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**Shopping Malls, Platforms and Consumer Search**

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# Shopping Malls, Platforms and Consumer Search \*

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## Abstract

We consider a general model of a market for differentiated goods, in which firms are located in marketplaces: shopping malls or platforms. There are search frictions between the marketplaces, but not within them. Marketplaces differ in their size. We show that consumers prefer to start their search from the largest marketplace and continue in the descending order of their size. We show that the descending search order is the only search order which can be a part of an equilibrium for any market configuration. Despite charging lower prices, firms at larger marketplaces earn higher profits, and under free entry all firms cluster at one place. If a marketplace determines the price of entry, the equilibrium marketplace size depends negatively on search costs.

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# 1 Introduction

In this paper we study markets for differentiated goods where firms sell their products in various marketplaces: shopping malls, online platforms, etc. Quite often sellers of similar products tend to concentrate in one location. For example, souvenir sellers in Oxford are mainly located on High Street. Several large fabric retailers in Moscow are located on Leninsky Prospekt. Restaurants tend to cluster in a few streets in city centers or in “food courts” in airports. In online markets, more and more retailers tend to join large selling platforms, like eBay or Amazon. Apparently, the losses from tougher competition are more than outweighed by the fact that these large marketplaces are the starting point of consumers’ search.

In this paper we consider a general industry structure with an arbitrary number of marketplaces of arbitrary sizes. We assume that there is a positive cost for searching between the marketplaces and zero search cost within each marketplace, although our results hold if search costs within each marketplace are the same and sufficiently low. Consumer search in our paper is directed by the size of each marketplace. Consumers prefer to sample larger marketplaces first. There are three forces which contribute towards this decision. Firstly, large malls offer a greater variety, thus providing a better expected match between a consumer and a product. Secondly, large concentration of firms under the same roof leads to strong competition and lower prices, making larger marketplaces even more attractive. Finally, the search order works as a self-fulfilling prophecy: as consumers expect lower prices in larger malls, they sample them first, and as a result the demand at larger marketplaces is more elastic, which reinforces lower prices. After consumers have sampled all the marketplaces of a certain size, they either move to lower-size places, or buy from a previously sampled retailer. This recall feature, which does not exist in random search models, is very typical for the kind of directed search protocol we have in our paper.

We also study the incentives of the firms to cluster in the same location, for example, a large shopping mall. It turns out that if the entry is free, all firms prefer to concentrate at the same place, meaning that they have tough competition and have to charge low prices.

The positive effect from search order dominates the negative effect from price competition. Naturally, such industry structure is socially efficient: search costs are minimal and low prices ensure maximum consumer participation. The situation changes, however, if the mall is allowed to choose its capacity and to charge a fixed fee for a retail space. In this case, the size of the mall critically depends on the search cost. If search friction is small, which is a common situation for online markets, as before, the largest marketplace absorbs all the firms in the industry. However, as search friction increases, the optimal size of the marketplace monotonically decreases, meaning that in off-line markets with brick-and-mortar stores and large search costs, intermediate levels of concentration should prevail.

Since consumer search in our paper is driven by the size of the marketplace, our paper makes a contribution to recent research on directed or ordered search and prominence<sup>1</sup>. An article by Arbatskaya (2007) first proposed a search protocol, in which consumers search in a certain specified order. She found that firms which are sampled earlier charge higher prices. This result was subsequently reversed in a sequence of papers which, as well as ours, build up on the model of differentiated products by Wolinsky (1986) and Anderson and Renault (1999). Zhou (2011) proposed a model of ordered search with differentiated products. In his paper any search order can be rationalized as in equilibrium earlier sampled firms set lower prices and due to Weitzman (1979) rule, so they should indeed be sampled first. This means that if there are  $N$  firms in the industry, there are  $N!$  possible equilibria (and even more if one allows for mixed search strategies). This situation arises because in Zhou (2011) there is only one force affecting the price – the elasticity of demand, which drops with each search round. In our paper the price sequence is increasing in search order as well, but there are additional forces contributing to this. As sampling order is defined by the size of the marketplace, first marketplaces are characterized by larger variety and higher competition, which pushes prices even further down. For some specific market configurations (partitions of the firms into marketplaces) other equilibria may exist. However, the equilibrium we characterize is the only robust equilibrium in the following sense: there are market configurations for

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<sup>1</sup>See Armstrong (2016) for a detailed discussion of this literature.

which the only optimal search order is decreasing in malls' size, and the equilibrium with the decreasing search order exists for any market configuration. Our paper is also related to the literature on prominence. In this literature firms are ranked by their prominence in search order, either exogenously as in Armstrong et al. (2009) or endogenously via position auction (Athey and Ellison (2011)), commission to salesman (Armstrong and Zhou (2011)) or stochastically via advertisement, as in Haan and Moraga-González (2011). We consider another natural reason for prominence – location in a larger mall or platform.

We show that although the competition in large marketplaces is very strong and prices are much lower than in smaller ones, firms earn higher profits in such places. The reason is that demand drops very fast with moving down to smaller malls. This result is similar to the one obtained in Zhou (2011), although harder to achieve in our case due to strong competition within large marketplaces. Our result implies that firms prefer to be located in the places with toughest competition, which is quite a counterintuitive conclusion for models without directed search.

Another contribution of our paper is that it brings together directed and random search in a very general framework: we consider equilibria in which marketplaces are sampled in descending order, but whenever consumers encounter multiple marketplaces of the same size, they search randomly. The only paper which looks at multiple prominent firms is Zhou (2009), but he only considers two groups of firms, when the first group is sampled before the second.

The closest paper to ours is Moraga-González and Petrikaitė (2013). In this paper, the authors consider incentives for firms to merge and, in the long run, sell multiple products under the same roof. They show that if the number of merged firms is not larger than ten and if search costs are sufficiently large, there is an equilibrium in which consumers prefer to start searching from the mall. An important difference with our model is that in Moraga-González and Petrikaitė (2013) prices are not set independently, but rather coordinated by a single parent entity. This means that unlike in our model, price in the mall is larger than in stand alone stores, making it harder to sustain the descending search order. Our results

are clear cut: grouping firms under the same roof increases competition, lowers prices and makes the descending search order attractive.

There is extensive literature on firms' concentration and clustering, pioneered by Eaton and Lipsey (1979), Stahl (1982) and Wolinsky (1983). Dudey (1990) considered a Cournot competition with search frictions and showed that there is an equilibrium when all firms cluster in the same marketplace. The closest papers to ours are Fischer and Harrington (1996) and Non (2010). Fischer and Harrington (1996) consider a model in which consumers search for a differentiated product and firms choose whether to locate in a single shopping mall or on the periphery. Due to the assumption of heterogeneous search costs, which are lower for the periphery stores than for the mall, they obtain that under free entry some but not all of the firms cluster. In our paper (if free entry assumption is made), this result is reversed due to search costs being independent of the mall size. Moreover, unlike in our paper, in Fischer and Harrington (1996) the incentives to cluster are higher for more differentiated products. We obtain that being in the first (the only one in the case of Fischer and Harrington (1996)) mall pays off a lot, as consumers rarely search beyond the largest marketplace. Another difference between the two papers is that we are able to deal with fully rational optimal stopping rule, while in Fischer and Harrington (1996) consumers naively assume an infinite number of stand alone stores while searching. Once we allow the marketplaces to charge a price for retail space, the industry structure similar to Fischer and Harrington (1996) can arise, provided that the search costs are sufficiently high. Thus, depending on parameters, either Dudey (1990) result of pure clustering, or Fischer and Harrington (1996) core-periphery result can arise in our model. Non (2010) also asks a question about the incentives of firms to price and locate in the malls in the presence of search frictions. She considers a homogeneous goods search model based on Stahl (1989) and, similarly to Fischer and Harrington (1996), works with the industry structure with just one large marketplace and a periphery of stand alone stores. She looks at equilibrium in mixed strategies, in which, like in our paper, stand alone stores tend to charge higher prices.

