

Dynamic Stochastic General Equilibrium Models with Money, Default and Collateral



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This dissertation is dedicated to my parents, loving wife and dearest son.

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Abstract

This D.Phil. dissertation investigates the areas in financial stability. The three comprising essays have a common ground: money, default and collateral in the theory of finance.

Chapter Two (co-authored with Prof. Dimitrios Tsomocos), which is titled “A Dynamic General Equilibrium Model to Analyse Financial Stability”, aims to refine and improve existing DSGE models in two ways. First, it incorporates hitherto neglected components such as endogenous default, money via cash-in-advance constraints and heterogenous banking sectors. Thus, in contrast to the New Keynesian approach, here it is liquidity and default that are the driving forces behind our results. Second, in focusing on both monetary policy and fiscal policy, it elucidates how interactions between the two policy arenas affect macroeconomic fluctuations, particularly in regard to financial stability. Through these refinements, we put forward the policy response necessary to achieve a stable financial system using a calibrated DSGE model.

Chapter Three, entitled “Monetary Policy in a Time of Natural Disaster”, investigates the appropriate monetary policy response to natural disasters in the DSGE framework. I develop a realistic model for financial turmoil by evaluating the impact of natural disasters on credit markets by including financial frictions such as endogenous default and liquidity constraints. I show that the standard Taylor rule (1993) response in models with money and default is to increase the nominal interest rate after a disaster shock. However, in fact an inflation-targeting policy (i.e. monetary contraction) is not compatible with mitigating financial fragility in the highly indebted economy with near-zero

interest rate, and arguably the ‘Taylor Principle’ does not hold in such as economy (e.g. Japan in 2011). Nevertheless, expansionary monetary policy induces a debt overhang even further.

Chapter Four, “Collateral, Default and Asset Prices”, uses a DSGE framework to put forward a model of how agents adjust their asset holdings in response to deflationary shocks. By introducing collateral constraints in the default decision, I capture some original features of the early debt-deflation literature, such as distress selling and instability. The estimated model successfully delivers a procyclical feedback loop for the default channel, which consists of foreclosure, high borrowing costs, inefficient capital allocation, and a further decrease in the output level. I investigated recessionary shocks inducing deflation in commodity and/or asset prices for monetary policy experiments. This, therefore, underlines the importance of monetary policy in restoring financial stability during a deflationary period.

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Chapter 1

Introduction

The banking sector clearly has a considerable effect on the real economy. The US Great Depression and Japan's recession in the 1990s are good historical examples. Closer to home, the ongoing global financial crisis beginning in late 2007 triggered a severe worldwide economic downturn. At its centre are the banks: subprime credit exposure and significant write-offs on asset-backed securities were the trigger for subsequent events. It is commonly held that financial frictions have a great influence on the business cycle, as credit markets and the real economy interact with each other. Policymakers have begun to highlight the importance of financial frictions in economic systems, beginning – at last – to treat those factors as a crucial part of their overall policy stance.

One of the key lessons of the crisis was that financial markets matter for macroeconomic developments and should therefore be taken into account when constructing macro models. This realisation resulted in a surge of interest in theoretical frameworks incorporating financial frictions. Models with imperfect financial markets, previously receiving little attention, dramatically entered the wider consciousness. They were used to answer

questions vital to policymakers, addressing the impact of financial shocks on the economy (Gerali et al., 2009; Brzoza-Brzezina and Makarski, 2011), the optimal monetary policy in the presence of financial frictions (Curdia and Woodford, 2009; De Fiore and Tristani, 2009; Carlstrom et al., 2010; Kolasa and Lombardo, 2011), the effectiveness of alternative monetary policy tools (Lombardo and McAdam, 2012) and the impact of capital regulations on the economy (Angelini et al., 2010). Moreover, financial frictions have recently been added into the models used for policy purposes at several central banks, including the Riksbank's RAMSES model (Christiano et al., 2011) and the European Central Bank's NAWM (Lombardo and McAdam, 2012).

The current literature mostly rests on two different approaches, both of which originate from before the crisis. One derives from the seminal 1997 paper by Kiyotaki and Moore, and introduces financial frictions via collateral constraints. Agents are heterogeneous in terms of their rate of time preference, meaning they can therefore be divided into lenders and borrowers. The financial sector intermediates between these groups and introduces frictions by requiring that borrowers provide collateral for their loans. As a result, frictions that directly affect the quantity of loans are introduced. Kiyotaki and Moore's (1997) original model has more recently been adapted by Iacoviello (2005), who introduced housing as collateral. Other recent applications relying on this framework include Calza et al. (2009), who focus on the effects of mortgage market characteristics on monetary transmission, and Gerali et al. (2009) and Brzoza-Brzezina and Makarski (2010), who use models with collateral constraints and monopolistic competition in the banking sector to explore how financial frictions affect monetary transmission within a credit crunch scenario. Similarly, Iacoviello and Neri (2010) use US data to generate a model with collateral constraints that allows them to study the effect of housing market shocks on the economy.

Bernanke and Gertler (1989) are behind the second stream of research. In their paper, financial frictions are incorporated into a general equilibrium model, an approach further developed by Carlstrom and Fuerst (1997) and then merged with the New Keynesian framework by Bernanke et al. (1999). This was the benchmark financial frictions model used throughout the 2000s. In this model, frictions arise because monitoring of the loan applicant is costly, causing an endogenous wedge to be driven between the lending rate and the risk-free rate. This means that financial frictions affect the economy via the prices of loans rather than via quantities, as is the case in the first approach using models based on collateral constraints. The external finance premium setup is used by Christiano et al. (2004) to analyse financial frictions in the Great Depression and by Christiano et al. (2011) to study the business cycle implications. Goodfriend and McCallum (2007) contribute by providing an endogenous explanation for steady state differentials between lending and money market rates. Furthermore, using a similar framework, Curdia and Woodford (2009) put forward an optimal monetary policy in the presence of time-varying interest rate spreads in a simple model with heterogeneous households.

While both of the approaches outlined above allow for the introduction of financial frictions into the macro model, the two models' propagation mechanisms can be significantly different. To develop a successful macro-financial framework it is crucial to gain a proper understanding of how price- and quantity-based frictions work. It is thus necessary to incorporate default in a general equilibrium framework as well as liquidity constraints. The possibility of default on any debt obligations underscores the necessity for liquidity constraints and the interaction of liquidity and default justifies fiat money being used as the stipulated mean of exchange. If it is not, the mere presence of a monetary sector (banking sector) without the possibility of default or any other financial friction in equilibrium may

become a veil with no effect on real trade and the final equilibrium allocation. Indeed, liquidity constraints are a minimal institutional arrangement needed to capture the fundamental aspects of money, not to mention how it interacts with default and thus affects the real economy.

Indeed, one of the key features of recent papers on financial fragility (Goodhart et al., 2006; Tsomocos, 2003) is the modelling of endogenous default. The inclusion of the possibility of default in general equilibrium models can be traced back to Shubik and Wilson (1977), with Dubey and Geanakoplos (1992) and Dubey et al. (2005) subsequently formally analysing default in models with and without uncertainty. In the Arrow-Debreu model, an implicit assumption is that all agents honour their obligations, meaning there is no possibility of default. However, when using models such as strategic market games (Shapley and Shubik, 1977), the introduction of minimal institutions – e.g. money, credit and default – becomes a logical necessity. In particular, Shubik and Wilson (1977) allow agents to choose their repayment rates; this means that equilibrium becomes compatible with partial or complete abrogation of agents' contractual obligations. If agents are not accountable for their repayments, the result is very predictable; they will rationally choose not to repay their debts. In response, we are thus led to introduce default penalties that constrain agents' choices of repayment. If these default penalties are infinite then the model is reduced to the standard Arrow-Debreu model – plus the added constraint, under uncertainty and incomplete markets, that no-one will borrow – whereas if these penalties are zero no equilibrium can be established, since there will be unbounded credit demand and zero credit supply. Shubik and Wilson (1977) therefore treat default continuously, allowing for partial default in equilibrium and thus providing a useful framework with which to analyse financial fragility as we encounter it in reality.

It was back at the turn of the last century when the general equilibrium theory was first systematically introduced by Walras (1900), later being significantly extended to incorporate uncertainty by Arrow and Debreu (1954) and Debreu (1959). It is now the theoretical framework upon which most microfoundation macroeconomic models are based, including textbook models such as those pioneered by Barro (1993) and McCandless and Wallace (1991).

Due to the limitations of space, I will not discuss the basic structure or features of general equilibrium models here. However, it is important to stress that such models are, by their very nature, purely real economies; by default, they involve no money, in any form, and all trades are simply exchanges of goods. In other words, the markets in the economy are entirely based upon barter. Many authors have argued that barter is inefficient and has a high cost as it demands a “double coincidence of wants” for trades to be successful. Moreover, it also requires the existence of $n(n - 1)/2$ markets (one for each pair of goods) in an economy with n goods. It was long since recognised that such problems can be eliminated by the use of money as a medium of exchange (Jevons, 1875), and hence monetary theorists have strived to incorporate money into general equilibrium models.

Nevertheless, this task is not straightforward. In particular, general equilibrium models have rational, optimising agents as their foundation, meaning we cannot arbitrarily impose the use of money on these agents without violating the spirit of such models. Instead, the agents must “voluntarily” choose to use money in order to solve their utility-maximisation problem. This raises deeper questions about why people use money in the real world, and even why money exists. Of course, a huge variety of answers to this question have been posited within the general equilibrium framework, some more satisfactory

than others. The result is a huge body of literature, mainly developed in the latter half of the previous century. The types of approaches considered are based on the introduction of money into the utility function, subsequently modelling the transactions demand for money through the use of cash-in-advance constraints, real resource costs, search theory and imperfect information. The latter two are found to be the most satisfactory from a general equilibrium point of view, but are less straightforward to implement analytically and perhaps serve only to motivate more refined uses of the other approaches.

Money in the utility function, pioneered by Sidrauski (1967), includes the use of money in the representative agent's utility function. We therefore assume that the utility function of the representative agent takes variables such as the flow of services yielded by money holdings and time-per-capita consumption (Walsh, 1998). What enters the utility function here is not the actual amount of currency held by the individual; instead, it is the command over goods represented by the money holding, or some measure of the transaction services, in terms of goods, that the money holding provides.

This approach is criticised by Walsh (1998), who holds that it uses assumptions to solve the problem inherent in the generation of a positive demand for money. In particular, it does not address the question of why money – particularly unbacked fiat money – actually yields utility. Instead, Walsh argues that the money-in-the-utility-function approach is something of a shortcut to a more fully specified model of the transactions technology that households face, which generates a positive demand for a medium of exchange. Clower (1967) writes of how, in a monetary economy, “money buys goods, goods buy money, but goods do not buy goods”. In other words, exchange is operationalised through a relation among commodities and the existence of monetary exchange is evidence that this relation

is asymmetric (Ostroy and Starr, 1990). This asymmetry needs to be captured by explicitly modelling the transactions demand for money.

Cash-in-advance constraints – an idea that originates with Clower (1967) – captures the role of money as a medium of exchange and specifically requires that money is used to purchase consumption goods. There is therefore an assumption that the representative agent faces a “cash-in-advance” constraint, which is in addition to their intertemporal budget constraint. This constraint is such that the agent’s real spending on consumption in any given period cannot exceed the amount of real money balances the agent carried into that period.

Under such a schema, when there is an opportunity cost to holding money (i.e. a positive nominal interest rate), then if the agent has no uncertainty about the future path of income it will hold only the amount of money that is exactly sufficient to finance their desired level of consumption. Furthermore, Walsh (1998) shows that that, in this model, money is just like any other asset; its value is equal to the present discounted value of the stream of returns it generates. In the case of money, these returns are liquidity services. If the cash-in-advance constraint is not binding, these liquidity services will have no value – and therefore money will have no value too.

Svensson (1985) and Lucas and Stokey (1987) are among the many thinkers who extend the cash-in-advance model to incorporate uncertainty. Doing so makes the model considerably more complex, but it also allows for simulation of the effects of different monetary policies (see, for example, Cooley and Hansen, 1989). Furthermore, in the case without uncertainty, the velocity of money is exactly equal to one at all times, a phenomenon clearly at odds with real-world experience. Introducing uncertainty generates a variable velocity of

money if money balances must be chosen prior to the resolution of the uncertainty. In this case, it may be that the desired level of consumption turns out to be less than the amount of the real money balances actually held, so that some money balances will be unspent. This reduces the velocity below one.

A criticism of the cash-in-advance model is that it is just as arbitrary as the practice of putting money in the utility function, making it subject to the very same failings as the model it was supposed to replace. Indeed, because it is an added constraint on the choices of agents it actually further reduces the exchange opportunities that are available given ordinary budget constraints (Ostroy and Starr, 1990). We must then ask whether this extra constraint is just another trick that allows money to be incorporated into general equilibrium models or whether it is a genuine feature of a monetary economy.

So, it has been suggested that the money-in-the-utility-function and cash-in-advance approaches are useful tricks for including money in general equilibrium models, as well as for ensuring a positive demand for money. Although useful from a practical modelling perspective, however, these approaches are somewhat arbitrary and difficult to justify with reference to the real world. Furthermore, it could be maintained that the real transaction costs approach is actually “functionally equivalent” to including money in the utility function.

The most satisfactory approach developed so far appears to be the use of search theory and imperfect information to explain the emergence of money as a medium of exchange, an approach that is undoubtedly less arbitrary and a better fit for the general equilibrium style of modelling. In such a model, the use of money arises as a result of the basic structure of the model world rather than as a result of arbitrary assumptions or restrictions. However,

the extra layer of complexity this entails is a further weight to be borne by models that are already complex by nature, which may in itself be enough to declare such an approach less useful than other methods for practically modelling applications.

The impacts of fiscal policies on financial stability are at the very heart of the current financial crisis. However, macroeconomic models are not very well suited to shedding light on the interactions between fiscal rules and monetary policy in regard to financial stability. The risk premium produced by standard macroeconomic models such as those by Christiano, Eichenbaum, and Evans (2005) or Smets and Wouters (2007) is too small and lacks the volatility needed to be truly representative (Rudebusch and Swanson, 2008). Furthermore, there is only limited support in the data for an approach that appends a term structure to these models (De Graeve, Emiris, and Wouters, 2009).

Similarly, standard asset pricing models targeted at the reproduction of risk premia typically lack some very important dimensions, such as plausible inflation dynamics, labour markets, or a meaningful fiscal sector. As den Haan (1995) shows, the key problem standard macroeconomic models face is to generate a co-movement which implies that bond prices are low in states where marginal utility, or “hunger”, is high. If bond prices are high in bad times, when marginal utility is high, long-term bonds provide a hedge against unforeseen movements in consumption. Indeed, there are some cases where the bond premium has even been negative (Backus, Gregory, and Zin, 1989). The bond premium puzzle also stems from the fact that most models fail to reflect the volatility of marginal utility and bond prices that is needed to explain the size of the risk premium observed in the data.

Empirical studies employing a standard VAR approach tend to show private consumption rising after a government spending shock (Fatás and Mihov, 2001; Blanchard and Perotti,

2002; and Perotti, 2007). In the theory, however, a government spending shock is considered to generate a negative wealth effect, which induces households to increase the labour supply and decrease consumption in a general equilibrium framework wherein all households are forward-looking (see Aiyagari et al., 1992; Baxter and King, 1993¹). As a result, recent DSGE studies on the macroeconomic effects of fiscal policy focus mainly on obtaining the crowding-in effect on consumption in initial periods, thus attempting to resolve this apparent conflict between the empirical evidence and the predictions of the model.

Works on the relationship between distortionary taxes and fiscal policy effectiveness in the context of DSGE models are rare. Bilbiie and Straub (2004) stress the importance of analysing distortionary taxation in DSGE models, stating that a lump-sum tax is unrealistic. They show that it is more difficult to obtain a positive consumption response after a government spending shock under distortionary taxation, which lowers after-tax wages. On the other hand, Linnemann (2004) identifies a crowding-in effect even in the presence of distortionary taxation, drawing evidence from the elastic labour supply and arguing that unemployment benefits help to widen the tax base. It should be noted that both studies see labour supply as critical, echoing the findings of Ludvigson (1998, 1999) and Jones (2002). This is to be expected given that fiscal policy effects in general equilibrium models are basically obtained through labour hour increases, as Aiyagari et al. (1992) and Baxter and King (1993) show.

¹These analyses are both based on neoclassical models. Linnemann and Schabert (2003) examine the fiscal policy effect in a simple New Keynesian DSGE model and find a decisive role for monetary policy. A recent study by Cogan et al. (2010) compares government spending multipliers (defined as the percentage change in output from a permanent increase in government spending equal to one percent of output) in an estimated medium-scale New Keynesian DSGE model with those in an old Keynesian model. They conclude that the multipliers are much smaller in New Keynesian models than in old Keynesian models, which they mainly attribute to the assumption of rational expectations. They also show that assumptions about monetary policy have a significant effect on the magnitude of the multipliers.

Chapter 2

A Dynamic General Equilibrium Model to Analyse Financial Stability

2.1 Introduction

History always provides substantial guidance on how to overcome current problems as well as the reasons behind them; this also applies to economic failure. We have seen that the banking sector can considerably affect the developments of the real economy in the U.S. Great Depression and the prolonged recession in Japan in the 1990s. In a similar vein, banks have come again under the spotlight since the onset of the financial turmoil in August 2007, as losses from subprime credit exposure and from significant write-offs on asset-backed securities raised concerns that a wave of widespread credit restrictions might trigger a severe economic downturn. Now it is commonly believed that financial factors have great influence in shaping the business cycle as credit markets and the real economy are likely to interact with each other. Policymakers have begun to highlight the importance

of financial factors in economic systems and these are treated as a customary part of the overall policy stance.

In times of financial turmoil, the government and economic agents seek a financially stable state but there is no general consensus on the definition of financial stability or financial fragility. Crockett (1997) noted that financial stability requires that the key institutions and the key markets are stable. Mishkin (1991) explained that financial instability occurs when shocks to the financial system interfere with information flows so that the financial system can no longer do its job of channelling funds to those with productive investment opportunities. Haldane et al. (2004) mentioned that financial instability is any deviation from the optimal saving-investment plan of the economy that is due to imperfections in the financial sector. Issing (2002) and Foot (2003) said that financial instability linked to financial market bubbles, or more generally, volatility in financial markets. However, none of the previous studies properly defines the measure to analyse financial stability. Thus, we follow the definition of financial instability provided by Goodhart et al. (2006), where financial fragility is characterised by reduced bank profitability and increased aggregate default. Increases in both banking sector vulnerability and aggregate default (lower repayment rates) are linked to welfare losses (agents' utilities).

DSGE models are the most popular tools used by academics and policymakers to identify the sources of macroeconomic fluctuations and to explain the effects of policy interventions. However, most workhorse DSGE models routinely employed in academia and policy institutions to study the dynamics of the main macroeconomic variables generally lack any interaction between financial markets and the rest of the economy. The introduction of financial frictions in a DSGE framework by Bernanke, Gertler and Gilchrist (1998) and

Iacoviello (2005) has started to fill this gap by introducing credit and collateral requirements and by studying how macroeconomic shocks are transmitted or amplified in the presence of these financial elements. These models assume that credit transactions take place through the market and do not assign any role to financial intermediaries such as banks. The recent financial crisis, however, demonstrated that serious limitations still exist in the current DSGE models (Goodhart et al. 2009). Therefore, refining and improving these models is essential for explaining financial stability.

We find two fundamental problems in current DSGE models: 1) failure to properly incorporate necessary components into the analysis financial stability; and 2) assumption of the limited role of fiscal policy (Gali 2003).

Without inclusion of all necessary elements in the model, the model cannot provide proper insight. It may, of course, be impossible to correctly mimic the real world in a model, but we can create a more realistic model by incorporating ignored elements. First, the conventional models have neglected the existence of default, a significant issue in the recent crisis. Second, they fail to conceptualise banks as heterogeneous entities with different portfolios, which is crucial for modelling the effects of interbank markets and contagious financial crises. Money, banks, and interest rates are key components of any robust model analysing financial stability. These ignored elements can be fully incorporated by expanding current DSGE models. This study intends to show how to do that and what implications can be drawn from such an improved model.

As an automatic stabiliser, fiscal policy can significantly assist or hinder monetary policy; however, existing studies have not paid sufficient attention to the interaction among policy instruments and highlight only a limited role for fiscal policy in financial stability, i.e.

no discussion about alternative tax schemes and government expenditures. With the fiscal deficit allowed, the fiscal authority can borrow money in the government bond market in order to finance its expenditure systematically. Coenen and Straub (2005) introduced distortionary taxes into the Smets and Wouters (2003) model but they modelled distortionary taxes in a time invariant manner. As a result, their model only captures the dynamics of the hypothetical lump-sum tax. In this study, two kinds of distortionary taxes, for consumption and income, are employed and their policy rules are also incorporated as their short-to-medium run effects are different to each other.

As suggested by Tsomocos (2003) and Goodhart et al. (2006), to provide an appropriate framework for financial stability analysis, we introduce two essential financial frictions: endogenous default and money via CIA constraints. The possibility of default on any debt obligations underscores the necessity of CIA constraints. The interaction of liquidity and default justifies fiat money as the stipulated mean of exchange. Otherwise, the mere presence of a monetary sector (banking sector) without the possibility of default or any other financial friction in equilibrium may become a veil without affecting real trade and final equilibrium allocation. Indeed, CIA constraints are a minimal institutional arrangement to capture the fundamental aspect of liquidity and how it interacts with default to affect the real economy.

Following Shubik and Wilson (1977) and Dubey et al. (2005), we modelled the endogenous default that arises as an equilibrium phenomenon, because agents are allowed to choose what fraction to pay from their outstanding debt. The cost of default is modelled by a penalty that reduces utility, the non-pecuniary default penalty, instead of directly reducing an individual's ability to borrow after s/he defaults on a loan obligation.

In sum, this study aims to improve standard DSGE models by (i) incorporating components such as endogenous default, heterogeneous banking sector and money, which enable us to capture the transmission mechanism of shocks via financial markets, and (ii) investigating the interactions between monetary and fiscal policy, and how their interactions produce variations in financial stability outcomes.

2.2 The Model

Two types of agents are considered in the private sector: household and firm. The former puts deposits into the wholesale bank and the latter uses bank loans from the retail bank to finance its capital investment. They trade two types of goods (consumption goods and capital goods) and labour services. The consumption goods are non-durable and produce utility, whilst capital goods are durable and produce consumption goods as well as utility.

Credit in the model economy is extended via financial intermediaries, banks. The banking sector is designed as described in Goodhart et al. (2009), and set up to include liquidity requirements. Two layers, wholesale banks and retail branches, are assumed in order to facilitate the formulation of the model. These two types of banks are heterogeneous and adequately represent the transmission channel in the interbank market; we assume there is limited market participation, i.e. the household only deposits into the wholesale bank not into the retail bank. In other words, we associate with each bank a particular borrower in order to support different lending rates among different banks.¹

The central bank conducts open market operations in the interbank market, and the fiscal authority intervenes in the economy through its fiscal rules such as adjusting fiscal expen-

¹For a microfoundation of our limited participation assumption, see Bhattacharya et al. 2007.

diture and controlling tax rates. Unlike other agents, these policy entities are modelled as strategic dummies rather than optimising their objective functions. A diagram of the economy's nominal flows is presented in Figure 2.1.

FIGURE 2.1

To model CIA constraints, we divide each period into two sub-periods, the beginning and the end of periods. At the end of each period, trades take place against a background of uncertainty about economic conditions (the state of nature) that will prevail in the future, i.e. at the beginning of the next period. Agents are, however, assumed to have rational expectations, and to know the likelihood of good or bad states occurring when they make their choices in the end of each period. At the beginning of each period, the actual economic conjecture is revealed and all the uncertainties are resolved.

Note that budget constraints for all agents are binding, because money is fiat and agents do not derive any utility from holding it. Thus, individuals do not hold any idle cash; instead they lend it out to someone who needs it. Figure 2.2 makes the timeline of the model explicit.

FIGURE 2.2

2.2.1 Private Sector

Household (α)

The household sells part of its capital endowment q_t^k , supplies labour q_t^n and allocates its wealth into an optimal portfolio, i.e. makes deposits into the wholesale bank m_t^d and

holding of government bonds m_t^α , in order to smooth life time consumption.

At the beginning of period t , the household receives sales income (by selling capital) $p_{t-1}^k q_{t-1}^k$ and wages $p_{t-1}^n q_{t-1}^n$, which are carried forward from period $t-1$, and also has income from its asset investment in period $t-1$, i.e. from deposits $\mathcal{R}_t^d(1+r_{t-1}^d)m_{t-1}^d$ and from holding government bonds $(1+r_{t-1}^f)m_{t-1}^\alpha$. In addition to these income streams, the household receives profit shares $\sum_{i \in \{\theta, \gamma, \delta\}} \Pi_{i,t}$. At the end of period t , the household uses these revenues to deposit m_t^d into the bank δ , buy government bonds m_t^α and purchase goods b_t^c . As the household has no liability in any financial market, there is no outflow of money at the beginning of period t . Similarly, as the household does not apply for any bank loans, there is no inflow of money at the end of period t . All of these transaction activities are summarised (2.2.1),

$$\begin{aligned} & m_t^d + m_t^\alpha + (1 + \tau_t^c)b_t^c \\ \leq & \mathcal{R}_t^d(1 + r_{t-1}^d)m_{t-1}^d + (1 + r_{t-1}^f)m_{t-1}^\alpha + \sum_{i \in \{\theta, \gamma, \delta\}} \Pi_{i,t} \\ & + (1 - \tau_{t-1}^s)(p_{t-1}^n q_{t-1}^n + p_{t-1}^k q_{t-1}^k) \end{aligned} \quad (2.2.1)$$

where τ_t^c , τ_t^s and $\mathcal{R}_t^d$ represent consumption tax rate (e.g. VAT), income tax rate and expected repayment rate on household's deposits, respectively.

The household is rich in every period as it has an endowment in labour n_t and in capital k_t whose log-linearised forms are modelled as AR(1) processes, (2.2.2) and (2.2.3). As all real and nominal variables do not grow and the economy is not subject to any current or expected shocks, we assume that the long run endowment streams of labour and capital are constants, i.e. $\bar{n}$ and $\bar{k}$.

$$\ln n_t = \rho_n \ln n_{t-1} + (1 - \rho_n) \ln \bar{n} + \varepsilon_t^n \quad (2.2.2)$$

$$\varepsilon_t^n \sim i.i.d. N(0, \sigma_n^2)$$

$$\ln k_t = \rho_k \ln k_{t-1} + (1 - \rho_k) \ln \bar{k} + \varepsilon_t^k \quad (2.2.3)$$

$$\varepsilon_t^k \sim i.i.d. N(0, \sigma_k^2)$$

where ρ_k and ρ_n stand for the AR(1) coefficients, and σ_k and σ_n represent their standard deviations.

We have introduced a capital accumulation law for the household similar to the one of the firm, (2.2.11). Capital stock K_t^α accumulates in the standard form as shown in (2.2.4), the law of motion for capital. Physical capital depreciates at the rate τ and the household refills its capital stock by $k_t - q_t^k$ in every period.

$$K_t^\alpha = (1 - \tau)K_{t-1}^\alpha + k_t - q_t^k \quad (2.2.4)$$

where q_t^k represents the capital supply to the firm and k_t refers to the capital endowment of the household. Thus, $k_t - q_t^k$ is the new capital added to the existing capital stock held by the household.

The household obtains utility not only by consuming goods ($c_t + \alpha_g g_t$) and holding physical capital ($k_t - q_t^k$), but also by having leisure ($n_t - q_t^n$), (2.2.5).² q_t^n stands for the labour supply to the firm and n_t refers to the labour endowment of the household. Hence, $n_t - q_t^n$ is the leisure enjoyed by the household in the same way as the capital held. In regard to

²There are many ways how we construct the utility function which consists of consumption goods, capital and labour. And the utility function in the model is standard, following Matteo Iacoviello 2011.

capital, we can think of capital in terms of buildings, so people can either gain utility from a house or rent out capital to firms in the form of factories, shops, etc.

$$\mathcal{U}(\cdot; \alpha, t) = \chi_c \ln(c_t + \alpha_g g_t) + \chi_k \ln K_t^\alpha + \chi_n \ln(n_t - q_t^n) \quad (2.2.5)$$

where χ_c , χ_n and χ_k stand for the preference factors for goods, leisure and capital, respectively. c_t is private consumption and g_t denotes public expenditures in real term, $g_t = G_t/p_t^c$. The parameter α_g accounts for the complementary/substituability between private consumption and public spending.³

As in the standard real business cycle (RBC) literature, we assume that the household is risk-averse and maximises the discounted sum of lifetime utility, (2.2.6)

$$\max_{\{b_t^c, q_t^n, q_t^k, m_t^d, m_t^\alpha\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta_\alpha^t \mathcal{U}(\cdot; \alpha, t) \quad (2.2.6)$$

where $E_0[\cdot]$ is the expected operator, conditioned on information available at time $t = 0$ and β_α is the subjective discount factor.

Firm (θ)

At the beginning of period t , the firm makes profit $\Pi_{\theta,t}$, (2.2.7), which is calculated as the difference between the amount of sales income $p_{t-1}^c y_{t-1}$ carried forward from the period

³If $\alpha_g \geq 0$, government spending substitutes for private consumption, with perfect substitution if $\alpha_g = 1$, as in Christiano and Eichenbaum (1997). In this case, a permanent increase in government spending has no effect on output and hours but reduces private consumption, through a perfect crowding-out effect. In the special case $\alpha_g = 0$, we recover the standard business cycle model, with government spending operating through a negative income effect on labor supply (see Christiano and Eichenbaum, 1997, Baxter and King, 1993). When the parameter $\alpha_g < 0$, government spending complements private consumption. Then, it can be the case (depending on the labor supply elasticity) that private consumption will react positively to an unexpected increase in government spending.

$t - 1$ and the amount it has to repay on its liabilities μ_{t-1}^l adjusted for its repayment rate v_t^l .

$$\Pi_{\theta,t} = p_{t-1}^c y_{t-1} - v_t^l \mu_{t-1}^l \quad (2.2.7)$$

where p_t^c and y_t are price of consumption goods and output level, respectively.

At the end of period t , the expenditure of the firm, which consists of payroll payment b_t^n and capital investment b_t^k , must be less than or equal to its liabilities $\mu_t^l(1 + r_t^l)^{-1}$ obtained from the corporate loan market. This budget set is explicitly described in (2.2.8)

$$b_t^n + b_t^k \leq \frac{\mu_t^l}{1 + r_t^l} \quad (2.2.8)$$

where r_t^l and μ_t^l represent interest rate of corporate loans and amount of money (corporate loans) offered to be repaid in the next period $t + 1$, respectively.

Output y_t is produced by means of accumulated capital stock K_t^θ and labour q_t^n presuming a constant return to scale technology represented by the Cobb-Douglas production function, (2.2.9)

$$y_t = a_t (K_t^\theta)^c (q_t^n)^{1-c} \quad (2.2.9)$$

where c and $1 - c$ are the output elasticities of capital stock and labour services, respectively.

The productivity factor a_t , which drives the output level, is modelled by (2.2.10) whose log-linearised form follows a simple AR(1) process.

$$\ln a_t = \rho_a \ln a_{t-1} + (1 - \rho_a) \ln \bar{a} + \varepsilon_t^a \quad (2.2.10)$$

$$\varepsilon_t^a \sim i.i.d N(0, \sigma_a^2)$$

where ρ_a is the smoothing coefficient of technology innovation. ε_t^a captures the *i.i.d.* shock and σ_a represents its standard deviation.

Capital investment is used to form physical capital, and it accumulates in the standard form as shown in (2.2.11), the law of motion for capital. Physical capital depreciates at the rate τ and the firm refills its capital stock by q_t^k in every period through new investment.

$$K_t^\theta = (1 - \tau)K_{t-1}^\theta + q_t^k \quad (2.2.11)$$

The firm maximises the discounted sum of its lifetime (real) profit adjusted by a non-pecuniary default penalty, (2.2.12)

$$\mathcal{U}(\cdot; \theta, t) = \frac{\Pi_{\theta, t}}{p_t^c} - \frac{\lambda^l}{2} \left[\frac{(1 - \nu_t^l) \mu_{t-1}^l}{p_t^c} \right]^2 \quad (2.2.12)$$

where we consider the profit maximising firm to be risk neutral. As the firm borrows from the corporate loan market and is allowed to repay partially, the non-pecuniary default penalty (which is quadratic) is considered. We discuss this in greater detail in 2.2.4, “Non-pecuniary Default Penalty”.

The optimisation formula of the firm is expressed as follows (2.2.13)

$$\max_{\{b_t^l, b_t^k, \nu_t^l, \mu_t^l\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta_\theta^t \mathcal{U}(\cdot; \theta, t) \quad (2.2.13)$$

where β_θ is the subjective discount factor. Although the dividend is distributed to the household, we presume that the firm manager has a different discount rate.

2.2.2 Banking Sector

The retail bank borrows money from the interbank market and extends a loan to the firm. The wholesale bank takes the deposit from the household and extends the loan to the retail bank. As our model contains limited market participation, the two banks interact with different agents in different markets. Other than that, these two banks and the central bank are the market participants in the interbank market.

Retail Bank (γ)

Dependent on which state actually occurs, the profit $\Pi_{\gamma,t}$ that the retail bank accrues at the beginning of period t is equal to the difference between the amount of money that it receives from its asset investment (i.e. extending corporate loans $\mathcal{R}_t^l(1 + r_{t-1}^l)m_{t-1}^l$ and holding government bonds $(1 + r_{t-1}^f)m_{t-1}^\gamma$) and the amount it has to repay on its liabilities μ_{t-1}^b adjusted for the repayment rate v_t^b , (2.2.14)

$$\Pi_{\gamma,t} = \mathcal{R}_t^l(1 + r_{t-1}^l)m_{t-1}^l + (1 + r_{t-1}^f)m_{t-1}^\gamma - v_t^b\mu_{t-1}^b \quad (2.2.14)$$

At the end of period t , the assets of bank γ , which consist of its credit extensions (corporate loans m_t^l and government bonds m_t^γ), must be less than or equal to its liabilities $\mu_t^b(1 + r_t^b)^{-1}$ obtained from the interbank market. The explicit budget constraint is described in (2.2.15)

$$m_t^l + m_t^\gamma \leq \frac{\mu_t^b}{1 + r_t^b} \quad (2.2.15)$$

The liquidity requirements mandate that banks hold a share of their assets in highly liquid government securities. As these securities provide a lower interest rate margin than

loans to the private sector, both retail and wholesale banks must hold the optimal amount needed to meet the requirements.⁴

As shown in Table 2.1, the retail bank's assets are divided between a common risky portfolio m_t^l provided to the firm as private sector loans and a highly liquid portfolio m_t^γ offered to the government.

TABLE 2.1

The highly liquid portfolio consists of government bonds rated risk free for regulatory purposes. The share of liquid and risky assets in the retail bank's portfolio is determined by ι_t^γ , (2.2.16). We discuss this in greater detail in 2.2.4, "Liquidity Requirement".

$$\iota_t^\gamma = \frac{m_t^\gamma}{m_t^l + m_t^\gamma} \quad (2.2.16)$$

The retail bank, an interbank net borrower, maximises the net present value of the flows of expected utilities, which are the quadratic functions of (real) profits. In the same manner as Goodhart et al. (2006), we assume that bank γ incurs a liquidity requirement violation penalty based on the liquidity adequacy ratio ι_t^γ that fixes the coverage ratio of risky assets as well as a non-pecuniary default penalty, (2.2.17)

$$\mathcal{U}(\cdot; \gamma, t) = \frac{1}{1 - a_\gamma} \left(\frac{\Pi_{\gamma,t}}{p_t^c} \right)^{1 - a_\gamma} - \frac{\chi_t^\gamma}{p_t^c} [\iota^\gamma - \iota_t^\gamma]^+ - \frac{\lambda^b}{2} \left[\frac{(1 - \nu_t^b) \mu_{t-1}^b}{p_t^c} \right]^2 \quad (2.2.17)$$

where a_γ represents the risk aversion coefficient of bank γ , χ_t^γ is the disutility caused by liquidity level, $[\iota^\gamma - \iota_t^\gamma]^+$ is the buffer for the liquidity requirement, and λ^b is the non-pecuniary default penalty set by the regulatory authority.

⁴Capital requirements and liquidity requirements are complementary in the sense that raising bank liquidity will also help to raise capital adequacy (Roget et al., 2011).

Formally, the optimisation problem of bank γ is as follows, (2.2.18)

$$\max_{\{v_t^b, \mu_t^b, m_t^l, m_t^\gamma\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta_\gamma^t \mathcal{U}(\cdot; \gamma, t) \quad (2.2.18)$$

where β_γ is the subjective discount factor which is different from β_α and β_θ .

Wholesale Bank (δ)

In the same manner as a retail bank, this bank makes profit, (2.2.19), at the beginning of each period t .

$$\Pi_{\delta,t} = \mathcal{R}_t^b(1 + r_{t-1}^b)m_{t-1}^b + (1 + r_{t-1}^f)m_{t-1}^\delta - v_t^d \mu_{t-1}^d \quad (2.2.19)$$

The budget set, (2.2.20), of bank δ is similar to that of bank γ except that it invests in, instead of borrowing from, the interbank market m_t^b and has its liabilities μ_t^d in the deposit market.

$$m_t^b + m_t^\delta \leq \frac{\mu_t^d}{1 + r_t^d} \quad (2.2.20)$$

Likewise, its liquidity ratio in the period t , as shown in (2.2.21), includes its credit extension in the interbank market.

$$l_t^\delta = \frac{m_t^\delta}{m_t^b + m_t^\delta} \quad (2.2.21)$$

Table 2.2 explicitly sets out the bank δ 's balance sheet including assets and liability.

TABLE 2.2

The risk-averse bank δ is a net lender in the interbank market and has a utility function in line with that of bank γ , (2.2.22)

$$\mathcal{U}(\cdot; \delta, t) = \frac{1}{1 - a_\delta} \left(\frac{\Pi_{\delta, t}}{p_t^c} \right)^{1 - a_\delta} - \frac{\chi_t^\delta}{p_t^c} [l^\delta - l_t^\delta]^+ - \frac{\lambda^d}{2} \left[\frac{(1 - v_t^d) \mu_{t-1}^d}{p_t^c} \right]^2 \quad (2.2.22)$$

where a_δ is the risk aversion coefficient of bank δ , χ_t^δ is the disutility caused by the liquidity level, $[l^\delta - l_t^\delta]^+$ represents the buffer for the liquidity requirement, and λ^d is the non-pecuniary default penalty set by the regulatory authority.

Bank δ 's optimisation formula is as follows (2.2.23):

$$\max_{\{v_t^d, \mu_t^d, m_t^b, m_t^\delta\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta_\delta^t \mathcal{U}(\cdot; \delta, t) \quad (2.2.23)$$

where β_δ is the subjective discount factor. It is presumed that $\beta_\gamma < \beta_\delta < \beta_\alpha$ hold; the borrowers are more impatient than the lenders.

2.2.3 Public Sector

Monetary Authority

In line with the standard Taylor rule (1993), we assume that the central bank sets the nominal interest rate r_t^b in the interbank market following a simple feedback rule in log-linearised form, (2.2.24)

$$\begin{aligned} r_t^b &= \phi_0 + \phi_\pi \pi_t + \phi_y \hat{y}_t + \varepsilon_t^r \\ \varepsilon_t^r &\sim i.i.d. N(0, \sigma_r^2) \end{aligned} \quad (2.2.24)$$

where ϕ_y and ϕ_π are the usual feedback coefficients on the output gap $\hat{y}_t$ and inflation π_t . Parameters ϕ_y and ϕ_π determine the aggressiveness of the monetary policy. ε_t^r captures *i.i.d.* shock and σ_r represents its standard deviation.

As the central bank controls the money supply through complying with the Taylor rule, the central bank supplies M_t to the interbank market to meet demand and target interest rate r_t^b in each period t . During this process, the central bank makes profits or losses (seigniorage costs) S_t (2.2.25). This revenue is used by government to finance a portion of fiscal expenditures besides collecting taxes.

$$S_t = M_{t-1} \left[\mathcal{R}_t^b (1 + r_{t-1}^b) - 1 \right] \quad (2.2.25)$$

Fiscal Authority

The fiscal authority purchases goods G_t , issues one-period bonds μ_t^f , and levies taxes⁵ on consumption T_t^c and income T_t^s , respectively. The fiscal authority is allowed to roll over the debt between periods, which means that fiscal surplus, as well as fiscal deficit, is allowed.⁶ The real flow budget constraints for the fiscal authority are expressed as follows, (2.2.26)

$$G_t + \mu_{t-1}^f \leq \frac{\mu_t^f}{1 + r_t^f} + S_t + T_{t-1}^c + T_{t-1}^s \quad (2.2.26)$$

Note that the budget constraint itself is a fiscal rule.⁷ Since we have four fiscal instruments,

⁵We introduce both consumption and income taxes as their short-to-medium run effects are different to each other. For example, increases in income tax result in lower output as the household supplies less capital and labour; instead, the household holds more capital and enjoys more leisure. However, the reverse holds in the case of increases in consumption tax. In addition, the impact on the government balance sheet is different as well, i.e. the short-to-medium run equilibrium path of government debt.

⁶Even though we only consider one-period bonds, the situation is the same for government securities with multiple maturities since we allow the debt to roll over.

⁷The fiscal rules allow partial debt finance, while debt is to be repaid through tax revenue over time. The speed of repayment is determined by a combination of the coefficients of the fiscal expenditures and tax rates.

we need as many as three fiscal rules other than budget constraint. Although there is little consensus in the literature on the formulation of fiscal rules, the rules in practice are designed to guarantee fiscal solvency in the model. Here, we assume that government spending follows a feedback rule that positively responds to output gaps, (2.2.27)

$$\ln G_t = \psi_g \ln G_{t-1} + \psi_y \ln p_t^c y_t + \psi_0 + \varepsilon_t^g \quad (2.2.27)$$

$$\varepsilon_t^g \sim i.i.d. N(0, \sigma_g^2)$$

where ψ_g is the smoothing coefficient of government spending and ψ_y is the feedback coefficient on the output gap. ε_t^g captures the *i.i.d.* shock in government spending and σ_g represents its standard deviation.

A consumption tax T_t^c is a tax on spending on goods and services. The term refers to a system with a tax base of the money spent on consumption. It typically takes the form of an indirect tax, such as value added tax. In our model, the household pays consumption tax when purchasing goods worth $p_t^c c_t$, (2.2.28)

$$T_t^c = \tau_t^c p_t^c c_t \quad (2.2.28)$$

An income tax T_t^s is levied on the income of the individual. It is collected on a pay-as-you-earn basis. In our model, the household pays the income tax; each household is liable for labour income tax $\tau_t^s p_t^n q_t^n$ and capital sales income tax $\tau_t^s p_t^k q_t^k$, (2.2.29)

$$T_t^s = \tau_t^s (p_t^n q_t^n + p_t^k q_t^k) \quad (2.2.29)$$

It would be perfectly possible to expand this exercise to include feedback rules in taxation, i.e. each tax rate would positively respond to the ratio of public debt to output. However,

two tax rates are modelled by exogenous process whose log-linearised forms follow AR(1) processes, (2.2.30) and (2.2.31), for examining the effects of transitory policy shocks.

$$\ln \tau_t^c = \rho_c \ln \tau_{t-1}^c + (1 - \rho_c) \ln \bar{\tau}^c + \varepsilon_t^c \quad (2.2.30)$$

$$\varepsilon_t^c \sim i.i.d. N(0, \sigma_c^2)$$

$$\ln \tau_t^s = \rho_s \ln \tau_{t-1}^s + (1 - \rho_s) \ln \bar{\tau}^s + \varepsilon_t^s \quad (2.2.31)$$

$$\varepsilon_t^s \sim i.i.d. N(0, \sigma_s^2)$$

where ρ_c and ρ_s are the AR(1) coefficients of tax rates. ε_t^c and ε_t^s capture the *i.i.d.* shocks in two tax rates. σ_c and σ_s represent their standard deviations.

In reality, we can consider the shocks which are not mean-reverting Markov process, i.e. the VAT changes from 15% to 20% in the UK. Our model allows for temporary as well as permanent shocks. The analysis in the paper only refers to temporary shocks (hence the use of the mean-reverting Markov process) in order to examine the non-trivial role of policy reaction and compare/contrast their impacts in the short-to-medium run.

2.2.4 Equilibrium Conditions

Market Clearing Conditions

There are seven active markets in the economy. In each market, prices are fully determined by supply and demand in equilibrium.

