

Impact of Wrong-Way Risk
on Margin Valuation Adjustment



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A thesis submitted in partial fulfillment of the requirements for the
M.Sc. in Mathematical Finance
Hilary 2021

Abstract

Valuation adjustments (XVA) have grown in popularity and importance since the Financial Crisis. In the post-Crisis environment, the value of a derivative can no longer be calculated simply as the risk-free price from by Black-Scholes theory, but other aspect, such as credit risk need to be accounted for as well. New regulations following the Crisis impose collateral requirements on over-the-counter derivatives, where initial and variation margin must be exchanged between trading counterparties. This introduces valuation adjustments that reflect the cost of funding for the derivative position. In particular, the Margin Valuation Adjustment (MVA) quantifies the cost of posting initial margin over the life of a trade.

Over the years, an XVA framework was established as market standard which neglects the correlation structure between the probability of default of the counterparty on the one hand, and the exposure or collateral measure on the other hand. Wrong-way risk, *i.e.*, the situation in which exposure is adversely correlated with the counterparty's default probability has increasingly become a concern for regulators, which started to give recommendations to financial institutions to identify and observe the wrong-way risk present in their portfolios.

This work combines the timely subjects of MVA and wrong-way risk by including wrong-way risk effects in MVA calculations. A model-independent approach is followed that has recently been applied to the calculation of regulatory and accounting Credit Valuation Adjustment and Funding Valuation Adjustment. Sample calculations are performed for plain-vanilla interest-rate swaps, cross-currency swaps, and European swaptions.

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1 Introduction

1.1 Regulation in the Aftermath of the Financial Crisis

In the aftermath of the Great Financial Crisis, the off-exchange *over-the-counter* (OTC) derivatives market has undergone large structural changes in the areas of trading, valuation, and regulation. Credit risk became the center of attention of financial market participants and regulators alike. Regulators committed to prevent pre-Crisis levels of credit risk from amassing again in the market and to increase transparency. The regulatory reforms that followed fundamentally transformed the financial marketplace. In the Basel III framework [1], bank capital bases were further strengthened as a buffer for losses and, in conjunction with leverage restrictions, disincentivised excessive risk taking. The OTC derivatives market in particular has seen significant changes. For non-centrally cleared derivatives, global margin requirement standards [2] were established by the Basel Committee on Banking Supervision. With the *European Market Infrastructure Regulation* (EMIR) [3], the European Union has implemented new rules for the OTC market in the EU. Under EMIR, entities must report every contract they enter into a trade repository in order to increase regulatory oversight. Furthermore, collateralisation is enforced for OTC derivatives, where variation margin must be exchanged between trading counterparties to cover mark-to-market moves in the asset prices. Under the new regulations, by the end of 2012 standardised contracts had to be cleared through central counterparties which, among other charges such as contributions to their default funds, require initial margin to cover their close-out risk.

1.2 Valuation Adjustments

Banks adopted a new view on the OTC market as well, and on how derivatives should be valued. Valuation adjustments, *i.e.*, the corrections to the risk-free price due to credit risk and related costs, gained in popularity in the years following the Crisis. Some of these adjustments already existed for some time before the Crisis. The *Credit Valuation Adjustment* (CVA) [4, 5, 6, 7], the discount to a risk-free price that takes into account the possible loss resulting from counterparty default, had been around since 2002 in some banks, but rapidly gained in popularity once the illusion was abandoned that large banks could not default. The move from unsecured trading to collateralized bilateral trading or even central clearing drastically reduced the counterparty risk so that other considerations, such as funding costs, gained in attention. As a consequence, valuation adjustments concerning funding were developed. The *Funding Valuation Adjustments* (FVA) represents the funding cost in the situation of collateral mismatch, in which a bank enters into an uncollateralised derivatives contract with a client while hedging the position with a collateralised counterparty in the interbank market. The *Margin Valuation Adjustment* (MVA) is one of the latest addition to the family of valuation adjustments. As the cost of posting initial margin for an OTC derivative, it symbolizes both the post-Crisis regulatory environment in which credit and funding costs have become major factors, as well as the banks' new perspective on derivatives valuation. Because MVA applies to derivatives that are cleared, and given the attempts to discourage unsecured trading, MVA is expected to become one of the most important valuation adjustments in the future.

1.3 Wrong-Way Risk

Initial margin and valuation adjustments are routinely calculated at banks today. The standard setup is to calculate initial margin using the ISDA SIMM [8, 9] methodology which employs current sensitivities to estimate the potential future exposure that defines the initial margin amount, but V@R-based methods are also both popular and in line with regulatory requirements. For the calculation of valuation adjustments, a framework has become market standard in which certain assumptions are made regarding the default probabilities of the counterparties involved [10]. For example, the highly unlikely simultaneous default of both parties to a trade is usually neglected in the calculations. Moreover, it is common practice to ignore the correlation between the default probability and the exposure. The situation in which default probability and exposure to a counterparty are adversely correlated is referred to as *wrong-way risk* (WWR). Including WWR into derivatives pricing is expected to grow in importance because regulators have started to focus on the adverse effects that WWR has on value of derivatives portfolios. For instance, the Basel III framework [1] strongly recommends banks to quantify their WWR.

Various methods have been developed to include WWR into CVA calculations (see Sec. 4.2). In 2020, a *model-independent* approach to include WWR into CVA and FVA calculations was proposed by Chris Kenyon, Mourad Berrahoui, and Benjamin Poncet [11]. The present work extends this approach to the calculation of margin valuation adjustments.

2 Exposure and Margin Simulation

2.1 Survival Probability and Hazard Rate

Valuation adjustments (see Sec. 3) are adjustments to the risk-free price due to the presence of credit risk, *i.e.*, the risk that one party of a trade defaults at some point in time. Some adjustments like CVA or DVA directly measure the potential loss due to the default of an entity, while other adjustments like ColVA or MVA have a more indirect connection to default events. In any case, survival probabilities naturally enter the considerations around these valuation adjustments. Default events are modelled by a stopping time τ_{def} which represents the time of default of an entity. For the information set \mathcal{F}_{t_0} at time t_0 , where the entity is still alive, the corresponding probability $\text{SP}(t_0, T | \mathcal{F}_{t_0})$ for survival up to time T is defined as

$$\text{SP}(t_0, T | \mathcal{F}_{t_0}) = \text{Prob}[\tau_{\text{def}} > T | \mathcal{F}_{t_0}]. \quad (1)$$

In the following, the filtration \mathcal{F}_{t_0} is dropped from the notation. The default process is modelled as a Poisson process with default rate $\lambda(t)$, referred to as *hazard rate*. This hazard rate may depend on time, but is non-stochastic. Given no prior default up to time t , the probability of default in $[t, t + \Delta t)$ is given by $\lambda(t)\Delta t$, *i.e.*,

$$\text{Prob}\left[\tau_{\text{def}} \in [t, t + \Delta t) \mid \tau_{\text{def}} \geq t\right] = \lambda(t)\Delta t. \quad (2)$$

In terms of the hazard rate, the survival probability can be written as

$$\text{SP}(t_0, T) = e^{-\int_{t_0}^T \lambda(s) ds}. \quad (3)$$

In the expressions for valuation adjustments, the term $\lambda(t)\text{SP}(t_0, t)\Delta t$ occurs frequently, which can be identified as the unconditional default probability in the interval $[t, t + \Delta t)$. This can be seen

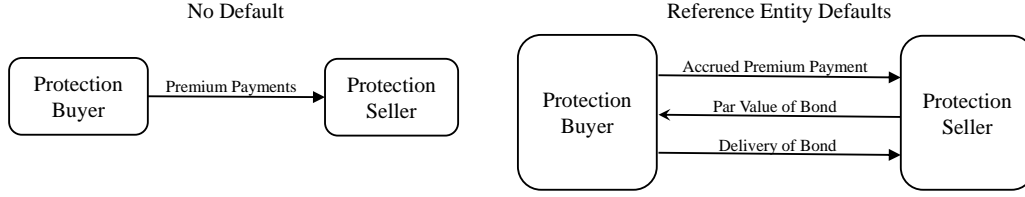


Figure 2.1: Mechanics of a credit default swap.

as follows. The unconditional default probability can be expressed as the probability difference $SP(t_0, t) - SP(t_0, t + \Delta t)$ of survival probabilities. This difference of survival probabilities can be written in terms of the hazard rate as

$$\begin{aligned}
& SP(t_0, t) - SP(t_0, t + \Delta t) \\
&= e^{-\int_{t_0}^t \lambda(s) ds} - e^{-\int_{t_0}^{t+\Delta t} \lambda(s) ds} \\
&= e^{-\int_{t_0}^t \lambda(s) ds} \left\{ 1 - e^{-\int_t^{t+\Delta t} \lambda(s) ds} \right\} \\
&\approx e^{-\int_{t_0}^t \lambda(s) ds} \left\{ 1 - e^{-\lambda(t)\Delta t} \right\} \quad (\lambda(t) \approx \text{constant over } \Delta t) \\
&= e^{-\int_{t_0}^t \lambda(s) ds} \left\{ 1 - \left[\sum_{k=0}^{\infty} \frac{(-\lambda(t)\Delta t)^k}{k!} \right] \right\} \quad (\text{Taylor expansion of exponential}) \\
&= e^{-\int_{t_0}^t \lambda(s) ds} \left\{ 1 - \left[1 + \frac{-\lambda(t)\Delta t}{1} + \mathcal{O}(\Delta t^2) \right] \right\} \quad (\text{expand to first order in } \Delta t \text{ only}) \\
&= \lambda(t) e^{-\int_{t_0}^t \lambda(s) ds} \Delta t \\
&= \lambda(t) SP(t_0, t) \Delta t.
\end{aligned} \tag{4}$$

2.1.1 Calibration

Default probabilities can either be calculated in the real-world measure or in the risk-neutral measure. Real-world probabilities are determined from historical data of default events, while risk-neutral probabilities are extracted from market quotes of credit-sensitive financial products. Risk-neutral default probabilities have been found to be significantly larger than their real-world counterparts. This is most likely due to liquidity concerns, pessimistic market attitudes, and default correlations priced into the quoted instruments [12].

This work follows the standard approach of using market-implied survival probabilities, by calibrating the hazard rates to spreads of *credit default swaps* (CDS) [13, 4]. CDS spreads represent the premium paid by the buyer of a CDS. The mechanics of a CDS is illustrated in Fig. 2.1. A CDS provides protection against a specific company named in the contract, the so-called *reference entity*. The protection buyer obtains the right to sell bonds issued by the reference entity at their face value in case of a *credit event* defined in the CDS [12]. As long as the reference entity does not default, the protection buyer makes regular premium payments (some percent of the insured notional) to the protection seller. In case of default of the reference entity, the protection buyer makes a final payment of the premium that has accrued up to the default date. In turn, the protection seller purchases the defaulted bonds from the protection buyer at par value. Therefore, the size of the CDS spread reflects the probability of the reference entity to default within the time period for which the CDS grants insurance. A bootstrapping method can be used in order to calculate market-implied

survival probabilities from the observed CDS spreads [4, 13, 6].

2.2 Exposure Measures

In the exposure simulation, a Monte Carlo engine generates N exposure paths $\Pi^{(i)}(t)$ under the risk-neutral measure \mathbb{Q} from the simulation start time t_0 until the simulation end time T . The specific nature of the exposure Π is not relevant at this point, but typical simulated exposures are given by the mark-to-market values of some portfolio with or without adjustments due to intermediate collateral payments.

By calculating the *expected exposure* (EE), the *expected positive exposure* (EPE), or the *expected negative exposure* (ENE), partial information about the collection of exposure paths can be represented in single exposure profiles. The EE profile is obtained by averaging the exposures $\Pi^{(i)}(t)$ at each point in time t ,

$$\text{EE}(t) = \mathbb{E}^{\mathbb{Q}}[\Pi(t) | \mathcal{F}_{t_0}] = \frac{1}{N} \sum_{i=0}^N \Pi^{(i)}(t), \quad (5)$$

whereas for the EPE profile, the exposures are floored at zero, leading to

$$\text{PE}(t) = \max\{\Pi^{(i)}(t), 0\} \quad (6)$$

$$\text{EPE}(t) = \mathbb{E}^{\mathbb{Q}}[\text{PE}(t) | \mathcal{F}_{t_0}] = \frac{1}{N} \sum_{i=0}^N \text{PE}(t). \quad (7)$$

Similarly, the ENE profile is obtained by capping the exposure at zero,

$$\text{NE}(t) = \min\{\Pi^{(i)}(t), 0\} \quad (8)$$

$$\text{ENE}(t) = \mathbb{E}^{\mathbb{Q}}[\text{NE}(t) | \mathcal{F}_{t_0}] = \frac{1}{N} \sum_{i=0}^N \text{NE}(t). \quad (9)$$

Figure 2.2 illustrates how these expected exposures arise from the exposure paths $\Pi^{(i)}(t)$.

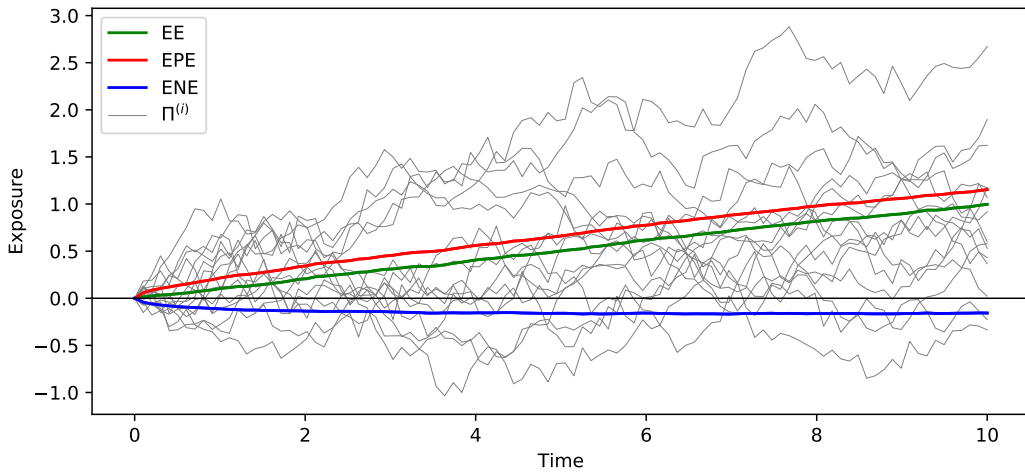


Figure 2.2: Illustration of how the exposure measures EE, EPE, and ENE arise from the simulation of exposure paths $\Pi^{(i)}(t)$ that follow a geometric Brownian motion with positive drift.

The *potential future exposure* (PFE) is a different class of exposure measure. It is a forward-looking measure that, for the information set \mathcal{F}_{t_0} at time t_0 , holds information about the potential exposure at some future point t in time. A common choice is to take the α -quantile of the exposure $\Pi(t)$, which corresponds to the *value at risk at the α level* ($V@R_\alpha$), *i.e.*,

$$\text{PFE}_\alpha(t) = V@R_\alpha(t|\mathcal{F}_{t_0}). \quad (10)$$

Evaluating $\text{PFE}_\alpha(t)$ involves estimating the probability density of the future exposure. For the results in this work, the Nadaraya-Watson kernel regression method [14] is applied for calculating PFE during a Monte Carlo simulation.

2.3 Variation Margin

Variation margin is a means to decrease the credit risk between two trading parties by reducing the exposure towards each other. This exposure reduction is achieved by means of exchanging *collateral* on a set of *collateral dates* $\{t_i^{\text{VM}}\}$. The collateral payments are designed such that they compensate the current exposure between the two counterparties at the collateral dates. This kind of collateral is referred to as *variation margin* (VM), and usually takes the form of cash. Under the uncleared margin requirements, VM may be rehypothecated and should therefore be treated as a significant source of income. In case of default, the collateral account is used to compensate the loss sustained by the counterparty not honoring its obligations.

Even if a trade is initiated at zero value, through market movements a significant exposure may arise over time. This is illustrated in the top panel of Fig. 2.3, which shows the time evolution of an uncollateralized exposure that vanishes at the trade date t_0 . The middle panel of Fig. 2.3 shows the same exposure subject to VM exchange. At the collateral dates $\{t_i^{\text{VM}}\}$ the exposure drops to zero, because the observed exposure is paid as VM to the collateral account shown in the bottom panel. Because the value of the trade decreases from t_2^{VM} to t_3^{VM} , some of the VM is paid back at t_3^{VM} . As is apparent from the middle panel, where the original uncollateralized exposure is indicated by the dashed gray line, the variation margin payments lead to an overall reduction of exposure. Because the exposure to the counterparty is zero at the margin dates, the VM account matches the portfolio in value at these dates.

The conditions under which margin is exchanged can be found in the trade's *credit support annex* (CSA). Typical parameters of CSA agreements, and the corresponding values used in this work, are listed in Table 2.1.

In theory, under certain assumptions (such as continuous exchange of VM, zero thresholds, cash collateral), VM can completely remove credit risk. In practice, however, besides non-zero thresholds one also needs to take into account the default process and the corresponding close-out risk. A typical default process is presented in Fig. 2.4. At some margin date t_i^{VM} , collateral is successfully exchanged for the last time. After that date, the counterparty experiences financial difficulties, and potentially misses the following collateral payment due at t_{i+1}^{VM} . In practice, it takes a few days until the default of the counterparty is officially declared at time τ_{def} . From that point, it again takes some time to unwind and re hedge the position at $t_{\text{close-out}}$. The time period between the last successful collateral exchange and the close-out of the portfolio is called the *margin period of risk* (MPOR) τ_{MPoR} , such that

$$t_{\text{close-out}} = t_{\text{last payment}}^{\text{VM}} + \tau_{\text{MPoR}}. \quad (11)$$

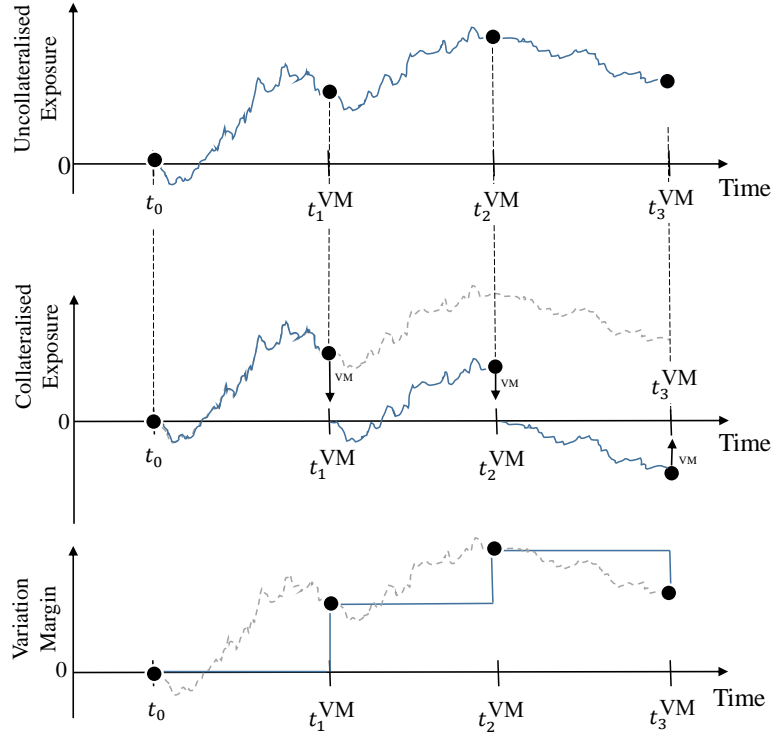


Figure 2.3: Illustration of the effect of variation margin. The top panel shows the uncollateralised exposure of some portfolio, starting at zero at the trade date t_0 . In the middle panel, VM payments are introduced at the margin dates $\{t_i^{\text{VM}}\}$ to compensate for changes in the mark-to-market value of the portfolio, leading to reduced credit risk (compare with the dashed grey line). The bottom panel presents the VM account, which grows and shrinks according to the current exposure at the margin dates. The fact that VM usually earns interest is neglected here.

