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# POSSIBLE ASTROPHYSICAL PROBES OF QUANTUM GRAVITY

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A satisfactory theory of quantum gravity will very likely require modification of our classical perception of space-time, perhaps by giving it a ‘foamy’ structure at scales of order the Planck length. This is expected to modify the propagation of photons and other relativistic particles such as neutrinos, such that they will experience a non-trivial refractive index even *in vacuo*. The implied spontaneous violation of Lorentz invariance may also result in alterations of kinematical thresholds for key astrophysical processes involving high energy cosmic radiation. We discuss experimental probes of these possible manifestations of the fundamental quantum nature of space-time using observations of distant astrophysical sources such as gamma-ray bursts and active galactic nuclei.

*“We finally turn to the most daunting problem of quantum gravity, the nearly complete lack of observational and experimental evidence that could point us in the right direction or provide tests for our models. The ultimate measure of any theory is its agreement with Nature; if we do not have any such tests, how will we know whether we are right?”*

S. Carlip (2001)[1]

## 1. Introduction

Quantum Mechanics, Special Relativity and General Relativity correspond to three vertices of Bronshtein’s ‘cube of theories’ constructed on axes labelled by the three dimensionful constants $c$, $\hbar$ and $G_{\rm N}$, which define the fundamental units of length, mass and time.[2] Each of these theories has been extremely successful in describing Nature, yet their union into a consistent (and comprehensible!) theory of quantum gravity still eludes us. Moreover it is often stated that the quantum effects of gravity may never be accessible to experiment because they would be manifest only on the tiny Planck length scale, $l_{\rm P} \equiv \sqrt{\hbar\, G_{\rm N}/c^3} \simeq 1.6 \times 10^{-33}$ cm (corresponding to the huge energy scale $M_{\rm P} \equiv \sqrt{\hbar\, c/G_{\rm N}} \simeq 2.2 \times 10^{-8}$ g or $1.2 \times 10^{19}$ GeV in units with $c = \hbar = 1$). However, gravity, being a non-renormalisable interaction in the language of quantum field theory, can perhaps leave a distinctive imprint at energies much lower than the Planck scale if it violates some fundamental symmetry of the effective low energy theory, resulting in qualitatively new phenomena (akin to the violation of parity in nuclear radioactive decay, at energies far below the true scale of the responsible weak interaction).

At present there are two main approaches to the quantisation of continuum gravity.[1] The first is the canonical programme for a non-perturbative and background-independent quantisation of general relativity, viz. the loop-gravity formalism.[3] The second is superstring theory,[4] and its non-perturbative D(irichlet)-brane extension.[5] Apart from these formal mathematical approaches there have been other discussions of the likely physical manifestations of quantum gravity, e.g of the possibility that quantum space-time has a 'foamy' structure,[6] in which Planck-size topological fluctuations resembling black holes with microscopic event horizons appear spontaneously out of the vacuum and subsequently evaporate back into it. These black-hole horizons have been viewed as providing an 'environment' that might induce quantum decoherence of apparently isolated matter systems.[7,8] In this picture Lorentz invariance (LI) would appear to be lost in the splitting between the matter system and the quantum-gravitational 'environment'; such a breaking of LI can be considered a property of the quantum-gravitational ground state, and therefore a variety of *spontaneous* breaking. This also occurs in the loop-gravity formalism, as demonstrated in the semi-classical limit.[9,10] In critical string theory there is no Planck scale space-time foam but in *non-critical* 'Liouville' string theory,[11] the link can be made by viewing D-branes as space-time defects, giving rise to a cellular structure in the space-time manifold;[12,13] Thus it seems not unreasonable to expect spontaneous violation of LI at high energies as a *generic* signature of quantum gravitational effects. Although a comprehensive theoretical framework may be lacking at present, such an exciting possibility is adequate motivation for a phenomenological approach to the question of whether such tiny violations can be observed.[14,15]

## 2. Quantum gravity and possible modifications to dispersion relations

In this talk I will discuss whether astrophysical observations can test possible LI violations due to the quantum structure of space-time. We are interested in signatures that are characterised by deviations from conventional quantum field theory, which would presumably be suppressed by some power of the Planck mass.[a] It appears that several such effects are at the edge of observability if the suppression is by just a *single* power of $M_{\mathrm{P}}$.

