[30] In a close paper Rantakari (2014) studies the problem of information acquisition when the decision maker faces a binary option (choosing one among two projects) each of which valuations is known to one of two different experts. The decision maker can invest in a signal that with some probability perfectly reveals the optimal project. The higher the probability, the higher the cost of the signal is. He shows that the acquisition of information leads to a loss of communication from the experts because coarser messages result in more precise signals. However, in contrast to this paper, the loss of communication would never arise if the same precision of information was chosen after each message.

4.2. Other Preferences

In Section 3, I focused the analysis on the case of quadratic-loss utilities. An intuition of how the results might change if we consider other functional forms for the preferences of the agents is as follows: Consider the following families of utility functions: $u^D(y,\theta) = -|y-\theta|^\rho$, $u^E(y,\theta,b) = -|y-(\theta+b)|^\varrho$ where $\rho, \varrho \geq 1$.[31] One can interpret ρ and ϱ as a measure of risk aversion since they measure the degree of concavity of $u^D(\cdot,\theta)$ and $u^E(\cdot,\theta,b)$ respectively. The case $\rho = \varrho = 2$ is equivalent to the quadratic-loss utilities studied before. The higher the ρ (ϱ) the more risk averse is the decision maker (expert).

In general, when $\varrho \neq 2$, the expected utility of the expert can no longer be written simply in terms of the expectation and the variance of the decision maker's action. However, it is useful to think of the information and risk effect in developing an intuition on these cases. Observe that the actions of the decision maker are completely independent of the preferences of the expert. Hence, if the preferences of the decision maker are fixed, it is enough to see how the preferences of the expert over lotteries change with risk aversion. Intuitively, as ϱ decreases, the expert is more tolerant of risk and the risk effect diminishes. As a result, larger intervals become more attractive, leading to even less communication in equilibrium. In contrast, as the expert's risk aversion increases, the risk effect becomes larger, reducing the impact of the information effect. For high enough risk aversion, it can even be the case that the risk effect outweighs the information effect leading to more communication in equilibrium. A numerical example is depicted below.

Example 2. *Consider an expert with preference parameter $\varrho = 6$ and bias $b = \frac{1}{4}$ who faces a decision maker with quadratic preferences ($\rho = 2$). Further, suppose that the decision maker has access to a Normal signal with $\sigma = 0.5$. Then the new equilibrium partition becomes $\{0, 0.0119, 1\}$. Hence in this case there is more information transmitted in equilibrium than under the uninformed case and obviously the decision maker is better off by the presence of the signal.*

Alternatively, we could fix the preferences of the expert and change the preferences of the decision maker. In this case, however, a change in the preferences of the decision maker changes the lotteries over actions and hence affects indirectly the preferences of the expert. Intuitively, a more risk averse decision maker is less sensitive to her private information because she dislikes the risk associated with the signal; in order for her to choose an action below (above) the middle of the interval, she needs to receive a lower (higher) signal than when she was less risk averse, so that she is more certain that the true state of the world is actually low (high).[32] In fact, an increase in the risk aversion of the decision maker has a similar effect on her actions to a decline in the accuracy of the signal structure. Hence, using the intuition of the comparative statics in Section 3, since the decision maker reacts less to her signal the more risk averse she is, the incentives to exaggerate are reduced and communication increases with the risk aversion of the decision maker. (Although the equilibrium partition is always less informative than the equilibrium in the CS framework.)

For the case where $\rho = \varrho$, namely $u^E(y,\theta,0) = u^D(y,\theta)$[33] my intuition is that although an increase in risk aversion smoothes the communication between the agents, the communication will still be worse compared to the canonical CS[34]. The reason is that, even if the expert is very risk averse, the fact that the decision maker is as risk averse as him implies that the lottery of actions will have little variance (the decision maker will not be very sensitive to her signal), and the information effect will still dominate the risk effect.

To sum up, as the risk aversion of both agents increases the communication between them improves, both because the risk effect of the expert is higher and hence he wants to even up the intervals, and because the decision maker is less sensitive to his signal and hence the information effect is not so strong. This intuition explains the surprising fact that the welfare of two risk averse agents, facing uncertainty, increases with their risk aversion!

[31] These families of utility functions were introduced under this context by Krishna and Morgan (2004).

[32] Notice, that although the conditional signal structure is symmetric, the posterior distribution of the state given a realisation of the signal is not symmetric. If the signal realisation is smaller than the middle of the interval, the posterior distribution is tilted to the left with a bigger tail on the right. A more risk averse decision maker dislikes this bigger tail and hence chooses an action closer to the middle of the interval. The symmetric argument applies to high realisations of the signal.

[33] This assumption was made for Proposition 1 and in the comparative statics study in CS.

[34] Observe that in CS, the risk aversion of the agents does not play any role when the prior distribution is uniform in [0,1] and the preferences are symmetric, as in this model. In fact, the equilibria in the CS are the same with any set of symmetric preferences (i.e. for any $\rho, \varrho \geq 1$).

5. Conclusion

This paper analyses the strategic information transmission from an expert to a decision maker who has access to private information. The private information has two effects on the incentives of the expert to communicate. On the one hand it reduces the implicit cost of being vague, because the decision maker is able to choose better actions. This damages communication (Proposition 3). On the other hand it introduces some risk from the point of view of the expert who is no longer able to correctly forecast the action of the decision maker. This helps communication because the expert wants to reduce the variance of the messages (Proposition 4). Theorem 2 says that for any symmetric signal structure there are always sufficiently biased experts such that the overall effect is that communication is no longer possible in equilibrium. For more general biases, I analyse two families of signals (Normal and Uniform signals) for which a comparative static result can be shown, the communication decreases with the precision of the signal. Finally it is shown through the examples that this loss of communication might lead to an overall loss in welfare so that the decision maker would be better off if she commits not to acquire information.

Appendix A. Appendix

Appendix A.1. Notation

The following notation will be used throughout the Appendix:

$g(\theta|\underline{a}, \overline{a}, s)$ denotes the decision maker's posterior distribution of θ given that she receives message $m = [\underline{a}, \overline{a}]$ and the realisation of the signal is s:

$$g(\theta|\underline{a}, \overline{a}, s) = \frac{f(s-\theta)}{\int_{\underline{a}}^{\overline{a}} f(s-t)dt}$$

$y(\underline{a}, \overline{a}, s)$ denotes the optimal action chosen by the decision maker when she receives message $m = [\underline{a}, \overline{a}]$ and the realisation of the signal is s:

$$y(\underline{a}, \overline{a}, s) = \arg\max_{y} \int_0^1 u(y-\theta)g(\theta|\underline{a}, \overline{a}, s)d\theta$$

$\hat{y}(\underline{a}, \overline{a}, \theta)$ denotes the expected decision maker's action from the point of view of an expert with type θ that sends message $m = [\underline{a}, \overline{a}]$:

$$\hat{y}(\underline{a}, \overline{a}, \theta) = \int_{\mathbb{R}} y(\underline{a}, \overline{a}, s)f(s-\theta)d\theta$$

$\hat{\sigma}^2(\underline{a}, \overline{a}, \theta)$ denotes the variance of the decision maker's actions from the point of view of an expert with type θ that sends message $m = [\underline{a}, \overline{a}]$:

$$\hat{\sigma}^2(\underline{a}, \overline{a}, \theta) = \int_{\mathbb{R}} (y(\underline{a}, \overline{a}, s) - \hat{y}(\underline{a}, \overline{a}, \theta))^2 f(s-\theta)d\theta$$

$U^E(\underline{a}, \overline{a}, \theta)$ denotes the expected utility of an expert with type θ that sends message $m = [\underline{a}, \overline{a}]$.