The usual choice for the MPoR is 10 business days. As indicated in Fig. 2.4, the portfolio may undertake a significant change in value over the MPoR. Because this change is not compensated by margin payments, this leads to close-out risk (see Ref. [15] for an extensive treatment of MPoR considerations).

As noted in the discussion of Fig. 2.3, immediately after the collateral exchange the VM account matches the mark-to-market value (MtM) of the portfolio, and starts to earn interest at some rate $r(t)$. Therefore, in case of default, where we evaluate our losses at time t , the variation margin available at that time t is given by the VM payment at $t - \tau_{\text{MPoR}}$, which is equivalent to

$$\text{VM}(t) = \frac{V(t - \tau_{\text{MPoR}})}{D_{\text{VM}}(t - \tau_{\text{MPoR}}, t)}, \quad (12)$$

where $D_{\text{VM}}(t - \tau_{\text{MPoR}}, t)$ is an appropriate discount factor.

From these considerations it follows that the exposure between two counterparties in presence of variation margin exchange is given by

$$\Pi^{\text{VM}}(t) = \text{MtM}(t) - \text{VM}(t). \quad (13)$$

Parameter	Description	This work
Threshold Amount	As a measure to reduce the number of collateral transactions, counterparties may agree on an exposure threshold below which no collateral is exchanged. For banks, this threshold is typically set to zero.	0
Minimum Transfer Amount	As a measure to reduce the number of collateral transactions, counterparties may agree on a minimum transfer amount of collateral that must be exceeded before the collateral is exchanged. This threshold is typically non-zero.	0
Independent Amount	If the counterparties agreed on the payment of an independent amount, this amount needs to be paid at the beginning of the contract in any case.	0
Call Frequency	The call frequency specifies the frequency at which collateral is exchanged (if threshold and independent transfer amounts are exceeded). Banks usually perform daily collateral exchange.	daily
Margin Period of Risk	Assumed time period between default and close-out (not a CSA parameter).	10 bd

Table 2.1: Relevant CSA parameters and the corresponding values used in this work.

2.4 Initial Margin

As discussed in Sec. 2.3, even for daily exchange of VM, there will be credit risk present due to the timespan of the default process, represented by the MPoR τ_{MPoR} . The purpose of *Initial Margin* (IM) is to cover the risk that arises from a non-zero MPoR.

Figure 2.4 illustrates two sources of risk during the default process, *i.e.*, the time between the last successful collateral payment and the close-out of the positions: missed payments by the counterparty and changes in MtM due to market movements. Figure 2.5 provides a closer look at this situation. Because the portfolio is no longer collateralized during the default process, the potential profit gained over the time interval $(t, t + \tau_{\text{MPoR}}]$ is at risk. Here, we denote as t the time of last successful collateral exchange, and it is assumed that two cashflows are due at dates t_1 and t_2 during the MPoR. From these considerations follows that the *Profit and Loss* (PnL) that is at risk is given by

$$\text{PnL}(t; \tau_{\text{MPoR}}) = D_r(t, t + \tau_{\text{MPoR}}) \text{MtM}(t + \tau_{\text{MPoR}}) - \text{MtM}(t) + \sum_i D_r(t, t_i) \text{CF}(t_i), \quad (14)$$

where the summation runs over all cashflows $\text{CF}(t_i)$ that occur in $(t, t + \tau_{\text{MPoR}}]$. Initial margin is designed to cover this potential loss of PnL. The overall exposure in presence of VM and IM

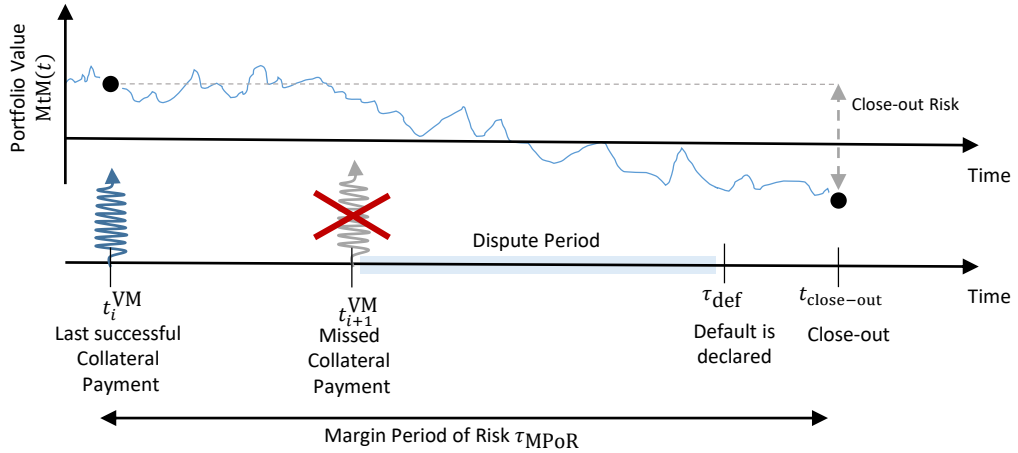


Figure 2.4: Illustration of the Margin Period of Risk.

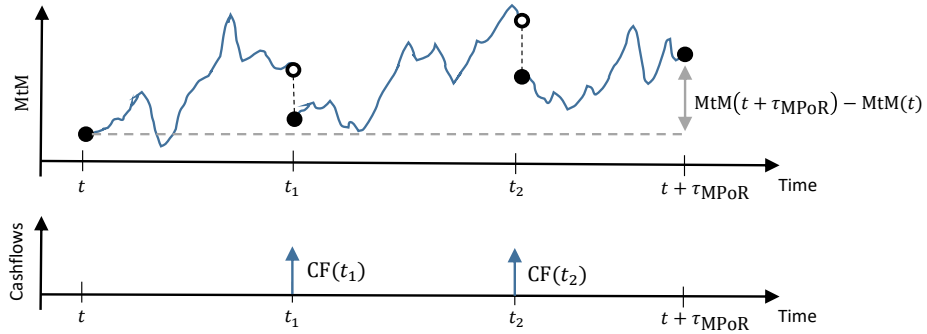


Figure 2.5: Contributions to initial margin. The MtM exhibits jumps at the cashflow dates that have the size of the cashflows. At risk is the MtM change from t to $t + \tau_{\text{MPoR}}$ as well as the cashflows during the MPoR.

exchange is then given by

$$\Pi^{\text{VM-IM}}(t) = \text{MtM}(t) - \text{VM}(t) - \text{IM}(t). \quad (15)$$

2.4.1 Modelling Initial Margin

Recalling the purpose of IM, which is to cover for potential losses during the MPoR, it becomes apparent that IM is a forward-looking quantity that needs to estimate the PnL over a time horizon of τ_{MPoR} . In its uncleared margin rules [2], the Basel Committee demands that the IM definition employed by a bank to be consistent with a one-tailed 99 per cent confidence interval over a 10-day horizon. Several methods have been proposed to achieve this goal.

The Committee itself has published a proposal in which the IM requirements for different asset classes are determined by a schedule of notional exposure percentages, see Table 2.2. While this schedule-based solution is rather simple conceptually, it is not very attractive to more sophisticated institutions due to its conservative nature. Also, criticism arose around difficulties in categorising the trades in a nettingset [16].

The *de facto* market standard is the *Standard Initial Margin Model* (SIMM) [8, 9] published by

Asset Class	Initial margin requirement (% of notional exposure)
Credit: 0-2 year duration	2
Credit: 2-5 year duration	5
Credit: 5+ year duration	10
Commodity	15
Equity	15
Foreign exchange	6
Interest rate: 0-2 year duration	1
Interest rate: 2-5 year duration	2
Interest rate: 5+ year duration	4
Other	15

Table 2.2: Initial margin schedule proposed by the Basel Committee, taken from Ref. [2].

the *International Swaps and Derivatives Association* (ISDA). At the heart of the SIMM methodology lies a delta-normal approximation of the 99%-V@R over a 10 day horizon. ISDA SIMM works with pre-defined risk- and product classes, and provides the correlation matrix between the risk factors to be used in the calculations. The IM amounts are calculated from the sensitivities of the risk factors, which serve as input to the method.

A conceptually more simple way, and the one followed in this work, is to directly calculate the 99%-V@R of the PnL without resorting to the approximations done in ISDA SIMM. In this case, the IM requirements are given by

$$\text{IM}(t) = \text{V@R}_{0.99} \left[\text{PnL}(t; \tau_{\text{MPoR}}) \middle| \mathcal{F}_t \right]. \quad (16)$$

Here, we use the IM implementation of Ref. [17]. The Nadaraya-Watson kernel regression is employed in order to obtain an estimate for the variance of $\text{PnL}(t; \tau_{\text{MPoR}})$ conditional on the mark-to-market value $\text{MtM}(t)$ of the form

$$\mathbb{E} \left[\text{PnL}^2(t; \tau_{\text{MPoR}}) \middle| \text{MtM}(t) \right] \approx \frac{\sum_{i=1}^N K_{h(t)} \left(\frac{\text{MtM}^{(i)}(t) - \text{MtM}(t)}{h} \right) \left[\text{MtM}^{(i)}(t) \right]^2}{\sum_{i=1}^N K_{h(t)} \left(\frac{\text{MtM}^{(i)}(t) - \text{MtM}(t)}{h} \right)}, \quad (17)$$

where $K_{h(t)}(x)$ are Gaussian kernel functions with a time-dependent bandwidth $h(t)$, and the summations run over all Monte Carlo paths. For a normal random variable \mathcal{X} , the value-at-risk at confidence level α can be calculated as

$$\text{V@R}_\alpha \left[\mathcal{X} \middle| \mathcal{F}_t \right] = \sqrt{\text{Var} \left[\mathcal{X} \middle| \mathcal{F}_t \right]} \Phi^{-1}(\alpha), \quad (18)$$

where Φ is the standard normal cumulative distribution function. Therefore, assuming that the distribution of $\text{PnL}(t; \tau_{\text{MPoR}})$ is normal for any given t , and replacing \mathcal{F}_t by $\text{MtM}(t)$, by virtue of Eq. (17) the following expression for the IM at time t and for path i is obtained

$$\text{IM}^{(i)}(t) \approx \sqrt{\frac{\sum_{i=1}^N K_{h(t)} \left(\frac{\text{MtM}^{(i)}(t) - \text{MtM}(t)}{h} \right) \left[\text{MtM}^{(i)}(t) \right]^2}{\sum_{i=1}^N K_{h(t)} \left(\frac{\text{MtM}^{(i)}(t) - \text{MtM}(t)}{h} \right)}} \cdot \Phi^{-1}(\alpha). \quad (19)$$

The *expected initial margin* (EIM) is needed in the calculation of the Margin Valuation Adjustment. It follows from averaging over the paths, *i.e.*,

$$\text{EIM}(t) = \mathbb{E}^{\mathbb{Q}}[\text{IM}(t)] = \frac{1}{N} \sum_{i=1}^N \text{IM}^{(i)}(t). \quad (20)$$

2.5 OTC Derivatives Clearing

After the Financial Crisis, the G20 pushed for new regulations of the financial sector [18]. As a cornerstone of these new regulations aimed at reducing systemic risk in the market, the Basel Committee proposed new margin requirements for the *over-the-counter* (OTC) derivatives market [2]. The *European Market Infrastructure Regulation* (EMIR) [3], which came into force 2013, requires clearing for standardised OTC derivatives through a *central counterparty* (CCP) [19]. A list of authorised central counterparties under EMIR can be found in Ref. [20]. Through the process of *contract novation*, CCPs act as intermediaries between the trading counterparties. This way, counterparties do no longer have direct exposure to each other, but only interact with the CCP. As required by the EMIR regulation, trading counterparties exchange VM and IM with their respective counterparty. As further lines of defense, CCPs often demand additional margins as well as member payments to their *default funds*. In cases where OTC derivatives are not cleared through a CCP, EMIR requires bilateral margining between the trade partners.

Variation and initial margin are usually exchanged on a daily basis via an end-of-day margin call, whenever they exceed their respective minimum transfer amount. According to Ref. [2], initial margin may not be rehypothecated except for some special cases covered in comment 5(iv) of Ref. [2]. Variation margin, on the other hand, is rehypothecatable. If a CCP is involved, IM is not transferred between the counterparties but held in a separate account at the CCP. This is because IM is designed to cover the credit risk the CCP faces, not the non-defaulting counterparty. For this work, the kind of clearing (bilateral or central) is not relevant, as the question of where the initial margin posted by the counterparty is held has only relevance for calculating the Collateral Valuation Adjustment (see Sec. 3.3), which is not covered in this work.

2.6 Portfolios

In this work, interest-rate swaps, cross-currency swaps, and swaptions are considered. Figures 2.6-2.9 present properties of the trades as well as their exposure profiles. The exposure calculations have been performed for the valuation date 2016/12/01. Details are given in the following sections.

2.6.1 Interest-Rate Swaps

As representatives of pure interest-rate derivatives, plain vanilla fix-for-float IR swaps are considered. In the examples, the fixed rate is received and the floating rate is paid. The floating rate is given by the 6-month Euribor. The notional is EUR 1 million. The fixed rate is always set to the par rate, *i.e.*, the rate which makes the initial value of the swap zero. In the calibration step, the par swap rate for the current simulation start date is determined and entered into the contract definition. Standard tenors as given in Ref. [21] are used for the fixed and floating payments. Maturities of 5, 10, and 30 years are considered.

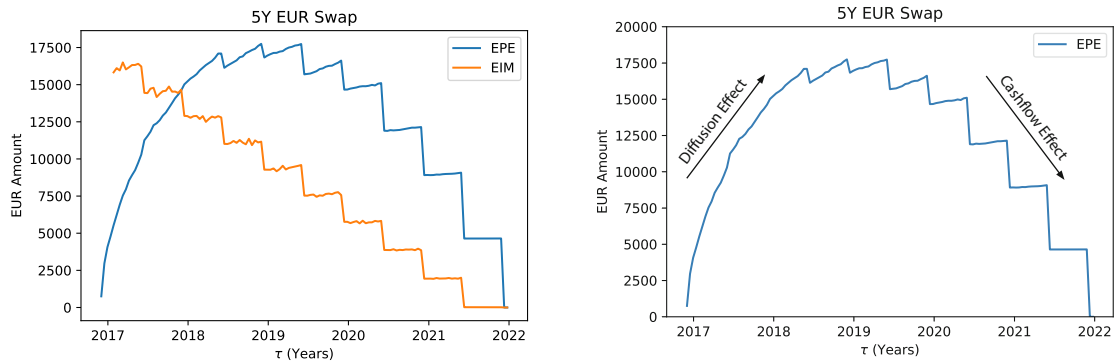


Figure 2.6: EPE, ENE, and EIM profiles for the plain vanilla fixed-for-float interest rate swap, initiated at the par rate. Right: Diffusion and cashflow effect for a 5-year fixed-for-float interest rate swap.

The EPE and EIM profiles of a 5-year EUR IR swap is presented in Fig. 2.6. Because the swap is initiated at the par rate, the exposure is zero in the beginning. Movements in the interest rate lead to a steep increase in EPE, where in the beginning even small movements in the interest rate can have a sizable effect on the value of the swap because of the large number of outstanding future cashflows. Because the maximum function in the definition of EPE cuts off contributions of paths which lead to a negative exposure, an increase in the EPE is observed. This effect is referred to as *diffusion effect*, see the right plot in Fig. 2.6. After the EPE maximum is reached at around half the maturity of the swap, the fact that less and less cashflows are outstanding which may contribute to the overall value of the swap, the EPE begins to decline until it reaches zero at the swap maturity. This effect is referred to as *cashflow effect*. Clearly visible are the cashflow dates at which the exposure exhibits jumps.

2.6.2 European Swaptions

A European swaption gives the holder the right to enter an interest rate swap at the given strike rate on a predetermined exercise date. The swaption considered in this work has an underlying fixed-for-floating receiver swap exchanging Euribor 6M semiannually for a fixed rate annually. The fixed rate will be determined as the par swap rate at the simulation start time. The swaption has an expiry of two years and the swap a maturity of 5 years. Here, a short position in the swaption is considered. Figure 2.7 illustrates the EPE and EIM profiles for this case. Because the bank has a short position in the swaption, the positive exposure is zero until the swaption maturity. Only if the counterparty exercises the swap and both parties enter a swap which may provide gains or losses for any of the trade partners, a positive exposure can arise. From the swaption maturity, the EPE profile shows the characteristics of a swaption. The EIM profile shows an increase from the trade date until the exercise date. This behaviour is due to the diffusion effect. After the exercise date, the cashflow effect sets in, where the EIM decreases over time due to the shrinking number of outstanding future cash flows.

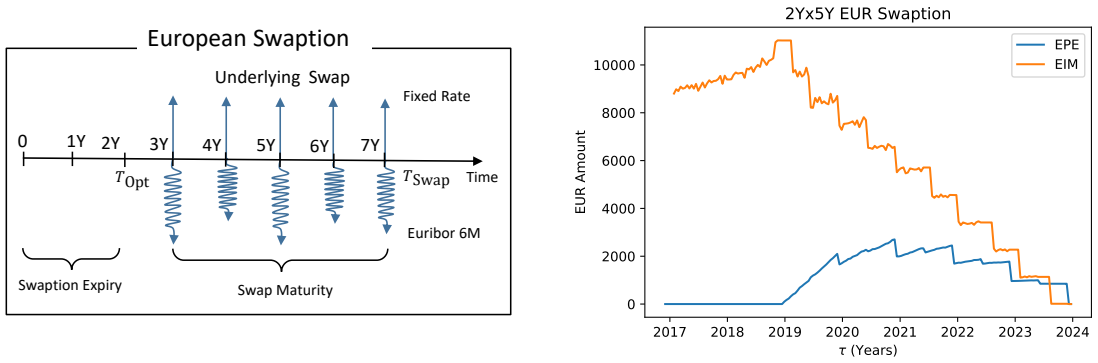


Figure 2.7: Left: European swaption considered in this work. The underlying swap exchanges 6M Euribor for the par swap rate determined at the simulation start date. The swaption has an expiry of 2 years and the swap a maturity of 5 years. Right: The exposure profiles for a short position in the swaption.