Finally, our paper is related to literature on platform competition, as any marketplace in

our paper can be viewed as a selling platform. However, unlike most of the literature (e.g. Armstrong (2006)), we abstract from the question of optimal platform pricing, and assume that a retail place at each marketplace is sold at a fixed price which does not affect the marginal costs of the firms. Paper by Wang and Wright (2016) has many common features with ours: they assume that the platform provides lower search costs and consumers decide whether to search via the platform or directly. However, there are a few notable differences. First of all, they consider a model with continuum of firms, which simplifies the analysis, but makes it impossible to analyze a general market structure with platforms of different sizes. Second, they allow for multi-homing and concentrate on the analysis of show-rooming and price parity practices, while we consider single home firms (see the discussion of incentives of firms to be multi-homed at the last section) and focus on the characterization of pricing and profits ranking in a general finite setting.

The rest of the paper is organized as follows. In Section 2 we describe the model. In Section 3 we derive the optimal stopping rule, characterize the market equilibrium and show its robustness. Section 4 provides results on incentives of the firms to join the largest marketplace, optimal trade space pricing and size of the shopping mall. Section 5 is devoted to the discussion of the role of our assumptions and conclusions.

## 2 Model

We consider an industry with  $N$  firms selling a differentiated product to consumers. Firms produce their products at zero marginal cost and compete in prices. There is a unit mass (continuum) of consumers in the market. Each consumer demands exactly one unit of the product. The utility of consumer  $i$  from buying a product of firm  $j$  is  $U_i = u_{ij} - p_j$ , where  $u_{ij}$  is a match value between consumer  $i$  and brand  $j$ , and  $p_j$  is a price charged by firm  $j$ . We assume that  $u_{ij}$ 's are distributed uniformly on  $[0, 1]$  and independently (both across consumers and across firms). When talking about the representative consumer, we will drop subscript  $i$ . We assume that if the consumer does not buy the product her utility is zero.

We assume that all firms sell their products in some marketplaces. These can either be on-line selling platforms or shopping malls. Of course, a firm might choose not to join any platform or mall, but rather sell its product via its own web-site or store. We refer to such firms as stand alone sellers or marketplaces of size one. We assume single homing, i.e. that each firm can sell in a single marketplace only. Suppose that marketplaces can be of  $K < N$  different sizes. Suppose that for any  $k = 1, \dots, K$  there are  $M_k$  marketplaces of size  $N_k$ ,  $\sum_{k=1}^K N_k M_k = N$ . We refer to a group of marketplaces of the same size as *cohort*  $k$ . We order cohorts in such a way that  $N_k > N_l$  for  $k < l$ , so  $N_1$  is the size of the largest marketplace(s), and  $N_K$  is the size of the smallest one(s).

Consumers are engaged in a costly search. We assume that once the consumer enters a marketplace she learns all the prices and match values in this marketplace without incurring any cost. If she, however, decides to leave the mall and search in another marketplace, we assume that she bears a search cost  $s \leq 1/8^2$  per marketplace. The first search is free<sup>3</sup>. We assume that consumers are aware of the size of each marketplace and can direct their search activity based on this information. We also make an assumption of costless return: consumers can come back to previously visited marketplaces for free. This assumption is natural for online shopping, where the main part of search cost is time and effort spent on getting acquainted with the interface of each seller, but is less plausible in case of physical stores, when search costs are mainly transportation costs. This assumption is common in consumer search literature, as it considerably simplifies the analysis, making reservation prices stationary (in our case, when searching within a single cohort), and, as Janssen and Parakhonyak (2014) suggest, should not make a qualitative difference in our analysis.

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<sup>2</sup>This condition guarantees that consumers prefer checking stand alone firms to leaving the market, even when this firm charges the monopoly price.

<sup>3</sup>This assumption is not restrictive: sufficiently small search costs guarantee that all consumers would like to search at least once. See Janssen et al. (2005) for detailed discussion of this assumption in homogenous goods setting.



## 3 Analysis

### 3.1 Optimal Stopping

We are going to construct an equilibrium in which consumers start searching from the cohort of the largest marketplaces (of size  $N_1$ ), then move to the second largest marketplaces and so on. Within each cohort we assume a random search procedure: as consumers cannot distinguish among the marketplaces of the same size, ex ante there is no reason to assume any specific search order. As we will show, there are two incentives to employ this search rule: in larger malls consumers expect to find a better match than in smaller ones, and it turns out that in equilibrium marketplaces that come first in the search order set lower prices than those which are visited later. Potentially, for some specific market configurations it is possible to find other equilibria. Say, if there are just two marketplaces of almost the same size, the price effect can be stronger than the variety effect and it might be possible to find an equilibrium with reversed search order. However, the proposed search rule, according to which consumers go from largest to smallest marketplaces, is the only one which is robust to any market configuration: for any vectors  $(M_1, \dots, M_K)$  and  $(N_1, \dots, N_K)$ , such that  $\sum_{k=1}^K M_k N_k = N$ , there is an equilibrium in which the largest-to-smallest search rule is optimal, while search rule which allows a different order fails to be a part of equilibrium for some  $M_k$  and  $N_k$ ,  $k = \overline{1, K}$  (see Proposition 3 for details).

In order to derive the optimal stopping strategy, we directly apply the result by Weitzman (1979). Suppose that consumers believe that price of each firm in each marketplace in cohort  $i$  is  $p_i^e$ , and that  $p_i^e \geq p_j^e$  for any  $i > j$ . Later we check that price monotonicity holds in equilibrium. As Weitzman (1979) showed, the optimal stopping rule is to sample options in descending order in their *reservation value* and terminate search as soon as one of the options delivers utility higher than the largest reservation value of all unsampled options. Let us define  $a_k$ ,  $k = \overline{1, K}$  as a solution to the following equation:

$$\int_{a_k}^1 N_k(u - a_k) u^{N_k-1} du = s \tag{1}$$

Thus, at  $a_k$  a searcher is indifferent between accepting  $a_k$  immediately and sampling additional  $N_k$  options at cost  $s$ . Define

$$z_k = a_k - p_k^e \quad (2)$$

$z_k$  is the reservation value for a marketplace of size  $k$  given the price belief  $p_k^e$ . Note that as  $p_i^e > p_j^e$  and  $a_i < a_j$  for  $i > j$ , we obtain that  $z_i < z_j$ . Then, direct application of Weitzman (1979) result allows us to formulate the following proposition.

