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COMMODITY TAXATION AS INSURANCE AGAINST PRICE RISK

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Abstract

The paper shows how commodity taxes can provide insurance
to consumers when the producer price is volatile. Specific and
ad valorem taxes have differing roles. The optimal specific tax
is positive when demand has some elasticity. The optimal *ad
valorem* rate is zero when demand is unit-elastic, negative
when demand is inelastic and positive for elastic demand.
When both types of taxes are used in general the specific tax is
positive and the *ad valorem* rate is negative. The model also
applies to problem in public utility regulation of determining
how retail prices should move with wholesale or fuel prices.

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1. Introduction

Commodity taxes can provide insurance to households when producer prices are volatile. This paper considers a model of optimal commodity taxation that analyzes the differing roles of specific and *ad valorem* taxes in insuring consumers against price risk. Two recent public policy issues motivate the analysis. In the summer of 2000 there were large protests in many European countries about the level of taxation of petrol and diesel. The immediate cause of the unrest had been a tripling of the world price of oil. Our model addresses the issue of the appropriate structure of taxation when the producer price, such as the world price of oil, is volatile. The results we derive imply that the tax structure used in the UK was providing only a small amount of insurance to consumers and this may have been the cause of the problem. The second issue concerns the regulation of public utilities facing volatile input prices. Electricity and natural gas suppliers typically face input prices that vary randomly because of fuel price changes, plant outages and demand shocks in neighbouring systems that raise the price of imports. In the case of the reformed electricity system in California the regulator fixed the retail price, so there was no pass-through of wholesale prices into retail prices. The model presented here suggests that this provided too much insurance to consumers. In contrast many regulators in other jurisdictions use fuel adjustment clauses to allow full pass-through of input prices into retail prices. As in the European petrol tax example full pass-through offers insufficient insurance to consumers.

There is a small literature on optimal taxation under uncertainty about real wages.[1] In Eaton and Rosen (1980) each household decides on labour supply before uncertainty about the real wage is resolved and faces both a lump-sum tax and a proportional tax on income. The optimal income tax rate is positive, despite the fact that households are identical *ex ante* and a lump-sum tax is available, because of the insurance that this provides.[2] A related paper that considers optimal commodity taxation when there is real wage risk is by Cremer and Gahvari (1995). They distinguish between committed and uncommitted goods. Committed goods, such as housing and hours of leisure, are chosen before the wage uncertainty is resolved, while spending on uncommitted goods is chosen subsequently. There is a role for

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[1] See Myles (1995, Chapter 7) for a general survey of taxation in the presence of risk.
[2] Anderberg and Andersson (2003) use a similar model to Eaton and Rosen (1980) and allow for the choice of education levels.

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commodity taxes to provide insurance against income risk, with higher tax rates on the goods that are uncommitted. In their model there is no difference between specific and *ad valorem* taxation since producer prices are fixed.

Another literature examines specific and *ad valorem* taxes – see Keen (1998) for a comprehensive survey and Delapilla and Keen (1993) for a formal model. A specific tax is a fixed amount per unit of the good, while an *ad valorem* tax is defined as a constant percentage of the producer's revenue. The literature focuses on the different characteristics the two types of indirect tax have when there is imperfect competition, since with certainty and perfect competition the two are equivalent. Skeath and Trandel (1994) prove that when a monopoly is subject to a specific tax a Pareto improvement is available by switching to an *ad valorem* tax. Myles (1996) shows how the two types of tax may be combined to improve outcomes in imperfectly competitive markets.

More closely related to our model is the analysis of the effects of randomness in the producer price. Keen (1998) shows that for tax revenue to be constant when there is a small change in the producer price the share of *ad valorem* taxes in total taxation should equal the price elasticity of demand. It is not clear, however, why stabilization of tax revenue should be an objective. Fraser (1985) examines the differences between the two types of taxation when the customer is a small, risk-averse firm (perhaps a farmer) who must pre-commit to output, but does not discuss optimal tax policy. Van Wijnbergen (1984) models the choice of tax rates when the price of an importable used in production, say oil, is volatile. Firms commit to their oil demand before the price of oil is revealed. Since the government's welfare function is a concave function of income it is optimal to use the tax system to set the consumer price constant. Our model differs in that households, not firms, are the consumers, and their choices are made after prices are known. Only in the special case of perfectly inelastic demand is optimal in our model for the consumer price to be stable.