The goods market clears when the amount of money offered for goods is exchanged for the quantity of goods provided for sale. Thus, (2.2.32) holds for $\forall t \in T$ whenever $b_t^c + G_t > 0$ and $y_t > 0$

$$p_t^c = \frac{b_t^c + G_t}{y_t} \quad (2.2.32)$$

The capital market clears when capital expenditure is exchanged for the amount of capital offered to sell. Thus, (2.2.33) holds for $\forall t \in T$ whenever $b_t^k > 0$ and $q_t^k > 0$

$$p_t^k = \frac{b_t^k}{q_t^k} \quad (2.2.33)$$

The labour market clears when labour cost is exchanged for the amount of labour provided. Thus, (2.2.34) holds for $\forall t \in T$ whenever $b_t^n > 0$ and $q_t^n > 0$

$$p_t^n = \frac{b_t^n}{q_t^n} \quad (2.2.34)$$

The deposit market clears when the amount of money offered to be repaid in the next period is exchanged for the amount of savings in the current period. Thus, (2.2.35) holds for $\forall t \in T$ whenever $\mu_t^d > 0$ and $m_t^d > 0$

$$1 + r_t^d = \frac{\mu_t^d}{m_t^d} \quad (2.2.35)$$

The interbank market clears when the amount of money promised to be repaid in the next period is exchanged for the credit extension in the current period. Thus, (2.2.36) holds for $\forall t \in T$ whenever $\mu_t^b > 0$ and $m_t^b + M_t > 0$

$$1 + r_t^b = \frac{\mu_t^b}{m_t^b + M_t} \quad (2.2.36)$$

The corporate loan market clears when the amount of money offered to be repaid in the next period is exchanged for the amount of money lent in the current period. Thus, (2.2.37) holds for $\forall t \in T$ whenever $\mu_t^l > 0$ and $m_t^l > 0$

$$1 + r_t^l = \frac{\mu_t^l}{m_t^l} \quad (2.2.37)$$

The government bond market clears when the amount of money that fiscal authority promises to repay in the next period is exchanged for the credit extension in the current period. Thus, (2.2.38) holds for $\forall t \in T$ whenever $\mu_t^f > 0$ and $m_t^\alpha + m_t^\gamma + m_t^\delta > 0$

$$1 + r_t^f = \frac{\mu_t^f}{m_t^\alpha + m_t^\gamma + m_t^\delta} \quad (2.2.38)$$

Rational Expectations

The rational expectations condition on loans implies that the lending agents are correct in their expectation about the fraction of loan that will be repaid. Conditions (2.2.39) and (2.2.40) show that household and retail banks are correct in their expectations about the fraction of loans, i.e. deposits and corporate loan extensions, that will be delivered to them.

$$\mathcal{R}_t^d = \begin{cases} \frac{p_t^d}{\mu_{t-1}^d} & \mu_{t-1}^d > 0 \\ \text{arbitrary}, & \mu_{t-1}^d = 0 \end{cases} \quad (2.2.39)$$

$$\mathcal{R}_t^l = \begin{cases} \frac{p_t^l}{\mu_{t-1}^l} & \mu_{t-1}^l > 0 \\ \text{arbitrary}, & \mu_{t-1}^l = 0 \end{cases} \quad (2.2.40)$$

Similarly, (2.2.41) explains that the central bank and wholesale bank are correct in their expectations about the fraction of interbank loans that will be delivered to them.

$$\mathcal{R}_t^b = \begin{cases} \frac{p_t^b}{\mu_{t-1}^b} & \mu_{t-1}^b > 0 \\ \text{arbitrary}, & \mu_{t-1}^b = 0 \end{cases} \quad (2.2.41)$$

$\mathcal{R}_t^i$, for $i \in \{d, l, b\}$, represents the expected repayment rate that will be realised in equilibrium since agents are all rational. Put differently, it is a macro variable that is taken as fixed when agents optimise and in equilibrium its expectation coincides with the realised outcome v_t^i . In addition, μ_{t-1}^i is the amount of money which the borrower promised to repay to the lender at period t . Since the borrower is allowed to repay partially, s/he only repays p_t^i ($0 \leq p_t^i \leq \mu_{t-1}^i$) as a result of solving her/his optimisation problem. Therefore, the rational expectations regarding the repayment rate is defined by (2.2.39), (2.2.40) and (2.2.41).

The lender perceives that $\mathcal{R}_t^i(1 + r_{t-1}^i)m_{t-1}^i$ is the repayment from the previous period deposits in his/her budget set, i.e. (2.2.1), (2.2.14) and (2.2.19), where the market clearing condition in the credit markets is defined by $\mu_t^i = (1 + r_t^i)m_t^i$, i.e. (2.2.35), (2.2.36) and (2.2.37). v_t^i is the repayment rate of borrower, i.e. firm, retail bank and wholesale bank. Since p_t^i is defined by $v_t^i\mu_{t-1}^i$ and μ_{t-1}^i is predetermined at period $t - 1$, deciding of v_t^i is equivalent to that of p_t^i for borrower.

Non-pecuniary Default Penalty

Borrowers deliver on their promise as a result of the existence of punishment for default. Default on secured debt results in the loss of the collateral. Default on unsecured debt generally brings pecuniary costs (such as a search cost in relation to new loans) or non-pecuniary penalties (such as reputation loss). For discussions on the consequences of default, refer to Dubey et al. (2005).

We introduce non-pecuniary default penalties so as to capture the reputational cost due to default. Moreover, such a modelling approach (i.e. a quadratic form of non-pecuniary

penalty) allows for time-varying consumption levels and for a positive correlation between the repayment rate and consumption. Put differently, we choose an increasing marginal cost of default. We discuss this in greater detail below.

Note that if the marginal cost of default were decreasing, then higher consumption would entail higher default since at some point the marginal cost would be lower than the marginal benefit of default. Thus, this would constitute a counterfactual default cost. Note also that having a default penalty in linear form will result in a constant consumption level. Suppose that we allow for a linear non-pecuniary penalty (i.e. when the borrowing agents repay partially, the corresponding non-pecuniary penalty is proportional to the amount of default). Since the coefficients for default penalties are constant, this implies no variations in the borrowing agent's inter-temporal consumption; this could be interpreted as a complete consumption smoothing. However, a constant level of consumption strongly violates and/or kills most of the dynamics. Therefore, we choose not to adopt a linear form.

By using a quadratic non-pecuniary penalty, we avoid the above issues. The first-order condition regarding borrowers' repayment rate allows for a varying consumption level and a positive correlation between consumption and the repayment rate.

Liquidity Requirement

Liquidity requirements affect the transmission of a negative shock to a bank's portfolio and balance sheet since banks have no incentive to raise their liquid asset holdings and, therefore, to raise their profitability without any regulation.

Banks can violate their liquidity requirement, subject to liquidity requirement violation penalties set by the regulator. In principle, each bank's effective liquidity ratios may not be

binding, (i.e. their values may be above the regulator's requirement), in which case they are not subject to any liquidity requirement penalty. However, in our calibration exercises, we assume for simplicity that each bank wants to keep a buffer above the required minimum, so that there is a non-pecuniary loss of reputation as the amount of highly liquid asset decreases; in this sense the ratios are always binding. Put differently, we assume that banks' self-imposed ideal liquid asset holdings are always above the actual values of all banks' liquidity ratios. Given this assumption, we can rule out corner equilibria and therefore focus our analysis entirely on well-defined interior solutions whereby banks violate their enhanced liquidity requirements. We assume that penalties are linear as a highly liquid asset declines from its ideal level.⁸

When an effective liquidity adequacy ratio is introduced, a bank chooses higher profitability by taking on more risk and/or raising interest rate spreads. In turn, these higher interest rates, which are charged to borrowers, will cause them to borrow less, thus reducing output, and to take on riskier investments (i.e. to plan to default more often). The benefit to financial stability of safer banks will be offset, to some extent, by both banks and bank borrowers selecting riskier portfolios, higher interest rates and, consequently, lower output. However, the introduction of liquidity adequacy ratios might still be a net benefit, depending on the likelihood of bank contagion, the probability of future shocks, etc. In practice, this adverse side-effect can be mitigated by relating liquidity adequacy ratios more closely to the relative riskiness of assets or by limiting the allowance to rise in interest rates (Hellman et al.).

⁸In practice, there will be some non-linearity as capital falls below its required minimum, but this is too complex to model at this stage.

2.3 Equilibrium Analysis

Our view, in general, is consistent with the long run money neutrality proposition that the RBC and New Keynesian literature suggests. However, our model obtains money non-neutrality in short run equilibrium unlike the RBC models, where neutrality always holds. Furthermore, in stark contrast to the New Keynesian approach, where short run non-neutrality is obtained through real frictions such as monopolistic competition and asymmetric information, in our framework, it is driven by the postulated transaction technology, subsequent transactions and investment demand for money. In other words, liquidity and default are the driving forces of our results.

Proposition 1. Fisher Effect

Suppose that the household spends money $b_t^c > 0$ for buying goods, deposits $m_t^d > 0$ into the bank δ , and invests $m_t^\alpha > 0$ in government bonds at $\forall t \in T$. Then, in any short run equilibrium, we have

$$\ln(1 + r_t^d) \approx \ln\left(\frac{\mathcal{U}'(c_t; \alpha)}{\beta_\alpha E_t \mathcal{U}'(c_{t+1}; \alpha)}\right) + \ln\left(\frac{1 + r_t^f}{1 + E_t r_{t+1}^f}\right) + \ln(E_t \pi_{t+1}) - \ln(E_t \mathcal{R}_{t+1}^d)$$

Proof. Appendix B □

The nominal interest rate is approximately equal to the real interest rate plus risk premia such as interest rate risk, inflation risk and default risk. The ‘Fisher Effect’ proposition explains that nominal price is linked to consumption; if nominal variables are affected, real variables are also affected real variables allocationally.

Proposition 2. Term Structure of Interest Rates

Suppose that the household invests in government bonds $m_t^\alpha > 0$ and market clearing conditions hold in the deposit $m_t^d > 0$, interbank $m_t^b > 0$, and corporate loan $m_t^l > 0$ markets. Then, in any short run equilibrium, we obtain

$$1 + r_t^f \approx (1 + r_t^d) \cdot E_t \mathcal{R}_{t+1}^d$$

$$1 + r_t^d \approx (1 + r_t^b) \cdot E_t \mathcal{R}_{t+1}^b$$

$$1 + r_t^b \approx (1 + r_t^l) \cdot E_t \mathcal{R}_{t+1}^l$$

Proof. Appendix B □

The ‘Term Structure of Interest Rates’ proposition explains that all the nominal interest rates are determined at the same time. Thus, together with proposition 1, we can conclude that nominal interest rates, real interest rates and inflation are all settled simultaneously.

Corollary. Money Non-Neutrality

Supposing that ‘Fisher Effect’ and ‘Term Structure of Interest Rates’ propositions hold, then money is not neutral. This suggests that the central bank can control the real economy, i.e. consumption, production, etc., through adjusting the interbank market rate.

The model deviates from neutrality of money.⁹ As the central bank increases the interbank market rate, this results in high financing costs in the corporate loan market (e.g. ‘Term

⁹Neutrality of money is the idea that a change in the stock of money affects only nominal variables in the economy such as prices, wages, and exchange rates, with no effect on real (inflation-adjusted) variables, like employment, real GDP, and real consumption. Neutrality of money is an important idea in classical economics and is related to the classical dichotomy, whereby nominal variables are determined independently of real variables and vice versa. It implies that the central bank can not affect the real economy (e.g. the number of jobs, the size of real GDP, the amount of real investment) by printing money via monetary policy. Instead, any increase in the supply of money is offset by a proportional rise in prices and wages. This assumption underlies some mainstream macroeconomic models (e.g. real business cycle models).

Structure of Interest Rates' proposition), lower levels of investment and a negative output gap. As a result, there is a real effect: (i) capital goods are redistributed from the firm to the household; (ii) labour demand decreases (i.e. through the Cobb-Douglas production function); and (iii) there is a negative output gap, which directly results in lower levels of consumption. Of course, we cannot say that the increased interbank market rate essentially hurts welfare because capital holdings as well as leisure increase even as consumption decreases, where the combination of capital, leisure and consumption are the constituents of the household's utility. The money non-neutrality (i.e. a redistribution effect) also holds in a complete market economy if there is a real effect that involves redistribution of wealth across lenders and borrowers.

We hasten to add that in the benchmark case if private monetary endowments are set equal to zero and there is no default then money, as expected, would be neutral. It is the wealth redistribution effects and the transaction technology via cash-in-advance constraints which generate money non-neutrality and determinacy of equilibrium.

Definition. No Arbitrage Conditions

Agents do not repay more than what they owe $\mathcal{R}_t^d, \mathcal{R}_t^b, \mathcal{R}_t^l \leq 1$, and they are not rewarded for defaulting on their obligations $\mathcal{R}_t^d, \mathcal{R}_t^b, \mathcal{R}_t^l \geq 0$. Consequently, endogenous default is compatible with the orderly function of the market economy.

Proposition 3. Interest Rate Spreads

Suppose that bank δ invests in government bonds $m_t^\delta > 0$, and market clearing conditions hold in deposit $m_t^d > 0$, interbank $m_t^b > 0$ and corporate loan $m_t^l > 0$ markets for $\forall t \in T$.

Suppose further that the liquidity requirements mandate banks: $c^\gamma, c^\delta > 0$ and $0 < \iota^\gamma, \iota^\delta < 1$.

1. Then in any short run equilibrium, we obtain

$$r_t^f < r_t^d < r_t^b < r_t^l$$

Proof. Appendix B □

Together with the ‘Fisher Effect’ proposition, we can conclude that interest rate spreads are fully determined by risk premia and liquidity requirements of banks.

Proposition 4. *On-the-Verge Conditions*

Suppose that ‘No Arbitrage Conditions’ hold. Then, in any equilibrium, we obtain

$$\begin{aligned} \mathcal{U}'(\Pi_{\theta,t}; \theta) &= \lambda^l (1 - v_t^l) \frac{\mu_{t-1}^l}{\pi_t} \\ \mathcal{U}'(\Pi_{\gamma,t}; \gamma) &= \lambda^b (1 - v_t^b) \frac{\mu_{t-1}^b}{\pi_t} \\ \mathcal{U}'(\Pi_{\delta,t}; \delta) &= \lambda^d (1 - v_t^d) \frac{\mu_{t-1}^d}{\pi_t} \end{aligned}$$

1. The borrower will default completely when the marginal gain for zero delivery of the asset s/he sold is higher than the marginal loss from default;
2. If at zero delivery the marginal utility gain is less than the marginal disutility from default then he will default up to the level where the marginal gain is equal to the marginal loss;
3. The borrower will deliver the obligation fully when the marginal gain for full delivery is lower than the marginal loss.

Proof. Appendix B

□

In our model, the default is endogenously determined by the agent. Each additional unit of income has a marginal value for the agent. On the other hand, not delivering an additional unit in accordance with one's contractual obligation and choosing to default incur a marginal penalty. When the marginal utility is higher than the marginal penalty, the agent decides to default on that additional unit of income. Thus, when the time comes to honour the contractual obligation, the borrower can default completely on her/his promise, default partially or deliver the obligation fully.

2.4 Calibration

There are 18 implied parameters that are estimated in a quarter. Some of them are taken straight from the existing literature: discount factors ($\beta_\alpha = 0.9963$, $\beta_\theta = 0.9844$, $\beta_\gamma = 0.9880$, $\beta_\delta = 0.9927$), depreciation rate of capital stocks ($\tau = 0.0300$) and capital elasticity for production ($c = 0.3333$). Risk aversion factors for both wholesale and retail banks are set by $a_\gamma = a_\delta = 1.0000$. Liquidity requirements for both types of banks are set by $\iota^\gamma = \iota^\delta = 0.3000$ and are higher than the steady state liquidity levels for generating the buffer in the utility function of banks. Other than these, 8 implied parameters such as preference factors, coefficients of liquidity buffer and non-pecuniary default penalties are calibrated endogenously. All of these are summarised in Table 2.3.

TABLE 2.3

There are 7 exogenous shocks and their percentage deviations are presumed to follow AR(1) processes. The steady state levels of both tax rates are set by $\bar{\tau}^c = \bar{\tau}^s = 0.3000$ and

we normalise productivity factor $\bar{a}$ to 1.0000. The steady state levels of endowments such as capital and labour are endogenously calibrated. The levels of transitory policy shocks in a steady state are set by 0.0000. AR(1) coefficients such as productivity factor, endowments and tax rates are presumed to be 0.8000 for generating shocks that persist for around 20 quarters. As monetary and fiscal policies have their own feedback rules, the transitory policy shocks are assumed to have no correlation with their own history. All of these are reported in Table 2.4.

TABLE 2.4

Four interest rates are directly calculated from the first order conditions: $\bar{r}^f = 0.0037$, $\bar{r}^d = 0.0062$, $\bar{r}^b = 0.0099$ and $\bar{r}^l = 0.0159$. For given $\bar{r}^d$ and $\bar{r}^l$, $\bar{\mathcal{R}}^d = 0.9976$ and $\bar{\eta}^\theta = 1.000$ are obtained from the first order conditions as well. Through calibrating $\bar{\eta}^\alpha = \bar{\eta}^\theta = \bar{\eta}^\gamma = \bar{\eta}^\delta$, we have four unknowns (c^γ , c^δ , $\bar{\mathcal{R}}^l$, $\bar{\mathcal{R}}^b$), and four equations. This leads to $c^\gamma = 0.0087$, $c^\delta = 0.0035$, $\bar{\mathcal{R}}^l = 0.9967$ and $\bar{\mathcal{R}}^b = 0.9974$. Other than these, all the endogenous variables are calculated through solving a set of simultaneous equations.

The economy is operating in the deterministic steady state of a competitive equilibrium in which endogenous variables are stationarised and reported in Table 2.5. Our steady state is in line with ‘No Arbitrage Conditions’ which simply means that agents do not repay more than what they owe and they are not rewarded for default on their obligation as $0 \leq \bar{\mathcal{R}}^d, \bar{\mathcal{R}}^b, \bar{\mathcal{R}}^l \leq 1$. In addition, four interest rates satisfy the ‘Interest Rate Spreads’ proposition, $\bar{r}^f < \bar{r}^d < \bar{r}^b < \bar{r}^l$, in our calibration exercise.

2.5 Quantitative Analysis

Fiscal policy involves changing government spending and tax rates. It involves a shift in the government's budget position. Monetary policy involves influencing the demand and supply of money, primarily through the use of interest rates. Monetary policy is most widely used for fine tuning the economy. However, monetary policy has its limitations. In serious recessions, we invariably need a combination of the two.

2.5.1 Assessing the Role of Monetary and Fiscal Policy

Fiscal Expansion

First, we examine the impact of expansionary fiscal shock on key macro-variables: financing cost, investment, labour supply, inflation rate, government debt, etc. In order to finance fiscal expenditure, the fiscal authority issues more gilts and this results in high financing cost in accordance with the 'Term Structure of Interest Rates' proposition. High financing cost depresses investment; a fiscal shock crowds out investment. The equilibrium path of labour supply decreases as well since the firm produces goods in an efficient manner, i.e. Cobb-Douglas production function. In sum, these result in a negative output gap together with inflation induced by low supply and high demand in the goods market. These are summarised in Figure 2.3.

FIGURE 2.3

Increasing Tax Rates

In order to examine the role of tax rules, we investigate the impulse response functions of key macro-variables with respect to two fiscal instruments, the consumption tax rate (Figure 2.4) and income tax rate (Figure 2.5). In terms of impact on the macroeconomy (capital supply, labour supply, private consumption, tax revenue and government debt), both fiscal instruments achieve more tax revenue, which is the first order effect. However, consumption tax rate has higher elasticity than income tax rate in terms of tax revenue. Considering the impact on government debt, there is a more pronounced difference between two. Through increasing the consumption tax rate, there is an improvement in fiscal accounts, but this does not hold for the income tax rate. High income tax reduces supplies of labour and capital; instead a household enjoys leisure and consumes more capital. This directly reduces output, hurts the household's income stream and results in reduction of taxable capacity in the end. However, an increasing consumption tax rate tells us the opposite story. To smooth the consumption over time, the household supplies more capital and labour services. This results in a positive output gap, more expenditure in the goods market and an increment in taxable capacity eventually. Nonetheless, we cannot say that the rise in consumption tax improves the household's welfare as the household gets utility not only from consumption but also from holding capital and enjoying leisure; both capital and leisure decrease in the household utility function.

FIGURE 2.4

FIGURE 2.5

Contractionary Monetary Policy

We investigate the effects of contractionary monetary policy in the benchmark economy, as illustrated in Figure 2.6. The key macro-variables are chosen such as investment, wage rate, output, inflation rate and asset price. Based on the 'Term Structure of Interest Rates' proposition, we can expect that the firm faces high financing cost and reduces the investment. As there is a low level of working capital, the demand for labour services decreases as well and this results in a low wage rate. In sum, these reduce output. In the case of a household, two frictions cause low expenditure on goods, i.e. a high deposit rate and low income streams. Although both demand and supply decrease in the goods market, low goods expenditure (demand) clouds out low production (supply) and results in the birth of deflation. As there is low investment (demand) with the same level of capital endowment, asset prices go down.

FIGURE 2.6

2.5.2 Policy Experiments

Productivity Shock

In Figure 2.7, we examine the impulse response functions of key macro-variables with respect to positive productivity shock. As expected, it directly increases output with the same order of magnitude and the positive supply effect in the goods market induces deflation. We can confirm that there are secondary effects as well. A high level of productivity also changes the optimal allocation of resources for production, thus the equilibrium path

of investment and labour services increases in the short run. However, this effect is not persistent even in the medium run (after the third quarter, it almost disappears) since it is dominated by the primary effects, deflation and a boom in asset prices. High demand (investment) with the same amount of resources (capital endowment) results in the boom in asset prices.

FIGURE 2.7

As we can see in Figure 2.8, a positive output gap results in expansionary fiscal policy and reduction in consumption tax revenue at the same time. As the optimal allocation of resource changes such as through an increment in investment and labour services, there is an increment in income tax revenue. However, this effect cannot work against low consumption tax revenue and high fiscal expenditure, and results in a substantial increment in public debt.

FIGURE 2.8

Figure 2.9 displays four panels, which are the components of the ‘Fisher Effect’ proposition. They show that the endogenous firm repayment rate generates a countercyclical risk premium that acts as an accelerator amplifying the productivity shock and stimulating employment and output. It is worth noting that the positive variation in the repayment rate is relatively weak when the fiscal authority adopts an active policy. In other words, the accelerator effect is diminished as the transmission channel of monetary policy is hurt by the aggressive fiscal policy. Moreover, there is a very important message in the credit markets; two equilibrium paths of the corporate loan rate move in opposite directions. Adopting

the standard Taylor rule, the central bank supplies the interbank loan with a lower interest rate. However, this does not properly equilibrate the demand effect in the government bond market when the fiscal authority pursues an active policy stance. Thus, the risk free rate increases (solid line) and it results in high financing cost of firms. In other words, the interest rate risk dominates inflation risk and default risk.

FIGURE 2.9

Figure 2.10 displays policy targets (inflation rate and output) and financial fragility measures (banks' profitability and repayment rates). Monetary policy targetting inflation performs better than that aiming at output as it induces less deviation for all measures in the figures. Of course, targetting output stimulates the economy, through more output, lower default risk in the corporate loan market and higher profitability in the banking sector. However, it causes relatively deep deflation and higher default risk in the interbank market.

FIGURE 2.10

Capital Endowment Shock

In Figure 2.11, we investigate the effects of positive capital endowment shock. This can take the form of finding a new mine, an endowment in heritage, revaluation of an existing asset, etc. As there is excess capital stock on hand, the household supplies more capital in equilibrium. This induces a high level of accumulated capital stock and leads to a positive output gap. Since the firm cannot produce more output efficiently only with more capital stock, it demands a high level of labour services as well. Thus, it results in a deflationary

equilibrium path since supply effect clouds out demand in the goods market. As marginal productivity of capital is a decreasing function, the excess supply (induced from positive capital endowment) pushes down asset prices.

FIGURE 2.11

As positive capital endowment yields a positive output gap, fiscal expenditure increases. Compared to the level of the increment in output, fiscal expenditure increases more in our simulation exercise ($\psi_y > 1$). Thus, it results in reduction of consumption tax revenue. However, an increment in capital endowment results in a higher level of household's income from sales of capital stock and supplies of labour services (Figure 2.11). This causes an increment of income tax revenue. Even though income tax revenue increases, it cannot cloud out the increment in fiscal expenditure and reduction in consumption tax revenue, which hurts the fiscal balance sheet and results in an increment in public debt. These are explicitly described in Figure 2.12.

FIGURE 2.12

As the fiscal authority changes its policy stance from passive (dashed line) to active (solid line), fiscal accounts respond more sensitively; fiscal expenditure increases, consumption tax revenue decreases and public debt rises. However, income tax does not react straightforwardly. Although the household's income (via capital sales and labour services) increases, the relative reduction in asset prices causes a reduction in income tax revenue for active fiscal policy (solid line) compared to passive fiscal policy (dashed line).

The four panels of Figure 2.13 display the constituents of the 'Fisher Effect' proposition. As discussed, there are deflation and a positive output gap with respect to the chosen shock.

Although the negative deviation of inflation measure is less than the positive deviation of output, the standard Taylor rule assigns more weight to inflation than output. Thus, the central bank takes the easy policy stance in the interbank market. Therefore, we can naturally expect that the risk free rate and corporate loan rate decrease; the liquidity injection in the money market forces the low financing cost for firms and the government. From these figures, we can conclude that three measures of corporate loan risk – the risk free rate, inflation and the repayment rate – move in the same direction. However, the repayment rate (default risk) has lower sensitivity compared to the other two constituents of corporate loan risk.

FIGURE 2.13

The role of fiscal policy in the credit market can be understood from those figures as well. The aggressive fiscal policy (solid line) induces lower levels of inflation risk and interest rate risk compared to the other policy (dashed line). In the case of the repayment rate, it is not so sensitive. However, all three measures support the ‘Fisher Effect’ proposition, in the same direction. This is due to the fact that whenever the fiscal authority aims to be more aggressive, the central bank’s policy intervention becomes weaker. This also supports the argument that policy coordination is important.

As we can see in Figure 2.14, monetary policy is crucial not only for policy targets themselves but also for financial fragility measures. The mis-specified policy rule (solid line) results in serious inflation and a distinctive reduction in output. Moreover, the conditions are sufficient to constitute a financially unstable regime: (i) lower bank profitability as an indicator of a recession in the real economy and (ii) increased default in the corporate loan

market as an indicator of increased volatility and risk-taking of retail banks, leading to financial instability. It is worth noting that the endogenous repayment rate of bank γ (solid line) acts as an accelerator that magnifies the adverse effects of positive capital endowment shock.

FIGURE 2.14

Labour Shock

Lastly, we check the effects of positive labour endowment in Figure 2.15. This can be thought as a high birth rate, opening the labour market to foreigners, etc. Overall, the effect of labour shock is similar to that of capital shock, even though one has more impact than the other. This is due to the higher elasticity of labour services in the Cobb-Douglas production function compared to capital stock. Unlike positive capital endowment shock, there is a persistent boom in asset prices. The high level of labour supply changes the equilibrium path of investment as the firm cannot produce more goods only with more labour services in an efficient manner. This results in more demand in the capital market and an asset price boom.

FIGURE 2.15

Positive labour endowment causes an increment in fiscal expenditure due to the positive output gap. Consumption tax revenue decreases as the total amount of taxable goods is reduced in the market. Increments in labour supply and investment result in a high level of household income and help the government income stream via the increment of income tax revenue. However, the increment in income tax revenue cannot cope with excess fiscal

expenditure and deterioration in consumption revenue. These features are all in line with those resulting from positive capital endowment. Of course, the impacts of positive labour endowment are more prominent than those of positive capital endowment as labour has higher elasticity than capital in the production function. These are shown in Figure 2.16.

FIGURE 2.16

In Figure 2.17, we can establish the interaction between two policy entities, the central bank and fiscal authority. For both simulation cases, it is assumed that the monetary authority does not change its policy rule. Considering the levels of increments in inflation and output, the central bank supplies more liquidity in the interbank market to follow the augmented Taylor rule. However, it does not push down the credit cost in the corporate loan market and government bond market when the fiscal authority maintains an aggressive policy stance. Even though the central bank pumps in more liquidity in the interbank market, it cannot properly mitigate the supply pressure of government bonds. Therefore, interest rate risk dominates inflation risk and default risk when the fiscal authority looks at the output gap very sensitively (solid line). Of course the typical story holds with a standard responsiveness coefficient of fiscal policy (dashed line); three risk measures in the 'Fisher Effect' proposition move in the same direction for explaining the low interest rate in the corporate loan market.

FIGURE 2.17

The role of monetary policy for positive labour endowment is very similar to that for positive productivity shock. Of course, the impact on inflation and output induced by labour

shock is less sensitive than that caused by productivity as the elasticity of labour is less than 1. Figure 2.18 shows that the endogenous firm repayment rate generates a counter-cyclical risk premium acting as an accelerator, amplifying the labour shock and stimulating employment and output. Again, we can confirm that the negative variation in the inter-bank repayment rate is very weak in impact compared to the positive variation in the firm repayment rate, meaning that bank default does not substantially affect the business cycle. Overall, the monetary policy targetting inflation performs better than that aiming at output as all measures in the figures move, resulting in more stable equilibrium paths.

FIGURE 2.18

2.5.3 Policy Response

Each public entity pursues its own policy goal. The fiscal authority aims to minimise the fluctuation in output by adjusting fiscal expenditures and the central bank tries to simultaneously control inflation and output by choosing the interest rate. Thus, the policy experiments are conducted in order to examine policy coordination with respect to two aspects, inflation and output.

Financial fragility is characterised by reduced bank profitability and increased aggregate default. An increase in both banking sector vulnerability and aggregate default is linked to welfare losses (agents' utilities), according to Goodhart et al. 2004. Based on these, we investigate two measures for financial (in)stability, i.e. bank profitability and the aggregate repayment rate.

Lastly, we extend the policy experiments to welfare analysis. By doing so we can investigate whether the policy targets and financial fragility measures are directly in line with

household utility or not.

In Figure 2.19, we can see that monetary policy is more important than fiscal policy in terms of achieving a stable equilibrium path of both inflation and output. If the central bank mis-specifies the policy rule (relaxing the inflation targetting, $\phi_\pi = 0.8$), it causes deep depression; this can be worse when it is combined with active fiscal policy ($\psi_y = 1.2$). For the chosen monetary policy (enhancing inflation targetting, $\phi_\pi = 1.5$), active fiscal policy can achieve a more stable equilibrium path for both inflation and output than passive fiscal policy. These are in line with welfare measures; of course, the role of monetary policy is not distinctive for welfare measures when fiscal expenditure does not react sensitively to output. Therefore, whenever there is a positive productivity shock, strong monetary policy aimed at inflation combined with active fiscal policy can be proper policy coordination. It is clear that a positive productivity shock causes a boom in the economy such as through a positive output gap and increment of banks' profitability. However, it can result in welfare loss without proper policy coordination and lead to vulnerable credit markets, i.e. reduction in aggregate repayment.

FIGURE 2.19

As can be seen in Figure 2.20, we find that there is a very limited role of fiscal policy. No matter how the fiscal authority reacts, the equilibrium path mainly depends on monetary policy. Specifically, if the central bank mis-specifies the policy rule (relaxing the inflation targetting, $\phi_\pi = 0.8$), it causes pronounced inflation as well as a negative output gap. However, those policy targets cannot properly capture gains and losses of welfare measures. Better policy coordination in terms of inflation and output is not in line with welfare

measures. Moreover, enhancing the inflation targetting monetary policy causes a vulnerable banking sector and reduction in the aggregate repayment rate in the economy. This drives us to conclude that credit risk is a crucial component for welfare.

FIGURE 2.20

When there is a positive labour shock, policy targets, financial fragility measures and welfare measures all behave similarly to a period of positive productivity shock. Enhancing inflation targetting monetary policy combined with active fiscal policy results in a stable equilibrium path for both inflation and output. This policy regime reduces vulnerability in the banking sector and is recommendable in terms of welfare measures as well.

FIGURE 2.21

In sections 2.5.1 and 2.5.2, we explain in detail other relevant experiments, e.g. a change in household capital and labour endowment, a change in monetary and fiscal policy, a change in productivity factor, etc. However, in this subsection we highlight some of the main lessons that can be drawn from them.

Agents that have more investment opportunities can deal with unexpected shocks more effectively by using their flexibility in quickly restructuring their investment portfolios. This allows them to transfer ‘negative externalities’ to other agents with a more restricted set of investment opportunities. This result has various implications. Among others, banks without well-diversified portfolios tend to follow a countercyclical credit extension policy in the face of an unexpected shock in the loan market. In contrast, banks that can quickly restructure their portfolio tend to reallocate their portfolio away from the loan market,

thus following a procyclical credit extension policy. This produces changes in a series of interest rates, and therefore the cost of borrowing for agents. This in turn produces a contagion effect on the real sector in the economy.

An improvement such as a positive productivity shock, which is concentrated in one part of the economy, can worsen that for others. The key reason for this lies in the fact that our model has heterogeneous agents and distributional effects therefore operate through various feedback channels among various sectors in the economy, which all are active in equilibrium.

2.6 Concluding Remarks

The economic system is complex and heterogeneous in reality and it has been one of the major tasks of economists to understand and seek a representative model of it. Economists have approached this with the rather simplified assumption that agents within the same sector are homogeneous as this allowed mathematically tractable solutions and produced sensible modelling results in certain cases. However it should be recognised that such approach does not provide insights into certain key features of the real economy. In particular, ignorance of heterogeneity and interactions between banks has led to a lack of power in financial fragility.

To overcome such weakness we aimed to integrate interactive channels in our model. More specifically our model can contribute to current literature in three ways: 1) heterogeneity is an essential component in the real economy and it was fully integrated in the model, especially heterogeneous banking sector to explain contagion mechanism and systemic risk

in the credit markets; 2) liquidity and endogenous default, i.e. minimum characteristics for financial stability analysis, are effectively modelled within the model; and 3) monetary and fiscal policies are analysed contemporaneously to assess complementary and substitute policies, to analyse optimal policy (N.B. constrained Pareto sub-optimality) and to fill out the gap between academics & central bankers by producing tractable models. The addition of these components inevitably increased the complexity of the model, but we tried to avoid excessive complexity by adopting an endowment economy with endowed households, i.e. no firms, no external sector, and no other financial intermediaries.

As an immediate follow up study we will match the model with empirical data as this will give us an idea of how realistic our model is as well as how to improve the model for empirical studies. Moreover, this study can be seen as a starting point of a long journey that will try to analyse financial fragility and evaluate various fiscal policy tools available as remedies. In carrying out future research we will stick to our principles and try to tackle the challenges to produce more illuminating explanations whilst reflecting reality.

2.7 Appendices

Appendix A. First Order Conditions

Household (α)

$$\frac{\partial \mathcal{L}(\alpha)}{\partial b_t^c} = 0 \quad : \quad \frac{\chi_c}{c_t + \alpha_g g_t} = \eta_t^\alpha (1 + \tau_t^c) - \beta_\alpha E_t \left(\eta_{t+1}^\alpha \pi_{t+1}^{-1} \right) \quad (2.A.1)$$

$$\frac{\partial \mathcal{L}(\alpha)}{\partial q_t^n} = 0 \quad : \quad \frac{\chi_n}{n_t - q_t^n} = \left(\frac{\partial y_t}{\partial q_t^n} + (1 - \tau_t^s) p_t^n \right) \beta_\alpha E_t \left(\eta_{t+1}^\alpha \pi_{t+1}^{-1} \right) \quad (2.A.2)$$

$$\frac{\partial \mathcal{L}(\alpha)}{\partial q_t^k} = 0 \quad : \quad \frac{\chi_k}{k_t - q_t^k} = \left(\frac{\partial y_t}{\partial K_t^\theta} + (1 - \tau_t^s) p_t^k \right) \beta_\alpha E_t \left(\eta_{t+1}^\alpha \pi_{t+1}^{-1} \right) \quad (2.A.3)$$

$$\frac{\partial \mathcal{L}(\alpha)}{\partial m_t^d} = 0 \quad : \quad \frac{\eta_t^\alpha}{1 + r_t^d} = \beta_\alpha E_t \left(\mathcal{R}_{t+1}^d \eta_{t+1}^\alpha \pi_{t+1}^{-1} \right) \quad (2.A.4)$$

$$\frac{\partial \mathcal{L}(\alpha)}{\partial m_t^f} = 0 \quad : \quad \frac{\eta_t^\alpha}{1 + r_t^f} = \beta_\alpha E_t \left(\eta_{t+1}^\alpha \pi_{t+1}^{-1} \right) \quad (2.A.5)$$

Firm (θ)

$$\frac{\partial \mathcal{L}(\theta)}{\partial b_t^n} = 0 \quad : \quad \eta_t^\theta p_t^n = \frac{\partial y_t}{\partial q_t^n} \beta_\theta E_t \left(\pi_{t+1}^{-1} \right) \quad (2.A.6)$$

$$\frac{\partial \mathcal{L}(\theta)}{\partial b_t^k} = 0 \quad : \quad \eta_t^\theta p_t^k = \frac{\partial y_t}{\partial K_t^\theta} \beta_\theta E_t \left(\pi_{t+1}^{-1} \right) + (1 - \tau) \beta_\theta E_t \left(\eta_{t+1}^\theta p_{t+1}^k \right) \quad (2.A.7)$$

$$\frac{\partial \mathcal{L}(\theta)}{\partial v_t^l} = 0 \quad : \quad \pi_t = \lambda^l (1 - v_t^l) \mu_{t-1}^l \quad (2.A.8)$$

$$\frac{\partial \mathcal{L}(\theta)}{\partial \mu_t^l} = 0 \quad : \quad \frac{\eta_t^\theta}{1 + r_t^l} = \beta_\theta E_t \left(\pi_{t+1}^{-1} \right) \quad (2.A.9)$$

Retail Bank (γ)

$$\frac{\partial \mathcal{L}(\gamma)}{\partial v_t^b} = 0 \quad : \quad \Pi_{\gamma,t}^{-a_\gamma} = \lambda^b (1 - v_t^b) \mu_{t-1}^b \pi_t^{-1} \quad (2.A.10)$$

$$\frac{\partial \mathcal{L}(\gamma)}{\partial \mu_t^b} = 0 \quad : \quad \frac{\eta_t^\gamma}{1 + r_t^b} = \beta_\gamma E_t \left(\Pi_{\gamma,t+1}^{-a_\gamma} \pi_{t+1}^{-1} \right) \quad (2.A.11)$$

$$\frac{\partial \mathcal{L}(\gamma)}{\partial m_t^l} = 0 \quad : \quad \frac{\eta_t^\gamma + c^\gamma \iota^\gamma}{1 + r_t^l} = \beta_\gamma E_t \left(\mathcal{R}_{t+1}^l \Pi_{\gamma,t+1}^{-a_\gamma} \pi_{t+1}^{-1} \right) \quad (2.A.12)$$

$$\frac{\partial \mathcal{L}(\gamma)}{\partial m_t^f} = 0 \quad : \quad \frac{\eta_t^\gamma - c^\gamma (1 - \iota^\gamma)}{1 + r_t^f} = \beta_\gamma E_t \left(\Pi_{\gamma,t+1}^{-a_\gamma} \pi_{t+1}^{-1} \right) \quad (2.A.13)$$

Wholesale Bank (δ)

$$\frac{\partial \mathcal{L}(\delta)}{\partial v_t^d} = 0 \quad : \quad \Pi_{\delta,t}^{-a_\delta} = \lambda^d (1 - v_t^d) \mu_{t-1}^d \pi_t^{-1} \quad (2.A.14)$$

$$\frac{\partial \mathcal{L}(\delta)}{\partial \mu_t^d} = 0 \quad : \quad \frac{\eta_t^\delta}{1 + r_t^d} = \beta_\delta E_t \left(\Pi_{\delta,t+1}^{-a_\delta} \pi_{t+1}^{-1} \right) \quad (2.A.15)$$

$$\frac{\partial \mathcal{L}(\delta)}{\partial m_t^b} = 0 \quad : \quad \frac{\eta_t^\delta + c^\delta \iota^\delta}{1 + r_t^b} = \beta_\delta E_t \left(\mathcal{R}_{t+1}^b \Pi_{\delta,t+1}^{-a_\delta} \pi_{t+1}^{-1} \right) \quad (2.A.16)$$

$$\frac{\partial \mathcal{L}(\delta)}{\partial m_t^f} = 0 \quad : \quad \frac{\eta_t^\delta - c^\delta (1 - \iota^\delta)}{1 + r_t^f} = \beta_\delta E_t \left(\Pi_{\delta,t+1}^{-a_\delta} \pi_{t+1}^{-1} \right) \quad (2.A.17)$$

Appendix B. Proofs

PROOF OF PROPOSITION 1

From the first order condition (2.A.4), we obtain

$$\frac{1}{1+r_t^d} = \beta_\alpha E_t \left(\frac{\eta_{t+1}^\alpha}{\eta_t^\alpha} \cdot \frac{1}{\pi_{t+1}} \cdot \mathcal{R}_{t+1}^d \right) \quad (2.A.18)$$

As $\bar{\tau}^c \gg \bar{r}^f$, $|\hat{\tau}_t^c| \ll 1$ and $|\hat{r}_t^f| \ll 1$, two first order conditions (2.A.1) and (??) yield

$$\mathcal{U}'(c_t; \alpha) \approx \frac{\eta_t^\alpha}{1+r_t^f} \cdot \tau_t^c \quad (2.A.19)$$

Substituting (2.A.19) into (2.A.18), and assuming that $\tau_t^c = \text{const.}$ for $\forall t \in T$, we obtain

$$\ln(1+r_t^d) \approx \ln \left(\frac{\mathcal{U}'(c_t; \alpha)}{\beta_\alpha E_t \mathcal{U}'(c_{t+1}; \alpha)} \right) + \ln \left(\frac{1+r_t^f}{1+E_t r_{t+1}^f} \right) + \ln(E_t \pi_{t+1}) - \ln(E_t \mathcal{R}_{t+1}^d)$$

PROOF OF PROPOSITION 2

Two first order conditions (2.A.4) and (??) can be expressed by

$$\begin{aligned} \frac{1}{1+r_t^d} &= \beta_\alpha E_t \left(\frac{\eta_{t+1}^\alpha}{\eta_t^\alpha} \cdot \frac{\mathcal{R}_{t+1}^d}{\pi_{t+1}} \right) \\ \frac{1}{1+r_t^f} &= \beta_\alpha E_t \left(\frac{\eta_{t+1}^\alpha}{\eta_t^\alpha} \cdot \frac{1}{\pi_{t+1}} \right) \end{aligned}$$

Thus, two equations lead to

$$1+r_t^f \approx (1+r_t^d) \cdot E_t \mathcal{R}_{t+1}^d \quad (2.A.20)$$

Rearranging two first order conditions (2.A.11) and (2.A.12), we obtain

$$(\eta_t^\gamma + c^\gamma \iota^\gamma) \cdot (1 + r_t^b) \approx \eta_t^\gamma \cdot (1 + r_t^l) \cdot E_t \mathcal{R}_{t+1}^l$$

As $0 < c^\gamma \iota^\gamma \ll \eta_t^\gamma$, we obtain

$$1 + r_t^b \approx (1 + r_t^l) \cdot E_t \mathcal{R}_{t+1}^l \quad (2.A.21)$$

Likewise, from two first order conditions (2.A.15) and (2.A.16), we obtain

$$(\eta_t^\delta + c^\delta \iota^\delta) \cdot (1 + r_t^d) \approx \eta_t^\delta \cdot (1 + r_t^b) \cdot E_t \mathcal{R}_{t+1}^b$$

As $0 < c^\delta \iota^\delta \ll \eta_t^\delta$, we obtain

$$1 + r_t^d \approx (1 + r_t^b) \cdot E_t \mathcal{R}_{t+1}^b \quad (2.A.22)$$

Thus, from (2.A.20), (2.A.21) and (2.A.22), we conclude

$$1 + r_t^f \approx (1 + r_t^d) \cdot E_t \mathcal{R}_{t+1}^d$$

$$1 + r_t^b \approx (1 + r_t^l) \cdot E_t \mathcal{R}_{t+1}^l$$

$$1 + r_t^d \approx (1 + r_t^b) \cdot E_t \mathcal{R}_{t+1}^b$$

PROOF OF PROPOSITION 3

Substituting (2.A.11) into (2.A.12), we obtain

$$\frac{\eta_t^\gamma + c^\gamma \iota^\gamma}{1 + r_t^l} = \frac{\eta_t^\gamma}{1 + r_t^b} \cdot E_t \mathcal{R}_{t+1}^l$$

As $c^\gamma \iota^\gamma > 0$ and $0 \leq \mathcal{R}_{t+1}^l \leq 1$, $\eta_t^\gamma + c^\gamma \iota^\gamma > \eta_t^\gamma \geq \eta_t^\gamma \cdot E_t \mathcal{R}_{t+1}^l$ holds. Thus, we obtain

$$r_t^b < r_t^l \quad (2.A.23)$$

Likewise, the following equation results from two first order conditions (2.A.15) and (2.A.17)

$$\frac{\eta_t^\delta - c^\delta(1 - \iota^\delta)}{1 + r_t^f} = \frac{\eta_t^\delta}{1 + r_t^d}$$

As $c^\delta(1 - \iota^\delta) > 0$, $\eta_t^\delta > \eta_t^\delta - c^\delta(1 - \iota^\delta)$ holds. Thus, we obtain

$$r_t^f < r_t^d \quad (2.A.24)$$

Two first order conditions (2.A.15) and (2.A.16) yield the following equation

$$\frac{\eta_t^\delta + c^\delta \iota^\delta}{1 + r_t^b} = \frac{\eta_t^\delta}{1 + r_t^d} \cdot E_t \mathcal{R}_{t+1}^b$$

