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BARGAINING MICROFOUNDATIONS FOR PRODUCTIVITY DISPERSION

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Bargaining Microfoundations for Productivity Dispersion

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Abstract

This paper analyses the implications of bargaining between buyers and sellers on the competitive outcome in a homogeneous good industry. Bargaining creates a competitive equilibrium in which some inefficient sellers coexist with efficient ones leading to productivity dispersion. Rival cost uncertainty then creates an endogenous distribution of productivities which shrinks if rival numbers grow - exactly paralleling current empirical findings. The ability to bargain results in list price dispersion but transaction price uniformity. The bargaining model is not observationally equivalent to Bertrand pricing with product differentiation as positive mark-ups are predicted as idiosyncratic seller cost shocks become small. This and other predictions of the bargaining model of competition are assessed against the empirical evidence. The insights are robust to search costs with a nonsequential search strategy where a pure strategy (no sales) price equilibrium is found. Further, the results extend to markets without bargaining if sellers post price matching guarantees.

Key Words: Bargaining, List prices, Transaction prices, Price-matching guarantees, Productivity Dispersion, Trade Liberalization

JEL Classification Numbers D24, D43, L11, F12

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1 Introduction

Bargaining is widespread, particularly in business to business markets. But does it have an appreciable effect on market outcomes or does competition through take it or leave it prices provide an acceptable reduced form approach? This model hopes to show how bargaining can provide a micro-foundation for the Bertrand paradox, productivity dispersion, and how rival numbers can lead to 'increased competitiveness' and hence alter the equilibrium level of productivity dispersion.

Many authors have confirmed that efficient firms don't undercut the inefficient leading to large and persistent productivity dispersion across a whole host of industries.[1] This evidence is recounted in the survey paper provided by Bartelsman and Doms (2000) for both low tech industries such as steel and beer, and high tech industries like telecoms and automobiles. The same survey paper notes that individual firms move only very slowly (over years) within the productivity distribution. Thus some inefficient firms survive for long periods of time and so productivity dispersion appears to be an equilibrium phenomenon. Why efficient firms don't drive inefficient firms out of the market and thus trigger average industry productivity growth is the subject of this paper.

Empirical evidence links greater rival numbers with lower productivity dispersion and increased productivity growth. That more rivals leads to lower productivity dispersion is shown by Waring (1996). As efficient firms drive out the inefficient, industry productivity grows and this has been empirically linked to rival numbers (Nickell (1996)). The tolerance efficient firms show the inefficient is costly to the economy; Disney et al. (2003) have noted that inefficient firms going bust with the business transferring to their efficient peers explains up to 90% of Total Factor Productivity (TFP) growth and half of Labour Productivity (LP) growth in their sample. So the act of driving out inefficient firms can explain up to 90% of productivity growth and this growth engine seems to be triggered, in part, by greater rival numbers. Thus explanations for the phenomenon of productivity dispersion are significant and must also explain the link with rival numbers.

The explanation I propose has two parts. Firstly, though sellers set list prices, they are not completely credible as take it or leave it prices: buyers are able to seek reductions, and firms leave themselves flexibility to match competitors' prices when a buyer demands. I would argue that this is true of most business to business markets, and in some retail markets (insurance, cars, estate agency). This causes list and transaction prices to rise above the Bertrand floor of marginal cost pricing: if a rival undercuts a seller through a posted list price, the list price becomes an outside option and so the transaction price at any firm will not exceed that price

[1] By large dispersion I refer to Bartlesman and Doms (2000) who report studies indicating 85^{th} to 15^{th} total factor productivity percentile ratios between 2:1 and 4:1 within various 4-digit SIC industries.

level. There is therefore little incentive for a consumer to go to one firm as opposed to any other. But this then removes the benefits of infinitesimally small price cuts and so breaks the Bertrand logic - ergo prices rise above marginal cost.[2] This first insight, akin to the well-known effects of price-matching guarantees, is demonstrated simply in Section 2.

The second part of the explanation is that sellers face uncertainty as to exactly how efficient their rivals are - that is, how low a price can they sustain if pushed? This uncertainty as to costs is a key part of a number of seminal analyses of industry dynamics (Jovanovic (1982), Hopenhayn (1992)).[3] This uncertainty creates a trade-off: price high, allowing inefficient firms to compete, and share a large pie with many players; alternatively price low, hopefully below rivals' costs, and take a larger (though uncertain) share of a smaller pie. Of course, if the list price is not lowered sufficiently then the rival will be able to match and profits will have been lost. The expected benefit of taking the latter approach is shown to increase as the number of potential competitors increases. This is because the information contained in the fact that a firm can compete at a given price level becomes negligible as rival numbers rise. This has the effect of lowering the expected proportion of rivals who can compete at low prices. Thus, unless there are sufficient competitors, or a firm's productivity advantage is sufficiently great, the efficient firms price high, inefficient firms stay in business, and so there is productivity dispersion.

This model thus proposes a concrete link between firm numbers and productivity dispersion. As driving inefficient firms out raises industry productivity over time, I am therefore also providing a link between rival numbers and productivity growth. The results are in fact broader still. The effect of bargaining is that outside options will be matched. In markets where sellers set price matching guarantees the same effect applies and thus my results hold. The model is described in Section 3 and the results derived in Sections 4 to 6.

Others have provided insightful reasons linking numbers of rivals to *individual firm* productivity growth - but if firms can set prices (that is if tight capacity constraints do not preclude the firms lowering their prices leaving a Cournot style situation), these alone do not provide an explanation for equilibrium productivity dispersion *across an industry*. That is, these explanations do not appear to explain why once a firm becomes efficient it tolerates the inefficient and fails to take their business in a Bertrand fashion.[4] If each firm's products in an industry are assumed to be sufficiently differentiated then productivity dispersion can be explained - though this explanation provides testably different predictions. Specifically, as the correlation in sellers'

[2] If a buyer leaves a seller then returns, she conveys bad news about her outside options - thus, in our model, buyers can't yo-yo between sellers to force prices down to cost.

[3] See also Athey et al. (2004), who consider collusion between sellers with private costs in which the costs are re-drawn from a distribution each time period.

[4] These findings include: better incentive schemes for managers the greater the number of competitors (Nalebuff and Stiglitz (1983) for explicit contract incentives and Meyer and Vickers (1995) for implicit incentives through career concerns); firms responding to a greater threat of bankruptcy under strong competition (Schmidt (1994)); and the incentives to conduct R&D (Aghion et al. (2002)).

costs increases (cost shocks become largely common) then this bargaining model of competition predicts that margins should converge to a strictly positive level above marginal cost, whereas Bertrand pricing with minor product differentiation would predict margins collapsing to zero.[5] Klemperer (1987) notes that switching costs in consumer markets may provide another explanation for productivity dispersion. I note that productivity dispersion can happen, and indeed may be expected, even in homogeneous goods markets, where firms bargain over the terms of trade.

Section 7 considers the robustness of the findings to a number of possible model extensions: bargaining costs; industry dynamics; and search costs plus a nonsequential search strategy so that not all buyers view the full set of list prices. Section 8 discusses the model's predictions and compares it to the available empirical evidence. Section 9 notes that this paper is tantamount to a new microtheoretic mechanism by which trade liberalization induces a reduction in productivity dispersion and hence productivity growth. Section 10 concludes.

To my knowledge this analysis is unique in combining oligopoly bargaining with list prices and firm cost heterogeneity. Bester (1988) considers a model with a continuum of sellers who bargain with buyers. The firms do not post list prices and so cannot hope to drive out any of their rivals. Bester (1993) considers oligopoly with bargaining and list prices, but all firms have the same cost by assumption at any given quality level and so the question of productivity dispersion cannot be addressed. The aim of understanding productivity dispersion as the result of competition is shared by this paper with others such as Melitz (2003), Hopenhayn (1992) and Jovanovic (1982). However these approaches consider continuums of firms each of which is a price taker or competing monopolistically. The approach here is a partial equilibrium one in which oligopoly pricing incentives are explicitly considered. Bargaining is key in my model as it transforms rivals' list prices into outside options for the buyers. Many have considered straight list pricing, with no bargaining, but with search costs. Perhaps Janssen and Moraga-Gonzalez (2004) is the most recent example, Varian (1980) and Stahl (1989) are two seminal contributions. These papers predict a mixed strategy pricing equilibrium which can be interpreted as a model of sales. In addition these models have identical firms assuming away productivity dispersion and its analysis. Bargaining leads to a pure strategy pricing equilibrium even with search costs so I do not predict random periods of price cutting in a business to business context.

2 A Simple Example: Bargaining and List Prices

The model's key features are apparent from one retail market which does involve the ability to bargain: the purchase of new and nearly new cars. Suppose that you are in the market for a car of a given brand. The manufacturers' websites will often allow a potential customer to see the

[5] Section 6.1 discusses in detail.

list prices of new and nearly new cars of any desired model in dealers across a desired geographic area. In addition the website will include colour, mileage information and sometimes also photos for nearly new cars. Thus there is no shortage of information available on list prices.[6] Suppose that a potential customer discovered the car of make and mileage they desired available in two locations: in their home town at £30,000 say and also in a much more distant town at the more reasonable £25,000. Casual empiricism would suggest that instead of trekking immediately to the more distant town, the customer would go first to the convenient garage and bargain over the price, quoting the low price as an outside option and not discussing any prices above the £25K level. Note immediately that we are including the realistic fact that just because a firm list prices lower than a rival, does not mean that it suddenly receives all the first customer visits. I propose that a bargained gap between list and transaction prices is a potentially better descriptor of business to business transactions as opposed to consumer facing transactions.

This short section therefore explores the simple economics of oligopoly bargaining: if sellers post list prices but then haven't got the credibility to refuse to bargain with buyers, then the simplest single-stage game model of competition leads to prices strictly above marginal cost and below the monopoly level and allows productivity dispersion to exist in equilibrium. This is not the outcome of collusion.

