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Novel method for the design and assessment of direct contact membrane distillation modules

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Highlights

- An Effectiveness-NTU (number of heat transfer units) method is developed for DCMD
- This method can be used to either size or rate co- and counter-current DCMD modules
- Production rate of DCMD module can be estimated within 6% accuracy
- The computational cost is minor compared with discretised models

Abstracts

An equivalent effectiveness-number of heat transfer units (ϵ -NTU_{MD}) method was developed for direct contact membrane distillation. Efficient performance rating and design sizing for individual DCMD modules can be rapidly made based upon limited experimental data. Using this method, the construction of a specific finite element model and their associated costs, involving both time and expenditure, are avoided. Instead the module performance or sizing requirements can be estimated efficiently using a set of expressions based on the conventional ϵ -NTU expressions used for the design of heat exchangers. The outlet temperatures are also predicted which is useful for the design of the overall DCMD process and module cascading networks. The ϵ -NTU_{MD} method was validated against an experimentally validated discretized model of a flat sheet DCMD module, built using MATLAB. A correction function is included in the ϵ -NTUMD method proposed which results in 100% of the data having an accuracy of better than 6%. Method validation was done for both co- and countercurrent flow, with a range of module dimensions, flowrates and membrane permeabilities.

1. Introduction

Membrane distillation (MD) has become a popular research area in recent years, particularly for desalination. It has the potential to tackle the shortage of potable water while using relatively small amounts of high grade energy; the main energy supply for MD is low grade heat. Compared to other established desalination processes, MD has the advantages of: (a) ambient operational pressure; (b) minimal chemical interactions; (c) 100% (theoretical) rejection of ions; and (d) ability to process highly concentrated feed. There are four commonly acknowledged MD configurations, namely, direct contact membrane distillation (DCMD), air-gap membrane distillation (AGMD), vacuum membrane distillation (VMD) and sweeping gas membrane distillation (SGMD). They are named accordingly to and differentiated by the media adjacent to the permeate side of the membrane. With the simplest design, the main disadvantage of DCMD is the relatively high conductive heat loss due to the direct contact between both fluids and the membrane. Compared to DCMD, the heat loss is reduced in AGMD but the stagnant air increases the mass transfer resistance and the transmembrane flux is then limited by diffusion across the air gap and evaporation through the pores. As a result the permeate flux of AGMD is significantly less than that of DCMD. In VMD, a low pressure or vacuum is applied on the permeate side for a higher permeate flux. Yet this sometimes gives rise to a more severe problem of membrane wetting compared to other configurations. An external

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condenser is required to collect the permeate in SGMD which is less desirable due to additional complexity and contributes to extra expense. Heat recovery is also difficult in this configuration. While each configuration has its merits and disadvantages, DCMD is the most considered configurations out of the four. It has the simplest design, with a higher permeate flux compared to AGMD, with less aggressive membrane wetting compared to VMD and without the complication of a condenser compared to SGMD.

The governing equations adopted for DCMD are very similar to those used in heat exchangers but instead of mere heat conduction across the fluids contact surface, an additional term is required as heat is also carried through the membrane via the permeate flux. With other membrane contactor processes the key performance indicators are usually: transmembrane flux, overall module permeation and separation ratio, but due to the thermal-driven nature of membrane distillation and the resultant coupled heat and mass transfer across the membrane, thermal efficiency is also an important parameter in design and evaluation of DCMD systems.

Due to its robustness, the effectiveness-number of transfer units ϵ -NTU method, popularised by Kays and London has for many years been the industrial standard for designing heat exchanger systems and networks. As a design tool, the ϵ -NTU method can be used to calculate the rate of heat transfer in heat exchangers when the temperature profile is unknown and thus there is insufficient information to obtain the log-mean temperature. It is also used to determine the required surface area of the heat exchanger for a fixed effectiveness and given set of inlet conditions. Recently, different researchers have used the backbone of the original version of ϵ -NTU, which originally only applies to heat exchangers, for various mass transfer systems. Sharqawy et al. has developed the effectiveness-mass transfer units (ϵ -MTU) for pressure retarded osmosis (PRO) membrane mass exchanger and similarly, Banchik et al. has developed ϵ -MTU for reverse osmosis (RO). Some key features of these two models are presented in Table 1. Others have also looked into the use of ϵ -NTU in dehumidification, which is a conjugate heat and mass transfer system.