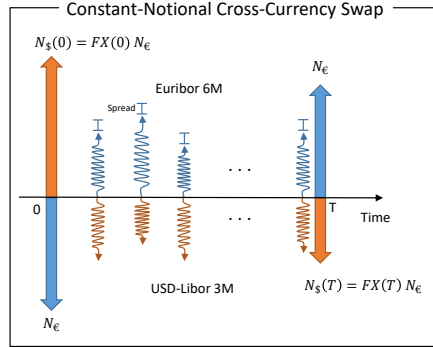


Figure 2.8: Mechanics of a constant-notional cross-currency swap.

2.6.3 Cross-Currency Swaps

In a *cross-currency* (XCCY) swap, floating interest rates such as Euribor and USD-Libor are exchanged on notionals in different currencies. This way a funding position in one currency can be transformed into a funding position in another currency. In order to make the trade fair at initiation, a constant spread is usually added to the leg of the less liquid currency. Notional exchanges take place at the beginning and at the maturity of the contract. In this work *constant-notional XCCY swaps* are considered, in which a constant EUR notional is exchanged for a notional in another currency in the beginning and the end of the contract at the foreign exchange rate prevailing at these dates. The mechanics of such a EUR-USD XCCY swap is illustrated in Fig. 2.8. In this work, the spread is set to zero, but the notional exchange at t_0 is adjusted in order to make the swap fair. The EPE profile of a 5-year EUR-USD XCCY swap is presented on the left in Fig. 2.9. Because the trade is fair initially, EPE starts out at zero. From there, it steadily increases over time due to the FX component in the final exchange of the large notional. In order to avoid such large exposures building up, the market also provides *resettable XCCY swaps*, in which the notionals are adjusted at the interest dates and compensation payments are made.

For XCCY swaps, there is an important exception regarding the calculation of initial margin. According to Ref. [22], the notional-exchange component of a cross-currency swap is exempted from

the bilateral initial margin requirements. In order to simulate this exemption, the initial margin amounts are calculated for XCCY swaps without notional exchange. The EPE and EIM profiles for such a 5-year EUR-USD XCCY swap without notional exchange is presented on the right in Fig. 2.9. For interest rate references, tenors etc., the standard market conventions of Ref. [21] are employed.

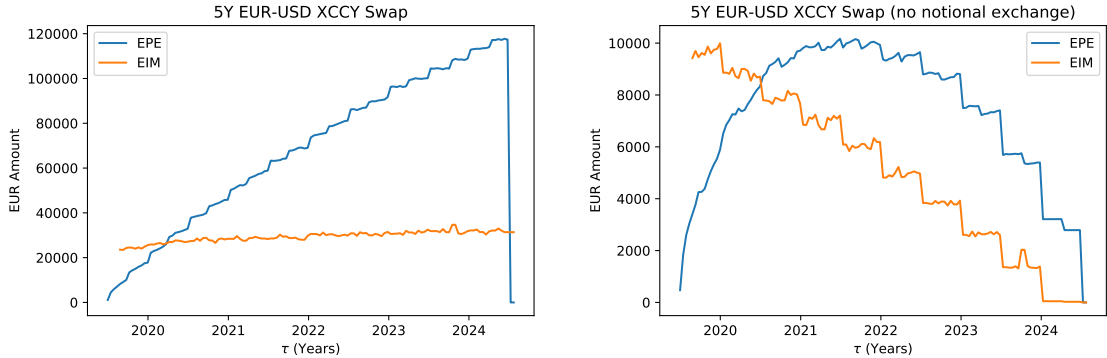


Figure 2.9: EPE and EIM profiles for the cross-currency swaps considered in this work. Left: Constant-notional cross-currency swap with notional exchange. For XCCY swaps, the notional in the foreign currency is adjusted at t_0 such that the price of the swap is zero. Right: Constant-notional cross-currency swap without notional exchange due to the FX exemption for initial margin.

3 Valuation Adjustments

3.1 Introduction

Over the past decade *valuation adjustments* (XVA) have significantly gained in importance because it was realised that standard pricing theory has become increasingly inadequate for representing the value of financial derivatives in the post-Crisis environment. Valuation adjustments are corrections to the risk-free value $V_{\text{risk-free}}(t)$ obtained from standard pricing theory,

$$V(t) = V_{\text{risk-free}}(t) + \sum_i \text{XVA}_i(t), \quad (21)$$

taking into account various aspects of today's derivative trading that is not captured by standard pricing. Since the Lehman bankruptcy in 2008, *credit risk* became a major concern for banks and regulators alike. Not surprisingly, the *Credit Valuation Adjustment* (CVA), representing the cost of hedging counterparty credit risk, was the first valuation adjustment to appear on stage. CVA is closely related to the *Debit Valuation Adjustment* (DVA), which represents the credit risk faced by the counterparty. The impact of funding the derivative position is captured in the *Funding Valuation Adjustment* (FVA). The cost or benefit from earning some yield on cash collateral is measured by the *Collateral Valuation Adjustment* (CoVA). The regulators' demand for capital allocation for a derivative position is evaluated by the *Capital Valuation Adjustment* (KVA). And finally, the requirement to post initial margin leads to benefits or costs captured by the *Margin Valuation Adjustment* (MVA). Currently, these are the most important members of the XVA family, see Table 3.1 for an overview.

Abbreviation	Name	Driver
CVA	Credit Valuation Adjustment	Credit risk faced by bank
DVA	Debit Valuation Adjustment	Credit risk faced by counterparty
FVA	Funding Valuation Adjustment	Funding cost of the derivative position
ColVA	Collateral Valuation Adjustment	Cost or benefit of cash collateral
KVA	Capital Valuation Adjustment	Impact of regulatory capital
MVA	Margin Valuation Adjustment	Funding cost of initial margin

Table 3.1: Family of valuation adjustments.

3.2 Credit Valuation Adjustment

The *Credit Valuation Adjustment* [4, 5, 6, 7] quantifies the risk due to counterparty default. A positive exposure towards the counterparty is at risk of (partial) loss if the counterparty defaults. CVA is calculated as the risk-neutral expectation of the present values of this potential loss

$$\text{CVA} = -\mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(\tau_{\text{def}}) D_r(t_0, \tau_{\text{def}}) \text{PE}(\tau_{\text{def}}) \right]. \quad (22)$$

In order to be precise, there exist at least two versions for the CVA: The *unilateral CVA* only takes into account the counterparty default, while the *bilateral CVA* also allows for a default of the bank itself. In this work, only the unilateral CVA is considered.

All the valuation adjustments mentioned in this work can be derived using the technique of *replication* [6]. For the unilateral CVA, an expression can be derived in terms of the positive exposure PE and survival probability SP_C of the counterparty as

$$\text{CVA} = -\int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) D_r(t_0, t) \text{PE}(t) \right] dt, \quad (23)$$

where the *loss given default* (LGD) of the counterparty C is defined in terms of the counterparty's recovery rate R_C as

$$\text{LGD}_C(t) = 1 - R_C(t). \quad (24)$$

A fact pointed out in Ref. [6] is that when going from Eq. (22) to Eq. (37), the implicit assumption is made that the expectation of the exposure (and other variables such as LGD) *at default* is equal to the *unconditional* expectation,

$$\mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) D_r(t_0, t) \text{PE}(t) \mid t = \tau_{\text{def}} \right] = \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) D_r(t_0, t) \text{PE}(t) \right]. \quad (25)$$

Furthermore, it is common practice to treat the recovery rate as constant and the hazard rate as deterministic, leading to the form

$$\text{CVA} = -(1 - R_C) \int_{t_0}^T \lambda_C(t) \text{SP}_C(t_0, t) D_r(t_0, t) \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{PE}(t) \right] dt. \quad (26)$$

While this expression of CVA is frequently cited, the assumptions that entered its derivation should be stressed and that the more complete version of CVA is given by Eq. (23) [7].

3.2.1 Discretized CVA Expression

In practical computations of XVAs using a finite simulation grid, the integral expressions are discretized with nodes matching the simulation dates. Straightforward discretization of the integral expression in Eq. (23) leads to

$$\text{CVA} \approx - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \lambda_C(t_i) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{PE}(t_i) \right] \Delta t_i \quad (27)$$

with $\Delta t_i = t_{i+1} - t_i$. From Eq. (4) it follows that

$$\lambda_C(t_i) \text{SP}_C(t_0, t_i) \Delta t_i = \text{SP}_C(t_0, t_i) - \text{SP}_C(t_0, t_{i+1}). \quad (28)$$

Therefore, the CVA expression in Eq. (27) becomes

$$\text{CVA} \approx - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \left(\text{SP}_C(t_0, t_i) - \text{SP}_C(t_0, t_{i+1}) \right) D_r(t_0, t_i) \text{PE}(t_i) \right]. \quad (29)$$

3.3 Margin Valuation Adjustment

As discussed in Sec. 2.5, OTC derivatives trading requires to post IM under the EMIR regulation. The cost (or benefit) of posting IM is quantified by the MVA. The two effects of providing IM are the cost of *funding* the IM payments over time, and the *yields* earned on the IM account, which can be summarized by the following formula

$$\text{MVA} = - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\left(\text{LGD}_B(t) \lambda_B(t) - s_I(t) \right) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) D_r(t_0, t) \text{IM}_B(t) \right] dt. \quad (30)$$

Because the hazard rate and loss given default can be linked to bank-issued zero-recovery zero-coupon bond prices [23], the term $\text{LGD}_B(t) \lambda_B(t)$ can be identified as the funding spread

$$s_F(t) = \text{LGD}_B(t) \lambda_B(t). \quad (31)$$

Therefore, the term involving LGD_B represents the funding cost of bank-posted IM. Furthermore, s_I in Eq. (30) refers to the spread over the risk-free rate earned by the IM account, and the corresponding term involving s_I consequently represents the benefit of posting IM. Because IM payments only take place while both the bank and counterparty have not yet defaulted, the cost and benefit terms are weighted with the corresponding survival probabilities.

It should be noted that in Eq. (30) only the IM posted by the bank appears, and not the IM posted by the counterparty. This way, this definition of MVA holds for both bilateral and central clearing. The effect of the IM posting of the counterparty is usually included in the definition of the ColVA

$$\text{ColVA} = - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\left(s_X(t) X(t) + r_{\text{IM}_C}(t) \text{IM}_C(t) \right) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) \right] dt, \quad (32)$$

where $X(t)$ refers to the collateral amount at time t , $s_X(t)$ refers to the corresponding funding spread, and $r_{\text{IM}_C}(t)$ to the yield earned by $\text{IM}_C(t)$.

3.3.1 Discretized MVA Expression

The discretization of the integral expression for MVA in Eq. (30) can be achieved analogous to the case of CVA. Straightforward discretization of the integral leads to

$$\begin{aligned}
\text{MVA} &\approx - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\left(\text{LGD}_B(t_i) \lambda_B(t_i) - s_I(t_i) \right) \text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{IM}_B(t_i) \right] \Delta t_i \\
&= - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t_i) \lambda_B(t_i) \text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{IM}_B(t_i) \right] \Delta t_i \\
&\quad + \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[s_I(t_i) \text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{IM}_B(t_i) \right] \Delta t_i, \tag{33}
\end{aligned}$$

where in the first term the hazard rate can again be eliminated in favor of a difference of survival probabilities

$$\lambda_B(t_i) \text{SP}_B(t_0, t_i) \Delta t_i = \text{SP}_B(t_0, t_i) - \text{SP}_B(t_0, t_{i+1}), \tag{34}$$

arriving at

$$\begin{aligned}
\text{MVA} &\approx - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t_i) \left(\text{SP}_B(t_0, t_i) - \text{SP}_B(t_0, t_{i+1}) \right) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{IM}_B(t_i) \right] \\
&\quad + \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[s_I(t_i) \text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{IM}_B(t_i) \right] \Delta t_i. \tag{35}
\end{aligned}$$

3.3.2 Constant-Spread Approximation

As stated in Ref. [24], initial margin typically earns little or no interest over the risk-free rate. This fact suggests to set the spread s_I on initial margin to zero in practical calculations, as is done, for instance, in Ref. [25]. This approach eliminates the second term in Eq. (35).

In the present work, the spread $s_I(t)$ will not be neglected, but treated as *constant*. This way, no sophisticated modelling of the spread is required, and also the correlation structure of the terms under the expectation is simplified, facilitating the incorporation of wrong-way risk in later chapters. This way, an estimate of the size of the contributions involving s_I to MVA can be obtained. Therefore, the expression for MVA used in the practical calculations in this work is given by

$$\begin{aligned}
\text{MVA} &\approx - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t_i) \left(\text{SP}_B(t_0, t_i) - \text{SP}_B(t_0, t_{i+1}) \right) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{IM}_B(t_i) \right] \\
&\quad + s_I \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i) D_r(t_0, t_i) \text{IM}_B(t_i) \right] \Delta t_i. \tag{36}
\end{aligned}$$

4 Wrong-Way Risk and Valuation Adjustments

4.1 Wrong-Way Risk

Wrong-way risk (WWR) becomes an increasingly relevant aspect in the quantification of credit risk. In their Basel III framework [1], the regulators acknowledge its importance by recommending that banks identify and address their wrong-way risk. Stress tests must be designed to identify

risk factors that are correlated with credit quality, and also the calculation of exposure amounts is affected by WWR [1]. In its *Letter to Richard Gresser* [26], the ISDA published the most popular and widely used definition of WWR. By that definition, wrong-way risk refers to a situation in which the exposure to a counterparty is adversely related to its probability of default. The ISDA distinguishes two types of WWR: *General* WWR is driven by macroeconomic relationships, such as interest rates or inflation levels. On the other hand, *specific* WWR emerges due to a situation that is specific to the counterparty. This situation might be a pending legal matter, a credit rating downgrade, and so on. Also, entering a very large trade with a counterparty, where meeting its obligations could lead to the counterparty's default, falls under specific WWR. This work will deal with general WWR, by taking into account historical correlations of market observables and default probabilities.

In this work, portfolios of interest rate and FX products are considered. The main drivers of the values for these portfolios (and consequently exposures) are interest rates and exchange rates. In order to verify whether or not WWR can be observed in the real-world for such portfolios, correlations between the survival probabilities and the interest rates for maturities ranging from 1 year up to 30 years are presented in Fig. 4.1. The survival probabilities have been bootstrapped from CDS spreads for the financial sector. The correlations have been calculated from daily market data over six-month periods. Sizeable correlations are clearly visible in Fig. 4.1, indicating that interest rates and survival probabilities do in fact have some dependence.

4.2 Including WWR in XVA Calculations

Various approaches of taking WWR into account for XVA calculations have been developed over the years, and a short overview follows. As discussed in Sec. 3.2, the dependence between the exposure and the default event is neglected if expectations conditional on default are replaced with their unconditional counterparts. A CVA formulation that still fully includes WWR is given by

$$\text{CVA} = - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) D_r(t_0, t) \text{PE}(t) \mid t = \tau_{\text{def}} \right] dt, \quad (37)$$

where the challenge lies in modelling the dependence between exposure and the credit events.

In a simple hazard rate model [27], a direct link between the conditional default probability and the exposure may be parametrically introduced by the relation

$$\lambda(t) = \ln \left[1 + \exp \left\{ a(t) + b(t) \text{PE}(t) \right\} \right]. \quad (38)$$

More advanced hazard rate models postulate stochastic processes for the hazard rate which are correlated to other processes (see, *e.g.* [28]).

Copula models, which impose an analytic form on the joint distribution for exposure and default, also have a long tradition of being employed in CVA calculations [29, 30, 31].

More recently, a sophisticated yet tractable method was proposed for capturing WWR in CVA calculations by means of infinite changes of measure [32].

4.2.1 Model-Independent Wrong-Way Risk for XVA

In this work a *model-independent* treatment of WWR in XVA calculations is employed that was introduced by Kenyon *et. al* [11]. The idea behind the approach is that general XVA expressions

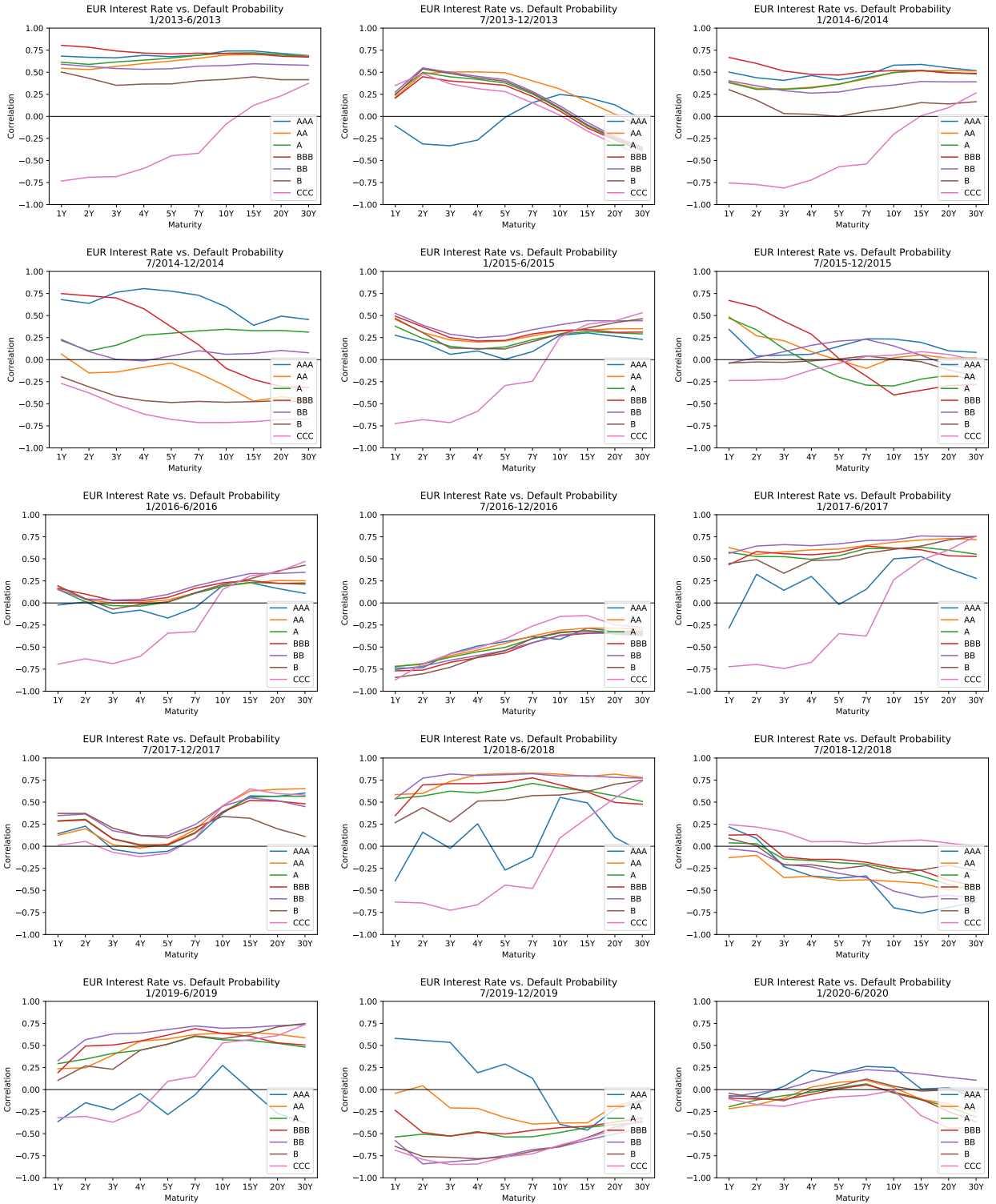


Figure 4.1: Correlations over six-month intervals between the EUR interest rate and the survival probabilities for financial companies with credit ratings ranging from AAA to CCC.

take the form

$$\text{XVA} = \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[A(t) \cdot B(t) \cdots Z(t) \right] dt. \quad (39)$$

The difficulty in Eq. (39) lies in modelling the joint distribution of all the quantities $A(t), B(t), \dots, Z(t)$. By iterative application of the relation

$$\mathbb{E}[XY] = \mathbb{E}[X] \mathbb{E}[Y] + \text{Corr}[X, Y] \sqrt{\text{Var}[X]} \sqrt{\text{Var}[Y]} \quad (40)$$

which follows immediately from the definitions of covariance

$$\text{Cov}[X, Y] = \mathbb{E} \left[\left(X - \mathbb{E}[X] \right) \left(Y - \mathbb{E}[Y] \right) \right] \quad (41)$$

and correlation

$$\text{Corr}[X, Y] = \frac{\text{Cov}[X, Y]}{\sqrt{\text{Var}[X]} \sqrt{\text{Var}[Y]}}, \quad (42)$$

a disentanglement of the quantities under the expectation may be achieved. At the same time, correlations appear in the expressions. By conveniently grouping quantities $A(t), B(t), \dots, Z(t)$ together, correlations between exposure and default quantities may be introduced. This is precisely the type of dependence that is relevant for capturing WWR. The challenge of this method lies in the calibration step, where the correlations and variances are calculated.