**Proposition 1.** *The optimal search rule is to start sampling from (one of) the marketplaces of size  $N_1$ . Suppose that after sampling several malls, the best sampled option gives utility  $\bar{U}$  and there are  $L$  unsampled marketplaces with the reservation value  $z$ , which is highest among remaining marketplaces. Then the optimal search rule is:*

1. *if  $\bar{U} < z$ , sample one of these marketplaces with probability  $1/L$ ;*
2. *if  $\bar{U} \geq z$ , buy at the firm which gives utility  $\bar{U}$ .*

The optimal stopping rule defined in Proposition 1 implies symmetry among the marketplaces with the highest reservation value. Of course, it is possible to consider the case of asymmetric optimal stopping rules (there are  $L!$  possibilities at each stage), but as there is no reason for consumers to prefer one mall over another, we stick to the symmetric version of the rule.

Note that Proposition 1 implies a descending, pyramidal search behaviour: first, the largest marketplaces are sampled, then, only when the top cohort is exhausted, consumers move a layer down to the next cohort and so on. The incentives to search larger malls first come from two sources: there are more options, so the expected quality of match is higher, and (correctly) expected prices are lower in larger malls. Another interesting feature of the optimal search behaviour is that there is a possibility of recall: consumers might go back to previously visited store before they sample all of the stores. If  $a_k - p_k > u_k - p_k > a_{k+s} - p_{k+s}$ , the consumer leaves the store in cohort  $k$  and if she was unlucky in sampling further  $s$  cohorts, she will come back to the store which gives her  $u_k$ . Now we are ready to move to the analysis of firms' behaviour.

### 3.2 Demand and Market Equilibrium

We start this section with derivation of the demand functions of the firms. We consider a firm in cohort  $k$  which charges a price  $\hat{p}_k$ , while any other firm in cohort  $j \leq K$  charges price  $p_j$ , and derive its demand. There are two sources of demand: fresh demand, i.e. consumers who visit the firm for the first time and decide to stop there, and returning demand, i.e. consumers who previously visited the firm, found the deal to be not attractive enough to stop, but decided to return later on. For purely expositional purposes, in our derivation of demand we make the following adjustment. For any firm  $j$  and any consumer  $i$ , there is a chance that firm  $j$  is located in the mall which is sampled by consumer  $i$  last among  $M_k$  malls in cohort  $k$ . If the net utility the consumer gets is sufficiently high for stopping even if this marketplace was not the last in the cohort ( $u_{ij} - \hat{p}_k \geq z_k$ ), we classify consumer  $i$ 's demand as fresh. If the deal was less attractive, but still sufficiently good for not moving to cohort  $k + 1$  ( $z_k > u_{ij} - \hat{p}_k \geq z_{k+1}$ ), we write such demand as returning (from the same cohort). In our derivations, whenever it is innocuous, we make an assumption that price deviations are relatively small, and thus the probabilities of events under consideration are strictly between zero and one. This allows us to reduce our notation considerably and does not have any material impact on the results.

Assuming that all other firms stick to the equilibrium pricing strategies, such that  $p_k < p_l$  for  $k < l$ , the consumer buys from a firm in cohort  $k$  which charges  $\hat{p}_k$  if the following conditions are met.

1. The consumer decides to search into cohort  $k$ , thus all  $\sum_{j=1}^{k-1} N_j M_j$  firms provide a match which is worse than the reservation value for cohort  $k$ , i.e.  $u - p_j < z_k = a_k - p_k^e$ ,  $j \leq k - 1$ . This happens with probability

$$\prod_{j \leq k-1} (z_k + p_j)^{N_j M_j}.$$

(which is set to 1 for  $k = 1$ ).

2. The firm we consider is reached during the search process in cohort  $k$ . If the firm is in marketplace  $j \leq M_k$ , it is reached if all other firms provide utility  $u - p_k < z_k$ . So, the

probability that a firm at a random marketplace in cohort  $k$  is reached equals to

$$\frac{1}{M_k} \sum_{j=1}^{M_k} (z_k + p_k)^{(j-1)N_k}.$$

Note that as the consumer randomizes between malls of the same size, the firm under consideration will be in  $j$ 'th ( $j = 1, \dots, M_k$ ) mall in her search order with probability  $1/M_k$ .

3. Finally, the consumer wants to buy from the firm under consideration, provided that she reached its marketplace. This implies that the consumer does not want to continue her search to further marketplaces ( $u - \hat{p}_k \geq z_k$ ) and all other firms at the same marketplace (remember, that the search is free within the marketplace) provide lower utility:  $u - \hat{p}_k \geq \max_{i \leq N_k-1} u_i - p_k$ , which gives the following expression for the probability:

$$\int_{z_k + \hat{p}_k}^1 (\min\{u - \hat{p}_k + p_k, 1\})^{N_k-1} du$$

Thus, the fresh demand can be written as

$$f_k(\hat{p}_k) \equiv \left[ \prod_{j \leq k-1} (z_k + p_j)^{N_j M_j} \right] \left[ \frac{1}{M_k} \sum_{j=1}^{M_k} (z_k + p_k)^{(j-1)N_k} \right] \cdot \int_{z_k + \hat{p}_k}^1 (\min\{u - \hat{p}_k + p_k, 1\})^{N_k-1} du \quad (3)$$

We denote

$$h_k \equiv \left[ \prod_{j \leq k-1} (z_k + p_j)^{N_j M_j} \right] \left[ \frac{1}{M_k} \sum_{j=1}^{M_k} (z_k + p_k)^{(j-1)N_k} \right]$$

Now consider a consumer, who sampled the firm charging  $\hat{p}_k$  at cohort  $k$  and decided to return back and buy after sampling cohort  $i$ . As this consumer reached cohort  $i$ , it must be the case that  $u - \hat{p}_k < z_i$ . As the consumer decided not to move to cohort  $i + 1$  and go back to the firm under consideration, it must be the case that  $u - \hat{p}_k > z_{i+1}$ . Moreover, the consumer returns to the firm which provides the best utility. Thus, the returning demand is

$$\sum_{i=k}^K \int_{z_{i+1}+\hat{p}_k}^{z_i+\hat{p}_k} \left[ \prod_{j \leq i, j \neq k} (u - \hat{p}_k + p_j)^{N_j M_j} \right] (u - \hat{p}_k + p_k)^{N_k M_k - 1} du$$

where we set  $a_{K+1} = p_{K+1} = 0$ , so the outside option gives the utility of zero. The expression for the returning demand can be rewritten as

$$r_k \equiv \sum_{i=k}^K \int_{z_{i+1}}^{z_i} \left[ \prod_{j \leq i, j \neq k} (u + p_j)^{N_j M_j} \right] (u + p_k)^{N_k M_k - 1} du \quad (4)$$

The returning demand does not depend on the firm's own price – this is a well known feature of the Wolinsky (1986) model with the uniform distribution of match values.

Total demand of a firm in cohort  $k$  can be rewritten as

$$D(\hat{p}_k) = r_k + h_k \int_{z_k+\hat{p}_k}^1 (\min\{u - \hat{p}_k + p_k, 1\})^{N_k - 1} du \quad (5)$$

The profit of a firm in cohort  $k$  can be written down as

$$\pi_k(\hat{p}_k) = \hat{p}_k D(\hat{p}_k) \quad (6)$$

Now we are ready to derive the market equilibrium of the model. In equilibrium, the profit-maximizing price  $\hat{p}_k$  must be equal to  $p_k$ , the equilibrium price of a firm located at a marketplace in cohort  $k$ .

