

Enhanced Constraints on Large-Scale Structure from Secondary CMB Anisotropies



Nick Koukoufilippas

Linacre College

University of Oxford

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*We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.*

– T. S. Eliot

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During my DPhil years, I led the British Astronomy Olympiad team to three international competitions, where our students earned numerous medals. In addition, I worked as an academic support worker for students with disabilities and as an academic mentor, roles that enriched my teaching experience and provided valuable insights into the academic community.

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Publications

The following is a list of articles that were published as a result of the research carried out for this thesis. Indicated in parentheses are the relevant chapters of the current thesis, where the papers are primarily discussed, where applicable.

- **Koukoufilippas N.**, Alonso D., Bilicki M., Peacock J. A., 2020. Tomographic measurement of the intergalactic gas pressure through galaxy-tSZ cross-correlations. *Monthly Notices of the Royal Astronomical Society*, 491, 5464. (Chapter 3)
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- Jago B., Ruiz-Zapatero J., García-García C., **Koukoufilippas N.**, Alonso D., 2023. The star formation history in the last 10 billion years from CIB cross-correlations. *Monthly Notices of the Royal Astronomical Society*, 520, 1895-1912.
- **Koukoufilippas N.**, Alonso D., Bilicki M., Peacock J. A., (*in prep.*). Tomographic reconstruction of structure growth and gas pressure at $z \lesssim 0.4$. (Chapter 4)
- LSST DESC Collaboration (+ **Koukoufilippas N.**), (*in prep.*). Core Cosmology Library Version-3: Accurate Predictions for Cosmology with LSST.

Abstract

The large-scale structure of the Universe encodes invaluable information about the fundamental cosmological parameters, the physics of structure formation, and the thermodynamic history of the Universe. In this thesis, we explore how secondary anisotropies of the Cosmic Microwave Background (CMB), particularly the thermal Sunyaev-Zeldovich (tSZ) effect and CMB lensing, can improve constraints on the large-scale structure. We develop a framework to cross-correlate tSZ maps from the *Planck* satellite with the distribution of galaxies at low redshift using tomographic bins with data from the 2MASS Photometric Redshift catalogue and WISE $\times$ SuperCOSMOS. These cross-correlations enable precise measurements of the bias-weighted gas pressure, $\langle bP_e \rangle$, and the hydrostatic mass bias parameter, $1 - b_H$, as a function of redshift.

This thesis is primarily based on two complementary studies employing galaxy clustering ($\delta_g \times \delta_g$), galaxy-tSZ cross-correlations ($\delta_g \times y$), and galaxy-CMB lensing cross-correlations ($\delta_g \times \kappa$) to constrain cosmological and thermodynamic parameters across six redshift bins ($z \in [0.1, 0.6]$).

In the first study, we use a combination of $\delta_g \times \delta_g$ and $\delta_g \times y$ to improve constraints on the thermal history of the Universe. We achieve $\sim 6\%$ precision on $1 - b_H$ across the six redshift bins, finding consistency with previous results and no evidence for significant redshift dependence. Our best-fit value of $1 - b_H = 0.75 \pm 0.03$ aligns well with joint analyses of *Planck* cluster counts and CMB anisotropies calibrated with CMB lensing. Additionally, our constraints on $\langle bP_e \rangle$, accurate to $\sim 10\%$ per bin, represent the most precise measurements to date, providing a robust test of baryonic feedback and models of energy injection.

The second study incorporates $\delta_g \times \kappa$ to enhance our tomographic analysis of structure growth and gas thermodynamics. Using CMB lensing as an additional tracer of large-scale structure, we constrain the amplitude of fluctuations of the matter power spectrum, σ_8 , to $\sim 6\%$ across all redshift bins, the hydrostatic mass bias, $1 - b_H$ to $\sim 18\%$, the bias-weighted average electron pressure, $\langle bP_e \rangle$

to $\sim 12\%$, the thermal energy density, Ω_{th} to $\sim 10\%$, as well as T_{AGN} , a single parameter quantifying the intensive thermodynamic properties of haloes. We perform multiple robustness checks to verify the stability of our models, and we report broad agreement with previous results in the literature.

This work builds on the use of secondary CMB anisotropies as a probe of non-linear physics, and of the interplay between dark matter and baryonic matter in haloes. We incorporate novel methods for combining tSZ data with other probes, in an attempt to refine models of halo bias and constrain σ_8 . The inclusion of $\delta_g \times \kappa$ helps to break degeneracies between key parameters of the theoretical framework used. Hence, we highlight the importance of secondary CMB anisotropies as a complementary tool for understanding the large-scale structure, offering new insights into the thermal evolution of the Universe, as well as the growth of structure.

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Chapter 1

Introduction

This thesis concerns itself with acquiring constraints on the evolution of structure in the Universe, enhancing our understanding of the interplay between galaxies and their surrounding dark matter haloes, and probing the thermal history of the Universe. These are achieved using a combination of statistical methods and software that help to accelerate the analysis of cosmological datasets.

This chapter introduces the theoretical background in Cosmology, which underpins the models used in the subsequent chapters to extract theoretical constraints from cosmological data sets.

1.1 Background Cosmology

1.1.1 The cosmological principle & Friedman equations

One of the fundamental assumptions of modern Cosmology is the *Copernican principle*, which states that we are not privileged observers in the Universe. It follows that our observations of the Universe should not deviate from what the average observer in the Universe would see.

This in turn gives rise to the *cosmological principle*, which states that on sufficiently large scales, the Universe is

- **homogeneous**, signifying that it is invariable under translations (i.e. regions look statistically identical);
- **isotropic**, signifying that it is invariable under rotations (i.e. there is no preferred ‘direction’).

These two simple assumptions lie at the heart of modern Cosmology, and, as there is no irrefutable observational evidence to suggest the contrary, we will accept them as ground

truth. The most general metric associated with the cosmological principle is the Friedman-Lemaître-Robertson-Walker (FLRW) metric [71],

$$-c^2 d\tau^2 = -c^2 dt^2 + a(t) d\Sigma^2, \quad (1.1)$$

where τ is the proper time, the 4-dimensional interval in spacetime, $d\Sigma$ is the spatial metric, described by 3 coordinates, and a represents the *scale factor*, a dimensionless number commonly normalised to 1 at present time, indicating the relative size of the Universe. Working in spherical coordinates and choosing a system of units where $c = 1$ the FLRW metric becomes

$$-d\tau^2 = -dt^2 + a(t) \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right), \quad (1.2)$$

where k is the constant *Euclidean curvature* of spacetime. For a Euclidean Universe, $k = 0$. For a closed Universe with spherical topology, $k > 0$. For an open Universe with hyperbolic topology, $k < 0$.

The force that dictates how the Universe evolves on the largest scales is gravity. The current most accurate theory of gravity is Einstein's *general relativity*, comprising a set of *field equations* describing the interplay between mass-energy and local spacetime geometry. Alexander Friedman solved the field equations of general relativity for the FLRW metric and derived the *Friedman equations* in 1922 [71]. These are

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho + \Lambda c^2}{3} - \frac{kc^2}{a^2}, \quad (1.3)$$

which describes how fast the Universe expands (or contracts) with cosmic time, and

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} + \frac{\Lambda c^2}{3} \right), \quad (1.4)$$

which describes the acceleration (or deceleration) of the Universe with cosmic time. In the Friedman equations k is the Euclidean curvature, Λ is the cosmological constant, ρ is the average density of the Universe which includes all mass-energy, and p is a measure of the pressure in the Universe (corresponding to the total mass-energy density). In the Friedman equations, we defined the time derivative of the scale factor normalised by the scale factor itself as the *Hubble parameter*, $H \equiv \dot{a}/a$. The Hubble parameter at present time is called the *Hubble constant*, H_0 and was first introduced in 1929 when Edwin Hubble discovered that the recessional velocities of galaxies are a linear function of their distance from us, d_H , something that became known as *Hubble's law* [102]:

$$v_r = H_0 d_H. \quad (1.5)$$

Using Hubble's law and the cosmological principle, we can infer that the Universe is expanding.

We often use the Doppler shift of the lines that appear on the spectrum of galaxies as a distance proxy. This is called *redshift* (or *blueshift* for approaching galaxies) and is defined as

$$z \equiv \frac{\lambda - \lambda_0}{\lambda_0}, \quad (1.6)$$

where λ and λ_0 refer to the observed and emitted wavelengths, respectively. For low-redshift galaxies receding from us at non-relativistic speeds, their redshift is simply their speed in units of the speed of light, $z = v_r/c$. Combining this approximation with Hubble's law (Eq. 1.5), we can relate redshift with the *Hubble distance*, d_H :

$$z = \frac{H_0 d_H}{c}. \quad (1.7)$$

The scale factor is related to redshift via $a = 1/(1+z)$.

For galaxies at higher redshifts or with significant peculiar velocities, relativistic corrections become important. In such cases, the full relativistic Doppler formula,

$$1+z = \sqrt{\frac{1+v_r/c}{1-v_r/c}}, \quad (1.8)$$

must be used to calculate the recessional velocity, rather than Eq. 1.6. Although these corrections are negligible for most low-redshift galaxies, they are essential for precise cosmological analyses and for accurately interpreting effects such as redshift-space distortions, which we discuss further in Section 1.4.3.

For a set of cosmological parameters, there is a one-to-one mapping between redshift and *comoving distance*, which is a notion of distance that follows the expansion of the Universe. It follows that objects that track the expansion of the Universe and have no proper motions (*Hubble flow*) have a fixed comoving distance from one another. Comoving distance is defined at a fixed time, which is conventionally chosen to be today (when $a = 1$).

We define the *critical density* as the average density required for the spatial curvature to be zero,

$$\rho_c \equiv \frac{3H^2}{8\pi G}. \quad (1.9)$$

Using this, we define the *density parameter* of a species as the partial ratio of the density of species x with respect to the critical density,

$$\Omega_x = \frac{\rho_x}{\rho_c}. \quad (1.10)$$

Finally, we define the following function, which is proportional to the expansion rate,

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}. \quad (1.11)$$

Using Eq. 1.11, the Hubble parameter at redshift z can be calculated using the present-day value as $H(z) = H_0 E(z)$.

There are a number of different distance definitions in Cosmology. We will briefly summarise each definition [96]. Fig. 1.1 showcases the most important distance definitions throughout cosmic history for a vanilla cosmology.

- **Hubble distance** is the distance implied by the recession velocity of a galaxy, assuming it follows Hubble flow. From Hubble's law, the recession velocity is given as $v_r = H_0 d_H$, so the Hubble distance is defined as

$$d_H = \frac{cz}{H_0}. \quad (1.12)$$

Often, it is useful to define it as the characteristic distance, at which the recession velocity of a galaxy following the Hubble flow is equal to the speed of light. It is then defined as

$$D_H \equiv \frac{c}{H_0}. \quad (1.13)$$

This definition is used to set a scale for the observable Universe. Fig. 1.1 uses the former definition, with d_H being a variable.

- **Proper distance** is the distance a ruler would measure between two observers who measure the same age of the Universe (simultaneous).
- **Line-of-sight comoving distance** is the proper distance divided by the scale factor. It is given by

$$D_C = D_H \int_0^z \frac{dz'}{E(z')}. \quad (1.14)$$

- **Transverse comoving distance** (or comoving angular distance) is defined such that the comoving distance between two observers at a fixed radial distance away from us (same redshift slice) separated by an angle θ is $\theta D_M(z)$, where D_M is the transverse comoving distance. It is given by

$$D_M = \begin{cases} D_H \frac{1}{\sqrt{\Omega_k}} \sinh \left(\sqrt{|\Omega_k|} \frac{D_C}{D_H} \right), & \Omega_k > 0 \\ D_C, & \Omega_k = 0 \\ D_H \frac{1}{\sqrt{\Omega_k}} \sin \left(\sqrt{|\Omega_k|} \frac{D_C}{D_H} \right), & \Omega_k < 0. \end{cases} \quad (1.15)$$

- **Angular diameter distance** is the ratio of the transverse diameter of an object to its angular diameter. Interestingly, for a vanilla cosmology it reaches a peak at $z \sim 1$, making objects beyond that redshift to appear larger in angular size. It is given by

$$D_A = \frac{D_M}{1+z}. \quad (1.16)$$

- **Luminosity distance** is used for objects of known intrinsic brightness (*‘standard candles’*). It assumes that light disperses using the inverse square law and is given by

$$D_L = \sqrt{\frac{L}{4\pi S}}, \quad (1.17)$$

where S is the object’s *bolometric flux*. It is related to the transverse comoving distance and the angular diameter distance as

$$D_L = (1+z)D_M = (1+z)^2 D_A. \quad (1.18)$$

- **Lookback time** is often useful as it expresses the interval between the emission of photons at redshift z and the present time. It is given by

$$t_{\text{lb}}(z) \equiv \int_0^z \frac{dz'}{(1+z')H(z')}, \quad (1.19)$$

so that $t_{\text{lb}}(0) = 0$ and $t_{\text{lb}}(z) \rightarrow t_0$ (the age of the Universe) as $z \rightarrow \infty$. While not a ‘distance’ in the usual sense, t_{lb} provides a direct mapping from redshift to cosmic time, and is especially helpful when discussing the different epochs of the Universe.

1.1.2 The standard model of Cosmology

Using the redshift of distant objects to conclude that the Universe is expanding ($\dot{a} > 0$) and in the absence of a cosmological constant Λ or any other form of exotic energy contribution to acceleration, the second Friedman equation (Eq. 1.4) predicts that the expansion should decelerate ($\ddot{a} < 0$) in a Universe filled with ordinary matter. This suggests the existence of a past singularity, assuming General Relativity remains valid and no component induces accelerated expansion, such as during inflationary periods.

The prevailing theory that describes cosmic history, cosmic expansion, and structure growth is the theory of the *Big Bang* [71, 119, 102]. It predicts a variety of observed phenomena and is backed by numerous independent observations. In short, the Big Bang theory predicts a growing spacetime, and thus a physical limit in the distance we can observe. Furthermore, it accurately predicts the final ratios of Hydrogen and Helium after nucleosynthesis has taken place, as well as the existence of the *Cosmic Microwave Background* (CMB) which corresponds to the point in time when photons decoupled from the previously ionised matter.

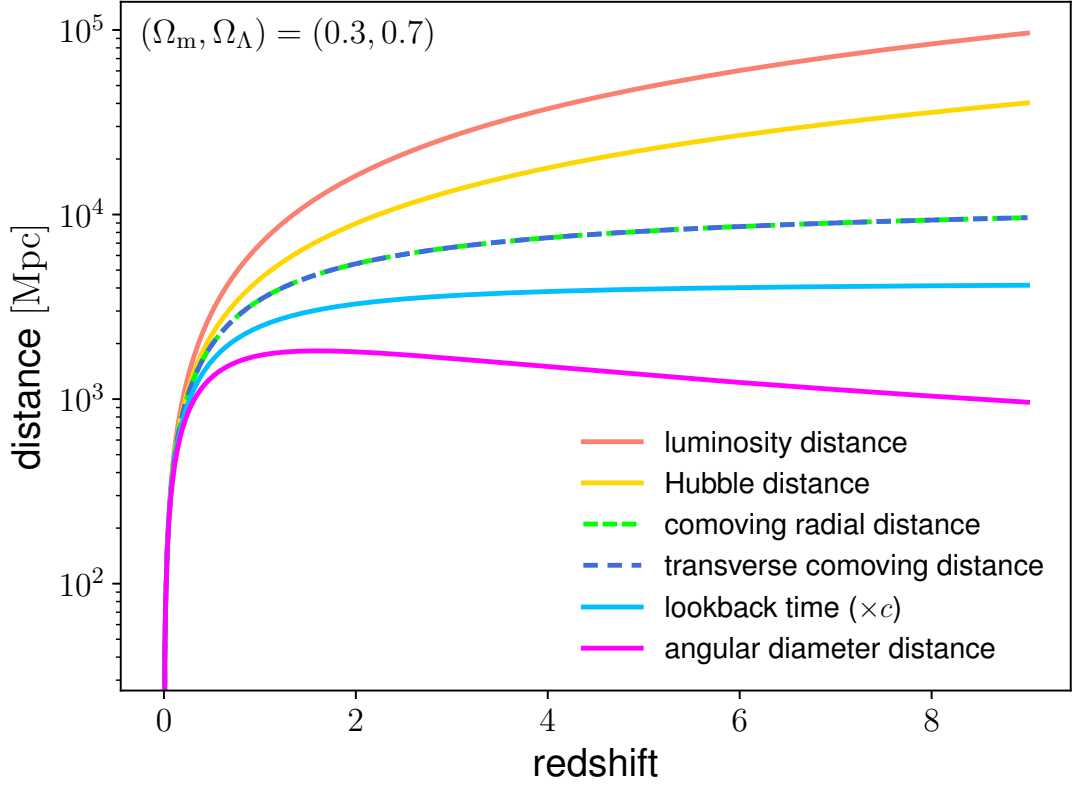


Figure 1.1: Comparison of the cosmological distance measures for a vanilla cosmology $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$. The lookback time (multiplied by the speed of light c) is also shown. Owing to $\Omega_k = 0$, the comoving radial distance for that particular cosmology is equal to the transverse comoving distance. In an open Universe ($\Omega_k > 0$), lines of sight at fixed angular separation subtend a *larger* physical distance than if space was flat (Eq. 1.15), so transverse distances exceed radial ones. In a closed Universe ($\Omega_k < 0$), space ‘wraps around’, and the same angle subtends a *smaller* separation than in flat space.

1.1.2.1 Mass-energy content & the equation of state

In Cosmology, the *equation of state* is used to describe the relation between density and pressure of the different energy components in the Universe, and characterise their evolution. We define a dimensionless quantity, $w \equiv p/\rho$, which is the ratio of pressure to density. Using the FLRW metric (Eq. 1.1) and replacing the equation of state into the Friedman equations we derive:

$$\rho \propto a^{-3(1+w)}, \quad (1.20)$$

where w is a constant describing the fluid characteristics of the energy component. The current standard model of Cosmology is dubbed the Λ CDM model.

‘ Λ ’ is the most abundant energy component in the Universe, representing dark energy, which is commonly modelled as a *cosmological constant*. Evidence for dark energy comes

from combining CMB measurements with low-redshift observations, suggesting that the Universe is flat. Since baryons and dark matter only account for $\sim 30\%$ of the *critical density* (the density required for the Universe to be spatially flat), a component with negative pressure is required to drive the observed acceleration. This is further independently supported by standard rulers in the large-scale structure, such as *Baryon Acoustic Oscillations* (BAO). In its simplest form, dark energy is modelled as having constant energy density and an equation of state $w = -1$, which corresponds to the energy density ρ remaining constant over time, as would be the case for a cosmological constant.

‘CDM’ stands for *cold dark matter*, which refers to dark matter that moves at non-relativistic speeds and does not interact with other particles with any other force (electromagnetism, strong, weak), except through gravity. Cold dark matter particles are both non-relativistic and non-interacting, so the pressure they exert on the dynamics of the Universe is negligible. Non-relativistic particles such as CDM have an equation of state $w = 0$, meaning their energy density scales with the cubic inverse of the scale factor, $\rho \propto a^{-3}$, as the Universe expands. This makes CDM a crucial component in explaining the formation of large-scale structure, as its gravitational influence drives the growth of structure without being impeded by interactions with other species.

The third most abundant mass-energy component of the Universe is *baryonic matter*, which is the ‘ordinary’ matter that we can see. Particles that constitute the standard model of particle physics make up all baryonic matter. Their equation of state is the same as that for cold dark matter.

Finally, today, a small fraction of the pie belongs to relativistic species, such as photons. The equation of state of these particles is $w = 1/3 \implies \rho \propto a^{-4}$. The early Universe went through the radiation-dominated era, where photons comprised most of the total mass-energy content. This period ended with the matter-radiation equality at a redshift of $z_{\text{eq}} \approx 3400$. In the very early Universe, neutrinos were highly energetic but with time they lost their energy; their equation of state changed as their w changed.

Fig. 1.2 shows the evolution of the mass-energy component pie chart for a vanilla cosmology. Where the curves intersect, we have the time of equality between two species. The intersection at high redshift denotes the time of the matter-radiation equality at $z \sim 3400$, and the intersection at low redshift denotes the time of the matter-dark energy equality at $z \sim 0.32$.

1.1.2.2 Big Bang Theory shortcomings & inflation

While the theory of the Big Bang makes a number of accurate predictions about the Universe we observe, it suffers from a number of problems related to the initial conditions. These

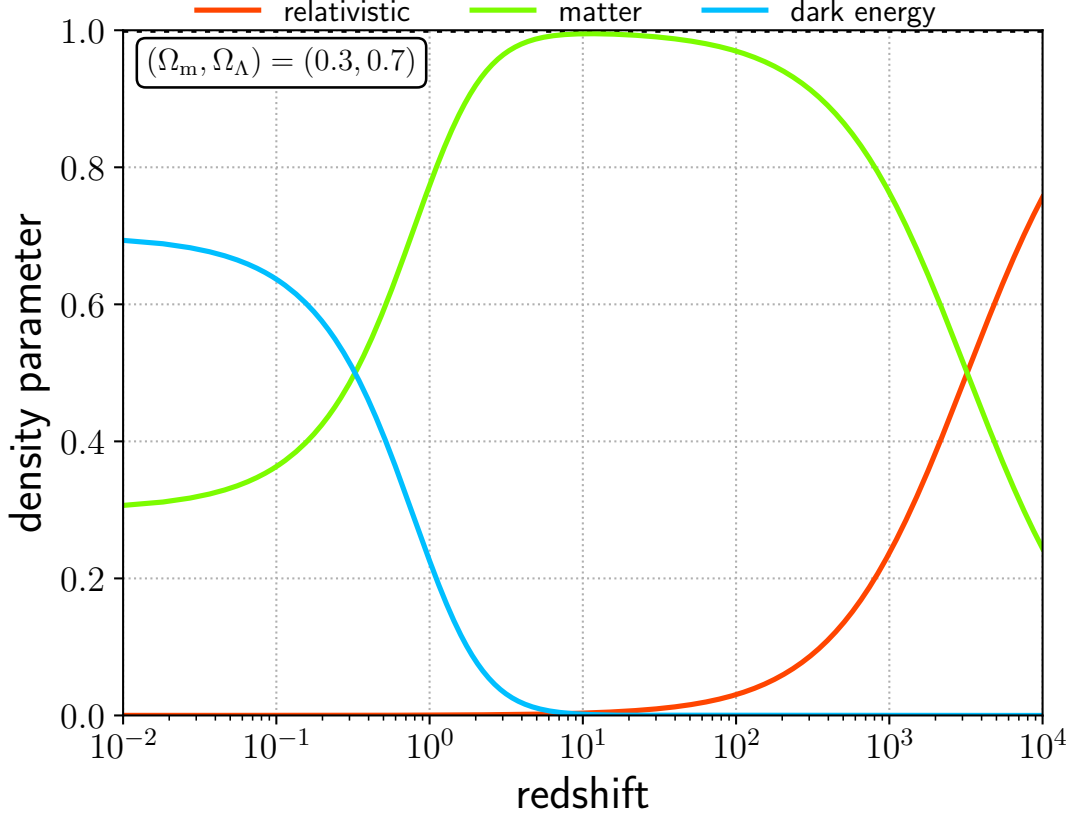


Figure 1.2: Density parameter, Ω_x , throughout cosmic time, for three species: relativistic (photons & massless neutrinos), matter (baryonic & dark), and dark energy.

are:

1. **The Horizon Problem:** CMB is observed to be homogeneous across the whole observed celestial sphere, except for small anisotropies with a relative amplitude of 10^{-5} . This level of homogeneity suggests that all points on the *surface of last scattering* were once in thermal equilibrium and thus causally connected. However, there are over 10^4 non-overlapping particle horizons on it.
2. **The Flatness Problem:** Measurements of the curvature of the Universe at late times indicate that the total mass-energy content is extremely close to the critical density, implying that the Universe is nearly flat today. This suggests that the initial curvature at early times must have been exceedingly close to zero, as the early Universe would need to be flat to 1 part in $\sim 10^{60}$ to remain so today. This poses a fine-tuning problem, unless a mechanism like inflation caused the Universe to accelerate and drive it toward flatness in its early stages.

3. **Magnetic Monopoles:** Grand Unification Theories (GUTs) predict the formation of magnetic monopoles when the Universe cooled to some symmetry breaking temperature. The predicted mass of these monopoles is $\sim 10^{14}$ higher than the total baryon mass and so it is puzzling that we have not observed them.

In 1980, Alan Guth proposed that if a period of inflationary expansion occurred in the very early Universe, it could tackle the aforementioned hurdles of the theory of the Big Bang [78, 79]. This period is called *inflation*. The required number of *e-foldings* to solve the horizon and flatness problems is ~ 60 . During inflation, the exponential expansion dilutes any initial curvature, driving the Universe towards flatness, while regions that were initially causally disconnected are stretched beyond the observable Universe, thereby explaining the observed large-scale homogeneity. Monopoles are also diluted to ~ 1 per particle horizon.

Most models of inflation invoke a scalar field, the “inflaton”. The most common picture is the so-called “slow-roll” inflation: the field starts at a large value of its potential (large compared with its kinetic energy). In these conditions, such a scalar field will drive a de-Sitter-like expansion (i.e. exponentially accelerated, driven by an effective cosmological constant), and inflation will proceed. As the field rolls down the potential, it gathers kinetic energy, although this process is slowed down by the Universe’s expansion, which acts as an effective friction force. Eventually, kinetic and potential energies become comparable, and the phase of accelerated expansion ends. Often the inflaton will end up oscillating around the minimum and, as it decays, it will transfer its energy to the other fields it is coupled to, reheating the Universe and producing all other particles.

Due to the quantum nature of the inflaton, we expect quantum fluctuations in its value, which lead to small inhomogeneities. As the field rolls down the potential, inflation will end at slightly different times at different points in the Universe. This transforms the quantum fluctuations in the inflaton into density inhomogeneities after inflation ends. Importantly, inflation is able to preserve these quantum fluctuations by expanding them to super-horizon scales, where they are frozen until inflation ends. Simple inflationary models predict that these fluctuations are *almost* scale-invariant, meaning that the amplitude of fluctuations is nearly constant across different length scales. The deviation from perfect scale invariance is characterised by the *spectral index* (n_s), which quantifies the tilt of the power spectrum of these primordial fluctuations.

For a quadratic potential in a *slow-roll* inflation we predict a spectral index of $n_s \sim 0.96$ which matches recent observations at 1σ [158]. If inflation is highly energetic, it is predicted to also leave a gravitational wave imprint in the early Universe. To date, this has not been observed. Since a number of single-field models predict a scale-invariant, Gaussian random

field of curvature perturbations, detecting non-Gaussianities on the CMB could rule out many simple inflationary models.

1.1.3 Chronological timescale

In this section, we briefly describe the most notable events that took place during cosmic evolution.

- **Baryogenesis:** As the inflaton field decays, the energy it contains is predicted to have been transformed into matter and antimatter. Matter and antimatter begin to annihilate via processes like $e^+ + e^- \rightarrow \gamma + \gamma$; however, due to a $1 : 10^9$ asymmetry in their relative abundances matter prevails in a Universe filled with radiation.
- **Electroweak phase transition:** As the Universe expands and cools below 100 GeV, the Higgs mechanism gives mass to particles.
- **QCD phase transition:** Quarks interact with gluons to form colourless triplets (*baryons*) or particle-antiparticle pairs (*mesons*).
- **Dark matter freeze out:** Dark matter decouples early on as it interacts very weakly with its environment.
- **Neutrino decoupling:** Next, neutrinos decouple at 0.8 MeV as they interact with the surrounding plasma through the weak interaction.
- **Electron-positron annihilation:** Electrons and positrons annihilate and the energy produced is transferred to radiation. This is the reason why the photon temperature is greater than the neutrino temperature.
- **Big Bang nucleosynthesis:** Next, the light elements are formed. These are Hydrogen and Helium, with trace amounts of Lithium.
- **Matter-radiation equality:** As shown in Fig. 1.2, around $z \sim 3400$ there is an equal abundance of radiation and matter. This is because, the energy of ultra-relativistic species (such as photons) drops as 10^{-4} while that of matter drops as 10^{-3} .
- **Recombination:** At some point around $z \sim 1100$ during the initial expansion the Universe cools down enough to effectively make the reaction $e^- + p^+ \rightarrow H + \gamma$ one-way. At that point, neutral hydrogen starts to form.

- **Photon decoupling:** As a result of recombination, photons no longer interact with free electrons, and can freely stream through space. We observe these (highly redshifted) photons today, as the *cosmic microwave background radiation*. The spherical shell surrounding us with a distance corresponding to that point in time is often called *surface of last scattering*. CMB temperature is uniform to 10^{-5} .
- **Reionisation:** As the first structures started to form, the temperature of the Universe was high enough to ionise the then neutral Hydrogen.
- **Matter - dark energy equality:** As shown in Fig. 1.2 at $z \sim 0.32$ dark energy becomes the dominant form of energy in the Universe, after matter has diluted enough. This again slowly drives exponential expansion.

1.2 Structure formation

The initial conditions for the evolution of density perturbations were set at the end of inflation. These minuscule perturbations seeded gravitational collapse, which led to structure formation early on in cosmic history.

We cannot model the evolution of individual density perturbations; rather, we look at the statistics (distributions and statistical averages) of these perturbations at different scales. These can be achieved through the 2-point correlation function (2PCF) or the Fourier-space equivalent, the power spectrum.

1.2.1 Newtonian perturbation theory

To describe the evolution of any field under gravity we need three quantities: its density, $\rho(\mathbf{r}, t)$, pressure $p(\mathbf{r}, t)$, and its velocity, $\mathbf{V}(\mathbf{r}, t)$. The fluid can then be propagated using

- the continuity equation, which describes mass conservation:

$$\partial_t \rho + \nabla_{\mathbf{r}}(\rho \mathbf{V}) = 0; \quad (1.21)$$

- the Euler equation, which derives from Newton's second law:

$$\partial_t \mathbf{V} + (\mathbf{V} \cdot \nabla_{\mathbf{r}}) \mathbf{V} + \frac{\nabla_{\mathbf{r}} p}{\rho} + \nabla_{\mathbf{r}} \Psi = 0, \quad (1.22)$$

- the Poisson equation, which relates matter to gravity:

$$\nabla_{\mathbf{r}}^2 \Psi = 4\pi G \rho. \quad (1.23)$$

Factoring in the expansion of the Universe, we can relate the physical coordinates $\mathbf{r}$ to the comoving coordinates $\mathbf{x}$ via $\mathbf{r} = a(t)\mathbf{x}$ and take the derivative with respect to $\mathbf{r}$ and $\mathbf{x}$:

$$\nabla_{\mathbf{r}} = \frac{1}{a} \nabla_{\mathbf{x}}. \quad (1.24)$$

Now we split each field into a homogeneous background plus a (small) perturbation. Explicitly,

$$\rho(t, \mathbf{x}) = \underbrace{\bar{\rho}(t)}_{\text{background density}} \left(1 + \underbrace{\delta(t, \mathbf{x})}_{\text{density contrast}} \right), \quad (1.25)$$

$$\mathbf{V}(t, \mathbf{x}) = \underbrace{\dot{a}(t) \mathbf{x}}_{\text{Hubble flow}} + \underbrace{\mathbf{v}(t, \mathbf{x})}_{\text{peculiar velocity}}, \quad (1.26)$$

$$\Psi(t, \mathbf{x}) = \underbrace{\bar{\Psi}(t)}_{\text{background potential}} + \underbrace{\psi(t, \mathbf{x})}_{\text{perturbation}}, \quad (1.27)$$

$$p(t, \mathbf{x}) = \underbrace{\bar{p}(t)}_{\text{background pressure}} + \underbrace{c_s^2(t) \bar{\rho}(t) \delta(t, \mathbf{x})}_{\text{adiabatic pressure perturbation}}. \quad (1.28)$$

Through these equations, we define the overdensity δ , as well as the peculiar velocity field. The other quantities are the peculiar Newtonian potential ψ , and the sound speed of the pressure waves $c_s = \sqrt{\partial_\rho p}$. For the continuity equation to be satisfied, we require that

$$\Psi = -\frac{1}{2} a \ddot{a} |\mathbf{x}|^2. \quad (1.29)$$

The perturbation equations 1.21, 1.22, 1.23 then become

$$\dot{\delta} + \frac{1}{a} \nabla \cdot ((1 + \delta) \mathbf{v}) = 0, \quad (1.30)$$

$$\dot{\mathbf{v}} + H \mathbf{v} + \frac{1}{a} (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{c_s^2}{a} \frac{\nabla \delta}{1 + \delta} + \frac{1}{a} \nabla \psi = 0, \quad (1.31)$$

$$\nabla^2 \psi = 4\pi G a^2 \bar{\rho} \delta, \quad (1.32)$$

where $\nabla \equiv \nabla_{\mathbf{x}}$. Under the assumption that all perturbations are small $|\delta|, |\mathbf{v}|, |\psi| \ll 1$, we can linearise these equations, which become:

$$\dot{\delta} + \frac{1}{a} \nabla \cdot \mathbf{v} = 0 \quad (1.33)$$

$$\dot{\mathbf{v}} + H \mathbf{v} + \frac{c_s^2}{a} \nabla \delta + \frac{1}{a} \nabla \psi = 0 \quad (1.34)$$

$$\nabla^2 \psi = 4\pi G a^2 \bar{\rho} \delta \quad (1.35)$$

The linear versions of the perturbation equations reduce the problem to solving a set of coupled linear differential equations. Simple analytical solutions for these can be found in particular regimes (e.g. during the matter-dominated era). For analytically intractable cases, the transfer function (see Section 1.2.3) for these perturbations can be solved numerically with Boltzmann codes such as CAMB [122] or CLASS [32], or it can be approximately calculated using a fitting function, such as the prescription of Eisenstein & Hu (1992) [66] or the BBKS formula [18].

1.2.2 The matter power spectrum

Building on the framework established in the previous section, we introduce the *matter power spectrum*, $P(k)$, which characterises the amplitude of matter density fluctuations as a function of wavenumber k (corresponding to a specific physical scale $\lambda \equiv 2\pi/k$). It is defined as the Fourier transform of the two-point correlation function $\xi(r)$, which measures the excess probability (over random) of finding a pair of matter particles separated by a distance r :

$$\xi(r) = \langle \delta(\mathbf{x})\delta(\mathbf{x} + \mathbf{r}) \rangle. \quad (1.36)$$

The 3-dimensional power spectrum $P(k)$ is then given by

$$P(k) = \int d^3r \xi(r) e^{-i\mathbf{k}\cdot\mathbf{r}}. \quad (1.37)$$

Alternatively, the power spectrum can be defined in terms of the expectation value of the Fourier-transformed overdensity field as

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') P(k), \quad (1.38)$$

where δ_D is the Dirac delta function. This formulation highlights that $P(k)$ captures the variance of the density fluctuations as a function of the wavenumber k . The Dirac delta function indicates that different wavenumbers are uncorrelated from each other, and is purely a consequence of the statistical isotropy of cosmological fields.

The power spectrum is a key ingredient in making theoretical predictions for multiple cosmological observables. Its shape and amplitude offer insights into various physical processes influencing structure formation.

The shape of the matter power spectrum is determined by the initial conditions of the Universe, which are assumed to be nearly Gaussian and scale-invariant (as predicted by inflationary models). These initial perturbations evolve under gravity and cosmic expansion, which govern the growth of the power spectrum.

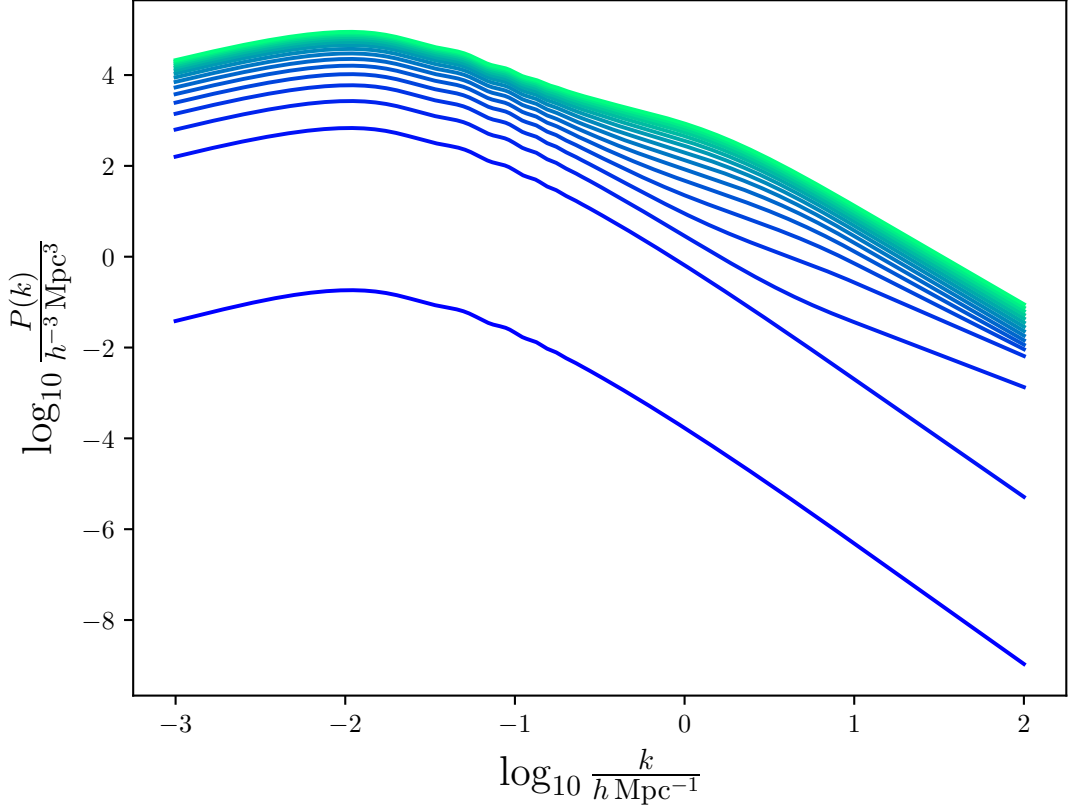


Figure 1.3: The matter power spectrum plotted qualitatively for a vanilla cosmology $(\Omega_m, \Omega_b, h, n_s, \sigma_8) = (0.30, 0.05, 0.67, 0.96, 0.81)$ over different cosmic times, in steps of 0.1 increasing scale factor $a \equiv 1/(1+z)$.

On large scales (small k) fluctuations are linear and $P(k)$ contains information about the primordial density perturbations. In the linear regime, where density contrasts are small ($|\delta| \ll 1$), the power spectrum can be evolved to time t from its initial form at time t_0 by

$$P(k, t) = P(k, t_0) \left(\frac{D(t)}{D(t_0)} \right)^2, \quad (1.39)$$

where $D(t)$ is the linear growth factor, arising from the solution to the linear perturbation theory equations outlined in the previous section.

There is a turnover around the scale of the horizon at the matter-radiation equality. The exact location of this turnover is sensitive to the matter density parameter Ω_m and the Hubble constant H_0 . Another distinctive feature is the presence of *baryon acoustic oscillations* (BAOs), which show up as small oscillations in the power spectrum. These oscillations are remnants of pressure waves in the photon-baryon fluid before recombination at $z \approx 1100$ and can be used as a *standard ruler* with a characteristic scale of approximately 150 Mpc.

On small scales (large k) the power spectrum is affected by non-linear gravitational collapse, which enhances overdensities, as well as baryonic effects [47]. Both processes lead to deviations from the predictions of linear theory. In this regime, we often resort to numerical simulations for accurate modelling.

We can place constraints on the matter power spectrum using different probes:

- Galaxy surveys, assuming that galaxies trace the underlying matter distribution, modulated by a bias factor b_g ;
- The CMB, which provides a snapshot at the time of recombinations and we can learn about the primordial power spectrum using temperature and polarisation anisotropies;
- Weak lensing is a direct probe of mass, and allows us to measure the matter power spectrum.

These observations are often combined to achieve precise constraints on the matter power spectrum and subsequently on the cosmological parameters and the models of structure formation.

1.2.3 Cosmological parameters

After the inflationary period, the (early) Universe was left with an initial distribution of density perturbations originating from quantum fluctuations during inflation. We quantify these perturbations using the *primordial power spectrum* to describe their distribution. It is expressed as

$$\Delta_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1}, \quad (1.40)$$

where A_s is the amplitude of the primordial power spectrum and the index describing its departure from scale invariance is n_s .

In most analyses we assume that the late-time matter power spectrum factorises neatly into a purely scale-dependent piece (the *transfer function*) times a purely time-dependent piece (the *linear growth factor*). That is,

$$P_m(k, z) = P_\zeta(k) T^2(k) D^2(z), \quad (1.41)$$

where $T(k)$ encapsulates all of the physics that “shapes” perturbations as they cross the horizon (radiation-matter equality, baryon-photon acoustic oscillations, Silk damping, etc.), and $D(z)$ is the scale-independent growth factor that describes how every Fourier mode grows (or decays) once it is well inside the horizon. The transfer function is calculated

by solving the perturbation equations and depends entirely on the cosmological parameters describing the background evolution of the Universe (which also influences the sound speed of the baryon-photon fluid at early times).

Why this separability can fail. When every species clusters in the same way, the growth rate is independent of scale, and the aforementioned factorisation is exact in linear theory. However, massive neutrinos spoil this clean split because they remain relativistic (and free-streaming) on large scales for longer than the cold components [99]. That introduces a small scale-dependent growth:

$$D(k, z) \neq D(z) \quad \text{for modes } k \gtrsim k_{\text{fs}}(z), \quad (1.42)$$

where $k_{\text{fs}}(z)$ is the neutrino free-streaming scale [120]. As a result, the true power spectrum is

$$P_{\text{m}}(k, z) = P_{\zeta}(k)T^2(k)D^2(z) \times (1 + \mathcal{O}(f_{\nu})) \rightarrow P_{\zeta}T^2(k)D^2(k, z) \quad (1.43)$$

in a more exact treatment, with $f_{\nu} \equiv \Omega_{\nu}/\Omega_{\text{m}}$, and where the scale-dependence has been absorbed into the growth factor $D(k, z)$.

Impact on halo model. Most halo model calculations assume strict k - z separability. If neutrino masses (or any other species with a scale-dependent growth) are non-negligible, then halo abundances and bias become slightly redshift- and scale-dependent in a way not captured by a single $D(z)$. To remedy this, we must use a generalised growth factor $D(k, z)$ from a Boltzmann code or incorporate an explicit suppression factor $S(k, z) \simeq 1 - 8f_{\nu}$ [170, 207]. In practice, for the small neutrino fractions allowed by current data ($f_{\nu} \lesssim 0.01$), the error from assuming a $P(k, z) \propto T^2(k)D^2(z)$ is $\mathcal{O}(0.1\%)$ on large scales and up to a few percent on quasi-linear scales [31]. Nonetheless, precision analyses (and future surveys) must include these scale-dependent corrections to avoid bias.

With these building blocks in place, the background expansion, spatial geometry, and growth of structure in an FLRW Universe can be described by a compact set of parameters. In addition to the primordial amplitude A_s and tilt n_s , we need:

- **Expansion rate:** The present-day Hubble parameter H_0 , often parameterised as dimensionless $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$.
- **Matter content:** The total (non-relativistic) matter density parameter,

$$\Omega_{\text{m}} = \Omega_{\text{c}} + \Omega_{\text{b}} + \Omega_{\nu}, \quad (1.44)$$

where Ω_c is cold dark matter, Ω_b is baryons, and Ω_ν is massive neutrinos. In particular, Ω_b plays a key role in setting the height of the acoustic peaks in the CMB and the scale of the BAO feature in large-scale structure.

- **Spatial curvature:** $\Omega_k \equiv 1 - \Omega_m - \Omega_\Lambda$, which governs whether the Universe is open ($\Omega_k > 0$), flat ($\Omega_k = 0$), or closed ($\Omega_k < 0$).
- **Dark energy:** At late times the dominant component is dark energy, typically parameterised by its present density Ω_Λ and its equation-of-state parameter $w \equiv p/\rho$. In the simplest cosmological constant model, $w = -1$ at all times; with more general models allowing $w(a)$ to evolve

$$w = w_0 + w_a(1 - a), \quad (1.45)$$

following the Chevallier-Polarski-Linder prescription [42].

Together, these parameters fully specify the Hubble expansion, the comoving distances, and the linear growth factor that underpin our observables.

1.3 The halo model

The perturbative treatment of matter inhomogeneities described in the previous section is able to describe the matter fluctuations on relatively small scales, where they are small. In order to describe the non-linear evolution of these fluctuations, as well as the statistics of other tracers of the Universe’s large-scale structure, we require more accurate and sophisticated approaches. These range from the computationally costly but formally correct approach of N -body simulations, in which the fluid equations are solved without approximations (at least when it comes to the gravitational evolution), to phenomenological approaches that exploit basic results from N -body simulations and combine them with more basic principles and analytical results derived from perturbation theory. In this thesis, we will make extensive use of one of these latter approaches, the so-called “halo model” [184]. We proceed to describe this approach in detail here.

1.3.1 Basic formulation

The halo model describes the spatial fluctuations of any quantity in terms of the contributions of all dark matter haloes, under the assumption that all matter in the Universe is contained in those haloes.

Let $U(r|M)$ be the profile of a given quantity as a function of the comoving distance r to the centre of a halo of mass M , and let $U(k|M)$ be its Fourier transform,

$$U(k|M) \equiv 4\pi \int_0^\infty dr r^2 \frac{\sin(kr)}{kr} U(r|M). \quad (1.46)$$

The halo model prediction for the cross-power spectrum between two such quantities, $P_{UV}(k)$, then consists of two contributions: the 2-halo term, which quantifies correlations between mass elements belonging to different haloes, and the 1-halo term, which quantifies correlations between points within the same halo:

$$P_{UV}(k) = P_{UV}^{1h}(k) + P_{UV}^{2h}(k). \quad (1.47)$$

Each of these can be estimated in terms of the Fourier-space profiles as:

$$P_{UV}^{1h}(k) = \int dM \frac{dn}{dM} \langle U(k|M) V(k|M) \rangle, \quad (1.48)$$

$$P_{UV}^{2h}(k) = \langle bU \rangle \langle bV \rangle P_L(k), \quad (1.49)$$

$$\langle bU \rangle(k) \equiv \int dM \frac{dn}{dM} b_h(M) \langle U(k|M) \rangle. \quad (1.50)$$

Here, $P_L(k)$ is the linear matter power spectrum, dn/dM is the halo mass function (comoving density of haloes per unit halo mass) and $b_h(M)$ is the halo bias function. We describe each of these ingredients below.