This may well be the case for the possible effects of a quantum-gravitational environment on the propagation of a massless particle such as a photon. At energies small compared to the Planck scale a series expansion of the usual dispersion relation in the small parameter $E/M_{\mathrm{P}}$ is justified, i.e. [14]

$$c^2p^2 = E^2\left[1 + \xi\left(\frac{E}{M_{\mathrm{P}}}\right) + \mathcal{O}\left(\frac{E^2}{M_{\mathrm{P}}^2}\right) + \ldots\right]. \qquad (1)$$

Such a relation with $\xi = +1$ is consistent with studies [18] of quantum deformations of space-time symmetries (viz. the Hopf extension of Poincaré algebra), if we identify the 'deformation parameter' $\kappa$ as the inverse of the Planck length and impose rotational symmetry (i.e. invariance under the O(3) subgroup of the SO(3,1) Lorentz

[a]Our approach is somewhat different from identifying all possible LI violating terms which can be added to the Standard Model at the *renormalisable* level; the possible phenomenology of such terms and current experimental bounds on them have been discussed in detail elsewhere.[16,17]

group).[19b] Such a deformed dispersion relation also arises in non-critical Liouville string theory, as has been demonstrated using the picture of D-brane recoil.[12] This is essentially because the action for such theories is proportional to the *square root* of the string tension (which has dimensions of the inverse of the fundamental length *squared*); by contrast in critical string theory the action is proportional to the string tension, consequently only corrections of $\mathcal{O}(E^2/M_{\rm P}^2)$ would be expected.[21c] Given however that the connection between critical string theory (in 10 dimensions) and our low energy world in 3+1 dimensions is still an open question, it would seem premature to dismiss the possibility of a dispersion term of $\mathcal{O}(E/M_{\rm P})$. It is more interesting to ask if we can test for such a term *experimentally* to obtain a constraint on theories. Somewhat unexpectedly this turns out to be possible.[14]

### 2.1. *Time-of-flight studies of radiation from distant objects*

Assuming the usual Hamiltonian equation of motion $\dot{x_i} = \partial H/\partial p_i$ to still be valid, the above dispersion relation (1) implies an *energy dependent* speed of light,

$$v = \frac{\partial E}{\partial p} \simeq c\left[1 - \xi\left(\frac{E}{M_{\rm P}}\right)\right]. \tag{2}$$

We observe that the effect (2) is easy to distinguish from dispersion in any field theoretical vacuum or plasma, which always *decreases* with increasing energy.[23] Thus when we study a source at distance $L$, we will see a quantum gravity induced *time delay* (for $\xi$ +ve) in the arrival of the more energetic photons as compared to the less energetic ones (differing in energy by an amount $\Delta E$) of order:

$$\Delta t \simeq \xi\frac{L}{c} \times \frac{\Delta E}{M_{\rm P}}. \tag{3}$$

An analogous effect arises in the semi-classical limit of loop gravity, if the gravitational degrees of freedom are assumed to be in a 'weave' state.[9]

The energy-dependent refractive index (2) would induce dispersion in the arrival times of photons emitted in a short pulse by an astrophysical source. It is also possible that different photons with the same energy (frequency) might travel at different velocities, as is suggested by higher-order studies in the D-brane approach to quantum gravity.[13] This would provide a second possible source of dispersion in a wave packet, beyond that associated with differing frequencies:

$$\delta t \sim \frac{\sqrt{\langle\sigma^2\rangle}}{L}, \tag{4}$$

where $\sigma^2$ denotes a quantum fluctuation about the specific classical background. Both these effects can be described in terms of quantum fluctuations in the light cone.[24,13] Moreover if the weave states of loop quantum gravity have definite parity then one also expects different propagation velocities for left- and right-handed

[b] A possible astrophysical test of $\kappa$-Poincaré deformations had been suggested earlier.[20]