**Theorem 2.** *There is a market equilibrium, such that each firm in cohort  $k$  sets a price implicitly defined by*

$$p_k = \frac{1 - a_k^{N_k}}{N_k} + \frac{r_k}{h_k} \quad (7)$$

with  $p_k \in \left[ \frac{1 - a_k^{N_k}}{N_k}, \frac{1 - a_k^{N_k}}{N_k} \frac{1}{1 - a_k} \right] \subset [0, \frac{1}{2}a_k]$ . In this equilibrium  $p_i < p_j$  for all  $i < j$  and consumers follow the descending optimal stopping strategy defined in Proposition 1.

This result shows that despite having larger demand, firms in larger marketplaces set lower prices. One reason is that demand in earlier malls is more elastic, as consumers have more options to be explored in the future, while in the latter stages it is apparent that

previous searches were not successful and there are fewer options left. Moreover, prices in larger marketplaces are driven down by stronger competition within the mall. This price monotonicity, together with the monotonicity of reservation utilities  $a_k$ , creates a very strong incentive for consumers to search in the descending marketplace size order. This result is similar to the one obtained in Zhou (2011). However, there is an important difference in our results. In his paper, *any* search order can be a part of equilibrium, as firms which are searched earlier always set lower prices. This is due to the fact that in his model the price monotonicity is driven only by the demand elasticity effect, but the competitive force is not present. This gives  $N!$  possible equilibria in pure search strategies and even more, if consumers decide to randomize over a certain subset of firms. In our case, the search order is determined not only by prices, but also by the number of options, which can be sampled at the marketplace. Furthermore, a larger marketplace could have lower prices even if it was not the first in the search order, purely because of stronger competition within a such marketplace. Thus, although for particular market configurations (typically with few marketplaces, which are of approximately equal size) one can construct an equilibrium where the search order is not descending in marketplace size, our equilibrium is the robust one. This means that for any given market configuration there is an equilibrium defined in Theorem 2, while equilibria with a non-descending order fail to exist in some market configurations, typically with large difference in the size of marketplaces. This intuition is formally captured by the following Proposition.

**Proposition 3.** *For any  $s > 0$  there exists a market configuration such that there is no equilibria in which smaller marketplaces are sampled before the larger ones.*

As only one equilibrium is robust to the market configuration, in further analysis we are going to concentrate on the properties of this equilibrium.

We know from Theorem 2 that prices are lower in larger marketplaces, but such marketplaces attract more consumers. Thus, the firms' profit ranking of the marketplaces is not immediately clear. The following Proposition claims, that the positive effect due to higher demand is stronger than the negative effect due to stronger competition.

**Proposition 4.** *In descending search order equilibrium firms in larger marketplaces attain higher profits, i.e.  $\pi_i > \pi_j$  for  $1 \leq i < j \leq K$ .*

Thus, any firm from a smaller marketplace would rather swap its place with a firm at the top size marketplace, meaning that firms prefer to be at the place with the toughest price competition. This somewhat counter-intuitive result is purely a consequence of the directed search protocol: gains from higher demand outweigh losses from stronger competition and larger demand elasticity.

Notably, as the search cost approaches zero, prices in our model and prices in Wolinsky (1986) model with  $N$  firms approach the same (positive) value. However, for  $s > 0$  the limiting properties deserve some extra discussion. From Theorem 2 it follows that prices approach zero as  $N_k$  approaches infinity:

$$\lim_{N_k \rightarrow \infty} p_k = 0$$

This result follows directly from the fact that the upper bound of  $p_k$  approaches zero as  $N_k$  approaches infinity. Thus, unlike in Wolinsky (1986) firms do not retain the market power as the number of firms in the marketplace increases without bound. The reason is that in our case the search cost within the marketplace is zero, which leads to exhaustive competition. However, if the number of firms in the market increases not via increasing the size of marketplaces, but rather via increasing the number of them, each price is still bounded away from zero. Therefore, the competitiveness of the market cannot be judged by merely looking at the number of firms, the market structure itself plays a crucial role here.

From the consumers' point of view, the optimal market configuration is the one when all firms are located in a single grand marketplace. This allows for savings to be made on the search costs, improves the expected quality of match and leads to lower prices, thereby increasing consumers' participation. The same applies to total welfare.

## 4 Industry Structure

Now we look at incentives of the firms to join a mall. The answer to this question depends on assumptions about the market for retail space. We start our analysis by assuming that firms can choose their location freely. In order to obtain clear-cut results, we are going to concentrate on a specific industry structure: we look at a single mall with a fringe of stand alone stores.

**Proposition 5.** *Suppose that the industry consists of one shopping mall and a finite number of stand alone stores, and there are  $L \geq 1$  potential entrants, which simultaneously decide whether to join a mall or enter as a stand alone firm. Then, for  $s > 0$ , all  $L$  firms prefer to join the shopping mall.*

Our result obtained in Proposition 4 suggests that being in the mall is more attractive. However, this result is valid only for a *fixed* industry structure. The entry affects profits of the stand alone stores, as well as firms located in the mall. Although for any entry location, firms in the mall earn higher profits, there is no guarantee that profits in the mall after a firm enters there would be higher than profits of stand alone stores after a firm enters as one of them. Proposition 5 establishes this ranking.

Our result is similar to Wolinsky (1983) for sufficiently small travel distance, or Dudey (1990), who, unlike us, considered Cournot competition. In contrast, Fischer and Harrington (1996) obtained equilibrium with just some proportion of firms willing to locate in the cluster, which is a consequence of their assumption that there are always some consumers who prefer to sample stand alone stores first. Moreover, they found that larger variety (relative to search friction) gives more incentives for firms to cluster. In our model, it is just the opposite. It can be easily verified that profits of stand alone and mall firms coincide at  $s = 0$  and profits of firms in the mall are increasing in  $s$ , while profits of stand alone firms decrease with  $s$ . As smaller  $s$  in our model corresponds to larger variety in Fischer and Harrington (1996), our results are exactly the opposite. In our model, larger search costs (or, equivalently, smaller variety) prevent consumers from searching beyond the largest marketplace.



Note that Proposition 5 implies that under free entry the equilibrium market configuration maximizes total welfare, both due to better match and lower number of searches, thereby saving on the cost friction.

