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Critical Noticing and its Consequences

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Abstract
Noticing is a momentary act. Enriching the possibility of noticing something specific in the future marks the beginning of an action labelled "professional development". Thus, what happens as a result of noticing is what matters for the teacher, but what really matters is what happens for the learner. Whether something noticed enables a passing remark, acts an object of reflection, stimulates a shift of attention, provides an action for pre-flection, or instantiates some theoretical construct, the consequence of noticing is much more complex than simply noticing, involving as it does the power of attention, and implicitly all other aspects of the human psyche. As a discipline, noticing encompasses all of these aspects. As a domain of research, noticing is sometimes divided into component parts, with distinctions between different kinds of things worth noticing and distinctions between different ways of behaving as a consequence of noticing. As a subject of research, the key issues about noticing are how it can be promoted, fostered, provoked and even directed, and what it enables. This paper re-integrates various disparate aspects of noticing appearing in the literature, in order to stress the complexity of sensitising oneself to notice, and informing one's future practice through noticing opportunities to act freshly. The reports in this issue are used as an indication of the range of things which people are encouraged to notice (be sensitised to notice) as part of their professional development and practice in mathematics education. This occasions reflection on the complexity of researching, not only what people notice and attend to, but how their noticing might be influenced as part of their professional development, and more importantly, what consequences of such noticing might be anticipated, such as how it might influence future practice.

Keywords Noticing · Discipline of noticing · Attention · Complexity · Sensitisation

Introduction

Let me begin with some observations which can be verified in personal experience and taken as definitions. They are briefly elaborated in the following section.

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1. The act of noticing is a momentary shift of attention, whether in focus or in form.
2. Attention to something noticed continues until overlaid or displaced by a subsequent shift of attention.
3. Noticing can occur with varying degrees of intensity of awareness.

- not even strong enough to remember, recall or recognise;
- only just strong enough to recognise in retrospect what someone else remarks upon what they noticed;
- strong enough to make a remark about it (to someone else or to oneself);
- strong enough to lead to making a record of some sort.

4. Noticing can be critical in several ways:

- judgmental or evaluative, usually negatively, as in "I wouldn't let that teacher in my classroom", or "I don't recognise what the author is claiming" (Jaworski, 1989; Pimm, 1993);
- pointing out potentially positive or negative aspects of what is noticed, which serve as turning points for an individual's awareness;
- significant for an educator who recognises some educational construct being instantiated;
- pertinent for a novice who needs to impress an educator–evaluator;
- significant for a practitioner who recognises a behaviour they would like to change or adopt;
- crucial for a practitioner who recognises they have a moment of choice in which to alter an established habit.

Being a momentary shift of attention, noticing lies at the core of expertise development, and as such, is a core issue for learners of any discipline such as mathematics, for teachers of any discipline, for coaches and leaders of professional development sessions, for educators, and for educators of educators. For example, Amador et al. (this issue) observe that.

> "A coach must carefully attend to and interpret teacher thinking to act responsively within each conversational moment, which parallels a teacher noticing student thinking to act responsively instead of habitually when interacting with students."

Indeed, noticing is not just of interest in education. It plays a role in the development from novice to expert behaviour in any human endeavour.

Elaboration

Noticing is a momentary act, either a brief shift of what is attended to, or a shift in the form of that attention, usually followed by a return to the previous focus. Among the various forms that attention can take, it is useful to discern.

holding wholes (or gazing, a much under-rated form of creative attention: Mason, 2025);
discerning details;
recognising relationships among discerned details;
perceiving properties as being instantiated;

reasoning on the basis of agreed properties, such as for example using properties as warrants to justify choices (Mason, 2003a).

Shifts between these can be observed when, for example, I encounter a noticeboard and I track how my attention shifts as a result of gazing at the board. Various details stand out, attracting attention, then fade as they are displaced. Despite William James' popular metaphor of a "stream of consciousness" (James 1890/1950), close examination suggests that a more accurate description of experience might be as a sequence of shifts of attention, of sharp moments of noticing which fade until displaced (Mason 2002). Experience is certainly recalled in fragments; on close inspection it is found to be fragmentary in nature (Mason, 1988). It appears to be continuous only when intellect–cognition later constructs a narrative linking fragments together (Norretranders, 1998). Close inspection of lived experience casts doubt on the cognition-based narrative that conscious interpretation followed by considered response is responsible much of the time. In most situations the body reacts first, emotions follow a close second, and cognition follows later (Mandler, 1989; Norretranders, 1998). Consequently, asking subjects to expose their interpretation and their choice of response is in danger of inviting fantasy.

Each moment of noticing begins when attention switches, when it is suddenly diverted to or captured by a fresh focus. That particular noticing ends when a fresh focus or noticing overlays and then displaces the current focus. Something noticed has longer-term impact or significance when it reaches an intensity which brings it to the surface of expression so as to be able to be remarked upon, and also when affect is affected, linked to desire or meaningfulness. With even greater intensity, something noticed may become the focus of later re-flection and possible pro-flection as a contribution to research or personal development (Mason 2002). Mindfulness (Williams & Penman, 2011), a translation of the Pali word for "awareness in the moment", is a modern recognition of ancient practices to be found in Zen (Shigematsu, 1981), Sufic (Shah, 1964, 1978), and Yogic (Ravindra, 2009) practices, and associated with noticing (Bennett, 1976), with being awake-to or present-to (*Rg Veda*: Rhadakrishnan 1953; *Bhagavad Gita*: Mascaró, 1962; *In Search of the Miraculous*: Ouspensky, 1950; *To Live Within*: Raymond, 1972), and significant phenomena associated with developing expertise such as the U-curve (McNeil, 2007).