4.3 Model-Independent Wrong-Way Risk for CVA

In order to include the model-independent WWR into CVA, the valuation adjustment is written in a form that corresponds to Eq. (39),

$$\text{CVA} = - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) D_r(t_0, t) \text{PE}(t) \right] dt. \quad (43)$$

As mentioned in Sec. 4.2.1, the expectations of the term $D_r(t_0, t) \text{PE}(t)$ related to the exposure, and the term $\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t)$ related to default probabilities are separated by means of Eq. (40), leading to

$$\begin{aligned} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) D_r(t_0, t) \text{PE}(t) \right] = \\ \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) \right] \mathbb{E}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{PE}(t) \right] \\ + \rho_{\text{CVA}}^{\mathbb{Q}}(t) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) \right]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{PE}(t) \right]}, \end{aligned} \quad (44)$$

where the correlation between the exposure and default quantities is given by

$$\rho_{\text{CVA}}^{\mathbb{Q}}(t) = \text{Corr}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t), D_r(t_0, t) \text{PE}(t) \right]. \quad (45)$$

Plugging Eq. (44) into Eq. (43), the working equations for WWR-CVA are obtained as

$$\begin{aligned} \text{CVA} = - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) \right] \mathbb{E}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{PE}(t) \right] dt \\ - \int_{t_0}^T \rho_{\text{CVA}}^{\mathbb{Q}}(t) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) \right]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{PE}(t) \right]} dt. \end{aligned} \quad (46)$$

It should be noted that the first term corresponds to an expression for CVA without WWR. Since Eqs. (46) and (43) are equivalent, the full effect of incorporating WWR into CVA must reside in the second term. This suggests the following identification of terms in Eq. (46)

$$\text{CVA} = \text{CVA}^{(\text{no WWR})} + \text{CVA}^{(\text{WWR only})} \quad (47)$$

where

$$\text{CVA}^{(\text{no WWR})} = - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) \right] \mathbb{E}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{PE}(t) \right] dt \quad (48)$$

$$\text{CVA}^{(\text{WWR only})} = - \int_{t_0}^T \hat{\rho}_{\text{CVA}}^{\mathbb{Q}}(t) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t) \lambda_C(t) \text{SP}_C(t_0, t) \right]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{PE}(t) \right]} dt. \quad (49)$$

4.3.1 Wrong-Way Risk for the Discretized CVA Expression

For the discretized case, the expression for CVA is given by Eq. (29),

$$\widehat{\text{CVA}} = - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \left(\text{SP}_C(t_0, t_i) - \text{SP}_C(t_0, t_{i+1}) \right) D_r(t_0, t_i) \text{PE}(t_i) \right], \quad (50)$$

where the value of the discretized CVA is denoted as $\widehat{\text{CVA}}$. Proceeding as for the integral expressions, the following expression for the discretized CVA is obtained

$$\widehat{\text{CVA}} = \widehat{\text{CVA}}^{(\text{no WWR})} + \widehat{\text{CVA}}^{(\text{WWR only})}, \quad (51)$$

where the terms $\widehat{\text{CVA}}^{(\text{no WWR})}$ and $\widehat{\text{CVA}}^{(\text{WWR only})}$ are defined as

$$\widehat{\text{CVA}}^{(\text{no WWR})} = - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \left(\text{SP}_C(t_0, t_i) - \text{SP}_C(t_0, t_{i+1}) \right) \right] \mathbb{E}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t_i) \text{PE}(t_i) \right] \quad (52)$$

$$\begin{aligned} \widehat{\text{CVA}}^{(\text{WWR only})} &= - \sum_{i=0}^{n-1} \hat{\rho}_{\text{CVA}}^{\mathbb{Q}}(t_i) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \left(\text{SP}_C(t_0, t_i) - \text{SP}_C(t_0, t_{i+1}) \right) \right]} \\ &\quad \times \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t_i) \text{PE}(t_i) \right]}. \end{aligned} \quad (53)$$

For deriving Eq. (53) it is noted that

$$\begin{aligned} &\sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \lambda_C(t_i) \text{SP}_C(t_0, t_i) \right] \Delta t_i} \\ &= \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \lambda_C(t_i) \text{SP}_C(t_0, t_i) \right] \Delta t_i^2} \\ &= \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \lambda_C(t_i) \text{SP}_C(t_0, t_i) \Delta t_i \right]} \\ &= \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \left(\text{SP}_C(t_0, t_i) - \text{SP}_C(t_0, t_{i+1}) \right) \right]}. \end{aligned} \quad (54)$$

The correlation between the exposure and default quantities is given for the discretized case as

$$\hat{\rho}_{\text{CVA}}^{\mathbb{Q}}(t_i) = \text{Corr}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_C(t_i) \left(\text{SP}_C(t_0, t_i) - \text{SP}_C(t_0, t_{i+1}) \right), D_r(t_0, t_i) \text{PE}(t_i) \right]. \quad (55)$$

4.4 Model-Independent Wrong-Way Risk for MVA

Starting from the general expression for the MVA

$$\text{MVA} = - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\left(\text{LGD}_B(t) \lambda_B(t) - s_I(t) \right) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) D_r(t_0, t_i) \text{IM}_B(t) \right] dt, \quad (56)$$

this formula is split into terms containing and not containing the initial margin spread $s_I(t)$. Furthermore, the quantities within the expectations are grouped according to the curly brackets in the following expression

$$\begin{aligned} \text{MVA} &= - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\left\{ \text{LGD}_B(t) \lambda_B(t) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) \right\} \left\{ D_r(t_0, t_i) \text{IM}_B(t) \right\} \right] dt \\ &\quad + \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\left\{ s_I(t) \right\} \left\{ \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) \right\} \left\{ D_r(t_0, t_i) \text{IM}_B(t) \right\} \right] dt \end{aligned} \quad (57)$$

$$\equiv \text{MVA}_1 + \text{MVA}_2. \quad (58)$$

These groups separate quantities related to exposures, survival probabilities, and IM spread.

The first term, here referred to as MVA_1 , can be treated precisely as for CVA, leading to

$$\begin{aligned} \text{MVA}_1 &= - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t) \lambda_B(t) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) \right] \mathbb{E}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{IM}_B(t) \right] dt \\ &\quad - \int_{t_0}^T \rho_{\text{MVA}_1}^{\mathbb{Q}}(t) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t) \lambda_B(t) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) \right]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t) \text{IM}_B(t) \right]} dt. \end{aligned} \quad (59)$$

The correlation between default and margin quantities is given by

$$\rho_{\text{MVA}_1}^{\mathbb{Q}}(t) = \text{Corr}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t) \lambda_B(t) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t), D_r(t_0, t) \text{IM}_B(t) \right]. \quad (60)$$

The second term, here referred to as MVA_2 , contains variables that can be divided into three categories: The initial margin spread, default probabilities, and the initial margin amount. Because there are three categories, Eq. (40) can be applied twice, leading to a choice of which categories will be grouped together in the first application of Eq. (40). As stated in Ref. [11], for three categories A, B, C , the different groupings lead to three equivalent expressions

$$\begin{aligned} \mathbb{E}[ABC] &= \mathbb{E}[A]\mathbb{E}[B]\mathbb{E}[C] + \mathbb{E}[A] \rho_{B,C} \sqrt{\text{Var}[B]} \sqrt{\text{Var}[C]} + \rho_{A,BC} \sqrt{\text{Var}[A]} \sqrt{\text{Var}[BC]} \end{aligned} \quad (61)$$

$$= \mathbb{E}[A]\mathbb{E}[B]\mathbb{E}[C] + \mathbb{E}[B] \rho_{A,C} \sqrt{\text{Var}[A]} \sqrt{\text{Var}[C]} + \rho_{B,AC} \sqrt{\text{Var}[B]} \sqrt{\text{Var}[AC]} \quad (62)$$

$$= \mathbb{E}[A]\mathbb{E}[B]\mathbb{E}[C] + \mathbb{E}[C] \rho_{A,B} \sqrt{\text{Var}[A]} \sqrt{\text{Var}[B]} + \rho_{C,AB} \sqrt{\text{Var}[C]} \sqrt{\text{Var}[AB]} \quad (63)$$

where a short-hand notation for the correlations was introduced. The expressions corresponding to choices Eq. (61)-(63) are given by (where time arguments have been dropped for readability)

$$\begin{aligned} &\mathbb{E}_{t_0}^{\mathbb{Q}} \left[\{s_I\} \{ \text{SP}_B \text{SP}_C \} \{ D_r \text{IM}_B \} \right] \\ &= \mathbb{E}_{t_0}^{\mathbb{Q}} [s_I] \mathbb{E}_{t_0}^{\mathbb{Q}} [\text{SP}_B \text{SP}_C] \mathbb{E}_{t_0}^{\mathbb{Q}} [D_r \text{IM}_B] \\ &\quad + \mathbb{E}_{t_0}^{\mathbb{Q}} [s_I] \text{Corr}_{t_0}^{\mathbb{Q}} [\text{SP}_B \text{SP}_C, D_r \text{IM}_B] \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} [\text{SP}_B \text{SP}_C]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} [D_r \text{IM}_B]} \\ &\quad + \text{Corr}_{t_0}^{\mathbb{Q}} [s_I, \text{SP}_B \text{SP}_C D_r \text{IM}_B] \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} [s_I]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} [\text{SP}_B \text{SP}_C D_r \text{IM}_B]}, \end{aligned} \quad (64)$$

$$\begin{aligned}
& \mathbb{E}_{t_0}^{\mathbb{Q}}[\{s_I\} \{SP_B SP_C\} \{D_r IM_B\}] \\
&= \mathbb{E}_{t_0}^{\mathbb{Q}}[s_I] \mathbb{E}_{t_0}^{\mathbb{Q}}[SP_B SP_C] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r IM_B] \\
&+ \mathbb{E}_{t_0}^{\mathbb{Q}}[SP_B SP_C] \text{Corr}_{t_0}^{\mathbb{Q}}[s_I, D_r IM_B] \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[s_I]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r IM_B]} \\
&+ \text{Corr}_{t_0}^{\mathbb{Q}}[SP_B SP_C, s_I D_r IM_B] \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[SP_B SP_C]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[s_I D_r IM_B]}, \tag{65}
\end{aligned}$$

and

$$\begin{aligned}
& \mathbb{E}_{t_0}^{\mathbb{Q}}[\{s_I\} \{SP_B SP_C\} \{D_r IM_B\}] \\
&= \mathbb{E}_{t_0}^{\mathbb{Q}}[s_I] \mathbb{E}_{t_0}^{\mathbb{Q}}[SP_B SP_C] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r IM_B] \\
&+ \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r IM_B] \text{Corr}_{t_0}^{\mathbb{Q}}[s_I, SP_B SP_C] \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[s_I]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[SP_B SP_C]} \\
&+ \text{Corr}_{t_0}^{\mathbb{Q}}[D_r IM_B, s_I SP_B SP_C] \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r IM_B]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[s_I SP_B SP_C]}. \tag{66}
\end{aligned}$$

There is no obvious reason for favouring one choice over another. In the constant-spread approximation, however, all expressions become identical, because the variance of s_I vanishes. The expression employed in this work is therefore given by

$$\begin{aligned}
& \mathbb{E}_{t_0}^{\mathbb{Q}}[\{s_I\} \{SP_B SP_C\} \{D_r IM_B\}] \\
&\approx s_I \mathbb{E}_{t_0}^{\mathbb{Q}}[SP_B SP_C] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r IM_B] + s_I \rho_{MVA_2}^{\mathbb{Q}} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[SP_B SP_C]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r IM_B]}. \tag{67}
\end{aligned}$$

with the correlation

$$\rho_{MVA_2}^{\mathbb{Q}}(t) = \text{Corr}_{t_0}^{\mathbb{Q}}[SP_B(t_0, t) SP_C(t_0, t), D_r(t_0, t) IM_B(t)]. \tag{68}$$

Consequently, MVA₂ reads

$$\begin{aligned}
MVA_2 &= + s_I \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}}[SP_B(t_0, t) SP_C(t_0, t)] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r(t_0, t) IM_B(t)] dt \\
&+ s_I \int_{t_0}^T \rho_{MVA_2}^{\mathbb{Q}}(t) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[SP_B(t_0, t) SP_C(t_0, t)]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r(t_0, t) IM_B(t)]} dt. \tag{69}
\end{aligned}$$

With this, the final form of the WWR-MVA in constant-spread approximation reads

$$\begin{aligned}
MVA &= - \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}}[LGD_B(t) \lambda_B(t) SP_B(t_0, t) SP_C(t_0, t)] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r(t_0, t) IM_B(t)] dt \\
&- \int_{t_0}^T \rho_{MVA_1}^{\mathbb{Q}}(t) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[LGD_B(t) \lambda_B(t) SP_B(t_0, t) SP_C(t_0, t)]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r(t_0, t) IM_B(t)]} dt \\
&+ s_I \int_{t_0}^T \rho_{MVA_2}^{\mathbb{Q}}(t) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[SP_B(t_0, t) SP_C(t_0, t)]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r(t_0, t) IM_B(t)]} dt \\
&+ s_I \int_{t_0}^T \mathbb{E}_{t_0}^{\mathbb{Q}}[SP_B(t_0, t) SP_C(t_0, t)] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r(t_0, t) IM_B(t)] dt \tag{70}
\end{aligned}$$

where the correlations are given by

$$\rho_{MVA_1}^{\mathbb{Q}}(t) = \text{Corr}_{t_0}^{\mathbb{Q}}[LGD_B(t) \lambda_B(t) SP_B(t_0, t) SP_C(t_0, t), D_r(t_0, t) IM_B(t)] \tag{71}$$

$$\rho_{MVA_2}^{\mathbb{Q}}(t) = \text{Corr}_{t_0}^{\mathbb{Q}}[SP_B(t_0, t) SP_C(t_0, t), D_r(t_0, t) IM_B(t)]. \tag{72}$$

4.4.1 Wrong-Way Risk for the Discretized MVA Expression

The final step towards a MVA formula that may be implemented for practical calculations is to discretize Eq. (70) in the usual manner. Again, the relation

$$\lambda_B(t_i) \text{SP}_B(t_0, t_i) \Delta t_i = \text{SP}_B(t_0, t_i) - \text{SP}_B(t_0, t_{i+1}) \quad (73)$$

is used in order to remove the hazard rate in favor of survival probability differences. The resulting expression reads

$$\begin{aligned} \widehat{\text{MVA}} &\approx - \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t_i) \left(\text{SP}_B(t_0, t_i) - \text{SP}_B(t_0, t_{i+1}) \right) \text{SP}_C(t_0, t_i) \right] \mathbb{E}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t_i) \text{IM}_B(t_i) \right] \\ &\quad - \sum_{i=0}^{n-1} \hat{\rho}_{\text{MVA}_1}^{\mathbb{Q}}(t_i) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t_i) \left(\text{SP}_B(t_0, t_i) - \text{SP}_B(t_0, t_{i+1}) \right) \text{SP}_C(t_0, t_i) \right]} \\ &\quad \quad \times \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t_i) \text{IM}_B(t_i) \right]} \\ &\quad + s_I \sum_{i=0}^{n-1} \hat{\rho}_{\text{MVA}_2}^{\mathbb{Q}}(t_i) \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[\text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i) \right]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t_i) \text{IM}_B(t_i) \right]} \Delta t_i \\ &\quad + s_I \sum_{i=0}^{n-1} \mathbb{E}_{t_0}^{\mathbb{Q}} \left[\text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i) \right] \mathbb{E}_{t_0}^{\mathbb{Q}} \left[D_r(t_0, t_i) \text{IM}_B(t_i) \right] \Delta t_i \end{aligned} \quad (74)$$

with correlations

$$\hat{\rho}_{\text{MVA}_1}^{\mathbb{Q}}(t_i) = \text{Corr}_{t_0}^{\mathbb{Q}} \left[\text{LGD}_B(t_i) \left(\text{SP}_B(t_0, t_i) - \text{SP}_B(t_0, t_{i+1}) \right) \text{SP}_C(t_0, t_i), D_r(t_0, t_i) \text{IM}_B(t_i) \right] \quad (75)$$

$$\hat{\rho}_{\text{MVA}_2}^{\mathbb{Q}}(t_i) = \text{Corr}_{t_0}^{\mathbb{Q}} \left[\text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i), D_r(t_0, t_i) \text{IM}_B(t_i) \right]. \quad (76)$$

4.5 Calibration

4.5.1 Exposure Simulation and Historical Calculations

In a typical XVA framework, in order to calculate valuation adjustments at some given valuation date t_0 , a Monte Carlo engine is used to generate mark-to-market values of the portfolio. From these mark-to-market values, exposures and collateral is calculated, from which the XVA are subsequently aggregated. Therefore, PE and IM directly follow from the simulation, and by averaging over paths the expectations $\mathbb{E}_{t_0}^{\mathbb{Q}}[D_r(t_0, t_i) \text{PE}(t_i)]$ and $\mathbb{E}_{t_0}^{\mathbb{Q}}[D_r(t_0, t) \text{IM}_B(t)]$ can be calculated. Variances of PE and IM can be calculated from the path statistics as well.

