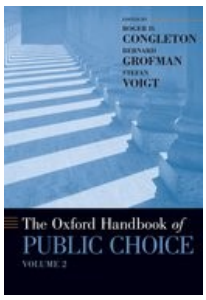

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CHAPTER

39 Precursors to Public Choice

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Abstract

This chapter reviews the many appearances, disappearances, and reappearances of axiomatic thought about social choice and elections since the era of ancient Greek democracy. Social choice is linked to the wider public-choice movement because both are theories of agency. Thus, just as the first public-choice theorists include Hobbes, Hume, and Madison, so the first social-choice theorists include Pliny, Llull, and Cusanus. The social-choice theory of agency appears in many strands. The most important of these are binary vs. nonbinary choice; aggregation of judgement vs. aggregation of opinion; and selection of one person vs. selection of many people. The development of social choice required both a public-choice mindset and mathematical skill.

Keywords: social choice, Pliny the Younger, Ramon Llull, Nicholas Cusanus, Condorcet paradox, Jury theorem, Borda count, C. L. Dodgson, E. J. Nanson

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39.1 Thinking like a (Public Choice) Economist

39.1.1 As a Frame of Mind

This chapter is about precursors to *social* choice:[1] that is, the paradoxes and mathematics of choice, as discovered, lost, rediscovered, lost again, and re-rediscovered numerous times before Arrow (1950, 1951). Some scholars, notably Buchanan (1954) and Tullock (1967a), have disputed whether social choice is really part of the public-choice movement. This chapter argues that it is, for two related reasons: that all of public choice, including social choice, is at heart a study of agency problems; and that the pre-Arrow pioneers of social choice were driven by the same motives as the pre-Buchanan and Tullock pioneers of public choice more broadly. They thought like economists.

In a well-known definition, public choice "uses economic tools to deal with traditional problems of political science" (Tullock 2008, abstract). Tullock precedes this encyclopedia definition, as he often did earlier, with the qualifying clause "By assuming that voters, politicians and bureaucrats are mainly self-interested." This is to confuse methods with assumptions. However, it has been a curiously fruitful confusion throughout the lifetime of public choice. Most of the developments reviewed in this chapter make no assumptions about self-interest. Many of the developments in choice theory before 1951 are simply applications of the mathematics of their day to a new problem. In the same encyclopedia entry whose abstract I have just quoted, Tullock (2008) contradicts Tullock (1967a):

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> In the 18th and 19th centuries a number of mathematicians (Condorcet, Borda, Laplace and Lewis Carroll) became interested in the mathematics of the voting process; their work was forgotten until Duncan Black rediscovered it (see, e.g., Black, 1958). Black can be called the father of modern Public Choice, which is in essence the use of economic tools to deal with the traditional problems of political science. (Tullock 2008, from body of entry)

Reconciling Tullock with Tullock, I aim to show that most of the precursors of public choice thought like economists, even in the centuries and millennia before economics was defined as a discipline. Pliny, Llull, and Cusanus, to be discussed, all considered the design of institutions in situations where they observed normal human behavior as having possibly perverse consequences.

In this sense, the greatest precursors of public choice have been the classical political theorists who thought like economists. Here are three well-known examples.

> Sixthly, the denial of marriage to priests serveth to assure this power of the Pope over kings. For if a king be a priest, he cannot marry and transmit his kingdom to his posterity; if he be not a priest, then the Pope pretendeth this authority ecclesiastical over him, and over his people. (Hobbes 1651, chap. 47)
>
> It is, therefore, a just political maxim, that every man must be supposed a knave: Though at the same time, it appears somewhat strange, that a maxim should be true in politics, which is false in fact. But to satisfy us on this head, we may consider, that men are generally more honest in their private than in their public capacity, and will go greater lengths to serve a party, than when their own private interest is alone concerned. Honour is a great check upon mankind: But where a considerable body of men act together, this check is, in a great measure, removed; since a man is sure to be approved of by his own party, for what promotes the common interest; and he soon learns to despise the clamours of adversaries. To which we may add, that every court or senate is determined by the greater number of voices; so that, if self-interest influences only the majority, (as it will always do) the whole senate follows the allurements of this separate interest, and acts as if it contained not one member, who had any regard to public interest and liberty. (Hume 1742, Essay I:vi)
>
> Ambition must be made to counteract ambition. . . . If men were angels, no government would be necessary. If angels were to govern men, neither external nor internal controls on government would be necessary. In framing a government which is to be administered by men over men, the great difficulty lies in this: you must first enable the government to control the governed; and in the next place oblige it to control itself. (Madison et al. 1788, #51)

In a Tullockian sense, Thomas Hobbes is the greatest of all precursors of public choice. The whole of *Leviathan* is an analysis of the rational course of action for a self-interested citizen. Not only that, but in common with David Hume, Adam Smith, and James Madison a century later, Hobbes's whole outlook is based on stripping away rival (especially pious) positive and normative accounts of behavior (see also

p. 799 Ferrero, chapter 37, this volume). After all, says Hobbes, "it is with the mysteries of our religion as with ↳ wholesome pills for the sick, which swallowed whole have the virtue to cure, but chewed, are for the most part cast up again without effect" (Hobbes 1651, chap. 32).

