ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

A NEW APPROACH TO THE ANALYSIS OF BUSINESS CYCLE TRANSITIONS IN A MODEL OF OUTPUT AND EMPLOYMENT

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Number 59

January 2001

Manor Road Building, Oxford OX1 3UQ

A New Approach to the Analysis of Business Cycle Transitions in a Model of Output and Employment

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September 16, 2000

Abstract

This paper proposes a new framework for the impulse-response analysis of business cycle transitions. A cointegrated vector autoregressive Markov-switching model is found to be a congruent representation of post-war US employment and output data. In this model some parameters change according to the phase of the business cycle which effects employment and output simultaneously. The long run dynamics are characterized by a cointegrating vector including employment, output and a trend as a proxy for technological progress and capital accumulation. Short-run and long-run dynamics are jointly estimated in a Markov-switching vector-equilibrium-correction model with three regimes representing recession, growth and high growth. For the analysis of the dynamics of output and employment, a new set of impulse-response exercises is considered.

Keywords: Business Cycles; Impulse-Response Analysis; Cointegration; Regime Shifts, Markov Switching; Labour Hoarding.

JEL classification: E32, E37, C32, E24

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[*] Financial support from the UK Economic and Social Research Council under grant L138251009 is gratefully acknowledged by the first author. We are grateful to Mike Artis, Giuseppe Bertola, Jörg Breitung, René Garcia, David Hendry, Søren Johansen, Massimiliano Marcellino, Mike Clements, Grayham E. Mizon, Alberto Musso, Thomas J. Sargent, Rolf Tschernig, two anonymous referees and an associated editor for useful comments and discussions. Helpful comments were received from conference participants at the NASM98, Montreal, the University of Warwick, the SED Meeting, Philadelphia, the ESEM98, Berlin, the Annual Meeting of EEA 98, Berlin, the MMF98, London, the Annual Meeting of the DStG, Lübeck, and seminar audiences at the Humboldt-University Berlin, the EUI, Florence, the University of Bielefeld, and the University of Dortmund.

1 Introduction:

Since the seminal paper by Sargent (1978) on adjustment costs for labour, there has been a wide literature on the dynamic adjustment of employment and its relationship with the business cycle (see the overview in Nickell, 1986). Within the framework setup by Sargent (quadratic adjustment costs for labour) part of the literature has tried to explain the asymmetric behaviour of employment throughout the cycle. Jaramillo, Schiantarelli and Sembenelli (1992) extend Sargent's framework in order to allow differences in firing and hiring cost according to the state of the business cycle. Burgess (1994) assumes symmetric adjustment costs, but the adjustment cost parameter is allowed to change along the business cycle. More specifically, this parameter is a function of hiring costs which in turn depend in the tightness of the market (the ratio between vacancies and unemployment). His estimation delivers the results that: (i) employment falls much faster than it rises following shocks of the same size but different sign, (ii) the effect of a given shocks to the long run level of employment is markedly different at different levels of employment and (iii) an asymmetric cycle results with the downswings in employment being sharper and deeper than the upswings. Using a different approach Huizinga and Schiantarelli (1992) justify the existence of asymmetries in the dynamics of employment along the cycle. They analyze an insider outsider model where the endogeneity of the reservation wage leads to asymmetric adjustment of employment to the steady state. In periods of mild recessions employment decreases gradually and the dynamics of employment is driven by quits. When facing severe recessions the dynamics of employment is first characterized by a sharp drop in employment made up of layoffs followed by a gradual decrease due to quits. On the contrary upswings, are characterized by a gradual increase in employment. Bertola and Bentolila (1990) present a model of linear adjustment costs where the size of the firing costs can produce asymmetries in the labour demand dynamics. They calibrate their model for four European countries analyzing the effects on employment of a change in regime. The two regime considered are 1961-1973 and 1975-1986, where the second regime is characterized by low growth and large firing costs relative to the first regime.

Most papers reviewed previously have tried to test the particular dynamics of the models by directly estimating the solution-equation of their maximization equations or implementing a calibration exercise. Our aim is different in this respect. We try to get a statistical model that offers a congruent representation of the data. We then analyze the dynamics and the impulse responses of the estimated model, and draw conclusions. More particularly we model employment and output in the USA for the period $1962q3$ to $1997q1$. When the economy is modeled considering some unobserved state variables, the Markov-switching time series model in the tradition of Hamilton (1989) has proven to be a very flexible tool. We use a cointegrated vector autoregressive Markov-switching model, where some parameters are allowed to change according to the states, which are governed by a discrete state Markov process. The states correspond to different regimes or phases of the business and employment cycle. The state dependent component is simultaneously estimated in a Markov-switching vector-equilibrium-correction model where short-run and long-run dynamics are jointly modelled. Employment and output are found to have a common cyclical component and the long run dynamics are characterized by a cointegrating vector including employment, output and a trend as a proxy for technological progress. A three regime model with changing intercept and variance turns out to be a good description of the data. The regimes correspond to recession, growth and high growth

We propose a set of exercises in order to analyze and learn about the dynamics of the variables modelled. In the simplest case of two regimes, say downswings and upswings of the business cycle, an interesting dynamic analysis would consist of studying the changes in the variables modelled in a switch from recession to boom. Furthermore one could investigate the response of the variables to a change from the ergodic probabilities to a sure state (say, boom or recession). In our case, with a error correcting term capturing the long run relationship, three regimes, and changing intercept and variance , the dynamics are even richer. We consider the response of output and employment to changes in regimes. The analysis extends to the short run as well as to the long run dynamics. We further analyze the response of employment and output to a shock in each of these variables. We do not orthogonalize the innovations. However, if one were interested in orthogonalized innovations then the changing variance between regimes would lead to regime-dependent response functions.

The paper proceeds as follows. Section 2 gives a statistical characterization of the classical cycle in output and employment with univariate Markov-switching models. The results suggest the existence of common component driving output and employment. Section 3 studies the cointegration properties of the system of variables and presents the results from a Markov-switching vector equilibrium correction model (MS-VECM) consisting of three phases of the business cycle and dynamic adjustments of employment to its equilibrium level. Section 4 discusses the impulse-response analysis of the MS-VECM. Finally, Section 5 concludes. The statistical foundations of our impulse-response analysis are discussed in the mathematical appendix.

2 The Co-movement of Employment and Output

Recently the potential nonlinearity of the business cycle has become a fast growing area of interest. Particularly with the Hamilton (1989) model of the US business cycle, Markov-switching vector autoregressions (MS-VARs) have been used as a fairly general approach to modelling time series subject to regime shifts and the inter-relationships between such series (see Krolzig, 1997 for an overview). This paper demonstrates the feasibility of the MS-VAR modelling approach for investigating the joint dynamics of output and employment during the business cycle.