Noticing in itself is fragmentary and frequent. Its importance lies in what it can contribute to providing practitioners with fresh choices of action to enact both in the present and in the future.

This paper is a commentary on and comparison of aspects of professional life, reported on as being noticed, in the papers in this issue. Mostly they arise as the focus of intentional noticing, which raises issues concerning what makes a moment of noticing critical according to these researchers, what actions they associate as a consequence, and how specific noticing may be promoted and provoked. These are placed in the general context of noticing as an action of the human psyche and how noticing can initiate and inform both personal and collective professional enquiry.

To notice something is to become aware, however briefly. Gattegno (1990; see also Young & Messum, 2011) extended the usual sense of the word *awareness* to link it to "that which enables action", thereby integrating both conscious and unconscious states. He observed that, for example, the soma initiates actions such as opening pores in the skin as temperature rises, increasing heart rate with increasing exercise or muscular challenge, and changing which nostril is dominant while breathing. These actions are initiated without my being conscious of them, yet my body is evidently "aware" of the need to act, and it initiates those actions. There are other actions which I only become

aware of as they happen, such as raising my voice in order to be heard, clearing my throat in order to speak, and getting out of bed in the morning, even though I am not usually conscious of the initiation of the action itself, only of the enacting of that action. I notice them happening "to me".

The problem which faces every teacher, educator, or professional development leader, was captured in the title of Norretranders' book: "The User Illusion" (Norretranders, 1998). His findings are in line with accumulated Eastern wisdom and backed up by neuroscientific evidence. He pointed out that conscious cognition is often under the mistaken opinion that it is in charge, when in fact the soma has already prepared a response to a stimulus (Yon et al., 2020), based on habits and experience built into psyche-coordinations in the past. Musculature is already poised to be enacted. This is how habits and established practices operate, usually below the level of consciousness. Occasionally there are actions which are considered and deliberated before being enacted, but these are far rarer than cognition likes to admit (Kahneman, 2012). All of these fall under the heading of awarenesses leading to actions.

Thus, interpreting what is noticed is most often a spontaneous act, and is in fact an integral part of that noticing, through metaphoric resonance with past experience or through metonymically surface-triggering of emotions. This is why noticing and acting upon that noticing involves previously formed coordinations of all aspects of the psyche: witness, emotion (affect), intellect (cognition), will, and behaviour (enaction) as well as attention (Mason & Metz, 2017).

Gattegno spoke about "educating awareness", meaning both internalising one or more actions, and sensitising myself to notice a situation in which that action might be appropriate. He claimed that it is awareness that can be educated, which he cast provocatively as "Only awareness is educable", in order to emphasise the central role of awareness (attention). It contrasts with "Only behaviour is trainable" which resonates with images to be found in Eastern literature such as the Upanishads (Radhakrishnan, 1953 p623). These in turn suggest a third "only": "Only emotion is harnessable" (Mason, 1994a, 1994b, 1994c) which contributes the affective dimension that energises and directs noticing. For example, carrying out a procedure or practice efficiently and effectively includes rehearsing the steps (training behaviour), but it is directed by educated awareness. The energy required to convert noticing into educating awareness and training behaviour comes from felt desire and disposition (harnessed emotion). Simply training behaviour tends to lead to inflexible actions, while simply educating conscious awareness leads to inert knowledge (Whitehead 1932), "knowing about actions" but not necessarily "knowing to act when appropriate" (Mason, 1998).

In education, the teacher initiates actions, whether discipline specific, psychologically or sociologically informed, or pedagogic in intention. The hope is that learners learn by engaging in the activity that results from the activity arising from initiated action. However, "one thing that people do not seem to learn from experience, is that they do not often learn from experience alone: something more is required" (Mason, 1994a). In order to maximise the possibility of really learning, that is educating awareness as well as training behaviour, it helps to develop personal narratives and inner incantations which trigger, accompany, and inform the relevant actions.

Having something pointed out, say by a teacher or educator, involves a momentary shift of attention, but is not often either stable or long-lasting by itself. The more fully the psyche is involved—cognition, affect, enaction, attention, will, and witness—the more likely it is that a momentary awareness will mature or evolve into an awareness that comes to the surface in-the-moment and provides access to a suitable action.

Two Roles for Noticing

There are two distinct roles or situations for working on noticing: noticing a fresh action enacted by someone else that one might want to enact for oneself at some time in the future and noticing a situation arising in-the-moment together with a possible non-habitual action becoming available to be enacted (constituting an educated awareness). Neither of these is straightforwardly simple.

When attending a seminar or workshop, it is usual to be fully immersed in the tasks and consequent activity, especially when mathematics is involved. Watching a video of a lesson, attention is most readily attracted to the (mathematical) content, especially if there are unfamiliar aspects. Consequently, it is not so obvious that participants will pay attention to pedagogical actions initiated by the leader, nor even to the presence of mathematical themes or the mathematical use of natural powers. Again, simply pointing these out, whether in midstream or during reflection, is not in itself usually sufficient to sensitise participants sufficiently so that they notice subsequent occurrences.

Solar et al. (this issue) make the same point: "… just because the teacher recognizes the Mathematical Point in the student contribution it does not mean that the students recognize it", and even when they do, it may not be with the intensity required to inform future practice. Suddenly noticing something as a teacher, it is tempting to ask a question intended to lead students to the same noticing, but this very often degenerates into "guess what's in teacher's mind". More subtle pedagogical actions are required.

