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RELATIONAL INCENTIVE CONTRACTS WITH PRIVATE INFORMATION

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Relational Incentive Contracts with Productivity Shocks[a]

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Abstract

This paper extends Levin's (2003) relational contract model by having not only the agent's cost of effort (agent's type), but also the value of that effort to the principal (principal's type) subject to i.i.d. shocks. When optimal effort is fully pooled across agent types for multiple principal types, it is also pooled across those principal types. When optimal effort separates some agent types for multiple principal types, efforts of those agent types may be separated across principal types. But then, somewhat perversely, some agent type's effort is decreasing in the principal's value of effort. When agent type is uniformly distributed, that applies to agent types with lower effort cost, so reducing the difference in effort between low and high effort cost types. This result extends to the principal's type being observed only by the principal if the marginal cost of effort to the agent is sufficiently convex.

Keywords: Relational incentive contracts, shocks, principal types, agent types
JEL codes: C73, D82, D86

1 Introduction

Relational incentive contracts have proved a fruitful way to study on-going economic relationships. See Malcomson (1999) for applications to employment, Malcomson (2013) for applications to supply relationships. Levin (2003) provides an elegant analysis of the implications of adding short-term shocks in an agent's cost of supplying effort to the basic relational incentive contract model in MacLeod and Malcomson (1989). But his analysis does not incorporate short-term shocks in the value of the agent's effort to the principal. This omission is important for practical applications because employers and downstream firms typically face fluctuations in the demand for their products. This paper aims to fill that gap.

In the basic relational incentive contract model, a principal employs an agent who provides effort to produce output. Neither effort nor output is verifiable by third parties, so there can be no legally enforceable performance-related pay. Incentives are instead provided by the (potentially infinite) repetition of the relationship — the gains from future continuation provide incentives for performance today. Levin (2003) calls this *dynamic enforcement*.

The hidden information model in Levin (2003) adds to this basic model i.i.d. shocks each period in the agent's cost of supplying effort (the agent's type) that are unobserved by the principal. Levin (2003) shows that, under specific assumptions discussed later, an optimal effort schedule takes one of three forms:

1. *first best:* effort for all agent cost types is first best;
2. *pooling:* effort is the same for all agent cost types;
3. *partial pooling:* there is a single interval of agent cost types (always including the lowest cost type) for which effort is the same; for other agent cost types, effort is strictly decreasing in cost; for all types, effort is below the first-best level.

The essential rationale is that, because there are only limited future gains from continuing the relationship with which to provide dynamic enforcement, the spread of rewards to the agent is limited. If the future gains are large enough, dynamic enforcement may permit first-best effort to be sustained for all agent types. But if the future gains are too small to permit that, it is optimal to pool some agent types. Under the assumptions in Levin (2003), the gain from separation is greatest for higher cost agent types, so pooling is of lower cost types.

The present paper adds to this model i.i.d. shocks each period in the value of the agent's effort to the principal (the principal's type). The implications are significant even when shocks to the principal are observed by the agent. When optimal effort for all agent types is pooled (case 2 above) for a set of principal types, it is also pooled across those principal types. Short-term shocks to the value of effort within that set have no effect on the agent's output even though it is efficient that they should. For agent types that are separated but not with first-best effort (case 3 above) for a set of principal types, those principal types are in general separated for some agent types. But, contrary to what one might expect, effort is *lower* for some principal type than for another principal type for which effort is *less* valuable. Thus separation of principal types is at the cost of less efficient effort for some types than if they were all pooled. For a uniform distribution of agent cost types, it is those with lower cost of effort who deliver less effort when effort is more valuable to the principal. Those with higher cost of effort deliver more effort.

These results have implications for employment and supply relationships. Employers and downstream firms typically face short-term shocks to demand. These shocks correspond to different principal types. If there is just one agent type, a demand shock has no effect on optimal agent effort as long as first-best effort remains unattainable — unless, that is, it is a negative shock sufficiently severe that first-best effort falls below the highest sustainable level. With first-best effort unattainable, a principal may respond to a positive shock by hiring more agents, or by increasing agent input along some verifiable dimension (such as overtime hours) but not by increasing agent input along an unverifiable dimension. Employment and/or overtime will then fluctuate more than they would if performance-related pay were legally enforceable.

With multiple agent types, any response to a positive shock results in *reduced* effort for some agent types, although increased effort for others. With uniformly distributed agent types, the reduction in effort is for lower cost agent types. But these have higher effort than higher cost agent types, so the difference in effort between the lowest and the highest cost agent types is reduced. Thus the interaction of multiple agent with multiple principal types results in practical implications that are significantly different than if no account is taken of short-term shocks faced by the principal.

The next section of the paper sets out the model. Section 3 analyses the limitations imposed by dynamic enforcement. Section 4 gives results on optimal relational contracts. Section 5 considers extensions of the basic model to the principal being privately informed about shocks to the value of the agent's effort and to those shocks not being i.i.d. Section 6 concludes. Proofs of propositions are in an appendix.

2 The model

The model is the same as the hidden information model in Levin (2003), with the addition of principal types that are i.i.d. draws each period.

The agent's type a_t in period t is an i.i.d. random draw from the distribution $F(a_t)$ with support $\left[\underline{a}, \overline{a}\right]$ and everywhere strictly increasing. It is privately observed by the agent. If in a relationship with the principal, the agent's payoff in period t is $W_t - c(e_t, a_t)$, where W_t is the payment to the agent in period t, $e_t \in [0, \overline{e}]$ is the agent's (non-verifiable) effort in period t and $c(e_t, a_t)$ is the cost to agent type $a_t \in \left[\underline{a}, \overline{a}\right]$ of providing that effort. If not in a relationship with the principal, the agent's payoff in period t is $\underline{u}$. The agent discounts the future with discount factor $\delta \in [0, 1)$.