To reach his conclusion that the rational citizen should hand his natural right of self-preservation to Leviathan, and covenant with every other citizen to do likewise, Hobbes must dispose of all claims to follow a higher authority than the state. In 1651 as now, many such claims are religiously driven. Hobbes, the public-choice economist, ruthlessly demolishes such claims as that heaven means living to eternity in a blessed place and hell means living to eternity in an accursed place (chap. 35), or that people who claim to know the word of God, or to perform miracles, should be followed (chaps. 36–37). Because his Leviathan must control the state religion, Hobbes devotes Book IV of *Leviathan* to what he regards as the most dangerous religions of his time: Presbyterianism and Roman Catholicism. His attack on the worldly claims of the Catholic Church (chap. 47 and the Review and Conclusion) is a magnificent piece of public-choice writing.

A century later, David Hume can be more open in his denial of religious motivation. Like Hobbes and Tullock, he assumes that citizens should be *treated as if* they were self-interested. He discusses "honor" as a check on cheating; and notes that the check of honor is weakened "where a considerable body of men act together" as a "party." Like his close friend of the Scottish Enlightenment Adam Smith, Hume wants to design institutions and bolster norms that promote sociable behavior and discourage cheating. Adam Smith's two great books, especially the first, his *Theory of Moral Sentiments* (Smith [1759] 1976), discuss the institutions and the interests that lead people to behave sociably, and that thus make moral sentiments effective and increase the wealth of nations.

At the College of New Jersey, President John Knox Witherspoon was a paradoxical man. Before leaving Scotland in 1768, he had been a bitter and satirical enemy of the deists and atheists of the Scottish Enlightenment. Hume and Smith supported the Moderate party in the Church of Scotland which, in their time, controlled the General Assembly to the exclusion of the strongly Calvinist Evangelical party to which Witherspoon belonged. With an established church, it mattered who had the levers of power because power in the church moderated power in the state. But on emigrating to the United States in the Revolutionary period, Witherspoon turned into a patriotic American radical, signer of the Declaration of Independence, and tutor of James Madison. No church, Presbyterian or other, was established in New Jersey. Witherspoon quickly moved from advocating establishment (with people like him in charge) to nonestablishment and freedom of religion. He appealed to his fellow Scots in America to rise up against the English governors' attempts to impose a regime or a religion (Witherspoon 1777).

In the same spirit, it seems, he taught the thinkers of the Scottish Enlightenment, including the man he accurately labeled as "the infidel Hume" (quoted in Noll 1989, 39) to his star student, James Madison. How deeply Madison absorbed Hume has been understood since Adair (1974, but written much earlier). How deeply he absorbed Smith is not so certain, but see McLean (2015). With his fellow-framer the Scot James
p. 800 Wilson, ↳ Madison offered to the Constitutional Convention a set of institutions in which ambition could be made to counteract ambition. He did not wholly succeed (see Dougherty, chapter 38, this volume). But in his numbers of *The Federalist*, he argued that the institutions provided for in the Constitution did indeed help ambition to counteract ambition. This permeates both (what has been argued to be) the sincere argument of Federalist #10, and the inconsistent but expedient argument of Federalist #51 (Kernell 2003; McLean 2003).

39.1.2 As a Way to Approach the Agency Problem: Market Failure vs. Voting Failure

Welfare economists before the modern public-choice movement tended to identify instances of market failure and propose regulation as a remedy (e.g., Pigou 1920). The public-choice response has taken various forms. Some externalities can be dealt with by assignment of rights (Coase 1960). Other scholars complain that the counterpart of market failure is voting failure: for instance, in the creation of, ultimately self-defeating, logrolling coalitions for rent-seeking objectives (Tullock 1967b; Krueger 1974). In a general sense, voting failure is agency failure. The representative has failed in some sense to be the true representative of the people. But what is "truly representative"? Does it even have a meaning? Since 1951, we have known that in some cases the answer is no, and that the "will of the people" is undefined or nonexistent. (For a vigorous statement, see Riker 1982, chap. 1.) Parts of that knowledge are ancient. It is those ancient discoveries that this chapter aims to explain and elucidate.

39.2 The Steps Toward Understanding Voting Failure

39.2.1 Binary Contexts—Sortition and Juries: Aristotle, Condorcet, and May

The tools of social choice include logic and algebra. There is a clue in the names. *Logic* derives from the Greek *logiké*, "pertaining to reasoning." *Algebra* derives from the Arabic *al-jabr*, which according to the *Oxford English Dictionary* "probably originally referred specifically to the method of solving quadratic equations by completing the square" (*OED* online, s.v. *algebra*, etymological note). We should expect logical thought in the West about choice to go back to the fifth century BCE, and algebraic thought in the West to go back, at furthest, to the thirteenth century CE, when Arabic mathematics began to drift westward. As we will see, that is exactly what the history shows. It would be fascinating to see if an algebraic understanding of
p. 801 choice paradoxes can be traced ↳ back to Arabic mathematics of any time between the eighth and thirteenth centuries CE, but no such evidence has yet been found.

Ancient Greek thinkers thought profoundly about democracy, but only in a binary context (Fleck and Hanssen, chapter 36, this volume). Voting, when it existed, was binary: for or against a proposition; for or against ostracizing (banishing) somebody from the city-state. Most institutions of Athenian democracy worked by lot (sortition); only technical experts were subject to election:

> The holders of all routine offices in the state are selected by lot except for the treasurer of the military funds, the controllers of the Theoric fund and the supervisor of the water supply. These are elected . . . all military officials are also elected. (Aristotle, *Const. Ath.* XLIII, quoted in Everson 1996, 244)

Plato and Aristotle, both enemies of Athenian democracy, described some of the ways it worked. For instance, the analysis of the *Constitution of the Athenians* attributed to Aristotle says that, after Pericles (*fl.* 430 BCE) introduced pay for jury service, "the quality of *dikastai* [jurors] declined, since it was always the ordinary people rather than the more respectable who took care to ensure that their names were included in the ballot for places on the juries" (Aristotle, *Const. Ath*. XXVII, quoted in Everson 1996, 231).