[c] Such $E^2/M_{\rm P}^2$ corrections are also found for the dispersion relation of transversely polarised photons due to graviton induced vacuum polarisation in 3+1 dimensions.[22]

polarised photons, viz. bi-refringence of the vacuum.[9] This possibility is however severely constrained already by observations of distant radio galaxies which exhibit detectable linear polarisation in their emissions;[25] the corresponding time delays for high energy photons in this model may then be too small to be observable. However a study of the propagation of spin 1/2 fermions in the semi-classical limit of loop quantum gravity shows that there would still be detectable time delays between neutrinos and photons from GRBs.[10d]

The figure of merit for such tests is

$$M_{\rm QG} \equiv \xi \frac{LE}{c\Delta t}, \tag{5}$$

where $M_{\rm QG}$ is the highest energy scale that can be probed using a source of photons of energy $E$ at distance $L$ which exhibits distinct structure on a time scale of order $\Delta\, t$. As we emphasised,[14] gamma-ray bursts (GRBs) have particularly large figures of merit. Some of them exhibit microstructures in their light curves of a millisecond or less, they are likely to emit $\gamma$ rays in the GeV or even TeV range, and many are now definitely known to be located at cosmological distances.[28] We estimated that GRB observations might already be sensitive to a quantum-gravity scale $M_{\rm QG} \sim 10^{16}$ GeV, and suggested that atmospheric Čerenkov telescopes (ACTs) would be able to improve on this sensitivity. The Whipple ACT group has applied this idea to observations of the active galactic nucleus (AGN) Mkn 421, establishing a lower limit $M_{\rm QG} > 4 \times 10^{16}$ GeV.[29] The potential of next-generation ACTs (such as VERITAS, MAGIC and HESS) for improving such limits has been assessed.[30] A similar lower limit has been inferred from observations of gamma-ray pulsars.[31] A careful analysis of present data on GRBs however finds a slightly weaker limit $M_{\rm QG} > 10^{15}$ GeV,[32e] but remarkably enough, space-borne experiments such as AMS and, particularly, GLAST are expected to definitively find (or rule out) the expected time delay for $M_{\rm QG} \sim M_{\rm P}$. Figure 1 demonstrates that GLAST (scheduled for launch in 2005) will be able to do this in just 2 years of operation!

### 2.2. *Modifications of relativistic kinematics due to LI violation*

Among the key considerations for the above experiments is the question of just how far can one see into the universe at high energies; for example at TeV energies, $\gamma$-rays are expected to be severely attenuated through pair-production on the intervening cosmic infra-red background (CIB) radiation (above a threshold energy $m_e^2/E_{\rm CIB} \sim$ 10 TeV) so that ACTs should see an exponential cutoff in AGN spectra at such energies.[35] The absence of the expected cutoff in the spectra of the relatively nearby sources Mkn 421 ($z = 0.031$) and Mkn 501 ($z = 0.033$) was thus interpreted as setting limits on the (otherwise poorly known) intensity of the CIB.[36] This radiation is created by stars and reprocessed by dust so its intensity is sensitive to the entire history of galaxy formation and can be estimated in specific models.[37]

[d] Such a time delay is also found [26] for the LI violating extension of the Standard Model;[16] this may not however be observable if the pulses are smeared out due to the dispersive effect (4).[27]
[e] A more severe limit of $M_{\rm QG} > 8.3 \times 10^{16}$ GeV was claimed to follow from observations of GRB 930131;[33] however this object has no measured redshift, hence a very uncertain distance.

