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Simon Cowan

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Department of Economics
Manor Road Building, Manor Road
Oxford, OX1 3UQ

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Simon Cowan*

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Abstract

The conditions for monopolization to be good for social welfare are examined. Social welfare can be higher when a monopoly sells to a monopoly, with double margins, than when a competitive industry sells to a downstream Cournot oligopoly with differing efficiency levels. This requires inverse demand to be sufficiently concave, and cannot hold when demand is convex. When there are no vertical issues an efficient monopoly can yield higher social welfare than an asymmetric Cournot duopoly as long as demand is log-concave. In general greater demand concavity increases the relative importance of the benefit of redistributing output to the efficient firm.

Keywords: monopoly, Cournot oligopoly, social welfare.
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*Department of Economics, University of Oxford, Manor Road Building, Oxford OX1 3UQ, UK. Email: simon.cowan@economics.ox.ac.uk. ORCID: 0000-0003-0704-2871. I am grateful to John Vickers for comments on a previous draft.

1 Introduction

This paper provides conditions for monopolization to raise social welfare because of the benefits of the redistribution of output to the most efficient firm. The shape of the inverse demand function matters. The more concave is inverse demand the more likely it is that monopolization will increase welfare. Three related issues are covered. First, we consider a competitive upstream industry selling to downstream oligopolists with different efficiency levels. Upstream monopolization causes the downstream industry to become a monopoly and double margins are introduced. Nevertheless a monopolist selling to a monopolist may be better for social welfare than a competitive industry selling to an oligopoly. Second, we abstract from vertical issues and consider a simple, classic, question: when will a monopoly with a low marginal cost be better for social welfare than an asymmetric duopoly? If demand is strictly log-concave and the cost difference is large enough then monopoly is better for social welfare. The more concave is inverse demand the smaller the cost difference needs to be for monopoly to yield higher welfare. Meanwhile social welfare is always lower with monopoly than duopoly for standard log-convex demand functions. Third, two well-known models are re-examined in the light of the analysis. Lahiri and Ono (1988) show that an increase in the marginal cost of an inefficient firm can induce higher social welfare, while Farrell and Shapiro (1990) examine the effect of an output-reducing merger on the welfare of consumers and non-participating firms.

The conditions under which upstream monopolization raises welfare are special but not paradoxical. Inverse demand must be strictly concave. The more concave is inverse demand the greater is the benefit from the reallocation of output. At the same time more concavity reduces the effect of an increased wholesale price on the retail price, i.e. the pass-through coefficient. The resulting impact on consumers of the retail price increase induced by upstream monopolization is reduced.

Section 2 contains the standard analysis of Cournot oligopoly when marginal costs differ. Section 3 provides conditions for a marginal increase in the wholesale price to have a positive effect on social welfare. Section 4 shows that upstream monopolization can raise welfare. Section 5 provides conditions for a low-cost monopoly to yield higher welfare than an asymmetric duopoly when there are no vertical issues. Section 6 reconsiders the models of Lahiri and Ono (1988) and Farrell and Shapiro (1990). Conclusions are in Section 7.

2 Downstream Cournot Competition

The downstream market has $n \geq 1$ retailers who are Cournot players. Retailer i buys quantity q_i and sells this to final consumers, paying the wholesale price w and incurring a marginal retail cost of $c_i \geq 0$. The retail costs differ across retailers. The simple average is $\bar{c} = \frac{1}{n}\sum_{i=1}^{n} c_i > 0$ and the variance is $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(c_i - \bar{c})^2 > 0$. Inverse demand for the downstream industry, $p(Q)$, where $Q = \sum_{i=1}^{n} q_i$ is total output, is strictly decreasing and twice differentiable. Assume that $p(0) > c_i$ for all i. Define $E(Q) \equiv -\frac{Qp''(Q)}{p'(Q)}$. This is the elasticity of the slope of inverse demand and will be called curvature. Demand is concave if $E \leq 0$, log-concave if $E \leq 1$, and convex if $E \geq 0$. Assume that $E(Q) < 2$ for all Q, which implies that marginal revenue for retailers in aggregate, $p(Q) + Qp'(Q)$, is decreasing in total output.

The profit of retailer i is $\pi_i(q_i, Q_{-i}) = (p(q_i + Q_{-i}) - c_i - w)q_i$ where $Q_{-i} = \sum_{j \neq i}^{n} q_j$ is the output of the other retailers. The first-order condition for firm i, taking others' outputs as given and assuming at this stage that $q_i > 0$ for all i, is

$$p(Q) + q_i p'(Q) - c_i - w = 0. \tag{1}$$

Firm i's profit function is concave in q_i. The second derivative of profit is $2p'(Q) + q_i p''(Q) = p'(Q)(2 - s_i E) < 0$, where $s_i \equiv \frac{q_i}{Q} < 1$ is i's market share. The Nash equilibrium in the retail market is characterized by the n first-order conditions of form (1). Adding these gives

$$np(Q) + Qp'(Q) = n(\bar{c} + w). \tag{2}$$

The left-hand side of (2) is strictly decreasing in Q if and only if $n + 1 - E > 0$, which holds because $n \geq 1$ and $E < 2$. It follows that the Cournot equilibrium is unique—see Kolstad and Mathiesen (1987), Gaudet and Salant (1991) and Vives (2001). The right-hand side of (2) shows that total output is a function of the average of the retail costs plus the wholesale price, and does not depend on the variation in retail costs. Bergstrom and Varian (1985) discuss this feature of Cournot oligopoly with different costs, which applies when the number of firms is unchanged.

3 The marginal effect of the wholesale price

Initially the upstream industry is perfectly competitive or, equivalently, there is pure Bertrand competition between at least two upstream suppliers. The wholesale price is equal to upstream marginal cost, which is assumed to be constant and, without loss of generality, equal to zero. When the upstream industry is monopolized the wholesale price is raised above zero, and allowance needs to be made for inefficient retailers to leave the market when their margins become negative. Assume that there is a single downstream firm with the lowest cost technology.