To see this, suppose that there are 2 sellers with known (by all) marginal production costs of $c_1 < c_2$. All buyers have known valuation v for one unit of the good and the population of buyers is normalised to 1. The sellers post list prices which are seen by the buyers.

Benchmark: Bertrand Pricing Firm 1 posts a list price of c_2 and only firm 1 is active. That is prices fall to marginal cost. There is no productivity dispersion.

List Prices and Bargaining Now suppose sellers post prices but then bargain with buyers. The bargaining process is standard Rubinstein bargaining with outside options (Shaked and Sutton (1984)). The bargaining will result in the parties splitting the pie between them unless there is a better outside offer in which case it is matched. Thus if firm 1 were to post some very low price p_1, then firm 2 would be forced to match this price in bargaining with its buyers and so all buyers would get p_1 from either seller - unless firm 2 couldn't compete at such a price in which case it drops out. Firm 2 (the inefficient firm) will never be able to drive firm 1 out and so will list price to not remove profits from the market. Firm 1's pricing decision thus becomes a choice between posting above and transacting at $\frac{v+c_1}{2}$ - the freely bargained level - or dropping list prices to c_2. In the former case both sellers compete (if firm 2 is not so inefficient that it can't compete even at this

[6] For a specific example visit www.skoda.co.uk and conduct used car searches. The specific website is http://www.skoda.co.uk/skoda3g/content/showroom/UsedCars.aspx?page=ucl1.aspx
A consumer can generate lists of all second hand cars sold through skoda dealers with list prices, mileage, colour and other details often including a photograph.

level, so $c_2 < \frac{v+c_1}{2}$) and buyers transact at $\frac{v+c_1}{2}$. In the latter case seller 2 is driven out and only seller 1 competes.

The profit tradeoff between these two causes seller 1 to price low if her cost advantage is sufficiently great, or to tolerate firm 2 and price high otherwise. So

$c_1 \leq \frac{4c_2-v}{3}$	Price at c_2	Marginal cost pricing - one seller active
$\frac{4c_2-v}{3} < c_1 < c_2$	Price at $\frac{v+c_1}{2}$	Price between marginal cost and monopoly level - both sellers active; market shared

Thus sellers being unable to refuse to bargain over list prices can explain transaction prices in excess of marginal cost and yet below the monopoly level without any collusive practice or intent.[7] Productivity dispersion exists as long as the efficient seller isn't 'too' efficient. As yet, leaving aside the need to formally define the model, I haven't explained why increasing rival numbers should lower the extent of productivity dispersion. This paper shows that such a prediction arises if sellers are uncertain as to their rivals costs.

3 The Model

N selling firms compete in a local market. The sellers' marginal costs are private information: each seller knows its own costs but not those of its rivals. Each seller's cost is independently and identically distributed on connected support $\Omega \subseteq \mathbb{R}_+$ according to continuous density function $f(\cdot)$.[8] This cost distribution is common knowledge. I require that $f(\cdot) > 0$ on its support. This cost density function may be formed from nationwide announcements or statistics while sellers in any given local market may have cost distributions slightly different from the national mean.[9]

The firms all sell a homogeneous good, and all buyers in the market desire one unit of the good which they all value at v. The population of buyers is normalised to 1. Once a buyer selects a seller, there is a process of bargaining in which the seller reveals her cost level to the buyer; thus I avoid the complications of asymmetric information bargaining games.[10] I require v to lie in the closure of Ω.

The model then proceeds as follows. At time $t = 0$ the sellers set their list prices, not knowing their rivals' current costs. These are real list prices - a firm which offers a list price of p is committed to selling the good for a price no higher than p. (This rules out fake prices in which

[7] Note that bargaining has the implication of creating behaviour very similar to all sellers advertising price matching guarantees.

[8] We use $F(\cdot)$ to denote the distribution function.

[9] For example, in the UK www.whatcar.co.uk provide estimates of 'fair', that is cost-reflecting, transaction prices for cars of any make, age and mileage. These are national estimates and may or may not reflect exact price and cost availability in any given local market.

[10] If sellers can delay their responses then full revelation is achieved even without this assumption (Admati and Perry (1987)).

capacity constraints or other impediments are subsequently presented to the consumer). The buyers then see the entire set of list prices.[11] This assumption will be relaxed by the analysis of search costs in Section 7.3.

The buyer then undertakes a process of bargaining to select the selling firm and final price which will be paid. Key to this process will be the fact that the list prices offered by all sellers will form an outside option when bargaining with any given seller. Once agreement is reached, the price is paid and the good delivered. We will see that the results are unaltered in substance if we substitute price matching guarantees for the bargaining phase.

The bargaining game itself is a synthesis of standard outside option bargaining (explained simply in Muthoo (1999), and used by Shaked and Sutton (1984)), combined with an adaptation of the Stole and Zwiebel (1996) model of many to one bargaining situations. A buyer decides on the order in which he would prefer to bargain with the sellers whose list prices she holds. In the equilibrium of the model she will be indifferent between the sellers and so is assumed to randomise to break the tie.[12] The first seller is then approached. Upon the commencement of bargaining the seller's costs is revealed. The buyer can either buy the good at the quoted list price or begin a process of alternating offer bargaining with a minimal lag of $\Delta = 0_+$ between offers.[13] I suppose that the players have a common discount rate.[14] When the buyer receives an offer, her options are to (i) accept the offer and buy at the agreed upon price, (ii) reject the offer and go straight to her outside option (bargaining with the next seller on her list) with no delay or (iii) reject the offer and wait Δ time periods later before making a counter-offer to the seller. The seller has no outside option and will continue to bargain with the buyer (though potentially giving no ground) as long as the buyer wishes. In the event that a buyer decides to break off negotiations with this seller then I assume that she cannot return to the same seller and resume bargaining, though she can return to purchase the good at the list price. This captures the idea that a return would carry information about bargaining failures elsewhere and so the same prices would no longer be available. It is this assumption which prevents the (arguably unrealistic) outcome whereby the buyer yo-yos between sellers and so forces prices down to marginal cost once two or more quotes are sourced.[15]

[11] The ease of seeing the list prices captures situations in which list prices can be ascertained by looking on the internet, browsing or ringing up a store. I would argue that business to business trade is well described in this way as individuals are employed to search initially quoted list prices.

[12] This randomisation captures buyers breaking the tie between sellers by unobservables such as first impressions of salesmen or ease of use of sales literature.

[13] That is I use the bargaining solution arrived at in the limit as time becomes continuous $(\Delta \searrow 0)$.

[14] This assumption is for convenience of exposition only. The analysis and results can be trivially altered to allow for differing discount rates and so unequal splits of the bargained pie.

[15] I have assumed, as is standard in outside option bargaining (Shaked and Sutton (1984)), that there is no delay in taking the outside option during the bargaining phase. For a discussion of the robustness of the model to this assumtpion see Section 7.1.

It is well known (see, for example, Muthoo (1999)) that the game between a given seller and the buyer has a unique subgame perfect equilibrium which can be paraphrased as: the players reach immediate agreement and split the pie evenly unless the buyer's outside option is binding (more generous), in this case the outside option is either matched if the seller can afford to, or the buyer moves to another store. This would therefore be the equilibrium between the last seller in the buyer's list and the buyer. Therefore, by induction the same equilibrium holds with the first seller.[16] We will see that the best list price will equal the outside option simplifying the buyer's task.

Note that buyers do not automatically go to the store with the lowest list price as a process of bargaining will ensue which will leave the buyer no worse off with any of the sellers. Perhaps realistically therefore, an ε price reduction does not win the whole market.

4 The Market Equilibrium

The list pricing decision of a seller with cost c, one of N rivals, is captured by the function $p_l(c, N) : \Omega \times \mathbb{N} \to \mathbb{R}_+$. Note that, at best, a seller with cost c expects to bargain down to a transaction price of $\frac{v+c}{2} < v$.[17] This fact implies that any list price above this level must be fully revealing of the seller's costs:

Lemma 1 *Any equilibrium list price $p_l(c, N) > \frac{v+c}{2}$ must be fully revealing of the seller's costs.*

Proof. Suppose not so that there can exist sellers with costs $c_1 < c_2$ who set a pooled list price of $p_l \gneqq \frac{v+c_1}{2}$. Seeing a list price of p_l buyers can only infer that they will be able to bargain down to $\min\left(p_l, \frac{v+c_2}{2}\right)$. Thus a very inefficient seller with costs $\kappa \in \left(\frac{v+c_1}{2}, \min\left(p_l, \frac{v+c_2}{2}\right)\right)$ would survive and take some business. Such a seller can exist as $f(\cdot) > 0$ on its support. But seller 1 could costlessly evict such a firm by list pricing at $\frac{v+c_1}{2}$ and would gain by an increase in demand. A contradiction to this equilibrium. ■

Thus list prices $p_l(c, N)$ are either set below $\frac{v+c}{2}$ or are fully revealing so that all buyers understand they are being offered a transaction at $\frac{v+c}{2}$ at this seller. We can proceed with any equilibrium, but any equilibrium other than list pricing at or below $\frac{v+c}{2}$ requires substantial buyer understanding and processing to allow inversion of the list price function. Thus, for expositional simplicity, I assume that sellers don't set prices greater than the maximum they can hope to charge: $(v$ + own cost$)/2$. This assumption could be removed by including the buyer inversion of $p_l(c, N)$ to extract $\frac{v+c}{2}$.

But this implies that all sellers in the market will set their list price at or below the freely bargained level of (v + seller's cost) / 2. This then implies that when a buyer visits a seller she

[16] The proof is identical to Stole and Zwiebel (1996, Theorem 2).