Now we turn our attention to a situation when a marketplace can charge a price for its floor space. Again, we assume that there is a single large shopping mall and a fringe of stand alone firms. Suppose, the mall can decide on the number of firms it accepts and the price it charges per trading place. Denote the number of firms in the mall by  $n$  and the number of stand alone firms by  $m$ . Denote  $\pi^M(n, m)$  and  $\pi^{SA}(n, m)$  a profit of a firm which operates in the mall or as stand alone, respectively. Suppose, the mall approaches  $n$  out of  $n + m$  firms and offers them a place at price  $p$ . There is an equilibrium in which  $n$  firms join the mall as long as

$$\pi^M(n, m) - \pi^{SA}(n - 1, m + 1) \geq p.$$

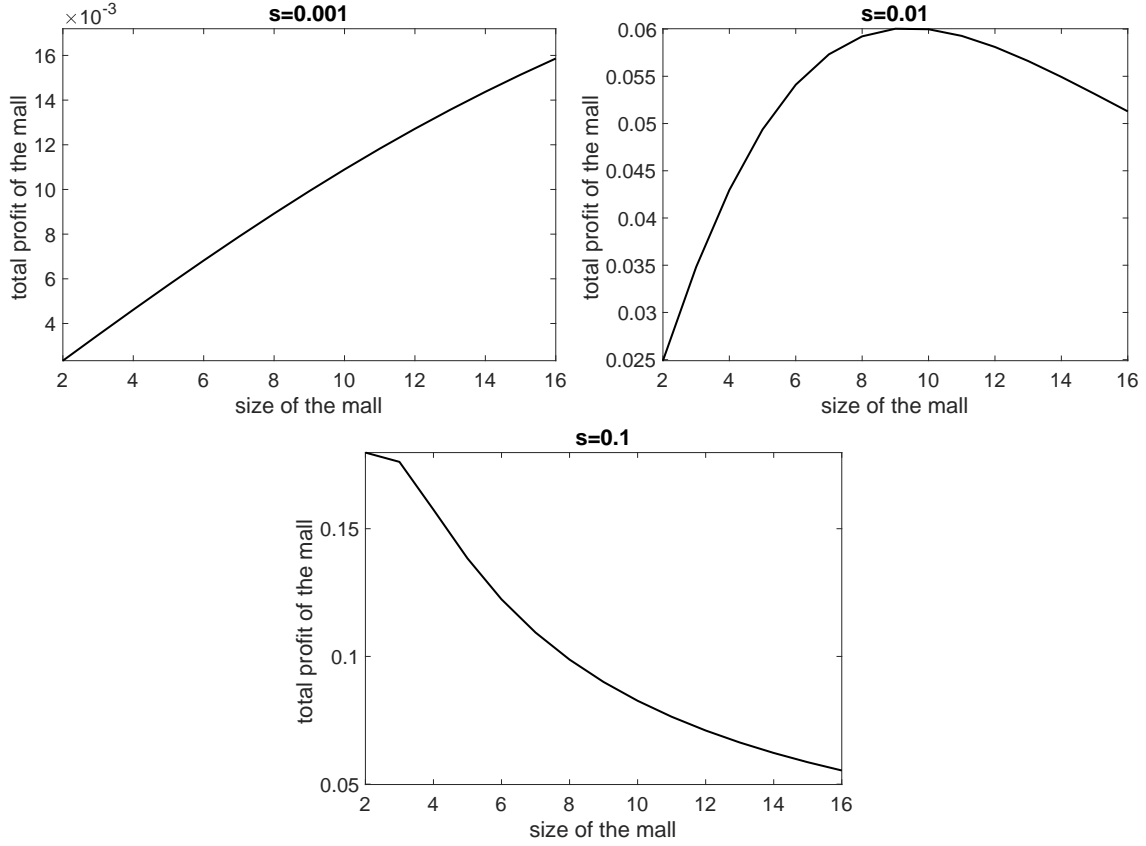
Therefore, the mall can collect a total profit of

$$\Pi^M = n [\pi^M(n, m) - \pi^{SA}(n - 1, m + 1)]$$

It turns out that the mall profit maximization problem is much harder than the one of a single firm. Partially this is due to the discrete nature of the strategic variable, namely the number of firms. Thus, we analyze the mall's problem numerically.

Figure 1 illustrates how the profit of the mall depends on the number of firms in the mall for various values of search cost  $s$ . It turns out that when the search frictions are small ( $s = 0.001$ ) the profit is increasing in the number of firms, therefore, the large marketplace tends to absorb the whole market. This happens because for small search costs many consumers search beyond the mall, which implies that the difference between the profits of firms inside and outside the mall are not that high, and moreover both profits are relatively low. In this case, adding an extra firm to the mall does not shift retail prices too much, therefore, the price charged for a trading spot is not sensitive to the number of firms in the mall. This situation is typical for online markets, when many sellers trade in the same marketplace, for example, eBay. If search costs are high ( $s = 0.1$ ) shopping malls tend to stay small.

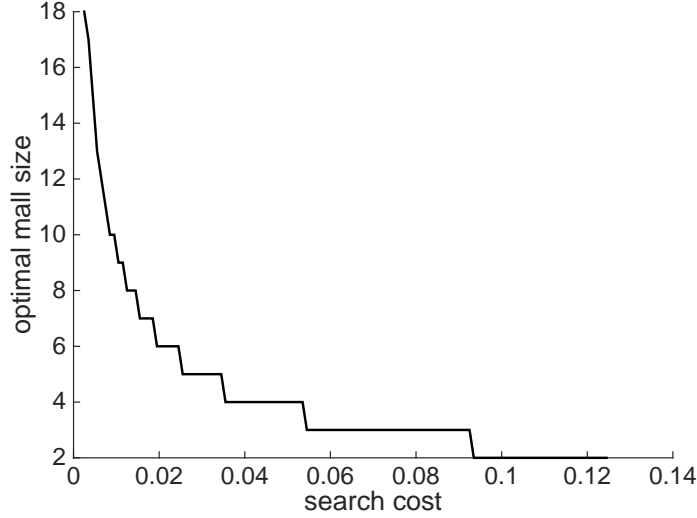
Figure 1: Mall profit as a function of its size, total number of firms equals to 20.



The reason is that high search costs protect firms inside the mall from outside competition. Thus, the retail prices in the mall are very sensitive to the number of firms, which means that the retail space price is very elastic. In this case, the mall just wants to keep its first position in the search order, which is achieved by  $n = 2$ . This situation is typical for markets with brick-and-mortar stores and high search or transportation costs. For moderate values of search costs, for example  $s = 0.01$  in Figure 1, malls tend to be of an intermediate size. Figure 2 summarizes the aforementioned discussion by depicting the optimal number of the firms in the mall depending on search costs.

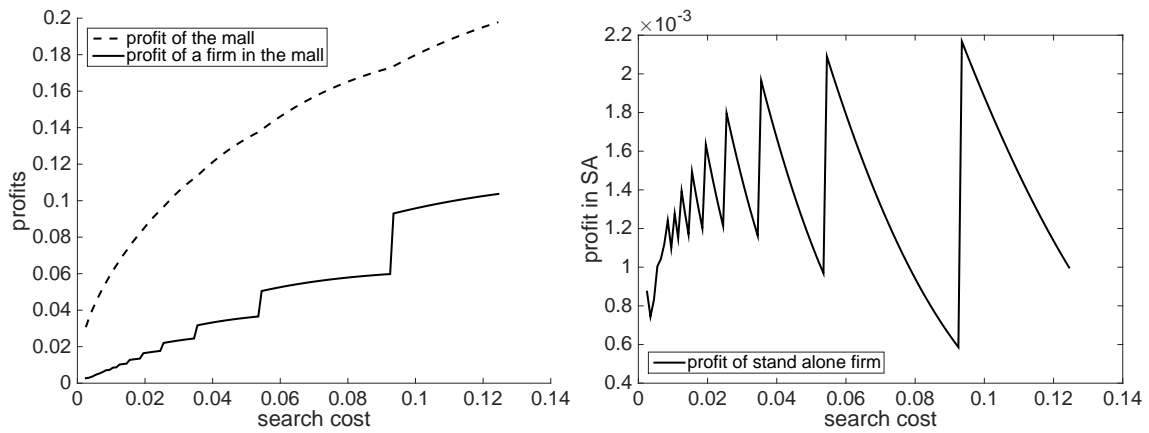
Note that in case of endogenous mall pricing, the negative effect of search frictions on social welfare has a new impact channel. In standard setting, higher search costs lead to higher prices and reduce the quality of matching, but in our case this is accompanied by