As $c^\delta \iota^\delta > 0$ and $0 \leq \mathcal{R}_{t+1}^b \leq 1$, $\eta_t^\delta + c^\delta \iota^\delta > \eta_t^\delta \geq \eta_t^\delta E_t \mathcal{R}_{t+1}^b$ holds. Thus, we obtain

$$r_t^d < r_t^b \quad (2.A.25)$$

Therefore, from (2.A.23), (2.A.24) and (2.A.25), we conclude

$$r_t^f < r_t^d < r_t^b < r_t^l$$

PROOF OF PROPOSITION 4

Supposing that $v_t^l, v_t^b, v_t^d > 0$ holds, then three first order conditions, (3.A.9), (2.A.10) and (2.A.14), yield

$$\begin{aligned}
\mathcal{U}'(\Pi_{\theta,t};\theta) &= \lambda^l(1-v_t^l)\frac{\mu_{t-1}^l}{\pi_t} \\
\mathcal{U}'(\Pi_{\gamma,t};\gamma) &= \lambda^b(1-v_t^b)\frac{\mu_{t-1}^b}{\pi_t} \\
\mathcal{U}'(\Pi_{\delta,t};\delta) &= \lambda^d(1-v_t^d)\frac{\mu_{t-1}^d}{\pi_t}
\end{aligned}$$

Appendix C. Figures

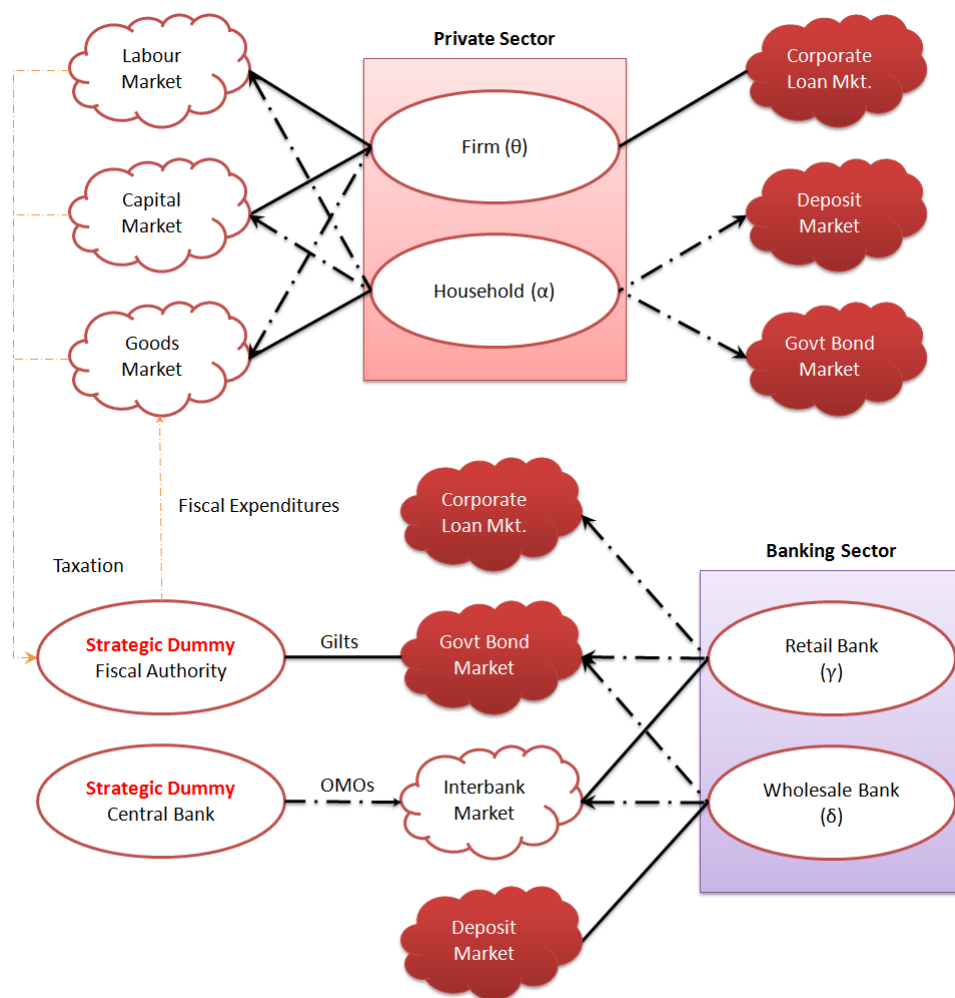


Figure 2.1: Nominal flows in the benchmark economy

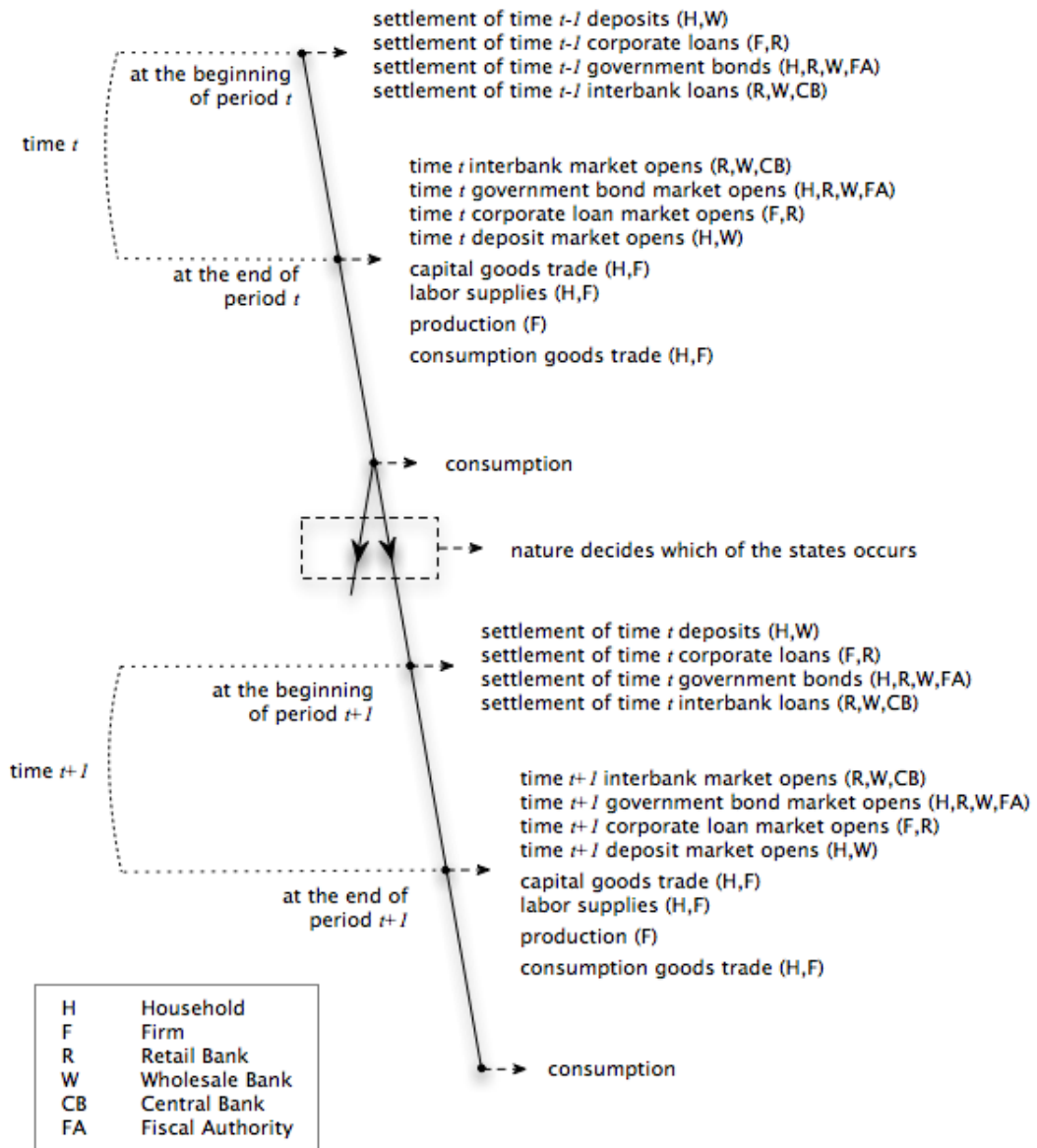


Figure 2.2: Time structure of the model

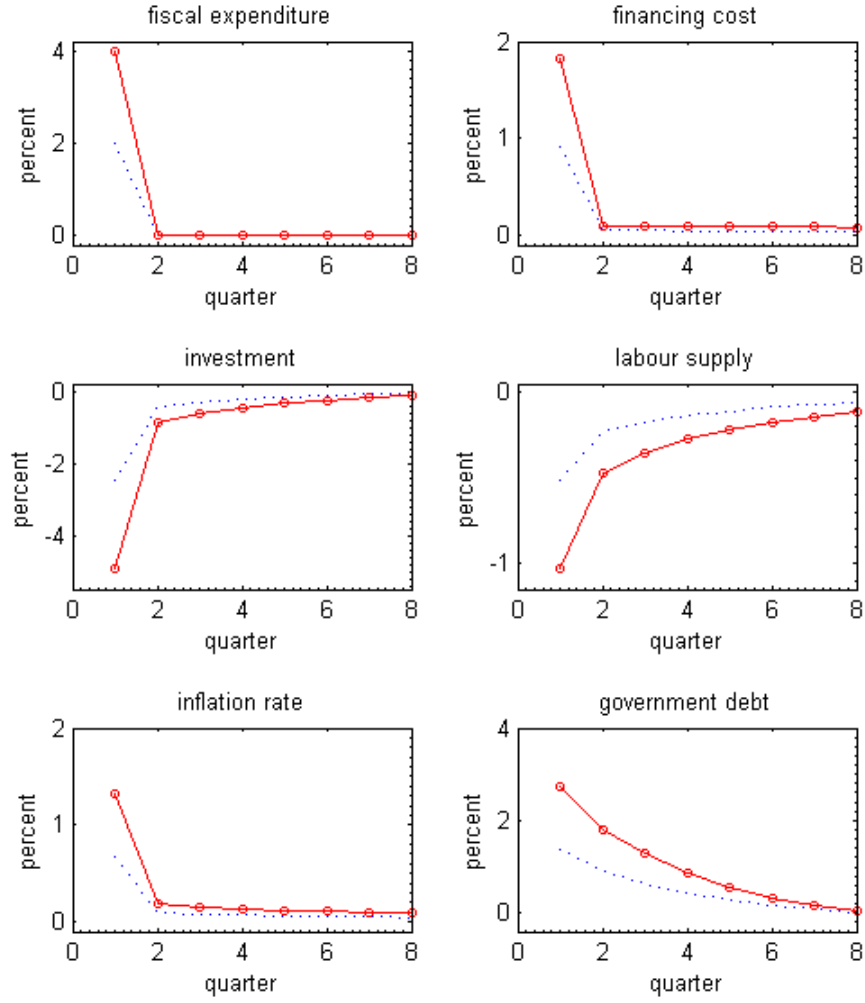


Figure 2.3: Response to one-time expansionary fiscal shock

(i) $\varepsilon_1^g = 0.02$ (dotted line) and (ii) $\varepsilon_1^g = 0.04$ (solid line) given that $\varepsilon_t^g = 0$ for $\forall t \geq 2$. Calculated under the assumption that $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

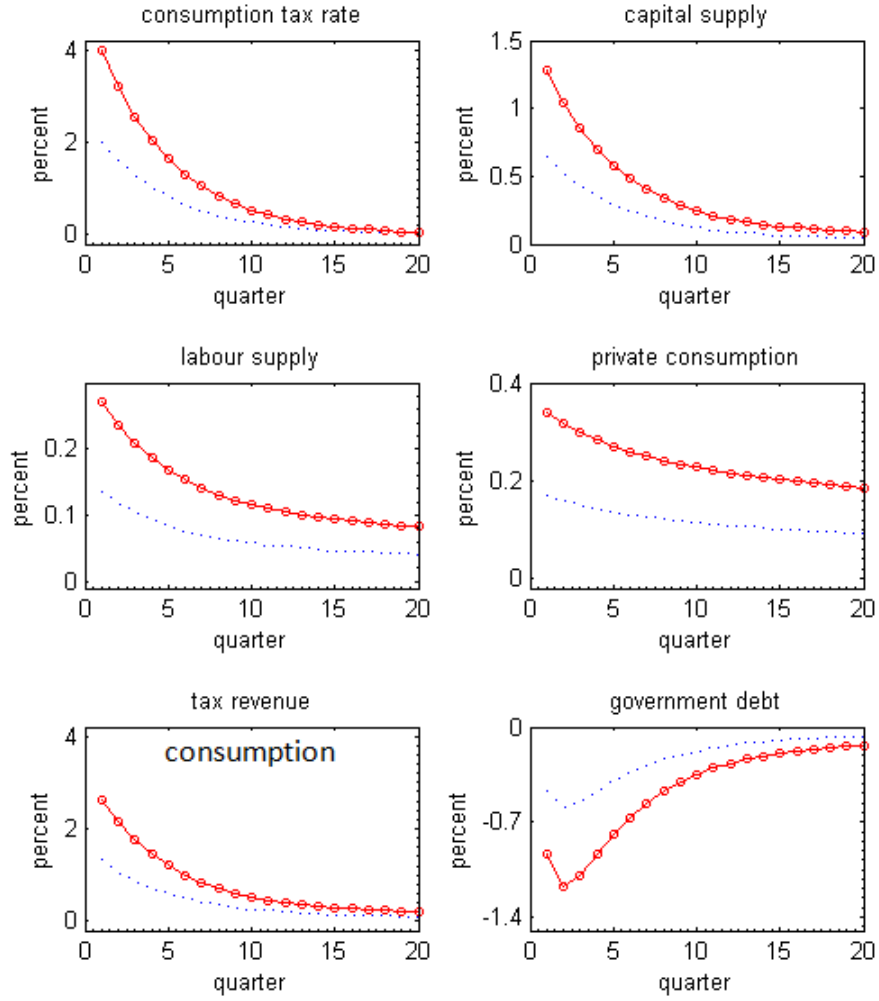


Figure 2.4: Response to one-time tight consumption tax shock

(i) $\varepsilon_1^c = 0.02$ (dotted line) and (ii) $\varepsilon_1^c = 0.04$ (solid line) given that $\varepsilon_t^c = 0$ for $\forall t \geq 2$. Calculated under the assumption that $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 0$ hold. In order to examine the role of tax shock, we assume that the fiscal authority does not adjust fiscal expenditure to output fluctuation in this simulation.

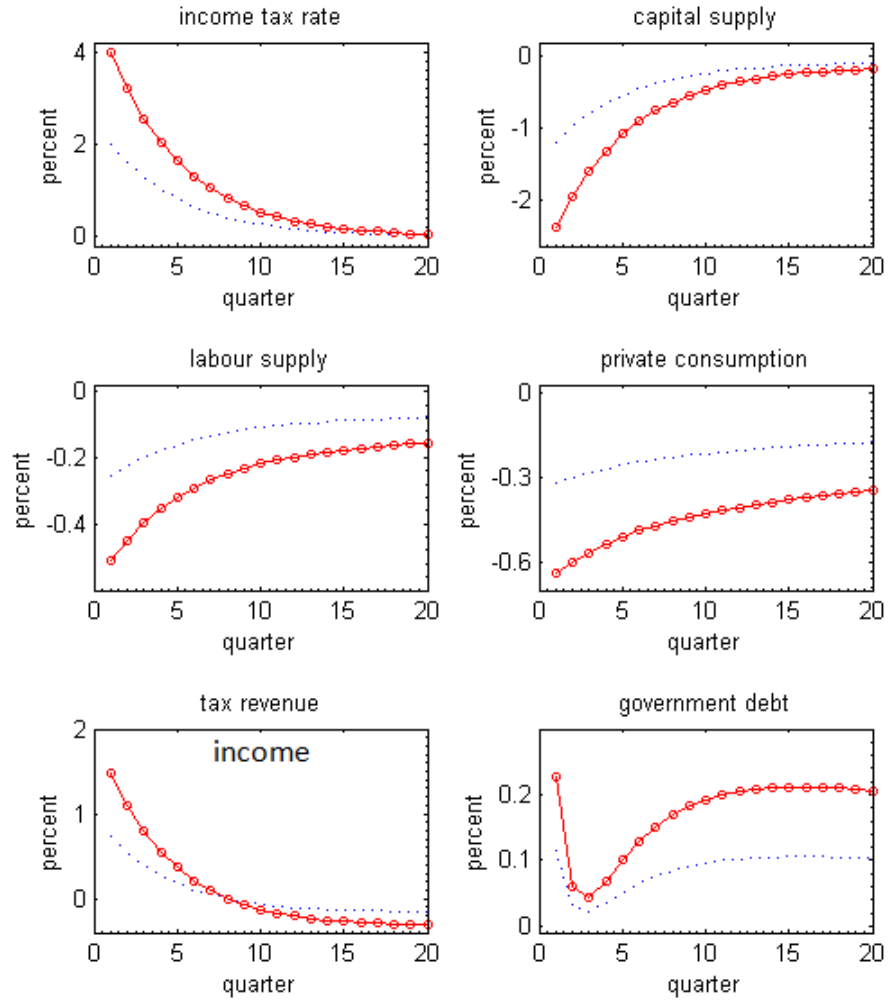


Figure 2.5: Response to one-time tight income tax shock

(i) $\varepsilon_1^s = 0.02$ (dotted line) and (ii) $\varepsilon_1^s = 0.04$ (solid line) given that $\varepsilon_t^s = 0$ for $\forall t \geq 2$. Calculated under the assumption that $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 0$ hold. In order to examine the role of tax shock, we assume that the fiscal authority does not adjust fiscal expenditure to output fluctuation in this simulation.

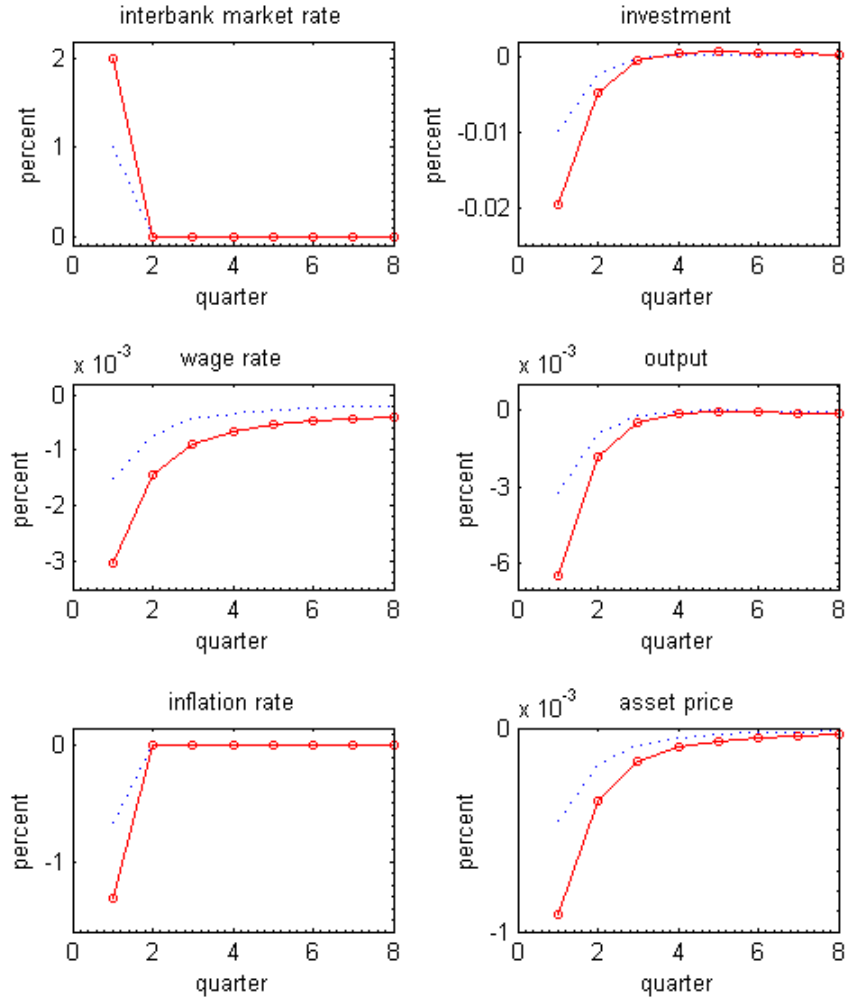


Figure 2.6: Response to one-time contractionary monetary shock

(i) $\varepsilon_1^r = 0.01$ (dotted line) and (ii) $\varepsilon_1^r = 0.02$ (solid line) given that $\varepsilon_t^r = 0$ for $\forall t \geq 2$. Calculated under the assumption that $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

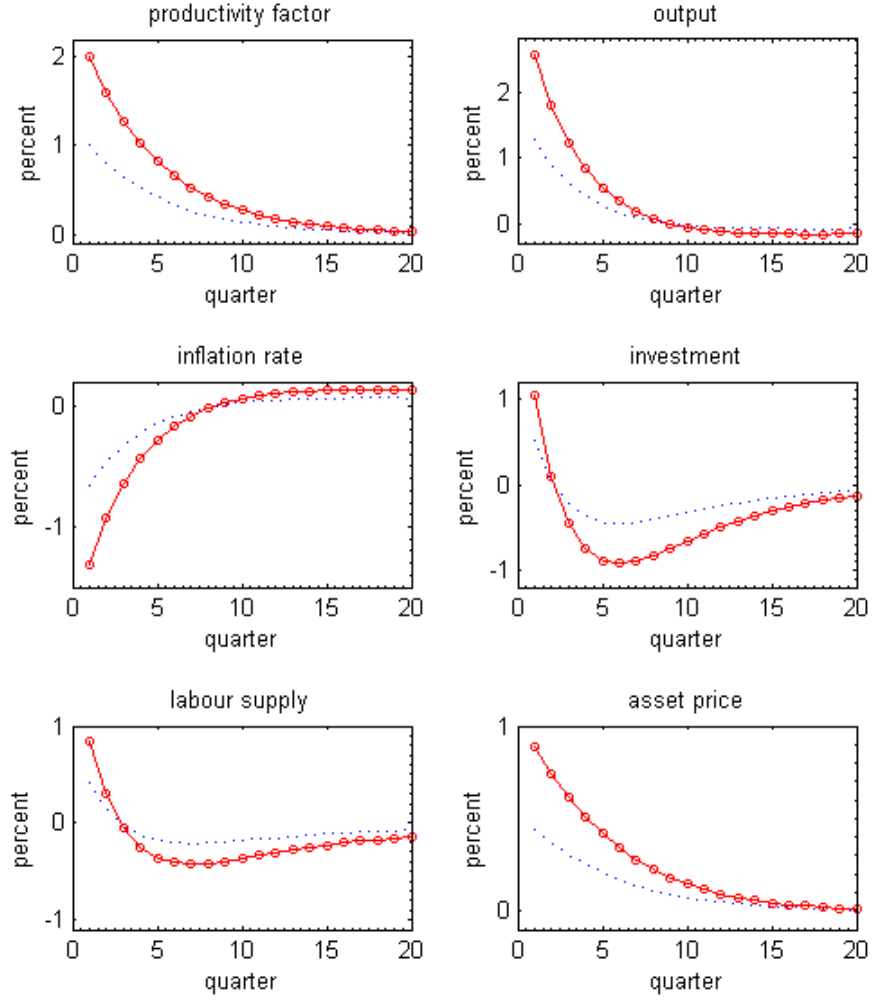


Figure 2.7: Response to one-time productivity shock

(i) $\varepsilon_1^a = 0.01$ (dotted line) and (ii) $\varepsilon_1^a = 0.02$ (solid line) given that $\varepsilon_t^a = 0$ for $\forall t \geq 2$. Calculated under the assumption that $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

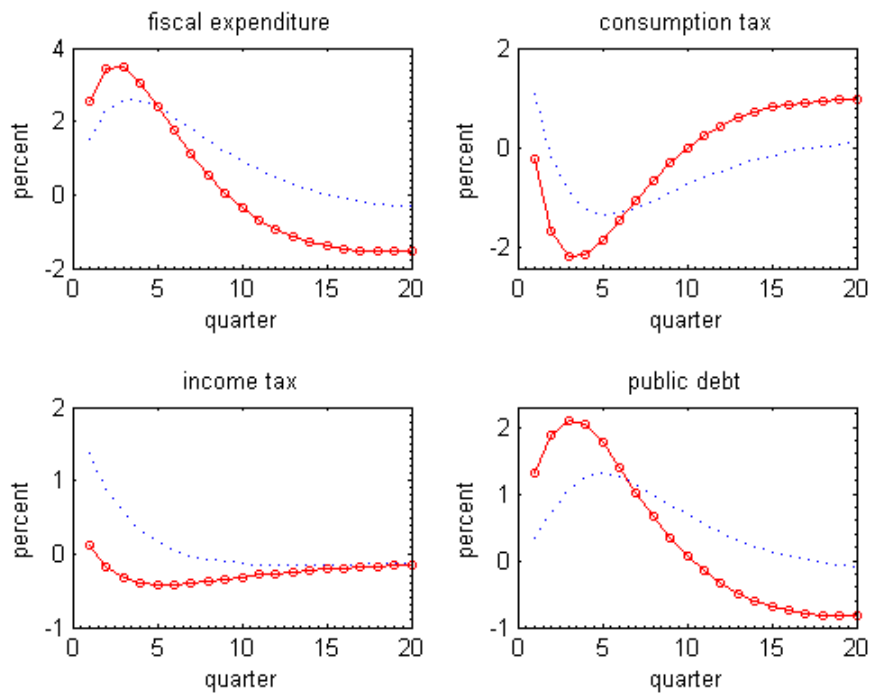


Figure 2.8: Response of fiscal account to one-time productivity shock

$\varepsilon_1^a = 0.01$ and $\varepsilon_t^a = 0$ for $\forall t \geq 2$. Calculated under two assumptions for fiscal policy: (i) $\psi_y = 1.2$ (dotted line) and (ii) $\psi_y = 2.6$ (solid line) given that $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

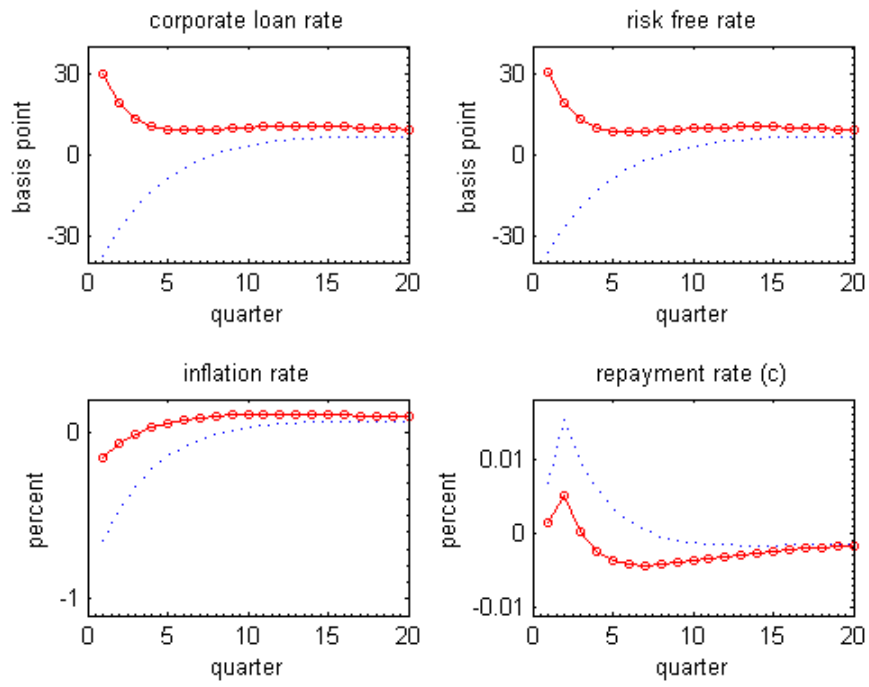


Figure 2.9: Response of credit risk to one-time productivity shock

$\varepsilon_1^a = 0.01$ and $\varepsilon_t^a = 0$ for $\forall t \geq 2$. Calculated under two assumptions for fiscal policy: (i) $\psi_y = 1.2$ (dotted line) and (ii) $\psi_y = 2.6$ (solid line) given that $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

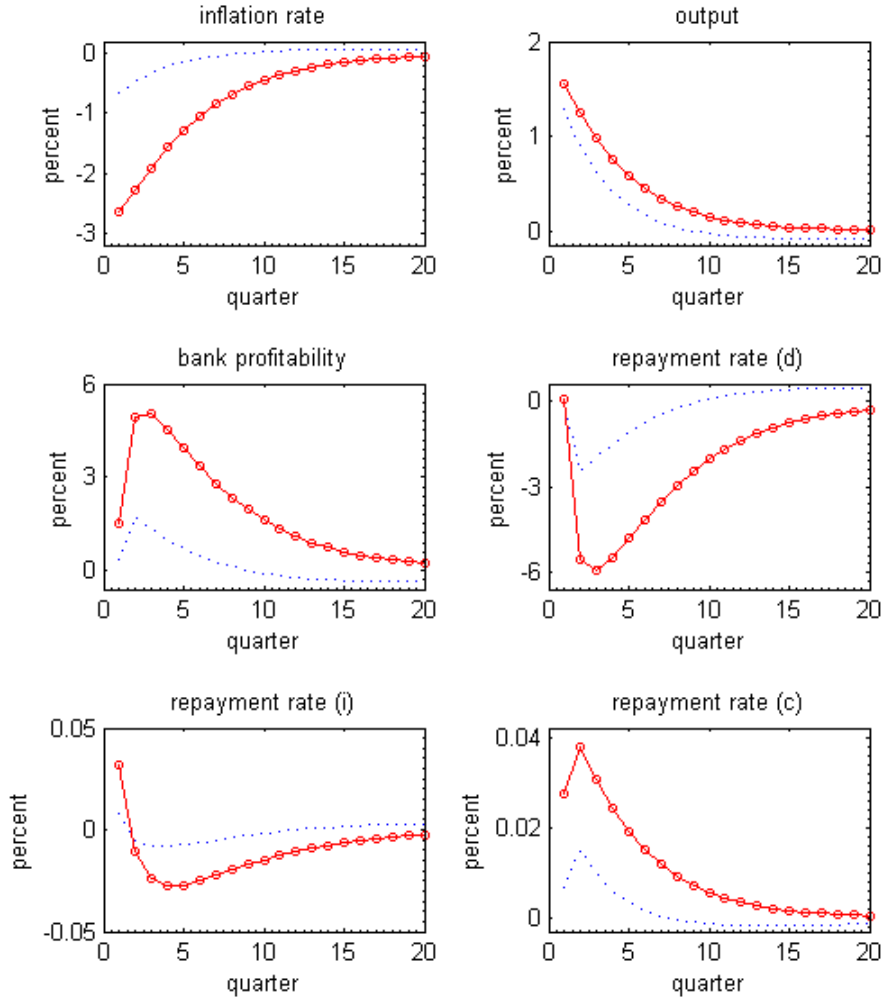


Figure 2.10: Response of policy targets to one-time productivity shock

$\varepsilon_1^a = 0.01$ and $\varepsilon_t^a = 0$ for $\forall t \geq 2$. Calculated under two assumptions for monetary policy: (i) $\phi_\pi = 1.5$ (dotted line) and (ii) $\phi_\pi = 0.5$ (solid line) given that $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

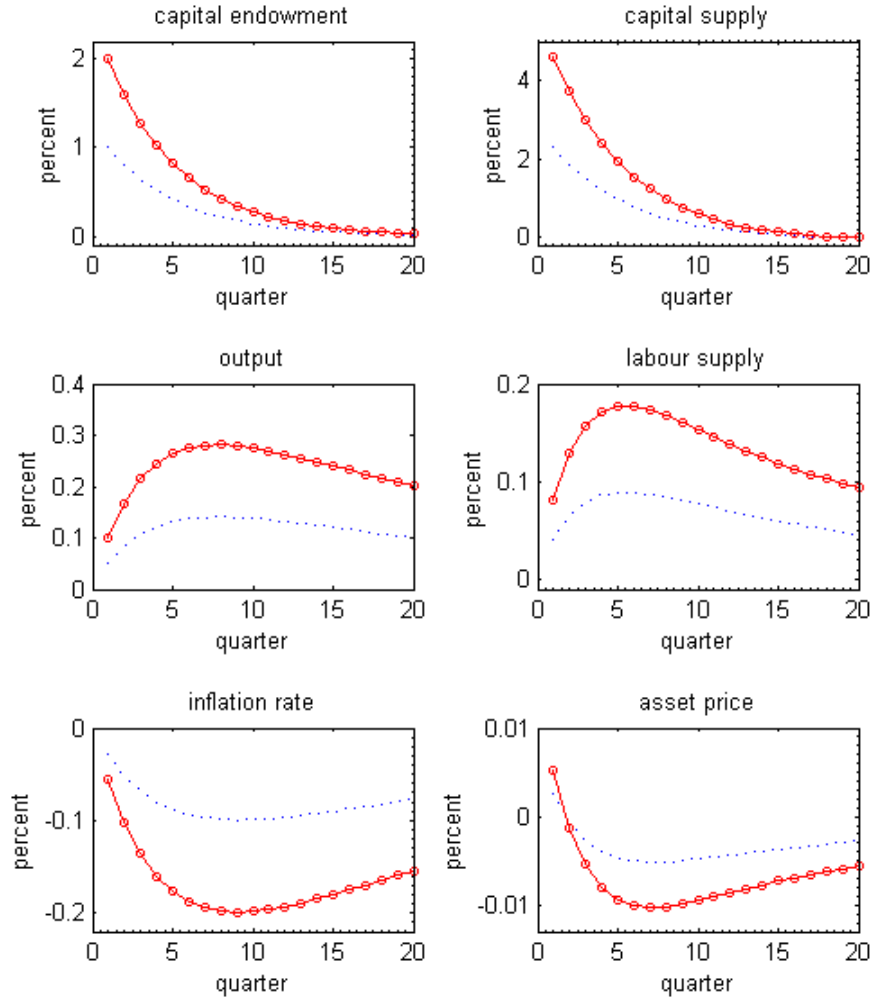


Figure 2.11: Response to one-time capital endowment shock

(i) $\varepsilon_1^k = 0.01$ (dotted line) and (ii) $\varepsilon_1^k = 0.02$ (solid line) given that $\varepsilon_t^k = 0$ for $\forall t \geq 2$. Calculated under the assumption that $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

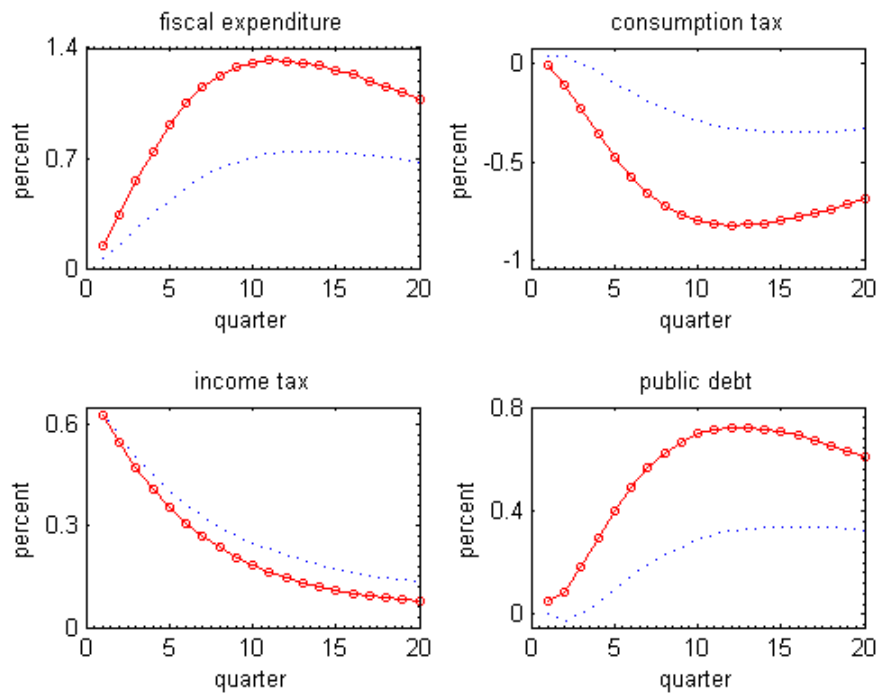


Figure 2.12: Response of fiscal account to one-time capital endowment shock

$\varepsilon_1^k = 0.01$ and $\varepsilon_t^k = 0$ for $\forall t \geq 2$. Calculated under two assumptions for fiscal policy: (i) $\psi_y = 1.2$ (dotted line) and (ii) $\psi_y = 2.6$ (solid line) given that $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

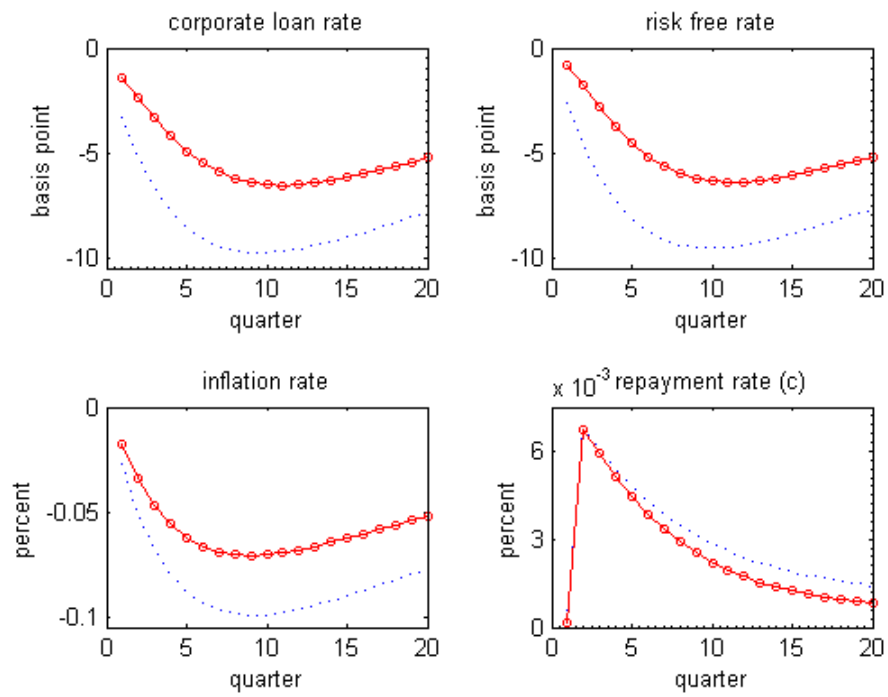


Figure 2.13: Response of credit risk to one-time capital endowment shock

$\varepsilon_1^k = 0.01$ and $\varepsilon_t^k = 0$ for $\forall t \geq 2$. Calculated under two assumptions for fiscal policy: (i) $\psi_y = 1.2$ (dotted line) and (ii) $\psi_y = 2.6$ (solid line) given that $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

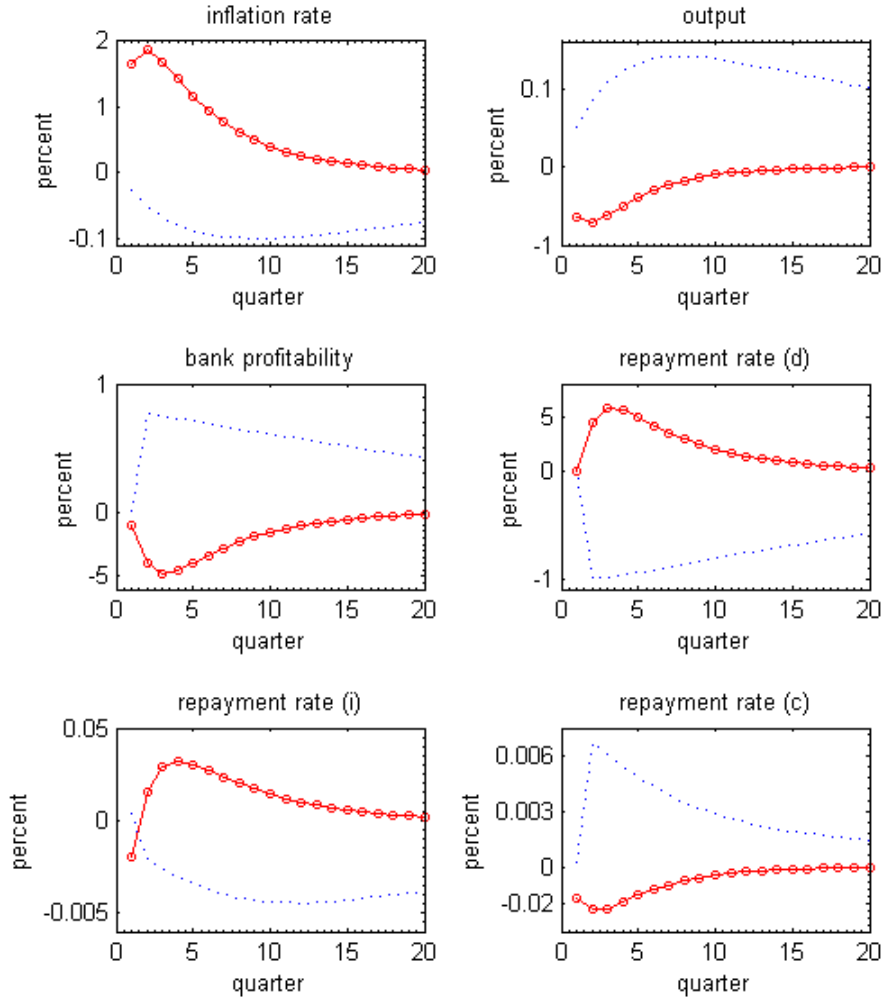


Figure 2.14: Response of policy targets to one-time capital endowment shock

$\varepsilon_1^k = 0.01$ and $\varepsilon_t^k = 0$ for $\forall t \geq 2$. Calculated under two assumptions for monetary policy: (i) $\phi_\pi = 1.5$ (dotted line) and (ii) $\phi_\pi = 0.5$ (solid line) given that $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

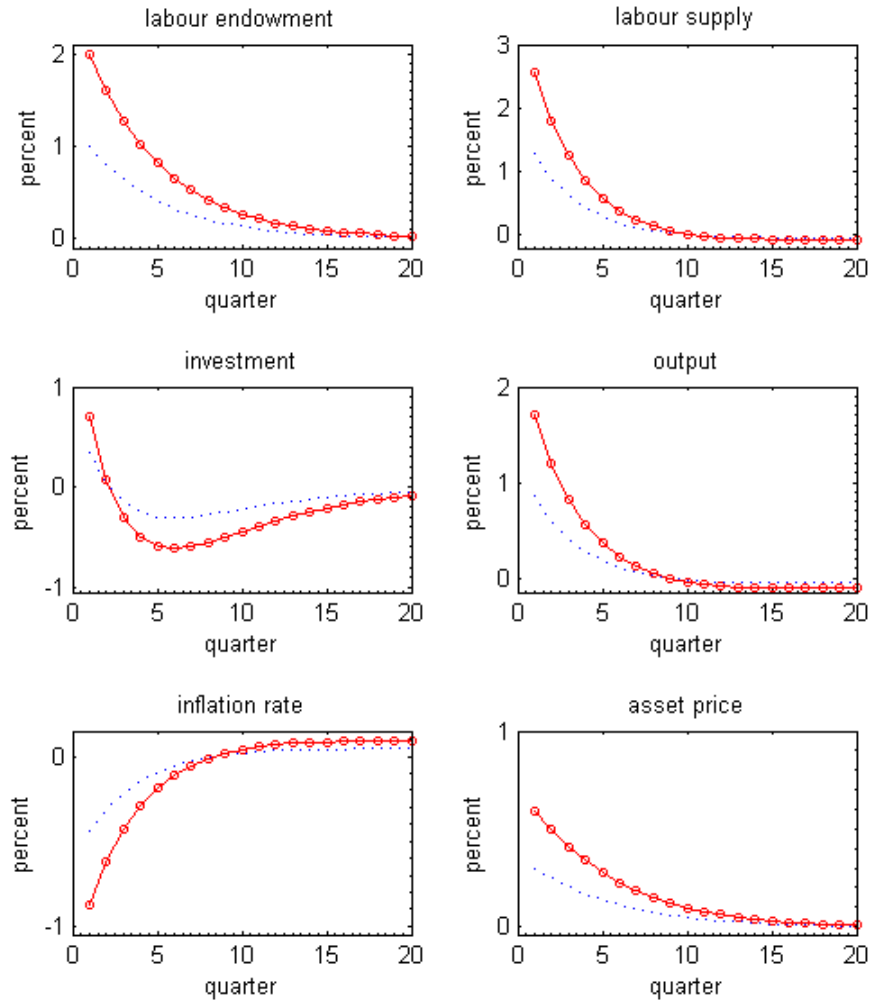


Figure 2.15: Response to one-time labour shock

(i) $\varepsilon_1^n = 0.01$ (dotted line) and (ii) $\varepsilon_1^n = 0.02$ (solid line) given that $\varepsilon_t^n = 0$ for $\forall t \geq 2$. Calculated under the assumption that $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

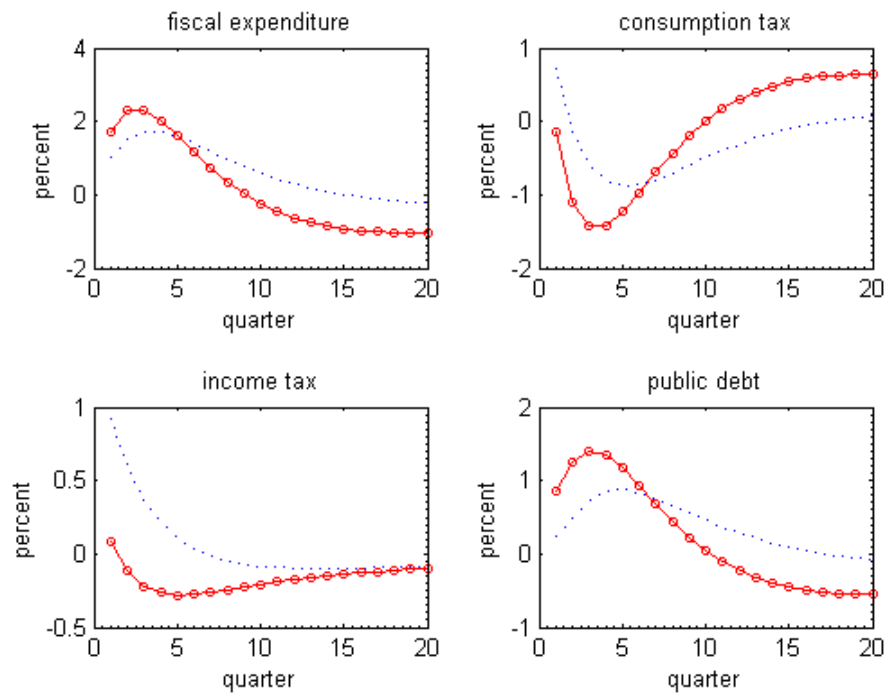


Figure 2.16: Response of fiscal account to one-time labour shock

$\varepsilon_1^n = 0.01$ and $\varepsilon_t^n = 0$ for $\forall t \geq 2$. Calculated under two assumptions for fiscal policy: (i) $\psi_y = 1.2$ (dotted line) and (ii) $\psi_y = 2.6$ (solid line) given that $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

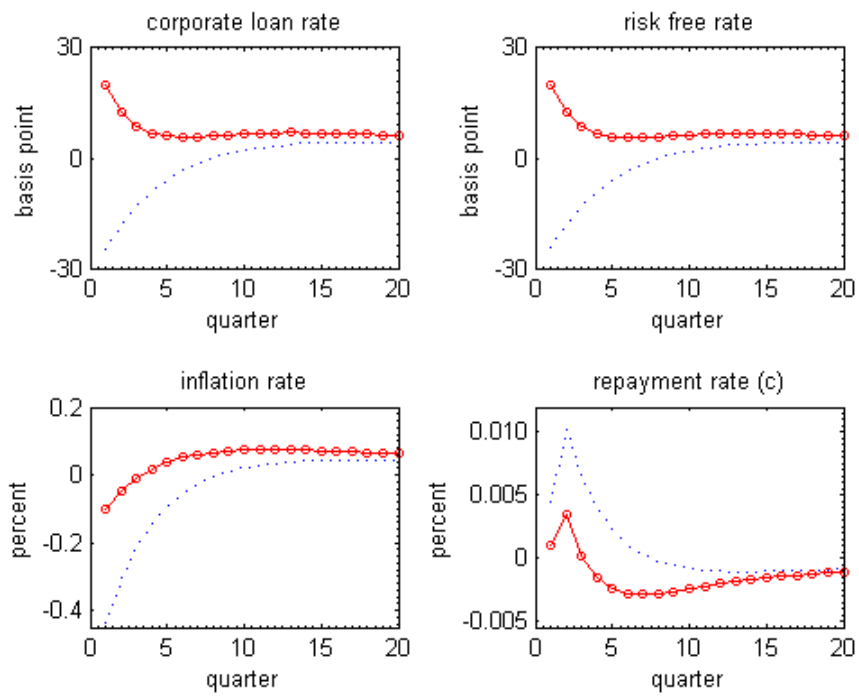


Figure 2.17: Response of credit risk to one-time labour shock

$\varepsilon_1^n = 0.01$ and $\varepsilon_t^n = 0$ for $\forall t \geq 2$. Calculated under two assumptions for fiscal policy: (i) $\psi_y = 1.2$ (dotted line) and (ii) $\psi_y = 2.6$ (solid line) given that $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\phi_g = 0.8$ hold.