The model presented here is one of optimal commodity taxation. The government chooses the three parameters of the tax system – the lump-sum tax, the specific tax and the *ad valorem* rate – to maximize expected utility of consumers subject to the constraint that expected tax revenue equals the revenue requirement. In the case of public utility regulation the equivalent problem for the regulator is to

choose the lump-sum charge and the intercept and slope terms of the linear function relating the retail price to marginal cost.

We make a minimal assumption about the cardinal properties of the utility function. The consumer is sufficiently risk-averse that the marginal utility of income increases with the price of the taxed good. This holds as long as relative risk aversion exceeds the income elasticity of demand. Having made this assumption all the main results that we present depend only on the ordinal properties of the utility function, in particular the value of the price elasticity of demand.

In general it is optimal to distort pricing in order to provide some insurance to consumers. With a specific tax the tax revenue collected falls when the consumer price rises as long as there is some elasticity of demand. The reduction in the tax payment below the expected level provides the consumer with some compensation for the price increase. With an *ad valorem* tax the consumer's payment to the government varies with the consumer price as long as the price elasticity of demand is not unity. The framework is one of second-best. Alternative insurance mechanisms, such as futures and insurance markets, are assumed to be unavailable. For household energy and fuel consumption this is a reasonable assumption since transactions costs prevent households from using the futures markets that do exist.[3] Similarly we make the plausible assumption that the government cannot make the tax parameters state-contingent. If it could then the government would optimally provide insurance through a state-contingent lump-sum tax without the need for any commodity taxation – see Cowan (2003) for an analysis of such first-best insurance in the context of utility regulation. With fixed commodity tax parameters the consumer price is a linear function of the producer price. Cowan (2003) also considers the general Ramsey pricing problem of choosing the function relating price to marginal cost when the function need not be linear.

In Section 2 we present the assumptions about consumers, discuss the way taxation operates and outline the compensated demand function that is relevant when there is price risk. Section 3 contains the optimal taxation analysis. We consider each type of commodity tax separately and then jointly. Concluding remarks are in Section 4.

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[3] See Gilbert (1985).

2. The consumer, taxation and compensated demand

2.1 The consumer

The consumer has exogenous fixed income of m and pays a lump-sum tax of A. The budget constraint is $pq + x \leq m - A$ where p is the consumer price of the taxed (or regulated) good, q is the quantity bought, and x is the quantity of the alternative good, which is untaxed and has a price of 1. Consumption decisions are made after p is known. Quantities bought of both goods are positive. Preferences are represented by the von Neumann-Morgenstern indirect utility function $V(p, m - A)$. Roy's Identity implies $V_p = -q(p, m - A)V_m$ with subscripts indicating partial derivatives and $q(p, m - A)$ denoting the uncompensated demand function. From the Slutsky equation the pure substitution effect is $q_p + qq_m$. In general this is non-positive, and we assume for simplicity that it is strictly negative. The consumer is not satiated and is risk-averse. The price elasticity of demand is $\varepsilon \equiv -pq_p/q$, the budget share of the taxed good is $s \equiv pq/(m - A)$ and the income elasticity of demand is $\eta \equiv (m - A)q_m/q$. We assume non-inferiority so uncompensated demand is strictly decreasing in the price. We shall, though, also derive results for the case of perfectly inelastic uncompensated demand, which has substitution and income effects that are both zero.

An important assumption is that $V_{mp} > 0$, so a rise in the price of the taxed good increases the marginal utility of income. To see why this is reasonable differentiate $V_p = -qV_m$ with respect to m to give the price elasticity of the marginal utility of income:

$$p\frac{V_{mp}}{V_m} = s(r - \eta) \qquad (1)$$

where $r \equiv -(m - A)V_{mm}/V_m$ is the coefficient of relative risk aversion. Equation (1) implies that the sign of V_{mp} is the same as the sign of $r - \eta$. We expect the income elasticity of demand to be around unity. Giuliano and Turnovsky (2000) summarize the empirical evidence about the value of relative risk aversion, and conclude that r is likely to be in the range $2 - 5$. Constantinides *et al.* (2002) use values for r of 4 and 6 in their simulations, and Anderberg and Andersson (2003) set $r = 4$. Gilbert (1985) uses a value of $r = 2.5$ in his estimates of the benefits of commodity price stabilization. Thus there is evidence that (1) is positive.