$V(a_{i-1}, a_i, a_{i+1}) = U^E(a_i, a_{i+1}, a_i) - U^E(a_{i-1}, a_i, a_i)$ denotes the difference in expected utility to the expert with type a_i between sending $m_{i+1} = [a_i, a_{i+1}]$ and $m_i = [a_{i-1}, a_i]$ In particular, the indifference condition (A_f) can be written as $V(a_{i-1}, a_i, a_{i+1}) = 0$.

All the previous functions will be indexed by the signal structure f whenever is necessary for clarity. Also, if needed, I will highlight the bias of the expert by explicitly adding the argument b to $U^E(\cdot)$ and $V(\cdot)$.

Finally, I will denote with subscripts the partial derivatives of a function.

Appendix A.2. Proof of Theorem 1

We first show that there is at most a finite number of distinct messages (messages inducing different distribution over θ), in a monotone partition equilibrium.

Lemma 1. *Given $b > 0$, the number of intervals sent in a partition equilibrium is finite. In particular, there is no separating equilibrium in the private information model.*

Proof of Lemma 1 The lemma follows as an immediate corollary of Lemma 2 and the fact that $[0, 1]$ is bounded. □

Lemma 2. *If $b > 0$ and $m = [\underline{a}, \overline{a}]$ is a message sent in a partition equilibrium with $\underline{a} > 0$, then $\overline{a} - \underline{a} \geq 2b$.*

Proof of Lemma 2: Suppose by way of contradiction that we could find a partition equilibrium in which message $m = [\underline{a}, \overline{a}]$ with $\underline{a} > 0$ and $\overline{a} - \underline{a} < 2b$ was sent. Then in particular $|\overline{a} - (\underline{a} + b)| < b = (\underline{a} + b) - \underline{a}$ implying that an expert with type $\theta = \underline{a}$ strictly prefers action $y = \overline{a}$ to action $y' = \underline{a}$. By continuity of preferences, there exists $\epsilon > 0$ such that $\underline{a} - \epsilon > 0$ and an expert with type $\theta' = \underline{a} - \epsilon$ strictly prefers $y = \overline{a}$ to $y' = \underline{a}$. Hence, by the concavity of the expert's preferences, all the actions $y(\underline{a}, \overline{a}, s)$, $s \in \mathbb{R}$ are preferred to $y' = \underline{a}$, which implies that type θ' strictly prefers message m to any interval message $m' \subset [0, \underline{a}]$, contradicting the view that m belongs to a partition equilibrium. □

Before moving to the proof of the theorem, I derive some previous results which will be used in the proof. Lemma 3 establishes some monotonicity properties of the decision maker's best action:

Lemma 3. *Given a message $m = [\underline{a}, \overline{a}]$, $y(\underline{a}, \overline{a}, s)$ is increasing in all its arguments and $\underline{a} \leq y(\underline{a}, \overline{a}, s) \leq \overline{a}$ for all $s \in \mathbb{R}$*

Proof of Lemma 3: $y(\underline{a}, \overline{a}, s)$ solves the first order condition:

$$\int_{\underline{a}}^{\overline{a}} u_1(y(\underline{a}, \overline{a}, s) - \theta) f(s - \theta) d\theta = 0 \tag{A.1}$$

Since $u_{11}(\cdot) < 0$ and $f(\cdot) \geq 0$, there exists a $\bar{\theta} \in (\underline{a}, \overline{a})$ such that $u_1(y(\underline{a}, \overline{a}, s) - \bar{\theta}) = 0$ and therefore:

$$u_1(y(\underline{a}, \overline{a}, s) - \underline{a}) < 0 \quad and \quad u_1(y(\underline{a}, \overline{a}, s) - \overline{a}) > 0 \tag{A.2}$$

Differentiating (A.1) with respect to its first argument and rearranging:

$$y_1(\underline{a}, \overline{a}, s) = \frac{u_1(y(\underline{a}, \overline{a}, s) - \underline{a}) f(s - \underline{a})}{\int_{\underline{a}}^{\overline{a}} u_{11}(y(\underline{a}, \overline{a}, s) - \theta) f(s - \theta) d\theta} > 0$$

where the inequality follows by (A.2) and $u_{11}(\cdot) < 0$. Analogously, differentiating (A.1) with respect to its second argument:

$$y_2(\underline{a}, \overline{a}, s) = -\frac{u_1(y(\underline{a}, \overline{a}, s) - \overline{a}) f(s - \overline{a})}{\int_{\underline{a}}^{\overline{a}} u_{11}(y(\underline{a}, \overline{a}, s) - \theta) f(s - \theta) d\theta} > 0$$

To show that $y(\underline{a}, \overline{a}, s)$ is increasing in s it is sufficient to prove that $U(y, s) = \int_{\underline{a}}^{\overline{a}} u(y - \theta) f(s - \theta) d\theta$ is supermodular in (y, s) (see Athey (2002)). Given $y' > y$, $U(y', s) - U(y, s) = \int_{\underline{a}}^{\overline{a}} (u(y' - \theta) - u(y - \theta)) f(s - \theta) d\theta$ which is increasing in s because $u(y' - \theta) - u(y - \theta)$ is increasing in θ by $u_{11}(\cdot) < 0$, and $f(s - \theta)$ is ordered in the FOSD (Milgrom, 1981). Therefore $U(y, s)$ is supermodular in (y, s) and $y(\underline{a}, \overline{a}, s)$ is increasing in s.
Finally, (A.2) and $u_{11}(\cdot) < 0$ imply that $\underline{a} \leq y(\underline{a}, \overline{a}, s) \leq \overline{a}$ for all $s \in \mathbb{R}$. □

Lemma 4 shows that given any monotone partition strategy $a_0 = 0 < a_1 < ... < a_N = 1$, $U^E(m_i, \theta)$ inherits the supermodulatiry of $u^E(y, \theta)$.

Lemma 4. *Consider a monotone partition strategy defined by $0 = a_0 < a_1 < ... < a_N = 1$, and denote by $m_i = [a_{i-1}, a_i]$, then $U^E(m_i, \theta)$ is supermodular in (i, θ).*

Proof of Lemma 4: By Lemma 3, $y(m_i, s) \equiv y(a_{i-1}, a_i, s)$ is increasing in i. Given that $u^E(y, \theta)$ is supermodular in (y, θ), $u^E(y(m_i, s), \theta)$ is supermodular in (i, θ) for all s. Since θ and s are affiliated, then $U^E(m_i, \theta) = \int_{\mathbb{R}} u^E(y(m_i, s), \theta) f(s - \theta) ds$ is supermodular in (i, θ). (See Athey (2002)). □

Lemma 5 shows that, given the symmetry of the setup, the decision maker's best response is completely determined by the length and the initial point of the interval sent and it is symmetric with respect to the mid-point of the interval.