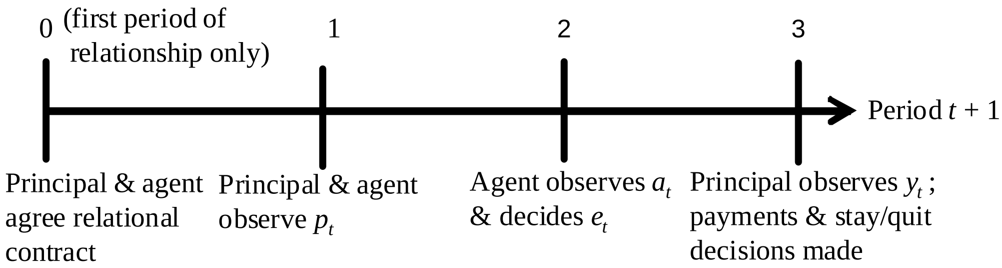

Figure 1: Timing of events in period t

The principal's type p_t in period t is an i.i.d. random draw from the set $P \subseteq \left[\underline{p}, \overline{p}\right]$ with distribution $G(p_t)$. This type is observed by both principal and agent. (Implications of it being observed only by the principal are discussed in Section 5.) If in a relationship with the agent, the principal's (unverifiable) payoff in period t is $y(e_t, p_t) - W_t$. If not in a relationship with the agent, the principal's payoff in period t is $\underline{\pi}$. The principal has the same discount factor δ as the agent.

Assumption 1 *For all $p \in P$ and $e \in [0, \overline{e}]$, $y(e, p)$ is: (1) strictly increasing in p for $e > 0$ with $y(0, p) = 0$, and (2) twice differentiable in e, with $y_1(e, p) > 0$ and strictly increasing with p, and $y_{11}(e, p) \leq 0$.*

For all $a \in \left[\underline{a}, \overline{a}\right]$, $c(0, a) = 0$ and, for all $e \in [0, \overline{e}]$, $c(e, a)$ is twice differentiable, with $c_2(e, a) > 0$ for $e > 0$, $c_1(e, a) > 0$, $c_{11}(e, a) \geq 0$, and $c_{12}(e, a) > 0$.

$y(e, p) - c(e, a)$ is strictly increasing in e for $e = 0$, strictly decreasing in e for $e = \overline{e}$, and strictly concave in e for all (p, a).

$\underline{s} \equiv \underline{\pi} + \underline{u} > 0$.

Assumption 1 is maintained throughout. The condition that $c_{12}(e, a) > 0$ ensures that the Spence-Mirrlees single crossing property holds. The other conditions ensure that first-best effort is strictly interior to $[0, \overline{e}]$ for all (p, a). That $\underline{s} > 0$ ensures that the relationship is not mutually beneficial if no output is produced.

The timing of events within each period is set out in Figure 1. Payments are as in Levin (2003). In each period t, there is a fixed payment w_t conditional only on the relationship being continued for period t by both parties at stage 3 of period $t - 1$ and on p_t. (For simplicity, the parties are assumed to be committed to paying w_t conditional on the p_t that occurs if the relationship is continued at stage 3 of period $t - 1$. This could be replaced without affecting the results in Sections 3 and 4 by the principal paying w_t between stages 1 and 2 of period t and the addition of constraints to ensure payment is incentive compatible.) There is also a bonus payment b_t that can be conditioned on the outcome at stage 3 of period t. None of these payments is restricted in sign or magnitude — there is no limited liability. Positive payments are from the principal to the agent, negative ones from the agent to the principal. To specify the history of the relationship at t, let $x^t = (x_1, x_2, \ldots, x_t)$ for any variable x_t. Then the public history at the end of period t is $h_t = \left(p^t, w^t, e^t, b^t\right)$. The agent's private history is a^t. Formally, a relational contract (denoted by C) is a pair of strategies determining w_t, e_t, and b_t, plus rules for continuing the relationship for each t, as functions of the history at t.

The joint payoff or surplus to the parties each period from being matched conditional on (e, p, a) is $s(e, p, a) = y(e, p) - c(e, a)$. First-best effort $e^*(p, a)$ maximizes this surplus and is given by

$$y_1\left(e^*(p, a), p\right) = c_1\left(e^*(p, a), a\right). \tag{1}$$

A relational contract cannot be enforced by going to court. It will, therefore, be carried out only if it is in the interest of both parties to do that, a characteristic referred to in the literature as *self-enforcing*. Levin (2003) formally defines a relational contract as "*self-enforcing* if it describes a perfect public equilibrium of the repeated game." The set of self-enforcing contracts is largest when the punishments for deviation are, as in Abreu (1988), the most severe available, which here corresponds to the deviating party receiving the future payoff that would result from the relationship ending.[1] Because the parties are risk neutral and can redistribute expected surplus in any way they like through the payment w_1 that is independent of effort, and thus has no incentive effect on effort, it is optimal for them to select an equilibrium contract that maximises expected surplus at the start of the relationship. Moreover, by a general argument in Levin (2003, Theorem 2) that applies to the model here, if an optimal contract exists, there are stationary contracts that are optimal. An optimal stationary contract depends only on current payoff-relevant information, so optimal effort and payment functions have the form $e_t = e(p_t, a_t)$, $w_t = w(p_t)$ and $b_t = b(p_t, e_t)$.