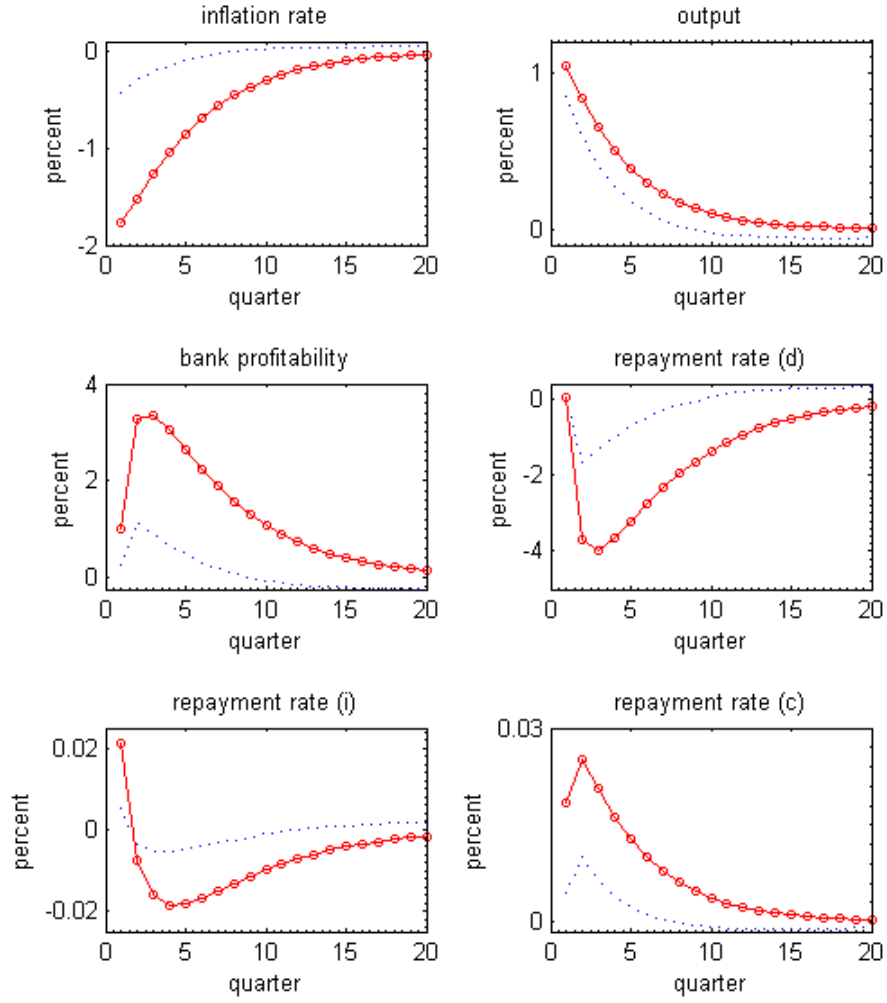


Figure 2.18: Response of policy targets to one-time labour shock

$\varepsilon_1^n = 0.01$ and $\varepsilon_t^n = 0$ for $\forall t \geq 2$. Calculated under two assumptions for monetary policy: (i) $\phi_\pi = 1.5$ (dotted line) and (ii) $\phi_\pi = 0.5$ (solid line) given that $\phi_y = 0.5$, $\psi_g = 0.8$ and $\psi_y = 1.2$ hold.

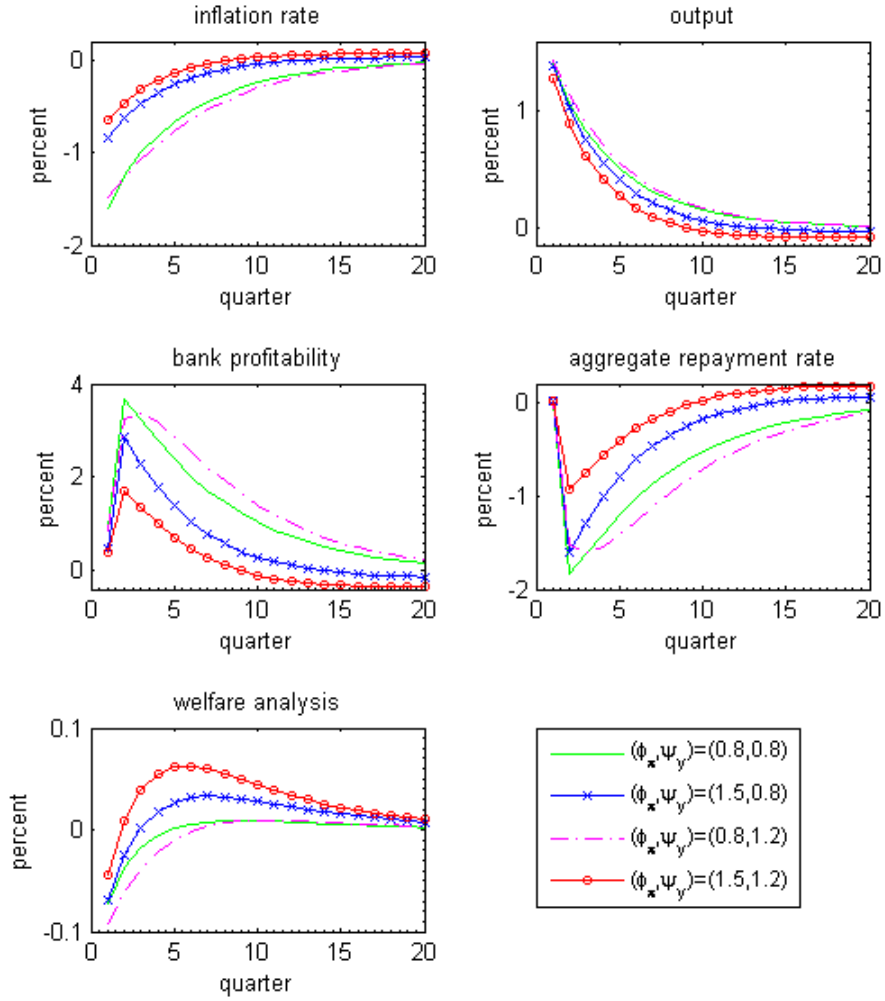


Figure 2.19: Response of welfare to one-time productivity shock

$\varepsilon_1^a = 0.01$ and $\varepsilon_t^a = 0$ for $\forall t \geq 2$. Calculated under four assumptions for monetary and fiscal policy such as (i) $(\phi_\pi, \psi_y) = (0.8, 0.8)$, (ii) $(\phi_\pi, \psi_y) = (1.5, 0.8)$, (iii) $(\phi_\pi, \psi_y) = (0.8, 1.2)$ and (iv) $(\phi_\pi, \psi_y) = (1.5, 1.2)$ given that $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

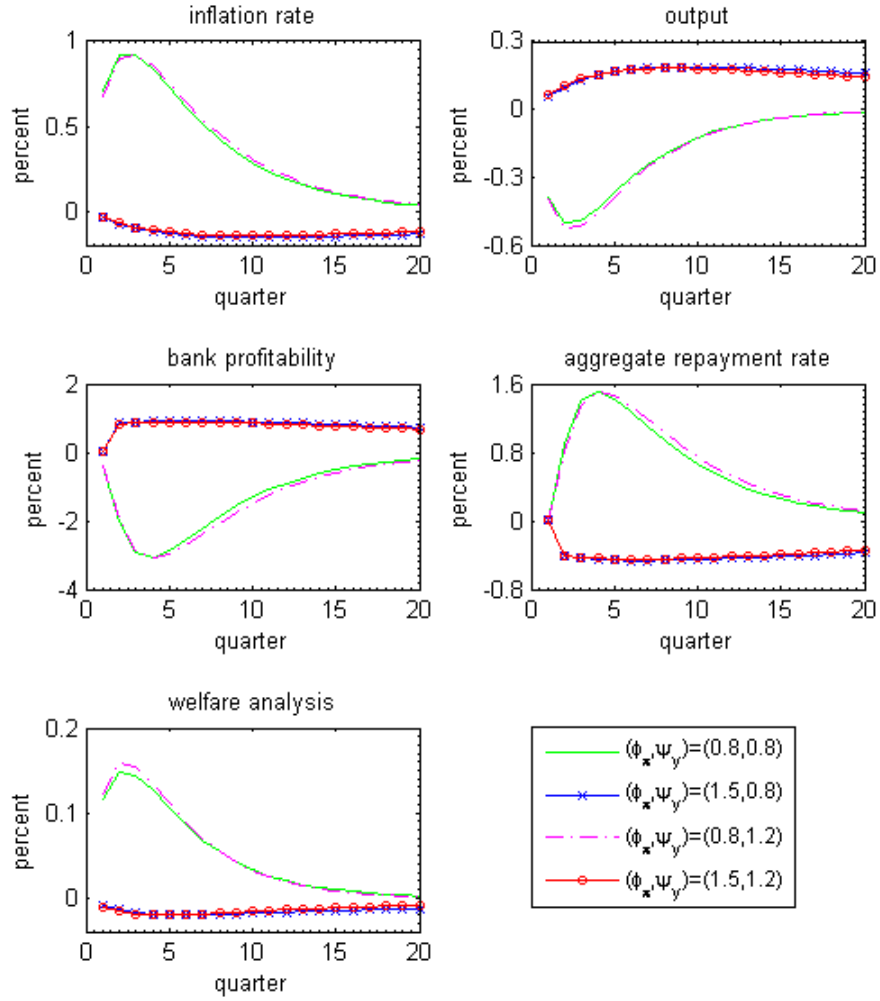


Figure 2.20: Response of welfare to one-time capital endowment shock

$\varepsilon_1^k = 0.01$ and $\varepsilon_t^k = 0$ for $\forall t \geq 2$. Calculated under four assumptions for monetary and fiscal policy such as (i) $(\phi_\pi, \psi_y) = (0.8, 0.8)$, (ii) $(\phi_\pi, \psi_y) = (1.5, 0.8)$, (iii) $(\phi_\pi, \psi_y) = (0.8, 1.2)$ and (iv) $(\phi_\pi, \psi_y) = (1.5, 1.2)$ given that $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

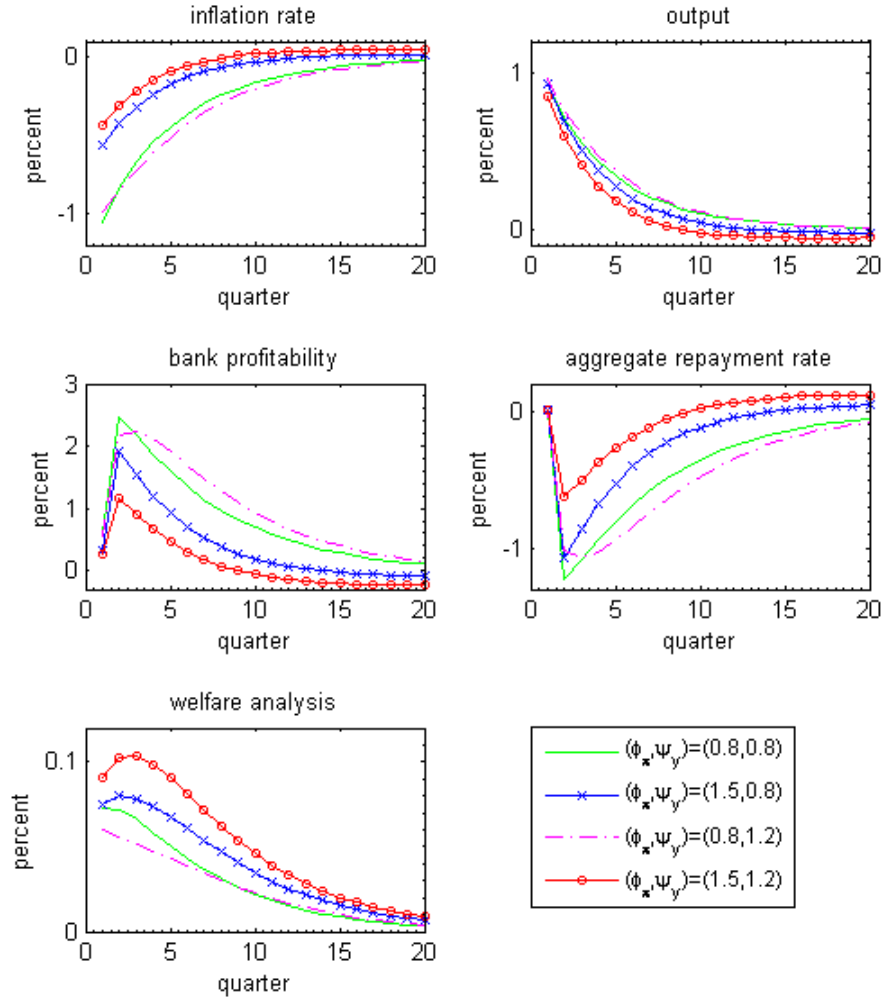


Figure 2.21: Response of welfare to one-time labour shock

$\varepsilon_1^n = 0.01$ and $\varepsilon_t^n = 0$ for $\forall t \geq 2$. Calculated under four assumptions for monetary and fiscal policy such as (i) $(\phi_\pi, \psi_y) = (0.8, 0.8)$, (ii) $(\phi_\pi, \psi_y) = (1.5, 0.8)$, (iii) $(\phi_\pi, \psi_y) = (0.8, 1.2)$ and (iv) $(\phi_\pi, \psi_y) = (1.5, 1.2)$ given that $\phi_y = 0.5$ and $\psi_g = 0.8$ hold.

Appendix D. Tables

Table 2.1: Bank γ 's balance sheet

Assets	Liabilities
Loans to firm m_t^l	Loans from interbank market μ_t^b
Liquid assets m_t^γ	Shareholder equity $\Pi_{\gamma,t}$

Table 2.2: Bank δ 's balance sheet

Assets	Liabilities
Interbank loans to retail bank m_t^b	Deposits from household μ_t^d
Liquid assets m_t^δ	Shareholder equity $\Pi_{\delta,t}$

Table 2.3: Implied parameters

parameter	value	parameter	value
a_γ	1.0000	ι^γ	0.3000
a_δ	1.0000	ι^δ	0.3000
α_g	1.0000	λ^d	1.1710
β_α	0.9963	λ^b	1.0607
β_θ	0.9844	λ^l	1.1555
β_γ	0.9880	τ	0.0300
β_δ	0.9927	χ_c	89.662
c	0.3333	χ_n	239.69
c^γ	0.0087	χ_k	60.256
c^δ	0.0035		

Table 2.4: Exogenous variables

parameter	value	parameter	value
$\bar{a}$	1.0000	ρ_a	0.8000
$\bar{k}$	10.481	ρ_k	0.8000
$\bar{n}$	703.10	ρ_n	0.8000
$\bar{\tau}^c$	0.3000	ρ_c	0.8000
$\bar{\tau}^s$	0.3000	ρ_s	0.8000

Table 2.5: Endogenous variables

parameter	value	parameter	value
$\bar{q}^n$	407.92	$\bar{\mathcal{R}}^d$	0.9976
$\bar{p}^n$	0.4750	$\bar{\mathcal{R}}^b$	0.9974
$\bar{b}^n$	193.76	$\bar{\mathcal{R}}^l$	0.9967
$\bar{q}^k$	4.6390	$\bar{\Pi}_\theta$	33.844
$\bar{p}^k$	13.880	$\bar{\Pi}_\gamma$	0.9977
$\bar{b}^k$	64.389	$\bar{\Pi}_\delta$	0.9989
$\bar{m}^\alpha$	165.28	$\bar{M}$	71.975
$\bar{m}^\gamma$	101.73	$\bar{T}^c$	50.422
$\bar{m}^\delta$	66.111	$\bar{T}^s$	77.445
$\bar{m}^d$	354.01	$\bar{\eta}^\alpha$	1.0000
$\bar{m}^b$	287.90	$\bar{\eta}^\theta$	1.0000
$\bar{m}^l$	258.15	$\bar{\eta}^\gamma$	1.0000
$\bar{\mu}^f$	334.35	$\bar{\eta}^\delta$	1.0000
$\bar{\mu}^d$	356.21	$\bar{t}^\gamma$	0.2827
$\bar{\mu}^b$	363.44	$\bar{t}^\delta$	0.1867
$\bar{\mu}^l$	262.25	$\bar{c}$	168.07
$\bar{r}^f$	0.0037	$\bar{g}$	127.16
$\bar{r}^d$	0.0062	$\bar{b}^c$	168.07
$\bar{r}^b$	0.0099	$\bar{G}$	127.16
$\bar{r}^l$	0.0159	$\bar{\pi}$	1.0000
$\bar{v}^d$	0.9976	$\bar{y}$	295.23
$\bar{v}^b$	0.9974	$\bar{K}^\theta$	154.63
$\bar{v}^l$	0.9967	$\bar{S}$	0.5236

Appendix E: Solution Method

There are n endogenous variables and k exogenous variables in the model. After the log-linearisation, endogenous variables are included as an element of vector X_t and the set of equations is expressed by the 2nd order matrix difference equation

$$BX_t = AE_t[X_{t+1}] + CX_{t-1} + DZ_t \quad (2.A.26)$$

where A , B and C are an n by n square matrix. D is an n by k matrix which includes the contagion mechanism between endogenous variables X_t and exogenous shocks Z_t .

Likewise, after the log-linearisation, exogenous variables are considered as a component of vector Z_t and the set of equations is summarised into the 1st order matrix difference equation

$$Z_t = RZ_{t-1} + \Theta_t \quad (2.A.27)$$

$$\Theta_t \sim N(0_{k \times 1}, \Xi_{k \times k})$$

where R is a k by k square matrix. Θ_t is the bundle of k exogenous shocks, which have the mean vector $0_{k \times 1}$ and the covariance matrix $\Xi_{k \times k}$, respectively.

Therefore, the problem turns into solving the quadratic matrix equation $AP^2 - BP + C = 0_{n \times n}$, which gives us the backward solution. After obtaining P , the forward solution is provided by calculating the matrix equation $Q = (B - AP)^{-1}D$.

Through a simple grid searching procedure of (2.A.26), I get the following matrix difference equation whose solution (equilibrium path) is unique and stable:¹⁰

¹⁰See Uhlig (1995) and Binder and Pesaran (1997) for detailed descriptions of the solution of the quadratic

$$X_t = PX_{t-1} + QZ_t \quad (2.A.28)$$

where P and Q have the dimension n by n and n by k , respectively.

By solving the set of matrix difference equations (2.A.27) and (2.A.28) iteratively, I obtain the simulation results.

Chapter 3

Monetary Policy in a Time of Natural Disaster

3.1 Introduction

Disasters have occurred all the time but been rare events. However, once they happen, they adversely influence regional, national and occasionally the global economy. For instance, the estimated damage from Hurricane Katrina, which struck the US Gulf coast, was over \$150 billion (Burton and Hicks, 2005). This was equivalent to approximately 0.4% of US capital stock and the disturbed economic activities in the affected region contributed to the decline of US GDP in the third quarter of 2005 (Keen and Pakko, 2011). The Great East Japan Earthquake caused more severe damage of around \$300 billion and this accounts for 1.8% of Japanese capital stock or 5.5% of national GDP in 2010. These two examples clearly show how much disasters can disrupt the macroeconomy, not to mention the loss of human capital and the degrading of the physical and mental health of the general population. Moreover, observed frequency of the catastrophes have doubled every ten years since 1960 and the size of negative economic impacts have also increased (Pelling, 2005;

Gassebner and Keck, 2010): this calls for more research to be devoted to this subject.

Toya and Skidmore (2007) empirically showed that countries with higher income, higher educational attainment, greater openness, more complete financial systems and smaller government are less vulnerable to disasters. This argument, on the other hand, suggests that the financial and governmental systems of a country have a strong association with the ability to cope with and prevent disaster damages. Horwich (2000) provides an excellent summary of probable economic impacts from the disaster: a decrease in output level due to the destruction of production means reduced supply and this increases the price level, reduces real income as well as saving and raises interest rates, which results in a fall in investment and finally in the governmental surplus and income. Disasters have important implications for fiscal and monetary policies (Benson and Clay, 2004) as they distort national economic environment and require collected efforts to mitigate any negative impacts from them.

Such disasters can be also interpreted as large macroeconomic shocks, as are wars and economic depressions. In a standard neoclassical model there are two ways to model those shocks: reduction in productivity factor and destruction of working capital. Moreover, the productivity factor plays an important role during economic depressions (Kehoe and Prescott, 2007). Although it is not well understood what causes fluctuations in productivity factor, large and persistent declines in production tend to be linked to poor government policies. The introduction of capital destruction deserves further discussion. In certain cases the literal interpretation of 'destruction' makes sense because those events physically destroy a large share of equipment and infrastructure. However, capital destruction is also indirectly caused too. For instance, it could reflect expropriation of capital holders (if the

capital is taken away and then not used as effectively). Alternatively, it could be a technological revolution that makes a large share of the capital worthless, or it could be also that even though physical capital is not literally destroyed, some intangible capital, such as matches between firms, employees and customers, is lost. Also there is a situation where demand shifts lead to capital idleness after a tragic event. For instance, factories which were built to produce luxury boats or private aircrafts might never be used at full capacity following a deep recession. From the point of view of the theory, what is important is that, realistically, the return on capital is low during a disaster and capital destruction generates this in a simple, tractable way.

This study aims to assess the macroeconomic impacts of disaster shocks by employing a DSGE approach and to draw policy implications in an endogenous framework. Commonly used tools in measuring disaster effects have been traditional input-output (I-O) and Computable General Equilibrium (CGE) models (Rose and Liao, 2005). However, Greenberg et al. (2007) and Rose (2004) claim that the DSGE has potential for the economic analysis of disasters as it provides simulation results based on explicit stochastic processes without substantial data requirements. Recently Keen and Pakko (2011) have used the DSGE to investigate the effective monetary policy responses to disasters. In our model a disaster is considered a negative externality that changes the equilibrium path of economic system and, unlike Keen and Pakko (2011) money, endogenous default and liquidity constraints act as key ingredients in our modelling process to reflect the credit crunch.

The classical macroeconomics archetype and its apparatus did not perform satisfactorily during the recent financial crisis. The micro-founded representative agent model embed-

ded features of the Real Business Cycle (RBC) and New Keynesian paradigms to guide policy makers and central banks towards identifying sources of economic fluctuations, to forecast and delineate the effects of policy interventions. However, the absence of liquidity, default and, more generally, financial frictions rendered it inadequate to address recent issues (Dubey et al., 2005; Shubik and Wilson, 1977). These phenomena are treated as general equilibriums in the dynamic stochastic environment; the current trend is to treat it by using an RBC representative agent model with added financial frictions. The financial frictions are often related to the fact that there is asymmetric information in the credit market, and therefore it generates some inefficiencies in price and allocation. Other types of frictions that are added into these models are related to price stickiness. Bernanke et al. (1998) include money and explain the financial accelerator consequences into the business cycle. In their framework, the main frictions are price stickiness, and costly state verification; these are the main sources of business fluctuations. Other applications include the analysis of the effects of those frictions on capital requirements, as in Covas and Fujita (2010) and Meh and Moran (2010). Kiyotaki and Moore (2004) include asymmetric information in the form of entrepreneur moral hazard and assume default as an out-of-the-equilibrium phenomenon, but this plays a reduced role in the budget constraint of the agents. They explain the transmission of financial shocks into the real economy by the existence of the asymmetric information and the liquidity constraints that this generates.

In this study the aggregate frictions are modelled by factors including money, default and liquidity constraints in equilibrium; I capture the first order effects of those frictions, which enables us to observe that the aggregate financial frictions are in reality sufficient to explain the dynamics of the nominal and real economy. The initial endeavours, by Leao (2006), Walque (2010) and Iacoviello and Neri (2010), which introduced those concepts into the

DSGE framework, did not simultaneously take into account liquidity, agent heterogeneity, money and default risk. Nevertheless, those models are valuable efforts in the development of a plausible explanation of the phenomena that I have observed after the credit crisis.

I have expanded their research by introducing one of the crucial elements, the liquidity constraint that agents face, into the DSGE framework. This should be emphasised because goods are not fully readily tradable. Acharya et al. (2005, 2009), Brunnermeier and Lasse (2009) and Vayanos (2004), have all studied liquidity within partial equilibrium models. In our model, liquidity will be modelled following Espinoza et al. (2009), Tsomocos (2003) and Goodhart et al. (2004). The extension to the dynamic framework is a direct and useful tool for assessing the impact of the financial and real shocks since it provides two important advantages. First, it provides the ability to monitor the impact of a liquidity shock in mid-run as well as short run. Second, the dynamic setting allows us to parameterise different liquidity environments (steady state values) and to examine how shocks impact the economic variables in each case.

The paper is structured as follows. Section 2 presents the empirical evidence of the macroeconomic effects of natural disasters in the Japanese economy. Section 3 proposes a relatively parsimonious model with financial frictions. Section 4 analyses the equilibrium, and Section 5 studies the quantitative properties. Section 6 extends the model by introducing additional shocks and studies the importance of disaster shocks through a structural estimation. Finally, Section 7 closes the study with a summary of findings and suggestions for future research.

3.2 Natural Disasters in the Japanese Economy

Japan has suffered significantly from natural disasters such as earthquakes, tsunamis and volcanoes because of its geographical location (it lies near the junction of the Eurasian and the Pacific plates). Earthquakes and volcanoes are caused by friction between tectonic plates, which can also create tsunamis.

The global economy contracted rapidly and severely in the wake of the Lehman shock. Economic conditions deteriorated sharply not only in the United States and Europe, which were the centre of the financial crisis, but also in Japan as well as in emerging economies especially in Asia, due to the fall in U.S. and European demand. Real GDP in Japan recorded a decline of more than 10% on an annualised basis, a larger drop than in the United States, for two consecutive quarters from the fourth quarter of 2008. Subsequently, Japan's economy picked up rapidly after levelling out around spring 2009, owing to the progress in global inventory restocking, the expansion of fiscal spending, and substantial monetary easing. In 2010, the pace of recovery slowed temporarily from summer through autumn, but the recovery trend remained intact until early 2011.

After the earthquake struck the Pacific coastal areas of eastern Japan, a devastating tsunami followed on March 11, 2011. It was one of the most severe natural disasters on record to hit Japan. The nuclear crisis caused by the earthquake made the situation even worse. The Japanese government has announced an emergency budget of almost 100 billion of US dollars for rebuilding. This is the Japan's largest reconstruction effort since World War II.

Immediately after the earthquake, the Bank of Japan (BOJ) swiftly took a range of measures, such as the provision of ample liquidity and the further enhancement of monetary

easing, focusing on three major aspects: maintaining the functioning of financial and settlement systems, ensuring the stability of financial markets, and supporting the economy. On March 14, 2011, the first business day after the disaster, the BOJ increased the amount of the Program, mainly of the purchases of risk assets and thereby prevented any deterioration in business sentiment from adversely affecting economic activity. In April, the BOJ decided to introduce a funds-supplying operation to provide financial institutions in disaster areas with longer-term funds in order to support their initial response efforts to meet demand for funds for restoration and rebuilding. It also decided to broaden the range of eligible collateral for money market operations.

The BOJ has also been doing its utmost to ensure financial market stability by making use of various funds-supplying operations. When strains in financial markets heighten, as seen in the case of the Lehman shock and the worsening of the European debt problem, financial institutions' funding conditions deteriorate, leading to a tightening of their lending attitudes.

To prevent this from happening, immediately after the earthquake in March 2011 the BOJ provided ample funds on an unprecedented scale, exceeding the amount provided immediately following the Lehman shock. Moreover, in response to the emergence of strains in U.S. dollar short-term funding markets, the BOJ re-established U.S. dollar funds-supplying operations in May 2010. It lowered interest rates on the operations as part of coordinated measures among six major central banks at the end of November 2011, when the European debt problem worsened. At the same time, the central banks also agreed to establish bilateral liquidity swap arrangements enabling the provision of liquidity in any of their currencies in addition to the already available U.S. dollar.

3.3 A Model with Money and Default

In our model, heterogeneous agents, such as capitalists and entrepreneurs, trade goods and capital. Goods are not durable and produce utilities, whilst capital is durable and is used for the production of goods. The capitalist produces capital in each period and the entrepreneur does not in any period. As the capitalist has no technology to store the capital stock and to produce goods, s/he consumes a part of it, sells the remaining to the entrepreneur and buys part of the entrepreneur's output (goods). Through these transaction activities, they correct the difference in their marginal utilities.

The central bank supplies the short-term loan to both agents in order to influence production, inflation, interest rate, etc. The long-term bond is an inter-temporal loan that the entrepreneur borrows from the capitalist as s/he needs to finance for investing on new capital stock. Figure 3.1 depicts these relationships amongst the agents.

FIGURE 3.1

In order to provide a realistic framework assessing the macroeconomic impacts of an external shock in times of financial turmoil, two essential financial frictions, such as money and default, are integrated into the model (Tsomocos, 2003; Goodhart et al., 2004). A negative externality, such as a disaster, hurts production processes (both goods and capital) and decreases existing capital stock (working capital). This externality distorts the income stream of liquidity constraint agents and propagates into a repayment problem on the long-term bond market eventually.

Figure 3.2 indicates the timeline, including the moments at which the various markets

meet. The transaction sequences are rigorously taken into account when the budget sets are described.

FIGURE 3.2

3.3.1 Model Setup

Capitalist

Capitalist starts period t with money on hands through applying short-term loan $\tilde{\mu}_{s,t}^\phi / (1 + r_{s,t})$, claiming back long-term bond $\mathcal{R}_t(1 + r_{l,t-1})\tilde{m}_{l,t-1} / (1 + \pi_t)$ and getting monetary endowment $\omega\tilde{T}_t$. With these revenues, this agent obtains new portfolio composition by extending the long-term bond $\tilde{m}_{l,t}$ as well as purchasing goods $\tilde{b}_{c,t}^\phi$ and capital $\tilde{b}_{k,t}^\phi$. The budget constraint, (3.3.1), summarises these transaction activities

$$\tilde{b}_{c,t}^\phi + \tilde{b}_{k,t}^\phi + \tilde{m}_{l,t} \leq \frac{\tilde{\mu}_{s,t}^\phi}{1 + r_{s,t}} + \mathcal{R}_t(1 + r_{l,t-1})\frac{\tilde{m}_{l,t-1}}{1 + \pi_t} + \omega\tilde{T}_t \quad (3.3.1)$$

where $\tilde{\mu}_{s,t}^\phi$, $r_{s,t}$, $r_{l,t}$, $\mathcal{R}_t$, π_t and ω mean short-term loan obligation, short-term loan rate, long-term loan rate, rational expectation on repayment rate, inflation rate and fraction of money endowment (seigniorage) for capitalist, respectively.

After the capital market closes, the sales income is realised and it is used for repaying her/his short-term loan subject to the cash-in-advance (CIA) constraint, (3.3.2)

$$\tilde{\mu}_{s,t}^\phi \leq \tilde{p}_{k,t}e_t \quad (3.3.2)$$

where $\tilde{p}_{k,t}$ stands for asset price and e_t represents capital supply.

Capitalist starts new capital production process in every period t , simply endowment.

Given the real price, this agent supplies capital in the market, (3.3.3)

$$\ln e_t = \rho_e \ln e_{t-1} + (1 - \rho_e) \ln \bar{e} + \varepsilon_{e,t} \quad (3.3.3)$$

$$\varepsilon_{e,t} \sim i.i.d. N(0, \sigma_e^2)$$

where ρ_e is the AR(1) coefficient of capital endowment. $\varepsilon_{e,t}$ captures *i.i.d.* shock and σ_e represents its standard deviation.

The amount of consumption is determined by how much s/he has purchased from the markets. As a result of her/his biddings, s/he consumes goods $\tilde{b}_{c,t}^\phi$ and holds capital k_t^ϕ in every period. We can think of capital in terms of buildings, so capitalist can either gain utility from house, or rent out capital to entrepreneur in form of factories, shops, etc.

The capitalist begins period t with capital stock passed from previous period $(1 - \delta)k_{t-1}^\phi$, where δ is the depreciation rate and k_{t-1}^ϕ is the capital stock owned in period $t - 1$. After buying new capital, the ultimate quantity of capital stock owned by the capitalist in period t is k_t^ϕ which is expressed as the following law of motion, (3.3.4)

$$k_t^\phi = (1 - \delta)k_{t-1}^\phi + \frac{\tilde{b}_{k,t}^\phi}{\tilde{p}_{k,t}} \quad (3.3.4)$$

The capitalist is risk-averse and maximises the discounted sum of life-time utilities from consuming goods and holding capital, (3.3.5)

$$\max_{\{\tilde{b}_{c,t}^\phi, \tilde{b}_{k,t}^\phi, \tilde{p}_{s,t}^\phi, \tilde{m}_{l,t}\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta^t \mathcal{U}(\tilde{b}_{c,t}^\phi, k_t^\phi; \phi) \quad (3.3.5)$$

Entrepreneur

The entrepreneur enters period t with short-term loan $\tilde{\mu}_{s,t}^\theta / (1 + r_{s,t})$, long-term bond $\tilde{\mu}_{l,t} / (1 + r_{l,t})$ and monetary endowment $(1 - \omega)\tilde{T}_t$. Afterwards, this agent starts reconstructing her/his portfolio. S/he invests $\tilde{b}_{k,t}^\theta$ to bid the capital stock, spends $\tilde{b}_{c,t}^\theta$ to purchase the goods, and uses $v_t \tilde{\mu}_{l,t-1} / (1 + \pi_t)$ to repay the previous bond. All of these transaction activities are described in the budget set, (3.3.6)

$$\tilde{b}_{c,t}^\theta + \tilde{b}_{k,t}^\theta + v_t \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t} \leq \frac{\tilde{\mu}_{s,t}^\theta}{1 + r_{s,t}} + \frac{\tilde{\mu}_{l,t}}{1 + r_{l,t}} + (1 - \omega)\tilde{T}_t \quad (3.3.6)$$

where $\tilde{\mu}_{s,t}^\theta$, $\tilde{\mu}_{l,t}$ and v_t are short-term loan obligation, long-term loan obligation and repayment rate, respectively.

After all the transactions are settled, the entrepreneur gets back her/his sales income y_t and repays her/his short-term loan. It is subject to the following CIA constraint, (3.3.7)

$$\tilde{\mu}_{s,t}^\theta \leq y_t \quad (3.3.7)$$

The entrepreneur starts period t with capital stock passed from previous period $(1 - \delta)k_{t-1}^\theta$, where δ is the depreciation rate and k_{t-1}^θ is the capital stock owned in period $t - 1$. After buying new capital, the ultimate quantity of capital stock owned by the entrepreneur in period t is k_t^θ which is expressed as the following law of motion, (3.3.8)

$$k_t^\theta = (1 - \delta)k_{t-1}^\theta + \frac{\tilde{b}_{k,t}^\theta}{\tilde{p}_{k,t}} \quad (3.3.8)$$

With the capital stock on hand, the entrepreneur goes into the production process. The goods production y_t follows a simple constant-return-to-scale fashion (AK production

function), (3.3.9)

$$y_t = a_t(k_t^\theta)^\alpha \quad (3.3.9)$$

$$\ln a_t = \rho_a \ln a_{t-1} + (1 - \rho_a) \ln \bar{a} + \varepsilon_{a,t} \quad (3.3.10)$$

$$\varepsilon_{a,t} \sim i.i.d. N(0, \sigma_a^2)$$

where α is the output elasticity of capital stock. a_t is the productivity factor whose log-linearised form is assumed to follow simple AR(1) process, (3.3.10). AR(1) coefficient ρ_a captures the persistency of productivity shock. Productivity shock $\varepsilon_{a,t}$, an *i.i.d.* random variable, follows normal distribution having zero mean and a standard deviation σ_a .

Entrepreneur maximises her/his quadratic utility function, not only by consuming goods but also by choosing the optimal level of default as s/he gets the non-pecuniary default penalty λ for her/his default decision, (3.3.11)

$$\max_{\{\tilde{b}_{c,t}^\theta, \tilde{b}_{k,t}^\theta, \tilde{\mu}_{s,t}^\theta, \tilde{\mu}_{l,t}^\theta, \nu_t\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \gamma^t \left[\mathcal{U}(\tilde{b}_{c,t}^\theta; \theta) - \frac{1}{2} \lambda \left\{ \frac{(1 - \nu_t) \tilde{\mu}_{l,t-1}}{1 + \pi_t} \right\}^2 \right] \quad (3.3.11)$$

Central Bank

In line with the standard Taylor rule (1993), I assume that the central bank sets the nominal interest rate $r_{s,t}$ in the short-term loan market following a simple feedback rule, (3.3.12)

$$r_{s,t} = \rho_s r_{s,t-1} + (1 - \rho_s)(\phi_0 + \phi_\pi \pi_t + \phi_y \hat{y}_t) + \varepsilon_{s,t} \quad (3.3.12)$$

$$\varepsilon_{s,t} \sim i.i.d. N(0, \sigma_s^2)$$

where ϕ_π and ϕ_y are usual feedback coefficients on inflation rate and output gap, and $\varepsilon_{s,t}$

is the transitory monetary policy shock, an *i.i.d.* random variable, having zero mean and a standard deviation σ_s .

As the central bank controls the money supply through accommodating the Taylor rule, it supplies $\tilde{M}_t$ into the short-term loan market to meet the demand and the target interest rate $r_{s,t}$ in each period t . During this process, the central bank may make profits $\tilde{T}_t$. This revenue is redistributed to each agent in forms of a lump-sum transfer, (3.3.13)

$$\tilde{T}_t = \frac{r_{s,t-1}\tilde{M}_{t-1}}{1 + \pi_t} \quad (3.3.13)$$

3.3.2 Equilibrium Conditions

Market Clearing Conditions

The short-term loan market clears when the amount of money $\tilde{\mu}_{s,t}^\phi + \tilde{\mu}_{s,t}^\theta$ that borrowers offer to repay in the end of period t is exchanged for the amount of money $\tilde{M}_t$ that the Central Bank extends in the beginning of period t . Thus, for $\forall t \in T$, whenever $\tilde{\mu}_{s,t}^\phi + \tilde{\mu}_{s,t}^\theta > 0$ and $\tilde{M}_t > 0$, the interest rate for the short-term loan $r_{s,t}$ is defined by (3.3.14),

$$1 + r_{s,t} = \frac{\tilde{\mu}_{s,t}^\phi + \tilde{\mu}_{s,t}^\theta}{\tilde{M}_t} \quad (3.3.14)$$

The long-term bond market clears when the amount of money $\tilde{\mu}_{l,t}$ that the entrepreneur offers to repay in the beginning of period $t + 1$ is exchanged for the amount of money $\tilde{m}_{l,t}$ that capitalist extends in the beginning of period t . Thus, for $\forall t \in T$, whenever $\tilde{\mu}_{l,t} > 0$ and $\tilde{m}_{l,t} > 0$, the interest rate for the long-term bond $r_{l,t}$ is defined by (3.3.15),

$$1 + r_{l,t} = \frac{\tilde{\mu}_{l,t}}{\tilde{m}_{l,t}} \quad (3.3.15)$$

The goods market clears when the amount of money $\tilde{b}_{c,t}^\phi + \tilde{b}_{c,t}^\theta$ that consumers offer to buy is exchanged for the quantity of goods y_t that the entrepreneur provides to sell. Thus, for $\forall t \in T$, whenever $\tilde{b}_{c,t}^\phi + \tilde{b}_{c,t}^\theta > 0$ and $y_t > 0$, the price of goods is defined by (3.3.16),

$$y_t = \tilde{b}_{c,t}^\phi + \tilde{b}_{c,t}^\theta \quad (3.3.16)$$

The capital market clears when the amount of money $\tilde{b}_{k,t}^\phi + \tilde{b}_{k,t}^\theta$ that consumers offer to buy is exchanged for the quantity of capital stock e_t that capitalist provides to sell. Thus, for $\forall t \in T$, whenever $\tilde{b}_{k,t}^\phi + \tilde{b}_{k,t}^\theta > 0$ and $e_t > 0$, the price of capital $p_{k,t}$ is defined by (3.3.17),

$$\tilde{p}_{k,t} = \frac{\tilde{b}_{k,t}^\phi + \tilde{b}_{k,t}^\theta}{e_t} \quad (3.3.17)$$

Rational Expectations

The rational expectations condition implies that a lending agent correctly knows the fraction of the loan that will be repaid. Condition (3.3.18) shows that the capitalist is correct in its expectations about the fraction of the long-term bond that will be delivered.

$$\mathcal{R}_t = \begin{cases} \frac{(1+\pi_t)\tilde{p}_{l,t}}{\tilde{\mu}_{l,t-1}} & \tilde{\mu}_{l,t-1} > 0 \\ \text{arbitrary}, & \tilde{\mu}_{l,t-1} = 0 \end{cases} \quad (3.3.18)$$

$\tilde{\mu}_{l,t-1}$ is the amount of money which borrower (θ) promised at period $t - 1$ to repay to lender (ϕ) at period t . Since borrower is allowed to repay partially, s/he only repays $\tilde{p}_{l,t}$ ($0 \leq (1 + \pi_t)\tilde{p}_{l,t} \leq \tilde{\mu}_{l,t-1}$) as a result of solving her/his optimisation problem. Thus, capitalist perceives that $\mathcal{R}_t(1 + r_{l,t-1})\tilde{m}_{l,t-1}/(1 + \pi_t)$ is the repayment from extending the long-term bonds in her/his budget set (3.3.1), where $\tilde{\mu}_{l,t-1} = (1 + r_{l,t-1})\tilde{m}_{l,t-1}$ is the market clearing condition in the long-term bonds market at period $t - 1$, (3.3.15).