2.2 Taxation

The producer price, c, is a continuous random variable with a known distribution on a positive support. In the utility regulation case c is the marginal cost. The tax receipt per unit (or the contribution to fixed costs) is $p - c$. The specific tax is t, the *ad valorem* rate is τ, and the resulting consumer price is:

$$p = t + (1 + \tau)c . \tag{2}$$

Equation (2) expresses p as an affine function of c.[4] In utility regulation the increase in p caused by a unit increase in c, i.e. $1 + \tau$, is known as the pass-through coefficient. This is below unity when the *ad valorem* rate is negative. Another useful measure of the sensitivity of p to c is the elasticity, $cp_c/p = (1 + \tau)c/p$, which is below 1 when $t > 0$.

Tax revenue, T, is:

$$T \equiv A + (p - c)q = A + tq + \tau cq = A + \frac{t}{1+\tau}q + \frac{\tau}{1+\tau}pq ,$$

where we have used (2) and the last expression holds as long as $\tau > -1$. Revenue depends on both the level of demand (through the specific tax) and on expenditure, pq, through the *ad valorem* tax. The effect of a rise in c on T is:

$$\frac{dT}{dc} = tq_p + \tau q(1 - \varepsilon) = q\left[-\frac{t}{p}\varepsilon + \tau(1 - \varepsilon)\right]. \tag{3}$$

From (3) we see that as long as there is some elasticity of demand and $t > 0$ the revenue from the specific tax falls as c rises. The effect on *ad valorem* revenue depends on whether demand is inelastic or elastic and on the sign of τ. Setting (3) equal to zero confirms that $dT/dc = 0$ when $\tau p/(t + \tau p) = \varepsilon$, i.e. the share of *ad valorem* revenue in total revenue equals the price elasticity.

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[4] In the UK the *ad valorem* rate is charged on top of the specific tax for fuel, alcohol and tobacco. This can be accommodated in equation (2) by defining $t \equiv (1 + \tau)t^*$ with t^* being the specific tax.

To illustrate (3) we use petrol tax data from the UK. At its peak in 2000 the retail price of petrol in the UK was 80 pence per litre. The excise duty was 48.82 pence and the Value Added Tax (VAT) rate was 17.5 percent, which was charged on top of the excise duty.[5] The producer price was thus 19.3 pence a litre. A reasonable estimate of the short-run price elasticity of demand for petrol is 0.3 (Glaister and Graham, 2000). These numbers and (3) imply that the point elasticity of tax revenue with respect to the producer price, $(c/T)dT/dc$, was -0.029. While the tax system was providing insurance against changes in the world oil price, the size of this effect was very small. A doubling of world oil prices would lead to a reduction in tax revenue of only 2.9 percent. The point elasticity remains very close to zero for a very wide range of values of c and is not sensitive to different estimates of the price elasticity.

2.3 The expenditure function and the compensated demand function

It is compensated demand functions that matter for assessing the cost of tax distortions. The following analysis of compensated demand with price risk parallels the discussion of wage risk in Cremer and Gahvari (1995) and in Anderberg and Andersson (2003). Expected utility is $E[V(t + (1 + \tau)c, m - A)]$ where $E[.]$ is the expectations operator. The expenditure function is the minimum net lump-sum income necessary to reach a given level of *expected* utility, $\bar{v}$, when the commodity tax parameters are t and τ. This function is $e(t, \tau, \bar{v})$, and is defined implicitly by:

$$E[V(t + (1+\tau)c, e(t, \tau, \bar{v}))] = \bar{v}.$$

Differentiating this and using Roy's Identity gives the partial derivatives of the expenditure function:

$$e_t = \frac{E[qV_m]}{E[V_m]}, \tag{4a}$$

$$e_\tau = \frac{E[cqV_m]}{E[V_m]}. \tag{4b}$$

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[5] This means that t in (2) is 1.175*48.82 = 57.36 pence.

Equation (4a) shows that the derivative of the expenditure function with respect to the specific tax is a weighted average of demand, with the marginal utility of income as the weights. Without risk (4a) is just the standard compensated demand function. Finally define the expected compensated demand function as

$$q^c(t,\tau,\overline{v}) \equiv E[q(t+(1+\tau)c, e(t,\tau,\overline{v}))]. \tag{5}$$

Differentiating (5) and using (4a) and (4b) gives the following Slutsky-type compensated demand derivatives:

$$q_t^c \equiv E[q_p] + \frac{E[qV_m]}{E[V_m]} E[q_m], \tag{6a}$$

$$q_\tau^c \equiv E[cq_p] + \frac{E[cqV_m]}{E[V_m]} E[q_m]. \tag{6b}$$

The derivative (6a) need not be negative in general. In the next section, though, we shall present a sufficient condition for (6a) to be negative.