Aiming to provide a similar ϵ -MTU method for DCMD, this paper adopts the derivation of the conventional ϵ -NTU method but with modifications which accommodate the conjugate heat and mass transfer in MD. In this present work, a set of ϵ -NTU expressions are developed specifically for DCMD. These ϵ -NTU expressions can provide good and robust estimations for the outlet temperatures, thermal efficiency of individual modules and the amount of transmembrane flux with a straightforward algorithm, reducing the time and computing cost required in cases when finite element models are constructed and used for design purpose. The method has been labelled as ϵ -NTU_{MD}.

2. Theory and analytical derivation of the ϵ -NTU_{MD} method for DCMD

Figure 1: Schematic drawings (a) DCMD countercurrent model (left) and (b) DCMD cocurrent model (right)

Table 1 Key features of the ϵ -MTU model for PRO and RO

Model	$\mathcal{E}$ -MTU for Ideal PRO [24]	$\mathcal{E}$ -MTU for RO [25]
Simplifications and assumptions	<ul style="list-style-type: none"> • Idealized model • Concentration polarization effects are neglected • Presser drop through flow channel is negligible • 100% salt rejection rate • Within the salinity range, the osmotic pressure follows van't Hoff's law, i.e. linearity between osmotic pressure and salinity • The PRO membranes can withstand arbitrary net driving pressures 	<ul style="list-style-type: none"> • Membrane permeability is constant and independent of feed salinity • Concentration polarization effects are incorporated via use of a dimensionless correction factor (Tables of correction factor are provided for feed concentration) • Hydraulic pressure drop along the flow channel is negligible • 100% salt rejection rate • Osmotic pressure follows van't Hoff's law, i.e. linearity between osmotic pressure and salinity⁽¹⁾
Key analogy and modification to conventional $\mathcal{E}$ -NTU for heat exchangers	<ul style="list-style-type: none"> • Instead of temperature difference, the driving potentials are the concentration and pressure differences • Effectiveness is expressed as the recovery ratio achieved and the maximum recovery ratio of the system • Four dimensionless groups were introduced to express the recovery ratio of the membrane 	<ul style="list-style-type: none"> • Instead of temperature difference, the driving potentials are the concentration and pressure differences • Effectiveness is expressed as the recovery ratio achieved and the maximum recovery ratio of the system • Four dimensionless groups were introduced to express the recovery ratio of the membrane • A correction factor is introduced into the model to allow for the effects of concentration polarization and nonlinearity between osmotic pressure and salinity
Model validation	<ul style="list-style-type: none"> • Results were validated using a numerical model that includes the nonlinear function for the osmotic pressure • Error percentage of approximately 5% reported for the calculations of recovery ratio and effectiveness. • Maximum error of 20.3% is reported when evaluating feed stream concentration in the case of power production using counterflow module 	<ul style="list-style-type: none"> • Results were compared with literature data, with the assumption that the mass transfer coefficient is held constant throughout the exchanger • With the incorporation of the correction factor, a mean error of 7.8% and a maximum of 29.3% were found

⁽¹⁾Only for initial development, adjustment were made to improve the model by introducing a correction factor

A few assumptions were adopted in the derivation of the ε -NTU_{MD} method; as with the work from MIT on PRO and RO [24, 25] the aim has been to develop a robust method. Thus for the designing or rating of a DCMD module the following approximations are reasonable:

- (a) Low salinity in the feed solution i.e. around that of seawater or lower. In this case, the effect of concentration polarization is negligible and chemical potential of water is approximated to unity (in reality the activity coefficient of seawater at 298K is 0.98)
- (b) Kinetic and potential energy effects are negligible
- (c) The effective membrane permeability, K_{eff} and overall heat transfer coefficient, U (equations (6) and (9) below) are, as suggested in , taken to be invariant with position. This is consistent with the classical approach to the design of heat exchangers.

Figure 1 shows the schematic drawings of the two DCMD configurations. The symbols used are conventional and a list is given at the end of the paper. The governing equations of the heat and mass transfer across the membrane of DCMD are given as:

$$q = h_f(T_f - T_{fm}) \quad (1)$$

$$q = h_p(T_{pm} - T_p) \quad (2)$$

$$q = N\lambda + \frac{k_m}{\delta}(T_{fm} - T_{pm}) \quad (3)$$

and the transmembrane flux, N , is given as:

$$N = \frac{K}{\delta}(P_{fm} - P_{pm}) \quad (4)$$

or in terms of temperature, as suggested in ,

$$N = K_{eff}(T_f - T_p) \quad (5)$$

where K_{eff} is given as:

$$K_{eff} = \frac{K^*}{\delta + \frac{k_m}{U_L} + \frac{\lambda K^*}{U_L}} \quad (6)$$

where

$$U_L = \left[\frac{1}{h_f} + \frac{1}{h_p} \right]^{-1} \quad (7)$$

and K^* is the membrane coefficient expressed in $\text{kg m}^{-2} \text{s}^{-1} \text{K}^{-1}$, as described in Eq. (A1) in the Appendix A1.