The Hamilton (1989) model of the US business cycle fostered a great deal of interest in the MS–AR model as an empirical vehicle for characterizing macroeconomic fluctuations, and there have been a number of subsequent extensions and refinements. Contractions and expansions are modelled as switching regimes of the stochastic process generating the growth rate of real GNP Δy_t:

$$\Delta y_t - \mu(s_t) = \alpha_1 \left(\Delta y_{t-1} - \mu(s_{t-1})\right) + \ldots + \alpha_4 \left(\Delta y_{t-4} - \mu(s_{t-4})\right) + u_t. \qquad (1)$$

The regimes are associated with different conditional distributions of the growth rate of real GNP, where the mean μ_1 is positive in the first regime (‘expansion’) and negative in the second

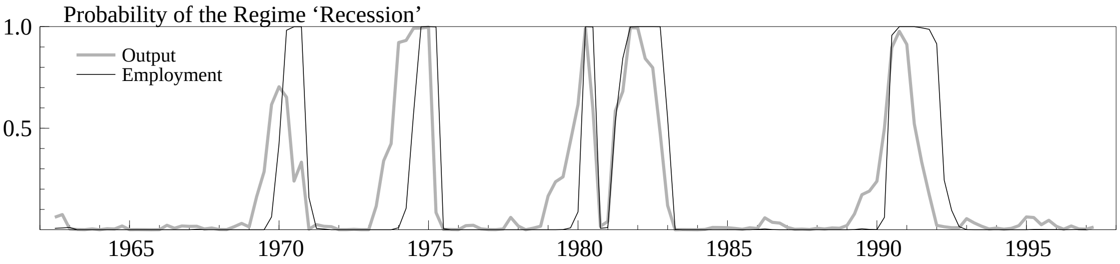

Figure 1 Output, Employment and the Business Cycle.

regime ('contraction'), $\mu_2 < 0$. The variance of the disturbance term, $u_t \sim \mathsf{NID}(0, \sigma^2)$, is assumed to be the same in both regimes.

The general idea behind this class of regime-switching models is that the parameters of a VAR depend upon a stochastic unobservable regime variable $s_t \in \{1, \dots, M\}$. The stochastic process generating the unobservable regimes is an ergodic Markov chain defined by the transition probabilities:

$$p_{ij} = \Pr(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i, j \in \{1, \dots, M\}. \tag{2}$$

By inferring the probabilities of the unobserved regimes conditional on an available information set, it is then possible to reconstruct the regimes. [1]

The original Hamilton model is a fourth-order autoregression fitted to the quarterly percentage change in US real GNP, Δy_t, from 1952 to 1984. In our case we use quarterly seasonally-adjusted data of US GNP and employment (total nonfarm payrolls: all employees) from $1962q3$ to $1997q1$. The associated regime probabilities are depicted in Figure 1 for the simplest possible case of a two-regime process: $\Delta y_t = \mu(s_t) + u_t$. The model for the growth in employment Δn_t is formulated analogously. [2] While the time series of GNP growth is much smoother than those of employment, the depicted series of the smoothed probabilities of being in the recessionary regime move together. As in Hamilton's original model, these univariate results are very close to the official NBER datings of the turning points of the US business cycle. For the estimation period, the macroeconomic fluctuations in the US were marked by the recessions dated by the NBER as $1970m1$ to $1970m11$, $1973m12$ to $1975m3$, the double-dip recession of $1980m2$ to $1980m7$ and $1981m8$ to $1982m11$, and the last recession from $1990m8$ to $1991m3$.

The contemporaneity of the regime shifts in the stochastic process of output and employment suggests an investigation of the business cycle in a simultaneous equation model of output y_t and employment n_t. The inference on dating the business cycle can be improved by considering the Markov-switching vector equilibrium model (MS-VECM) introduced in Krolzig

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[1] Maximum likelihood (ML) estimation of the model is based on a version of the Expectation-Maximization (EM) algorithm discussed in Hamilton (1990) and Krolzig (1997). All the computations reported in this paper were carried out with the MSVAR class for Ox 2.10, see Krolzig (1998) and Doornik (1999).

[2] We present the results of these certainly non-congruent model only for descriptive purposes.

(1997). This approach reflects the definition of the business cycle as the comovement of macroeconomic series. There is one unobserved state variable driven by an ergodic Markov process that is common to all series. Our conclusion of the results so far is to model employment and output simultaneously. We will show later the need to allow for a third regime to overcome the problems with the original Hamilton model as two regimes might be restrictive given the different pattern of growth over the last decades.

A Markov-switching vector equilibrium correction model is a vector equilibrium model with shifts in some of the parameters:

$$\Delta x_t = \nu(s_t) + \alpha(s_t)\left[\beta' x_{t-1} - \gamma t\right] + \sum_{k=1}^{p-1} \Gamma_i(s_t)\Delta x_{t-k} + u_t, \tag{3}$$

where the innovations u_t are conditionally Gaussian, $u_t|s_t \sim \mathsf{NID}(\mathbf{0}, \Sigma(s_t))$. As in (2), the unobservable regime variable s_t is governed by a Markov chain with a finite number of states defined by the transition probabilities p_{ij}. In the tradition of Hamilton (1989), researcher focus on shifts to the intercept $\nu(s_t)$ in studies of the business cycle.

The MS-VECM model is closely related to the notion of multiple equilibria in dynamic economic theory. Henceforth, each regime is characterized by an attractor of the system defined by the drift $\delta(s_t)$ and the long-run equilibrium $\mu(s_t)$:

$$\Delta x_t - \delta(s_t) = \alpha\left[\beta' x_{t-1} - \mu(s_t) - \gamma t\right] + \sum_{k=1}^{p-1} \Gamma_i\left[\Delta x_{t-k} - \delta(s_t)\right] + u_t, \tag{4}$$

In (4), both Δx_t and $\beta' x_t$ are expressed as deviations about their regime and time-dependent means $\delta(s_t)$ and $\mu(s_t)$.

Consider, for example, a bivariate model with just one lag, where the long-term relation is determined by the cointegration vector $\beta' = (1 : -1)$ and the long-run equilibrium $\mu(s_t) = \mathsf{E}\left[x_{1t} - x_{2t} - \gamma(t-1)\right]$. Each regime m is associated with a particular attractor (μ_m, δ_m^*) given by the equilibrium growth rate δ_m^* and the equilibrium mean μ_m

$$\begin{bmatrix} \Delta x_{1t} - \delta^*(s_t) \\ \Delta x_{2t} - \gamma - \delta^*(s_t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} (x_{1t-1} - x_{2t-1} - \mu(s_t) - \gamma t) + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}. \tag{5}$$

Changes in the state s_t of the business cycle are associated with shifts in the common equilibrium growth rate and in the equilibrium mean $\mu(s_t)$.

For an ergodic and irreducible Markov chain, as will be assumed in the following, regime shifts are persistent (if $p_{ij} \neq p_{ii}$ for some i, j) but not permanent (if $p_{ii} \neq 1$ for all i). MS-VECMs exhibit equilibrium as well as error correction mechanisms: in each regime disequilibria are adjusted by the *vector equilibrium correction mechanism*. Since the regimes themselves are generated by stationary irreducible Markov chain, the *errors* arising from regime shifts themselves are *corrected* towards the stationary distribution of the regimes.

The assumed properties of the Markov chain have important implications for the analysis of the long-run properties of the system. Cointegrated systems with Markovian regime shifts can be characterized as non-Gaussian cointegrated VARs of infinite order. This property of cointegrated MS-VAR processes allows one to base the cointegration analysis of such data generating processes on procedures available for infinite order VARs. This is the basic idea of Krolzig (1996) who proposed a limited information approach to cointegration analysis using a pure finite-order VAR approximation of the underlying DGP without modelling the Markov-switching of the first stage. In the second stage, conditional on the cointegration matrix, the remaining structural parameters are estimated.

3 Empirical Results

In the following we apply the Markov-switching vector equilibrium model to the US postwar data. First we investigate the cointegration properties of the system of variables. Then we present the results of a Markov-switching vector equilibrium model where the business cycle consists of three phases as well as dynamic adjustments by employment and output to its equilibrium level.