The elections that Aristotle (or his research assistant) describes are binary—that is, for jury decisions, "those chosen to supervise the voting give each member of the jury two tokens, one pierced and one blocked" (*Const. Ath.* LXVIII, in Everson 1996, 262). None of the three known principal writers on Athenian democracy, viz., Aristotle, Plato, and Thucydides, discusses voting among more than two options. Thucydides's most famous set piece is the Mytilene debate, in which the Athenian assembly changes its

mind and reverses the previous day's decision to massacre every male inhabitant of Mytilene. Thucydides ends his account: "at the show of hands the votes were nearly equal. However the motion of Diodotus [to call off the massacre] was passed" (Thucydides, Book III, quoted in Warner 1954, 222).

From ancient Greece to the present day, logic is good at analyzing binary decisions, especially whether something is true or false. An obvious context is jury decision. Jurors face an essentially binary decision: Is the accused guilty of the crime as charged or not? This strand of choice theory developed with modern probability theory in the seventeenth and eighteenth centuries CE. Where choice is truly binary, Condorcet's (1785) Jury theorem leads to a neat answer, which has spawned a huge literature of its own. As a piece of mathematics, the Jury theorem is beautifully simple. Assuming a probability p ($p > 0.5$) that each juror has correctly perceived whether the accused is guilty or not, the majority required to be confident that the jury's verdict is correct is a simple function of p and $(a - b)$, where a is the number of jurors in the majority and b is the number of jurors in the minority (Austen-Smith and Banks 1996; McCannon 2015).

The other classic result in binary social choice is due to May (1952): that the unique procedure to satisfy anonymity, neutrality, universal domain, and positive responsiveness is simple majority rule. May's result,
p. 802 along with Huntington's (1938) rediscovery of ↳ the importance of independence of irrelevant alternatives, Arrow's (1950, 1951) theorem, and Black's (1948, 1958) median voter theorem, marks the dawn of modern social choice. As all these scholars knew, binary choice leads to existence theorems. Nonbinary choice does not.

39.2.2 The Nonbinary Problem from Pliny to Arrow

If social choice is about the problems of nonbinary decision, then the first known social-choice theorist is Pliny the Younger, writing in circa 105 CE. A member of the Senate of Rome, Pliny was a keen parliamentarian, obsessed with matters of procedure and smugly pleased at his mastery of them. For instance, he speaks with pride of the Senate's unanimous adoption of the secret ballot (*Epistulae* III. 20, included in Radice 1969, 106). Pliny's *Epistula* VIII.14 became known in social choice when one of its modern pioneers (Farquharson 1969) rediscovered it (but see also Lagerspetz 2014). Farquharson used it as a running example of strategizing under binary and ternary procedure.

The debate concerned the freedmen of Afranius Dexter, who had been found dead. All senators, we may infer, assumed that the freedmen were complicit in the murder or suicide, and the options have been handily labeled *A*cquit, *B*anish, or *C*ondemn them to death (Farqhuarson 1969, annexure). Pliny was himself the leader of the acquitters. The acquitters formed a plurality but not a majority of the Senate. Pliny argued for a ternary vote—that each faction should stand in a different corner of the room, such that the largest party (his own) would then carry the day. His strategizing was matched by the leader of the condemners. When Pliny's procedure carried the day, the leader of the condemners led his troops out of their corner to stand with the banishers. Banishment won the day. On the information given by Pliny, plus a reasonable assumption of single-peakedness, banishment was the Condorcet winner: it beat both acquittal and condemnation in pairwise comparison. It carried the day once the manipulation and counter-manipulation by Pliny and his enemy had canceled out (Pliny, *Epistula* VIII.14, referenced in McLean and Urken 1995, 67–70; Farquharson 1969, 57–60, 64–67; Riker 1986, 79–88).

Pliny thinks like an economist. He would have understood Hume's recommendation to treat each voter as if he were a knave. But he does not think like an algebraist. That had to await the first Western mathematician to apply the new mathematics to the problems of social choice: Ramon Llull. The context was: How do humans discern the will of God when a nonbinary choice must be made—of a pope or an abbot, for example?

As Christianity became the official religion of the Roman Empire (from the 4th century CE), the parliamentary traditions of Pliny began to percolate into religious decision-making, but only through a

thick cloth. There is an element of majority rule in the decision-making of the Church councils that determined what was orthodoxy and what was heresy, but there was also violence and intimidation (MacMullen 2006). The difficulty for majority rule in the Western Church, even in binary contexts, came to
p. 803 focus on one Latin phrase: *maior et sanior pars* ("the larger and sounder part"). The ↳ trouble was, predictably, that every losing faction insisted that, although not *maior*, it was *sanior*—perhaps because it had a better understanding of the will of God, perhaps because it comprised older and therefore, by presumption, holier churchmen. There was a tendency for *sanior* to be (mis)read as *senior* (Colomer and McLean 2001).