3 Self-enforcing contracts

The agent's type is not observed by the principal. So, for effort function $e(p, a)$ to be part of a self-enforcing contract, agent type a must prefer $e(p, a)$ to $e\left(p, a'\right)$ for $a' \neq a$. Conditions for this are standard in the literature on one-period models when output is verifiable, as a result of which the principal can commit to performance-related payments. In a stationary relational contract with the agent's type an i.i.d. draw each period, agent type a can mimic any other agent type in period t without consequences for payoffs in future periods. Thus those standard results transfer straightforwardly to such a relational contract conditional on the parties actually making the performance-related payments specified in it. Theorem 4 in Levin (2003) provides conditions for the parties to make those payments when there is only one principal type. The first result here is a straightforward extension of that theorem to multiple principal types that are observed by both the agent and the principal. To state it, let $s = E_{p,a}\left[s(e(p, a), p, a)\right]$ denote the expected surplus each period, before types are realized, from a relationship with

[1] As Levin (2003) notes, this does not require the relationship actually to end. Equally effective is for the continuation equilibrium following such a deviation to be one in which the deviating party receives payoff equal to its outside payoff.

effort function $e(p,a)$. For stationary relational contract C, let

$$u(p,a) = w(p) + b(p, e(p,a)) - c(e(p,a), a), \quad \text{for all } (p,a). \tag{2}$$

Then $u = E_{p,a}\left[u(p,a)\,|C\right]$ and $\pi = s - u$ are the expected payoffs each period of the agent and the principal respectively if both adhere to the contract.

Proposition 1 (Levin (2003), Theorem 4 extended) *An effort schedule $e(p,a)$ that generates expected surplus s can be implemented by a stationary relational contract if and only if (i) $e(p,a)$ is non-increasing in a for all (p,a) and (ii)*

$$\frac{\delta}{1-\delta}\left(s - \underline{s}\right) \geq c\left(e\left(p, \underline{a}\right), \underline{a}\right) + \int_{\underline{a}}^{\overline{a}} c_2(e(p,\tilde{a}), \tilde{a})\, d\tilde{a}, \quad \textit{for all } p \in P. \tag{3}$$

To see why conditions (i) and (ii) arise, consider first a one-period model with verifiable output. For that case, a necessary and sufficient condition for there to exist a contract for which an agent type prefers the effort specified for it to that specified for any other agent type is $e(p,a)$ non-increasing in a for all (p,a), see Laffont (1989, Chapter 10).[2] Moreover, from a general result in Milgrom and Segal (2002, Corollary 1), for any such contract the agent's payoff satisfies

$$u(p,a) = u(p,\overline{a}) + \int_{a}^{\overline{a}} c_2(e(p,\tilde{a}), \tilde{a})\, d\tilde{a}, \quad \text{for all } (p,a). \tag{4}$$

With verifiable output, deviation to an effort not designated in the contract for any agent type is deterred by a sufficiently severe monetary penalty. With the relational contract model used here, output is unverifiable so deviation to such "off the equilibrium path" efforts must be deterred in some other way. The most severe penalty available is for the agent to receive payment $w(p)$, but no bonus, for the current period and payoff $\underline{u}$ for all future periods. Then the most attractive deviation is $e = 0$ because that incurs no cost of effort. Deterrence of such deviations thus corresponds to

$$u(p,a) + \frac{\delta}{1-\delta}\left(u - \underline{u}\right) \geq w(p), \quad \text{for all } (p,a). \tag{5}$$

Also with output unverifiable, performance-related payments cannot be legally enforced, so it must be in the interests of the parties to pay them. Specifically, it must be in the interest of principal type p to continue the relationship, including paying positive bonuses when specified by the contract, and of agent type a to continue the relationship, including paying negative bonuses when specified by the contract. Conditions for these are

$$\frac{\delta}{1-\delta}\left(\pi - \underline{\pi}\right) \geq \max\left[0, b(p, e(p,a))\right], \quad \text{for all } (p,a), \tag{6}$$

$$\frac{\delta}{1-\delta}\left(u - \underline{u}\right) \geq \max\left[0, -b(p, e(p,a))\right], \quad \text{for all } (p,a). \tag{7}$$

[2] The proof in Laffont (1989) assumes $e(p,a)$ is continuously differentiable in a but the result is more general, see Milgrom and Segal (2002, Footnote 10).

Finally, the future surplus from continuing the relationship must be sufficient to ensure that conditions (4), (5), (6) and (7) can all be satisfied. Adding the agent's future payoff gain $\delta\left(u-\underline{u}\right)/(1-\delta)$ to both sides of (2), noting that $u=s-\pi$ and $\underline{u}=\underline{s}-\underline{\pi}$, and substituting for $u(p,a)$ from (4) gives

$$\begin{aligned} u(p,\overline{a})+\int_{a}^{\overline{a}} c_2(e(p,\tilde{a}),\tilde{a})\,d\tilde{a}+\frac{\delta}{1-\delta}\left(u-\underline{u}\right) \\ = w(p)+b(p,e(p,a))-c(e(p,a),a) \\ +\frac{\delta}{1-\delta}\left(s-\underline{s}-\pi+\underline{\pi}\right), \quad \text{for all } (p,a). \quad (8)\end{aligned}$$

Substitution from (5) for $a=\overline{a}$ and (6) into (8) then gives the requirement

$$\frac{\delta}{1-\delta}\left(s-\underline{s}\right)\geq c(e(p,a),a)+\int_{a}^{\overline{a}} c_2(e(p,\tilde{a}),\tilde{a})\,d\tilde{a}, \quad \text{for all } (p,a). \qquad (9)$$

With $c_1>0$ and $e(p,a)$ non-increasing in a, this condition is tightest for $a=\underline{a}$. That implies condition (3) in the proposition is necessary. The proof in the appendix shows that, if the conditions in the proposition are satisfied, there exists a stationary payment schedule such that all incentive constraints are satisfied.

