Teachers' perceptions regarding the use of manipulatives within the secondary mathematics classroom and a study into the effective provision of initial teacher training in the use of physical based tasks to aid pupil learning

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Abstract

Teachers' perceptions regarding the use of manipulatives within the secondary mathematics classroom and a study into the effective provision of initial teacher training in the use of physical based tasks to aid pupil learning.

Research has shown manipulatives to be a useful tool within the mathematics classroom, enabling pupils to better connect abstract mathematical concepts with a concrete understanding. However, their use varies between school, teacher and stage of learning, with the majority of use occurring in primary schools. Research also implies that many teachers are ill-equipped to use manipulatives effectively, unsure how to facilitate the forming of connections; either using them to little effect or dismissing their use altogether. This study set out to answer three research questions:

- What are teachers' perceptions of the role of manipulatives in secondary mathematics education?
- In what ways and to what extent are secondary mathematics teachers equipped to use manipulatives within the classroom?
- How can the use of manipulatives be effectively integrated into a secondary initial teacher education course?

The literature review focuses on manipulatives (within teaching and teacher education), concrete to abstract mathematical understanding and teachers' beliefs, knowledge and growth. Data was collected through a survey (150 participants) and more in-depth semi-structured interviews of 5 mathematics teachers, with the main findings presented under the headings of the three research questions. Findings show teachers to hold predominantly positive beliefs regarding the potential usefulness of manipulatives within secondary schools. However, these belief seem to contradict teachers' practice. The data suggests teachers to be lacking in confidence and knowledge on how best to use manipulatives, with some feeling obliged to use them but lacking the adequate skills. There is a distinct lack of education in the effective use of manipulatives and most teachers would value further professional development in this area, recognising manipulatives' potential value. Recommendations are made on the effective embedding of manipulatives into an initial teaching programme, focusing on the barriers to using manipulatives effectively which were most prevalent in the data, and suggestions for further research.

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List of Abbreviations

IT	Initial Teacher
ITY	Initial Teaching Year
PT	Primary Teacher
SMT	Secondary Mathematics Teacher
TE	Teacher Educator
KQ	Knowledge Quartet

Chapter 1: Introduction

This study sets out to investigate teachers' beliefs and practice regarding the use of manipulatives in secondary education and how the use of manipulatives can be better integrated into an initial teacher (IT) education course. This chapter describes the rationale behind the study and introduces the research questions used.

1.1. Rationale

The use of manipulatives in mathematics education is not new. Artefacts show that early civilisations used objects such as beans or stones to aid mathematical calculations and over time these have developed into the multitude of sophisticated manipulatives seen in classrooms today (Golafshani, 2013). Although Pestalozzi, in the nineteenth century, is often cited as the first person to strongly advocate the use of hands-on learning (his motto: 'Learn by head, heart and hand') (Sowell, 1989), the modern use of manipulatives can be traced back to Piaget (1952) and his persuasion that children require interactions with models and instruments to make the invisible visible, enabling them to grasp mathematical concepts (Golafshani, 2013). Marshall & Swan (2008:338) refer to an ancient proverb which they say provided the basis often used to justify the use of manipulatives in the 1960's:

I hear and I forget
I see and I remember
I do and I understand.

However, Marshall & Swan believe, based on their own experience and observations, that (primary) teachers use mathematics manipulatives convinced that their use automatically equals understanding, in line with the mantra above. They found that teachers did not question the effect or value of the manipulatives used nor how to create connections between the manipulatives and mathematical concepts. Research seems to suggest (see Chapter 2) that when manipulatives are embedded within teaching practice, and used by teachers confident and able in their use, that

manipulatives can help both primary and secondary pupils to better understand and apply mathematical principles. Swan & Sparrow (2004) argue that teachers play a key role in helping pupils connect ideas, between the concrete and the abstract, and that for effective manipulative use a fourth line should be added to the proverb:

I talk it and I connect.

In my own experience, manipulatives are becoming ever more prevalent in secondary mathematics departments (although not necessarily used within the classrooms) driven by an unchallenged assumption that their use will miraculously end all pupil conceptual misunderstandings, and pushed by the mathematics manipulatives industry. However, I have witnessed teachers struggling or nervous when encouraged to use manipulatives or purposefully avoiding them all together. Often I have seen manipulatives in mathematics cupboards, left unused. As Ball (1992:47) puts it:

'Delivering boxes of plastic links, wooden cubes, and pattern blocks is insufficient to affect the practice of mathematics teaching and learning.'

In the UK there is presently no set curriculum when it comes to teacher education. There are a number of routes to becoming a qualified teacher (Whiting et al., 2018) and once qualified there is no additional compulsory education, though most schools would expect their staff to participate in an aspect of continuous professional development. Because of this lack of a consistent curriculum, there is no guarantee that a teacher will ever receive education in the use of manipulatives in their initial teaching year (ITY) or beyond. If the use of manipulatives can be as effective and significant as some research suggests, then it could be argued that the use of manipulatives should be a fundamental part of all IT training programmes. Yet, in my experience, manipulatives were never referenced within any part of my ITY and I did not first observe their effective use until many years into my teaching career. Now, with my duel-role as a teacher and teacher educator (TE), I am interested to know whether my own experience of feeling ill-equipped in how to use manipulatives effectively is a common experience of other teachers. Specifically, I would like to know how the use

of manipulatives can be integrated successfully into an initial education course so that I, and others, can ensure teachers are better equipped from the start to take full advantage of the resources available to them.

Contrary to research showing that manipulatives can be a useful tool across all ages (see Chapter 2), my anecdotal experience aligns with Marshall & Swan (2008) and Gilbert & Bush (1988) who found that teachers manipulative use decreases with age of pupil. I am interested to know why this contradiction exists and whether it is connected to teachers' beliefs and perceptions of the use of manipulatives in secondary schools. If so, then maybe teachers' beliefs and perceptions could be shifted. If it is not due to teacher's beliefs and perceptions, then maybe other reasons for the decline in use with age can be identified and targeted.

1.2. Research Questions

I have condensed the areas of interest described above, regarding manipulatives in secondary schools and teacher education, into three research questions which shape the remainder of this study:

1) What are teachers' perceptions of the role of manipulatives in secondary mathematics education?

2) In what ways and to what extent are secondary mathematics teachers (SMTs) equipped to use manipulatives within the classroom?

3) How can the use of manipulatives be effectively integrated into a secondary initial teacher education course?

These research questions have steered the investigation, influencing the literature read and reviewed, the data collected and analysed and the recommendations made. A description of the research methods used can be found in Chapter 3.

Chapter 2: Review of Literature

Having established the research questions which define this study, this chapter presents a review of the literature relevant to the enquiry, enabling later comparisons with the findings to be made. The literature review is structured under the following headings:

- **What are 'manipulatives'?** – *How does the literature define what manipulatives are? Is this consistent?*
- **Concrete to abstract mathematical understanding** – *What is the link between the concrete manipulative and the abstract mathematical concept? What is the reasoning behind the use of manipulatives?*
- **The effect of manipulatives in mathematics education** – *What does the literature say about the effect manipulatives have on learning, including in a secondary context?*
- **Manipulatives in teacher education** – *Does the literature identify a need for manipulatives to be included in teacher education? If so how, what is said on how that should be done?*
- **Teacher growth** – *How do teachers, including prospective teachers, grow in their practice i.e. in their use of manipulatives?*
- **Teacher beliefs** – *What is the relationship between teachers' beliefs and practice? What is said in the literature regarding teacher's beliefs and use of manipulatives?*
- **Teacher Knowledge** – *How is teacher knowledge described in the literature? What is said in the literature about developing a teacher's knowledge regarding the use of manipulatives?*

The chapter concludes with a brief discussion of the effect of the reviewed literature on the remainder of the study.

2.1. What are 'manipulatives'?

Although the use of 'manipulatives' in mathematics education is often discussed within literature, the definition of what a manipulative is is not always consistent. Generally, manipulatives are

considered a form of mathematical representation used in teaching and learning, and the word itself suggests the term relates to something which can be 'manipulated' or moved for a purpose. Ball (1992) notes that manipulatives are another name for 'concrete materials' or 'concrete objects', suggesting that the three terms can be used interchangeably. Kennedy (1986: 6) uses the term manipulatives to describe 'objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children in hand-on learning', following this definition with one by Young (1983: 12) which talks of a connection between the use of physical objects and the representation of abstract mathematical ideas. Examples of physical objects, which are often categorised as manipulatives include everyday items such as money, sweets or lolly-sticks, or resources specifically designed to be used as learning tools such as tiles, rods or base-ten blocks (Kennedy, 1986).

Nührenbörger and Steinbring (2008: 163), when elaborating on the theoretical perspectives of how manipulatives can be used as a learning tool with prospective teachers, have a broader definition of what a manipulative is. They understand the term to mean:

> 'working and visualisation materials on the one hand, and visual charts and diagrams as a means to represent mathematical knowledge as manipulatives on the other hand.'

This definition not only encompasses physical objects that students can touch and move around, but also diagrammatic representations of these manipulatives. For example, a child may use eight counters and split them into two piles when working out that four is half of eight, or they may draw eight circles, or marks, and split them into two 'piles' by shading in four. Here, one might argue, the drawn circles are a diagrammatic representation of a manipulative, or that the drawing is a form of manipulative in its own right.

There are also discrepancies in the literature as to whether instruments of measure (such as pairs of compasses, protractors or rulers) are classified as manipulatives or not. Nührenbörger (2001) makes the argument that an individual object can be classed as a manipulative or a non-manipulative

depending on *how* the object is being used. Taking the example of a ruler, Nührenbörger is convinced that a measurement tool can only be described as a manipulative when it becomes a tool of the mind in contrast to a technical tool. This means that the term manipulative does not describe the physical objects used but instead the way in which they are used. In contrast, Gordon (1996), examining the use of manipulatives by 129 secondary teachers in the state of Virginia, included protractors, compasses and rulers as types of manipulative in the study. Consequently, the data and findings of the study do not take into account how these instruments of measure are used but just that they have been used. Most studies within the literature have not regarded these instruments of measure as manipulatives. Therefore, one must consider the definition of manipulative used by the authors when directly comparing findings from different studies.

Increasingly in recent years, a sub-category of manipulatives is mentioned in literature: virtual manipulatives. An initial definition of a virtual manipulative by Moyer et al. (2002:373) was revised by Moyer-Packenham & Bolyard (2006:1.8) due to the changes in technology, and evolved into "an interactive technology-enabled visual representation of a dynamic mathematical object". For example, pictures of blocks on a screen that someone can move around as if the blocks were in front of them, to solve a mathematical problem. The advantages of virtual manipulatives are they are often free to find on the internet, they can be used on a large screen at the front of a classroom making it easy for everyone to see the example and they do not involve the inconvenience of handing out many objects to a class. However, along with the limitation of the availability of technology, there are those that would not class these virtual representations as actual manipulatives. Instead, they 'simulate' manipulatives and are much more abstract (Durmus et al., 2006). This is because they do not possess all of the characteristics of a manipulative. For example, blocks on a screen are not physical objects, nor do they appeal to the sense of touch. Moving objects around on a screen with a mouse does not have the same physical impact as 'playing' with the objects in front of you, as each move must be deliberate, whilst when in physical form, pupils are able to use both hands and 'try things out' quickly and easily (Moyer, 2001). Although Brown (2007),

through her comparative study involving 48 participants, found that both concrete and virtual manipulatives enhanced learning, the results of her investigation found that those pupils taught using concrete manipulatives out-performed those taught using the virtual kind. However, there are a number of differences within Brown's experiment which may have contributed to this result, including the difference in starting ability of the two groups of sixth-grade children, and the types of manipulatives used/represented (fraction bars vs pattern blocks).

2.2. Concrete to abstract mathematical understanding

Piaget is often referred to in the defence of manipulatives and cited to have shown that young people require concrete experiences to be able to learn (Ball, 1992). Other notable educational theorists have also linked the learning of abstract mathematical ideas to an effective tangible and concrete introduction. Dienes (1960) found that those children who were introduced to mathematical concepts through the use of manipulatives were better able to integrate the everyday world they knew with the abstract mathematical world. Bellonio (2012) has similarly written about the importance of manipulatives in helping children transition between a concrete and abstract level of understanding.

Bruner (1966) is credited with the conception of the 'enactive-iconic-symbolic' modes of representation theory which form the foundation for many mathematical instructional practices around the world (Leong *et al.,* 2015). Although the 'modes' are often given different names, the idea that children learn best through a scaffolding of actions, images and finally abstract symbols is one which is prevalent in many teaching models. One popular model is the concrete-pictorial-abstract model often associated with teaching in Singapore. Leong *et al.* (2015) provide a brief history of this model of teaching in Singapore, commenting on how the concrete-pictorial-abstract approach widely became a key mathematical teaching strategy in the 1980's and is regularly mentioned in official curricular documents published by their Ministry of Education. Within this

model, which has become increasingly popular elsewhere in the world (Leong *et al.,* 2015), teachers act as facilitators who guide pupils through the three stages by scaffolding, open-dialogue and gradual replacement of the representations (Ministry of Education, 2012). Leong *et al.* (2015) attributes part of the positive impact of the concrete-pictorial-abstract model on mathematics learning in Singapore to the way in which the model is embedded and used by the entire mathematics education community. This means there is no contradiction between textbooks, curriculum or teaching within different schools or stages of learning. Leong *et al.* also comments that importantly the model is taught in pre-service mathematics teaching courses.

However, in the West, often methods of teaching will vary depending on teacher or setting. A recurring concern within the literature is the failure of teachers to successfully facilitate the transition between the concrete and the pictorial or abstract phase of their learning. Hodgen *et al*. (2018) emphasise the power manipulatives can have in enabling pupils to engage in mathematical ideas, though they identify that this is dependent on the teacher drawing attention to the connections between the manipulatives and the mathematical ideas they represent. They go on to give a stark warning that (p11):

> *"whilst learners need extended periods of time to develop their understanding by using manipulatives, using manipulatives for too long can hinder learners' mathematical development".*

There seems to be widespread agreement that for pupils to excel in their mathematical thinking, at some point they need to move on from a purely concrete understanding to one which is able to comprehend, use and apply increasingly abstract ideas. Ball (1992) is extremely critical of teachers' ability to help pupils make these connections. Giving examples in the form of vignettes, Ball discusses the common practice of using manipulatives to explore a particular topic but then assuming the child has an understanding of the mathematical concept because they are able to replicate the procedure the teacher has modelled. Ball disapproves of the way in which teachers

often ‘tick the box’ by using manipulatives but then move on quickly to symbolic representations with no reference or connection to the physical representation previously used.

Whilst there is some agreement that these connections need to be made to make the use of manipulatives more effective, how this should be done is less documented. The [American] National Mathematics Advisory Panel (2008:29) reports that:

> *“Students must eventually transition from concrete (hands-on) or visual representations to internalized abstract representations. The crucial steps in making such transitions are not clearly understood at present and need to be a focus of learning and curriculum research.”*

2.3. The effect of manipulatives in mathematics education

Hattie’s (2019) ongoing research, based on almost 1200 (at present) meta-analyses, synthesises 252 influences on the learning outcome of pupils and provides a calculated effect size for each one. This allows the different factors to be ranked and compared based on Hattie’s research and analyses, where a score of 0.4 is equivalent to an average year’s progress. The use of manipulative materials in mathematics ranks 150, with a score of 0.3, suggesting that the use of manipulatives can make a significant impact on the learning outcome of pupils. These findings echo those of Suydam & Higgins (1977) whose comprehensive meta-analysis of studies related to manipulative use with primary aged children found that using concrete materials resulted in greater achievement when compared to not using them. Sowell (1989) acknowledged the work of Suydam & Higgins, as well as other reviewers, who also concluded that manipulative use is effective with younger children (such as Fennema (1972), Friedman (1978) and Wilkinson (1974)) and went one step further to include studies conducted within a secondary/college setting in her meta-analysis of the use of manipulatives in 60 studies. Sowell’s results showed that long-term use of concrete manipulatives by *knowledgeable* teachers resulted in an improvement in pupils’ achievements and attitudes towards mathematics across all ages.

However, some have criticised Hattie's work (see Slavin, 2018) for seeking to isolate factors from each other and simplifying the complexity of teaching too much, expressing concern for an over-reliance on the effectiveness of individual factors. Likewise, there are studies which appear to contradict those which conclude favourably towards an increased use of manipulatives. Gerling and Wood (1976), having compiled and analysed 103 research studies into the use of manipulatives in mathematics learning, found that the studies (from between 1970 and 1975) were inconclusive on the effectiveness of their use. More recently, the [American] National Mathematics Advisory Panel (2008) stated that the evidence regarding the usefulness of manipulatives in the learning of mathematics was 'tenuous at best', whilst the NCTM 'standards' (1991) hold manipulatives as important, but no more important than other mathematical tools. However, it can be noted that rarely do these studies take into account the knowledge or beliefs of the teacher, related to their use of manipulatives. Indeed, often the studies simply compare the learning of those that used manipulatives and those that did not without acknowledging *how* the manipulatives were used. Ball (1992) points out that there is often a lack of talk in literature regarding how to use manipulatives. Instead manipulatives are seen as a 'magical fix'. She argues that teachers hear of the benefits of manipulatives but are without the opportunity to develop their thinking of them as a pedagogical tool. Nührenbörger & Steinbring (2008) share the concerns of Ball, claiming there is a thought that giving pupils manipulatives to 'play' with whilst teaching a concept will automatically lead to an increase in pupil knowledge. When it does not, it may then be reported that the manipulatives have little or no effect. Ball (1992) argues that teachers need to be educated in manipulatives for their use to be truly effective.

There is also an argument that the effectiveness of manipulatives may be dependant not only on how the objects are used, but who with. For example, manipulatives are shown to be valuable as a mediating tool for pupils with visual impairments or pupils with intellectual disabilities and/or disorders, in the forming of mathematical concepts (De Fátima Alvaristo et al., 2020), however this does not mean that only these pupils would benefit from their use. Kablan (2016) found that the

advantages of manipulatives on a pupil's learning depended greatly on the preferred learning style of the pupil. Those who were 'hands-on' learners, preferring concrete examples, benefitted by the teacher's inclusion of manipulatives into a lesson. Whilst those who were more abstract learners did not benefit, but nor did their learning suffer. Jackson (1979) sets out to make the reader aware of a number of misconceptions relating to hands-on mathematics and the use of manipulatives. One misconception he writes about is the belief that manipulatives are more useful with primary aged or lower attaining pupils than with secondary or higher attaining. Nührenbörger and Steinbring (2008) acknowledge that manipulatives are predominantly used within primary schools, and use is much more seldom within a secondary context. However, though most studies into the use of manipulatives seem to take place in a primary setting, those such as Sowell's (1989) study often show that manipulatives are also effective with older pupils when used. It should be noted though that pupils' age does not always directly correlate with attainment. Sowell's meta-analyses does not take into account the ability of the pupils being worked with nor the level of mathematics being worked on. It could be that where manipulatives were effective in a secondary setting, the pupils were in fact working at a 'primary-level' of mathematical ability.