1.3.2 Ingredients

The halo model aims to relate the statistics of an observable quantity to an underlying field in the large-scale structure. For concreteness, we present the halo model in terms of the matter density, and we describe its application to other physical quantities in Chapter 3.

1.3.2.1 Halo mass function

In 1974, Press & Schechter developed a formalism to predict the abundance of galaxy clusters and other virialised structures [163]. In its basic formulation, this formalism remains a crude approximation, but it has two main advantages: (i) its assumptions are relatively simple, making it easy to use in many models; (ii) it provides a qualitatively accurate picture that can be easily extended to capture more complicated models.

The assumptions it makes are:

1. the overdensity field is linear and Gaussian at early times;

2. a ‘halo’ is formed when the local (linearly-extrapolated) overdensity exceeds the threshold of spherical collapse, $\delta_c \approx 1.686$.

Let us now consider the linearly-extrapolated overdensity field δ_L , which we filter using a top-hat window function with comoving radius R' . Since the overdensity of the patch exceeds the threshold δ_c somewhere, at some larger radius R the local overdensity is equal to δ_c . Thus, the formed halo has a mass

$$M = \frac{4\pi}{3} R^3 \rho_{M,0} = 1.16 \times 10^{12} \Omega_m \left(\frac{R}{h^{-1} \text{Mpc}} \right)^3 (\text{M}_\odot h^{-1}), \quad (1.51)$$

where R and $\rho_{M,0}$ are measured in comoving coordinates.

For a Gaussian field δ_R (e.g. linear overdensity filtered over a comoving scale R) with mean and standard deviation $(0, \sigma_R)$, the fraction of space where the field is above a threshold δ_c is

$$P(\delta_R > \delta_c) = \int_{\delta_c}^{\infty} \frac{d\delta_R}{\sqrt{2\pi\sigma_R^2}} \exp\left(-\frac{\delta_R^2}{2\sigma_R^2}\right) = \frac{1}{2} \left(1 - \text{erf}(\nu_R/\sqrt{2})\right), \quad (1.52)$$

where $\nu_R \equiv \delta_c/\sigma_R$, the rescaled threshold. Therefore, using Eq. 1.52, the fractional volume of the Universe that has now collapsed to form a halo is

$$F(> M) = P(\delta_R > \delta_c) = \frac{1}{2} \left(1 - \text{erf}\left(\frac{\nu}{\sqrt{2}}\right)\right), \quad (1.53)$$

with $\nu \equiv \frac{\delta_c}{\sigma(M)}$, and $\sigma(M) \equiv \sigma_R$ is the standard deviation of the density field smoothed over a scale R , in terms of the linear matter power spectrum.

If we plug in the numbers to compute the collapsed fraction of matter we see that the Press-Schechter formalism underestimates the collapsed fraction of matter by a factor of 2, as it does not account for the contribution from haloes of a given scale living within underdensities on larger scales. This issue is resolved by introducing a ‘fudge factor’ of 2, effectively doubling the collapsed fraction,

$$F(> M) = 2P(\delta_R > \delta_c). \quad (1.54)$$

From this fractional volume, we can then derive the differential of the halo abundance, or the *halo mass function* (HMF), by considering the contribution to $F(> M)$. Since halo masses span several orders of magnitude and the collapsed mass follows a logarithmic distribution, we express these differential counts in terms of logarithmic mass intervals:

$$f(M)dM = -\frac{dF(> M)}{d \ln M} d \ln M, \quad (1.55)$$

where $f(M)$ is the fraction of mass in haloes of mass M per unit $\ln M$. The comoving number density of haloes with mass in the interval $[M, M + dM]$ is thus

$$n(M)dM = \frac{\rho_{M,0}}{M} f(M) dM. \quad (1.56)$$

It is also useful to work with the *cumulative mass function*

$$n(> M) \equiv \int_M^\infty n(M') dM'. \quad (1.57)$$

Despite its simplicity and qualitative accuracy, the Press-Schechter mass function has some limitations. It overpredicts the abundance of low-mass haloes, and underpredicts the abundance of high-mass haloes when compared to numerical simulations. This discrepancy mainly arises due to the simplifying assumptions the formalism inherently contains, such as spherical collapse and the top-hat filter.

To address these shortcomings, more sophisticated models have been developed. One such improvement is the *Sheth-Tormen halo mass function* [181], which incorporates an ellipsoidal collapse model and introduces a fitting formula to better match the simulation data:

$$f(\nu) = A \sqrt{\frac{2\alpha}{\pi}} \left(1 + \left(\frac{\nu^2}{\alpha} \right)^{-p} \right) \nu \exp \left(-\frac{\alpha \nu^2}{2} \right), \quad (1.58)$$

where A , α , and p are parameters calibrated from simulations, typically chosen to be $(A, \alpha, p) \approx (0.322, 0.707, 0.3)$. Here, ν is defined as above, $\nu \equiv \delta_c / \sigma(M)$.

Finally, the universality of the halo mass function (its invariance under redshift and cosmology when expressed in terms of ν) has been a topic of exploration. While early results suggested a high degree of universality, more recent studies have identified dependencies on cosmological parameters and redshift, particularly at the extremes of the mass function [194, 195, 35, 57]. These variations underscore the need for consistent mass definitions and halo identification methods, such as the *Friends-of-Friends* (FoF) and *Spherical Overdensity* (SO) approaches [112]:

1. **Friends-of-Friends** (FoF): Groups particles based on a specified linking length, identifying haloes as clusters of neighbouring particles at a distance l (called the *linking length*). The linking length is parameterised as the mean inter-particle distance in the simulation, and usually $b \approx 0.2$;
2. **Spherical Overdensity** (SO): Centres on density peaks and defines haloes by growing a sphere until the enclosed mass reaches a pre-determined overdensity Δ , expressed

in terms of a reference density. The choice of reference density is usually the critical (ρ_c), or the mean matter (ρ_m) density. The enclosed mass is then

$$M = \frac{4\pi}{3} \Delta \bar{\rho} R_\Delta^3. \quad (1.59)$$

The associated radius is found in terms of physical (not comoving) coordinates. Another common choice of reference density is the virial density. When haloes virialise, their mean density is $\Delta_{\text{vir}} \approx 178 \Delta_m$. Therefore, a common choice for Δ_m is 200. Larger values of the reference overdensity probe the inner regions of the halo. This is typical of X-ray studies (where $\Delta = 500$ is commonly used).

In defining halo abundances, it is crucial to specify whether the reference density is evaluated at the present day or at the halo's own redshift. In the FoF and SO methods discussed above, one must choose a *reference density*

$$\rho_{\text{ref}}(z) = \begin{cases} \rho_c(z) = \frac{3H^2(z)}{8\pi G}, \\ \bar{\rho}_m(z) = \Omega_m(z) \rho_c(z), \end{cases} \quad (1.60)$$

where $H(z)$ is the Hubble parameter at redshift z and $\Omega_m(z)$ is the matter fraction at that epoch. Eq. 1.59 is then modified for the reference density ρ_{ref} as

$$M = \frac{4\pi}{3} \Delta \rho_{\text{ref}}(z) R_\Delta^3, \quad (1.61)$$

with both Δ and the evolving $\rho_{\text{ref}}(z)$ evaluated at the halo's formation redshift [112]. Using the redshift-dependent reference density ensures that the *physical* overdensity criterion remains consistent across cosmic time, and it is this choice that must be matched when comparing mass functions or converting between different M_Δ definitions.

Mass definition translation. Accurate modelling of halo properties requires a consistent mass definition across all components of our analysis (the mass function, the mass-concentration relation, and the halo profiles). In our work in Chapters 3 and 4, we adopt a ρ_c -based mass definition with an overdensity of $\Delta = 500c$, which is a natural choice for tSZ studies. This is because tSZ data, through the thermal Sunyaev-Zeldovich effect, primarily probes the inner regions of clusters where the gas is hot and dense, and the generalised NFW (GNFW) pressure profile is calibrated at Δ_{500c} (e.g. [14]).

However, many of the widely used mass functions, such as those from [194] and [195], and the mass-concentration relations like that of [62], are calibrated using alternative mass definitions (e.g. Δ_{200m} , Δ_{200c} , Δ_{vir}). To maintain consistency in our modelling, it is necessary to translate these alternative definitions into a baseline 500c mass definition.

To overcome this issue, we follow the procedure used by [155] and [36]: our baseline mass definition is ρ_c -based with $\Delta = 500$ (or any other target overdensity), as used in [14]. At each redshift, we translate this into a Δ value for a ρ_M -based definition, which we use to compute the mass function from the parameterisations of [194, 195]. We also re-derive the concentration-mass relation of [62] for a ρ_c -based $\Delta = 500$ from their $\Delta = 200$ parameterisation by integrating the NFW profile to the corresponding halo radius. Within the redshifts covered by our analysis, we find that this is well fit by

$$c_{500c}(M, z) = A \left(\frac{M}{M_{\text{pivot}}} \right)^B (1+z)^C, \quad (1.62)$$

which is the empirical form factor often used when parameterising the M - c relation, discussed further in Section 1.3.2.3.

Translating the [62] concentration relation to be Δ_{500c} -based, we find $M_{\text{pivot}} = 2.7 \times 10^{12} M_\odot$ and $(A, B, C) = (3.67, -0.0903, -0.51)$. This result is used in Chapter 3 as our fiducial model, and in Chapter 4 as one of our robustness checks.

1.3.2.2 Halo bias function

Another ingredient commonly found in clustering models of haloes is the *halo bias*. It refers to the relationship between the clustering of haloes and that of the underlying matter distribution and links the non-linear halo distribution to the linear matter overdensity field.

Peak-background split framework. This framework [110, 49] provides a theoretical basis to understand the concept of halo bias. The density field is decomposed into a long-wavelength background density (δ_b), representing the large-scale environment, and a short-wavelength peak density (δ_h), representing the small-scale fluctuations that ultimately collapse to form haloes.

Haloes are more likely to form in regions where $\delta_b > 0$, as the background overdensity effectively reduces the threshold for collapse (denser regions). This leads to enhanced clustering of haloes in overdense regions, introducing a bias in their spatial distribution.

Conditional mass function & Lagrangian bias. We can now predict the conditional or environmental mass function (i.e. the density of haloes conditioned on the density of their environment). We express this as a function of the collapse threshold as

$$n(M, \delta_c | \delta_b) = n(M, \delta_c - \delta_b) \simeq n(M, \delta_c) \left(1 + b_h^L(M) \delta_b \right), \quad (1.63)$$

where $n(M, \delta_c)$ is the global mass function. The mass function is Taylor-expanded to first-order for small δ_b in the last equality, and the quantity $b_h^L(M)$ is defined as the *Lagrangian halo bias*:

$$b_h^L(M) \equiv -\frac{d \log n(M)}{d \delta_c} = -\frac{\nu}{\delta_c} \frac{g'(\nu)}{g(\nu)}. \quad (1.64)$$

Normalisation. To ensure consistency with the total matter distribution, the effective bias of all haloes must satisfy

$$b_M = \frac{\int_0^\infty dM M b_h(M) n(M)}{\int_0^\infty dM M n(M)} = 1, \quad (1.65)$$

or

$$\int_0^\infty dM M b_h(M) n(M) = \rho_{M,0}. \quad (1.66)$$

This imposes a normalisation on the bias function:

$$\int \frac{d\nu}{\nu} g(\nu) b_h(\nu) = 1 + g(\nu=0) - g(\nu \rightarrow \infty) = 1, \quad (1.67)$$

where, in the second equality, we assume b_h is derived from the mass function. The normalisation condition is therefore satisfied if $g(\nu)$ drops to zero at $\nu=0$ and $\nu=\infty$.

Comparison with simulations. Simulations by Tinker et al. [195] provide updated fitting functions for halo bias across a range of masses and redshifts. Their results confirm the evolution of bias with redshift.

1.3.2.3 The halo profile

The final key ingredient of the halo model is the halo density profile, describing the distribution of matter around haloes of different masses. Empirically, from N-body simulations, it is known that the dark matter density profile is well-described by the so-called Navarro-Frenk-White (NFW) density profile [142],

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)\right)^2}, \quad (1.68)$$

where r_s is the scale radius.

The mass-concentration (M - c) relation describes how the concentration parameter, c , of dark matter haloes depends on their mass, M , and redshift, z . The concentration parameter

quantifies the density profile of haloes, and is often modelled using the NFW profile (Eq. 1.68). The concentration parameter is then defined as

$$c = \frac{R_{\text{vir}}}{r_s}, \quad (1.69)$$

where R_{vir} and r_s are the virial radius and a scale radius, respectively. Note that in the numerator one may use any overdensity index, probing different parts of a halo. In the definition above, we use the virial radius as this is a common choice in the literature.

Empirical studies and simulations suggest that c decreases with halo mass and redshift, mainly due to the hierarchical nature of structure formation. A common parameterisation of the M - c relation is [62]

$$c = A \left(\frac{M}{M_{\text{pivot}}} \right)^B (1+z)^C, \quad (1.70)$$

where A is the normalisation, M_0 is a pivot mass, and B , C describe the dependence with mass and with redshift, respectively. These parameters are usually empirically fitted from N-body simulations. In Chapters 3 and 4 we have used the mass-concentration relation described in [62].

The M - c relation plays a crucial role in understanding the evolution of haloes, and their impact on observables such as clustering. For instance, any deviations in the M - c relation due to baryonic effects might propagate to the final cosmological parameter constraints.

1.3.3 Corrections

Although the formalism of the halo model is very powerful in cosmological inference, its very assumptions lead to a lack of accuracy in certain regimes [15].

The halo model is known to underpredict the transition region between the 2-halo and 1-halo-dominated regimes [15, 138]. There are several ways to account for this inaccuracy and smooth out the transition region. One of them is to use simulations of the matter power spectrum in order to fudge an approximate correction factor for these scales. For example, using the revised HALOFIT (HF) prescription for the matter-matter power spectrum can be used to correct the predicted halo model (HM) power spectrum by a scale-dependent factor [190]:

$$R(k) \equiv \frac{P_{\text{HF}}(k)}{P_{\text{HM}}(k)}. \quad (1.71)$$

It is important to note that this factor is only accurate for the power spectrum for which the HALOFIT was modelled, and it will depart for quantities that do not exactly trace cold matter.

All in all, we wish to smooth out the transition regime of the halo model to account for more complex correlations arising from neither pairs of different haloes, nor strictly one individual halo. Mead et al. (2020) [137] suggest a one-parameter departure from the standard halo model formalism to smooth the overall power:

$$\tilde{P}_{\text{HM}}(k) = (P_{1h}^\alpha + P_{2h}^\alpha)^{1/\alpha}, \quad (1.72)$$

where α controls the sharpness ($\alpha > 1$) or smoothness ($\alpha < 1$) of the transition region. For one additional parameter, we can make α a time-dependent quantity via $\alpha = \alpha_0 + \alpha_p (a - a_*)$, where a_* is a pivot scale factor.

The problem with the 1-halo to 2-halo transition is likely associated with the relatively simple model used to describe the clustering of dark-matter haloes in the 2-halo term. Other approaches, derived from N -body simulations, have also been proposed in the literature [139].

As a consequence of conservation of mass and momentum, the 1-halo term must increase with k^4 at large scales. However, the standard halo model predicts that the 1-halo term tends to a constant at those scales, in absence of negative density perturbation in its theoretical formalism. To correct for this unphysical contribution to the overall power spectrum, Mead et al. (2020) [137] suggest the following scale-dependent modification for the 1-halo term:

$$P_{1h}(k) \rightarrow P_{1h}(k) \frac{(k/k_*)^4}{1 + (k/k_*)^4}. \quad (1.73)$$

This departure introduces another free parameter (k_*), while preserving the $\propto k^4$ behaviour of the 1-halo term at large scales. As with α , this parameter can also become time-dependent with the addition of another pivot parameter $k_{*,p}$.

There are more corrections described in the HMCode paper [137], which will not be discussed here as they depart from the scope of this thesis.

1.4 Observables

1.4.1 Projected anisotropies

The main cosmological observables we will use in this thesis can be described as a projected quantity $u(\hat{\theta})$ related to a three-dimensional field $U(\mathbf{r})$ through some radial kernel $W_u(\chi)$:

$$u(\hat{\theta}) = \int d\chi W_u(\chi) U(\chi\hat{\theta}). \quad (1.74)$$

Any projected quantity can be decomposed into its spherical harmonic coefficients $u_{\ell m}$, the covariance of which is the so-called angular power spectrum,

$$\langle u_{\ell m} v_{\ell' m'}^* \rangle \equiv C_{\ell}^{uv} \delta_{\ell \ell'} \delta_{mm'}. \quad (1.75)$$

Below the overall projection in Eq. 1.74, each tracer $u(\hat{n})$ is built by integrating a three-dimensional field $U(\mathbf{r})$ along the line of sight, with a kernel $W_u(\chi)$ that encodes where that tracer is most sensitive. In other words, $W_u(\chi)$ is the *window function* in radius: it peaks at redshifts that contribute most to our observable and falls off elsewhere. Before diving into the individual tracers, Fig. 1.4 shows all individual kernels used in this thesis, plotted against comoving radial distance χ .

We typically choose $W_u(\chi)$ so that

$$\int_0^{\chi_*} d\chi W_u(\chi) = 1 \quad \implies \quad u \sim U \quad (1.76)$$

up to an overall amplitude; for kernels that carry physical units (e.g. the Compton-y kernel) the integral will equal the prefactor with units.

Because both the tSZ and CMB-lensing kernels are relatively broad in comoving distance, their window functions vary slowly compared to the oscillatory Bessel functions that appear in the exact angular power spectrum integrals. As a result, the Limber approximation (discussed in the paragraphs below) is extremely accurate for these tracers over all multipoles of interest.

The angular power spectrum can be related to the 3D power spectrum of the associated 3D fields P_{UV} via:

$$C_{\ell}^{uv} = \int d\chi \frac{W_u(\chi) W_v(\chi)}{\chi^2} P_{UV} \left(k = \frac{\ell + 1/2}{\chi}, z(\chi) \right). \quad (1.77)$$

Note that Eq. 1.77 is only valid in the *Limber approximation* [124, 111], which is accurate for broad radial kernels. This approximation is adequate for the kernels and scales we will explore here.

It is worth emphasising that Eq. 1.77 is an approximation. Calculating the theoretical prediction for angular power spectra exactly is generally a slow operation, since it involves integrating over highly oscillatory spherical Bessel functions $j_{\ell}(k\chi)$. For sufficiently broad radial kernels, which is the case for all observables we explored in this thesis. The Limber replaces these spherical Bessel functions with a delta function:

$$j_{\ell}(k\chi) \rightarrow \sqrt{\frac{\pi}{2\nu}} \delta(\nu - k\chi), \quad (1.78)$$

where $\nu \equiv \ell + 1/2$, and leads to Eq. 1.77.

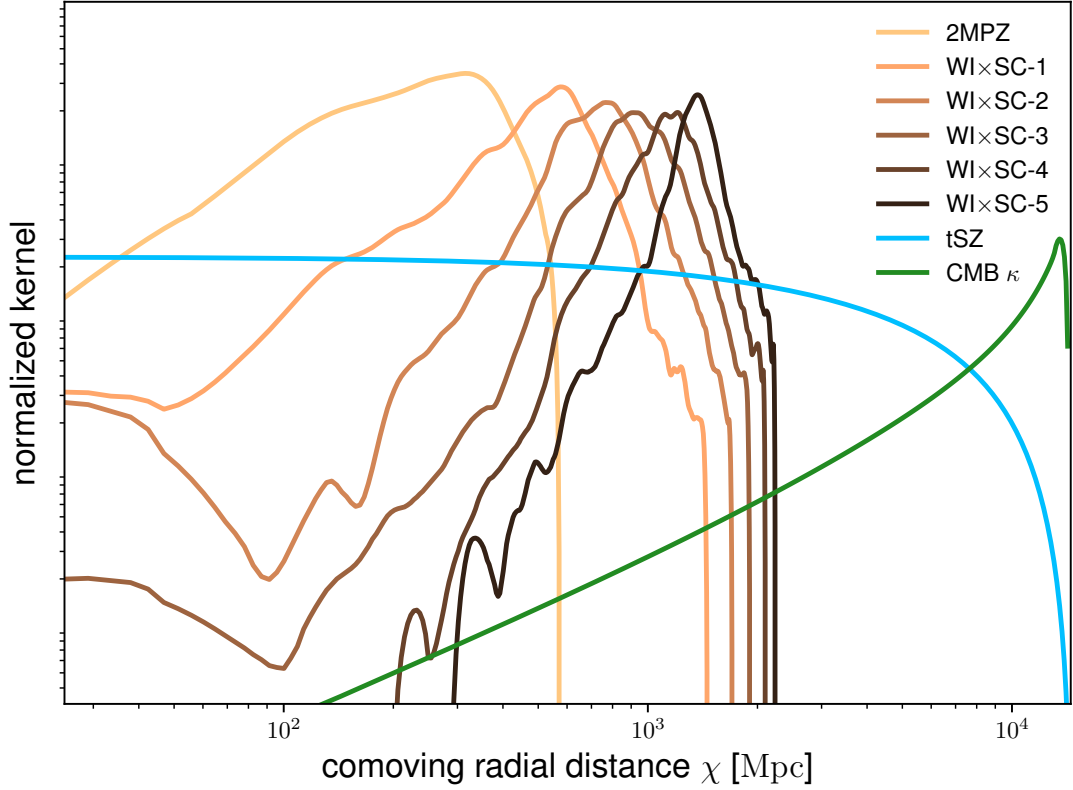


Figure 1.4: Comoving distance window functions for our main tracers. The six coloured curves in shades of copper show the galaxy density kernels $W_g(\chi)$ for each tomographic bin. The blue curve shows the tSZ Compton- y kernel $W_y(\chi)$, which is broad and sensitive to gas pressure. The green curve shows the CMB lensing convergence kernel $W_\kappa(\chi)$, which spans the entire line of sight. The broadness of both W_y and W_κ justifies our use of the Limber approximation in computing their angular power spectra.

In Eq. 1.77, the 3D power spectrum is analogously defined as the covariance of the Fourier-space 3D quantities:

$$\langle U(\mathbf{k})V^*(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P_{UV}(k). \quad (1.79)$$

We model the 3D power spectrum using the halo model, which we describe in the next section.

In the next sections, we introduce the most relevant projected tracers of the large-scale structure we use.

1.4.2 Weak lensing and CMB lensing

Matter gives rise to gravitational fields which distort the path light rays take to reach us. This translates to a coordinate transform between the nominal coordinates of galaxies and

the observed ones, as well as to a change in the observed ellipticity of galaxies. Measuring and correlating the shapes of galaxies allows us to infer the underlying matter field in the Universe through the so-called “cosmic shear” effect. However, we can also study lensing through its effect on the CMB, an effect called *CMB lensing* [121, 19].

Lensing smooths out the CMB power spectrum, producing new non-Gaussian signals and *B*-mode polarisation. At the same time, each photon is independently lensed multiple times, which makes the lensing signal dominate in comparison to other effects. It also changes the shape and size of the acoustic peaks of the power spectrum. More interestingly, lensing leads to a non-Gaussian feature in the observed CMB map, correlating the observed primordial fluctuations on different angular scales in a characteristic manner. This can be exploited to make maps of the CMB lensing potential [121], which in turn can be used to probe the large-scale structure between recombination and today.

The lensing potential has the form

$$\Phi_L(\hat{\theta}) = -2 \int_0^{\chi_*} d\chi \psi(\chi \hat{\theta} - \eta_0 - \chi) \frac{\chi_* - \chi}{\chi_* \chi}, \quad (1.80)$$

where χ is the comoving distance, and ϕ is the Newtonian (Weyl) potential. It is also common to work with the CMB lensing *convergence* $\kappa(\hat{\theta})$ [121], defined as the angular Laplacian of the lensing potential $\kappa \equiv \nabla_{\hat{\theta}}^2 \Phi_L / 2$. Lensing convergence can be directly connected with the matter overdensity δ_m via

$$\kappa(\hat{\theta}) = \int d\chi W_\kappa(\chi) \delta_m(\chi \hat{\theta}), \quad (1.81)$$

where the CMB lensing kernel is

$$W_\kappa(\chi) \equiv \frac{3}{2} H_0^2 \Omega_m \frac{\chi}{a(\chi)} \frac{\chi_* - \chi}{\chi_*} \Theta(\chi_* - \chi), \quad (1.82)$$

with χ_* the comoving distance to the last-scattering surface.

1.4.3 Galaxy clustering

By measuring the positions of galaxies in the sky and their redshifts, we obtain a three-dimensional map of their distribution, which is an indirect tracer of the underlying matter field. This information can be cross-correlated with other cosmological observables such as the CMB or weak lensing maps. Since the galaxy redshifts select specific epochs in cosmic history, the cross-correlation effectively picks out contributions from those redshifts, enabling tomographic measurements of the target quantity.

In its simplest incarnation, the galaxy overdensity field is related to the underlying matter field via a factor called *galaxy bias* (b_g):

$$P_{\text{gg}}(k, z) = b_g^2 P_{\text{mm}}(k, z), \quad (1.83)$$

where indices gg and mm indicate clustering and matter-matter correlations, respectively.

Clustering suffers from a number of systematics. Firstly, as galaxies (like everything else) do not follow the homogeneous Hubble flow, their measured redshifts usually have an intrinsic scatter due to their peculiar motions. For example, a bound system (i.e. a galaxy) seems elongated in redshift space (this effect is called *Fingers of God*), and a collapsing system seems squashed in redshift space (this effect is called the *Kaiser effect*). The name given to these kinds of distortions is *redshift space distortions* (RSDs), illustrated in Fig. 1.5. As a consequence of RSDs, the power spectrum is no more isotropic. Because RSDs are caused by the local gravitational field of the observed galaxies, they can be used to constrain the growth rate, $f = d \ln G / d \ln a$, as well as to test theories of gravity [110].

The observed galaxy number counts can also be affected by magnification bias, which changes the number of observed number counts due to lensing magnification.

These two effects are negligible for the galaxy samples explored here, due to the relatively large redshift errors associated with photometric redshifts, and to the low redshifts covered by our samples. We will neglect them in what follows.

In this work we explore the projected overdensity of galaxies, δ_g , related to the three-dimensional overdensity Δ_g via

$$\delta_g(\hat{\theta}) = \int dz p(z) \Delta_g(\chi(z)\hat{\theta}, z), \quad (1.84)$$

where $p(z)$ is the redshift distribution of the sample. The radial kernel associated with galaxy clustering is thus proportional to the redshift distribution of the sample under study:

$$W_g(\chi) = H(z(\chi)) p(z(\chi)). \quad (1.85)$$

1.4.4 Thermal Sunyaev-Zeldovich effect

Sunyaev & Zeldovich showed in 1972 ([188]) that radiation from the CMB is affected by hot gas in the Universe. On their way to us, photons interact with energetic electrons and are boosted to higher energies via inverse-Compton scattering, distorting the background view of the CMB. This signal has a characteristic spectral energy distribution (SED) and can be easily separated from other processes distorting the background light. Picking up this signal allows us to examine the gas properties of the foreground clusters.

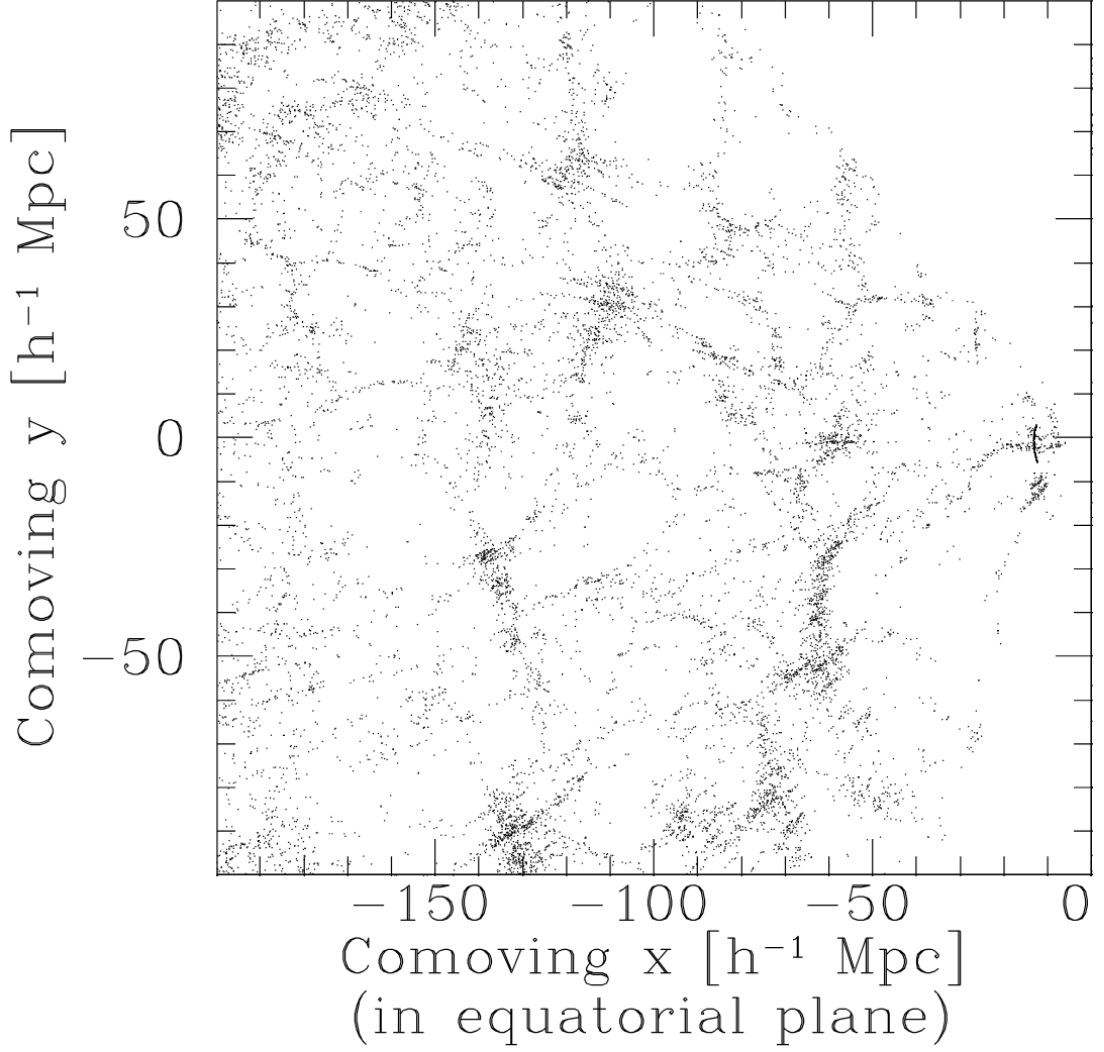


Figure 1.5: Illustration of the effect of redshift-space distortions (‘fingers of god’) in a part of an equatorial slice in the SDSS sample of galaxies [193].

The thermal Sunyaev-Zeldovich (tSZ) effect is given by the Compton- y parameter as

$$y(\hat{\theta}) = \frac{\sigma_T}{m_e c^2} \int \frac{d\chi}{(1+z)} P_e(\chi \hat{\theta}), \quad (1.86)$$

where $P_e = n_e T_e$ is the electron pressure (n_e and T_e are the electron density and temperature respectively), σ_T is the Thomson scattering cross-section for the electron, and m_e is the electron mass. The $1+z$ factor exists because of the bandwidth smearing due to redshift and comes from converting the physical pressure (energy density) into comoving units. For a fully ionised gas, the electron pressure is directly related to the total thermal gas pressure through $P_{\text{th}} = P_e \frac{(8-5Y)}{(4-2Y)}$, where Y is the helium mass fraction (with $Y \simeq 0.24$).

From Eq. 1.86, it is clear that we can link tSZ with the statistics of the pressure in the

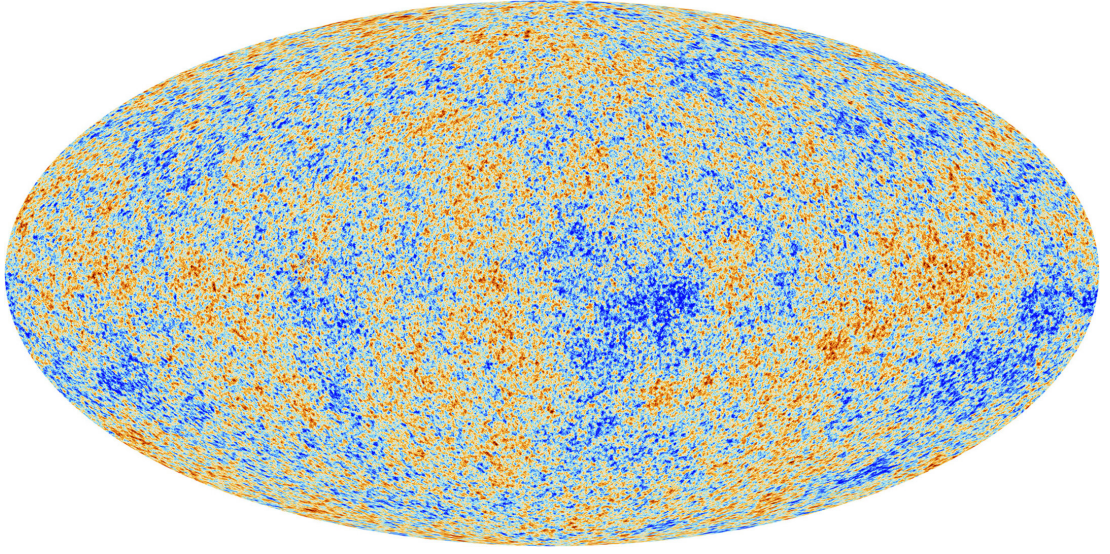


Figure 1.6: The cosmic microwave background. Hot spots differ from cold spots by 10^{-5} .

large-scale structure. The radial kernel associated with the Compton- y map is then

$$W_y(\chi) = \frac{\sigma_T}{m_e c^2} \frac{1}{1+z}. \quad (1.87)$$

1.4.5 Other probes of structure

1.4.5.1 The cosmic microwave background

As briefly described in Section 1.1.3, the cosmic microwave background arises from the surface of last scattering as photons decouple from the sea of free electrons, after the latter combine with protons to form neutral hydrogen. Photons that stream freely in the Universe are received uniformly from the entire sky today, and, as they have lost energy due to cosmic expansion, they are observed in the microwaves having a temperature of 2.7 K.

Temperature fluctuations on the last scattering surface comprise the CMB primary anisotropies. These have an amplitude of 10^{-5} relative to the mean temperature of the CMB surface (2.72 K). The spectrum of the CMB approximates very well a blackbody spectrum, and the statistics of the temperature fluctuations are governed by the Boltzmann equation, as well as by the evolution of the photon energy density during recombination.

To study which effects play an important role when we observe the CMB, we make the assumption that recombination was instantaneous, at a point in time when the temperature dropped below a certain threshold. Recombination occurred rapidly, so this first assumption is not restrictive. Then, expressing the peak frequency of the blackbody spectrum as

$\nu \propto k_B T/h$, we estimate that the temperature fluctuation is

$$\frac{\delta T}{\bar{T}} = \frac{\delta \nu}{\bar{\nu}}, \quad (1.88)$$

where the observed fluctuations are related to the unperturbed ones via $x = \bar{x} + \delta x$.

Now, using Eq. 1.88 into the equation for the redshift propagation equation for the perturbed FLRW metric, and discarding terms attributed to the gravitational potential and the peculiar velocity of the observer, which only give rise to the monopole and dipole ($\ell = 0, 1$), we derive the following equation when we use the Stefan-Boltzmann law, $\rho \propto T^4$:

$$\frac{\delta T}{T}(\hat{\mathbf{n}}) = \left(\frac{\delta_\gamma}{4} + \psi - \hat{\mathbf{n}} \cdot \mathbf{v} \right) (\eta_{\text{rec}}, \chi_{\text{rec}} \hat{\mathbf{n}}) + \int_{\eta_{\text{rec}}}^{\eta_0} d\eta (\phi' + \psi')(\eta, \chi \hat{\mathbf{n}}), \quad (1.89)$$

where $\delta_\gamma = 4\delta T/T|_{\text{intrinsic}}$ is the radiation energy overdensity.

The observed fluctuations thus result from four different effects: (i) the intrinsic fluctuations produced at recombination (δ_γ), (ii) the Sachs-Wolfe effect (ψ), (iii) the integrated Sachs-Wolfe effect (integral term), and (iv) the Doppler term ($\hat{\mathbf{n}} \cdot \mathbf{v}$). The late-time integrated Sachs-Wolfe effect, although detected to a high significance via cross-correlations [150], does not play an important role in the CMB power spectrum, with the main contribution arising from the radiation epoch.

1.4.5.2 Kinematic Sunyaev-Zeldovich effect

The kinematic Sunyaev-Zeldovich effect (kSZ) also entails inverse Compton scattering of CMB photons, similarly to the tSZ. However, the CMB photons are not scattered off of hot gas; rather, they scatter off of moving gas in the Universe. This allows us to build maps of *peculiar velocity* (line-of-sight velocity relative to the Hubble flow). The difference in the observed temperature of the CMB has to be separated from the tSZ and is related to peculiar velocity v_{pec} as

$$\Delta T_{\text{kSZ}} = -T_{\text{CMB}} \frac{v_{\text{pec}}}{c} \tau, \quad (1.90)$$

where τ is the optical depth of the moving gas [91, 84].

1.4.5.3 Cosmic Infrared Background

Among the three major extragalactic backgrounds most often studied in work on the large-scale structure, the CIB has the second-largest integrated intensity [60]. It arises from the combined IR emission of all galaxies over cosmic time, peaking at $\sim 200 - 3000$ GHz. It is currently the one most poorly understood, primarily because of all the underlying processes that occur at scales much smaller than the typical size of a halo, which produce that light.

For example, it is prevalent where star formation occurs, and where vast dust clouds absorb UV light from new stars and re-emit it in the infrared band.

The light intensity in the infrared is

$$I_\nu(\hat{\mathbf{n}}) = \int d\chi \frac{j_{(1+z)\nu}(\chi\hat{\mathbf{n}})}{4\pi(1+z)}, \quad (1.91)$$

where j_{ν_e} is the specific intensity (emitted power per unit comoving volume per unit bandwidth). The $1+z$ factor exists because of the bandwidth smearing due to redshift. The emitted frequency is related to the observed one via a Doppler transformation as $\nu_e = (1+z)\nu$. The radial kernel of the CIB is

$$q(\chi) = \frac{1}{4\pi(1+z)}. \quad (1.92)$$

Cross-correlations with the CIB may be used to further our understanding of this background, as well as important astrophysical processes, such as the physics and history of star formation. For example, using techniques such as the ones we will discuss in Chapters 3 and 4, one can measure the mean star-formation rate density weighted by halo bias at different redshifts. We did so in [107], although we do not include this paper in this thesis.

1.4.5.4 Baryon Acoustic Oscillations

Photons in the primordial plasma, during the initial phases of cosmic expansion, were strongly coupled to baryons via Thomson scattering. This phenomenon caused a wave of outward pressure, which was counteracted by inward pressure caused by gravity. In turn, this led to oscillations in the baryon density. As the Universe expanded and electrons combined with protons in the plasma to form neutral hydrogen, photons began to stream freely and decoupled from baryons. The acoustic oscillations then froze in place and started to only evolve with the Hubble flow. The density peaks then became the seeds where the first clusters started to form through gravitational collapse [4, 210].

After decoupling from photons, baryons kept moving outward, conserving momentum, up to the baryon drag epoch. The wavelength of the baryon acoustic oscillations (BAOs) is given by

$$r_s(z_{\text{drag}}) = \int_0^{t(z_{\text{drag}})} c_s (1+z) dt, \quad (1.93)$$

where c_s is the speed of sound in the primordial plasma, and is a function of Ω_b and h .

To that end, the maximum distance baryons could travel outside of the centre of a gravitationally collapsing region provides us with a standard ruler with which we can characterise cosmic expansion. BAOs can be detected both on the CMB and on the large-scale structure [210].

1.5 Current & future experiments

Several current and upcoming cosmological missions aim to study the large-scale structure and evolution of the universe, shedding light on fundamental questions about its composition, origins, and fate. Examples of these cutting-edge missions include:

- **Planck Satellite (2009–2013):** Launched by the European Space Agency (ESA), the Planck satellite provides high-precision measurements of the CMB temperature and polarisation anisotropies. Its results have been instrumental in refining the standard cosmological model, testing alternative cosmological models, and constraining the cosmological parameters.
- **Atacama Cosmology Telescope (ACT) (2007–2022):** Located in the Atacama Desert, this ground-based CMB experiment measured small-scale anisotropies in the CMB. Its data complemented Planck, improving constraints on cosmological parameters, neutrino masses, and dark matter.
- **Dark Energy Survey (DES) (2013–2019):** Conducted using the Dark Energy Camera on the Blanco Telescope in Chile, DES mapped hundreds of millions of galaxies to study the effects of dark energy on the expansion of the universe. Its results include significant constraints on cosmological parameters and studies of large-scale structure through galaxy clustering and weak gravitational lensing.
- **DESI (Dark Energy Spectroscopic Instrument, 2021–ongoing):** Operating from Kitt Peak National Observatory, DESI is creating a 3D map of the universe by measuring redshifts of millions of galaxies. It aims to improve constraints on the dark energy equation of state, and to test theories of modified gravity.
- **Euclid Satellite (2023–ongoing):** Launched by ESA, Euclid is dedicated to understanding the nature of dark energy and dark matter. By combining weak lensing and galaxy clustering measurements, Euclid maps the geometry of the universe over the past 10 billion years, offering unprecedented insights into cosmic acceleration.
- **South Pole Telescope (SPT) (Ongoing):** Operating from Antarctica, SPT is designed to study the CMB with high resolution, particularly focusing on small-scale anisotropies. It has contributed to constraints on inflation, neutrino physics, and galaxy cluster properties through the Sunyaev-Zeldovich effect.

- **Kilo-Degree Survey (KiDS):** Using the Very Large Telescope (VLT) in Chile, KiDS focuses on weak gravitational lensing and galaxy clustering to study dark matter distribution and cosmic evolution. Its results are highly complementary to DES and Euclid.
- **Hyper Suprime-Cam (HSC, Subaru Telescope):** Located in Hawaii, HSC conducts wide-field imaging surveys to study large-scale structure, galaxy evolution, and weak lensing. Its high-resolution imaging provides valuable data for understanding dark energy and dark matter.
- **SPHEREx (2025):** A NASA mission to conduct an all-sky spectral survey, SPHEREx will probe large-scale structure, inflation, and cosmic history by studying the near-infrared background.
- **Vera C. Rubin Observatory (formerly LSST, Early 2020s–2030s):** The Rubin Observatory, under construction in Chile, will undertake the Legacy Survey of Space and Time (LSST). It will produce a vast dataset spanning billions of galaxies, enabling studies of dark matter distribution, supernovae, and time-domain astrophysics.
- **Nancy Grace Roman Space Telescope (Late-2020s):** Developed by NASA, the Roman Space Telescope will perform wide-field infrared surveys, exploring dark energy, exoplanets, and more. Its high-resolution imaging and spectroscopy capabilities will complement Euclid and ground-based surveys, deepening our understanding of the growth of cosmic structures.
- **Simons Observatory (2025–ongoing):** Located in the Atacama Desert, this project will advance studies of the CMB through temperature and polarization measurements. The observatory is designed to constrain inflationary models and improve our understanding of fundamental physics.
- **CMB-S4 (Planned for Late 2020s):** A next-generation ground-based experiment, CMB-S4 will consist of telescopes in Antarctica and Chile to measure the CMB with unmatched sensitivity. It will probe inflation, neutrino physics, and the matter-energy content of the universe.
- **SKA (Square Kilometre Array, 2020s–2030s):** The SKA will be the world’s largest radio telescope, enabling studies of neutral hydrogen distribution across cosmic time. It will address questions about reionization, dark matter, and gravitational waves.

- **LiteBIRD (Planned for late 2020s or early 2030s):** A Japanese-led space mission aiming to measure CMB polarization with extreme precision to detect primordial B-modes, potentially revealing evidence of inflation.

1.6 Overview of the thesis

This thesis describes the work undertaken throughout my doctoral studies. Chapter 2 provides a summary of the statistical methods often used in cosmological analyses. Chapter 3 describes measurements of the thermal history of the Universe in the last ~ 5 Gyr. Chapter 3 contains published work ([115]). Chapter 4 describes measurements of structure growth in the Universe in the last ~ 5 Gyr. Chapter 4 contains work which is in preparation for publication, similar to [215]. Chapter 5 concludes this thesis.

A large portion of my work focused on improving the Core Cosmology Library (CCL, [46] & *v2 in prep.*), which is the theoretical predictions pipeline of the LSST Dark Energy Science Collaboration (DESC). The relevant work has been omitted from this thesis as it diverges from its main scope. A paper that will describe much of this work, presenting the latest version of CCL, is currently in preparation, with me as one of the primary co-authors.



Figure 1.7: Artistic celebration of the Dark Energy Spectroscopic Instrument (DESI) year-one data, showing a slice of the larger 3D map that DESI is constructing during its five-year survey. By mapping objects across multiple periods of cosmic history with extremely high precision, DESI is allowing astronomers to make unprecedented measurements of dark energy and its effect on the accelerating expansion of the Universe. Credit: NSF/NOIRLab

Chapter 2

Statistical Methods in Cosmology

Modern cosmology mainly focuses on deriving constraints for a set of parameters through observations of the large-scale structure, where a wealth of cosmological information is contained. Small departures from homogeneity are often made evident at late times, and thus, we can derive the original set of cosmological parameters through observations of the underlying fields and their tracers in the Universe, analogous to pinpointing their value today and turning the clock backward to determine their value at early times.

Because of the sheer diversity of galaxies in the Universe and the complexity of sub-galactic-scale astrophysics, cosmology relies on statistical methods to extract the cosmological parameters. Observing large numbers of galaxies allows us to derive meaningful statistics about the distribution of matter and energy in the Universe, despite uncertainties in small-scale physics. In this context, the entire set of cosmological observables can be considered a single realisation of a stochastic process determined by the underlying cosmological parameters. As a result, the study of cosmological perturbations is inherently tied to the mathematics of random fields.

2.1 N-point statistics

As previously outlined, cosmological observations are predominantly analysed by employing statistical methods. These methods provide significant insight about the large-scale clustering of galaxies, and higher-order statistics probe the small-scale details of the cosmic web [211, 59, 148]. The term ‘N-point statistics’ refers to the number of map-level quantities being correlated. For example, with 2-point statistics, the pairwise correlation of fields at different points is analysed.