3. Optimal taxation

3.1 The general problem

In choosing its tax parameters the government ensures that expected tax revenue from all three sources equals the revenue requirement, R, which may be zero. What happens when actual revenue differs from R? In the wage risk models, such as Eaton and Rosen (1980), Cremer and Gahvari (1995) and Anderberg and Andersson (2003), there is no aggregate risk and the population is sufficiently large that actual tax revenue always equals expected tax revenue. In our model we must have additional agents who are prepared to take on the risk and act indirectly as insurers to the consumers. In the utility regulation interpretation of the model we suppose that shareholders are risk-neutral and do not consume the regulated good. When operating profits differ from the fixed cost the shareholders receive the excess and pay out to cover any deficit. Since shareholders' expected income is unaffected, and they are not directly affected by price changes because they do not consume the good, their welfare is unchanged. In the tax case the role of the shareholders may be taken either

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by a different set of taxpayers who are risk-neutral and do not consume the taxed good, or by financial markets (perhaps the world capital market).

The government chooses the three tax parameters, t, τ and A, to maximize expected utility subject to the constraint that expected tax revenue equals the revenue requirement and subject to (2).[6] The Lagrangian for the government's problem is

$$L = E[V(p, m-A)] + \lambda[A + tE[q] + \tau E[cq] - R]$$

where $E[.]$ is the expectations operator. The first-order conditions for t, τ and A are:

$$L_t = E[V_p] + \lambda\big[E[q] + tE[q_p] + \tau E[cq_p]\big] = 0 \tag{7}$$

$$L_\tau = E[cV_p] + \lambda\big[tE[cq_p] + E[cq] + \tau E[c^2 q_p]\big] = 0 \tag{8}$$

$$L_A = -E[V_m] + \lambda\big[1 - tE[q_m] - \tau E[cq_m]\big] = 0. \tag{9}$$

We assume that the second-order sufficient conditions hold. Before examining the general solution to (7)-(9) we consider the special cases where only one type of excise tax is used.

3.2 Specific taxation

Suppose first that the government uses specific taxation only (combined with lump-sum taxation). Equations (7) and (9) imply (using Roy's Identity and the definition of the covariance):

$$tq_t^c \equiv t\left[E[q_p] + \frac{E[qV_m]}{E[V_m]}E[q_m]\right] = \frac{Cov(q, V_m)}{E[V_m]}. \tag{10}$$

This equation captures the trade-off between distorting the price and insurance. A rise in t from zero causes an efficiency loss but provides an insurance benefit. The efficiency loss, represented by the left-hand side, arises because the price increase

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[6] In the case of public utility regulation the agency chooses the retail price as an affine function of marginal cost, subject to the constraint that expected operating profit covers the fixed cost, R.

cuts demand, but at the same time there is some compensation on average because expected tax revenue is raised, enabling the lump-sum tax to be reduced. The left-hand side is the cost of the tax, which is proportional to the derivative of expected compensated demand defined by (6a). Note that when there is no risk this term is the pure substitution effect. At the same time the covariance on the right-hand side is zero so equation (10) confirms that $t = 0$ when there is no risk. There is no role for commodity taxation in a certain world when a lump-sum tax is available. Similarly when there is price risk but the consumer does not mind it because $V_{mp} = 0$ the optimal tax is zero.

The sign of the compensated demand derivative in (10) is ambiguous in general. Eaton and Rosen (1980) show that with wage uncertainty an income-compensated increase in wage taxation does not necessarily reduce labour supply (because the compensation is in expected and not actual terms). In the appendix we show that a sufficient condition for this term to be negative is that the price elasticity of demand is independent of income. Any demand function that can be written in the multiplicatively separable form $q = g(p)h(m - A)$, where $g'(p) < 0$ and $h'(m - A) \geq 0$, satisfies this condition. Examples are demand functions with zero income effects, demands derived from utility functions representing homothetic tastes and iso-elastic demand functions.

The insurance term can be explained in the following way.[7] The first-order effect of an increase in t is to raise tax revenue by $E[q]$. This means that the lump-sum tax, A, can be cut by this amount, and the expected utility from the reduction in A is $E[V_m]E[q]$. The rise in t also has a direct effect on expected utility of $E[V_p] = -E[qV_m]$ because it raises the consumer price. The net effect is then $E[V_m]E[q] - E[qV_m] = -Cov(q, V_m)$. To find the sign of the covariance note from (2) that a rise in c raises p one-for-one, which increases marginal utility and cuts demand given the assumptions that $V_{mp} > 0$ and $q_p < 0$. Thus the covariance is negative, and we have:

Proposition 1. *When the price elasticity of demand is independent of income the optimal specific tax is positive.*

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[7] See Cremer and Gahvari (1995) for a similar explanation of the covariance term in their model of specific commodity taxation with income risk.