Default probabilities are usually not simulated. Rather, the default probability term structure bootstrapped from current CDS quotes is employed. Because of this, the probabilities are deterministic and do not depend on the Monte Carlo paths. Variances of these probabilities over the paths obviously vanish, as well as the correlations. Thus, another approach is needed in order to estimate the term structures involving quantities that are deterministic in the exposure simulation.

Here, the procedure outlined in Ref. [11] is followed. In order to obtain the variances and correlations involving the default probabilities, a number of historical calculation is performed. Table 4.1 lists all relevant terms for performing WWR-CVA and WWR-MVA calculations, and how this terms are determined: Either directly from the exposure simulation, or from the historical calculations. In Sec. 4.5.2 a detailed description of the calibration procedure is given.

Term	Expression	Source
Exposures		
Positive Exposure	PE	Simulation
Initial Margin	IM _B	Simulation
Correlations		
Default-PE	Corr[LGD _C λ _C SP _C , D _r PE]	Historical
Default-IM	Corr[LGD _B λ _B SP _B SP _C , D _r IM _B]	Historical
Probability-IM	Corr[SP _B SP _C , D _r IM _B]	Historical
Variations		
PE	Var[D _r PE]	Simulation
IM	Var[D _r IM _B]	Simulation
Default	Var[LGD _C λ _C SP _C]	Historical
Default	Var[LGD _B λ _B SP _B SP _C]	Historical
Default	Var[SP _B SP _C]	Historical

Table 4.1: Quantities relevant for CVA and MVA calculations and the way they are determined. Terms involving survival probabilities are calculated from historical data.

4.5.2 Calibration Procedure

The correlations that appear in the expressions for WWR-CVA and MVA always measure the dependence between some simulated measure \mathcal{M} and some non-simulated quantity \mathcal{P} involving probabilities. In the calculations, the correlation term structure is required over the full simulation horizon, *i.e.*,

$$\left\{ \rho(t) = \text{Corr}[\mathcal{M}_t, \mathcal{P}_t] : t \in [t_0, T] \right\}. \quad (77)$$

These correlations will be calculated for all dates t_i of the simulation time grid. At this point in the discussion, an interpretation should be given to $\rho(t)$. If a trade is initiated at the simulation start date t_0 and matures at T , then $\rho(t)$ measures the dependence of the quantities \mathcal{M}_t and \mathcal{P}_t a time interval $t - t_0$ after the trade was initiated. It is therefore reasonable to introduce a time measure τ that is *relative* to the state of the trade. In the example given, such a measure could be the lifetime of the trade

$$\text{Corr}[\mathcal{M}_\tau, \mathcal{P}_\tau], \tau \in [0, H] \quad (78)$$

where H is the simulation horizon, which should correspond to the maturity of the trade.

For given τ , a set of data pairs

$$\left\{ (\mathcal{M}_\tau^{(1)}, \mathcal{P}_\tau^{(1)}), (\mathcal{M}_\tau^{(2)}, \mathcal{P}_\tau^{(2)}), \dots, (\mathcal{M}_\tau^{(n)}, \mathcal{P}_\tau^{(n)}) \right\} \quad (79)$$

is required in order to estimate the correlation Eq. (78). The data pairs in Eq. (79) are obtained from performing n exposure simulations over the horizon H , each starting at a different date $t_0^{(k)}$. Each simulation is calibrated to the time- $t_0^{(k)}$ market data. Therefore, the calibration includes market information over the time interval $\mathcal{T}_{\text{calib}} = [t_0^{(1)}, t_0^{(n)}]$. The portfolio is held constant for these simulations, *i.e.*, the internal state of the portfolio is the same for all of these runs. For instance, the

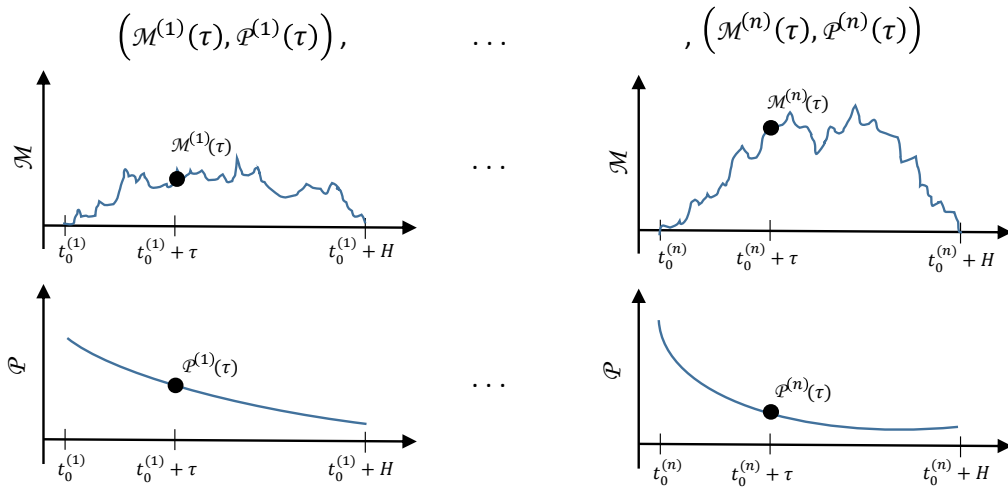


Figure 4.2: Calibration of the model-independent WWR-XVA method. For given time variable τ , data pairs of exposures and probabilities are calculated by performing historical calculations using a portfolio that is held constant. These data pairs are subsequently used to calculate correlations.

vanilla interest-rate swaps that are considered in later chapters, are always initiated at the respective simulation start date $t_0^{(k)}$ and at the respective time- $t_0^{(k)}$ fair swap rate. This procedure is illustrated in Fig. 4.2. Besides correlations, the data pairs in Eq. (79) can also be used to calculate the variance of \mathcal{M}_τ and \mathcal{P}_τ .

5 Results

5.1 Computational Parameters

For the calculations performed in this work, the C++ pricing library "MoCo" [33] from d-fine GmbH is used as the starting point for the implementation. MoCo provides an Excel interface that allows to set up financial objects such as interest-rate curves, swaption volatility surfaces, trades, and to run exposure simulations with subsequent XVA aggregation. In order to perform the historical calculations, all steps needed to be automated to avoid the manual labor of setting up hundreds of calculations.

The calibration interval is traversed using a two-week time grid, resulting in 104 grid points. This time spacing gives a sufficient number of data points for the calculation of statistical quantities such as correlations and variances. For each calculation, a hybrid IR-FX Hull-White model (HWnFX) is calibrated using market data for the respective valuation date. The FX correlations that are needed as input for the HWnFx model are determined using time series going back one year from the respective date for which the model is calibrated. The individual interest-rate Hull-White models are calibrated to co-terminal swaptions with maturities up to the calibration horizon. For products with maturities up to 10 years, this horizon is 15 years, for the 30-year products it is set to 31 years.

For the exposure and collateral simulation, a two-week time grid is employed as well. This choice results in 120 grid points for 5-year contracts up to 720 grid points for the 30-year contracts. Monte Carlo paths in the range from 1000 up to 5000 are employed, depending on the computational effort,

see Sec. 5.2.2. Analytical pricers are used to calculate the contract values at the simulation dates. For the collateral calculations, the CSA parameters from Table 2.1 are employed.

A recovery rate of 0.4 is assumed throughout.

All exposure calculations and XVA aggregation are performed for the valuation date 2020/07/01.

5.2 Calibration

5.2.1 Calibration Interval

For the calibration interval a sufficiently large time period of at least a year should be chosen. Different choices lead to calibration to different market environments. Wrong-way risk calculations are most likely performed for risk-management purposes, for which a pessimistic view on the market is appropriate in order to obtain conservative risk estimates. Using real-world market data, the ideal time period is of course given by the Financial Crisis from 2008-2012. The size and volatility of CDS spreads are regarded as an indicator for financial turmoil, so the choice of calibration interval can be guided by these considerations. In this work, due to limited availability of market data, the period from 2013-2016 is employed. During these years, increased volatility of CDS spreads can in fact be observed, as shown in Fig. 5.1. However, these CDS spreads and volatilities are far from their Crisis levels. In order to simulate a time period of financial stress, a crisis-scenario will also be considered later on, in which the 2013-2016 CDS spreads are scaled by a factor of 20.

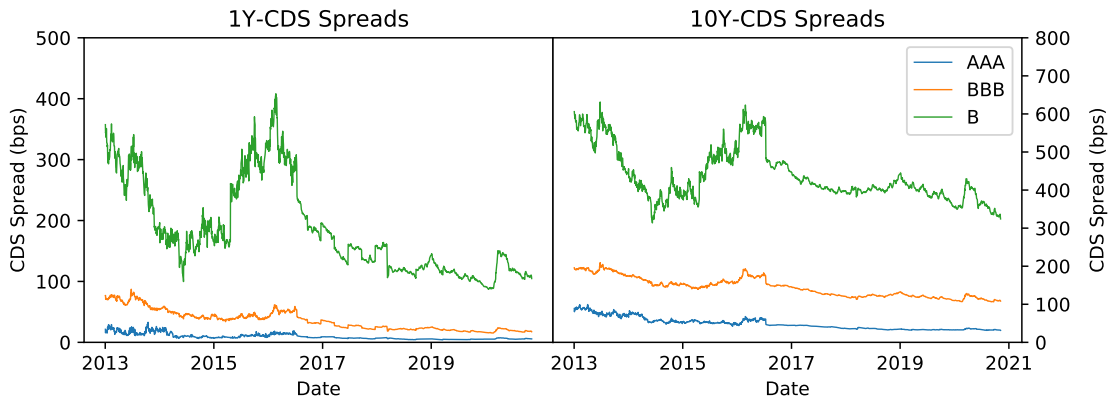


Figure 5.1: CDS spreads from the time period 2013 - 2021. Increased CDS spreads and volatility is observed from 2013 - 16.

5.2.2 Monte Carlo Paths

Due to the large number of historical calculations that need to be performed, the calibration procedure is computationally involved. In order to reduce the computational cost, the number of Monte Carlo paths in the exposure simulation is reduced during the calibration calculations. The top panel of Fig. 5.3 presents for the 5-year EUR interest-rate swap the EPE and IM_B variances for 1000, 2500 and 5000 Monte Carlo paths. Only small differences are found, in particular between the results for 2500 and 5000 paths. The differences in the correlations for CVA and MVA are not visible on the employed scale (not shown in Fig. 5.3). Finally, variances of the probabilities do not depend on the number of Monte Carlo paths, because they are determined solely from historical data. Therefore,

calibration calculations will usually be performed with 2500 paths. For very long-running products with 30 years maturity, however, 1000 paths will be employed in order to reduce the computational effort.

5.2.3 Correlations and Variances

The middle panel of Fig. 5.3 presents for the 5-year EUR swap the correlations $\hat{\rho}_{CVA}^Q$, $\hat{\rho}_{MVA_1}^Q$, and $\hat{\rho}_{MVA_2}^Q$ for various combinations of bank and counterparty ratings (the notation in the plot legend is *bank rating/counterparty rating*). In this particular case, the correlations are particularly pronounced for the AAA/CCC and CCC/BBB ratings pairs. Other combinations show correlations of other magnitude or signs. Overall, the correlations are significantly different from zero.

The bottom panel of Fig. 5.3 shows the various variances of the default-related quantities needed for WWR-CVA and WWR-MVA calculations. The standard deviation of $SP_B SP_C$ is of the order of 10^{-2} . This quantity represents the variance of the product of survival probabilities at the various simulation dates. Because the survival probabilities must be 1 at the simulation start date, the variance of $SP_B SP_C$ starts at zero and increases with time. For $\tau \rightarrow \infty$ the variances will tend to zero again, but not at the short time period considered in Fig. 5.3.

The two other standard deviations in the bottom panel involving the hazard rate contain Δt and consequently depend on the simulation grid. The simulation time grid has a spacing of 2 weeks, and therefore the two remaining standard deviations correspond to the quantities

$$SP_C(t_0, t) - SP_C(t_0, t + 2 \text{ weeks}) \quad (80)$$

and

$$\left(SP_B(t_0, t) - SP_B(t_0, t + 2 \text{ weeks}) \right) SP_C(t_0, t). \quad (81)$$

Given the short time span between the simulation dates, these variances are expected to be rather small, and indeed they are found to be of the order of 10^{-4} .

It is noteworthy in Fig. 5.3 that the correlations $\hat{\rho}_{MVA_2}^Q$ for the rating pairs AAA/CCC and CCC/BBB are almost identical, as well as for the rating pairs A/BB and BB/AAA. Recalling the definition of $\hat{\rho}_{MVA_2}^Q$,

$$\hat{\rho}_{MVA_2}^Q(t_i) = \text{Corr}_{t_0}^Q \left[SP_B(t_0, t_i) SP_C(t_0, t_i), D_\tau(t_0, t_i) IM_B(t_i) \right], \quad (82)$$

this observation can be understood if these rating pairs behave in a similar way. Evidence for this explanation can also be found in the $SP_B SP_C$ variance plot in Fig. 5.3, where the product of survival probabilities $SP_B SP_C$ for these rating pairs show very similar variances.

For the short European swaption, the positive exposure is zero until the option is exercised, as can be seen in Fig. 5.6. Therefore, the variance of PE is zero up to this point and, consequently, the correlation $\hat{\rho}_{CVA}^Q$ is not defined at these times. For calculating WWR-CVA, $\hat{\rho}_{CVA}^Q$ can be set to an arbitrary value up to the option exercise, because it will be multiplied with the zero variance of the exposure. This issue does not exist for long positions in swaptions, because in this case the positive exposure is non-zero already before the option exercise.

5Y EUR Swap

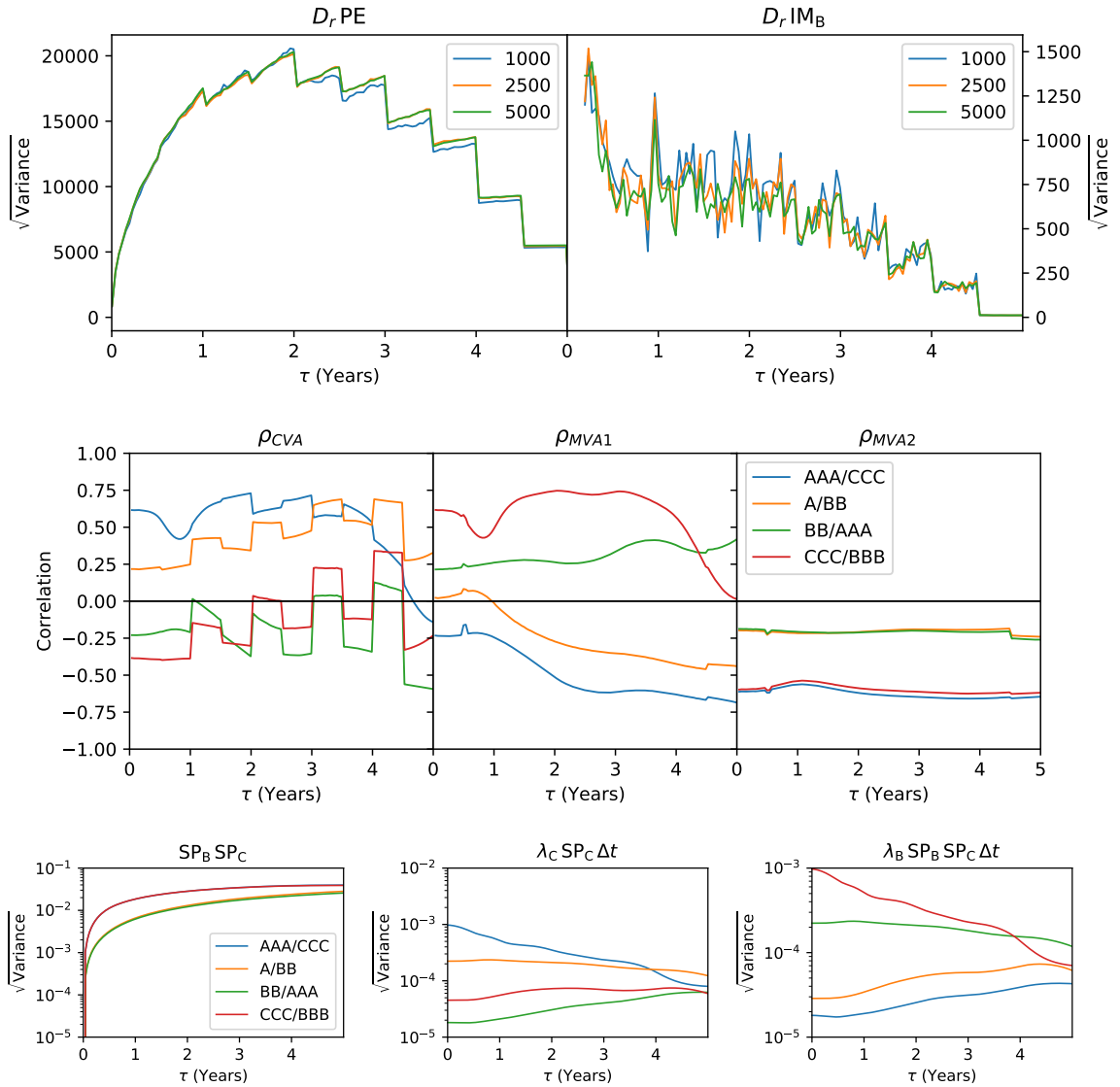


Figure 5.3: Calibration results for the 5-year EUR interest-rate swap. Top panel: Standard deviations of positive exposure and initial margin for various Monte Carlo paths. For 2500 paths, the results are sufficiently converged for calibration purposes (other quantities show no or only very little dependence on the Monte Carlo paths). Middle panel: Correlations $\hat{\rho}_{CVA}^Q$, $\hat{\rho}_{MVA_1}^Q$, and $\hat{\rho}_{MVA_2}^Q$ for various combinations of bank and counterparty ratings. Different rating combinations produce correlations of different magnitudes and signs. Bottom panel: Standard deviations of the relevant quantities involving survival probabilities. The quantities that contain the hazard rate depend on survival probability differences over the time spacing of the simulation grid, which is 2 weeks in this case.