It should be noted that, although analysing higher-order statistics (e.g. correlation between multiple galaxies) can provide us with a wealth of information on the clustering of galaxies, the computation is quite resource-heavy, and often approximations that simplify

the calculation have to be used. These approximations come at the cost of discarding part of the information content for the clustering of galaxies.

2.1.1 Power spectra

Correlation functions and power spectra provide a direct means of comparing observations with theoretical models. In particular, angular power spectra are a natural choice for characterizing tracers in wide-field surveys, as they are computed directly from the observed angular positions of objects and naturally incorporate the geometry of the sky.

Computational methods have emerged to make the measurement of these angular power spectra feasible for large datasets, balancing accuracy and efficiency. Accurate measurements require accounting for noise, survey geometry, and other observational effects, which also play a key role in the estimation of the covariance of these tracers. The covariance quantifies uncertainties in the measurements and helps determine how different modes or scales are correlated, forming a crucial component of cosmological parameter inference. Taking advantage of these statistical tools, we can extract cosmological information encoded in the large-scale distribution of tracers and refine our understanding of the structure and evolution of the Universe.

The fluctuations of projected quantities $u(\hat{\theta})$ first presented in Eq. 1.74 can be decomposed into spherical harmonic coefficients $u_{\ell m}$ which form the basis for analyses of the angular power spectrum:

$$u(\hat{\theta}) = \sum_{\ell m} u_{\ell m} Y_{\ell m}(\hat{\theta}). \quad (2.1)$$

As presented in Eq. 1.75, for an isotropic field, the covariance of these coefficients is diagonal in ℓ and m , and so the angular power spectrum C_{ℓ}^{uv} is defined as $\langle u_{\ell m} v_{\ell m}^* \rangle = C_{\ell}^{uv} \delta_{\ell\ell'} \delta_{mm'}$. In essence, the angular power spectrum quantifies the variance of fluctuations on a given angular scale ℓ .

Measuring two-point functions from cosmological fields is often a prohibitively expensive computational task for high-resolution data, since the brute-force calculation scales as $\ell_{\max}^6$ (or $\ell_{\max}^4$ for some optimal algorithms), with $\ell_{\max}$ being the maximum multipole.

For this reason, power spectra are typically measured in cosmology using pseudo- C_{ℓ} algorithms (e.g. [94]) which scale similarly to the spherical harmonic transforms as $\ell_{\max}^3$. These algorithms have been adjusted to also work with arbitrary contaminants in scalar fields, and they can also measure E - and B -mode power spectra. Pseudo- C_{ℓ} algorithms work by first computing the raw (‘pseudo’) power spectrum from the observed map, which includes the effects of partial sky coverage (e.g. masking). These effects artificially couple different multipoles, introducing mode mixing. The pseudo- C_{ℓ} is then related to the true

power spectrum through a *mode-coupling matrix* $\mathbf{M}$ which encodes the impact of the survey geometry (and other contaminants).

For an incomplete sky coverage, a given field observed on the sphere, $\tilde{u}(\hat{\theta})$, can be modelled as a product of the true underlying field u , and a sky mask w ,

$$\tilde{u}^{\text{obs}}(\hat{\theta}) = w(\hat{\theta}) u(\hat{\theta}). \quad (2.2)$$

In the simplest scenario, the mask w is a binary map ($w \in 0, 1$) selecting the pixels in the sky that have been observed. More generally, w can be designed to optimally up- or down-weight different regions according to the inverse of their variance. Through the convolution theorem, the spherical harmonic transform of the observed field is a convolution of the harmonic transforms of the true field and the mask. Provided the mask and true field are uncorrelated, this translates into a similar result for the ensemble average of the observed power spectra $\tilde{C}_\ell^{uv}$,

$$\tilde{C}_\ell^{uv} = \sum_{\ell'} M_{\ell\ell'}^{uv} C_{\ell'}^{uv}, \quad (2.3)$$

where C_ℓ^{uv} is the true underlying power spectrum. $M_{\ell\ell'}^{uv}$ is the mode-coupling matrix, which depends solely on the masks of both fields and which can be computed analytically.

Roughly speaking, the pseudo- C_ℓ approach is then based on estimating M^{uv} and inverting it to yield an unbiased estimate of the power spectrum. The mode-coupling matrix can then be computed from the mask, and its inversion allows full recovery of the true power spectrum. Computing this matrix is often non-trivial, and regularisation techniques (discussed further in Section 2.1.2) are employed to ensure numerical stability in matrix inversion.

For galaxy autocorrelation, the pseudo- C_ℓ method requires an additional step to subtract the shot noise bias, which arises due to the discrete and finite sampling of galaxies in a survey. Shot noise introduces a scale-independent contribution to the power spectrum, effectively adding a constant offset. Since this is a Poisson-type noise (having to do with quantised ‘packets’ of data) it is proportional to the inverse number density of galaxies, n^{-1} , so the local noise variance is given by $\sigma_n^2 = 1/\bar{n}_\Omega$, where $\bar{n}_\Omega$ is the mean surface number density in units of sr^{-1} .

HEALPix ([76]) is one of the most popular packages to generate and operate on maps, and NaMaster ([9]) is a popular framework that unifies pseudo- C_ℓ computations and other operations on fields.

2.1.2 Covariance

Covariance matrices are a crucial part of cosmological data analysis, encapsulating the uncertainties and correlations between different data points or bins. In the context of large-scale structure (LSS) and cosmic microwave background (CMB) analyses, the covariance matrix provides a statistical description of how the measurements at different scales or angular separations are related to each other. Understanding and accurately estimating the covariance is vital for correctly determining parameter constraints and uncertainties.

2.1.2.1 Covariance from jackknife resampling

A common approach to derive the covariance matrix directly from the data is through techniques like *jackknife resampling*. The jackknife method involves dividing the data into a number of subsets, sequentially leaving out one subset and recalculating the statistical measure of interest (e.g. the power spectrum). This process is repeated for each subset, and the covariance is estimated from the variation between the different resampled estimates.

It is computed as

$$\mathbf{C}_{\text{JK}}(i, j) = \frac{N_{\text{JK}} - 1}{N_{\text{JK}}} \sum_{k=1}^{N_{\text{JK}}} (x_i^k - \bar{x}_i)(x_j^k - \bar{x}_j), \quad (2.4)$$

where N_{JK} is the number of jackknife samples, x_i^k is the measurement in the k -th jackknife resample, and $\bar{x}_i$ is the mean over all jackknife resamples. Finally, the factor $\frac{N-1}{N}$ accounts for the reduced number of degrees of freedom after resampling.

Estimating the covariance with jackknife resampling can be particularly useful for data-limited datasets, and it is especially powerful for quantifying statistical uncertainties in galaxy surveys, where there is a high degree of spatial correlation due to clustering and cosmic variance.

This work includes estimating the covariance using jackknife resampling to update the main diagonal of the covariance matrix and avoid underestimating the uncertainties arising from the data covariance matrix. This ensures that any off-diagonal correlations do not introduce artificially tight constraints to the error bars, and remains a conservative choice to estimate cosmological and astrophysical parameters.

2.1.2.2 Regularisation

Although the jackknife method is able to provide an estimate of the size of the measurement uncertainties in a model-independent way, it has some drawbacks. There are only so many jackknife regions of reasonable size that can be selected; therefore, the estimated covariance

is noisy at some level. Typically, the number of independent realisations used to estimate a sample covariance matrix should be at least one order of magnitude larger than the size of the data vector. Furthermore, since the footprint associated with the removal of each jackknife region is different from the overall footprint, this method is unable to recover the mode-coupling associated with the map geometry, by construction.

One would thus have to regularise the calculated covariance matrix to prevent it from becoming noisy or ill-conditioned (which would in turn lead to unstable or biased parameter fits). Typically, the techniques used for regularisation are

- **Shrinking the covariance matrix:** The matrix is ‘shrunk’ to a more stable form (a diagonal matrix), reducing the impact of noise. Covariance shrinkage does *not* diagonalise the matrix; instead it mixes the noisy estimate with a much better-conditioned “target” matrix (often the diagonal of the sample covariance, or even the identity). Concretely, one replaces

$$\mathbf{C}_{\text{JK}} \rightarrow (1 - \lambda)\mathbf{C}_{\text{JK}} + \lambda\mathbf{T}, \quad (2.5)$$

with $0 \leq \lambda \leq 1$ chosen to minimise the mean squared error (e.g. the Ledoit-Wolf estimator [118]). Here $\mathbf{T}$ might be the diagonal of $\mathbf{C}_{\text{JK}}$, or a scaled identity matrix. By pulling the small, noisy off-diagonal elements toward zero, shrinkage dramatically improves invertibility and reduces variance in the inverse covariance, without discarding all correlations.

- **Principal component analysis (PCA):** By identifying the most significant modes of the covariance matrix, PCA allows us to reduce the dimensionality of the data while retaining most of the variance. Principal component truncation *does* begin by diagonalising the sample covariance via

$$\mathbf{C} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^T. \quad (2.6)$$

One then discards eigenmodes whose eigenvalues λ_i lie below some threshold (i.e. sets those $\lambda_i \rightarrow 0$) and reconstructs a lower-rank covariance

$$\mathbf{C}_{\text{PCA}} = \sum_{i=1}^{N_{\text{kept}}} \lambda_i \mathbf{e}_i \mathbf{e}_i^T. \quad (2.7)$$

This throws away the noisiest directions entirely, reducing dimensionality and stabilising the matrix inversion, but at the cost of losing some true correlations if the cutoff is too aggressive.

Regularisation techniques ensure the robustness of the final parameter constraints by stabilising the inversion of the covariance matrix. However, one must take care to ensure that genuine correlations are not discarded in the process.

2.1.2.3 Building the covariance matrix

In cosmological analyses, in order to verify and improve the estimate of the covariance matrix, a second analytical estimator is often used. Firstly, the analytical covariance matrix is computed following what is broadly outlined in [116]. The covariance receives two main additive contributions, from the so-called *disconnected* and *connected* trispectra. The disconnected part is essentially the covariance matrix estimated under the assumption that all fields are Gaussian. In the absence of any sky masks, it is given by

$$\mathbf{C}^G(C_\ell^{uv}, C_{\ell'}^{wz}) = \delta_{\ell\ell'} \frac{C_\ell^{uv} C_\ell^{vz} + C_\ell^{uz} C_\ell^{vw}}{2\ell + 1}. \quad (2.8)$$

A sky mask introduces non-zero coupling between different ℓ modes. To account for these, we can use the method introduced in [64] and implemented in the NaMaster library [9] which has been shown to be an excellent approximation for LSS data [73].

We then compute the connected (i.e. non-Gaussian) contribution to the covariance matrix using the halo model as the 1-halo trispectrum [114, 52, 189, 100], given by

$$\mathbf{C}^{\text{NG}}(C_\ell^{uv}, C_{\ell'}^{wz}) = \int d\chi \frac{W_u(\chi)W_v(\chi)W_w(\chi)W_z(\chi)}{4\pi f_{\text{sky}} \chi^6} \times T_{uvwz}^{1h}\left(k = \frac{\ell + 1/2}{\chi}\right), \quad (2.9)$$

where

$$T_{uvwz}^{1h}(k, k') \equiv \int dM \frac{dn}{dM} \langle U(k|M)V(k|M)W(k'|M)Z(k'|M) \rangle.$$

The total covariance matrix is then $\mathbf{C} = \mathbf{C}^G + \mathbf{C}^{\text{NG}}$. We shall note at this point that calculating the connected term requires knowledge of the true parameters of our model, which we do not know a priori. To circumvent this issue, we may proceed as in Ando et al. [12], and estimate the covariance matrix in a two-step process. First, we obtain the best-fit parameters using only the Gaussian covariance, where the power spectra are directly computed from the data, and then we use those parameters to calculate the non-Gaussian contribution (as well as to recalculate the Gaussian contribution using the best-fit prediction for the power spectra).

In summary, the fiducial covariance matrix is obtained by combining jackknife resampling with the analytical computation of the covariance matrix. This gives us a more realistic estimate of the error bar sizes of the fitted parameters. The final covariance is therefore

$$\mathbf{C}_{ij} = \mathbf{C}_{ij}^{\text{ana}} \sqrt{\frac{\mathbf{C}_{ii}^{\text{JK}} \mathbf{C}_{jj}^{\text{JK}}}{\mathbf{C}_{ii}^{\text{ana}} \mathbf{C}_{jj}^{\text{ana}}}}, \quad (2.10)$$

where $\mathbf{C}^{\text{JK}}$ and $\mathbf{C}^{\text{ana}}$ are the jackknife and analytical covariance matrices respectively, and the subscripts i and j label the individual data points (or bins) in our data vector

$\mathbf{d} = \{d_1, d_2, \dots, d_N\}$. Thus, $\mathbf{C}_{ii}^{\text{JK}}$ and $\mathbf{C}_{jj}^{\text{JK}}$ are the *diagonal* elements of the jackknife covariance for bins i and j respectively. In this process, one also has to verify that the resulting covariance matrices are well-behaved (i.e. they are invertible, and their eigenvalues have a reasonable dynamical range).

To summarise, this hybrid approach is necessary since the analytical covariance captures the full off-diagonal structure (arising from mode coupling and survey geometry), but it can underestimate the true error bars if, for instance, unmodelled systematics or non-Gaussianities are present. To that end, the jackknife covariance provides a more data-driven scale for the diagonal uncertainties, but its off-diagonal elements are typically noisy (and may even make the matrix singular).

2.1.2.4 Covariance and inference

The covariance matrix plays a central role in parameter estimation, as it is used to construct the likelihood function that quantifies how well the theoretical model fits the data. Typically, we define the likelihood $\mathcal{L}$ to be

$$\mathcal{L} \equiv p(\mathbf{d}|\vec{\theta}) \propto \exp\left(-\frac{1}{2}(\mathbf{d} - \mathbf{t}(\vec{\theta}))^T \mathbf{C}^{-1}(\mathbf{d} - \mathbf{t}(\vec{\theta}))\right), \quad (2.11)$$

where $\mathbf{d}$ is the data vector, $\mathbf{t}$ is its theoretical prediction, depending on a set of parameters $\vec{\theta}$, and $\mathbf{C}^{-1}$ is the inverse of the covariance matrix (also called the *precision matrix*). If the covariance is not accurately estimated, it will lead to either under- or over-estimation of the confidence intervals for the cosmological parameters. Thus, ensuring that the covariance matrix is accurate is as crucial as the modelling of the theoretical predictions [81, 82].

Often, working with the *log-likelihood* is easier because it returns more reasonable numbers and the algorithm becomes more stable. This is a trivial one-to-one mapping, particularly as we define likelihood as an exponential to some power. Thus, in practice, we aim to maximise the log-likelihood. This can be easily achieved by employing a root-finding algorithm.

By using a combination of analytically computed data covariance and covariance estimated through jackknife resampling, we ensure that the uncertainties are properly quantified, leading to robust cosmological constraints.

2.2 Cosmological inference

Broadly, inference aims to fit models to existing data. In Cosmology, our aim is to describe the Universe at the largest scales adequately, and with as few parameters as possible, in order to acquire a better grasp of the processes underlying its evolution.

There are two schools of interpretation of statistical data in order to derive conclusions regarding experiments in Physics. One is the frequentist approach, and the other one is the Bayesian approach. The former is used in fields such as Particle Physics, where a certain experiment can be repeated multiple times, and thus information is extracted from the sample statistics of a series of repeated experiments. This is not the case with cosmology, where the stochastic process which has derived our Universe has only happened once and will never happen again: there is only one realisation of the Universe, and we ought to use that one only.

Owing to the limited supply of data in the sky, we will henceforth be describing the Bayesian data analysis approach.

2.2.1 Bayesian analysis

In the Bayesian framework, we quantify probability as a belief that a certain model adequately fits a certain data set. Bayes' theorem states that

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}, \quad (2.12)$$

where $p(A|B)$ is the probability of A given B .

Interpreting A as the parameters of our model and B as our data, we can break down this equation into its different terms:

- $p(A|B)$ is the posterior distribution function, and what a typical cosmological analysis seeks to find. From this distribution we derive the statistical properties of the estimated parameters.
- $p(B|A)$ is the likelihood function. This is constructed before the analysis and consists of a forward model allowing us to compute theoretical predictions, given a set of parameters values that go into the model.
- $p(A)$ is the prior probability distribution function, which encompasses our prior knowledge about the inferred parameters. The prior could be derived either by previous independent measurements, or by estimates of the posterior distribution.
- $p(B)$ is the (Bayesian) evidence, which is the probability of getting an observable B over the entire set of values for A .

A typical working scenario in cosmology consists of a theoretical model (such as the Λ CDM model) which describes a quantity (such as the matter power spectrum) in the Universe. This model has a set of free parameters, $\vec{\theta}$, (such as the relative densities of cold

dark matter, baryons, or the Hubble constant), which we attempt to fit to an observed data vector $\mathbf{d}$. Applying the Bayesian formalism to the posed question, we get

$$p(\vec{\theta}|\mathbf{d}) = \frac{p(\mathbf{d}|\vec{\theta})p(\vec{\theta})}{p(\mathbf{d})}, \quad (2.13)$$

where we aim to find the parameter set $\vec{\theta}_{\text{bf}}$ which best fits the data, as well as the posterior uncertainties on these parameters, given our prior knowledge.

2.2.2 Likelihood

The likelihood is the quantity describing the probability of acquiring a set of data $\vec{\theta}$ given a set of parameters in our model,

$$\mathcal{L}(\vec{\theta}) \equiv p(\mathbf{d}|\vec{\theta}). \quad (2.14)$$

We see that the likelihood is a function of $\vec{\theta}$, though it is not its probability distribution as it is normalised over $\mathbf{d}$.

The next step to constructing the likelihood is to create an estimator of θ . A typical choice is the *maximum likelihood estimator* (MLE). The MLE assumes that the parameter set that best fits the data vector is that which maximises the likelihood.

In the Bayesian framework, rather than simply maximising the likelihood, we combine it with a prior distribution $P(\vec{\theta})$ to obtain the posterior distribution via Bayes' theorem (Eq. 2.13). The posterior then encapsulates our updated knowledge about the parameters, having considered the data. Therefore, instead of relying on a single point estimate like the MLE, Bayesian inference typically samples the posterior distribution (e.g. via Markov Chain Monte Carlo methods, discussed in the next sub-section) to obtain confidence intervals for the parameters.

2.2.3 Markov Chain Monte Carlo

Finding the model parameters that best fit the measured data is a relatively simple task: a minimisation problem. However, calculating the uncertainties requires exploration of the probability space for different parameter combinations, which can be quite laborious. To that end, Markov Chain Monte Carlo (MCMC) techniques are used to adequately sample the posterior probability distribution. MCMCs are particularly useful in high-dimensional parameter spaces where grid-based methods become computationally expensive.

In cosmology, MCMC techniques are widely used to extract constraints from observational data by efficiently sampling from the parameter space, determining the likelihood of various astrophysical models.

The most common MCMC method is the *Metropolis-Hastings algorithm*, which constructs a Markov chain (a stochastic process of sequential events) whose equilibrium distribution is the desired posterior. The algorithm proceeds as follows:

1. Start with an initial set of parameter values θ_0 ;
2. Propose a new set of parameters θ' by drawing from a proposal distribution $q(\theta'|\theta)$;
3. Compute the acceptance ratio

$$A = \min \left(1, \frac{p(\mathbf{d}|\theta')p(\theta')q(\theta|\theta')}{p(\mathbf{d}|\theta)p(\theta)q(\theta'|\theta)} \right); \quad (2.15)$$

4. Accept θ' as the new state of the Markov chain with probability A ; otherwise, keep the current state θ .

After many iterations, the Markov chain will sample from the posterior distribution, providing a comprehensive view of the likelihood landscape, and allowing marginalisation over nuisance parameters, as well as placing constraints on the inferred parameters.

From the steps outlined above, it is evident that the Markov chain requires several iterations to adequately sample from a distribution that resembles the posterior. Formally, we say that the Markov chain ‘burns in’. A key part of inference requires locating where the Markov chain is burning-in and discarding those steps when estimating the posterior, or the estimation will be biased until the chain has converged.

Another important aspect of inference is ensuring that the Markov chains have sufficiently explored the posterior distribution. Once this is achieved, we formally say that the Markov chain is ‘converged’, and the MCMC algorithm may be terminated. Convergence is typically assessed through the *Gelman-Rubin statistic* R [75], which compares the variance within chains to the variance between different chains. For $R \approx 1$, we assume convergence.

More powerful variants of the Metropolis-Hastings algorithm are widely used in cosmology. These are the *Hamiltonian Monte Carlo* (HMC) [61, 37], and the *No-U-Turn Sampler* (NUTS) [95], which are implemented in tools like *emcee* [70], *cobaya* [196], and *Stan* [40]. The efficiency of these algorithms enables us to constrain cosmological parameters by combining datasets from different sources, such as the CMB and LSS.

2.2.4 Fisher analysis

Fisher matrix analysis is a powerful tool in cosmology for obtaining parameter constraints and understanding the sensitivity of cosmological observations to various parameters. Unlike MCMC, which samples the full posterior, Fisher analysis provides an approximation

to the likelihood hypersurface around the best-fit parameters. This allows for an efficient estimation of parameter uncertainties, particularly useful in the early stages of an analysis, and helps us to explore whether a particular experiment can constrain parameters before building it (understand its impact).

Taking the likelihood as a starting point, the value of parameters that maximise it is given by

$$\left. \frac{\partial \mathcal{L}}{\partial \vec{\theta}} \right|_{\vec{\theta}_{\text{MLE}}} = 0, \quad (2.16)$$

where the partial derivative of the likelihood is evaluated when the parameters return the *maximum likelihood estimate* (MLE).

Taylor-expanding the log-likelihood version of Eq. 2.16 at an initial guess $\vec{\theta}_0$, we get

$$\left. \frac{\partial \ln \mathcal{L}}{\partial \vec{\theta}} \right|_{\vec{\theta}_{\text{MLE}}} = \left. \frac{\partial \ln \mathcal{L}}{\partial \vec{\theta}} \right|_{\vec{\theta}_0} + \left. \frac{\partial^2 \ln \mathcal{L}}{\partial \vec{\theta}^2} \right|_{\vec{\theta}_0} (\vec{\theta}_0 - \vec{\theta}_{\text{MLE}}) + \mathcal{O}((\vec{\theta}_0 - \vec{\theta}_{\text{MLE}})^2). \quad (2.17)$$

If we expand so that $\vec{\theta}_0$ is near the maximum likelihood parameters $\vec{\theta}_{\text{MLE}}$, the first derivative will be near zero. The second derivative (second term in the RHS) then gives us a direct measure of how a small change in the parameter values affects the likelihood value, which can be used to measure the errors of the inferred parameters.

We define the Fisher information matrix as the ensemble average (over signal and noise) of the matrix making up the second-order derivative of the Taylor expansion (the curvature matrix). It quantifies how much information a given dataset provides about the cosmological parameters. It is defined as

$$\mathbf{F} \equiv \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \vec{\theta}^2} \right\rangle, \quad (2.18)$$

where the ensemble average is taken to represent the mean over all realisations of the data.

We can then obtain the uncertainties using the covariance matrix, which is given by the inverse of the Fisher information matrix. It is important to note that in deriving the expression for the Fisher matrix, we have assumed $\vec{\theta}_0 \simeq \vec{\theta}_{\text{MLE}}$ so that the first derivative of the Taylor expansion is zero. When this condition is satisfied, the Fisher matrix cannot tell us anything about the best-fit parameters; it merely tells us the error associated with those parameters, so it relies on independent (e.g. root-finding) information on the best-fit parameter values.

Chapter 3

Measurements of Thermal History

Modern observational cosmology has reached a stage where constraints from most current and future datasets are limited by astrophysical systematic uncertainties, i.e. our lack of detailed understanding of the small-scale physics behind the luminous components of the Universe, galaxies and gas [e.g. 199, 180, 69, 65, 138, 177, 101, 178, 48]. These uncertainties limit the cosmological insights we can extract from high-precision datasets, including galaxy clustering, cosmic shear measurements, and even the CMB itself. The limiting factor for cosmological constraints from cluster number counts is the uncertainty in the mass-observable relation [205, 202, 179, 87, 132, 151, 133, 155, 56, 33]. Cosmic shear measurements are strongly affected by sub-grid baryonic physics, and by uncertainties in the processes by which galaxies acquire correlated intrinsic alignments [e.g. 41, 128, 93, 109, 197, 89, 131, 172]. Understanding the impact of baryons on the matter power spectrum requires better knowledge of the distribution of gas in haloes. Finally, even though the clustering pattern of galaxies is one of the cosmological observables with the highest signal-to-noise ratio, the cosmological constraints that can be extracted from it are severely limited by our incomplete understanding of the galaxy-dark matter connection [see e.g. 209, and references therein]. Even the analysis of the temperature anisotropies in the Cosmic Microwave Background (CMB), arguably the cleanest cosmological observable, is currently limited by the impact of astrophysical foregrounds on small scales [98, 125, 160].

One of the primary roadblocks in observational cosmology is the *mass-to-observable* relation, which links observable quantities, like galaxy clustering or tSZ maps, to the underlying mass distribution. The uncertainty in this relation affects cosmological constraints derived from cluster number counts, weak lensing, and studies of the LSS. In addition, other small-scale processes, such as baryonic effects (feedback mechanisms, small-scale hydrodynamics, star formation) affect the smaller scales of the matter power spectrum, introducing further uncertainty. Addressing these challenges requires using novel statistical

techniques and cross-correlations of different tracers, which can encompass information from multiple observational probes.

The CMB secondary anisotropies, in particular the thermal and kinematic Sunyaev-Zeldovich effects [tSZ and kSZ respectively, 188], as well as the gravitational lensing of CMB photons, have gained popularity as a means to address these issues [22, 23]. These effects are relatively clean probes of some of the physical quantities that need to be understood in order to mitigate the impact of astrophysical uncertainties: the matter and gas densities, the gas pressure, and the velocity field. Since these observables are also sensitive to cosmology, their combination with large-scale structure data can be extremely powerful at disentangling astrophysical and cosmological parameters. This has been explored by a large number of groups: CMB lensing data have been used to constrain the bias of different tracers of the matter distribution [e.g. 83, 7, 1, 146], as well as to calibrate the measurement of galaxy shapes in cosmic shear analyses [1]. The tSZ effect has been used in cross-correlation with galaxy clustering and weak lensing data to determine the physical properties of the diffuse gas, as well as to potentially improve constraints on the amplitude of matter fluctuations [201, 92, 127, 97, 16, 8, 129, 55, 191, 192, 144, 130]. The kSZ has been used in cross-correlation with galaxy clustering to constrain the growth of structure and the gas density profile around haloes [174, 153, 91, 185, 53].

tSZ is caused by inverse Compton scattering of CMB photons by hot electrons, and so serves as an excellent probe of cosmic thermal history. Since the tSZ signal directly traces the integrated electron pressure along the line of sight, it provides insights into the thermal energy content of clusters.

The relationship between the tSZ signal and cluster mass plays a pivotal role in cosmology as it helps us calibrate the mass-to-observable relation. The *Planck* collaboration makes use of the following relation that links the Compton- y parameter to cluster mass [161]:

$$E^{-\beta}(z) \frac{D_A^2(z) \bar{Y}_{500}}{10^{-4} \text{Mpc}^2} = Y_* \left(\frac{h}{0.7} \right)^{\alpha-2} \left(\frac{(1 - b_H) M_{500}}{6 \times 10^{14} \text{M}_\odot} \right)^\alpha, \quad (3.1)$$

where $E(z) \equiv H(z)/H_0$, D_A is the angular diameter distance, Y_* , α , β are fitted parameters calibrating the relation, and M_{500} , Y_{500} are the cluster mass and the magnitude of the Compton- y parameter, respectively. The index 500 signifies the reference overdensity Δ at which these quantities have been measured. Typically tSZ probes the inner regions of a cluster better, so in tSZ studies a reference overdensity of 500 times the critical density of the Universe is often used. Most importantly, cluster masses are linked to the Compton- y parameter via a scaling factor $(1 - b_H)^\alpha$. This factor includes b_H , the so-called *hydrostatic*

mass bias, which accounts for the difference between the observationally deduced M^{obs} and true M^{true} cluster mass, respectively, such that

$$M^{\text{obs}} = (1 - b_{\text{H}}) M^{\text{true}}. \quad (3.2)$$

Cosmological and astrophysical parameters are very sensitive to the mass bias, and thus it is crucial that this quantity is adequately characterised [92].

This chapter explores the cross-correlation between galaxy clustering data and maps of the tSZ Compton- y parameter, making use of existing data from the *Planck* collaboration [161] and a set of photometric galaxy surveys [28, 29]. The availability of redshift information allows us to place constraints on the cosmic evolution of the thermal gas pressure and the thermal energy in haloes, as well as to examine any redshift dependence of the mass bias for tSZ cluster studies. This is a relevant topic given the current mild tensions between SZ cluster number counts and CMB primary anisotropies [155, 56, 33, 218], which could be caused by the assumptions made to model the y -mass relation. Conversely, under the assumption that the relation between mass and gas pressure is well understood, the tSZ effect can be thought of as a mass tracer, which can be used to break degeneracies with the galaxy bias parameter.

3.1 Methods

3.1.1 Halo profiles

In order to describe the electron pressure in a halo, we use the generalised NFW profile (GNFW) described in [14] and used in the *Planck* tSZ cluster analysis [155]. This profile takes the form:

$$P_e(r) = P_* p(r/r_{500c}), \quad (3.3)$$

where r_{500c} is the cluster radius enclosing an overdensity of 500 times the critical density (see Section 1.3.2.1). The normalisation P_* is given by

$$P_* = 6.41 \left(1.65 \text{ eV cm}^{-3} \right) h_{70}^{8/3} \left(\frac{h_{70}(1 - b_{\text{H}})M_{500c}}{3 \times 10^{14} \text{ M}_{\odot}} \right)^{0.79}, \quad (3.4)$$

where $h_{70} = H_0/(70 \text{ km s}^{-1} \text{ Mpc}^{-1})$, and M_{500c} is the halo mass enclosed by r_{500c} . The GNFW form factor is

$$p(x) = (c_P x)^{-\gamma} [1 + (c_P x)^{\alpha}]^{(\gamma-\beta)/\alpha}, \quad (3.5)$$

with $(\alpha, \beta, \gamma, c_P) = (1.33, 4.13, 0.31, 1.81)$. We must note that other pressure profiles have been proposed in the literature (e.g. [21]), but we choose this parameterisation in order to be able to relate our measurement of $(1 - b_{\text{H}})$ to the results of [155].

The quantity $1 - b_{\text{H}}$ in Eq. 3.4 parameterises our lack of knowledge about the precise relation between mass and pressure in clusters. This factor is also commonly referred to as the ‘hydrostatic mass bias’ since it was originally defined to account for the fraction of halo mass not in hydrostatic equilibrium missed by X-ray observations. Since this parameter also encapsulates other sources of bias in the X-ray-based mass estimates, we instead refer to it as the *mass bias*. Numerical simulations have constrained the mass deficit to be around 20% (i.e. $1 - b_{\text{H}} \simeq 0.8$: [20]; [143]), although it is known that a smaller value is necessary in order to fully reconcile CMB primary constraints and SZ cluster counts [155]. This is a central point of our discussion in Section 3.5.2.1.

Within the halo model description, and assuming a log-normal y -mass relation, the pressure profile cumulants are given by:

$$\langle u_y(k|M) \rangle = P_e(k), \quad (3.6)$$

$$\langle u_y^2(k|M) \rangle = P_e^2(k) e^{\sigma_{\ln Y}^2}, \quad (3.7)$$

where $P_e(k)$ is the Fourier transform of the GNFW profile and $\sigma_{\ln Y} = 0.173 \pm 0.023$ is the intrinsic logarithmic scatter in the y -mass relation [155]. Note that, since we do not make use of the tSZ auto-correlation, we do not use the second-order cumulant of the pressure profile in our analysis. However, since we analyse the galaxy-tSZ correlation, we need to model the covariance between the galaxy overdensity and pressure profiles. For simplicity, we adopt a one-parameter model:

$$\langle u_g(k|M)u_y(k|M) \rangle = (1 + \rho_{gy})\langle u_g(k|M) \rangle \langle u_y(k|M) \rangle, \quad (3.8)$$

where the free parameter ρ_{gy} determines the sign and magnitude of the correlation between galaxy abundance and pressure. Marginalising over this parameter has the added advantage of removing any sensitivity of our final constraints to $1 - b_{\text{H}}$ on the details of the cross-spectrum model in the 1-halo regime, where both parameters are completely degenerate.

3.1.2 Halo occupation distribution

To model the galaxy overdensity, Δ_g , we use a Halo Occupation Distribution (HOD) model [25, 217, 200], as prescribed by [216]. The HOD models the galaxy content of dark matter haloes as being made up of central and satellite galaxies. Centrals lie at the centre of the halo, while satellites are distributed according to a profile $u_s(r|M)$. Haloes can have zero or one central, and the mean number of centrals for a halo of mass M is modelled as a smoothed step function,

$$\langle N_c(M) \rangle = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\log(M/M_{\min})}{\sigma_{\ln M}} \right) \right]. \quad (3.9)$$

In our fiducial scenario we assume that satellites can only be formed if a halo has a central and has a mass larger than some threshold M_0 . In that case, the average number of satellites follows a power law of the form:

$$\langle N_s(M) \rangle = N_c(M) \Theta(M - M_0) \left(\frac{M - M_0}{M'_1} \right)^{\alpha_s}. \quad (3.10)$$

For simplicity, we fix M_0 to $M_{\min}$, $\sigma_{\ln M} = 0.15$ and $\alpha_s=1$, as in [12], leaving only two free parameters: $M_{\min}$ and M'_1 . Coupling M_0 and $M_{\min}$ allows for all haloes containing a central to also contain satellites, and conversely, for all haloes containing satellites to necessarily contain a central. This assumption breaks down in cases such as recent major mergers, where centrals may not be immediately established.

Besides their mean values, we also need to specify the statistics of N_c and N_s . Following standard practice [200], we assume N_c to have a Bernoulli distribution with probability $p = \langle N_c \rangle$, and N_s to be Poisson-distributed.

Putting everything together, the moments of the galaxy overdensity Fourier profile are [e.g. see section 2.2 of 200]:

$$\langle u_g(k) \rangle = \bar{n}_g^{-1} [\langle N_c \rangle + \langle N_s \rangle u_s(k)], \quad (3.11)$$

$$\langle |u_g(k)|^2 \rangle = \bar{n}_g^{-2} [\langle N_s \rangle^2 u_s^2(k) + 2\langle N_s \rangle u_s(k)], \quad (3.12)$$

where the mean number density $\bar{n}_g$ is

$$\bar{n}_g \equiv \int dM \frac{dn}{dM} (\langle N_c \rangle + \langle N_s \rangle), \quad (3.13)$$

where we have suppressed the mass dependence of all quantities for brevity.

Finally, we assume that the satellites follow the matter distribution, and therefore $u_s(k|M)$ is given by a truncated Navarro, Frenk & White profile [141]:

$$u_s(k|M) = \left[\log(1 + c_\Delta) - \frac{c_\Delta}{(1 + c_\Delta)} \right]^{-1} \quad (3.14)$$

$$\begin{aligned} & [\cos(q) (\text{Ci}((1 + c_\Delta)q) - \text{Ci}(q)) \\ & + \sin(q) (\text{Si}((1 + c_\Delta)q) - \text{Si}(q)) \\ & - \sin(c_\Delta q)/(1 + c_\Delta q)], \end{aligned}$$

where $q \equiv kr_\Delta/c_\Delta$, r_Δ and c_Δ are the halo radius and concentration defined in Section 1.3.2.3, and $\{\text{Ci}, \text{Si}\}$ are the cosine and sine integrals.

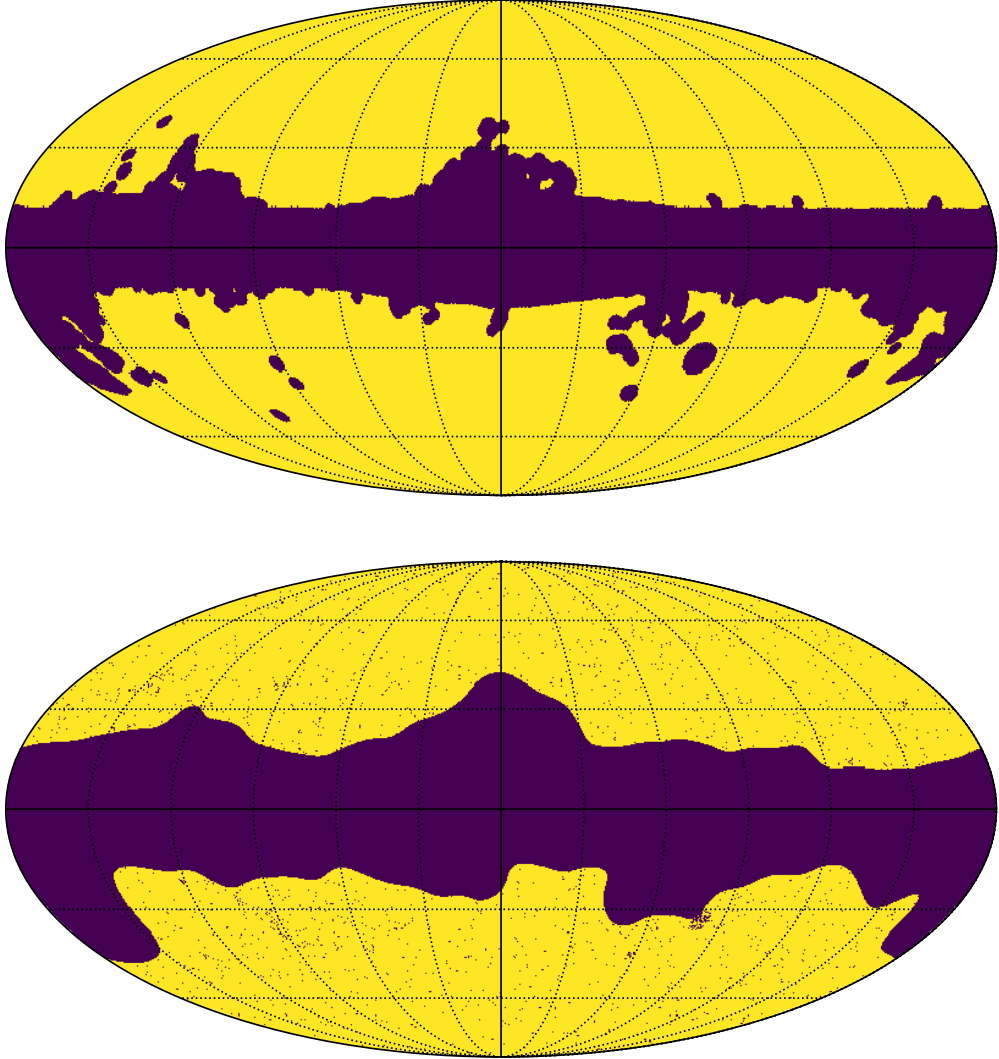


Figure 3.1: *Top*: sky mask used for the Compton- γ map, corresponding to a sky fraction $f_{\text{sky}} = 0.59$. *Bottom*: mask used for the 2MPZ and WISE galaxy surveys, corresponding to a sky fraction $f_{\text{sky}} = 0.68$. The product of both masks leaves a usable sky fraction $f_{\text{sky}} \simeq 0.58$.

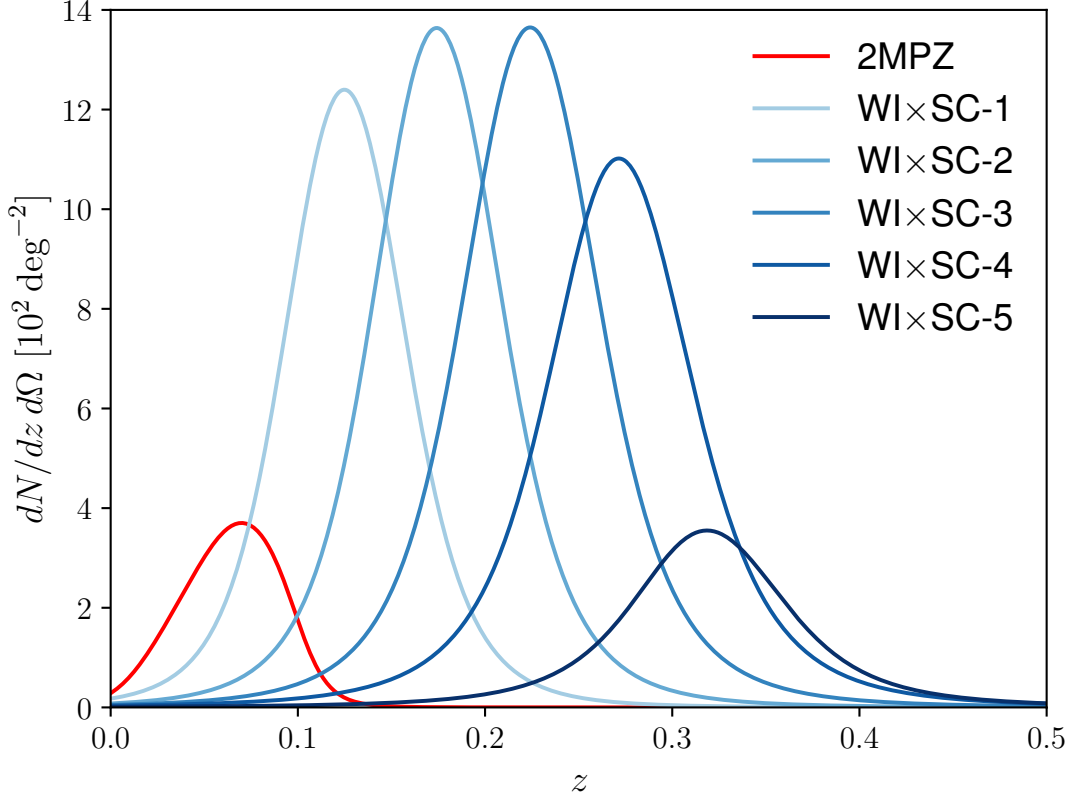


Figure 3.2: Fiducial redshift distributions of the different galaxy samples used in this analysis. See Table 3.1 for further details.

Sample	$[z_{\text{ph},i}, z_{\text{ph},f}]$	$\bar{z}$	$\bar{n}_g [\text{deg}^{-2}]$	ℓ_{max}
2MPZ	[0.00, 0.10]	0.07	25.5	280
WI×SC-1	[0.10, 0.15]	0.13	106	540
WI×SC-2	[0.15, 0.20]	0.18	126	745
WI×SC-3	[0.20, 0.25]	0.23	136	945
WI×SC-4	[0.25, 0.30]	0.27	118	1130
WI×SC-5	[0.30, 0.35]	0.32	41	1310

Table 3.1: Galaxy samples used in this analysis, corresponding to the subsample of the 2MPZ survey used in our analysis, and five tomographic redshift bins of the WI×SC survey. The second, third and fourth columns list the photometric redshift interval defining the sample, its mean redshift and its number density respectively. The largest multipole used in the analysis of each sample (corresponding to a comoving scale of $k_{\text{max}} \sim 1 \text{ Mpc}^{-1}$) is shown in the last column.

3.2 Data

3.2.1 Galaxies

We make use of two low-redshift photometric redshift (photo- z) catalogues, the 2MASS Photometric Redshift catalogue [2MPZ, 28] and the WISE $\times$ SuperCOSMOS catalogue [WI $\times$ SC, 29]. Both samples were created by cross-matching full-sky imaging surveys, and photo- z 's were subsequently computed for all the included sources. Their broad photometric coverage, together with adequate spectroscopic calibration data, allows for well-constrained photo- z 's, with minimal mean bias, relatively low scatter, and a small number of outliers.

2MPZ was constructed by cross-matching the extended-source catalogue from the 2 Micron All-Sky Survey [2MASS, 182, 106] with the photographic plates of SuperCOSMOS [80, 147] and the photometry of the Wide-field Infrared Survey Explorer (WISE: [212]). After applying an apparent magnitude cut $K_s < 13.9$ (using the Vega system) to achieve uniformity, 2MPZ includes over 940,000 sources observed in 8 bands: SuperCOSMOS's optical (B, R, I), 2MASS's near-infrared (J, H, K_s) and WISE's mid-infrared ($W1, W2$). Photometric redshifts were extracted using the neural network code ANNz [51] trained on a large spectroscopic sample from overlapping surveys. The resulting photo- z 's have a typical error $\sigma_z \simeq 0.015$, and the sample has a median redshift $z \simeq 0.08$ (see also [17] for a more detailed analysis of the 2MPZ photo- z properties). As demonstrated in Section 3.5, due to the low redshift of this sample, the cross-correlation with the y map is dominated by the 1-halo term for 2MPZ, and little is gained by subdividing it into narrower redshift bins. Therefore, we use 2MPZ as a single tomographic sample, up to a photometric redshift of $z_{\text{photo}} = 0.1$.

A deeper sample is obtained by ignoring the 2MASS data and cross-matching WISE and SuperCOSMOS only. After removing the sources already contained in 2MPZ, the resulting catalogue, WI $\times$ SC, is ~ 3 times deeper, contains ~ 20 million sources and reaches up to redshift $z \sim 0.4$, with a median redshift of ~ 0.2 . The photometric redshifts are less accurate, given the poorer photometric coverage ($B, R, W1, W2$), with a mean error $\sigma_z/(1+z) \simeq 0.035$. We divide the WI $\times$ SC sample into 5 redshift bins, corresponding to photo- z intervals of equal width $\delta z_{\text{photo}} = 0.05$ in the range $0.10 < z_{\text{photo}} < 0.35$. Details regarding each of these redshift bins are given in Table 3.1, and the corresponding redshift distributions are shown in Fig. 3.2. We estimate fiducial redshift distributions for each tomographic sample as described in [146], and we discuss the marginalisation over uncertainties in these distributions in Section 3.3.