Lemma 5. *Consider $0 \leq h \leq 1$ and $0 \leq a \leq 1 - h$. If $u(\cdot)$ and $f(\cdot)$ are symmetric:*

1. $Pr(\theta \in [a, a + h)|s) = Pr(\theta \in [0, h)|s - a)$.
2. $Pr(\theta \in [0, h)|\frac{h}{2} - s) = Pr(\theta \in [0, h)|\frac{h}{2} + s))$.
3. $g(\frac{h}{2} - \theta|0, h, \frac{h}{2} - s) = g(\frac{h}{2} + \theta|0, h, \frac{h}{2} + s)$.
4. $y(a, a + h, s) = a + y(0, h, s - a)$.
5. $y(0, h, \frac{h}{2} + s) - \frac{h}{2} = \frac{h}{2} - y(0, h, \frac{h}{2} - s)$. *In particular* $y(0, h, \frac{h}{2}) = \frac{h}{2}$.

Proof of Lemma 5: All the results are immediate implications of the symmetry of the functions and a change in variable.
1. $Pr(\theta \in [a, a + h)|s) = \int_a^{a+h} f(s - \theta) d\theta = \int_0^h f(s - a - \theta) d\theta = Pr(\theta \in [0, h)|s - a)$.
2. $Pr(\theta \in [0, h)|\frac{h}{2} - s) = \int_0^h f(\frac{h}{2} - s - \theta) d\theta = \int_0^h f(h - \theta - (\frac{h}{2} + s)) = \int_0^h f(\theta - (\frac{h}{2} + s)) d\theta = Pr(\theta \in [0, h)|\frac{h}{2} + s))$.
3. $g(\frac{h}{2} - \theta|0, h, \frac{h}{2} - s) = \frac{f(s-\theta)}{Pr(\theta \in [0,h)|\frac{h}{2} - s)} = \frac{f(\theta - s)}{Pr(\theta \in [0,h)|\frac{h}{2} + s)} = g(\frac{h}{2} + \theta|0, h, \frac{h}{2} + s)$.

4. $0 = \int_a^{a+h} u_1(y(a, a+h, s) - \theta) f(\theta - s) d\theta = \int_0^h u_1(y(a, a+h, s) - a - \theta) f(\theta - (s-a)) d\theta$ therefore $y(a, a+h, s) - a$ solves $\int_0^h u_1(y-\theta) f(\theta - (s-a)) d\theta = 0$ which implies that $y(0, h, s-a) = y(a, a+h, s) - a$.
5. $0 = \int_0^h u_1(y(0, h, \frac{h}{2} + s) - \theta) f(\theta - (\frac{h}{2} + s)) d\theta = \int_0^h -u_1(h - y(0, h, \frac{h}{2}) - \theta) f(\frac{h}{2} - s - \theta) d\theta$ and therefore $y(0, h, \frac{h}{2} - s) = h - y(0, h, \frac{h}{2})$.
Finally, using this equation for $s = 0$, $y(0, h, \frac{h}{2}) = \frac{h}{2}$. □

Lemma 6 derives some properties of the function $V(\cdot)$ that will be used in the proof of Proposition 6 and Proposition 1.

Lemma 6. *If $0 \leq a_{i-1} < a_i < a_{i+1} \leq 1$ and $V(a_{i-1}, a_i, a_{i+1}) = 0$, then $U_1^E(a, a_i, a_i) > 0$ and $V_1(a, a_i, a_{i+1}) < 0$ for all $a \in [0, a_i)$, $U_2^E(a_i, a, a_i) < 0$ and $V_3(a_{i-1}, a_i, a) < 0$ for all $a \in [a_{i+1}, 1]$, and $V(a_{i-1}, a_i, a) > 0$ for all $a \in [a_i, a_{i+1})$.*

Proof of Lemma 6: By the concavity of $u(\cdot)$ and Lemma 3,

$$U_1^E(a, a_i, a_i) = \int_{\mathbb{R}} u_1(y(a, a_i, s) - (a_i + b)) \frac{\partial y}{\partial a}(a, a_i, s) f(s - a_i) ds > 0 \tag{A.3}$$

so $U^E(a, a_i, a_i)$ is strictly increasing in a for all $a \leq a_i$ and hence $V_1(a, a_i, a_{i+1}) < 0$ for all $a \in [0, a_i]$. Assumption A and the fact that $V(a_{i-1}, a_i, a_i) > 0$ (which follows by $U_1^E(a, a_i, a_i) < 0$ for all $a \in [0, a_i]$ and $V(a_{i-1}, a_i, a_{i+1}) = 0$) entails that $U_2^E(a_i, a, a_i) < 0$ and $V_3(a_{i-1}, a_i, a) < 0$ for all $a \in [a_{i+1}, 1]$, and $V(a_{i-1}, a_i, a) > 0$ for all $a \in [a_i, a_{i+1})$. □

The following proposition uses Assumption A to prove a stronger version of the monotonicity condition (M) in CS which in particular ensures that there is at most one partition of size N satisfying (A_f).

Proposition 6. *If $\hat{a}_0 < \hat{a}_1 < ... < \hat{a}_N$ and $\tilde{a}_0 < \tilde{a}_1 < ... < \tilde{a}_N$ are two partial partitions satisfying (A_f) with $\hat{a}_0 = \tilde{a}_0$ and $\hat{a}_1 > \tilde{a}_1$, then $\hat{a}_i - \hat{a}_{i-1} > \tilde{a}_i - \tilde{a}_{i-1}$ for all $i \geq 1$.*

Proof of Proposition 6: Denote by $h_{i+1} = a_{i+1} - a_i$ and $h_i = a_i - a_{i-1}$. By Lemma 5, for all $a \in [h_i, 1 - h_{i+1}]$, $U^E(a_i, a_{i+1}, a_i) = U^E(a, a+h_{i+1}, a)$ and $U^E(a_{i-1}, a_i, a_i) = U^E(a-h_i, a, a)$. Hence $V(a_{i-1}, a_i, a_{i+1})$ is a function only of the length of the intervals h_i and h_{i+1} and not of the location of the intervals. Denote this function as $\tilde{V}(h_i, h_{i+1})$. Given h, define $\phi(h)$ as the positive number, if one exists, which solves $\tilde{V}(h, \phi(h)) = 0$. (If this equation does not have a solution then I will consider that $\phi(h) = +\infty$). By Assumption A, there is at most one solution to this equation and therefore $\phi(h)$ is a well defined function of h. Proposition 6 is then reduced to prove that $\phi(h)$ is increasing in h. Totally differentiating $\tilde{V}(h, \phi(h)) = 0$ with respect to h we have:

$$\phi'(h) = \frac{-U_1^E(a-h, a, a)}{U_2^E(a, a+\phi(h), a)} > 0$$

where the inequality follows by Lemma 6. □

Proof of Theorem 1: The proof of the first statement of the theorem follows the proof of Theorem 1 of CS using the function $U^E(m, \theta) = \int_{\mathbb{R}} u^E(y(m, s), \theta) f(s - \theta) ds$ instead of $u^E(y(m), \theta)$. I reproduce the proof here for completeness. I start by proving that there exists an integer $N_f(b)$, such that for every N, $1 \leq N \leq N_f(b)$, there exists a partition of size N satisfying the arbitrage condition (A_f).