3.1 Cointegration Analysis

By following the two-stage procedure proposed in Krolzig (1996), the cointegration properties of the data are studied within a linear vector autoregressive representation using maximum likelihood techniques. The analysis is based on the existence of a finite-order VARMA representation for MS-VAR models. The first stage of the maximum likelihood procedure involves approximating the VARMA with a finite-order VAR model and applying Johansen's maximum likelihood procedure (see Johansen, 1995). In the second stage, conditional on the estimated cointegrated matrix, the remaining parameters of the vector equilibrium correction representation of the MS-VAR process are estimated with the EM algorithm.

Johansen's cointegration analysis was applied to a VAR with five lags with the trend entering the cointegration space as follows:[3]

$$x_t = \nu + \sum_{i=1}^{5} A_i x_{t-i} - \alpha\gamma t + u_t, \tag{6}$$

where $x_t = [\, y_t : n_t]'$.

The results of the cointegration tests are shown in Table 1 with the trace and maximal eigenvalue test statistics. The tests support the acceptance of one cointegration rank.

Under the hypothesis that output is weakly exogenous for the long-run equilibrium $\alpha_1 = 0$, the LR test accepts the restricted model at the marginal level of 0.0666, $\chi^2(1) = 3.3659$. Furthermore, the LR-test statistic $\chi^2(1) = 2.5374\ [0.1112]$ permits the restriction of the cointegration space to the trend-adjusted per-capita output[4]

$$y_t - n_t = \mu + 0.1089t. \tag{7}$$

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[3] In line with the suggestion of Krolzig (1996), the VARMA was approximated by a pure VAR model of order $p^* = \max\{\arg\min_p AIC, T^{1/3}\}$. The Akaike information criterion (AIC) and the Schwarz criterion (SC) supported a VAR(5).

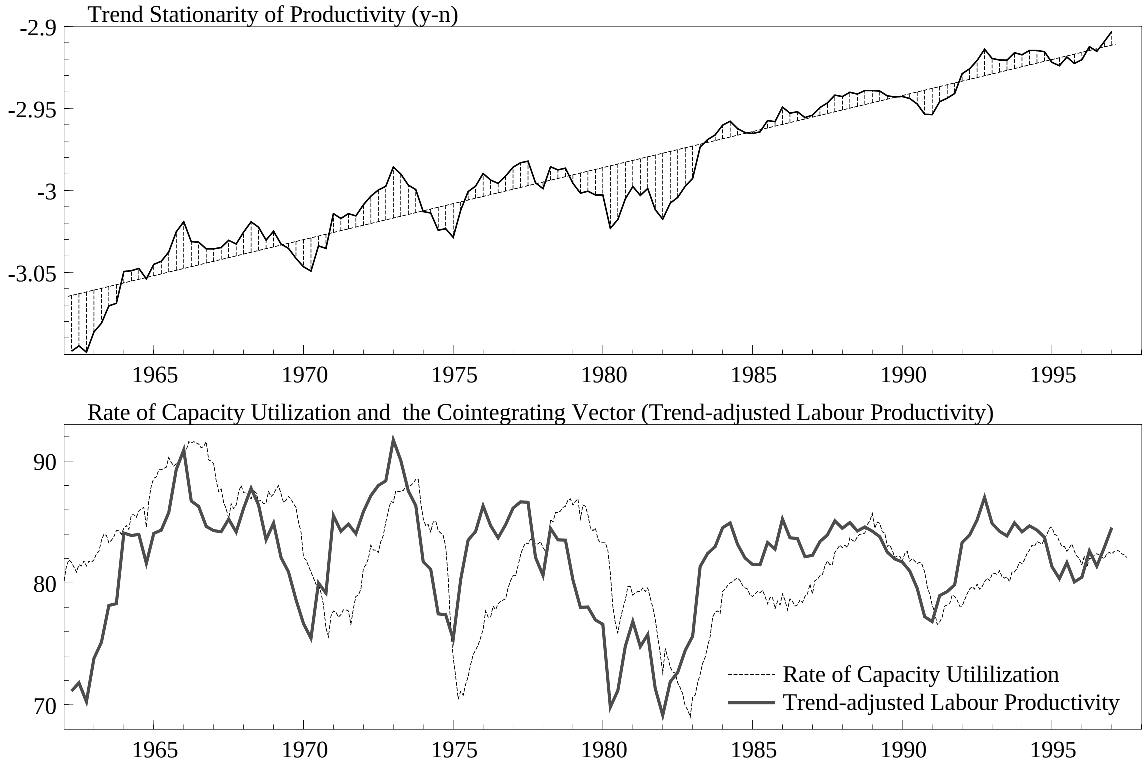
[4] The trend stationarity of $y_t - n_t$ has also been established by unit-root tests from $1962q3$ to $1997q1$ (see figure 2). Under inclusion of a constant and a trend, augmented Dickey-Fuller tests reject a unit root at 5% consistently for lags of 1 to 5.

Table 1 Johansen Cointegration Likelihood Ratio Test.

	Maximal Eigenvalue Test			Trace Test		
Ho:rank=r	-Tlog(1-μ)	T-nm	95%	-T $\sum$ log(.)	T-nm	95%
$r = 0$	21.17*	19.67*	19.0	32.68**	30.37*	25.3
$r \leq 1$	11.51	10.69	12.3	11.51	10.69	12.3

** Significant at 1% level, * Significant at 5% level.

In economic terms, the cointegrating combination $z_t = y_t - n_t - \gamma t - \mu$ corrects per-capita output for its positive trend over the post-war period. The trend reflects the long-run trend growth of labour productivity due to the accumulation of physical and human capital as well as technical progress.


Trend Stationarity of Productivity (y-n)

Rate of Capacity Utilization and the Cointegrating Vector (Trend-adjusted Labour Productivity)

Figure 2 Cointegration Results.

Note that the equilibrium correction mechanism corresponds to the labour hoarding theory (Solow, 1964) which considers employment as a stock which is partially adjusted towards the optimal labour demand of the firms. Thus the cointegrating variable $y_t - n_t$ can be interpreted as a labour-based measurement of *capacity utilization*. This interpretation is supported by figure 2 which shows that the cyclical behaviour of the cointegration combination goes in line with the published rate of capacity utilization. These results support the evidence of Sbordone (1996) for US manufacturing sectors where labor adjustment costs are found to be important for the reaction of firms to cyclical movements in macroeconomic activity; and agree with the conclusions of previous investigations of the observed procyclical behaviour of productivity (see, inter alia, Rotemberg and Summers, 1990, and Burnside, Eichenbaum and Rebelo, 1980).

For a cointegration rank $K-1$ of the K-dimensional system, there is only one stochastic trend in the system and $K-1$ linearly combinations of the data which are stationary. Thus the stochastic trend of the system is given by output. While output is weakly exogenous for the long-run equilibrium, employment adjusts towards equilibrium:

$$\begin{aligned} \Delta n_t &= \nu_1 + \underset{.0148}{0.0602} \cdot \left(y_{t-1} - n_{t-1} - \underset{.0074}{0.1089}\, t \right) + \ldots + \varepsilon_{1t} && (8) \\ \Delta y_t &= \nu_2 + \ldots + \varepsilon_{2t}. && (9) \end{aligned}$$

The VAR(p) representation has been considered so far as a finite pure VAR approximation of the VARMA representation of an MS-VAR processes. In the next section the cointegrating vectors are used within the Markov-switching vector equilibrium model.

3.2 The US Business Cycle and Net Job Creation / Destruction

In the class of MS(3)-VECM(p) models with shifts in the intercept ν and the variance Σ, $p=1$ is the outcome of AIC model selection procedures for the lag length p. By applying the two-stage procedure proposed in Krolzig (1996), the cointegration results from the last section are used on this stage of our analysis:

$$\Delta x_t = \nu(s_t) + \Gamma_1 \Delta x_{t-1} + \alpha z_{t-1} + u_t, \quad u_t \sim \mathsf{NID}(0, \Sigma(s_t)) \qquad (10)$$

where $z_{t-1} = y_{t-1} - n_{t-1} - \tilde{\gamma} t$ has been normalized such that $\mathsf{E}[z_t] = 0$.