Fig. 3.3 presents the galaxy overdensity maps for the six tomographic bins used in this analysis, as well as in the structure growth analysis of Chapter 4. These maps have been

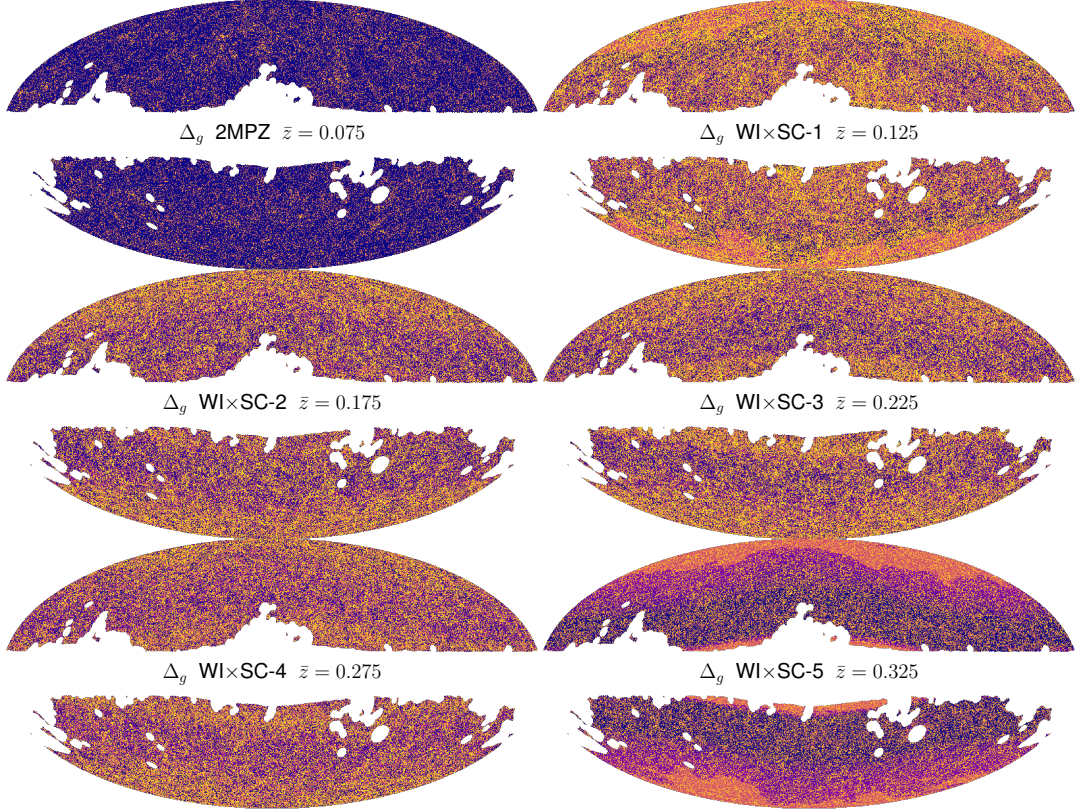


Figure 3.3: Galaxy overdensity maps for the six tomographic bins used in this analysis. The galactic mask from Fig. 3.1 has been applied to each map. Logarithmic rescaling has been applied individually to enhance visual contrast.

masked using the galactic mask shown in Fig. 3.1. To enhance visual contrast, each map has been individually rescaled using a logarithmic stretch.

Both 2MPZ and WI×SC suffer from different levels of contamination from Galactic and observational systematics. The most relevant Galactic systematic is stellar contamination, particularly for WI×SC [214]. Besides avoiding regions of high dust and stellar contamination using the sky mask described in [146] (see bottom panel of Fig. 3.1), we correct for the effects of stellar contamination by correcting for a smooth non-linear relation between galaxy and star density (also described in [146]). After doing so, three potential sources of systematic uncertainty remain: residual stellar contamination, modulation of the galaxy density due to Galactic dust reddening, and modulation due to zero-point fluctuations in the photographic plates used by SuperCOSMOS. We address the first two (dust and stars) by deprojecting them at the map level as described in Section 3.4.1.1. We address the contamination from fluctuations in the SuperCOSMOS plates by modelling it at the power spectrum level, which we describe in Section 3.4.1.2.

We depart from the approach used in [146] in that we do not additionally employ the SDSS photometric catalogue [24] as a galaxy tracer. At redshifts $z < 0.35$, 2MPZ and WISE have wider-angle coverage than SDSS, which results in a larger signal-to-noise ratio of the $\delta_g \times y$ cross-correlation; therefore, SDSS could only prove useful at redshifts $z > 0.35$. This is, however, where the Dark Energy Survey has already provided constraints [144], offering in particular well-constrained redshift distributions for their mass tracers (luminous red galaxies). In the case of the SDSS photo- z sample, redshifts are difficult to calibrate in the $z > 0.35$ regime. This is because there are no sufficiently complete overlapping wide-angle spectroscopic datasets in order to do it robustly for the purposes of our work (see Section 3.3). This is also briefly discussed in Section 2.1 of [146], although the uncertainties on the SDSS redshift distributions were of minor importance for the final results there. This is not any more the case in this work, as we have verified.

3.2.2 Compton- y parameter

We make use of the Compton- y parameter maps made public by the *Planck* collaboration [161]. These maps were generated using different flavours of the Internal Linear Combination method [68, 208]. In a simplified description, the ILC technique selects the linear combination of all frequency channels that preserves the spectrum of the source one wishes to map, minimising the map-level variance. The refined versions of the ILC method used by *Planck* further optimise the linear weights on different scales and different regions of the map, and project out sources with known spectra that are likely to cause significant contamination. In particular, *Planck* has released two y maps, extracted using the MILCA [104] and NILC [165] variations of the ILC technique. Both methods deproject CMB contamination through its well-known spectrum, but differ in the methods used to calculate the optimal scale-dependent and spatially-varying linear weights.

The MILCA and NILC maps have been found to be in good agreement in different studies, although the NILC map has a higher noise level on large scales [161]. We thus use the MILCA map as our fiducial Compton- y map but repeat our analysis on the NILC map as part of our systematics analysis. We use a fiducial mask for the y maps based on a combination of the *Planck* 60% Galactic mask and the union of the HFI and LFI point source masks (see top panel of Fig. 3.1).

Finally, in order to evaluate the level of contamination from extragalactic dust in the $\delta_g \times y$ cross-correlation, we make use of the HFI 545 GHz map, as described in Section 3.5.3.1.

Fig. 3.4 presents the Compton- y parameter tSZ maps computed using the MILCA and NILC methods, respectively (top two panels), as well as the HFI 545 GHz map of

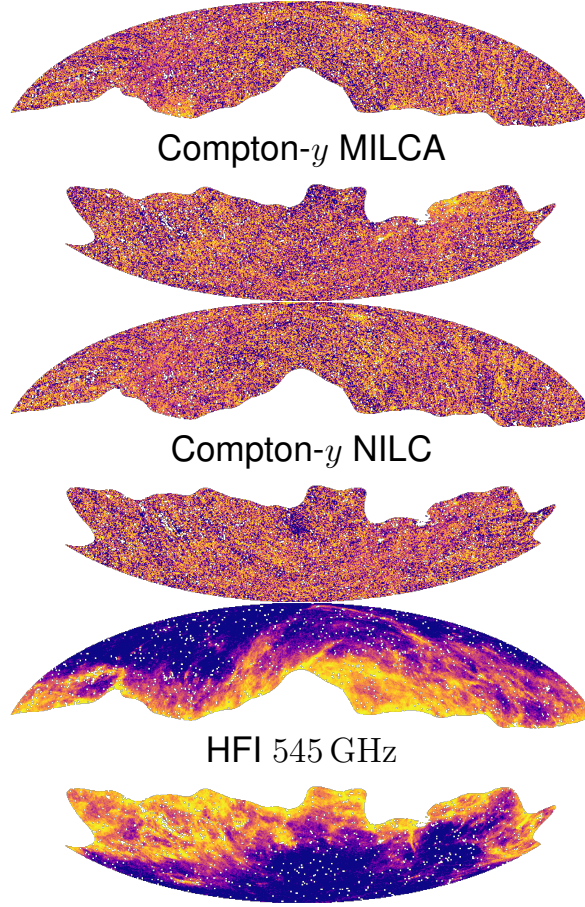


Figure 3.4: Comparison of the Compton- y parameter maps and the extragalactic dust map. The top panel shows the MILCA y map and the middle panel the NILC y map, while the bottom panel displays the HFI 545 GHz map of extragalactic dust. All maps have been masked using the Galactic mask from Fig. 3.1 and have their colour scales logarithmically stretched individually to highlight structural details and enhance visual contrast.

extragalactic dust (bottom panel). The Galactic mask shown in Fig. 3.1 has been applied, and the colour scale has been stretched logarithmically to enhance visual contrast.

Fig. 3.4 presents a comparison of the Compton- y parameter maps produced by the MILCA and NILC methods (top two panels) alongside the HFI 545 GHz map of extragalactic dust (bottom panel). In this figure, the Galactic mask shown in Fig. 3.1 has been applied, and the colour scale for each map has been logarithmically stretched individually to enhance visual contrast. These maps are used to assess the level of extragalactic dust contamination in the $\delta_g \times y$ cross-correlation.

3.3 Redshift distributions

Accurate redshift distributions are critical for cosmological analyses, especially when performing tomographic measurements on the LSS. As the redshift distributions underpin the accuracy of the final parameter constraints, we need precise calibration of the photometric redshifts (z_{photo}) using spectroscopic datasets.

For each redshift bin we estimate fiducial redshift distributions using the method described in [146]. In short, a true-redshift distribution is estimated from the distribution of photometric redshifts for a given bin using a model for the conditional photo- z distribution:

$$p(z) = \int dz_{\text{photo}} p(z|z_{\text{photo}}) p(z_{\text{photo}}), \quad (3.15)$$

where the model for $p(z|z_{\text{photo}})$ was calibrated using overlapping spectroscopic data. Although the spectroscopic coverage of 2MPZ and WISPC is high compared to other photometric redshift surveys, the resulting redshift distributions are not infinitely precise, and therefore we need to account for any uncertainty in them that could affect our measurement.

Accurate estimates of the true underlying redshift distributions of galaxies play a pivotal role if we want to obtain accurate theoretical predictions. Photometric redshifts are generally calibrated so that the distribution of true z at a given z_{photo} is unbiased; however, we still need to quantify the scatter in order to appropriately broaden the calculated $p(z)$. In particular, our constraints on $1 - b_{\text{H}}$ are indeed very sensitive to the width of the redshift distributions. This is because wider redshift distributions, which lead to lower projected clustering amplitudes, affect the galaxy auto-correlation, $\delta_g \times \delta_g$, while the $\delta_g \times y$ cross-correlation is almost insensitive to the width.

We introduce an additional parameter, w_z , that stretches the redshift distribution while preserving unit total probability. Concretely, given a fiducial distribution $p_{\text{fid}}(z)$ with mean redshift $\bar{z}$, w_z is implemented as

$$p(z) \propto p_{\text{fid}}\left(\bar{z} + \frac{z - \bar{z}}{w_z}\right), \quad (3.16)$$

where the proportionality constant is fixed by making sure that $p(z)$ integrates to unity. We make w_z a free parameter that we marginalise over with a top-hat prior $0.8 < w_z < 1.2$. This prior is significantly larger than the actual expected uncertainty in the width of the redshift distributions. To be precise, we estimate that the parameters (e.g. Gaussian mean and variance) of the conditional photo- z distribution $p(z|z_{\text{photo}})$ used to calculate the true redshift distributions (Eq. 3.15), are known to the level of 1%. When propagated into an uncertainty on the width w_z of $p(z)$, this corresponds to an uncertainty of the same order

($\sim 0.9\%$). Thus, the margins of the assumed top-hat priors are wide enough to encompass any lack of precision in the number density distribution.

Another possible source of systematic uncertainty is the effect of biased photometric redshifts (i.e. where $\langle z|z_{\text{photo}} \rangle \neq z_{\text{photo}}$). The ANNz method used in [146] should guarantee unbiased photo- z 's, as it aims to minimise the mean residual,

$$\langle z_{\text{spec}} - z_{\text{photo}} \rangle \quad (3.17)$$

over the training set, i.e. it is trained to satisfy

$$\langle z_{\text{spec}} | z_{\text{photo}} \rangle \approx z_{\text{photo}} \quad (3.18)$$

on average. In the ideal case where the spectroscopic training set is fully representative of the photometric sample in both colour and magnitude space, and when overfitting is avoided, this delivers zero mean bias in each z_{photo} bin. Of course, this can never be achieved exactly in practice because real spectroscopic catalogues are neither complete nor perfectly representative (e.g. they may undersample faint or very red galaxies). It would be possible to account for this form of uncertainty by treating $\bar{z}$ as a free parameter, as is usually done in photometric cosmic shear analyses. Our results, however, are more sensitive to the distribution widths, and we find that this simple parameterisation is able to describe our data sufficiently well. A more detailed analysis of the associated photometric redshift systematics is carried out in Chapter 4.

3.4 Systematics analysis

3.4.1 Map contaminants

3.4.1.1 Map-level deprojection

For small levels of contamination, the impact of a given systematic on an observed sky map can be modelled at the linear level:

$$\mathbf{m}_{\text{obs}} = \mathbf{m}_{\text{true}} + \epsilon \mathbf{t}. \quad (3.19)$$

Here $\mathbf{m}_{\text{obs}}$ and $\mathbf{m}_{\text{true}}$ are vectors corresponding to the observed sky map and the true underlying quantity we wish to map respectively, $\mathbf{t}$ is a template map describing the contaminant (e.g. a map of the Galactic dust fluctuations), and ϵ is an unknown amplitude. In order to fully account for the effects of this contaminant one has to build a likelihood for $\mathbf{m}_{\text{obs}}$ using the Eq. 3.19 as a model (together with a model for $\mathbf{m}_{\text{true}}$) and marginalise over ϵ . As shown in [67] and [9], within the pseudo- C_ℓ framework, this can be done exactly by

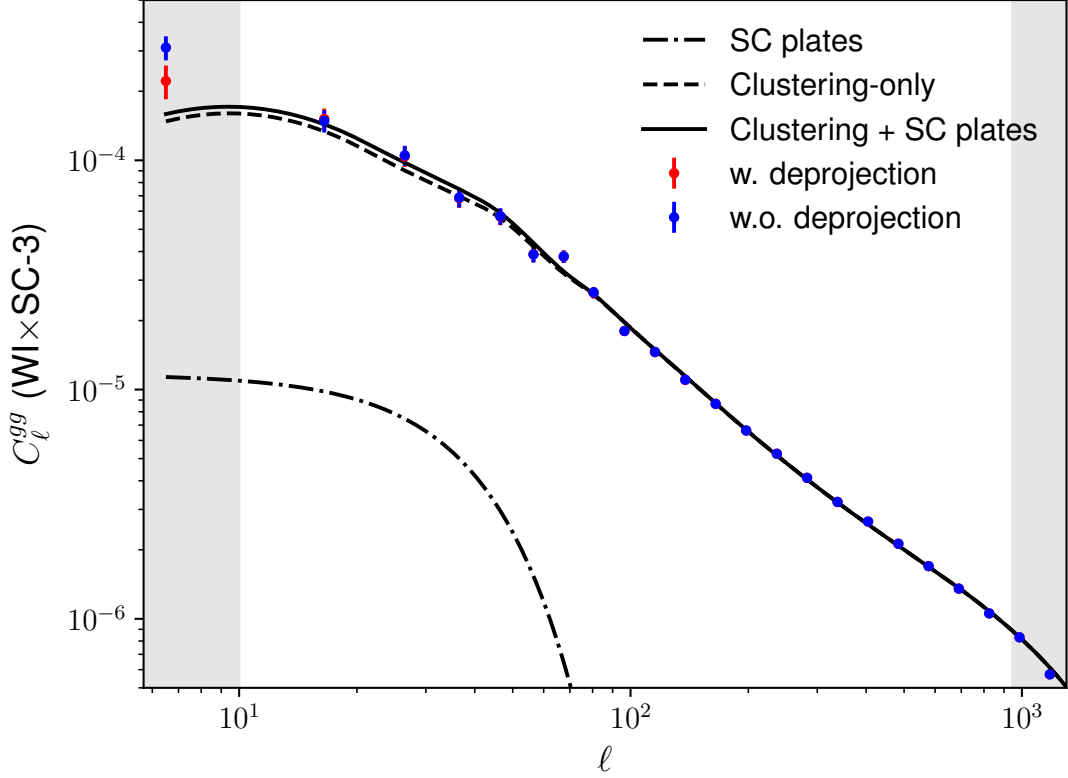


Figure 3.5: Summary of the different sources of systematic contamination affecting the galaxy auto-correlations for the particular case of the third WI×SC bin. The red and blue points show the measurements before and after deprojecting a dust and a star template. The dot-dashed line shows the best-fit contamination from SuperCOSMOS plate fluctuations that explains the residuals of the data with respect to the best-fit HOD-only model (dashed line). The total combined prediction is shown as the solid line. The light grey bands show our scale cuts.

projecting $\mathbf{m}_{\text{obs}}$ onto the subspace perpendicular to $\mathbf{t}$ or, in other words, by ‘deprojecting’ $\mathbf{t}$. The loss of modes due to deprojection then needs to be taken into account when estimating the power spectrum, which can be done analytically.

For our analysis, we deproject two systematic templates from the galaxy and y maps. We create a reddening template using the dust map of [175], and remove its associated contamination from the y maps and from all the galaxy overdensity maps. We also generate a star density template from the WISE data, and remove its associated fluctuations from all the galaxy overdensity maps. The effects of this deprojection are illustrated in Fig. 3.5.

Fig. 3.6 presents maps of the two main contaminants: Galactic dust reddening, $E(B-V)$, and stellar contamination in the WI×SC tomographic bins. The dust reddening map, shown in the top left panel, is based on the extinction estimates of [176], while the remaining

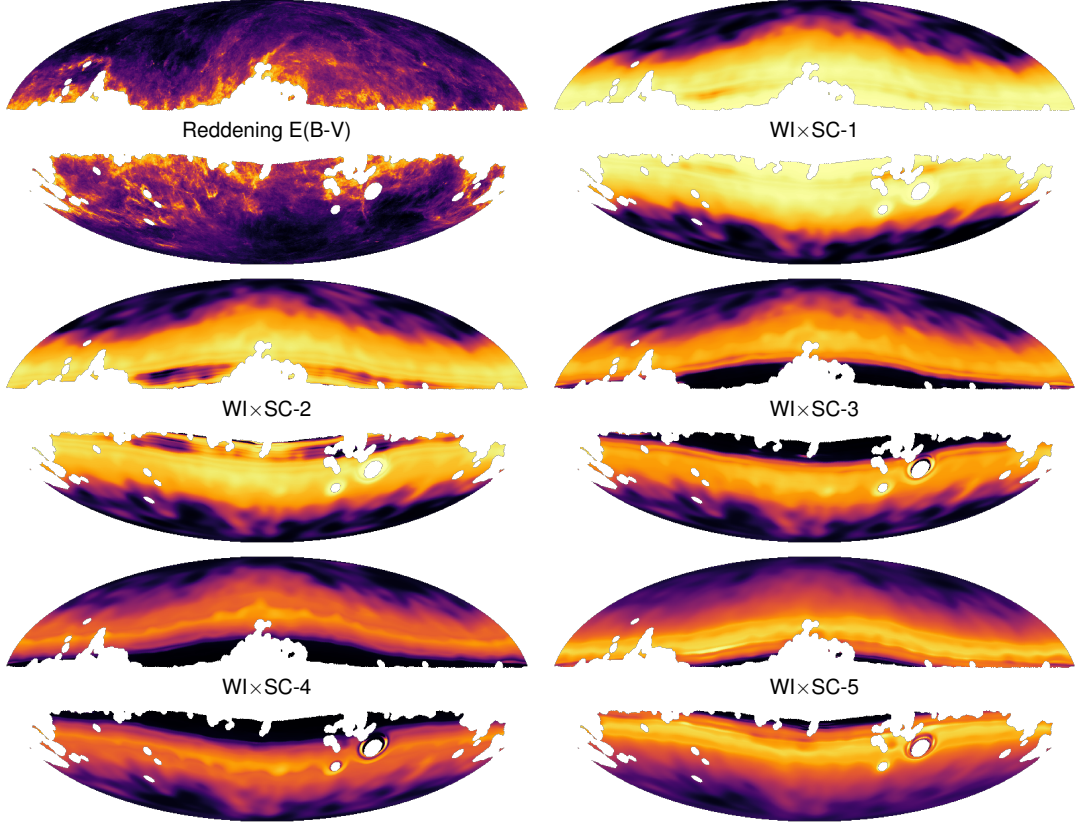


Figure 3.6: Maps of systematic contaminants affecting the galaxy overdensity maps. **Top left:** Galactic dust reddening map, $E(B - V)$, from [176], used as a proxy for extinction. **Remaining panels:** stellar contamination maps for the five tomographic bins. These templates are used in NaMaster to deproject systematics from the analysis.

panels display the stellar contamination maps for each tomographic bin. These maps serve as input templates for systematics deprojection using NaMaster [9]. These contaminant maps have also been used in our analysis in Chapter 4.

It is worth noting that in our work we do not re-derive the stellar contamination maps from scratch but rather adopt the publicly released templates produced in [29] and [147]. Extended source candidates were first selected via the SuperCOSMOS profile statistic $\eta > 2.5$ and then cleaned with a latitude-dependent $W1 - W2$ colour cut. Purity is quantified by cross-matching with SDSS at $|b| > 30^\circ$, while residual stellar contamination is mapped as a function of WISE source density and photometric redshift slices, and subtracted per pixel.

3.4.1.2 C_ℓ -level deprojection

We model the systematic fluctuations in the number density of sources in the WI×SC sample caused by variations in the zero-point of the SuperCOSMOS photographic plate exposures as Gaussian random variations within the footprint of each exposure with a variance σ_{plate}^2 :

$$\delta_{\text{plate}}(\hat{\boldsymbol{\theta}}) = \sum_p \delta_p S_{\text{plate}} W(\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}_p), \quad (3.20)$$

where the sum runs over all exposures, δ_p is the fluctuation in exposure p (with $\langle \delta_p^2 \rangle = \sigma_{\text{plate}}^2$), S_{plate} is the footprint area of each exposure and $W(\hat{\boldsymbol{\theta}})$ is the plate window function. Assuming a roughly homogeneous coverage of the sky, the power spectrum of these fluctuations is given by

$$C_\ell^{\text{plate}} = S_{\text{plate}} \sigma_{\text{plate}}^2 |W_\ell|^2, \quad (3.21)$$

where W_ℓ is the harmonic transform of the plate window function. SuperCOSMOS used photographic plates covering an area of $5^\circ \times 5^\circ$. The corresponding window function can be roughly approximated as $|W_\ell|^2 = \exp[-(\ell \theta_{\text{plate}})^2/12]$, where $\theta_{\text{plate}} = 5\pi/180$ for 5-degree plates.

The contamination from plate fluctuations can therefore be accounted for as an additive contribution to the model describing the galaxy auto-spectrum, proportional to a template $T_\ell \equiv |W_\ell|^2$ with a free amplitude $A \equiv S_{\text{plate}} \sigma_{\text{plate}}^2$ that is marginalised over. Since A is a linear parameter, this can be done analytically in a pre-processing step by modifying the inverse covariance matrix of the galaxy auto-spectra as follows [169]:

$$\mathbf{C}^{-1} \rightarrow \mathbf{C}^{-1} - \frac{\mathbf{C}^{-1} \mathbf{T} \cdot \mathbf{T}^T \mathbf{C}^{-1}}{\mathbf{T}^T \mathbf{C}^{-1} \mathbf{T}}, \quad (3.22)$$

where $\mathbf{T}$ is a vector containing the template T_ℓ for all the scales used in the analysis. The dot-dashed line in Fig. 3.5 shows the shape of the plate template T_ℓ normalised by the best-fit value of the parameter A that explains the residuals of the data with respect to the best-fit HOD-only model (shown as a dashed line). The plate fluctuations contribute to $\sim 10\%$ of the signal on scales $\ell \lesssim 30$.

3.4.1.3 Scale cuts

After deprojecting dust and stars at the map level, and the imprint of the SuperCOSMOS plates at the power spectrum level, we still observe an unacceptably large amount of power on scales $\ell < 10$ in all galaxy auto-correlations involving WI×SC. These may be due to residual stellar contamination that is not simply removed with a linear template [214], or inaccuracies in our treatment of the SuperCOSMOS fluctuations. To avoid biasing

our results, we therefore remove the lowest bandpower from all galaxy auto-correlations involving WI×SC.

On small scales, the simple halo model prescription used to describe the non-linear power spectra and covariance matrices may not be sufficiently accurate. Therefore, we impose a cut on angular multipoles ℓ larger than the typical physical scale of a halo, $\ell_{\max} = k_{\max} \bar{\chi} - 1/2$, where $k_{\max} = 1 \text{ Mpc}^{-1}$, and $\bar{\chi}$ is the mean comoving radial distance in each redshift bin. The corresponding values of $\ell_{\max}$ in each bin are given in Table 3.1.

3.4.2 Covariance estimation

We compute the covariance as described in Section 2.1.2. A first estimate of the covariance is made using jackknife resampling. The common footprint covered by the masks of both the galaxy overdensity and the Compton- y maps is divided into N_{JK} regions of roughly equal area. Each region is masked in turn, and the power spectrum is computed using the remaining available footprint. The covariance is then given by Eq. 2.4 as

$$\mathbf{C} \left(C_{\ell}^{uv}, C_{\ell'}^{wz} \right) = \frac{N_{\text{JK}} - 1}{N_{\text{JK}}} \sum_{n=1}^{N_{\text{JK}}} \Delta C_{\ell}^{uv, (n)} \Delta C_{\ell'}^{wz, (n)}, \quad (3.23)$$

where $\Delta C_{\ell}^{uv, (n)}$ is the difference between the power spectrum estimated when removing the n -th jackknife region and the power spectrum averaged over all jackknife regions. We use $N_{\text{JK}} = 461$ jackknife regions defined as HEALPix [77] pixels with resolution $N_{\text{side}} = 8 = 7.3^{\circ}$.

The number of independent realisations using jackknife resampling is circa one order of magnitude larger than the size of the data vector (maximum size 50 elements), so we expect numerical stability.

We then proceed as described in Section 2.1.2 to calculate the full covariance matrix. Fig. 3.7 verifies that the first- and second-order diagonals of the covariance matrix for the second WI×SC redshift bin are similar irrespective of the covariance estimation method used.

3.4.3 Priors

In order to connect our measurements with the posterior distribution of the model parameters $\vec{\theta}$, we assume that the measured power spectra follow a Gaussian likelihood (Eq. 2.11):

$$-2 \ln p(\mathbf{d}|\vec{\theta}) = \chi^2 \equiv (\mathbf{d} - \mathbf{t}(\vec{\theta}))^T \mathbf{C}^{-1} (\mathbf{d} - \mathbf{t}(\vec{\theta})), \quad (3.24)$$

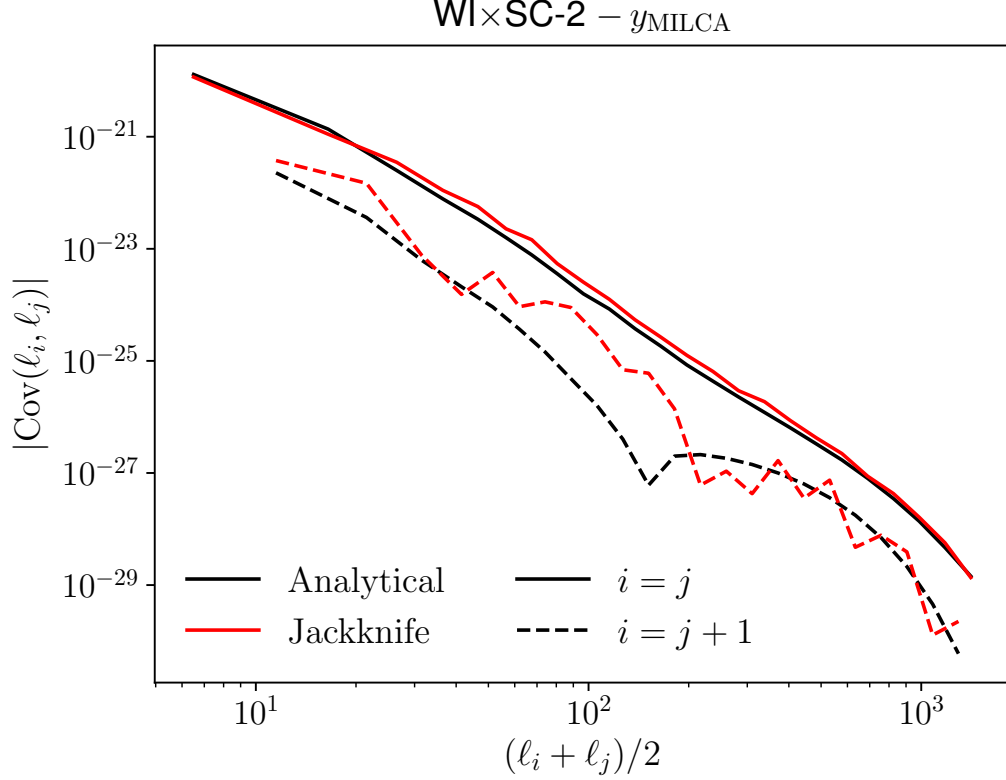


Figure 3.7: Diagonals of the covariance matrix for the $\delta_g \times y$ cross-correlation in the second WIxSC redshift bin. Black and red lines show the analytical and jackknife covariances respectively. The solid and dashed lines show the zeroth-order ($i = j$) and first-order ($i = j + 1$) diagonals respectively. We find compatible results from both methods.

where $\mathbf{d}$ is a vector of power spectrum measurements, $\mathbf{t}(\vec{\theta})$ is the theory prediction for $\mathbf{d}$ with parameters $\vec{\theta}$ and $\mathbf{C}$ is the covariance matrix described in the previous sections.

We explore the likelihood of each redshift bin separately. For each bin, our data vector $\mathbf{d}$ includes two sets of power spectrum measurements, corresponding to the autocorrelation of the galaxy overdensity and to its cross-correlation with the Compton- y map, $\mathbf{d} = (C_\ell^{gg}, C_\ell^{gy})$. We compute all power spectra in the range of multipoles $2 < \ell < 1535$, binned into the bandpowers described in Table 3.2. For a given redshift bin, we only include those bandpowers that satisfy the scale cuts described in Section 3.4.1.3.

For each redshift bin, our theoretical model has five free parameters: $\log_{10} M_{\min}/M_\odot$, $\log_{10} M'_1/M_\odot$, $1 - b_H$, ρ_{gy} , and a nuisance width parameter w_z . The first two parameters effectively fit the galaxy bias and small-scale amplitude in the galaxy autocorrelation, while $1 - b_H$ and ρ_{gy} are then constrained by including the tSZ cross-correlation. We fix all cosmological parameters to the best-fit values in [158]: $(\Omega_c h^2, \Omega_b h^2, h, \sigma_8, n_s) = (0.119, 0.0224, 0.6766, 0.8102, 0.9665)$. In our fiducial scenario, we adopt the mass func-

b	ℓ_{ini}^b	ℓ_{end}^b	b	ℓ_{ini}^b	ℓ_{end}^b
1	2	12	13	180	216
2	12	22	14	216	258
3	22	32	15	258	308
4	32	42	16	308	369
5	42	52	17	369	441
6	52	62	18	441	527
7	62	74	19	527	629
8	74	88	20	629	752
9	88	106	21	752	899
10	106	126	22	899	1074
11	126	151	23	1074	1284
12	151	180	24	1284	1535

Table 3.2: Power spectrum bandpowers used in this work. The b -th bandpower includes all integer multipoles $\ell_{\text{ini}}^b < \ell < \ell_{\text{end}}^b$. The bandpower edges listed here correspond roughly to linearly-spaced bands between $\ell = 2$ and $\ell = 52$, and logarithmic bands afterwards.

tion parameterisation of [194]. We have used the halo model bias described in [195] throughout our analysis; in our fiducial case, and when exploring departures from it. We impose the following top-hat priors on the free parameters:

$$10 \leq \log_{10} M_{\text{min}}/M_{\odot} \leq 16, \quad (3.25)$$

$$10 \leq \log_{10} M'_1/M_{\odot} \leq 16, \quad (3.26)$$

$$0 \leq (1 - b_{\text{H}}) \leq 0.99, \quad (3.27)$$

$$-1 \leq \rho_{gy} \leq 1, \quad (3.28)$$

$$0.8 \leq w_z \leq 1.2. \quad (3.29)$$

We sample the resulting posterior distributions using the Markov chain Monte Carlo method (MCMC) as implemented in the `emcee` software package [70]¹. We use the Core Cosmology Library² [46] in our theory calculations.

3.5 Results & discussion

3.5.1 Power spectra and covariances

We estimate the galaxy auto-power spectrum, the galaxy-tSZ cross-spectrum, and their covariance matrix for each of the redshift bins shown in Fig. 3.2 using the methods described

¹<https://emcee.readthedocs.io/en/v2.2.1/>.

²<https://github.com/LSSTDESC/CCL>

Sample	$\bar{z}$	$1 - b_H$ (best fit)	$1 - b_H$ (68% C.L.)	$\langle bP_e \rangle$ [meV cm ⁻³]	$\chi^2/\text{d.o.f.}$	PTE(χ^2)
2MPZ	0.07	0.66	$0.45^{+0.29}_{-0.10}$	$0.064^{+0.022}_{-0.074}$	0.91	0.59
WI×SC-1	0.13	0.88	$0.83^{+0.07}_{-0.06}$	$0.172^{+0.024}_{-0.028}$	1.23	0.18
WI×SC-2	0.18	0.84	$0.80^{+0.05}_{-0.05}$	$0.187^{+0.025}_{-0.015}$	0.82	0.76
WI×SC-3	0.23	0.81	$0.81^{+0.04}_{-0.04}$	$0.203^{+0.021}_{-0.019}$	1.01	0.46
WI×SC-4	0.27	0.74	$0.78^{+0.05}_{-0.05}$	$0.194^{+0.014}_{-0.026}$	0.82	0.78
WI×SC-5	0.32	0.75	$0.78^{+0.04}_{-0.05}$	$0.225^{+0.025}_{-0.019}$	1.29	0.09

Table 3.3: Summary table presenting our main results. The first two columns list our 6 tomographic bins and their mean redshift. Columns 3 and 4 show the best-fit value of the mass bias $1 - b_H$ and their 1D peak value and 68% confidence interval respectively. Column 5 shows the peak value and 68% confidence interval of the bias-averaged thermal pressure (Eq. 4.14) in each bin. Finally, columns 6 and 7 show the reduced χ^2 and associated probability to exceed, indicating that we find a good fit in all cases.

in Section 3.1. The resulting measured power spectra and errors are shown in red in Fig. 3.8, together with their best-fit halo model prediction in black, decomposed into its 1-halo and 2-halo contributions (burgundy and blue respectively). The bottom part of each panel shows the residuals with respect to the best-fit prediction normalised by the 1σ errors. The grey bands cover the data points not used in the analysis due to scale cuts.

Fig. 3.9 shows the correlation matrix of the combined data vector (C_ℓ^{gg}, C_ℓ^{gy}) for each redshift bin (where the correlation matrix r_{ij} is related to the covariance $\mathbf{C}_{ij}$ as $r_{ij} = \mathbf{C}_{ij} / \sqrt{\mathbf{C}_{ii}\mathbf{C}_{jj}}$). At low redshifts, 2MPZ shows strong correlations between different scales, mostly caused by the non-Gaussian contribution to the covariance matrix (Eq. 2.9). These become less relevant at higher redshifts, where non-linear effects are weaker, and where the radial projection pushes the non-linear scale into larger multipole values.

Overall, we find a good agreement between theory and data over the scales used in this analysis. We discuss this agreement and the associated scientific results in the following sections.

3.5.2 Fiducial results

3.5.2.1 Tomographic measurement of the mass bias

We use the measured power spectra to constrain the free parameters of the model described in Section 3.1, with the main aim of providing an alternative measurement of the mass bias ($1 - b_H$) as a function of redshift. Fig. 3.10 shows an example of the posterior parameter contours for our 5 free parameters, $\{M_{\min}, M'_1, b_H, \rho_{gy}, w_z\}$, in the WI×SC-3

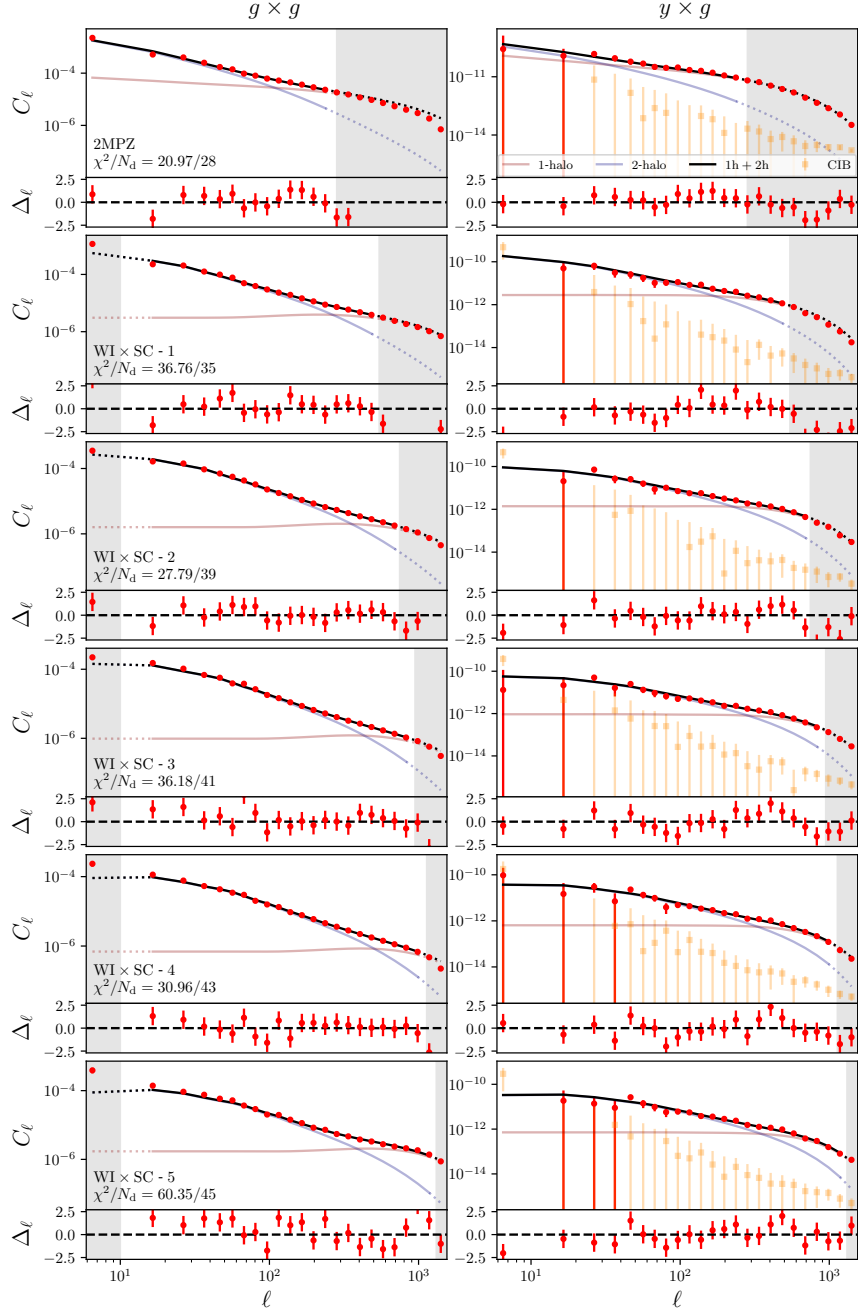


Figure 3.8: Measured galaxy auto-spectra (left column) and $\delta_g \times y$ cross-spectra (right column) for the 6 galaxy redshift bins shown in Fig. 3.2 in ascending order. Each panel shows the measurements (red points with error bars) and best-fit predictions (solid black lines) decomposed into their 1-halo and 2-halo contributions (burgundy and blue lines). The right column additionally shows the estimated contamination from extragalactic dust (yellow squares with error bars), which is found to be negligible. The grey bands indicate the scales removed from the analysis. The bottom part of each panel shows the difference between data and theory normalised by the 1σ uncertainties.

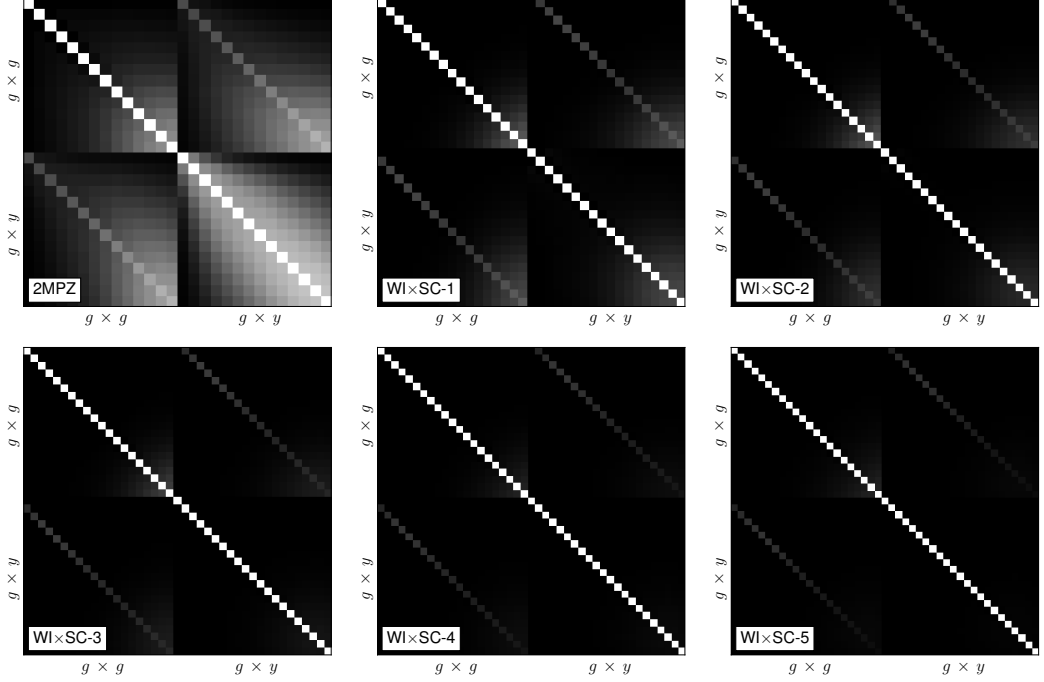


Figure 3.9: Correlation matrices for the 6 redshift bins used in the analysis. Each covariance matrix consists of 4 sub-matrices, corresponding to the covariances of C_ℓ^{gg} and C_ℓ^{gy} (block diagonals), as well as their cross-covariance.

sample ($z \sim 0.23$). As the figure shows, there are strong degeneracies between $M_{\min}$, M'_1 and w_z . This is easy to understand, given that all of these parameters affect the overall amplitude of the galaxy auto-correlation. Since we do not probe scales where the 1-halo term is fully resolved, we do not break the degeneracy between $M_{\min}$ and M'_1 , which regulate the abundance of centrals and satellites, and therefore the constraint on these parameters mostly comes from the 2-halo amplitude (i.e. the galaxy bias), and the 1-halo shot-noise level. On the other hand, the width of the redshift distribution also has a strong impact on the amplitude of the angular power spectrum at all scales due to projection effects. Since a measurement of the galaxy bias from the galaxy autocorrelation is effectively used to constrain b_H from the galaxy-tSZ cross-correlation, the mass bias also shows some degeneracy with the HOD parameters and w_z , albeit at a more moderate level. More interestingly, the mass bias parameter, b_H , shows a visible degeneracy with ρ_{gy} . This is also expected: the effects of ρ_{gy} and b_H on the 1-halo term of the galaxy-tSZ power spectrum are completely degenerate, and thus a free ρ_{gy} ensures that any information obtained on b_H comes entirely from the 2-halo contribution.

The corresponding constraints on $1 - b_H$ for all redshift bins are shown in the main panel of Fig. 3.11 as black circles with error bars. These values, which are also given explicitly in

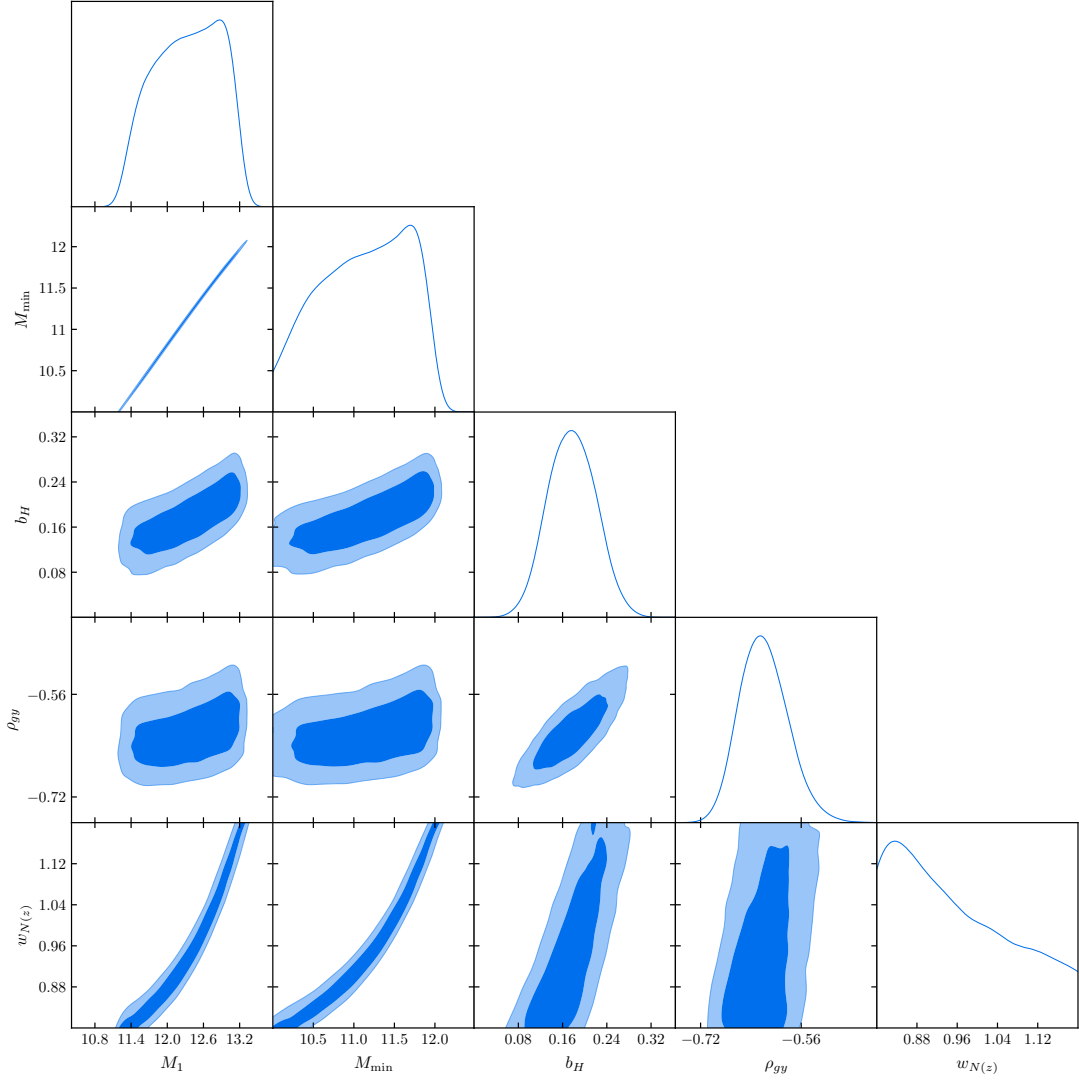


Figure 3.10: 68% and 95% contours for the model parameters in the third WI×SC redshift bin.