Figure 2: Optimal mall size as a function of search cost, total number of firms equals to 20.



the decrease of the mall size, thereby the shrinkage of the competitive part of the market. Profits exhibit a non-monotone behaviour. As  $s$  increases, the profit of a firm in the mall increases, while the profit of a stand alone firm goes down. At some point, the mall finds it optimal to reduce its size, so there is less competition, which means that profits of both types of the firms go up (except for the firm which leaves the mall). As  $s$  goes further, this process, as shown on Figure 3, repeats itself.

Figure 3: Mall profit, profit of a firm in the mall and stand alone firm depending on  $s$ ,  $N = 20$ .



## 5 Discussion and Conclusions

In this section we are going to discuss the role of our assumptions and economic implications of our analysis.

In our model we assumed that the match values are distributed uniformly. As illustrated in Moraga-González and Petrikaitė (2013), this assumption is innocuous. The main purpose it serves is that the returning demand does not depend on the firm’s price, which considerably simplifies derivations, but does not have any material impact either on search or on pricing strategies.

The second important assumption is that there is no search friction when consumers search within the mall. Some modelling settings necessarily require positive search friction at the shopping malls or platforms either due to the assumption of homogenous products (Non (2010)) or continuum of firms (Wang and Wright (2016)). In our case, however, this assumption is not crucial and our results obtained for zero search cost would be qualitatively similar if there was a positive cost of sampling a firm within each marketplace, provided that it is (i) less than the search cost between the marketplaces and (ii) either constant or decreasing in size of the marketplace. As the search cost between the malls is higher than within each mall, it is guaranteed that consumers do not come back or move to the next marketplace unless they sample all the firms in the mall. Thus, the model does not differ from ours in any significant way. The only distinction is that when the search cost within the mall is positive, the price set by the firms does not approach zero as the number of firms at this marketplace approaches infinity. This is a well known result from Wolinsky (1986).

The third assumption we made is perfect recall. This assumption ensures that the reservation price is constant for any cohort. As Janssen and Parakhonyak (2014) show, the presence of returning costs makes search strategies time and path dependent, and the equilibrium price in Wolinsky (1986) model non-monotonically depends on the returning costs. As returning costs create extra monopoly power, the prominence is more important in a model with returning costs than in a model with free re-visits. Moreover, the price monotonicity in marketplace size is reinforced: malls coming later in search order both understand

that previous matches were relatively more unattractive than what would be without returning friction and also realize that consumers are less likely to go back due to the presence of returning cost, which implies a less elastic demand.

The forth assumption is single-homing, i.e. the setting in which each firm can sell at one marketplace only. Note that if a firm is located in marketplace  $i$  it does not gain anything from being present in marketplace  $j > i$ , as its match value was discovered by all the customers who reach step  $j$  already back on step  $i$ . Moreover, stronger competition leads to lower prices in cohort  $i$ , so customers would never buy at store in cohort  $j$ . Thus, the only gain a firm can get from multi-homing is when it locates multiple stores in cohort of the largest mall size. In this case, the probability that the consumer hits a store of such a firm increases.

We also concentrated on equilibrium in which consumers search randomly within each cohort of marketplaces. This allows us to bring together directed and random search in the same framework. If one assumes that malls in each cohort are sampled in some prescribed order, the results would be very similar to those in Zhou (2011): firms in marketplaces sampled first set lower prices and earn higher profits. As Proposition 3 suggests, this specific order can be safely introduced only within one cohort but might not be extended for different mall sizes, as in this case the optimality of this order might be violated.

Finally, we considered the platform pricing problem of a very special form. As long as marketplaces charge firms a fixed fee (whatever way it is determined), our analysis from Section 4 holds true. This is a typical situation for brick-and-mortar shopping malls. Online platforms usually charge either per click (consumer visit) or per transaction. As it follows from Wang and Wright (2016), per transaction fees effectively shift the prices of firms on the platform upwards, as they increase the marginal cost. Thus, as long as all marketplaces charge the same fee, our analysis holds. However, if larger marketplaces set a higher per-transaction price, the monotonicity of prices is not guaranteed and, thus, the optimality of the search order requires an extra check. However, as any marketplace has strong incentives to be as high as possible in the search order and larger marketplaces have an advantage

in achieving this due to better expected match, one can expect that the descending search order would still be optimal.

Relying on the aforementioned assumptions, our paper provides a surprisingly tractable framework for analysis of general market structures while taking into account search frictions. We concentrate on the unique “robust” equilibrium, i.e. an equilibrium which exists for all industry structures. The search order in this equilibrium is determined by three forces working in the same direction: lower prices due to demand elasticity, lower prices due to a larger number of competitors and larger variety. Our entry results accommodate both central (see Dudey (1990)) and core-periphery (see Fischer and Harrington (1996)) structures, which naturally arise from the same framework for various parameter values. Our result that the mall size is decreasing with search friction explains arguably larger concentration in on-line markets than in its off-line counterparts. The framework developed in this paper can be used for the analysis of various questions, like platform pricing, sequential entry, development of the socially optimal search technology, etc.

## Appendix: Proofs

**Proof of Theorem 2.** We construct our proof in several steps. In step one, we derive the profit-maximizing prices given the behaviour of the consumers and obtain an expression similar to (7), while not yet imposing that the equilibrium beliefs about prices are correct. In step two, we show that this expression implies the monotonicity of prices, which means that firms’ behaviour is aligned with consumers’ beliefs and from this point onwards we can impose  $p_k^e = p_k$  and obtain (7). In step three we show that if prices satisfy (7) they must belong to a compact convex set. In step four we guarantee the existence of equilibrium by applying the Brouwer fixed point theorem.