The estimated parameters of the MS(3)-VECM(1) model (10) using data from $1962q3$ to $1997q1$ are presented in Table 2. The transition matrix is given by

$$\mathbf{P} = \begin{bmatrix} 0.8305 & 0.0513 & 0.0299 \\ 0.0350 & 0.9487 & 0.0680 \\ 0.1346 & 0.0001 & 0.9020 \end{bmatrix}.$$

where $p_{ij} = \Pr(s_t = i | s_{t-1} = j)$. The regimes are persistent with an average recessions duration of one and a half year.

The resulting regime probabilities are given in Figure 3. Regime 1 depicts very precisely the recessions of 1970, 1973/74, 1979/80 and 1990. Regime 2 represents normal growth, while regime 3 characterizes high-growth episodes after recessions.

Note that regime 3 can only be observed until 1985, which might indicate a structural change in the phase structure of the business cycle. Expansions after 1985 (regime 2) are characterized by a lower mean growth rate and reduced volatility of macroeconomic fluctuations. We will discuss the implications of these findings for the properties of recession recoveries in the next section (see figure 7).

Compared to a linear VECM(1) in table 2 with a log-likelihood of -148.86, the LR-test rejects the linearity hypothesis significantly (68.26) even by invoking the upper bound of Davies (1977), (1987). Furthermore the AIC (with 2.02 vs. 2.28) and the HQ criterion (2.26 vs. 2.38) are in favour of the non-linear VECM. Note that the adjustment coefficients have barely changed; so that the two-stage procedure seems to be justified.

Table 2 ML Estimation Results.

	MS(3)-VECM(1)		MS(3)-VAR(1)		VECM(1)	
	Δy_t	Δn_t	Δy_t	Δn_t	Δy_t	Δn_t
Regime-dependent intercepts						
ν_1	-0.2119	-0.0522	-0.1960	-0.1427		
	.2769	*.1115*	*.2617*	*.1110*		
ν_2	0.6203	0.2368	0.6519	0.2052	0.4464	0.1853
	.1143	*.0428*	*.1144*	*.0481*	*.1268*	*.0484*
ν_3	1.1914	0.4256	1.2170	0.3613		
	.2022	*.0807*	*.1903*	*.0761*		
Autoregressive coefficients						
Δy_{t-1}	-0.0019	0.0295	0.0066	0.0572	0.2072	0.1001
	.0960	*.0355*	*.0930*	*.0347*	*.1113*	*.0425*
Δn_{t-1}	0.1111	0.5357	0.0483	0.5956	0.1972	0.5468
	.1692	*.0598*	*.1622*	*.0637*	*.2075*	*.0792*
Adjustment coefficients						
α	-0.0038	0.0666			-0.0031	0.0653
	.0645	*.0225*			*.0674*	*.0257*
Regime 1: Variance						
Δy	1.0861	0.3979	1.0855	0.3893		
Δn	*.8468*	0.2033	*.8334*	0.2010		
Regime 2: Variance						
Δy	0.1826	0.0229	0.1827	0.0240	0.7307	0.2174
Δn	*.3940*	0.0184	*.3970*	0.0201	*.7797*	0.1063
Regime 2: Variance						
Δy	0.6055	0.1461	0.6009	0.1383		
Δn	*.6403*	0.0860	*.5939*	0.0902		
LogLik		-114.73		-122.67		-148.86
AIC/HQ	2.02	2.26	2.11	2.32	2.28	2.38
	Erg.Prob	Duration	Erg.Prob	Duration		
Regime 1	0.2050	5.898	0.2073	5.996		
Regime 2	0.5131	19.472	0.5003	18.223		
Regime 3	0.2819	10.205	0.2924	10.050		

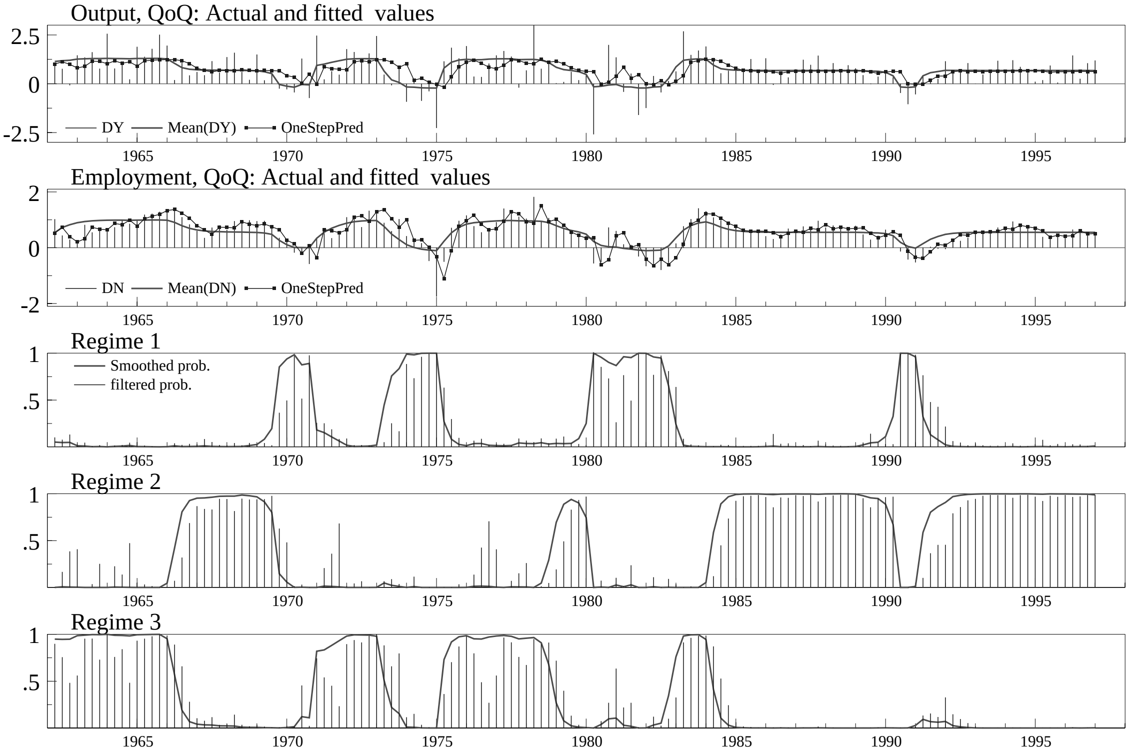


Figure 3 The MS(3)-VECM(1) Model.

Testing for the number of regimes in an MS-VAR model is a difficult enterprise.[5] In our case, economics can help. If we consider a two-regime model , we end up with an MS(2)-VECM(1) model exhibiting two roughly symmetric regimes.The regime probabilities of the two-regime model reveal for the seventies a quite high uncertainty regarding the identification of the underlying regime. Altogether the model seems to pick up more historical episodes with a different volatility of economic growth than the business cycle itself. Thus a three-regime model is required to extract information about the state of the business cycle.