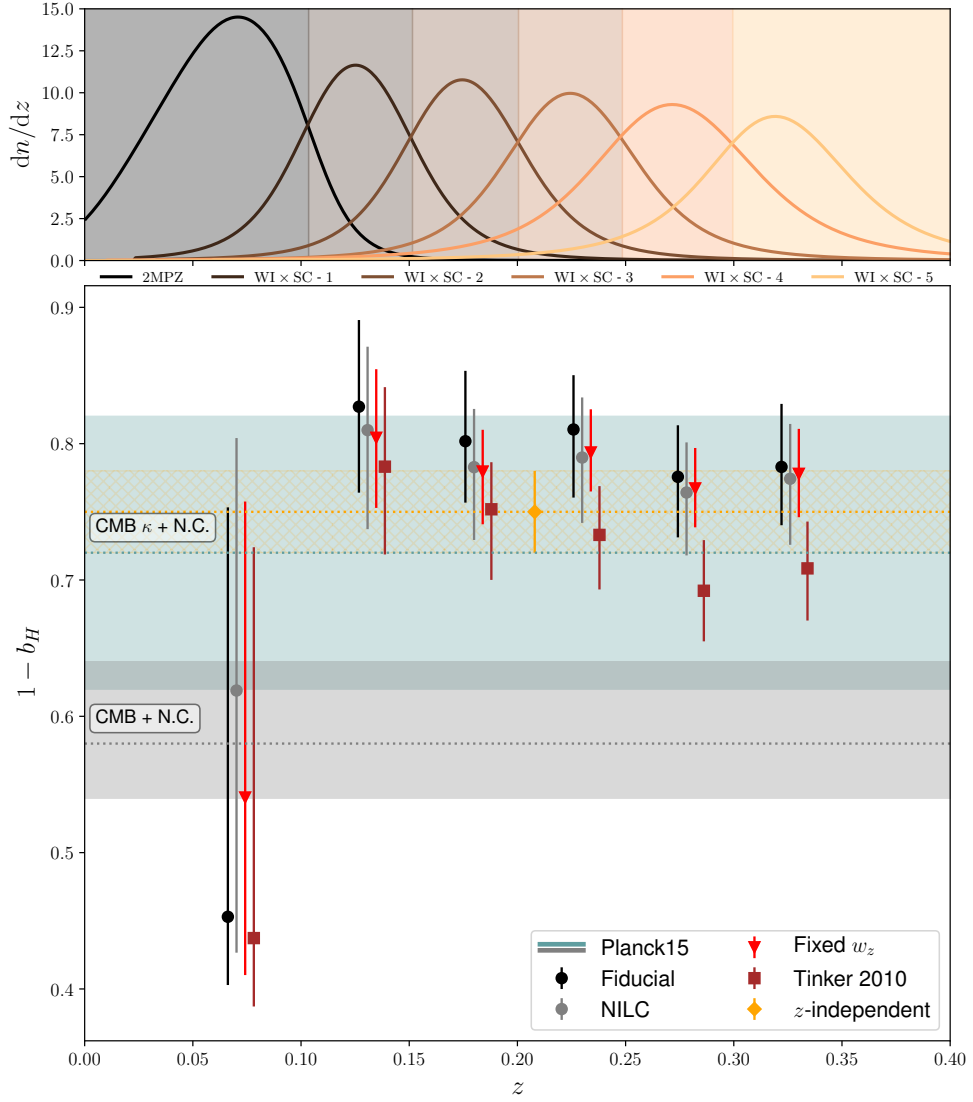


Figure 3.11: Summary figure showing our constraints on the tSZ mass bias $1 - b_H$. Our fiducial constraints are shown as black circles with error bars, and are centred at the mean redshift of each sample. The grey circles show the same constraints found using the NILC y map, which are in good agreement with our fiducial results. The red downward-pointing triangles show the constraints found fixing the photo- z width parameter, w_z . They are in good agreement with our fiducial measurements, with $\sim 20\%$ smaller error bars. The burgundy squares show the results found using the 2010 Tinker mass function, which are systematically biased low. The orange diamond and the mesh show our combined, redshift-independent constraint on $1 - b_H$. For comparison, the grey and turquoise bands show the constraints on the mass bias found by combining cluster counts and CMB primary [155] as well as the constraints using CMB lensing to calibrate cluster masses [218], respectively. The top panel shows the normalised redshift distributions for the six redshift bins, for reference.

column 4 of Table 3.3, correspond to the peak of the 1-dimensional distribution of $1 - b_{\text{H}}$ and to the equal-probability values encompassing a total probability of 0.68. We report our results on departures from our fiducial model (other data points in Fig. 3.11) in Section 3.5.3.

In Table 3.3, column 3 shows the maximum likelihood value of $1 - b_{\text{H}}$. Columns 6 and 7 of the same table list the χ^2 values for each sample divided by the number of data points (approximately equal to the reduced chi-squared statistic),

$$\chi_{\text{red}} \equiv \frac{\chi^2}{N_{\text{dof}}} \approx \frac{\chi^2}{N_{\text{d}}} \quad (3.30)$$

and their associated probability-to-exceed (PTE), defined as

$$\text{PTE} = \int_{\chi^2}^{\infty} P(\chi^2 | N_{\text{dof}}) d\chi^2, \quad (3.31)$$

i.e. the probability that a random realisation of the model would give a chi-squared at least as large as observed. In the absence of systematic errors, the PTE should be uniformly distributed between 0 and 1, with values below 0.05 indicating a poor fit and values near unity sometimes suggesting over-estimated errors. In our case, all six tomographic bins have

$$0.09 \lesssim \text{PTE} \lesssim 0.78, \quad (3.32)$$

showing that none of the fits are in significant tension with the data. Taken together with the reduced chi-squared values clustering around unity, this demonstrates that our fiducial model described in Section 3.1 provides an adequate description of the measured cross-correlations in every redshift bin. The top panel of Fig. 3.11 shows the normalised redshift distributions of the different bins used in this analysis and can be used to visually assess the level of correlation between the different measurements.

We see that, while we are able to measure $1 - b_{\text{H}}$ to a reasonable accuracy ($\sim 5 - 10\%$) in all of the WI×SC redshift bins, the sensitivity for the 2MPZ sample is much poorer. This is not entirely unexpected: even though the cross-correlation between the Compton- γ map and the 2MPZ catalogue yields the highest signal-to-noise ratio of all the samples used here, the low-redshift range covered by this sample implies that the signal is strongly dominated by the 1-halo term. This can be seen in the top right panel of Fig. 3.8. The constraining power of the 2MPZ cross-correlation degrades significantly since, as we have already described, we can only obtain reliable constraints on $1 - b_{\text{H}}$ from the 2-halo contribution.

Our results are in agreement with the estimate of a redshift-independent $1 - b_{\text{H}}$ found by [218] by calibrating cluster masses with CMB lensing ($1 - b_{\text{H}} = 0.71 \pm 0.10$), shown in Fig. 3.11 as a turquoise semi-transparent band. In turn, the grey band in the same figure

shows the constraints on $1 - b_{\text{H}}$ found by combining tSZ cluster counts and the TT CMB power spectrum measured by *Planck* [155] ($1 - b_{\text{H}} = 0.58 \pm 0.04$). This is the value of $1 - b_{\text{H}}$ needed to simultaneously explain the amplitude of density perturbations predicted by the CMB and the abundance of massive clusters. This corresponds to a $\sim 3 - 4\sigma$ tension with our results.

We can however use our results to quantify whether approximating $1 - b_{\text{H}}$ to be constant with redshift is supported by the data. To do so, we combine our six measurements under the assumption that they correspond to the same redshift-independent quantity. We do so by finding the quantity $\bar{b}_{\text{H}}$ that minimises the χ^2 :

$$\chi^2 = \sum_{i,j=1}^6 (b_{\text{H},i} - \bar{b}_{\text{H}}) \mathbf{C}_{b,ij}^{-1} (b_{\text{H},j} - \bar{b}_{\text{H}}), \quad (3.33)$$

where $b_{\text{H},i}$ is the mass bias measured in the i -th redshift bin, and $\mathbf{C}_b$ is the covariance matrix of these measurements. Since the galaxy samples used in this analysis have significant redshift overlap, the off-diagonal elements of $\mathbf{C}_b$ cannot be ignored. We estimate $\mathbf{C}_b$ through jackknife resampling: we use the power spectra measured in each jackknife region described in Section 3.4.2 to estimate the best-fit value of $b_{\text{H},i}$ for each of them, and then calculate the covariance through Eq. 2.4. Since $\bar{b}_{\text{H}}$ is a linear parameter, its best-fit and standard deviation can be found analytically as:

$$\bar{b}_{\text{H}} = \frac{\sum_{ij} \mathbf{C}_{b,ij}^{-1} b_{\text{H},i}}{\sum_{ij} \mathbf{C}_{b,ij}^{-1}}, \quad \sigma(\bar{b}_{\text{H}}) = \left(\sum_{ij} \mathbf{C}_{b,ij}^{-1} \right)^{-1/2}. \quad (3.34)$$

At this point, it is worth acknowledging that this estimate of $\bar{b}_{\text{H}}$ is only strictly consistent if the posterior distribution of the $b_{\text{H},i}$ is Gaussian. We find that, with the exception of the 2MPZ sample, which has a comparatively small statistical power, the marginalised distributions in all redshift bins are sufficiently well-behaved that this is a reasonable approximation (e.g. see 1-dimensional distribution for b_{H} in Fig. 3.10). Our combined constraint on $\bar{b}_{\text{H}}$ following this procedure is:

$$1 - \bar{b}_{\text{H}} = 0.75 \pm 0.03. \quad (3.35)$$

More interestingly, the χ^2 value associated with this measurement is $\chi^2 = 2.1$, which has an associated PTE $\simeq 0.8$. We therefore find that the assumption of a constant mass bias with redshift is compatible with our measurements. This agrees with the results of [45], who used the SZ effect to explore the cosmic thermal history. It also agrees with the results of [171], found using tSZ-selected clusters. The preferred value of $1 - \bar{b}_{\text{H}}$ found also agrees

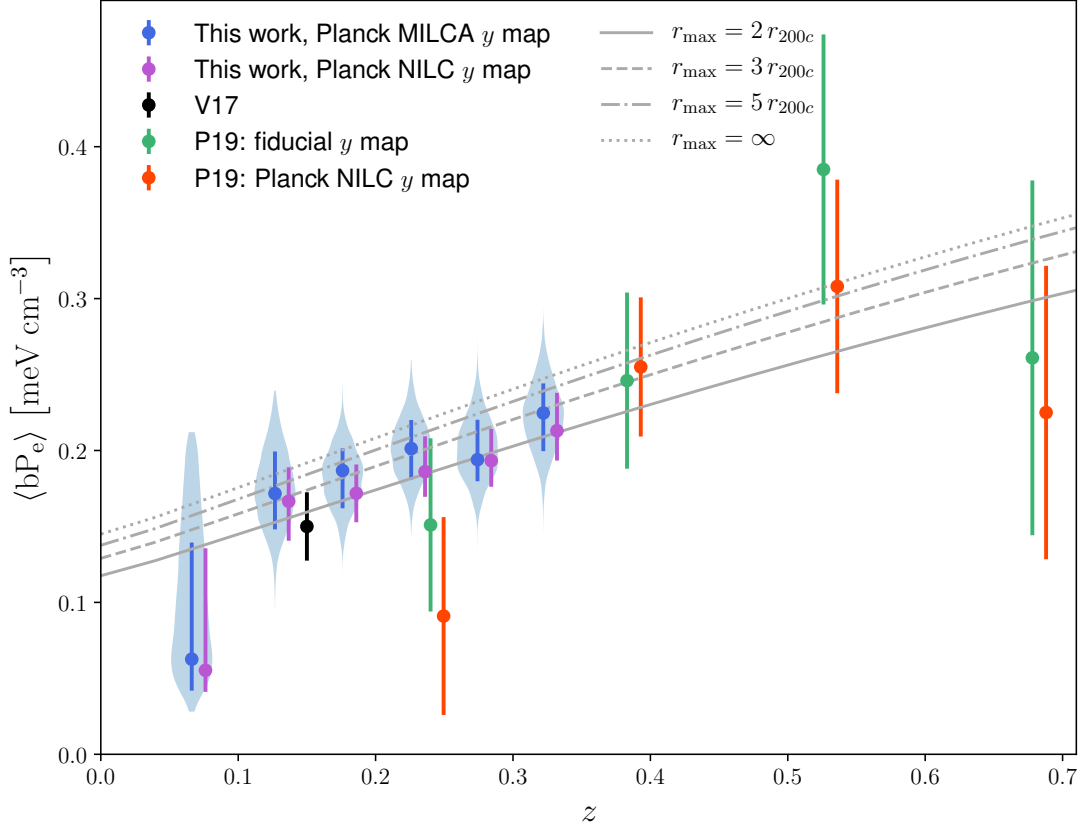


Figure 3.12: Summary figure showing our constraints on the bias-weighted thermal pressure $\langle bP_e \rangle$. Our measurements are shown as blue circles with error bars and a ‘violin’ background. The circle and error bars show the peak of the 1D posterior distribution and 68% confidence interval, while the violins show the full 1D posterior distribution. We also plot our measurements using the *Planck* NILC y map, in violet. The grey lines show the predictions of the shock-heating model of [21] for different values of the r_{max} threshold cluster radius (see legend). For comparison, the figure also shows the constraints on the same parameter found by [206] (black) and [144] (red and green).

with the results of [129] in cross-correlation with galaxies at very low redshift, and those of [130] using weak-lensing cross-correlations at higher z .

While the best-fit value of $1 - b_{\text{H}} \simeq 0.8$ is in broad agreement with the expectation from hydrodynamical simulations [27], the estimate from CMB lensing mass calibration [218], and other direct calibration efforts [e.g. 183, 63], which seem to prefer smaller missing mass fractions ($1 - b_{\text{H}} \simeq 0.8$), our results are in slight tension with the value of $1 - b_{\text{H}}$ found by the *Planck* number counts analysis in combination with primary CMB anisotropies. A possible resolution of the mild tension between tSZ cluster counts and CMB data could be the existence of new physics modifying the late-time expansion and structure growth. However, most of these modifications (e.g. a one-parameter excursion where the dark

energy equation of state takes a larger value $w \sim -0.7$) would make the existing tension in the value of the local expansion rate between CMB and local measurements [166] worse. A careful study of all possibilities, invoking both non-standard physics and more sophisticated astrophysical models, together with improved datasets, is therefore necessary before this tension can be fully resolved.

3.5.2.2 Tomographic measurement of thermal gas pressure

So far we have presented our constraints in terms of the mass bias parameter $1 - b_H$, since this is the main source of systematic uncertainty in the cosmological analysis of cluster number counts carried out by *Planck*. However, the physical interpretation of this parameter (the fraction of missing mass estimated from X-ray measurements under the assumption of hydrostatic equilibrium) is not directly related to the physical process that allows us to constrain it through the cross-correlation of y and δ_g : the fact that galaxy density and pressure trace the same underlying dark-matter fluctuations. In this sense, a more direct observable is the bias-weighted pressure $\langle bP_e \rangle$. This quantity can be interpreted both as the relation between large-scale matter and pressure fluctuations and as the halo-bias-weighted thermal energy density of all haloes at a given redshift [22]. It has been measured at low redshifts by [206] making use of galaxy groups, and at higher redshifts by the Dark Energy Science Collaboration [144]. The redshift range $0.1 \lesssim z \lesssim 0.4$, containing a large fraction of the tSZ sources detected by *Planck* [154], has been so far fairly unconstrained tomographically through this type of measurements.

We derive constraints on $\langle bP_e \rangle$ from our data by reprocessing our Monte-Carlo chains, computing $\langle bP_e \rangle$ at each sample to find its 1-dimensional posterior distribution. The results are listed in Table 3.3 and shown in Fig. 3.12, together with our measurements using the *Planck* NILC y map, and the measurements of [206] and [144]. Our results are in good qualitative agreement with the trend of these previous measurements, as well as with the predictions of the shock heating models of [21]. In these models, the thermal energy entering Eq. 4.14 is estimated by integrating the pressure profile of [21] up to a radius $r_{\max} = N r_{200c}$. As a visual aid to evaluate the agreement of our results with these models, the predictions for $N = 2, 3, 5$ and ∞ are shown as solid, dashed, dot-dashed, and dotted grey lines respectively in Fig. 3.12. These results are the most precise measurement of this quantity to date. The main factors that contribute to the improved constraining power are the larger amplitude of the tSZ signal towards low redshifts and the high density of tracers in the 2MPZ and W1×SC samples.

3.5.3 Systematics analysis

3.5.3.1 tSZ systematics

No component separation method is perfect, and the MILCA Compton- y map used in our fiducial analysis is known to suffer from small levels of contamination from various other astrophysical components. The most relevant for this analysis is the presence of Galactic and extragalactic dust. As described in Section 3.4.1.1, we remove contamination from Galactic dust at the map level in both y and δ_g . The extragalactic component, the so-called Cosmic Infrared Background (CIB), is, however, a more relevant concern, given that it traces the large-scale structure, and is therefore statistically correlated with both of our observables (δ_g and y). Since the CIB is most relevant near the peak of star formation ($z \sim 2$), we expect this contamination to be small, but it must be quantified carefully.

To do so, we follow the same method used in [206]. We model the CIB contamination in the y map as:

$$y_{\text{obs}}(\hat{\theta}) = y_{\text{true}}(\hat{\theta}) + \epsilon_{\text{CIB}} c(\hat{\theta}), \quad (3.36)$$

where y_{obs} and y_{true} are the observed and true Compton- y maps, ϵ_{CIB} is a free parameter, and $c(\hat{\theta})$ is a template for the CIB emission. For our analysis, we used the 545 GHz map released by *Planck* as a proxy for CIB emission. The different cross-correlations between y_{obs} and both δ_g and c are then given by:

$$C_{\ell}^{yc,\text{obs}} = C_{\ell}^{yc} + \epsilon_{\text{CIB}} C_{\ell}^{cc}, \quad (3.37)$$

$$C_{\ell}^{gy,\text{obs}} = C_{\ell}^{gy} + \epsilon_{\text{CIB}} C_{\ell}^{gc}, \quad (3.38)$$

where C_{ℓ}^{cc} , C_{ℓ}^{yc} , and C_{ℓ}^{gc} are the auto-correlation of the CIB, and its intrinsic cross-correlations with y and δ_g respectively. Since C_{ℓ}^{gc} can be estimated directly by cross-correlating our galaxy overdensity maps with the 545 GHz map, the only remaining step to quantify the contamination to the $\delta_g \times y$ cross-correlation is to estimate ϵ_{CIB} . To do so, we measure the cross-correlation between the MILCA y map and the 545 GHz map and fit a model of the form of Eq. 3.37, with a single free parameter ϵ_{CIB} , and C_{ℓ}^{yc} and C_{ℓ}^{cc} given by the best-fit models for the CIB auto- and cross-correlation provided by [152, 156]. As reported in [8], this procedure yields an estimate of the CIB contamination,

$$\epsilon_{\text{CIB}} = (2.3 \pm 6.6) \times 10^{-7} (\text{MJy/sr})^{-1}. \quad (3.39)$$

For this exercise, in order to reduce the noise variance from the Galactic component of the 545 GHz map, the cross-correlations $C_{\ell}^{yc,\text{obs}}$ and C_{ℓ}^{gc} were estimated using *Planck*'s 20% Galactic mask. Additionally, all uncertainties were estimated using the jackknife method described in Section 2.1.2.

Using this measurement of ϵ_{CIB} and the estimate of C_ℓ^{gc} from the cross-correlation with the 545 GHz map, Fig. 3.8 shows, in the right panels, the estimated level of contamination from the CIB in our $\delta_g \times y$ cross-correlation. In all cases, the contamination is small, at the level of $\sim 1\%$ of the signal, and therefore can be neglected in our analysis.

As an additional test for systematics, we have also repeated our full analysis replacing the MILCA y map with the NILC map released by *Planck*. Different component-separation methods are sensitive to different types of contamination, and this exercise is therefore both a necessary consistency check and a way to reinforce our conclusion that the level of CIB contamination is negligible. The resulting measurements of $1 - b_{\text{H}}$ are shown in Fig. 3.11 next to the fiducial measurements, as grey circles with error bars. The results are in very good agreement with our fiducial analysis.

3.5.3.2 Photometric redshift uncertainties

We have quantified the impact on our results of the uncertainties on the redshift distributions of the different samples used here by introducing the free width parameter w_z with a 20% top-hat prior. In order to study the impact of these uncertainties on our results, we have recomputed the constraints on $1 - b_{\text{H}}$ fixing w_z to its fiducial value of 1. The results are shown in Fig. 3.11 as red downward-pointing triangles. We observe that the only effect of allowing w_z to vary is to increase the final uncertainties on $1 - b_{\text{H}}$ by about 20%. The final best-fit value of $1 - b_{\text{H}}$ does not change significantly, and the posterior distribution of w_z is always peaked around the fiducial value of 1 (see e.g. bottom-right panel of Fig. 3.10).

3.5.3.3 Mass function parameterisation

Another source of systematics in our measurement is the theoretical uncertainty due to our choice of halo mass function in the halo model. Our fiducial mass function is the parameterisation of [194], which we use in order to be able to make a direct comparison with the results of [155]. To explore the dependence on the choice of mass function parameterisation, we perform our analysis again using the updated parameterisation of [195]. Fig. 3.11 shows the final constraints on $1 - b_{\text{H}}$ in this case as burgundy squares, which are consistently lower by $\sim 2\sigma$. We find that this is caused by a redshift-dependent systematic offset between the two mass function parameterisations which reaches the level of 10-20% in the range $0 < z < 0.4$. This is shown explicitly in Fig. 3.13, where it is evident that this offset is present for two mass function parameterisations: [195] and [57]. Since the relative differences of both models with that of [194] are of the same order ($\sim 10\%$ enhancement), we expect the constraints on $1 - b_{\text{H}}$ using [57] to be similar to those shown in Fig. 3.11 as burgundy squares. We have not explored any offsets potentially caused

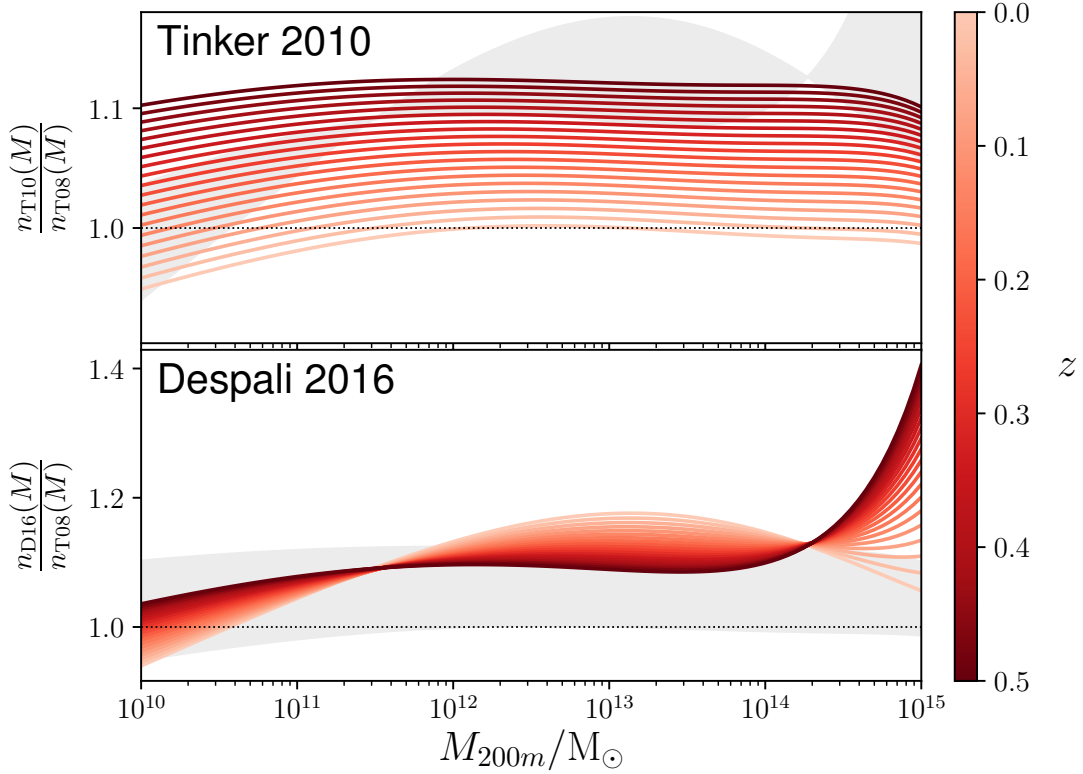


Figure 3.13: Ratio between the mass function parameterisations of [195] and [57] with the parameterisation of [194] in the range of redshifts relevant to this analysis. A systematic offset of about 10-20% between them can be observed, which causes the lower value of $1 - b_H$ found using the [195] parameterisation (see Fig. 3.11).

by our choice of halo bias parameterisation (having adopted the one described in [195] throughout). Since the aim of this paper is not to characterise the theoretical uncertainties on the combination of halo mass function and halo bias, we leave the study of this issue for future work and limit ourselves to using the parameterisation of [194] in order to match previous analyses [e.g. 155, 36, 12, 218, 130]. It should be pointed out, however, that these systematics should not affect our results of the halo model bias for the Compton- y parameter, $\langle bP_e \rangle$, as the latter is mainly affected by the relative amplitudes of the $\delta_g \times \delta_g$ and $\delta_g \times y$ power spectra, and is therefore robust to the mass function we use.

3.5.3.4 Galaxy clustering model

To verify that our results are robust against the details of the HOD model used to parameterise the galaxy-matter connection, we have repeated the analysis for extended versions of our baseline model. We explore two such extensions:

- We include the threshold width for centrals $\sigma_{\text{ln}M}$ as an additional free parameter with a broad top-hat prior, instead of fixing it to the value used by [12].
- We decouple central and satellite galaxies, allowing for haloes without a central galaxy to contain satellites. This implies removing the factor $N_c(M)$ in Eq. 3.10 and introducing M_0 (the mass threshold to have satellites) as a new free parameter that is not linked to M_{min} .

In both cases, we have confirmed that the constraints derived on $1 - b_{\text{H}}$ do not deviate significantly from our fiducial results and that our data are not able to constrain the new free HOD parameters introduced by each extension ($\sigma_{\text{ln}M}$ and M_0 respectively). We therefore conclude that our results are insensitive to the specifics of the model used to characterise the clustering of galaxies, which is well described in the range of scales studied here by our fiducial 2-parameter HOD model .

3.6 Conclusion

Cross-correlating maps of the tSZ Compton- y parameter with tomographic measurements of the projected galaxy distribution allows us to study the redshift evolution of the thermal gas pressure, since we expect both pressure and galaxies to trace the same underlying matter inhomogeneities. We have measured the $\delta_g \times y$ cross-correlation to very high significance using public y maps made available by the *Planck* collaboration [161] and the 2MPZ and WIXSC galaxy catalogues [28, 29] in six photometric redshift bins covering the redshift range $z \lesssim 0.4$.

Combining this measurement with a measurement of the galaxy auto-correlation allows us to break the degeneracy between the two parameters that relate δ_g and y to the matter fluctuations: the galaxy bias, which we model effectively using a 2-parameter HOD prescription, and the mass bias parameter $1 - b_{\text{H}}$; and thus enables us to make a redshift-dependent measurement of $1 - b_{\text{H}}$. The results, shown in Table 3.3 and Fig. 3.11, agree well with the calibration of tSZ cluster masses with CMB lensing measurements [218], and with similar measurements of $1 - b_{\text{H}}$ made by [129] through the same type of cross-correlation at lower redshifts. Nevertheless, we observe a slight tension with the measurements of $1 - b_{\text{H}}$ made by *Planck* through the combination of cluster number counts and CMB primary anisotropies [155]. More importantly, our tomographic measurement allows us to study the possible redshift dependence of the mass bias, an important ingredient in the cosmological analysis of cluster abundances. Within our uncertainties, we do not find any statistical evidence for a redshift dependence of $1 - b_{\text{H}}$, in agreement with previous analyses [171].

Perhaps more interestingly, our measurements can be interpreted as constraints on the bias-weighted mean gas pressure $\langle bP_e \rangle$, the equivalent of the large-scale galaxy bias for the y parameter. This quantity is directly related to the energetics of gas in haloes and can be used to constrain different heating models. Our results, shown in Table 3.3 and Fig. 3.12, agree well with previous measurements of the same quantity [206, 144], as well as with shock-heating models [21]. This result is also the most precise measurement of the large-scale correlation between δ_g and y to date.

The main sources of systematic uncertainty in our analysis are contamination of the galaxy clustering auto-correlation by stars, Galactic dust, and other observing conditions, the contamination from CIB emission in the y map, and uncertainties in the galaxy redshift distribution due to the use of photometric redshifts. We address the galaxy clustering systematics by deprojecting templates for dust and stellar contamination at the map level, by deprojecting the expected contamination from zero-point fluctuations in the SuperCOSMOS photographic plates at the power spectrum level, and by masking out the largest scales ($\ell < 10$) in the galaxy clustering auto-correlation. We use the 545 GHz *Planck* map as a tracer of CIB to quantify the level of contamination in the $\delta_g \times y$ cross-correlation, and find it to be negligible (as could be expected given the relatively low redshift of our sample compared with the peak of star formation, $z \sim 2$). Finally, we determine that the most relevant form of systematic associated with uncertainties in the redshift distributions is that associated with the width of $p(z)$. To address this, we add a new parameter to our model, w_z , which describes the distribution widths. We marginalise over w_z with a 20% prior, which we have determined to be significantly larger than the expected uncertainty on the true width. This has the effect of degrading our constraints on $1 - b_H$ and $\langle bP_e \rangle$ by $\sim 20\%$ without modifying the best-fit value of either quantity significantly.

We must also note that our parameter constraints have been derived for a fixed cosmological model, corresponding to the best-fit parameters found by *Planck* [158]. The most significant consequence is the fact that our constraints are strongly degenerate with the amplitude of matter fluctuations, parameterised by σ_8 . Therefore, the agreement between our constraints on $1 - b_H$ and those found by *Planck* [155] can be thought of as a proof of the consistency between the properties of the galaxy distribution in 2MPZ and W1xSC and the CMB anisotropies. However, our results regarding the redshift evolution of $1 - b_H$ (or lack thereof), and the agreement of $\langle bP_e \rangle$ with existing heating models, are only possible thanks to the tomographic cross-correlation with the galaxy density fluctuations. Robust joint constraints on cosmological and astrophysical parameters could be achieved by a combined analysis of galaxy clustering, Compton- y maps and gravitational lensing data (either from cosmic shear or CMB lensing observations). We leave this analysis for future work.

The scientific yield of these types of observations will increase significantly with data from current and near-future experiments, such as the Atacama Cosmology Telescope [54] or the Simons Observatory [5] on the CMB side, and the Vera C. Rubin Observatory [126] or the Euclid satellite [117] in terms of galaxy clustering and cosmic shear. The main advances will come in the form of lower-noise and higher-resolution y and CMB lensing maps, as well as a dense ($\sim 30 \text{ arcmin}^{-2}$) sampling of the galaxy distribution to much higher redshifts ($z \lesssim 2$). This increase in sensitivity, however, will have to be accompanied by a much better control of systematics such as contamination from CIB and other sources, galaxy clustering systematics, or photo- z uncertainties.

Chapter 4

Multi-Probe Tomography of Thermal Energy & Growth

Modern observational cosmology has reached a stage where the precision of many constraints is limited by systematics and astrophysical uncertainties. These arise primarily from our incomplete understanding of small-scale baryonic physics, including the distribution of galaxies, their feedback mechanisms, and their interplay with dark matter. As outlined in Chapter 3, one of the primary challenges is the *mass-to-observable* relation, which links observable quantities such as galaxy clustering or tSZ maps to the underlying mass distribution. The uncertainty in this relation affects the cosmological constraints we derive from cluster number counts and other studies of the large-scale structure.

In Chapter 3 we demonstrated how combining CMB secondary anisotropies with large-scale structure data allows us to probe the intergalactic gas pressure. These cross-correlations are critical in order to break degeneracies between astrophysical and cosmological parameters. Building upon that work, this chapter jointly explores galaxy clustering ($\delta_g \times \delta_g$), and the cross-correlations of galaxy positions with tSZ ($\delta_g \times y$), as well as the galaxy positions with CMB lensing ($\delta_g \times \kappa$), emphasising the potential of those probes to calibrate the mass bias ($1 - b_H$) in studies of the tSZ, and constrain cosmic evolution.

4.1 Breaking the mass bias-cosmology degeneracy

Astrophysical systematics, particularly the mass bias in tSZ studies, play a central role in limiting the precision of cosmological constraints. As previously mentioned, the mass bias quantifies the difference between the true cluster mass M^{true} and the observationally inferred mass M^{obs} , $M^{\text{obs}} = (1 - b_H)M^{\text{true}}$.

This bias propagates into the mass-to-observable relation used to derive cosmological parameters, introducing degeneracies that hinder precise measurements of e.g. the ampli-

tude of matter fluctuations (σ_8), the matter density (Ω_m). Eq. 3.1 shows an example of how cluster mass is inferred via a scaling relation. In turn, the mass bias directly affects the inferred values of σ_8 and Ω_m , and even small deviations in its characterisation can significantly shift cosmological constraints.

This degeneracy is mainly caused by the correlated impact of these two parameters on the observed tSZ signal. While $1 - b_H$ modulates the normalisation of the scaling relation, σ_8 governs the overall amplitude of matter fluctuations, which directly influences cluster abundance (and thus the tSZ power spectrum). As a result, varying these parameters produces similar observational effects, making it challenging to disentangle their individual contributions. At some fixed σ_8 , a larger value of $1 - b_H$ shifts the inferred halo masses upward, leading to reduced gas pressure profiles and matter clustering amplitudes. Conversely, if we fix $1 - b_H$, increasing σ_8 enhances the overall amplitude of the cross-power spectra. Assuming $T_X(k, z)$ is the transfer function of some observable X , this can be expressed in terms of the observable 3D power spectra:

$$P_{XY}(k, z) \propto \sigma_8^2 b_X b_Y T_X(k, z) T_Y(k, z), \quad (4.1)$$

where $X, Y \in g, y, \kappa$ are pairs of tracers, and b_X is the bias associated with tracer X .

Breaking this degeneracy is important in order to derive robust constraints on σ_8 and therefore improve our understanding of the growth of structure. Joint analyses of multiple probes achieve this because each tracer has a distinct dependency on σ_8 and $1 - b_H$. For example, the $\delta_g \times y$ cross-power spectrum probes the thermodynamics of baryons via the pressure profile $P_e(r)$, which depends on halo mass and redshift (Eq. 3.3). In contrast, $\delta_g \times \kappa$ is sensitive to the total matter distribution, irrespective of any baryonic effects.

In this chapter we aim to break this degeneracy by introducing cross-correlations of galaxy positions with another independent probe, CMB lensing convergence ($\delta_g \times \kappa$). Here, we rely on the fact that galaxy bias, b_g , and lensing convergence trace distinct aspects of the underlying matter distribution, offering an ‘orthogonal’ constraint on cosmological parameters. At the same time, the cross-correlation between galaxy positions and tSZ maps also exploits the connection between galaxy bias and the underlying matter density field. By combining these datasets, we attempt to disentangle the contributions of astrophysical systematics from cosmological parameters, thereby enhancing our ability to jointly constrain $1 - b_H$ and σ_8 .

Note that in this chapter, we focus specifically on the constraints on σ_8 , leaving a detailed analysis of its interplay with Ω_m to future work. We also investigate the redshift dependence of the thermal energy content of haloes, as well as the mass bias. This analysis is particularly relevant given the ongoing tensions between tSZ cluster counts and CMB

primaries, which may indicate unaccounted systematics in the y -mass relation. Hence, this work demonstrates a pathway in order to mitigate astrophysical uncertainties and to enhance the cosmological insights gained from high-precision datasets, in light of future experiments.

4.2 Overview of the cosmological tensions

4.2.1 The Hubble tension

The Hubble constant, H_0 , which quantifies the current rate of expansion of the Universe, is perhaps the most widely known parameter in all of cosmology. However, measurements of H_0 with different methods have revealed a significant discrepancy, often referred to as the *Hubble tension*.

Observations based on the early Universe, such as the CMB, yield lower values of H_0 , while late-Universe probes, such as distance-ladder measurements of supernovae calibrated by Cepheid stars, result in higher values.

The CMB is a clean cosmological probe that provides a direct determination of H_0 , interpreted under the widely-accepted Λ CDM cosmological model. Measurements using the *Planck* satellite [6] yield a value of

$$H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (4.2)$$

This value is derived from the angular size of the sound horizon at the time of last scattering, which depends on the matter density Ω_m , baryon density Ω_b , and the speed of sound in the primordial plasma.

The relation between H_0 and the comoving size of the sound horizon r_s at the time of last scattering is given by

$$r_s = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (4.3)$$

where c_s is the speed of sound in the primordial plasma, and $H(z)$ is the Hubble parameter.

In contrast, H_0 measurements at late times (e.g. the SH0ES project, [167]) use Cepheid-calibrated Type Ia supernovae to measure distances, and they yield a value of

$$H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (4.4)$$

Calibrating distances using Cepheid stars relies on the *cosmic distance ladder* where distances are calculated in a step-wise manner, and nearer objects are used to calibrate the tools used to measure far distances. This method is independent of any assumptions

about the early Universe (cosmological model) but relies on precise calibrations of Cepheid variable stars and supernova luminosities.

Late-time methods measure H_0 directly through

$$D_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{dz'}{E(z')}, \quad (4.5)$$

where $D_L(z)$ is the luminosity distance and $E(z) \equiv H(z)/H_0$. Here, any mismatch in calibrating $d_L(z)$ would translate into a discrepancy in H_0 .

The discrepancy between early- and late-time measurements currently stands at over 5σ , strongly suggesting either unknown systematic errors in one or both methods, or new physics beyond the standard Λ CDM model.

4.2.2 The σ_8 tension

The parameter σ_8 describes the amplitude of matter density fluctuations on scales of $8 h^{-1}$ Mpc and also plays a crucial role in quantifying the growth of LSS in the Universe. Under the widely-accepted Λ CDM model, σ_8 is tightly coupled to matter density Ω_m , via the growth of structure. Similar to the Hubble tension, a persistent discrepancy is observed when we infer this parameter using early-Universe probes (e.g. the CMB) versus when measuring it with late-time data (e.g. weak lensing and galaxy clustering).

The *Planck* collaboration [6] determines σ_8 from the temperature and polarisation anisotropies in the CMB. Since this parameter is degenerate with Ω_m , the inferred value of the parameter combination $S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$ is

$$S_8 = 0.834 \pm 0.016 \quad (4.6)$$

when additional constraints based on baryon acoustic oscillations are excluded. This value relies on the assumption that Λ CDM governs evolution from recombination to the present.

In contrast, measurements from the late-time Universe that probe the growth of LSS generally find lower values of S_8 . For example, surveys using weak lensing [88, 2] report

$$S_8 = 0.776 \pm 0.017 \quad [\text{KiDS}], \quad (4.7)$$

$$S_8 = 0.773 \pm 0.020 \quad [\text{DES}]. \quad (4.8)$$

These values are derived from galaxy clustering and cosmic shear measurements, which depend on the distribution of matter in the late Universe.

The σ_8 tension is currently at the $2 - 3\sigma$ level, and suggests a discrepancy between the growth of structure inferred from the early Universe and the one measured directly at low redshifts. As with the Hubble tension, this tension might be a result of systematic effects

(e.g. unaccounted-for biases in measurements, redshift distributions, etc.) or it could hint at new physics.

The mathematical formulation for calculating the value of σ_8 is

$$\sigma_8^2 = \int_0^\infty \frac{k^3}{2\pi^2} P(k) |W(kR)|^2 dk, \quad (4.9)$$

where $P(k)$ is the matter power spectrum, $W(kR)$ is the top-hat window function in Fourier space (which partially defines σ_8 as a quantity), and $R = 8 h^{-1}$ Mpc. Resolving the σ_8 tension is critical in order to test the validity of the much-established cosmological model, the Λ CDM.

Recent works by [11] and [164] provide evidence that refined non-linear modelling may offer a solution to the S_8 tension. In these studies, the authors demonstrate that by incorporating more accurate treatments of non-linear gravitational evolution and baryonic feedback, the inferred value of S_8 is brought into closer alignment with the one derived from CMB observations. Their work suggests that the tension may be primarily driven by systematic uncertainties and limitations in the current modelling of small-scale physics, rather than by fundamentally new physics. This highlights the need for continued improvement of the non-linear models and data calibration in order to fully resolve the discrepancies between early- and late-time measurements.

4.3 Methods

4.3.1 Redshift distributions

The number densities of galaxies are central to calculating the projected galaxy overdensity. They are also not trivial to estimate accurately. In Section 3.3, we mitigated this lack of accuracy by introducing a shape parameter w_z that stretched the redshift distribution. We noted in Section 3.5.3.2 that the uncertainty of w_z propagated to an uncertainty of the same order in our final $(1 - b_H)$ constraints.

Since in this work we also wish to constrain cosmological parameters, we re-derive the galaxy number densities using *weighted direct calibration* (DIR) [123, 90]. Details of this method can be found in the references provided, but we briefly describe the process here for completeness.

All of the galaxies in the 2MPZ and WIXSC catalogues have photometric estimates of their redshift. However, only a sub-sample of those galaxies also have a spectroscopic measurement of their redshift. In performing DIR, we assume that the redshift distribution of the entire sample of galaxies can be approximated by constructing an appropriately

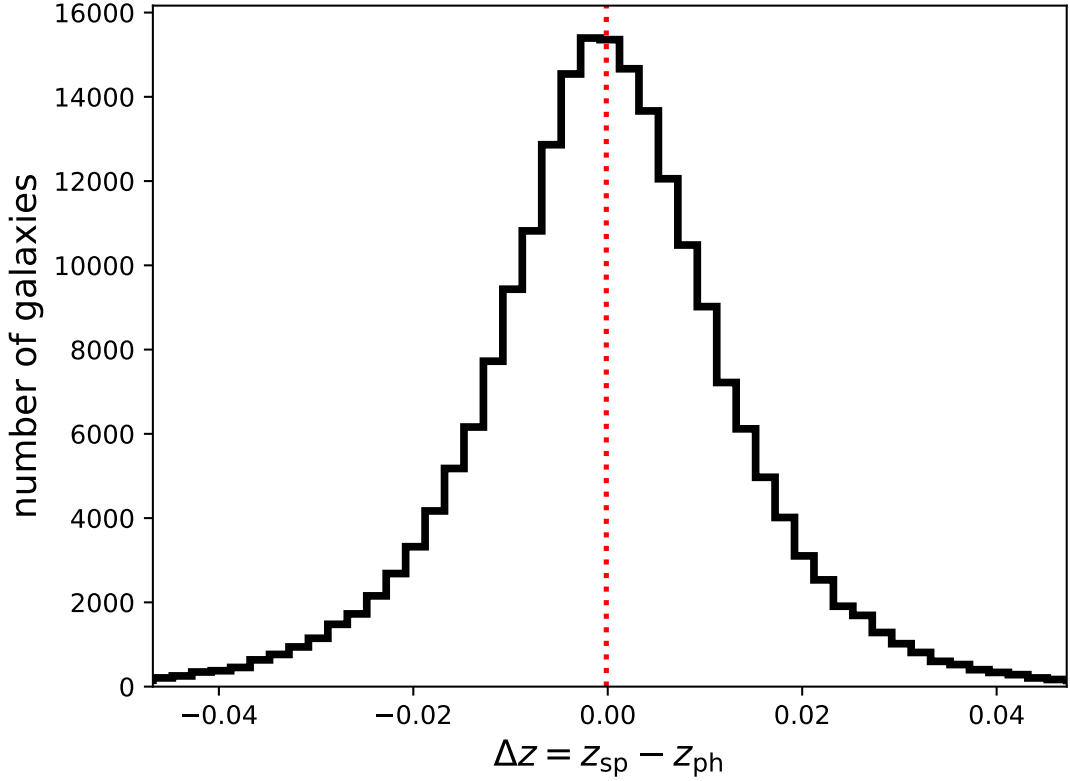


Figure 4.1: Histogram of the difference between photometric redshift estimates and spectroscopic (true) redshifts for the sub-sample of galaxies where both measurements are available, in the first redshift bin (2MPZ) of our analysis. The scatter highlights the limitations of uncalibrated photometric redshifts.

weighted histogram of the redshift distribution of the high-fidelity subsample. The problem then reduces to one of calculating the weight of each galaxy in the sub-sample. Note that we already have photo- z 's, which were derived via neural nets using a sub-sample with spectroscopy as a training set [30].

Photometric redshifts, while useful, suffer from inherent inaccuracies compared to spectroscopic redshifts due to limited photometric bands and colour-space degeneracies. To illustrate this, Fig. 4.1 shows a histogram of the difference $\Delta z = z_{\text{phot}} - z_{\text{spec}}$ for the sub-sample with both measurements available, in the first redshift bin of our analysis (2MPZ bin). The characteristic scatter reflects both random errors and systematic biases in estimating z_{phot} , emphasising that relying solely on photometric redshifts without calibration would compromise the accuracy of redshift distributions, particularly for precision-dependent analyses such as cross-correlations.