The notion of *scaffolding* and *fading* (Seeley-Brown et al., 1989) provides one extended pedagogical action to assist this process. An alternative formulation of the same idea is provided by the trio of action types *directed–prompted–spontaneous* (Love & Mason, 1992), which suggests that over time, initially direct question-prompts (such as What do you know? or Say What You See, or Have you got an example?) are gradually softened so as to become less and less direct prompts to action. They turn into more general, less focused, indirect prompts (such as What did I suggest last time? or What worked last time?) until learners are able to initiate the action for themselves spontaneously (they have educated their awareness). Vygotsky (1978; van den Veer & Valsiner, 1991) would surely have described this as working within the Zone of Proximal Development.

Even when an action has been thoroughly rehearsed in several contexts, it does not always come to the surface immediately, which is why gazing is such an important form of attention. It provides space and time for things to come to the surface, whether mathematical or pedagogic in nature. Teaching a lesson or leading a professional development session gives little time for gazing, although initiating the pedagogic action of inviting participants to talk in pairs is one way of releasing a little time to allow a follow-up action to become available.

When planning a lesson, it is common experience to intend to enact some particular pedagogic action, only for that action not to surface during the session itself. Despite planning intentions, remembering to enact it, noticing an opportunity to enact it, and actually enacting it, all take effort. What tends to happen is to notice in retrospect that an opportunity has been missed. Considerable effort may be required over time to move retrospective noticing into prospective noticing, that is, noticing an opportunity just before or just as it becomes relevant. The Discipline of Noticing (Mason 2002) articulates methods for doing this, based on practices articulated in various near and far eastern sources.

The papers in this issue are mainly concerned with studying either what other people notice, or whether they notice what the educator–researcher intends them to notice, and there are major issues with this endeavour.

Educators' Issues

As an educator, leading in-service or pre-service development sessions, it is tempting to want participants to notice what I am sensitised to notice. But long experience, not to say relevant literature is clear that deliberately trying to change others is at the very least, difficult, and perhaps ethically questionable in any case. This is a fundamental paradox of teaching, because as a teacher I expect learners to grow, develop, and mature as disciplined thinkers, yet I have discovered that I cannot do the learning for my students. What I *can* do as a teacher–educator is offer experiences through which participants may come to have doubts about their own practice and at the same time, recognise possible alternative actions to enact in the future. However, it is entirely up to the individual to make use of what they notice, energised by the intensity of their own desire, but independent of the educator's intensity of desire. For example, among her reflections, on teaching mathematics, Mary Boole (Tahta, 1972) recognised "teacher lust" as an endemic tension: the desire to explain.

Of major concern to teacher educators and leaders of professional development, as well as to groups of collaborating teachers, is *what* teachers are currently sensitised to notice, and this constitutes the primary focus of much of current noticing-oriented research. In the next section I turn to the question of *what* is noticed, and what might be worth noticing. I then consider how teachers might be provoked or sensitised to notice other aspects of professional practice, and how that noticing might influence current and future actions. Then I reiterate the case that effective professional development necessarily goes beyond simply noticing.

What is Noticed?

There is a vast range of "things worth noticing" in any domain of expertise: from survival of classroom management issues, to caring for learners and caring for mathematics (Watson, 2021), from their own practices to the practices of their learners.

What someone expresses about what they notice is at best only a partial indication of what they are currently sensitised to notice, which in turn is influenced by social and psychological aspects of the situation as manifested in their personal "selves" or psyche-coordinations (Mason & Metz, 2017), and by cultural aspects manifested in the material resources available. For example, an unfamiliar cultural context may sensitise me at first to physical aspects of the situation, while in a familiar cultural context, other concerns such as for equity may sensitise me to social aspects of teacher–student and student–student interaction or to gender disparities. A focus on caring for students may sensitise me to psychological aspects of teacher–student interactions involving mutual trust and respect, and a focus on caring for the discipline of mathematics may sensitise me to the care taken to bring student attention to mathematical relationships.

The anthropological adage "absence of evidence is not evidence of absence" is highly pertinent. Just because no remark is made about something, it does not follow that it was not noticed, only that it did not reach a level of remarkableness in the presence of other things noticed. Simply reporting that someone did or did not (appear) to notice something that the speaker/writer/observer notices, is of little significance in

itself. Furthermore, as with most educational research, the more precise the descriptions which constitute data, the more that is revealed about the researcher's sensitivities, propensities, and proclivities (Mason 2002).

The question of what is noticed is valid for teachers asking about the learners with whom they are working (on-line, in person, asynchronously), although the papers in this issue in the main ask the question about teachers in pre-service and in-service education. For some it is simply "what catches learners' attention"; for others it is "in what way might behaviour be influenced as a consequence"; for yet others it is more to do with whether what is noticed is most relevant to influencing learners' experience, or to contributing to the author's research endeavours. In each case we learn something of what the researcher(s) deem to be of significance and hence by implication, critical.