Condition (3) is central to the analysis that follows. The underlying economics is that, to induce the parties to a contract that is not legally enforceable to carry it out, they must gain sufficiently from having the relationship continue. The joint gain from having the relationship continue is the surplus it generates over what the parties would get if they separated, which is the left-hand side of (9). For a contract to be self-enforcing, that joint gain must be sufficient to provide the incentives required for each agent type to choose the effort specified in the contract given by the right-hand side of (9). That condition is most stringent for agent type $\underline{a}$ who, because effort must be non-increasing in type, is to choose the highest effort. Thus the critical constraint in (9) is that for $\underline{a}$, which is just (3).

4 Optimal contracts

It follows from Proposition 1 that optimal effort solves

$$\begin{aligned} &\max_{e(.,.)} s=E_{p,a}\left[s(e(p,a),p,a)\right] \text{ subject to} \\ &\frac{\delta}{1-\delta}\left(s-\underline{s}\right)\geq c\big(e\big(p,\underline{a}\big),\underline{a}\big)+\int_{\underline{a}}^{\overline{a}} c_2(e(p,\tilde{a}),\tilde{a})\,d\tilde{a}, \text{ for all } p, \qquad (10) \\ &e(p,a) \text{ non-increasing in } a, \text{ for all } (p,a). \end{aligned}$$

An immediate consequence is that, because first-best effort $e^*(p,a)$ given by (1) is decreasing in a, optimal effort for given p is $e(p,a)=e^*(p,a)$ unless (10) is binding for p. The next proposition gives some other consequences.

Proposition 2 *Suppose the first-best effort schedule, $e^*(p,a)$ for all $a \in [\underline{a}, \overline{a}]$, is not attainable for some $p \in P$.*

1. *The first-best effort schedule is not attainable for any $p' \in P$ with $p' > p$.*
2. *If the first-best effort schedule is not attainable for $p' \in P$, $e(p,a) > e(p',a)$ optimal for all a in some interval implies $e(p,a') < e(p',a')$ optimal for some a' outside that interval.*
3. *Let $P' \subseteq P$ denote the set of p for which optimal $e(p,a)$ is independent of a for all $a \in [\underline{a}, \overline{a}]$. Then $e(p,a)$ is independent of (p,a) for all $p \in P'$ and $a \in [\underline{a}, \overline{a}]$.*

Proposition 2 follows from the constraint (10), which is just condition (3) for a contract to be self-enforcing. The left-hand side of (10) is the future surplus from continuing the relationship over what the parties would get if they separated. It is this that limits the efforts that are self-enforcing for all a. It is independent of the current value of p because p is an i.i.d. draw each period, so there is a fixed budget for inducing effort for different agent types that is the same for all p. With $c_1, c_{12} > 0$, the right-hand side of (10) is increasing in effort for given (p,a). So if more of the surplus is used to induce effort from some a for p than for p', there is less of it to use for inducing effort for other a for p than for p'. Part 1 follows because first-best effort is increasing in p so, if (10) is binding for p, it is certainly binding for $p' > p$. Moreover, when (10) is a binding constraint, higher effort for p than for p' for some agent type implies lower effort for p than for p' for some other agent type, as stated in Part 2. An implication is that, although first-best effort is everywhere increasing in p, optimal effort is not. Part 3 states that, if effort is pooled over all a for different p, it is also pooled over these p. It follows because, with pooling over all a, the integral on the right-hand side of (10) takes the value $c(e(p,\overline{a}), \overline{a}) - c(e(p,\underline{a}), \underline{a})$. With the left-hand side independent of p and complete pooling over a, this implies $e(p,a)$ the same for these p.

In oligopoly models with demand shocks such as Rotemberg and Saloner (1986) and Hanazono and Yang (2007), legally unenforceable (collusive) relationships are also sustained by the gains from continuing the relationship in the future. In those models, high demand increases the short-run gain from reneging on a given collusive price so, when demand shocks are i.i.d., equilibrium price needs to be lower to reduce that gain. In contrast, here the incentive to renege on a given effort schedule for a is independent of p — the combined incentive compatibility condition (10) depends on p only through $e(p,a)$. However, the incentive to renege on the first-best effort schedule, which shifts upwards with p, is increasing with p. It is this that drives Part 1 of Proposition 2.

Proposition 2 has implications for the response to shocks. Suppose there is just one type of agent which, without loss of generality, can be taken as $\underline{a} = \overline{a}$. Then (10) implies that effort is the same for all principal types for which first-best effort is unattainable. Moreover, if first-best effort is unattainable for type p, effort is the same for all $p' > p$ as for p. Then a favourable shock has no effect on the agent's output. In contrast, an unfavourable shock reduces the agent's output if it is sufficiently severe to correspond to a p for which first-best effort is attainable. With multiple agent types, Part 2 of Proposition 2 shows that, if optimal effort is not pooled across all principal types, then it is decreasing in p for some agent type even though higher p corresponds to higher value of effort. Thus separation of principal types is at the cost of less efficient effort for some type pairs than if they were all pooled. So an obvious question is whether it is ever optimal to separate principal types when the first-best effort schedule is unattainable. To explore this, consider the following assumptions.

Assumption 2 (Levin (2003)) *$P \subset \left[\underline{p}, \overline{p}\right]$ consists of a finite number of discrete points. For all $a \in \left[\underline{a}, \overline{a}\right]$,*

1. *$F(a)$ is concave and admits a density;*
2. *$c_{112}(e, a), c_{122}(e, a) \geq 0$ for all $e \in [0, \overline{e}]$;*
3. *optimal $e(p, a)$ is continuous in a for all $p \in P$.*

The finite number of possible values for p allows the use of the optimal control techniques used in Levin (2003). The requirements (1)–(3) are used in Levin (2003) to derive the results in the Introduction. These ensure that the optimal contracting problem is concave and would ensure full separation of agent types in a one-period model with verifiable effort.