**Step 1: optimal price** By differentiation of (6) with respect to  $\hat{p}_k$  we obtain

$$\frac{\partial \pi_k}{\partial \hat{p}_k} = D_k(\hat{p}_k) - \hat{p}_k h_k \left[ (\min\{z_k + p_k, 1\})^{N_k-1} + \int_{z_k + \hat{p}_k}^1 (N_k - 1) (\min\{u - \hat{p}_k + p_k, 1\})^{N_k-2} \mathbb{I}_{u - \hat{p}_k + p_k < 1} du \right] = 0, \quad (8)$$

where  $\mathbb{I}_{u - \hat{p}_k + p_k < 1}$  is an indicator function, which takes value of 1 if  $u - \hat{p}_k + p_k < 1$  and zero otherwise. In equilibrium,  $\hat{p}_k = p_k$ , and thus

$$D_k(p_k) - p_k h_k \left( (\min\{z_k + p_k, 1\})^{N_k-1} + 1 - (\min\{z_k + p_k, 1\})^{N_k-1} \right) = r_k + h_k \left( \frac{1 - (z_k + p_k)^{N_k}}{N_k} \right) - p_k h_k = 0,$$

which gives

$$p_k = \frac{1 - (z_k + p_k)^{N_k}}{N_k} + \frac{r_k}{h_k}. \quad (9)$$

**Step 2: monotonicity of prices.** Note that for the search order described in Proposition 1 and any price vector  $p$  it must be the case that  $h_{k+1} \leq h_k$ , as  $h_k$  is the probability that the consumer reaches cohort  $k$  in her search. Write the price difference as

$$\begin{aligned} p_{k+1} - p_k &= \frac{1 - (z_{k+1} + p_{k+1})^{N_{k+1}}}{N_{k+1}} - \frac{1 - (z_k + p_k)^{N_k}}{N_k} + \frac{r_{k+1}}{h_{k+1}} - \frac{r_k}{h_k} \\ &> \frac{1 - (z_{k+1} + p_{k+1})^{N_{k+1}}}{N_{k+1}} - \frac{1 - (z_k + p_k)^{N_k}}{N_k} + \frac{r_{k+1} - r_k}{h_k}. \end{aligned} \quad (10)$$

Now, we can rewrite

$$\begin{aligned}
r_{k+1} - r_k &= \sum_{i=k+1}^K \int_{z_{i+1}}^{z_i} \left[ \prod_{j \leq i, j \neq k+1} (u + p_j)^{N_j M_j} \right] (u + p_{k+1})^{N_{k+1} M_{k+1} - 1} du \\
&\quad - \sum_{i=k}^K \int_{z_{i+1}}^{z_i} \left[ \prod_{j \leq i, j \neq k} (u + p_j)^{N_j M_j} \right] (u + p_k)^{N_k M_k - 1} du \\
&= (p_k - p_{k+1}) \underbrace{\sum_{i=k+1}^K \int_{z_{i+1}}^{z_i} \left[ \prod_{j \leq i, j \neq k, k+1} (u + p_j)^{N_j M_j} \right] (u + p_k)^{N_k M_k - 1} (u + p_{k+1})^{N_{k+1} M_{k+1} - 1} du}_A \\
&\quad - \underbrace{\int_{z_{k+1}}^{z_k} \left[ \prod_{j \leq k-1} (u + p_j)^{N_j M_j} \right] (u + p_k)^{N_k M_k - 1} du}_B.
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{B}{h_k} &= - \frac{\int_{z_{k+1}}^{z_k} \left[ \prod_{j \leq k-1} (u + p_j)^{N_j M_j} \right] (u + p_k)^{N_k M_k - 1} du}{\left[ \prod_{j=1}^{k-1} (z_k + p_j)^{N_j M_j} \right] \left[ \frac{1}{M_k} \sum_{j=1}^{M_k} (z_k + p_k)^{(j-1)N_k} \right]} \\
&\geq - \frac{\int_{z_{k+1}}^{z_k} \left[ \prod_{j \leq k-1} (z_k + p_j)^{N_j M_j} \right] (z_k + p_k)^{N_k(M_k-1)} (u + p_k)^{N_k-1} du}{\left[ \prod_{j=1}^{k-1} (z_k + p_j)^{N_j M_j} \right] \left[ \frac{1}{M_k} \sum_{j=1}^{M_k} (z_k + p_k)^{(j-1)N_k} \right]} \\
&= - \frac{(z_k + p_k)^{N_k(M_k-1)}}{\frac{1}{M_k} \sum_{j=1}^{M_k} (z_k + p_k)^{(j-1)N_k}} \int_{z_{k+1}}^{z_k} (u + p_k)^{N_k-1} du \\
&\geq - \int_{z_{k+1}}^{z_k} (u + p_k)^{N_k-1} du = \frac{(z_{k+1} + p_k)^{N_k}}{N_k} - \frac{(z_k + p_k)^{N_k}}{N_k}.
\end{aligned}$$

By plugging this back into (10), we obtain

$$\begin{aligned}
p_{k+1} - p_k &> \frac{1 - (z_{k+1} + p_{k+1})^{N_{k+1}}}{N_{k+1}} - \frac{1 - (z_k + p_k)^{N_k}}{N_k} \\
&\quad - \frac{(z_k + p_k)^{N_k}}{N_k} + \frac{(z_{k+1} + p_k)^{N_k}}{N_k} + (p_k - p_{k+1}) \frac{A}{h_k} \\
&= \frac{1 - (z_{k+1} + p_{k+1})^{N_{k+1}}}{N_{k+1}} - \frac{1 - (z_{k+1} + p_k)^{N_k}}{N_k} + (p_k - p_{k+1}) \frac{A}{h_k}. \quad (11)
\end{aligned}$$



Now, as  $\frac{1-(z_{k+1}+p_{k+1})^{N_{k+1}}}{N_{k+1}}$  is a decreasing function of  $N_{k+1}$ <sup>4</sup>, and  $N_k > N_{k+1}$ , we get that

$$\begin{aligned} p_{k+1} - p_k &> \frac{1 - (z_{k+1} + p_{k+1})^{N_k}}{N_k} - \frac{1 - (z_{k+1} + p_k)^{N_k}}{N_k} + (p_k - p_{k+1}) \frac{A}{h_k} \\ &= \sum_{j=0}^{N_k} \binom{N_k}{j} z_{k+1}^j [p_k^{N_k-j} - p_{k+1}^{N_k-j}] + (p_k - p_{k+1}) \frac{A}{h_k}, \end{aligned}$$

which can hold only if  $p_{k+1} > p_k$ . Thus, prices must be monotone. Finally, by plugging  $p_k^e = p_k$  into (9) we obtain (7).

**Step 3: Price range.** Firstly, since  $r_k(p)/h_k(p) > 0$ , we immediately get that  $p_k \geq \frac{1-a_k^{N_k}}{N_k}$ . Then, we can write

$$\begin{aligned} r_k &= \sum_{i=k}^K \int_{a_{i+1}-p_{i+1}}^{a_i-p_i} \left[ \prod_{j \leq i, j \neq k} (u + p_j)^{N_j M_j} \right] (u + p_k)^{N_k M_k - 1} du \\ &< \sum_{i=k}^K (a_i - p_i - a_{i+1} + p_{i+1}) \left[ \prod_{j \leq i, j \neq k} (a_i - p_i + p_j)^{N_j M_j} \right] (a_i - p_i + p_k)^{N_k M_k - 1} \\ &< \sum_{i=k}^K (a_i - p_i - a_{i+1} + p_{i+1}) \left[ \prod_{j=1}^{k-1} (a_i - p_i + p_j)^{N_j M_j} \right] (a_i - p_i + p_k)^{N_k M_k - 1} \\ &< \sum_{i=k}^K (a_i - p_i - a_{i+1} + p_{i+1}) \left[ \prod_{j=1}^{k-1} (a_k - p_k + p_j)^{N_j M_j} \right] a_k^{N_k M_k - 1} \\ &< (a_k - p_k) \left[ \prod_{j=1}^{k-1} (a_k - p_k + p_j)^{N_j M_j} \right] a_k^{N_k M_k - 1}, \end{aligned}$$

where the first inequality comes from the fact that the integrand is an increasing function, the second inequality is obtained from the fact that all terms are smaller than one (as we construct an equilibrium with  $p_i < p_j$  for  $i < j$ ), the third one follows from the monotonicity of  $p_k - a_k$  and the last one comes from expanding the sum and using  $a_K - p_K = 0$ . By cancelling the product terms we obtain