First, note that, by equation A.3, $U^E(a, a_i, a_i)$ is strictly increasing in a. Consider a strictly decreasing partial partition $a_0 > a_1 > ... > a_i$ which satisfies (A_f). By the monotonicity of $U^E(a, a_i, a_i)$, there can at most be one value $a_{i+1} < a_i$ satisfying (A_f).[35]
Define $K(a) \equiv \max\{i :$ there exists $0 \leq a_i < .... < a_2 < a < 1$ satisfying $(A_f)\}$. By Lemma 2, $K(a)$ is finite, well defined and uniformly bounded. Hence $sup_{a\in[0,1)} K(a) \in [0, 1)$ is achieved at a point $\bar{a}$ and $N_f(b) < \infty$. It remains to be proven that for each $1 \leq N \leq N_f(b)$ there is a partition a satisfying (A_f). Denote $a^{K(a)}$ the decreasing partial partition of size $K(a)$ satisfying (A_f) and such that $a_1^{K(a)} = a$. The partition changes continuously with a and therefore $K(a)$ is locally constant and can at most change by one at a discontinuity. Finally $K(0) = 1$, so $K(a)$ takes on all integer values between one and $N_f(b)$.

[35]In CS the authors use a symmetric argument with strictly increasing partial partitions. The reason I use decreasing partitions is that, given that $b > 0$ the expected utility of an expert of type t_i when he sends message $m = [t, t_i]$ strictly decreases as t decreases. For increasing partitions, the monotonicity is harder to prove because there are actions on both sides of the expert's peak. (This is the role of Assumption A2, although it is not necessary for this stage.).

Now, given Lemma 4, $U^E(m_i, \theta)$ is supermodular in (i, θ) and hence the arbitrage condition (A_f) is also sufficient for the equilibrium. Note that to show this first statement I have only used that (θ, s) are affiliated, $f(s \mid \theta)$ is log-concave and $u(y, \theta)$ is supermodular in (y, θ) and concave in y. Hence existence of monotone partition equilibria follows without the symmetry and uniformity assumptions.

For the second statement of Theorem 1, let a be a partition which supports an equilibrium, and let $h_i = a_i - a_{i-1}$ and $h_{i+1} = a_{i+1} - a_i$. Suppose that $h_{i+1} \leq h_i$, then for all $s \in \mathbb{R}$, $y(a_i, a_{i+1}, a_i + s) - a_i \leq y(a_i, a_i + h_i, a_i + s) - a_i = a_i - y(a_i - h_i, a_i, a_i - s) = a_i - y(a_{i-1}, a_i, a_i - s)$, where the inequality follows because $h_{i+1} \leq h_i$ and Lemma 3 and the equality follows by Lemma 5. But then if $b > 0$, an expert with type a_i strictly prefers $y(a_i, a_{i+1}, a_i + s)$ to $y(a_{i-1}, a_i, a_i - s)$ for all s, and since $f((a_i + s) - a_i) = f((a_i - s) + a_i)$ by the symmetry of $f(\cdot)$, $U^E(m_{i+1}, a_i) > U^E(m_i, a_i)$, contradicting the equilibrium condition.

Finally, the proof of the third statement of the theorem mimics the proofs of Theorem 3 and 5 of CS using Proposition 6 in the place of condition (M). □

Appendix A.3. Proof of Proposition 1

Proof of Proposition 1: Suppose that for any $0 \leq a_{i-1} \leq a_i \leq a_{i+1} \leq 1$ such that $V_f(a_{i-1}, a_i, a_{i+1}) = 0$, we have $V_{f'}(a_{i-1}, a_i, a_{i+1}) > 0$.[36] First I prove that if $a(K)$ and $a'(K)$ are two partial partitions of size K satisfying (A_f) and $(A_{f'})$ respectively, with $a_0(K) = a'_0(K)$ and $a_K(K) = a'_K(K)$ then $a_i(K) > a'_i(K)$.

The proof is made by induction on the size of the partition K. If $K = 1$ the statement is vacuous. Suppose $K > 1$ and the statement is true for all $K' = 1, .., K - 1$. Suppose by way of contradiction that $a_j(K) \leq a'_j(K)$ for some $j = 1, ..., K - 1$. Suppose further that j is the highest index for which this inequality is satisfied and hence $a_i(K) > a'_i(K)$ for all $j < i < K$. Define ${}^x a \equiv ({}^x a_0, {}^x a_1, ..., {}^x a_j)$ the partial partition which satisfies (A_f) such that ${}^x a_0 = a_0(K)$ and ${}^x a_1 = x$. By definition ${}^{a_1(K)} a_j = a_j(K) \leq a'_j(K)$. By the continuity of ${}^x a$ in x there exists an $\bar{x} \geq a_1(K)$ such that ${}^{\bar{x}} a_j = a'_j(K)$ and by Proposition 6, ${}^{\bar{x}} a_i \geq a_i(K)$ for all $0 < i < j$. Denote by $\bar{a} \equiv {}^{\bar{x}} a$. By Assumption A, there exists a unique $\bar{a}_{j+1} > \bar{a}_j$ such that $V_f(\bar{a}_{j-1}, \bar{a}_j, \bar{a}_{j+1}) = 0$. By the condition of the Proposition, $V_{f'}(\bar{a}_{j-1}, \bar{a}_j, \bar{a}_{j+1}) > 0$. By Proposition 6 $\bar{a}_{j+1} \geq a_{j+1}(K) > a'_{j+1}(K)$, and hence using the fact that $\bar{a}_j = a'_j(K)$ and Lemma 6:

$$V_{f'}(\bar{a}_{j-1}, a'_j(K), a'_{j+1}(K)) > 0 \tag{A.4}$$

At the same time, applying the induction hypothesis to $(\bar{a}_0, ..., \bar{a}_j)$ and $(a'_0(K), ..., a'_j(K))$, $a'_i(K) < \bar{a}_i$ for all $0 < i < j$. But then using Lemma 6,

$$V_{f'}(\bar{a}_{j-1}, a'_j(K), a'_{j+1}(K)) < V_{f'}(a'_{j-1}(K), a'_j(K), a'_{j+1}(K)) = 0$$

which contradicts (A.4) and establishes the result.

Finally, let $a'(N_{f'}(b))$ be the partition equilibrium of f' private information model of size $N_{f'}(b)$. Let $\bar{a}$ be the partial partition satisfying (A_f) such that $\bar{a}_1 = a'_1(N_{f'}(b))$, then by Proposition 6 and the previous result, $\bar{a}_i < a'_i(N_{f'}(b))$. In particular, $\bar{a}$ is at least of length $N_{f'}(b)$. Hence $N_f(b) \geq N_{f'}(b)$ □

Appendix A.4. Proof of Proposition 2

Lemma 7 states that an expert with type $\theta = a$, sending message $[a, a + h]$ with $h \geq 4b$, strictly prefers the lottery induced by the decision maker, than the action $y = a + \frac{h}{2}$ for sure.