2.4. Manipulatives in teacher education

Nührenbörger & Steinbring (2008) describe manipulatives as a pedagogical tool which is only effective with appropriate use. To use a tool effectively, the user (teacher) needs to be appropriately skilled. Therefore, ensuring teachers are well equipped is vital to ensuring the use of concrete materials is an effective and beneficial use of precious classroom time. Ball (1992:47) recognises that teachers need much more support if they are to 'make possible the wise use of manipulatives', pointing out that many teachers themselves were not taught using manipulatives and therefore find it difficult to represent or model mathematical ideas in this way.

Thompson (1994) notes that studies into the effectiveness of the same type of manipulative (base-ten blocks) have differing conclusions depending on which study you read. For example, Labinowicz (1985) and Resnick & Omanson (1987) found that upper primary aged pupils either struggled using the manipulatives or that the objects had little impact on their learning. In contrast, Fuson & Briars (1990) and Wearne & Hiebert (1988) reported 'consistent success' in the pupils' use of the blocks and the impact on their understanding. Thomson's opinion is that these contrasting results must be due to the differences in teacher instruction and the way in which the objects are used. He concludes that manipulatives can be an effective aid for teaching, though to achieve the most benefit teachers must ask themselves "What do I want my students to understand?" not "What do I want my students to do?". This emphasis on manipulatives' effectiveness being dependant on the instruction of the teacher is echoed by a number of authors, including Ball (1992) who is extremely critical of teachers' assumptions that pupils will make connections between the concrete materials and the abstract mathematical concepts without any due regard for the instruction or language used by the teacher. Heddens (1997) cautions that manipulatives must be used with care by teachers, so as not to cause pupils to compartmentalise mathematics into two categories: manipulative and symbolic. What is clear is that teachers' instruction is important in relation to manipulatives and therefore teachers need to be educated in this area early on in their careers, preferably in their ITY.

Kamina & Iyer (2009) found, in their work with pre-service elementary teachers, that these teachers often struggled to transfer knowledge from the use of manipulatives to symbolisation and abstraction. In their role as TEs, they started to use a model where different types of manipulatives where introduced separately towards the start of their course. This model involved three steps: scaffolding, exploration and abstraction and was used to establish a routine to help the pre-service teachers with their understanding and transfer of knowledge whilst emphasising the effective use of manipulatives. The example they used in their paper involved introducing a manipulative and direct questioning to guide teachers to think about the qualities of the manipulative, an open-ended task

used in their exploration phase and a closed task where teachers had to reflect on what they had learnt and apply it without the use of the physical manipulatives.

Whiting et al. (2018) discuss the numerous routes into teaching currently in England, where teachers can receive a differing experience, curriculum or level of education depending on the route taken. The different curriculums means there is no guarantee that an initial mathematics teacher will gain experience in the use of manipulatives. There seems to be a gap within the literature available concerning how equipped new teachers are or feel, from their IT training year, for the effective use of manipulatives with their pupils.

Nührenbörger & Steinbring (2008) argue that manipulatives have two roles to play in teacher education. First, teachers (this includes prospective teachers) need to be educated in what manipulatives are, how they can be effectively used and be taught how to analyse pupils' progress and learning processes when using manipulatives. The emphasis on being able to analyse pupils progress is to counteract the 'naïve' beliefs that manipulatives have an automatic positive effect on pupils learning and that mathematical truths can be directly 'seen' by pupils through their use (Ball, 1992). Secondly, Nührenbörger & Steinbring endorse the use of manipulatives as a learning tool by the teachers themselves to increase their own 'teacher knowledge'. Nührenbörger & Steinbring regard it as essential that the use of manipulatives is not taught as a subject matter but must instead evolve through ongoing interactive learning.

Although Nührenbörger & Steinbring (2008) do not expand on the applied steps teachers need to be educated in when using manipulatives, Suydam & Higgins (1976) offer a number of suggestions on how manipulatives can be best utilised to produce greater mathematical achievement, providing a practical list of aspects in which teachers need to be adequately educated. These points were generated through their thorough meta-analyses into effective manipulative use (Durmus & Karakirik (2006)) and can be found in Appendix A.

Ball (1992) reasons that an increased amount of support is needed for teachers to be able to use manipulatives effectively and to really understand where their merit and pitfalls lie. Ball refers to a lack of talk around manipulatives which means experienced and prospective teachers do not have the opportunity to think about them as a pedagogical tool and are often ill-equipped to be able to invent their own examples and uses. This is directly linked to an argument that providers of popular teaching materials often make substantial claims that pupils' eyes will be instantly opened to mathematical truths through their manipulatives and examples (Ball, 1992), persuading schools to pay for their innovative resources. Instead, Suydam & Higgins (1976) suggest that the simplest of manipulatives should be used and that teachers should be proficient in linking them to all different examples and aspects of mathematics learning.

On Hattie's (2019) impact on pupil learning list mentioned above, somewhat surprisingly, IT training programmes rank 211, with a score of 0.12, while professional development programmes rank 117 with a score of 0.41. What is not clear with the low scoring IT training programmes is whether this score depicts the impact an initial teaching programme has intrinsically or whether this score depicts the choosing of one programme over another. What is clear from this data is that when a teacher participates in professional development, it has a profound impact on pupils' learning.

When researching how one might successfully educate pre-service teachers in the use of manipulatives, one must also consider how teachers themselves learn, to make best use of this professional development. Whilst considering how teachers learn, I have reviewed what the literature says about teacher growth, relating this to teacher beliefs and knowledge and specifically beliefs and knowledge regarding the use of manipulatives.

2.5. Teacher Growth

Clarke & Hollingsworth's (2002) 'interconnected model of professional growth' (see Appendix B) was designed to model the cyclic process through which teachers' educational impact improves, or the

teachers 'grow'. This model claims that teachers' growth depends on enactment and reflection by the teacher, using these two processes to connect four distinct domains: the external domain, personal domain, domain of practice and domain of consequence. Clarke & Hollingsworth applied this model to a number of different contexts and found it useful in relating the domains and describing teachers' growth pathways. Within this model, a change in practice can occur from enacting upon a change in the teacher's knowledge or beliefs, or from the impact of an external source or salient outcome. Therefore, the model would suggest that a teacher's practice could change independently of whether their beliefs or knowledge have changed, e.g. by imitating a task observed within a professional development programme (external source) without understanding or changing their beliefs on its use. The model would also suggest that a teacher could have a change in beliefs or knowledge but without enacting these changes there would be no impact on their classroom practice. Therefore, for teachers to truly 'grow', teacher growth must incorporate both a change in the personal domain and the domain of practice. Using this model, one can interpret the role of the TE as someone who is required to provide external stimuli whilst actively encouraging the teachers they work with to be continuous reflectors, providing them with the tools they need to enact their change in knowledge or beliefs.

To maximise teacher growth, McIntyre (1993), along with others, argues that the interplay between theory and practice is vital. McIntyre emphasises the essentiality of theory as both content and process- that is that early teacher development should involve gaining technical knowledge and the skill to be able to reflect upon their own practice. Some IT education schemes such as the Oxford Internship model base themselves upon this idea, setting out to minimise the gap between theory and practice and emphasising their equal importance (Hayward, 1997). This course has a cyclic structure, where new teachers are able to put into practice the theory they have learnt in university and in their school placements, before being tasked to explicitly reflect on the outcomes. This structure is compatible with Clarke & Hollingsworth's (2002) model in that reflection and enactment are closely interconnected and used to promote teacher growth.

2.6. Teacher Beliefs

It is evident within literature that there is a correlation between a teacher's beliefs and their practice, found being discussed in relation to general pedagogy as well as a teacher's use of manipulatives explicitly (Golafshani, 2013). However, the extent to which they are related and, more specifically, the impact beliefs have on a teacher's practice is often debated (Beswick, 2012; Forgasz & Leder, 2008). Those agreeing with Sztajn (2003) are so convinced that a teacher's practice must be consistent with the teacher's beliefs that they cast doubt over any research methods which conclude otherwise. In contrast, Beswick (2012) found there may be a misalignment between a teacher's beliefs and practice. Beswick's study was small though, only including eight teachers, and so may not be viewed as statistically significant. However, the theme of beliefs and practice misaligning can also be found elsewhere in literature. Swan (2006) acknowledged that a teacher's beliefs and practice may not directly correlate, though stated that it is when teachers become aware of this misalignment that the most valuable development in teacher growth will occur. This in itself fits in well with Clarke & Hollingsworth's (2002) model which, as discussed above, implies that enacting one's beliefs will result in a change in practice. Forgasz & Leder (2008) in their review of literature relating to teachers' beliefs and practice between 1997 and 2006, found that beliefs have a significant impact on teachers' practice, although there is scope for a misalignment to occur. The literature available on beliefs sheds light on why this misalignment may occur, for instance it may be that the day-to-day challenges faced by teachers may constrain their practice, leading to a contradiction between their practice and beliefs (Swan, 2006). Therefore, one cannot simply take what a teacher 'does' as what they 'believe', and a change in practice may not always need a change in knowledge or beliefs but simply the removal of a barrier. Forgasz & Leder (2008) also found that the definition used for a teacher's beliefs differed so much between some studies that it was impossible to make accurate comparisons. Therefore, it is important when talking of a teachers'

beliefs of manipulatives to define what one means by this. Within this study 'beliefs' are taken to mean a teacher's perceived beliefs (discussed further on in this chapter).

Nührenbörger & Steinbring (2008) citing Ball (1992) found that prospective teacher's beliefs about manipulatives are so deep-rooted and often naïve: deep-rooted in the sense that they are well established and hard to change, and naïve in that teachers often hold 'simplistic' beliefs, either that manipulatives are only useful for certain groups of children/topics or that giving a pupil manipulatives to use will instantly enlighten them and be a cure to their mathematical struggles. Nührenbörger & Steinbring discuss these specific beliefs in terms of explicit and implicit conceptions, and make the observation that these conceptions are often predefined by the teacher's own experiences and history concerning manipulatives. If a teacher believes that manipulatives are there solely to make a topic more fun, or that they are most useful as a distraction tool when teachers have no other ideas of their own, or that they are a waste of time, then inadvertently pupils will start to use manipulatives for play, resulting in no to little learning, or they will learn to use them in a procedural and rote-style way (Hiebert & Wearne, 1992; Moyer & Jones, 2004). A misalignment between Canadian curriculum advice on the use of manipulatives and teachers' practice was attributed to teachers' attitudes and beliefs towards their use (Golafshani, 2013). Golafshani reported that results from a small-scale study indicated that improving teachers' confidence in the use of manipulatives had a positive impact on their beliefs and practice. He recommends that professional learning communities, such as workshops and teacher collaboration, should be better utilised to encourage and promote the effective use of manipulatives, suggesting this will have a positive effect on teachers' beliefs. Although this study was small, it did highlight the need for more research into not only how beliefs affect practice with manipulatives, but how a teacher's attitude towards their use can be shifted. According to Clarke & Hollingsworth (2002) an individual teacher's growth pathway may not necessarily be dependent on a change in beliefs occurring, instead the teacher may require a change in knowledge.

2.7. Teacher Knowledge

Over the last 30 years there have been a number of different frameworks developed to describe the knowledge required by mathematics teachers (see Ball, Thames, & Phelps, 2008; Baumert & Kunter, 2013; Ernest, 1989; Fennema & Franke, 1992; O'Meara, 2011; Rowland et al., 2009). However, as observed by Koponen et al. (2019), these frameworks have evolved from the work of Shulman, and particularly his commonly referenced paper: 'Those Who Understand Knowledge Growth in Teaching' (1986). In his work, Shulman categorises content knowledge into three classes:

- *Subject matter content knowledge* - describing a teacher's knowledge of the mathematics they are teaching. This, Shulman argues, includes knowledge of how different areas of the subject connect and should go beyond the knowledge required by the pupil.
- *Curricular knowledge* – describing knowledge of the sequencing of topics and how the mathematics taught fits in with the mathematics taught in previous and following years. This should also include how familiar a teacher is with material available to them.
- *Pedagogical content knowledge (PCK)* – describing the knowledge needed to be able to translate subject matter knowledge to the pupil, including knowledge of types of representation, teaching methods and any preconceptions or difficulties a pupil may face.

Although each type of knowledge is important for effective teaching, researchers debate the importance placed on each one. Teacher subject matter knowledge ranks low (136 out of 150) on Hattie's (2019) scale of effect, whilst factors which could be classified under the heading of PCK (such as meta-cognitive strategies, rank 14) score a lot higher. The year's worth of research conducted by Ball & Bass (2002) also concluded that those teachers who 'knew more mathematics' were not necessarily the ones whose pupils made the most progress, showing that a teacher's PCK was imperative to their efficacy.

Koponen et al. (2019) note that a number of models have arisen to better describe and analyse a teacher's PCK and that subsequently researchers have diverged on what PCK actually is. A summit in 2012 (Carlson & Gess-Newsome, 2013) set about to create a 'consensus model of PCK', including related vocabulary and assessment. However, Gastaldo et al. (2017) found just five years later that there was still disagreement within literature on its exact meaning and use, which depended on the model chosen. This incongruity highlights how complex the concept of PCK is and therefore how difficult it is for a single model to fully capture its complexity.

One notable framework used to model PCK is known as the 'Mathematical Knowledge for Teaching' framework and was developed by Ball et al. (2008). This framework has been used by researchers across the world, specifically to analyse mathematics teachers' knowledge (Koponen et al., 2019). However, a main flaw of this egg-shaped model (see Appendix C for full model) is the lack of interconnections between the different domains. Concluding a study into 18 Finnish teachers' perceptions of the knowledge needed to be an effective mathematics teacher, Koponen et al. (2019) found there to be connections and hierarchies between the domains. For example, to most effectively make connections and use relevant examples (SCK domain), teachers required knowledge of different mathematical theories (CCK domain). Where this hierarchal approach is not explicitly expressed in the framework of Ball et al. (2008), it is a defining feature of a framework known as the 'Knowledge Quartet' (KQ), introduced by Rowland, Hucksteep & Thwaites (2005).

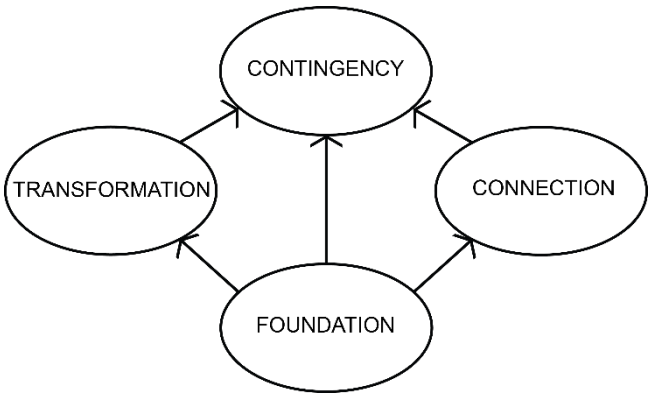


Figure 1: Representation of the Knowledge Quartet

Rowland et al. (2005) developed the KQ by analysing 24 recorded lessons taught by pre-service teachers. They came up with 18 different codes to analyse the lessons, each code referring to a different aspect of PCK, and then further organised these codes into four groups which became the domains of the KQ. These domains were arranged in a hierarchal formation, where the bottom layer (foundation knowledge) feeds into both the middle layer (transformation and connection knowledge, known as the knowledge-in-action domains) and the top layer (contingency knowledge, or knowledge-in-interaction). The middle layer also feeds into the top layer. A brief description of each of the domains follows:

- *Foundation Knowledge* – This is the knowledge possessed by the teacher and is not dependent on whether it is put to use or not. It includes knowledge of pedagogy, and mathematics.
- *Transformation Knowledge* – This is the knowledge revealed through a teacher's explanations, choice of representation and how they make their foundation knowledge accessible to pupils.
- *Connection Knowledge* – This is the knowledge demonstrated through the mathematical and curricular connections made within planning and teaching of lessons.
- *Contingency Knowledge* – This is the ability of the teacher to respond to unexpected or unplanned situations aptly; the knowledge needed to be able to appropriately deviate from the lesson plan when needed.

This model is particularly helpful as it highlights the connections between the different domains of knowledge. Rowland et al. (2005) promoted the framework as a tool to be used between prospective teachers, TEs and mentors when discussing the prospective teacher's knowledge for teaching. The framework is clear and is extremely simple yet effective.

Prospective teachers need opportunities to develop all aspects of teacher knowledge during their university education (Sandir, 2016) and should be exposed to situations where they are required to make decisions regarding which of the many available materials to use to aid the development of certain mathematical concepts. This includes opportunities involving manipulative material. An (2004) examined teachers' PCK, focusing on how mathematics teachers identified and addressed/corrected pupils' misconceptions. When reviewing this study, Kilpatrick (2006:258) comments on the participants in An's study having limited success when 'bridging from manipulative materials to mathematical ideas'. This area of PCK, which could be described as connection knowledge, is important if teachers are to utilise the pedagogical impact of manipulatives with their pupils. To be confident in their use of manipulatives, teachers need to first have the foundation knowledge of what manipulatives are, including examples of how they can be used. They also need to have the connection knowledge mentioned above, as well as transformation knowledge of how a mathematical concept can be represented and explored using concrete materials. Following the model of the KQ, this will allow teachers to develop their contingent knowledge thus being confident that they will be able to address issues which may arise and use manipulatives fluently within the classroom.

When thinking about how an aspect of pedagogy can be addressed within an initial teaching course, Zaslavsky & Sullivan (2011) claim that having a TE set a classroom style task for the prospective teachers to work on and then reflect upon can make a significant contribution to teachers' learning. An example of this would be a TE using manipulatives with prospective teachers. Nührenbörger & Steinbring (2008:159), discussing the use of manipulatives in teacher education, describe how manipulatives can be used to enhance both a teacher's mathematical knowledge and their PCK. First, they describe how using manipulatives with teachers/prospective teachers helps the teacher learn what it is like to use manipulatives, as well as enabling them to construct interpretations of the manipulatives to explain mathematical knowledge whilst also allowing the teacher to gain deeper mathematical understanding. This use of manipulatives is shown diagrammatically in Figure 2. For

example, the use of manipulatives may give a teacher a different perspective of a mathematical concept they had learnt algorithmically in the past and had difficulty explaining *why* the steps work.