Substitution of Eq. (1), (2) and (5) into (3) and rearrangement yields:

$$q = \left(\frac{K_{eff}\lambda + \frac{k_m}{\delta}}{1 + \frac{k_m}{\delta h_p} + \frac{k_m}{\delta h_f}} \right) (T_f - T_p) \quad (8)$$

Defining

$$U = \left(\frac{\delta}{k_m} + \frac{1}{h_p} + \frac{1}{h_f} \right)^{-1} \quad (9)$$

and

$$\alpha = U \left(1 + \frac{K_{eff}\lambda}{\frac{k_m}{\delta}} \right) \quad (10)$$

$$q = \alpha(T_f - T_p) \quad (11)$$

Conservation of energy determines the temperature changes in the feed and permeate flows along the module. It follows that as:

$$dT_f = \frac{-q dA}{\dot{m}_f c_f} \quad (12)$$

$$dT_p = \frac{-q dA}{\dot{m}_p c_p} \quad (13)$$

Combining Eq. (12) and (13) then substituting Eq. (11) yields the differential equation below:

$$d(T_f - T_p) = -\alpha(T_f - T_p)dA \left(\frac{1}{\dot{m}_f c_f} + \frac{1}{\dot{m}_p c_p} \right) \quad (14)$$

For a flat sheet membrane module with uniform flow channels along the module, the NTU expressions in Table 2 and Table 3 are obtained by solving Eq. (14) with the boundary conditions given below. The resultant expressions are given alongside the standard heat exchanger expressions. For the standard heat exchanger reference is made to hot and cold streams, and for DCMD to feed and permeate streams. Do note that capital C's refer to heat capacity rate (the product of mass flowrate and specific heat capacity) as is standard in ϵ -NTU analysis. In the case of DCMD, both feed and permeate have similar fluid properties and the specific heat capacities c_f and c_p are similar.

Cocurrent flow:

$$\text{At } A = 0 \quad T_f - T_p = T_{f,in} - T_{p,in} \quad (15)$$

$$\text{At } A = A_t \quad T_f - T_p = T_{f,out} - T_{p,out} \quad (16)$$

Countercurrent flow:

$$\text{At } A = 0 \quad T_f - T_p = T_{f,in} - T_{p,out} \quad (17)$$

$$\text{At } A = A_t \quad T_f - T_p = T_{f,out} - T_{p,in} \quad (18)$$

where A_t denotes the total surface (membrane) area.

3. Applications of the new ϵ -NTU_{MD} expressions for DCMD

From the set of modified ϵ -NTU_{MD} expressions for DCMD one can straightforwardly determine either the stream outlet temperatures when the module size is known, or the membrane area required for the desired overall heat transfer across the membrane. However as the estimation of the value of transmembrane flux is of primary concern and the main performance indicator in MD systems it is important that design tools can (i) determine the overall permeate production of a given DCMD membrane module and (ii) estimate the size a module given a desired production rate. To achieve (i) and (ii), an accurate estimation of the membrane module thermal efficiency, $\eta_{mod.}$ is required. Now,

$$\dot{m}_{production} = \frac{Q \eta_{mod.}}{\lambda} \quad (19)$$

Typically, the thermal efficiency of a DCMD module is given by:

$$\eta = \frac{N\lambda}{Q} \quad (20)$$

And substituting Eq. (5) and (10) into Eq.(20) gives:

$$\eta = \frac{K_{eff}\lambda(1 + \frac{k_m}{\delta h_p} + \frac{k_m}{\delta h_f})}{K_{eff}\lambda + \frac{k_m}{\delta}} \quad (21)$$

Although there is a very modest change in temperature profile along the module, this work has assumed a constant $\eta_{mod.}$. An accurate estimation of the overall module efficiency, $\eta_{mod.}$, can be obtained by taking the average of the η obtained at both ends of the module. The validity of assuming a constant energy efficiency $\eta_{mod.}$ along the module will be commented upon in section 6. Details of the method for obtaining the value of K_{eff} are included in the Appendix A1.