Furthermore, at least one nonbinary decision needed to be taken whenever a pope died, was abducted, or became incapable: the election of his successor. Elections of pairs of popes by different factions of the Church provoked the intervention of Roman troops in 366 and 418, thus putting the Church under the protection of the state, from which it needed to escape. The emperor Honorius ruled in 420 CE that if two popes were elected, neither would be valid and a new election would be called, in which divine judgment, as revealed by unanimity (*divinum judicium et universitatis consensus*), would be required. It is not hard to guess whose *universitatis consensus* was likely to prevail if the emperor had soldiers and the Church did not.

To become independent, the Church had to shake off the state by ensuring that the state lost the opportunity to impose its form of unanimity on anyone else's. Learning by doing, the Church changed the rules for electing a pope several times, until it settled on a two-thirds majority rule, which is now, centuries on, known to have interesting stability properties (Caplin and Nalebuff 1988). The Church shook off secular control under Gregory VII (reigned 1073–1085). For a century thereafter, it maintained a fiction that election was always unanimous. Several procedures to manufacture unanimity were implemented, known as "acclamation," "scrutiny," and *compromissum* (in Roman law, "submission to arbitration"; http://thelawdictionary.org/compromissum/). Elections by "acclamation" were rare and probably intimidated. The formula *maior et sanior pars* solved nothing. The crucial breakthrough was the adoption of two-thirds majority rule by Pope Alexander III in 1179.

The qualified majority requirement produced stability, but increased risks of deadlock. The electors in 1216, 1241–1243, 1261, 1265, and 1268–1270 took months to reach a decision, having to resort to commissions in several cases. In 1216 and 1241–1243, the civil authorities of Rome locked up the cardinals until they produced a result. On the second occasion, the head of civil administration locked them up in an old, unhygienic building, guarded by police, and after two years, threatened to have the corpse of the dead pope exhumed and shown publicly in full papal regalia. In 1270, a crowd besieged the cardinals in the episcopal palace, removed its roof, and allowed nothing but bread and water to be sent in. A new pope was elected on this occasion by *compromissum* after a record vacancy of 34 months (Vauchez 1990, 522–523). Thus, the two-thirds rule produced efficacious and stable outcomes, at the price of long delays in decision-making. This is now recognized as a classic trade-off in social choice.

The stalemates led Pope Gregory X to adopt a new procedure known as the Conclave (Latin: "with key"), in 1274. It aimed to obtain a quick decision, and to prevent strategic maneuvering in the election of the pope. The cardinal electors were to lead a life in common in a single room, with no communication to the outer world. Food was to be supplied to them through a guarded window; the menu was restricted from the fourth
p. 804 day on and was reduced to bread, water, and wine after the ninth day. The cardinals ↳ received no income until they reached a collective decision. These provisions created strong and increasing incentives for them to agree. Some cardinals fell ill and some died in Conclave, precipitating agreement among the remaining participants (especially if they suspected that there was infectious disease in the room). All side payments, coercion, or explicit pacts between cardinals were forbidden, under penalty of excommunication and annulment of the election; they must keep silence during the election and afterward. These rules made exchanges and formation of large coalitions very difficult and often promoted agreed outcomes on the basis of the immediate, apparent appeal of some candidate. Similar institutions had been established in the

Dominican constitution of 1228, as well as in communes such as Venice and Piacenza, in 1229 and 1233, respectively (Ruffini Avondo 1925; Ullmann 1972).

The first papal election under this procedure, in 1276, was made in one day. The succeeding popes suspended the application of this procedure, whereupon long delays reappeared: more than 7 months in 1277, 6 months in 1281, almost 11 months in 1288, and 27 months in 1292–1294. This evidence moved the pope elected in 1294, Celestine V, to reestablish the Conclave. Successful short Conclaves have become normal since then, including in 2006 and 2013. Celestine V was enabled to enforce a rule because of his publicly known utility function. He was an elderly hermit who plainly did not want to be pope. After enforcing the rules for Conclaves, he resigned again to become a hermit: the last papal resignation until 2013 (Colomer and McLean 2001; McLean and Lorrey 2006).

So long as a stable system for papal elections was in place, most other Church appointments could simply be made by or on behalf of the pope ("supreme pontiff"). But this did not help the monastic orders. This problem fascinated the greatest medieval voting theorist, the Catalan mathematician and Christian apologist Ramon Llull (c.1232–1315). Llull lived on the Balearic Islands, on the frontier between Christian and Islamic civilizations; he was also familiar with Jewish civilization. After his conversion to devout Christianity in this thirties, he devoted his life and Art ("Art" is a term of art in Llull's writing, standing for a grand organized scheme of knowledge), to missionary efforts in which he tried to persuade anyone who would listen that Christianity was correct and that Judaism and Islam were both false. To this end, he made several missionary journeys to North Africa, which he survived. He also visited centers of Christian intellectual life, such as the University of Paris, where he complained that nobody understood him because of his "Arabic way of speaking" (Llull 1299; McLean and Urken 1995, 17).

Llull was fascinated by combinatorics, which he likely learned from contemporary Arab mathematicians—particularly with combinations of 2 from $n > 2$. Some of his applications seem wild. But his three known contributions to social choice are entirely sane. Two of them were rediscovered by McLean and London (1990), the third by Hägele and Pukelsheim (2001). The corpus is now readable in facsimile and in English, French, or German at www.math.uni-augsburg.de/stochastik/llull/welcome.html.