Consider the relaxed optimal contract problem without the constraint $e(p, a)$ non-increasing in a. Optimal effort for this problem, $e^R(p, a)$, satisfies the first-order condition with respect to $e(p, a)$

$$s_1\left(e^R(p, a), p, a\right) dG(p)\, F'(a) \left(1 + \frac{\delta}{1-\delta} \sum_{\tilde{p} \in P} \mu(\tilde{p})\right) - \mu(p)\, c_{12}\left(e^R(p, a), a\right) = 0, \text{ for } a \in \left[\underline{a}, \overline{a}\right], p \in P, \quad (11)$$

where $\mu(p)$ is a non-negative Lagrange multiplier attached to constraint (10) for p. (The multipliers $\mu(\tilde{p})$ for $\tilde{p} \neq p$ also enter because increasing the surplus s relaxes the constraint (10) for all p.) With Assumptions 1 and 2, $e^R(p, a)$ is unique for given $\mu(p)$ and decreasing in a. Moreover, $e^R(p, a) < e^*(p, a)$ for all a for p such that $\mu(p) > 0$ because $e^*(p, a)$ satisfies $s_1(e^*(p, a), p, a) = 0$.

Proposition 3 (Levin (2003), Theorem 5 extended) *Suppose Assumptions 2 holds. If a relationship is potentially mutually beneficial, the optimal effort schedule $e(p, a)$ takes one of three forms for each $p \in P$:*

1. *Pooling:* $e(p, a)$ *is the same for all* $a \in [\underline{a}, \overline{a}]$.

2. *Partial pooling: For some* $\hat{a}(p) \in (\underline{a}, \overline{a})$,

$$e(p, a) = \begin{cases} e^R(p, \hat{a}(p)), & \text{for } a \in [\underline{a}, \hat{a}(p)), \\ e^R(p, a), & \text{for } a \in [\hat{a}(p), \overline{a}], \end{cases}$$

where $\hat{a}(p)$ *satisfies, for some* $\mu(p) > 0$ *and* $\mu(\tilde{p}) \geq 0$ *for all* $\tilde{p} \in P$,

$$\int_{\underline{a}}^{\hat{a}(p)} s_1(e^R(p, \hat{a}(p)), p, a)\, F'(a)\, da = \frac{\mu(p)}{\left(1 + \frac{\delta}{1-\delta} \sum_{\tilde{p} \in P} \mu(\tilde{p})\right) dG(p)} c_1(e^R(p, \hat{a}(p)), \hat{a}(p)). \quad (12)$$

3. *First best:* $e(p, a)$ *is first best for all* $a \in [\underline{a}, \overline{a}]$.

In either second-best scenario, $e(p, a) < e^*(p, a)$ *for all* $a \in [\underline{a}, \overline{a}]$.

The proof of Proposition 3 (in an online appendix) is essentially the same as in Levin (2003) but extended to multiple principal types. The intuition is also the same. For any type pair (p, a) for which optimal effort is strictly decreasing in a, the requirement that optimal effort is non-increasing in a is not a binding constraint. For any such type pair, therefore, optimal effort can be chosen pointwise, so $e(p, a) = e^R(p, a)$ given by (11), which satisfies the requirement that effort is non-increasing in a. If (10) is not binding for p, $\mu(p) = 0$ and (11) then implies effort for that p is first best. But if (10) is not satisfied by the first-best effort schedule for p, $\mu(p) > 0$ so (11) implies effort strictly less than first best. In that case, as explained in Levin (2003), it is optimal to pool agent types with low cost of effort when effort cost satisfies Assumption 2. Then the cutoff $\hat{a}(p)$ between pooled and separated types satisfies (12) if there exists an $\hat{a}(p) \in (\underline{a}, \overline{a})$ that does this for $\mu(\tilde{p})$ such that (10) holds with equality for all $\tilde{p} \in P$. If there is no such $\hat{a}(p)$, all agent types are pooled for that p.

The first possibility in Proposition 3 is that optimal effort pools all agent types for some principal types. From Part 3 of Proposition 2, optimal effort then also pools over those principal types. The next result considers optimal effort that separates some agent types for some principal types, the second possibility in Proposition 3.

Proposition 4 *Under Assumption 2, suppose* $y(e, p) = pe$ *and* $c(e, a) = a\hat{c}(e)$, *with* $\underline{p}, \underline{a} > 0$. *Suppose also there exists an agent type* $\hat{a} < \overline{a}$ *at which optimal effort* $e(p, a)$ *is strictly decreasing for two distinct principal types* p' *and* p'' *satisfying* $p' < p''$. *Consider* a' *and* a'' *such that* $\hat{a} \leq a' < a'' \leq \overline{a}$. *Then the optimal effort schedule satisfies*

$$e(p', a'') \leq e(p'', a'') \text{ implies } e(p', a') < e(p'', a'); \quad (13)$$

$$e(p', a') \geq e(p'', a') \text{ implies } e(p', a'') > e(p'', a''). \quad (14)$$

Furthermore, $e(p', a') < e(p'', a')$ *implies*

$$e(p', a'') < e(p'', a'') ; \tag{15}$$

$$p''e(p'', a') - p'e(p', a') > p''e(p'', a'') - p'e(p', a'') , \text{ if } \hat{c}'''(e) \leq 0 \text{ for } e \in [0, \overline{e}] . \tag{16}$$

Proposition 4 shows that it is optimal to have some separation among principal types for which the first-best effort schedule cannot be attained when output and cost of effort are multiplicative in principal and agent type, respectively. (Conditional on that, effort can without loss of generality be measured so that output is pe, as in the economically important case in which p is the price at which the product of effort is sold.) Under these conditions, Part 2 of Proposition 2 is not empty — there really are cases in which optimal effort decreases with principal type even though the value of that effort increases with principal type. Proposition 4 also establishes that the effort schedules for different p have a single-crossing property, the difference between them retaining the same sign for agent types above some level and gives results on how the difference in output $pe(p, a)$ for different p changes with a.