$v_{l,t}$ is the repayment rate of borrower, i.e. entrepreneur. Since $p_{l,t}$ is defined by $v_{l,t}\mu_{l,t-1}$ and $\mu_{l,t-1}$ is predetermined at period $t - 1$, deciding of $v_{l,t}$ is equivalent to that of $p_{l,t}$ for borrower. The repayment rate $v_{l,t}$ is used instead of amount of default or repayment in the entrepreneur's budget set (3.3.6) and her/his optimisation problem (3.3.11).

3.4 Equilibrium Analysis

Our view, in general, is consistent with the long run money neutrality proposition that the RBC and the New Keynesian literature suggests. However, our model obtains money non-neutrality in the short run equilibrium unlike the RBC models where neutrality always holds. Furthermore, in stark contrast to the New Keynesian approach where short run non-neutrality obtains through real frictions, such as monopolistic competition and asymmetric information, in our framework, it is driven by the postulated transaction technology, subsequent transactions and demand for money. In other words, money and default are the driving forces of our results.

Proposition 5. *Fisher Effect*

Suppose that the market clearing condition holds in the long-term bond market ($\tilde{m}_{l,t} > 0$). Suppose further that both agents (capitalist and entrepreneur) have some money left over the moment that goods market opens ($\tilde{b}_{c,t}^\phi, \tilde{b}_{c,t}^\theta > 0$). Then, at any short run equilibrium, I have

$$\begin{aligned}\ln(1 + r_{l,t}) &\approx \ln\left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta)}{\gamma E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\theta; \theta)}\right) + \ln(1 + E_t \pi_{t+1}) \\ \ln(1 + r_{l,t}) &\approx \ln\left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\phi; \phi)}{\beta E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\phi; \phi)}\right) + \ln(1 + E_t \pi_{t+1}) + \ln(E_t \mathcal{R}_{t+1}^{-1})\end{aligned}$$

Proof. Appendix B □

The nominal interest rate is approximately equal to the real interest rate plus expected inflation and risk premium. The Fisher Effect explains that the nominal price is linked to consumption, affecting nominal variables as well as real variables allocationally.

Proposition 6. *Term Structure of Interest Rates*

Suppose that the entrepreneur is financed by both a short-term loan ($\tilde{\mu}_{s,t}^\theta > 0$) and a long-term bond ($\tilde{\mu}_{l,t}^\theta > 0$), and has money to buy goods ($\tilde{b}_{c,t}^\theta > 0$) and to invest capital ($\tilde{b}_{k,t}^\theta > 0$). Then, at any short run equilibrium, I get

$$\tilde{p}_{k,t} = \frac{1}{1 + r_{s,t}} \frac{\partial y_t}{\partial k_t^\theta} + \frac{1}{1 + r_{l,t}} (1 - \delta) E_t [\tilde{p}_{k,t+1} (1 + \pi_{t+1})]$$

Proof. Appendix B □

The term structure of interest rates explains that both the short-term loan rate and the long-term bond rate are determined at the same time. Thus, together with proposition 1, I can conclude that nominal interest rates, real interest rate and inflation are all settled simultaneously.

Corollary 7. *Money Non-Neutrality*

If propositions 1 and 2 hold then money is not neutral. This, in turn, implies that the Central Bank can control the real economy, i.e. consumption, production, etc., through adjusting the nominal interest rate in the short-term loan market.

Proposition 8. *Existence of Default in the Steady State*

Suppose that market clearing condition holds in the long-term bond market ($\tilde{m}_{l,t} > 0$). With the assumption that the borrower is more impatient than the lender, $\gamma < \beta$, there exists default in the steady state ($0 < \bar{\mathcal{R}} < 1$).

This ensures that agents do not repay more than what they owe or that they are not rewarded for defaulting on their obligations. Consequently, endogenous default is compatible with the orderly function of the market economy.

Proof. Appendix B □

Proposition 9. *On the Verge Condition*

Suppose that there exists default ($\nu_t > 0$) and entrepreneur has money to buy goods ($\tilde{b}_{c,t}^\theta > 0$). Then, at any short run equilibrium, I obtain

$$\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta) = \lambda(1 - \nu_t) \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t}$$

1. The borrower will default completely when the marginal gain for zero delivery of the asset s/he has sold is higher than the marginal loss from default,
2. If at zero delivery the marginal utility gain is less than the marginal disutility from default then s/he will default up to the level that the marginal gain is equal to the marginal loss,
3. The borrower will deliver fully when its marginal gain for full delivery is lower than the marginal loss.

Proof. Appendix B

□

In our model, the default is endogenously determined by the agent. Each additional unit of income has a marginal value for the agent. On the other hand, not delivering an additional unit in accordance with the contractual obligation and choosing to default incurs a marginal penalty. When the marginal utility is higher than the marginal penalty, the agent decides to default on that additional unit of income. Thus, when the time comes to honour the contractual obligation, the borrower can either default completely on her/his promise, default partially or deliver the obligation fully.

3.5 Quantitative Analysis

To evaluate the quantitative effects of productivity and capital endowment shocks, I construct a series for those shocks using some of the model restrictions (as described below). The macroeconomic effects are then captured by the responses of the model to the shocks. Through the simulation of the model I am able to show that those shocks are important, not only for capturing the dynamics of real quantities but also for the dynamics of financial flows.

Before proceeding, two points about the nature of the exercise and the results need to be clarified. First, the finding that the shocks have played an important role in the real quantities does not mean that other shocks are not important. The exercise is not designed to replicate exactly the empirical series of interest. Second, the fact that I abstract from other shocks does not bias the results since the approach I use to identify those shocks is independent of how many shocks are added to the model.

3.5.1 Parameterisation

The parameters can be grouped into three sets. The first set includes parameters that can be calibrated using steady state targets, some of which are typical in the literature. The second includes parameters that cannot be calibrated using steady state targets, initiating the dynamics of the economy. The third comprises parameters able to prescribe the reaction function of the central bank's monetary policy.

Parameters Set with Steady State Targets

The period in the model is a quarter. I set $\beta = 0.975$ (per year) and selected the target inflation rate $\bar{\pi} = 0$, implying that the annular steady state return from holding risk free assets is 2.56 percent. The parameter in the production function is set to $\alpha = 0.5$ and the depreciation rate to $\delta = 0.12$ (per year). The mean value of productivity $\bar{a}$ is normalised to 1. These values are standard and the quantitative properties of the model are not particularly sensitive to this group of parameters.

The interest rate on short-term loans is set to $\bar{r}_s = 0.005$ (per year), a near-zero policy rate. The interest rate on long-term bonds is chosen to have a steady state value $\bar{r}_l = 0.06$ (per year), generating a sufficiently large interest rate spread. Afterwards, the entrepreneur's discounting factor and the steady state level of repayment are directly calculated from the first-order conditions, (3.A.1) and (3.A.7), such as $\gamma = 0.9434$ (per year) and $\bar{v} = 0.9676$ (per year), respectively.

For the share of consumption goods, I consider that two types of agents consume half of production $\bar{b}_c^\phi = \bar{b}_c^\theta$. The seigniorage is distributed to each agent equally at every period

$\omega = \frac{1}{2}$. The capital endowment is set such that the total amount of capital stock in the steady state is constant. More specifically, the amount of depreciation corresponding to $\bar{k}$ is $\delta\bar{k}$ and it is presumed that the capitalist consumes $\bar{c}_k$. Thus, it is set $\bar{e} = \delta\bar{k} + \bar{c}_k$ so that depreciated and consumed capital is replenished in the steady state.

In addition, I use numerical methods which solve the set of simultaneous equations. Thus, I presume that the economy is operating in the deterministic steady state of a competitive equilibrium; implied parameters are summarised in Table 3.1 and endogenous variables are stationarised and reported in Table 3.2. Our steady state guarantees the existence of default, which means that agents do not repay more than they owe and are not rewarded for default.

TABLE 3.1

TABLE 3.2

Parameters that Cannot be Set with Steady State Targets

The parameters that cannot be set with steady state targets are those determining the stochastic properties of the shocks. Of course, in a steady state equilibrium the stochastic properties of the shocks do not matter. Therefore, I use an alternative procedure to construct a series of productivity and capital endowment shocks.

For the productivity variable a_t , I follow the standard Solow residuals approach. Using the AK production function (3.3.9), I derive

$$\hat{a}_t = \hat{y}_t - \alpha \hat{k}_t \quad (3.5.1)$$

where $\hat{a}_t$, $\hat{y}_t$ and $\hat{k}_t$ are the percentage or log-deviations from the deterministic trend. Given the value of α and the empirical series for $\hat{y}_t$ and $\hat{k}_t$, I construct the $\hat{a}_t$.

For output I use gross domestic product (GDP) deflated by Consumer Price Index (CPI); GDP is real GDP from the Cabinet Office (Quarterly Estimates of GDP). The $\hat{y}_t$ series used in (3.5.1) is constructed by linear detrending the log of y_t over period 1996.I-2012.II.

Capital stock is measured as ‘Tangible fixed assets all industries’ minus ‘Finance and insurance’ from the Cabinet Office (Preliminary Quarterly Estimates of Gross Capital Stock of Private Enterprises). Then, the $\hat{k}_t$ series is constructed through applying the same procedure as output - deflating the capital stock using CPI and detrending the log of k_t .

Likewise, capital endowment is directly measured as Gross Fixed Capital Formation (GFCF) deflated by CPI from the Federal Reserve Economic Data (JPNGFCFQDSMEI). In the same manner as $\hat{y}_t$ and $\hat{k}_t$, the $\hat{e}_t$ sequence is made by detrending the log of e_t .

I verify a condition ex-post: after constructing the series for the shocks, I feed the shocks into the model. Notice that I do not directly force any endogenous variable to perfectly match an individual data series.

After constructing the series for productivity and capital endowment over the period 1996.I-2012.II, I estimate the autoregressive systems, (3.3.3) and (3.3.10), in the forms of (3.5.2) and (3.5.3). These forms are chosen to have AR(1) coefficients and standard deviations generated by the model over the period 1996.I-2012.II equal to the empirical ones. The full set of parameters is given in Table 3.3.

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t} \quad (3.5.2)$$

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{e,t} \quad (3.5.3)$$

TABLE 3.3

Now that I have described the procedure used to construct the series for productivity and capital endowment, it should be clear that these series do not depend on the number of shocks included in the model. No matter how many shocks I add to the model, (3.5.2) and (3.5.3) will not be affected. Thus, given empirical measurements for e_t , I would directly generate the same series for capital endowment shocks. Similarly, given the observable variables k_t and y_t , I would generate the same series for productivity shocks.

Parameters that Prescribe the Monetary Policy

The Taylor rule has been a popular tool to assess a monetary policy stance. It is viewed as a reaction function of a central bank, which determines the policy nominal interest rate if the GDP gap and the deviation of the inflation rate from the target inflation rate are given. The simple version of the Taylor rule can be defined as (3.3.12).

If the nominal interest rate is regressed on the constant term, the GDP gap and the inflation deviation, the coefficients ϕ_y and ϕ_π are obtained. In this paper, I estimate the Taylor rule by using the data from the last 16 years in Japan and calculate the fitted value of the policy nominal interest rate (i.e. the target interest rate) during the 2000s.

I use the inflation rate for the CPI from the Federal Reserve Bank of St. Louis. For the short-term interest rate, I use the uncollateralised overnight call rate from the BOJ. For the GDP gap (the log difference of the GDP and the potential GDP), I use sequence $\hat{y}_t$ constructed by linear detrending.

The result of estimation of the Taylor rule is shown in the following table.

TABLE 3.4

The result shows that the coefficient of the inflation rate is not different from zero and this means that the BOJ has not been taking affirmative action against inflation. The ‘Taylor Principle’, $\phi_\pi > 1$, does not hold. However, the coefficient of output gap is greater than zero and this implies that the BOJ targets output stabilisation: ‘Learning Against Wind’, $\phi_y > 0$.

To estimate the residual of the Taylor rule, I follow a similar approach but use the enforcement constraint (3.3.12) under the assumption that this constraint is always binding. The variable $\varepsilon_{s,t}$ is determined as residual using the empirical series for $r_{s,t}$, π_t and $\hat{y}_t$. Of course, the validity of the procedure depends on the validity of the assumption that the enforcement constraint is always binding.

3.5.2 Findings

Real Shocks and Policy Response

The top three panels of Figure 3.3 plot the variables $\hat{a}_t$, $\hat{e}_t$ and $r_{s,t}$. The bottom three panels plot the innovation $\varepsilon_{a,t}$, $\varepsilon_{e,t}$ and $\varepsilon_{s,t}$.

FIGURE 3.3

It is important to point out that the macroeconomic effects of productivity and capital endowment are mostly driven by the unexpected changes in $\varepsilon_{a,t}$ and $\varepsilon_{e,t}$, not the level of these variables. A high value of $r_{s,t}$ may have moderate effects on investment and output if

the tightening has not taken place shortly, that is, if the economy has had time to adjust to the higher interest rate. This helps us understand the role of monetary policy in the recent crisis, where the unexpected tightening in $\varepsilon_{s,t}$ has been the largest since the mid-1990s.

By using the estimations in Table 3.4, I calculate the fitted value of the short-term interest rate (i.e. the target interest rate); the result is shown in the figures (two right panels in Figure 3.3). In these figures, there is persistent easy monetary policy in the mid-2000s, i.e. $\varepsilon_{s,t} < 0$. In fact, the BOJ implemented a quantitative monetary easing policy in addition to a zero interest rate policy from 2001 to 2006.

Policy Intervention and Economic Resilience

I show first the dynamics induced by the series of productivity and capital endowment shocks. The monetary policy is presumed to follow the benchmark Taylor rule. To study the dynamics of the model induced by the constructed series of shocks and the policy response, I conduct the following simulation. Starting with initial values of $\hat{a}_{1996,I}$ and $\hat{e}_{1996,I}$, I feed the innovations into the model and compute the responses for key real and financial variables. Although I use the actual sequence of shocks, they are not perfectly anticipated by the agents. They forecast future values of a_t and e_t using the autoregressive systems, (3.5.2) and (3.5.3).

Figure 3.4 plots the responses of inflation rate, output, financing cost, asset price, default rate and loan to value (LTV) ratio to the sequence of productivity and capital endowment shocks. With those shocks, the dynamics of inflation rate and output are quite close to the data. In particular, I see a boom in output until the late 1990s. Furthermore, those shocks generate sharp drops in output in the recessions in the early 2000s and the late 2000s as

decreases in price lead to lower production, which in turn leads to lower income streams and demand, which leads to further decreases in price, and thus a deflationary spiral.

FIGURE 3.4

The performance of the model with money and default in response to productivity and capital endowment shocks mainly relies on the impact that these shocks have on the demand for long-term bonds. As shown in the panels, those shocks generate large fluctuations in the default rate and LTV ratio. LTV ratio is one of the key risk factors that lenders assess when qualifying borrowers because the risk of default, as well as the level of indebtedness, are always at the forefront of lending decisions. Therefore, as the LTV ratio of a loan increases, the qualification guidelines for issuing long-term bonds become much stricter. Thus, I observe that financing cost moves in the same direction as the LTV ratio.

However, the default rate does not explain much of the financing cost since its elasticity is lower than the real interest rate's. This means that the level of indebtedness works as a crucial factor for financing cost compared to the default risk itself because the economy is in the liquidity trap and the monetary policy is not properly transmitted into the real economy. Also, they generate large drops in asset prices during the recessions and an upward trend during the mid-1990s and 2000s. Asset prices explain the default rate reasonably well as the value of assets reveals the level of repayment ability.

The model with money and default captures the dynamics of the financial flows, as shown in the panels of Figure 3.4. The series generated by the model broadly mimics the main features of the empirical series for financial variables. Of course, I would not expect this parsimonious model to fit the data perfectly. In particular, the volatility of inflation rate is

somewhat lower than the data in the early 2000s, and higher in the mid-2000s. However, the model continues to predict sharp drops in output during each of the major recessions. In particular it captures most of the output decline observed in the recent crisis. I close this section by observing that the financing cost depends on the real interest rate as well as the inflation rate, short-term loan rate and default rate, the 'Fisher Effect'. Over the whole simulation period the financing cost is mainly driven by the real interest rate and adjusted by the inflation rate, short-term loan rate and default rate.

3.5.3 Sensitivity

Productivity and Capital Endowment Shocks

Figure 3.5 shows the response of key macro-variables to one-time shocks such as productivity and capital endowment. For clarity, this picture, as well as the ones following, assumes that no other shock occurs. The response of quantities is similar to that of the RBC model: investment rises as the entrepreneur desires to accumulate more capital, and inflation persists as the demand effect crowds out the supply resulting from the low interest rate in the short-term loan market (equivalently easy monetary policy). The entrepreneur faces low financing cost in the long-term bond market as the monetary policy is properly transmitted into the long-term bond market for the chosen shocks. The asset price is high on impact, reflecting the sensitivity of demand effects to shocks considered.

FIGURE 3.5

Moreover, the default risk is not immune to these shocks; the endogenous repayment rate generates a procyclical risk premium which cannot act as an accelerator amplifying the

productivity and capital endowment shocks and stimulating output. This is mainly due to the fact that the economy is in the highly indebtedness; the default risk cannot properly affect the financing cost as the high level of liquidity (low short-term loan rate) and low real interest rate wipe out the accelerator channel. Thus, the return on long-term bonds (the financial cost) does not mirror the default rate. This is the situation where the principle dynamics of credit risks do not apply; although the rational lender provides long-term bonds with a high interest rate whenever s/he expects high default risk, this strategy does not work.

Alternative Specification of the Monetary Policy

As can be seen in Figure 3.6, the policy experiments are done for the estimated inflation-targeting coefficient of the Taylor rule and for the standard value of it. Irrespective of whether the monetary authority assigns a positive or negative coefficient to the inflation rate, there is no obvious impact on the equilibrium path of financing cost; the monetary policy is properly transmitted into a long-term bond market in any case. The main actors helping the transmission channel of the monetary policy are the real interest rate and the inflation rate. As the central bank aims to press the inflation rate down, the entrepreneur adjusts her/his investment plan and demand for long-term bonds. This mitigates financial fragility measures; the default rate induced by inflation-targeting monetary policy moves towards a lower level of equilibrium than the other one as caused by the estimation. High consumption leads to low marginal utility gains from the default 'On-the-Verge Condition'. However, it turns out that there is a high default rate for both policy experiments. This, in turn, implies that the equilibrium path of marginal utility gain is crowded out by the level of indebtedness, and thus cannot properly explain the optimal default decision.

FIGURE 3.6

3.6 Extended Model with Natural Disasters

I presume that a natural disaster has strongly destructive characteristics (Barro, 2009; Gourio, 2009; Keen and Pakko, 2009) and an economy does not have control over its occurrence; in other words, it is external to the economy. First, it disrupts productivity temporarily and hurts capital endowment in the short to medium run. Second, it destroys an economically relevant share of the working capital. Thus, I can model it as a transitory negative externality.

3.6.1 Model Setup

Technology is an economy-wide factor that enters the production function multiplicatively. The overall technology level A_t comprises the typical productivity factor a_t whose log-linearised form follows a first-order autoregressive process, (3.5.2), and an additional component related to the disaster variable's state. The disaster shock also affects capital endowment, (3.5.3). As it takes time to rebuild the facilities such as power plants, etc., the disaster shock hurts capital endowment in a short to medium run. These are described by (3.6.1) and (3.6.2)

$$\log A_t = \log a_t - \zeta_a \mathcal{D}_t \quad (3.6.1)$$

$$\log e_t = \rho_e \log e_{t-1} - \zeta_e \mathcal{D}_t + \varepsilon_{e,t} \quad (3.6.2)$$

where ζ_a and ζ_e are the reduction in productivity factor and capital endowment following

a disaster. Here, I take ζ_a and ζ_e to be the parameters. $\mathcal{D}_t$ is 1 if there is a disaster at time t (with probability p_t) and zero otherwise (with probability $1 - p_t$).

The disaster shock directly influences the accumulated capital stock. A disaster, when it strikes, occurs at the beginning of period t before production begins. The non-destroyed capital k_t^h , (3.3.8), which is available for use is redefined by (3.6.3)

$$k_t^h = (1 - \delta)k_{t-1}^h + \frac{\tilde{b}_{k,t}^h}{\tilde{p}_{k,t}} - \zeta_k \mathcal{D}_t \bar{k}^h \quad h \in \{\phi, \theta\} \quad (3.6.3)$$

where ζ_k represents the contribution of disaster shock to existing capital stock. As ζ_k is a parameter, the quantitative role will be discussed.

In line with (3.3.9), AK production function is redefined by (3.6.4) considering the impact of a disaster described in (3.6.1), (3.6.2) and (3.6.3)

$$y_t = A_t (k_t^\theta)^\alpha \quad (3.6.4)$$

where A_t is productivity factor considering the disaster effects. Thus, $A_t = a_t$ and (3.3.9) hold in non-disaster state.

Last, one crucial element is the persistence and volatility of movements in the probability of disaster. I assume that the log of the probability follows an AR(1) process¹, (3.6.5)

$$\begin{aligned} \ln p_t &= \rho_p \ln p_{t-1} + (1 - \rho_p) \ln \bar{p} + \varepsilon_{p,t} \\ \varepsilon_{p,t} &\sim i.i.d. N(0, \sigma_p^2) \end{aligned} \quad (3.6.5)$$

¹Although this equation allows the probability to be greater than one, I approximate this process with a finite Markov chain, which ensures that $0 < p_t < 1$ for all $t \geq 0$ (Gourio 2009).

where ρ_p is the AR(1) coefficient of probability of disaster. $\varepsilon_{p,t}$ captures *i.i.d.* shock and σ_p represents its standard deviation.

3.6.2 Estimation

One essential part of the calibration is the probability and size of disaster. I assume that $\zeta_a = \zeta_e = \zeta_k \approx 0.05$ and the probability is 0.017 per year on average. These numbers are motivated by the evidence in Barro (2006) who reports this unconditional probability. Barro actually uses the historical distribution of sizes of disaster.

Whether one should model a disaster as capital destruction or reduction in productivity and capital endowment is an important question. Clearly some disasters, e.g. in South America since 1945, or Russia 1917, affected productivity and capital endowment. On the other hand, World War II led in many countries to massive physical destruction and losses of human capital. It would be interesting to gather further evidence on disasters and measure ζ_a , ζ_e and ζ_k directly. This is beyond the scope of this paper. Here I concentrate on the parsimonious benchmark case $\zeta_a = \zeta_e = \zeta_k$. In section 6.4, I discuss an alternative calibration with $\zeta_k = 0$, which generates many of the same results, provided that there are reductions in productivity and damage to capital endowment.

The parameter $\bar{p}$ is picked so that the average probability is 0.017 per year, and I set $\rho_p = 0.92$ and the unconditional standard deviation $\sigma_p / \sqrt{1 - \rho_p^2} = 1.85$, which allows the model to fit the volatility reasonably well (Gourio, 2011). On top of this benchmark calibration, I will also present results from different calibrations (no disasters, constant probability of disaster, and in section 6.3 more extensions) to illustrate the sensitivity of the results.

3.6.3 Findings

Implications for Inflation-Targeting

I chose a parameterisation for the benchmark equilibrium of the model that allowed the central bank to follow actions consistent with an inflation-targeting strategy. However, it is arguable whether countries whose policy rate is close to zero still aim at stabilising the inflation rate or not. The estimated Taylor rule gives us the intuition that the BOJ already departs from the ‘Taylor Principle’. Specifically it notes that there is a negative inflation rate while asset prices go up when the central bank operates the inflation-targeting monetary policy for positive productivity and capital endowment shocks, as can be seen in Figure 3.6. Therefore, in those states of nature LTV ratios improve, mainly due to upward pressure in asset price and results in relatively lower default through central bank targets to stabilise the inflation rate. These are confirmed by the policy analysis in Figure 3.7. As the central bank’s policy is in line with the ‘Taylor Principle’, there are improvements in the default, LTV ratio and inflation rates. In other word, the financial markets respond more robustly to the considered shocks. However, there is an adverse effect. Inflation-targeting policy ends up with high volatility in output level for the chosen shocks.

FIGURE 3.7

However, the above story does not hold when there is a disaster shock. As can be seen in Figure 3.8, the monetary policy in line with the ‘Taylor Principle’ improves the volatility of inflation rate and LTV ratio but allows for high fluctuation in output level. These are all we can see for the real shocks cases in Figure 3.7. But the default rate tells us a different story.

Although the initial equilibrium is driven by supply shocks (natural disaster), it is possible to assess the contribution of monetary policy to financial instability through the simulation exercises. The simulation results show that an inflation-targeting policy promotes financial instability by increasing fluctuation in default and output in the bad states of nature, such as hitting by natural disaster; departure from the ‘Taylor Principle’ (if effective) reduces the deviation of default in the long-term bond market and improves the volatility of output.

FIGURE 3.8

Central banks around the world operate under an inflation-targeting regime, whereby the short-term interest rate is set to stabilise the rate of inflation of goods and services and output gap. However, the current financial crisis has reminded us all that, in addition to achieving price stability, central banks are responsible for maintaining financial stability. One of the problems of central banking is that these two objectives may often be in conflict. This model portrays this disjuncture. Hence, I argue that by taking into account asset prices to conduct monetary policy (e.g. by widening the targeted CPI to include an appropriate measure of asset prices), the central bank can contribute to financial stability.

For positive productivity and capital endowment shocks, inflation-targeting monetary policy out-performs the other one (estimated policy), which departs from ‘Taylor Principle’, in terms of the inflation rate, default and LTV ratio, allowing more deviation in output level. Moreover, through accommodating asset prices in the monetary policy, the central bank can coordinate more robust financial markets in terms of further reducing LTV ration and default rate. These are all crucial when the economy is in the highly indebtedness with near-zero interest rate and in times of financial turmoil.

As negative externality (natural disaster) hits the economy, the monetary policy departing from the 'Taylor Principle' out-performs the standard policy in terms of the financial fragility measures: output and default rate. By introducing a new policy target (asset prices), there is significant improvement on financial fragility measures. However, there are adverse effects as well, such as high volatility of inflation rate and large uncertainty in LTV ratio. Specifically, the economy's level of indebtedness increases.

One of the reasons why central banks are subject to trade-off price and financial stability is the fact that they have only one instrument, the short-term interest rate. Thus, the objective of financial stability could be better achieved by the development and application of separate instruments designed for that purpose. This proposal is fully developed by Brunnermeier et. al (2009), who stress the importance of designing a counter-cyclical regulatory mechanism aimed at reducing the systemic risks that are threatening to financial stability.

Similarly, the results show that systemic regulation of financial institutions could be more effective than monetary policy at promoting financial stability. Although I do not model a regulatory institution, the simulations show that implementing measures to force the lender and the borrower to behave more prudently ex-ante, reduces uncertainty in default and output level without negatively affecting households' welfare.

An Increase in the Probability of a Disaster

The important shock in this paper is the shock to the probability of disaster, i.e. an increase in perceived risk. Figure 3.9 presents the responses to an unexpected increase in the probability of disaster: the higher risk leads to a reduction in capital endowment as well as working capital. Output decreases sharply mainly due to a prompt reduction in total

factor productivity, even though the capital stock adjusts slowly. An increase in the probability of disaster also pushes down asset prices, as demand for investment decreases and this crowds out a reduction in a capital supply. This is due to the fact that the borrower in the long-term bond market faces high financing cost and this results in increasing the opportunity cost for new investment.

FIGURE 3.9

The policy prescription for the disaster shock is reducing the liquidity supply since low production level increases the price level and may result in inflation. However, the equilibrium path of inflation rate significantly depends on monetary policy, i.e. whether the central bank works against the ‘Taylor Principle’ or not. In this simulation exercise, the monetary policy departing from the ‘Taylor Principle’ ends up with deflation and the other (the standard Taylor rule) leads to inflation. Qualitatively, these dynamics (other than inflation rate) are similar to each other, but the quantitative results are quite different. To illustrate this clearly, Figure 3.9 superimposes the responses to a shock to the probability of disaster for the estimated monetary policy (dotted line) and for the standard Taylor rule (solid line). The response of two monetary policies on impact is similar, other than for inflation rate: the one (estimated) induces deflation and the other causes inflation. The monetary policy in line with the ‘Taylor Principle’ significantly amplifies the effect of risk shocks in financial markets, such as lower asset price, higher financing cost and deeper indebtedness even though the actual monetary contraction is not significantly different.

Because disaster risk increases, risk premia (opportunity cost) rise as the economy enters recession: the difference between asset prices and returns on long-term bonds becomes larger. The model hence generates the required negative correlation between credit

spreads and investment output. More generally, the model implies, consistent with the data, that risk premia are larger in recessions. Last, the default probability falls slightly as the disaster risk goes up, although a decrease in output leads to a reduction in consumption and results in high marginal utility gain for default decision. This is mainly due to high indebtedness. The counter-cyclical property of default only works when the level of indebtedness and the inflation rate do not work against it, Proposition 5. Hence, the observed increase in spreads creates an increase in the long-term bond risk premium, rather than an increase in the probability of default, a result consistent with the empirical findings of Gilchrist and Zakrajsek (2012).

3.6.4 Sensitivity

Figure 3.10 illustrates the impact of a disaster when the central bank follows a standard Taylor rule ($\phi_\pi = 1.50$ and $\phi_y = 0.50$). This is equivalent to 6.0 and 2.0 on annualised percent changes of inflation rate and output, respectively. The shock is calibrated to deliver a 5.0 percent fall in productivity factor, a 5.0 percent fall in capital endowment persisting over 10 years and a 5.0 percent fall in existing capital stock. The negative shock to the capital stock, productivity factor and capital endowment prompts the lowering of output and the raising of prices simultaneously.

FIGURE 3.10

The larger output decline lowers entrepreneurs' income, giving them fewer resources to invest in capital stock. In the period after the disaster, the prompt return of productivity to its pre-disaster level permits entrepreneurs to increase output, which lifts their income

and enables them to increase their investment in physical capital. That process continues for a number of periods as the capital stock is slowly reconstructed. In the longer term, the protracted rebuilding of the capital stock is associated with below-trend output and persistent, above-trend paths for inflation.

The endogenous response of monetary policy to inflation rate and output drives the movements in the liquidity operation. The policy reaction to higher inflation after a disaster puts upward pressure on the interest rate of short-term loans, while the decline in output generates downward pressure. In the standard Taylor rule calibration, the inflation rate effect dominates, so that a decrease in money supply is the prescribed monetary policy response. The initial decrease in the liquidity injection is around eight percent, while it rises only by about six percent with the return of long-term bonds. Destruction of the capital stock also increases future asset prices, which causes the equilibrium real interest rate to rise. Finally, the gradual reconstruction of the capital stock keeps inflation and inflation expectations above their steady states for an extended period of time.

As can be seen in Table 3.4, there is no clear evidence supporting that the BOJ still follows the Taylor Principle. Moreover, 50 percentile of the coefficient of the inflation target is estimated by negative value -0.0216, whose magnitude is also very small compared to the estimated coefficient of output target 0.0999. In order to examine the role of monetary policy for disaster shock in liquidity trap, I conduct the simulation exercise with the estimated values of the Taylor rule coefficients ($\phi_\pi = -0.0922$ and $\phi_y = 0.0999$). Instead of using the 50 percentile value for the coefficient of inflation target, I use the 0.5 percentile value to amplify the effect of estimated monetary policy. All of these are shown in Figure 3.11.

FIGURE 3.11

Departing from the ‘Taylor Principle’ means that the BOJ wants a booming economy rather than one with suppressed inflation. Therefore, this policy coordination initiates more investment and increases the level of working capital compared to following a standard Taylor rule. This directly increases the relative output level as well. The increased level of output decreases the real interest rate as the marginal utility of consumption relatively decreases. It causes the downward pressure of lower financing costs and results in a lower opportunity cost on asset prices. However, lower financing costs as well as lower asset prices induce a high level of indebtedness, as shown in the LTV ratio.

To evaluate the contribution of disaster shock to working capital, two cases of simulations are carried out for both policy experiments: the standard Taylor rule response in Figure 3.10 and the estimated Taylor rule response in Figure 11. As it directly hurts the amount of working capital, there is more pronounced and persistent declines in output. Thus, there is more deviation in the central bank’s reaction function (its liquidity operation) and this results in a vulnerable state of the economy in terms of inflation rate, LTV ratio, etc. However, the assumption that there is no impact of disaster shock on working capital ($\zeta_k = 0$) captures many of the similar results as long as there are reductions in productivity and capital endowment.

3.7 Concluding Remarks

Once the damage from Hurricane Katrina became apparent, the media and financial markets speculated that the Federal Reserve might ease policy by delaying an expected 25 basis point increase in the federal funds rate. Three weeks later at its next meeting, however, the Federal Reserve decided to maintain its pre-Katrina policy stance and raise the

federal funds rate by 25 basis points. The Great East Japan Earthquake tells us the different story. After the earthquake struck, the BOJ promptly reduced the “Call Rates, Uncollateralised Overnight/Average” from 0.085 to 0.062 and persisted in this monetary stance even though they already had a near-zero interest rate. Moreover, the inflation increased during this period. This paper examines the appropriate monetary policy response to a natural disaster such as Hurricane Katrina and the Great East Japan Earthquake.

Our findings suggest that, in most circumstances, the monetary authority should increase its nominal interest rate target after a natural disaster that reduces productivity, disrupts capital endowment and destroys some working capital. When monetary policy is conducted using a standard Taylor-style rule, the higher inflation effect dominates the lower output effect, such that the endogenous policy response to a disaster is a rise in the nominal interest rate. Thus, the monetary policy response to a natural disaster entails an increase in the nominal interest rate. However, this is not line with mitigating the financial fragility, i.e. aggregate default, especially in a highly indebted economy with near-zero policy rate. Considering the recent financial turmoil, the conventional inflation-targeting monetary policy cannot be a prescription. This is because tight monetary policy can result in drying out liquidity and an inactive market economy; monetary contraction after a natural disaster increases default and deepens the resulting recession. Nevertheless, expansionary monetary policy increases the debt overhang. I summarise this in greater detail in Figure 3.12.

FIGURE 3.12

The reaction of monetary policy to any recurring shock should be evaluated in terms of systematic responses to infrequent events, not as discretionary responses to random

shocks. Individuals observe policy actions and form expectations about similar future events. These expectations must be endogenised within economic models in order to provide robust policy analysis. The findings from our disaster-scenario framework show that a rigorous model-based approach to policy analysis sometimes generates prescriptions that are at odds with the prevailing public opinion expressed in the popular press. This parsimonious setup is fairly tractable, and this quality allows it to derive some analytical results and, importantly, makes it easy to embed into richer models.

Many of the ad-hoc measures following a natural disaster are implemented via fiscal policy, and are intended to: (i) spread the pain of the disaster over the entire population (providing ex-post insurance); and (ii) rebuild the infrastructure destroyed by the disaster. However, this paper aims to investigate the appropriate monetary policy response to natural disasters in the dynamic stochastic general equilibrium (DSGE) framework; unlike with fiscal policy, there is little research investigating the monetary policy response in times of disaster shock. I will investigate that point in a companion paper and compare and contrast the effect of fiscal policy with the effect of monetary policy (which is the main effort of this paper).

3.8 Appendices

Appendix A: First Order Conditions

$$\frac{\partial L(\phi)}{\partial \tilde{m}_{l,t}} = 0 \quad \frac{\eta_{1,t}^\phi}{1 + r_{l,t}} = \beta E_t \left(\frac{\eta_{1,t+1}^\phi \mathcal{R}_{t+1}}{1 + \pi_{t+1}} \right) \quad (3.A.1)$$

$$\frac{\partial L(\phi)}{\partial \tilde{\mu}_{s,t}^\phi} = 0 \quad \eta_{2,t}^\phi = \frac{\eta_{1,t}^\phi}{1 + r_{s,t}} \quad (3.A.2)$$

$$\frac{\partial L(\phi)}{\partial \tilde{b}_{c,t}^\phi} = 0 \quad \eta_{1,t}^\phi = \mathcal{U}'(\tilde{b}_{c,t}^\phi; \phi) \quad (3.A.3)$$

$$\frac{\partial L(\phi)}{\partial \tilde{b}_{k,t}^\phi} = 0 \quad \mathcal{U}'\left(\frac{\tilde{b}_{k,t}^\phi}{\tilde{p}_{k,t}}; \phi\right) = \tilde{p}_{k,t}(\eta_{1,t}^\phi - \eta_{2,t}^\phi) \quad (3.A.4)$$

$$\frac{\partial L(\phi)}{\partial \eta_{1,t}^\phi} = 0 \quad \tilde{b}_{c,t}^\phi + \tilde{b}_{k,t}^\phi + \tilde{m}_{l,t} = \frac{\tilde{\mu}_{s,t}^\phi}{1 + r_{s,t}} + \mathcal{R}_t(1 + r_{l,t-1}) \frac{\tilde{m}_{l,t-1}}{1 + \pi_t} + \omega \tilde{T}_t \quad (3.A.5)$$

$$\frac{\partial L(\phi)}{\partial \eta_{2,t}^\phi} = 0 \quad \tilde{\mu}_{s,t}^\phi = \tilde{p}_{k,t} e_t \quad (3.A.6)$$

$$\frac{\partial L(\theta)}{\partial \tilde{\mu}_{l,t}^\theta} = 0 \quad \frac{\eta_{1,t}^\theta}{1 + r_{l,t}} = \gamma E_t \left(\frac{\eta_{1,t+1}^\theta}{1 + \pi_{t+1}} \right) \quad (3.A.7)$$

$$\frac{\partial L(\theta)}{\partial \tilde{\mu}_{s,t}^\theta} = 0 \quad \frac{\eta_{1,t}^\theta}{1 + r_{s,t}} = \eta_{2,t}^\theta \quad (3.A.8)$$

$$\frac{\partial L(\theta)}{\partial \nu_t} = 0 \quad \eta_{1,t}^\theta = \lambda(1 - \nu_t) \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t} \quad (3.A.9)$$

$$\frac{\partial L(\theta)}{\partial \tilde{b}_{c,t}^\theta} = 0 \quad \eta_{1,t}^\theta = \mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta) \quad (3.A.10)$$

$$\frac{\partial L(\theta)}{\partial \tilde{b}_{k,t}^\theta} = 0 \quad \eta_{1,t}^\theta = \frac{\alpha y_t \eta_{2,t}^\theta}{\tilde{p}_{k,t} k_t^\theta} + \gamma(1 - \delta) \frac{E_t \left(\tilde{p}_{k,t+1} \eta_{1,t+1}^\theta \right)}{\tilde{p}_{k,t}} \quad (3.A.11)$$

$$\frac{\partial L(\theta)}{\partial \eta_{1,t}^\theta} = 0 \quad \tilde{b}_{c,t}^\theta + \tilde{b}_{k,t}^\theta + \nu_t \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t} = \frac{\tilde{\mu}_{s,t}^\theta}{1 + r_{s,t}} + \frac{\tilde{\mu}_{l,t}}{1 + r_{l,t}} + (1 - \omega) \tilde{T}_t \quad (3.A.12)$$

$$\frac{\partial L(\theta)}{\partial \eta_{2,t}^\theta} = 0 \quad \tilde{\mu}_{s,t}^\theta = y_t \quad (3.A.13)$$

Appendix B: Proofs

PROOF OF PROPOSITION 1

Substituting (3.A.3) into (3.A.1), I get

$$\frac{\mathcal{U}'(\tilde{b}_{c,t}^\phi; \phi)}{1 + r_{l,t}} = \beta E_t \left(\mathcal{U}'(\tilde{b}_{c,t+1}^\phi; \phi) \cdot \frac{R_{t+1}}{1 + \pi_{t+1}} \right)$$

Through log-transformation, I can derive

$$\ln(1 + r_{l,t}) \approx \ln \left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\phi; \phi)}{\beta E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\phi; \phi)} \right) + \ln(1 + E_t \pi_{t+1}) + \ln(E_t \mathcal{R}_{t+1}^{-1})$$

Likewise, substituting (3.A.10) into (3.A.7), I obtain

$$\frac{\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta)}{1 + r_{l,t}} = \gamma E_t \left(\mathcal{U}'(\tilde{b}_{c,t+1}^\theta; \theta) \cdot \frac{1}{1 + \pi_{t+1}} \right)$$

Through log-transformation, I can derive

$$\ln(1 + r_{l,t}) \approx \ln \left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta)}{\gamma E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\theta; \theta)} \right) + \ln(1 + E_t \pi_{t+1})$$

PROOF OF PROPOSITION 2

The marginal productivity of physical capital is defined by

$$\frac{\partial y_t}{\partial k_t^\theta} = \alpha \frac{y_t}{k_t^\theta}$$

Thus, plugging this into (3.A.11) with (3.A.8) and (3.A.10), I can drive

$$\tilde{p}_{k,t} = \frac{1}{1 + r_{s,t}} \frac{\partial y_t}{\partial k_t^\theta} + \frac{1}{1 + r_{l,t}} (1 - \delta) E_t [\tilde{p}_{k,t+1} (1 + \pi_{t+1})]$$

PROOF OF PROPOSITION 4

Since endogenous variables are not changed in the steady state, (3.A.1) and (3.A.7) can be transformed into the following equations

$$\begin{aligned}\frac{1}{1 + \bar{r}_l} &= \frac{\beta \bar{\mathcal{R}}}{1 + \bar{\pi}} \\ \frac{1}{1 + \bar{r}_l} &= \frac{\gamma}{1 + \bar{\pi}}\end{aligned}$$

where $\bar{x}$ means the long-run equilibrium value of x_t , i.e. $\bar{x} = \lim_{t \rightarrow \infty} x_t$.

As the borrower is more impatient than the lender, i.e. $\gamma < \beta$, I can conclude that there exists default in the steady state, $0 < \bar{\mathcal{R}} < 1$.