Lemma 7. *If $h \geq 4b$, $U^E(a, a + h, a) > -(a + \frac{h}{2} - (a + b))^2$*

Proof of Lemma 7: By Lemma 5 it is enough to prove it for $a = 0$.

$$\begin{aligned}
&U^E(0, h, 0) + (\tfrac{h}{2} - b)^2 = \textstyle\int_{\mathbb{R}} [-(y(0, h, s) - b)^2 + (\tfrac{h}{2} - b)^2] f(s) ds \\
&= \textstyle\int_{\mathbb{R}} (\tfrac{h}{2} - y(0, h, s))(\tfrac{h}{2} - 2b + y(0, h, s)) f(s) ds \\
&= \textstyle\int_{s>0} \left[(\tfrac{h}{2} - y(0, h, \tfrac{h}{2} - s))(\tfrac{h}{2} - 2b + y(0, h, \tfrac{h}{2} - s)) f(\tfrac{h}{2} - s) + \right. \\
&\qquad \left. (\tfrac{h}{2} - y(0, h, \tfrac{h}{2} + s))(\tfrac{h}{2} - 2b + y(0, h, \tfrac{h}{2} + s)) f(\tfrac{h}{2} + s) \right] ds \\
&= \textstyle\int_{s>0} ((\tfrac{h}{2} - y(0, h, \tfrac{h}{2} - s)) \left[(\tfrac{h}{2} - 2b + y(0, h, \tfrac{h}{2} - s)) f(\tfrac{h}{2} - s) - (\tfrac{h}{2} - 2b + y(0, h, \tfrac{h}{2} + s)) f(\tfrac{h}{2} + s) \right] ds
\end{aligned}$$

where the third equality follows by dividing the signal space at $\frac{h}{2}$, and the last equality follows by the symmetric properties of the functions (see Lemma 5). The first factor in the last integral is always positive,

[36] The opposite case is symmetric.

so it is sufficient to show that the second factor is positive as well. Recall that for quadratic-loss preferences $y(0,h,s) = \int_0^h \theta g(\theta|0,h,s)d\theta$. This together with the fact that $g(h|0,h,\frac{h}{2}+s) = \frac{f(\frac{h}{2}-s)}{\int_0^h f(t-(\frac{h}{2}+s))dt}$, $g(h|0,h,\frac{h}{2}-s) = \frac{f(\frac{h}{2}+s)}{\int_0^h f(t-(\frac{h}{2}-s))}$ and $\int_0^h f(t-(\frac{h}{2}+s))dt = \int_0^h f(t-(\frac{h}{2}-s))dt$, imply that:

$$\begin{aligned}
&(\tfrac{h}{2} - 2b + y(0,h,\tfrac{h}{2}-s))f(\tfrac{h}{2}-s) - (\tfrac{h}{2} - 2b + y(0,h,\tfrac{h}{2}+s))f(\tfrac{h}{2}+s)\\
&= \int_0^h f(t-\tfrac{h}{2}+s)dt \left(\int_0^h (\theta + \tfrac{h}{2} - 2b)\left[g(\theta|0,h,\tfrac{h}{2}-s)g(h|0,h,\tfrac{h}{2}+s) - g(\theta|0,h,\tfrac{h}{2}+s)g(h|0,h,\tfrac{h}{2}-s)\right]d\theta\right)\\
&> 0
\end{aligned}$$

where the inequality follows because $h \geq 4b$ (so $\theta + \frac{h}{2} - 2b \geq 0$) and the fact that s and θ are affiliated. □

Using a similar argument it can be shown that the expert with type $\theta = a+h$ (the upper bound of the interval), sending message $[a, a+h]$, strictly prefers the lottery induced by the decision maker, than the action $y = a + \frac{h}{2}$ for sure (this is true for any length h).

Lemma 8. $U^E(a, a+h, a+h) > -(a + \frac{h}{2} - (a+h+b))^2$

Proof of Lemma 8: By Lemma 5 it is enough to prove it for $a = 0$.

$$\begin{aligned}
&U^E(0,h,h) + (\tfrac{h}{2}+b)^2 = \int_{\mathbb{R}}[-(y(0,h,s)-(h+b))^2 + (\tfrac{h}{2}+b)^2]f(s-h)ds\\
&= \int_{\mathbb{R}}(y(0,h,s) - \tfrac{h}{2})(\tfrac{3h}{2} + 2b - y(0,h,s))f(s-h)ds\\
&= \int_{s>0}\left[(y(0,h,\tfrac{h}{2}+s) - \tfrac{h}{2})(\tfrac{3h}{2} + 2b - y(0,h,\tfrac{h}{2}+s))f(s-\tfrac{h}{2}) + \right.\\
&\qquad \left.(y(0,h,\tfrac{h}{2}-s) - \tfrac{h}{2})(\tfrac{3h}{2} + 2b - y(0,h,\tfrac{h}{2}-s))f(s+\tfrac{h}{2})\right]ds\\
&= \int_{s>0}(y(0,h,\tfrac{h}{2}+s) - \tfrac{h}{2})\left[(\tfrac{3h}{2} + 2b - y(0,h,\tfrac{h}{2}+s))f(s-\tfrac{h}{2}) - (\tfrac{3h}{2} + 2b - y(0,h,\tfrac{h}{2}-s))f(s+\tfrac{h}{2})\right]ds
\end{aligned}$$

where the third equality follows by dividing the signal space at $\frac{h}{2}$, and the last equality follows by Lemma 5. The first factor in the integral is always positive, so it is sufficient to show that the second factor is positive as well. Using the same argument as above:

$$\begin{aligned}
&(\tfrac{3h}{2} + 2b - y(0,h,\tfrac{h}{2}+s))f(\tfrac{h}{2}-s) - (\tfrac{3h}{2} + 2b - y(0,h,\tfrac{h}{2}-s))f(\tfrac{h}{2}+s)\\
&= \int_0^h f(t-\tfrac{h}{2}+s)dt \left(\int_0^h (\tfrac{3h}{2} + 2b - \theta)\left[g(\theta|0,h,\tfrac{h}{2}+s)g(h|0,h,\tfrac{h}{2}+s) - g(\theta|0,h,\tfrac{h}{2}-s)g(h|0,h,\tfrac{h}{2}-s)\right]d\theta\right)\\
&> 0
\end{aligned}$$

where the inequality follows because $\frac{3h}{2} + 2b - \theta > 0$ always and s and θ are affiliated. □

Proof of Proposition 2: It follows as an immediate corollary of Lemma 7 and Lemma 8 given that whenever $0 \leq a_{i-1} \leq a_i < a_{i+1} \leq 1$ satisfy (A_{CS}) then $a_{i+1} - a_i \geq 4b$. □

Appendix A.5. Proof of Theorem 2

Proof of Theorem 2: To prove the first statement of the Theorem, observe that Lemma 7 implies that $V_f(0,0,4b) > 0$, that is, an expert with type $\theta = 0$ strictly prefers to send message $[0, 4b]$ to perfectly reveal that the state is 0. Note that the function V_f depends implicitly on b in a continuous way. With some abuse of notation, consider b as a fourth argument of V_f. Since for $b = \frac{1}{4}$, $V_f(0,0,1,\frac{1}{4}) > 0$ and $V_f(\cdot)$ is continuous in b, there exists a $\bar{b} < \frac{1}{4}$ such that $V_f(0,0,1,b) > 0$ for all $b \in (\bar{b}, \frac{1}{4}]$. By Lemma 6 $V_f(0,0,a,b) > 0$ for all $a \in [0,1]$, and by Proposition 1 there is no information transmitted in equilibrium.