With a positive specific tax the elasticity of the consumer price with respect to the producer price is below unity. The lump-sum tax, A, is set below R to maintain budget balance in expected terms.

Proposition 1 has an intuitive explanation. A small specific tax causes an efficiency distortion that is of second-order importance. At the same time it provides some insurance. When the producer price rises, and the consumer price increases by the same amount, the consumer's tax payment to the government falls. This reduction in the tax payment provides some compensation to the consumer for the price increase. This compensation is valuable when the marginal utility of income rises with the price so a positive specific tax is better than no tax. In the special case of zero uncompensated elasticity the covariance is zero. The utility function is a concave function of residual income, i.e. $V(m - A - c - t)$ for $q = 1$. The specific tax acts in exactly the same way as the lump-sum tax and thus cannot provide insurance.

3.3 Ad valorem taxation

When an *ad valorem* tax is the only form of commodity taxation equation (2) implies that the consumer price is proportional to the producer price and the mark-up is $(p - c)/p = \tau/(1 + \tau)$. Equations (8) and (9) and $c = p/(1 + \tau)$ imply that:

$$\frac{\tau}{1+\tau}\left[E[p^2 q_p] + \frac{E[pqV_m]}{E[V_m]}E[pq_m]\right] = \frac{Cov(pq, V_m)}{E[V_m]}. \tag{11}$$

The left-hand side of (11) is the mark-up multiplied by another substitution-type effect. This is not, though, the same as the derivative of compensated demand with respect to the *ad valorem* rate defined by (6b). The right-hand side is the insurance term. When there is no risk the term in square brackets on the left-hand side is $p^2(q_p + qq_m)$ which is negative. Of course with no risk the right-hand side is zero so $\tau = 0$.

We need some conditions to ensure that the term in square brackets on the left-hand side of (11) is strictly negative. Essentially this term is negative holds as long as the income effect, q_m, is not too large. One sufficient condition is that preferences are represented by a Cobb-Douglas utility function. Cobb-Douglas utility implies that $\varepsilon = 1$ and $\eta = 1$ and both pq and $s \equiv pq/(m - A)$ are constant. The square-bracketed term on the left-hand side of (11) is $pq(s - 1) < 0$ since $s < 1$. Similarly when there is no

income effect in demand the term is clearly negative. An example that is especially useful is when demand is iso-elastic, so $q(p) = p^{-\varepsilon}$. From know on we shall focus on the cases of Cobb-Douglas preferences and constant demand elasticity with no income effect.

The insurance term in (11) depends on the covariance of expenditure with the marginal utility of income.[8] This can be interpreted in a similar way to the insurance term for the specific tax. Starting from $\tau = 0$, the cost of a rise in τ is $- E[V_p p_\tau] = - E[cV_p] = E[cqV_m]$. The rise in the *ad valorem* rate increases the cost of buying q for a particular value of c by cq, and the utility cost is cq multiplied by the marginal utility of income. This is then averaged across all possible values of c. The marginal benefit comes from the extra tax revenue that allows a cut in the lump-sum tax, and the expected utility from this is $E[V_m]E[cq]$. Thus the net benefit is:

$$E[V_m]E[cq] - E[cqV_m] = - Cov(cq, V_m).$$

The covariance in (11) is $Cov(pq, V_m) = Cov(cq, V_m)/(1 + \tau)$. The sign of $Cov(pq, V_m)$ is straightforward to determine since demand has been assumed to be iso-elastic. When $\varepsilon = 1$ expenditure is constant as p varies and thus the covariance is zero. The covariance is positive when $\varepsilon < 1$, because expenditure increases with the price, and negative when $\varepsilon > 1$.

Proposition 2. *When an* ad valorem *tax is the only form of commodity tax used the optimal rate has the same sign as $\varepsilon - 1$ for the cases of Cobb-Douglas preferences and iso-elastic demand with no income effect*.

There is a clear intuition for Proposition 2. When $\varepsilon = 1$ the effect of a rise in c on the commodity tax payment is zero, since *ad valorem* tax revenue is $(\tau/(1 + \tau))pq$ and pq is constant. Thus the *ad valorem* tax provides no insurance. With inelastic demand the tax payment rises with c when τ is positive, and falls with c when τ is negative. Since consumers value insurance they want the tax payment to fall when p rises, so the optimal value of τ is negative for $\varepsilon < 1$. A positive rate is set when demand is elastic.