PROOF OF PROPOSITION 5

Assuming that (i) default exists ($\nu_t > 0$) and (ii) the entrepreneur has some money left over the moment that goods market opens ($\tilde{b}_{c,t}^\theta > 0$) both for $\forall t \in T$, then (3.A.9) and (3.A.10) hold for any short run equilibrium. From the two first order conditions, I can directly derive

$$\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta) = \lambda(1 - \nu_t) \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t}$$

Appendix D: Figures

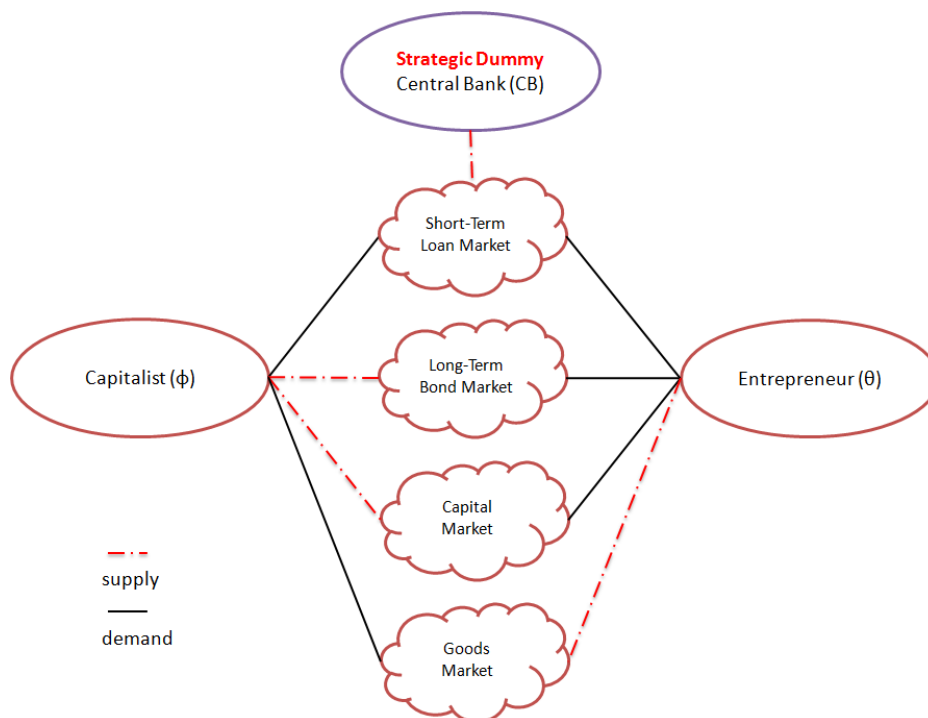


Figure 3.1: Nominal flows in the benchmark economy

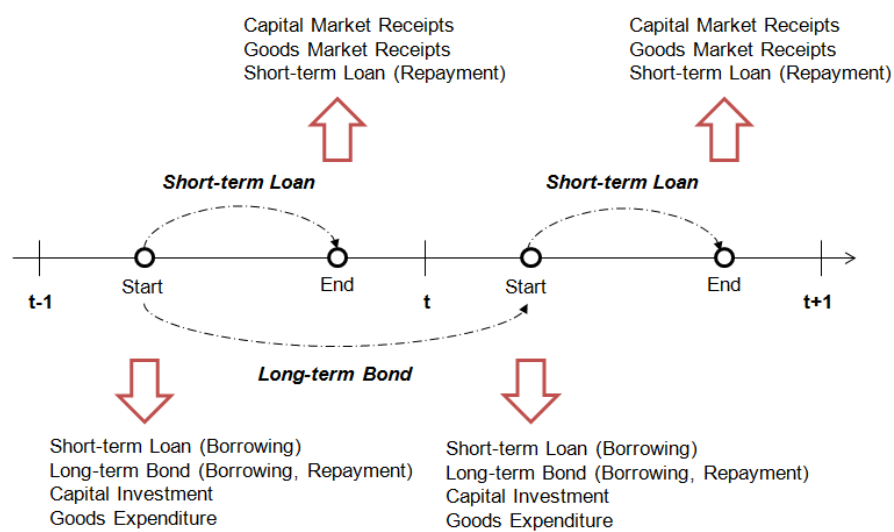


Figure 3.2: Time line of the model

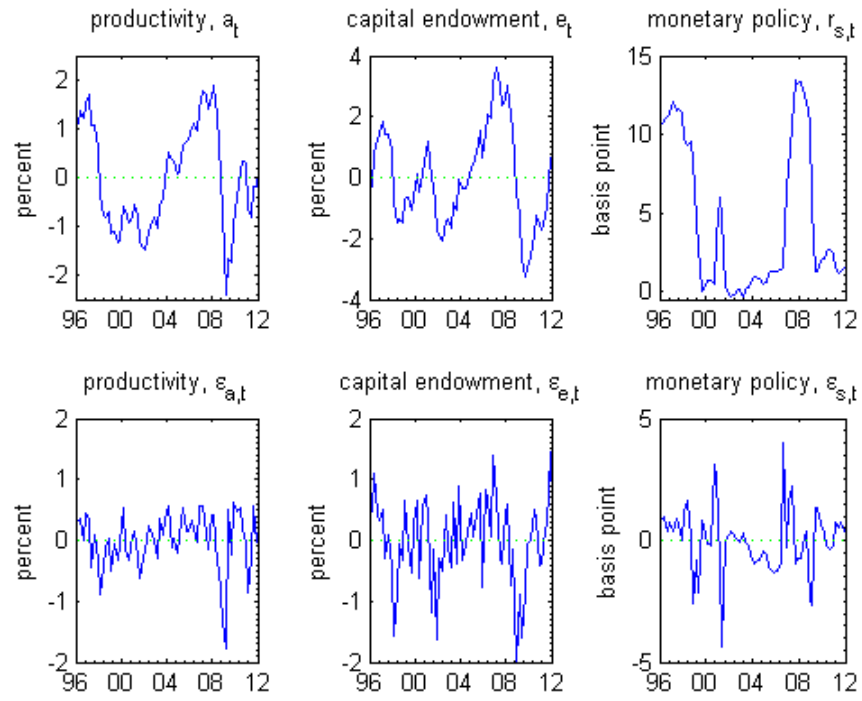


Figure 3.3: Time series of exogenous variables

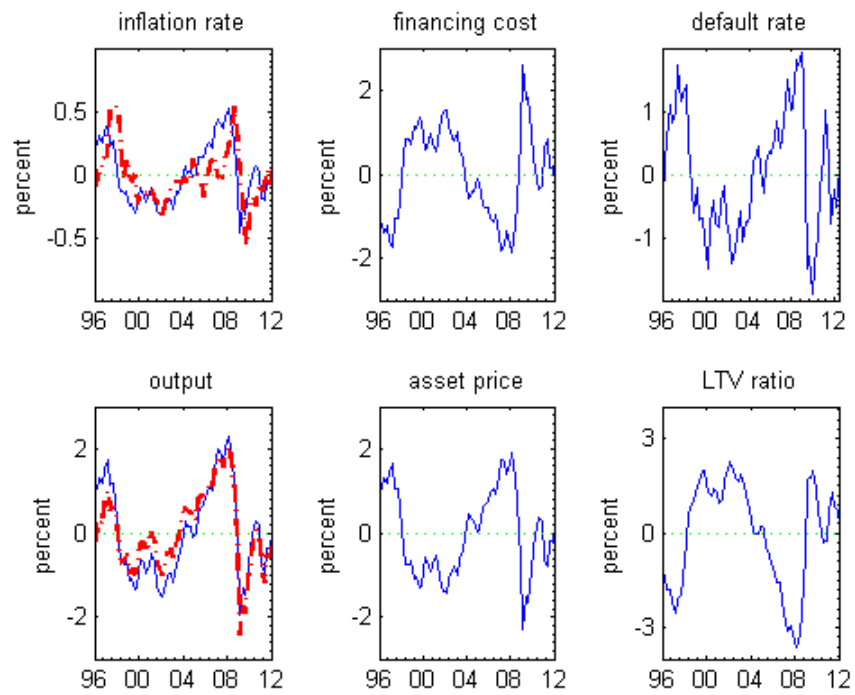


Figure 3.4: Time series of endogenous variables

The red dotted line shows the empirical data (inflation rate and output only) and the blue solid line represents the model responses.

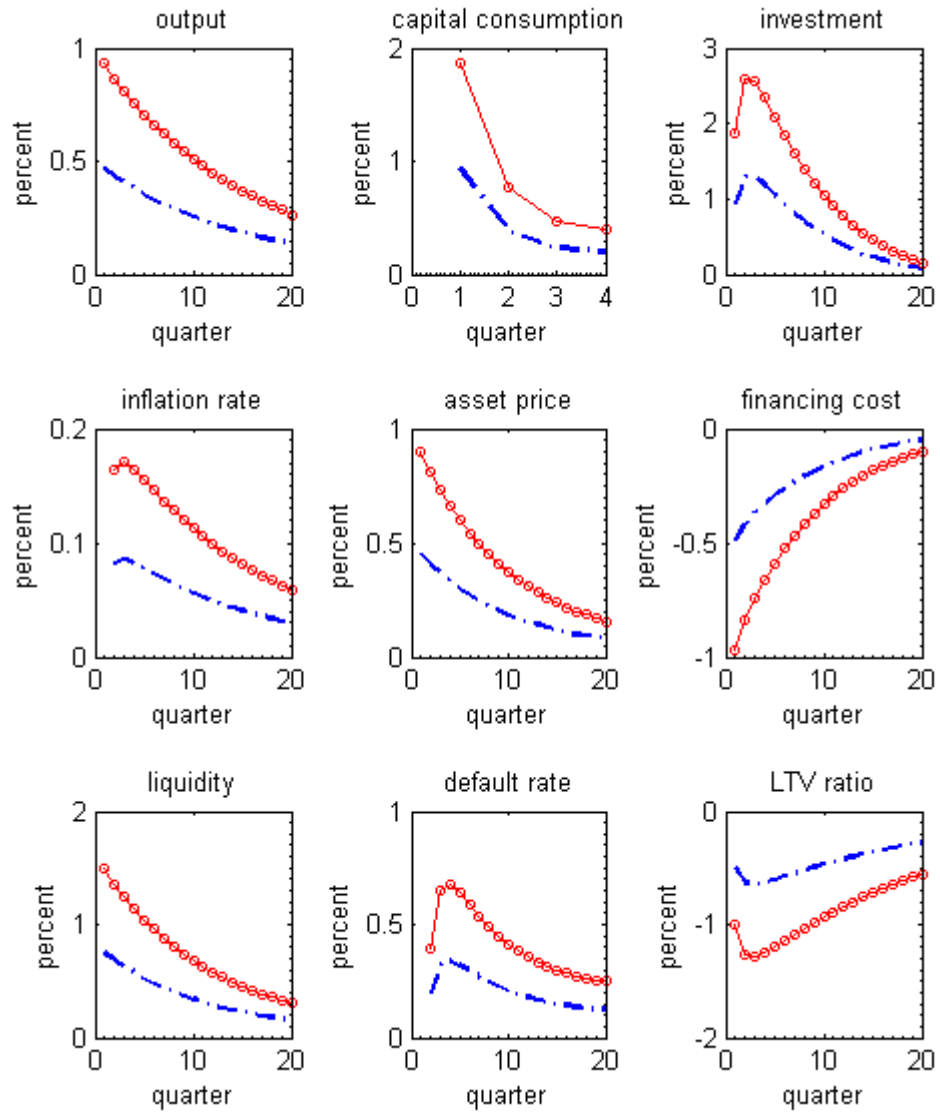


Figure 3.5: Response to one-time real shocks

This figure shows impulse response to unanticipated productivity and capital endowment shocks in the first quarter. The size of the shocks is normalized such that it leads to (i) 0.5 percent ($\varepsilon_{i,1} = \sigma_i$ and $\varepsilon_{i,t} = 0$ for $t \geq 2$ and $i \in \{a, e\}$; blue dotted line) and (ii) 1 percent ($\varepsilon_{i,1} = 2\sigma_i$ and $\varepsilon_{i,t} = 0$ for $t \geq 2$ and $i \in \{a, e\}$; red solid line) increases in real output on impact under the estimated monetary policy. Paths denote different simulation cases.

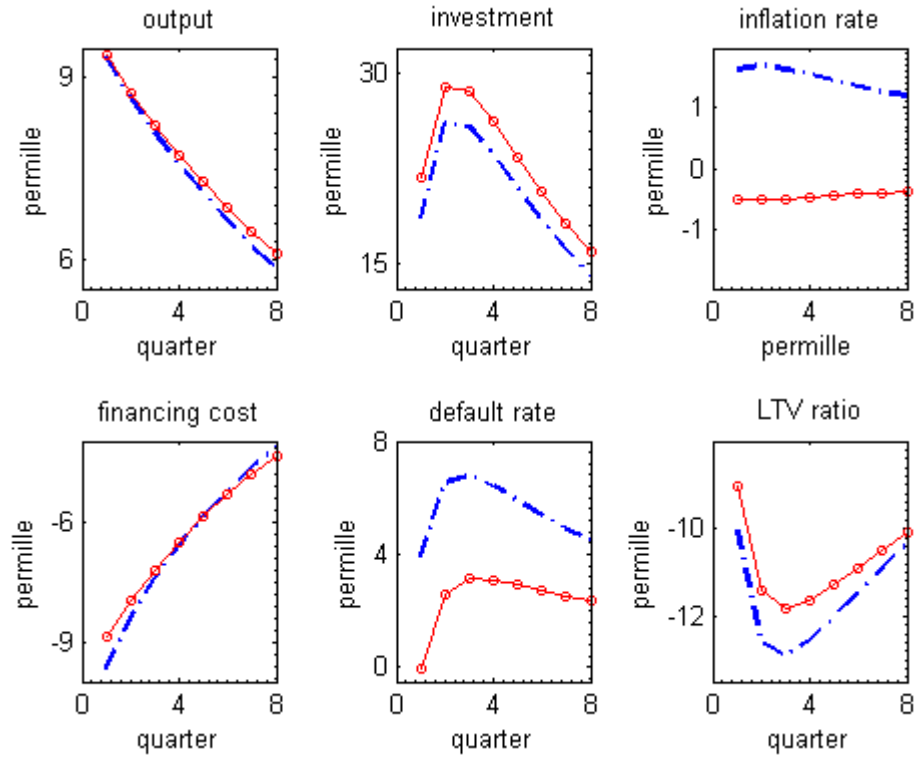


Figure 3.6: Policy experiments to one-time real shocks

This figure shows impulse response to unanticipated productivity and capital endowment shocks in the first quarter. The size of the shocks is normalized such that it leads to 1 percent ($\varepsilon_{i,1} = 2\sigma_i$ and $\varepsilon_{i,t} = 0$ for $t \geq 2$ and $i \in \{a, e\}$) increase in real output on impact under the two monetary policy regimes (i) the estimated monetary policy (blue dotted line) and (ii) the standard Taylor rule (red solid line). Paths denote different policy experiments.

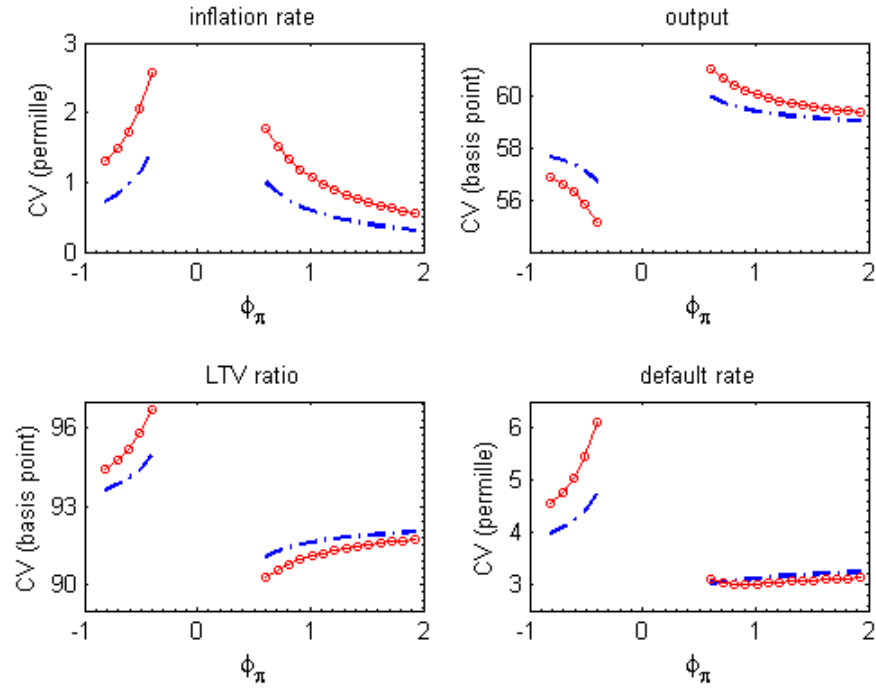


Figure 3.7: Policy analysis for one-time real shocks

This figure shows policy analysis result to unanticipated productivity and capital endowment shocks in the first quarter. The size of the shocks is normalized such that it leads to 1 percent ($\varepsilon_{i,1} = 2\sigma_i$ and $\varepsilon_{i,t} = 0$ for $t \geq 2$ and $i \in \{a, e\}$) increase in real output on impact under the two monetary policy regimes (i) the monetary policy aiming to stabilise inflation rate and output gap (blue dotted line) and (ii) the monetary policy targeting asset price as well (red solid line).

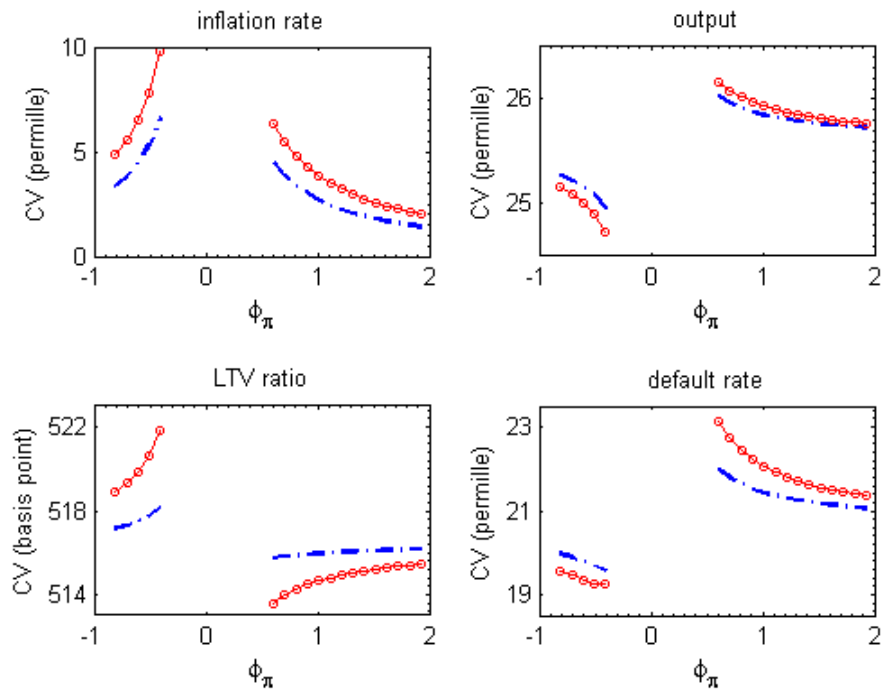


Figure 3.8: Policy analysis for one-time disaster shock

This figure shows policy analysis result to unanticipated disaster shock in the first quarter ($\mathcal{D}_1 = 1$ and $\mathcal{D}_t = 0$ for $t \geq 2$) under the two monetary policy regimes (i) the monetary policy aiming to stabilise inflation rate and output gap (blue dotted line) and (ii) the monetary policy targeting asset price as well (red solid line).

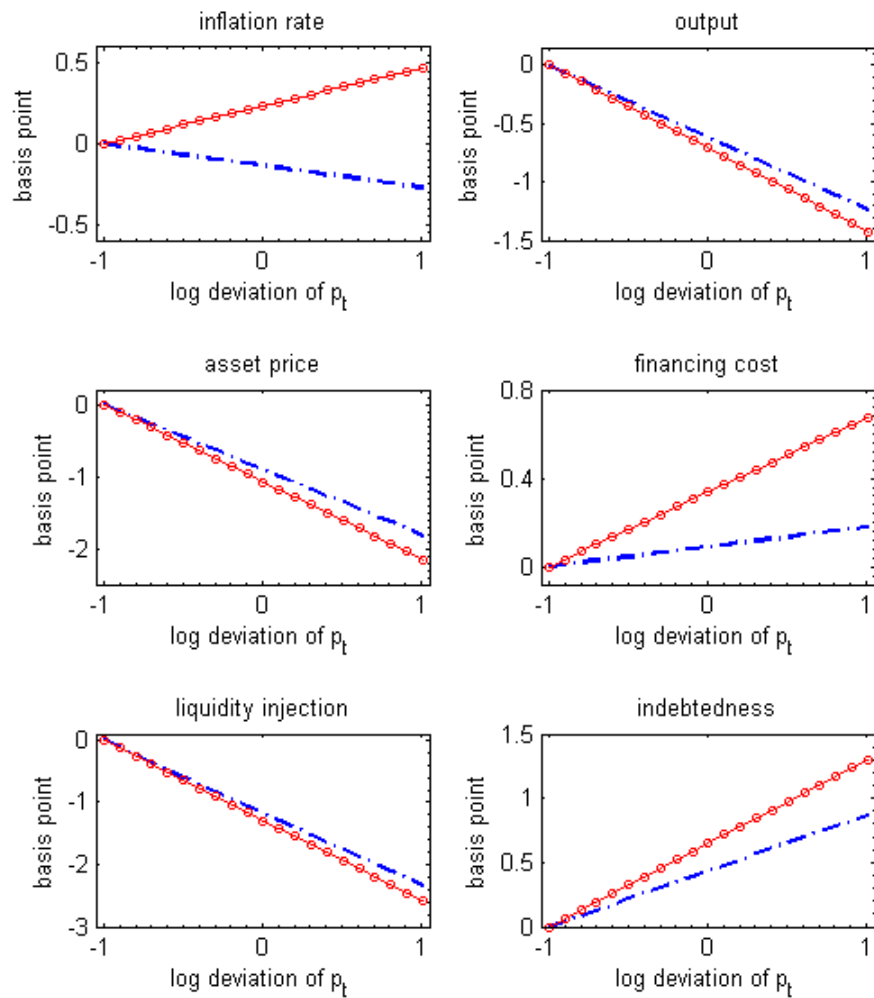


Figure 3.9: An increase in the probability of a disaster

The blue dotted line shows the simulation result of the estimated monetary policy and the red solid line represents that of the standard Taylor rule.

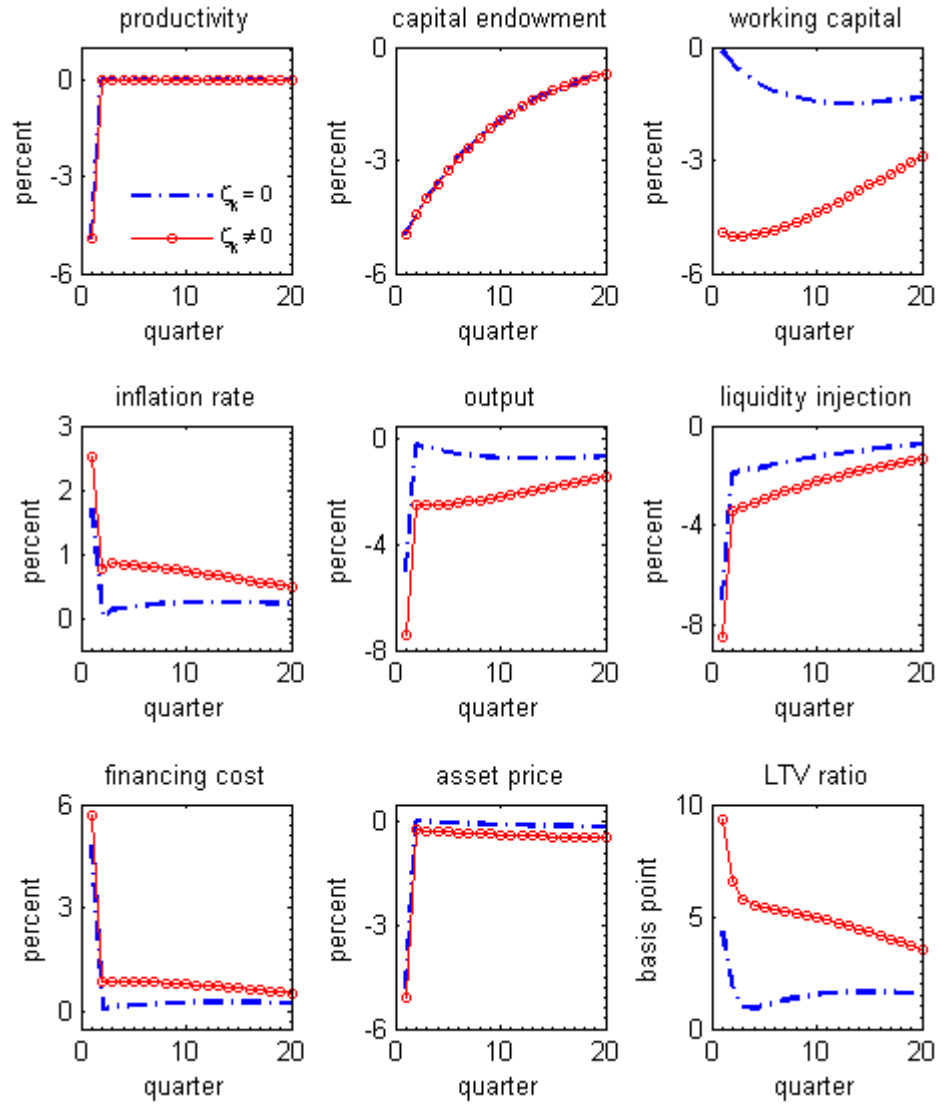


Figure 3.10: Response to a disaster shock with the standard Taylor rule

This figure shows impulse response to an unanticipated disaster shock in the first quarter. The effects of disaster shock are calibrated such that it leads to 5.0 percent decrease in productivity factor, 5.0 percent decrease in capital endowment persisting 10 years and 5.0 percent decrease in existing capital stock on impact under the standard Taylor rule. The blue dotted line shows the result of $\zeta_k = 0$ and the red solid line represents that of $\zeta_k \neq 0$.

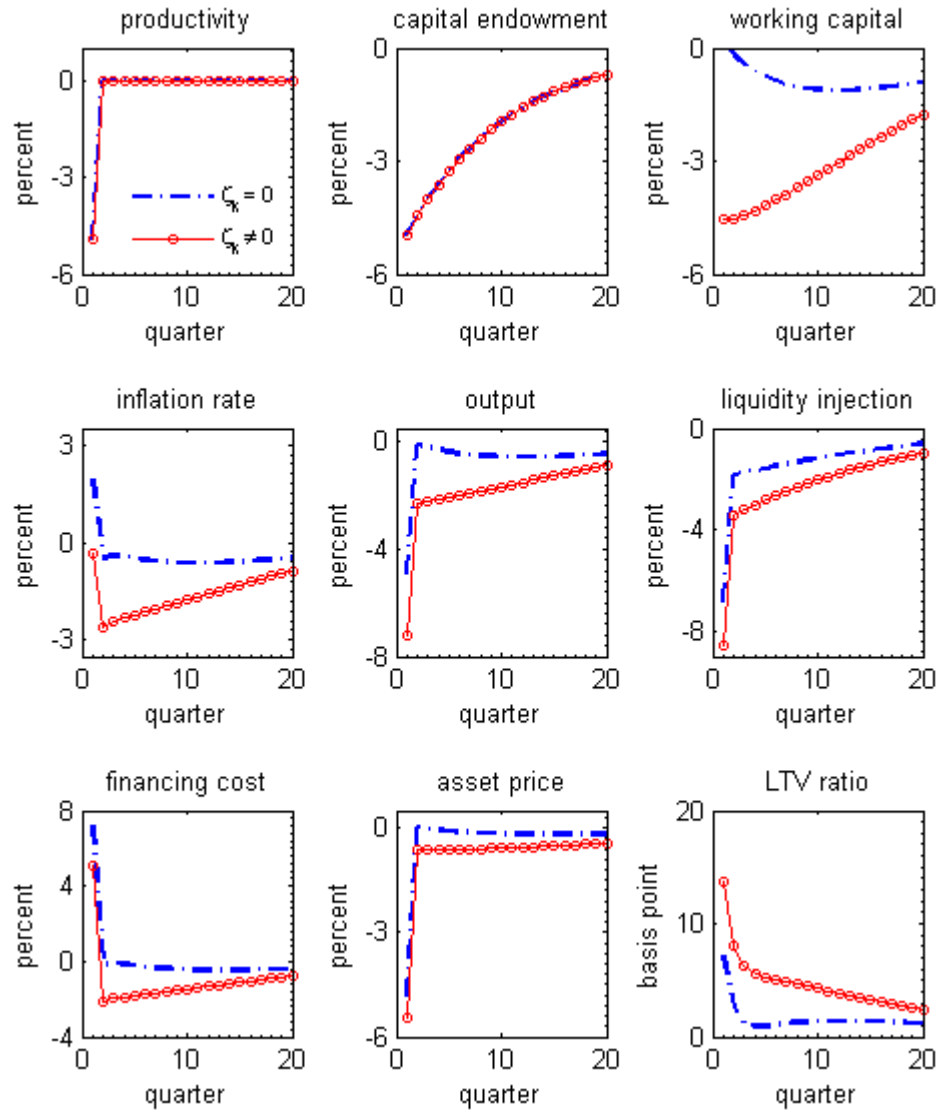


Figure 3.11: Response to a disaster shock with the estimated monetary policy

This figure shows impulse response to an unanticipated disaster shock in the first quarter. The effects of disaster shock are calibrated such that it leads to 5.0 percent decrease in productivity factor, 5.0 percent decrease in capital endowment persisting 10 years and 5.0 percent decrease in existing capital stock on impact under the estimated monetary policy. The blue dotted line shows the result of $\zeta_k = 0$ and the red solid line represents that of $\zeta_k \neq 0$.

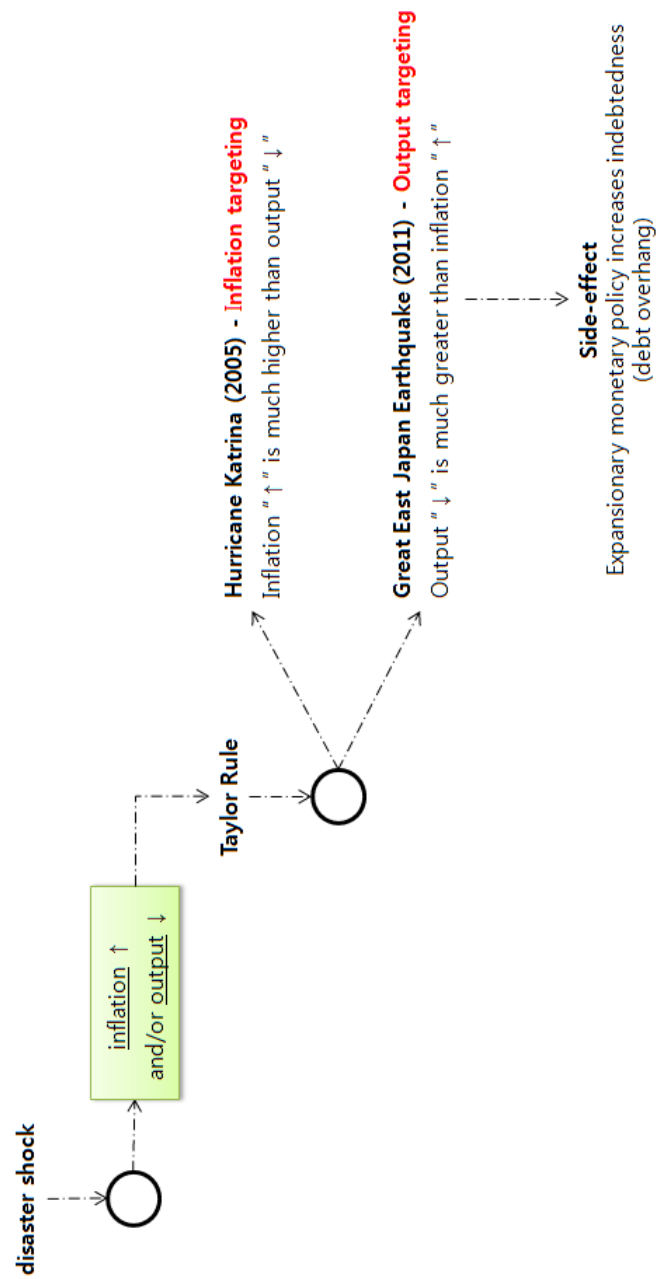


Figure 3.12: Summary of the Model and Conclusion

Appendix E: Tables

Table 3.1: Implied parameters

parameter	value	parameter	value
α	0.5000	χ	0.0008
β	0.9937	ω	0.5000
γ	0.9855	$\bar{\pi}$	0.0000
δ	0.0300	$\bar{a}$	1.0000
λ	0.0415	$\bar{e}$	13.887

Table 3.2: Endogenous variables

parameter	value	parameter	value
$\bar{b}_c^\phi$	7.6069	$\bar{M}$	25.524
$\bar{b}_c^\theta$	7.6069	$\bar{T}$	0.0309
$\bar{b}_k^\phi$	5.1726	$\bar{m}_l$	381.11
$\bar{b}_k^\theta$	5.1726	$\bar{v}$	0.9918
$\bar{\mu}_s^\phi$	10.345	$\bar{\mathcal{R}}$	0.9918
$\bar{\mu}_s^\theta$	15.214	$\bar{r}_s$	0.0012
$\bar{\mu}_l$	386.72	$\bar{r}_l$	0.0147
$\bar{\eta}_1^\phi$	0.1315	$\bar{p}_k$	0.7449
$\bar{\eta}_2^\phi$	0.1313	$\bar{c}_k$	6.9437
$\bar{\eta}_1^\theta$	0.1315	$\bar{y}$	15.214
$\bar{\eta}_2^\theta$	0.1313	$\bar{k}$	231.46

Table 3.3: Stochastic properties of the shocks

$$\hat{a}_{t+1} = \text{const.} + \rho_a \hat{a}_t + \varepsilon_{a,t+1}$$

	parameter	standard error	p-value
const.	-0.0001	0.0006	> 0.10
AR(1) coeff.	0.8976	0.0547	< 0.01
RMSE	0.0046		
R-square	0.8136		

$$\hat{e}_{t+1} = \text{const.} + \rho_e \hat{e}_t + \varepsilon_{e,t+1}$$

	parameter	standard error	p-value
const.	0.0003	0.0009	> 0.10
AR(1) coeff.	0.9018	0.0560	< 0.01
RMSE	0.0071		
R-square	0.8078		

Table 3.4: Coefficients of the Taylor rule

$$r_{s,t} = \rho_s r_{s,t-1} + (1 - \rho_s)(\phi_0 + \phi_\pi \pi_t + \phi_y \hat{y}_t) + \varepsilon_{s,t}$$

	parameter	standard error	p-value
const.	0.0003	0.0003	> 0.10
AR(1) coeff.	0.9013	0.0445	< 0.01
inflation rate	-0.0216	0.0271	> 0.10
output gap	0.0999	0.0208	< 0.01
RMSE	0.0001		
R-square	0.9274		

Chapter 4

Collateral, Default and Asset Prices

4.1 Introduction

The Real Business Cycle (RBC) paradigm has been widely used to study economic recessions, with intuition suggesting that they are caused by adverse productivity shocks. The role of monetary policy is to stabilise prices and the financial sector concomitantly works to facilitate the transfer of funds from creditors to debtors and the production sector. There has been a general assumption in the literature that explicit modelling of banking activity is not necessary as it is commonly held that monetary policy and its effect on credit extension are neutral in equilibrium.

However, the recent financial crisis and its adverse effects on the real economy strongly support the argument that a new methodological approach is required. The RBC literature simply cannot adequately address the issues arising from this crisis; in short, it is difficult, if not impossible, to take the position that a recession preceded the large number of mortgage defaults and the subsequent pessimism in the banking sector that resulted in the

credit crunch. Such a stance would certainly require a nontrivial degree of irrationality, given how optimistic expectations had been prior to the crisis. Moreover, traditional Dynamic Stochastic General Equilibrium (DSGE) models display similar problems, since they pay little regard to financial frictions, heterogeneity and, most importantly, the possibility of endogenous default.

Nevertheless, it was mortgage defaults and the adverse effect on banks' capital that caused the economic slowdown and the near financial meltdown. This view of economic recessions can be traced back to 1933, when Irving Fisher put forward his Debt Deflation theory of Great Depressions. His analysis was based on two fundamental principles: over-indebtedness and deflation. He argued that over-indebtedness can precipitate deflation in future periods and a subsequent liquidation of collateralised debt, as borrowers would rather default than honour their contractual obligations. Debt is denominated in nominal terms and therefore it is constant, although the value of the collateral that secures this debt is dependent on market forces and monetary aggregates in the economy. The main mechanism here is the decrease in the relative price of collateralised durable goods with respect to the numeraire in the economy. This leads to debt liquidation and thus to fire sales, which serve to suppress the value of the collateral even further. Hence, the initial deflationary pressures are exacerbated, leading to even higher levels of default and, ultimately, to lower output. This then exacerbates the initial deflationary pressure, which in the end is the primary cause of output reduction.

This paper is intended to open a dialogue with a separate strand in the literature, one that argues that financial crises and in particular defaults on financial contracts can lead to economic recessions. Bernanke (1983) established that the Great Depression is better

explained when one explicitly models banking behaviour and he introduces the concept of the balance sheet channel in relation to monetary policy. Bernanke and Gertler (1989) modelled a collateral-driven credit constraint that introduces strong informational asymmetries to their model, whereby a firm is only able to obtain fully collateralised loans. As a result, the value of the firm's assets has to be equal to or greater than the value of the loan. However, because of the scarcity of assets and capital, the amount of credit available to the firm shrinks in the presence of deflationary pressures on the prices of its assets. An external finance premium is the result, which then increases with a decrease in the relative price of capital. In turn, an increase in the cost of capital will result in a decrease in the marginal product and a reduction in GDP. Bernanke and Gertler (1989) show that GDP and investment do not depend only on fundamentals and productivity; they are also dependent on the soundness of a firm's financial situation. This is clearly an important source of financial instability. I argue that informational asymmetries are not the crucial factor entailing that the financial situation of firms results in GDP contraction. Rather, the possibility of positive default and asset liquidation can start a chain reaction that undermines the financial system and results in lower production. More elaborate models on the finance premium can be found in Holmstrom and Tirole (1997), Bernanke, Gertler and Gilchrist (1998), Aiyagari and Gertler (1999), Kocherlakota (2000), Bolton and Freixas (2000), and Cooley, Marimon and Quadrini (2004).

Bernanke (1983) concludes that monetary forces alone are quantitatively insufficient to explain the depth and persistence of the Great Depression. Nevertheless, the collapse of the financial system is crucial and can be attributed to higher defaults and, more importantly, to overly pessimistic predictions in regard to future deliveries. Bernanke is opposed to Friedman and Schwartz (1963), who found a high positive correlation between money

supply and output, thus concluding that the decline in the money stock was a substantial factor in the Depression. Bernanke and Gertler (1995) instead claim that Friedman and Schwartz treat the transmission of monetary policy as a “black box” and instead show the existence of a credit channel. Moving the debate forward, the Fisherian approach combines both lines of thought, as a decrease in the money supply can lead to over-indebtedness, deflation, higher default and ultimately a financial crisis. When the money supply decreases, over-indebtedness and deflation are usually close behind. The crucial question is: can deflation in asset prices used as collateral trigger default and consequently a financial crisis? In order to answer this, I propose a “debt deflation” channel of monetary policy.

In doing so, I design a dynamic general equilibrium model for the study of debt-deflation. The model is based on Lin, Tsomocos and Vardoulakis (2010), and the operation of monetary policy affects the real sector via a previously unexplored “debt-deflation” channel. Their paper is a theoretical exploration of the monetary policy transmission mechanism, and is based on a two-period general equilibrium framework. However, it is my contention that a dynamic setting is necessary in order to examine the propagation mechanism of deflationary shocks.

My aim is to build a dynamic model to show a “debt-deflation” channel, demonstrating how monetary policy interacts with trade, asset price, default, investment and aggregate output. The main feature is the interconnection of liquidity and default. The possibility of default and of a “fire-sale” serves as an amplification mechanism for recessionary shocks. It should be noted that, rather than assuming default as an off-the-equilibrium phenomenon, I explicitly model an economy where default is positive. Meanwhile, by assessing investment pattern and output variations, I contend that investment inefficiency –

in addition to reduced investment levels – is the cause of output fluctuation.

The main difficulty in constructing such a model is the issue of how to incorporate “discontinuous default” in a dynamic model. Following Dubey, Geanakopolas and Shubik (2005), two ways of modelling default in a general equilibrium framework have generally been used: “continuous default” and “discontinuous default”. In the former, in-equilibrium agents repay their obligations partially, after which penalties corresponding to the amount of default are imposed. In a “discontinuous default”, on the other hand, liabilities are backed by collateral; thus, when agents choose not to fulfil their debt payments, they default completely and the entire collateral is surrendered. Papers by Martinez and Tsomocos (2011) and Ahn and Tsomocos (2012) represent the beginning of work to introduce “continuous default” into DSGE models. In their model, agents are allowed to default partially in a steady state. However, this paper represents the first attempt to bring “discontinuous default” into the DSGE framework.

The paper proceeds as follows. Section 2 presents the model and section 3 discusses the equilibrium. Then, section 4 describes the calibration of the model and section 5 discusses the quantitative analysis. The principal focus here will be: (i) how monetary policy and real shocks cause default and result in debt-deflation and (ii) what the constrained Pareto sub-optimal monetary policy can be for the chosen deflationary shocks in terms of financial stability measures. Section 5 concludes, while proofs can be found in the Appendix.

4.2 Analytical Framework

It is taken as a given that there are two types of agents in the economy: rational agents and the strategic dummy. The former takes the form of rich household and poor household,

and the latter is the Central Bank. The rich household owns an intermediate goods firm, a final goods firm and a commercial bank, while the poor household only has another final goods firm. All the subsidiaries – meaning the intermediate goods firm, two final goods firms and a commercial bank – are assumed to seek to maximise their shareholders' life time utility functions. Heterogeneous households trade two types of goods – final goods (commodity) and intermediate goods (capital) – to correct for their different endowments and marginal productivities. The commodity is non-durable and produces utilities and the capital is durable and produces commodities. The poor household (or, more precisely, the final goods firm owned by the poor household) applies collateralised loans in order to finance capital investment and the rich household (or the commercial bank owned by the rich household) extends inter-temporal loans, which are secured by the capital (collateral) owned by the debtor; the entire analysis of this chapter rests on the assumption that debt is not indexed to the collateral value. Thus, the effect of liquidity mainly depends on collateral value via the default channel. The Central Bank extends short-term loans to both households in order to influence production, inflation, interest rates, and the like. All of the transactions take place in the markets and all use money. The details are outlined in Figure 4.1.

FIGURE 4.1

In order to introduce money into the general equilibrium framework, cash-in-advance constraint is considered. To do this, each time period is divided into two sub-periods: the beginning of the period and the end of the period. To smooth consumption streams over time, two types of intelligent agent apply short-term loans at the beginning of each period, which are then repaid at the end of that period. It should be noted that budget constraints for all agents are binding because money is fiat and agents obtain no utility from holding it. Consequently, individuals do not hold any idle cash, but rather lend it out to someone who needs it. Figure 4.2 illustrates the time structure of the model.

FIGURE 4.2

4.2.1 Model Setup

Rich Household (h^ϕ)

Given that the rich household is assumed to be in possession of an intermediate goods firm, a final goods firm and a commercial bank, the money inflow and outflow of those affiliated companies all serve to reveal this household's budget constraints. During each period t , the following series of events unfolds: This household starts period t with money on hand by applying for a short-term loan $\tilde{\mu}_{s,t}^\phi / (1 + r_{s,t})$ and claiming back a collateralised loan $R_t(1 + r_{l,t-1})\tilde{m}_{l,t-1} / (1 + \pi_t)$. With these revenues, this agent obtains a new portfolio composition that involves extending the collateralised loan $\tilde{m}_{l,t}$, purchasing final goods $\tilde{b}_{c,t}^\phi$ and investing in intermediate goods $\tilde{b}_{k,t}^\phi$. The budget constraint, (4.2.1), summarises these transaction activities

$$\tilde{b}_{c,t}^\phi + \tilde{b}_{k,t}^\phi + \tilde{m}_{l,t} \leq \frac{\tilde{\mu}_{s,t}^\phi}{1 + r_{s,t}} + R_t \frac{\tilde{m}_{l,t-1}(1 + r_{l,t-1})}{1 + \tilde{\pi}_t} \quad (4.2.1)$$

After the intermediate goods (capital) and final goods (commodity) markets close, the sales income (e.g. from commodity $y_{c,t}^\phi$ and from capital $\tilde{p}_{k,t}(y_{k,t} + q_{k,t})$) is realised and a seigniorage cost results: $\omega \tilde{T}_t$. This revenue is used to repay the short-term loan $\tilde{\mu}_{s,t}^\phi$, subject to the cash-in-advance constraint (4.2.2)

$$\tilde{\mu}_{s,t}^\phi \leq y_{c,t}^\phi + \tilde{p}_{k,t}(y_{k,t} + q_{k,t}) + \omega \tilde{T}_t \quad (4.2.2)$$

where $q_{k,t}$ is the amount of collateral liquidated in period t when there is a default (see the section “Rational Expectations” for further details).

The amount of consumption is the result of bidding in the final goods market in every period t . The rich household is risk averse, and therefore maximises the sum of discounted utilities, (4.2.3)

$$\max_{\{\tilde{b}_{c,t}^\phi, \tilde{b}_{k,t}^\phi, \tilde{\mu}_{s,t}^\phi, \tilde{m}_{l,t}\}_{t=0}^\infty} U(\phi) = E_0 \sum_{t=0}^\infty \beta^t \ln \tilde{b}_{c,t}^\phi \quad (4.2.3)$$

where the parameter $\beta \in (0, 1)$ is a subjective discount factor for the rich household and E_0 represents a mathematical expectation operator at $t = 0$.

Poor Household (h^θ)

The poor household is presumed to own one final goods firm, so this final goods firm’s money flow is included in poor household’s budget sets. This household enters period t

with a short-term loan $\tilde{\mu}_{s,t}^\theta / (1 + r_{s,t})$ and a long-term bond $\tilde{\mu}_{l,t} / (1 + r_{l,t})$. Subsequently, this agent starts to reconstruct her/his portfolio. S/he invests $\tilde{b}_{k,t}^\theta$ to bid the capital, spends $\tilde{b}_{c,t}^\theta$ to purchase the commodity and uses $v_t \tilde{\mu}_{l,t-1} / (1 + \pi_t)$ to repay previously held bonds. All of these transaction activities are described in the budget set, (4.2.4)

$$\tilde{b}_{c,t}^\theta + \tilde{b}_{k,t}^\theta + v_t \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t} \leq \frac{\tilde{\mu}_{s,t}^\theta}{1 + r_{s,t}} + \frac{\tilde{\mu}_{l,t}}{1 + r_{l,t}} \quad (4.2.4)$$

After these transactions are all settled, this household gets back her/his sales income $y_{c,t}^\theta$, receives a monetary endowment $(1 - \omega) \tilde{T}_t$ and repays the short-term loan $\tilde{\mu}_{s,t}^\theta$. This is subject to the following cash-in-advance constraint, (4.2.5)

$$\tilde{\mu}_{s,t}^\theta \leq y_{c,t}^\theta + (1 - \omega) \tilde{T}_t \quad (4.2.5)$$

The poor household is also assumed to be risk averse, and thus maximises the life-time utility function by smoothing the consumption streams over time, (4.2.6)

$$\max_{\{\tilde{b}_{c,t}^\theta, \tilde{b}_{k,t}^\theta, v_t, \tilde{\mu}_{s,t}^\theta, \tilde{\mu}_{l,t}\}_{t=0}^\infty} U(\theta) = E_0 \sum_{t=0}^\infty \gamma^t \ln \tilde{b}_{c,t}^\theta \quad (4.2.6)$$

where the parameter $\gamma \in (0, 1)$ is a subjective discount factor for the poor household and E_0 represents a mathematical expectation operator at $t = 0$. These are in line with what we have seen in regard to the rich household.