The LR test statistic for the hypothesis $\Sigma_1 = \Sigma_2 = \Sigma_3$ is asymptotically $\chi^2(6)$ as the number of regimes is unaltered under the null and clearly rejected with $\chi^2(6)$= 46. It might be worth noting that the implications of a model without regime-dependent variances are broadly comparable to our model. We also checked for the significance of the vector equilibrium correction mechanism by estimating an MS-VAR in differences. The LR statistic $\chi^2(2) = 15.86$ is very much in favour of the Markov switching vector equilibrium model (see Table 2). It is worth noting that the similarity of the regime classifications supports the importance of

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[5] Conventional testing approaches are not applicable due to the presence of unidentified nuisance parameters under the null of linearity (that is, the transition probabilities) and because the scores associated with parameters of interest under the alternative may be identically zero under the null. Formal tests of the Markov-switching model against linear alternatives employing the standardized LR test designed to deliver (asymptotically) valid inference have been proposed by Hansen (1992, 1996) and Garcia (1998). The extension of Hansen's approach to our model seems to be impossible to implement computationally (see Ang and Bekaert, 1995) and is certainly beyond the scope of this paper. Furthermore it delivers only a bound on the asymptotic distribution of the standardized LR test. The test is conservative, tending to be under-sized in practice and of low power.

shifts in the common trend of the system.

We found that the statistical model is a congruent representation of the structure of the data. The errors associated with the MS(3)-VECM(1) model are plotted in Figure 4. The figures on the left report one-step *predicted errors* for the first differences of output and employment. The prediction errors $\Delta x_t - \mathsf{E}\left[\Delta x_t | X_{t-1}\right]$ based on the information set $X_{t-1} = \{x_{t-1,} x_{t-2}, \ldots, x_0\}$ show the break in the volatility of the process in $1984q4$ (as dated by the MS-VECM). The *smoothed standard errors* $\tilde{u}_{kt}/\sigma_{kt}$ on the right side are corrected for the effects of regime shifts: $\sum_{m=1}^{3} \left\{ \left(\Delta x_t - \mathsf{E}\left[\Delta x_t | s_t = m, X_{t-1}\right]\right) \mathsf{Pr}(s_t = m | X_T) \right\}$ and provide an inference of the Gaussian innovation process. In contrast, the one-step prediction errors also consist of the innovations to the regimes and the errors in reconstructing the regime. The statistical properties of the smoothed and predicted errors are visualized in Figure 5. Remarkable is the normality of the standardized (smoothed) errors and non-normality of the predicted errors as assumed in the MS-VECM. There is no strong autocorrelation left in the errors.

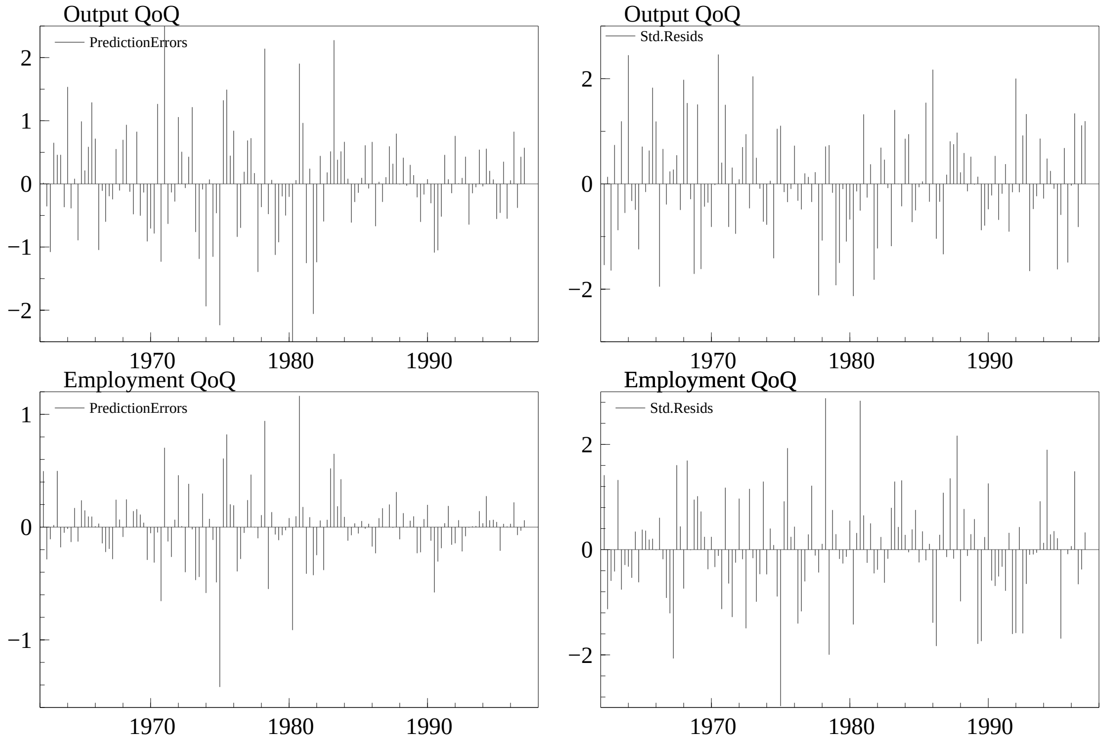

Figure 4 Smoothed and Predicted Errors in the MS(3)-VECM(1) Model.

4 Impulse-Response Analysis

There has been some recent interest in the impulse-response analysis of non-linear models. Beaudry and Koop (1993) investigated the persistence of output innovations when output is modelled in a non-linear fashion. They demonstrated that previous results by Campbell and Mankiw (1987) are biased. Their results show that the persistence of positive innovations had been underestimated whereas the persistence of negative innovations has been overestimated.

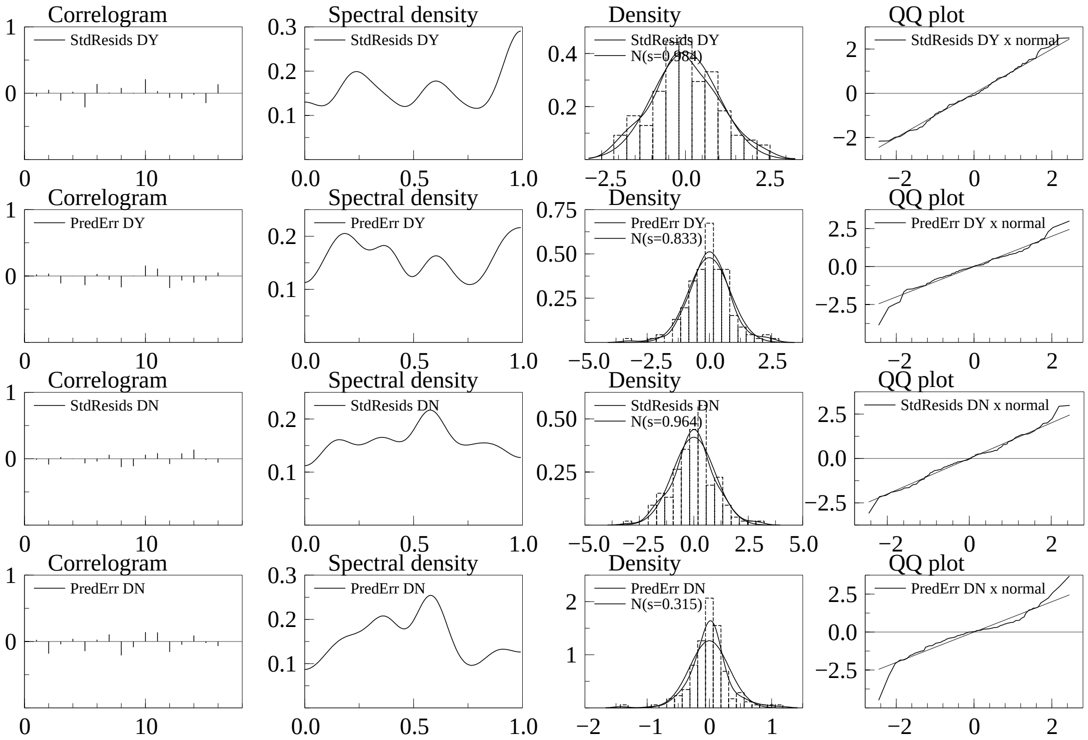

Figure 5 Statistical Properties of the Smoothed and Predicted Errors.