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[8] Besley (1988) presents a model of reimbursement health insurance where the optimal reimbursement rate depends on the covariance of health expenditure with the marginal utility of income.

In Section 2.1 we noted the assumption that the alternative good is not taxed. What happens when all goods are subject to the same rate of *ad valorem* taxation, as in a system with a uniform VAT rate? Such taxation does not provide insurance. Let the uniform VAT rate be τ. The government's budget constraint is $A + \tau E[cq + x] = R$. The consumer's budget constraint is $(1+\tau)[cq + x] = m - A$ and thus total expenditure is constant when c changes and $E[cq + x] = cq + x = (m - A)/(1+\tau)$. Solving the two budget constraints gives $cq + x = m - R$, so the tax rate has no effect on the consumer's choices. Intuitively because total tax revenue is constant as c changes the tax system is not providing any insurance. It is only differential taxation that provides insurance.

When the uncompensated elasticity is zero indirect utility is $V(m - A - p) = V(m - A - (1 + \tau)c)$ which is increasing and concave because of non-satiation and risk-aversion. A specific tax is of no use when $\varepsilon = 0$, but an *ad valorem* subsidy provides a first-best outcome.

Proposition 3. *When the price elasticity is zero the optimal* ad valorem *rate is* $- 1$, *and the consumer receives full insurance against price risk.*

Proof. The revenue constraint is $A + \tau E[c] = R$. Substituting this into expected utility gives: $E[V(m - R + \tau E[c] - (1 + \tau)c)]$. The first-order condition for the choice of τ is $E[V'(.)(E[c] - c)] = 0$. The covariance of marginal utility with the producer (and consumer) price should be zero. This is ensured by setting $\tau = - 1$ so that $V'(.)$ is constant for all values of c. The second-order condition holds since the consumer is risk averse.

The consumer is fully insured against price risk since the price is constant at zero. The government buys the good at a price of c and lets consumers purchase at a zero price, though they pay for this through an increase in the lump-sum tax of $E[c]$.

Which is better, specific or *ad valorem* taxation? A revealed preference argument shows that a specific tax is better when the price elasticity of demand is unity. When $\varepsilon = 1$ the optimal *ad valorem* rate is 0. With specific taxation the government could achieve the same result by setting $t = 0$. But Proposition 1 shows that it is optimal to choose a positive specific tax. *Ad valorem* taxation, however, is

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preferable when $\varepsilon = 0$. In this case a specific tax has the same effects as a lump-sum tax and can provide no insurance. An *ad valorem* tax rate of -1 fully insures consumers on fair terms, which is valuable since they are risk-averse.

3.4 Combining specific and ad valorem taxes

Before presenting the general analysis we look first at full price stabilization. When the elasticity of demand is zero a stable price is optimal. Does this generalize to the case of a demand elasticity that exceeds zero? The government stabilizes the consumer price by setting $\tau = -1$. Solving (7) and (9) while setting $\tau = -1$ gives $t = E[c]$. It follows that $A = R$, and the two commodity taxes do not raise any net revenue.

Does the consumer benefit from complete price stabilization with the consumer price set equal to the expected producer price, compared to the case where both commodity taxes are zero? There is a standard answer to this question. The consumer benefits if and only if indirect utility is concave in p. Differentiating $V_p = -qV_m$ with respect to p, and using (1), gives

$$\frac{-pV_{pp}}{V_p} = \varepsilon - p\frac{V_{mp}}{V_m} = \varepsilon + s(\eta - r) \tag{12}$$

as the measure of relative preference for price risk (see Turnvosky *et al.*, 1980). This is the difference between the price elasticity of demand and the price elasticity of marginal utility. The expression in (12) has the same sign as V_{pp}. A fixed price is preferred to a random price when the relative risk aversion coefficient, r, is high enough to make (12) negative. A necessary (but not sufficient) condition for this is that $V_{mp} > 0$.

A fixed price might or might not be preferable to letting the producer price always equal the consumer price, depending on the sign of (12). When, however, there is some elasticity of demand some pass-through is optimal.

Proposition 4. *When there is some elasticity of demand the consumer price should not be constant.*

Proof. Set $t = E[c]$ and $\tau = -1$ in (7) and (9) and examine the first-order condition for τ, (8), noting that q is constant. Equation (8) becomes:

$$L_\tau = -V_m q_p Var(c).$$ (13)

Equation (13) is strictly positive when $q_p < 0$, so a rise in the *ad valorem* rate above –1 will increase expected utility.