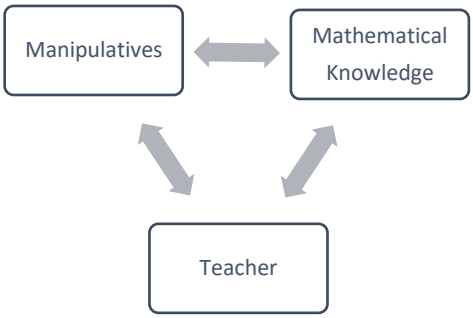


Figure 2: Manipulatives and Teacher Mathematical Knowledge

Secondly, Nührenbörger & Steinbring (2008:159) describe how manipulatives can be used to aid teachers' PCK; the knowledge of how pupils learn using manipulatives and how they can be taught using them. Figure 3 is a replica of Figure 2, but here mathematical knowledge has been replaced with 'teacher knowledge' of the relationship between manipulatives, mathematical understanding and the pupil. By using manipulatives themselves, teachers may develop a better understanding of how manipulatives can be used with pupils. This understanding, or knowledge, touches on all four domains of the KQ and may show that using manipulatives themselves may help teachers to develop the knowledge needed to effectively use them in their own practice.

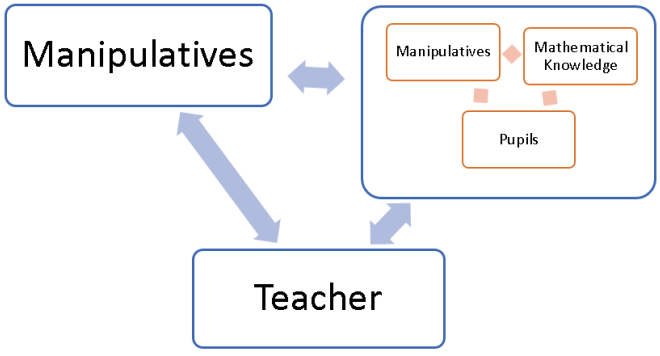


Figure 3: Manipulatives and PCK

2.8. Impact on this study

I found it interesting how the term 'manipulatives' could be interpreted differently by different people. Within this study I planned to look at how the participants defined manipulatives and what this revealed of their perceptions/ beliefs regarding their use. Having collected participants definitions, I introduced them to a definition of manipulatives to be used for the remaining questions, to enable direct comparisons to be made. This definition used an amalgamation of those by Kennedy (1986: 6) and Young (1983:12):

'Manipulatives are physical objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children in hand-on learning of mathematics'.

I have chosen a definition that does not encompass virtual manipulatives, as these are often treated separately to physical manipulatives within the literature. However, I allowed scope for teachers to discuss the use of virtual manipulatives within my questioning.

The literature highlighted the need to define what is meant when using the term 'teacher beliefs'. For this study I have taken the term 'beliefs' to mean teachers' *perceived* beliefs – their own perceptions of what they believe. This is because (as discussed in Chapter 3) I was unable to carry out my own observations of practice (due to pandemic-restrictions), and therefore could not analyse teachers' enacted beliefs. A teacher's *actual* beliefs, if different from their enacted or perceived, are extremely complex (Valcke et al., 2010) especially when considering whether a teacher acknowledges these beliefs or not, and it was not the purpose of this study to probe a teacher's unacknowledged beliefs. Instead, I have focused on what the teachers' descriptions of their beliefs (regarding the use of manipulatives) and their descriptions of their actions reveal and how a change in these beliefs may need to be enacted to result in a change in practice and growth. I have also looked at whether there is a misalignment between a teacher's beliefs and practice and, if so, tried to explore what may be causing this. Looking at whether the misalignment is due to the teacher being unable to enact their beliefs as a result of, for example, a lack of knowledge or a barrier in

their setting, has enabled me to make suggestions as to how this misalignment may be targeted within a teacher education context.

I was convinced by the literature that manipulatives can be an effective pedagogical tool, even within a secondary setting involving complex mathematical concepts, and that their prevalence in IT education courses is vital in ensuring teachers are skilled in their effective use and do not share the misconceptions of some. This literature review has given me an insight into the issues regarding the use of manipulatives in teaching, especially the lack of use in secondary schools and the effect of teachers' beliefs on their use. I also gained a good foundation of literature and research to which I could compare the findings of my investigation (see Chapter 4) and an understanding of how beliefs and knowledge are intertwined to contribute to teacher learning and growth.

Chapter 3: Methodology

Due to the COVID-19 global pandemic, this study had to be adapted mid-project to account for the short-notice government-enforced regulations and guidelines, including social distancing and school and university closures. Ultimately this forced the study to change course and shift focus. This chapter provides an outline of the final methodology used in the study, whilst also describing the adaptions which had to be made.

3.1. Methodological framework

The study was initially designed to be a piece of action-research, fitting the definition of Cohen & Manion (1994:185) as being 'a small-scale intervention in the functioning of the real world and a close examination of the effects of such an intervention'. The research was to follow an iterative grounded theory process (Cohen et al., 2007:185), moving backwards and forwards between the data collected and a theory of how teachers could be better equipped to use manipulatives. This would have meant collecting data to support a design for an intervention (addressing the use of manipulatives), implementing this design, collecting data to analyse the impact of the intervention and using this data to redesign the intervention. This would have carried on in a cyclic manner until the data collected and theories matched (Cohen et al., 2007:185). Appendix D sets out the original planned stages of this study, against the eight steps described by McNiff (2002:71) as a model of action research. This research would have involved collaboration with colleagues from the IT programme I lead. However, this was unfeasible as discussed below.

The pandemic-related restrictions meant that completing/repeating the action-research cycle was not possible. Therefore, the study evolved to focus more heavily on the first two research questions. This meant conducting further detailed research into the role of manipulatives, secondary mathematics teachers' (SMTs') perspectives and gaps in teachers' education, and to then present recommendations as to what effective SMT education in the use of manipulatives could look like.

Although there was a change of focus in the design of the research, the study still followed a grounded theory approach in that the theories generated are rooted in the data which has been systematically collected and analysed (Cohen et al., 2007:491).

The study was redesigned as a piece of mixed-method research, where the mixed methods were a triangulation between survey, interview and observation. These three methods were chosen to provide both qualitative and quantitative data, reducing the opportunity for bias from a single method approach (Denzin, 1989) and enabling an increased understanding of the issues involved. The study was sequential in design (Sandelowski, 2003) in that quantitative data was collected and analysed preceding the qualitative aspects of the project. This approach allowed a substantial amount of data to be collected in the form of the survey, with the hope of providing patterns and trends regarding the use of manipulatives before collecting more in-depth data in the form of interviews/observations, allowing me to explore the contexts and beliefs which may be behind these trends. A complementary design was used, meaning the interview data was used to clarify and elaborate on the results from the survey (Onwueghuzie & Teddie, 2003). This may give an insight into the reasoning behind the teachers' practice and therefore how IT education could be adapted to most effectively prepare teachers in operative use of manipulatives, if indeed needed.

Further into the research it became apparent that observations would also not be possible due to the prolonged school closures. In addressing this, questions which gave teachers the opportunity to demonstrate their practice were included in the interviews.

The following section on data-collection methods briefly addresses the rationale and process for each of the methods used.

3.2. Data-collection Methods

Having defined my initial research questions, my data-collection methods and instruments were specifically designed to obtain data which would address these questions. My initial design included four research questions, which were later condensed into the three found in Chapter 1 (due to the pandemic -restrictions). Table 1 maps out the data I considered relevant to each question. Following the table, I offer the rationale behind each data-collection method plus a description of instruments and processes taken. *It should be noted that the data described in italics in Table 1 was not possible to collect due to the pandemic-restrictions discussed above.*

Table 1: Research questions and corresponding data

Research questions	Data
What are teachers' perceptions of the role of manipulatives in secondary mathematics education?	- **Survey of secondary mathematics and primary school teachers** (Data used to analyse the different perceptions and beliefs dependant on year group taught, teaching experience and topics taught.) - **Semi-structured interviews** (Data used to further explore the perceived beliefs/perceptions of a smaller sample of SMTs on the use and role of manipulatives in secondary education, their own use with manipulatives and their own educative experience regarding manipulatives.)
In what ways and to what extent are secondary mathematics teachers (SMTs)	- **Survey of secondary mathematics and primary school teachers** (Data used to analyse how confident SMTs feel using manipulatives based on age range taught, route/methods

equipped to use manipulatives within the classroom?	of IT education, teaching experience, the comparison between beliefs and practice. This data can be compared with that of primary teachers (PTs).) - **Semi-structured interviews** (Focus on the confidence of SMTs in their use with manipulatives, the knowledge they possess, limitations they face.) - ***Lesson observations*** *(Data used to analyse how manipulatives are being used in the secondary classroom and whether teachers are equipped with the knowledge to use them effectively. Data used to compare the knowledge demonstrated with the perceived confidence.)* - ***Teacher education programme observations*** *(Observe teacher education sessions to collect data on how teachers are being equipped to use manipulatives within the classroom. Observe both secondary and primary to compare how the use of manipulatives are integrated into the courses and the impact these have on the teachers' knowledge and beliefs. Compare with methods used in literature.)*
How can the use of manipulatives be effectively integrated into a secondary IT education course?	- **Data Above** (Data collected for the first two research questions should influence the design of the intervention (along with literature)) - **Survey of secondary mathematics and primary school teachers**

	(Data used to analyse whether effective integration of manipulatives in a teacher education course should target teachers' beliefs, knowledge or both. How have teachers' been educated in the use of manipulatives and what was most 'effective'?) - **Semi-structured interviews** (Data used to explore further how manipulatives were integrated into teacher education courses and what effect this had on the teachers' confidence and use and views on what methods are most effective.)
[Originally a fourth research question was included. This was later removed due to the circumstances] ***In what ways and to what extent does this intervention have an impact on teachers' use of manipulatives?***	- ***Notes and design of intervention*** *(Data used to analyse the intervention session designed against the opportunity for teacher growth, in accordance with Clarke & Hollingsworth's (2002) model for change as well as opportunity for teachers to increase their knowledge as categorised by the knowledge quartet (Rowland et al., 2005))* - ***Post-intervention interviews*** *(To assess change in beliefs/knowledge/practice due to intervention)* - ***Post-intervention observations*** *(To assess change in knowledge/practice due to intervention)*

3.2.1. Survey

Rationale

I decided to use an online questionnaire-style survey to compile data from a large number of teachers, giving an insight into the perceptions and uses of manipulatives generally, before getting

more detailed views of the interviewees. I wanted to capture the general practice of teachers as I thought this would better serve the purpose of my first and second research questions. Had I just used interview data, I would not have known whether the views held by those interviewed were typical or anomalous. By triangulating the two data collection methods I could enhance the credibility of the research whilst providing a more comprehensive understanding of the issues revealed (Salkind, 2010). Using an online platform ensured I could maintain anonymity, reach participants easily, make certain questions mandatory and efficiently collate the data produced. However, limitations meant that I was unable to control whether a participant completed the survey multiple times and/or were truthful in their responses (Bloch et al.,2011). Nevertheless, due to the nature of the survey and the anonymity attached, I decided these risks were small and acceptable.

Design and Process

Once I had finalised my research questions I considered what information would be helpful to better understand and compare teachers' views of manipulatives, potential gaps in teachers' knowledge and what effective teacher education on the use of manipulatives may look like. I then drafted both open and closed questions to provide relevant quantitative and qualitative information before piloting the survey with a colleague (who was not counted as a participant). I was careful not to include emotive or misleading questions so as not to invalidate the responses. Through reviewing the pilot I decided to change the wording and formatting of some parts of the survey. For example, I asked participants to describe in their own words the definition of the term 'manipulatives' but, to ensure participants were interpreting the following questions' terminology in the same way, on the following page I gave the participants a definition which I asked them to adopt from there on. I formatted the survey so it was mandatory to give a definition before being able to move onto the next page and see the one I had provided. This was to eliminate participants being influenced by the

wording used, which was an amalgamation of the definitions formed by Kennedy (1986) and Young (1983) (see section 2.8).

Once I had made these changes I used a virtual snowball sampling technique (Baltar & Brunet, 2012) to distribute the survey, by sending the final version (see Appendix E for the full survey) to teaching contacts and asking them to share it with mathematics teachers they knew. In the title of the survey and in my instructions to those I sent it to, I sought to make it clear that the survey was for teachers who taught mathematics, regardless of age taught, as I was interested in how the responses of those who taught secondary would compare with those who taught other age groups. The first question in the survey asked what the participants primary role was in mathematics education, including an option for 'other'. This meant that where one participant completed the survey, but answered that they were a geography teacher and did not teach mathematics, I was able to disregard the rest of his responses as not relevant to the study.

I chose to use the virtual snowball sampling method because a) this had the potential to reach many more mathematics teachers than I personally know and from different teaching circles(Cohen at al.,2007), b) this method allowed for targeting of individual mathematics teachers through my contacts and c) I thought teachers more likely to complete the survey if it had been sent to them individually rather than an online advert asking for participants. I also decided to limit the number of responses to the survey to 150. I chose this number as I thought it was a large enough sample size to be able to provide significant data (one person represents less that 1%) but also small enough that I would have enough time to analyse the responses, especially the responses to open-ended questions. It took approximately three days to receive 150 valid completed surveys. The survey comprised of multiple choice questions, questions where the participant could choose more than one option, questions requiring ranking of multiple options and open questions where the participant was required to type an answer. At the end the survey asked whether the participant wanted to comment or elaborate further on any aspect of the survey.

3.2.2. Semi- structured Interviews

Rationale

Having chosen to conduct a survey to gather both quantitative and qualitative data from a large group of participants, though useful in providing a wide picture of the role manipulatives play in teaching, I knew this data would not provide a rich insight into teachers' experiences, beliefs and knowledge behind their responses. To obtain this rich data, different in-person approaches were considered (such as structured interviews, unstructured interviews and focus groups) as the direct contact between interviewer and interviewee allows for gestures, body language, expressions, tone and pauses to add to the rich pool of data (Wilkinson & Birmingham, 2003). However, a semi-structured interview approach was decided most suitable. This is because semi-structured interviews allow the interviewer to probe and explore the interviewees' responses with follow-up questions which can lead to clarity and a good quantity of useful data (Wilkinson & Birmingham, 2003). This data collection method is dynamic, flexible and has been proven to be effective in providing an insight into teachers' knowledge and beliefs (Tyson, 1991). This was especially suited to the research questions I was answering as it would allow me to explore not only the teachers' use of manipulatives but also their beliefs, knowledge and educational experiences behind them.

Design and Process

Having received 150 survey responses, I propositioned six participants to be interviewed. I chose six participants because I thought that would be the maximum number I would be able to analyse within my time constraints. The criteria I used to select these participants were:

1. They had given permission (on the survey) to be contacted and had left their contact details.

2. They had expressed an interest (on the survey) in taking part in further research.
3. They were currently employed as a secondary mathematics teacher.

Having filtered the participants to find all those that matched the criteria above, I selected six by ensuring I had a spread of experience and workplaces. I chose not to look at the participants survey data before the interview or during the selection process as I did not want their responses to influence who I picked. I had also informed all survey participants that their responses would be kept anonymous to increase the chance of honest answers. Therefore, I ensured I kept the contact details separate from the rest of the results so that no link could be made. Five out of the six I contacted were able to arrange a suitable time to be interviewed.

Due to the on-going government regulations on social distancing, interviews had to take place via a video call. The limitations to this were that I was unable to read the interviewees' body language in the way I might have been able to if meeting in person, though this was more possible than if it had just been an audio call. It also meant that I could not ask questions using 'props' in the way I might have done in person. For example, within the interview I asked the teachers to *describe* how they may use manipulatives to model a certain question. If I had been interviewing the person in the same room, I would have asked the teacher to *show* me how they would have used the manipulatives. I expect that the teachers would have found answering that question easier with the manipulatives in front of them, in line with the literature on the use of manipulatives, instead of just picturing it in their mind. The advantages to interviewing over a video link were that I was able to interview teachers who lived across the UK, not just within a close proximity. This meant that I was able to interview teachers from a spread of schools. I was also, with each of the participants permission, able to easily video the interview, so that when analysing I had the exact record of what I could hear and see of the interview at the time. Although I am unable to prove this, I think that video calling may also have made the participants more relaxed and less intimidated during the

interview as they were sat in their own familiar surroundings. Indeed all participants seemed relaxed and open throughout the interviews.

To ensure my interviews provided data directly relevant to my research questions I created an interview guide (Appendix F), in line with the recommendations of Drever (1995). Structuring the interview guide in sections which related to my research questions ensured I gave all interviewees the same opportunities to answer my open-ended questions while keeping my follow-up questions focused and relevant. I included a section at the start of the guide with 'simple' questions relating to the interviewees background, encouraging the participants to feel comfortable. The questions were written having reviewed the collective responses from the survey, using the data to decide which areas to explore further. I piloted these questions on a willing participant who was not one of the five participants, and as a result made some minor changes including eliminating some questions and changing the order of others to ensure a more coherent flow of ideas.

The following is a brief background of the five interviewees. Each one has been given a pseudonym.

> *Teacher A* completed her PGCE four years ago and is currently the KS4 maths lead in a secondary academy. Before entering the teaching profession, Teacher A studied for a degree in mathematics at university then worked in industry for approximately five years.
>
> *Teacher B* worked as part of the support staff at a specialist primary school for children with specific educational needs before obtaining a degree in social policy. She then worked for a year as a key worker in a *secondary* school before completing a secondary mathematics PGCE last year. She is currently employed as a mathematics teacher.
>
> *Teacher C* started a computer science degree but changed to study accounting and finance when he found the course wasn't what he expected. He then completed a mathematics enhancement course before completing a PGCE year last year. He now currently works part time as a SMT.

Teacher D worked as a lawyer before retraining 8 years ago and is now head of mathematics at a secondary school. Teacher D has been involved in running a number of seminars at mathematics conferences relating to different aspects of teaching.

Teacher E is currently head of the mathematics department at a secondary and 6th form academy. She qualified as a mathematics teacher 13 years ago having completed her year on the 'Schools Direct' programme. Teacher E's original degree was in meteorology though she did not work in this field before entering teaching.

3.3. Analysis

I started my analysis by focusing on the survey data. I organised the data using percentages and graphs, including comparative bar charts, box plots and pie charts. There was a substantial amount of data available and therefore I had to ensure my analysis was kept relevant to the research questions.. To do this, I systematically went through each question and noted how the data related to each of the three questions. I then went through and used graphical methods to triangulate the values from different questions. I.e. Did, how often and which type of manipulatives used corelate with the confidence of the teacher? Were the confidence levels of teachers' dependant on how they had been educated in the use of manipulatives? Did teachers views on the relevance of manipulatives have an impact on the year groups they used them with? Having explored what the data revealed, I calculated confidence levels on the results I had and used tests of two proportions involving hypothesis testing to make credible comparisons (Triola, 1992). Open questions in the survey, asking participants to describe in their own words what manipulatives are and to express their views on the use of manipulatives were analysed by searching for key and recurrent words and themes. This was done by reading through the answers and noting the key themes. I then coded each answer depending on which theme it best fitted. This was was used to give simple indications of the types of answers given. These findings are reported alongside the other results in Chapter4.