Situational aspects such as posture, gesture, and tone during talk, specifically about number-lines: Jazby, Ochoa, Chan & Drie (this issue);
Aspects of argumentation: focus on structural or dialogic details that the action intends to address, and on cognitive, social, or affective specific details of argumentation as discernible in transcripts of teacher–student interaction, leading to discerning three types of teacher actions: directly contributing argument components, asking questions that elicit parts of arguments, and using other supportive actions: Ayalon & Nama (this issue)
Aspects of argumentation: teachers beginning to notice that an observed teacher chose not to validate student remarks, and providing possible reasons for this pedagogic choice: Solar (this issue).
Students' mathematical thinking as observed in video, using software to draw attention to posture, and the importance of explicit guidance to enrich this experience: Larison, Sherin & Richards (this issue);
Video used to elicit teachers' noticing for equitable mathematics instruction so as to inform the design of professional development intended to provoke teachers expanding their classroom noticing practices: van Es, Munzer, Barnhart & Alvaraez (this issue);
Principles of teaching, learning, coaching, and other classroom aspects: Amador, Gillespie, Kruger, Hanan, Choppin & Ritter (this issue);
Incidents that can be exploited mathematically as providing 'teaching moments': Stockero, Zoest, Leatham & Peterson (2015); Ayalon *et al.*. (this issue);
How sensitivity to notice can develop and change over time: Rotem (this issue);
How sensitivity to notice varies between people: Amador *et al.* (this issue);
What sparks off noticing (ignites, focuses, and centres the noticing of a coach): Amador *et al.* (this issue);

while.

Choy (this issue) focuses on ways in which explicit discussion of incidents and of what was attended to both before and after these incidents can influence future behaviour;
Coles and Helliwell (this issue) critique the rational-analytic approach to noticing as a three-stage process of noticing, interpreting, and acting, by taking an enactivist stance which identifies knowledge with action; one consequence is that noticing, interpreting and acting are all of a piece. They highlight particularly the role of the affective domain.

In most cases, examples are provided in the form of extracts from transcripts or descriptions of instances from classrooms. But there are many different things to notice, indeed many different types of things to notice. Other possibilities include.

Specific discipline related actions such as imagining and expressing, specialising and generalising, conjecturing and convincing: (Mason, Burton & Stacey 1982/2010); Gardiner, 1987; Cuoco et al., 1996)
Specific pedagogic actions such as getting learners to talk in pairs before contributing to whole class discussion; stimulating the construction of personal narratives and own-explanations (Chi, Bassok, Lewis, Reiman & Glaser 1989); promoting gazing before acting; specialising in order to recognise patterns of relationships; …
Recognition of connections with other topics;
Attempts to use different media (words, diagrams, images, …) when expressing what is noticed;
The use of particular technical language (including the difference between gargling and meaning Mason 2006 p59); adopting classroom rubric including a conjecturing atmosphere Mason et al., 1982/2010);
Showing respect for someone struggling to express their thinking;
Use of labels with attention to what is labelled and how those labels are used (Ingram, Mason & Planas in press);

Solar et al. (this issue) use a variant of the observe–interpret–respond triad, preferring Perceive-Interpret-Decide so that they include not just cognitive aspects of argumentation, but also student support, student cognitive activation, and classroom management. There are so many aspects that could be noticed, could be developed as sensitivities to notice, that the experience could be overwhelming, to the extent of posing an obstacle to action.

The list is almost endless, because what the individual is sensitised to notice can develop and change over time, as was the case for the teacher in Rotem's report, and varies between teachers as was the case in Amador's report. Furthermore, as Coles et al. observe, what is noticed and the actions it triggers are "all of a piece", or as Stockero et al. put it, "iterative, dynamic, and relational". Interpretation is almost always integral to the noticing. It is influenced by personal judgements and evaluations of actions enacted and of consequent learner activity, as well as personal ideals on the one hand, and institutional requirements on the other. As a caring profession, teaching is necessarily a balancing act between care that the subject discipline is respected and furthered, and care that the learners have positive and effective experiences. That is why the most appropriate criteria for useful noticing and consequent action is enrichment or enhancement of students' experience of mathematics as a discipline.

Provoking Noticing

Of concern to many of the authors in this issue and in similar writings under the heading of Noticing is the range of pedagogical choices during professional development sessions which seem to contribute to promoting and provoking noticing of particular kinds of incidents. In Alan Schoenfeld's review of the Discipline of Noticing (Schoenfeld, 2003), and again in his commentary on Mathematics Teacher Noticing (Schoenfeld, 2010), he asked what is worth noticing and what needs to be noticed in order to improve practice. As he put it,

> "… noticing is important. But, if a group of teachers set on their own on a program of noticing, what might be the focus of their noticing?"
>
> and.
>
> "Noticing operates within the teacher's zone of proximal development. What the teacher notices, and thus what he or she works on, may depend very heavily on what the environment offers (or doesn't)."

Thus, Rotem observes that "novice teachers (NTs) focus mainly on survival and on resolving discipline and classroom management issues. It takes time for them to begin thinking about their own teaching practices and student learning". Teacher educators and professional development leaders have their own quire different concerns and so quite naturally tend to try to sensitise their clients to those aspects of professional practice. Unfortunately, this seems to be leading to the emergence of a hierarchy of valued sensitivities which in some cases is even being used to gauge a person's suitability for or readiness for teaching (Weyers, König, Scheiner, Santagata & Kaiser 2023). For example,

> "The decomposition of what it means to notice into … four levels illuminates *what* and *how* more advanced forms of noticing should happen and elevates critical aspects of noticing as related to the decomposition of the practice. Having teacher noticing broken down in this way is important to understand a trajectory of noticing and to explain what noticing actually entails for teachers; we argue for a parallel approach for coach noticing" [Amador *et al. (*this issue)].

Once a hierarchy or preferred trajectory is chosen, educating awareness is likely to turn into training in behaviour. As novice teachers and in-service teachers work out what the leader is looking for, they are likely to turn to more and more performance-driven behaviour. The ever-present didactic tension hangs over every session: the more explicit a teacher is about the behaviour expected, the easier it is for the learners to display (perform) that behaviour without generating it from themselves (from their awareness) (Brousseau, 1984, 1997; Mason, 1994b). The tension applies equally to mathematics classrooms and to professional development, especially where accreditation is an issue. This tension stems from the implicit didactic contract: if learners undertake the tasks they are set, the resulting activity is assumed to bring about the required learning; teachers are responsible for providing appropriate tasks and supervising appropriate activity; learners are responsible for fully engaging in suggested activity (Brousseau, 1997). When the cult of performance dominates construal, human power and potential can be severely blocked and over looked, because it ignores the psychological reality of how humans function.