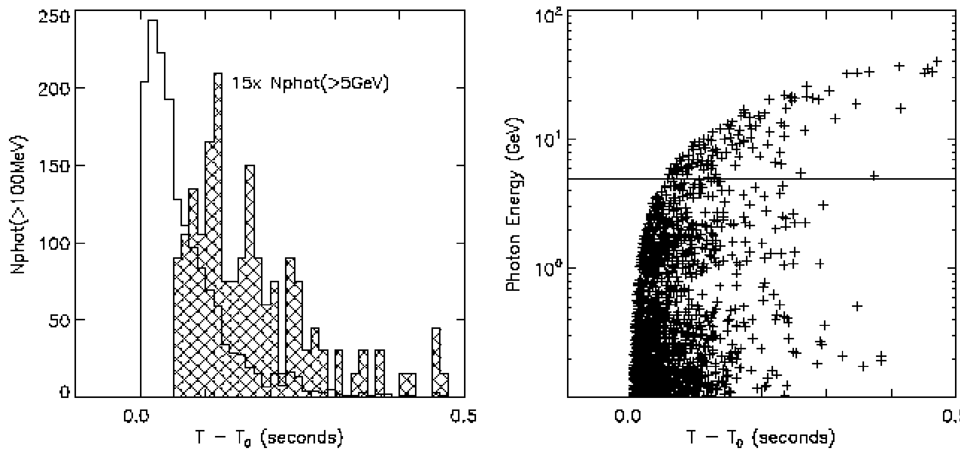


Fig. 1. The expected sensitivity of GLAST to the quantum gravity induced dispersion in Eq.(3) using observations of gamma-ray bursts.[34] The left panel shows the simulated composite pulse from 20 bright GRBs observed over 2 years, with energy threshold of 0.1 GeV (open histogram) and 5 GeV (cross-hatched histogram). The right panel shows the arrival time as a function of the photon energy; the expected dispersion is clearly seen above a 5 GeV threshold (horizontal line).

When the first direct measurements of the CIB were made, notably by the DIRBE instrument on COBE,[38] the fluxes proved surprisingly to be *above* the limits set by the above argument. It had been pointed out however [39,40,41] that the LI violation implicit in the dispersion relationship (1) would affect the kinematics of the attenuation process $\gamma\gamma \to e^+e^-$ such that there is *no* physical threshold for this process, hence no attenuation of TeV energy $\gamma$-rays even from very distant sources, independently of the intensity of the CIB.[f] This possibility was promoted as a solution of the 'IR-TeV $\gamma$-ray crisis'[45] created by the non-observation of a sharp cutoff in the spectrum of Mkn 501 upto $\sim 21$ TeV by the HEGRA ACT.[46] Their point is made in Figure 2 which shows an updated version of this argument.[47] The left panel shows a compendium of CIB observations with 3 reference curves drawn to indicate the range of possibilities — curve 1 is a 'nominal' fit, curve 2 is close to the expectations from modelling of galaxy formation,[37] while the 'extreme' curve 3 is drawn to match the highest CIB fluxes reported (which may have suffered contamination from zodiacal light). The right panel shows the intrinsic $\gamma$-ray spectrum of Mkn 501 reconstructed from the HEGRA observations.[46] Although the observed spectrum does bend down, particularly after reducing the intensity in the highest energy bin (down to the starred point) at 21.45 TeV to allow for possible spillover from lower energies,[48] making allowance for the expected attenuation by the CIB (especially at $\sim 100\,\mu$m) still requires a rather unphysical turnup in the intrinsic spectrum at $\sim 10$ TeV.[45] This is so even for the 'nominal' fit (curve 1), although the turnup is not as sharp as when the 'extreme' fit (curve 3) is used. There is no anomaly if

[f] There is in fact no change in the threshold energy condition for the $\kappa$-deformed Poincaré algebra,[18] but there are significant corrections to the kinematics above threshold.[42,44]

curve 2 reflects the true CIB and this is indeed close to estimates based on galaxy evolution models.[49,50] However such estimates are necessarily model-dependent and have many uncertainties so it seems unreasonable to conclude on this basis alone that there is no problem.[51,52] Clearly the issue will be resolved with further observations of even more distant AGN, e.g. H1426+428 ($z = 0.129$); present observations are in fact consistent with attenuation by the CIB with *no* need for LI violation.[53]