[17] The Rubinstein or Nash bargaining pie sharing result.

will be unable to bargain the seller down beyond her list price unless there is explicitly available a lower list price elsewhere. Summarising we have the result that:

Lemma 2 *Each buyer's outside option is the minimum posted list price.*

We can therefore turn to the question of whether seller A, with cost of c, would ever seek to list price below the freely bargained level of $\frac{v+c}{2}$. Though the standard Bertrand price cutting logic breaks down (Section 2) there can still be a benefit to lowering list prices as they force firms with marginal costs at or above this level to exit the market for this product and so the smaller pie is shared between fewer players.[18] Exactly how many players are left will, of course, be a random variable as sellers' costs are private information. The first tool necessary is to note the perhaps expected result, that the list price chosen increases weakly with the seller's cost.

Lemma 3 $\frac{d}{dc} p_l(c, N) \geq 0$

Proof. Suppose that a seller knew the costs and hence list price choices of its rivals. The seller will set a list price at $\frac{v+c}{2}$, or perhaps lower if she can remove some rivals. Thus the list price choice will be drawn from a finite set $\left\{p^t, p^{t+1}, p^{t+2}, p^3, ..., p^T, \frac{v+c}{2}\right\}$ where p^i denotes the highest price at which only i firms remain active; $p^t := c$ so that a price at her cost leaves t firms active and $p^i < p^{i+1}$.[19] If p^i is the optimal price choice then $\frac{p^i - c}{p^j - c} \geq \frac{i}{j}$ for all $j \neq i$. But if the seller's costs were to rise to $\widetilde{c}$ then we have $\frac{p^i - \widetilde{c}}{p^k - \widetilde{c}} > \frac{p^i - c}{p^k - c} \geq \frac{i}{k}$ for all $k < i$ as $p^i > p^k$. Therefore the seller would choose a weakly higher price if her costs were higher. Therefore the seller's best response list price is weakly rising in her cost for every realisation of other costs amongst her rivals. That is raising (weakly) the list price is a dominant strategy in the face of rising costs and so $p_l(c, N)$ must be weakly increasing in c also. ■

Therefore the lowest posted list price gives the buyer's outside option and hence transaction price (Lemma 2) and a given seller will only be pivotal in setting the transaction price if she transpires to have the lowest marginal cost amongst the sellers (Lemma 3). Given the bargaining (price matching) nature of competition this is enough for the seller to set her list price for the eventuality that she is the lowest cost seller in the market. This is because at a list pricing equilibrium a more efficient rival will be willing to accept a lower industry transaction price in an attempt to evict inefficient rivals (as evidenced by the proof of Lemma 3) - hence there is no incentive to undercut more efficient rivals even if their list price choice were known. The next lemma states this formally and a second algebraic proof is provided in the appendix.

Lemma 4 *The list price equilibrium has each seller setting price to maximise their expected profits conditional on being the lowest cost seller.*

[18] Business is not taken off sellers whose marginal cost is below the list price posted as they respond to their buyers in the bargaining phase.

[19] To simplify we suppose all rivals have different costs. The proof is trivially extendable if any subset of rivals have equal costs.

The remainder of this section now proves the following proposition which gives the pricing equilibrium in the market.

Proposition 5 *A firm of marginal cost c, one of N potential rivals, will choose a list price $p_l(c, N)$ such that*

$$p_l(c, N) = \arg \max_{p \in \left[c, \frac{v+c}{2}\right]} [p - c] \cdot \frac{1 - (1 - G(p))^N}{G(p)} \quad \textit{where } G(p) = \frac{F(p) - F(c)}{1 - F(c)}$$

The transaction price in the market will be at the minimum realised list price.

Thus suppose that seller A say sets a list price of $\widetilde{c}$ and finds herself as having the lowest cost in the market. In this case only firms with marginal costs below $\widetilde{c}$ will be able to compete. Given that A is the lowest cost firm available, all other sellers must have costs distributed according to $G(\widetilde{c}) = \frac{F(\widetilde{c}) - F(c)}{1 - F(c)}$. Let $\alpha = G(\widetilde{c})$ be the probability that a randomly picked firm can compete at a price of $\widetilde{c}$, given that A (with costs of c) is the lowest cost seller in the market. Let L be the random variable giving the (total) number of firms with marginal costs below $\widetilde{c}$ - that is including seller A. Thus, conditional on being the lowest cost seller in the market the expected value of setting a list price of $\widetilde{c}$ is $(\widetilde{c} - c) E\left(\frac{1}{L}\right)$. It is the expected value of $\frac{1}{L}$ which needs to be determined.

Lemma 6 *If a firm has marginal costs below $\widetilde{c}$, and L is the realised number of firms with marginal costs below $\widetilde{c}$ out of the N in the market, then setting $\alpha = G(\widetilde{c})$,*

$$E\left(\frac{1}{L}\right) = \frac{1 - (1 - \alpha)^N}{\alpha N}$$

Note that, given a firm is low cost, $\lim_{\alpha \to 0} E\left(\frac{1}{L}\right) = 1$ and $\lim_{\alpha \to 1} E\left(\frac{1}{L}\right) = \frac{1}{N}$ as would be expected. This result, proved in the appendix, is key in the firm's pricing decision as low cost firms ultimately compare getting a smaller share of a high profit market (high prices) with getting a larger share of a low profit market.

The economic implications of Lemma 6 arise from the fact that $E\left(\frac{1}{L}\right)$ decreases at a slower pace than $\frac{1}{N}$ as N, the number of competitors, grows. This provides an increasing incentive to price low in an attempt to drive inefficient rivals out as the number of competitors increases. This is illustrated by the following example:

Example 7 *Suppose firm A sees the probability a randomly chosen rival could survive at transaction prices of $\widetilde{c}$ as $\frac{1}{2}$.*
When competing in a market of 3 firms, then no price reduction leads to A serving $\frac{1}{3}^{rd}$ of the market whilst a reduction to $\widetilde{c}$ leads to A expecting to serve 0.58 $\left(= E\left(\frac{1}{L}\right)\right)$ of the market.
Now suppose that entry or some other cause results in A competing in a market of 6 firms. Now no price reduction results in A serving $\frac{1}{6}^{th}$ of the market, only half the size it was serving when

one of three. But a reduction of price to $\widetilde{c}$ results in A expecting to serve 0.33 of the market. This is greater than half of 0.58. Thus the price reduction has become relatively more attractive to A as the firm numbers have risen.

As Lemma 6 is so key to the way in which oligopolies with list prices and bargaining function, we explore the intuition behind the lemma before proceeding. The lemma follows as A knows her cost and knows that she can compete at a price of $\widetilde{c}$ - otherwise she wouldn't be considering posting it. But then the random variable L has an expectation raised above αN. That is, as far as A is concerned it is the variable $L-1$ which is a Binomially distributed random variable with mean $(N-1)\,\alpha$ and variance $(N-1)\,\alpha\,(1-\alpha)$. Using the normal (denoted $\mathcal{N}$(mean, st.dev)) approximation to the binomial we therefore have that

$$\frac{L}{N} \sim \mathcal{N}\left(\frac{N-1}{N}\alpha + \frac{1}{N}, \sqrt{\frac{(N-1)\,\alpha\,(1-\alpha)}{N^2}}\right) \tag{1}$$

Now note that, unsurprisingly, as the number of rivals N increases, the variance of $\frac{L}{N}$ declines (as $N \geq 2$). However what is key is that as N increases the *mean of* $\frac{L}{N}$ *declines also.* This is because the information contained in the fact that A can compete at $\widetilde{c}$ has an increasingly negligible effect on $E\left(\frac{L}{N}\right)$ at large N. Therefore, when firm rivals are low in number then $\frac{L}{N}$ has high variance around a large mean. By contrast, when rival numbers are large $\frac{L}{N}$ has low variance, and so is concentrated, around a low mean. The inverse of this random variable is therefore expected to be large when firm numbers are large and decline as firm numbers decline. This is the content of lemma 6 which explores $E\left(\frac{1}{L/N}\right)$ but exactly without using the normal approximation.

The tools are now in place to prove Proposition 5.

Proof of Propostion 5. We have noted that the list price must lie in $\left[c, \frac{v+c}{2}\right]$. We have noted that the list price will be chosen to maximise expected profits conditional on the seller being pivotal: that is having the lowest marginal cost in the sample (Lemma 4). The proposition then follows from Lemma 6. ■

Corollary 8 *Price setting firms will post list prices strictly above marginal cost.*

Proof. Let us denote the objective function of the seller, derived from Proposition 5, as $\pi\,(p)$. The first order condition for profit maximisation evaluated at a list price of c is then

$$\lim_{p\to c}\frac{d\pi}{dp} = \lim_{p\to c}\frac{1-(1-G\,(p))^N}{G\,(p)} = \lim_{p\to c} N\,(1-G\,(p))^{N-1} = N > 0$$

That is a seller will always prefer to post a list price strictly above her marginal cost of c. ■

Hence prices lie above marginal costs due to the ability to bargain with outside options - inefficient firms therefore survive, share the market, and so we have productivity dispersion.

Thus we have a very straightforward new solution to the Bertrand paradox which does not require any search costs or capacity constraints.[20] Secondly, this model predicts list price dispersion, but with uniform transaction prices. We therefore note in passing that there is thus no a priori need to explain (list) price dispersion by invoking buyer heterogeneity such as zero vs. positive search costs (Stahl (1990)), or informed vs. uninformed consumers (Varian (1980), Salop and Stiglitz (1977)).

Price Matching Guarantees If there was no bargaining phase, but sellers posted list prices and price matching guarantees, then clearly Lemma 2 would immediately hold. The proof of Lemmas 3, 4 and 6 are unaffected. Therefore proposition 5 holds but with the list price now lying anywhere in the range $p \in (c, v]$. Thus all the results of the paper follow trivially to markets with price matching guarantees.