The overall algorithm and method validation Figure 2 and 3 show respectively the algorithms to rate and size a DCMD module using the ϵ -NTU_{MD} expressions. It is shown in Figure 3 that the calculation of the membrane area needed for a specified amount of permeate includes an extra step; this step verifies whether such demand is possible for the operating conditions supplied. Mathematically the step is the one where one checks whether $Q_{max} > Q_{req.}$.

From Eq.(19), the heat required is given by:

$$Q_{req.} = \frac{\dot{m}_{production}\lambda}{\eta} \quad (22)$$

Figure 2 Flowchart for DCMD ‘rating’ calculations. The ϵ -NTU_{MD} method is used to obtain the permeate flowrates given inputs that include membrane area.

Figure 3 Flowchart for DCMD design calculations; using the ϵ -NTU_{MD} method to obtain the required membrane area for a specified production rate.

It is worth highlighting that the quality of results mainly depends upon having a suitable value for K^* and good approximation of U_L . The assumptions and other approximations used in this work have a much less significant impact to the accuracy of the method.

4. Determination of the correction function

A correction function is introduced to enhance the accuracy of the ϵ -NTU_{MD} method. Literature suggested that a higher feed flowrate has a greater influence on transmembrane flux compared with the effect of the same change in flowrate of the permeate side. This means that an improvement in h_f will have a greater beneficial effect than one in h_p . Now this method is insensitive to the location of the individual values of the two heat transfer coefficients and the correction function reduces the resulting relatively minor but consistent offset. More discussions and explanations regarding this are included in the result and discussion section.

The proposed expression for the correction factor is:

$$\xi = \mathcal{H}_1 \left(\frac{h_f}{h_p} \right)^{\mathcal{H}_2} \quad (23)$$

Where $\mathcal{H}_1$ and $\mathcal{H}_2$ are constants to be found and the correction factor is to be incorporated into the method as shown in Eq. (24). This is of similar nature to the correction for heat transfer correlations provided by Sieder and Tate which takes the form $(\mu_b/\mu_w)^{0.14}$, wherein the two terms are the bulk and wall viscosity respectively.

$$\dot{m}_{\text{production}} = \frac{Q\eta_{\text{mod.}}}{\lambda} \xi \quad (24)$$

Using the algorithms described in the last section, the initial permeate production (without incorporating the error function) were computed and compared with the result obtained from the discretized, numerical model described in the next section. The error function is then introduced. Excel Solver was used to evaluate the values of $\mathcal{H}_1$ and $\mathcal{H}_2$ that give the minimum sum of percentage errors across all the data.

The expressions obtained for the error functions for co- and counter-current modules were:

$$\xi_{\text{cocurrent}} = 1.0 \left(\frac{h_f}{h_p} \right)^{0.125} \quad (25)$$

$$\xi_{\text{countercurrent}} = 0.96 \left(\frac{h_f}{h_p} \right)^{0.111} \quad (26)$$

Table 2 Comparison for countercurrent flow of the expressions for the conventional case and the modified case for DCMD.

Flow condition	Conventional ϵ -NTU	Modified ϵ -NTU _{MD} for DCMD	
$m_{h,in}c_h > m_{c,in}c_c$ $m_{f,in}c_f > m_{p,in}c_p$ Countercurrent flow	$\epsilon = \frac{Q}{Q_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_c(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_c(T_{h,in} - T_{c,in})}$	$\epsilon = \frac{Q}{Q_{\max}} = \frac{C_f(T_{f,in} - T_{f,out})}{C_{\min}(T_{f,in} - T_{p,in})} = \frac{T_{p,out} - T_{p,in}}{T_{f,in} - T_{p,in}}$	(27)
	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{c,in} c_c}{\dot{m}_{h,in} c_h}$	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{p,in} c_p}{\dot{m}_{f,in} c_f} = \frac{T_{f,in} - T_{f,in}}{T_{p,out} - T_{p,in}} = \frac{C_p}{C_f}$	(28)
	$NTU = \left(\frac{UA_t}{C_{\min}} \right) = \left(\frac{UA_t}{C_c} \right)$	$NTU_{MD} = \left(\frac{\alpha A_t}{C_p} \right) = \left(\frac{K_{eff}\lambda + \frac{k_m}{\delta}}{1 + \frac{k_m}{\delta h_p} + \frac{k_m}{\delta h_f}} \right) \left(\frac{A_t}{C_p} \right)$	(29)
	$\epsilon = \frac{1 - \exp [(-NTU)(1 - C_r)]}{1 - C_r \exp [(-NTU)(1 - C_r)]}$	$\epsilon = \frac{1 - \exp [(-NTU)(1 - C_r)]}{1 - C_r \exp [(-NTU)(1 - C_r)]}$	(30)
$m_{c,in}c_c > m_{h,in}c_h$ $m_{p,in}c_p > m_{f,in}c_f$ Countercurrent flow	$\epsilon = \frac{Q}{Q_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_h(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_h(T_{h,in} - T_{c,in})}$	$\epsilon = \frac{Q}{Q_{\max}} = \frac{C_p(T_{p,out} - T_{p,in})}{C_{\min}(T_{f,in} - T_{p,in})} = \frac{T_{f,in} - T_{f,out}}{T_{f,in} - T_{p,in}}$	(31)
	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{h,in} c_h}{\dot{m}_{c,in} c_c}$	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{f,in} c_f}{\dot{m}_{p,in} c_p} = \frac{T_{p,out} - T_{p,in}}{T_{f,in} - T_{f,out}} = \frac{C_f}{C_p}$	(32)
	$NTU = \left(\frac{UA_t}{C_{\min}} \right) = \left(\frac{UA_t}{C_h} \right)$	$NTU_{MD} = \left(\frac{\alpha A_t}{C_f} \right) = \left(\frac{K_{eff}\lambda + \frac{k_m}{\delta}}{1 + \frac{k_m}{\delta h_p} + \frac{k_m}{\delta h_f}} \right) \left(\frac{A_t}{C_f} \right)$	(33)
	$\epsilon = \frac{1 - \exp [(-NTU)(1 - C_r)]}{1 - C_r \exp [(-NTU)(1 - C_r)]}$	$\epsilon = \frac{1 - \exp [(-NTU)(1 - C_r)]}{1 - C_r \exp [(-NTU)(1 - C_r)]}$	(34)