How does an increase in the wholesale price affect social welfare at the margin? From (2) the effect of w on total output is negative,

$$\frac{dQ}{dw} = \frac{n}{p'(Q)(n+1-E)} < 0 \tag{3}$$

and the effect on the retail price, the cost pass-through coefficient, is positive,

$$\frac{dp}{dw} = p'(Q)\frac{dQ}{dw} = \frac{n}{n+1-E} > 0. \tag{4}$$

As demand becomes more concave, that is E is a more negative number, the pass-through coefficient falls.

The effect of the wholesale price on the output of an individual retailer is written here as the derivative of total output, $\frac{dQ}{dw}$, multiplied by the fraction of the total output reduction that is attributable to firm i:

$$\frac{dq_i}{dw} = \frac{dQ}{dw}\left(\frac{1}{n} + E\left(s_i - \frac{1}{n}\right)\right). \tag{5}$$

Equation (5) implies that if demand is linear ($E = 0$), or if there is no retail cost variation so $s_i = \frac{1}{n}$, each firm reduces output by the same amount. When, however, demand is nonlinear and there is retail cost variation the effect of the wholesale price increase on a firm's output depends on whether demand is convex or concave. If demand is strictly convex then more efficient firms, with $s_i > \frac{1}{n}$, reduce their outputs by more than the less efficient firms, as $\frac{1}{n} + E\left(s_i - \frac{1}{n}\right) > \frac{1}{n}$. With convex demand inefficient firms may even raise their outputs if $E > 1$.[1] The redistribution of output effect goes the wrong way when demand is strictly convex. Seade (1985) notes that when $E > 0$ a common cost increase, such as an excise tax, penalizes efficient firms—see also Fevrier and Linnemer (2004) and Van Long and Soubeyran (1997). With strictly concave demand efficient firms reduce their outputs by less than inefficient firms. In fact an efficient firm will raise its output if its market share is large enough.

Proposition 1. *An increase in the wholesale price induces retail firm i to raise its output if $s_i > \frac{1-E}{-En}$ while demand is strictly concave.*

Proof. The condition implies that $\frac{1}{n} + E\left(s_i - \frac{1}{n}\right) < 0$ and thus that (5) is positive. □

[1] A firm will increase its output when demand is strictly log-convex if its market share is small enough with $s_i < \frac{E-1}{En}$.

For example if $E = -5$ and $n = 5$ firm i raises its output in response to a wholesale price increase if $s_i > 0.24$. Market shares with equally efficient firms would equal 0.2 in this example, so it is plausible that a firm could be efficient enough to increase its output. If, counter-factually, the other firms were to keep their outputs constant then firm i would reduce q_i. This is the direct effect of the wholesale price increase. The equilibrium response of the other firms, however, is to cut their joint output, and firm i responds to this by raising its output. With concave demand outputs are strategic substitutes. The derivative in (5) can be written as

$$\frac{dq_i}{dw} = \frac{1}{p'(Q)(2 - s_i E)} - \frac{(1 - s_i E)}{2 - s_i E}\frac{dQ_{-i}}{dw}$$

where the first term is the direct effect and $-\frac{(1-s_iE)}{2-s_iE} = \frac{dq_i}{dQ_{-i}} < 0$ is the slope of i's best-response to the other firms' output.[2] The more concave inverse demand is, other things equal, the larger the increase in a firm's output when the other firms reduce their collective output. Proposition 1 holds when the strategic substitutes effect offsets the direct effect.

When a wholesale price increase induces firm i to raise its output the combined retail and wholesale profit associated with firm i, $(p(Q) - c_i - w)q_i + wq_i = (p(Q) - c_i)q_i$, increases. The wholesale price increase causes $p(Q)$ to rise, so both $p(Q) - c_i$ and q_i increase. This profit measure increases even if q_i falls as long as the proportional fall in q_i is smaller than the proportional increase in $p(Q) - c_i$.

The effect of the wholesale price increase on total costs, $\sum_i c_i q_i$, using (5), is

$$\sum_i c_i \frac{dq_i}{dw} = \frac{dQ}{dw}\left(\bar{c} - E\left(\bar{c} - \sum_i c_i s_i\right)\right) = \frac{dQ}{dw}\left(\bar{c} - \frac{E\sigma^2}{p(Q) - \bar{c} - w}\right). \quad (6)$$

The expression $\bar{c} - \sum_i c_i s_i$ is the difference between the simple average and the weighted average of the marginal costs and is positive because lower-cost retailers have higher market shares. The appendix shows that $\bar{c} - \sum_i c_i s_i = \frac{\sigma^2}{p(Q)-\bar{c}-w}$. Call $\frac{dQ}{dw}\frac{E\sigma^2}{p(Q)-\bar{c}-w}$ the benefit from the redistribution of output.

Social welfare, W, is consumer surplus plus total profits or, equivalently, direct utility, $U(Q)$, less total costs: $W = U(Q) - \sum_i c_i q_i$. Over the relevant outputs $U(Q)$ is increasing and concave, and its derivative equals the price. Direct utility falls as the wholesale price rises because total output falls. The effect on direct utility of a wholesale price increase is $U'(Q)\frac{dQ}{dw} = p(Q)\frac{dQ}{dw} < 0$. The effect on welfare will be positive if the reduction in total costs offsets this. Subtracting the change in costs, from (6), gives the marginal welfare effect

[2] To obtain this version of the output effect write the first-order condition using $Q = q_i + Q_{-i}$.

$$\frac{dW}{dw} = \frac{dQ}{dw}\left[p(Q) - \bar{c} + \frac{E\sigma^2}{p(Q) - \bar{c} - w}\right]. \tag{7}$$

The marginal welfare effect has two parts. The first part, $\frac{dQ}{dw}(p(Q) - \bar{c})$, is negative. This captures the standard feature that an output reduction when price exceeds marginal cost reduces welfare. The second part, $\frac{dQ}{dw}\frac{E\sigma^2}{p(Q)-\bar{c}-w}$, is the benefit of the redistribution of output and is positive if and only if $E < 0$.

The expression for the marginal welfare effect in (7) still applies when the wholesale price has risen far enough that only the most efficient retailer remains. At that point the variance of marginal costs is zero, $\bar{c}$ equals the marginal cost of the most efficient retailer, and the welfare effect is negative.[3] The first welfare result may now be stated.