5 Rival Numbers and Productivity Dispersion

Section 4 provided an explanation for productivity dispersion: sellers list price above marginal cost and inefficient sellers have positive market share, though at a uniform transaction price. Importantly the uncertainty as to rivals' costs creates a link between each firm's pricing decision and the number of rivals. We have noted that the link between firm numbers and productivity parameters is well established empirically. This section explores what the link is in this bargaining model of competition. We first begin with the following Lemma whose proof is in the appendix.

Lemma 9 *For any distribution function F, if $c_1 < c_2$ then $\frac{1-(1-F(c_1))^N}{1-(1-F(c_2))^N}$ is increasing in N.*

This lemma then allows us to establish the main result in this section.

Proposition 10 *The list price selected by a firm of marginal cost c, denoted $p_l(c, N)$, is monotonically declining in the number of rivals N.*

This proposition begins to highlight how competition, measured as the number of rivals, has the implications on firm behaviour commonly observed.

Proof. Proposition 5 implies that

$$[p_l(c,N) - c] \cdot \frac{1 - (1 - G(p_l(c,N)))^N}{G(p_l(c,N))} \geq [p - c] \cdot \frac{1 - (1 - G(p))^N}{G(p)} \; \forall p \in \left(c, \frac{v+c}{2}\right] \quad (2)$$

[20] Spulber (1995) has noted that basic Bertrand price setting when firms have private costs will result in prices slightly greater than marginal cost. However, Bertrand pricing would predict that only one firm, the lowest list pricing one, survives. This would imply that there should be no productivity dispersion at all.

Now consider a higher number of potential rivals, $\widetilde{N} > N$. We wish to show that $p_l\left(c, \widetilde{N}\right) \leq p_l(c, N)$. This is true if

$$[p_l(c,N) - c] \cdot \frac{1 - (1 - G(p_l(c,N)))^{\widetilde{N}}}{G(p_l(c,N))} > [p - c] \cdot \frac{1 - (1 - G(p))^{\widetilde{N}}}{G(p)} \quad \forall p > p_l(c,N) \tag{3}$$

which follows if

$$\frac{1 - (1 - G(p_l(c,N)))^{\widetilde{N}}}{1 - (1 - G(p))^{\widetilde{N}}} > \frac{p - c}{p_l(c,N) - c} \cdot \frac{G(p_l(c,N))}{G(p)} \quad \forall p > p_l(c,N) \tag{4}$$

But we note from (2) that if $\widetilde{N} = N$ then (4) is satisfied with $>$ replaced by $\geq$. The desired result then follows from Lemma 9. ■

We have shown that all active firms will lower their prices (weakly) as the number of rivals increases. Recall that the market is shared between all remaining active sellers as bargaining allows those with higher list prices to lower their transaction prices. It quickly follows that the maximal productivity dispersion in an industry is reduced as the number of rivals increases. To allow meaningful comparative statics, the results are expressed conditional on the most efficient realised seller in the market having a cost of $\underline{c}$. Thus:

Corollary 11 *Conditional on the most efficient realised firm having cost $\underline{c}$, the maximal extent of productivity dispersion in an industry decreases as the rival numbers increases.*

Proof. The maximal extent of productivity dispersion, conditional on $\underline{c}$ is$[\underline{c}, p_l(\underline{c}, N)]$: this is the maximal range of active sellers' costs. By Proposition 10 this range decreases as the number of rivals, N, increases. ■

Thus a benefit of augmenting one's view of buyer-seller interaction by introducing bargaining is that an explanation is provided for the observed link between productivity dispersion and firm numbers. As firm numbers increase, firms price more aggressively in seeking to limit the number of competitors and so the maximal extent of productivity dispersion declines.[21] For the (price setting) sellers remaining, the market is shared at a common transaction price.

6 Cost Distributions and List Prices

In this section we explore which sellers will act to lower their prices given a particular distribution of costs amongst potential sellers. The first result notes that sellers at the expensive end of the cost distribution have no incentive to lower their prices:

Proposition 12 *If a seller has cost such that $f(p)$ is decreasing in p for all $p > c$ then this seller's list price is set at the upper level of $\frac{v+c}{2}$: that is the freely bargained level.*

[21] For a discussion of how these results extend to a dynamic model see Section 7.2.

Proof. As $f(p)$ is decreasing in p for $p > c$ we must have $F(p)$ being concave on $[c, \sup \Omega]$. This implies that $F(p) \geq F(c) + (p-c) f(p)$. But this is equivalent to noting that the quotient $\frac{p-c}{G(p)}$ is increasing in p. The proof now follows by contradiction. Thus suppose that the optimal list price (p_l) for this seller is strictly less than $\frac{v+c}{2}$. It then follows that

$$[p-c]\left\{\frac{1-(1-G(p))^N}{G(p)}\right\} < [p_l - c]\left\{\frac{1-(1-G(p_l))^N}{G(p_l)}\right\} \; \forall p \in \left(p_l, \frac{v+c}{2}\right]$$

But $\frac{p-c}{G(p)} > \frac{p_l-c}{G(p_l)}$ and so we must have

$$(1-G(p_l))^N < (1-G(p))^N$$

But $p > p_l \Rightarrow G(p_l) < G(p)$ and so we have a contradiction. ■

Proposition 12 follows as profits become monotonically increasing in price. A seller would like to set their list prices as close to the freely bargained level as possible. The only reason to set a list price below this level is that some rivals might be evicted. Thus consider a list price p below the freely bargained level. If the list price is raised by ε then ε more will be made on every unit sold. However, there is a chance that a rival who would have been evicted survives to take some business. The probability of no such rival existing is $\{1-[G(p+\varepsilon)-G(p)]\}^{N-1}$ which is approximately equal to $[1-\varepsilon g(p)]^{N-1}$. Expanding the bracket, the probability of such a rival existing is therefore $\frac{\varepsilon(N-1)}{1-F(c)} f(p)$ which is declining in p. Therefore when comparing raising prices by 2ε as opposed to ε, the gain per unit sold doubles, while the probability that a rival survives grows but at a decreasing rate as $f(p)$ is declining. Hence larger prices rises are preferred to small ones. Finally note that a price rise above cost is always desired (Corollary 8), and so the list price moves to the boundary: the freely bargained level.

To derive greater insight into which firms will ever try and evict their inefficient rivals, suppose that the density of sellers' costs is a continuous unimodal function. Thus we can divide the support of f, Ω, into two regions, (Ω_+, Ω_-). The density function is increasing on Ω_+ and decreasing on Ω_-. Hence the distribution function (F) is convex on Ω_+ and concave on Ω_- with a point of inflexion lying at the boundary. Finally note that associated with any point $c \in \Omega_+$, there is a unique connected range of points with infimum $\omega_c \in \Omega_-$ such that the chord between $(c, F(c))$ and $(\omega_c, F(\omega_c))$ is tangential to the distribution function at ω_c.[22] Analysis of the pricing decision leads to the following result, proved in the appendix.

[22] The existence of ω_c is clear as given a point c in Ω_+, one can always draw a line and allow its gradient to shrink until it just touches the curve $F(.)$. As F is convex on Ω_+, and increasing in Ω_-, this point of tangency must lie in Ω_-. For uniqueness note that if there were two such points then they would both lie in Ω_- and would contradict the concavity of F here unless all the intermediate points also lay on the same chord. (That is the gradient of $F(\cdot)$ is a constant over a range and exactly equals the gradient of the chord).

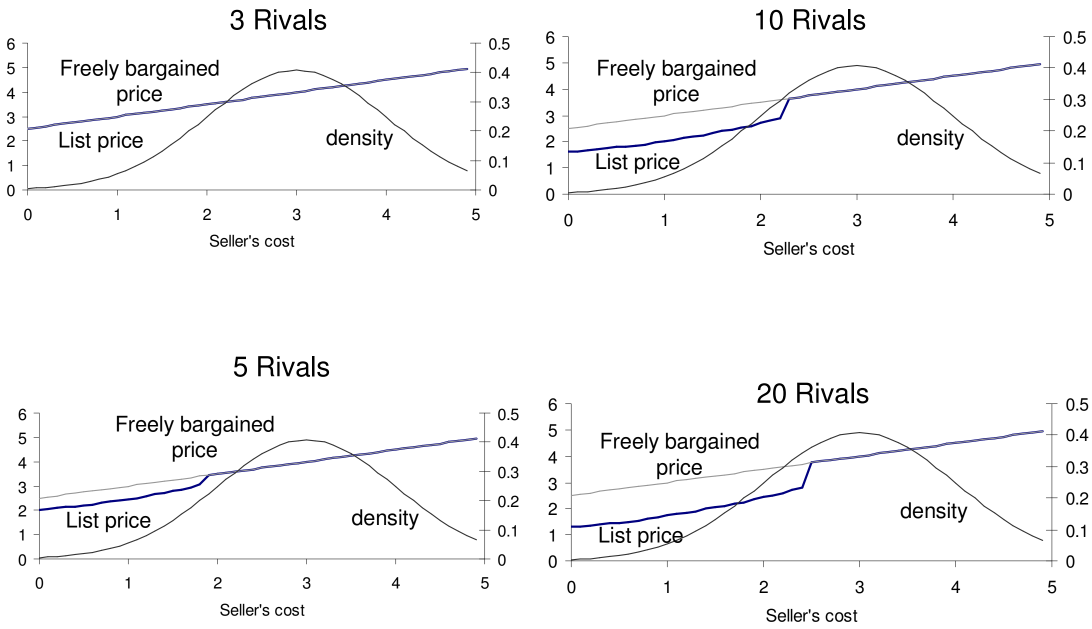

Figure 1: Numerical analysis in which sellers' costs are distributed normally according to $\mathcal{N}(3,1)$ adjusted by truncation at 0 and 5. Thus $v=5$ and $\Omega=[0,5]$. The freely bargained price level is given by $\frac{v+c}{2}$, the optimal list price is $p_l(c,N)$. The rival numbers, $N\in\{3,5,10,20\}$. The left axis gives the price and the right the probability density.