Two of the texts are in Latin, one in Catalan. The electoral methods proposed are not identical. However, since the discovery of the third (but earliest) document *Artificium electionis personarum* ("A Method of
p. 805 Electing Representatives") in the Vatican Library, ↳ it has become clear that the common idea behind all three is that of the Copeland (1951) method: *Make exhaustive pairwise comparisons among the set of candidates. Elect the candidate with the largest number of pairwise victories in the tournament. If there is more than one, hold a run-off election.* The Copeland method is in the family of Condorcet methods (Luce and Raiffa 1957, 358).

The last to be written, but first to be rediscovered, of Llull's three statements, *De arte eleccionis*, dated Paris 1299, is the weakest. It proposes an amendment, rather than an exhaustive tournament, which has the property that a top cycle would not be revealed, whereas in an exhaustive tournament it would. However, it was this text that provided the route into Llull's thought for the next pioneer, Nicholas Cusanus (and, nearly six centuries later, for McLean and London 1990).

Nicholas of Cusa/Cusanus (1401–1464) was a German churchman and mathematician. His contribution to social choice comes in his *De concordantia catholica* (1434; social choice section translated in McLean and Urken 1995; full English version in Sigmund 1991). He proposes a system of electing a Holy Roman emperor which is exactly the Borda count, scored for n candidates as 1 for the lowest candidate up to n for the highest. His library contains the unique known manuscript of Llull's text of 1299, in what the librarians believe is Cusanus's hand. By Cusanus's time, Llull was suspected by the Church authorities of having been too clever by half. His mystical doctrines had been formally condemned by the pope in 1376. However, his works were still studied in Padua, where Cusanus came across them at the dawn of Italian humanism. The ambiguous

wording of Llull's 1299 text could suggest that he was proposing a Borda rather than a Copeland scheme (depending on whether the instruction to the scrutineers is read as "sum votes" [i.e., Borda] or "sum wins" [i.e., Copeland]. See McLean and Urken 1995, 18). But even if Cusanus read Llull's scheme to be what we now call the Borda rule, he had an independent reason for proposing it. Llull's schemes involve open voting among members of a religious community who have to live with each other and with the result, and whose life depends on mutual trust. Cusanus's scheme involves secret voting among interest-holders (German princes who are Electors of the Holy Roman emperor) coming together just once to elect an emperor whom they might or might not obey. Llull's rules seem closer to those used by medieval monastic orders (McLean and Lorrey 2006), and Cusanus's, to the Eurovision Song Contest, which also uses Cusanus (Borda) voting.

Thus, by the fifteenth century CE, social-choice theory had been applied to two choice problems:

- Selecting the leaders of a self-governing religious community.
- Electing political leaders in an electoral body comprising mutually suspicious and self-interested electors.

Both address agency problems. In the first, humans are imperfectly informed agents of the will of God. In the second, electors are the highly imperfect agents of their population, of whom the Holy Roman emperor is thus an even more imperfect agent. ↳ However, after Cusanus, choice theory was lost again, as far as we know, until the French Enlightenment.

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In 1770, the military engineer Jean-Charles de Borda made an oral presentation to a meeting of the French Royal Academy of Sciences, outlining a new voting procedure. The minutes of the meeting record no comments or discussion. The permanent secretary of the academy, the Marquis de Condorcet, was an enemy of Borda. Their enmity turned on two related conflicts: between pure and applied science, and between independence and patronage when selecting academicians.

Borda was a loyal servant of the state. His skills as a civil engineer were useful to Louis XVI, and later to the one and indivisible French Republic, to whose service he switched seamlessly, surviving the Revolution and the Terror of 1789–1794. The state, which paid for the Academy of Science, thought it appropriate that those it found useful should become academicians.

Condorcet's withering assessment of Borda was uncovered in the 1880s:

> [H]e talks in Academy meetings and likes nothing better than to waste his time drawing up prospectuses, examining machines, etc. . . . [R]ealizing that he was eclipsed by other mathematicians, he abandoned mathematics for petty experiments (*la physicaille*) . . . some of his papers display some talent, although nothing follows from them and nobody has ever spoken of them or ever will. (Condorcet to A-R-J Turgot, 1775, quoted in Henry 1883, 214–215; our translation)

Why then did Condorcet, as secretary of the academy and the person in charge of its publications, resurrect Borda's paper between 1781 and 1784? Borda was serving with the French navy in the Caribbean, fighting in alliance with George Washington's Continental Army in the American War of Independence. His paper was published in the proceedings of the Academy of Science for the year 1781 (which appeared three years late).

Borda's paper (translated, with Condorcet's editorial comment, in McLean and Urken 1995, 81–89) is a model of modest clarity. He proposes the Borda rule, under which voters cast ballots ranking all the candidates, and each candidate is awarded a score of a for each ballot on which he is ranked in last place, incrementing by b ($b > 0$) per higher rank. The candidate with the highest total score is elected. In what we have labeled "Borda's equivalence theorem," Borda shows that his method can be implemented from a table

of exhaustive pairwise comparisons among the candidates. If *a* is set at 0 and *b* at 1, then each candidate's Borda score is exactly his number of votes in pairwise contests with each of the other candidates. He goes on to prove that, as the number of voters increases, the support for the plurality winner required to guarantee that he is also the Borda winner approaches unanimity, "and this surprising result justifies the way in which one of the Northern nations [viz., Poland] elects its kings." The unanimity rule for electing Polish kings led, unsurprisingly, to weakness, and hence to the invasion and carving up of Poland between neighboring powers (Borda 1784, quoted in McLean and Urken 1995, 89).

p. 807 Condorcet's introduction to Borda's paper is superficially polite, and the published version omits an incomplete sarcastic comment to be found in Condorcet's manuscript (details in McLean and Urken 1995, 83n1). He resurrected Borda's paper in order to rebut it in his own, vast *Essai sur l'application de l'analyse . . .* (*An Essay in the Application of Analysis to the Probability of Majority Decisions*: Condorcet 1785). The main theme of the *Essai* is the Jury theorem, discussed earlier. For binary inference, the Jury theorem is perfectly sound; but until Black (1958), it obscured the rival revolutionary idea that is struggling to get out in the *Essai*: that of the paradox of voting, and Condorcet's proposed solution.