Beyond Simple Noticing

What is the point of noticing something during an incident in a classroom, whether it be some detail, some relationship between details, or that some general mathematics education construct is being instantiated? As a teacher, it may be something that has surprised or caught my attention; it might enrich my sense of one or more learners' mathematical, professional, or social maturation; it might be something that one or more learners do or say which I want to invite them to modify; it might be something that I was pleased about in my response or in some pedagogical action which I initiated; or it could be something I wish to avoid in future, by enacting some alternative action instead. As a teacher–educator it might be something that I suspect a novice teacher has not noticed, so I am moved to try

to bring it to their attention, or it might appear to have potential for initiating a productive discussion around some issue in mathematics education, perhaps related to the literature. As a researcher it might be something that could serve as data for a research project or at least serve as an indicator of something worth pursuing in more detail.

"To notice" on its own is insufficient. By itself it is too weak an experience to play any significant role in the development of practice or in the appreciation and comprehension of issues in mathematics education. As Stockero et al. observe, collecting accounts of what different people notice is simply collecting noticings. What matters is whether they contribute to productive discussion by shedding light on the current focus, rather than providing fodder for evaluative judgements.

As one of a string of noticings, it may enrich someone's sense of the complexity of the situation; as a critical noticing, it may lead to re-flection, and perhaps even pro-flection and a change of habit in the future, as in the Discipline of Noticing (Mason 2002). This applies to whatever aspect of professional life is the focus of attention, such as to students learning mathematics (or any other subject) as well as to professional development.

This is re-articulated by van Es and Sherin (2025; 2008) who describe the point of noticing as "(a) identifying what is important in a teaching situation; (b) using what one knows about the context to reason about a situation; and (c) making connections between specific events and broader principles of teaching and learning" (p. 573).

Weyers, König, Scheiner, Santagata & Kaiser (2023) in their systematic review of research reports concerned with noticing spanning 2019 to 2024, quote König et al. (2022) and Santagata et al., (2021) as distinguishing 4 types or drivers of noticing: cognitive-psychological, socio-cultural, discipline-specific, and expertise-related. However, as Coles et al. *(*this issue) suggest, and as a closer reading of the Discipline of Noticing reveals, this may not be the most apposite description of how action follows noticing. It may be more useful to acknowledge that something about a moment of noticing may resonate with past experiences, including cognitively based labels and felt emotions, and that this may trigger habitual coordinations in the psyche between behaviour (action), emotion (affect), intellect (cognition), attention, will and witness (Mason & Metz, 2017). Concentrating on what people notice without acknowledging and working with the whole of peoples' psyche misses the role of noticing and attention in the action of teaching and the experience of learning. A fruitful noticing is like an epiphany, a sudden realisation (making real) of a possibility that moments before was not available.

The impetus and desire to discern and recount differences, despite being the current trend in mathematics education research reports, is likely, as a sociocultural awareness would suggest, to focus only on certain details while overlooking others. For example, describing the Discipline of Noticing as discipline-specific overlooks the fact that it applies, at the very least, to any caring profession, and more generally to any engagement of the psyche. It is referred to by professionals in many professions, including nursing, medical education, veterinary science, business management, language learning, and even dance, as well as teacher education and to education in mathematics, in science and more generally.

Critical and Productive Noticing

Although not commented on explicitly, most researchers involve groups of teachers in discussions based on considering descriptions, transcripts, or even videos of teaching incidents. Such incidents may then be labelled and may emerge as focal points for reflection,

leading to fresh or enhanced comprehension and opening up of situations for enacting fresh pedagogic actions. As such they can become pivotal, or critical in an individual's development. Thus, post-paration[1] or re-flection can lead to pre-paration or pro-flection for in-the-moment insights and action. The significance of noticing emerges as a fresh collection of pedagogic or mathematical actions to enact when relevant situations arise and are recognised as such.

For example, Larison et al. (this issue) found that online video-based professional development designs can indeed cultivate teachers' noticing of students mathematical thinking in a video, in valuable ways, particularly when providing participants with foci for what to attend to, and probably, how to attend to it, as well.

Some authors refer to "critical noticing" or "productive noticing", but this retains much of the ambiguity in the use of the word "critical" indicated earlier. Is it critical because it was remembered "in tranquillity", because it led to an unfamiliar but productive action, because it aligns with or fits the observer's concerns, or because it was unexpected, epiphanic, even shocking, in some way? For example, Coles & Helliwell (this issue) highlight the role of affectivity, suggesting that a felt response would be a key marker that the incident is worth dwelling in and exploring. To be considered critical, or pivotal, surely noticing has to lead to some non-habitual action, and not just once. That raises the question of what constitutes productive noticing. Choy (this issue) points to the ongoing gap between what teachers learn during teacher preparation programs and what they need to be able to do during actual classroom practice. He recognises that what is noticed that may prove critical or productive for teachers can happen before or after a session, during preparation for or reflection on, as well as in-the-moment. Indeed, the significance of an incident may only emerge later, after discussion with colleagues or from a growing awareness of the recurrence of similar situations. Perhaps it is what follows the noticing that determines criticality or productiveness, what discussions and other prompts for reflection are acted upon, and what actions are enacted in the future that initiate or amplify change in the teacher's pedagogic actions.