Proposition 13 *Suppose that the distribution of sellers' costs is a unimodal function. If a seller with cost c lowers her list price below the freely bargained level, then her cost must lie in Ω_+ and the chosen list price will lie in (c,ω_c).*

Figure 1 depicts these insights through a numerical example. Note that increasing rival numbers lowers the list prices of the most efficient firms and so would act to reduce productivity dispersion. Specifically, in Figure 1, no firm seeks to lower its prices below the freely bargained level with only 3 competitors, the most efficient lower their prices with 5 rivals and the extent of price reductions grows as the rival numbers increase.

6.1 Cost Correlation and Equilibrium Markups

This section explores how the equilibrium markups predicted by this bargaining model of competition vary with the correlation between rivals' costs. To analyse this question we suppose that each seller's marginal costs are given by $c=\kappa+\lambda\varepsilon$. κ captures the common component, known to all when list prices are set.[23] The industry cost asymmetry arises from the idiosyncratic cost

[23] κ may itself be random - but this doesn't alter any of the analysis as the common cost component is commonly known.

shock ε where $E(\varepsilon) = 0$ and ε is drawn from density $h(\cdot)$ with distribution $H(\cdot)$. The industry is parameterised by $\lambda \geq 0$ which captures the extent of correlation between seller's costs: In the extreme of $\lambda = 0$ all sellers have the same costs and so the correlation is perfect.

Under bargaining the following proposition shows that as the sellers' costs become increasingly correlated, markups converge on a positive amount. This is because if sellers are certain that almost all rivals will have costs much as they do, then there is little benefit in pricing low and trying to drive any of the rivals out. Thus list prices rise to the freely bargained level. This stands in marked contrast to the predictions of Bertrand pricing models which would see markups vanishing as cost correlation grew. The result is also in contrast to the economy modelling of Melitz (2003) in which markups remain constant for any distribution of costs by the assumption of CES utility functions for consumers. The formal result, captured in the following proposition, is proved in the appendix.

Proposition 14 *Suppose individual seller costs are given by $c = \kappa + \lambda\varepsilon$ where ε is private information to the seller and is a random variable independently drawn from a known distribution with $E(\varepsilon) = 0$. Then, as λ falls to 0, the bargaining model of competition has markups converging to $\frac{v-\kappa}{2} > 0$ while they vanish with Bertrand pricing.*

A leading explanation for productivity dispersion is that sellers set prices in a Bertrand fashion but are differentiated slightly. These prices are assumed to be credibly offered as take it or leave it prices. The result above offers an empirical prediction which differs from a model of Bertrand pricing. In an industry in which costs are closely correlated so that most variation is common across sellers then the bargaining model of competition predicts markups above marginal cost for the active sellers. This is not the case with Bertrand pricing in which markups above marginal cost would be predicted to vanish.

7 Robustness of the Model to Extensions

We have studied a model of bargaining in oligopoly markets and noted that it can explain a link between productivity dispersion and rival numbers. I believe that this static model provides insight into a more complicated dynamic world. This section seeks to explore how extensions to the model would affect the results. Section 8 will explore the empirical evidence for the model.

7.1 Little Disadvantage to Pricing High

A key feature of a market with bargaining is that the incentive to price low is severely weakened as compared to the Bertrand case without any tacit collusion or threat of punishment in future rounds. This is because buyers are aware that any price will be matched and so will be happy to randomise as to the sellers they purchase from. But a market may contain some buyers, say proportion μ, who dislike bargaining or incur costs in taking an outside option. The measure μ of consumers will go to the lowest list pricing seller. The optimal pricing for these μ consumers can be found from the techniques of Holt (1980)[24] and sellers have an incentive to undercut their rivals - though unsure of their costs. However, for the other $1-\mu$ buyers the analysis here stands. Thus the sellers' objective function is a weighted average of undercutting rivals to (with some probability) win μ of the market versus pricing to optimally evict rivals for $1-\mu$ of the market. The insights here therefore stand and become increasingly dominant as μ becomes small. I would argue that μ is indeed likely to be small as in normal business to business interactions buyers may maintain negotiations with more than one seller to ensure that a switch to a rival can be made quickly, and with minimal cost, in the event of price disagreement.

Further, the analysis stands when embedded in a dynamic context. Consider the repeated version of the game, formally introduced in Section 7.2 following, where costs do not remain constant. Suppose in addition that buyers did incur some delay cost in taking the bargained outside option. The seller would, in this setting, still exactly match the lowest posted list price in bargaining or withdraw for the period when unable to do so. Any other strategy would mean the seller was never visited when she wasn't the most efficient firm and she still shares the market when she is the most efficient firm. Therefore the analysis described in the static game is the equilibrium of behaviour in a dynamic setting.

7.2 From Static to Dynamic Analysis

We note that in this model initially quoted list prices may be fully revealing of current costs while rival cost uncertainty was used to provide a link between productivity dispersion and rival numbers. Does the model therefore stand in a dynamic setting? Before addressing this point directly, two points are in order. Firstly, firms often sell multiple products and multiple versions of each product; quotes are then provided for a given buyer's specific request. It would be difficult for a seller to monitor initial quotes for every combination of product, and the underlying costs would vary over time as capacity utilisation changed. Thus the dynamic concerns expressed may be weaker than at first appears. Secondly, Section 7.3 will introduce search costs whereby buyers solicit initial quotes from a subset of the sellers. (Evidence exists that such a model is particularly suited to estate agency for example). Here even if a seller learns about his rivals' current costs for one contract, he can expect to be drawn against a different subset of sellers next time and so the rival cost uncertainty effects described remain.

Nevertheless, we turn our attention to an extended model in which the market interaction is repeated. If, using the assumptions in Athey et al. (2005) to explore dynamic collusion,

[24] Spulber (1995) explicitly considers Bertrand pricing with asymmetric costs but assumes a downward sloping demand curve. The model here with buyers having valuation v is more amenable to analysis using the methodology Holt (1980) proposes.

costs are redrawn each period, then the analysis provided here extends trivially. A weaker assumption is that information as to costs today provides information as to the likely costs tomorrow. Costs are unlikely to be entirely constant as they depend upon, among other things, changing unobservables such as capacity utilisation, staff learning by doing, staff motivation and undisclosed input deals with suppliers. Thus we need to explore the effect of increased information as to rivals' costs upon the predictions.

Suppose that sellers are drawn from known cost classes $\mathcal{C}_1, ..., \mathcal{C}_S$. Each period the N sellers spread over cost classes $\mathcal{C}_i$ have costs redrawn from distribution F_i; we assume that there are K_i such sellers and sellers do not exit the game entirely if they are unable to compete in any period. Thus we suppose that list prices have identified, with no uncertainty, which cost class a rival is drawn from, though there is some variation around the mean on a period by period basis. The broad analysis described above remains robust. To see this suppose that a seller with cost c from class $\mathcal{C}_1$ considers setting a list price of $\widetilde{c}$ to maximise its profits from that period of competition. As before it would not be optimal to undercut a more efficient firm as they benefit more from any reduction in the number of rivals (Lemma 3). Hence pricing is set to be optimal in the event that c is the lowest cost seller. If L is the number of firms who can compete at a price of $\widetilde{c}$ then $L-1$ is a random variable: it is the sum of a number of independent binomial random variables. Using the normal approximation to the Binomial we have

$$L-1 \sim \mathcal{N}\left(\left[\sum_{i=1}^{S} K_i G_i\left(\widetilde{c}\right)\right] - G_1\left(\widetilde{c}\right), \left[\left(\sum_{i=1}^{S} K_i G_i\left(\widetilde{c}\right)\left(1-G_i\left(\widetilde{c}\right)\right)\right) - G_1\left(\widetilde{c}\right)\left(1-G_1\left(\widetilde{c}\right)\right)\right]^{\frac{1}{2}}\right)$$

where $G_i\left(p\right)$ is the distribution of costs in class $\mathcal{C}_i$ given that all costs are greater than c. To allow comparative statics as N (the number of rivals) changes, suppose that the proportion of sellers in each cost class remains constant so that $\frac{K_i}{N}$ is constant for all i. In this case the random variable $\frac{L}{N}$ again has a variance declining in N and a mean which falls in N also. Thus the previous analysis stands: sellers will price more aggressively the greater the number of rivals.[25]

Over the longer term sustained low pricing can encourage permanent exit (subject to the option value of staying in). Analysis of temporally strategic pricing in this situation would require a model of dynamic learning in which price setting is used to try and deduce a rival's cost class, whilst the rival potentially sold below cost for a time to try and disguise her true cost. How this might play out is a topic for ongoing research.

[25] Suppose, in the above setting, that sellers who cannot compete in a given period exit and that sellers are not intertemporally strategic so that prices are set to maximise profits in each period. Then intitially, with N sellers, pricing is aggressive and so a number of sellers will exit. As the number remaining declines the aggressiveness of the pricing declines (Proposition 10) so that the number of sellers does not generally drop to only one. (Section 2 shows that productivity dispersion can remain even when costs are known). Hence our model is compatible with productivity dispersion *persistence*.

7.3 Search Costs

Hitherto we have assumed that buyers receive quotes from all of the sellers. Others (Varian (1980), Janssen and Moraga-Gonzalez (2004)) have considered the effect of there existing a subset of the buyers with some positive search costs so that these buyers only request a very few quotes. Varian notes, in a model without any bargaining, that this results in a mixed strategy pricing equilibrium and he interprets this, quite appealingly, as a model of 'sales'. However, in business to business interactions the prediction of sales is more controversial. This section shows that in our model where sellers can bargain after posting list prices, a pure strategy price equilibrium does exist even if buyers search nonsequentially and incur search costs: There are no 'sales'. In addition more search leads, broadly, to lower productivity dispersion as might be expected.