Fig. 4.2 directly gives the quantity of interest, which is $p(z_{\text{spec}} | z_{\text{phot}})$. We again illustrate this only for the first redshift bin (2MPZ). The advantage of repeating the calibration

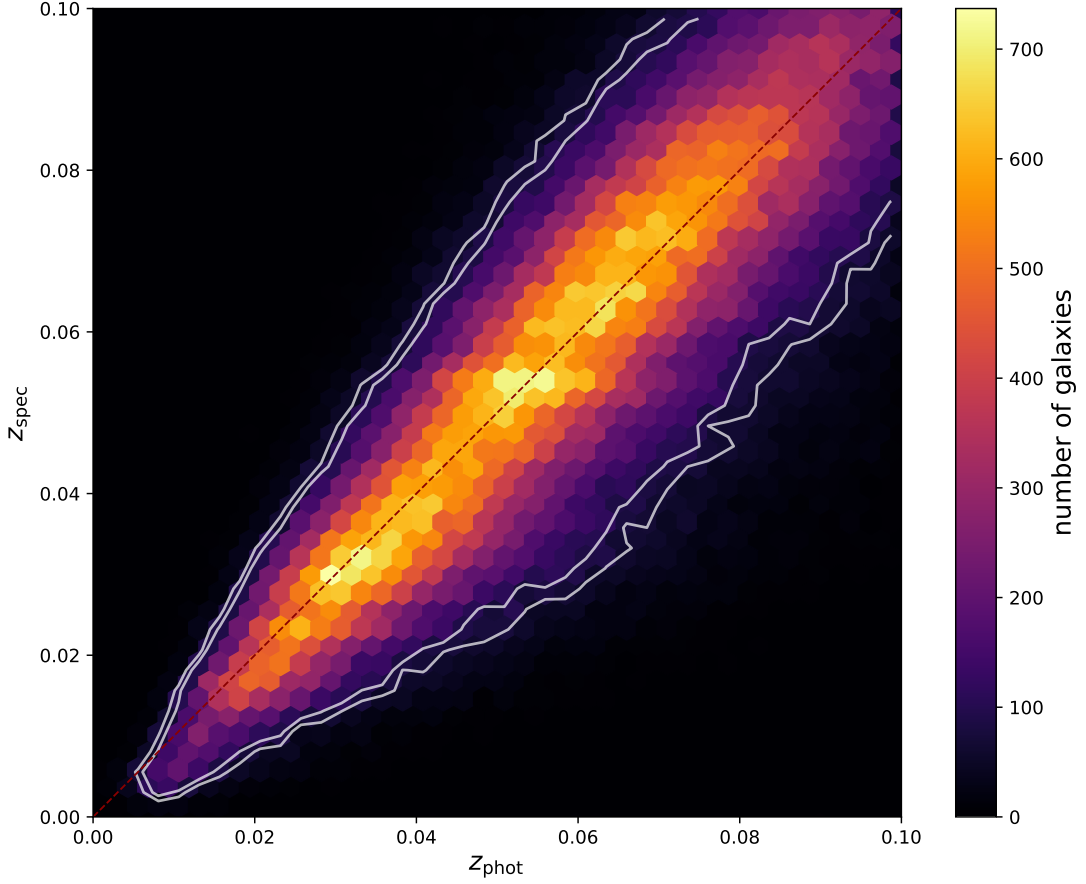


Figure 4.2: Binned plot of the spectroscopic redshift, z_{spec} against the photometric redshift z_{phot} predicted using ANNz. This is only for the subsample of galaxies, the spectroscopic redshift of which is known, within the first redshift bin used in our analysis.

of the redshift distributions using the same data lies in the fact that the results are extremely sensitive to calibration, and thus it is advantageous to have a transparent and direct method where it is easier to understand the source of uncertainties, unlike ANNz.

Below, we describe the calibration algorithm step-by-step. It relies critically on a *k-d tree* data structure to accelerate its nearest-neighbour searches. A *k-d tree* (short for *k*-dimensional tree) is a binary, axis-aligned partitioning of a *k*-dimensional feature space (here, the multi-band photometric magnitudes) built by recursively splitting the dataset along one coordinate at each node. By cycling through the dimensions (e.g. $J, H, K, W1, W2, B, R, I$) and dividing at median values, the tree creates a hierarchy of hyper-rectangles. When querying the tree for the nearest neighbours of a spectroscopic object, only the branches whose bounding hyper-rectangles could contain closer points need to be explored, reducing the complexity from $\mathcal{O}(N)$ to $\mathcal{O}(\log N)$ per query. In practice, we query the nearest neighbours to compute the DIR re-weighting factors.

To calculate the weights, we proceed as follows:

1. We split the 2MPZ and WI×SC catalogues in bins of photometric redshift as defined in Table 3.1, and apply the galactic mask shown at the top of Fig. 3.1. The steps following are performed separately for each redshift bin.
2. Using the apparent magnitudes of the galaxies in each photometric band, we construct two k-d trees: one containing all the galaxies in the redshift bin (*big tree*), as well as one containing only the sub-sample of galaxies with measured spectroscopic redshifts (*small tree*). Galaxies in the 2MPZ catalogue have magnitude information in 8 bands ($J, H, K, W1, W2, B, R, I$) while galaxies in the WI×SC catalogue have magnitude information in 4 bands ($W1, W2, B, R$).
3. For each galaxy in the small tree we wish to sample a representative hypervolume in the multidimensional colour space. We choose to sample the $N_{\text{neigh}} = 20$ nearest neighbours, and record the Euclidean distance of the furthest neighbour.
4. Querying the big tree, we count the total number of galaxies, $N_{\text{photo-z}}$, contained within the (maximum) recorded distance.
5. The weight of each galaxy in the sub-sample is then simply the relative number density in the colour-space vicinity of the galaxies in the two trees, normalised by the tree sizes:

$$w_i = \frac{\text{len}(\text{sub-sample})}{\text{len}(\text{sample})} \times \frac{N_{\text{photo-z}, i}}{N_{\text{neigh}, i}}. \quad (4.10)$$

A critical condition for this method to work is for the weights to be unbiased; hence, we require that the following conditions are met:

- **Colour-space tracing:** The galaxies in the sub-sample trace the galaxies in the entire sample, in colour-space, so that each multidimensional voxel containing N_{neigh} galaxies defines a single redshift value. In contrast, when galaxies in the sub-sample do not trace the galaxies of the entire sample, the furthest neighbours will be so far out that redshift values from adjacent voxels will leak into the voxel we query in the big tree.
- **Photometric band sufficiency:** There are enough photometric bands so that each voxel uniquely defines a redshift value.

In our case, the first requirement is satisfied because of the relatively large fraction of galaxies with measured spectroscopic redshifts. In Table 4.1 we list the fraction of galaxies in each redshift constituting our training sample (i.e. the spectroscopic redshift is known).

Note that in general it is not obvious that these conditions are met. Shortcomings of sample incompleteness have been extensively studied (e.g. [86]). Even with a large redshift

Sample	$N_{\text{spec}}/N_{\text{tot}}$	Sample	$N_{\text{spec}}/N_{\text{tot}}$
2MPZ	0.455	WI×SC-3	0.024
WI×SC-1	0.080	WI×SC-4	0.028
WI×SC-2	0.041	WI×SC-5	0.076

Table 4.1: Fraction of galaxies with measured spectroscopic redshift which are used as the training sample to perform DIR.

fraction – which is only actually a few % – there will be subregions of colour space that are poorly sampled. For instance, if redshifts of blue galaxies are rarer, then the 20th nearest neighbour will most likely be in the red direction and so $\bar{z}$ will be biased.

Another concern arises from the fact that we are working in colour-space. For a given colour, a faint galaxy is likely to be more distant. So if the spectroscopic training sample is brighter than the main catalogue, that is a potential problem. The ANN estimates should avoid that in principle because we use magnitudes rather than colours as the input to the ANNz code.

In particular, we explore the second assumption (band sufficiency) and the relationship between photometric magnitudes and redshifts through *Uniform Manifold Approximation and Projection* (UMAP) [136], a dimensionality reduction technique that maps high-dimensional data into a lower-dimensional latent space.

The full feature space includes the available magnitudes, the colours (difference between magnitudes, e.g. B-R), as well as a colour index that captures the SED shape, which we define as

$$\text{CI}_{\text{SED}} = \frac{B + R}{W_1 + W_2}, \quad (4.11)$$

where we use the magnitudes at the edges of the spectrum to get a measure of the skewness (or slope) of the SED of galaxies. CI_{SED} essentially captures how the relative contribution of optical light (from the *B* and *R* bands) and mid-infrared light (from the *W1* and *W2* bands) changes with z_{spec} . Fig. 4.3 demonstrates how CI_{SED} yields good separability of three redshift bins, separating the sample using the 0.1 and 0.9 quantiles in z_{spec} .

Fig. 4.4 presents a scatter plot of the first two UMAP components, where points have been colour-coded with respect to spectroscopic redshift. The hue indicates the qualitative difference between regions of different spectroscopic redshifts in the full parameter space (hence in the UMAP latent space). To emphasise the different redshift regions in a robust manner, quantile rescaling with a target uniform distribution has been applied to the spectroscopic redshifts, so that the impact of marginal outliers is reduced. There is a fine divide separating the 2MPZ from the WI×SC catalogues, which is entirely a UMAP artefact, owing to the different local densities of galaxies in the full parameter space defined

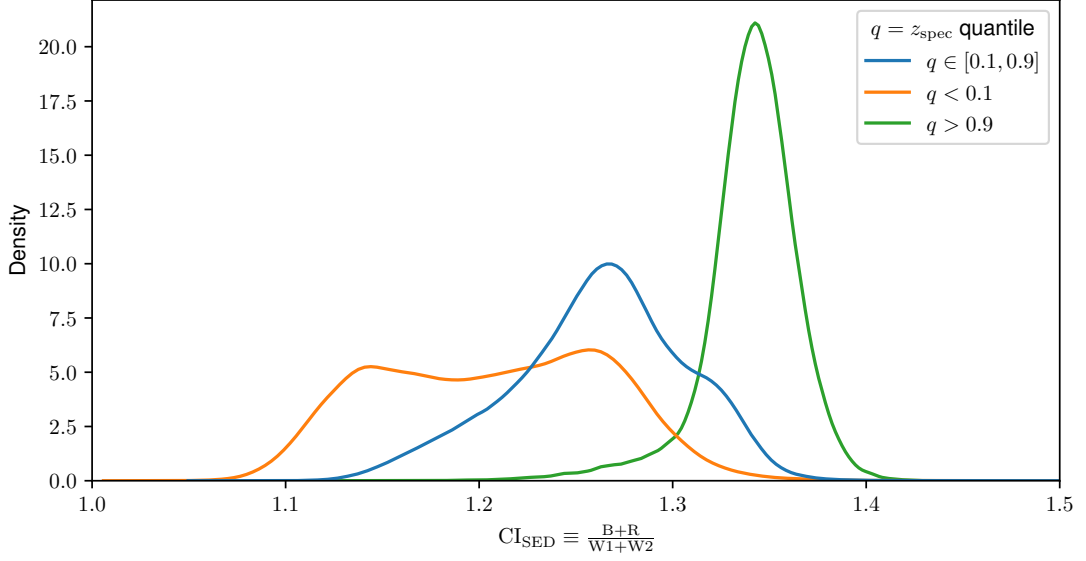


Figure 4.3: KDEs of the SED colour index CI_{SED} for galaxies split into three redshift bins defined using the 0.1 and 0.9 quantiles as bin edges: low-, mid-, high- z_{spec} . CI_{SED} captures the relative contribution of optical light (from the B and R bands) to mid-infrared emission (from the $W1$ and $W2$ bands), thus providing a measure of the SED slope. It demonstrates good separability of the redshift bins, indicating its effectiveness in characterising how the light distribution shifts between the optical and the infrared with redshift.

by the two catalogues. To that end, we address this by increasing the effective minimum distance between embedded points to $d_{\text{min}} = 1.0$ in order to suppress local structure and highlight global structure. Nevertheless, we can observe that galaxies of similar redshift live in adjacent regions in parameter space (as well as in the UMAP latent space). The DIR algorithm exploits this: galaxies nearby in colour-space (and thus in the latent space) have similar redshifts, allowing the spectroscopic sub-sample to represent the full sample when appropriately weighted.

To quantify the error introduced due to the limited number of photometric bands we perform jackknife resampling, each time excluding $1/100^{\text{th}}$ of the galaxies in each redshift bin, and repeat steps (ii) to (v) to estimate the resampled redshift distribution. We then run a χ^2 -minimisation subroutine to calculate the value of the shape parameter w_z needed for the resampled distribution to best approximate the fiducial DIR distribution. The distribution of w_z is the prior on our DIR redshift distributions. We note that this is ~ 200 times narrower than the prior we used in Chapter 3, so throughout our analysis we fix $w_z = 1$ and use the fiducial DIR redshift distributions as truth, effectively eliminating them as a systematic.

Recent work by [146] has emphasised the importance of accurately modelling the conditional photo- z error distribution $p(\delta z)$ (also shown in Fig. 4.1) when calibrating

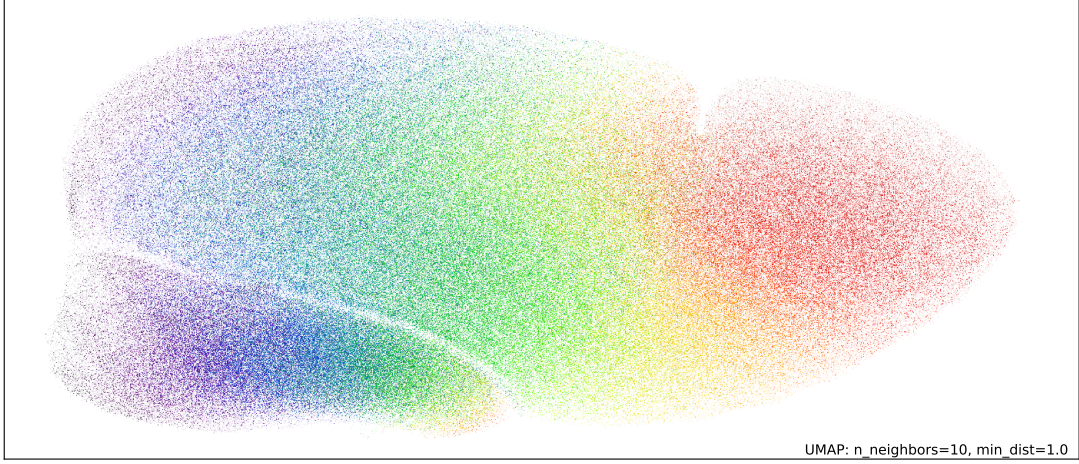


Figure 4.4: Scatter plot of the galaxies projected in the first two components of the UMAP latent space [136]. The data points are coloured qualitatively (quantile rescaling has been applied as a preprocessing step) according to their spectroscopic redshift. The fine divide separates the 2MPZ from the WIXSC catalogues, which is entirely a UMAP artefact, owing to the different local densities of the two catalogues in parameter space. This has partly been addressed by increasing the effective minimum distance between embedded points to $d_{\min} = 1.0$ in order to suppress local structure and highlight global structure of the full dataset. Overall, the UMAP transformation and projection highlights that galaxies at similar redshifts reside nearby in the full parameter space (as well as in the UMAP latent space).

redshift distributions. To that end, Fig. 4.5 presents a comparison between the different versions of the redshift distributions. The dashed blue curve (2mpz_v1) shows the redshift distribution assuming a Gaussian distribution of photo- z residuals ($\delta z = z_{\text{spec}} - z_{\text{phot}}$):

$$p(\delta z) \propto \exp\left(-\frac{(\delta z)^2}{2\sigma^2(z)}\right), \quad (4.12)$$

with the scatter $\sigma(z)$ typically evolving linearly with redshift (e.g. $\sigma(z) = \sigma_0(1+z)$). However, this Gaussian model fails to capture the non-Gaussian wings present in the data. Hence, the dashed orange curve (2mpz_v2) is computed using a modified Lorentzian form:

$$p(\delta z) \propto \left(1 + \frac{(\delta z)^2}{2as^2}\right)^{-a}, \quad (4.13)$$

where the parameters a and s are fitted to the calibration sample. For the 2MPZ sample [28] found that the Gaussian model yields $\sigma_{\delta z} \approx 0.014$, while the Lorentzian parameters are $(a, s) \approx (2.93, 0.012)$.

In Fig. 4.5 we also plot the (uncalibrated) distribution of photometric redshifts (solid purple), where the sharp cutoff due to our sample selection can be seen at $z = 0.1$. The solid orange line presents the distribution for the spectroscopic sub-sample. Finally, the

DIR redshift distribution, computed by re-weighting the spectroscopic sample, is shown as the green curve, where the error bars are determined via jackknife resampling using the unbiased standard deviation $\sqrt{\frac{N}{N-1}(\Delta z)^2}$. Additionally, the lower subplot shows the offset Δz for each jackknife realisation relative to the fiducial DIR distribution, characterising the relative uncertainty in our calibration.

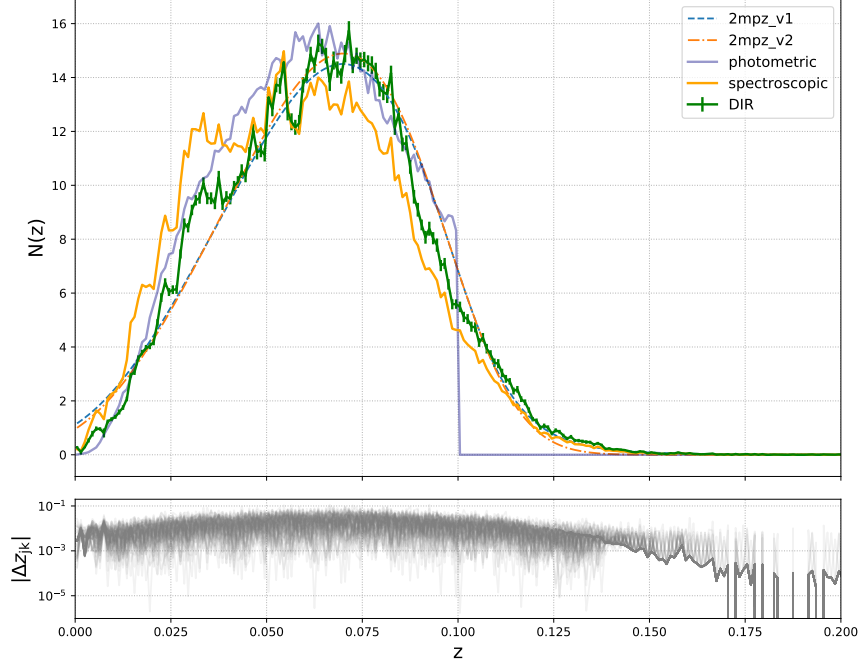


Figure 4.5: Comparison of the redshift distributions for the 2MPZ sample in the first photometric redshift bin ($z_{\text{phot}} < 0.1$). The dashed blue curve represents $N(z)$ computed assuming a Gaussian model for the photo- z residuals. The dashed orange curve shows $N(z)$ derived using a modified Lorentzian model for the error distribution. The solid purple line displays the photometric redshift distribution (truncated at $z = 0.1$), while the solid orange line shows the distribution of the spectroscopic sub-sample. The green curve represents the final DIR-calibrated redshift distribution, with error bars computed via jackknife resampling, using the unbiased standard deviation of the jackknife realisations. The lower subplot presents the offsets Δz for each jackknife realisation relative to the fiducial DIR distribution.

Finally, we use a low-pass Savitzky-Golay filter [173] to suppress the high-frequency scatter in the final redshift distributions, which are presented in Fig. 4.7. The effect of smoothing our inferred redshift distributions with a low-pass filter is shown in Fig. 4.6.

It is worth noting that for our analysis $N(z)$ is not actually what we want. For any cross-correlation methodology, one needs the distribution of true comoving distances corresponding to a given tomographic slice. However, due to the peculiar velocities of galaxies, we cannot simply take the distribution of redshifts and apply a homogeneous cosmology.

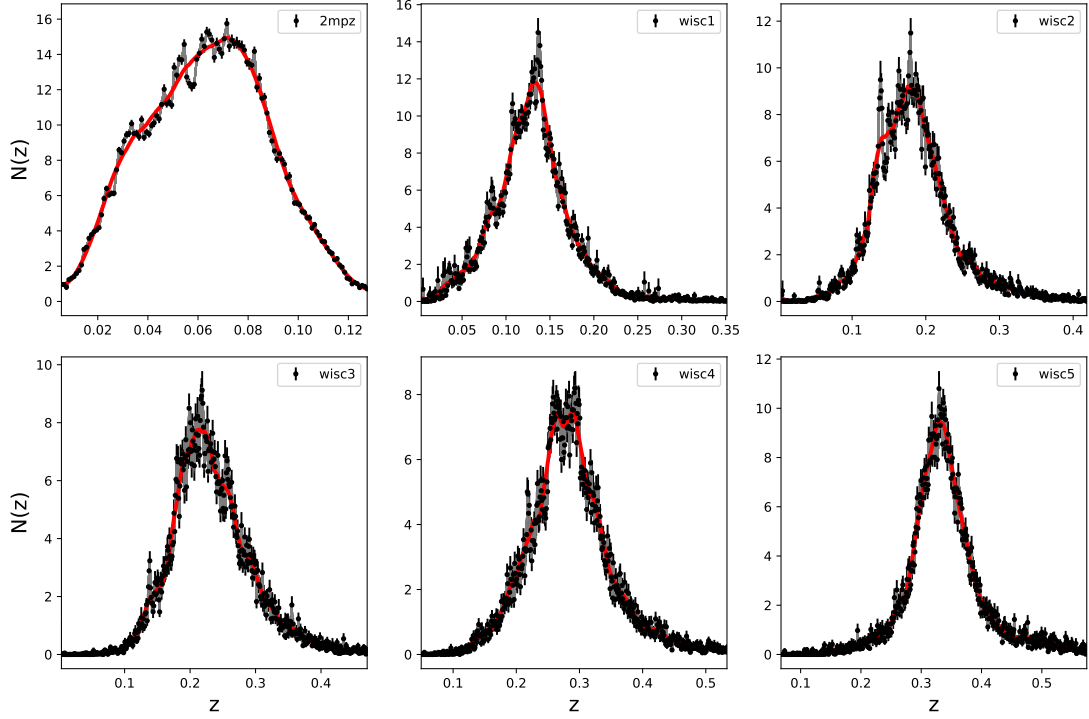


Figure 4.6: DIR-inferred redshift distributions (grey curves) using the adjusted weights in calibration (black data points with error bars) and final redshift distributions used in this analysis, after smoothing using a low-pass Savitzky-Golay filter (red curves). The six subplots represent the different photometric redshift bins used in our analysis.

If, for example, we selected an infinitesimal slice in spectroscopic redshift, that would in fact be a Gaussian selection in radius (due to the intrinsic scatter of peculiar velocities of galaxies). Therefore, the slices need to be thick enough in space in order for RSDs to be negligible.

For completeness, Fig. 4.8 illustrates the sky distribution and radial extent of galaxies with measured spectroscopic redshifts. The top panel presents a histogram of the number of galaxies in each (hexagonal) bin, N_g , where we have combined all redshift bins used in this analysis (2MPZ and WI×SC). The Galactic foreground mask has been applied, and the density values are mapped logarithmically to enhance visual contrast. The bottom panel displays a polar scatter plot of galaxies up to a redshift of $z_{\max} = 0.14$ (and a lookback time of 1.83 Gyr assuming *Planck* Cosmology), using the same merged catalogue. This projection, shown along right ascension, includes only galaxies within a narrow slice of thickness 1° along the celestial equator, allowing for a clear visualization of the large-scale structure traced by the spectroscopic sample. Galaxies are qualitatively coloured according to their $B - R$ colour index.

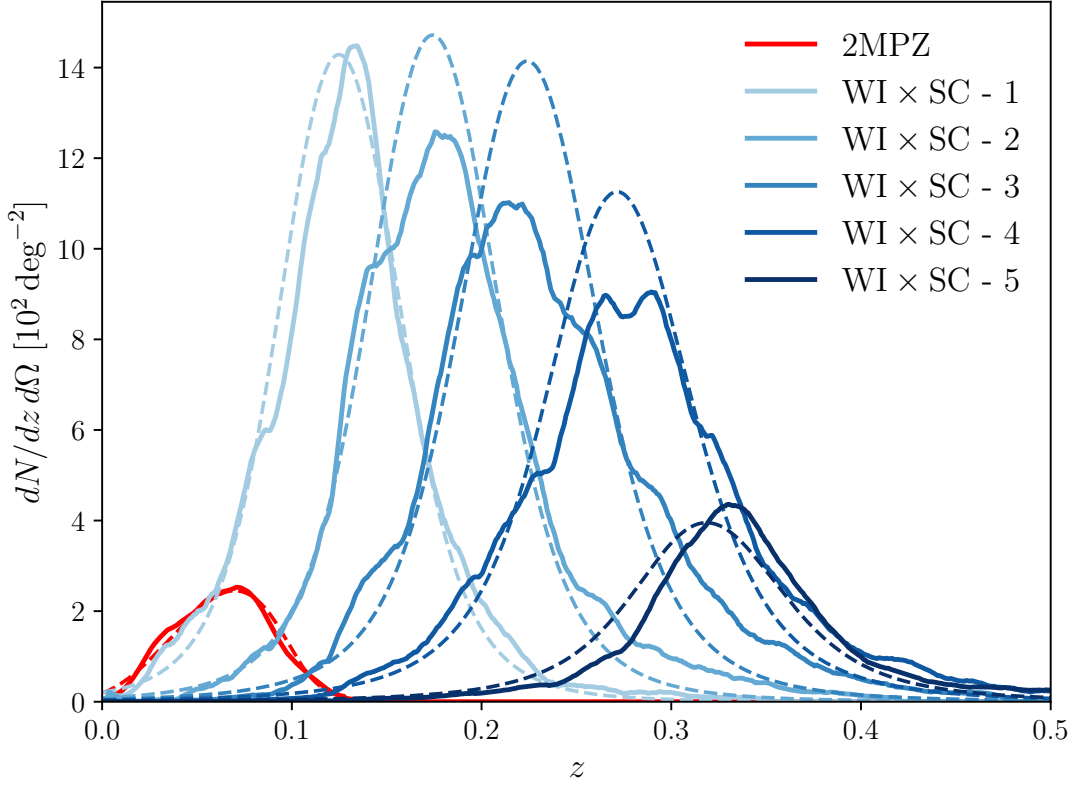


Figure 4.7: Redshift distributions of the different galaxy samples. Solid lines present the redshift distributions used in this analysis, after DIR was performed on the galaxy samples. For comparison, dashed lines present the redshift distributions used in Chapter 3. See Table 3.1 for further details.

4.3.2 Theory

Our galaxy-halo connection follows exactly the 5-parameter HOD of [217] as laid out in Chapter 3. In particular,

$$\langle N_c(M) \rangle, \langle N_s(M) \rangle, \langle u_g(k) \rangle, \langle |u_g(k)|^2 \rangle$$

are all defined in Section 3.1.2. Here we fix all “standard” parameters ($\sigma_{\ln M}$, α_s , etc.) to the values of Chapter 3, and leave only $M_{\min}$ and M'_1 free. Again, see Section 3.1.2 for full details.

We adopt the same GNFW pressure profile and its Fourier-space cumulants that were introduced in Chapter 3 ([14]). In brief,

$$P_e(r) = P_* p(r/r_{500c}), \quad \langle u_y(k|M) \rangle = P_e(k), \quad \langle u_y^2(k|M) \rangle = P_e^2(k) e^{\sigma_{\ln Y}^2},$$

where all parameters appear in Section 3.1.1.

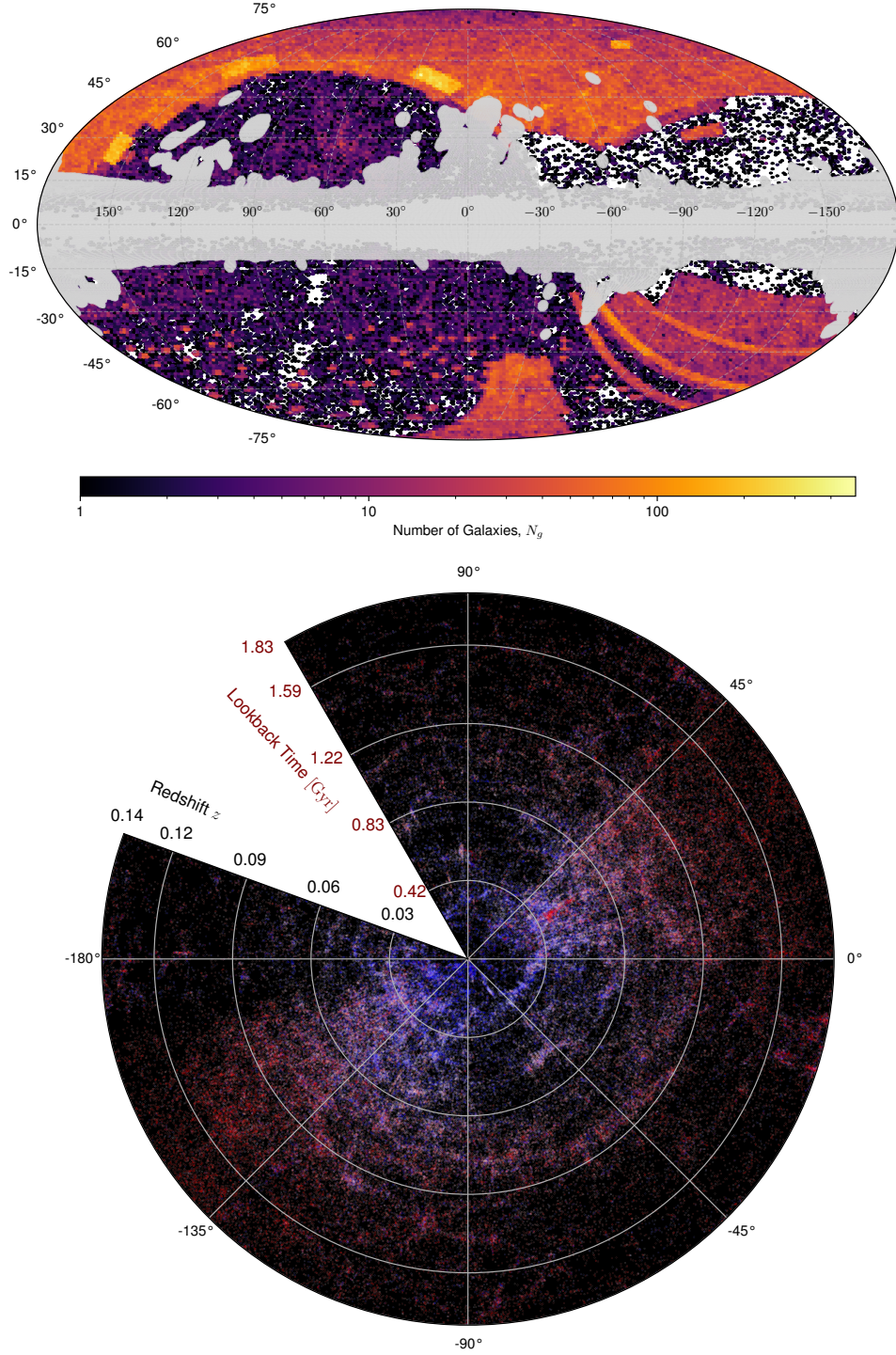


Figure 4.8: Sky distribution of galaxies with spectroscopic redshift measurements. **(Top):** Histogram of the number of galaxies in each (hexagonal) bin, N_g , combining all redshift bins used in this analysis, with a logarithmic scaling applied. **(Bottom):** Polar projection of galaxies up to $z_{\text{max}} = 0.14$ (lookback time $t = 1.83$ Gyr assuming *Planck* Cosmology), shown in a thin 1° slice along the celestial equator. Galaxies are qualitatively coloured according to their $B - R$ colour index.

It is also worth exploring the halo model bias (Eq. 1.50) for the Compton- y parameter, as introduced in Section 3.5.2.2. At $k \rightarrow 0$ it is given by:

$$\begin{aligned}\langle bP_e \rangle &= \int dM \frac{dn}{dM} b_H(M) \int_0^\infty dr 4\pi r^2 P_e(r|M) \\ &= \frac{4-2Y}{8-5Y} \int dM \frac{dn}{dM} b_H(M) E_T(M),\end{aligned}\quad (4.14)$$

where $E_T(M)$ is the thermal energy in a halo of mass M [206, 144]. A measurement of $\langle bP_e \rangle$ can therefore be related to the thermodynamics of gas inside haloes. We also express our measurements in terms of this parameter in Section 4.4.2.

It is worth noting that while the HOD model already incorporates scatter in N_g (the number of galaxies per halo), this scatter may not fully capture all sources of decorrelation between different profiles. The scatter in N_g from the HOD model [216] mainly accounts for the stochasticity in galaxy numbers, given a halo mass. However, other physical processes can lead to additional decorrelation between the galaxy distribution and the tSZ or CMB lensing profiles. For instance, gas pressure (hence tSZ) may be affected by non-linear physics originating from baryons, such as AGN feedback, cooling, and shocks. These can introduce fluctuations which depend on the specific galaxy population. At the same time, the distribution of the electron pressure might have its own scatter and/or might deviate from our assumption of a spherically symmetric halo profile [108]. This motivates our departure from the standard halo model prescription.

Of course, those effects could be carefully derived from first principles, but the complexity and current uncertainties in modelling baryonic processes [20] would make it difficult to accurately predict the exact level of decorrelation. In this analysis, we make use of the cross-correlation of galaxies with tSZ and CMB lensing. As two different profiles need not be fully correlated, we diverge from the standard halo model and adopt our approach from Section 3.1.1, introducing a one-parameter departure $(1 + \rho_{uv})$ which plays the role of a nuisance parameter and captures any residual scatter that is not already described by the HOD:

$$\langle uv \rangle = \langle u \rangle \langle v \rangle + (\text{higher-order terms}). \quad (4.15)$$

In particular, we assume

$$\langle u(k|M)v(k|M) \rangle = (1 + \rho_{uv}) \langle u(k|M) \rangle \langle v(k|M) \rangle. \quad (4.16)$$

The difference here is that our fiducial model uses one nuisance parameter for each pair of cross-correlated quantities. When cross-correlating galaxies with tSZ, marginalising over this parameter has the added advantage of removing any sensitivity of our final constraints

to $1 - b_{\text{H}}$ on the details of the cross-spectrum model in the 1-halo regime, where both parameters are completely degenerate. Therefore, a value of $\rho_{uv} = 0$ would indicate that the two profiles are fully correlated, while $\rho_{uv} < 0$ permits some degree of decorrelation, reducing the 1-halo covariance term.

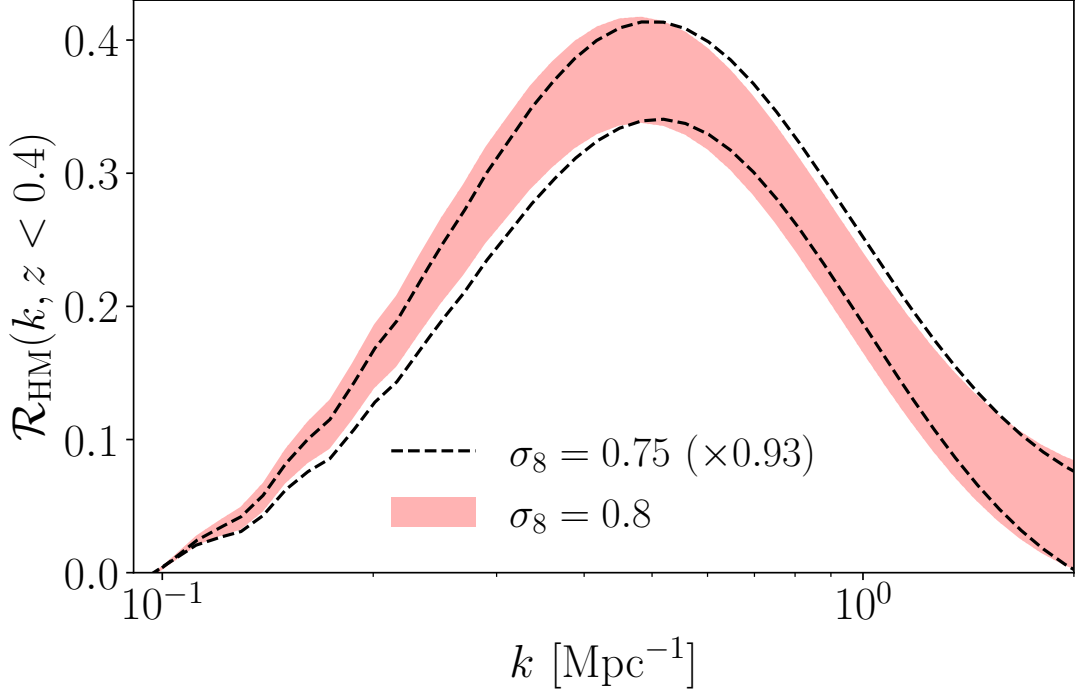


Figure 4.9: Halo model correction factor $\mathcal{R}_{\text{HM}}(k, z)$ in the range of redshifts $0 < z < 0.4$ for a standard Λ CDM cosmology with $\sigma_8 = 0.8$ (red band), and for a cosmology with $\sigma_8 = 0.75$ scaled by a factor 0.93. The shape of the correction is approximately universal, its cosmology dependence is mild, and can be largely absorbed via a multiplicative factor A_{HM} .

Halo radii r_{Δ} are usually defined as the size of the sphere containing a given mass (Eq. 1.59). In order to avoid unnecessarily complicating our analysis, we use the same mass definition (i.e. choice of Δ and ρ_*) for all the ingredients in our modelling of the profiles. Nonetheless, while the GNFW profile is calibrated to Δ_{500c} (where c denotes critical density), the mass functions of [194, 195] are only provided for ρ_{m} -based mass definitions. To overcome this issue, we follow the procedure used by [155] and [36]: our baseline mass definition is ρ_{c} -based with $\Delta = 500$, as used by [14]. At each redshift, we translate this into a Δ value for a ρ_{m} -based definition, which we use to compute the mass function from the parameterisations of [194, 195]. It is worth noting that using a reference overdensity of 500c corresponds to an overdensity of ~ 1666 , which is far from the common

choice of 200 used for consistency with FoF halo finders. Δ_{500c} only probes the very central part of the halo.

From [105] we use the Δ_{500c} -based mass-concentration relation, which is fitted using the Uchuu N-body simulations. We test the robustness of the constraints by re-running the fiducial chains using the integrated NFW re-derivation of the mass-concentration relation described in [62] (as we did in Chapter 3). We discuss the impact of our choice of mass function parameterisation and mass-concentration relation on our final parameter constraints in Section 4.4.4.

A well-known shortcoming of the halo model in its simplest incarnation, described in Section 1.3, is its inability to produce accurate predictions on scales corresponding to the transition between the 1-halo and 2-halo-dominated regimes [138]. This is likely to be caused by the overly simplistic model used to describe the clustering of haloes through a linear bias parameter [140]. This is illustrated in Fig. 4.9, which shows the ratio

$$\mathcal{R}_{\text{HM}}(k, z) \equiv \frac{P_{\text{HF}}(k, z)}{P_{\text{HM}}(k, z)} - 1, \quad (4.17)$$

where $P_{\text{HF}}(k)$ is the matter-matter power spectrum computed using the HALOFIT prescription of [190], and $P_{\text{HM}}(k)$ is the halo model prediction described here. The figure shows this ratio for a standard Λ CDM cosmology with $\sigma_8 = 0.8$, and for a cosmology with $\sigma_8 = 0.75$ after scaling $\mathcal{R}_{\text{HM}}$ by a constant factor of 0.93.

Thus, to mitigate the inaccuracy of the halo model in the transition regime, and to absorb any cosmological dependence in this correction, our final model of the 3D power spectrum will be:

$$P_{UV}(k) = [1 + A_{\text{HM}} \mathcal{R}_{\text{HM}}^{\text{fid}}(k)] \left(P_{UV}^{1h}(k) + P_{UV}^{2h}(k) \right), \quad (4.18)$$

where $\mathcal{R}_{\text{HM}}^{\text{fid}}(k)$ is computed for a fixed cosmology corresponding to the best-fit *Planck* Λ CDM model. We explore the impact on our final constraints of fixing $A_{\text{HM}} = 1$, and of marginalising over it as a free parameter. For simplicity, in our fiducial model we choose the value of A_{HM} to be the same irrespective of which two profiles are being correlated.

Another shortcoming of the halo model is that it over-predicts the 1-halo power at large scales as it only accounts for positive density perturbations of haloes. We follow [137] and modify the 1-halo term to suppress its large-scale power:

$$P_{UV}^{1h}(k) \rightarrow P_{UV}^{1h} \frac{(k/k_*)^4}{1 + (k/k_*)^4}, \quad (4.19)$$

where k_* is the pivot wavenumber. At small scales, this transformation gives the uncorrected 1-halo term. We fix $k_* = 0.01 \text{ Mpc}^{-1}$ throughout the analysis.

4.3.3 Data analysis methods

Much of the analysis presented here follows closely that of Chapter 3. We present a summary of this analysis here, emphasising the new elements introduced in this paper, and point the reader to [115] for further details.

As in Chapter 3, we make use of two low-redshift photometric redshift (photo- z) catalogues, the 2MASS Photometric Redshift catalogue [2MPZ, 28] and the WISE $\times$ SuperCOSMOS catalogue [WI $\times$ SC, 29]. These catalogues were produced from the cross-match of three optical/infrared surveys [2MASS, 182, 106, 80, 147, 212], achieving a high density of sources, broad area coverage, and accurate photometric redshifts, making them ideal for cross-correlation studies with other projected tracers of the large-scale structure.

We use the same six tomographic samples employed in Chapter 3. The first bin includes all sources in 2MPZ with photometric redshifts in the range $z \in [0, 0.1]$. The subsequent 5 redshift bins are derived from WI $\times$ SC, using sources in photometric redshift bins of width $\Delta z = 0.05$ in the range $z \in [0.1, 0.35]$. The most relevant properties of these bins are summarised in Table 3.1.

Galaxy overdensity maps were generated at resolution $N_{\text{side}} = 1024$ using the HEALPix framework [77] for each of the 6 redshift bins described above. We follow the same procedure described in Chapter 3 (see also [146]) to minimise the impact from Galactic and other sky systematics on this mask. This involves subtracting a smooth, local, non-linear relation between galaxy and star density, linearly deprojecting a Galactic extinction template at the map level, and modelling the contribution from zero-point fluctuations in the SuperCOSMOS photographic plates at the power-spectrum level. As described in Chapter 3, the impact of these systematics is negligible in the range of scales ($\ell > 10$) used here. In addition to this, we use an angular mask that removes the regions of higher dust and stellar contamination (see top panel of Fig. 4.10, and Chapter 3 for more details).

Fig. 4.11 shows the *Planck* 2015 Minimum Variance (MV) CMB lensing convergence map (R2). The MV map is produced by optimally combining temperature and polarization data using quadratic estimators to minimize noise, thereby providing the best available estimate of the integrated mass distribution along the line of sight [157]. We note that the *Planck* team has also published the SZ-deprojected CMB lensing convergence map, which is processed to remove tSZ contamination, so as not to bias the lensing reconstruction in regions with strong tSZ signals. While both maps trace the same underlying mass distribution, the SZ-deprojected map typically exhibits reduced tSZ contamination at the expense of slightly higher noise levels. In Section 4.4.4, we compare cosmological constraints obtained using both maps to assess the impact of residual foreground contamination. As with the tSZ maps,

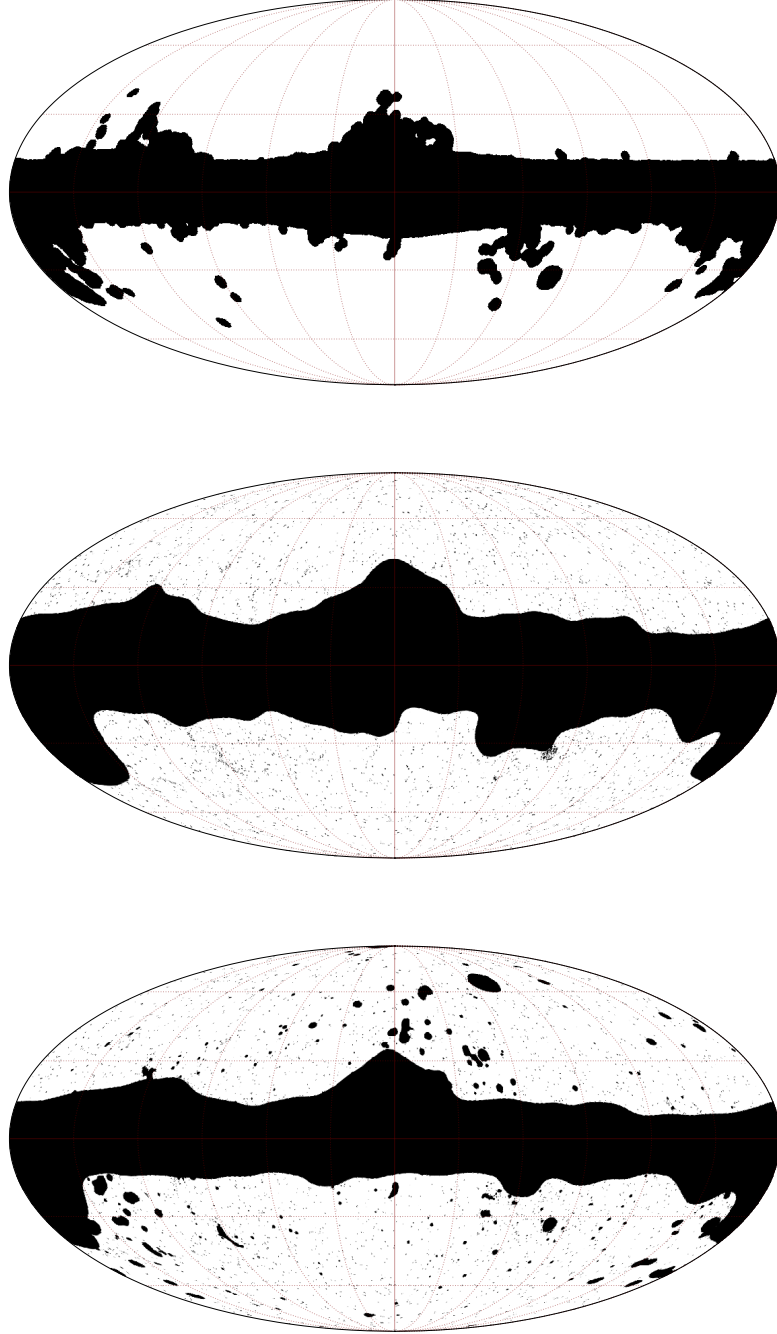


Figure 4.10: *Top*: Mask used for the 2MPZ and WIXSC galaxy surveys, corresponding to a sky fraction $f_{\text{sky}} = 0.68$. *Middle*: Sky mask used for the Compton-y map, corresponding to a sky fraction $f_{\text{sky}} = 0.59$. *Bottom*: Mask used for the *Planck* lensing convergence map, corresponding to a sky fraction $f_{\text{sky}} = 0.67$. The product of all masks leaves a usable sky fraction $f_{\text{sky}} \simeq 0.57$.