Proposition 2. *If demand is convex everywhere then the marginal effect on welfare is negative for all wholesale prices, so monopolization of the upstream industry reduces social welfare.*

Proof. With $E \geq 0$ and $n \geq 2$ the marginal welfare effect is the sum of a negative term and a non-positive term and so is negative. When $n = 1$, so only the most efficient retailer is in the market, the welfare effect is $\frac{dQ}{dw}(p(Q) - \bar{c}) < 0$ where $\bar{c}$ is now the lowest marginal cost. $\square$

Proposition 2 implies that a necessary condition for welfare to rise with upstream monopolization is that demand is not convex everywhere. In particular if $E < 0$ the marginal welfare effect in (7) is the sum of a negative term and a positive term, giving the possibility that the marginal welfare effect is positive for $n \geq 2$.

Assume now that $E < 0$ everywhere. Three factors, curvature, E, and the variance and simple average of the marginal costs, σ^2 and $\bar{c}$, determine the sign of the marginal welfare effect. In each case start with a marginal welfare effect that is negative.

First, the marginal welfare effect will be positive for inverse demand that is sufficiently concave. Suppose that at the equilibrium total output and price there is a concave transformation of inverse demand. The position and slope of inverse demand at the initial total output are preserved, so each firm's output and the equilibrium price are unchanged by the transformation. Malueg (1994) shows how curvature affects monopoly outcomes using the same device. For a sufficiently negative value

[3] At wholesale prices where less efficient retailers drop out the welfare function may have sharp points and so would not be differentiable at those points. The welfare function is, however, continuous and integrable.

of E the marginal welfare effect in (7) will be positive. The average margin $p(Q) - \bar{c}$ is unaffected by the concave transformation, while $\frac{E\sigma^2}{p(Q)-\bar{c}-w}$ becomes more negative.

Second, suppose that the variance of marginal costs, σ^2, increases while $\bar{c}$ remains constant. For this effect to work the number of retailers is assumed to be unchanged. The average margin, $p(Q) - \bar{c}$, remains constant, because total output does not depend on the variance of costs, by the result of Bergstrom and Varian (1985), and $E(Q)$ is also unchanged. A high enough value of σ^2 ensures that the redistribution of output effect becomes sufficiently positive that the marginal welfare effect is positive.

Third, suppose that $\bar{c}$ increases while σ^2 remains constant. Again assume that the number of firms remains unchanged. Now assume also that E is constant. The retail price increases as $\bar{c}$ rises, with the same pass-through coefficient as in (4). Because demand is concave pass-through is below 1, and the average margin, $p(Q) - \bar{c}$, falls. Thus $p(Q) - \bar{c}$ is a smaller positive number, and $\frac{E\sigma^2}{p(Q)-\bar{c}-w}$ is a more negative number, and for both reasons the expression in square brackets in (7) falls. At a high enough value of $\bar{c}$ the expression in square brackets falls below zero and the welfare effect becomes positive.

4 Upstream Monopolization and Welfare

Upstream monopolization can increase welfare. Direct calculation is used rather than integration of marginal welfare effects, because the latter will change sign if upstream monopolization induces downstream monopolization.

Inverse demand has constant curvature of -8 and is $p(Q) = 1.2 - (Q^9 - 1)/9$. There is one efficient retailer with cost $c_1 = 0.8$ and there are four inefficient retailers with cost $c_2 = 1.05$. The simple average of the costs is $\bar{c} = 1$ and the variance is $\sigma^2 = 0.01$. The Cournot total output is 1, with the output and share of the efficient firm being 0.4. The Cournot price is 1.2. At the initial wholesale price of zero the marginal effect of a higher wholesale price, given in (7), is positive. Upstream monopolization induces the inefficient retailers to leave the market. The upstream monopolist sets a wholesale price of $w = 0.46$ and the downstream monopolist sets a retail price of $p(Q) = 1.306$ with monopoly output of 0.7103.[4] The ratio of the downstream margin to the upstream margin, $(p(Q)-w-c_1)/w$, equals 1/10. Adachi and Ebina (2014) show generally that this ratio equals $1/(2-E)$. Monopolization

[4] The wholesale pricing problem is to maximize $wQ(p(w))$ where $Q(p)$ is direct demand and $p(w)$ is the monopoly retail price as a function of the wholesale price. The monopoly retail price, conditional on w, is $p(w) = (9 \times 1.2 + 1 + w)/10$, and the monopoly wholesale price is $w = (9 \times (1.2 - 0.8) + 1)/10 = 0.46$.

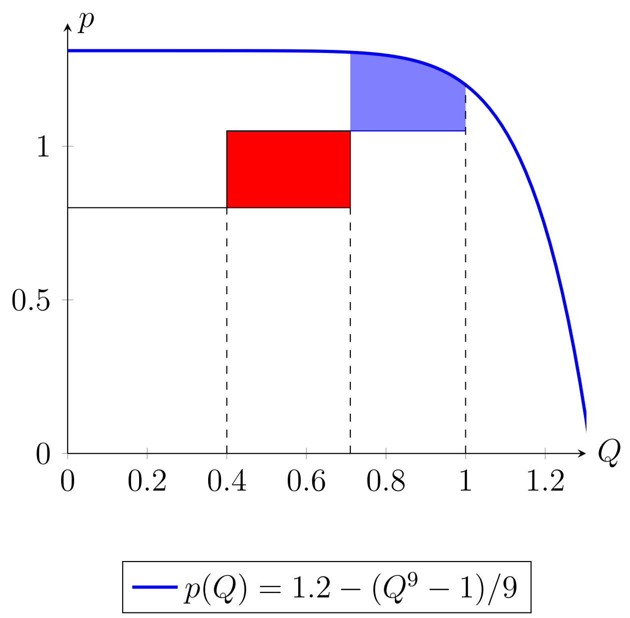

Figure 1: Welfare-Increasing Upstream Monopolization

induces double margins but the effect of the additional margin is small because inverse demand is very concave and thus cost pass-through is low.