Having analysed the survey data, I wrote and conducted the interviews as described above. I used thematic analysis (Braun & Clarke, 2006) to methodically analyse the emerging themes from the rich interview data. Using a table (see Appendix G) I made a summary of each participant's answers to the primary interview questions. I then reviewed the interviews at a later date by re-watching the recordings and revised the summaries as needed, making a separate note of any quotes which I thought were particularly relevant or noteworthy. I then used this grid to compare and contrast the emerging themes, which are reported in the following chapter.

I chose to present my findings under a framework of the research questions. Doing this meant I was able to triangulate the different data sources and discuss where the data converged or diverged whilst ensuring all findings discussed were relevant to the aims of the study.

3.**4. Ethical considerations**

Before beginning the research I familiarised myself with the British Educational Research Association's Ethical Guidelines (2011) and ensured I considered the ethical implications at each step of the study. When conducting the online survey I made participants aware of the purpose of the study, that their data would be kept anonymous and that participants were under no obligation to take part or complete the survey. I used an online platform which did not record participants names or contact details unless they ticked a box giving their permission to be contacted to take part in a further interview if selected. Those contact details which were provided were separated from the survey results to preserve anonymity.

The interviews took place using online video/audio software and participants were asked for permission (which they all gave) to record the interview to aid my analysis. I made it clear that these recordings would not be shown to any third party nor would they be used for any other research. Participants were offered a copy of the notes and summaries I had written on their interviews. None of the interviewees requested this. Participants were also assured that their identities would be kept

anonymous, including their places of work, and therefore when referred to throughout this study the interviewees have been assigned a letter. I chose not to interview anyone that had previously participated in the IT education programme I lead. This is because, where I was interviewing participants on their previous experience (including that in their ITY) I did not want the participants to feel uncomfortable or pressured into giving false answers or saying what they thought I wanted to hear.

Although ethical considerations had been made regarding observations, because these were unable to go ahead, no steps needed to be taken to ensure ethical standards were kept in this area. Similarly, ethical considerations regarding the intervention part of the original research plan were also no longer relevant.

3.5. Summary

- The COVID-19 pandemic meant that changes to the research had to be made.
- The focus of the research is on the current perceptions, practice and education regarding teachers' use of manipulatives, and where this can be improved.
- The study aims to recommend ways in which pre-service teachers can be better prepared to use manipulatives effectively in the secondary classroom.
- The study is in the form of a mixed methods piece of research, where both qualitative and quantitative data collected from surveys and interviews are triangulated and analysed systematically.
- Ethical considerations have been made at each step of the research.

Chapter 4: Results and Discussion of Findings

This chapter presents an overview and discussion of the findings from the survey and interviews, presented under the headings of the three research questions.

4.1. Research Question 1: What are teachers' perceptions of the role of manipulatives in secondary mathematics education?

The data collected from the survey of 149 teachers, 89 secondary mathematics teachers (SMTs) and 59 Primary Teachers (PTs), when combined with the more in-depth data from the semi-structured interviews conducted, begins to paint a picture of the perceptions regarding the use of manipulatives in secondary education today. Having reviewed the data which relates to this research question, I have presented a discussion of the findings under the following three sub-headings: Teachers' perception of what manipulatives are; SMTs use of manipulatives; SMTs' perceptions of the usefulness of manipulatives.

Teachers' perceptions of what manipulatives are

Within the survey, teachers were asked if they were familiar with the term 'manipulatives' when referring to mathematics education. 81% of PTs and 74% of SMTs said they were. At first it seems this data shows that PTs are more likely to recognise the term 'manipulatives' than SMTs. However, calculating the statistical significance of the two results, using a Z-test calculation and a P-value threshold of 0.05, resulted in a P-value of 0.17 (significance level of 83%) showing that the difference in values between the PTs and SMTs cannot be classified as statistically significant (being >0.05). Therefore, the data cannot be taken to show that SMTs are less likely to be familiar with the term than PTs. However, this did indicate that 26% of SMTs were not familiar with the term which may highlight a lack of education on the concept or that it is not in everyday use in many schools.

I was interested in whether the number of years of teaching experience had an effect on whether an SMT was familiar with the term. If it were that those who had been in the profession the longest were more likely to be familiar with the term than those newer to teaching, this could indicate that teachers are more likely to have 'discovered' manipulatives later in their teaching career suggesting that their use is not being included in initial teaching programmes. Or, it could mean that teachers used to be better familiarised with manipulatives during their ITY but no longer are. To explore this, I compared the spread in years of teaching experience between those teachers who were not familiar with the term manipulatives, with the spread in experience of the whole sample of SMTs. The results are shown in Figure . Looking at the box plots, they are almost identical, with a matching range and interquartile range and a very similar median (less than a year's difference in the median years of teaching experience).

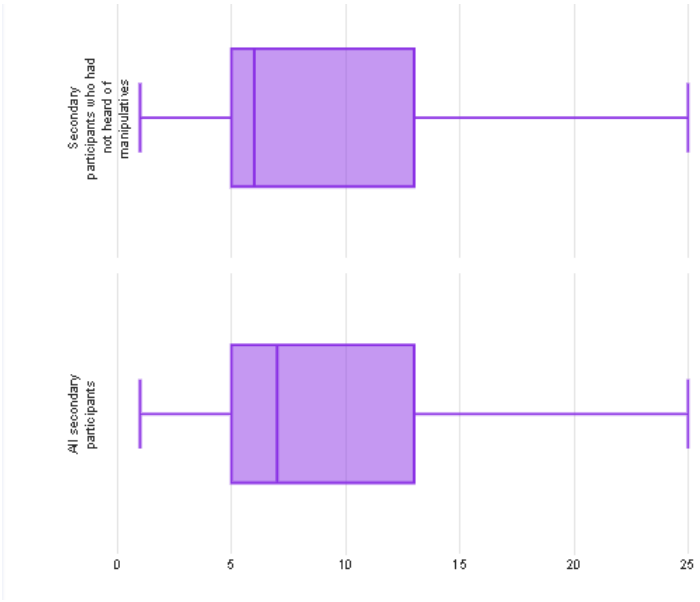


Figure 4: Box plots comparing the experience in years of SMTs who were not familiar with what manipulatives are with the experience of the whole sample

This striking similarity, almost identicality, suggests that the number of years of teaching experience has no impact on familiarity with what manipulatives are. It should be noted that not knowing what

manipulatives are does not mean they are not being used. It most probably suggests that the teacher has not received education in the merits or uses of manipulatives, but could mean the teacher is using manipulatives without being familiar with the term. Therefore, for the remainder of the survey questions participants were given a definition and an example of what may be classed as a manipulative.

Those teachers who said they were familiar with the term manipulatives were then asked to give their own definition of the word. The results revealed something of teachers' perceptions of what manipulatives are. All of the PTs and all but 3 of the SMTs focused on the physical aspect of manipulatives, using words such as: tangible, practical, solid objects, physical, hands-on, touch, moveable and concrete. 2 SMTs referred to manipulatives as visual or pictorial, without any reference to their tangible nature and the third simply referred to them as 'something to make maths more fun'. The majority of SMTs referred to manipulatives being used to help struggling pupils and/or support understanding of mathematical concepts, with 3 SMTs writing about concrete-pictorial-abstract learning. This shows that when talking about manipulatives almost all the teachers familiar with the term had an understanding which aligned with that of Kennedy (1986) and Young (1983). 6 (out of the 62 SMTs answering this question) wrote about manipulatives being there to help pupils with basic numeracy, some talking about counting or number-bonds. However, most other teachers did not define manipulatives as a resource only suitable for 'basic concepts', and no PTs limited their use to a single area of mathematics in their definitions.

There was no mention by any of the teachers surveyed of virtual manipulatives, leaving the impression that these are not thought of as a type of manipulative or that teachers are less familiar with them. However, three of the teachers interviewed did talk of virtual manipulatives. When asked to discuss the definition of manipulatives that had been used in the survey, these three broadly agreed with the statement but believed there was too much emphasis on manipulatives being tangible. Teachers C and D aligned with A's statement that "it's the visual representation about

manipulatives that is the most important" and said that they used virtual manipulatives in their classrooms. D's school had provided all pupils with a tablet which they were encouraged to use manipulatives on. Teacher D commented that it was much quicker and avoided the hassle distributing and collecting small objects. However, he did say that:

> "There are some manipulatives that just don't have the same impact on a screen… like rods and blocks which represent 1s, 10s, 100s etc. Children appreciate the different sizes of them when handling them. They just have more impact in real life than on a screen."

Teacher D's experience here supports the argument of Moyer (2001) that although useful, virtual manipulatives and those physical do not have the same impact as each other. Teacher C said he uses virtual manipulatives on the screen at the front of the classroom. Sometimes he does this because he thinks the visual representation is useful for pupils to see but does not always have the time in his teaching programme to allow the pupils to actively participate themselves. This practice reveals an underlying belief that the time spent using manipulatives is not equal in worth to the time that could be used on other methods of teaching.

SMTs use of manipulatives

To gain an insight into teachers' perceptions of how often they use manipulatives, I asked survey participants to choose an option, from the following, which best described their use with different pupil ages: *Never, Rarely, Sometimes, Often, Most of the Time, Always* and *Not Applicable*. Due to having a mixture of SMTs and PTs, and teachers having varying experience teaching different ages (meaning the frequencies were different for each age range), I converted the frequencies into percentages to be able to compare the results. I combined the primary years into infants (Years R-2) and juniors (Years 3-6) because the focus of the research is predominantly on secondary use of manipulatives and there is less need for a breakdown of the primary years. Having reviewed the

data, I found the six categories made it difficult to see the trends clearly. To overcome this I categorised the data into three classes: *Infrequent* (merging the data from *Never* and *Rarely*), *Periodically* (merging *Sometimes* and *Often)* and *Frequent* (*Most Of The Time* and *Always*). The results of this question, in these modified categories, are presented in Figure using a comparative bar chart.

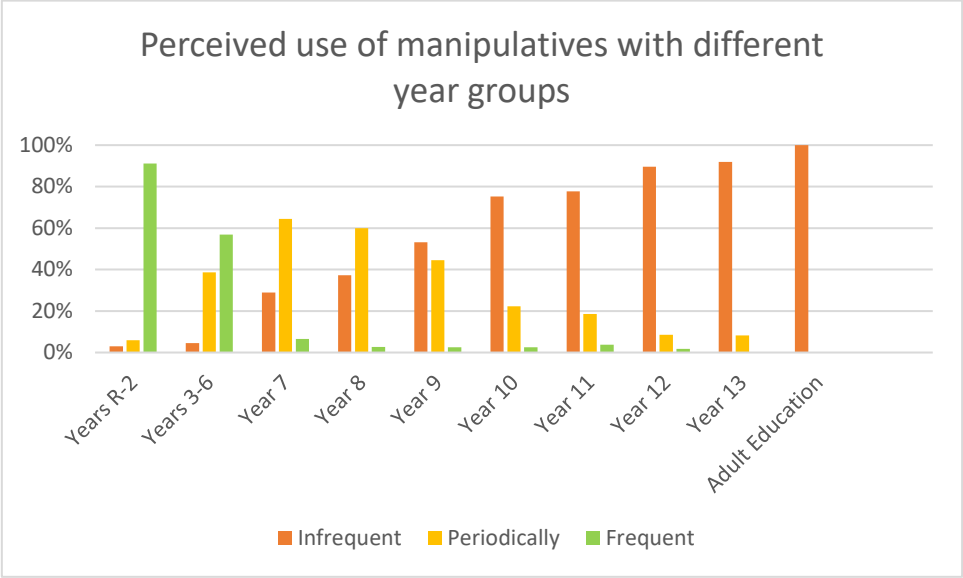


Figure 5: Comparison of teachers' use of manipulatives with different year groups

What is noticeable from the chart is that the use of manipulatives becomes more infrequent as the age range increases. 97% of those teaching infant aged pupils claimed to use manipulatives frequently or periodically, with only 3% claiming to use them infrequently. Whilst at the other extreme, 100% of those teaching adult education and >90% of those teaching sixth form claim that they either *never* or *rarely* (infrequently) use manipulatives when teaching mathematics. In contrast to the infrequent proportion, the proportion of those using manipulatives frequently decreases with age of pupils. However, it is not a steady decline, instead there seems to be a sharp decrease in those using manipulatives *always* or *most of the time* between the higher years of primary school and those teaching the lower years of secondary school.

One limitation of the data collected is it does not provide a measure of the actual use of manipulatives and therefore cannot be used to make claims regarding so. Instead, it provides indications based on teachers' *perceptions* of their use. Teachers may perceive frequency of use differently and so give different answers even if their use of manipulatives is the same. However, the number of participants in the survey means that indications of trends in frequency can be seen, as highlighted in the obvious trend in the decrease in use with age. This data appears to confirm the earlier findings of Marshall & Swan (2008) and even earlier Gilbert & Bush (1988) who found that the use of manipulatives reduces as the year group increases. Another limitation of this data is that is does not reveal the reasons *why* the use of manipulatives seems to decline.

All those interviewed, except Teacher D, confirmed the trend that manipulatives are being used mostly in lower years and/or with lower attaining pupils as a type of intervention. Teacher A said that her school had recently tried to incorporate manipulatives into the scheme of work but only for year 7 classes. The department were trying to follow a concrete-pictorial-abstract scheme and using manipulatives to introduce new topics. However, teacher A said that these were often not referred to when moving on to pictorial/abstract representation and she did not feel she had been adequately trained in how to link the three forms. She feared that some in her department were using manipulatives to "tick a box" and were still teaching by procedure, though the pupils got to "play with some blocks" at the start of each topic. Teacher D, however, said that he used to use manipulatives to introduce topics but his use with them had become "less linear and more fluid", commenting that he will dip in and out of using manipulatives with more confidence now. Teacher D attributed this confidence to his own research into using manipulatives, driven by his own interest.

SMTs' perceptions of the usefulness of manipulatives

To understand why and how SMTs use manipulatives, teachers were asked to select from a list of reasons why they have used manipulatives in the past (they could select more than one option). In

the same way, they were also asked to select from a list of mathematical topics in which they would be most likely to use manipulatives. The data showed that teaching shape, space and geometry (69%), followed closely by ratio and proportion (67%) and basic numeracy (64%) were the most popular topics to use manipulatives with. 6% of SMTs said they would not use manipulatives at all, whilst 0% PTs selected this option. Typically, higher level and more complex areas of mathematics had the fewest selections with calculus and mechanics scoring 4% and 2% respectively. This aligns with the previous data that showed that teachers were less likely to use manipulatives with higher years. However, it is interesting because mechanics is an applied area of mathematics used to describe real-life scenarios. Therefore, one might assume it's an area where the use of concrete objects in the teaching and learning of the subject is easier. However, it should be noted that until recently mechanics was an optional module in the A-level curriculum and therefore fewer teachers may have experience teaching it. This may have affected the data when it comes to mechanics specifically, though other data collected (as discussed previously) does suggest that manipulatives are not used as often when teaching these higher level topics.

Another notable observation is that a much higher proportion of PTs compared to SMTs claimed they are likely to use manipulatives with problem solving tasks (82% compared to 42%) and investigations (72% compared to 37%). This is noteworthy because research has shown that investigative and problem solving style tasks are important in developing mathematical habits of mind which are often lacking in pupils taught mathematics in secondary schools (Cuoco *et al.,* 1996). Cuoco *et al.* (1996) describe and argue that these habits of mind need to be encouraged to develop within pupils. For example, pupils need to be capable in "tinkering" and "guessing" without fear of not reaching a solution straight away. Problem solving, investigations and exploratory learning have no fixed mathematical content nor procedures, and therefore require the teacher to be proficient in facilitating tasks where pupils are encouraged to try out and organise their own ideas (Ponte, 2008). Manipulatives have been shown to be an extremely useful tool in aiding pupils in these types of task (Moyer & Bolyard, 2002), however teachers themselves need to be skilled in their use so that they

can encourage pupils to use manipulatives to model their ideas without simply imitating a set of steps or procedures. This is something Ball (1992) points out is a skill lacking in teachers. The data suggests that SMTs are either using investigative style tasks less than their primary counterparts, or that they are not confident in how to incorporate manipulatives into these tasks, or that they do not think manipulatives useful here.

SMT's were asked to list the reasons for using manipulatives if/when they use them. 10% said they never used manipulatives, and 55% said they used manipulatives because they thought them an effective learning tool. That means that 35% of SMTs are using manipulatives with their pupils for reasons other than being an effective aid to learning. This would suggest that these teachers' have little belief in the usefulness of manipulatives to facilitate learning, and therefore may not be investing the time or effort into using them to their full potential. 56% of SMTs said they chose to use manipulatives because pupils find them fun, interesting or engaging, whilst 30% said they just wanted to try something new. 11% said they were required to use manipulatives by their place of work and had no choice. Another popular reason included being useful to check pupils understanding quickly and easily. Low-scoring reasons included differentiation (2%) and it being an easy learning task to plan (1%). This data reveals that 22% of the SMTs are not *choosing* to use manipulatives, either fulfilling a requirement or not using them at all. If manipulatives are not being used purposefully by teachers who view them as an effective learning tool and therefore use them as such, then they may not be being utilised as an effective and important aspect of the concrete-pictorial-abstract learning cycle.

Participants were asked their beliefs on the value of manipulatives in secondary education and given an open text box to answer. The results highlight the divide in teachers' opinions which seemed to fall into one of three categories. These categories are found below, with illustrative quotes from the survey and the number of responses that were classed within that category. The remaining respondents did not answer the question.

- Those who believed manipulatives had little or no value in secondary education. *Approximately 14% (12 people).*

  "They go in and out of fashion."

  "[Manipulatives are] a useful tool in primary but little value in secondary".

- Those who thought they had their place, but only with younger years or children of lower attainment. *Approximately 25% (21 people).*

  "I feel they are best used with younger or lower attainment groups."

  "Useful for less able students and students in lower years such as year 7 doing basic concepts."

- Those who thought manipulatives were valuable used with all years and across attainment levels. *Approximately 48% (40 people).*

  "[An] essential part of teaching for deep understanding…even for older pupils e.g. yr 11."

  "I think manipulatives can be useful for any ages group or ability. They would just be used in different ways. They can be really helpful in problem solving lessons. I learnt a lot from seeing them being used in primary and brought it back to my department."

Of those who endorsed the use of manipulatives across all ages and stages, many referred to concrete-pictorial-abstract learning. Two participants said that they believed in the value of manipulatives, but needed more training in how to use them. One participant commented that "manipulatives are important for developing students' mathematical flexibility" i.e. being able to see and interpret concepts in different ways." This observation aligns with the arguments of Cuoco et al. (1996) and the need for developing pupils' mathematical habits of mind.