The argument underlying Proposition 4 is as follows. By Part 2 of Proposition 3, if optimal effort is strictly decreasing in a for given p, it is given by $e^R(p, a)$ defined in (11). Moreover, it is strictly decreasing in a for that p at all higher a. So optimal effort is given by (11) for all $a \in [\hat{a}, \overline{a}]$. Thus it must be that

$$\frac{s_1(e(p, a) , p, a) \, F'(a)}{c_{12}(e(p, a) , a)} = \frac{s_1(e(p, a'') , p, a'') \, F'(a'')}{c_{12}(e(p, a'') , a'')}, \text{ for } a \in [\hat{a}, a''] , p \in \{p', p''\} . \tag{17}$$

That (13) and (14) hold follows from use in (17) of the functional forms specified in the proposition. The results (15) and (16) follow from manipulation of (11).

Proposition 4 has the following implications for agent types for which effort is strictly decreasing in a. If optimal effort is decreasing in p at $\hat{a}$, it is decreasing in p for all higher a. If, on the other hand, optimal effort is increasing in p at $\hat{a}$, it is increasing in p for all lower a. Moreover, if optimal effort is increasing in p at $\hat{a}$, it is also increasing in p for all higher a and the difference in output as p increases decreases with a.

A uniform distribution of agent types satisfies 2 of Assumption 2. The next result is more specific about how the optimal effort schedule changes with p for that case.

Proposition 5 *Under Assumption 2, suppose* $y(e, p) = pe$, $c(e, a) = a\hat{c}(e)$, *with* $\underline{p}, \underline{a} > 0$, *and* $F(a)$ *is uniform. Then, if there is partial pooling of agent types for* p' *and for* $p'' > p'$*:*

1. $\hat{a}(p') < \hat{a}(p'')$*;*

2. $e(p', a) > e(p'', a)$ *for* $a \in [\underline{a}, \hat{a}(p')]$*;*

3. $e(p', a) < e(p'', a)$ *for* $a \in [\hat{a}(p''), \overline{a}]$.

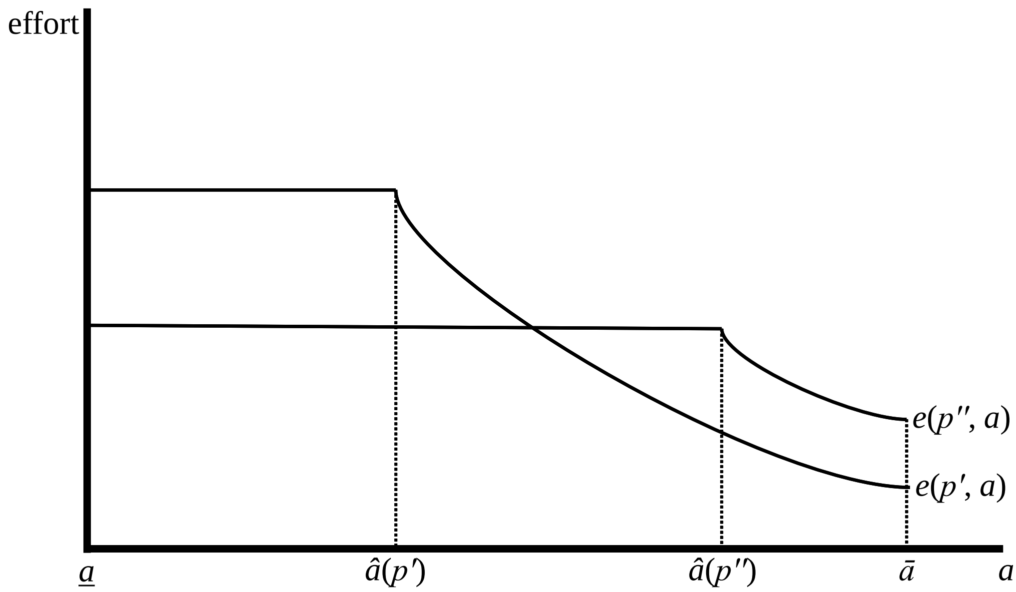

Figure 2: Effort for $p'' > p'$ when agent type is uniformly distributed

Proposition 5 establishes that the single-crossing property in Proposition 4 takes the form illustrated in Figure 2. Recall that $e(p, a)$ is constant for $a \in [\underline{a}, \hat{a}(p)]$. For $p'' > p'$, effort is more valuable to the principal. Then more of the agent cost types with lower cost of effort are pooled for p'' than for p' because, by Part 1, $\hat{a}(p'') > \hat{a}(p')$. For agent cost types $a \leq \hat{a}(p')$, effort is lower for p'' even though it is more valuable to the principal. (This does not imply that output $pe(p, a)$ is lower because $p'' > p'$, nor that earnings are lower.) In contrast, agent cost types $a \geq \hat{a}(p'')$ have higher effort when effort is more valuable to the principal. The crossover point is between $\hat{a}(p')$ and $\hat{a}(p'')$. Thus, in this case, the general result from Proposition 2 takes the form that effort more valuable to the principal results in less effort for agent types with lower cost of effort and more effort for those with higher cost of effort. So a positive shock to productivity reduces the range of effort across agent types. It does not however necessarily reduce the dispersion in output $pe(p, a)$. With p'', output is the same for all $a \in [\underline{a}, \hat{a}(p'')]$, so it is obvious from Figure 2 that dispersion of output over this interval is less than for p'. But, from (16) in Proposition 4 applied to $a = \hat{a}(p'')$, the difference in output between $\hat{a}(p'')$ and $a'' \in (\hat{a}(p''), \bar{a}]$ is greater with p'' than with p' when $\hat{c}'''(e) \leq 0$.