Thus the Californian solution to input price risk of a constant retail price is sub-optimal. It can be argued that regulation in this case was providing too much insurance to consumers.

We now turn to the general solution to the optimal tax problem. Combining (9) with (7) and (8) and using (2) to substitute for c gives the following equations characterizing the solution (as well as the revenue constraint):

$$t\left[E[q_p] + \frac{E[qV_m]}{E[V_m]}E[q_m]\right] + \tau\left[E[cq_p] + \frac{E[qV_m]}{E[V_m]}E[cq_m]\right] = \frac{Cov(q,V_m)}{E[V_m]}$$ (14)

$$t\left[E[pq_p] + \frac{E[pqV_m]}{E[V_m]}E[q_m]\right] + \tau\left[E[cpq_p] + \frac{E[pqV_m]}{E[V_m]}E[cq_m]\right] = \frac{Cov(pq,V_m)}{E[V_m]}.$$ (15)

In (14) the term multiplying t is the derivative of expected compensated demand, q_t^c, defined in (6a). In (15) the term multiplying t is $tq_t^c + (1+\tau)q_\tau^c$. We first take the case where the consumer has Cobb-Douglas preferences, so expenditure on the taxed good is constant and the price and income elasticities are both 1. Equation (15) then implies

$$tE[q] + \tau E[cq] = 0$$ (16)

because the covariance is zero when expenditure is constant, and the income effect q_m is proportional to q. Equation (16) says that the revenue derived from one commodity tax is used to finance an equivalent subsidy (in expected terms) through the other form of commodity tax. The revenue constraint then implies $A = R$. Using (16) to simplify (14) gives:

$$tE[q_p] + \tau E[cq_p] = \frac{Cov(q,V_m)}{E[V_m]}.$$ (17)

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The left-hand side of (17) is the marginal effect on tax revenue of an increase in the consumer price. Solving (16) and (17) gives:

Proposition 5. *With Cobb-Douglas preferences the specific tax is positive, the* ad valorem *rate is negative, and the lump-sum tax equals the revenue requirement.*

Proof.
See the appendix.

As before the specific tax is positive because this provides insurance to the consumer in the face of a varying producer price. The negative sign of the *ad valorem* rate is perhaps more surprising, since we know from (3) that when the price elasticity of demand is unity the *ad valorem* rate does not affect directly the sensitivity of tax revenue to the producer price. What the *ad valorem* subsidy does do, however, is to make the consumer price less sensitive to the producer price than when only a specific tax is used, and this is valuable to the consumer. A specific tax on its own does not affect the pass-through coefficient, but an *ad valorem* subsidy does cut the pass-through coefficient below one.

When the demand function is characterized by no income effect and a constant elasticity of demand below unity we have the following result:

Proposition 6. *When the constant price elasticity is below 1 and there is no income effect the specific tax is positive, the* ad valorem *rate is negative and the lump-sum tax exceeds the revenue requirement.*

Proof.
See the appendix.

Apart from the fact that $A > R$ the qualitative features of this result are the same as when $\varepsilon = 1$. There should be a positive specific tax and an *ad valorem* subsidy to provide insurance. Since demand is inelastic the *ad valorem* subsidy directly provides insurance.

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4. Conclusion

The paper has shown that there is a role for both types of commodity taxation in providing insurance against producer price fluctuations. In energy and fuel supply producer prices are typically volatile, and our "back-of-the-envelope" calculation suggests that the UK system of petrol taxation provided little insurance to consumers, while the regulators of the Californian electricity system gave consumers too much insurance.

From a theoretical point of view the paper extends the literature on taxation and insurance to allow for price risk. We also show that specific and *ad valorem* taxes have rather different roles as insurance providers when the producer price varies. There are several possibilities for future research. First, demand-side risk could be modelled. At present the model has exogenous producer prices. If marginal production cost is increasing and demand is subject to stochastic shifts then producer prices will fluctuate endogenously. Commodity taxation might serve a rather different role with demand risk and endogenous producer prices. Second, we could investigate empirically the risk-sharing properties of existing commodity taxes and pass-through arrangements for regulated utilities.