Total welfare is 0.35 with upstream competition and downstream oligopoly. With upstream monopolization total welfare is higher at 0.3627.[5] The welfare change can be divided into two parts. First, because monopolization reduces total output there is a loss of welfare. Total output falls by the difference between oligopoly output and monopoly output (here by $1 - 0.7103$). Utility is lower but the lost output was relatively expensive to supply under oligopoly. This loss of welfare is illustrated by the blue (or lightly-shaded) area in Figure 1. Second, there is an increase in welfare because of the reduced cost of producing the extra units of output that the most efficient retailer sells rather than the inefficient retailers, represented by the red (or dark shaded) rectangle in Figure 1. This is the increase in output of the efficient firm, $0.7103 - 0.4$, times the reduction in the marginal cost of producing those units, $1.05 - 0.8$. The second effect is stronger than the first.

The functions are chosen so that a concave transformation of inverse demand, represented by a more negative value of E, does not affect the outputs or the price in the Cournot equilibrium. As E becomes more negative the loss, represented by the blue area, becomes smaller for two reasons: inverse demand becomes even flatter up to the Cournot output of 1, and the monopoly output increases. The increase in monopoly output also increases the width of the welfare gain represented by the red area. In the limit as inverse demand approaches a rectangle the loss term becomes

[5]To calculate welfare use the fact that consumer surplus, as a function of output, is $Q^{10}/10$.

close to zero while the gain from reallocating output approaches its upper bound, which is the cost saving if the monopoly output were to equal 1.

5 When is monopoly better than duopoly?

When will monopoly pricing by an efficient firm yield higher welfare than a Cournot duopoly where the efficient firm competes with an inefficient rival? This is a classic "market power versus efficiency" question. Here there are no vertical issues. The analysis in Section 2 applies with $n = 2$ and a wholesale price of 0. Initially there are two Cournot firms with different marginal costs and positive outputs. Then the industry is monopolized with the more efficient firm being the only producer. Again demand curvature matters. In particular if demand is strictly log-concave, with $E < 1$, there is a range of values of the cost difference for which monopoly yields higher social welfare than duopoly. The key insight here is that this range is larger the more concave is inverse demand.

The cost difference could be large enough that the inefficient duopolist sets an output of zero so the Cournot outcome is the same as the monopoly one. The interesting cost differences are smaller. In the language of the research and development literature the cost differences will be non-drastic, with the monopoly price of the efficient firm being above the marginal cost of the inefficient one.

Let firm 1 be the efficient firm and firm 2 be the inefficient one, with $c_1 < c_2$. The monopolist chooses the price to maximize $(p - c_1)Q(p)$, where $Q(p)$ is direct demand, with first-order condition $Q(p) + (p - c_1)Q'(p) = 0$. The solution is the monopoly price, $p^m(c_1)$. The assumption that $E < 2$ implies that (i) monopoly profit as a function of price is strictly quasi-concave and (ii) monopoly profit as a function of output is strictly concave. The monopoly cost pass-through coefficient depends on curvature alone and is $\frac{dp^m}{dc_i} = \frac{1}{2-E}$, which is the special case of (4) when $n = 1$. When demand is strictly log-concave, so $E < 1$, pass-through is below 1. Pass-through equals or exceeds 1 when demand is log-convex.

The simple average of the costs, $\bar{c} = \frac{c_1+c_2}{2}$, is fixed. Let $x \equiv \frac{c_2-c_1}{2}$ be half the cost difference, so $c_1 = \bar{c} - x$, $c_2 = \bar{c} + x$ and x measures the distance between a firm's marginal cost and the simple average. The variance of the marginal costs is x^2, so x is also the standard deviation. Write the monopoly price as a function of x only, so $p(x) \equiv p^m(c_1) = p^m(\bar{c} - x)$, with $p'(x) = -\frac{dp^m}{dc_1}$. The maximum value of x, called y, is defined as the value at which the inefficient firm's cost equals the efficient firm's monopoly price: $\bar{c} + y = p(y)$. Assume that $\bar{c} - y \geq 0$ so the efficient firm's cost at the maximum value of x is non-negative.

Monopoly output as a function of x is $Q(p(x))$. Output increases as x rises with

$$\frac{d}{dx}Q(p(x)) = Q'(p)p'(x) = \frac{Q(p(x))}{p^m(c_1) - c_1}\frac{dp^m}{dc_1} > 0,$$

where the second equality uses the monopoly first-order condition. We explore what happens for values of $x \in [0, y]$, so the cost difference is non-drastic. The sum of consumer surplus and profits is used as the welfare measure. Consumer surplus is $V(p)$, with $V'(p) = -Q(p)$ by Roy's identity for quasi-linear utility. Monopoly welfare is

$$W^m(x) = V(p(x)) + (p(x) - \bar{c} + x)Q(p(x)).$$

An increase in x reduces marginal cost and the price and increases consumer surplus. At the margin the effect on consumer surplus is $\frac{d}{dx}V(p(x)) = Q(p(x))\frac{dp^m}{dc_1} > 0$. The effect on monopoly profit of an increase in x equals $Q(p(x))$. The envelope theorem implies that the total effect on monopoly profit equals the direct effect, which is the reduced cost of producing the given output. The derivative of monopoly welfare with respect to x is thus

$$\frac{dW^m}{dx} = Q(p(x))\left(\frac{dp^m}{dc_1} + 1\right).$$

Because $\bar{c}$ is constant the result of Bergstrom and Varian (1985) implies that in the Cournot equilibrium total output, Q^d, and the price, p^d, are unaffected by changes in $x \in [0, y]$. The superscript "d" denotes a variable under duopoly. It is important that, by construction, when x takes its maximum value the duopoly outcome is the same as the monopoly outcome for $c_1 = \bar{c} - y$. As $\bar{c}$ is fixed total duopoly output, for every value of x, is $Q^d = Q(p(y))$, and the duopoly price is the monopoly price with the lowest possible marginal cost: $p^d = p(y)$.

The individual duopoly outputs depend on x. The first-order conditions, (1), imply that

$$q_1(x) = \frac{p^d - \bar{c} + x}{-p'(Q^d)},$$
$$q_2(x) = \frac{p^d - \bar{c} - x}{-p'(Q^d)}$$

so the difference in duopoly outputs is proportional to x:

$$q_1(x) - q_2(x) = \frac{2x}{-p'(Q(p(y)))} = \frac{xQ(p(y))}{y},$$

using the fact that $Q^d = Q(p(y))$, the monopoly first-order condition and the definition of y. The difference in outputs is zero when there is no cost difference, and equal to monopoly output with the lowest marginal cost when the cost difference is at its maximum.