30Y EUR Swap

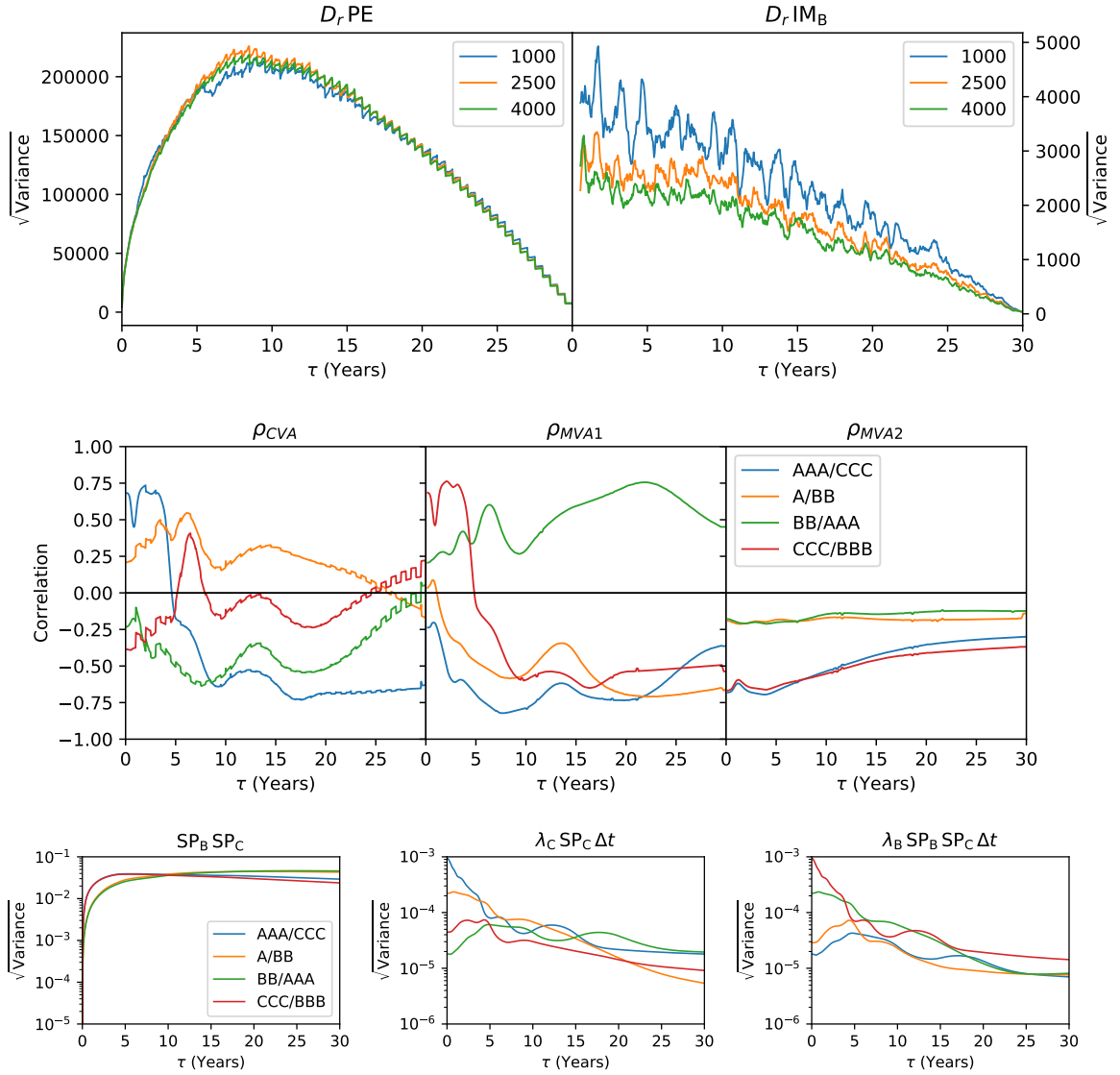


Figure 5.2: Calibration results for the 30-year EUR interest-rate swap.

5Y EUR-USD XCCY Swap

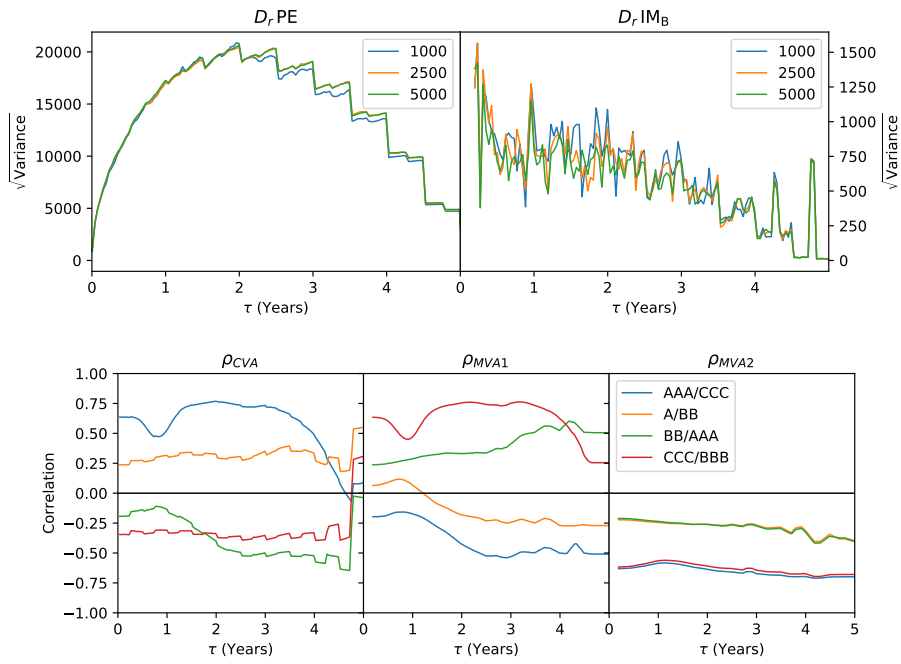


Figure 5.4: Calibration results for the 5-year EUR-USD cross-currency swap without notional exchange.

30Y EUR-USD XCCY Swap

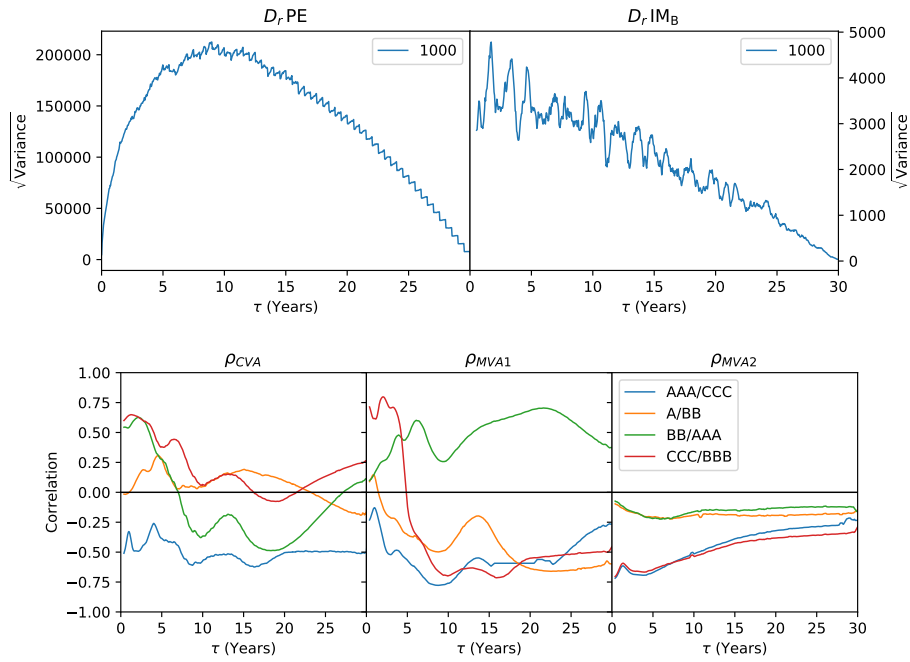


Figure 5.5: Calibration results for the 30-year EUR-USD cross-currency swap without notional exchange.

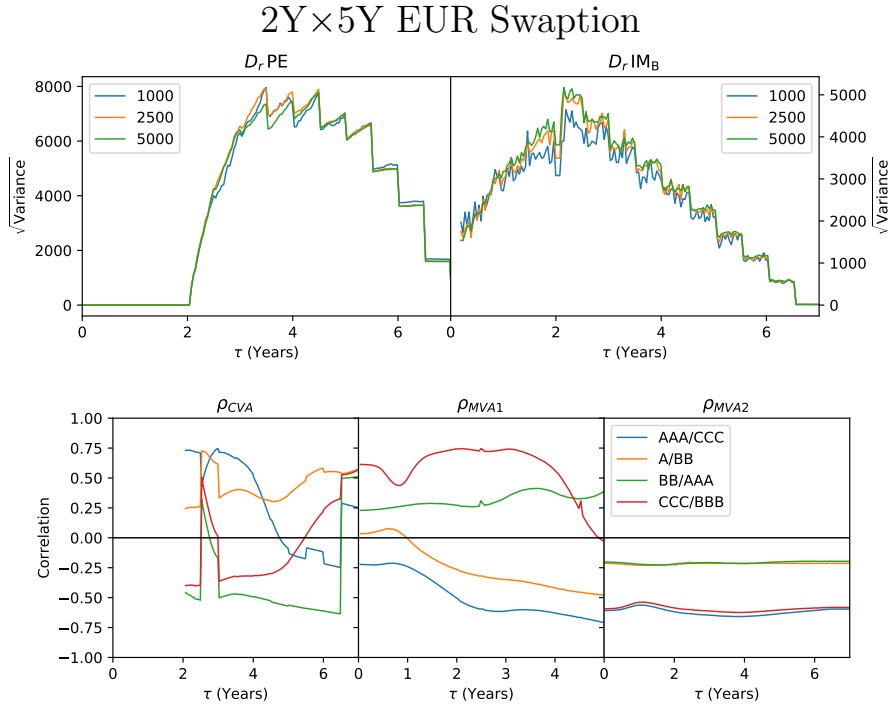


Figure 5.6: Calibration results for the short 2Y×5Y European EUR swaption. Up to the swaption maturity, PE is zero and ρ_{CVA}^Q is not defined.

5.3 Credit Valuation Adjustment

5.3.1 Interest-Rate Swaps

The numerical results for the uncollateralised unilateral CVA of the 5-year plain-vanilla EUR interest-rate swap are presented in Table 5.1. All results are quoted in basis points of the trade notional.

As expected, the no-WWR contribution becomes more negative with deteriorating credit quality of the counterparty, ranging from -0.48 for a counterparty with AAA rating to -16.23 for a counterparty with CCC rating. For the WWR contribution, the situation is more complex, and one cannot expect the contribution to be a monotonic function of credit quality.

Considering the CCC case, it is apparent from Fig. 5.3 (recalling the notation of the plot legend as *bank rating / counterparty rating*, the CCC case is given by the blue curve) that $\rho_{CVA}^Q(t)$ is positive and also rather large (around 0.5) for practically the whole life of the swap. Therefore, this is a clear case of WWR, and the WWR contribution to CVA is negative and sizeable. The WWR contribution is about 14 per cent of the no-WWR term.

For the AAA case (green curve in Fig. 5.3) on the other hand, $\rho_{CVA}^Q(t)$ is mostly negative, but not as large in magnitude as the CCC case. This is a *right-way risk* (RWR) scenario, and the WWR-only contribution to CVA is in fact positive. Here, a receiver swap is considered. For the case of a payer swap, the sign of the correlations would flip, and a WWR scenario becomes a RWR scenario and vice versa. Although the magnitude of the correlation is about half of the CCC case, the WWR-only effect is much smaller for the AAA case than for CCC. This is due to the fact that

the variance of the conditional default probabilities are much smaller for the AAA rating than for CCC, as can be seen from the $\lambda_C \text{SP}_C \Delta t$ plot in Fig. 5.3. This is a general observation: While there is typically strong *correlation* between exposure and default probability, the *covariance* is rather small due to the small variance of the probability quantities.

5Y EUR Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-0,48	-0,70	-1,11	-1,80	-3,55	-7,51	-16,23
WWR only	0,07	-0,08	0,00	0,08	-0,95	-1,18	-2,28

10Y EUR Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-4,72	-7,15	-10,11	-15,64	-26,30	-28,28	-33,45
WWR only	0,84	0,17	0,12	-0,04	-3,83	-3,42	-4,37

30Y EUR Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-56,19	-83,41	-110,38	-154,33	-213,98	-294,04	-299,17
WWR only	12,93	7,60	5,22	1,89	-14,24	-15,30	-2,87

Table 5.1: CVA results for the 5-year plain-vanilla EUR swap. The results are quoted in basis points of the respective notional.

For the 30-year swap, the long-term structure of the correlations becomes important. As shown in Fig. 5.2, for the CCC-rating $\rho_{CVA}^Q(t)$ changes sign after about 5 years and transitions from sizeable positive values to sizeable negative values of about -0.75 for the remaining 25 years of the swap. From this behaviour alone a positive contribution to CVA is expected. However, at the same time, the contributions after 5 years are suppressed by the small variance of the conditional probability, which decreases over time, such that the most important contribution to the CCC-rating WWR-only term stems from the first few years of the life of the trade. This is why the WWR-only contribution to the 30-year swap is very similar to the contribution to the 5-year swap. For the AAA rating, the variance of the conditional default probability is larger in the time span where the PE variance is also largest, leading to a rather large WWR- (or rather right-way risk) contribution to CVA.

5.3.2 Cross-Currency Swaps

For the cross-currency swap, the situation is analogue. The results can be found in Table A.1 in the Appendix. Because the FX exemption is taken into account, the exposure profiles are similar to that of a regular interest-rate swap. Furthermore, the variance of the default probabilities is identical for all products. The behaviour of $\rho_{CVA}^Q(t)$ is also similar, except for the CCC rating, where the correlation is positive over the life of the trade.

5.3.3 European Swaptions

For the European swaption, the observations are again similar to the 5-year EUR swap. The numerical results are presented in Table B.1 in the Appendix.

5.4 Margin Valuation Adjustment

There are four contributions to MVA, where two contributions come from each the expressions with and without WWR. For each of these cases, there is one contribution that involves s_I . In the following, the contributions that depend on s_I will be quoted for $s_I = 10$ bp. Because the IM spread is constant, the results for other choices of s_I can simply be obtained by rescaling the quoted results for $s_I = 10$ bp.

In order to facilitate the discussion of the numerical results, Table 5.2 lists the individual contributions to MVA together with the names by which they are referred to in the text.

Term	Expression
$MVA^{(1)}$	$-\int \mathbb{E}_{t_0}^{\mathbb{Q}}[\text{LGD}_B \lambda_B \text{SP}_B \text{SP}_C] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r \text{IM}_B] dt$
$MVA^{(2)}$	$+s_I \int \mathbb{E}_{t_0}^{\mathbb{Q}}[\text{SP}_B \text{SP}_C] \mathbb{E}_{t_0}^{\mathbb{Q}}[D_r \text{IM}_B] dt$
$MVA_{\text{WWR}}^{(1)}$	$-\int \rho_{\text{MVA}_1}^{\mathbb{Q}} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[\text{LGD}_B \lambda_B \text{SP}_B \text{SP}_C]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r \text{IM}_B]} dt$
$MVA_{\text{WWR}}^{(2)}$	$+s_I \int \rho_{\text{MVA}_2}^{\mathbb{Q}} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[\text{SP}_B \text{SP}_C]} \sqrt{\text{Var}_{t_0}^{\mathbb{Q}}[D_r \text{IM}_B]} dt$

Table 5.2: Contributions to WWR-MVA and the names by which they are referred to in the text.

5.4.1 Interest-Rate Swaps

The numerical MVA results for the 5-year EUR swap are presented in Table 5.3, and in the following the aspects of WWR-MVA are mainly discussed based on this example.

Because it represents the *cost* of funding IM, the term $MVA^{(1)}$ is generally negative. For deteriorating counterparty credit quality, the magnitude of $MVA^{(1)}$ decreases. This can be understood that from a IM-funding point of view, a counterparty default is beneficial because in that case the bank may stop posting IM and saves the associated costs. In the expression for $MVA^{(1)}$, this fact is taken into account by the counterparty survival probability under the integration sign. In Fig. 5.7 on the left, the survival probabilities for various credit ratings are presented over a time horizon of 30 years. However, this beneficial effect due to deteriorating counterparty credit quality is rather small compared to varying the credit rating of the bank.

The magnitude of $MVA^{(1)}$ increases with deteriorating credit quality of the bank. This effect is driven by the large changes in the conditional default probabilities $\lambda_B \text{SP}_B \Delta t$ with deteriorating credit quality, as presented in Fig. 5.7 on the right (the conditional default probability for the CCC rating has been rescaled in order to fit in the plot).

The term $MVA^{(2)}$ represents the returns earned on the IM account. In Table 5.3, the results involving the IM spread are quoted for $s_I = 10$ bp. The results for other funding spreads can be obtained by scaling the quoted results accordingly. Because $MVA^{(2)}$ measures the *benefit* from posting IM, this benefit decreases with both the bank and counterparty credit quality, since the IM account stops earning returns as soon as one party defaults. This is clear from the expression for $MVA^{(2)}$, where both survival probabilities SP_B and SP_C appear under the integration sign.

5Y EUR Swap

		MVA Term 1						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	-0,25	-0,25	-0,25	-0,25	-0,25	-0,24	-0,20
	AA	-0,37	-0,37	-0,37	-0,37	-0,36	-0,35	-0,29
	A	-0,59	-0,59	-0,59	-0,59	-0,58	-0,57	-0,47
	BBB	-0,97	-0,97	-0,97	-0,96	-0,96	-0,93	-0,77
	BB	-1,96	-1,96	-1,96	-1,95	-1,93	-1,87	-1,55
	B	-4,68	-4,68	-4,67	-4,65	-4,61	-4,49	-3,74
	CCC	-20,38	-20,37	-20,35	-20,32	-20,24	-20,00	-17,75

		MVA Term 2						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,200	0,199	0,199	0,199	0,197	0,194	0,165
	AA	0,199	0,199	0,199	0,198	0,197	0,194	0,165
	A	0,199	0,199	0,199	0,198	0,197	0,193	0,164
	BBB	0,199	0,198	0,198	0,198	0,197	0,193	0,164
	BB	0,197	0,197	0,197	0,197	0,195	0,192	0,163
	B	0,194	0,194	0,193	0,193	0,192	0,188	0,160
	CCC	0,165	0,165	0,164	0,164	0,163	0,160	0,137

		MVA WWR Term 1						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,0052	0,0052	0,0052	0,0051	0,0051	0,0051	0,0050
	AA	0,0004	0,0005	0,0004	0,0003	0,0006	0,0008	0,0016
	A	0,0042	0,0042	0,0041	0,0040	0,0042	0,0044	0,0051
	BBB	0,0107	0,0108	0,0107	0,0104	0,0109	0,0111	0,0122
	BB	-0,0265	-0,0262	-0,0264	-0,0266	-0,0244	-0,0232	-0,0172
	B	-0,0442	-0,0438	-0,0440	-0,0443	-0,0416	-0,0398	-0,0314
	CCC	-0,1541	-0,1534	-0,1537	-0,1541	-0,1486	-0,1451	-0,1274

		MVA WWR Term 2						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,000022	0,000009	0,000018	0,000039	-0,000058	-0,000118	-0,000393
	AA	0,000009	-0,000003	0,000006	0,000026	-0,000069	-0,000128	-0,000401
	A	0,000018	0,000006	0,000015	0,000035	-0,000060	-0,000120	-0,000393
	BBB	0,000039	0,000026	0,000035	0,000055	-0,000040	-0,000100	-0,000373
	BB	-0,000058	-0,000069	-0,000060	-0,000040	-0,000128	-0,000184	-0,000439
	B	-0,000118	-0,000128	-0,000120	-0,000100	-0,000184	-0,000238	-0,000481
	CCC	-0,000393	-0,000401	-0,000393	-0,000373	-0,000439	-0,000481	-0,000673

Table 5.3: Results for the different contributions to MVA for the 5-year interest-rate swap. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

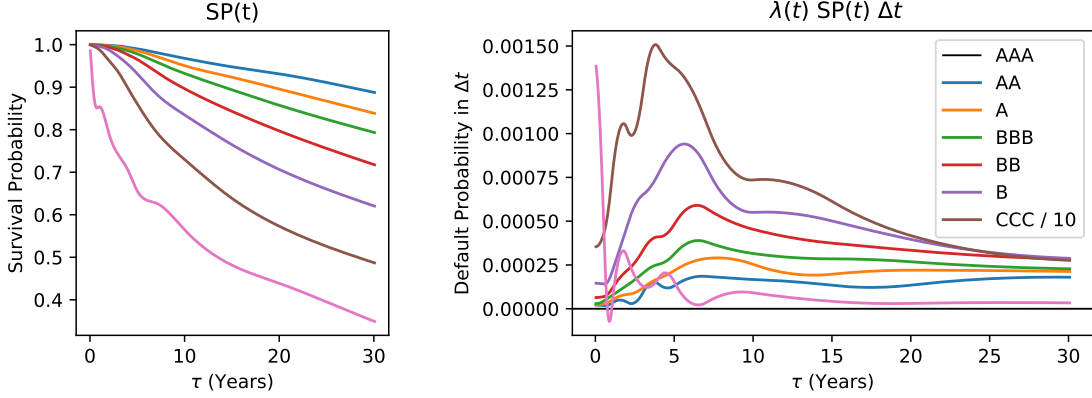


Figure 5.7: Left: Survival probabilities over 30 years for various credit ratings. Right: Conditional default probabilities for default in $[t, t + \Delta t)$ with $\Delta t = 2$ weeks. The CCC default probability has been divided by 10 in order to fit the plot scale.