The paradox of voting, or Condorcet's paradox, is just that when there are at least three voters and at least three options, there may be a cycle (the term was coined a century later by Lewis Carroll, but is used here for expository clarity). In the minimum case, a "Latin square" preference profile with voters 1, 2, and 3 and candidates A, B, and C, such that voter 1 prefers A to B to C, voter 2 prefers B to C to A, and voter 3 prefers C to A to B, produces the cycle A > B > C > A where > is read as "wins a majority against." Transitive individual preferences have generated an intransitive majority preference relationship.

Condorcet's solution is to look for what we now call the "Condorcet winner," viz., the candidate who beats each of the others in pairwise comparison. If there is a cycle such that no Condorcet winner exists, he proposes to break it by a procedure so obscure that it was not plausibly characterized until Young (1988).

The reason that the theory of social choice struggles to escape from the shadow of the analytically distinct Jury theorem was first spotted by Duncan Black (1958, 169–171). The Jury theorem shows that for a large jury making a binary decision, each juror's probability of correctness need only be slightly above the random point of 0.5 for the majority decision to have an acceptably low statistical probability of error. So far, so good, for the Jury theorem. But, embarrassingly for Condorcet, for probabilities of correctness only slightly greater than 0.5, the Borda winner is a better guide to the probably correct outcome than is the Condorcet winner (Black 1958, fig. 161). Black comments drily, "The situation is decidedly awkward for Condorcet's approach" (170). This awkwardness leads Condorcet away from probabilism toward social choice.

The 1785 *Essai* contains a deeply obscure passage that Black thought might prefigure some kind of independence axiom. Three years later, Condorcet thought of an example that shows with crystal clarity that the Borda rule violates what we now label "independence of irrelevant alternatives" (IIA). Discussing a three-candidate example (see table 39.1) involving Peter, Paul, and Jack, in which Peter is the Condorcet winner and Paul is the Borda winner, Condorcet writes:

> But how is it that Paul is not the clear winner[?] . . . The points [Borda] method confuses votes comparing Peter and Paul with those comparing either Peter or Paul to Jack and uses them to judge the relative merits of Peter and Paul. As long as it relies on irrelevant factors to form its judgement,
> p. 808 it is bound to lead to error. (adapted from translation in McLean and Urken 1995, 34) ↳

Table 39.1 Independence of Irrelevant Alternatives in 1788. The Borda Rule Violates IIA: Condorcet's Example

# of voters	30	1	29	10	10	1
Favorite	A	A	B	B	C	C
Middle	B	C	A	C	A	B
Least-liked	C	B	C	A	B	A

Source: McLean and Urken 1995.

A = Peter, B = Paul, C = Jack

Pairwise comparisons:

A > B (41/40)

A > C (60/21)

B > C (69/12)

Borda count ($a = 0$, $b = 1$):

A = 101

B = 109

C = 33

So, the fact that the Borda count violates IIA was discovered before 1788, but forgotten again for most of the intervening years. On being shown a draft of this translation in 1990, Kenneth Arrow expressed surprise at being scooped by almost 200 years, although he rightly pointed out that Condorcet fails to formalize his IIA axiom (personal communication).

A few mathematicians and policy analysts understood what Condorcet was talking about. Condorcet himself died heroically in the Terror of 1794. But one of his baroque voting procedures, designed to select Condorcet winners when they existed, had been implemented in Geneva. It did not work, and its failings, such as sometimes failing to select an extant Condorcet winner, were analyzed by the Genevan mathematician Simon Lhuilier. A Spanish mathematician, José Isidoro Morales (cf. Ródanes 2016), read in a newspaper report that the Institut de France (the successor of the Academy of Sciences, reconstituted after the Terror) had adopted the Borda rule for the election of academicians. He thought up, *ab initio*, a justification for the Borda rule, deeming it suitable for elections of people but not for votes on propositions, which he sent to the Institut in 1797. There it lay unread (as far as we know) for almost two centuries. A later officer of the Institut, P. C. F. Daunou, had been an associate of Condorcet before the Terror. The Institut introduced the Borda rule for electing new academicians in 1796. It abandoned it (despite Morales) in 1800, following an intervention by the emperor Napoleon. The replacement system, although devised by a commission including Laplace, was defective, Daunou shows. He also confirms a remark of Borda himself, who had said
p. 809 "My election method is only for honest men." Voters had found out how to ↳ manipulate his rule by placing the most dangerous rivals to their favorite candidates at the bottom of their lists. This is a practical implication of Borda's violation of IIA. Daunou's work proves that it was discovered very quickly. (Texts and further details are in McLean and Urken 1995, 38–41, 151–276).