Koop, Pesaran and Potter (1996) offer a more general analysis of impulse responses in nonlinear models, and introduce the concept of generalized impulse response. The generalized impulse response differs from the traditional impulse response because of the conditional information set used in the dynamic analysis (that is, the type of shocks and the history of the variables).

These previous analyses have mainly focused on the response of the system to Gaussian innovations. We introduce here the dynamic analysis for the case when the system is subject to business cycle transitions. As in Koop *et al.* (1996), our proposed methodology does take into account the shock and the history of the variables: The history is represented by the given state from which we shock the system whereas the nature of the shock is given by the specific state to which we move to.

One of the advantages of this new methodology is that non-Gaussian innovations (say, changes in the phase of the cycle) might be what some economists have in mind when they refer to “cyclical shocks”; that is, investigating the dynamics of some variables in the transition from boom to bust. While the impulse responses with regards to the Gaussian innovations are based on the time invariant linear VAR operator, the impulse responses with regards to transitions of the state variables depend on the properties of the VAR as well as on the properties of the hidden Markov chain. The appendix discusses how these latter responses can be derived from the stationary state space representation of the MS-VECM. Furthermore, our impulse response analysis is free from scaling criticisms.

Our dynamic analysis will extend to:

- The response of output and employment to a one percentage innovation in each of the variables. These correspond to the graphs in figure 6. These reveal that output represents the stochastic trend of the system, and employment is partially adjusted after shocks.[6]

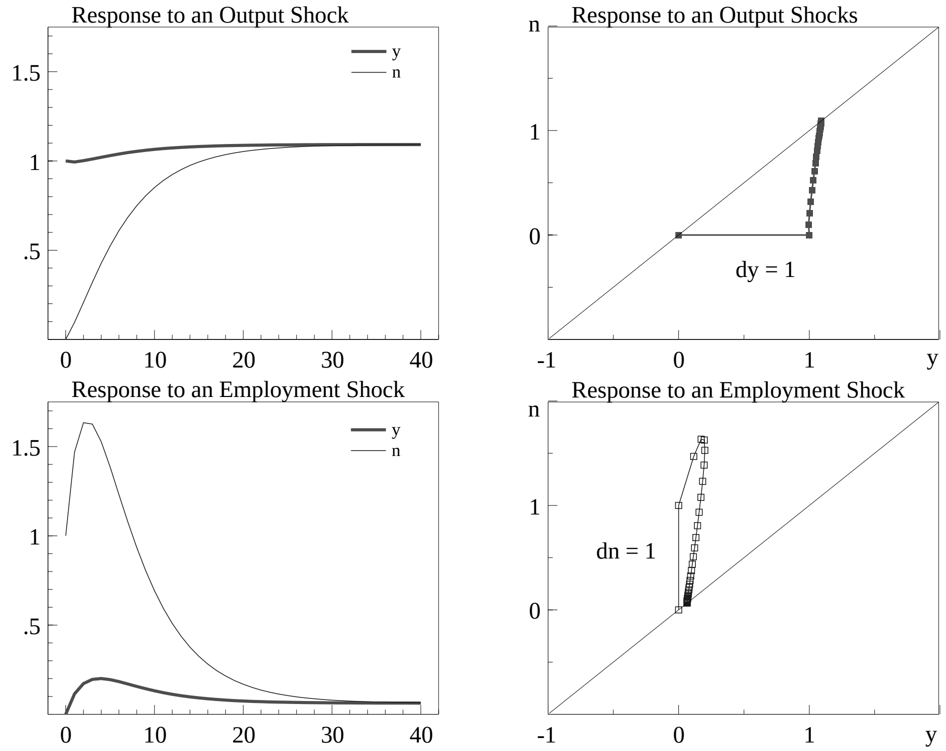

Figure 6 The Response to the Gaussian Innovations.

- The dynamics of output and employment when we move from the ergodic distribution to a sure state, say recession. This corresponds to the graphs placed in figure 8. These measure the costs of recessions in terms of output and employment. It demonstrates the slump of the economy and sudden response to changes in the state of the business cycle.
- The study of the path of output and employment when there is a change in regime such as from recession to growth, from recession to high growth, growth to recession or any other shift between the existing regimes. This corresponds to the graphs placed in figure 7, which show positive net job creation in cases of recovery via the high-growth regime 3 and the less job growth in cases of a recovery via the normal-growth regime 2. These effects are due to (i) changes in the current state and hence changes to the conditional expectation of a future regime and (ii) autoregressive transmission of intercept shifts. This offers an explanation for the fairly large differences in the effects of the two types of recoveries which are due to the estimated transition probability of moving from regime 2 to the high-growth regime 3 which is close to zero, $p_{23} \approx 0$. Note that the impulse-response functions provide an average response of output and employment to a change

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[6] We have not orthogonalized the innovations. Further state dependent responses could be obtained if orthogonalization had been used. This is due to the fact that our data is characterized by a changing variance among regimes.

in regime conditionally to all economic policies used since 1962.

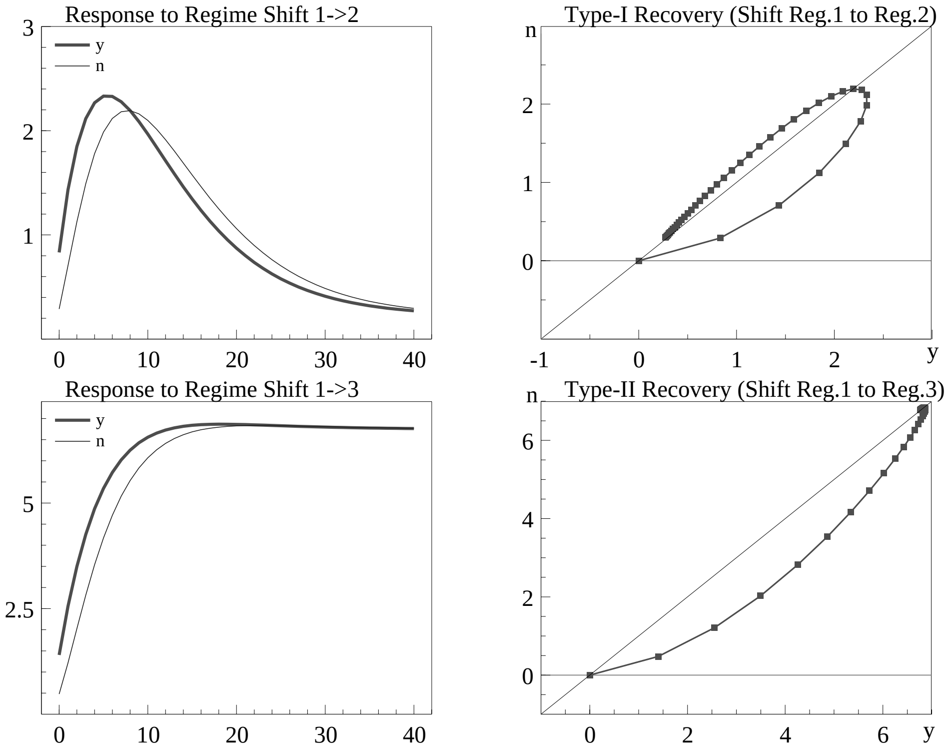

Figure 7 The Two Types of Recoveries.