Intermediate Goods Firm (f_k^ϕ)

This firm enters a new capital (intermediate goods) production process, (4.2.7), in every period t , simply endowment.¹ Given the real price, this agent supplies capital stock $y_{k,t}$ in the market:

$$\ln y_{k,t} = \rho_k \ln y_{k,t-1} + (1 - \rho_k) \ln \bar{y}_k + \varepsilon_{k,t} \quad (4.2.7)$$

$$\varepsilon_{k,t} \sim i.i.d. \quad N(0, \sigma_k^2)$$

where ρ_k is the AR(1) coefficient of capital endowment. $\varepsilon_{k,t}$ captures the *i.i.d.* shock and σ_k represents its standard deviation.

Final Goods Firms (f_c^ϕ and f_c^θ)

There are two final goods firms: f_c^ϕ and f_c^θ . The first is owned by the rich household and the second by the poor household. Both produce homogeneous final goods (commodity) following simple AK production functions, which are identical but still have their own working capital and technology (productivity factor). This can be rendered as:

$$y_{c,t}^\phi = z_t^\phi (k_t^\phi)^\alpha \quad (4.2.8)$$

$$y_{c,t}^\theta = z_t^\theta (k_t^\theta)^\alpha \quad (4.2.9)$$

¹The capital endowment is set such that the total amount of working capital in the steady state is constant. In other words, the amount of depreciation corresponding to $\bar{k}^\phi + \bar{k}^\theta$ is defined by $\delta(\bar{k}^\phi + \bar{k}^\theta)$. Thus, it is set $\bar{y}_k = \delta(\bar{k}^\phi + \bar{k}^\theta)$, with the result being that depreciated capital is replenished in the steady state.

where α is the output elasticity of working capital and z_t^h , for $h \in \{\phi, \theta\}$, is the total factor productivity (TFP) presumed to follow the AR(1) process

$$\ln z_t^\phi = \rho_\phi \ln z_{t-1}^\phi + (1 - \rho_\phi) \ln \bar{z}^\phi + \varepsilon_{z,t}^\phi \quad (4.2.10)$$

$$\varepsilon_{z,t}^\phi \sim i.i.d. \quad N(0, \sigma_\phi^2)$$

$$\ln z_t^\theta = \rho_\theta \ln z_{t-1}^\theta + (1 - \rho_\theta) \ln \bar{z}^\theta + \varepsilon_{z,t}^\theta \quad (4.2.11)$$

$$\varepsilon_{z,t}^\theta \sim i.i.d. \quad N(0, \sigma_\theta^2)$$

The final goods firm f_c^ϕ goes into period t with working capital passed from the previous period $(1 - \delta)k_{t-1}^\phi$; δ is the depreciation rate and k_{t-1}^ϕ is the capital owned in period $t - 1$. The level of new investment is determined by how much has been purchased from the market. As a result of the biddings, s/he invests in $\tilde{b}_{k,t}^\phi / \tilde{p}_{k,t}$ in every period t . Thus, the ultimate quantity of working capital k_t^ϕ owned by the firm in period t is defined following the law of motion, (4.2.12)

$$k_t^\phi = (1 - \delta)k_{t-1}^\phi + \frac{\tilde{b}_{k,t}^\phi}{\tilde{p}_{k,t}} \quad (4.2.12)$$

The law of motion for capital accumulation in relation to a final goods firm f_c^θ is assumed to be the same as that of a final goods firm f_c^ϕ , (4.2.12). However, this firm loses collateral $q_{k,t}$ based on her/his default decision. In the end, the ultimate quantity of the capital stock owned by final goods firm f_c^θ in period t is k_t^θ , expressed as the following law of motion: (4.2.13)

$$k_t^\theta = (1 - \delta)k_{t-1}^\theta + \frac{\tilde{b}_{k,t}^\theta}{\tilde{p}_{k,t}} - q_{k,t} \quad (4.2.13)$$

Commercial Bank (b^θ)

The commercial bank is owned by the rich household and extends the collateralised loans to the final goods firm f_c^θ (or, equivalently, the poor household). Thus, this agent extends the new loans $\tilde{m}_{l,t}$ and gets back $R_t(1 + r_{l,t-1})\tilde{m}_{l,t-1}/(1 + \pi_t)$ in every period, something that is especially true at the beginning of each period. These money flows are explicitly shown up the rich household's budget set (4.2.1). As a result of his/her business, the profit of the commercial bank, $\Pi_{b,t}$, is realised as follows:

$$\Pi_{b,t} = R_t \frac{\tilde{m}_{l,t-1}(1 + r_{l,t-1})}{1 + \pi_t} - \tilde{m}_{l,t} \quad (4.2.14)$$

As the bank is wholly owned by the rich household and there is no other constraint being considered (such as capital adequacy ratio and/or liquidity requirements), it is considered to be the portfolio of the rich household. However, we can relax this assumption by enriching the model along the lines of Goodhart et al. 2006, Tsomocos 2003, etc.

Monetary Authority

It is assumed that the Central Bank supplies the short-term loans to both rich and poor households through open market operations (OMOs). These liquidity injections must exit the system (with accrued interest) when borrowing households repay their obligations. Consequently, I model the Central Bank's monetary base as changes to the level of expansionary OMOs, giving the following:

$$\frac{\tilde{M}_t(1 + \pi_t)}{\tilde{M}_{t-1}} = e_{M,t} \quad (4.2.15)$$

$$\ln e_{M,t} = \rho_M \ln e_{M,t-1} + (1 - \rho_M) \ln \bar{e}_M + \varepsilon_{M,t} \quad (4.2.16)$$

$$\varepsilon_{M,t} \sim i.i.d. \ N(0, \sigma_M^2)$$

where ρ_M is the AR(1) coefficient of money supply in the short-term loan market and $\varepsilon_{M,t}$ is the transitory monetary policy shock, an *i.i.d.* random variable, having a zero mean and a standard deviation σ_M .

As the Central Bank conducts OMOs, it supplies $\tilde{M}_t$ into the short-term loan market to meet the demand seen in each period t . During this process, the Central Bank may make a profit. This revenue is redistributed to each household in the form of a lump-sum monetary transfer, (4.2.17)

$$\tilde{T}_t = r_{s,t} \tilde{M}_t \quad (4.2.17)$$

4.2.2 Equilibrium Conditions

Market Clearing Conditions

The goods market clears when the amount of money that consumers (households) offer to buy is exchanged for the quantity of consumption goods that producers (final goods firms) provide to sell. Thus, for $\forall t \in T$, whenever $\sum_{h \in \{\phi, \theta\}} \tilde{b}_{c,t}^h$ and $\sum_{h \in \{\phi, \theta\}} y_{c,t}^h > 0$,

$$y_{c,t}^\phi + y_{c,t}^\theta = \tilde{b}_{c,t}^\phi + \tilde{b}_{c,t}^\theta \quad (4.2.18)$$

The capital market clears when the amount of money that investors (final goods firms) offer to buy is exchanged for the quantity of capital that producer (intermediate goods

firm) and commercial bank provide to sell. Thus, for $\forall t \in T$, whenever $\sum_{h \in \{\phi, \theta\}} \tilde{b}_{k,t}^h$ and $y_{k,t} + q_{k,t} > 0$,

$$\tilde{p}_{k,t} = \frac{\tilde{b}_{k,t}^\phi + \tilde{b}_{k,t}^\theta}{y_{k,t} + q_{k,t}} \quad (4.2.19)$$

The collateralised loan (long-term bond) market clears when the amount of money that the poor household (the final goods firm owned by the poor household) offers to repay in the beginning of period $t + 1$ is exchanged for the amount of money that the rich household (the commercial bank owned by the rich household) extends in the beginning of period t . Thus, for $\forall t \in T$, whenever $\tilde{\mu}_{l,t}$ and $\tilde{m}_{l,t} > 0$,

$$1 + r_{l,t} = \frac{\tilde{\mu}_{l,t}}{\tilde{m}_{l,t}} \quad (4.2.20)$$

The short-term loan market clears when the amount of money that borrowers (households) offer to repay in the end of period t is exchanged for the amount of money that the Central Bank extends in the beginning of period t . Thus, for $\forall t \in T$, whenever $\sum_{h \in \{\phi, \theta\}} \tilde{\mu}_{s,t}^h$ and $\tilde{M}_t > 0$,

$$1 + r_{s,t} = \frac{\tilde{\mu}_{s,t}^\phi + \tilde{\mu}_{s,t}^\theta}{\tilde{M}_t} \quad (4.2.21)$$

Rational Expectations

When the borrower repays the collateralised loan, her/his default decision depends on the relative value between the nominal amount of the loan obligation $\mu_{l,t-1}$ and the time t value of the collateral $p_{k,t}c_{k,t-1}$, where $c_{k,t-1}$ serves as collateral for the loan obligation

and $p_{k,t}$ is the (nominal) price of capital stock. If s/he expects that the collateral is less valuable, i.e. $\mu_{l,t-1} \geq p_{k,t}c_{k,t-1}$, the borrower rationally chooses to default rather than repaying the loan. When this happens, the creditor seizes the collateral and liquidates it in the market. If the reverse is true, i.e. $\mu_{l,t-1} < p_{k,t}c_{k,t-1}$, the borrower makes a full repayment to the creditor. This indicates that there is no partial default decision. In other words, the decision is binary, i.e. the agent either defaults or does not default. However, the creditor does not lose all of her/his cash flow even when there is a default as s/he can liquidate the seized collateral. This entails the existence of an effective repayment rate of the collateralised loan. Therefore, it is assumed that the borrower renegotiates with the creditor and pays back $R_t\mu_{l,t-1}$. R_t is the rational expectation of the effective payment and this can be expressed as follows:

$$R_t = \min \left(1, \frac{p_{k,t}c_{k,t-1}}{\mu_{l,t-1}} \right) \quad (4.2.22)$$

where the borrower puts collateral $c_{k,t}$ at time t and it cannot be more than what s/he owns k_t^θ at that time. Thus, the collateral seized by the creditor at time t is bounded by $0 \leq c_{k,t} \leq k_t^\theta$.

Afterwards, the creditor liquidates the part of collateral $q_{k,t}$ to get paid the money that s/he loses due to borrower's default decision. As both entities are rational, the amount of liquidation is equal to that of the creditor's loss considering the short-term loan rate $r_{s,t}$:

$$(1 + r_{s,t})(1 - R_t)\mu_{l,t-1} = p_{k,t}q_{k,t} \quad (4.2.23)$$

If the debt were indexed then agents would never default. Provided that there was only one goods, if this were the case, inflationary effects would be removed and, therefore,

wealth effects would be inconsequential. However, if there were more than one goods present in the economy, as it is the case in our model, then a natural question arises with respect to which goods will serve as an index. This observation is important since with more than one goods not only inflation causes wealth effects but also relative prices. Hence, a fully indexed debt contract would require information about agents' preferences.

4.3 Equilibrium Analysis

Our view, in general, is consistent with the long run money neutrality proposition that the RBC and NK literature suggests. However, the model obtains money non-neutrality in short run equilibrium unlike the RBC models, where neutrality always holds. Furthermore, in stark contrast to the NK approach, where short run non-neutrality is obtained through real frictions such as monopolistic competition and asymmetric information, in our framework, it is driven by the postulated transaction technology, subsequent transactions and investment demand for money. In other words, liquidity and default are the driving forces of our results.

Definition 10. *No Arbitrage Condition*

Assume that $\tilde{\mu}_{l,t}$ and $\tilde{m}_{l,t} > 0$ for $\forall t \in T$. Presume, more that $\sum_{h \in \{\phi, \theta\}} \tilde{b}_{k,t}^h$ and $y_{k,t} + q_{k,t} > 0$ for $\forall t \in T$. Then, at any short run equilibrium,

$$R_t = \min \left(1, \frac{(1 + \pi_t) \tilde{p}_{k,t} c_{k,t-1}}{\tilde{\mu}_{l,t-1}} \right)$$

Agents do not repay more than what they owe $R_t \leq 1$, or that they are not rewarded for defaulting on their loan obligations $R_t \geq 0$ (these are equivalent to the transversality

conditions). Consequently, endogenous default is compatible with the orderly functioning of the market economy.

Proposition 11. *Fisher Effect*

Suppose that for some $h \in \{\phi, \theta\}$, $\tilde{b}_{c,t}^h$ and $\tilde{b}_{c,t+1}^h > 0$ for $\forall t \in T$. Suppose, further that h has some money left over the moment that the long-term loan comes due at $\forall t \in T$. Then, at any short run equilibrium,

$$\begin{aligned}\ln(1 + r_{l,t}) &\simeq \ln \left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\phi; \phi)}{\beta E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\phi; \phi)} \right) + \ln(1 + E_t \pi_{t+1}) + \ln E_t R_{t+1}^{-1} \\ \ln(1 + r_{l,t}) &\simeq \ln \left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta)}{\gamma E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\theta; \theta)} \right) + \ln(1 + E_t \pi_{t+1}) + \ln E_t v_{t+1}^{-1}\end{aligned}$$

Proof. Appendix B □

The nominal interest rate (collateralised loan) is approximately equal to real interest rate plus expected inflation and a risk premium. Fisher effect explains that nominal prices are linked to consumption; affecting nominal variables also affects real variables allocationally.

Proposition 12. *Quantity Theory of Money*

At any short run equilibrium, if $r_{s,t} > 0$ then the aggregate income for $\forall t \in T$, namely the value of all sales income, is equal to the total money supply provided by the central bank.

$$\tilde{M}_t = y_{c,t}^\phi + y_{c,t}^\theta + \tilde{p}_{k,t}(y_{k,t} + q_{k,t})$$

Proof. Appendix B □

If nominal interest rate (short-term loan) is positive, the real velocity of money is endogenous and nominal changes of short-term loan rate affect both prices and quantities.

Proposition 13. *Term Structure of Interest Rates*

At any short run equilibrium, $\forall t \in T$

$$\tilde{b}_{c,t}^\theta + \tilde{b}_{k,t}^\theta + v_t \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t} = \frac{y_{c,t}^\theta + (1 - \omega)\tilde{T}_t}{1 + r_{s,t}} + \frac{\tilde{\mu}_{l,t}}{1 + r_{l,t}}$$

Proof. Appendix B □

Term structure of interest rates explain that the nominal interest rates, i.e. short-term loan rate and long-term bond rate, are determined at the same time. Thus, together with Proposition 1, we can conclude that nominal interest rates, real interest rate and inflation are settled simultaneously. The term structure of interest rates is affected by aggregate liquidity and default, because interest rates price-in anticipated default rates (default premium).

Corollary 14. *Money Non-Neutrality*

Suppose that Propositions 1 and 2 hold, then money is not neutral which suggests that the Central Bank can control the real economy, i.e. consumption, through adjusting the short-term loan rate.

Proposition 15. *Interest Rate Effect on Aggregate Productivity*

Suppose that $\bar{k}^\phi = \bar{k}^\theta$ in steady state and $y_{k,t} = \text{const.}$ for $\forall t \in T$, than $r_{l,t} \neq \bar{r}_l$ for $\exists t \in T$ causes an inefficient capital allocation, i.e. $k_t^\phi \neq k_t^\theta$, and results in welfare losses in the economy.

$$\bar{y}_c = \bar{y}_c^\phi + \bar{y}_c^\theta \geq y_{c,t}^\phi + y_{c,t}^\theta = y_{c,t} \text{ for } \exists t \in T$$

Proof. Appendix B □

Proposition 4 implies that, when financing costs increase, the capitalist will end up with more working capital than before, whilst the reverse applies for the entrepreneur. As a result, capital stock is redistributed from the entrepreneur to the capitalist. Hence, we can see that the interest rate has non-trivial effect on aggregate productivity of the economy. Put differently, the entrepreneur is more productive (efficient) than the capitalist in the end. Thus, the total output decreases due to the inefficient allocation of capital and results in welfare losses.

Proposition 16. *Price Wedge*

At any short run equilibrium, if $r_{s,t} > 0$ then the aggregate income for $\forall t \in T$, specifically the value of all sales income, is less than the total expenditure provided by the consumers and investors.

$$\frac{y_{c,t}^\phi + y_{c,t}^\theta + \tilde{p}_{k,t}(y_{k,t} + q_{k,t})}{1 + r_{s,t}} \leq \tilde{b}_{c,t}^\phi + \tilde{b}_{c,t}^\theta + \tilde{b}_{k,t}^\phi + \tilde{b}_{k,t}^\theta$$

Proof. Appendix B □

Since agents must borrow money to purchase/invest and interest rates (short-term loan) are positive, there is a wedge between selling and purchasing prices.

4.4 Calibration

The period in the model is a year and I set $\beta = \gamma = 0.975$. The parameter in the production function is set to $\alpha = 0.33$ and the depreciation rate to $\delta = 0.12$. The seigniorage cost is equally redistributed to each agent in forms of monetary endowment, $\omega = 0.5$, at every period. These values are standard and the quantitative properties of the model are not particularly sensitive to this group of parameters; implied parameters are summarised in Table 4.1.

TABLE 4.1

I selected the target inflation rate $\bar{\pi} = 0.03$, implying that the annular steady state return from holding risk free assets is 5.64 percent. It is presumed that there exists default in steady state, i.e. $\bar{v} = 0.985$, and the short-term loan rate is set to $\bar{r}_s = 0.02$. Afterwards, I use numerical methods to solve the set of simultaneous equations. I assume that the economy is operating in the deterministic steady state of a competitive equilibrium; endogenous variables are stationarised and reported in Table 4.2.

TABLE 4.2

It is assumed that two final goods firms hold the same amount of working capital, i.e. $\bar{k}^\phi = \bar{k}^\theta$, in steady state so as to examine the capital allocation effects described in Proposition 4. I assign 0.985 to the fraction of repayment rate, $\bar{R}$, in line with the 'No Arbitrage Condition'. The steady state endogeneously calculated satisfies $0 \leq \bar{c}_k \leq \bar{k}^\theta$, the lower

and upper bound of collateral. Figure 4.3 explicitly shows the calculation errors of 25 simultaneous equations² in steady state.

FIGURE 4.3

4.5 Quantitative Analysis

The purpose of this section is twofold: (i) to describe how the endogenous variables of the model respond to deflationary shocks; and (ii) to assess the impact of introducing collateral constraint into the default decision in a DSGE framework. Thus, I analyse the effects of a benchmark economy with respect to contractionary monetary shocks and other externalities inducing recession with debt-deflation, i.e. negative capital endowment shocks and unbalanced (negative) technology innovation shocks, resulting in foreclosure, high borrowing costs, capital reallocation, and a decrease in output levels.

4.5.1 Contractionary Monetary Shock

First, I describe the role of collateral constraints and their interaction with liquidity (monetary policy) and default. Then, in the RBC tradition, I test whether the model is able to reproduce some important stylised facts, especially the “Fisher Effect” and the “Term Structure of Interest Rates”. I finally examine the effects of contractionary monetary shocks in terms of financial stability.

²There are 30 endogeneous variables. However, I calibrate the model satisfying $\bar{v} = \bar{R} = 0.985$, $\bar{\pi} = 0.03$, $\bar{r}_s = 0.02$ and $\bar{e}_M = 1.03$. Thus, I use 25 simultaneous equations for calculating the steady state. Specifically, I drop out 4 AR(1) processes and rational expectations condition $\bar{v} = \bar{R}$.

Role of Collateral Constraint

The decision to default is endogenous and depends on the Loan-To-Value (LTV) ratio. As can be seen from Figure 4.4, the relative value of collateral depends on monetary policy; a contractionary monetary policy causes deflation in asset prices and a higher LTV ratio, resulting in foreclosure and higher borrowing costs. Therefore, the collateral constraints work as a transmission channel of monetary policy into the real economy, flowing through defaults.

FIGURE 4.4

If the collateralised loan rate is not contingent on the ex-post realisation of repayment, i.e. a pre-determined lending rate, shocks hitting the economy tend to have a more muted effect relative to the benchmark scenario. This reflects the less pronounced interactive effects between borrowers' net worth and the financial markets. This mitigates the macroeconomic amplification implied by the existence of collateral and default. However, turning to the specification with collateral constraints, the borrower has more limited access to the loan market. Thus, the immediate macroeconomic impact of an adverse shock (i.e. the contractionary monetary policy) on the borrower is more pronounced. This comes about mainly via the more restrictive lending implied by borrowers being bound by their collateral values.

Stylised Facts

As expected, the result is exactly the same regardless of whether the Central Bank uses its base money or the short-term loan rate as its monetary policy instrument; the Cen-

tral Bank decreases its base money and allows the short-term loan rate to be determined endogenously. Figure 4.5 summarises the key variables in response to a decrease in the Central Bank's base money. Given a higher short-term loan rate, with all other things being held constant, a commercial bank encounters pressure to decrease its loan supplies (i.e. an endogenous credit supply contraction). This portfolio adjustment on the part of a commercial bank produces a positive pressure on lending rates for collateralised loans.

FIGURE 4.5

Moreover, a commercial bank rationally anticipates that its reduced credit extension would decrease the overall supply of credit in the economy even further, thus causing deflation and the probability of default to eventually worsen. This is because a lower aggregate credit supply not only directly decreases both agents' liquidity but also decreases their income stream in the subsequent period. As was anticipated from the "Interest Rate Effect on Aggregate Productivity", there is misallocation of resources (capital) as well, which causes the reduction in output level. Thus, the expected rate of return from extending inter-temporal loans decreases, implying that the willingness to supply more credit falls even further.

Given lower expected output, a final goods firm owned by poor household demands fewer loans. This results in a negative pressure on the lending rates offered by the commercial bank. However, this "crowding-out" effect is dominated by the corresponding positive pressure from the reduced credit supply. Thus, we observe that the lending rates of long-term bonds rise. I also find that there is a negative feedback loop inducing a lower short-term loan rate (endogenous sustainability) if the Central Bank's OMOs follow a standard

Taylor rule, with lower expected output and deflation working together to stabilise (lower) the short-term loan rate.

Figure 4.6 depicts the yield curve of collateralised loans, i.e. the geometrical averages over time, wherein the debtor is allowed to roll over the debt obligation. As can be seen in the figure, the yield curve is inverted (i.e. the short-term yields are higher than the medium- to long-term yields). The yield of the two-year bond stands around at 2.4 basis points higher than the steady state, but only around 0.6 basis points for the five-year bond. The market's anticipation of falling interest rates causes this incident. This is consistent with the argument that strongly inverted yield curves have preceded economic depressions.

FIGURE 4.6

Financial Stability Analysis

Financial fragility is characterised by reduced bank profitability and increased aggregate default. Increases in both banking sector vulnerability and aggregate default (lower repayment rates) are linked to welfare losses (agents' utilities) (Goodhart et al. 2004). Accordingly, I investigate two measures for financial stability, i.e. bank profitability and repayment rates, in Figure 4.7.

FIGURE 4.7

A contractionary monetary policy reduces the repayment rate of collateralised loans as well as decreasing the bank's profit. The one is mainly due to deflation in asset price, following the "Quantity Theory of Money", and the other is caused by the fact that increased

interest rate spreads are dominated by the lower repayment rates. In other words, bank profit has higher elasticity with respect to repayment rates than with respect to interest rate spreads. Thus, we can say that the financial market becomes a more fragile regime as a result of a contractionary monetary policy. Note that, in general, contractionary monetary policy does not necessarily entail lower bank profitability.

4.5.2 Externalities Inducing Recession

I consider two kinds of shock as a source for inducing recession with debt-deflation: (i) negative capital endowment shocks; and (ii) unbalanced (negative) productivity shocks.

Negative Capital Endowment Shock

As can be seen in Figure 4.8, I investigate the effects of a negative capital endowment shock persisting for around 10 years, i.e. $\rho_k = 0.60$, $\varepsilon_{k,1} = -0.001$ and $\varepsilon_{k,t} = 0$ for $t \geq 2$. This could result from disaster shocks³ or an increase of inefficiency in capital production⁴, as well as from many other sources. As capital production is reduced, the intermediate goods firm (or, equivalently, the rich household) supplies less in equilibrium, with the result being a higher level of asset price. This increase in asset prices causes downward pressure on the LTV ratio, i.e. default risk. However, the lower level of aggregate output (which is a result of the lower level of working capital) increases the marginal utility of default, which in turn crowds out the descending force default risk. In other words, the counter-cyclical risk premium is persistent throughout the simulation periods. As a creditor has a rational

³For example, an earthquake or a tsunami could disable a power plant and thus compromise electricity production persistently and over a long period.

⁴During a recession, idle working capital may increase, restricting production of luxury goods due to there being a lower demand for such goods.

expectation of a lower repayment rate, the borrower faces a higher interest rate in regard to their collateralised loans; higher financing costs result in lower levels of investment, with outputs thus falling even further.

FIGURE 4.8

Figure 4.9 shows the inverted yield curve. Figure 4.8 makes clear the risks associated with that yield curve; there is high level of default risk like foreclosure and LTV ratio. This is the risk associated with a borrower going into default (not making full payments as promised), which can be described in terms of the risk premium. As there is no interest rate risk (i.e. the Central Bank's policy stance remains unchanged), the inflation risk is dominated by the default risk in the "Fisher Effect"; the rate of default increases more than deflation in the commodity market. This initiates financial accelerator effects, which amplify the shock in the real economy. As debtors rationally expect a default risk, they consequently adjust their portfolio to smooth the consumption stream, therefore trying to reduce their balance sheet. Even after the shock recedes, it takes a longer time to achieve stable financial systems due to the symbiotic relationship between the real and financial sectors.

FIGURE 4.9

The negative capital endowment shock reduces both bank profitability and repayment rates, as can be seen in Figure 4.10. The latter (repayment rate) decreases mainly due to the increased level of marginal utility of default (On-the-Verge Conditions); as the short-term money supply is presumed to be constant, the poor household demands more inter-period

loans and this increased debt level (debt overhang) dominates the increases in asset price. Since bank profit has higher elasticity with respect to repayment rates than in regard to interest rate spreads, bank profitability decreases too. Thus, we can confirm that negative capital endowment shocks result in financial instability as well as departures from standard policy objectives (such as targeting inflation and stabilising output levels).

FIGURE 4.10

As bank profitability drops in spite of the fact that the face value of the collateral increases in real terms during negative capital endowment shocks (an indication of a debt overhang), debt forgiveness is one of the tactics the bank to resolve the problem. It can be modelled by introducing an exogenous variable (i.e., one of policy instruments) in the optimisation problem, specifically the partial forgiveness of debt to slow or stop the growth of the debt owed to the bank. This model provides the baseline framework which can indeed be extended in the way mentioned.

Unbalanced (Negative) Productivity Shock

The model is here calibrated with the same productivity factor and capital elasticity for both final goods firms in the steady state. To generate an unbalanced productivity shock with debt-deflation, a negative productivity shock is only assigned to the final goods firm owned by the rich household. This persists for around 10 years, i.e. $\rho_z^\phi = 0.60$, $\varepsilon_{z,1}^\phi = -0.001$ and $\varepsilon_{z,t}^\phi = 0$ for $t \geq 2$. This entails that the rich household can produce fewer commodities than before but can retain the same level of working capital. Therefore, as well as aggregate output decreasing, the asset price also decreases (deflation) due to the

lower demand effect, in that a decreased productivity factor for a rich household's final goods firm means a lower demand of capital for another final goods firm, thus equalising the marginal productivities between two firms. The result is in the end lower investment, which causes the higher default risk and higher interest rate in regard to the collateralised loans because the LTV ratio is directly linked to the asset price, with the increased level of foreclosures also being an indicator. Moreover, higher financing costs entail lower levels of investment, which further results in a lower price level in regard to capital. Figure 4.11 illustrates the above:

FIGURE 4.11

As long-term yields fall below short-term yields, an inverted yield curve occurs, which is shown in Figure 4.12. Under unusual circumstances like a negative productivity shock, long-term investors are content to settle for lower yields if they think the economy will slow or even decline in the future. Campbell's (1986) work demonstrates that an inverted yield curve accurately forecasts US recessions. In addition to potentially signalling an economic decline, inverted yield curves also imply a belief on the part of the market that inflation will remain low. This is because, even in the event of a recession, a low bond yield will still be offset by low inflation. However, in the case at hand, the particular inverted nature of the yield curve is mainly due to the default risk caused by the high level of foreclosure and increased LTV ratio (the associated risk factors are shown above in Figure 4.11). In the short run, the yield curve steepens as the spread between long- and short-term interest rates decreases; therefore, long-term bond prices decrease in comparison to short-term bonds. Changes in the yield curve stem from bond risk premiums and expectations in regard to future interest rate increases.

FIGURE 4.12

Like the financial stability measures shown in Figure 4.10 during negative capital endowments, both financial stability measures such as repayment rates and banks' profitability move towards a fragile regime, as can be seen in Figure 4.13. However, the repayment rate decreases mainly due to the decreases in the asset price, which represents a marked contrast to the case of negative capital endowments. As bank profit has higher elasticity in regard to the repayment rate compared to interest rate spreads – which is like before – profitability falls in the banking sector. The conclusion from this is that the fragile financial markets are a result of the recessionary shock and the deflation in asset prices headed by the unbalanced negative productivity shock.

FIGURE 4.13

Optimal Monetary Policy

The Central Bank controls the supply of money in the short-term loan market (equivalently, the short-term loan rate) in such a way as to achieve a target output level while also keeping inflation stable. However, whether or not these goals are consistent with financial stability measures is not clear, especially in times of recession with deflation in commodity and/or asset prices.

In this section, the optimal monetary policy reactions during recessionary disturbances are investigated. The first assumption here is that the ad-hoc stabilisation goal is to minimise the quadratic loss function of fluctuations in bank profitability, (4.5.1). It is also assumed that the Central Bank seeks to minimise fluctuations in the repayment rate, (4.5.2).

$$\min_{e_{M,1}} \sum_{t=0}^{\infty} \beta^t \Delta_{p,t}^2 \quad (4.5.1)$$

$$\min_{e_{M,1}} \sum_{t=0}^{\infty} \beta^t \Delta_{v,t}^2 \quad (4.5.2)$$

where $\Delta_{p,t}$ and $\Delta_{v,t}$ are the log-deviation of bank profit and repayment rate, respectively. $e_{M,1}$ represents the ratio of the money supply at time $t=1$, in other words the monetary policy reaction at a time when externality (i.e. shocks) is affecting the economy.

Equations (4.5.1) and (4.5.2) are ex-post policy measures in regard to bank profitability and repayment rate since financial fragility is characterised by reduced bank profitability and increased aggregate default (Goodhart et al. 2006, Tsomocos 2003): increases in both banking sector vulnerability and aggregate default (lower repayment rates) are linked to welfare losses (agent utility).

In times of negative capital endowments, when the economy's working capital is reduced, aggregate output also decreases, which results in a lower supply in the commodity market. However, this inflationary pressure (low supply) is less important than the deflationary pressure (low demand), given that households have a rational expectation of lower income. Together, the negative output gap and deflation work as a negative pressure on the short-term loan rate in a case in which the Central Bank follows the Taylor principle, i.e. (i) inflation targeting and (ii) leaning against the wind (output stabilisation). In such a case, the Central Bank pursues an expansionary monetary policy during a negative capital endowment shock.

In Figure 4.14, the two loss functions defined in (4.5.1) and (4.5.2) with respect to monetary policy reactions are plotted in times of negative capital endowments. This confirms that an inflation-targeting monetary policy (in this case monetary expansion) increases the level

of vulnerability in the banking sector (as bank profitability decreases significantly) but reduces the level of default. Therefore, the effect of monetary policy targeting on inflation is ambiguous in terms of financial stability measures. Moreover, that negative loss of stability is relatively substantial in terms of its effect on bank profitability, particularly when compared to the gains of stability on default. Therefore, a monetary policy focused on inflation does not exactly achieve the optimal policy regime in terms of financial stability.

FIGURE 4.14

The unbalanced (negative) productivity shock results in: (i) deflation in asset prices; and (ii) inflation in commodity prices. The former is due to lower demand effects in the asset market and the latter to lower supply effects in the commodity market. Thus, if the Central Bank aims to set the short-term loan rate with higher elasticity in regard to inflation rather than output gap, it supplies a reduced level of liquidity in the short-term loan market, i.e. a monetary contraction. This causes a reduction of the base money in the short-term market, with the effect being that the lending rates in regard to collateralised loans simultaneously increase even further. In other words, this chain reaction motivates the commercial bank to adjust its balance sheet by borrowing fewer short-term loans and extending smaller amount of long-term loans. As a result of low credit supply pressure, investment reduces and income streams worsen. Thus, the commercial bank adopts riskier investment strategies and raises interest rate spreads to maintain its profitability. In turn, the higher interest rates mean final goods firms reduce their investments, pushing down the price level in the asset market and detrimentally affecting repayment rates. Moreover, as the bank's profitability has a higher elasticity on repayment than interest rate spreads, the monetary contraction leads to a more vulnerable banking sector (reduced bank profitability). Therefore,

this policy prescription (monetary contraction) results in a more fragile regime in terms of financial stability measures such as bank profitability and repayment rates.

As Figure 4.15 shows, what I have explained above is actually obvious given that the repayment rate directly depends on the collateral value (asset price) and that lower repayment rates mean reduced bank profitability. Figure 4.15 shows two loss functions as defined in (4.5.1) and (4.5.2) with respect to monetary policy reactions, which are plotted in the light of an unbalanced (negative) productivity shock. From the figures, it is confirmed that monetary contraction in this case increases levels of vulnerability in the banking sector as well as reducing repayment rates. Therefore, the effect of monetary policy targeting on inflation clearly results in fragile financial systems in terms of financial stability measures during the time of an unbalanced (negative) productivity shock.

FIGURE 4.15

Interest Rate Effect on Aggregate Productivity

I make the assumption that both of the final goods firms have the same total productivity factor and output elasticity of capital. Furthermore, I calibrate that the initial allocation (i.e. the steady state) of working capital is equal between the two producers, $\bar{k}^\phi = \bar{k}^\theta$. This is done in order to test the capital reallocation effect of the monetary policy. An increase in the interest rate prevents the liquidity-constrained buyer purchasing as much capital as before, meaning that capital is redistributed between the two; the rich household thus acquires more working capital than before. In return, this allocation of capital changes the marginal productivities of both producers, which results in a lower output level. The

(tightly) liquidity-constrained producer is more productive and produces less, while the other is less productive and produces more.

If the optimality condition $k_t^\phi = k_t^\theta = k_t^+$ does not hold, a capital misallocation effect occurs in the productivity of the modelled economy. This is due to the decreasing marginal productivity in the AK production function, i.e. concavity $\partial^2 y_{c,t}^i / \partial k_t^i < 0$ for $i \in \{\phi, \theta\}$. This capital misallocation leads to lower effective working capital k_t^- , which is less than k_t^+ in the end as described (4.5.3) - (4.5.6).

$$k_t^\phi + k_t^\theta = 2k_t^+ \quad (4.5.3)$$

$$y_t^\phi + y_t^\theta \leq 2y_t^+ \quad (4.5.4)$$

$$y_t^- \leq y_t^+ \quad (4.5.5)$$

$$k_t^- \leq k_t^+ \quad (4.5.6)$$

A reduction in the Central Bank's base money leads to the price deflation of collateral (capital), and means in the end that the default risk increases. Higher levels of default mean foreclosures, higher borrowing costs, inefficient investment (capital misallocation) and a decrease in output level. In this section, I focus on a contractionary monetary shock and assess the default channel's effect on output using Fisher's debt deflation theory. This can also be applied when there is a recessionary shock with deflation, given that the role of monetary policy is to correct the capital allocation.

4.6 Concluding Remarks

By including collateral constraints, liquidity constraints and agents' heterogeneity in my model, I seek to overcome some of the limitations of the current DSGE models as a tool

for analysing financial stability; 1) collateral constraints as a default channel, 2) liquidity constraints via CIA constraints and 3) heterogeneous agents to generate redistribution of wealth effect cross lenders and borrowers. In order to analyse the role of these frictions, I compared the effects of a contractionary monetary policy across different aspects, including the role of collateral constraints, the interest effect in the real economy, stylised facts and financial stability.

In addition, I investigated recessionary shocks inducing deflation in commodity and/or asset prices such as a negative capital endowment shock or an unbalanced (negative) productivity shock. The one induces deflation in the commodity price and the other in the asset price, with the result being a procyclical feedback loop in the default channel; foreclosure, high borrowing costs, inefficient capital allocation and a decrease in output level. All of these come from the default channel (collateral constraints); whenever the collateral value is less than the loan obligation, there is an increase in default, foreclosure and liquidation.

Our results suggest that agent heterogeneity is essential when assessing the real effects of monetary policy because agents depend on the part of the economy directly affected, i.e. capital reallocation. Furthermore, unlike as occurs within the New Keynesian framework, in models incorporating financial frictions such as liquidity and default, interest rates are determined in the markets and the price level of the economy is determined endogenously (fully flexible price).

The numerical exercise suggests that, during recessionary shocks, an inflation-targeting monetary policy is not optimal in regard to financial stability measures. Moreover, the debt-deflation feature is expected despite there being no proper policy intervention. There-

fore, I conclude that policy interventions are essential for a sustainable economy, not only in relation to typical policy targets such as stabilising output and targeting inflation but also financial stability measures that focus on such issues as bank profitability and repayment rates.

This model and its results are part of an ongoing research project. Due to the limitations of time and length, I have not modelled a sophisticated interbank loan market, bailout or asset purchases, or covered the reaction function of policy makers, in particular the Central Bank and the Fiscal Authority. Nevertheless, this model will be extended in these ways and the findings will be presented in future studies.

4.7 Appendices

Appendix A: First Order Conditions

$$\begin{aligned}
L(\phi) &= E_0 \sum_{t=0}^{\infty} \beta^t [\ln \tilde{b}_{c,t}^{\phi} + \eta_{1,t}^{\phi} \{ \frac{\tilde{\mu}_{s,t}^{\phi}}{1+r_{s,t}} + R_t \frac{\tilde{m}_{l,t-1}(1+r_{l,t-1})}{1+\tilde{\pi}_t} - \tilde{b}_{c,t}^{\phi} - \tilde{b}_{k,t}^{\phi} - \tilde{m}_{l,t} \} \\
&\quad + \eta_{2,t}^{\phi} \{ y_{c,t}^{\phi} + \tilde{p}_{k,t}(y_{k,t} + q_{k,t}) + \omega \tilde{T}_t - \tilde{\mu}_{s,t}^{\phi} \}] \\
L(\theta) &= E_0 \sum_{t=0}^{\infty} \gamma^t [\ln \tilde{b}_{c,t}^{\theta} + \eta_{1,t}^{\theta} \{ \frac{\tilde{\mu}_{s,t}^{\theta}}{1+r_{s,t}} + \frac{\tilde{\mu}_{l,t}^{\theta}}{1+r_{l,t}} - \tilde{b}_{c,t}^{\theta} - \tilde{b}_{k,t}^{\theta} - \nu_t \frac{\tilde{\mu}_{l,t-1}^{\theta}}{1+\pi_t} \} \\
&\quad + \eta_{2,t}^{\theta} \{ y_{c,t}^{\theta} + (1-\omega) \tilde{T}_t - \tilde{\mu}_{s,t}^{\theta} \}]
\end{aligned}$$

$$\frac{\partial L(\phi)}{\partial \tilde{b}_{c,t}^{\phi}} = 0 \quad \eta_{1,t}^{\phi} = \frac{1}{\tilde{b}_{c,t}^{\phi}} \quad (4.A.1)$$

$$\frac{\partial L(\phi)}{\partial \tilde{b}_{k,t}^{\phi}} = 0 \quad \tilde{p}_{k,t} \eta_{1,t}^{\phi} = \beta(1-\delta) E_t \left(\tilde{p}_{k,t+1} \eta_{1,t+1}^{\phi} \right) + \alpha \frac{y_{c,t}^{\phi}}{k_t^{\phi}} \eta_{2,t}^{\phi} \quad (4.A.2)$$

$$\frac{\partial L(\phi)}{\partial \tilde{\mu}_{s,t}^{\phi}} = 0 \quad \eta_{2,t}^{\phi} = \frac{\eta_{1,t}^{\phi}}{1+r_{s,t}} \quad (4.A.3)$$

$$\frac{\partial L(\phi)}{\partial \tilde{m}_{l,t}} = 0 \quad \frac{\eta_{1,t}^{\phi}}{1+r_{l,t}} = \beta E_t \left(\frac{R_{t+1} \eta_{1,t+1}^{\phi}}{1+\pi_{t+1}} \right) \quad (4.A.4)$$

$$\frac{\partial L(\theta)}{\partial \tilde{b}_{c,t}^{\theta}} = 0 \quad \eta_{1,t}^{\theta} = \frac{1}{\tilde{b}_{c,t}^{\theta}} \quad (4.A.5)$$

$$\frac{\partial L(\theta)}{\partial \tilde{b}_{k,t}^{\theta}} = 0 \quad \tilde{p}_{k,t} \eta_{1,t}^{\theta} = \gamma(1-\delta) E_t \left(\tilde{p}_{k,t+1} \eta_{1,t+1}^{\theta} \right) + \alpha \frac{y_{c,t}^{\theta}}{k_t^{\theta}} \eta_{2,t}^{\theta} \quad (4.A.6)$$

$$\frac{\partial L(\theta)}{\partial \nu_t} = 0 \quad \eta_{1,t}^{\theta} = \alpha \eta_{2,t}^{\theta} \frac{y_{c,t}^{\theta}}{\tilde{p}_{k,t} k_t^{\theta}} (1+r_{s,t}) \quad (4.A.7)$$

$$\frac{\partial L(\theta)}{\partial \tilde{\mu}_{s,t}^{\theta}} = 0 \quad \eta_{2,t}^{\theta} = \frac{\eta_{1,t}^{\theta}}{1+r_{s,t}} \quad (4.A.8)$$

$$\frac{\partial L(\theta)}{\partial \tilde{\mu}_{l,t}^{\theta}} = 0 \quad \frac{\eta_{1,t}^{\theta}}{1+r_{l,t}} = \gamma E_t \left(\frac{\nu_{t+1} \eta_{1,t+1}^{\theta}}{1+\pi_{t+1}} \right) \quad (4.A.9)$$

Appendix B: Proofs

PROOF OF PROPOSITION 1

Two first-order conditions, (4.A.1) and (4.A.4), lead

$$\frac{\mathcal{U}'(\tilde{b}_{c,t}^\phi; \phi)}{1 + r_{l,t}} = \beta E_t \left(\frac{R_{t+1} \mathcal{U}'(\tilde{b}_{c,t+1}^\phi; \phi)}{1 + \pi_{t+1}} \right) \quad (4.A.10)$$

Two first-order conditions, (4.A.5) and (4.A.9), result in

$$\frac{\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta)}{1 + r_{l,t}} = \gamma E_t \left(\frac{\nu_{t+1} \mathcal{U}'(\tilde{b}_{c,t+1}^\theta; \theta)}{1 + \pi_{t+1}} \right) \quad (4.A.11)$$

After the log-linearisation of (4.A.10) and (4.A.11), I can derive the following two equations,

$$\ln(1 + r_{l,t}) \simeq \ln \left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\phi; \phi)}{\beta E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\phi; \phi)} \right) + \ln(1 + E_t \pi_{t+1}) + \ln E_t R_{t+1}^{-1} \quad (4.A.12)$$

$$\ln(1 + r_{l,t}) \simeq \ln \left(\frac{\mathcal{U}'(\tilde{b}_{c,t}^\theta; \theta)}{\gamma E_t \mathcal{U}'(\tilde{b}_{c,t+1}^\theta; \theta)} \right) + \ln(1 + E_t \pi_{t+1}) + \ln E_t \nu_{t+1}^{-1} \quad (4.A.13)$$

PROOF OF PROPOSITION 2

Summing up two liquidity constraints, (4.2.2) and (4.2.5), I get

$$\tilde{\mu}_{s,t}^\phi + \tilde{\mu}_{s,t}^\theta = y_{c,t}^\phi + y_{c,t}^\theta + \tilde{p}_{k,t}(y_{k,t} + q_{k,t}) + \tilde{T}_t \quad (4.A.14)$$

Plugging the market clearing condition in the short-term loan market (4.2.21) and the lump-sum monetary transfer (4.2.17) both into (4.A.14), I can drive

$$\tilde{M}_t = y_{c,t}^\phi + y_{c,t}^\theta + \tilde{p}_{k,t}(y_{k,t} + q_{k,t}) \quad (4.A.15)$$

PROOF OF PROPOSITION 3

Substituting the liquidity constraint (4.2.5) into budget constraint (4.2.4), I can derive

$$\tilde{b}_{c,t}^\theta + \tilde{b}_{k,t}^\theta + v_t \frac{\tilde{\mu}_{l,t-1}}{1 + \pi_t} = \frac{y_{c,t}^\theta + (1 - \omega)\tilde{T}_t}{1 + r_{s,t}} + \frac{\tilde{\mu}_{l,t}}{1 + r_{l,t}} \quad (4.A.16)$$

PROOF OF PROPOSITION 4

As output elasticity of working capital is defined by $0 < \alpha < 1$, both production functions, $y_{c,t}^\phi$ and $y_{c,t}^\theta$, are concave. Moreover, as $y_{k,t} = \text{const.}$, $k_t^\phi + k_t^\theta = \bar{k}^\phi + \bar{k}^\theta$ for $\forall t \in T$. Thus, $r_{l,t} \neq \bar{r}_l$ for $\exists t$ causes $k_t^\phi \neq k_t^\theta$ (Corollary. Money Non-Neutrality) and results in welfare losses in the economy, $\bar{y}_c^\phi + \bar{y}_c^\theta \geq y_{c,t}^\phi + y_{c,t}^\theta$.