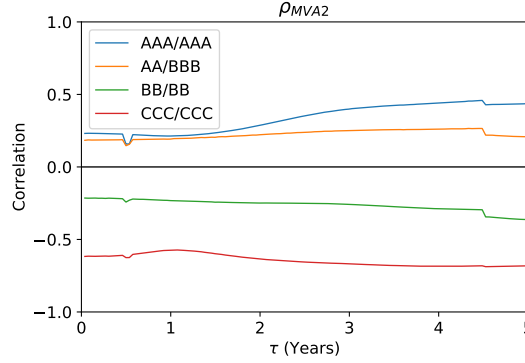


Figure 5.8: Correlations $\rho_{MVA_1}^Q$ for the 5-year EUR interest-rate swap and various combinations of credit ratings.

For the first WWR contribution $MVA_{WWR}^{(1)}$, the sign of the contribution depends on the correlation $\rho_{MVA_1}^Q$. As in the case of $MVA^{(1)}$, where also the term $\lambda_B SP_B SP_C$ appears, the results change little with the counterparty rating. The more significant changes occur in the direction of the bank rating, which can be explained analogously to $MVA^{(1)}$. As can be seen in Fig. 5.3, for the 5-year EUR swap $MVA_{WWR}^{(1)}$ is negative for good bank credit ratings and positive for bad ratings. This explains the observed sign change in Table 5.3 when going from bank rating BBB to BB.

The sign of the second WWR term $MVA_{WWR}^{(2)}$ also depends on the correlation $\rho_{MVA_2}^Q$ appearing in the corresponding expression. Because Fig. 5.3 only shows rating pairs with negative correlations, the correlations corresponding to additional rating pairs are presented in Fig. 5.8.

It is apparent from Table 5.3 that $MVA_{WWR}^{(2)}$ is three to four orders of magnitude smaller than $MVA_{WWR}^{(1)}$. Recalling the discrete expressions of these terms,

$$MVA_{WWR}^{(1)} = - \sum_{i=0}^{n-1} \hat{\rho}_{MVA_1}^Q(t_i) \sqrt{\text{Var}_{t_0}^Q \left[\text{LGD}_B(t_i) \left(SP_B(t_0, t_i) - SP_B(t_0, t_{i+1}) \right) SP_C(t_0, t_i) \right]} \quad (83)$$

$$\times \sqrt{\text{Var}_{t_0}^Q \left[D_r(t_0, t_i) \text{IM}_B(t_i) \right]}$$

$$\text{MVA}_{\text{WWR}}^{(2)} = s_I \sum_{i=0}^{n-1} \hat{\rho}_{\text{MVA}_2}^{\text{Q}}(t_i) \sqrt{\text{Var}_{t_0}^{\text{Q}}[\text{SP}_B(t_0, t_i) \text{SP}_C(t_0, t_i)]} \sqrt{\text{Var}_{t_0}^{\text{Q}}[D_r(t_0, t_i) \text{IM}_B(t_i)]} \Delta t_i, \quad (84)$$

this observation can be explained as follows. While the correlations that enter both terms are presented of similar size in Fig. 5.3, it should be noted that in deriving Eq. (83) the Δt from discretizing the integral has been absorbed into $\hat{\rho}_{\text{MVA}_1}^{\text{Q}}$, which is the quantity shown in Fig. 5.3. For the second WWR-MVA contribution $\text{MVA}_{\text{WWR}}^{(2)}$ in Eq. (84), Δt is still present as a small factor. Furthermore, Eq. (84) involves the small IM spread $s_I = 10$ bp. This results in $\text{MVA}_{\text{WWR}}^{(2)}$ being of negligible size when compared to the other MVA contributions, and this term can safely be dropped from the calculations. Furthermore, given that the no-WWR term involving s_I is typically significantly larger than the WWR contributions, the constant-spread approximation should be reverted in the no-WWR term whenever the dynamics of s_I is known. Therefore, the following expression provides a good approximation to WWR-MVA

$$\begin{aligned} \text{MVA} \approx & - \int_{t_0}^T \left(\text{LGD}_B(t) \lambda_B(t) - s_I(t) \right) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t) \mathbb{E}_{t_0}^{\text{Q}}[D_r(t_0, t) \text{IM}_B(t)] dt \\ & - \int_{t_0}^T \rho_{\text{MVA}_1}^{\text{Q}}(t) \sqrt{\text{Var}_{t_0}^{\text{Q}}[\text{LGD}_B(t) \lambda_B(t) \text{SP}_B(t_0, t) \text{SP}_C(t_0, t)]} \sqrt{\text{Var}_{t_0}^{\text{Q}}[D_r(t_0, t) \text{IM}_B(t)]} dt, \end{aligned} \quad (85)$$

where the first line is simply the regular no-WWR MVA, and the second line corresponds to the non-negligible WWR correction.

As was already discussed for the CVA results, the WWR contributions to the MVA are generally smaller than the no-WWR contributions, despite sizeable correlations between IM and default probabilities. This is again due to the small variances in the probabilities in the calibration interval. As discussed in Sec. 5.5, Calibrating the method to a time period with higher volatility in the default probabilities (*i.e.*, the CDS spreads) will lead to larger WWR contributions to CVA and MVA.

As final observation it is noted that while the no-WWR contributions to CVA and MVA are of similar size, the WWR contributions are significantly smaller for MVA. As can be seen in Fig. 2.6, EPE and EIM are of similar size for the 5-year EUR swap, and therefore the no-WWR contributions to CVA and MVA are expected to be similar as well. For the WWR contributions it is noted that not the EPE and EIM enter the expressions, but the *variances* of PE and IM. Here, Fig. 5.3 shows that the variance of IM is significantly smaller than the variance of PE by a factor of 10. This is because IM is defined as a quantile, which shows less variance than the exposure measure.

The numerical results for the 10-year and 30-year EUR swaps can be found in Tables C.1 and C.2 in the Appendix. As expected, the MVA increases in magnitude with the maturity of the swap, because IM needs to be posted over a longer time period. The WWR contributions $\text{MVA}_{\text{WWR}}^{(1)}$ and $\text{MVA}_{\text{WWR}}^{(2)}$ roughly scale with the maturity of the swap. However, the terms $\text{MVA}^{(1)}$ and $\text{MVA}^{(2)}$ show a stronger increase in magnitude. This is because they directly involve the survival probabilities SP_C and/or SP_B , which change rapidly over time for bad credit ratings.

5.4.2 Cross-Currency Swaps

The numerical MVA results for the 5-year, 10-year, and 30-year EUR-USD cross-currency swap can be found in the Appendix in Tables D.1-D.3. Again, because the exposure and IM profiles are

similar, the results for the cross-currency swap without notional exchange are analogue to the results for the interest-rate swap in the previous section.

5.4.3 European Swaption

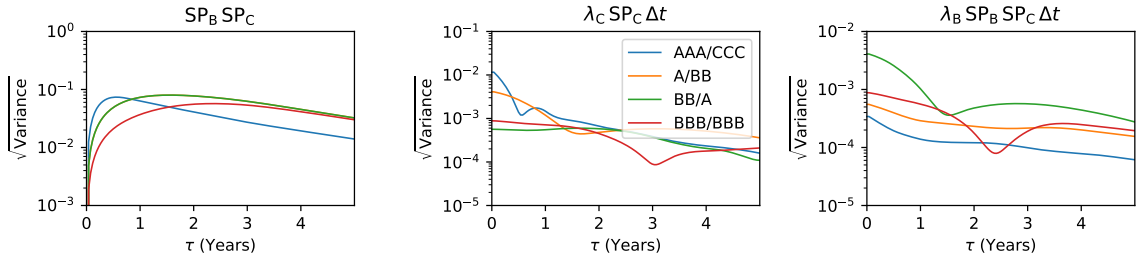
For the European 2Y×5Y EUR-USD swaption, the results are again very similar to the case of the 5-year EUR interest rate swap. In this case the variances of PE and IM are comparable, and consequently the WWR-contributions to CVA and MVA are of comparable size.

5.5 Crisis Scenario

In the previous section it was argued that despite of clear correlations between probabilities and exposures, the WWR contributions to the valuation adjustments are rather small due to small variances in the survival probabilities. Calibrating over a time period with more financial turmoil and thus, larger variation in the probabilities, should consequently result in larger WWR contributions. In order to simulate a financial crisis scenario, the CDS spreads are scaled by a factor of 20 in the calibration procedure. This has an obvious impact on the variances of the survival probabilities. Figure 5.9 shows the resulting variances of the probabilities over a 5-year and 30-year horizon. Comparing to the original variances for $SP_B SP_C$ shown in Fig. 5.3, the desired effect of increasing the variances is achieved for maturities up to 5 years. From there, the variances fall off rapidly. Therefore, the simple crisis scenario may not be quite appropriate for increasing the $SP_B SP_C$ volatility over larger maturities. For the other probability variances, the crisis scenario increases the variances by about a factor 10.

Because the correlations between exposures and survival probabilities should not be affected much by scaling the CDS spreads, the computationally expensive correlations from the calibration

Probability Variances over 5 Years



Probability Variances over 30 Years

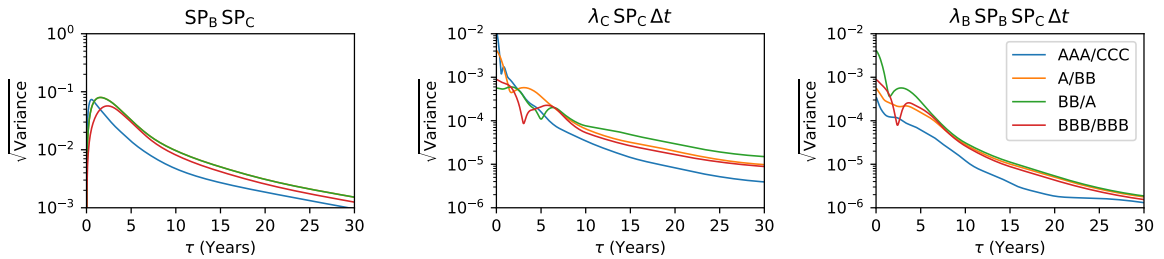


Figure 5.9: Variances of survival probabilities in the crisis scenario.

5Y EUR Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-0,48	-0,70	-1,11	-1,80	-3,55	-7,51	-16,23
WWR only	0,90	-0,83	-0,05	0,96	-3,44	-3,83	-5,81

30Y EUR Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-56,19	-83,41	-110,38	-154,33	-213,98	-294,04	-299,17
WWR only	38,67	16,52	13,32	6,15	-36,03	-33,13	-28,72

Table 5.4: CVA results for the 5-year and 30-year plain vanilla EUR swap in the crisis scenario. The results are quoted in basis points of the respective notional.

using unmodified CDS spreads can be re-used and only the variances of the survival probabilities need to be recalculated and replaced. The variances of exposure and initial margin can be re-used as well. Since the variances of the probabilities can be directly calculated from the market quotes, no Monte Carlo simulation is required for this. The XVA are then calculated using the updated variances of the probabilities.

In the following, only the interest-rate swaps are considered, because the previous section has shown that the other portfolios exhibit similar results. Figure 5.4 presents the results for the 5-year EUR interest-rate swap. Because only the probability variances in the calibration step are affected by the crisis scenario, and the XVA are calculated with unmodified market data from the valuation date, the no-WWR CVA results are unchanged. The WWR-CVA results, however, increase as expected significantly in magnitude, for good credit ratings even exceeding the magnitude of the no-WWR contribution. For bad credit ratings, the crisis-scenario WWR contributions still are twice as large as before.

For the MVA, the no-WWR contributions are also identical. The WWR contributions also show an increase of about a factor of 10, as presented in Table 5.5. In the crisis scenario, the WWR contribution $MVA_{WWR}^{(1)}$ goes up to about 5% of the no-WWR term $MVA^{(1)}$, but $MVA_{WWR}^{(2)}$ still remains negligible.

For the 30-year swap, the observations are similar. The CVA results are also displayed in Table 5.4. Unlike the 5-year case, the WWR contribution does not exceed the no-WWR term in magnitude. This is because the crisis scenario has its largest effect on the probability variances over short maturities, where the variances fall off exponentially with time, suppressing WWR-CVA contributions from higher maturities. Using a more realistic crisis scenario might produce larger WWR effects for long-running portfolios.

5Y EUR Swap

MVA WWR Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,057	0,055	0,051	0,044	0,038	0,035	0,027
	AA	-0,016	-0,017	-0,017	-0,019	-0,014	-0,013	-0,008
	A	0,018	0,018	0,014	0,010	0,014	0,014	0,014
	BBB	0,097	0,097	0,093	0,086	0,091	0,087	0,078
	BB	-0,184	-0,183	-0,180	-0,175	-0,158	-0,144	-0,113
	B	-0,290	-0,290	-0,286	-0,282	-0,269	-0,242	-0,192
	CCC	-0,843	-0,839	-0,835	-0,823	-0,793	-0,764	-0,599

MVA WWR Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,00029	0,00009	0,00019	0,00037	-0,00033	-0,00054	-0,00076
	AA	0,00009	-0,00007	0,00003	0,00023	-0,00038	-0,00057	-0,00076
	A	0,00019	0,00003	0,00011	0,00029	-0,00033	-0,00053	-0,00074
	BBB	0,00037	0,00023	0,00029	0,00042	-0,00020	-0,00040	-0,00066
	BB	-0,00033	-0,00038	-0,00033	-0,00020	-0,00046	-0,00057	-0,00071
	B	-0,00054	-0,00057	-0,00053	-0,00040	-0,00057	-0,00064	-0,00071
	CCC	-0,00076	-0,00076	-0,00074	-0,00066	-0,00071	-0,00071	-0,00067

Table 5.5: Results for the WWR contributions to MVA for the 5-year interest-rate swap in the crisis scenario. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

30Y EUR Swap

MVA WWR Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,470	0,430	0,388	0,302	0,300	0,252	0,180
	AA	0,083	0,079	0,053	0,004	0,064	0,033	0,024
	A	0,160	0,166	0,141	0,095	0,162	0,134	0,105
	BBB	0,371	0,394	0,379	0,358	0,449	0,431	0,379
	BB	-0,980	-0,957	-0,925	-0,869	-0,764	-0,668	-0,477
	B	-1,442	-1,422	-1,399	-1,356	-1,248	-1,115	-0,847
	CCC	-4,131	-4,109	-4,097	-4,055	-3,881	-3,745	-2,927

MVA WWR Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,0088	0,0046	0,0045	0,0040	-0,0022	-0,0032	-0,0045
	AA	0,0046	0,0020	0,0021	0,0023	-0,0024	-0,0032	-0,0044
	A	0,0045	0,0021	0,0023	0,0024	-0,0020	-0,0029	-0,0041
	BBB	0,0040	0,0023	0,0024	0,0026	-0,0013	-0,0022	-0,0036
	BB	-0,0022	-0,0024	-0,0020	-0,0013	-0,0025	-0,0029	-0,0037
	B	-0,0032	-0,0032	-0,0029	-0,0022	-0,0029	-0,0031	-0,0036
	CCC	-0,0045	-0,0044	-0,0041	-0,0036	-0,0037	-0,0036	-0,0034

Table 5.6: Results for the WWR contributions to MVA for the 30-year interest-rate swap in the crisis scenario. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

6 Summary

Since the Financial Crisis, valuation adjustments have become an increasingly important aspect of derivative pricing. The need for taking into account credit risk as well as a new regulatory environment in which OTC trades are required to be collateralized, sparked the development of many valuation adjustments such as CVA or MVA. In the standard XVA framework employed in most banks the correlation structure between exposures and default probability is neglected. However, regulators have started to focus on the impact that wrong-way risk has on financial institutions.

In this work, the model-independent approach of including WWR into XVA calculations as proposed by Kenyon *et al.* [11] is extended to the calculation of MVA. In the derivation of MVA, the IM spread s_I is treated as constant. This significantly simplifies the derivation and still allows to obtain estimates for the size of the individual contributions involving s_I to MVA. Sample calculations are performed for plain-vanilla interest-rate swaps, cross-currency swaps, and swaptions, with maturities ranging from 5 years to 30 years. The method is calibrated to the time period from 2013 to 2016, and is subsequently used to calculate uncollateralized unilateral CVA as well as MVA.

The numerical calculations indicate that the WWR contributions are of the order of a few percent of the no-WWR results. The main reason is that for the chosen calibration interval, the variances of the probabilities are small. Calibrating to a time period with higher financial stress is expected to result in larger WWR effects. For MVA, the contribution involving s_I is negligible, which suggests to drop this term and to use Eq. (85) as a good approximation to WWR-MVA. While no-WWR results are of similar size for CVA and MVA, the WWR contributions are significantly smaller for MVA. This is because the variance of the initial margin is lower than for the positive exposure.

Although CDS spreads and volatilities in the calibration time period are relatively high compared to 2020 levels, they are still far from the Financial Crisis-levels from 2008-2012. Therefore, a simple financial crisis scenario is considered in which the CDS spreads are scaled by a factor of 20. As expected, increased WWR contributions are observed, which may even surpass the no-WWR values.