Table 3 Comparison for cocurrent flow of the expressions for the conventional case and the modified case for DCMD.

Flow condition	Conventional ε -NTU	Modified ε -NTU for DCMD
$m_{h,in}C_h > m_{c,in}C_c$ $m_{f,in}C_f > m_{p,in}C_p$ Cocurrent flow	$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_c(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_c(T_{h,in} - T_{c,in})} \quad (35)$	$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_f(T_{f,in} - T_{f,out})}{C_{\min}(T_{f,in} - T_{p,in})} = \frac{T_{p,out} - T_{p,in}}{T_{f,in} - T_{p,in}} \quad (35)$
	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{c,in} C_c}{\dot{m}_{h,in} C_h}$	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{p,in} C_p}{\dot{m}_{f,in} C_f} = \frac{T_{f,in} - T_{f,out}}{T_{p,out} - T_{p,in}} = \frac{C_p}{C_f} \quad (36)$
	$NTU = \left(\frac{UA_t}{C_{\min}} \right) = \left(\frac{UA_t}{C_c} \right)$	$NTU_{MD} = \left(\frac{\alpha A_t}{C_p} \right) = \left(\frac{K_{eff}\lambda + \frac{k_m}{\delta}}{1 + \frac{k_m}{\delta h_p} + \frac{k_m}{\delta h_f}} \right) \left(\frac{A_t}{C_p} \right) \quad (37)$
	$\varepsilon = \frac{1 - \exp [(-NTU)(1 + C_r)]}{1 + C_r}$	$\varepsilon = \frac{1 - \exp [(-NTU)(1 + C_r)]}{1 + C_r} \quad (38)$
$m_{c,in}C_c > m_{h,in}C_h$ $m_{p,in}C_p > m_{f,in}C_f$ Cocurrent flow	$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_h(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_h(T_{h,in} - T_{c,in})} \quad (39)$	$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_p(T_{p,out} - T_{p,in})}{C_{\min}(T_{f,in} - T_{p,in})} = \frac{T_{f,in} - T_f}{T_{f,in} - T_{p,in}} \quad (39)$
	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{h,in} C_h}{\dot{m}_{c,in} C_c}$	$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{f,in} C_f}{\dot{m}_{p,in} C_p} = \frac{T_{p,out} - T_{p,in}}{T_{f,in} - T_{f,out}} = \frac{C_f}{C_p} \quad (40)$
	$NTU = \left(\frac{UA_t}{C_{\min}} \right) = \left(\frac{UA_t}{C_h} \right)$	$NTU_{MD} = \left(\frac{\alpha A_t}{C_f} \right) = \left(\frac{K_{eff}\lambda + \frac{k_m}{\delta}}{1 + \frac{k_m}{\delta h_p} + \frac{k_m}{\delta h_f}} \right) \left(\frac{A_t}{C_f} \right) \quad (41)$
	$\varepsilon = \frac{1 - \exp [(-NTU)(C_r + 1)]}{1 + C_r}$	$\varepsilon = \frac{1 - \exp [(-NTU)(1 + C_r)]}{1 + C_r} \quad (42)$

5. Numerical DCMD model

To determine the accuracy of the approximate analytical result given by the numerical DCMD model was developed. The numerical model was discretised into elements and outputs from each element including temperature and mass flowrate were used as inputs for the next element. For each element, the four DCMD simultaneous equations, equations (1)-(4), were solved using MATLAB *fsolve*, a simultaneous equation solver for coupled non-linear equations. Our model was validated against experimental results in our recent publications [13, 14].