Total welfare with duopoly is

$$\begin{aligned} W^d &= V(p^d) + (p^d - c_1)q_1 + (p^d - c_2)q_2 \\ &= V(p^d) + (p^d - \bar{c})Q^d + x(q_1(x) - q_2(x)) \\ &= V(p(y)) + (p(y) - \bar{c})Q(p(y)) + \frac{x^2 Q(p(y))}{y} \\ &= W^m(y) - \frac{(y^2 - x^2)Q(p(y))}{y}. \end{aligned}$$

where the second version uses the definition of x, the third version uses the expression for the difference in duopoly outputs and the facts that $Q^d = Q(p(y))$ and $p^d = p(y)$, and the fourth version adds and subtracts $yQ(p(y))$. Duopoly welfare is increasing in the variance of costs, x^2. The effect of x on duopoly welfare is

$$\frac{dW^d}{dx} = \frac{2xQ(p(y))}{y},$$

which is twice the difference in firm outputs.

Define the difference between monopoly and duopoly welfare as

$$\begin{aligned} \Delta(x) &\equiv W^m(x) - W^d(x) \\ &= W^m(x) - W^m(y) + \frac{(y^2 - x^2)Q(p(y))}{y}. \end{aligned}$$

The values of $\Delta(x)$ at 0 and y are immediate. Social welfare is lower with monopoly than with duopoly when $x = 0$, so $\Delta(0) < 0$. There is no cost difference, so the given output is efficiently produced, and the duopoly output is above the monopoly level. Second, when $x = y$ social welfare with monopoly equals that with duopoly. Thus $\Delta(y) = 0$.

The marginal effect of x on $\Delta(x)$ is the difference between the two marginal welfare effects defined above:

$$\Delta'(x) = \frac{dW^m}{dx} - \frac{dW^d}{dx} = Q(p(x))\left(\frac{dp^m}{dc_1} + 1\right) - \frac{2xQ(p(y))}{y}.$$

At $x = 0$ this is positive and equal to $Q(p(0))\left(\frac{dp^m}{dc_1} + 1\right)$. Locally the welfare difference becomes less negative as x rises above 0. At the maximum cost difference the marginal effect is:

$$\Delta'(y) = Q(p(y))\left(\frac{dp^m}{dc_1} - 1\right) = Q(p(y))\left(\frac{E-1}{2-E}\right),$$

where $E = E(Q(p(y)))$. The sign of the effect here depends on whether monopoly pass-through is above or below 1. It is negative if demand is strictly log-concave,

$E < 1$, so pass-through is strictly below 1. The marginal welfare effect is zero or positive if demand is log-convex.

When demand is strictly log-concave there always exists a set of values of x below y for which monopoly yields higher welfare than duopoly. At y the welfare difference is zero. The welfare difference function is differentiable and thus continuous. With strictly log-concave demand $\Delta'(y) < 0$ so the welfare difference in the neighborhood to the left of y falls from a positive number towards zero as x moves towards y. Thus there is a region where the welfare difference is positive.

Proposition 3. *Suppose that demand is strictly log-concave. Monopoly welfare is above that with asymmetric duopoly for values of the cost difference measure close to the maximum.*

Now make the weak assumption that pass-through is constant or increasing, so $\frac{d^2p^m}{dc_1^2} \geq 0$. Many demand functions are strictly log-concave and imply that cost pass-through is weakly increasing—see Fabinger and Weyl (2018), Appendix J. Examples include demands for which E is constant and below 1 and probit and logistic demands. For these demand functions the result in Proposition 3 has more structure. The key is that the welfare difference, $\Delta(x)$, is strictly concave. Concavity implies that there is an unique value $\tilde{x} \in (0, y)$ at which the welfare difference is zero. For higher values satisfying $\tilde{x} < x < y$ the welfare difference is strictly positive. As x moves close to its maximum value the welfare difference remains positive but is closer to zero.

Proposition 4. *Suppose that demand is strictly log-concave and that monopoly cost pass-through is weakly increasing in marginal cost. There exists a unique value $\tilde{x} \in (0, y)$ such that monopoly yields strictly higher welfare than duopoly for all $x \in (\tilde{x}, y)$.*

Proof. First, strict concavity of $\Delta\colon [0, y] \to \mathbb{R}$ is proved. The second derivative of $\Delta(x)$ is

$$\Delta''(x) = \left(\frac{dp^m}{dc_1} + 1\right) \frac{dp^m}{dc_1} \frac{Q(p(x))}{p^m(c_1) - c_1} - 2\frac{Q(p(y))}{y} - Q(p(x))\frac{d^2p^m}{dc_1^2}$$

The first two terms are together negative because $\left(\frac{dp^m}{dc_1} + 1\right) \frac{dp^m}{dc_1} < 2$ by strict log-concavity, $Q(p(x)) \leq Q(p(y))$ by the fact that $Q(p(x))$ is increasing in x and $p^m(c_1) - c_1 > y$ because $p^m(c_1) - c_1 - y = p(x) - (\bar{c} + y) + x = p(x) - p(y) + x > 0$. By assumption $\frac{d^2p^m}{dc_1^2} \geq 0$ so the third term is negative. We know that $\Delta(0) < 0, \Delta'(0) > 0, \Delta(y) = 0$, and, by strict log-concavity, $\Delta'(y) < 0$. With strict concavity of $\Delta(x)$ it follows that $\Delta(\tilde{x}) = 0$ at a unique $\tilde{x} \in (0, y)$ and that for all $x \in (\tilde{x}, y)$, $\Delta(x) > 0$. $\square$