The data from this question shows the majority of teachers claim they believe manipulatives are useful throughout secondary school, however this does not match the data which showed teachers' perceptions of their use with different year groups. Therefore, this misalignment between the teachers' beliefs and their practice must be due to either a practice-based or a knowledge-based

effect, or to other barriers. This is something looked at further in this chapter under research question 2. The data also shows that although many teachers do not believe that manipulatives can be used with all age ranges and mathematical levels, there are teachers who not only believe this but also put it into practice and claim to see the advantages. This difference may be due to lack of experience or training in how best to use the object, something referred to by participants. Again, this will be explored under research question 3.

There are a number of interesting issues that have arisen from analysing the survey data in reference to research question 1. Although there were far fewer teachers interviewed, their ability to express their views on the use of manipulatives in increased depth sheds further light on the differing perspectives of SMTs. Teacher D perceived manipulatives to be useful to all pupils, stating they were equally as useful for higher attainers, especially in making and exploring their own mathematical conjectures. Teacher A said she had only heard of manipulatives in the last 2 years and has heard and seen more and more evidence that they can be extremely impactful. However, she recognises that there "needs to be a shift" in her own teaching to reflect this. Teacher C thought that mathematics could be split into 2 groups- abstract and functional- and that manipulatives are only useful for functional maths. He thought this is why they were more useful for lower years/attainers where they are "learning more functional bits of maths". Both Teacher B and E felt that they could be useful for a wider range of children than they currently use them with, however felt they lacked the knowledge and time to implement change in their current teaching programmes. Teacher B summed this up by saying "I think they would be beneficial [for older pupils] if I knew how to use them. But I don't have the time to invest in how to use them with KS4 and why force it if they can just do it symbolically?".

4.2. Research Question 2: In what ways and to what extent are SMTs equipped to use manipulatives within the classroom?

Without being able to observe the SMTs use of manipulatives, the findings to this research question mainly reveal teachers own perspectives on how equipped they are. However, those interviewed were also given a task which is also discussed. The findings and discussion are organised under the following subheadings: How equipped SMTs perceive themselves to be; Interviewees' response to given task; The perceived barriers to the effective use of manipulatives; SMTs' response to additional education on the use of manipulatives.

How equipped SMTs perceive themselves to be

When plotting the difference between how equipped SMTs feel using manipulatives after their ITY and how equipped they feel now (from the survey data), against the number of years they have been teaching, there was no correlation. The results indicate this growth is not merely a bi-product of increased experience in teaching *per se*, as one would therefore expect to see a positive correlation between the two variables. Instead there seem to be other contributing factors. Notably 20% of participants recorded no increase in how equipped they feel since they started teaching, with 2 participants stating their confidence had decreased. There was a wide spread of years' experience of these participants.

However, the overall confidence of the SMTs had increased (see Figure 6), with a median confidence level rising from a score of 4 up to a 7 (1 being – not equipped at all, 10 being – extremely confident in how to use manipulatives effectively), with 50% of the data now falling between the 6 and 8 rating. At the same time, <25% of SMTs felt very well equipped, with a score of 9 or 10.

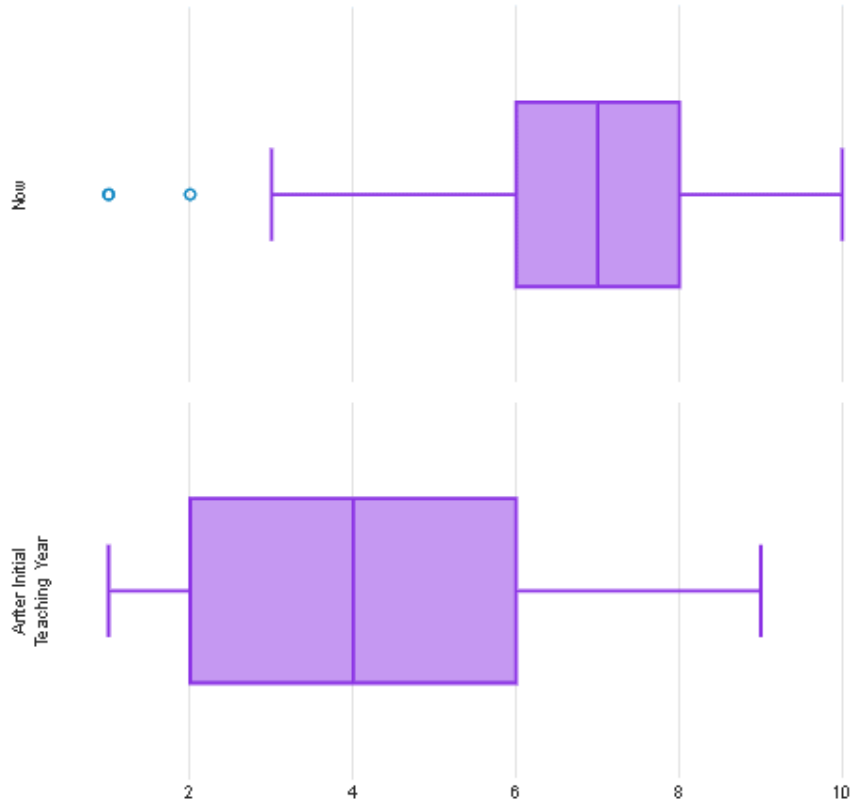


Figure 6: Boxplots showing the spread of how equipped to teach manipulatives SMTs feel now and after their initial teacher education year.

This data shows that half of teachers would have rated themselves at a 4 or under in how equipped they were to use manipulatives at the end of their ITY. This is significant as it indicates that initial teaching programmes, on the whole, are not preparing teachers in the effective use of manipulatives in the classroom. However, teachers are generally feeling more equipped once they have been teaching a while, indicating that they are picking up skills and tools by other means. All teachers interviewed indicated that they had not had any experience of manipulatives within their ITY, except one who had observed a teacher using them as an intervention measure in a school where children were taught in mixed ability sets. All interviewed teachers claim that they were not equipped at all to use manipulatives when they started teaching. Teacher C mentioned that those who had seemed more confident to use manipulatives were those whose training had been in primary years initially, or those who had studied a PGCE focusing on ages 7-14. Teacher D felt a lot more equipped now due to their own research and years of using and trying different approaches

with manipulatives. Teacher B had recently done some online training but felt unconfident, stating that her head of department had bought lots of manipulatives for the department to use but that no one knew how to do this and so they had gone mostly unused.

Interviewees' response to given task

Within the interview I gave the teachers a task, as way of allowing the teachers to demonstrate their contingent knowledge (Rowland et al., 2005) in relation to their use of manipulatives and how equipped they are now to apply their knowledge of manipulatives to an unseen example. I introduced this task after discussing how equipped the teacher's feel, as I did not want the results of this task to affect that discussion. I chose to ask the teachers if they could think of a way in which concrete objects, or pictorial representations, could be used to model the division of fractions:

$$1\frac{3}{4} \div \frac{1}{2}$$

I chose this expression as Ball (1992) mentions that she used a similar task, using this question, with a number of college students. Ball found that only a small proportion were able to model in this way, with some stating it was impossible. The division of fractions is commonly taught in schools by procedure (often the phrase 'keep, change, flip' is used to help pupils remember the steps) and in my own experience pupils not understanding the reasoning behind these steps can hinder their mathematical understanding, especially when studying mathematics at a higher level. Children (and adults) often confuse dividing by a half with dividing by 2. By using manipulatives, the question becomes more visual and, often to many, less intimidating and simpler.

Of the five teachers interviewed, only one (teacher D) was able to demonstrate how they would model this using manipulatives and was able to explain why this would have been an effective learning tool. The fact that only one teacher was able to demonstrate good contingent knowledge

during the interview, enforced by his well exercised transformation knowledge, confirms Ball's (1992:47) opinion that:

'Teaching with manipulatives is not just a matter of pedagogical strategy and technique. Few well-educated adults - not just teachers - can devise or use legitimate representations for many elementary mathematical concepts.'

This again highlights that many pupils are being taught by teachers who are not confident in their own knowledge and use of manipulatives and therefore these pupils are unlikely to benefit from the understanding manipulatives can facilitate.

The perceived barriers to the effective use of manipulatives

SMTs were asked if they would like to use manipulatives more in their teaching and what, if any, are the barriers that stop them using manipulatives more in their lessons. Only 8% said they would not like to use manipulatives more (with some of these reporting that they feel they already use them as a regular feature of their teaching). 65% said yes, they would like to use manipulatives more, with 26% answering maybe. This data, showing that the majority of teachers want, or are at least open to using manipulatives more, indicates that teachers hold beliefs that manipulatives are beneficial, although these perceived benefits may take different forms between teachers. Therefore, it is not predominantly that teachers 'need persuading' that manipulatives are beneficial, instead it is other factors which are holding back their use, let alone their effective use. This data complements Ball's (1992) claims that teachers believe in the effectiveness of manipulatives, but that there are other factors which may prohibit their use.

When asked what SMTs perceived the main barriers to furthering their use of manipulatives were, a number of reasons were given with most participants giving more than one reason. However, there

were six reasons which had the most frequent level of response (listed here with the % of SMTs who selected the response):

- Availability of resources and objects (52%)
- The more advanced the mathematics is, the trickier it is to know how to relate the abstract with the concrete (43%)
- There is not enough time/opportunity allowed within the scheme of work (38%)
- Manipulatives are a distraction and can lead to problems in behaviour (32%)
- I do not feel sufficiently trained in how to use manipulatives effectively (31%)
- The hassle and time consumed sourcing, distributing and collecting in resources (29%)

This list gives an insight into the extent to which teachers are equipped to use manipulatives and where they perceive the barriers lie, and may be used to indicate areas which teacher education in the use of manipulatives might target. Although some of the lower ranking barriers could be described as belief (e.g. manipulatives are not useful for older year groups), most of the six higher ranking barriers could be classified as a lack in teacher knowledge. For example, using the classifications of the knowledge quartet (Rowland et al., 2005), finding it hard to use manipulatives with higher topics may reveal a lack in transformation and/or connection knowledge. This will be explored further under research question 3.

The teachers interviewed all mentioned aspects of the 6 barriers listed above, although there was a greater emphasis, in general, on how unconfident they felt due to lack of training and lack of time to do their own research into manipulative use. Three of the teachers mentioned how their schemes of work were so tightly scheduled that they did not have the time to try new things out. There was also an emphasis on teaching for the GCSE exam and the pressures that teachers are under to 'get the grades'. This pressure, for most of the interviewed teachers, meant that often they had to knowingly sacrifice mathematical understanding for procedural recall as it is 'quicker to teach someone the steps to get marks than it is to make sure they understand what they are doing' (Teacher E). Teacher

E thought this may be why pupils often struggled when starting A-level mathematics. Teacher B mentioned that she felt that senior leaders, even those with a mathematical background, would not know or understand the rationale behind the use of manipulatives and that if she was observed using them, they would react negatively. This may reiterate the idea that many believe manipulatives to be only appropriate for young or low attaining pupils.

SMTs' response to additional education on the use of manipulatives

When asked in the survey if SMTs would want additional training in how to use manipulatives effectively, the results were split into identical proportions as the responses to the question on whether they would like to use manipulatives more in their teaching (Figure). This shows that only 8% of the teachers perceive they need no further education on the topic, either because they already feel fully equipped or because they see little advantage in increasing or improving their use of manipulatives. Therefore, 92% of SMTs either want or may want education on how to use manipulatives effectively indicating that SMTs perceive the extent to which they are equipped to use manipulatives could be improved. All teachers interviewed expressed a desire for further education in this area, with all saying they thought it should be an area included in IT education programmes. What effective education on the use of manipulatives should involve is looked at under research question 3.

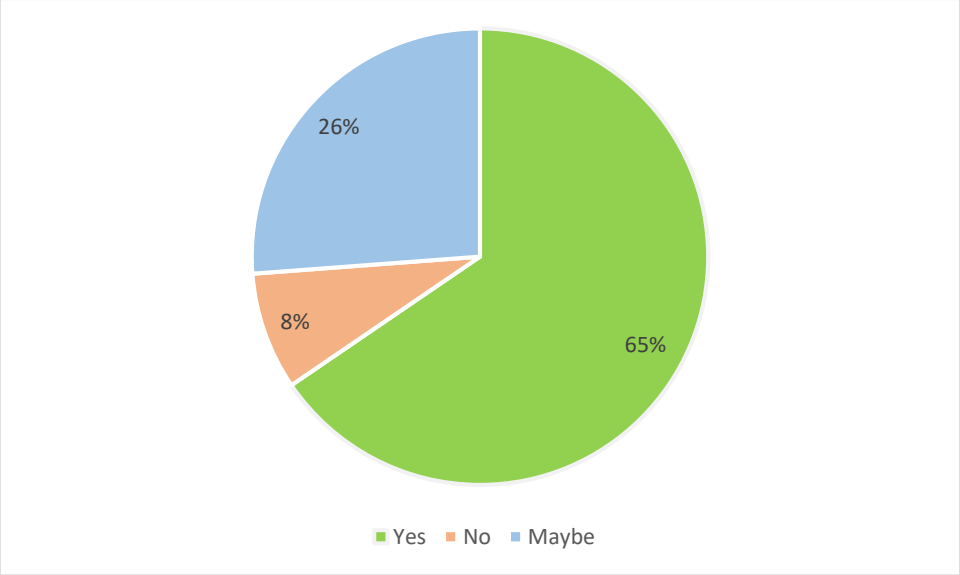


Figure 7: SMTs' responses when asked if they would like further training in the use of manipulatives

4.3. Research Question 3: How can the use of manipulatives be effectively integrated into a secondary IT education course?

This section explores what the survey and interview data, along with the literature reviewed, reveals about how effective use of manipulatives could be impactfully integrated into an initial secondary programme. Recommendations on how these findings could be used to implement changes to an initial teaching course are presented in the final chapter. The section on the third research question is split into two subheadings: The effect of different training routes on how equipped SMTs feel; Ways of integrating the use of manipulatives into a secondary initial teaching programme.

The effect of different training routes on how equipped SMTs feel

By looking at the correlation between the routes into teaching of the survey participants and how equipped after their ITY the SMTs felt in the use of manipulatives, the mean and the median self-assessed scores were almost identical between those teachers who had taken a PGCE route into teaching compared to those who had followed a more schools-based route (such as the Graduate

Teacher Programme, Schools Direct, TeachFirst or School Centred Initial Teacher Training). These averages were in line with the averages of the whole SMT sample, with a mean of 4.1 and a median of 4, indicating that the route into teaching had no impact on how equipped these teachers were to use manipulatives effectively when starting their careers. Instead of being dependant on the route into teaching, how equipped teachers felt seemed to be more dependent on the TE running/designing the course they were on and/or the attitude towards manipulatives of the mentors and teachers they worked closely with during that year. This became especially apparent during the interviews. Teacher E commented that manipulatives were not discussed at all during her training year (a PGCE course), because she thought either her tutor did not consider it or because they deemed other teaching tools more significant. Teacher D, who took a schools-based training route, mentioned that the specific teacher/s he worked closely with did not use manipulatives and therefore he did not. However, he thought that a schools-based route meant he had to do a lot of research into teaching methods himself, undirected, and that was how he eventually became interested in the use of manipulatives years after qualifying.

Ways of integrating the use of manipulatives into a secondary initial teaching programme

Figure breaks down the way in which the SMT participants learnt how to use manipulatives during their ITY. The largest section of the pie chart shows that 27% of SMTs did not study the use of manipulatives at all, while very few had manipulatives embedded throughout their course or even benefitted from stand-alone sessions (20% and 5% respectively). Those that learnt about the use of manipulatives often did through their own initiative (i.e. through their own trial and error) or through the fortune of being placed in schools where teachers or mentors used them.

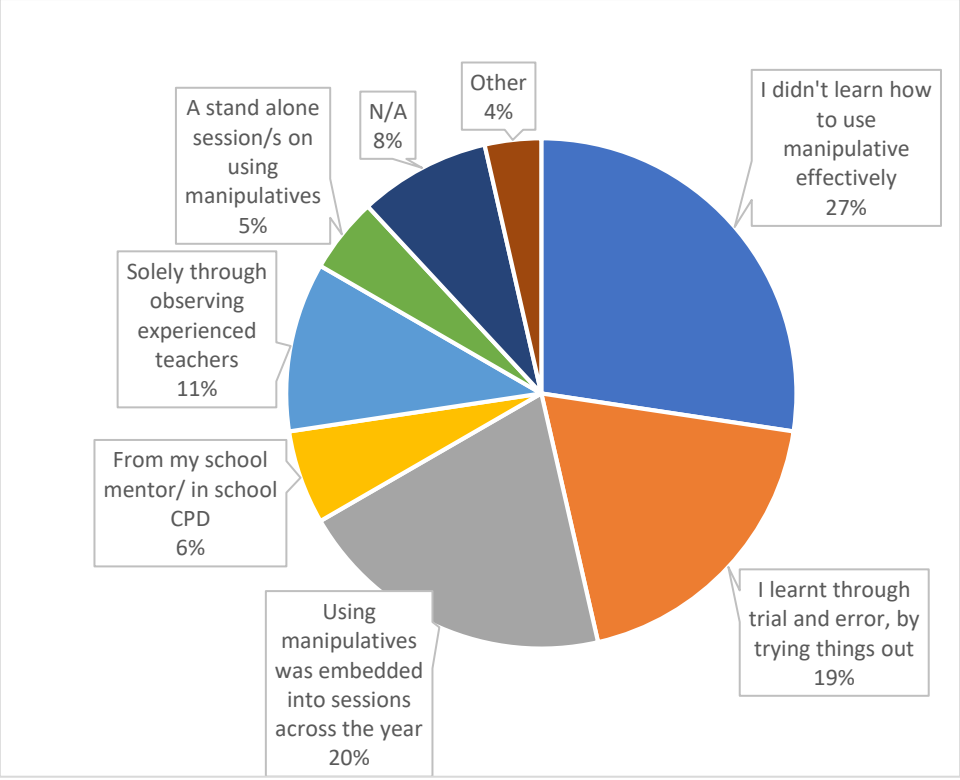


Figure 8: How SMTs learnt to use manipulatives effectively during their initial teaching year

A comparison was made between the ways in which SMTs had been exposed to using manipulatives effectively during their initial teaching year with how equipped they had felt in their use by the end of that year. The average (mean) self-assed score for how equipped SMTs were was 4.1, with the average score of those who said they did not learn about manipulatives on their course being 2.0. Most average scores for the ways in which SMTs had been taught were around the 4 mark, with negligible differentiation between the scores. However, the average score for how equipped teachers felt after a training course where manipulatives were embedded throughout the year gave a score of 6.7, significantly higher than any other. This indicates that embedding manipulatives throughout a course is the most effective way of including manipulatives in an ITY, more effective than stand-alone sessions, learning solely through mentors or leaving it to the student-teacher to trial themselves without instruction.