Did any Americans understand Condorcet's revolutionary social mathematics in the era of writing and arguing about the U.S. Constitution? Thomas Jefferson was American minister in Paris from 1784 to 1789. He quickly made links with Enlightenment intellectuals, including Condorcet. He sent a copy of the *Essai* to James Madison with instructions to pass it on to Governor Edmund Randolph of Virginia. Madison had the *Essai* on his desk for nine days before passing it on to Randolph (Madison to Randolph, August 2, 1788, in

Stagg 2010). Various scholars have scrutinized the writings of Jefferson, Madison, and Alexander Hamilton for signs of understanding Condorcet. Jefferson for sure, and quite possibility either or both Madison and Hamilton, understood some of Condorcet's probabilistic reasoning. The evidence is clearest in the case of Jefferson's letter to Madison, sent on September 6, 1789, and anthologized as "The earth belongs to the living" (e.g., in Peterson 1984, 959–964). There are also Condorcetian echoes in Madison's Federalist #10 and Hamilton's Federalist #68. In #10, Madison speaks of republican government as maximizing "the probability of a fit choice" in the selection of a president. In #68, defending the Electoral College, Hamilton commends the college as offering "a moral certainty that the office of President will seldom fall to the lot of any man who is not in an eminent degree endowed with the requisite qualifications." Like Madison, he uses the word "fit" to describe the person selected by the Electoral College to be president.

These are intriguing echoes. In particular, Hamilton's phrase "moral certainty" looks like a translation of Condorcet's *certitude morale*—a key concept of the *Essai*. When the probability of an incorrect choice has been reduced to an acceptably low level (which Condorcet derives from life tables to be 1/144,768), we have a *certitude morale* that the outcome is correct. But Hamilton's use of the phrase may be a coincidence. And no group analyzing these documents has convincingly shown that any U.S. Framer understood the social-choice component of Condorcet's reasoning. In the one certain connection, Madison rejects a request from the go-between Filippo Mazzei to translate Condorcet's unicameralist *Lettres d'un bourgeois de New Haven à un citoyen de Virginie*, writing

> I did not translate the comment on the federal Constitution, as you wished, because I could not spare the time as well as because I did not approve the tendency of it. . . . If your plan of a single Legislature etc. as in Pena. were adopted, I sincerly [*sic*] believe that it would prove the most deadly blow ever given to republicanism. (Madison to Mazzei, Dec. 10, 1788, cited by McLean and Hewitt 1994, 66)

The controversy is still open. Some believe that there is no crossover from Condorcet to the Americans (McLean and Urken 1992, 1995; McLean and Hewitt 1994); others that there is (Schofield 2006, 2013).

p. 810 After Daunou, social choice went back to sleep. Even mathematicians as sharp as P.-S de Laplace and A. A. Cournot showed no sign of understanding their predecessor Condorcet's revolutionary discoveries. The story jumps forward to the 1870s. Both Lewis Carroll (C. L. Dodgson, 1832–1898) and Edward Nanson (1850–1936) were English mathematicians who may have learned about Condorcet from Todhunter (1865). But if Todhunter offered them their way into social choice, it was in spite of himself:

> [T]he theory of elections fares badly at his hands. He has only gone a short distance when he exclaims in regard to one of its more obvious and entirely well-founded theorems: 'Unfortunately these propositions are not consistent with each other.' (Black 1958, 161, citing Todhunter 1865, 375).

Todhunter had missed the essential problem in social choice—cycling—when it was under his nose.

This is not the only reason for regarding Dodgson's rediscovery of the paradoxes of social choice as entirely his own work. In the late 1940s, Black visited Christ Church, Oxford, whose library, directly opposite Dodgson's college rooms, would have been his first port of call. Black found that the crucial volume of the *Histoire et Mémoires de l'académie des sciences*, dated 1781 and published in 1784, was uncut. This was the volume containing Borda's paper and Condorcet's commentary. Nobody had ever opened it (Black 1958, 193–194). Nor have they yet, as the library kindly allows the present writer to use it from time to time as a show-and-tell in lectures on Dodgson and on voting theory.

Dodgson wrote three short and totally original pamphlets on social choice (Dodgson 1873, 1874, 1876, reproduced in McLean and Urken 1995, 279–297), in the contexts of electing fellows (faculty) in his Oxford college or deciding controversial architectural questions. At first he adopts the Borda rule. He then discards it, having found out its manipulability (at least in the context of academicians electing new colleagues). He introduces the term "cycle" and the matrix notation that is still useful for computing Condorcet or Borda winners. In Dodgson (1876), he proposes a Condorcet rule with a cycle-breaking procedure: "When the issues . . . have been reduced to a single cycle, the Chairman shall inform the meeting how many alterations of votes each issue requires to give it a majority over every other separately" (290).

This compound procedure is now known as the Dodgson rule.

Dodgson's introduction to the cycle-breaking module makes it sound trivial. It is not at all trivial, as Dodgson's worked examples (Dodgson 1876, figs. 3–4, 9–10) warn the reader. Bartholdi et al (1989) title their article on the computability of the Dodgson rule "Voting Schemes for Which It Can Be Difficult to Tell Who Won the Election," with the comment, "We think Lewis Carroll would have appreciated the idea that a candidate's mandate might have expired before it was ever recognized" (Bartholdi et al. 1989, 161).

Dodgson's work on proportional representation was equally original, although Black initially (but see Black 1996) failed to recognize this. It was written in (and for) a strong two-party system, as in Victorian Britain.
p. 811 In discussing the class of limited-voting ↳ schemes (in which there are m seats to fill in a district and each voter has $n < m$ votes), Dodgson uses game-theoretic reasoning to prove that in equilibrium, the single-non-transferable vote (SNTV) is no less likely to yield a proportional result than the much better-known single-transferable vote (STV), but without the latter's technical defects. That he reasoned in this way several decades before a notation for game theory even existed is quite striking. His result anticipates much later results on equilibrium derived independently by Cox (1994).