For Rotem & Ayalon (2022, 2023), *critical events* can be defined as "moments during which students' mathematical thinking becomes apparent and can provide opportunities for teachers to delve more deeply into the mathematics discussed in the lesson" (p. 2). For example, Ayalon et al. (this issue) focus on how teachers identify pivotal moments in a written classroom scenario that can be leveraged to enhance argumentation, which they label as Critical Events Associated with Argumentation (CEAs). They consider how their teachers interpret these opportunities, and the teaching alternatives that are then proposed for fostering argumentation. Put another way, teacher awareness can contribute to explicit opportunities for learners to educate their mathematical awareness.

There are other acronyms for much the same construct, such as Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOST), used by Leatham et al. (2015). Stockero et al. (this issue) use this to observe that "in order to support teachers in enacting responsive teaching, it is important to have a way to distinguish high-leverage student contributions from among the many contributions available to a teacher". This involves the teacher using a coordinated combination of moves to engage the class in collaboratively developing a sense-making argument about the significant mathematics that has arisen. Stockero et al. make the really important observation that it is not just what

[1] Paration: "act of obtaining for use", root of "preparation": obtaining in advance for use.

people notice that matters, but what they notice that is pertinent to, or sheds light on, the current focus of discussion.

Educator's Core Task

For me the core task for educators involves not simply attracting novice and in-service teachers to notice certain aspects of professional practice, but assisting them in educating their awareness so as to have access to correspondingly appropriate actions, and most importantly, supporting them in noticing in-the-moment an opportunity to enact that action. I suggest that the question of what happens as a result of noticing is by far the most important. It is in many ways the most difficult, as well.

Teachers enter the classroom with some sense, however explicitly or implicitly articulated, of what they hope learners will attend to, thus educating their awareness by internalising mathematical and pedagogical actions. Consequently, their focus is likely to be on these aspects, with other aspects perhaps rising to the surface if they are particularly striking, and within the sensitivities of the teacher. Since the very act of teaching is an intervention in the attention of learners, studying what the teacher says and does, what pedagogical actions are initiated and carried through, indicates at least some of what the teacher is sensitised to, attending to, and in what ways. However, it reveals little about its effect on learners. Similarly, coaches and professional development leaders enter a session or an interview with some sense, again varying between the implicit and the explicitly articulated, of what it would be worthwhile for the person to be attending to. But it is very difficult to discern what the long-term effect is.

Changing my own behaviour is hard enough; attempting to change other people's behaviour, to altering their sensitivities to notice, is fraught with ethical, moral and practical issues. Why should my propensities and sensitivities displace the legitimate and ongoing concerns of others?

Again, what comes to mind is the Gattegno-based three-onlys: "only behaviour is trainable", "only awareness is educable", and "only emotion is harnessable". In other words, the energy to change oneself comes from appropriately aroused emotions. It is not the leader but the individual who educates their own awareness by internalising possible actions alongside sensitisation to notice an opportunity to act freshly. Thus, my reply to Schoenfeld's questions in his review was along the lines that all that *can* be worked on at any moment is what one is currently sensitised to notice (Mason, 2003a, 2003b). A necessary but not sufficient condition is to be in the presence (synchronously or asynchronously) of someone exhibiting those sensitivities and displaying corresponding actions, instantiating the claim made by Vygotsky (1978) that higher psychological processes are encountered first in the behaviour of others.

For example, Amador et al. (this issue) use three vignettes of teachers being coached. Having asked participants what was noticed about the coaching moves and the teaching in each vignette, they report not only on what a teacher and what a coach appeared from their remarks to notice, but what teacher educators noticed about relationships between what in the vignette the teacher appeared to notice and what the coach drew attention to. As a result, they learn a lot about the teachers', the coaches', and the educators' sensitivities.

Stockero et al. (this issue) recognise that a single instance of noticing is too simplistic a notion to capture the ongoing noticing required to keep a mathematical discussion with students productive and focussed: "just because the teacher recognizes the Mathematical Point in the student contribution does not mean that the students do."

Solar (this issue) found that teachers not only noticed a pedagogic action used by the teacher being studied (not validating student responses), but began offering justifications for this choice.

Rotem et al. (2024) found that using critical incidents in the university discussions enabled the prospective teachers to link students' thinking with the teacher's teaching practices while supporting their reflection using classroom evidence. However, they did not have evidence for whether this was sufficient to enable fresh actions to be enacted in subsequent teaching.

Larison et al. *(*this issue) note that in promoting student noticing of students' mathematical thinking researchers have studied the use of video, student work, classroom observations, and case studies. Larison and colleagues made use of self-filmed video of teachers' own classrooms, finding that focusing on just a few students was more productive than whole class filming. Having been asked to tag three significant incidents in each video, and to engage in discussion with colleagues based on their filming, they found that teachers emphasised what students could and did do rather what they could not. They were led to think beyond correctness, and to being open to alternative interpretations.

van Es, Munzer, Barnhart, and Alvarez (this issue) responded to teachers reporting that watching video of classrooms and engaging in a "noticing interview" made them more aware of how their classroom noticing was shaped by their personal and social histories, narratives of mathematics education and broader theories. In their paper van Es et al. set out to discern how video extracts of teaching incidents might contribute to teacher learning specifically about equity issues. This in turn informed the design of professional learning to provoke teachers expanding their classroom noticing practices. Multiple viewings, and multiple opportunities to notice, mark and record observations offers opportunities to deepen and enrich the detail and the range of what is noticed (van Es et al.*: this issue*).