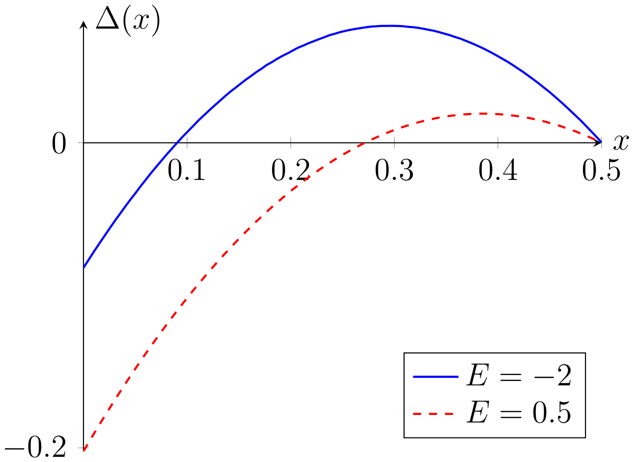

Figure 2: The difference in welfare for $E = -2$ and $E = 0.5$

Figure 2 illustrates $\Delta(x)$ for a constant $E = -2$ and for a constant $E = 0.5$, both with the same $y = 0.5$ and with duopoly output of $Q(p(y)) = 1$. It is notable that the set of values of x for which monopoly is better than duopoly is considerably larger with $E = -2$. More concavity increases the possibility that there is a welfare gain from monopolization. Formally let inverse demand be $p(Q) = a - (Q^{1-E} - 1)/(1-E)$ for $E < 1$. Set $a = \bar{c} + 0.5$ so that the duopoly price equals a and is independent of curvature. It follows that duopoly output is 1. The monopoly price is $p(x) = ((1-E)a + 1 + \bar{c} - x)/(2-E)$, and, with $a = \bar{c} + 0.5$, this implies that $y = 0.5$. The welfare difference is a function only of E and x and is

$$\Delta(x) = \left(\frac{3-E}{2-E}\right)\left(\frac{3-E+2(1-E)x}{2(2-E)}\right)^{\frac{2-E}{1-E}} - \left(\frac{3-E}{2-E}\right) + \frac{1}{2} - 2x^2.$$

For $E < 1$ this is a strictly concave function, and the value of $x < y$ at which $\Delta(x) = 0$ can be shown numerically to be increasing in E.

What happens when demand is log-convex? Consider the two standard classes of log-convex demands: exponential demand, for which $E = 1$, and constant elasticity demand, with elasticity $\varepsilon > 1$, for which $E = 1 + \frac{1}{\varepsilon}$.

Proposition 5. *Suppose demand is exponential or has a constant elasticity greater than* 1*. Social welfare is higher with duopoly when both firms have positive outputs than with monopoly pricing by the efficient firm.*

Proof. See the Appendix. □

The common feature of the demand functions covered by Proposition 5 is that $\Delta'(x) > 0$ for $x \in [0, y)$. As $\Delta(0) < 0$ and $\Delta(y) = 0$, and $\Delta(x)$ is continuous, it follows that $\Delta(x) < 0$ for $x \in [0, y)$. Proposition 2 in Lahiri and Ono (1988) states that "Under Cournot oligopoly, national welfare increases if a firm with a sufficiently low share is removed from the market." An implication of Proposition 5 is that Lahiri and Ono's result does not hold under the conditions stated.[6] Elimination of an inefficient duopolist that has a positive output always reduces welfare with these log-convex demand functions.

The comparison in this section is between a duopoly and a monopoly. When there are more than two firms elimination of an inefficient firm may raise welfare with log-convex demands. For example suppose there are three firms with $c_1 = \bar{c} - x, c_2 = \bar{c}$ and $c_3 = \bar{c} + x$ and that demand is exponential. At a sufficiently high value of x, below the level at which the firm with the highest cost would exit anyway, social welfare is higher without this firm.

In the case of vertical monopolization a necessary condition for welfare to increase is that $E < 0$. Here double margins are not an issue. Welfare can increase with monopolization under the weaker condition that $E < 1$, and it will increase if the cost difference is large enough.

6 The welfare effects of cost changes and mergers

This section considers the models of Lahiri and Ono (1988) and Farrell and Shapiro (1990) in the light of the above analysis. It confirms the conjecture that greater concavity of demand increases the benefits of the redistribution of output when total output falls. Lahiri and Ono (1988) show that a reduction in marginal cost could harm welfare if the firm whose cost is reduced is sufficiently small. The reason is that other firms with higher margins cut their outputs in response. Consider the effect on welfare of a marginal increase in a firm's cost (say that of firm k). The general expression for this effect, which we look to be positive, is

$$\frac{dW}{dc_k} = \sum_{i=1}^{n} (p(Q) - c_i) \frac{dq_i}{dc_k} - q_k. \tag{8}$$

The direct effect of the cost increase, before any responses, is negative and is $-q_k$. The initial output of firm k costs more to produce. The summation term is the output response of each firm multiplied by that firm's margin. This could outweigh the direct effect if firm k's margin, $p(Q)-c_k$, is small, which will imply that q_k is also small. The extra cost of k's output has a minor negative effect on welfare, and the loss from reduced output of k, $(p(Q) - c_k)\frac{dq_k}{dc_k}$, is small, while there are substantial gains in welfare from increases in the outputs of firms with higher margins.

[6] Lahiri and Ono (1988) assume that $1 - s_i E > 0$ for all i, which is effectively the assumption that $E < 1$. Thus they do not consider log-convex demand.

Lahiri and Ono (1988) assume that outputs are strategic substitutes, which in our notation is $1 - s_i E > 0$. This implies that a firm's marginal revenue falls as the output of the other firms increases, and is also the Hahn stability condition (Hahn, 1962). A small generalization is possible: the analysis that follows does not require the strategic substitutes assumption to hold for each firm. The earlier assumption that $E < 2$ is not necessary here, so instead assume directly that $n + 1 - E > 0$ to guarantee uniqueness.[7]

The effects of the cost increase on firm k's output and the other firms' outputs may be written as

$$\begin{aligned}\frac{dq_k}{dc_k} &= -\frac{(1 - s_k E)}{p'(Q)(n+1-E)} + \frac{1}{p'(Q)};\\ \frac{dq_j}{dc_k} &= -\frac{(1 - s_j E)}{p'(Q)(n+1-E)},\ j \neq k.\end{aligned}$$

The Herfindahl Index, defined as the sum of the squared market shares, $H \equiv \sum_{i=1}^{n} s_i^2$, is the standard measure of concentration. The output derivatives and the fact that the margin is $p(Q) - c_i = -q_i p'(Q)$ imply that (8) is

$$\frac{dW}{dc_k} = Q\left(\frac{1 - EH}{n+1-E} - 2s_k\right). \tag{9}$$

This is positive if

$$s_k < \frac{1 - EH}{2(n+1-E)}.$$

Expression (9) is new: it is consistent with the analysis of Lahiri and Ono (1988) but is not given in their article. A necessary condition for a positive welfare effect is that the weighted average of $\{1 - s_i E\}$ is positive: $\sum_{i=1}^{n} s_i(1 - s_i E) = 1 - EH > 0$, so on "average" there are strategic substitutes. Note that $\frac{1-EH}{2(n+1-E)}$ is increasing as E falls, for a given value of the Herfindahl.[8] More concavity makes it more likely that the welfare effect in (9) is positive.