PROOF OF PROPOSITION 5

Summing up two budget constraints, (4.2.1) and (4.2.4), I obtain

$$\frac{\tilde{\mu}_{s,t}^\phi + \tilde{\mu}_{s,t}^\theta}{1 + r_{s,t}} = \tilde{b}_{c,t}^\phi + \tilde{b}_{k,t}^\phi + \tilde{b}_{c,t}^\theta + \tilde{b}_{k,t}^\theta \quad (4.A.17)$$

Plugging two liquidity constraints, (4.2.2) and (4.2.5), into (4.A.17), I can derive

$$\frac{y_{c,t}^{\phi} + y_{c,t}^{\theta} + \tilde{p}_{k,t}(y_{k,t} + q_{k,t})}{1 + r_{s,t}} \leq \tilde{b}_{c,t}^{\phi} + \tilde{b}_{c,t}^{\theta} + \tilde{b}_{k,t}^{\phi} + \tilde{b}_{k,t}^{\theta} \quad (4.A.18)$$

Appendix C: The Complete Model Equations

I summarise here the (log-linearised) equations describing the equilibrium of the model.

There are 26 equations explaining the evolution of endogenous variables, (4.A.19) - (4.A.44),

and 4 equations expressed by AR(1) process, (4.A.45) - (4.A.48).

$$\hat{\eta}_{1,t}^\phi + \hat{b}_{c,t}^\phi = 0 \quad (4.A.19)$$

$$-\frac{\bar{p}_k \bar{\eta}_1^\phi}{\alpha} (\hat{p}_{k,t} + \hat{\eta}_{1,t}^\phi) + \beta(1 - \delta) \frac{\bar{p}_k \bar{\eta}_1^\phi}{\alpha} E_t(\hat{p}_{k,t+1} + \hat{\eta}_{1,t+1}^\phi) + \frac{\bar{y}_c^\phi \bar{\eta}_2^\phi}{\bar{k}^\phi} (\hat{y}_{c,t}^\phi + \hat{\eta}_{2,t}^\phi - \hat{k}_t^\phi) = 0 \quad (4.A.20)$$

$$-\hat{\eta}_{1,t}^\phi + \hat{\eta}_{2,t}^\phi + \hat{R}_{s,t} = 0 \quad (4.A.21)$$

$$E_t(\hat{R}_{t+1} + \hat{\eta}_{1,t+1}^\phi - \hat{\Pi}_{t+1}) - \hat{\eta}_{1,t}^\phi + \hat{R}_{l,t} = 0 \quad (4.A.22)$$

$$\hat{\eta}_{1,t}^\theta + \hat{b}_{c,t}^\theta = 0 \quad (4.A.23)$$

$$-\frac{\bar{p}_k \bar{\eta}_1^\theta}{\alpha} (\hat{p}_{k,t} + \hat{\eta}_{1,t}^\theta) + \gamma(1 - \delta) \frac{\bar{p}_k \bar{\eta}_1^\theta}{\alpha} E_t(\hat{p}_{k,t+1} + \hat{\eta}_{1,t+1}^\theta) + \frac{\bar{y}_c^\theta \bar{\eta}_2^\theta}{\bar{k}^\theta} (\hat{y}_{c,t}^\theta + \hat{\eta}_{2,t}^\theta - \hat{k}_t^\theta) = 0 \quad (4.A.24)$$

$$-\hat{\eta}_{1,t}^\theta + \hat{\eta}_{2,t}^\theta + \hat{y}_{c,t}^\theta - \hat{p}_{k,t} - \hat{k}_t^\theta + \hat{R}_{s,t} = 0 \quad (4.A.25)$$

$$-\hat{\eta}_{1,t}^\theta + \hat{\eta}_{2,t}^\theta + \hat{R}_{s,t} = 0 \quad (4.A.26)$$

$$E_t(\hat{v}_{t+1} + \hat{\eta}_{1,t+1}^\theta - \hat{\Pi}_{t+1}) - \hat{\eta}_{1,t}^\theta + \hat{R}_{l,t} = 0 \quad (4.A.27)$$

$$\bar{y}_c^\phi \hat{y}_{c,t}^\phi + \bar{y}_c^\theta \hat{y}_{c,t}^\theta - \bar{b}_c^\phi \hat{b}_{c,t}^\phi - \bar{b}_c^\theta \hat{b}_{c,t}^\theta = 0 \quad (4.A.28)$$

$$\bar{p}_k(\bar{y}_k + \bar{q}_k) \hat{p}_{k,t} + \bar{p}_k \bar{y}_k \hat{y}_{k,t} + \bar{p}_k \bar{q}_k \hat{q}_{k,t} - \bar{b}_k^\phi \hat{b}_{k,t}^\phi - \bar{b}_k^\theta \hat{b}_{k,t}^\theta = 0 \quad (4.A.29)$$

$$\hat{\mu}_{l,t} - \hat{m}_{l,t} - \hat{R}_{l,t} = 0 \quad (4.A.30)$$

$$\bar{R}_s \bar{M} \hat{R}_{s,t} + \bar{R}_s \bar{M} \hat{M}_t - \bar{\mu}_s^\phi \hat{\mu}_{s,t}^\phi - \bar{\mu}_s^\theta \hat{\mu}_{s,t}^\theta = 0 \quad (4.A.31)$$

$$-\bar{b}_c^\phi \hat{b}_{c,t}^\phi - \bar{b}_k^\phi \hat{b}_{k,t}^\phi - \bar{m}_l \hat{m}_{l,t} + \frac{\bar{\mu}_s^\phi}{\bar{R}_s} (\hat{\mu}_{s,t}^\phi - \hat{R}_{s,t}) + \bar{R} \frac{\bar{m}_l \bar{R}_l}{\bar{\Pi}} (\hat{R}_t + \hat{m}_{l,t-1} + \hat{R}_{l,t-1} - \hat{\Pi}_t) = 0 \quad (4.A.32)$$

$$\bar{y}_c^\phi \hat{y}_{c,t}^\phi + \bar{p}_k(\bar{y}_k + \bar{q}_k) \hat{p}_{k,t} + \bar{p}_k \bar{y}_k \hat{y}_{k,t} + \bar{p}_k \bar{q}_k \hat{q}_{k,t} + \omega \bar{T} \hat{T}_t - \bar{\mu}_s^\phi \hat{\mu}_{s,t}^\phi = 0 \quad (4.A.33)$$

$$-\bar{b}_c^\theta \hat{b}_{c,t}^\theta - \bar{b}_k^\theta \hat{b}_{k,t}^\theta - \bar{v} \frac{\bar{\mu}_l}{\bar{\Pi}} (\hat{v}_t + \hat{\mu}_{l,t-1} - \hat{\Pi}_t) + \frac{\bar{\mu}_s^\theta}{\bar{R}_s} (\hat{\mu}_{s,t}^\theta - \hat{R}_{s,t}) + \frac{\bar{\mu}_l}{\bar{R}_l} (\hat{\mu}_{l,t} - \hat{R}_{l,t}) = 0 \quad (4.A.34)$$

$$\bar{y}_c^\theta \hat{y}_{c,t}^\theta + (1 - \omega) \bar{T} \hat{T}_t - \bar{\mu}_s^\theta \hat{\mu}_{s,t}^\theta = 0 \quad (4.A.35)$$

$$-\bar{k}^\phi \hat{k}_t^\phi + (1 - \delta) \bar{k}^\phi \hat{k}_{t-1}^\phi + \frac{\bar{b}_k^\phi}{\bar{p}_k} (\hat{b}_{k,t}^\phi - \hat{p}_{k,t}) = 0 \quad (4.A.36)$$

$$-\bar{k}^\theta \hat{k}_t^\theta + (1 - \delta) \bar{k}^\theta \hat{k}_{t-1}^\theta + \frac{\bar{b}_k^\theta}{\bar{p}_k} (\hat{b}_{k,t}^\theta - \hat{p}_{k,t}) - \bar{q}_k \hat{q}_{k,t} = 0 \quad (4.A.37)$$

$$-\hat{y}_{c,t}^\phi + \hat{z}_t + \alpha \hat{k}_t^\phi = 0 \quad (4.A.38)$$

$$-\hat{y}_{c,t}^\theta + \hat{z}_t + \alpha \hat{k}_t^\theta = 0 \quad (4.A.39)$$

$$\hat{R}_t - \hat{v}_t = 0 \quad (4.A.40)$$

$$\hat{R}_{s,t} - \frac{\bar{v}}{1 - \bar{v}} \hat{v}_t + \hat{\mu}_{l,t-1} - \hat{\Pi}_t - \hat{p}_{k,t} - \hat{q}_{k,t} = 0 \quad (4.A.41)$$

$$\hat{v}_t + \hat{\mu}_{l,t-1} - \hat{\Pi}_t - \hat{p}_{k,t} - \hat{c}_{k,t-1} = 0 \quad (4.A.42)$$

$$\hat{M}_t - \hat{M}_{t-1} + \hat{\Pi}_t - \hat{e}_{M,t} = 0 \quad (4.A.43)$$

$$\hat{T}_t - \frac{\bar{R}_s}{\bar{R}_s - 1} \hat{R}_{s,t} - \hat{M}_{t-1} + \hat{\Pi}_t - \hat{e}_{M,t} = 0 \quad (4.A.44)$$

$$-\hat{y}_{k,t} + \rho_k \hat{y}_{k,t-1} + \varepsilon_{k,t} = 0 \quad (4.A.45)$$

$$-\hat{z}_t^\phi + \rho_\phi \hat{z}_{t-1}^\phi + \varepsilon_{z,t}^\phi = 0 \quad (4.A.46)$$

$$-\hat{z}_t^\theta + \rho_\theta \hat{z}_{t-1}^\theta + \varepsilon_{z,t}^\theta = 0 \quad (4.A.47)$$

$$-\hat{e}_{M,t} + \rho_M \hat{e}_{M,t-1} + \varepsilon_{M,t} = 0 \quad (4.A.48)$$

where I redefine $\Pi_t = 1 + \pi_t$, $R_{s,t} = 1 + r_{s,t}$ and $R_{l,t} = 1 + r_{l,t}$. The log-linearisation form of each equation is defined by $\hat{\Pi}_t = \frac{\bar{\pi}}{1 + \bar{\pi}} \hat{\pi}_t$, $\hat{R}_{s,t} = \frac{\bar{r}_s}{1 + \bar{r}_s} \hat{r}_{s,t}$ and $\hat{R}_{l,t} = \frac{\bar{r}_l}{1 + \bar{r}_l} \hat{r}_{l,t}$ respectively; $\hat{x}_t$ represents the log-deviation of x_t from the steady state $\bar{x}$.

Appendix D: Figures

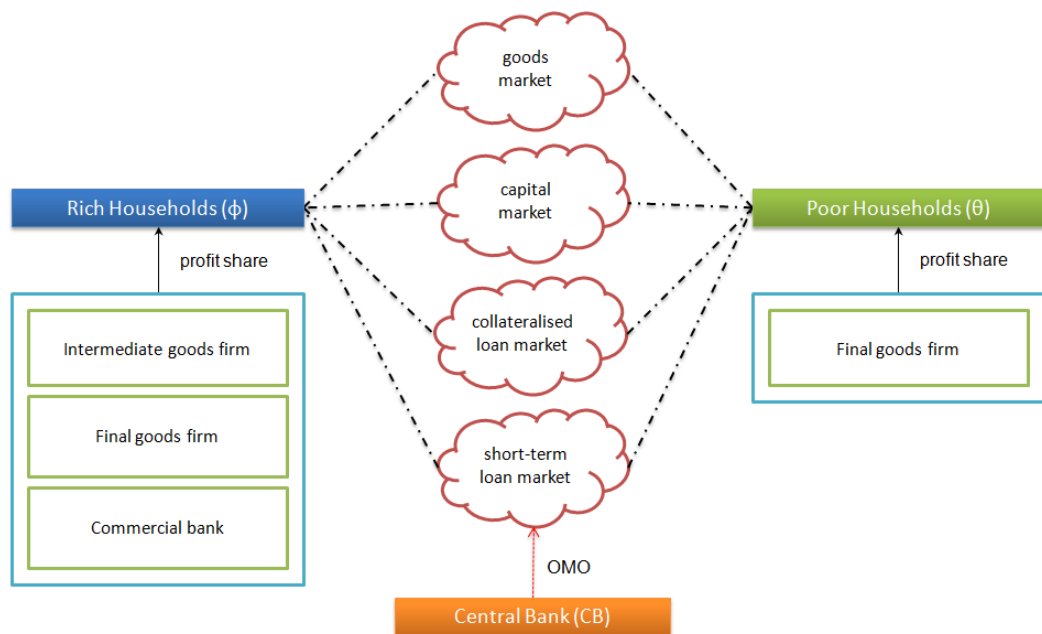


Figure 4.1: Nominal flows in the benchmark economy

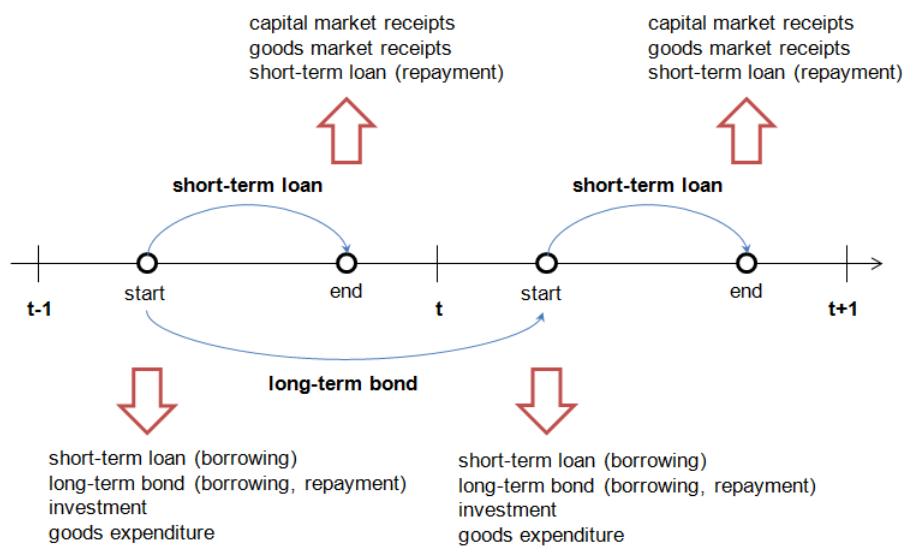


Figure 4.2: Time structure of the model

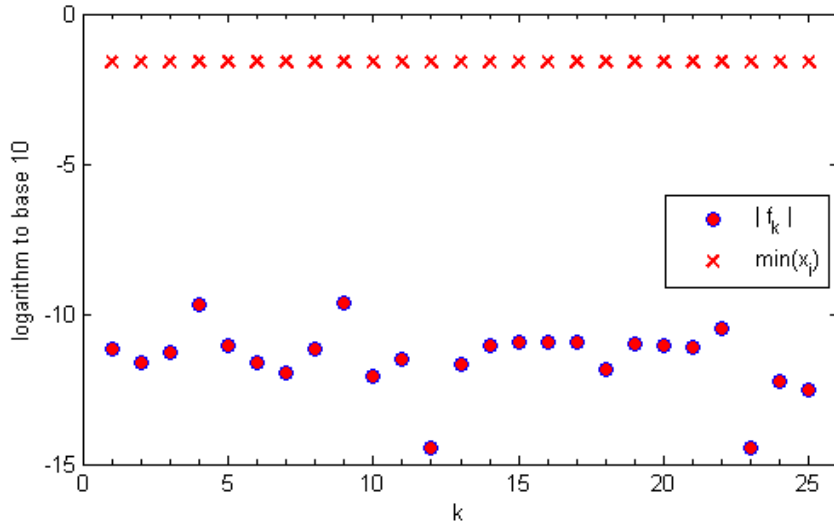


Figure 4.3: Calculation errors of 25 simultaneous equations

x_i represents the endogeneous variable in steady state for $i = 1, 2, \dots, 25$. $f_k = 0$ is the k^{th} simultaneous equation of steady state for $k = 1, 2, \dots, 25$. As $\forall |f_k| \leq 10^{-9}$ and $\min(x_i) \simeq 10^{-2}$, the steady state in this model economy is reasonably stable.

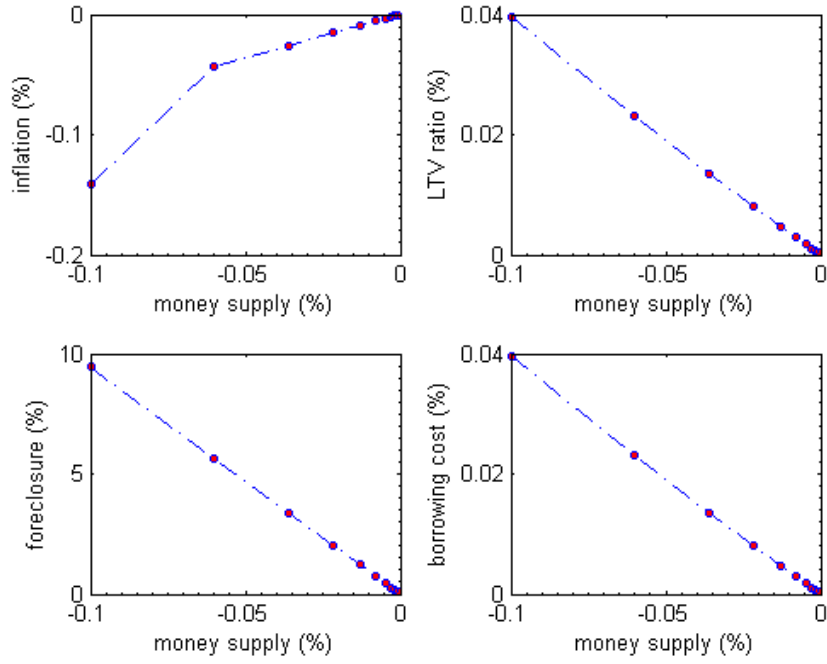


Figure 4.4: Transimission channel of monetary contractions

One-time contractionary monetary shock is considered such as $\varepsilon_{M,1} = -0.001$ and $\varepsilon_{M,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of money supply is set by $\rho_M = 0.60$ and it is equivalent to the monetary contractions for 10 years.

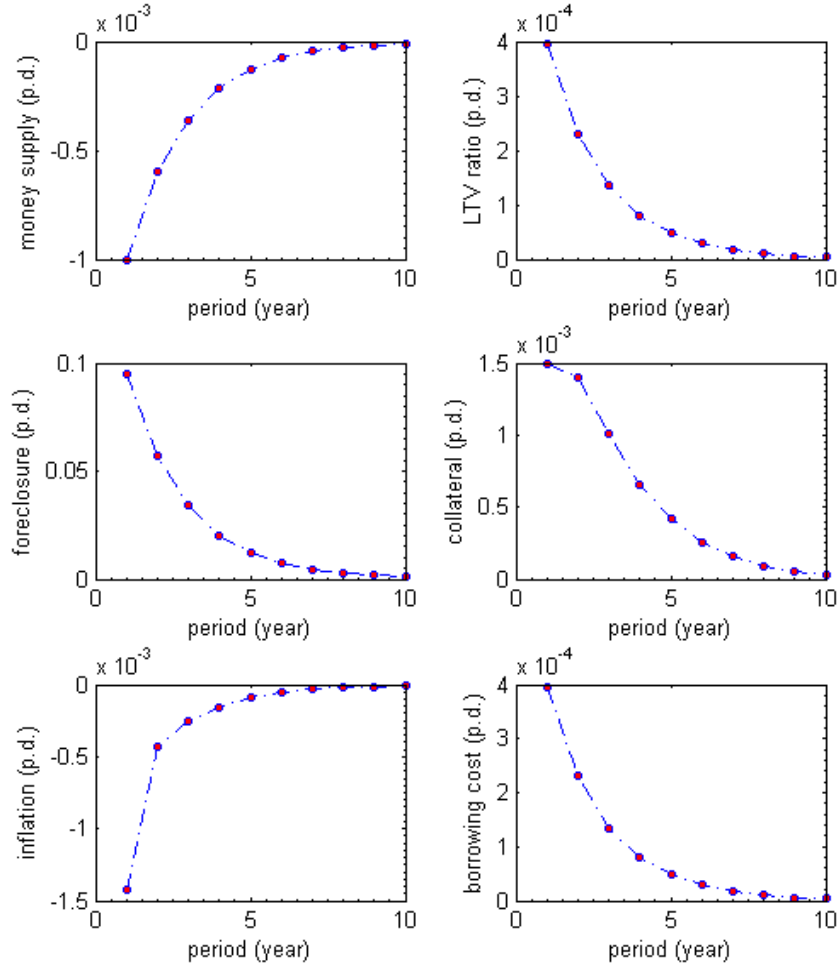


Figure 4.5: Debt-deflation channel of monetary contractions

One-time contractionary monetary shock is considered such as $\varepsilon_{M,1} = -0.001$ and $\varepsilon_{M,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of money supply is set by $\rho_M = 0.60$ and it is equivalent to the monetary contractions for 10 years.

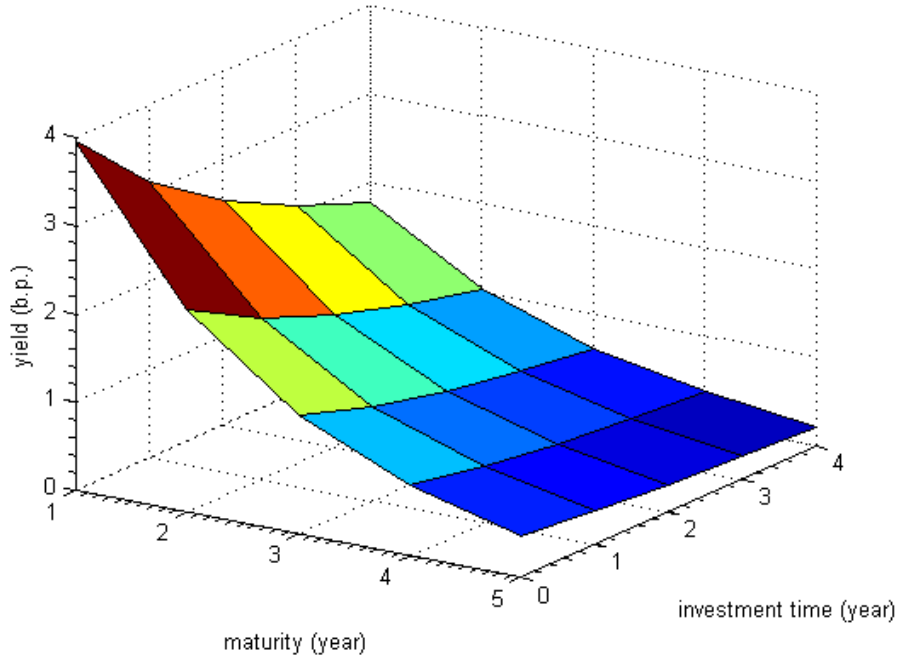


Figure 4.6: Yield curve and monetary contractions

One-time contractionary monetary shock is considered such as $\varepsilon_{M,1} = -0.001$ and $\varepsilon_{M,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of money supply is set by $\rho_M = 0.60$ and it is equivalent to the monetary contractions for 10 years.

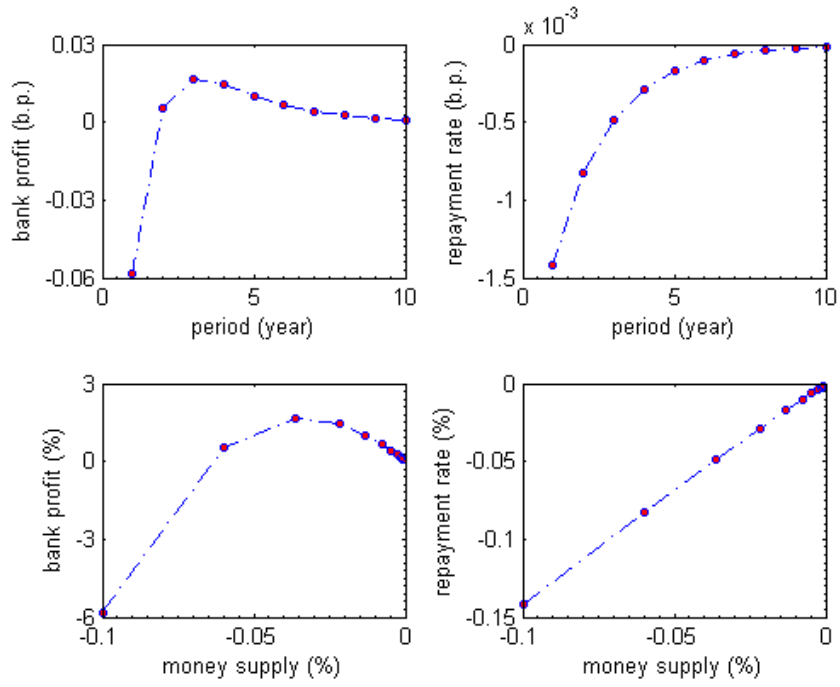


Figure 4.7: Financial stability measures during the monetary contractions

One-time contractionary monetary shock is considered such as $\varepsilon_{M,1} = -0.001$ and $\varepsilon_{M,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of money supply is set by $\rho_M = 0.60$ and it is equivalent to the monetary contractions for 10 years.

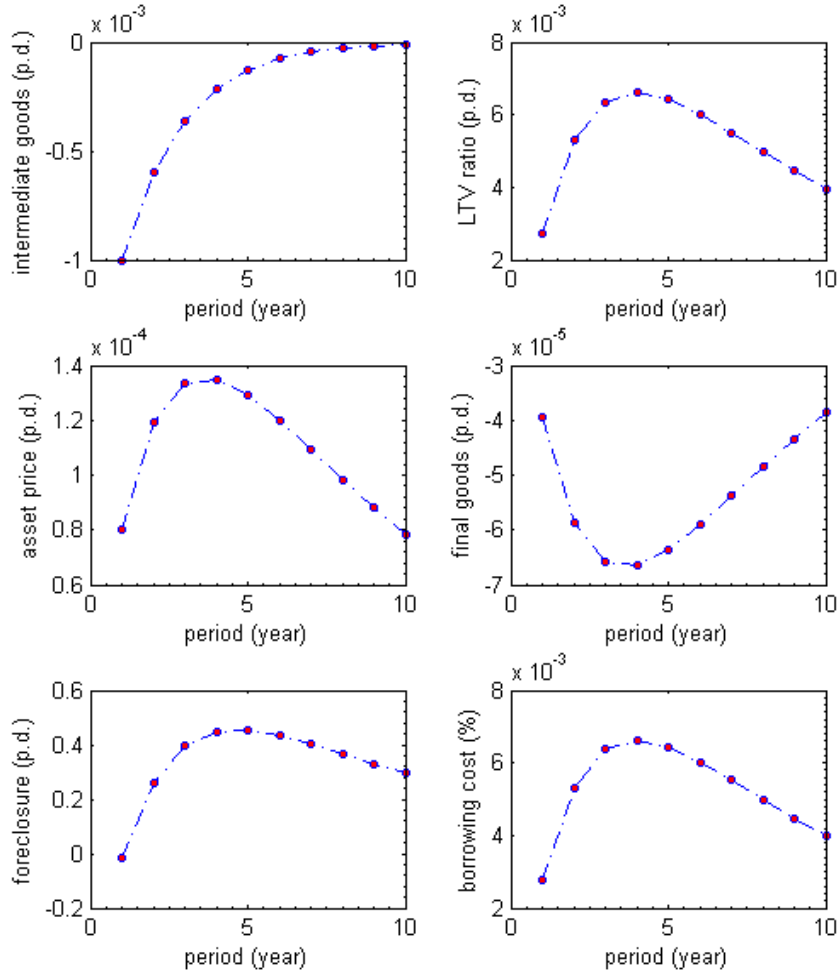


Figure 4.8: Debt-deflation channel of negative capital endowments

One-time negative capital endowment shock is considered such as $\varepsilon_{k,1} = -0.001$ and $\varepsilon_{k,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of capital endowment is set by $\rho_k = 0.60$ and it is equivalent to the negative capital endowments for 10 years.

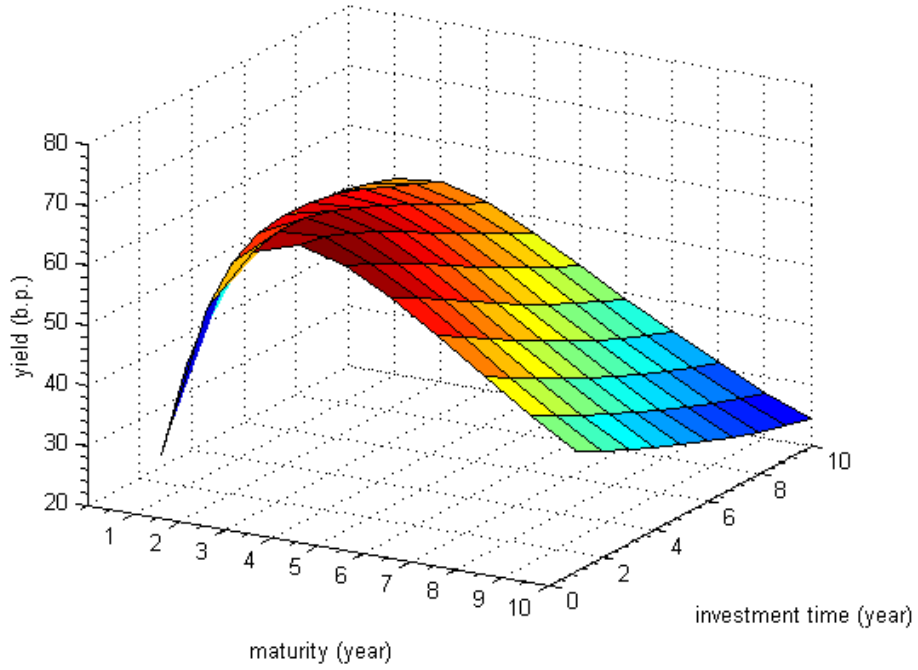


Figure 4.9: Yield curve and negative capital endowments

One-time negative capital endowment shock is considered such as $\varepsilon_{k,1} = -0.001$ and $\varepsilon_{k,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of capital endowment is set by $\rho_k = 0.60$ and it is equivalent to the negative capital endowments for 10 years.

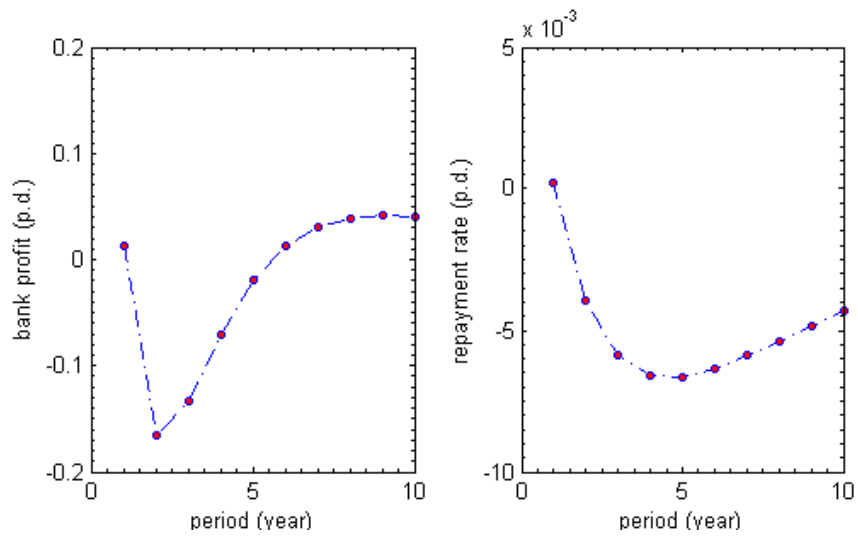


Figure 4.10: Financial stability measures during negative capital endowments

One-time negative capital endowment shock is considered such as $\varepsilon_{k,1} = -0.001$ and $\varepsilon_{k,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of capital endowment is set by $\rho_k = 0.60$ and it is equivalent to the negative capital endowments for 10 years.

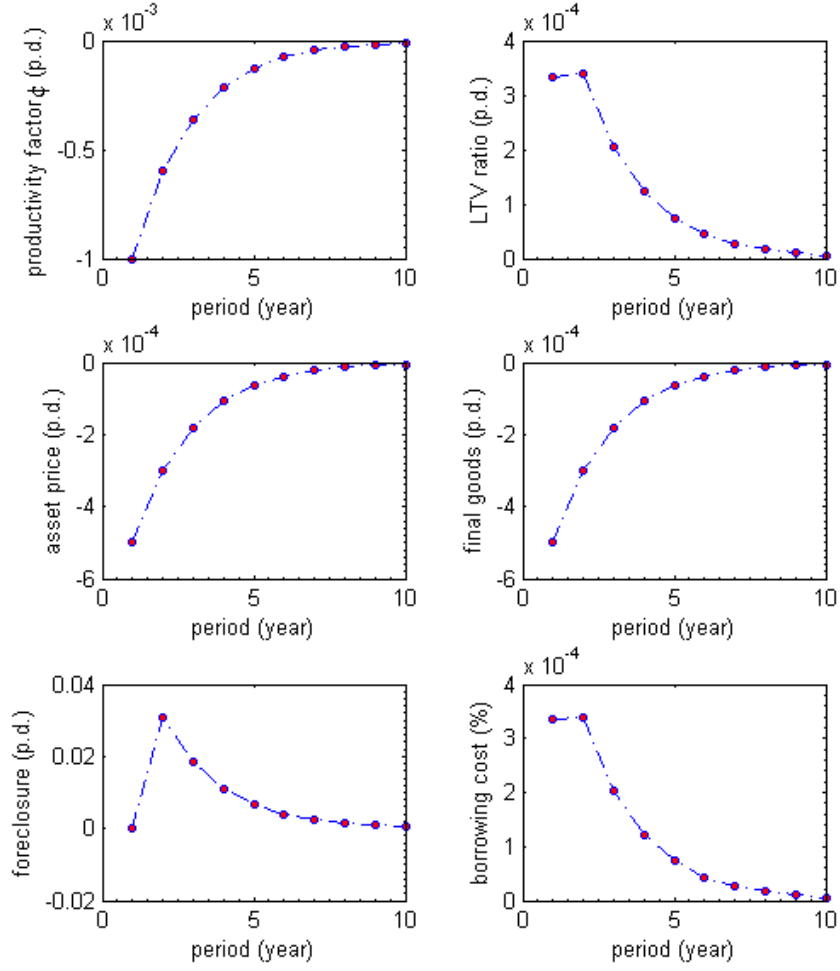


Figure 4.11: Debt-deflation channel of unbalanced productivities

One-time unbalanced productivity shock is considered such as $\varepsilon_{z,1}^{\phi} = -0.001$ and $\varepsilon_{z,t}^{\phi} = 0$ for $\forall t \geq 2$. AR(1) coefficient of productivity is set by $\rho_z^{\phi} = 0.60$ and it is equivalent to the unbalanced (negative) productivities for 10 years.

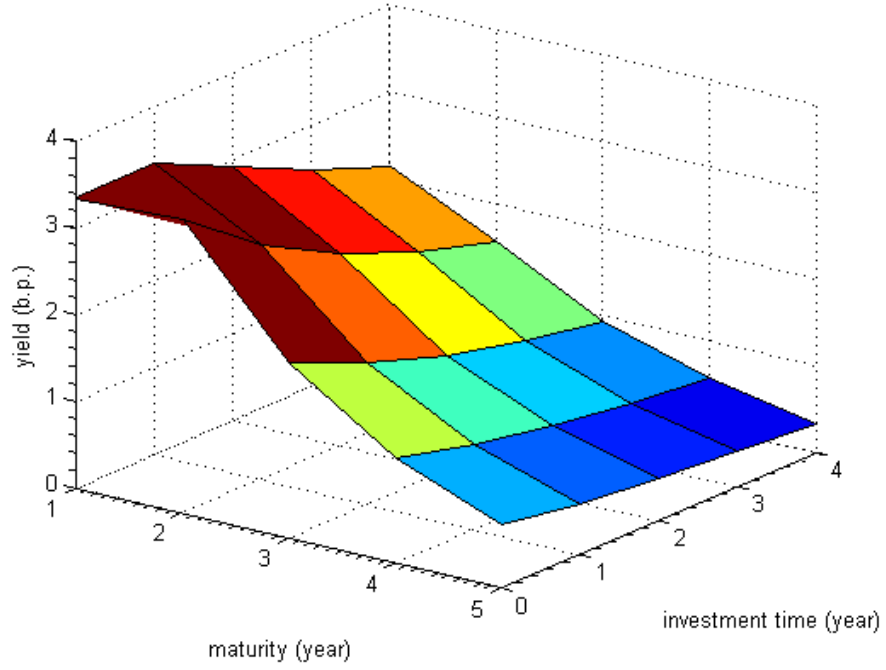


Figure 4.12: Yield curve and unbalanced productivities

One-time unbalanced productivity shock is considered such as $\varepsilon_{z,1}^\phi = -0.001$ and $\varepsilon_{z,t}^\phi = 0$ for $\forall t \geq 2$. AR(1) coefficient of productivity factor is set by $\rho_\phi = 0.60$ and it is equivalent to the unbalanced (negative) productivities for 10 years.

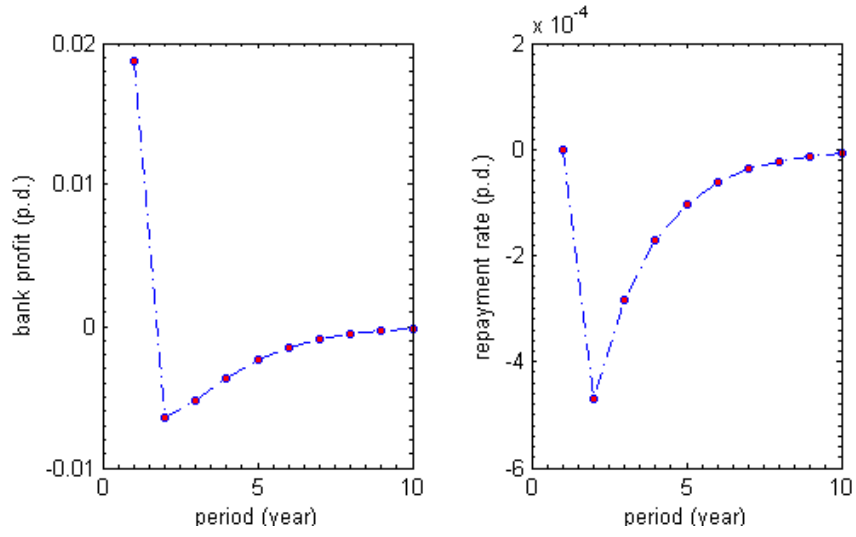


Figure 4.13: Financial stability measures during unbalanced productivities

One-time unbalanced productivity shock is considered such as $\varepsilon_{z,1}^{\phi} = -0.001$ and $\varepsilon_{z,t}^{\phi} = 0$ for $\forall t \geq 2$. AR(1) coefficient of productivity factor is set by $\rho_{\phi} = 0.60$ and it is equivalent to the unbalanced (negative) productivities for 10 years.

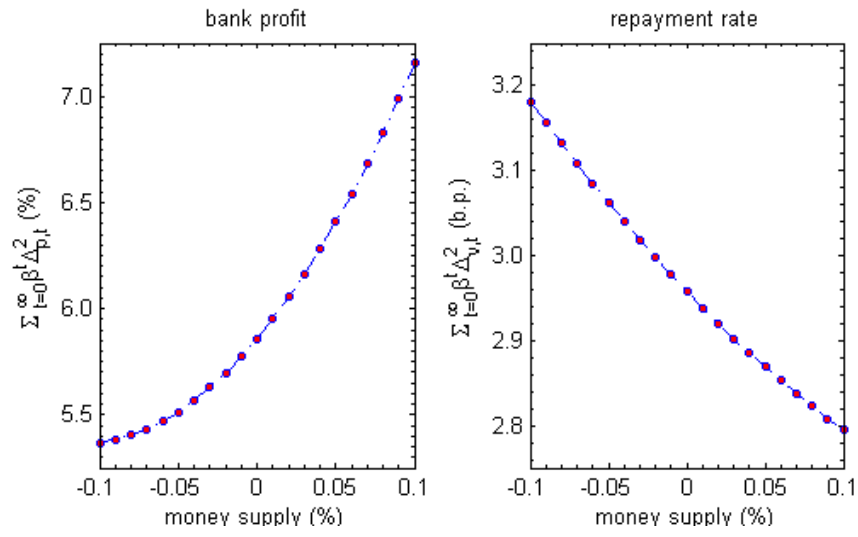


Figure 4.14: Monetary policy analysis during negative capital endowments

One-time negative capital endowment shock is considered such as $\varepsilon_{k,1} = -0.001$ and $\varepsilon_{k,t} = 0$ for $\forall t \geq 2$. AR(1) coefficient of capital endowment is set by $\rho_k = 0.60$ and it is equivalent to the negative capital endowments for 10 years.

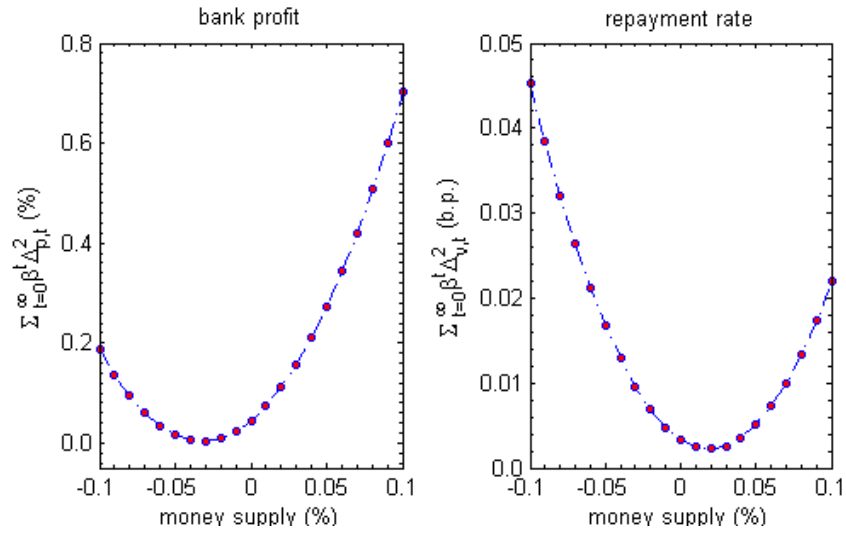


Figure 4.15: Monetary policy analysis during unbalanced productivities

One-time unbalanced productivity shock is considered such as $\varepsilon_{z,1}^{\phi} = -0.001$ and $\varepsilon_{z,t}^{\phi} = 0$ for $\forall t \geq 2$. AR(1) coefficient of productivity factor is set by $\rho_{\phi} = 0.60$ and it is equivalent to the unbalanced (negative) productivities for 10 years.

Appendix E: Tables

Table 4.1: Implied parameters

parameter	description	value
α	output elasticity of working capital	0.3300
β	discount factor for rich household	0.9750
γ	discount factor for poor household	0.9750
δ	depreciation rate of physical capital	0.1200
ω	fraction of money endowment for rich household	0.5000

Table 4.2: Endogenous variables in steady state

parameter	value	parameter	value
$\bar{b}_c^\phi$	1.0141	$\bar{k}^\phi$	0.8630
$\bar{b}_c^\theta$	0.8942	$\bar{k}^\theta$	0.8630
$\bar{b}_k^\phi$	0.0378	$\bar{c}_k$	1.5551
$\bar{b}_k^\theta$	0.0466	$\bar{v}$	0.9850
$\bar{\mu}_s^\phi$	1.0585	$\bar{R}$	0.9850
$\bar{\mu}_s^\theta$	0.9741	$\bar{T}$	0.0399
$\bar{\mu}_l$	0.5933	$\bar{\eta}_1^\phi$	0.9861
$\bar{y}_c^\phi$	0.9541	$\bar{\eta}_2^\phi$	0.9668
$\bar{y}_c^\theta$	0.9541	$\bar{\eta}_1^\theta$	1.1183
$\bar{q}_k$	0.0242	$\bar{\eta}_2^\theta$	1.0964
$\bar{m}_l$	0.5532	$\bar{M}$	1.9927
$\bar{\pi}$	0.0300	$\bar{y}_k$	0.2071
$\bar{p}_k$	0.3648	$\bar{z}^\phi$	1.0017
$\bar{r}_s$	0.0200	$\bar{z}^\theta$	1.0017
$\bar{r}_l$	0.0725	$\bar{e}_M$	1.0300