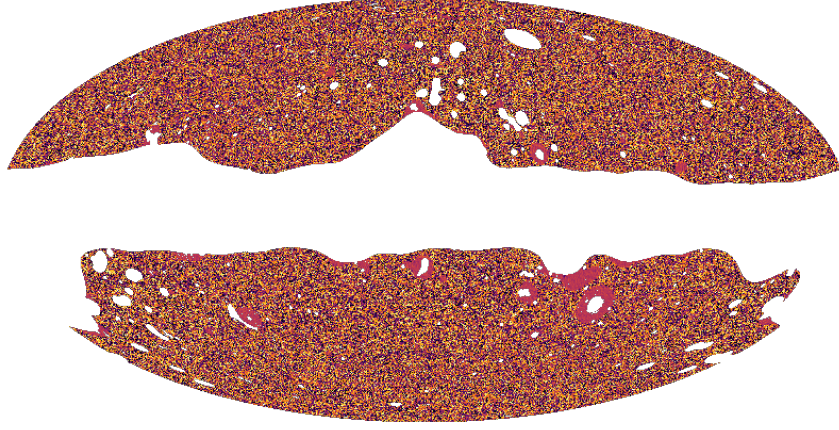


Figure 4.11: *Planck* 2015 [157] Minimum Variance CMB lensing convergence map (R2). The *Planck* Galactic mask has been applied, and the values have been stretched logarithmically to highlight variations in the integrated mass distribution.

the *Planck* Galactic mask has been applied to mitigate contamination from foregrounds, and a logarithmic colour stretch has been used to enhance the dynamic range of the signal.

In order to connect our measurements with the posterior distribution of the model parameters $\vec{\theta}$, we assume that the measured power spectra follow a Gaussian likelihood (Eq. 2.11):

$$-2 \ln p(\mathbf{d}|\vec{\theta}) = \chi^2 \equiv (\mathbf{d} - \mathbf{t}(\vec{\theta}))^T \mathbf{C}^{-1} (\mathbf{d} - \mathbf{t}(\vec{\theta})), \quad (4.20)$$

where $\mathbf{d}$ is a vector of power spectrum measurements, $\mathbf{t}(\vec{\theta})$ is the theory prediction for $\mathbf{d}$ with parameters $\vec{\theta}$ and $\mathbf{C}$ is the covariance matrix described in the previous sections.

Note that in principle, the covariance matrix $\mathbf{C}$ is also parameter dependent, in which case $\ln \mathcal{L}$ is not just $-\frac{\chi^2}{2}$. We have ignored this and instead use the 2-step process of computing the covariance matrix outlined in Section 2.1.2. In fact, a parameter-independent covariance matrix has been proven to be an exceptionally good approximation in all realistic scenarios, including the kind of cross-correlation analysis we employ in this study [113]. Therefore, the effect of a parameter-dependent covariance matrix can be fully ignored both in terms of accuracy and precision of the recovered cosmological constraints.

As in Chapter 3, we study each redshift bin separately. In this case, the data vector includes the three power spectra ($C_\ell^{gg}, C_\ell^{gy}, C_\ell^{gk}$) up to the maximum multipole $\ell_{\max}$ listed in Table 3.1. Note that, as in Chapter 3, we impose a large-scale cut $\ell_{\min} = 10$ to guard against the impact of sky systematics in the galaxy overdensity and CMB lensing maps.

For each redshift bin, our fiducial model contains six free parameters: $1 - b_{\text{H}}$, σ_8 , $\log_{10} M_{\min}/M_\odot$, $\log_{10} M'_1/M_\odot$, ρ_{gy} , and A_{HM} . Our main aim is constraining σ_8 and the mean gas pressure, parameterised by $1 - b_{\text{H}}$. $M_{\min}$ and M'_1 control the amplitude of galaxy

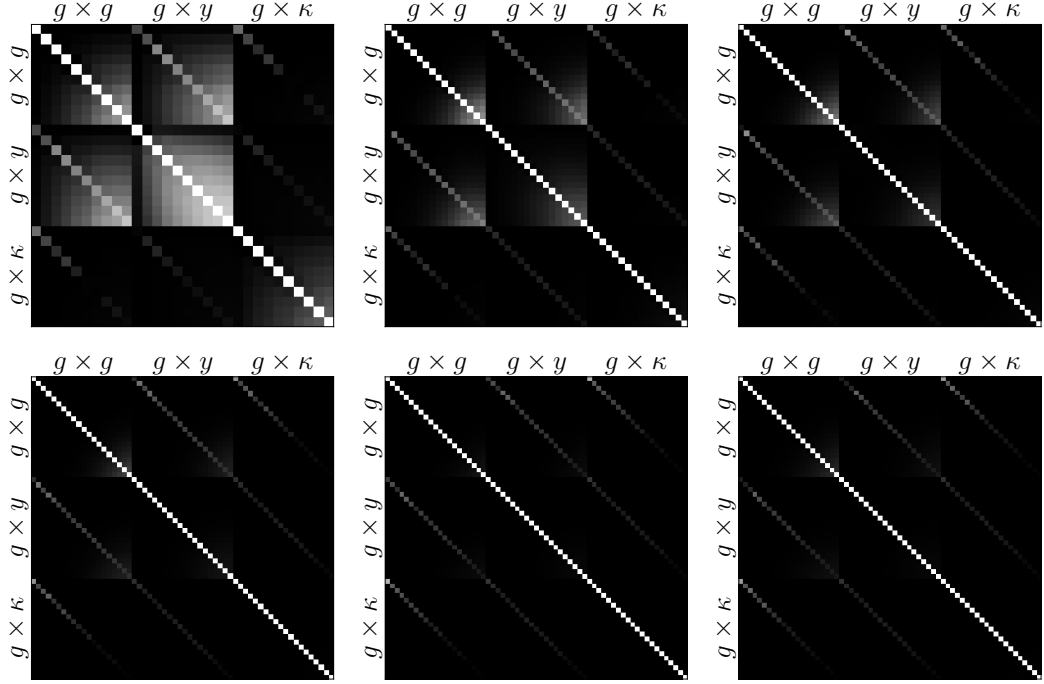


Figure 4.12: Correlation matrices for the 6 redshift bins used in the analysis. Each covariance matrix consists of 9 sub-matrices, corresponding to the covariances of C_ℓ^{gg} , C_ℓ^{gy} , and $C_\ell^{g\kappa}$ (block diagonals), as well as their cross-covariance. The correlation matrix is defined as $r_{ij} = C_{i,j} / \sqrt{C_{i,i}C_{j,j}}$, where $C_{i,j}$ denotes the covariance between i and j .

b	ℓ_{ini}^b	ℓ_{end}^b	b	ℓ_{ini}^b	ℓ_{end}^b
1	2	12	15	256	306
2	12	22	16	306	365
3	22	32	17	365	436
4	32	42	18	436	521
5	42	52	19	521	622
6	52	62	20	622	743
7	62	74	21	743	887
8	74	88	22	887	1059
9	88	105	23	1059	1265
10	105	126	24	1265	1510
11	126	150	25	1510	1804
12	150	179	26	1804	2154
13	179	214	27	2154	2571
14	214	256	28	2571	3072

Table 4.2: Power spectrum bandpowers used in this work. The b -th bandpower includes all integer multipoles $\ell_{\text{ini}}^b \leq \ell < \ell_{\text{end}}^b$. The bandpower edges listed here correspond roughly to linearly-spaced bands between $\ell = 2$ and $\ell = 52$, and logarithmic bands afterwards.

Parameter	Prior
$1 - b_{\text{H}}$	$\mathcal{U}(0.005, 1.150)$
σ_8	$\mathcal{U}(0.2, 1.5)$
$\log_{10} M_{\text{min}}/M_{\odot}$	$\mathcal{U}(8, 15)$
$\log_{10} M'_1/M_{\odot}$	$\mathcal{U}(8, 15)$
ρ_{gy}	$\mathcal{U}(-1, 1)$
A_{HM}	$\mathcal{U}(0.1, 2.0)$

Table 4.3: Top-hat priors for the fiducial parameters used in the analysis.

clustering, ρ_{gy} determines the level of correlation between galaxy density and pressure inside each halo, and A_{HM} allows us to marginalise over the uncertain correction to the halo model prediction on intermediate scales (see Section 1.3.3). We fix all other cosmological parameters to the best-fit values in [159]: $(\Omega_c h^2, \Omega_b h^2, h, n_s) = (0.119, 0.0224, 0.6766, 0.9665)$. As in Chapter 3, our fiducial model adopts the [194] mass function parameterisation, the halo bias of [195], but makes use of the concentration-mass relation described in [105]. The fiducial top-hat priors used for each of the free parameters are shown in Table 4.3.

Besides this fiducial set of analysis choices, we explore variations in the set of free parameters, the combinations of power spectra included in **d**, scale cuts, mass function and halo concentration models, and parameter priors. These are described in Section 4.4.4.

We sample the posterior distributions using the Markov chain Monte Carlo method (MCMC) as implemented in Cobaya¹ [196]. Theory calculations were carried out using the Core Cosmology Library² [46]. We use the BACCO³ [13] emulated Boltzmann code to compute the linear power spectrum component of the halo model.

4.4 Results

4.4.1 Measurements and goodness of fit

We measure the auto-angular power spectrum of galaxy number density $\delta_g \times \delta_g$, and the cross-angular power spectra combinations with the tSZ and CMB lensing secondary anisotropies: $\delta_g \times y$ and $\delta_g \times \kappa$, and estimate the covariance of the data as described in Section 2.1.2. Fig. 4.12 shows the correlation matrix for all redshift bins used in our analysis. The non-Gaussian contributions are most noticeable in the lowest redshift bins, causing correlations between different bandpowers, and become irrelevant at higher redshifts. We

¹<https://github.com/CobayaSampler/cobaya>

²<https://github.com/LSSTDESC/CCL>

³<https://bacco.dipc.org>

Sample	$\bar{z}$	$\sigma_8(z)$	$1 - b_H$	$\langle bP_e \rangle$ [meV cm ⁻³]	Ω_{th} [$\times 10^{-8}$]	χ^2/N_d	PTE(χ^2)
2MPZ	0.06	$1.01^{+0.08}_{-0.07}$	$0.39^{+0.08}_{-0.10}$	$0.10^{+0.03}_{-0.03}$	$1.24^{+0.30}_{-0.30}$	16/33	0.99
WI×SC-1	0.13	$0.74^{+0.05}_{-0.05}$	$0.75^{+0.11}_{-0.19}$	$0.16^{+0.02}_{-0.02}$	$1.48^{+0.14}_{-0.15}$	72/54	0.05
WI×SC-2	0.18	$0.70^{+0.04}_{-0.05}$	$0.80^{+0.13}_{-0.15}$	$0.17^{+0.01}_{-0.02}$	$1.35^{+0.10}_{-0.11}$	60/57	0.36
WI×SC-3	0.23	$0.65^{+0.03}_{-0.03}$	$0.92^{+0.11}_{-0.13}$	$0.20^{+0.01}_{-0.01}$	$1.34^{+0.08}_{-0.08}$	82/63	0.05
WI×SC-4	0.28	$0.60^{+0.01}_{-0.03}$	$1.09^{+0.11}_{-0.06}$	$0.21^{+0.01}_{-0.01}$	$1.22^{+0.07}_{-0.06}$	55/64	0.78
WI×SC-5	0.34	$0.69^{+0.04}_{-0.03}$	$0.74^{+0.09}_{-0.12}$	$0.23^{+0.02}_{-0.02}$	$1.21^{+0.08}_{-0.10}$	73/65	0.23

Table 4.4: Summary table presenting our main results (median and 68% confidence interval around the median) for each redshift bin. The first two columns list our 6 tomographic bins and their mean redshift. Column 3 tabulates the best-fitting values for σ_8 . Column 4 tabulates the best-fitting values for the mass bias $1 - b_H$. Column 5 tabulates the best-fitting values for $\langle bP_e \rangle$. Column 6 tabulates the best-fitting values for Ω_{th} . Column 7 presents the total χ^2 of each bin and the number of fitted data points. Column 8 tabulates the χ^2 survival function for that particular summary statistic (i.e. probability that we obtain results at least as extreme as the ones presented in this manuscript). A high probability suggests an adequate fit for that particular redshift bin.

sample the posterior using MCMC and obtain the best-fit parameters and corresponding χ^2 value. We do so for all three cross-correlations simultaneously, but separately for each tomographic bin. Fig. 4.13 presents the power spectrum measurements as red circles with error bars, and the best fit (parameters maximising the posterior) for each bin (black curve), decomposed into the individual 1-halo (uncorrected) and 2-halo contributions.

Below each cross-correlation subplot we show the spread of the data around the best-fitting curve, normalised by the error of each data point (in units of σ). For each tomographic bin, we include the total χ^2 value, along with the number of data points in the combined data vector of the bin, in the first row. These results are also individually reported in Table 4.4. Overall we find good agreement between the best-fit theory predictions and the measured power spectra, with reasonable probability-to-exceed values (PTE > 0.05) for all redshift bins. Details on the quality of the fits, as well as the significance of the fitted parameters, are discussed in the sections below. When reporting parameter constraints we present the median and 68% confidence interval around the median unless otherwise stated.

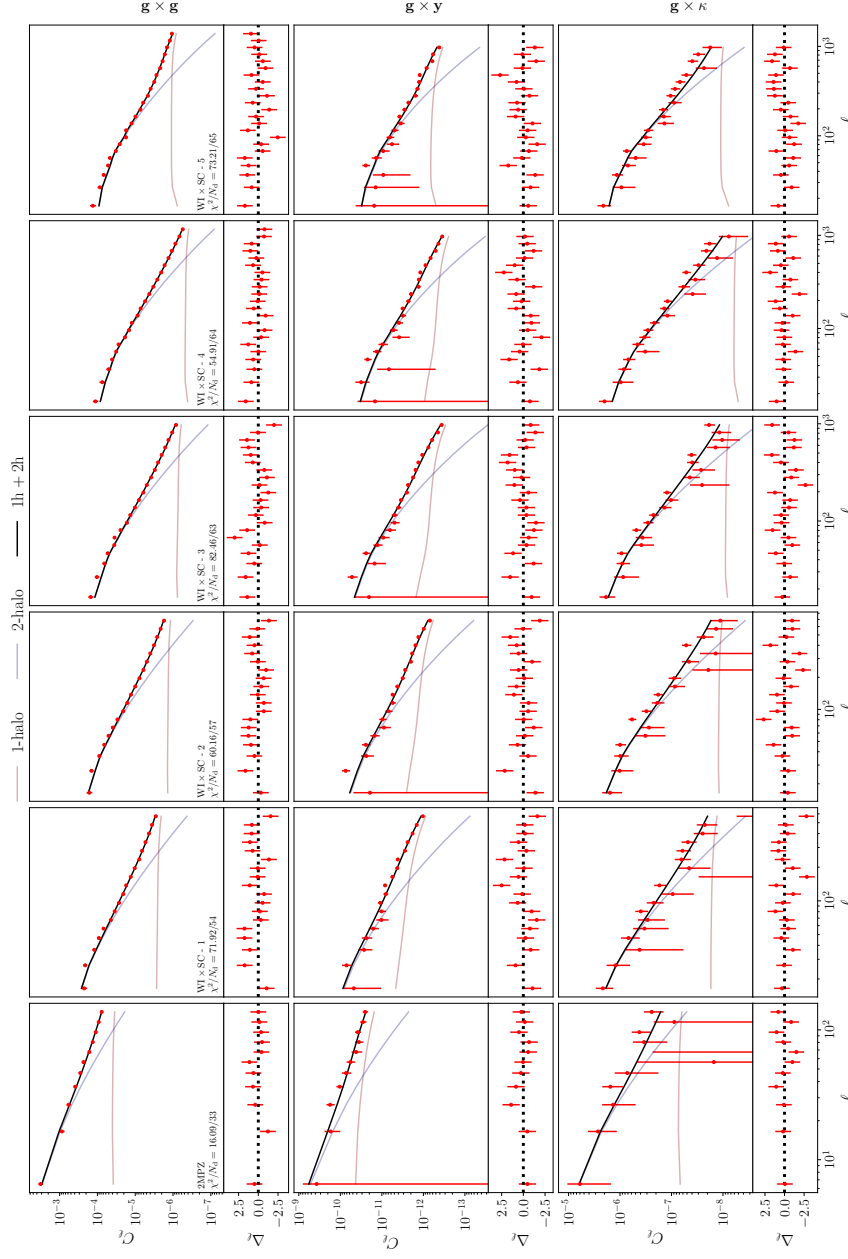


Figure 4.13: χ^2 -minimising auto- and cross- angular power spectra (solid black curves) showcased along with the measurements (red circles). The 1- and 2-halo individual contributions to the total angular power spectrum are also shown in pink and cyan, respectively. The inset subplots below the main fits present the offset of the data points from the fitted curve in units of χ^2 . For each tomographic bin, a summary statistic with the total χ^2 value for that bin and the number of data points fitted, is also shown in the main plot. The rows show $\delta_g \times \delta_g$, $\delta_g \times y$, and $\delta_g \times \kappa$, respectively. We impose an angular scale cut corresponding to a linear scale of 0.5 Mpc^{-1} for the 2MPZ bin, and 1.0 Mpc^{-1} for the W1xSC bins.

For completeness, we include the full triangle plot for the posterior distribution of the WI×SC-2 bin in Fig. 4.14. The black contours show our results for the fiducial run, while the burgundy contours show the contours after setting a Gaussian prior on $1 - b_{\text{H}} \sim G(0.73, 0.10)$. We discuss the posterior distributions as well as the departures from our fiducial run in Sections 4.4.2, 4.4.3 and 4.4.4.

4.4.2 Thermal energy

The model parameter characterising the gas thermodynamics is the mass bias, $1 - b_{\text{H}}$, which accounts for the fraction of mass missed by X-ray studies when assuming hydrostatic equilibrium [218]. In Fig. 4.15 we present constraints on the value of the mass bias at late times from a joint fit to the three cross-correlations in each redshift bin. Our fiducial results, where we marginalise over all other parameters without informative priors, are shown in black and are also included in Table 4.4. Along with our results, we include measurements made with different parameter priors. These are separated on the x axis by a small amount to make them easier to distinguish. The brown points show the measurements after imposing a Gaussian prior on $1 - b_{\text{H}}$ following the results of [149], while the green points show the measurements after fixing σ_8 to the best-fit value measured by *Planck*. For reference, we also show two benchmark measurements of $1 - b_{\text{H}}$ obtained from *Planck* data. The green band presents an estimate of a constant $1 - b_{\text{H}}$ from [218] obtained by calibrating cluster masses with CMB lensing ($1 - b_{\text{H}} = 0.71 \pm 0.10$). The grey band shows the constraint from [155] obtained using tSZ cluster counts and the TT CMB power spectrum ($1 - b_{\text{H}} = 0.58 \pm 0.04$).

Overall, we find that our fiducial constraints on $1 - b_{\text{H}}$ are broadly compatible with those found in Chapter 3, albeit with significantly broader error bars due to the uncertainty on σ_8 and its degeneracy with $1 - b_{\text{H}}$. The qualitative results of Chapter 3 hold, nevertheless: the value of $1 - b_{\text{H}}$ we find after breaking its degeneracy with σ_8 using CMB lensing is broadly compatible with that found in hydrodynamical simulations and cluster mass calibration studies. Moreover, we do not find any evidence of evolution in its value as a function of redshift. We justify this by measuring the reduced chi-squared χ_{red}^2 statistic (and its associated PTE) over a constant fit to the measurements of b_{H} with Gaussian priors. The resulting χ_{red}^2 is ~ 3 , which, for approximately 5 degrees of freedom, yields $\text{PTE} \approx 0.7$.

It is worth noting, however, that the value of $1 - b_{\text{H}}$ obtained in the WI×SC-4 bin ($1 - b_{\text{H}} = 1.09^{+0.11}_{-0.06}$) is significantly higher than the other bins and than the expectations from previous studies. This is caused, again, by the degeneracy between $1 - b_{\text{H}}$ and σ_8 , and the preference of the $\delta_g \times \delta_g$ and $\delta_g \times \kappa$ power spectra in that bin for a lower value of σ_8 . We discuss this in more detail in Section 4.4.3. Likewise, the fiducial measurement in the 2MPZ bin is lower than previous measurements, due to the preference for a higher σ_8 .

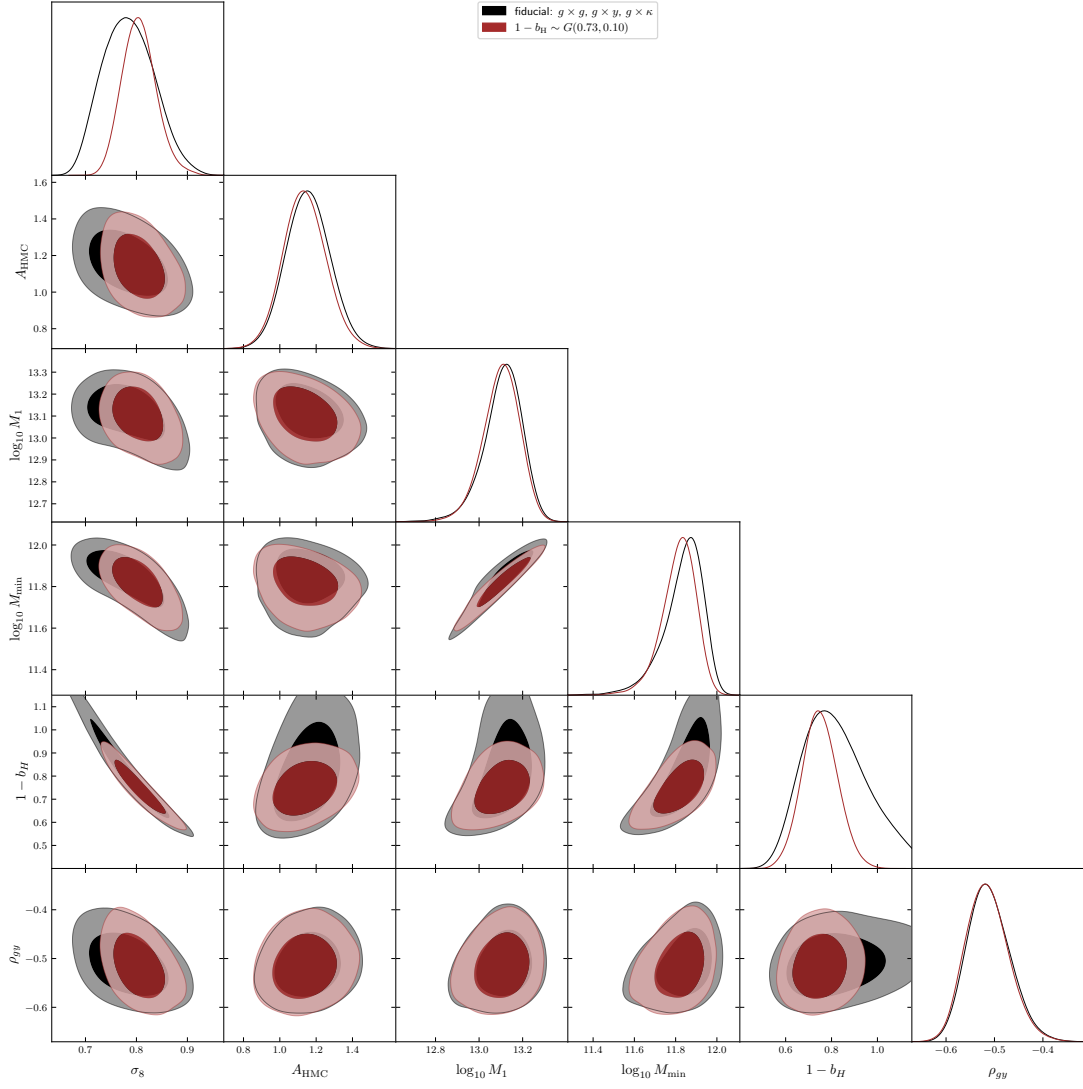


Figure 4.14: Triangle plot of the full posterior distribution for two of our models: [black] the fiducial model (3×2 -pt) with uninformed (top-hat priors) on the parameters, and [burgundy] a similar run, this time with a Gaussian prior on the mass bias, $1 - b_{\text{H}} \sim G(0.73, 0.10)$.

in this bin. This latter measurement, however, is more sensitive to systematic uncertainties, as discussed below.

We note that in our analysis we chose to only free up σ_8 while fixing Ω_m and h to the best-fit values found by *Planck* in order to reduce the computational complexity and focus on the degeneracy between σ_8 and the mass bias $1 - b_H$. Although it is common in similar analyses to free up both Ω_m and σ_8 , since these parameters are strongly degenerate through their influence on the amplitude of the matter power spectrum, in our case we do not allow Ω_m or $\Omega_m h^3$ to vary to reduce the dimensionality of the explored parameter space and speed up convergence of the MCMCs.

Moreover, given the tensions of these parameters and the possibility of non-standard gravity affecting Ω_m and h independently, we argue that focusing on σ_8 serves as a proof-of-principle demonstration of the constraining power of the probes we use in this analysis, given the degeneracy between σ_8 and $1 - b_H$. Extensions of this analysis could allow an additional parameter combination, $\Omega_m h^3$, to vary in order to fully explore the parameter space and achieve a more accurate characterisation of the constraining power of the used tracers. Nevertheless, Ω_m can be independently constrained using other cosmological probes, as we discuss in Section 4.4.3.

Imposing a prior, either on $1 - b_H$ from cluster mass calibration measurements (brown), or by fixing σ_8 to the preferred Λ CDM value found by *Planck* (green), has the effect of moving the posterior towards a constant $1 - b_H$ compatible with that obtained by [218, 149], as was found in Chapter 3 and [43]. This is a somewhat surprising result, since a much lower value of this parameter was found by [155] to be needed in order to reconcile tSZ cluster counts with primary CMB data from *Planck*. A solution to this conundrum could be the potential dependence of $1 - b_H$ on halo mass, as discussed in [168], which would affect cluster number counts and the tSZ power spectrum more than the $\delta_g \times y$ cross-correlation studied here. Overall, the constraints found when fixing σ_8 are compatible with those found by K20 (Chapter 3), albeit with $\sim 54\%$ smaller error bars. This is mostly due to the more reliable $N(z)$ calibration used here allowing us to reduce the uncertainty on the redshift distribution width, which was found in Chapter 3 to be degenerate with $1 - b_H$. Some additional constraining power is brought about by the $\delta_g \times \kappa$ power spectrum tightening the constraints on the HOD parameters.

Although $1 - b_H$ is the parameter used in our model to quantify thermal pressure, the quantity directly probed by these cross-correlations is the bias-weighted mean pressure $\langle bP_e \rangle$ (Eq. 4.14). Fig. 4.16 shows our tomographic measurements of this parameter for the same three choices of prior described above. These values shown in the figure were obtained as a derived parameter from the MCMC chains for each value of $(1 - b_H, \sigma_8)$.

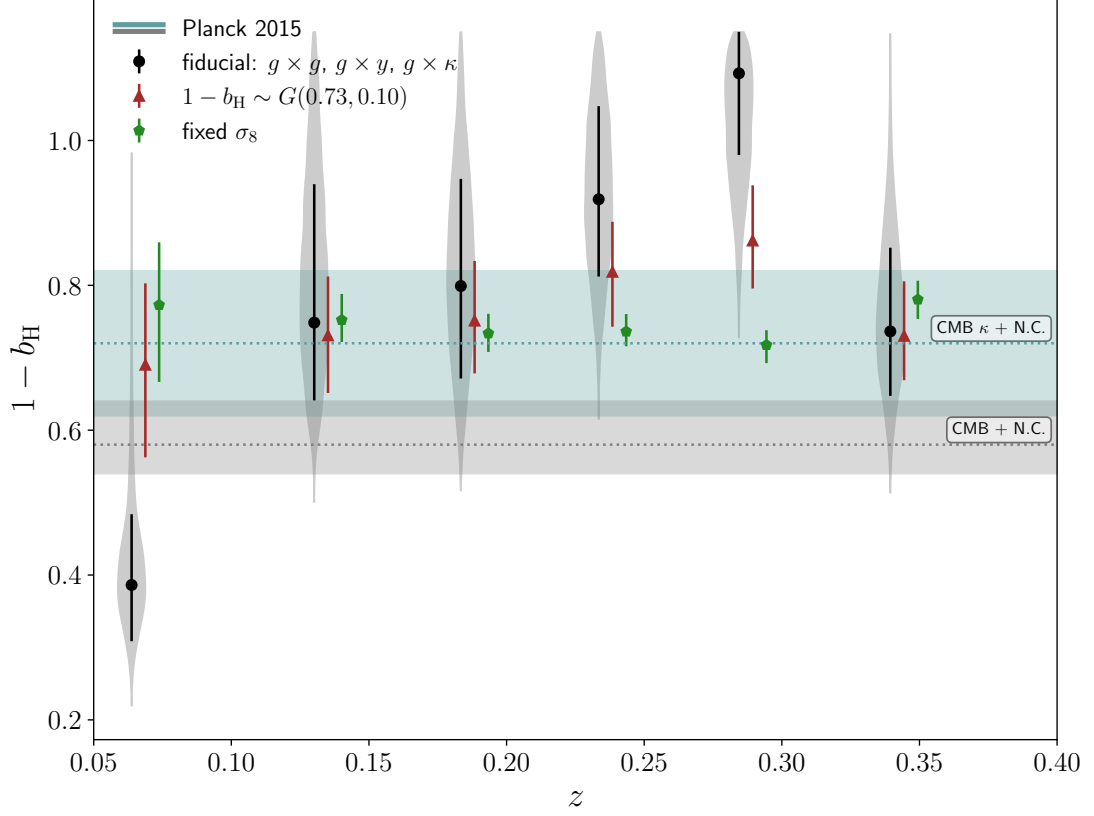


Figure 4.15: Summary figure showing our constraints on the tSZ mass bias $1 - b_H$, for three models with different prior choices, which are offset in redshift by a small amount for readability. The fiducial model presents tomographic constraints from auto- and cross-correlations of galaxy number density, tSZ and CMB lensing (black circles). For completeness, we show the marginalised posterior for this parameter in the violin background. In the second model, we replace the top-hat prior on $1 - b_H$ by a Gaussian prior, following bibliography on past measurements of this quantity [149] (brown triangles). Finally, we fix σ_8 in the third to roughly follow our work in [115], with the difference being the inclusion of the $\delta_g \times \kappa$ cross-correlation (green pentagons). We specifically note the high measured value of $1 - b_H$ in the WI×SC-4 bin. Relevant discussion is included in Section 4.4.2 of this thesis. The teal and grey bands show the constraints on the mass bias found by combining cluster counts and CMB primary [155] as well as the constraints using CMB lensing to calibrate cluster masses [218], respectively.

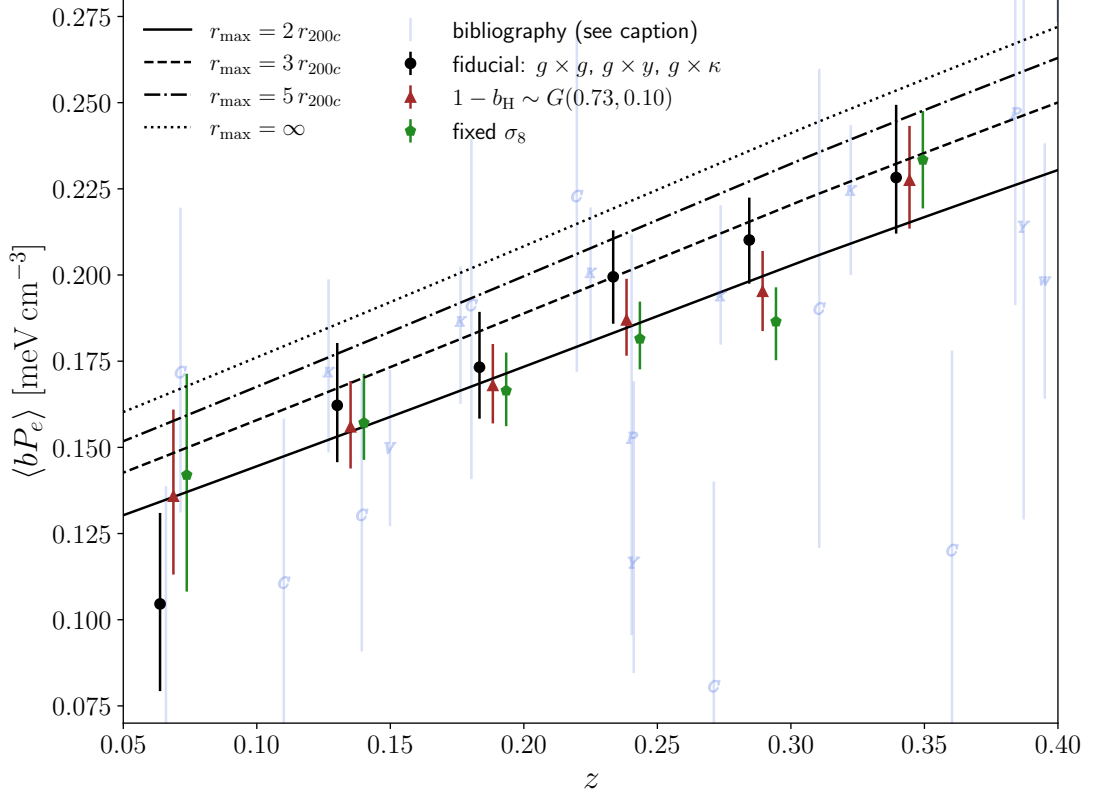


Figure 4.16: Summary figure showing our constraints on the bias-weighted thermal pressure $\langle bP_e \rangle$. As with Fig. 4.15, we present the measurements using models with three different prior choices: the fiducial model with a wide top-hat prior on $1 - b_H$ (black circles), a considerably narrower Gaussian prior on $1 - b_H$ following other measurements, and a model where we fix σ_8 to the best-fitting value measured by *Planck*. Error bars show the 68% confidence interval around the median of the posterior in each redshift bin. We compare our results with predictions from shock-heating models of the pressure profile presented in [21] for different values of the r_{max} threshold cluster radius (see legend). Further, we compare with other measurements of this quantity (washed-blue data with error bars and author initials as markers) from [145], [45], [215], [115], [206], [201].

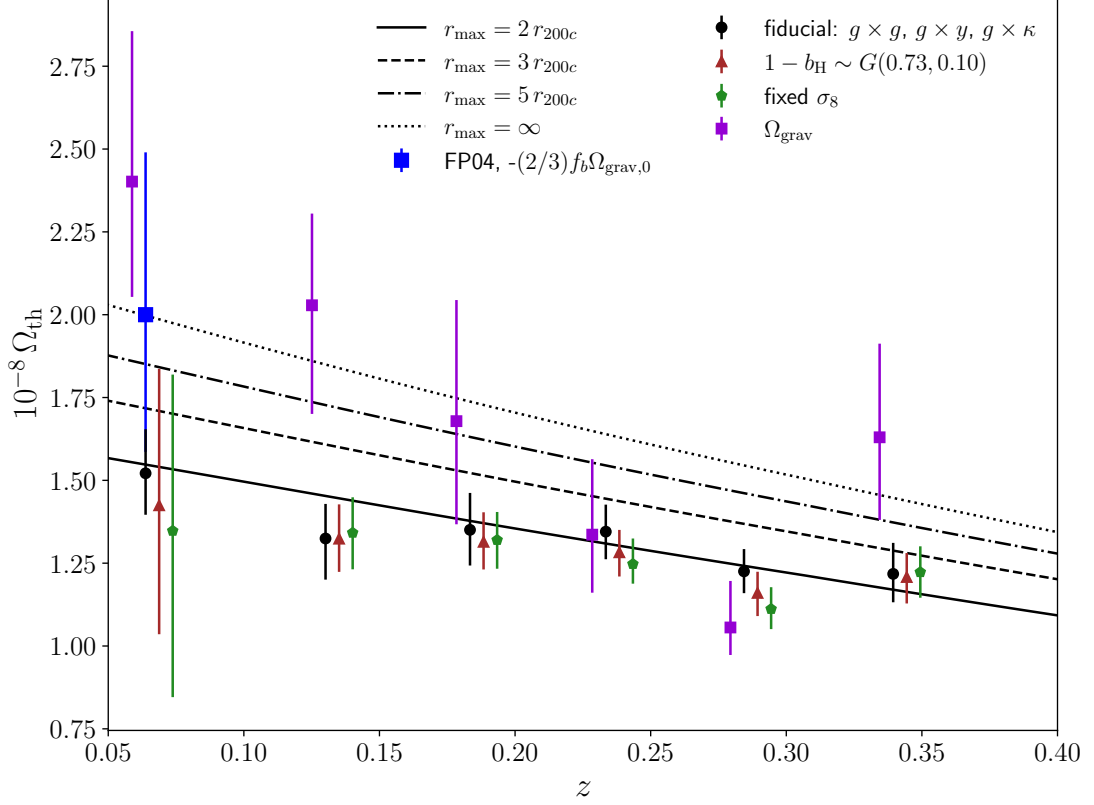


Figure 4.17: Summary figure showing our constraints on the fractional thermal energy density in the late-time universe. Along with our fiducial model with a wide top-hat prior on $1 - b_{\text{H}}$ (black circles), we also show our measurements having imposed a considerably narrower Gaussian prior on $1 - b_{\text{H}}$, following other measurements (brown triangles), and fixing σ_8 to the best-fitting value measured by *Planck* (green pentagons). Error bars show the 68% confidence interval around the median of the posterior in each redshift bin. We compare our results with predictions of Ω_{th} using the pressure profile presented in [20]. The blue square is a reference to compare with the assumption that haloes at $z = 0$ are thermalised. This measurement, which is also included in [43] and [44], comes from [72], who measure the gravitational binding energy of haloes, $\Omega_{\text{grav},0}$. If haloes are thermalised, we would expect $\Omega_{\text{th}} = -(2/3)f_{\text{b}}\Omega_{\text{grav}}$, where f_{b} is the baryon fraction. For reference, we also include the measurements of the magnitude of $-\Omega_{\text{grav}}$. As Ω_{grav} does not depend on the thermal properties of haloes, the error bars quantify the uncertainty arising from our constraints on σ_8 .

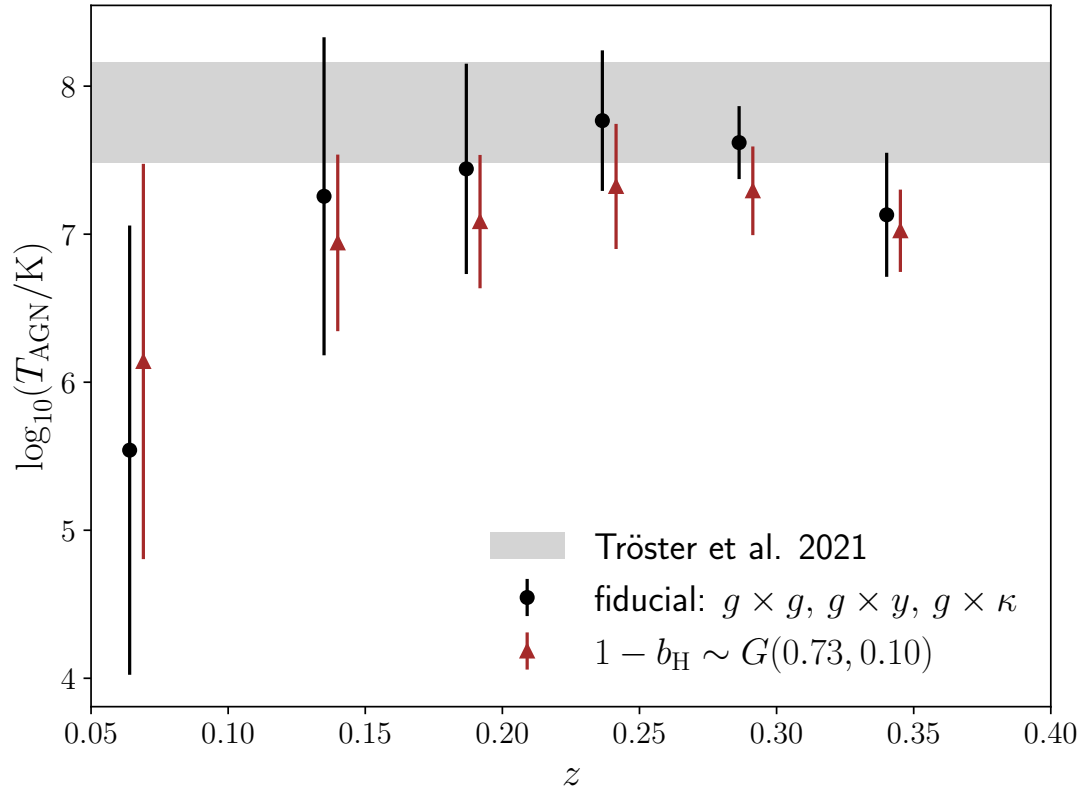


Figure 4.18: Measurements of the AGN temperature parameter T_{AGN} , characterising the strength of feedback in the BAHAMAS simulations. Specifically, T_{AGN} quantifies the temperature at which gas is heated by the central AGN. We derive these measurements from the values of $\langle bP_e \rangle$ obtained in our analysis. The grey band shows the value measured by [198] from cross-correlations between cosmic shear data and SZ maps.

Our fiducial results are shown as black circles, the results imposing an informative prior on $1 - b_{\text{H}}$ are shown in brown triangles, and the results after fixing σ_8 are shown in green pentagons. The connection between this parameter and thermodynamic quantities (thermal energy) has made it an interesting target for the community in the last few years. The fainter blue data points show results found by other groups within the same redshift range [145, 43, 115, 206, 215, 201], (see figure caption for details). Our results are in good agreement with these measurements, as well as with theoretical predictions from hydrodynamical simulations based on shock heating. These are shown as black lines and were computed by integrating the pressure profile of [20] to different radii. The results shown in the Figure agree well with these shock-heating models.

Following [45], we also translate our measurements into constraints on the fractional thermal energy density in haloes, $\Omega_{\text{th}} \equiv \rho_{\text{th}}/\rho_{\text{crit}}$. Here, $\rho_{\text{c}} = 1.054 \times 10^4 h^2 \text{ eV cm}^{-3}$ is the critical density at $z = 0$, and the comoving thermal energy density can be computed as

$$\rho_{\text{th}}(z) = \int dM \frac{dn}{dM} \frac{E_T(M)}{(1+z)^3}. \quad (4.21)$$

As described in Section 4.3.2, the thermal energy of a halo E_T can be computed from the electron pressure profile as

$$E_T(M) = \int dr 4\pi r^2 P_{\text{th}}(r|M) = \frac{8-5Y}{4-2Y} \langle P_e(k \rightarrow 0|M) \rangle.$$

The resulting measurements (for the same three prior choices presented before) are shown in Fig. 4.17. We include theoretical predictions using the mean pressure profile described in [20]. Our measurements are qualitatively in agreement with previous measurements of Ω_{th} by [43] and [44]. It must be noted, however, that, while $\langle bP_e \rangle$ is a data-driven quantity, robust to modelling uncertainties, this is not the case for Ω_{th} . Inferring Ω_{th} from $\langle bP_e \rangle$ requires making assumptions about both halo bias and the mass dependence of the electron pressure profile. While both quantities can be calibrated with numerical simulations, uncertainties on $b_h(M)$ alone are at the $\sim 10\%$ level [213], which is comparable with the size of our measurement uncertainties.

Notably, if at a particular redshift all haloes have been thermalised, we would expect the total thermal energy contained inside of them to be related to their gravitational binding energy as

$$\Omega_{\text{th}} = -\frac{2}{3} f_{\text{b}} \Omega_{\text{grav}}, \quad (4.22)$$

where $f_{\text{b}} = 0.157$ is the cosmic baryon mass fraction. We report that our results are in agreement (within 1σ) with predictions of Ω_{th} using the pressure profile presented in [20].

Moreover, following [72], we also compute Ω_{grav} using an integration over the power spectrum which characterises the gravitational binding energy of dark matter:

$$\Omega_{\text{grav}} = -\frac{3\Omega_{\text{m}}^2 H_0^2}{8\pi a c^2} \int_0^\infty dk P(k, a). \quad (4.23)$$

It is important to note that Ω_{grav} is inherently negative, reflecting the bound structures in the Universe, while Ω_{th} is positive, representing the thermal energy content of gas.

In Fig. 4.17, we compare Ω_{m} with Ω_{grav} . We note that if the conversion factor $-\frac{2}{3}f_b$ is applied to Ω_{grav} , the two quantities should align in both magnitude and sign, facilitating a direct comparison. In our analysis, we have chosen not to correct for this pre-factor, but we simply reverse the sign of Ω_{grav} to provide a notion of the magnitudes of the respective quantities on the same scale. Directly comparing the magnitudes of these two quantities also allows us to characterise the efficiency of thermalisation in large-scale structure [43].

Finally, we use our measurements to place constraints on intensive (as opposed to extensive) thermodynamic properties of gas in haloes. For this, we employ the HMx model of [137]. HMx is a halo-model-inspired formalism that is able to reproduce the matter and electron-pressure power spectra from the BAHAMAS suite of hydrodynamic simulations [135] by varying a single “AGN temperature” parameter governing the strength of feedback processes. To produce constraints on T_{AGN} , we post-process our MCMC chains and find the value of T_{AGN} that reproduces the measured biased-weighted pressure $\langle bP_e \rangle$ at each step (since this is the least model-dependent quantity inferred in this analysis). The resulting constraints on $\log_{10} T_{\text{AGN}}$ are shown in Fig. 4.18. Our results are in broad agreement with the measurement of [198] from cross-correlations between cosmic shear data and SZ maps. Using a Gaussian prior on $1 - b_{\text{H}}$ degrades this agreement slightly, making our constraints consistently lower by about 1σ ($\log_{10} T_{\text{AGN}}/\text{K} \sim 7.3$). This could point toward weaker feedback than predicted by other observations [134]. Note, however, that the constraints of [198] were obtained from the cross-correlation of SZ maps and cosmic shear at significantly higher redshifts and that our inference of T_{AGN} is done indirectly via the measured $\langle bP_e \rangle$.