Compared to the robust ε -NTU_{MD} method, the numerical model accounts for the nonlinear relationship between vapour partial pressure and temperature. It also includes allows the variation in thermal efficiency along the membrane module. The percentage error of the ε -NTU_{MD} method is calculated with respect to the output of the MATLAB discretised numerical model and given by:

$$\text{error} = \left| 1 - \frac{\varepsilon\text{-NTU}_{\text{MD}} \text{ result}}{\text{numerical result}} \right| * 100\% \quad (43)$$

In order to evaluate the accuracy of the robust ε -NTU_{MD} method, various design and operating conditions of DCMD systems were investigated and the outputs of the ε -NTU_{MD} method were compared with the outputs from the validated numerical model. A comparison of the MATLAB model and the NTU_{MD} model with experimental result is given in the Appendix A2. It is evidenced that the NTU_{MD} model achieves as good a degree of accuracy as the MATLAB model.

The simulation conditions are referred and compared to a base case defined by the values given in Table 4. Symbols are defined at the end of the paper.

Table 4 List of values used for the base case simulation

Parameters for base case	Assigned value
K_0	$8.6 \times 10^{-7} \text{ kg m}^{-1} \text{ s}^{-1} \text{ Pa}^{-1}$
G_{f0}	$3 \times 10^{-3} \text{ m}$
G_{p0}	$3 \times 10^{-3} \text{ m}$
$\dot{m}_{f0}$	0.025 kg s^{-1}
$\dot{m}_{p0}$	0.025 kg s^{-1}
T_{fin}	358 K
T_{pin}	298 K

Figure 4 Drawing of the cross section of one of the membrane channel (not to scale)

Both the feed and permeate channels have a width of 10 cm (depicted in Figure 4) and the membrane area is changed by changing the length of the module. For both the ε -NTU_{MD} and the numerical models, the heat transfer coefficients are calculated using Eq. (43), given in , for laminar flow in channels:

$$h = \frac{k}{D} 0.332 Re^{\frac{1}{2}} Pr^{\frac{1}{3}} \quad (43)$$

Whilst the validation was specifically for flat sheet systems the ε -NTU_{MD} method is, as with heat transfer theory, equally valid for any co-current or counter-current units. Just as the conventional ε -NTU method does not need to distinguish between shell-and-tube and plate heat transfer systems so we do not need to distinguish between flat plate and hollow fibre systems when deriving the equations; what counts is whether the flow is co-current or counter-current.

6. Result and discussions

In the plots below (Figures 5-9) the solid lines are simulated results obtained from our validated MATLAB discretised DCMD model whilst the outputs from the ε -NTU_{MD} method are given as symbols. It will be observed that the ε -NTU_{MD} provides excellent estimates of the performance of DCMD modules across a wide range of operating conditions. Thus the assumption of assuming a constant energy efficiency η_{mod} along the module is vindicated.

Figure 5 (A-left) and (B-right) Evaluation of ε -NTU_{MD} method: variation of module production rate with membrane area with changing membrane permeability. Solid lines are simulated result obtained from MATLAB discretised DCMD module.

Figure 6 (A-left) and (B-right) Evaluation of ε -NTU_{MD} method: variation of module production rate with membrane area with changing mass flowrate. Solid lines are simulated result obtained from MATLAB discretised DCMD module.

Figure 7 (A-left) and (B-right) Evaluation of ε -NTU_{MD} method: variation of module production rate with membrane area with changing membrane flowrate ratio. Solid lines are simulated result obtained from MATLAB discretised DCMD module.

In ε -NTU heat transfer analysis it is traditional to present the results as plots of efficiency (ε) versus the Number of Transfer Units (NTU) but for DCMD module design it is more appropriate to present the output

as module production rate versus membrane area because these are the design parameters of concern. In Figure 5-Figure 7 it is shown that in spite of changes membrane permeability, feed and permeate mass flowrates and ratio, the ϵ -NTU_{MD} method provides good estimations, comparable to the output from the numerical model. While the difference in the results between the two methods increases gradually as the production rate and the membrane area increase, the percentage error is consistently small, as shown in Figure 8. The accuracy in all cases considered was better than $\pm 6\%$ with an average of 2.3%.