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<sup>4</sup> Note, that this expression can be written as  $\frac{1-a^N}{N}$ , which is decreasing as long as  $a^N \ln a^N > 1 - a^N$ , which always holds as  $\ln(1/a^N) < 1/a^N - 1$ .

$$\frac{r_k(p)}{h_k(p)} \leq \frac{(a_k - p_k)a_k^{N_k M_k - 1}}{\frac{1}{M_k} \sum_{j=1}^{M_k} a_k^{(j-1)N_k}} = (a_k - p_k) \frac{M_k a_k^{N_k M_k - 1} (1 - a_k^{N_k})}{1 - a_k^{N_k M_k}}.$$

Now, as  $\frac{xa^{x-1}}{1-a^x}$  is a decreasing function of  $x$  for  $a < 1$ , the right hand side of the expression above attains its maximum at  $N_k M_k = 1$ . Thus, we obtain

$$\frac{r_k(p)}{h_k(p)} \leq (a_k - p_k) \frac{1 - a_k^{N_k}}{N_k(1 - a_k)}.$$

Now, we get that

$$p_k = \frac{1 - a_k^{N_k}}{N_k} + \frac{r_k}{h_k} \leq \frac{1 - a_k^{N_k}}{N_k} \frac{1}{1 - a_k}.$$

Next note that

$$(a_k - p_k) \frac{1 - a_k^{N_k}}{N_k(1 - a_k)} \leq a_k - p_k.$$

This inequality can be obtained from the monotonicity of  $\frac{1 - a_k^{N_k}}{N_k}$  proven in footnote 4 and plugging  $N_k = 1$ . Therefore,  $p_k \leq \frac{1}{2}a_k \leq \frac{1}{2}$ .

**Step 4: the fixed point.** Using the result from the previous step, we know that prices are bounded by  $\left[\frac{1 - a_k^{N_k}}{N_k}, \frac{1}{2}a_k\right] \subset [0, \frac{1}{2}]$ . Thus, the system of equations (7) defines a continuous (as  $r_k$  and  $h_k$  are continuous and  $h_k > 0$ ) bounded function mapping a compact convex set  $[0, \frac{1}{2}]^K$  onto itself, which, by Brouwer's theorem, has a fixed point.

**Proof of Proposition 3.** Consider a market configuration with one large mall of size  $N$ , and a stand alone store. Suppose that there is an equilibrium in this market such that consumers start their search from the stand alone store. Recall that in this case we define the reservation values as

$$\int_{a_1}^1 (u - a_1) du = s, \quad \int_{a_2}^1 N(u - a_2) u^{N-1} du = s.$$

Note, that as the first order conditions in this case are still determined by (7), from Theorem 2 it follows that  $p_1 > 1 - a_1 > 0$  (as  $s > 0$ ) and  $p_2 \leq \frac{1 - a_2^N}{N} \frac{1}{1 - a_2}$ , which implies  $p_2$  approaches zero as  $N$  approaches infinity. Moreover,  $a_2 > a_1$  for any  $N > 1$ . Thus, for  $N$  large enough expected surplus from sampling the mall is larger than from sampling a stand alone store:  $a_2 - p_2 > a_1 - p_1$ , and therefore starting a search from the stand alone store is irrational.

**Proof of Proposition 4.** Write down the profit of a firm in cohort  $k$  as  $\pi_k(p_k, p_{-k}) = p_k D_k(p_k, p_{-k})$  where  $p_{-k}$  are equilibrium prices played by other firms, including those in the same cohort. Then, as  $p_k$  is the optimal price, we obtain

$$\pi_k(p_k, p_{-k}) = p_k D_k(p_k, p_{-k}) > p_{k+1} D_k(p_{k+1}, p_{-k}).$$

Now, due to the descending search order, we get  $D_k(p_{k+1}, p_{-k}) > D_{k+1}(p_{k+1}, p_{-k})$ . Hence,

$$\begin{aligned} \pi_k(p_k, p_{-k}) &= p_k D_k(p_k, p_{-k}) > p_{k+1} D_k(p_{k+1}, p_{-k}) > p_{k+1} D_{k+1}(p_{k+1}, p_{-k}) \\ &> p_{k+1} D_{k+1}(p_k, p_{-k}), \end{aligned} \quad (12)$$

where the last inequality follows from  $p_k < p_{k+1}$  and the fact that demand in cohort  $k+1$  is increasing in prices of firms in cohort  $k$ . Finally, one can easily spot that the right hand side of (12) is just a profit of a firm in cohort  $k+1$ , which charges the equilibrium price  $p_{k+1}$ , which completes the proof.

**Proof of Proposition 5.** Suppose that there are  $N$  firms located in the mall and  $M$  stand alone firms. First, note that at  $s = 0$  profits and prices of the firms located in and outside the mall are the same. Also note that by the envelope theorem we have  $\frac{\partial \pi_k}{\partial s} = p_k \frac{\partial D_k}{\partial s}$ . Then, in equilibrium

$$D_2 = (a - p_2 + p_1)^N \frac{1 - a^M}{M} + \int_0^a (u + p_1)^N (u + p_2)^{M-1} du$$

and therefore

$$\frac{\partial D_2}{\partial a} = (a + p_1)^N (a + p_2)^{M-1} + N(a - p_2 + p_1)^{N-1} \frac{1 - a^M}{M} - (a - p_2 + p_1)^N a^{M-1} > 0$$

as  $a + p_1 > a - p_2 + p_1$  and  $a + p_2 > a$ . As  $\text{sign} \frac{\partial D_k}{\partial a} = -\text{sign} \frac{\partial D_k}{\partial s}$ , profits of stand alone firms are decreasing in  $s$ .

Similarly,

$$D_1 = \int_{a-p_2+p_1}^1 u^{N-1} du + \int_{p_1}^{a-p_2+p_1} u^{N-1} (u + p_2 - p_1)^M du.$$

Then,

$$\frac{\partial D_1}{\partial a} = -(a - p_2 + p_1)^{N-1} + (a - p_2 + p_1)^{N-1} a^M < 0$$

and thus  $\frac{\partial D_1}{\partial s} > 0$ , so the profit of a firm in the mall is increasing in  $s$ . Note that these results on monotonicity of profits hold for arbitrary  $N$  and  $M$ . Moreover, firms' profits at  $s = 0$  do not depend on the split of the number of firms between the mall and the fringe. Thus, for any  $N_1 + M_1 = N + M = N_2 + M_2$  we have

$$\pi_1(s; N_1, M_1) > \pi_1(0; N_1, M_1) = \pi_2(0; N_2, M_2) > \pi_2(s; N_2, M_2).$$

If a group of  $L$  firms is entering, set  $N_1 = N_2 + L$  and  $M_1 = M_2 - L$ . This completes the proof.

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