Nanson was plucked from Cambridge in 1875, when the infant University of Melbourne was stranded without a mathematician. Nanson went there as professor of mathematics and stayed there for the rest of his life. Like Dodgson, he wrote about both majority rule and proportional representation. On majority rule, Nanson (1883), unlike Dodgson, had read the French classics. It remains a mystery *how* he read them. There were no library copies in Melbourne. Did he carry them from Cambridge? He proposes a Condorcet-Borda hybrid rule, differing from Dodgson's. There is a (very) small literature on the properties of Nanson's rule (Niou 1987; Fishburn 1990). However, it was implemented in the Universities of Adelaide and Melbourne for some years for elections of faculty members to university committees. The Melbourne version was programmed in COBOL by the university's registrar in the 1960s (McLean 1996), but both universities have since abandoned the rule.

On proportional representation, Nanson was active when the Commonwealth of Australia's constitution was being written (1892–1900) and several state constitutions rewritten. Australia is well known as a laboratory for electoral systems, being in particular the place where both alternative vote (AV; preferential voting) and STV have been used for the longest continuous times. However, these electoral systems were mostly adopted by politicians who perceived partisan advantages in them, and not because of any mathematical arguments advocated by Nanson (McLean 1996).

39.2.3 Agency Through Representation: Two Incompatible Conceptions

The principal–agent conception of electoral representation is but one of two, both etymologically sound, both with medieval origins. The oldest meaning of the verb "to represent" in the *Oxford English Dictionary*, attested since 1390, is:

> To assume or occupy the role or functions of (a person), typically in restricted, and usually formal situations; to be entitled to speak or act on behalf of (a person, group, organization, etc.); (in later use esp.) to act or serve as the spokesperson or advocate of. (*OED* online, "represent," v.t., sense 1a; www.oed.com/view/Entry/162991#eid25855688)

I represent you if I stand for you in a place where you cannot: for example, if I am your lawyer, with expert
p. 812 knowledge that you lack; or if there are many of you, who cannot all ↳ attend Parliament, so I attend it as your representative. This is the principal–agent conception. You are the principal(s), and I am your agent.

But almost as old (attested from 1400) is sense 8b: "b. To bring clearly and distinctly before the mind or imagination; to describe, evoke, conjure; to imagine, conceptualize." This is the sense in which a painting or a play re-presents a character. When applied to a large body of people, it leads to the "microcosm" conception of representation. In the French and American Revolutions, the Comte de Mirabeau and John Adams came (independently, I think) to the same idea: that in some sense, the assembly should be a microcosm of the people who elected it (cited in McLean 1991, 173).

Etymologically, both senses are perfectly valid. But they are incompatible. One person cannot be a microcosm of many, as the many comprise different genders, different ages, different ethnicities, and have different interests and values. For legislative elections, the microcosm conception points to proportional representation; the principal–agent conception points to single-member districts.

39.2.4 Judgments vs. Preferences

Aggregating judgments and aggregating preferences are two different things. In principle, all the jurors want the same thing, which is to find out the unknown truth: Did the accused commit the crime as charged, or not? There should be no difference between conservative and socialist jurors.

A less pure example is selecting somebody for a job. All members of the selection committee want the best person for the job. But they may have different ideas as to the qualities of the ideal candidate, so it is not a pure case of judgment aggregation.

A mass election is quite different. Some voters are moved by interests; some by ideology; many by both. Interests and ideologies differ. There is no sense in which the voters can be said to be groping toward an unknown truth. For this reason, even Black (1958) was unable to see the full relevance of Condorcet's work.

39.2.5 Electing One vs. Electing Many

The third dimension for judging systems asks: Is this an election of one person, or of more than one? Obviously, the concept of proportional representation has no meaning if there is only one post to fill. A president cannot be male, female, black, white, rich, poor, straight, gay … in the same proportions as the population. There is only one of him (or her). To elect a multi-member body, such as a parliament, then, the principal–agent and microcosmic conceptions point in different ways. As already noted, the former points to single-member districts, so that each group of principals knows for sure who is their agent. The latter points to proportional representation, which cannot be achieved in single-member districts, for the reason just given.

p. 813 Hence, there are two distinct reasons to say that there is no best electoral system. One is rooted in Arrow and social choice. The other is that the question is not fully specified. It should be "What is the best electoral system *for the purpose in hand?*"

39.3 Concluding Thoughts: Why Did It Take So Long?

The advances in social and public choice reviewed in this chapter required a range of skills and perspectives. First, they required the tools of logic and algebra. Logic was studied in ancient Greece; algebra was not. The most stunning intellectual feats of the pre-Arrow era are: the first use of matrix notation and algebraic symbols by Llull in the 1280s; the adaptation of the notation for permutations and combinations, by Condorcet in 1785; and the use of game-theoretic reasoning by Dodgson in his writings on proportional representation in the 1880s, some 50 years before the notation of game theory was first set down. Second, they required people who thought like economists when they came to study institutional design. That description fits many of the conventional classical writers on politics, including Hobbes, Hume, Smith, and Madison. Third, for their ideas to be implemented, they also had to be understood by institutional designers themselves. That is why we have focused on the intellectual history of Madison (and in other writing, of James Wilson) at the U.S. Constitutional Convention.

These three requirements do not often coexist. Hence, the strange and episodic history detailed in this chapter.

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