This subsection adds further to the realism of the model and is, for example, particularly well suited to the estate agency market (analysed in the UK by Office of Fair Trading (2004)). The stylised facts of this market in the UK are that: (i) around half of buyers request initial quotes from more than one seller; (ii) those who do shop around and negotiate extract a 14% reduction on average; (iii) there is no evidence of 'sales' rather sellers using set initial price points. The bargaining model of competition with search tallies well with these results.

We use the Janssen and Moraga-Gonzalez non-sequential search framework and augment our model by supposing that buyers request $q \in \{1, 2, ..., N\}$ initial quotes with probability $h(q)$. The determination of the sellers' objective function proceeds much as before. Thus list prices (now denoted $p_l(c, N; h)$) can be found to be monotonic in costs (Lemma 3) and sellers price to maximise their profits conditional on having the lowest cost of the sample they are drawn in as it is never optimal to undercut the transaction price choice of a more efficient rival (Lemma 4). Given a buyer picks q quotes then a given seller is picked with probability $\frac{q}{N}$.[26] In a sample of q picked firms, Lemma 6 gives the expected share of the market a seller receives for a given price conditional on having the lowest cost. Thus analogously to Proposition 5 a pure strategy price equilibrium exists and we have:

Proposition 15 *A seller with a cost of c will choose a list price of $p_l(c, N; h)$ such that*

$$p_l(c, N; h) = \arg \max_{p \in \left(c, \frac{v+c}{2}\right]} [p - c] \sum_{q=1}^{N} h(q) \cdot \frac{1 - (1 - G(p))^q}{G(p)} \quad \textit{where } G(p) := \frac{F(p) - F(c)}{1 - F(c)}$$

[26] For example, the probability I am picked out of N when the buyer seeks 3 quotes is given by:

$$\frac{1}{N} + \frac{N-1}{N}\frac{1}{N-1} + \frac{N-1}{N}\frac{N-2}{N-1}\frac{1}{N-2} = \frac{3}{N}$$

where the first term on the left is the probability I am picked first, the second term is the probability I am picked second and the third term is the probability I am not picked in positions 1 or 2 but picked 3^{rd}.

This formulation allows us to establish an important caveat to the importance of rival numbers: it is not the number of rivals per se, but rather the number of rivals any firm may be competing against for a given seller, which dictates pricing strategy. Thus

Corollary 16 *The list price choice depends upon the quote sizes which occur with positive probability and not on the rival numbers N.*

This is immediate from Proposition 15 as the price which maximises the right hand side is independent of N if $h(N) = 0$. Thus though larger N may mean a given seller is sampled less frequently, she prices to maximise her profits in the event that she is sampled.

It might be suspected that the more searching conducted by the population, the lower the expected prices and the less the productivity dispersion in the market. In this vein we return to the search models exemplified by Varian (1980) in which there exist a proportion λ of buyers who do not search and only request one quote, while $(1 - \lambda)$ of the population do request quotes from all the sellers in the market. In this case we note:

Proposition 17 *As λ declines, the equilibrium list price of a seller with cost c declines also.*

Thus the more consumers who search the whole market the lower the equilibrium price and thus the smaller is the productivity dispersion in the market. This conforms well with intuition as a seller drawn by one of the λ proportion buyers is guaranteed a sale while incentives to try and evict inefficient rivals only exist for $1 - \lambda$ of the population of buyers. So a large λ leads to sellers pricing as if they will be the only firm in their sample - they therefore price high. Given the empirical evidence provided by Disney et al (2003) that productivity growth is strongest when inefficient firms are removed by their efficient rivals, encouraging full search by buyers would seem to be an important tool in improving industry productivity. Likewise, one would expect productivity dispersion to be comparatively larger (other things - such as number of rivals - equal) in markets were a substantial subset of buyers did not request quotes from all the sellers in the market.

We now turn to the implications of increasing rival numbers for productivity dispersion. In general the message of this paper remains: increasing numbers of rivals causes sellers to lower their list prices.

Proposition 18 *If buyers search the whole market with probability $1 - \lambda$ and just request one quote otherwise, then list prices are weakly decreasing in rival numbers.*

Thus the insight we describe - that bargaining over list prices plus uncertainty as to rivals' costs provides a possible explanation for the observed link between increased rival numbers and reduced productivity dispersion - is robust to the introduction of search costs as described.

8 Predictions and Evidence

I hope this paper illustrates that, in theory, productivity dispersion and above cost pricing can be explained by firms uncertain of their rivals' costs bargaining with buyers over their list prices or initial price quotes. The unresolved question is whether this explanation is empirically important. This section describes the predictions the model makes and recounts some of the evidence which exists in support:

8.1 Model predictions

In any given homogeneous good market well described by this model (I argue business to business interactions especially), this model predicts that:

- Transaction prices differ from list prices for most sellers - the exception being the most efficient seller which sets the prevailing transaction price through its list price.
- There exists transaction price uniformity across sellers, subject to any search cost inspired limitations in the quotes sourced by buyers.
- There exists list price dispersion.
- There exists productivity dispersion.
- Productivity dispersion declines as rival numbers rise.
- Markups above marginal cost are positive even if sellers' costs are highly correlated.

There is a further prediction:

- Without any innate efficiencies of scale, larger markets should have larger firms which are more efficient on average.

To see this note that if sellers were all the same size then larger markets would see more rivals competing and so Proposition 10 implies that the more efficient sellers would price lower in the larger markets, drive out their inefficient rivals and so the average firm size will rise as will the average productivity (lower marginal costs).

8.2 Evidence

The evidence for productivity dispersion and its link to firm numbers was the starting point of this research. References were given in the introduction and this model is reported because it tallies with these findings. One further empirical link to report here concerns the deregulation of telecoms in the US between the 1960s and 1980s. Olley and Pakes (1996) show that the act of allowing private firms to connect to the public network came close to doubling the number of rivals with the result that low productivity plants exited rapidly and industry productivity grew - precisely as predicted here.

Next, the concomitant prediction linking greater market demand and larger more productive sellers has very recently received careful scrutiny in the U.S. ready-mixed concrete industry by Syverson (2004). The author is concerned with evaluating the effect of transport costs on productivity dispersion and to this end explores the *density* of demand throughout a region[27] as against the productivity dispersion of sellers. Demand density and market size are correlated - strongly if most demand actually arises from within the metropolitan area as opposed to evenly dispersed over the whole geographic market area. Syverson does find the link between demand density (and so I allege market size) and average productivity that this model describes. It is not my aim to displace Syverson's explanation for this - only to provide a second explanation which may work alongside. This bargaining explanation may have something to add as to the extent that building demand is concentrated in cities then though Syverson's spatial transport cost explanation for the results would be weakened, the list prices as outside option explanation given here would not be. Thus Syverson's finding that more dense markets have more efficient and larger sellers is compatible with this model.

As concerns the predictions involving transaction prices, it is a truism in economics that transaction (as opposed to list) prices are hard to observe. Perhaps this explains the paucity of published data concerning transaction prices. Nevertheless, there is some evidence for list prices differing appreciably from transaction prices. Firstly Gordon (1973) showed that list prices for capital goods (e.g. machines), when aggregated into a wholesale price index, moved much less flexibly than transaction prices derived from Census data. A similar conclusion is reached by Coleman (2003, pages 27,28) posted on the FTC website. The data used is confidential and is only identified as deriving from an industrial products industry. I have been unable to locate any published evidence as to whether there exists transaction price uniformity (subject to search costs) or not. However the Office of Fair Trading (2004) study on estate agency explicitly finds that some buyers do negotiate after requesting multiple quotes (§4.47/48), confirming the applicability of the bargaining model of competition in at least this market.

Finally the presence of list price dispersion is well documented generally (see the references in Joskow and Waterson (2004)). It has also been observed explicitly in the business to business sector which, it might be argued, is prone to bargaining: Abbott (1992) using manufacturing census data in the US.

[27] Proxied by construction workers per square mile.

9 A Microtheoretic Mechanism For The Benefits Of Trade Liberalization

The work we have already completed extends immediately to provide a microtheoretic mechanism by which trade liberalization can reduce productivity dispersion and hence increase productivity growth. Consider a homogeneous good industry in a closed economy consisting of N rivals. There may be quota limits restricting foreign firms, or tariffs which cause the foreign firms to be so inefficient they are unable to compete for sales. If these restrictions were lifted then the number of rivals in the industry will increase from N to $\widehat{N}$. But Proposition 10 makes clear that increasing the number of rivals in a market will push the equilibrium closer to a low list price level in which the efficient firms strive to drive out the inefficient firms as they attempt to share a less profitable market amongst fewer rivals. Thus our model would predict that productivity dispersion should fall (Corollary 11) leaving the more efficient sellers, and so industry productivity should rise after trade liberalization. What is more, this result would hold as long as the entering firms were of comparable efficiency with the home firms - they do not need to be more efficient. Also observe that the actual volume of demand supplied by external entrants would underestimate the positive effect of trade liberalization as much of the benefit would come from the competitive response of the native sellers.

The link between trade liberalization and productivity has some empirical backing in developing countries. Ferreira and Rossi (2003) provide such evidence for Brazil, Chand and Sen (2002) also in an analysis of Indian trade liberalization. Tybout (2000) provides a comprehensive survey of the impact of trade liberalization on manufacturing in developing countries. He notes that empirically trade liberalization is associated with rising efficiency levels, and protected industries exhibit heightened productivity dispersion - all strongly compatible with this model. This model therefore provides a microtheoretic foundation for why increasing rival numbers through trade liberalization may 'increase competitiveness'. This is not the only channel through which benefits to trade liberalization may flow as Melitz (2003) discusses.