It is worthwhile emphasizing that, in the literature, there is some confusion about Markov-switching models and non-linearities. The non-linearities in our model come from the fact that the mean and variance of the process are state dependent. Yet all regimes share the same autoregressive parameters. The logical conclusion is that the response of output and employment to a change from regime two to regime one is the mirror image (but with negative coefficients) of the response of the variables of a change from regime one to regime two. If the autoregressive parameter had changed then this would not have been the case.

However, business cycle asymmetries can be observed by looking at the path of output and employment when moving from the steady-state probabilities to a sure regime, as shown in the graphs in figure 8: The conditional expectation in expansions is not the mirror image of the conditional expectation in recessions indicating the asymmetry of the business cycle (see Clements and Krolzig, 1998, for statistical asymmetry tests). In order to get more insights, these have been graphed in the employment-output space in order to illustrate the important differentiated dynamics that are in play in the US economy for our period of analysis.

The asymmetries are better grasped if we look at the time path of the disequilibrium and the common trend growth in figure 9. The MS-VECM in Table 2 exhibits contemporaneous regime shifts in the drift and the mean, leading to the unrestricted shifts of the intercept term. To clarify the roles of growth changes and shifts in equilibrium means, we separate out in Figure 9 the effects of regime shifts into changes in the disequilibrium $\beta' x_t = y_t - n_t$ and the common growth rate $\beta'_{\perp} \Delta x_t \propto (\Delta y_t + \Delta n_t)$. This analysis again reveals the contemporaneity

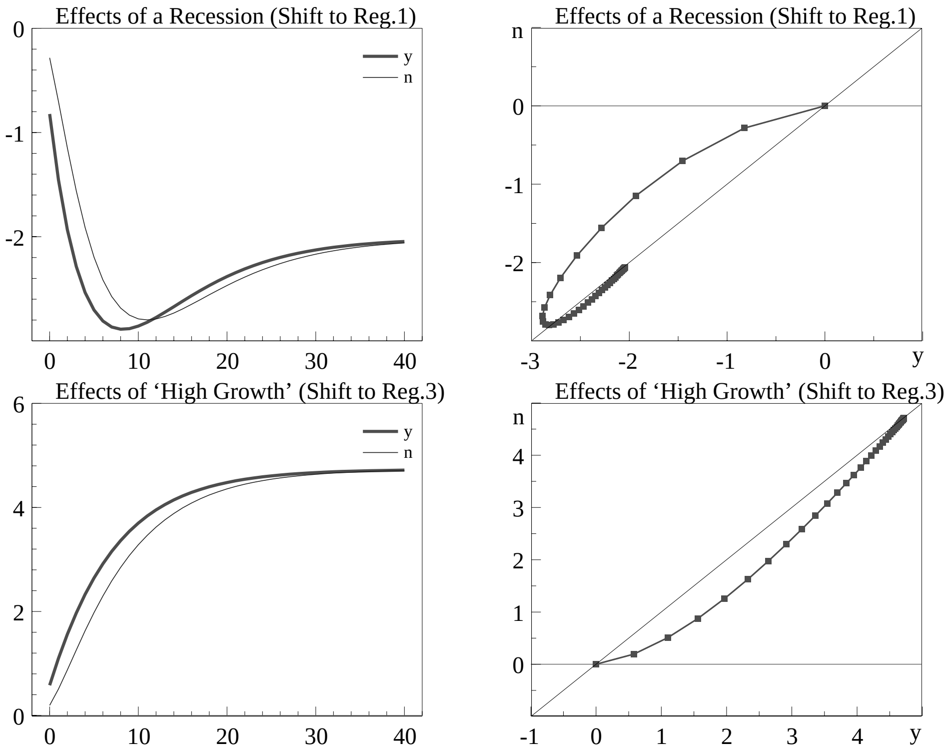

Figure 8 The Response to Regime Shifts.

of changes in the employment-output relation and the business cycle.

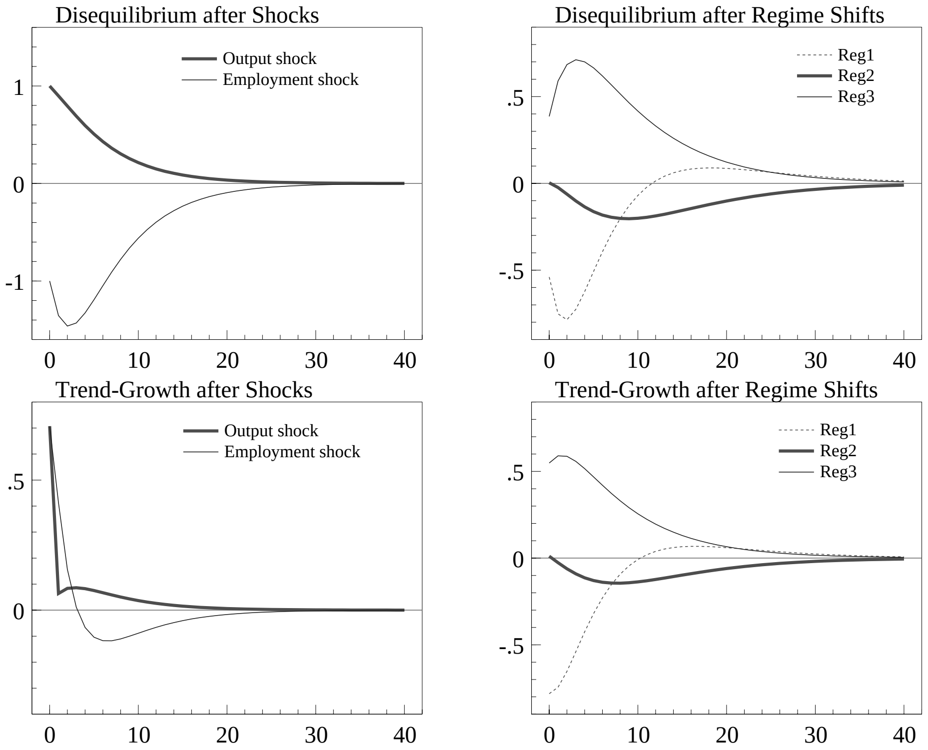

Figure 9 Equilibrium and Growth Adjustments.

5 Conclusions

In this paper we have presented a new approach to the impulse-response analysis of business cycle transitions. Within its framework the shocks to the economy are seen as regime shifts of the business cycle, i.e. shifts in the state of the unobserved variables, rather than the traditional Gaussian innovations in the observed variables. The proposed set of impulse response experiments is likely to improve the understanding of the dynamics of the macroeconomy. In the following we will illustrate the general applicability of the new approach by discussing some suitable extensions for analyzing certain features of the labour market.

The model presented in this paper was found to be a well specified statistical measurement system of the dynamics of output and employment in the US over the last thirty years. Short and long-run dynamics had been considered as well as the possibility of a shifting intercept (shifts in the equilibrium mean and the mean rate of trend growth) driven by a common Markov chain. The model could be easily extended to integrate equilibrium unemployment and job creation – job destruction processes, similar to Blanchard and Diamond (1989) and Storer (1996). Blanchard and Diamond (1989) estimate a matching function with constant returns to scale. The unemployment-vacancy ratio for the period of analysis varies between 0.5 and 0.9, which would lead to different estimates for the parameter of the matching function. The unemployment-vacancy ratio or tightness of the market is likely to depend on the state of the cycle and hence a better characterization could be used with a model similar to that presented in this paper. They then analyze a three variables vector autoregression with the labour force, vacancies and unemployment as the relevant variables. The objective is to analyze the Beveridge curve (the relationship between unemployment and vacancies). The dynamics are generated by innovations characterized as shocks to the labour force, vacancies and unemployment. A Markov-switching model of the joint process of labour force, unemployment and vacancies seems to be a well suited tool for this analysis. Interesting insights could be obtained if the dynamic analysis was extended to the response of the variables to changes from recession to booms, or used to analyze shifts from the ergodic probabilities to a specific regime.