The condition for (9) to be positive is straightforward to apply. Three examples follow. First, if demand is linear, so $E = 0$, welfare rises with a cost increase for a firm whose share is below $1/2(n + 1)$. Zhao (2001) obtains this result for linear demand by direct calculation. Second, Lahiri and Ono (1988) show that a cost increase must reduce welfare when the firms are identical. With symmetry $H = \frac{1}{n}$ and $s_k = \frac{1}{n}$. The welfare effect is positive if and only if $n + 2 - E < 0$, which cannot hold because $n + 1 - E > 0$. Third, in line with Proposition 5 above, when there is

[7] The assumption that $E < 2$ would be needed if the industry were to be monopolized. Here we focus on the effect of marginal changes in cost without altering industry structure.

[8] The derivative of this expression with respect to E has the same sign as $1 - Hn - H < 0$ because $H > \frac{1}{n}$.

a duopoly and demand is exponential or has a constant elasticity the welfare effect cannot be positive. This can be shown easily for exponential demand. Using the same notation as in Section 5, so $x = c_2 - \bar{c}$, the share of the inefficient firm is $s_2 = \frac{1}{2} - x$ and the Herfindahl index is $H = \frac{1}{2} + 2x^2$ for $x \in [0, 0.5]$. The condition for a cost increase for the inefficient firm to raise welfare in this case is $\frac{1}{2} - x < \frac{1}{8} - \frac{x^2}{2}$, which does not hold for $x \in [0, 0.5]$.

Subtracting and adding $\bar{c}$ within the margins in (8) yields an alternative expression for the effect of the cost increase:

$$\frac{dW}{dc_k} = \left(p(Q) - \bar{c} + \frac{E\sigma^2}{p(Q) - \bar{c}}\right)\frac{dQ}{dc_k} + Q\left(\frac{1}{n} - 2s_k\right)$$

The effect of the cost increase on total output, $\frac{dQ}{dc_k} = \frac{1}{p'(Q)(n+1-E)}$, is negative. This is multiplied by a familiar term that is the same as the expression in square brackets in (7) for a zero wholesale price. The welfare effect is positive if E is sufficiently negative and firm k's share is below $1/2n$. Concavity of demand is not, however, required for the welfare effect to be positive.

Farrell and Shapiro (1990) consider the welfare effects of a merger in a Cournot model. The merger is assumed to increase the joint profits of the merging firms. The focus is on mergers that reduce total output but may increase the total welfare of consumers and non-merging firms combined. The effect of a "marginal" merger is assessed. The welfare of consumers and the non-merging firms, with O denoting the set of firms outside the merger, is consumer surplus plus the outsiders' profits, $Z(Q) = V(p(Q)) + \sum_{i \in O}(p(Q) - c_i)q_i$. The effect of an increase in total output on consumer surplus plus the outsiders' profit is

$$Z'(Q) = n(p(Q) - \bar{c})\left(\sum_{i \in I} s_i - \sum_{i \in O} s_i + E\sum_{i \in O} s_i^2\right), \tag{10}$$

where I denotes the set of merging firms or the insiders (see the appendix for the derivation of (10)). This has the same sign as the expression in large brackets, which is the difference between the share of the insiders, $\sum_{i \in I} s_i$, and the share of the outsiders, $\sum_{i \in O} s_i$, plus E times the contribution of the outsiders to the Herfindahl index. When $Z'(Q)$ is negative a reduction in total output increases Z. The expression in large brackets is

$$\sum_{i \in I} s_i - \sum_{i \in O} s_i(1 - s_i E),$$

so a necessary condition for this to be negative is that for the outsiders outputs are, on average, strategic substitutes: $\sum_{i \in O} s_i(1 - s_i E) > 0$. Note that by definition $\sum_{i \in I} s_i + \sum_{i \in O} s_i = 1$.

With linear demand a small merger that reduces total output raises the welfare of consumers and the non-merging firms combined if the share of the outsiders is larger than the share of the insiders. The general expression is more likely to be negative, other things equal, the more concave demand is: for given market shares the expression to be signed decreases as E falls. The more concave demand is the greater the increase in the output of the outsiders in response to the reduction in output of the insiders.

7 Conclusion

The reallocation of output effect is important in determining the sign of welfare effects in oligopoly. This paper provides a comprehensive analysis of the conditions under which monopolization by an efficient firm will raise welfare. The key is demand curvature. With highly concave demand the cost savings from the reallocation of output can imply that a monopoly selling to a monopoly with double margins yields higher social welfare than a competitive industry selling to an oligopoly. With strictly log-concave demand social welfare can be higher with an efficient monopoly than with a duopoly when there are no vertical issues, and this effect is more pronounced the more concave inverse demand is.

Throughout we have considered situations where total output falls. Suppose, instead, that total output increases, perhaps because an industry-wide tax is reduced or a production subsidy is offered. The analysis suggests that it will be when demand is strictly convex that there will be a positive reallocation of output effect, and this will reinforce the positive effect of higher output.

There are several reasons for being cautious about applying the analysis of this paper to policy contexts. First, antitrust policymakers often have the objective of maximizing consumer surplus and in the models in this paper consumer surplus is always lower with monopoly because total output falls. Second, arguments for monopolization might be improperly used by special interests seeking to entrench unwarranted market power.