One can thus conclude that a novel method for efficient performance rating and design sizing of DCMD modules has been developed. The ϵ -NTU_{MD} method is based upon well developed chemical engineering principles and thus its success is not surprising. The inclusion of the correction function avoids the systematic error that would occur without its inclusion; compare Figure 8 with Figure 9. However it is emphasised that even without the correction factor the percentage difference between the ϵ -NTU_{MD} method and the standard only reaches a maximum of 15.5% with 95% of the results being within $\pm 10\%$ to the data obtained from the numerical model. As this is for a wide ratio of the ratio of heat transfer coefficients it confirms that the basis of the ϵ -NTU_{MD} method is sound.

Figure 8 Percentage difference between the MATLAB discretized model and the ϵ -NTU_{MD} method presented in Figure 5-Figure 7.

Figure 9 Percentage difference between the MATLAB discretized model and the ε -NTU_{MD} method when the correct function is excluded.

The effect of the approximations in our model are mitigated by the inclusion of the correction factor which specifically corrects for fact that the ε -NTU_{MD} method uses the overall heat transfer coefficient and does not distinguish between the slightly difference contribution levels of the individual heat transfer coefficients as previously mentioned. The reason for this is readily understandable; a change of one degree Centigrade in membrane surface temperature has more effect on the hot side than on the cold side because of the non-linear dependency of water vapour pressure on temperature given in Eq. (A3).

The assumption of negligible concentration polarization and constant thermal efficiency are readily justified and uncertainties in the prediction of production rate for the ‘rating’ case, and membrane area for the ‘design’ case, of the ε -NTU_{MD} method will be much more dependent upon the uncertainties in obtaining input values for U and K^* than on the approximation inherent in the ε -NTU_{MD} method itself.

That the outlet temperatures are predicted by the ε -NTU_{MD} method is useful for the design of the overall DCMD process and module cascading networks.

7. Concluding remarks

An equivalent ε -NTU_{MD} approach was introduced for both concurrent and countercurrent DCMD. It was validated against an experimentally validated discretized model of a flat sheet DCMD module built using MATLAB. For the ε -NTU_{MD} method the set of equations were derived in a manner analogous to that used for heat exchanger systems. Future users can use the set of equations given in either Table 2 or Table 3 (depending on module configuration) together with the algorithms provided in Figure 2 or 3 to obtain accurate estimations of either expected performance or required size. With the introduction of a correction function that accounts for the unequal contribution to DCMD performance by the two heat transfer

coefficients, the accuracy to all cases considered was better than $\pm 6\%$. Even without the correction function, 95% of the results were within $\pm 10\%$ to the data obtained from the numerical model.

This ε -NTU_{MD} method for DCMD can be applied in a similar fashion as the conventional ε -NTU method for heat exchanger design and networks. Compared to other design tools such as discretized or finite element models, this method combines robustness, rigorousness and versatility. It is suitable for any preliminary calculations, allowing one to ‘have a feel’ regarding sizing or throughput requirements and output estimations. Uses could range from the design of experiments to the construction of a more detailed model or to the designing or scaling up of a full-scale DCMD system. Moreover, any changes in operating conditions or the use of a membrane with a different permeability can be assessed very readily and easily using this set of ε -NTU_{MD} expressions. It is, for example, straight forward to explore the effect upon performance of changing T_f .