4.4.3 Tomographic reconstruction of structure growth

The most notable difference between this analysis and Chapter 3 is that we include the cross-correlation of galaxy number density with CMB lensing. Having CMB lensing as a tracer of large-scale structure allows us to sample it at particular redshifts, via the kernel of the galaxy distribution, and thus tomographically measure its properties. By probing the large-scale structure we can infer the value of the matter energy density, Ω_{m} , or the fluctuation amplitude of the power spectrum, σ_8 . Because Ω_{m} is well constrained from

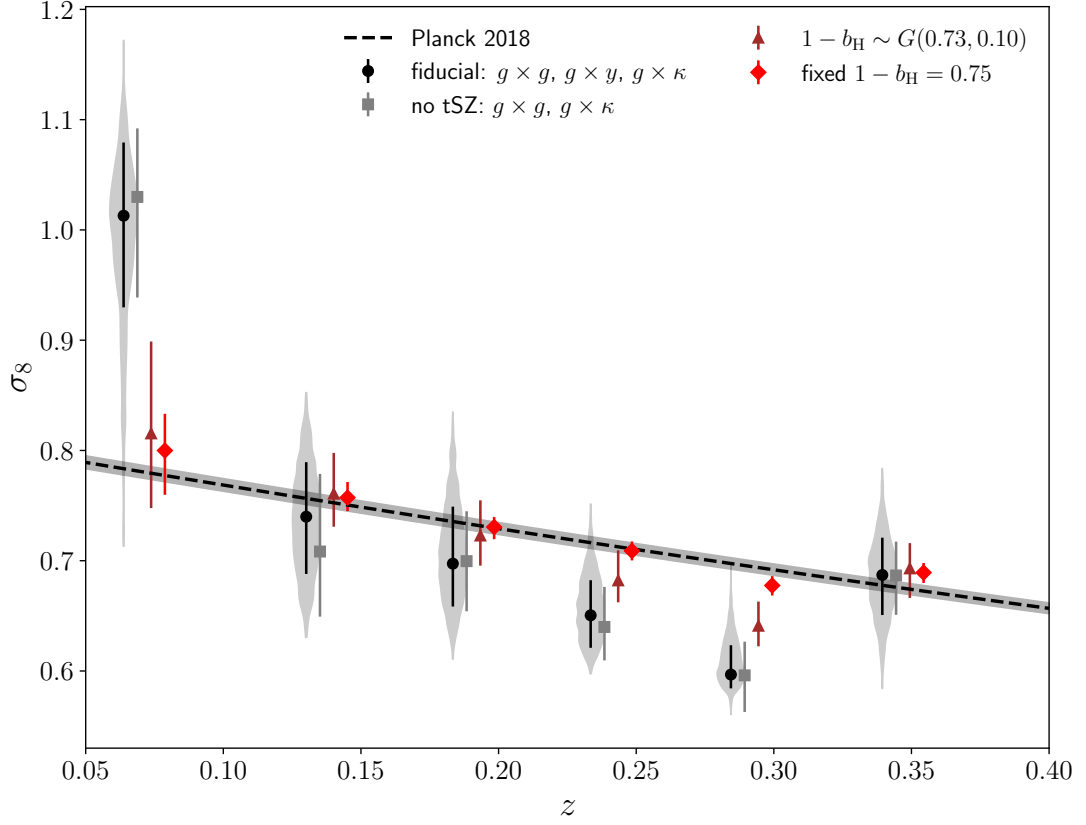


Figure 4.19: Summary figure showing our late-time constraints on σ_8 . Error bars show the 68% confidence interval of the posterior distribution around the median value. The violin background shows the marginalised posterior distribution. We present four models in the figure, which are separated by a small amount in redshift, for readability. The fiducial model (black circles) shows the constraints using all three auto- and cross-correlations of galaxies, tSZ, and CMB lensing, with a wide top-hat prior on $1-b_H$. The second model (grey squares) shows the constraints by omitting tSZ. In the third model (brown triangles), following other measurements of the mass bias, we impose a considerably narrower Gaussian prior on $1-b_H$. In the fourth model (red diamonds), we fix $1-b_H$ to the best-fitting value in Chapter 3, effectively removing a free parameter. We compare our measurements with *Planck* (dashed line).

other cosmological probes (CMB [6], BAO [50], SNe [3]), we choose to fix Ω_m to the best-fit value from *Planck* and have σ_8 freely vary in order to investigate the extent to which low-redshift galaxies, as well as tSZ and CMB lensing, can constrain the evolution of structure growth.

Fig. 4.19 shows our results for σ_8 measured tomographically across the six redshift bins employed in this analysis. We measure σ_8 using four different prior choices. We include the value of σ_8 inferred by *Planck* in the Figure (dashed line). With black circles, we show our fiducial model which includes all three auto- and cross-correlations: $\delta_g \times \delta_g$, $\delta_g \times y$, and $\delta_g \times \kappa$. Not having imposed a strict prior on σ_8 , we report that while our tomographic measurements broadly agree with the evolution of σ_8 assuming *Planck* cosmology, we find a systematically lower σ_8 in the redshift range $\sim [0.23, 0.32]$ (WI×SC bins 2 and 3), and a higher value at very late times ($z \sim 0.07$). Recent work on the properties of the large-scale structure also reports this tension at a redshift of $z \sim 0.3$ [88, 85, 74].

We note that, by omitting tSZ from our analysis we roughly recover the same constraints on σ_8 (grey squares), suggesting that, while our tSZ measurements have very high signal-to-noise, the growth rate of large-scale structure is mostly constrained by CMB lensing, which is a tracer with considerably lower signal-to-noise.

As with the thermal history models, next, we choose to impose a Gaussian prior on the mass bias, $1 - b_H \sim \text{Gauss}(0.73, 0.10)$, based on [149]. Doing so, we note that (i) we improve the constraining power by $\sim 20\%$ (ii) the measurements of σ_8 are more in line with those inferred by *Planck*. These are shown in Fig. 4.19 as brown triangles. It is thus evident that, due to the strong degeneracy between $1 - b_H$ and σ_8 , a $\sim 15\%$ measurement of $1 - b_H$ (following the current best estimates) significantly improves on the constraining power of CMB lensing alone, by including SZ information.

The fact that the constraints on σ_8 are sensitive to the prior on $1 - b_H$ suggests that acquiring a more accurate measurement of $1 - b_H$ would significantly improve our constraints on structure growth. Doing so would require us to improve our understanding of gas thermodynamics and feedback well beyond the current state-of-the-art, or to obtain external constraints on the hydrostatic bias (e.g. from X-ray data).

As an example of the effect that a better understanding of $1 - b_H$ could have, we show explicitly the posterior probability of the WI×SC-4 redshift bin, where we measure the largest offset from *Planck*, in Fig. 4.20, for our fiducial model, and the model where we impose a Gaussian prior on $1 - b_H$. That redshift bin shows a preference for lower σ_8 and higher $1 - b_H$, compared to *Planck*. However, assuming that $1 - b_H$ lies within the Gaussian prior range, brings the marginalised distributions of both parameters more in line to the *Planck* values.

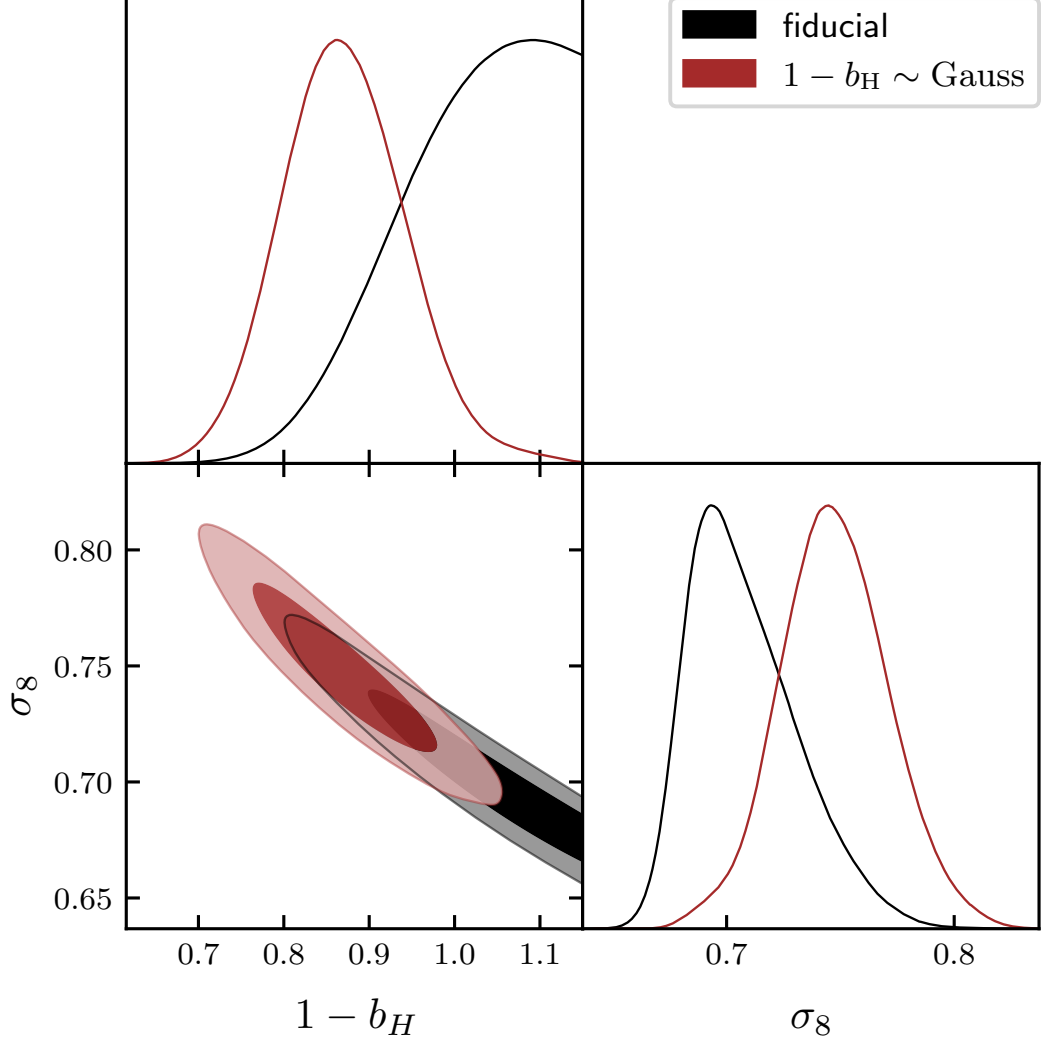


Figure 4.20: The σ_8 vs. $1 - b_H$ slice of the posterior distribution in the WI×SC-4 redshift bin. We present the fiducial model in black contours, and have overlaid the model where we narrow down the prior on $1 - b_H$ by imposing a Gaussian prior on that quantity. We note that while the contours broadly agree with one another, in the fiducial model with an uninformed prior on $1 - b_H$, a higher value of $1 - b_H$ and a lower value of σ_8 is preferred. Both quantities control the clustering amplitude of the cross-correlations.

Unsurprisingly, given this discussion, fixing $1 - b_{\text{H}}$ dramatically improves the constraints we can infer on σ_8 . To exemplify this, in Fig. 4.19, we choose to fix $1 - b_{\text{H}} = 0.75$, and show the resulting constraints as red diamonds. Due to the high signal-to-noise ratio of the $\delta_g \times y$ cross-correlation, which now drives the constraining power, these are the most precise measurements of σ_8 we could potentially achieve with these data. In this case, we effectively use the tSZ map as another, significantly more sensitive, CMB lensing map. These constraints are a factor $\sim 5 - 6$ smaller than those found from $\delta_g \times \kappa$, and comparable with the uncertainty on σ_8 achieved by *Planck*. This highlights the need to improve our understanding of the thermodynamics of cosmic gas, developing more accurate models and exploring alternative gas probes.

4.4.4 Robustness

In this section we show the degree of sensitivity of our final parameter constraints on $\langle bP_e \rangle$ (Fig. 4.21) and σ_8 (Fig. 4.22) to the specific assumptions that have gone into our fiducial model. We depart from the ingredients we used in our fiducial model in the following ways:

1. **Black circles:** The fiducial, 6-parameter model with wide uniform priors on all parameters, outlined in Table 4.3.
2. **Plum diamonds:** In this case we fix the correction factor of the halo model semilinear regime to $A_{\text{HM}} = 1.0$, thus multiplying the halo model prediction by the ratio $\text{HALOFIT}/\text{halo model}$ for the best-fitting cosmological parameters of *Planck* in all auto- and cross-correlations. We do not observe any significant difference in our constraints when we fix the halo model correction factor for the semilinear regime.
3. **Yellow crosses:** We decouple the halo model correction factor, allowing each auto- and cross-correlation to vary freely within the priors outlined in Table 4.3. While we expect the decoupled corrections to describe more accurately the semilinear regime in the halo model, since, different tracers of the large-scale structure would have different correction factors, we observe that the results we obtain are consistent with the fiducial model, where we assume a unified correction factor.
4. **Pink pluses:** We use the *Planck* SZ-deprojected map of CMB lensing convergence in place of the *minimum-variance* (MV) map we use in our fiducial model. This map was constructed from a CMB temperature map in which the contribution from SZ sources has been explicitly nulled, reducing the level of SZ contamination that could affect this cross-correlation. We do not observe any significant difference in

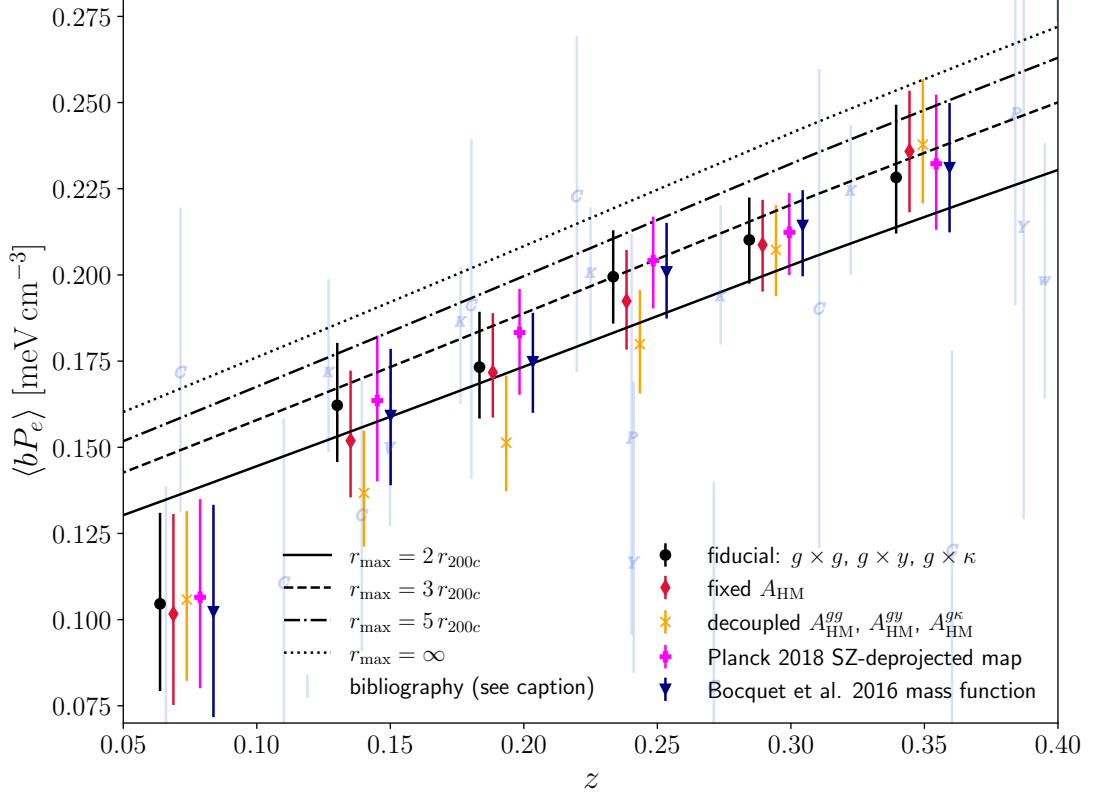


Figure 4.21: Summary figure showing measurements of $\langle bP_e \rangle$ for slight alterations to the parameterisation of our fiducial model, in an attempt to check the robustness of our fiducial measurements. The fiducial model and the comparison with shock-heating models are the same as in Fig. 4.16. Additionally, we present four different tests. In plum diamonds, we remove A_{HM} as a free parameter, effectively fixing the correction of the halo model semilinear transition region to 1, for all auto- and cross-correlations. In yellow crosses, we decouple A_{HM} for each auto- and cross-correlation, allowing correlations between different tracers to have their value for the halo model correction amplitude. In pink pluses, we repeat our fiducial analysis using the SZ-deprojected map from *Planck*. In navy triangles, we repeat our fiducial analysis using a different halo mass function parameterisation: the one described in [34]. All tomographic measurements are in broad agreement for the different robustness checks, suggesting that our modelling choices do not affect our constraints on $\langle bP_e \rangle$.

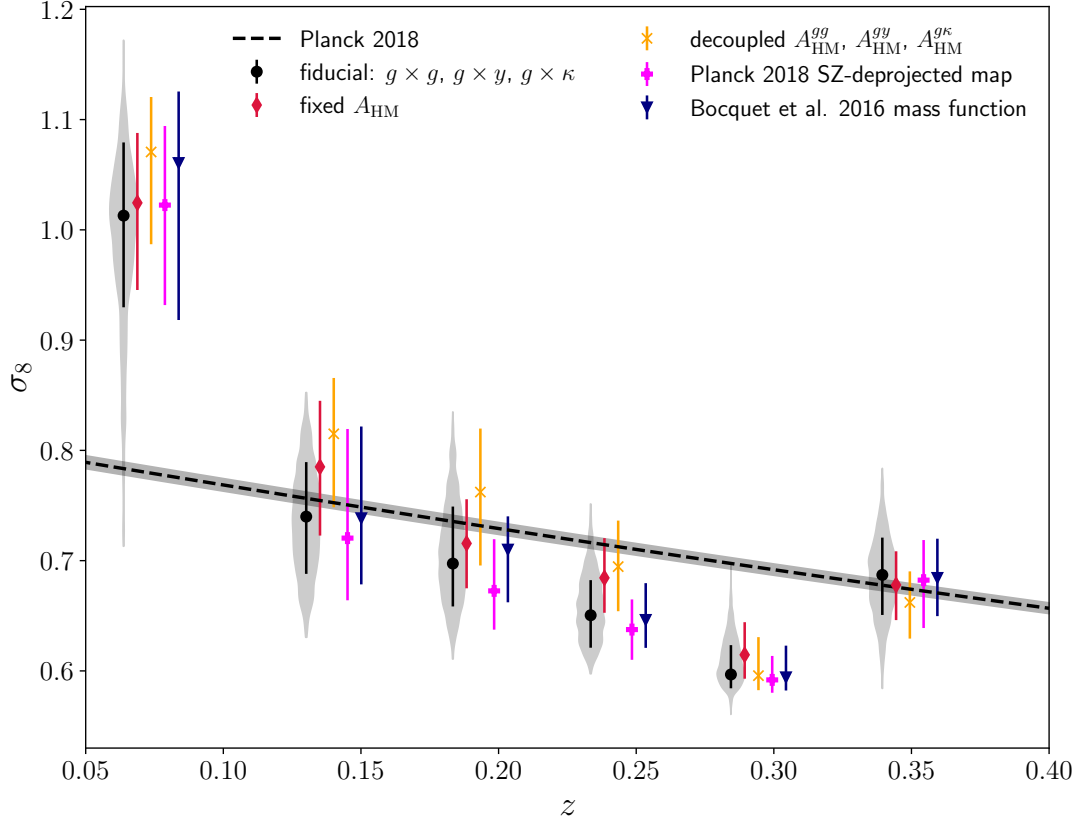


Figure 4.22: Summary figure showing measurements of σ_8 for slight alterations to the parameterisation of our fiducial model, in an attempt to check the robustness of our fiducial measurements. The fiducial model and the comparison with the σ_8 measurement from *Planck* are the same as in Fig. 4.19. As per Fig. 4.21, we present four different tests. In plum diamonds, we remove A_{HM} as a free parameter, effectively fixing the correction of the halo model semilinear transition region to 1, for all auto- and cross-correlations. In yellow crosses, we decouple A_{HM} for each auto- and cross-correlation, allowing correlations between different tracers to have their value for the halo model correction amplitude. In pink pluses, we repeat our fiducial analysis using the SZ-deprojected map from *Planck*. In navy triangles, we repeat our fiducial analysis using a different halo mass function parameterisation: the one described in [34]. All tomographic measurements are in broad agreement for the different robustness checks, suggesting that our modelling choices do not affect our constraints on σ_8 .

our final parameter constraints, implying that our fiducial analysis is not sensitive to this contamination.

5. **Navy triangles:** We use the halo mass function parameterisation described in [34] instead of the one described in [194] which we used in our fiducial model. We observe no significant change in our final parameter constraints. Nevertheless, as the choice of mass function plays a central role in the halo model, we investigate this further by using another two mass functions prescriptions. We discuss these later.
6. **Scale cuts:** We investigated the role of the imposed scale cuts by re-running our fiducial analysis this time with setting $k_{\text{max}} = 0.5 \text{ Mpc}^{-1}$ (instead of $k_{\text{max}} = 1.0 \text{ Mpc}^{-1}$) for the 2MPZ and WI×SC-4 redshift bins, the constraints of which seem to be affected by our choice of priors the most. We observe a small change for 2MPZ, and choose to implement a larger scale cut for this redshift bin in particular, throughout our analysis, since this redshift bin lies at the lowest redshifts (so it is the most affected by non-linearities), and it corresponds to a significantly different galaxy sample (due to the matching of 2MASS sources). Furthermore, it may be affected from correlation between the CMB lensing mask and the LSS as the masked tSZ clusters in it have low redshifts. We observe no change in our final parameter constraints for the WI×SC-4 redshift bin. These results are not shown in the figure for simplicity.

In addition to the aforementioned checks, we run the fiducial analysis with another two halo mass function parameterisations: the one described in [195] and the one described in [58]. We observe a considerable difference in the posterior probability, which we choose to best showcase with the corner plot of the WI×SC-3 redshift bin for the total of four different halo mass functions with which we ran different analyses. This is presented in Fig. 4.23, where the different shades of blue indicate (from darkest to lightest) the mass functions of [194], [34], [195], and [58].

We observe that, while the marginalised posterior probabilities of $1 - b_{\text{H}}$ and σ_8 are not significantly affected by our choice of mass function parameterisation, there is a $\sim 4\sigma$ maximum tension between the contours in probability space. The mass function described in [58] is the one that differs the most, but this figure highlights the importance of having an accurate mass function parameterisation. In our fiducial model, we choose to use the mass function of [194] in line with K20 (Chapter 3), and other similar analyses [107, 215, 155, 45], as we observe agreement between this mass function and that of [34] which has been calibrated for $\Delta = 500c$ halo mass definitions.

The differences in the posterior contours obtained with various mass function parameterisations are not unexpected. In Chapter 3, we demonstrated how the choice of mass

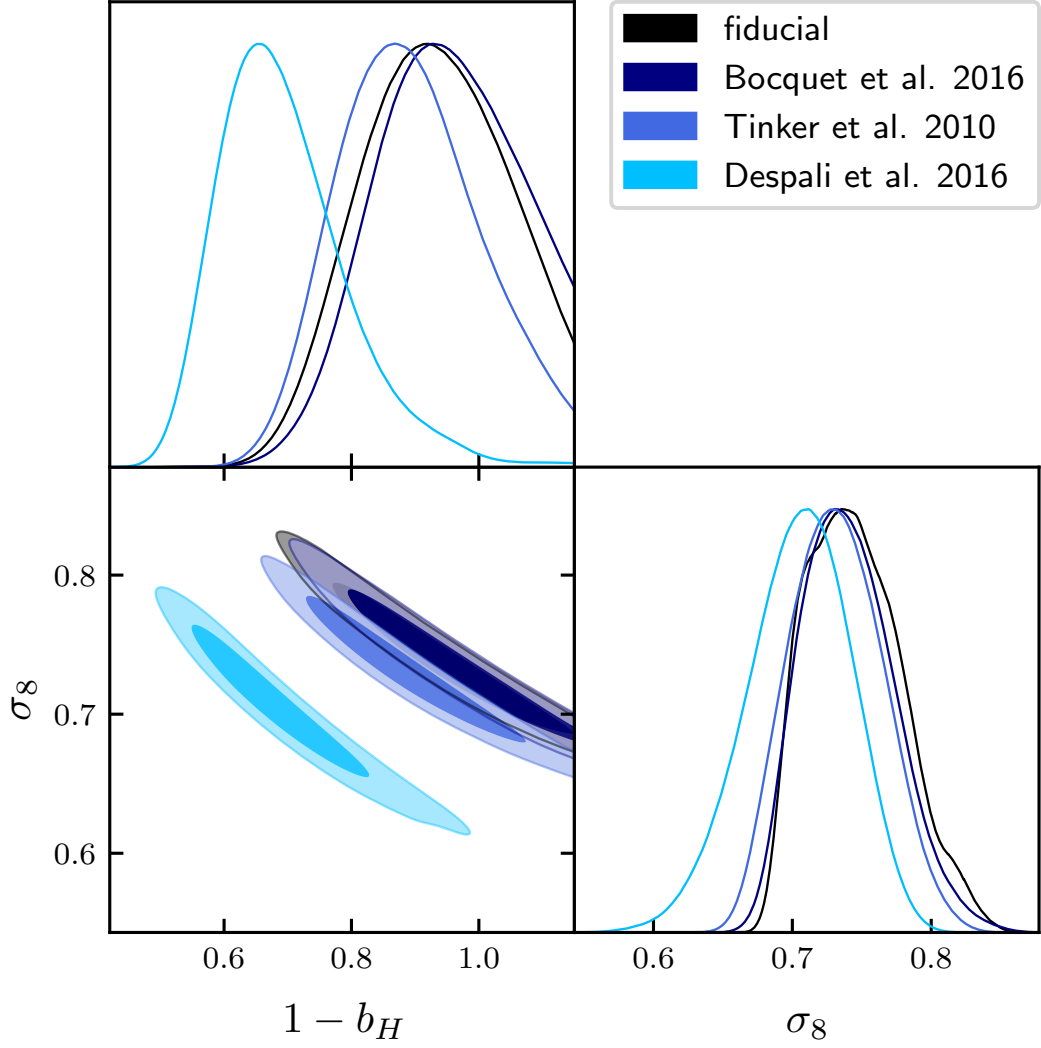


Figure 4.23: The σ_8 vs. $1 - b_H$ slice of the posterior probability distribution for the WI×SC-3 redshift bin, for models departing from our fiducial model using a different halo mass function parameterisation. Our fiducial model uses the mass function outlined in [194]. The three overlaid contours show the different constraints we obtain when we use the mass function prescriptions of [34], [195], and [58]. While the marginalised posteriors on each individual parameter are not in mutual tension, the probability-space distance between the different contours highlights the need for an accurate mass function parameterisation.

function can significantly alter the inferred parameter values. For example, as shown in Fig. 3.11, replacing the fiducial mass function of [194] with that of [195] leads to a lower inferred value of $1 - b_{\text{H}}$. Moreover, Fig. 3.13 illustrates that while these two mass functions exhibit only modest differences, the one from [57] deviates even further from the others.

As discussed in Section 1.3.2.1 via the Press-Schechter formalism, the differential halo mass function is exponentially sensitive to $\sigma(M)$ (and thus σ_8):

$$n(M)dM \propto \exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right), \quad (4.24)$$

which implies that even small changes in σ_8 can result in large variations in the predicted number of high-mass haloes. Consequently, to reconcile the over-prediction of high-mass haloes by the mass function of [57] with observations, a lower value of σ_8 is required compared to that inferred using the [194] mass function.

This sensitivity underscores the importance of selecting an accurate mass function parameterisation. In our fiducial model, we adopt the mass function of [194] which we find to be in good agreement with the calibration of [35] for $\Delta = 500c$ mass definitions.

4.5 Conclusion

In this chapter, we have presented a comprehensive tomographic analysis of structure growth at $z < 0.4$ using the following cross-correlations of secondary anisotropies of the CMB: $\delta_g \times \delta_g$ (galaxy clustering), $\delta_g \times y$ (galaxies with tSZ), and $\delta_g \times \kappa$ (galaxies with CMB lensing). By combining these power spectra, we have been able to break the degeneracy between the hydrostatic mass bias $1 - b_{\text{H}}$ and the amplitude of matter fluctuations σ_8 . This approach has enabled us to extract constraints not only on the growth of the large-scale structure, but also on the bias-weighted average electron pressure $\langle bP_e \rangle$, and, indirectly, on the AGN feedback temperature T_{AGN} and the thermal energy density Ω_{th} across six tomographic redshift bins.

Notwithstanding potential systematic uncertainties and degeneracies, our fiducial results demonstrate the constraining power of cross-correlation analyses of secondary CMB anisotropies. In particular, incorporating the galaxy-CMB lensing ($\delta_g \times \kappa$) cross-correlation has allowed us to pin down the normalisation of the matter power spectrum, thereby constraining σ_8 . Imposing an informative prior on $1 - b_{\text{H}}$ based on external calibrations of cluster masses reduces the posterior uncertainty on σ_8 by $\sim 20\%$. We have demonstrated that the tSZ signal contains a wealth of information about structure growth. This highlights the importance of accurately modelling the small-scale hydrodynamics of haloes, mainly

involving interactions of baryons and feedback mechanisms in order to break the degeneracy between $1 - b_{\text{H}}$ and σ_8 .

Our measurements highlight intriguing deviations in certain redshift bins (notably the WI×SC-4 bin) that may either point to residual systematics in mass calibration, or further complexities in the galaxy-dark matter connection. To that end, our results underscore the potential of combining multiple cosmological probes to disentangle astrophysical uncertainties from fundamental cosmological parameters.

Looking ahead, the methodology we employ here is particularly promising for the era of precision Cosmology. With next-generation missions (see Section 1.5) such as DESI, Euclid, LSST, CMB-S4, and the SKA, datasets with unprecedented quality and volume have started to become available. These surveys will not only enable tighter constraints on σ_8 , but will also offer the opportunity to extend these analyses to higher redshifts and smaller scales. Furthermore, improvements in redshift calibration, through techniques such as weighted direct calibration (DIR) and other advanced machine learning algorithms, can further mitigate systematic uncertainties.

In summary, our work demonstrated that cross-correlations of secondary CMB anisotropies can provide a robust pathway for probing structure growth and the thermal properties of haloes. By breaking the parameter degeneracies and extracting meaningful constraints from existing data, we lay the basis for future analyses that will further refine our understanding of cosmic evolution and the physics driving structure formation.

Chapter 5

Conclusions

5.1 Summary of this thesis

This thesis has explored new avenues to enhance our understanding of large-scale structure through novel statistical techniques, combining cross-correlations of secondary CMB anisotropies and large-scale structure tracers. In Chapter 1, we laid the groundwork by reviewing the essential elements of modern Cosmology – from background theory and structure formation to the halo model and the most important cosmological observables – while also outlining the landscape of current and upcoming experiments. This comprehensive overview established the context and motivation for our work in an era where precision Cosmology is increasingly challenged by astrophysical systematics.

Building on this foundation, Chapter 2 introduced the statistical methods used in contemporary cosmological analyses. We discussed n -point statistics, power spectra, covariance estimation methods, and detailed our approach to cosmological inference through Bayesian analysis, likelihood evaluation, and parameter-space sampling techniques such as MCMC and the Fisher matrix. These tools enable us to rigorously quantify uncertainties and derive robust constraints on cosmological parameters.

In Chapter 3, we applied these statistical methods to measure the thermal history of the Universe. By cross-correlating galaxy positions with maps of the thermal Sunyaev-Zeldovich effect (tSZ), we constrained the evolution of thermal gas pressure $\langle bP_e \rangle$ as well as the hydrostatic mass bias $1 - b_H$ across multiple low-redshift tomographic bins. Our results, which are broadly consistent with previous studies and hydrodynamical simulations, demonstrate the power of combining multiple observational probes to mitigate astrophysical uncertainties.

Chapter 4 extended this analysis by incorporating an additional cross-correlation between galaxy positions and CMB lensing. This allowed us to break the degeneracy between

$1 - b_{\text{H}}$ and the amplitude of matter fluctuations σ_8 , thereby providing tomographic measurements of structure growth. Despite some tensions in specific redshift bins, our work demonstrates that, given adequate understanding and modelling of the small-scale hydrodynamics of haloes, we can achieve considerable constraining power on the cosmological parameters. These findings also contribute to the broader discourse on cosmological tensions and highlight the potential of cross-correlations of secondary CMB anisotropies in resolving long-standing discrepancies in cosmological parameter measurements.

Overall, this thesis demonstrates that using secondary CMB anisotropies and precise calibration techniques such as weighted direct calibration (DIR) of photometric redshifts, we can significantly improve constraints on both astrophysical and cosmological parameters. Our approach not only refined the measurements of quantities such as σ_8 , $1 - b_{\text{H}}$, and $\langle bP_e \rangle$, but also paved the way for robust tests of the Λ CDM model in the context of precision Cosmology.

The results presented in this thesis set the stage for further investigations. In the following section, we outline potential pathways for future work that will make use of current and upcoming surveys, and advanced methodologies to build upon what is presented in this thesis.

5.2 Future work

The results presented in this thesis open the door to several promising avenues for future research. Future work would need to refine the current theoretical framework and address some of the outstanding problems in Cosmology.

First and foremost, there is significant potential to improve the estimation of redshift distributions. This work explored two ways in which this can be achieved: a one-parameter extension model that artificially broadens or tightens the redshift distribution, and a machine learning approach which achieved precise estimates of redshifts using weighted direct calibration of colour space. Future efforts could explore more advanced techniques that capture the complex relationships between photometric and spectroscopic data, thus reducing the uncertainties in cosmological parameter inference.

To that end, expanding the tomographic approach to include higher redshifts would enable a deeper understanding of the evolution of the quantities measured and presented in this thesis. Particularly when probing σ_8 as a means for structure growth, performing tomography at higher redshifts could allow us to potentially explore the origin of the σ_8 tension. Data from surveys like Euclid and the upcoming Rubin Observatory’s LSST will provide us with an unprecedented data volume (and quality), which would allow us

to significantly reduce the largest source of uncertainty in cosmological analyses: the estimation of redshift distributions.

With the new generation of cosmological surveys, this high quality and volume of data will allow us to not only constrain astrophysical parameters, but also to test departures from the standard Λ CDM model. Testing non-standard models could reveal whether the challenges we face today in Cosmology hint at new physics beyond Λ CDM.

Thus, going forward, **it will be crucial to analyse observations from multiple cosmological probes consistently**, while remaining open to theoretical models that best describe the data. Progress in Cosmology is likely to be driven by tackling the current challenges through observational advances, which will reveal the extent to which existing theoretical models, such as Λ CDM, remain valid.

The inclusion of additional tracers of the LSS in future studies will enable us to refine our understanding of Physics. Observables such as the CIB and X-ray surveys, when integrated into cross-correlation analyses, can provide independent measurements on key parameters like σ_8 and offer a more comprehensive view of the underlying astrophysics. Each tracer brings unique insights into the structure and evolution of the Universe, helping to break parameter degeneracies that limit current analyses.

Incorporating multiple tracers, of course, comes at the cost of having to accurately model their associated systematics (observational biases and model uncertainties). To address these issues, Cosmology needs a standardised framework that consistently handles the systematics of all probes under consideration. Such a unified approach will accelerate cosmological inference, giving us the opportunity to get more robust and precise constraints on the parameters, as well as potentially uncovering new physics beyond the standard cosmological model.

5.3 The current status of Cosmology

Cosmology has long transitioned to its new era, often referred to as the *precision cosmology* era, where observations across the electromagnetic spectrum have allowed for unprecedented accuracy in the measurement of the fundamental cosmological parameters. The Λ CDM model has emerged as the standard framework for describing the evolution of the Universe, and is supported by a wealth of independent and complementary evidence from early- and late-time observations.

Mapping the CMB by missions such as COBE, WMAP, and Planck, has been arguably the most significant milestone in modern Cosmology. The measurements these probes have provided have allowed us to make robust estimations of some key quantities for the

Universe, such as its total matter content (through Ω_m), its total baryon content (through Ω_b), its total dark energy content (through Ω_Λ), as well as its expansion rate (through H_0). The newest results from Planck, as well as the efforts of multiple groups around the world, represent the culmination of this effort, delivering a precise snapshot of the Universe at the epoch of recombination ($z \sim 1100$).

At the same time, surveys of the LSS, such as SDSS and DES, have mapped the distribution of galaxies and matter at late times. These surveys probe the growth of structure and expansion history, offering independent constraints on the cosmological parameters. Baryon acoustic oscillations and redshift-space distortions serve as additional, independent probes of the expansion history and the growth of structure.

Measurements of Type Ia supernovae have long established that the Universe is expanding in an accelerated fashion. These measurements provided us with the first direct evidence for the existence of dark energy. This discovery, combined with measurements of galaxy clustering and weak lensing, has enhanced our understanding of the mass-energy contents of the Universe. We have come to realise that most of the Universe is made of dark energy and dark matter, which we understand very little; with only a small fraction consisting of ordinary, baryonic matter.

Apart from the contents of the Universe, these datasets have further enabled us to place constraints on other physical characteristics, such as the Universe’s age and geometry. While Λ CDM provides a remarkably consistent description of these observations, some emerging tensions and phenomena we have not managed to disentangle yet suggest that our understanding remains incomplete. These challenges motivate ongoing efforts to refine and expand our theoretical framework.

Cosmology is undergoing a computational revolution, with alternative methods for MCMC and advancements in differentiable Cosmology promising to significantly accelerate analysis pipelines. Traditional MCMC approaches, while robust, can become computationally prohibitive when dealing with high-dimensional parameter spaces (typical of multi-probe analyses). Methods such as *Hamiltonian Monte Carlo* (HMC) and its variants, which exploit gradients of the likelihood function to navigate the parameter space more efficiently, can drastically reduce the number of required iterations to converge [37, 26]. These techniques are particularly suited for Cosmology, where datasets are large, and models often involve high-dimensional priors and non-linear parameter dependencies.

Moreover, the advent of differentiable Cosmology, which makes use of automatic differentiation (auto-diff), is transforming likelihood-based inference. By constructing fully differentiable forward models and likelihood functions, cosmological parameters can be

optimised or sampled at great speed and accuracy. This approach not only accelerates inference, but it also integrates well with several machine learning techniques, enabling joint modelling of datasets. Differentiable frameworks such as JAX-COSMO [39] make it possible to compute gradients of likelihoods with respect to cosmological parameters in real time, streamlining inference and enabling faster exploration of the parameter space.

These advancements are crucial for next-generation cosmological surveys, where computational efficiency will be essential to interpret data that is increasing both in precision and in volume. Together, alternative MCMC methods and differentiable Cosmology have the potential to address current bottlenecks in cosmological analyses, accelerating the advancement of Cosmology and providing insights into the nature of the Universe.

5.4 Λ CDM

As discussed earlier, the Λ CDM is the cornerstone of modern Cosmology, encapsulating the simplest theoretical framework that successfully describes the evolution of the Universe. This model is built under the assumptions of General Relativity and the cosmological principle (at sufficiently large scales, the Universe is homogeneous and isotropic).

The model contains six key parameters:

- matter density, Ω_m ;
- baryon density, Ω_b ;
- dark energy density, Ω_Λ ;
- (either) the amplitude of the primordial density fluctuations, A_s ;
- (or) the amplitude of matter density fluctuations at a scale of $8 h^{-1}\text{Mpc}$, σ_8 ;
- the scalar spectral index, n_s ;
- the optical depth to reionisation, τ .

These parameters, when constrained by observations, define a Universe that is 13.8 billion years old, spatially flat, and composed of dark energy ($\sim 68\%$), cold dark matter ($\sim 27\%$), and baryonic matter accounting for the remaining $\sim 5\%$.

Λ CDM has been so successful because it can explain a broad array of observational data. It naturally accounts for the formation of large-scale structure via the growth of the small initial perturbations that inflation seeded. It can also accurately characterise the acoustic peaks in the CMB temperature power spectrum, the baryon acoustic oscillations in

galaxy clustering, and the accelerated expansion of the Universe as inferred from Type Ia supernovae.

The cosmological constant as the existing form of dark energy (Λ), although hidden from our direct view (we can only infer its existence) offers a simple mechanism for the observed acceleration of the expansion. At the same time, cold dark matter (CDM), composed of non-relativistic particles, explains the observed clustering of galaxies and the formation of structures on different scales.

In the Λ CDM model, the evolution of the Universe is governed by the Friedman equations, which describe the relationship between the expansion rate ($H(z)$), the energy density of the different components, and the curvature.

Despite its simplicity and success in describing a plethora of observations, Λ CDM faces some challenges. Firstly, it does not provide a fundamental explanation about the nature of dark matter and dark energy. Furthermore, the cosmological constant Λ raises questions about fine-tuning, as the observed value is orders of magnitude smaller than what quantum field theory predicts to be the value of vacuum energy.

Nevertheless, Λ CDM remains the benchmark against which new models are tested, providing a robust framework that continues to guide progress in Cosmology, since its solidification with data from the COBE satellite in 1992, and the first results from WMAP in 2003.

5.5 Challenges

As mentioned above, while the Λ CDM has been remarkably successful in explaining a wide range of cosmological observations, some persistent challenges and tensions hint at either the existence of complicated systematics or new physics.

Most prominently, the Hubble tension, where the value of H_0 inferred by the early Universe is at a 5σ disagreement with the one measured at late times, suggests either unaccounted-for systematics or the need for reassessment of the entire Λ CDM model.

Similarly, the tension involving the σ_8 parameter, which is currently at the 2.5σ level, could indicate deviations in the growth of cosmic structure, point to possible modifications in how dark matter clusters at late times, or even new physics.

In addition to these observational tensions, some challenges with our understanding of the Universe remain. The nature of dark matter is still unknown, with a range of candidates from weakly interacting massive particles (WIMPs) to axions, sterile neutrinos, etc. Experiments aimed at direct detection have not been fruitful as we have no conclusive evidence for the existence of dark matter particles. At the same time, computer simulations

experience discrepancies from observations. Namely, there are missing satellite galaxies in clusters, and the density profile of low-mass galaxies is different (core-cusp problem).

Just like dark matter, dark energy and the existence of a cosmological constant remain an unsolved puzzle. The observed value of Λ is many orders of magnitude smaller than the equivalent quantity (vacuum energy) predicted using quantum field theory. This raises questions of fine-tuning, and/or whether there exists a dynamical form of dark energy (e.g. quintessence).

Furthermore, the very assumption of a homogeneous and isotropic Universe may break down at certain scales or epochs. To that end, anomalies on the CMB such as the cold spot or the hemispherical power asymmetry challenge the assumption of statistical isotropy posed by Λ CDM.

Finally, transitioning into galactic Astrophysics, tensions between galaxy formation models and observations (mismatches in the abundance, distribution, morphology) highlight how important it is that we establish a solid understanding of baryonic physics and its interplay with dark matter. These challenges underline the need for more advanced simulations, enhanced understanding of the feedback processes, and finding novel observational probes.

5.6 Alternatives

The challenges of Λ CDM have motivated the exploration of alternative models of the Universe. These alternatives seek to address specific problems or provide a more comprehensive explanation of cosmological phenomena.

5.6.1 Dark energy

One prominent class of alternatives involves modifications to the nature of dark energy. Dynamical models, such as quintessence [38] or k-essence, replace the cosmological constant Λ with a scalar field that varies with time. This alleviates any fine-tuning issues associated with Λ . Other models, like coupled dark energy [10] or interacting dark energy, allow for interactions between dark energy and dark matter, introducing additional dynamics that could affect structure formation or expansion history.

5.6.2 Dark matter

In the realm of dark matter, alternatives to cold dark matter (CDM) have been proposed to address small-scale structure problems. Warm dark matter [204], fuzzy dark matter

[103], or self-interacting dark matter introduce additional physical properties that modify the clustering behaviour at small scales, which could resolve any discrepancies we see in observations.

5.6.3 Modified gravity

Modified gravity theories provide another path for explaining an accelerated Universe without the existence of dark energy. Models like $f(R)$ gravity [186], Brans-Dicke theory, and Modified Newtonian Dynamics (MOND) attempt to alter General Relativity at cosmological scales. Many of these families of models face extremely stringent constraints from cosmological tests of gravity.

5.6.4 Resolving tensions

Solutions to the Hubble and σ_8 tensions often extend Λ CDM by introducing new physics, such as early dark energy [162], decaying dark matter [203], or changes to neutrino properties. These extensions typically modify the physics of the early Universe and/or the growth of cosmic structures.

5.6.5 Other cosmologies

Finally, alternative cosmologies, such as bouncing or cyclic Universes, have been proposed as replacements for the inflationary paradigm [187]. While these models remain speculative, some offer potential solutions to conceptual issues like the initial singularity or the observed large-scale anomalies on the CMB surface.

5.7 Final remarks

This thesis represents my, and my collaborators’ efforts to address key questions in modern Cosmology, making use of novel statistical techniques and state-of-the-art data to advance our understanding of the Universe. Constraining the small-scale astrophysics (the interactions of galaxies with one another, and their interplay with the feedback mechanisms) ‘gastrophysics’ we can do two things: (i) characterise them in order to learn more about what is happening at those small scales; (ii) ‘reverse’ their effect in order to clean up and differentiate the cosmological signal from other systematics. This thesis presented some of the most stringent constraints to date on some key astrophysical parameters: the hydrostatic mass bias ($1 - b_H$) and the bias-weighted average electron pressure in clusters $\langle bP_e \rangle$, which

enabled us to put constraints on the growth of structure at late times through σ_8 . These contributions underscore the power of cross-correlation analyses (of secondary anisotropies) in breaking degeneracies and improving parameter constraints.

Looking forward, two things are certain: the challenges are vast, but the opportunities are also vast. The standard Λ CDM model, while remarkably successful, continues to face scrutiny as tensions persist between early- and late-time observational datasets. The search for answers will depend on novel theoretical frameworks, more precise measurements, and the development of standardised methodologies capable of integrating multiple cosmological probes.

As we enter the era of next-generation cosmological surveys, future research will be driven by the volume of new data we acquire. I hope that some of the methods and analysis insights used here will lay the foundation for future research, in an effort to enhance our understanding of the Universe. Cosmology now stands at the threshold of potentially transformative discoveries, and it is an exciting time to contribute to this ever-evolving field.

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