Appendix

Retail market shares and the cost of retailing

The first-order condition (1) implies that the output of firm i is $q_i = (p(Q) - c_i - w)/(-p'(Q))$ and its market share is $s_i = (p(Q) - c_i - w)/(-Qp'(Q)) = (p(Q) - c_i - w)/n(p(Q) - \bar{c} - w)$ using (2). Subtracting and adding $\bar{c}$ within the numerator gives

$$s_i = \frac{1}{n} + \frac{\bar{c} - c_i}{n(p(Q) - \bar{c} - w)}.$$

It follows that

$$\sum_i c_i s_i = \frac{\sum_i c_i}{n} + \frac{\sum_i c_i(\bar{c} - c_i)}{n(p(Q) - \bar{c} - w)} = \bar{c} - \frac{\sigma^2}{p(Q) - \bar{c} - w}.$$

Proof of Proposition 5

(i) Exponential demand is $Q(p) = e^{(a-p)/b}$ for $b > 0$. Without loss of generality set $a = 0$ and $b = 1$ so $Q(p) = e^{-p}$. For this demand function $E = 1$ so $\Delta'(y) = 0$. The monopoly price is $p(x) = \bar{c} - x + 1$ and $y = 0.5$. Output is $Q(p(x)) = e^{-\bar{c}-1+x}$. Consumer surplus is $V(p) = e^{-p} = Q(p)$ so monopoly welfare is $W = Q(p)(1 + p - \bar{c} + x) = 2Q(p(x))$. The welfare difference can be calculated directly and is

$$\Delta(x) = 2e^{-\bar{c}-1}\left(e^x - (0.75 + x^2)e^{0.5}\right)$$

$\Delta\colon [0, 0.5] \to \mathbb{R}$ is a concave and increasing function that reaches its maximum value of 0 at $y = 0.5$. Thus $\Delta(x) < 0$ for $x < y$.

(ii) Constant elasticity demand is $Q(p) = Ap^{-\varepsilon}$ for $A > 0$ and $\varepsilon > 1$. Without loss of generality set $A = 1$ and $\bar{c} = 1$. The monopoly price is $p(x) = \frac{\varepsilon}{\varepsilon-1}(1 - x)$. Monopoly pass-through is $\frac{dp^m}{dc_1} = \frac{\varepsilon}{\varepsilon-1} > 1$ so $\Delta'(y) > 0$. The maximum value of the cost difference measure is $y = \frac{1}{2\varepsilon-1}$, and $1 - y = 2\frac{(\varepsilon-1)}{2\varepsilon-1}$. Note that $\frac{dp^m}{dc_1} + 1 = \frac{2\varepsilon-1}{\varepsilon-1}$. The derivative of the welfare difference is

$$\begin{aligned}
\Delta'(x) &= Q(p(x))\frac{2\varepsilon - 1}{\varepsilon - 1} - \frac{2x}{y}Q(p(y)) \\
&= 2\left(\frac{Q(p(x))}{1 - y} - \frac{xQ(p(y))}{y}\right).
\end{aligned}$$

$\Delta'(x)$ is itself strictly convex in x because $Q(p(x))$ is strictly convex in x: $Q(p)$ is strictly convex and $p(x)$ is linear. If $\Delta'(x)$ is decreasing or increasing everywhere then, because $\Delta'(0) > 0$ and $\Delta'(y) > 0$, $\Delta'(x) > 0$ over $[0, y]$ and thus $\Delta(x) < 0$ for $x < y$. Alternatively $\Delta'(x)$ reaches a minimum in $(0, y)$. If the value of $\Delta'(x)$ at the minimum is positive then $\Delta'(x) > 0$ everywhere. An interior minimum satisfies $\Delta''(\tilde{x}) = 0$ and

$$\Delta'(\tilde{x}) = \frac{2Q(p(y))}{y}\left(\frac{1-(\varepsilon+1)\tilde{x}}{\varepsilon}\right).$$

This is positive if the expression in large brackets is positive. There are two cases. First, when $\varepsilon \geq 2$ this holds because $1-(\varepsilon+1)\tilde{x} > 1-(\varepsilon+1)y = \frac{\varepsilon-2}{2\varepsilon-1} \geq 0$. Second, for $\varepsilon \in (1,2)$ $\Delta'(x)$ always reaches an interior minimum. Solving $\Delta''(\tilde{x}) = 0$ gives

$$\tilde{x} = 1 - \left(\frac{\varepsilon}{2(\varepsilon-1)}\left(\frac{2(\varepsilon-1)}{2\varepsilon-1}\right)^{\varepsilon}\right)^{\frac{1}{\varepsilon+1}}.$$

Over $(1,2]$ the expression $\frac{1-(\varepsilon+1)\tilde{x}}{\varepsilon}$ is a decreasing function of ε. It follows that for all $\varepsilon \in (1,2]$ the minimum value of $\Delta'(x)$ is strictly positive. Thus $\Delta'(x) > 0$ for $x \in [0,y]$. The welfare difference is negative at $x = 0$ and then increases continuously until it reaches its maximum of zero at y.

The Farrell-Shapiro derivative

The welfare of consumers and the non-merging firms, with O denoting the set of firms outside the merger, is consumer surplus plus the outsiders' profits

$$\begin{aligned} Z(Q) &= V(p(Q)) + \sum_{i\in O}(p(Q)-c_i)q_i \\ &= V(p(Q)) - p'(Q)\sum_{i\in O} q_i^2 \end{aligned}$$

where the second equation follows from the firms' first-order conditions. Differentiating the first-order condition (1) gives the effect of an increase in total output on the output of firm i:

$$\frac{dq_i}{dQ} = -(1 - s_i E).$$

The marginal effect of higher total output on Z is

$$\begin{aligned} Z'(Q) &= -Qp'(Q) - p''(Q)\sum_{i\in O} q_i^2 - 2p'(Q)\sum_{i\in O} q_i\frac{dq_i}{dQ} \\ &= n(p(Q)-\bar{c})\left(\sum_{i\in I} s_i - \sum_{i\in O} s_i + E\sum_{i\in O} s_i^2\right) \end{aligned}$$

where I denotes the set of merging firms and the aggregate first-order condition implies that $n(p(Q)-\bar{c}) = -Qp'(Q)$.

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