5 Extensions

This section considers two extensions of the model analysed in the preceding sections. The first is to p_t being observed only by the principal. The second is to persistence in the shocks p_t.

5.1 Only principal observes p_t

For this extension the timing of events for period t in Figure 1 is changed so that, at stage 1, the principal privately observes p_t and claims to the agent that its value is $\hat{p}_t$ not necessarily equal to p_t. Corresponding to the assumption in the preceding sections that the parties are committed to paying w_t conditional on the p_t that occurs if the relationship is continued at stage 3 of period $t-1$, they are now committed to paying w_t conditional on $\hat{p}_t$. For truthful revelation of p_t, the relational contract must then be such that it is in the principal's interest to announce $\hat{p}_t = p_t$. The following result extends Proposition 1 to this case when p_t has interval support.

Proposition 6 *Suppose $G(p)$ is everywhere strictly increasing on $P = \left[\underline{p}, \overline{p}\right]$ and the realization p_t is information private to the principal. Then, for an effort schedule $e(p,a)$ that generates expected surplus s to be implemented by a stationary relational contract, it is necessary that conditions (i) and (ii) of Proposition 1 hold and, for all $p \in \left[\underline{p}, \overline{p}\right]$,*

$$\int_{\underline{a}}^{\overline{a}} y_2\left(e\left(\hat{p}, a\right), p\right) dF(a) \text{ is non-decreasing in } \hat{p} \tag{18}$$

when evaluated at $\hat{p} = p$. It is sufficient that conditions (i) and (ii) of Proposition 1 hold and that (18) holds for all $\hat{p}, p \in \left[\underline{p}, \overline{p}\right]$.

Condition (18) in Proposition 6 is the incentive compatibility condition for the principal that corresponds to the incentive compatibility condition for the agent that $e(p,a)$ is non-increasing in a. It does not, however, require $e(p,a)$ non-decreasing in p because a_t is unknown at the time the principal announces $\hat{p}_t$ and so the principal decides what to announce on the basis of the expected payoff given the distribution $F(a)$. Thus it is not inconsistent with $e(p,a)$ decreasing in p for some $p \in P$ which, as established in Part 2 of Proposition 2, is required to satisfy incentive compatibility for the agent if the first-best effort schedule is not attainable and $e(p,a)$ is not to be independent of p. Indeed, since the proof of Proposition 2 depends only on (3) being a binding constraint, the results in that proposition extend to the case in which p is information private to the principal for any set of p for which (3) is a binding constraint on optimal effort choice.

For Proposition 6, it is assumed that the parties are committed to paying $w\left(\hat{p}_t\right)$ conditional on announcement $\hat{p}_t$, which requires that neither can quit the relational contract without further payment immediately after $\hat{p}_t$ is announced. An alternative very much in the spirit of relational contracts is that $w\left(\hat{p}_t\right)$ is paid between stages 1 and 2 of period t but that either party can quit the relationship until $w\left(\hat{p}_t\right)$ is paid, though still forgoing the payoffs $\underline{u}$ and $\underline{\pi}$ for period t as a result of deciding to continue the relationship at stage 3 of period $t-1$. The relational contract must then ensure that both parties prefer to continue the relationship immediately after $\hat{p}_t$ is announced. This is not restrictive when p_t is observed by both parties. But it may be when p_t is observed only by the principal because of the rent the principal must receive to induce truthful revelation. The next result concerns this formulation.

Proposition 7 *Suppose $y(e, p) = p\hat{y}(e)$, with $\underline{p} > 0$. Then Proposition 6 holds even if either party can quit the relationship without further payment after $\hat{p}_t$ is announced.*

Whether Proposition 3 (and Propositions 4 and 5 that depend on it) can be extended to the case in which only the principal observes p_t is more problematic. One approach is to treat p_t as a draw from the whole interval $\left[\underline{p}, \overline{p}\right]$ and impose the additional constraints on the optimal control problem. But that gives rise to an optimal control problem with state variables in the two dimensions p and a that is not covered by the standard results used in Levin (2003).

An alternative adopted here is to look for conditions under which the solution to the problem without the additional constraints in fact satisfies those constraints. Proposition 6 applies to an interval of types p, not the finite number used to apply standard optimal control techniques in the proof of Proposition 3. However, the sufficient conditions for principal incentive compatibility for P an interval of types used in Proposition 6 and expressed in terms of (18) are also sufficient for a finite number of distinct types drawn from the same interval $\left[\underline{p}, \overline{p}\right]$. If $\hat{p} = p$ is incentive compatible for type p when any $\hat{p} \in \left[\underline{p}, \overline{p}\right]$ can be chosen, it remains incentive compatible for type p when only a subset of $\left[\underline{p}, \overline{p}\right]$ can be announced. This approach can be used to establish the following result.