I was also interested in whether teachers' views on how they thought manipulatives could be best integrated into a course matched with the evidence from the survey on which ways were most

effective. If so, this would suggest that teachers have an insight into how they would best learn using manipulatives and therefore it is worth consulting them when making recommendations on implementing changes to a course. With the majority of teachers claiming they would (or may) like further training on the effective use of manipulatives, teachers were asked how they thought the effective use of manipulatives could be best integrated into an IT education programme. (Figure). The results showed that the majority of SMTs viewed embedding throughout the year as the most effective way of learning about their use. 10% of SMTs did not think manipulatives needed to be better integrated into IT education programmes, which is roughly in line with the 8% who did not want to use or have additional training on manipulatives. This data shows that teachers' beliefs on what would be most impactful correlated with the data which showed that embedding manipulatives throughout the year was the most effective means of feeling equipped to use manipulatives in the classroom. However, this does not mean that the other methods would not be impactful at all, and embedding throughout a year may involve a mixture of other methods.

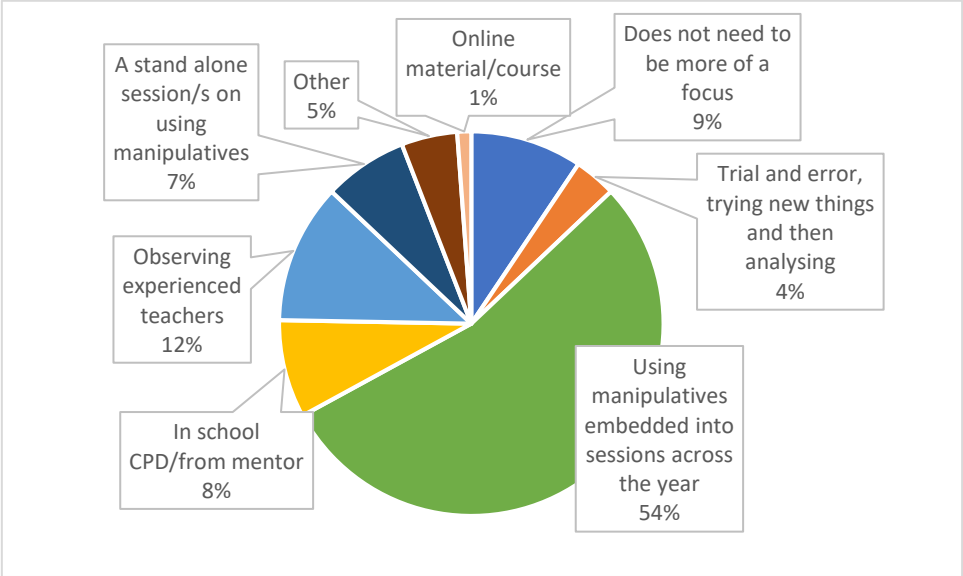


Figure 9: SMTs perceptions of how manipulatives could be better integrated into an initial teaching course

All teachers interviewed thought that manipulatives should be embedded throughout an IT education course, however their views on how this should be done differed. All teachers discussed a course with a PGCE structure in mind (involving a mix between sessions in university and time in schools). Teacher A thought that a session at the start of the year should be dedicated to manipulatives so that teachers knew of the benefits and arguments for and against using them early on. She also thought that manipulatives should always be present during university sessions so that they could be easily integrated into discussions and practice. Teacher B thought there should be opportunity to observe experienced and expert teachers in the field and opportunities when looking at different aspects of the mathematics syllabus to discuss examples of how manipulatives may be a beneficial learning tool in that area. Teachers C and E also talked about the importance of collaboration and how student teachers could work together to research, implement and present their experiences on using manipulatives to each other throughout the year. Teacher D thought that giving student teachers practical tasks that involved designing activities using manipulatives would enforce an awareness of their presence and give student teachers an opportunity to discuss what went well and how the design of their activity could be improved.

Clarke & Hollingsworth's (2002) model of teacher growth implies that teachers need to both enact and reflect to grow as teachers, and relates both a change in beliefs or knowledge with a change in practice. This growth could only be minimally achieved with a standalone session. This is because there would be limited scope to enact change, reflect on the change and repeat the process. This supports the idea of embedding manipulatives throughout a course to help equip teachers for their use. To ensure that the embedding does aid teachers' knowledge of how to use manipulatives effectively, the recommendations made in the next chapter take into account research on teacher growth, knowledge and beliefs, as well as the findings from this chapter.

Chapter 5: Conclusions and Implications

This chapter begins with a summary of the conclusions drawn from the investigation, organised under the three research questions. Recommendations are then made as to how initial teaching programmes might better incorporate the effective use of manipulatives. The chapter closes with a discussion of the potential impact of the study, its limitations and scope for further research.

5.1. Summary of findings

Research Question 1: What are teachers' perceptions of the role of manipulatives in secondary mathematics education?

The findings from the data collected revealed a great deal about the perceptions of the teachers within this study with regards to the use of manipulatives in secondary schools. The data suggested that teachers were familiar with what manipulatives are and generally held positive beliefs regarding the potential impact they can have on the learning of mathematics within a secondary context, with almost half (48%) of those surveyed and all of those interviewed believing that manipulatives can be impactful across all year groups and attainment levels. However, this belief was not evident in their self-evaluated practice. Manipulative use dramatically decreased as the age of pupil increased, in line with older studies which have taken place (Marshall & Swan (2008); Gilbert & Bush (1988)). The perception of those interviewed was that this is common practice within schools. SMTs perceived their use of manipulatives to be mostly in mathematical areas which are regarded as more 'functional', or easy to represent in concrete form (i.e. ratio and proportion) rather than those areas regarded more abstract (such as algebra). Four out of the five interviewees mentioned the use of virtual manipulatives, suggesting that their use is becoming an increasingly popular alternative to traditional physical objects.

Research Question 2: In what ways and to what extent are SMTs equipped to use manipulatives within the classroom?

By the end of their ITY, SMTs did not feel well equipped for teaching mathematics using manipulatives, with a median score of 4 (1 being – not equipped at all, 10 being – extremely confident in how to use manipulatives effectively), indicating that initial teaching programmes are not preparing teachers for their use. The data implies this has been the case for many years, as how equipped teachers felt did not seem to be influenced by when they began teaching. Those interviewed were able to elaborate on this in more detail and conveyed that manipulatives had been rarely mentioned within their own IT education and that, in their opinions, SMTs felt unconfident and lacking in skills, knowledge and time to use manipulatives effectively. How equipped teachers perceived themselves to be seemed to increase once teachers had begun their careers. However, few SMTs (25%) would score themselves 8 or greater on a scale of how equipped they now feel to use manipulatives effectively, with 92% of SMTs answering yes or maybe to a hypothetical offer of more training in this area. This was the same proportion of those who answered yes or maybe to whether they would like to use manipulatives more in their teaching. Only one of the five teachers interviewed was able to use manipulatives to devise a representation of a fractional division question, which supports Ball's (1992) claim from almost three decades ago, that few teachers hold this ability. When questioned on the barriers SMTs felt prevented teachers using manipulatives effectively, six reasons became prevalent. One of these reasons, 'not feeling sufficiently trained in the use of manipulatives', was explored further with the interviewees, where it became evident that teachers felt lacking in how to connect the concrete with the abstract, and using manipulatives for higher level mathematics. In answer to research question 2, the data suggests that SMTs are equipped to a small extent to use manipulatives effectively, especially during their ITY. Teachers seem to be lacking more in knowledge of manipulative use than in the belief that manipulatives can be an impactful tool. This again supports Ball's (1992) argument that teachers often pin much 'hope' on the effect of manipulatives, but are lacking in knowledge of how to use them impactfully.

Research Question 3: How can the use of manipulatives be effectively integrated into a secondary IT education course?

The data showed the route into teaching had no significant impact on how equipped SMTs felt to use manipulatives effectively at the end of their initial teaching course. What seemed to have an impact, according to the interview data, was the attitude towards the use of manipulatives of the TEs and mentors they worked closely with. The data revealed that many SMTs either did not learn about manipulatives or learnt through their own initiative during their ITY, with few teachers on a course where manipulatives were embedded throughout. Those that had manipulatives embedded throughout the year reported feeling more equipped in their use, compared with those that either did not learn about the use of manipulatives or had them introduced another way. This indicated that embedding manipulatives throughout a course is the most effective way of including them in an ITY. The majority of SMTs surveyed viewed embedding manipulatives throughout an ITY to be the most effective way to learn about their use. This matched the results of the survey, indicating that teachers had a good insight into how they best learn. Views of those interviewed on how manipulatives could be better integrated into an ITY varied and included discussions, practical tasks, observations and teaching by example. Embedding the use of manipulatives throughout the year fits well with Clarke & Hollingsworth's (2002) model of teacher growth, as it would allow for continuous reflection and enactment between the four different domains.

5.2. Recommendations

The misalignment between teachers' beliefs and their practice was highlighted under the findings of research question 1. This misalignment draws attention to the barriers hindering teachers' use of manipulatives from being consistent with their reported beliefs. This research has shown there to be a lack in teacher knowledge in all four domains of the knowledge quartet (Rowland et al., 2005) in respect of the use of manipulatives. Therefore it is recommended that an initial teaching course

targets each four of these areas: Teachers' foundational knowledge (their perception of what manipulatives are and how they can be used); Teachers' transformation knowledge (how to represent a mathematical problem through the use of manipulatives); Teachers' connection knowledge (how to connect the concrete to the abstract); Teachers' contingent knowledge (the ability to form examples and use manipulatives to represent unplanned scenarios).

Practical suggestions of how a TE could embed the use of manipulatives into an initial teaching course can be found in **Error! Reference source not found.**, corresponding to the six barriers most frequently cited within the data. It should be noted that there were responses where minimal use of manipulatives (e.g. with older pupils) was found to align with beliefs that manipulatives would be ineffective when used for higher level topics or older pupils. In this case, many of the suggestions in Table 2 may also be useful in prompting a change in beliefs as well as knowledge.

Table 2: Recommendations of changes to initial teaching programmes

Barrier	Recommendation
- Availability of resources and objects	- , Suydam & Higgins (1976) advocate the simplest of manipulatives should be used. The TE should consider demonstrating and promoting the use of easily available objects as manipulatives (such as paperclips, pens, beads) or objects which can be made by the teacher/pupil (such as fraction strips or algebra tiles out of paper or card) ensuring that the student teachers are equipped to use the same manipulatives in multiple ways. - Although the TE cannot determine the resources available to the teacher once they are working in schools, the course provider can liaise with placement schools to ensure/recommend they have resources available to the student teacher.
- The more advanced the mathematics is, the trickier it is to know	- TE to research and use tasks where the student teacher can experience using manipulatives themselves whilst exploring higher level topics facilitated by the TE. This is inline with the

how to relate the abstract with the concrete	views of Nührenbörger & Steinbring (2008) and Zaslavsky & Sullivan (2011). - Include manipulatives in any Subject Knowledge Enhancement course student teachers are required to complete before they being their ITY, so they become familiar and are used from the very start. - Set collaborative lesson planning tasks for higher end topics where using manipulatives is required. Use these lesson plans as a basis for discussion, demonstration and peer learning. - Ensure there is a section on the course lesson planning proforma where student teachers are prompted to think about what, if any, manipulatives can/will be used in the lesson they are planning.
- There is not enough time/opportunity allowed within the scheme of work	- TE to focus on lesson planning from a scheme of work. When using an example, show how manipulatives can be used alongside/instead of other tasks, not necessarily as an additional task. - By emphasising the concrete-pictorial-abstract aspect of mathematical ideas, discuss how ensuring pupils are secure in their foundation knowledge can save time in the future (the basis for mathematics mastery) (DfE, 2020).
- Manipulatives are a distraction and can lead to problems in behaviour	- When focusing on behaviour for learning on the initial teaching course, including expectations and boundaries, integrate this with the use of manipulatives. - Use videos to watch and discuss how a teacher may deal with/promote certain behaviours when using manipulatives. - During mandatory primary experience, ensure the student teachers observe and reflect upon the PT's use of manipulatives and pupils' behaviour. One aim of this would be to reassure SMTs that when manipulatives are embedded in a curriculum, the novelty (which may lead to poor behaviour choices) can wear off quickly.

- I do not feel sufficiently trained in how to use manipulatives effectively	- TE to research effective use of manipulatives before including in their course, bringing in 'expert' teachers if necessary. Liaise with primary TEs on the matter. - The use of manipulatives to be included on the student teachers' initial self-evaluation audit (or equivalent) and reviewed regularly throughout the year with the student teacher to ensure progress in this area is being made. - Run manipulative training for mentor teachers. - TE to consider using the practical suggestions of , Suydam & Higgins (1976) (found in Appendix A) as a framework for teachers.
- The hassle and time consumed in sourcing, distributing and collecting resources	- Explore idea (including positives and limitations) of having some central manipulatives available to pupils at all times, so that pupils are used to using them when needed and can initiate their use. - Promote sharing of examples/ideas/plans between student teachers so that they finish their ITY with an array of resource ideas. - Observe/reflect/discuss how to deploy resources whilst minimising disruption to the lesson.

For changes to an initial teaching programme to be effectual in facilitating a change in beliefs, knowledge and practice, and therefore teacher growth, the TE should promote a culture of reflection and enaction (Clarke & Hollingsworth, 2002), embedding manipulatives throughout the course.

5.3. Limitations, suggestions for future research and impact

A significant limitation of this study was the inability to observe practice and therefore findings were based on teachers' perceptions instead of demonstrations (due to current pandemic restrictions). Had observations been allowed, recommendations could be made more specifically to the needs of

the teacher conveyed by their practice not just their perceptions. The other significant limitation was the inability to implement and evaluate any changes to an initial teaching programme. It is recommended that further research implements the recommendations from this study into an initial teaching course and analyses the effect of these changes. It is expected that this would result in amendments/additions to the recommendations which may then be implemented and analysed, forming an action-research cycle.

For further research into this area I would also suggest observations of how current initial teaching programmes, including those for PTs, incorporate manipulatives. Again, this was a limitation caused by the current circumstances. It has also arisen from the study that further research into the use of virtual manipulatives by teachers and TEs would be beneficial, as the prevalence of technology within classrooms increases. Other areas which have arisen during this study, and may offer opportunity for further research, into teachers' use of manipulatives with pupils with specific educational needs, and the comparison between PTs and SMTs use of investigative style tasks. Collaboration between researchers may also result in richer findings and would help prevent any subjective interpretations of data, which is a risk of working alone.

This study has the potential to have a significant impact on the way in which manipulatives are integrated into initial teaching programmes, and therefore the way in which future generations of teachers use manipulatives within the secondary classroom. My hope is this work will form a foundation for future research and begin to bridge the gap between SMTs' belief in the potential of manipulatives and the current lack of use in secondary classrooms. For this impact to take place, I plan on disseminating these finding more widely throughout the mathematics teacher education community, whether through professional publications or conferences. Within my own role as a TE, the study has impacted the way in which I plan my programme and seek to incorporate the effective use of manipulatives through the recommendations made above. Not only have I learnt more about SMTs' perceptions, and use of manipulatives, but I have become increasingly aware of the impact a

lack in knowledge can have on the alignment between a teacher's beliefs and practice, highlighting the potential impact a TE can have through their planning and choice in course content.

Bibliography

(BERA), British Education Research Association. (2011). *Ethical Guidelines for Educational Research.* London: BERA.

Akkan, Y. (2012). VIRTUAL OR PHYSICAL: In-service and Pre-Service Teacher's Beliefs and Preferences on Manipulatives. *Turkish Online Journal of Distance Education, 13*(4).

An, S. (2004). *The middle path in math instruction: Solutions for improving math education.* Lanham, MD: Rowman & Littlefield Education.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator, 16*(2), 18-18, 46-47.

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis, & E. Simmt (Ed.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (pp. 3-14). Edmonton AB: CME.

Ball, D. L., Thames, M. A., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Baltar, F., & Brunet, I. (2012). Social research 2.0: virtual snowball sampling method using Facebook. *Internet Research, 22*(1), 57-74.

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and pro- fessional competence of teachers* (pp. 25-48). New York: Springer US.

Bellonio, J. L. (2012). *Multi-Sensory Manipulatives in Mathematics: Linking the Abstract to the Concrete.* Curricular Resources, Yale-New Haven Teachers Institute, Human Intelligence: Theories and Developmental Origins. Retrieved March 23, 2020, from https://teachersinstitute.yale.edu/curriculum/units/2001/6/01.06.12/8

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127-147.

Bloch, A., Phellas, C., & Seale, C. (2011). Structured methods: Interviews, questionnaires and observation. In *Researching Society and Culture* (3rd ed.). London: Sage Publications Ltd.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101.

Brown, S. E. (2007). *Counting Blocks or Keyboards? A comparative analysis of concrete versus virtual manipulatives in elementary school mathematics concepts.* Marygrove College. Retrieved from https://www.scribd.com/document/39698567/09

Carlson, J., & Gess-Newsome, J. (2013). The PCK Summit Consensus Model and Definition of Pedagogical Content Knowledge. *The Symposium Reports from the Pedagogical Content Knowledge (PCK) Summit, ESERA Conference 2012.* Colorado Springs.

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*.

Cohen, L., & Manion, L. (n.d.). *Research Methods in Education* (4th ed.). London: Routledge.

Cohen, L., Morrison, K., & Manion, L. (2007). *Research Mathods in Education* (6th ed.). Taylor& Francis Routledge.

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of Mind: an Organizing Principle for Mathematics Curriculum. *Journal of Mathematical Behaviour, 15*, 375–402.

De Fatima Alvaristo, E., Da Silva, S., Mamcasz Viginheski, L. V., & Pilatti, L. A. (2020). The use of manipulative didactic material to teach concepts related to the treatment of information for visually impaired students. *Acta Scientiae, 22*(2), 105-121.

Denzin, N. (1970). *The Research Act in Sociology.* London: Butterworth.

DfE, (. f. (2020). *PRINCIPLES AND PRACTICE Maths Mastery In Further Education.* London: Pearson Education Limited.

Dienes, Z. P. (1960). *Building up mathematics.* London: Hutchinson Education Ltd.

Drever, E. (n.d.). *Using semi-structured interviews in small scale research.* Edinburgh: Council for Research in Education.

Durmus, S., & Karakirik, E. (2006). Virtual Manipulatives in Mathematics Education: A Theoretical Framework. *Turkish Online Journal of Educational Technology, 5*(1).

Ernest, P. (1989). The knowledge, belieifs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*(1), 13-33.

Fennema, E. H. (1972). Models and mathematics. *Arithmetic Teacher, 18*, 635-640.

Fennema, E., & Franke, L. M. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.

Forgasz, H., & Leder, G. (2008). Beliefs about mathematics and mathematics teaching. . In *In Knowledge and Beliefs in Mathematics Teaching and Teaching Development.* Rotterdam: Sense.

Friedman, M. (1978). The manipulative materials strategy: The latest Pied Piper? , 9, 78-80. *Journal for Research in Mathematics Education, 9*, 78-80.