Finally, for the second statement suppose that the conditional distribution of the signal belongs to a parametrised family $\{f^\lambda(\cdot|\theta), \lambda \in (0,\infty)\}$, where λ represents the precision[37] of the signal, and such that in the limit, when $\lambda \to \infty$, it corresponds to the degenerate distribution in θ. Then the second statement follows by the fact that as $\lambda \to \infty$ the conditional distribution $G(\theta|s)$ converges to the degenerate distribution on s. And hence, there is a precision λ^b, such that the lottery induced by message $[0,1]$ is preferred by the expert with type $\theta = 0$ and bias b to the constant action $y = 0$. Namely, $V_{f^\lambda}(0,0,1,b) > 0$, and by Lemma 6, $V_{f^\lambda}(0,0,a,b) > 0$ for all $a \in [0,1]$, so there can be no information transmitted in equilibrium. □

[37] One signal is more precise than another if the latter is a mean preserving spread of the former.

Appendix A.6. Proof of Proposition 3

The following results are used for the proof of Proposition 3. Lemma 9 transfers the symmetric properties of the best response established in Lemma 5 to the expected best response.

Lemma 9. *Given a message $m = [\underline{a}, \overline{a})$, and a type θ, the expected action of the decision maker $\hat{y}(\underline{a}, \overline{a}, \theta)$ satisfies the following properties:*

1. *$\hat{y}(\underline{a}, \overline{a}, \theta)$ is increasing in all its arguments and $\underline{a} < \hat{y}(\underline{a}, \overline{a}, \theta) < \overline{a}$*
2. *$\hat{y}(a, a+h, \theta) = a + \hat{y}(0, h, \theta - a)$.*
3. *$\hat{y}(0, h, \frac{h}{2} + \theta) - \frac{h}{2} = \frac{h}{2} - \hat{y}(0, h, \frac{h}{2} - \theta)$. In particular $\hat{y}(0, h, \frac{h}{2}) = \frac{h}{2}$.*

Proof of Lemma 9: All the results are immediate implications of Lemma 5, Lemma 3 and a change in variable.
1. It is a direct implication of Lemma 3 and the fact that s and θ are affiliated.
2. $\hat{y}(a, a+h, \theta) = \int_{\mathbb{R}} y(a, a+h, s) f(s-\theta) ds = \int_{\mathbb{R}} a + y(0, h, s-a) f(s-\theta) ds =$
$= a + \int_{\mathbb{R}} y(0, h, s) f(s - (\theta - a)) ds = a + \hat{y}(0, h, \theta - a)$.
3. $\hat{y}(0, h, \frac{h}{2} + \theta) - \frac{h}{2} = \int_{\mathbb{R}} y(0, h, s) f(s - (\frac{h}{2} + \theta)) ds - \frac{h}{2} =$
$= \int_{\mathbb{R}} h - y(0, h, h-s) f(s - (\frac{h}{2} + \theta)) ds - \frac{h}{2} =$
$= \frac{h}{2} - \int_{\mathbb{R}} y(0, h, s) f(\frac{h}{2} - \theta - s) ds = \frac{h}{2} - \hat{y}(0, h, \frac{h}{2} - \theta)$.
Finally using this equation for $\theta = 0$, $\hat{y}(0, h, \frac{h}{2}) = \frac{h}{2}$. □

Lemma 10 is the key result for Proposition 3. It states that as the length of the interval increases, the distance between the CS action and the expected action from the point of view of the boundary type increases.

Lemma 10. $\frac{\partial}{\partial h}(h/2 - \hat{y}(0, h, 0)) > 0$

Proof of Lemma 10: By Lemma 9.3, $\hat{y}(0, h, \theta) + \hat{y}(0, h, h - \theta) = h$. Totally differentiating this equation with respect to h:

$$\hat{y}_2(0, h, \theta) + \hat{y}_2(0, h, h - \theta) + \hat{y}_3(0, h, h - \theta) = 1$$

where all the terms on the left hand side are positive by Lemma 9.1. It is therefore enough to show that if $\theta < h/2$ then $\hat{y}_2(0, h, \theta) \leq \hat{y}_2(0, h, h - \theta)$ since this would imply that $\hat{y}_2(0, h, \theta) < 1/2$ for all $\theta < h/2$, and in particular that $h/2 - \hat{y}(0, h, 0)$ is increasing in h.
First note that given quadratic loss utilities, $y(0, h, s) = \int_0^h \theta \frac{f(\theta - s)}{\int_0^h f(t-s)dt} d\theta$, and therefore:

$$y_2(0, h, s) = \int_0^h (h - \theta) \frac{f(h-s) f(\theta - s)}{(\int_0^h f(t-s)dt)^2} d\theta = \int_0^h (h - \theta) g(h|0, h, s) g(\theta|0, h, s) d\theta$$

and therefore if $s > 0$:

$$\begin{aligned} y_2(0, h, \tfrac{h}{2} + s) - y_2(0, h, \tfrac{h}{2} - s) &= \int_0^h (h - \theta) \big[g(h|0, h, \tfrac{h}{2} + s) g(\theta|0, h, \tfrac{h}{2} + s) \\ &\quad - g(h|0, h, \tfrac{h}{2} - s) g(\theta|0, h, \tfrac{h}{2} - s) \big] d\theta \\ &= \int_0^h (h - \theta) \big[g(h|0, h, \tfrac{h}{2} + s) g(h - \theta|0, h, \tfrac{h}{2} - s) \\ &\quad - g(h|0, h, \tfrac{h}{2} - s) g(h - \theta|0, h, \tfrac{h}{2} + s) \big] d\theta \\ &> 0 \end{aligned} \tag{A.5}$$

where the equality follows by Lemma 5.3 and the inequality follows because $g(\theta|\cdot, s)$ is log-supermodular in (θ, s) (recall that θ and s are affiliated).
Finally, if $\theta < \frac{h}{2}$,

$$\begin{aligned} &\hat{y}_2(0, h, \theta) - \hat{y}_2(0, h, h - \theta) = \int_{\mathbb{R}} y_2(0, h, s)(f(s - \theta) - f(s - h + \theta)) ds \\ &= \int_{s > 0} \left(y_2(0, h, \tfrac{h}{2} + s) - y_2(0, h, \tfrac{h}{2} - s) \right) \left(f(\tfrac{h}{2} + s - \theta) - f(\tfrac{h}{2} + s - (h - \theta)) \right) ds \leq 0 \end{aligned}$$

where the second equality follows by dividing the signal space at $h/2$, and the inequality follows because the first term is always positive by (A.5) and the second is negative whenever $\theta < \frac{h}{2}$. □

Proof of Proposition 3: If $\hat{y}(m_{i+1}, a_i) \leq a_i + b$, then by Lemma 9.1 $a_i < \hat{y}(m_{i+1}, a_i) \leq a_i + b$ and $\hat{y}(m_i, a_i) < a_i$. So clearly $(\hat{y}(m_{i+1}, a_i) - (a_i + b))^2 \leq b^2 < (\hat{y}(m_i, a_i) - (a_i + b))^2$. This together with the fact that $a_i + b$ is equidistant to $y_{CS}(a_{i-1}, a_i)$ and $y_{CS}(a_i, a_{i+1})$, implies that the information effect for message m_{i+1} is greater than for message m_i.

Suppose now that $\hat{y}(m_{i+1}, a_i) > a_i + b$. In this case comparing the distance between the expected actions and the expert's peak is equivalent to comparing the distance between the expected actions and the respective CS actions. The bigger the distance between the expected action and the CS action, the closer is the expected action to the expert's peak and hence the bigger is the information effect.