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List of symbols

Symbol	Meaning	Units
A	Area	m^2
C	Heat capacity rate	$\text{J K}^{-1} \text{s}^{-1}$
C_r	Heat capacity ratio	-
G	Channel gap	m
K	Membrane permeability	$\text{kg m}^{-1} \text{s}^{-1} \text{Pa}^{-1}$
K^*	Membrane coefficient	$\text{kg m}^{-2} \text{s}^{-1} \text{K}^{-1}$
K_{eff}	Effective membrane permeability	$\text{kg m}^{-1} \text{s}^{-1} \text{K}^{-1}$
P	Saturated vapour pressure	Pa
N	Mass flux across membrane	$\text{kg m}^{-2} \text{s}^{-1}$
Q	Overall heat transfer	W
T	Temperature	K
U	Overall heat transfer coefficient, Eq. (9)	$\text{W m}^{-2} \text{K}^{-1}$
U_L	Combined heat transfer coefficient, Eq. (7)	$\text{W m}^{-2} \text{K}^{-1}$
c	Specific heat capacity	$\text{J kg}^{-1} \text{K}^{-1}$
h	Film heat transfer coefficient	$\text{W m}^{-2} \text{K}^{-1}$
k_m	Thermal conductivity	$\text{W m}^{-1} \text{K}^{-1}$
$\dot{m}$	Mass flowrate	kg s^{-1}
q	Heat flux	W m^{-2}
α	MD overall heat transfer coefficient	$\text{W m}^{-1} \text{K}^{-1}$
ε	effectiveness	-
δ	Membrane thickness	m
λ	Heat of vaporization	J kg^{-1}
η	Thermal efficiency	-
$\eta_{\text{mod.}}$	Module thermal efficiency	-
Subscripts		
0	Simulation base case	
avg	Average value	
b	Bulk	
c	Cold fluid side	
f	Feed side (bulk)	
fm	Feed side adjacent to membrane	
h	Hot fluid side	
in	Condition at inlet of membrane module	
max	Maximum value	
min	Minimum value	
p	Permeate side (bulk)	
pm	Permeate side adjacent to membrane	
out	Condition at outlet of membrane module	
t	Total	

Appendices

A1 Determination of average K_{eff} for ε -NTU_{MD} method

K_{eff} , is seen to depend on both the hydrodynamics, which affects U_L , and membrane properties. Generally this is obtained experimentally, but can be estimated when the membrane permeability, K or K^* (with the unit of $\text{kg m}^{-1}\text{s}^{-1}\text{Pa}^{-1}$ or $\text{kg m}^{-1}\text{s}^{-1}\text{K}^{-1}$ respectively) and the heat transfer coefficients are known.

For the new methodology one needs a value of K^* , but as the transmembrane flux is driven at the fundamental level by the difference in vapour partial pressure, values of K may well be the information to hand or indeed estimated. The two quantities are related as follows:

$$K^* \Delta T = K \Delta P \quad (\text{A1})$$

where the terms ΔT and ΔP refer to the conditions across the membrane itself, *not* bulk conditions i.e. $\Delta T = T_{fm} - T_{pm}$.

Due to the non-linearity of the vapour partial pressure and temperature relationship, the value of K^* can be estimated using Eq. (A2) provided one has an appropriate value for $\frac{dP}{dT}$.

$$K^* = K \frac{dP}{dT} \quad (\text{A2})$$

where the value of K solely dependent on the properties of the hydrophobic membrane.

By fitting a forth order polynomial to the Antoine equation in the temperature range 290 to 340K, the water vapour partial pressure and its differential with respect to temperature can be expressed as follows:

$$P = 0.00093687T^4 - 1.059T^3 + 452.75T^2 - 86690T + 6267900 \quad (\text{A3})$$

with differentiation giving:

$$\frac{dP}{dT} = 0.00374738T^3 - 3.177T^2 + 905.5T - 86690 \quad (\text{A4})$$

Then from Eq. (A5) K^* can be evaluated for any temperature range when the value of K is known.

$$K^* = \frac{K}{2} \left(\left. \frac{dP}{dT} \right|_{T_f} + \left. \frac{dP}{dT} \right|_{T_p} \right) \quad (\text{A5})$$

Finally, using K^* , K_{eff} is computed using Eq. (7).

A2 Model validation and comparison with experimental data

Whilst our MATLAB model had already been validated against experimental results in recent publications [13, 14], and could therefore serve as an appropriately sophisticated benchmark for the evaluation of the NTU_{MD} model, it was decided that it would be reassuring for some readers if a comparison of the MATLAB model and the NTU_{MD} model with experimental results would be included herein. This provides explicit as well as implicit validation and serves as an example of how to use the NTU_{MD} method. The experimental data used was obtained from where the membrane thickness is given as 150 microns and the feed and permeate velocities are 1.75 ms⁻¹.

Firstly for a fixed K values of K^* can be obtained using the procedure in Appendix A1. Then secondly with $k_m=0.089 \text{ Wm}^{-1}\text{K}^{-1}$ and appropriate values of the heat transfer coefficients K_{eff} is obtained using equations (6) and (7) given in the main text. It is evidenced in Figure A1 that the NTU_{MD} model achieves as good a degree of accuracy as the MATLAB model.

Figure A1 Comparison of NTU_{MD} model, MATLAB model and data from [34]. The plot is of permeate flux in kgm⁻²h⁻¹ vs feed inlet temperature.

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