Fuson, K. C., & Briars, D. J. (1990). Using a Base-Ten Blocks Learning/Teaching Approach for First-and Second-Grade Place-Value and Multidigit Addition and Subtraction. *Journal for Research in Mathematics Education, 21*, 180-206.

Gastaldo, Brunno & Castro, Pablo & Homem-de-Mello,, Gastaldo, B., Castro, P., Homem-de-Mello, P., & Henrique Leal, S. (2017). Representations of the PCK before and after the Summit. *ESERA conference.* Dublin.

Gerling, M., & Wood, S. (1976). *Literature review:Research on the use of manipulatives in mathematics learning (PMDC Technical report 13).* Washington, DC: National Science Foundation.

Gilbert, R., & Bush, W. (1988). Familiarity, availability and the use of manipulative devices in mathematics at the primary level. *School Science and Mathematics, 94*(6), 459-469.

Golafshani, N. (2013). Teachers' Beliefs and Teaching Mathematics with Manipulatives. *Canadian Journal of Education, 36*(3), 137-159.

Hattie, J. (2012). *Visible learning for teachers.* New York & London: Routledge.

Hattie, J. (2019, July 18). *Updated research*. Retrieved from Visible Learning Plus: www.visiblelearningplus.com

Hayward, G. (1997). Principles for school focused initial teacher education: some lessons from the Oxford internship Scheme. In T. Allsop, & A. Benson (Eds.), *Mentoring for Science Teachers.* Buckingham: Oxford University Press.

Heddens, J. (1986). Bridging the gap between the concrete and the abstract. *Arithmetic Teacher, 33*(6), 14-17.

Heddens, J. W. (1997). Improving mathematics teaching by using manipulatives. *Theses*. Kent State University. Retrieved from http://www.fed.cuhk.edu.hk/~fllee/mathfor/edumath/9706/1%EE%80%80hedden.html

Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education, 23*, 98-122.

Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). *Evidence for review of mathematics teaching: Improving mathematics in key stages two and three: Evidence review.* London: Education Endowment Foundation.

Jackson, R. (1979). Hands--on Math: Misconceptions and Abuses. *Learning, 7*, 76-78.

Kablan, Z. (2016). The effect of manipulatives on mathematics achievement across different learning styles. *Educational Psychology, 36*(2), 277-296.

Kamina, P., & Iyer, N. (2009). From Concrete to Abstract: Teaching for Transfer of Learning when Using Manipulatives. *NERA Conference Proceedings 2009.*

Kennedy, L. (1986). A rationale. *Arithmetic Teacher, 33*(6), 6-7,32.

Kilpatrick, J. (2006). A golden means to teaching mathematics effectively: A review of the middle path in math instruction: Solutions for improving math education. *Journal for Research in Mathematics Education, 37*(3), 256-259.

Koponen, M., Asikainen, M. A., Viholainen, A., & Hirvonen, P. E. (2019). Using network analysis methods to investigate how future teachers conceptualize the links between the domains of teacher knowledge. *Teaching and Teacher Education*, 79, 137–152.

Labinowicz, E. (1985). *Learning from Children. New Beginnings for Teaching Numerical Thinking.* Menlo Park, Calif: Addison-Wesley Publishing Co.

Leong, Y. H., Ho, W. K., & Cheng, L. P. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator, 16*(1), 1-18.

Marshall, L., & Swan, P. (2008). Exploring the Use of Mathematics Manipulative Materials: Is It What We ink It Is? *Proceedings of the EDU-COM 2008 Sustainability in Higher Education: Directions for Change*, (pp. 338-350).

McIntyre, D. (1993). Theory, theorizing and reflection in initial teacher education. In J. C. Gates (Ed.), *Conceptualizing Reflection in Teacher Development* (pp. 39-52). Abingdon: RoutledgeFalmer.

McNiff, J., & Whitehead, J. (2002). *Action Research:Principles and Practice* (2nd ed.). London: RoutledgeFalmer.

Ministry of Education. (2012). *O-level teaching and learning syllabus.* Singapore: Author.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*, 175-197.

Moyer, P. S., & Jones, M. G. (2004). Controlling choice: Teachers, students, and manipulatives in mathematics classrooms. *School Science and Mathematics, 104*, 16-31.

Moyer, P., Bolyard, J., & Spikell, M. (2002). What are virtual manipulatives? *Teaching Children Mathematics, 8*(6), 371-377.

Moyer-Packenham, P., & Bolyard, J. (2016). Revisiting the definition of a virtual manipulative. In p. Moyer-Packenham (Ed.), *International perspectives of teaching and learning with manipulatives* (pp. 3-25). Switzerland: Springer International Publishing.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel. .* Washington, DC:: U.S. Department of Education. .

NCTM. (1991). *Professional standards for teaching mathematics.* Reston,VA: NCTM.

Nührenbörger, M. (2001). Insights into children's ruler concepts - Grade-2-students' conceptions and knowledge of length measurement and paths of development. In M. van den Heuvel-Panhunzen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology ofMathematics Education. 3*, pp. 447-454. Utrecht: Freudenthal Institute.

Nührenbörger, M., & Steinbring, H. (2008). Manipulatives as tools in teacher education. In D. Tirosh, & T. Wood (Eds.), *International handbook of mathematics teacher education* (pp. 157-181). Rotterdam: Sense.

O'Meara, N. (2011). *Improving mathematics teaching at second level through the design of a model of teacher knowledge and an intervention aimed at developing teachers' knowledge.* Theses. Retrieved from https://ulir.ul.ie/bitstream/handle/10344/3254/OMeara_2011_teaching.pdf.

Onwuegbuzie, A., & Teddlie, C. (2003). A framework for analyzing data in mixed methods research. In A. Tashakkori, & C. Teddlie (Eds.), *Handbook of Mixed Methods in Social and Behavioral Research* (pp. 351-383). Thousand Oaks, CA: Sage.

Piaget, J. (1952). *The Origins of Intelligence in Children .* New York: W. W. Norton& Co.

Ponte, J. P. (2008). Exploring and investigating in mathematics teaching and learning. *Proceedings of the 11th International Congress on Mathematical Education*, (pp. 208-228). Monterrey, Mexico.

Resnick, L., & Omanson, S. (1987). Learning to Understand Arithmetic. In R. Glaser (Ed.), *Advances in Instructional Psychology* (Vol. 3, pp. 41-95). Hillsdale, N.J.: Lawrence Erlbaum Associates.

Rowland, T., Hucksteep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.

Salkind, N. J. (2010). *Encyclopedia of research design* (Vol. 1). Thousand Oaks, CA: SAGE Publications.

Sandelowski, M. (2003). Tables or tableaux? The challenges of writing and reading mixed methods studies. In A. Tashakkori, & C. Teddlie (Eds.), *Handbook of Mixed Methods in Social andBehavioral Research* (pp. 321-350). Thousand Oaks, CA: Sage.

Sandir, H. (2012). Investigating Preservice Mathematics Teachers' Manipulative Material Design Processes. *Eurasia Journal of Mathematics, Science and Technology Education, 12*, 2103-2114.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Sowell, E. (1989). Effects of Manipulative Materials in Mathematics Instruction. *Journal for Research in Mathematics Education, 20*(5), 498-505.

Stigler, J., & Hiebert, J. (2004). Improving Mathematics Teaching. *Educational Leadership, 61*, 12-17.

Suydam, M. N., & Higgins, J. l. (1976). *Review and Synthesis of Studies of Activity-Based Approaches to Mathematics Teaching, Final Report.* NIE Contract No. 400-75-0063.

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, 58-70.

Swan, P., & Sparrow, L. (2004). Hands on — Heads on: using manipulatives in the classrooom. In B. Tadich, S. Tobias, C. Brew, Beatty B, & P. Sullivan (Ed.), *Towards excellence in mathematics* (pp. 516-525). Proceedings of the 41st Annual conference of the Mathematics Association of Victoria.

Sztajn, P. (2003). Adapting reform ideas in different mathematics classrooms: Beliefs beyond mathematics. *Journal of Mathematics Teacher Education, 6*, 53-75.

Taylor, S. J., & Bogdan, R. (1984). *Introduction to qualitative research methods: The search for meanings* (2nd ed.). New York: Wiley Interscience Publication.

Thompson, P. W. (1994). Concrete materials and teaching for mathematical understanding. *Arithmetic Teacher, 41*(9), 556-558.

Triola, M. F. (1992). *Elementary statistics* (5th ed.). Reading, Massachusetts: Addison-Wesley.

Tyson, P. (1991). Talking about lesson planning: The use of semi-structured interviews in teacher. *Teacher Education Quarterly, 18*(3), 87-96.

Valcke, M., Sang, G., Rots, I., & Hermans, R. (2010). Taking Prospective Teachers' Beliefs into Account in Teacher Education. In P. Peterson, E. Baker, & B. McGaw (Eds.), *International Encyclopedia of Education* (3rd ed., pp. 622-628). Elsevier.

Wearne, D., & Hiebert, J. (1988). A Cognitive Approach to Meaningful Mathematics Instruction: Testing a Local Theory Using Decimal Numbers. *Journal for Research in Mathematics Education, 19*, 371-384.

Whiting, C., Whitty, G., Black, P., Hordern, J., Parfitt, A., Reynolds, K., & Sorenson, N. (2018). Diversity and complexity: Becoming a teacher in England in 2015–2016. *Review of Education, 6*(1), 69-96.

Wilkinson, D., & Birmingham, P. (2003). *Using Research Instruments:A guide for Researchers.* London: RoutledgeFalmer.

Wilkinson, J. (1974). In W. M. Fitzgerald, & J. L. Higgins (Eds.), *Mathematics laboratories: Implementation, research and evaluation* (pp. 35-60). Columbus: ERIC Center for Science, Mathematics, and Environmental Education.

Young, S. (1983). How Teacher Educators Can Use Manipulative Materials with Preservice Teachers. *Arithmetic Teacher, 31*, 12-13.

Zaslavsky, O., & Sullivan, P. (2011). Setting the Stage: A Conceptual Framework for Examining and Developing Tasks for Mathematics Teacher Education. In *Constructing knowledge for teaching secondary mathematics : Tasks to enhance prospective and practicing teacher learning.* (pp. 1-22). New York: Springer.

Appendices

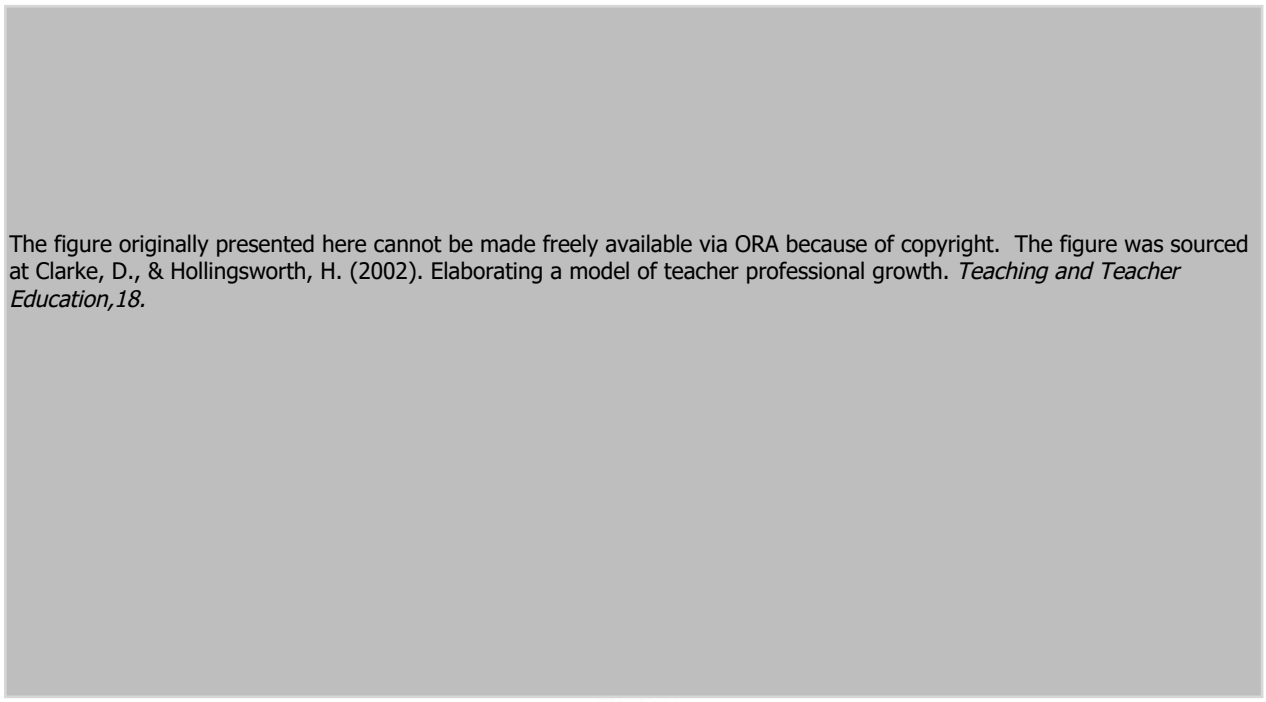
Appendix A

Practical aspects of using manipulatives teachers should be educated in according to Suydam & Higgins (1976), and listed by Durmus & Karakirik (2006).

1. Manipulative materials should be used frequently in a total mathematics program in a way consistent with the goals of the program.
2. Manipulative materials should be used in conjunction with other aids, including pictures, diagrams, textbooks, films, and similar materials.
3. Manipulative materials should be used in ways appropriate to mathematics content, and mathematics content should be adjusted to capitalize on manipulative approaches.
4. Manipulative materials should be used in conjunction with exploratory and inductive approaches.
5. The simplest possible materials should be employed.
6. Manipulative materials should be used with programs that encourage results to be recorded symbolically.

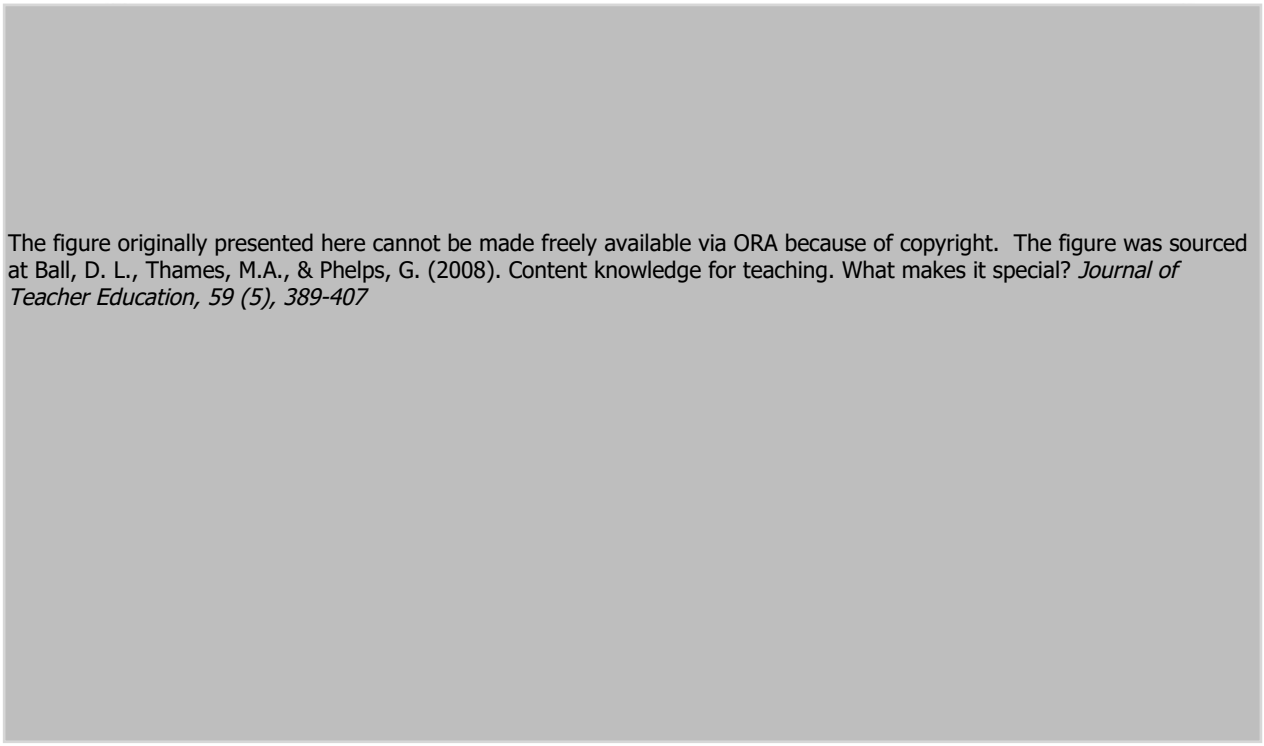
Appendix B

The Interconnected Model of Professional Growth (Clarke & Hollingsworth, 2002).