What makes a moment or a noticing critical (such as watching a video, reflecting on noticing something in a personal incident) depends on many factors, such as for whom it may be critical, and with what consequences. It may be critical in the sense of having significance, perhaps even being profession-changing, through opening up new possibilities for action not previously available (Tripp, 1993). It may be critical in the sense that it raises social or psychological questions about previous and current practice, inducing uncertainty, confusion or discomfort. It may be significant for the observer even though not apparently for the observed teacher, and it may be critical for a learner, even without the teacher (or observer) being aware of that significance.

Changing one's teaching practices go beyond adopting a set of core practices in teaching (Ball & Cohen, 1999). I conjecture that it is the Being (Heidegger, 1927/1962) of the teacher or leader which sets the tone, animates the culture, and enculturates learners, not just the behavioural practices. This is particularly significant when, as in the case of van Es et al. (this issue), the aim is to promote work on equitable teaching, which, because of its subtlety, may require some explicit attention from an external observer.

Setting Out to Notice Oneself

Setting out to notice specific situations in which a fresh action might usefully be enacted is a lonely path. The basic natural human power required is to imagine myself in a future situation in which you would like to act differently to habit, and to bring as much of the psyche (especially affect but also cognition, attention, will and witness) as intensely into play as possible. In the Discipline of Noticing, this is called

pre-paration. When situations are only retrospectively noticed (post-paration), another round of pre-paration is indicated, until the noticing becomes timely (parative), enabling an alternative action to be enacted.

Taking every opportunity, such as describing to colleagues moments of retrospective noticing, and when available, noticing in-the-moment, not only contributes to pre-paration, but enriches both my own sense of appropriate situations, and that of colleagues as well. Developing more refined techniques to bring retrospective noticing into noticing in-the-moment would be worthy of further research, making use of and acknowledging the variety of teachers' experiences.

Reacting immediately to a shift in my own attention is as much riddled with habit as acting without noticing. An important but often overlooked facet and power of the human psyche is that of gazing, of holding a focus of attention without immediately reacting or analysing. Sometimes this happens in sleep, or during some activity in which the ordinary aspects of the psyche are otherwise engaged, like taking a shower, riding a bicycle, queuing at a bus stop, or rambling in the countryside.

Noticing as a cognitive act is accompanied by access to different affective energies. Sometimes something noticed quickly fades from memory; sometimes it can be re-awakened, resonated by some remark that someone else makes; sometimes there is sufficient intensity to enable a remark about it to be made to someone else; sometimes the intensity is sufficient to prompt a written record being made as part of an observation or research diary.

A Different Discourse

When a learner says or does something, a window on what they may be attending to mathematically is cracked open. When a teacher makes an intervention initiating some pedagogical action, a window on what they are attending to is similarly prised open. In both cases what a researcher can observe is behaviour, including physical action, posture, gesture, voice tones, eye movements, and the like. From this, abductions are made as to the foci of attention and perhaps even as to possible forms of that attention (Mason, 2003a). Describing what is discerned is one thing (giving an account-of). It is quite another to then succumb to the temptation to interpret, analyse, and generally account-for what has been discerned. This is where research can all too easily part company with practice. Spun narratives by researchers may not actually contribute to the professional development of individuals. Just because some specific behaviour is sought or desired, its presence does not guarantee that it is a sufficiently educated awareness to be an action that will be available to be enacted in the future, and its absence does not indicate absence of noticing, only that it has not risen to the intensity required to initiate action.

Using the term *awareness* in the sense of Gattegno (1990), to mean "that which enables action" may make it easier to come closer to the lived experience of teaching or leading professional development sessions, or indeed participating in mathematics lessons and professional development sessions. Something about a situation resonates with past experience and brings a possible action to the surface ready to be enacted. Integrating that action into one's general functioning is no easy matter: sometimes it comes about quite naturally, and sometimes it requires numerous pre-flectional preparation in order for the requisite noticing to take place just before old habits kick in.

Researching What Others Notice

Finding evidence of opportunities arising for others to choose to act freshly is of course difficult to research., as noted by Rotem et al. Educators observing classroom teaching usually notice multiple possibilities for alternative actions, whereas for the teacher, immersed in the action, the view is necessarily more restricted. The reports in these chapters are in many cases edging towards this difficult task of researching what others notice, how, why, and to what effect. I personally choose to be satisfied by evidence that others find in their own experience and the experiences shared with others, as I do, that development and enrichment is possible.

Researchable Issues

So much passes us by, so many aspects of a situation lie well below the surface of consciousness, that it can be difficult to trap specific shifts of attention, and even more difficult to develop sensitivity to notice specific kinds of things. It is not surprising therefore that Teacher Noticing as a research topic has been guided by a rational, materialistic, but ultimately unrealistic perspective in which teacher action is based on explicitly interpreting what is noticed, and only then deciding how to respond, in alignment with Shulman who defined teaching as “an act of reason”, which “continues with a process of reasoning” to culminate in a series of pedagogical actions (Shulman, 1987 p. 13).

I disagree fundamentally. My extensive observations lead me to a very different sense of the human psyche, as do the observations of both Jazby et al. *(*this issue) and Coles et al. (this issue). For me, teaching is a sequence of acts: initiation of pedagogically intended actions in which and through which learners (and teacher) may experience a transformation of not only what they attend to, but how they attend to it, and not just in-the-moment, but again in the future.