10 Conclusion

The ability of buyers to bargain raises both list and transaction prices above the Bertrand level. The same is true of price matching guarantees. Uncertainty as to rivals' costs makes efficient firms price more aggressively as the number of rivals rises. Thus bargaining over list prices/initial quotes (or price matching guarantees) and rival cost uncertainty provides a new explanation for the link between larger competitor numbers and lower productivity dispersion which has been empirically determined. It also provides a new microtheoretic mechanism linking trade liberalization with productivity improvements. I offer this explanation as most applicable to business to business interactions in which bargaining is common place. The results are robust to nonsequential search strategies with search costs. The results are consistent with available evidence on productivity dispersion, list price dispersion and the gap between transaction and list prices. Further the results are not observationally equivalent to the predictions of product differentiation models.

I believe that this static analysis provides an insight into productivity dispersion which is robust to a dynamic viewpoint. In the long run entry by more efficient rivals would be expected (Jovanovic (1982)), and this would represent a route to substantial industry productivity growth. But in the medium to short term, empirical analyses such as Disney et al. (2003) indicate that industry productivity grows fastest when these new efficient firms displace their now inefficient rivals. This paper provides an explanation as to why this process might be so slow.

A Further Proofs

Proof of Lemma 4. This can be proved by using now standard tools as displayed by Holt (1980). We suppress the rival numbers and denote the pure strategy list price equilibrium by $p_l : \Omega \to [0, v]$. This is a weakly increasing function by Lemma 3. We can define a weakly increasing inverse correspondence $\gamma : [0, v] \to \Omega$. Consider seller i with cost c_i. Let $y = \min_{j \neq i} c_j$. The minimum other bid is then given by $p_l(y)$. Select any inverse function $\gamma : [0, v] \to \Omega$ such that $\gamma \circ p_l(c_i) = c_i$.[28] Thus seller i can set the transaction price by bidding a price $b_i < p_l(y)$ iff $\gamma(b_i) < y$. Let $H(y)$ be the probability distribution of the lowest cost amongst the $N-1$ other rivals with density $h(y)$. Finally let $L(x)$ be the number of (sufficiently low cost) sellers who can compete at a price of x - a random variable.

Seller i will not set the transaction price if she bids b_i with probability $H(\gamma(b_i))$. In this case profits may still be made and the expected profit to seller i is given by

$$H(\gamma(b_i)) \int_{y=\gamma(c_i)}^{\gamma(b_i)} (p_l(y) - c_i) E\left(\frac{1}{L(p_l(y))} \middle| c_{\min \neg i} = y \right) \frac{h(y)}{H(\gamma(b_i))} dy \tag{5}$$

where we have used the fact that the density of the minimum rival cost, conditional on it lying below $\gamma(b_i)$, is given by $\frac{h(y)}{H(\gamma(b_i))}$.[29] However, with probability $1 - H(\gamma(b_i))$ seller i will set the lowest list price and so makes an expected profit of

$$[1 - H(\gamma(b_i))] \int_{y=\gamma(b_i)}^{\sup \Omega} (b_i - c_i) E\left(\frac{1}{L(b_i)} \middle| c_{\min \neg i} = y \right) \frac{h(y)}{[1 - H(\gamma(b_i))]} dy \tag{6}$$

The bid b_i is chosen to maximise the sum of (5) and (6), and at an equilibrium $\gamma(b_i) = c_i$ and $p_l(c_i) = b_i$. If b_i is changed slightly then the probability of having the lowest list price versus

[28] We can use any such inverse as when two different costs set the same list price which transpires to be lowest posted price, it is irrelevant which one we label as having 'set' the transaction price and which one not - the transaction price and profits are unchanged.

[29] If $y \leq \gamma(c_i)$ then seller i will be unable to compete and fails to make any profits at all.

not is altered, but the expected profits in either case are unchanged - the market is shared at a transaction price arbitrarily close to b_i as the buyers bargain.[30] Thus the first order condition for optimality at an equilibrium collapses to

$$\int_{y=c_i}^{\sup\Omega} \frac{d}{db_i}\left(b_i - c_i\right) E\left(\left.\frac{1}{L\left(b_i\right)}\right| c_{\min\neg i} = y\right) h\left(y\right) dy = 0$$

which is the condition for maximising the expected profit conditional upon being the lowest cost operator as required. ■

Proof of Lemma 6. We immediately note that by definition, given that one firm is low cost, it calculates that considering the other $N-1$ firms

$$E\left(\frac{1}{L}\right) = \left(1-\alpha\right)^{N-1} + \frac{1}{2}\alpha\left(1-\alpha\right)^{N-2}\begin{pmatrix} N-1 \\ 1 \end{pmatrix} + \frac{1}{3}\alpha^2\left(1-\alpha\right)^{N-3}\begin{pmatrix} N-1 \\ 2 \end{pmatrix}$$
$$+\cdots+\frac{1}{n}\alpha^{n-1}\left(1-\alpha\right)^{N-n}\begin{pmatrix} N-1 \\ n-1 \end{pmatrix}+\cdots+\frac{1}{N-1}\alpha^{N-2}\left(1-a\right)\begin{pmatrix} N-1 \\ N-2 \end{pmatrix}+\frac{1}{N}\alpha^{N-1}$$

Now note that

$$\frac{1}{n}\begin{pmatrix} N-1 \\ n-1 \end{pmatrix} = \frac{1}{n}\frac{(N-1)!}{(N-n)!\,(n-1)!} = \frac{1}{N}\frac{N!}{(N-n)!n!} = \frac{1}{N}\begin{pmatrix} N \\ n \end{pmatrix}$$

and so we consider expanding $\frac{1}{\alpha N}\left[\alpha + (1-\alpha)\right]^N$ to give

$$\begin{aligned}\frac{\left[\alpha+(1-\alpha)\right]^N}{\alpha N} &= \frac{1}{\alpha N}\left\{\begin{array}{c} (1-\alpha)^N + \alpha(1-\alpha)^{N-1}N + \alpha^2(1-\alpha)^{N-2}\begin{pmatrix} N \\ 2 \end{pmatrix} \\ +\alpha^3(1-\alpha)^{N-3}\begin{pmatrix} N \\ 3 \end{pmatrix} + \cdots + \alpha^n(1-\alpha)^{N-n}\begin{pmatrix} N \\ n \end{pmatrix} \\ +\cdots+\alpha^{N-1}(1-\alpha)N + \alpha^N \end{array}\right\} \\ &= E\left(\frac{1}{L}\right) + \frac{(1-\alpha)^N}{\alpha N}\end{aligned}$$

which then gives the result. ■

[30] To spell out this step, note that if $p_l(c)$ is strictly increasing over an open set containing c_i then γ is continuous on this set and so the result follows as directional derivatives can be taken directly. Suppose $p_l(c)$ is constant over a range of costs containing c_i. By Lemma 3, given any constellation of rivals costs, c_i will set a weakly higher (lower) price than rivals with lower (higher) costs and so if c_i is not at the edge of the range of constant list prices then $p_l(c_i)$ will be optimal. Thus suppose that $p_l(c) = p_l(c_i)\,\forall c \in [c_i, c_i+\delta)$ for some small δ, might c_i wish to lower her bid? If p_l is strictly increasing on some set $(c_i - \delta^\dagger, c_i]$ for some small $\delta^\dagger$ then γ is continuous in this range and again directional derivatives can be taken so the result follows. Finally then suppose that p_l is constant on $\left(c_i - \delta^\dagger, c_i\right)$ and again on $(c_i, c_i+\delta)$. If $c_i's$ bid is lowered slightly then the probability of setting the transaction price is unaffected as the new bid is above those of lower cost rivals, but the profit conditional on winning is altered and so again the result holds.

Proof of Lemma 9. Recall that we have $c_1 < c_2$. Differentiating the quotient $\frac{1-(1-F(c_1))^N}{1-(1-F(c_2))^N}$ with respect to N gives an expression equal in sign to an expression which we denote by $S(c_1, c_2)$ where

$$\begin{aligned} S(c_1, c_2) = & -\left[1-(1-F(c_2))^N\right][1-F(c_1)]^N \ln(1-F(c_1)) \\ & +\left[1-(1-F(c_1))^N\right][1-F(c_2)]^N \ln(1-F(c_2)) \end{aligned} \qquad (7)$$

we are asked to show that $S(c_1, c_2) > 0$ when $c_1 < c_2$.

It is clear that $S(c_1, c_1) = 0$. Keeping c_1 fixed we have

$$\frac{dS(c_1, c_2)}{dc_2} = Nf(c_2)[1-F(c_2)]^{N-1} \underbrace{\left\{ \begin{array}{c} [1-F(c_1)]^N \left[\ln \frac{1}{(1-F(c_1))} - \ln \frac{1}{(1-F(c_2))}\right] \\ +\ln \frac{1}{(1-F(c_2))} - \frac{1}{N} + \frac{1}{N}[1-F(c_1)]^N \end{array} \right\}}_{\Sigma(c_1,c_2)}$$

We thus see that $\left.\frac{dS(c_1,c_2)}{dc_2}\right|_{c_2=c_1} =_{\text{sign}} \ln \frac{1}{(1-F(c_1))} - \frac{1}{N} + \frac{1}{N}(1-F(c_1))^N$. But note that the graph of $y = \frac{1}{N}\left[\ln \frac{1}{x} - 1 + x\right]$ is positive for $x \in (0,1)$, and equals 0 at $x = 1$, implying that $\left.\frac{dS(c_1,c_2)}{dc_2}\right|_{c_2=c_1} \geq 0$. That is, $\Sigma(c_1, c_1) \geq 0$. If we could show that $\Sigma(c_1, c_2) > 0$ for $c_2 > c_1$ then $\frac{dS(c_1,c_2)}{dc_2} > 0$ for $c_2 > c_1$ and thus as $S(c_1, c_1) = 0$ we would have shown that $S(c_1, c_2) > 0$ when $c_1 < c_2$ as required.