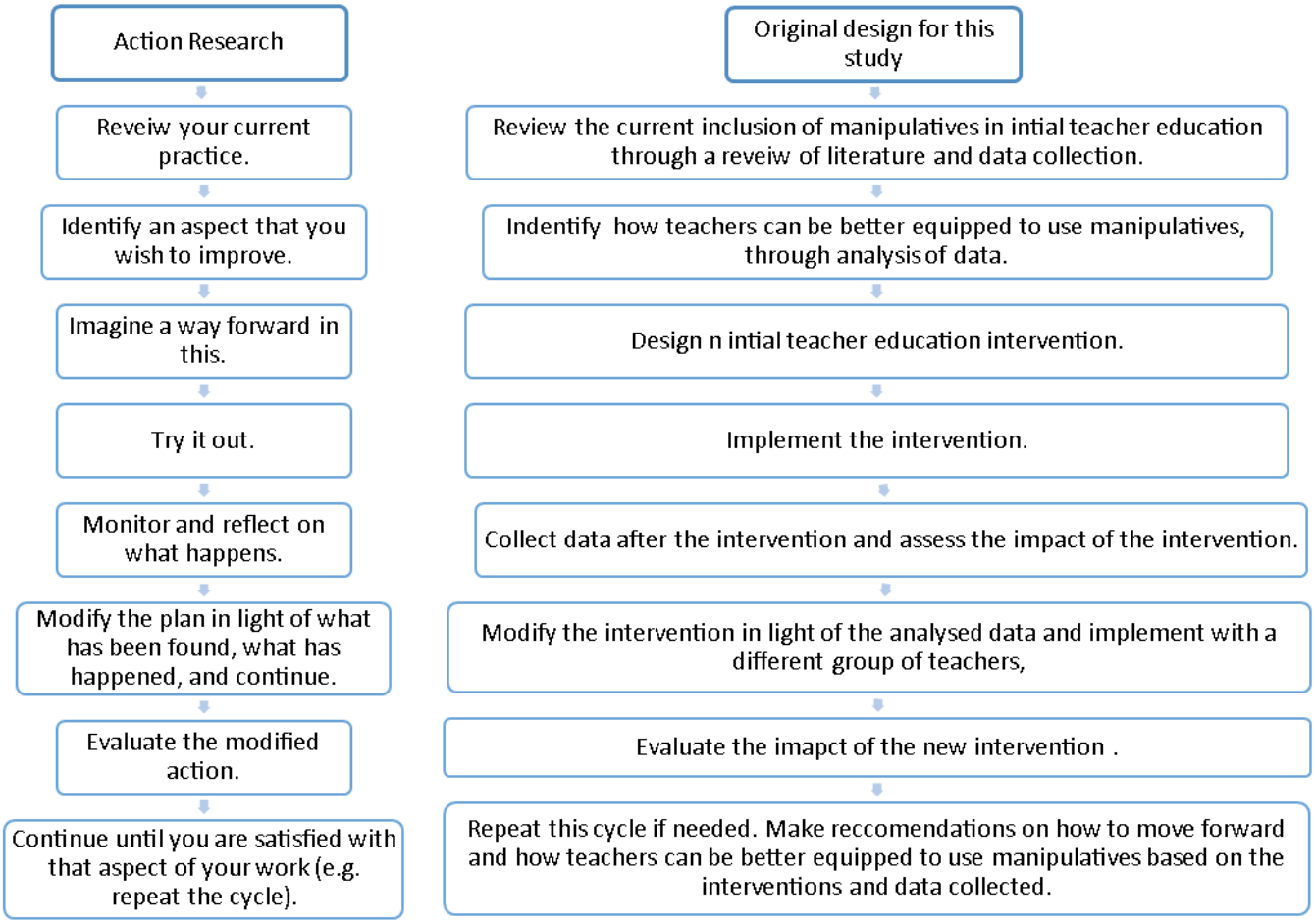
Appendix C

Domains of Mathematical Knowledge for Teaching model (Ball et al., 2008).



Appendix D

Comparison between model of action research (McNiff, 2002:71) and original design for this study.



Appendix E

Copy of survey questions.

Use of Manipulatives in Mathematics Education

Survey for teachers of mathematics (primary or secondary). By participating you are agreeing to your answers being used in a Masters dissertation, however all data will be anonymised. If you are happy to be interviewed and/or take part in some action research then please leave your contact details. In this case your answers will be linked to your further contribution, however in the write-up, if referred to, you will be given a pseudonym.

The research is on the use of manipulatives in mathematics education and the role initial teacher education courses have in equipping teachers to use manipulatives effectively.

Thank you in advance for taking part!
* Required

1. What is your primary role in mathematics education? *

Mark only one oval.

- ◯ Secondary Mathematics Teacher
- ◯ Primary Teacher (where I teach mathematics as part of the syllabus)
- ◯ Secondary Teacher Educator (e.g. University Tutor)
- ◯ Primary Teacher Educator (e.g. University Tutor)
- ◯ Secondary Teaching Assistant
- ◯ Primary Teaching Assistant
- ◯ Secondary Trainee Teacher
- ◯ Primary Trainee Teacher
- ◯ Other: _____

2. How many years have you been in your role (by the end of this academic year)?

Mark only one oval.

- ◯ 1
- ◯ 2
- ◯ 3
- ◯ 4
- ◯ 5
- ◯ 6
- ◯ 7
- ◯ 8
- ◯ 9
- ◯ 10
- ◯ 11-15
- ◯ 16-20
- ◯ 20+

3. Have you heard of the term 'manipulatives' used in the context of mathematics education?

Check all that apply.

- [ ] Yes
- [ ] No

4. If 'yes' can you briefly describe in your own words what 'manipulatives' are?

Use Of Manipulatives In Your Teaching

For the remaining questions the term 'manipulatives' is used to describe physical objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children in hand-on learning of mathematics.

A selection of possible 'manipulatives' used within a mathematics classroom.

5. How often do you use manipulatives as part of your regular activity with the following year groups? *

Mark only one oval per row.

	N/A	Never	Very Rarely	Sometimes	Often	Most of the time	Always
Primary Infant School (Reception - Year 2)	◯	◯	◯	◯	◯	◯	◯
Primary Junior School (Year 3 - Year 6)	◯	◯	◯	◯	◯	◯	◯
Year 7	◯	◯	◯	◯	◯	◯	◯
Year 8	◯	◯	◯	◯	◯	◯	◯
Year 9	◯	◯	◯	◯	◯	◯	◯
Year 10	◯	◯	◯	◯	◯	◯	◯
Year 11	◯	◯	◯	◯	◯	◯	◯
Year 12	◯	◯	◯	◯	◯	◯	◯
Year 13	◯	◯	◯	◯	◯	◯	◯
With Adults (i.e. University)	◯	◯	◯	◯	◯	◯	◯

6. Which of the following mathematical areas are you most likely to use manipulatives to teach? (You can choose more than one area)

Check all that apply.

- ☐ Basic Numeracy
- ☐ Logic and reasoning
- ☐ Algebraic manipulation
- ☐ Shape, space and geometry
- ☐ Data and statistics
- ☐ Problem solving
- ☐ Calculus
- ☐ Ratio and Proportion
- ☐ Investigations
- ☐ None
- ☐ Option 11

Other: ☐ _____

7. Which of the following manipulatives have you used in a classroom within the last 2 years?

Check all that apply.

- [ ] Dice
- [ ] Counters / beads
- [ ] Blocks
- [ ] Tangrams
- [ ] Beads on string
- [ ] Fraction tiles
- [ ] Cuisenaire rods
- [ ] 3d shapes
- [ ] Other: _____

8. When you have used manipulatives in the past, what were the main reasons behind this decision?

Check all that apply.

- [ ] I was expected/told to by my school/department/mentor
- [ ] I wanted to try something new
- [ ] I thought it would engage the children/ be fun
- [ ] I thought it the most effective way to teach a topic
- [ ] I thought it would appeal to children's different learning preferences
- [ ] It was an easy lesson for me to plan
- [ ] It would help children consolidate their learning
- [ ] It was a useful way for me to check understanding visually
- [ ] Other: _____

9. What stops you using manipulatives more in your lessons? (You can select more than one answer)

Check all that apply.

- [ ] Availabilty of resources and objects
- [ ] The hassle and time wasted sourcing, distributing and collecting in resources
- [ ] They are a distraction and can lead to behavioural issues
- [ ] I do not feel sufficiently trained in how to use manipulatives most effectivly
- [ ] There is not enough time/oppertunity allowed within the scheme of work
- [ ] I don't see much benefit to their use
- [ ] I prefer to teach in different ways
- [ ] The more advanced maths gets the more tricky it is to relate abstract concepts to physical maniuplatives
- [ ] Manipulatives should be a last resort when children are unable to solve a problem in their head or on paper alone
- [ ] My department/school would not approve
- [ ] I already use manipulatives enough
- [ ] Other: _____

10. If it were more possible, would you like to use manipulatives more in your role?

Mark only one oval.

- ◯ Yes
- ◯ No
- ◯ Maybe

11. In your opinion what role does the use of manipulatives have in the teaching of mathematics today? (You may wish to comment on the use with different age ranges, how important or helpful they are/are not etc.)

_____

_____

12. What route of initial teacher training did you take (or are you taking)?

Mark only one oval.

- ◯ PGCE
- ◯ Schools Direct
- ◯ Teach First
- ◯ Apprenticeship
- ◯ Assessment only
- ◯ I am an unqualified teacher
- ◯ Other: _____

13. During your initial teacher training year/s, how were you taught to use manipulative effectively in your own teaching?

Mark only one oval.

- ◯ A stand alone session/s on using manipulatives
- ◯ Using manipulatives was embedded into sessions across the year
- ◯ From my school mentor/ in school CPD
- ◯ I learnt through trial and error, by trying things out
- ◯ I didn't learn how to use manipulative effectively
- ◯ N/A
- ◯ Solely through observing experienced teachers
- ◯ Online course/session
- ◯ Other: _____

14. How might initial teacher education courses equip teachers more to be able to use manipulatives more effectively?

Mark only one oval.

- A stand alone session/s on using manipulatives
- Using manipulatives embedded into sessions across the year
- In school CPD
- I don't think it needs to more of a focus
- Observing experienced teachers
- Online course/session
- Directed discussion with peers
- Trial and error, trying new things and then analysing
- Other: _____

15. How equipped did you feel to teach using manipulatives by the end of you initial teacher training?

Mark only one oval.

	1	2	3	4	5	6	7	8	9	10	
Not equipped at all											Confident in how to use manipulative effectively

16. How equipped to use manipulatives in teaching do you feel now?

Mark only one oval.

	1	2	3	4	5	6	7	8	9	10	
Not equipped at all											Confident in how to use manipulatives effectively

17. If you had the chance, would you value additional training on how to use manipulatives more effectively?

Mark only one oval.

- Yes
- No
- Maybe

Thank you so much for taking part in my research!

19. If you answered yes above, please leave your email address or mobile number. This will not be passed on to anyone else nor used for any other purpose than to contact you regarding research for this dissertation.

_____

Appendix F

Copy of semi-structured interview questions.

1. **Background**

- Can you please describe how you became a maths teacher?
- How would you describe your view of mathematics and teaching mathematics?
- Can you describe the way in which you learnt mathematics?

2. **What are teachers' perceptions of the role of manipulatives in secondary mathematics education?**

- Do you agree/disagree with this definition of a manipulative? Would you add anything?
  'physical objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children in hand-on learning of mathematics'
- What role do manipulatives have in your school and in your own teaching? Do think this is typical?
- Have you seen a change during your teaching in the use of manipulatives? Who do you think drove this change?
- Do you think manipulatives are helpful in teaching mathematics in secondary and if so how?
- Do you think they are/would be beneficial to all or specific pupils?
- What are the limitations of using manipulatives?

3. **In what ways and to what extent are secondary mathematics NQTs equipped to use manipulatives within the classroom?**

- Tell me about you experiences with manipulatives up to your NQT year?
  To what extent do you think your initial teacher education equipped you to use manipulatives?
  How confident did you feel (by the end of your NQT year) and do you feel now in using manipulatives?
- Has the manipulatives you used changed/increased/decreased?
- How might you use manipulatives to model 1 ¾ divided by ½ ?

4. **i) How can I effectively integrate the use of manipulatives into an initial teacher training course?**

- Are there particular aspects/areas of using manipulatives that you feel lacking in knowledge?
- What education have you had in the use of manipulatives and would you consider this effective?
- What do you think effective education in the use of manipulatives would look like?

Appendix G

- *Summary of interview data.*

	Definition of manipulatives	**Role of manipulatives in own teaching**	**Usefulness of manipulatives**
Teacher A	The visual representation is the most important. Physical counters are easier to manipulate, move around and practice physically before deciding on the representation. A protractor is not as it's too specific. Virtual manipulatives are counted as the same if everyone has access.	School: mastery KS3, theory is used every few lessons. When you introduce a topic first to get that physical representation Emphasis on exploration before defining a concept. Don't teach ks3 much, when I did, I was getting cubes out a lot more. I have with yr 11 bottom set if needed. If a pupil doesn't get it it's I turn to Ms. I don't plan to use them though. Lessons have been planned for KS3 including the use of manipulatives. Teachers who are more confident in their practice and more reflective tend to use them more. Feel that those who have taught longer are not as reflective so find it harder to see the purpose. Newly trained are more susceptible to new ideas, not stuck in ways or proven.	Manipulative scene: there has been a real push in the last few year. I hadn't heard the word manipulative until the last 2 years. There needs to be more of a shift in my own teaching to using manipulatives more. Time and curriculum restraints. Kids learn in different ways. I need to use more confident to use them as a tool. If I don't explore them more I won't know where to use them better. Generally viewed for younger children it is more appropriate. Even though I think some of KS 4 would benefit. Would be beneficial if I knew how to use them. I don't have the time to invest in how to use it with ks4. Why force them to use it if they can they can do symbolically
Teacher B	Don't agree with the appeal to the senses as Ms don't need to appeal to smell or taste and maybe not even touch though not sure, otherwise agree with definition given.	HOD bought loads of manipulatives but we don't feel we have had training on them or explored how to use them and feel really unconfident. Manipulatives are seen as a tool for children who can't access written method. But I think this should change as I have done seen them used better. KS2 only use for low ability. Physical, pictorial, abstract (mastery) SOW doesn't allow for it. Want to bring in new ideas.	Can't say yes or no [to whether useful]. Can see potential from the recent online training I've done but haven't seen in practice.
Teacher C	Virtual manipulatives count as well as physical. We are using the virtual instead. Teacher led, or on white board. Compass' etc are not manipulatives as it can't be played and rearranged etc.	Used across the school but generally used with lower ability or support aids. Embedded in yr 7 and 8. An hour each week spent discussing which manipulatives would be relevant for the lessons ahead. The HOD leads the hour. All had mastery training . In a session with	They have a real place, especially at the start of topics or at the start of the concept. But don't have enough time. At the start of the year introducing Ms it was hard and tedious, by the end not all pupils need the directive. We needed to train the students in how to use them. Had

	it's equipment instead. It has a specific purpose	mastery trainers they went through 3 or 4 manipulatives though was very quick.	them out every team meeting for the year. Its still not natural for the teachers but yr 7s seem to be more natural using now.
Teacher D	Happy with the definition given. A couple of years ago I came across different problems involving not being able to buy different manipulatives. But started using online manipulatives. I didn't use to think these were manipulatives but now also think it could be something on a screen. Virtual manipulatives were able to be used by the pupils as children had their own chrome books. Disadvantages depends to type of manipulatives. Algebra tiles and prime factor tiles you could use in the same way. Blocks have different sizes and the children appreciate the size difference. They have more impact.	Some teachers would regularly use but only as intervention or with low sets. Tried over the last few years at using them with higher order concepts. Starting point, then moving on to pictorial and then abstract. Now more fluid and used to dpo into not just as a starting point. Not so linear now, more fluid. Taken quite a bit of time to get teachers confident. Manipulatives were not part of any of the teachers TE. Only teachers who'd had confidence were those with training of lower attaining. Linking more to pictorial/abstract. A couple of activities L had used but didn't feel she had got the most out of it. So that's what sparked her interest. Attended events on cuisinnaire and algebra tiles Was able to come up with M example to model the fractional question given.	M can be useful to all pupils but they are often seen with lower order maths because the ground work hasn't' been done. Equally as useful for high attainers to develop their own conjectures and explore these.
Teacher E	I agree with the definition, though wouldn't have written it like that I think it works.	WE are starting to teach maths mastery though at the moment there doesn't seem to be much buy in from the dept and so it seems like a tick box exercise. I want to use them as have heard good things about how they help to develop deeper knowledge but don't have the time to invest myself.	I have read and heard they can be really useful. I think maybe It depends on the kind of teacher you are. Maybe you need to be a bit creative and not afraid if things going wrong. They use them in primary school and I think pupils seem to enjoy maths more at primary so maybe that is a factor.

	Limitations of manipulatives	**Manipulatives and Initial Teaching Year/CPD**	**Manipulatives in a ITY**
Teacher A	Time restraints learn how to use, not having them, syllabus, training, exam drive, expectation from SLT.	CPD: Not at school, bar modelling had one session on. Mastery day – not helpful. 3 or 4 session delivered by the mastery lead- quite prescriptive to a question. Videos on the maths mastery website – more theory on where they come from ITY: Not at all. If you had a strong pedagogy you could probably relate Equipped after ITY: 0 Equipped now: 3	Aspect feel lacking in: Higher tier content. Less basic. Statistics. Thoughts on M in ITY: More present in teacher training. Embedded in school CPD time discussing ideas. PGCE something at the start of the year a couple of sessions. Make sure they are on the table looking at different topics. Embedded throughout.
Teacher B	I didn't know how to use them or training or opportunities on how t use them. I think that SLT would just think I was playing. Or if I take the risk and use but it doesn't pay off then I will have to explain myself when they don't do as well in a test.	ITY: Weren't mentioned at Uni, talked about 3d shapes that was it. Wasn't a trend then. CPD: 2 white rose training and one other online course, 10 min from HOD. Everything else has been my own research. (maths hub) info out there 0, now still not that confident, but I have optimism. I feel about a 5 as I have potential. I feel like If I got observed they would think I am playing.	Aspect feel lacking: feel I would like to know double sided counters and algebra tiles ITY needs to observe great quality teaching in manipulatives. This isn't happening because schools are so flaky on how to use it.
Teacher C	Time constraint (handing out and collecting in every lesson) and pressure of exams. A lot of effort to change all the lesson plans I have developed.	ITY: During PGCE didn't come across at all. PGCE year, 1st placement has mixed ability sets and used at a support tool with hose struggling or SEN. More targeted. The only reason we use them this year is because they are defined within the new curriculum. CPD: 1 training day, and expert come in and work with the department on 2 days and weekly meetings when you discuss in advance and one online session. Day was effective, someone coming in was OK but no more effective than our group meetings	Aspect feel lacking: Seen used effectively in number and none of the other areas. In ITY : opportunity to discuss it, work collaboratively and learn with/from each other.. Find different ways to use it for this topic. Maths specific training . Embedded throughout not just as a standalone. Practical example of teacher being used, or videos to discuss. Explore the misconceptions etc.

		I wasn't at all confident, now we have had that hour a week, feeling more confident.	You can't assume the kids will just get it, if they don't then it can just be more confusing and the teachers need to be ready for them
Teacher D	Budget to have sufficient quantities. Time – 60 mins isn't enough time. SOW and curriculum needs to be developed to allow time for the use of manipulatives. I.e. not recovering things so much.	CPD: None given to us. Done my own. Did a lot of online CPD, i.e. use of algebra tiles. ITY: None at all - if you had a strong pedagogy you could probably relate yourself but this was left to you. Didn't come up in NQT – 0, maybe a 3 now.	Lacking in: Regularly finding new things, now using with higher attaining but still using for number, algebra, proportion. Not really shape and space. Understand why they don't because a lot of schools don't use routinely. Giving opportunity to explore using them. I.e. giving research and asking them to design a lesson would give them awareness of what they are.
Teacher E	Although it might pay off it seems a lot slower than just teaching the procedure and there is so much to fit in the GCSE and A level that we have to rush through. It's probably why the students often struggle when in yr 12, because they only have surface knowledge due to timings.	ITY: manipulatives not mentioned at all, I think because the tutor and my mentor didn't ever use them and therefore didn't think about it. It's a shame really because those teachers wont use them either and the cycle goes round. CPD: The trust school is in has told us to use maths mastery but no really supported us in that. W e had a day of CPD but it was so fast and didn't give us a chance to explore the manipulatives, just lots of example questions how to use them. I cam away not very confident. 0 then, 2 now.	Encourage student teachers to work together to research or design lessons with manipulatives. I am a fan of getting student teachers to team teach.