Further extensions could consider modelling job creation and job destruction as in Davis and Haltiwanger (1992). These authors used data for job creation and job destruction in the manufacturing industry from 1979 to 1983. They found that job creation is procyclical and job destruction is countercyclical. The large flows in and out of employment are well explained by the high rates of job creation and destruction. Blanchard and Diamond (1990) reinforce this point by stressing that ”movements in employment seem to be associated with much larger fluctuations in job destruction that in job creation. Recessions are associated with large increases in job destruction and only small decreases in job creation. And, while direct evidence exists only for manufacturing, the indirect evidence suggests that, if anything, the asymmetry is even stronger for the economy as a whole”. Mortensen and Pissarides (1994) analyze the job creation job destruction process with a matching model where there is common price component to all the firms that varies probabilistically. This common price component captures the business cycle. They analyze the dynamics when there is anticipation of the aggregate productivity change. From a statistical point of view this would correspond to our impulse response analysis of a change from one regime of boom to recession. Interestingly their simulation exercise for the USA for the period 1947-1991 is implemented with a Markov transition matrix with three states, the same number of states that we identify in the data for the period 1962-1997. Thus as a by-product of our estimation results we can produce a transition matrix which is consistent with the data and might help to sharpen the conclusions which can be drawn from the theoretical analysis. The type of model and dynamic analysis that we have introduced seems particularly useful for deriving stylized equilibrium employment facts. This would permit the testing of the explanatory power of a wide range of labour market models that exist in the current literature. The application to some matching models is straightforward. Modelling vacancies and unemployment, and analyzing their dynamics, is another interesting avenue for further research.

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Mathematical Appendix: Impulse-Response Analysis

This appendix presents the mathematical background for the impulse-response analysis presented in the paper. Consider the MS(M)-VECM($p-1$) model

$$\Delta x_t = \mathbf{M}\xi_t + \alpha\beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \ldots + \Gamma_{p-1}\Delta x_{t-p+1} + u_t, \tag{11}$$

where $\mathbf{M} =[\nu_1 : \cdots : \nu_M]$. The corresponding MS($M$)–VAR($p$) representation is given by

$$x_t = \mathbf{M}\xi_t + A_1 x_{t-1} + \ldots + A_p x_{t-p} + u_t, \tag{12}$$

where $A_1 = \mathsf{I}_K + \alpha\beta' + \Gamma_1$ and $A_j = \Gamma_j - \Gamma_{j-1}$ for $1 < j \leq p$ with $\Gamma_p = \mathbf{0}_K$.

To derive the impulse-response functions we use the stacked MS(M)-VAR(1) representation of MS(M)-VAR(p) processes of an MS(M)-VAR(p) process. Denote $\mathbf{x}_t = (x_t', \ldots, x_{t-p+1}')'$, then equation (12) can be rewritten as:

$$\mathbf{x}_t = \mathbf{H}\xi_t + J\mathbf{A}\mathbf{x}_{t-1} + u_t, \tag{13}$$

where $\mathbf{A} = \begin{bmatrix} A_1 & \ldots & A_{p-1} & A_p \\ \mathsf{I}_K & & \mathbf{0} & \mathbf{0} \\ & \ddots & & \vdots \\ \mathbf{0} & & \mathsf{I}_K & \mathbf{0} \end{bmatrix}$ is a $(Kp \times Kp)$ matrix, $\mathbf{H} = \begin{bmatrix} \mathbf{M} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} = \iota_1 \otimes \mathbf{M}$ is a $(Kp \times M)$ matrix and $J = \begin{bmatrix} \mathsf{I}_K & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} = \iota_1' \otimes \mathsf{I}_K$ is a $(K \times Kp)$ matrix.

The state-space representation is completed by the VAR(1) representation of the Markov chain (see Hamilton, 1994):

$$\xi_{t+1} = \mathbf{F}\xi_t + v_t, \tag{14}$$

where $\xi_t$is the unobservable $(M \times 1)$ state vector consisting of the indicator variables $I(s_t = m)$ for $m = 1, \ldots, M$ and v_t is a martingale difference sequence.

Hence the expectation of y_{t+h} conditional upon $\{u_t, \xi_t, Y_{t-1}\}$ is given by:

$$\mathbf{x}_{t+h|t} = \mathbf{H}\xi_{t+h|t} + J\mathbf{A}\mathbf{x}_{t+h-1|t} \tag{15}$$

where the conditional expectation of ξ_{t+h} is

$$\xi_{t+h|t} = \mathbf{F}^h \xi_t. \tag{16}$$

Based on this representation the following types of analysis are feasible: a) Corresponding to the impulse response analysis in linear Gaussian VARs we can look at the response of the system to shocks arising from the Gaussian innovations in each of the variables. b) Impulse responses to changes in regimes can be also considered where we can distinguish between (i) the study of the path of the variables when there is a change in regime such as a shift from regime one to two or any other shift between the existing regimes, (ii) the dynamics when we move from the ergodic distribution to a sure state, say regime one.

The response of output and employment to shocks arising from the Gaussian innovations to the variables.

According to the impulse response analysis in time-invariant linear VARs we have that

$$\frac{\partial x_{t+h}}{\partial u_{jt}} = J\mathbf{A}^h \iota_j, \tag{17}$$

where ι_j is the j^{th} column of the identity matrix. If the variance-covariance matrix Σ_u is regime-dependent the standardized and orthogonalized impulse-responses also become regime-dependent:

$$\frac{\partial x_{t+h}}{\partial \varepsilon_{jt}} = J\mathbf{A}^h \mathbf{D}(\xi_t)\iota_j, \tag{18}$$

where $u_t = \mathbf{D}(\xi_t)\varepsilon_t$ and $\mathbf{D}(\xi_t)$ is a lower triangular matrix resulting from the Choleski decomposition of $\Sigma_u(\xi_t) = \mathbf{D}(\xi_t)\mathbf{D}(\xi_t)'$.

The study of the path of output and employment when there is a change in regime.

The effects of regime shifts can be measured as the reaction of x_{t+h} to the information that $s_t = j$, considered as a shift from the unconditional distribution $\overline{\xi}$,

$$dx_{t+h} = \; J\left(\sum_{k=0}^{h} \mathbf{A}^k \mathbf{H}\mathbf{F}^{h-k}\right)\left(\iota_j - \overline{\xi}\right). \tag{19}$$

When there is a shift from regime m to regime j then the responses of the system are given by:

$$dx_{t+h} = J\left(\sum_{k=0}^{h} \mathbf{A}^k \mathbf{H}\mathbf{F}^{h-k}\right)\left(\iota_j - \iota_m\right). \tag{20}$$

In both cases, the dynamics are generated by (i) changes of the current state, (ii) changes to the conditional expectation of future regimes, and (iii) the autoregressive transmission of intercept shifts.

More complicated models might consider changing parameters in the autoregressive part of the system and the response analysis should take that into account. Though we concentrate on impulse response analysis, an extension to error variance decomposition analysis could be a fruitful exercise.