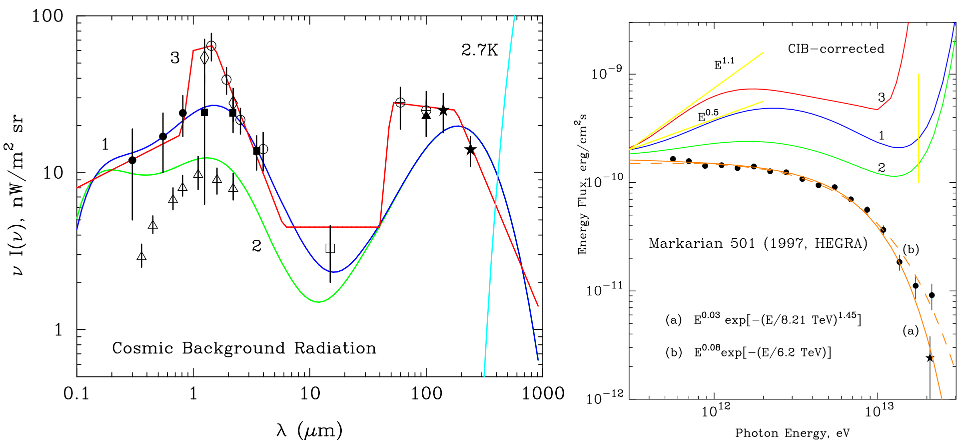


Fig. 2. Reconstruction [47] of the emission spectrum of Mkn 501 from TeV $\gamma$-ray observations (right panel),[46,48] after allowing for attenuation by the observed cosmic infrared background (left panel). The vertical line at 17 TeV indicates the point upto which the spectrum is mesured reliably.

Similarly it has been noted that photopion production of ultra high energy cosmic rays (UHECR) on the cosmic microwave background (CMB), $p\gamma \to \Delta(1232) \to \pi^+ \ldots$, which ought to result in a 'GZK' cutoff [54] of their energy spectrum above $\sim 5 \times 10^{10}$ GeV (if they are indeed protons of extragalactic origin), may be evaded by LI violation at high energies.[55,56,57,58,40] This is an interesting possibility given that the UHECR spectrum is seen to extend well beyond the GZK cutoff while their sky distribution is consistent with isotropy, suggestive of cosmologically distant sources.[59] However there are plausible alternative explanations for this, for example such UHECRs may originate *locally* in the halo of our Galaxy, thus evading GZK energy losses while still being quasi-isotropic.[52,60] The ongoing Pierre Auger experiment will soon provide improved determinations of the composition, spectral shape and anisotropy of UHECRs which will critically test such explanations.

The existence of 'threshold anomalies' is thus rather controversial from the observational point of view. As a cautionary theoretical note, the assumption of energy conservation implicit in recent analyses of scattering kinematics [39,40,41,42,43] may not in fact be valid — e.g. in the space-time foam picture energy is conserved only in a *statistical* sense.[7,27,61] In this picture there is no connection between the TeV $\gamma$-ray and the UHECR puzzles which require e.g. very different values of the LI violating parameter.[62] This is in contrast to the claim [43,63] that both of these puzzles, as well as an alleged 'anomaly' in the development of cosmic ray air showers,[64g] can all be traced to the same quantum gravity induced dispersion effect (1). This underlines the limitations of a purely 'phenomenological' approach — without explicit guidance from a theory of quantum gravity, it is difficult to be certain of the effects of possible LI violation in different experimental contexts.

Alternatively one can simply parameterise every possible LI violating term that can be added to the low energy effective theory, with LI assumed to hold in the 'preferred' frame identified by the (absence of a) dipole anisotropy in the CMB.[17] In such schemes, particles in any other frame may move faster or slower than light (in contrast to the space-time foam picture where only *subluminal* motion is possible [13]). This suggests other interesting astrophysical tests e.g. if the maximum attainable speed of photons $c_\gamma$ exceeds the maximum attainable speed of electrons $c_e$ then photons of high enough energy would be unstable against decay $\gamma \rightarrow e^+e^-$; if the reverse were true then one can have Čerenkov radiation $e \rightarrow e\gamma$ *in vacuo*.[66,17] Since we do observe photons of energy upto $\sim 20$ TeV from distant AGN [46] and infer the existence of electrons with energies upto $\sim 100$ TeV (from observations of non-thermal X-rays and TeV $\gamma$-rays in some supernova remnants,[67] restrictive limits can be set on such LI violating effects, in addition to those discussed earlier concerning possible 'threshold anomalies' [58,66,17] and pion stability.[64] A systematic analysis of such constraints has recently been undertaken [68,69] in the framework of the quantum gravity inspired dispersion relation (1), but allowing for a different value of $\xi$ for photons than for other particles (the correspondance to the alternative LI violating formalism [17] can be made [40] by setting $\xi_i\, E/M_{\rm P}$ equal to $c_\gamma^2 - c_i^2$). These analyses deduce joint constraints on the LI violating parameters from a careful examination of the relativistic kinematics of all 'forbidden' processes (assuming energy conservation) and find that a large region of parameter space is ruled out as seen in Figure 3.[68] This shows the allowed region in $\xi - \eta$ space, where $\xi$ is the same quantity as defined for photons in Eq.(1) but now defined with *opposite* sign, and $\eta$ is defined similarly for electrons. Thus Eq.(1) would suggest $\xi = \eta = -1$ as has been considered in previous work.[14,39,40,45] We see that this is *not* ruled out, which is unsurprising since in this case there is no 'superluminal' motion.