Due to the large number of historical calculations that need to be performed, the calibration of the method is computationally expensive. Recalling that the need for the historical calculations arises because the survival probabilities are not stochastic in the exposure simulation, future work may consider stochastic evolutions of the survival probability. In this case, the probability variances and correlation with exposures and initial margin can directly be determined from the Monte Carlo paths during the exposure simulation. The resulting method still requires calibration of the probability process, but this calibration no longer involves expensive exposure simulations. Furthermore, calibration to different financial regimes is facilitated, and could even be parameterised in order to cover both regular and crisis regimes in a single framework.

This work employs a V@R-based approach of calculating the initial margin amounts. While such approaches are convenient because all the required information is provided by the exposure simulation, future work may look into determining initial margin using the ISDA SIMM methodology [8, 9], as it has become the market standard for IM calculations today. For this, derivatives of the trades with respect to various risk factors need to be calculated and the ISDA SIMM methodology needs to be implemented. However, including SIMM in MVA calculations is computationally expensive. Therefore, approximation methods such as Ref. [24] can be employed.

Appendices

A WWR-CVA Results for Cross-Currency Swaps

5Y EUR-USD Cross-Currency Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-0,52	-0,74	-1,18	-1,90	-3,76	-7,94	-16,87
WWR only	0,17	0,05	0,15	0,25	-0,71	-1,01	-2,57

10Y EUR-USD Cross-Currency Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-4,94	-7,48	-10,58	-16,37	-27,53	-45,89	-59,21
WWR only	1,75	1,31	1,27	0,87	-2,32	-2,78	-5,44

30Y EUR-USD Cross-Currency Swap

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-56,12	-83,20	-110,17	-154,03	-213,95	-295,06	-304,26
WWR only	3,70	2,23	-3,89	-5,60	-5,42	-6,87	23,26

Table A.1: CVA results for the EUR-USD cross-currency swap without notional exchange. The results are quoted in basis points of the notional.

B WWR-CVA Results for Swaptions

European 2Y×5Y EUR Swaption

	Counterparty Rating						
	AAA	AA	A	BBB	BB	B	CCC
No WWR	-0,15	-0,22	-0,32	-0,51	-0,93	-1,54	-1,70
WWR only	0,10	0,06	0,08	0,04	-0,24	-0,20	-0,26

Table B.1: Wrong-way risk CVA results for the European 2Y×5Y EUR swaption. The results are quoted in basis points of the notional.

C WWR-MVA Results for EUR Swaps

10Y EUR Swap

MVA Term 1

Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	-2,02	-2,01	-2,00	-1,97	-1,91	-1,78	-1,41
	AA	-3,06	-3,04	-3,02	-2,97	-2,87	-2,68	-2,12
	A	-4,48	-4,46	-4,43	-4,37	-4,23	-3,95	-3,13
	BBB	-7,11	-7,08	-7,03	-6,93	-6,72	-6,28	-4,99
	BB	-12,93	-12,87	-12,78	-12,63	-12,27	-11,52	-9,16
	B	-24,76	-24,67	-24,52	-24,27	-23,67	-22,40	-17,97
	CCC	-59,56	-59,46	-59,30	-59,02	-58,34	-56,72	-48,64

MVA Term 2

Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,81	0,81	0,81	0,80	0,78	0,75	0,61
	AA	0,81	0,81	0,80	0,80	0,78	0,75	0,61
	A	0,81	0,80	0,80	0,79	0,78	0,74	0,61
	BBB	0,80	0,80	0,79	0,79	0,77	0,74	0,60
	BB	0,78	0,78	0,78	0,77	0,76	0,72	0,59
	B	0,75	0,75	0,74	0,74	0,72	0,69	0,57
	CCC	0,61	0,61	0,61	0,60	0,59	0,57	0,47

MVA WWR Term 1

Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,027	0,026	0,026	0,026	0,026	0,025	0,024
	AA	0,014	0,014	0,014	0,013	0,015	0,015	0,018
	A	0,018	0,018	0,018	0,017	0,019	0,020	0,023
	BBB	0,020	0,020	0,020	0,019	0,024	0,025	0,032
	BB	-0,069	-0,068	-0,068	-0,068	-0,057	-0,052	-0,030
	B	-0,091	-0,090	-0,090	-0,091	-0,078	-0,072	-0,045
	CCC	-0,279	-0,276	-0,277	-0,279	-0,261	-0,253	-0,211

MVA WWR Term 2

Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,0002	0,0002	0,0002	0,0002	-0,0003	-0,0005	-0,0015
	AA	0,0002	0,0001	0,0001	0,0002	-0,0003	-0,0005	-0,0015
	A	0,0002	0,0001	0,0002	0,0002	-0,0003	-0,0005	-0,0015
	BBB	0,0002	0,0002	0,0002	0,0003	-0,0003	-0,0005	-0,0014
	BB	-0,0003	-0,0003	-0,0003	-0,0003	-0,0007	-0,0009	-0,0017
	B	-0,0005	-0,0005	-0,0005	-0,0005	-0,0009	-0,0010	-0,0018
	CCC	-0,0015	-0,0015	-0,0015	-0,0014	-0,0017	-0,0018	-0,0023

Table C.1: Results for the different contributions to MVA for the 10-year interest-rate swap. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

30Y EUR Swap

MVA Term 1
Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	-21,94	-21,50	-20,99	-20,13	-18,70	-16,36	-12,67
	AA	-33,34	-32,67	-31,90	-30,59	-28,41	-24,84	-19,24
	A	-45,75	-44,86	-43,84	-42,10	-39,18	-34,37	-26,63
	BBB	-67,54	-66,29	-64,84	-62,36	-58,17	-51,23	-39,75
	BB	-103,38	-101,63	-99,60	-96,10	-90,12	-80,03	-62,25
	B	-166,69	-164,21	-161,28	-156,23	-147,46	-132,38	-103,65
	CCC	-263,28	-260,50	-257,28	-251,66	-241,94	-224,78	-184,98

MVA Term 2
Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	5,91	5,81	5,70	5,50	5,17	4,62	3,64
	AA	5,81	5,71	5,60	5,41	5,09	4,55	3,59
	A	5,70	5,60	5,49	5,31	4,99	4,47	3,53
	BBB	5,50	5,41	5,31	5,13	4,83	4,34	3,42
	BB	5,17	5,09	4,99	4,83	4,56	4,10	3,24
	B	4,62	4,55	4,47	4,34	4,10	3,71	2,94
	CCC	3,64	3,59	3,53	3,42	3,24	2,94	2,34

MVA WWR Term 1
Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,122	0,122	0,121	0,117	0,122	0,118	0,113
	AA	0,059	0,063	0,062	0,061	0,082	0,083	0,094
	A	0,048	0,053	0,053	0,052	0,079	0,083	0,100
	BBB	0,032	0,039	0,039	0,041	0,079	0,089	0,120
	BB	-0,231	-0,217	-0,216	-0,210	-0,139	-0,115	-0,033
	B	-0,300	-0,286	-0,285	-0,279	-0,200	-0,171	-0,074
	CCC	-0,617	-0,608	-0,611	-0,615	-0,546	-0,526	-0,417

MVA WWR Term 2
Counterparty Rating

		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,0048	0,0037	0,0035	0,0031	-0,0024	-0,0040	-0,0082
	AA	0,0037	0,0028	0,0026	0,0023	-0,0029	-0,0043	-0,0083
	A	0,0035	0,0026	0,0024	0,0021	-0,0029	-0,0043	-0,0082
	BBB	0,0031	0,0023	0,0021	0,0018	-0,0029	-0,0042	-0,0080
	BB	-0,0024	-0,0029	-0,0029	-0,0029	-0,0064	-0,0072	-0,0098
	B	-0,0040	-0,0043	-0,0043	-0,0042	-0,0072	-0,0078	-0,0100
	CCC	-0,0082	-0,0083	-0,0082	-0,0080	-0,0098	-0,0100	-0,0112

Table C.2: Results for the different contributions to MVA for the 30-year interest-rate swap. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

D WWR-MVA Results for Cross-Currency Swaps

5Y EUR-USD Cross-Currency Swap

MVA Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	-0,27	-0,27	-0,26	-0,26	-0,26	-0,25	-0,21
	AA	-0,39	-0,39	-0,39	-0,39	-0,39	-0,38	-0,31
	A	-0,63	-0,63	-0,63	-0,62	-0,62	-0,60	-0,49
	BBB	-1,03	-1,03	-1,02	-1,02	-1,01	-0,98	-0,81
	BB	-2,08	-2,07	-2,07	-2,06	-2,04	-1,98	-1,63
	B	-4,92	-4,91	-4,90	-4,88	-4,84	-4,71	-3,91
	CCC	-21,12	-21,11	-21,09	-21,06	-20,97	-20,71	-18,37

MVA Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,208	0,207	0,207	0,207	0,205	0,201	0,171
	AA	0,207	0,207	0,207	0,206	0,205	0,201	0,171
	A	0,207	0,207	0,207	0,206	0,205	0,201	0,171
	BBB	0,207	0,206	0,206	0,206	0,204	0,200	0,170
	BB	0,205	0,205	0,205	0,204	0,203	0,199	0,169
	B	0,201	0,201	0,201	0,200	0,199	0,195	0,166
	CCC	0,171	0,171	0,171	0,170	0,169	0,166	0,142

MVA WWR Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,004	0,004	0,004	0,004	0,004	0,004	0,004
	AA	-0,002	-0,002	-0,002	-0,002	-0,001	-0,001	0,000
	A	0,002	0,002	0,002	0,002	0,002	0,002	0,003
	BBB	0,008	0,008	0,008	0,008	0,009	0,009	0,010
	BB	-0,033	-0,033	-0,033	-0,033	-0,031	-0,029	-0,023
	B	-0,052	-0,052	-0,052	-0,052	-0,049	-0,047	-0,038
	CCC	-0,161	-0,160	-0,160	-0,161	-0,154	-0,151	-0,133

MVA WWR Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,00001	0,00000	0,00001	0,00002	-0,00009	-0,00015	-0,00041
	AA	0,00000	-0,00001	-0,00001	0,00001	-0,00010	-0,00016	-0,00042
	A	0,00001	-0,00001	0,00000	0,00002	-0,00009	-0,00015	-0,00042
	BBB	0,00002	0,00001	0,00002	0,00004	-0,00007	-0,00013	-0,00040
	BB	-0,00009	-0,00010	-0,00009	-0,00007	-0,00017	-0,00023	-0,00048
	B	-0,00015	-0,00016	-0,00015	-0,00013	-0,00023	-0,00029	-0,00052
	CCC	-0,00041	-0,00042	-0,00042	-0,00040	-0,00048	-0,00052	-0,00071

Table D.1: Results for the different contributions to MVA for the 5-year EUR-USD cross-currency swap without notional exchange. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

10Y EUR-USD Cross-Currency Swap

MVA Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	-2,05	-2,04	-2,02	-2,00	-1,93	-1,80	-1,43
	AA	-3,10	-3,08	-3,06	-3,01	-2,91	-2,72	-2,15
	A	-4,54	-4,52	-4,49	-4,43	-4,29	-4,01	-3,18
	BBB	-7,21	-7,17	-7,12	-7,03	-6,81	-6,37	-5,06
	BB	-13,10	-13,04	-12,96	-12,80	-12,43	-11,68	-9,29
	B	-25,09	-25,00	-24,85	-24,60	-23,99	-22,70	-18,21
	CCC	-60,43	-60,33	-60,16	-59,87	-59,18	-57,55	-49,36

MVA Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,82	0,82	0,82	0,81	0,79	0,76	0,62
	AA	0,82	0,82	0,82	0,81	0,79	0,76	0,62
	A	0,82	0,82	0,81	0,80	0,79	0,75	0,62
	BBB	0,81	0,81	0,80	0,80	0,78	0,75	0,61
	BB	0,79	0,79	0,79	0,78	0,77	0,73	0,60
	B	0,76	0,76	0,75	0,75	0,73	0,70	0,58
	CCC	0,62	0,62	0,62	0,61	0,60	0,58	0,48

MVA WWR Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,020	0,020	0,020	0,020	0,020	0,020	0,020
	AA	0,008	0,008	0,008	0,007	0,010	0,011	0,014
	A	0,008	0,009	0,008	0,008	0,011	0,013	0,017
	BBB	0,011	0,011	0,011	0,011	0,016	0,019	0,026
	BB	-0,086	-0,084	-0,084	-0,084	-0,071	-0,065	-0,041
	B	-0,115	-0,113	-0,113	-0,113	-0,098	-0,090	-0,061
	CCC	-0,295	-0,293	-0,294	-0,295	-0,275	-0,266	-0,224

MVA WWR Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,00016	0,00009	0,00010	0,00014	-0,00047	-0,00073	-0,00165
	AA	0,00009	0,00002	0,00004	0,00008	-0,00052	-0,00077	-0,00168
	A	0,00010	0,00004	0,00005	0,00009	-0,00050	-0,00076	-0,00166
	BBB	0,00014	0,00008	0,00009	0,00013	-0,00045	-0,00071	-0,00160
	BB	-0,00047	-0,00052	-0,00050	-0,00045	-0,00095	-0,00116	-0,00193
	B	-0,00073	-0,00077	-0,00076	-0,00071	-0,00116	-0,00135	-0,00205
	CCC	-0,00165	-0,00168	-0,00166	-0,00160	-0,00193	-0,00205	-0,00254

Table D.2: Results for the different contributions to MVA for the 10-year EUR-USD cross-currency swap without notional exchange. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

30Y EUR-USD Cross-Currency Swap

MVA Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	-21,73	-21,29	-20,79	-19,93	-18,52	-16,20	-12,54
	AA	-33,02	-32,35	-31,59	-30,29	-28,13	-24,59	-19,05
	A	-45,30	-44,42	-43,41	-41,68	-38,78	-34,02	-26,36
	BBB	-66,87	-65,62	-64,19	-61,73	-57,58	-50,70	-39,33
	BB	-102,30	-100,57	-98,55	-95,09	-89,16	-79,17	-61,57
	B	-164,89	-162,42	-159,52	-154,51	-145,82	-130,89	-102,47
	CCC	-260,12	-257,36	-254,17	-248,60	-238,97	-221,97	-182,64

MVA Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	5,85	5,75	5,64	5,44	5,11	4,57	3,60
	AA	5,75	5,65	5,54	5,35	5,03	4,50	3,55
	A	5,64	5,54	5,43	5,25	4,94	4,42	3,49
	BBB	5,44	5,35	5,25	5,07	4,78	4,29	3,38
	BB	5,11	5,03	4,94	4,78	4,51	4,06	3,21
	B	4,57	4,50	4,42	4,29	4,06	3,67	2,91
	CCC	3,60	3,55	3,49	3,38	3,21	2,91	2,31

MVA WWR Term 1

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,155	0,154	0,154	0,150	0,157	0,155	0,147
	AA	0,081	0,085	0,087	0,085	0,114	0,121	0,131
	A	0,039	0,045	0,047	0,048	0,085	0,099	0,120
	BBB	0,011	0,020	0,024	0,027	0,081	0,104	0,143
	BB	-0,303	-0,286	-0,279	-0,271	-0,175	-0,128	-0,023
	B	-0,442	-0,424	-0,416	-0,406	-0,297	-0,239	-0,107
	CCC	-0,672	-0,663	-0,662	-0,669	-0,583	-0,541	-0,408

MVA WWR Term 2

		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,0064	0,0051	0,0044	0,0038	-0,0035	-0,0068	-0,0115
	AA	0,0051	0,0040	0,0033	0,0028	-0,0040	-0,0071	-0,0116
	A	0,0044	0,0033	0,0027	0,0022	-0,0044	-0,0073	-0,0117
	BBB	0,0038	0,0028	0,0022	0,0018	-0,0044	-0,0071	-0,0114
	BB	-0,0035	-0,0040	-0,0044	-0,0044	-0,0089	-0,0109	-0,0137
	B	-0,0068	-0,0071	-0,0073	-0,0071	-0,0109	-0,0123	-0,0146
	CCC	-0,0115	-0,0116	-0,0117	-0,0114	-0,0137	-0,0146	-0,0158

Table D.3: Results for the different contributions to MVA for the 30-year EUR-USD cross-currency swap without notional exchange. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

E WWR-MVA Results for European Swaption

European 2Y×5Y EUR Swaption

		MVA Term 1						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	-0,45	-0,45	-0,44	-0,44	-0,43	-0,41	-0,33
	AA	-0,68	-0,68	-0,67	-0,67	-0,65	-0,62	-0,50
	A	-1,05	-1,05	-1,05	-1,04	-1,02	-0,97	-0,77
	BBB	-1,70	-1,69	-1,69	-1,67	-1,64	-1,56	-1,25
	BB	-3,35	-3,34	-3,33	-3,30	-3,24	-3,09	-2,48
	B	-6,95	-6,94	-6,91	-6,87	-6,75	-6,48	-5,24
	CCC	-19,43	-19,40	-19,37	-19,30	-19,14	-18,68	-15,96

		MVA Term 2						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,237	0,236	0,235	0,234	0,231	0,224	0,184
	AA	0,236	0,236	0,235	0,234	0,231	0,223	0,184
	A	0,235	0,235	0,234	0,233	0,230	0,223	0,184
	BBB	0,234	0,234	0,233	0,232	0,229	0,222	0,183
	BB	0,231	0,231	0,230	0,229	0,226	0,219	0,181
	B	0,224	0,223	0,223	0,222	0,219	0,212	0,175
	CCC	0,184	0,184	0,184	0,183	0,181	0,175	0,145

		MVA WWR Term 1						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,0627	0,0625	0,0619	0,0604	0,0607	0,0598	0,0590
	AA	0,0269	0,0276	0,0266	0,0251	0,0306	0,0315	0,0408
	A	0,0518	0,0525	0,0513	0,0494	0,0549	0,0560	0,0655
	BBB	0,0646	0,0660	0,0646	0,0624	0,0725	0,0756	0,0930
	BB	-0,1966	-0,1920	-0,1940	-0,1955	-0,1647	-0,1509	-0,0832
	B	-0,2509	-0,2457	-0,2484	-0,2514	-0,2149	-0,1971	-0,1110
	CCC	-0,5954	-0,5876	-0,5911	-0,5953	-0,5373	-0,5080	-0,3630

		MVA WWR Term 2						
		Counterparty Rating						
		AAA	AA	A	BBB	BB	B	CCC
Bank Rating	AAA	0,00039	0,00021	0,00033	0,00052	-0,00086	-0,00144	-0,00426
	AA	0,00021	0,00003	0,00016	0,00035	-0,00101	-0,00158	-0,00436
	A	0,00033	0,00016	0,00028	0,00047	-0,00089	-0,00146	-0,00424
	BBB	0,00052	0,00035	0,00047	0,00065	-0,00069	-0,00127	-0,00404
	BB	-0,00086	-0,00101	-0,00089	-0,00069	-0,00189	-0,00239	-0,00486
	B	-0,00144	-0,00158	-0,00146	-0,00127	-0,00239	-0,00286	-0,00518
	CCC	-0,00426	-0,00436	-0,00424	-0,00404	-0,00486	-0,00518	-0,00686

Table E.1: Results for the different contributions to MVA for the European 2Y×5Y EUR swaption. The results are quoted in basis points of the notional. The IM spread is set to $s_I = 10$ bp.

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