What matters is the socio-psychological state surrounding and informing the noticing. Put another way, what matters are the coordinations of affect, cognition, and behaviour, with consequences in attention, will, and witness (Mason & Metz, 2017). The way the parts of the psyche are coordinated constitute patterns which are both experienced and observed in behaviour, affect, and cognition, as actions. These actions come about from awarenesses, where the term "awareness" is used in the technical sense introduced earlier, in which an awareness is "that which enables action". So, attention directed to some detail in a situation resonates with (metaphorically) or triggers (metonymically) a connection to one or more actions, one of which gets enacted. When a teacher is awake to, or present to, the situation, there is a moment of a sense of freedom; otherwise, the action is the working out of a previously determined automaticity or habit. The Discipline of Noticing provides a method for analysing this in more detail.

Shulman also described teaching as “ways of talking, showing, enacting, or otherwise representing ideas so that the unknowing can come to know, those without understanding can comprehend and discern, and the unskilled can become adept” (Shulman, 1987 p14) (quoted in Choy this issue), which can be interpreted much less mechanistically and more in line with theorists such as Lev Vygotsky (1978), that learning comes about (initially at least) through being in the presence of disciplined Being, whether synchronously or asynchronously.

Larison et al. note that a matter of significance, in addition to what is noticed and with what effect, is whose noticing is taken up as a focus, especially where equity and diversity are concerned. Getting others to notice something specific almost always entails a reciprocity or recursion in which it behoves the noticing director to notice their own pedagogic choices.

What aspects or features of a mathematical task contribute to learners' productive noticing in the future? How do expectations associated with particular learners influence what is noticed in their behaviour? What are effective ways of using perceived errors as "teaching points" or MOSTs in the sense of Stockero et al., or CEAs in the sense of Ayalon et al.?

By Way of Summary: Exploiting Noticing

Noticing is the initial act which enables an action to become available to be enacted. The whole point of studying noticing is to prompt learners to educate relevant awarenesses effectively. Thus, there is noticing a potential action, perhaps when reading some research literature, reading a classroom account, attending a professional development session, observing another teacher, or reflecting on one's own recent experience. Noticing also plays an important role when preparing a lesson, or in-the-moment in a lesson, in which what is noticed is an opportunity in-the-moment to make use of some as yet non-habitual action. It may take several or even many occasions for noticing-in-the-moment to precede the activation of a habit and so activating a different action. Even when someone desires to alter a habit, to enact a fresh action or to alter even slightly an established practice, making the change is often no easy matter.

What is required is.

Sensitivity to notice a situation, to become aware, even if only subconsciously
An alternative action sufficiently internalised not to require too much attention to enact it
Sufficient intensity of awareness to trigger an associated action

The Discipline of Noticing, as well as constituting a method of researching my own practice, an articulation of a means of supporting changes in behaviour. It enhances the chances that noticing will happen in time (as distinct from in retrospect) in order to being an alternative internalised action to the surface.

When observing teaching, much like when observing mathematical thinking, the scope and depth of noticing opportunities tends to be richer and broader than the opportunities noticed by the teacher immersed in teaching. There are multiple reasons for this of course. More extensive experience, experience of a broader range of familiar pedagogic (or mathematical) actions, and most centrally, the tunnel-vision effect of being immersed in action rather than observing "from a distance".

Humberto Maturana (1988) remarked that "Everything that is said is said by an observer". Making a remark about something noticed translates the speaker from immersion in action to reflection upon action (Schön, 1983, 1987). One of the implications is that the act of speaking/writing emerges as description. The more that description uses technical labels, the more it is imbued with judgement and evaluation, and so the more it accounts-for what is observed. The more abstractly labelled a description, the less likely it is to serve as a basis for recognising a relevant situation as it develops in practice, whereas the more precise a description, minimising judgement and emotion (an account-of), the more likely it will be to serve as trigger for an awareness and associated action in the midst

of practice, when it is needed. The greater the element of accounting-for, the less it is possible to agree on the accuracy and appropriateness of the description, and so the less reliable is any associated action which is resonated or triggered. Furthermore, the greater the accounting-for the less reliable are the descriptions as contributions to research.

One subtlety of Maturana's "observation" includes, for example, the difference between acting out of habit (a basic form of noticing), and "marking", that is, being able to remark on something noticed, which often means some sort of inner monologue concerning what has been noticed and what opportunities for action that might afford. Making distinctions, labelling situations, and actions calls upon language, which turns the noticing into an observer's utterance. A positive aspect of labelling is that it is the mechanism whereby the awareness makes action become available to be enacted. A negative aspect of labelling is that it channels the effect of noticing into already established coordinations of the psyche, limiting possible cognitions, affect, and action.

What then is it Critical to Notice?

The articles in this issue vary quite considerably in what constitutes critical noticing, which is not surprising because it depends on critical-for-whom and critical-for-what-purpose, with what consequences. This ranges from *critical for passing a test* to *critical for future development*, and ultimately, because it is within mathematics education, to *critical for the efficiency and effectiveness of students' experience of mathematics.* It may be critical in the sense of having significance, perhaps even being profession-changing, through opening up new possibilities for action not previously available (Tripp, 1993). It may be critical in the sense that it raises social or psychological questions about previous and current practice, inducing uncertainty, confusion, or discomfort. It may be significant for the observer even though not apparently for the observed teacher, and it may be critical for a learner, even without the teacher (or observer) being aware of its significance.

So, criticality in the sense of essential or pivotal depends on the ambient pedagogy and ethos. As long as it remains based in the educator–researcher's world of experience, it is likely to pass some learners by, because it is what teachers themselves are sensitised and sensitisable to that matters, helping them to convert sensitivity to notice into educated awarenesses which enable actions that contribute to learners' experience of mathematical thinking.

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