Using Lemma 9.2 and 9.3, the distance between the expected actions and the CS actions can be written as a function which depends only on the length of the intervals:

$$\begin{aligned} y_{CS}(m_{i+1}) - \hat{y}(a_i, a_{i+1}, a_i) &= \tfrac{a_i + a_{i+1}}{2} - \hat{y}(a_i, a_{i+1}, a_i) = \tfrac{h_{i+1}}{2} - \hat{y}(0, \tfrac{h_{i+1}}{2}, 0) \\ \hat{y}(a_{i-1}, a_i, a_i) - y_{CS}(m_i) &= \hat{y}(a_{i-1}, a_i, a_i) - \tfrac{a_{i-1} + a_i}{2} = \hat{y}(0, h_i, h_i) - \tfrac{h_i}{2} = \tfrac{h_i}{2} - \hat{y}(0, \tfrac{h_i}{2}, 0) \end{aligned} \tag{A.6}$$

where $h_{i+1} = a_{i+1} - a_1$ and $h_i = a_i - a_{i-1}$.

Since $h_{i+1} > h_i$, then to conclude that the information effect for message m_{i+1} is greater than for message m_i it is enough to show that $\frac{h}{2} - \hat{y}(0, h, 0)$ increases with h, which is proved in Lemma 10. □

Appendix A.7. Proof of Proposition 4

Lemma 11 is used in the proof of Proposition 4. It establishes some useful symmetric properties to the variance of the decision maker's actions:

Lemma 11. *Given a message $m = [\underline{a}, \overline{a})$, and a type θ, the variance of the actions of the decision maker $\hat{\sigma}^2(\underline{a}, \overline{a}, \theta)$ satisfies the following properties:*

1. $\hat{\sigma}^2(a, a+h, \theta) = \hat{\sigma}^2(0, h, \theta - a)$.
2. $\hat{\sigma}^2(0, h, \frac{h}{2} + \theta) = \hat{\sigma}^2(0, h, \frac{h}{2} - \theta)$.

Proof of Lemma 11: All the results are immediate implications of Lemma 5, Lemma 9 and a change in variable.

1. $\hat{\sigma}^2(a, a+h, \theta) = \int_{\mathbb{R}} (y(a, a+h, s) - \hat{y}(a, a+h, \theta))^2 f(s - \theta) ds =$
$\int_{\mathbb{R}} (y(0, h, s - a) - \hat{y}(0, h, \theta - a))^2 f(s - \theta) ds =$
$\int_{\mathbb{R}} (y(0, h, s) - \hat{y}(0, h, \theta))^2 f(s - (\theta - a)) ds = \hat{\sigma}^2(0, h, \theta - a)$.
2. $\hat{\sigma}^2(0, h, \frac{h}{2} + \theta) = \int_{\mathbb{R}} (y(0, h, s) - \hat{y}(0, h, \frac{h}{2} + \theta))^2 f(s - (\theta + \frac{h}{2})) ds =$
$\int_{\mathbb{R}} (y(0, h, h - s) - \hat{y}(0, h, \frac{h}{2} - \theta))^2 f(s - (\theta + \frac{h}{2})) ds =$
$\int_{\mathbb{R}} (y(0, h, s) - \hat{y}(0, h, \frac{h}{2} - \theta))^2 f(\frac{h}{2} - \theta - s) ds = \hat{\sigma}^2(0, h, \frac{h}{2} - \theta)$. □

Proof of Proposition 4: Using Lemma 11 the risk effect for a boundary type can be written as a function of the size of the interval sent only: $\hat{\sigma}^2(a_i, a_{i+1}, a_i) = \hat{\sigma}^2(0, h_{i+1}, 0)$ and $\hat{\sigma}^2(a_{i-1}, a_i, a_i) = \hat{\sigma}^2(0, h_i, 0)$. Hence to compare the risk effect of sending m_i versus m_{i+1} it is enough to show that $\frac{\partial}{\partial h} \hat{\sigma}^2(0, h, 0) > 0$. $\frac{\partial}{\partial h} \hat{\sigma}^2(0, h, 0) = 2 \int_{\mathbb{R}} (y_2(0, h, s) - \hat{y}_2(0, h, 0)) f(s) ds > 0$. This follows because by (A.5), when h increases, the distance between the average action and the decision maker's actions when the signal realisations $s < \frac{h}{2}$ increases. Note that $s < \frac{h}{2}$ are more likely than $s > \frac{h}{2}$ given $\theta = 0$. □

Appendix A.8. Proof of Proposition 5

Proof of Proposition 5: Denote by $MB(h)$ the marginal benefit of acquiring the signal for a decision maker that receives a message (interval) of size h. By Persico (2000), $MB'(h) > 0$. Consider $\pi = \{0, a, 1\}$, with $0 < a < \frac{1}{2}$ the monotone partition equilibrium in the case of a free signal. Denote by $\underline{c}_f = MB(a)$ and $\overline{c}_f = MB(1 - a)$. Consider now that the cost of acquiring the signal is $c \in [\underline{c}_f, \overline{c}_f]$. Fixing the equilibrium partition π, the decision maker would acquire the signal upon hearing the upper interval but would not invest if the lower interval was received. Hence,

$$\begin{aligned} U_f^E(a, 1, a) - U_{CS}^E(0, a, a) &= U_f^E(a, 1, a) - U_f^E(0, a, a) + U_f^E(0, a, a) - U_{CS}^E(0, a, a) \\ &= U_f^E(0, a, a) - U_{CS}^E(0, a, a) > 0 \end{aligned}$$

where the second equality follows by (A_f), and the inequality is due to Proposition 2. Hence, there exists a type $0 < a' < a$ that solves $U_f^E(a', 1, a') - U_{CS}^E(0, a', a') = 0$.

The same argument can be used to show that there is always less communication with information acquisition than in the baseline CS model. □

Appendix B. Uniform Private Information Model

Suppose that the conditional distribution of the signal is uniform on $[\theta-\delta,\theta+\delta]$. Recall that the optimal action in this model is:

$$y_\delta(\underline{a},\overline{a},s)=\frac{\max\{\underline{a},s-\delta\}+\min\{\overline{a},s+\delta\}}{2}$$

If $\overline{a}-\underline{a}\leq 2\delta$ the expectation and the second moment of the decision maker's actions from the point of view of the expert are given by:

$$\begin{aligned}\hat{y}_\delta(\underline{a},\overline{a},\theta) &= \tfrac{\underline{a}+\overline{a}}{2}+\tfrac{1}{8\delta}(\overline{a}-\underline{a})(2\theta-\overline{a}-\underline{a})\\ E_\delta(y^2|\underline{a},\overline{a},\theta) &= \tfrac{(\underline{a}+\overline{a})^2}{4}+\tfrac{1}{24\delta}[(\theta+\overline{a})^3-(\theta+\underline{a})^3-3(\underline{a}+\overline{a})^2(\overline{a}-\underline{a})]\end{aligned}$$