Appendix

We show that $E[q_p] + E[qV_m]E[q_m]/E[V_m] < 0$ when $q = g(p)h(m - A)$. The following chain of inequalities and equations proves the claim.

$$0 > E[q_p + qq_m] = E[q_p] + E[q]E[q_m] + Cov(q, q_m)$$
$$\geq E[q_p] + E[q]E[q_m]$$
$$> E[q_p] + E[q]E[q_m] + \frac{Cov(q, V_m)}{E[V_m]}E[q_m] = E[q_p] + \frac{E[qV_m]}{E[V_m]}E[q_m].$$

The first inequality holds because the substitution effect is negative. The equation is a consequence of the definition of the covariance. When $q = g(p)h(m - A)$ the covariance is $Cov(q, q_m) = Cov(f(p)h(m - A), f(p)h')$ which is non-negative when $h' \geq 0$, i.e. when demand is not inferior. Thus the covariance is non-negative and the second inequality holds. The third inequality follows from the assumptions of non-inferiority, and $V_{mp} > 0$ and the fact that uncompensated demand is downward-sloping, so $Cov(q, V_m) < 0$ and $E[q_m] \geq 0$. The final equation is again a consequence of the definition of the covariance.

Proof of Proposition 5.

Solving (16) and (17) simultaneously gives $t^* = -E[cq]Cov(q, V_m)/\Delta E[V_m]$ where $\Delta \equiv E[q]E[cq_p] - E[cq]E[q_p]$. Using (2) to substitute for c and $q = s(m - R)/p$ gives $(1 + \tau)\Delta = -E[pq]E[q_p] + E[q]E[pq_p] = E[pq]E[q/p] - \{E[q]\}^2 = E[q^2] - \{E[q]\}^2 = Var(q)$. This implies $t^* = -E[cq]Cov(q, V_m)(1 + \tau)/Var(q)E[V_m]$. Using $c = (p - t)/(1 + \tau)$ and rearranging gives

$$t^* = -\frac{s(m - R)Cov(q, V_m)}{Var(q)E[V_m] - E[q]Cov(q, V_m)} > 0$$

and

$$\tau^* = \frac{E[q]Cov(q, V_m)}{Var(q)E[V_m] - E[q]Cov(q, V_m)} < 0.$$

The signs follow from the fact that $Cov(q, V_m) < 0$ so the denominator for both expressions is positive. □

Proof of Proposition 6.

With $q_m = 0$ equation (15) is $tE[pq_p] + \tau E[cpq_p] = Cov(pq, V_m)/E[V_m]$. Using the fact that $q = p^{-\varepsilon}$ and so $pq_p = -\varepsilon q$ the equation becomes

$$tE[q] + \tau E[cq] = -\frac{Cov(pq, V_m)}{\varepsilon E[V_m]}. \tag{A1}$$

The left-hand side of (A1) is expected revenue from the two commodity taxes, and the right-hand side is negative because $\varepsilon < 1$ implies that the covariance is positive. Thus $A > R$. It then follows that a sufficient condition for $\tau < 0$ is that $t > 0$. Solving (14) and (A1) simultaneously gives

$$t = \frac{(1+\tau)}{E[pq]E[q/p] - \{E[q]\}^2}\left(-\frac{E[cq]Cov(q, V_m)}{\varepsilon E[V_m]} + E\left[\frac{cq}{p}\right]\frac{Cov(pq, V_m)}{\varepsilon E[V_m]}\right). \tag{A2}$$

Since $\varepsilon < 1$ $Cov(pq, V_m)$ is positive and the term in large brackets is positive. To show that $t > 0$ we need to prove that $E[pq]E[q/p] - \{E[q]\}^2 > 0$. Using $q = p^{-\varepsilon}$ we can write this in terms of q as:

$$\begin{aligned}
E[pq]E[q/p] - \{E[q]\}^2 &= E[q^{(\varepsilon-1)/\varepsilon}]E[q^{(1+\varepsilon)/\varepsilon}] - \{E[q]\}^2 \\
&= E[q^{(\varepsilon-1)/\varepsilon}]E[q^{(1+\varepsilon)/\varepsilon}] - \{E[q]\}^{(\varepsilon-1)/\varepsilon}\{E[q]\}^{(1+\varepsilon)/\varepsilon}
\end{aligned} \tag{A3}$$

For $\varepsilon > 0$ the function $q^{(1+\varepsilon)/\varepsilon}$ is convex in q, and thus from Jensen's inequality $E[q^{(1+\varepsilon)/\varepsilon}] > E[q]^{(1+\varepsilon)/\varepsilon}$. For $\varepsilon < 1$ the function $q^{(\varepsilon-1)/\varepsilon}$ is also convex, so $E[q^{(\varepsilon-1)/\varepsilon}] > E[q]^{(\varepsilon-1)/\varepsilon}$. It follows that (A3) is strictly positive. Thus $t > 0$ and, from (A1), $A > R$ and $\tau < 0$. $\square$

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