For this final part we note that

$$\frac{d}{dc_2}\Sigma(c_1, c_2) = \left[1-[1-F(c_1)]^N\right] \frac{f(c_2)}{1-F(c_2)} > 0$$

and therefore as $\Sigma(c_1, c_1) \geq 0$ and $c_2 > c_1$ we must have $\Sigma(c_1, c_2) > 0$ completing the proof. ■

Proof of Proposition 13. We see from Proposition 12 that any seller who lowers her list price below the freely bargained level cannot have cost lying in Ω_- and so will have cost $c \in \Omega_+$: the upswing part of the density function. Next note that if $p > c$ and both lie in Ω_+ then, by the convexity of F in this region, we must have $F(p) < F(c) + (p-c) f(p)$ which implies that $\frac{(p-c)g(p)}{G(p)} > 1$. This remains true in the region (c, ω_c) where ω_c is defined so that $\frac{(\omega_c - c)g(\omega_c)}{G(\omega_c)} = 1$. If p lies to the right of ω_c however and a fortiori in Ω_- then $F(p) \geq F(c) + (p-c) f(p)$, otherwise ω_c could be drawn from two disconnected ranges contradicting the concavity of $F(\cdot)$ on Ω_-. In this case $\frac{(p-c)g(p)}{G(p)} \leq 1$.

To complete the proof note that if a seller with cost c doesn't price at $\frac{v+c}{2}$ then they must be at an interior solution to the maximisation problem given in Proposition 5. The first order condition for p_l can then be written:

$$\frac{d}{dp}\left[(p-c)\frac{1-(1-G(p))^N}{G(p)}\right]_{p=p_l} = 0$$

$$\left\{1-(1-G(p))^N + \frac{(p-c)g(p)}{G(p)}\left[G(p)N(1-G(p))^{N-1} - 1 + (1-G(p))^N\right]\right\}_{p=p_l} = 0$$

Hence

$$\frac{(p_l - c)\, g(p_l)}{G(p_l)} = \frac{1 - (1 - G(p_l))^N}{1 - (1 - G(p_l))^N - G(p_l)\, N\, (1 - G(p_l))^{N-1}} > 1 \tag{8}$$

and so $p_l < \omega_c$. ■

Proof of Proposition 14. We consider the pricing of a seller with cost draw of $\underline{\varepsilon}$ so that overall marginal costs are given by $\kappa + \lambda\underline{\varepsilon}$. Maintaining the notation of the model, the distribution of costs in the population, $F(c)$ is then given by $H\left(\frac{c-\kappa}{\lambda}\right)$. Thus $G(\kappa + \lambda\varepsilon) = \frac{F(\kappa+\lambda\varepsilon) - F(\kappa+\lambda\underline{\varepsilon})}{1 - F(\kappa+\lambda\underline{\varepsilon})} = \frac{H(\varepsilon) - H(\underline{\varepsilon})}{1 - H(\underline{\varepsilon})}$ which is independent of λ. The seller would therefore select a list price such that

$$p_l = \kappa + \lambda \cdot \arg \max_{\varepsilon \in \left[0, \frac{v-\kappa+\lambda\underline{\varepsilon}}{2\lambda}\right]} \lambda(\varepsilon - \underline{\varepsilon}) \underbrace{\frac{1 - (1 - G(\kappa + \lambda\varepsilon))^N}{G(\kappa + \lambda\varepsilon)}}_{\substack{\text{Independent} \\ \text{of } \lambda}} \tag{9}$$

If the seller decides to lower her list price below the freely bargained level then the ε selected will satisfy a first order condition which is independent of λ. Denote the optimal such choice as ε'. The argument of the arg max expression evaluated at ε' clearly tends to 0 as λ tends to 0.

Suppose instead that the seller decides to set her price at the freely bargained level. The price selected is $\kappa + \lambda\overline{\varepsilon}$ chosen so that this equals $\frac{v+\kappa+\lambda\underline{\varepsilon}}{2}$. Now note that

$$G(\kappa + \lambda\overline{\varepsilon}) = \frac{H\left(\frac{\underline{\varepsilon}}{2} + \frac{v-\kappa}{2\lambda}\right) - H(\underline{\varepsilon})}{1 - H(\underline{\varepsilon})} \to 1 \text{ as } \lambda \searrow 0$$

The argument of (9) in this case tends to $\lambda(\overline{\varepsilon} - \underline{\varepsilon}) = \frac{v-\kappa}{2} - \frac{\underline{\varepsilon}}{2}\lambda >> 0$. Thus for small λ setting the freely bargained price dominates setting the best reduced price. The mark up in these cases converges to $\frac{v-\kappa}{2}$ from below - the freely bargained markup when $\lambda = 0$.

In a Bertrand pricing setting, where costs are unknown, the analysis of Holt (1980) shows that the price chosen by a firm with cost of c is given by

$$p^{bert}(c) = c + \frac{\int_{r=c}^{v} 1 - J(r)\, dr}{1 - J(c)}$$

where $J(\cdot)$ is the distribution of the minimum of $N-1$ cost draws. Then note that $J(r) = 1 - \left[1 - F\left(\frac{r-\kappa}{\lambda}\right)\right]^{N-1}$ and so

$$p^{bert}(\kappa + \lambda\underline{\varepsilon}) = \kappa + \lambda\underline{\varepsilon} + \frac{\int_{r=c}^{v} \left[1 - F\left(\frac{r-\kappa}{\lambda}\right)\right]^{N-1} dr}{[1 - F(\underline{\varepsilon})]^{N-1}}$$

$$p^{bert}(\kappa + \lambda\underline{\varepsilon}) - (\kappa + \lambda\underline{\varepsilon}) = \frac{\lambda}{[1 - F(\underline{\varepsilon})]^{N-1}} \int_{s=\underline{\varepsilon}}^{\frac{v-\kappa}{\lambda}} [1 - F(s)]^{N-1} ds$$

and so the Bertrand price tends to cost as λ tends to zero as required as the integral term is bounded above as $F(v) = 1$. ■

Proof of Proposition 17. Proposition 15 gives us that sellers will choose a list price $p_l(c, N; \lambda)$ so that

$$\frac{[p - c]}{[p_l(c, N; \lambda) - c]} \le \left\{\lambda + (1 - \lambda)\frac{1 - (1 - G(p_l(c, N; \lambda)))^N}{G(p_l(c, N; \lambda))}\right\} \Bigg/ \left\{\lambda + (1 - \lambda)\frac{1 - (1 - G(p))^N}{G(p)}\right\} \quad \forall p.$$

The proof follows by showing that as λ falls, the inequality continues to hold for all $p > p_l(c, N; \lambda)$. That is, the right hand side is decreasing in λ. To see this note that any quotient $\frac{\lambda+(1-\lambda)A}{\lambda+(1-\lambda)B}$ is decreasing in λ iff $B < A$. Thus the result follows if

$$\frac{1-(1-G(p))^N}{G(p)} < \frac{1-(1-G(p_l(c,N;\lambda)))^N}{G(p_l(c,N;\lambda))} \text{ for } p > p_l(c,N;\lambda) \tag{10}$$

But consider the function $\frac{1-(1-x)^N}{x}$ on $(0,1)$. This function has derivative equal in sign to

$$Nx(1-x)^{N-1} - 1 + (1-x)^N$$

This takes the value of 0 when $x = 0$, and -1 when $x = 1$ and itself has a negative derivative and so is a decreasing function. Therefore $\frac{d}{dx}\frac{1-(1-x)^N}{x} < 0$. But this implies that (10) holds completing the proof. ■

Proof of Proposition 18. At the optimal list with N rivals (p_l) we must have

$$[p-c]\left\{\lambda + (1-\lambda)\frac{1-(1-G(p))^N}{G(p)}\right\} \leq [p_l - c]\left\{\lambda + (1-\lambda)\frac{1-(1-G(p_l))^N}{G(p_l)}\right\} \ \forall p \tag{11}$$

This inequality will continue to hold when N is changed to $N+1$ rivals if

$$[p-c](1-G(p))^N \leq [p_l - c](1-G(p_l))^N \tag{12}$$

We wish to show that this must be the case for $p > p_l$. The proof now proceeds by contradiction. Thus suppose there exists some price $p \in \left(p_l, \frac{v+c}{2}\right)$ which violates (12). Combining with (11) we then have that at this p

$$\frac{G(p)\left\{\lambda G(p_l) + (1-\lambda)\left[1-(1-G(p_l))^N\right]\right\}}{G(p_l)\left\{\lambda G(p) + (1-\lambda)\left[1-(1-G(p))^N\right]\right\}} \geq \frac{p-c}{p_l-c} > \frac{(1-G(p_l))^N}{(1-G(p))^N}$$

Or re-expressing that

$$\frac{\lambda G(p_l) + (1-\lambda)\left[1-(1-G(p_l))^N\right]}{G(p_l)(1-G(p_l))^N} > \frac{\lambda G(p) + (1-\lambda)\left[1-(1-G(p))^N\right]}{G(p)(1-G(p))^N} \tag{13}$$

for some $p > p_l$. But this final expression is increasing in p providing the contradiction. To see this differentiate the right hand side of (13) with respect to p to give

$$\frac{g(p)N\lambda}{(1-G(p))^{N+1}} - \frac{(1-\lambda)\left\{g(p)(1-G(p))^N - Ng(p)G(p)(1-G(p))^{N-1}\right\}}{G(p)^2(1-G(p))^{2N}} + \frac{(1-\lambda)}{G(p)^2}g(p)$$

the first term is positive. The second two terms can be combined to give

$$\frac{(1-\lambda)g(p)}{G(p)^2(1-G(p))^{N+1}}\left\{(1-G(p))^{N+1} - 1 + G(p) + NG(p)\right\}$$

This is also positive as the braced term is positive for $G(p) \in (0,1)$. To see this final step consider the function $(1-x)^{N+1} - 1 + x + Nx$. This takes the value 0 when $x = 0$ and has derivative $(N+1)\left[1-(1-x)^N\right]$ which is positive if $x \in (0,1)$. Thus (13) is a contradiction for any $p > p_l$ and the proof is complete. ■

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