## 3. Conclusions

Our modest proposal [14] that a possible effect of quantum gravity on the propagation of photons may be amenable to observation, has triggered quite a burst of activity and been generally well received by theorists.[70] However, as already remarked, one needs a concrete formalism to do further work, in particular to study the possible effects of the associated spontaneous LI violation for relativistic kinematics. So far this has been done mainly in the framework of the Liouville string theory model of space-time foam,[13,27] the semi-classical limit of loop quantum gravity,[9,10,71] the

[g]It is stated [63] that the observed development of cosmic ray air showers can be explained "by assuming that ultra-high-energy neutral pions are much more stable than low energy ones". However the cited source [64] does not claim any such anomaly; the authors simply obtain a bound on possible LI violating effects.[17] A subsequent analysis [65] notes on the contrary that the dispersion effect (1) would imply *unacceptable* changes in the development of the em cascade — however this conclusion is based on assuming energy conservation which may be invalid as noted above.

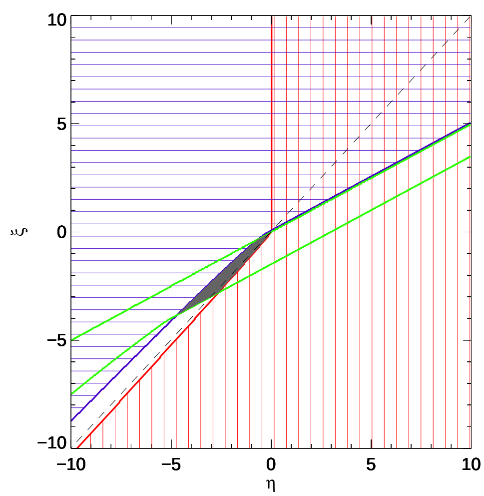


Fig. 3. Constraints [68] on the LI violating parameters for photons ($\xi$) and electrons ($\eta$) (defined as in Eq.(1) but with *opposite* sign). The horizontally ruled region is excluded by the absence of vacuum photon decay and the vertically ruled region by the absence of vacuum Čerenkov radiation. The threshold for attenuation of cosmic $\gamma$-rays on CIB photons (of energy 0.025 eV) is raised from 10 to 20 TeV in moving from the upper diagonal line to the lower one. The dashed line is $\xi = \eta$.

deformed $\kappa$-Poincaré algebra,[19,44h] and in a model for quantum energy-momentum uncertainties in space-time foam.[62] As these unconventional ideas attract increasing attention in the community, new phenomenological tests are being suggested. For example it has been noted that if energetic particles can travel faster than gravitons then they would emit *gravitational* Čerenkov radiation at an unacceptable rate.[75] Moreover the dispersive effects of space-time foam, if *flavour dependent*, would induce neutrino oscillations with parameters in conflict with experiment.[76] The associated light cone fluctuations may also distort the observed Planckian spectrum of the CMB.[77] The non-critical Liouville string model has been shown to escape these constraints,[78] but the consequences for other models remain to be worked out and confronted with observations. What is clear is that there is indeed a phenomenology of quantum gravity and that this is a most exciting time for its practitioners.

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[h]The recent ‘Doubly Special Relativity’ constructions [72,73] also fall in this category.[74]

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