If $\overline{a}-\underline{a}>2\delta$, the expectation and second moment of the decision maker's actions are:

$$\hat{y}_\delta(\underline{a},\overline{a},\theta)=\begin{cases}\frac{\delta+\underline{a}+\theta}{2}+\frac{1}{8\delta}(\underline{a}-\theta)^2 & if\quad \theta<\min\{\underline{a}+2\delta,\overline{a}-2\delta\}\\ \frac{\underline{a}+\overline{a}}{2}+\frac{1}{8\delta}(\overline{a}-\underline{a})(2\theta-\overline{a}-\underline{a}) & if\quad \overline{a}-2\delta<\theta<\underline{a}+2\delta\\ \theta & if\quad \underline{a}+2\delta<\theta<\overline{a}-2\delta\\ \frac{\theta+\overline{a}-\delta}{2}+\frac{1}{8\delta}(\theta-\overline{a})^2 & if\quad \theta\geq\max\{\underline{a}+2\delta,\overline{a}-2\delta\}\end{cases}$$

$$E_\delta(y^2|\underline{a},\overline{a},\theta)=\begin{cases}\frac{1}{24\delta}[4(\underline{a}+\delta)^3+4(\theta+\delta)^3-(\underline{a}+\theta)^3] & if\quad \theta<\min\{\underline{a}+2\delta,\overline{a}-2\delta\}\\ \frac{1}{24\delta}[4(\underline{a}+\delta)^3-4(\overline{a}-\delta)^3+(\theta+\overline{a})^3-(\underline{a}+\theta)^3] & if\quad \overline{a}-2\delta<\theta<\underline{a}+2\delta\\ \theta^2+\frac{\delta^2}{3} & if\quad \underline{a}+2\delta<\theta<\overline{a}-2\delta\\ \frac{1}{24\delta}[(\overline{a}+\theta)^3-4(\overline{a}-\delta)^3-4(\theta-\delta)^3] & if\quad \theta\geq\max\{\underline{a}+2\delta,\overline{a}-2\delta\}\end{cases}$$

Given quadratic-loss utilities, $U^E_\delta(\underline{a},\overline{a},\theta,b)=-E_\delta(y^2|\underline{a},\overline{a},\theta,b)+2\hat{y}_\delta(\underline{a},\overline{a},\theta)-(\theta+b)^2$. In particular, denoting by $h_i=a_i-a_{i-1}$ and $h_{i+1}=a_{i+1}-a_i$, the expected utilities of an expert with type $\theta=a_i$ who sends message $[a_{i-1},a_i]$ and $[a_i,a_{i+1}]$ are respectively:

$$\begin{aligned}U^E_\delta(a_{i-1},a_i,a_i,b) &= \begin{cases}-(\frac{h_i}{2}+b)^2+\frac{1}{12\delta}h_i^3+\frac{b}{4\delta}h_i^2 & if\quad h_i\leq 2\delta\\ -\delta b-\frac{\delta^2}{3}-b^2 & if\quad h_i>2\delta\end{cases}\\ U^E_\delta(a_i,a_{i+1},a_i,b) &= \begin{cases}-(\frac{h_{i+1}}{2}-b)^2+\frac{1}{12\delta}h_{i+1}^3-\frac{b}{4\delta}h_{i+1}^2 & if\quad h_{i+1}\leq 2\delta\\ \delta b-\frac{\delta^2}{3}-b^2 & if\quad h_{i+1}>2\delta\end{cases}\end{aligned}\tag{B.1}$$

Appendix B.1. Proof of Theorem 3

For the proof of Theorem 3 I use the following results:

Lemma 12. *Let a be a monotone partition equilibrium of the δ-Uniform Private Information model. Suppose that $a_{i+1}-a_i<2\delta$, then $V_{\delta'}(a_{i-1},a_i,a_{i+1})>0$ for all $\delta'<\delta$.*

Proof of Lemma 12: By Theorem 1, $h_{i+1}\equiv a_{i+1}-a_i>a_i-a_{i-1}\equiv h_i$, and hence, $h_{i+1}<2\delta$ implies $h_i<2\delta$. Since $V_\delta(a_{i-1},a_i,a_{i+1})=0$, for $\delta'\in(\frac{h_{i+1}}{2},\delta)$:

$$\begin{aligned}V_{\delta'}(a_{i-1},a_i,a_{i+1}) &= V_{\delta'}(a_{i-1},a_i,a_{i+1})-V_\delta(a_{i-1},a_i,a_{i+1})\\ &= (\tfrac{1}{12\delta'}-\tfrac{1}{12\delta})(h_{i+1}^2(h_{i+1}-3b)-h_i^2(h_i-3b))\end{aligned}\tag{B.2}$$

which is positive for $\delta'<\delta$ as long as $h_{i+1}>3b$. Note that as δ goes to infinity, the signal becomes uninformative resulting in the CS setup where $h^{CS}_{i+1}=h^{CS}_i+4b\geq 4b$. Therefore, by (B.2), as the signal becomes more informative, the required h_{i+1} which makes $\theta=a_i$ indifferent between m_i and m_{i+1} becomes larger, implying that $h_{i+1}\geq 4b>3b$ always holds, and thus $V_{\delta'}(a_{i-1},a_i,a_{i+1})>0$. If $\delta'<\frac{h_i}{2}$, then by (B.1), $V_{\delta'}(a_{i-1},a_i,a_{i+1})>0$. □

Consider now the case $a_{i+1}-a_i>2\delta$. Observe that it cannot be that $a_i-a_{i-1}>2\delta$ as well, because in that case by (B.1) the expert with type a_i strictly prefers m_{i+1}. Since by Theorem 1, intervals in equilibrium are increasing in size, the only interval which might be larger than 2δ is the last one. The following remark summarizes this argument.

Remark 1. *For any equilibrium partition* $a = \{0 = a_0 < a_1 < ... < a_{N-1} < a_N\}$ *of the* $\delta - PI$ *model,* $h_i = a_i - a_{i-1} < 2\delta$ *for* $1 \leq i \leq N-1$.

Lemma 13. *Suppose that* $a_i - a_{i-1} < 2\delta < 1 - a_i$ *and* $V_\delta(a_{i-1}, a_i, 1) = 0$ *then* $V_{\delta'}(a_{i-1}, a_i, 1) > 0$ *for* $\delta' < \delta$.

Proof of Lemma 13: By (B.1), $V_\delta(a_{i-1}, a_i, a_{i+1}) = \delta b - \frac{\delta^2}{3} - b^2 + (\frac{h_i}{2} + b)^2 - \frac{1}{12\delta}h_i^3 - \frac{b}{4\delta}h_i^2$. Taking the derivative with respect to δ:

$$\begin{aligned} \frac{\partial}{\partial\delta}V_\delta(a_{i-1}, a_i, a_{i+1}) &= b - \frac{2\delta}{3} + \frac{1}{12\delta^2}h_i^3 + \frac{b}{4\delta^2}h_i^2 \\ &< b - \frac{2\delta}{3} + \frac{1}{12\delta^2}(2\delta)^3 + \frac{b}{4\delta^2}(2\delta)^2 = 0 \end{aligned}$$

where the inequality follows because, by assumption $h_i < 2\delta$. $V_\delta(\cdot)$ decreasing in δ combined with $V_\delta(a_{i-1}, a_i, 1) = 0$ implies $V_{\delta'}(a_{i-1}, a_i, 1) > 0$ for $\frac{h_i}{2} < \delta' < \delta$. If $\delta' < \frac{h_i}{2} < \frac{h_{i+1}}{2}$, by (B.1), $V_{\delta'}(a_{i-1}, a_i, 1) > 0$. □

Proof of Theorem 3: The theorem is a direct implication of Lemmas 12, 13, Remark 1 and Proposition 1. □

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Declaration: I hereby declare that no changes have been made to the current accepted version other than those requested above.

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April 30 2024

Declaration of interest: None.

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