Proposition 8 *Suppose there is partial pooling of agent types for all $p \in P$. Then Proposition 5 holds when the principal's type is private information if the cost of effort function $\hat{c}(e)$ has the property*

$$\frac{\hat{c}'(e)\,\hat{c}'''(e)}{2\hat{c}''(e)} \geq 1, \text{ for all } e \in [0, \overline{e}]\,. \tag{19}$$

Condition (19) requires that the third derivative of the cost of effort function $\hat{c}(e)$ is sufficiently large. The reason this condition is sufficient can be seen as follows. With partial pooling of agent types for all $p \in P$ as in Proposition 5, the constraint (10) holds with equality for all p. The left-hand side of (10) is independent of p. From Figure 2, $e\left(p, \underline{a}\right)$ is decreasing in p and, hence, so is $c\left(e\left(p, \underline{a}\right), \underline{a}\right)$. Thus the integral in (10) must be increasing in p. For continuous p that would, with $c(e, a) = a\hat{c}(e)$ and the notation $\psi(a; p) = \hat{c}'(e(p, a)) \,/ \int_{\underline{a}}^{\overline{a}} \hat{c}'(e(p, a))\, da$, imply

$$\int_{\underline{a}}^{\overline{a}} e_1(p, a)\, \psi(a; p)\, da > 0, \text{ for all } p. \tag{20}$$

Under the conditions of Proposition 5, $y(e, p) = pe$ and $F(a)$ is uniform, so (18) becomes

$$\int_{\underline{a}}^{\overline{a}} e_1(p, a)\, \frac{1}{\overline{a} - \underline{a}} da \geq 0, \text{ for all } p. \tag{21}$$

Comparing the two conditions (20) and (21) corresponds to comparing the expected value of given $e_1(p, a)$ over a for given p for the distribution $\psi(a; p)$ and the uniform distribution. From Figure 2, $e(p, a)$ is non-increasing for all a, and strictly decreasing for some a, and so

$\hat{c}'(e(p,a))$, and hence $\psi(a;p)$, have that same property. Thus the distribution in (21) stochastically dominates that in (20) in the second-order sense. So, by a standard result, the expression on the left-hand side of (21) is larger than that on the left-hand side of (20), and hence condition (18) is satisfied, if $e_1(p,a)$ is non-decreasing for all a and strictly increasing for some a. Condition (19) is sufficient to ensure this is the case. Moreover, although sufficient, it is far from necessary — it ensures that just one of the two positive terms in $e_{12}(p,a)$ dominates the single negative term. Thus it is plausible that the same conclusion holds under a wider set of functions for the cost of effort.

5.2 Persistence in p_t

This extension has the distribution of p_t changed to $G(p_t|p_{t-1})$ when p_t is observed by the agent as well as the principal. Then the left-hand side of (3) is not independent of the value of p on the right-hand side. Because the proof of Proposition 2 is based entirely on the constraint (3), Part 1 continues to hold as long as persistence in p is not so strong that the left-hand side of (3) increases with p faster than the right-hand side when $e(p,a) = e^*(p,a)$ for all (p,a). But Parts 2 and 3 do not then follow directly from (3) being a binding constraint. Moreover, the implications for Propositions 3, 4 and 5 are not straightforward. Further research is needed to establish conditions under which these continue to hold.

6 Conclusion

The results obtained here extend the relational incentive contract model in Levin (2003) with i.i.d. shocks each period in the agent's cost of effort (agent type) to the principal also facing i.i.d. shocks each period in the value of the agent's effort (principal type).

In relational incentive contract models, effort and output are unverifiable, so legal enforcement of performance pay is not available. It is replaced by dynamic enforcement that relies on the parties not wanting the relationship to end because of the gains from continuing it in the future. Because those future gains are limited, dynamic enforcement imposes constraints on the efforts that can be sustained for each pair of types. Obviously, if first-best effort for every type pair satisfies those constraints, it is optimal. The interesting cases for dynamic enforcement are, therefore, when first-best effort is not attainable for every type pair.

The main results derived for that case are as follows. If first-best effort is not attainable for a principal type, it is not attainable for any principal type for which agent effort is more valuable. For such types, effort for a given principal type is either (1) the same for all agent types or (2) the same for an interval of those agent types with the lowest cost of effort and strictly decreasing in agent type for all higher cost types. For principal types to which the first of these applies, effort for all agent types is the same — effort is pooled across those principal types. For principal types to which the second applies, effort generally differs between principal types for some agent type for which effort is strictly decreasing in type. In particular, effort must (somewhat perversely) be lower for some agent type when the principal values effort more highly. For agent cost types uniformly distributed, it is those with lower cost of effort who deliver less effort when effort is more valuable to the principal. Those with higher cost of effort deliver more effort. This conclusion holds even when the principal's type is observed only by the principal if the marginal cost of effort to the agent is sufficiently convex. The interaction of multiple principal types with multiple agent types studied here thus has implications significantly different than if no account is taken of short-term shocks faced by the principal.

Appendix A Proofs

Proof of Proposition 1. Necessity follows from the argument in the text. Sufficiency is shown by specifying payment schedules $w(p)$ and $b(p,e)$ that ensure (4), (5), (6) and (7) are satisfied for any $e(p,a)$ that is (i) non-increasing in a for all (p,a) and (ii) satisfies (3). For given $b(p,0)$ for each p, set $b(p,\tilde{e}) = b(p,0)$ for $\tilde{e} \notin \{e(p,a)\}_{a\in[\underline{a},\overline{a}]}$ and

$$b(p,e(p,\overline{a})) = b(p,0) + c(e(p,\overline{a}),\overline{a}) \geq b(p,0)$$
$$b(p,e(p,a)) = b(p,0) + c(e(p,a),a) + \int_a^{\overline{a}} c_2(e(p,\tilde{a}),\tilde{a})\,d\tilde{a}, \text{ for all } a \leq \overline{a}.$$

With $e(p,a)$ non-increasing in a and $c_1 > 0$, $b(p,e(p,a))$ is non-increasing in a. Consider $b(p,0) \leq 0$ such that $b\big(p,e\big(p,\underline{a}\big)\big) \geq 0$ for each p.

By construction of $b(p,e(p,a))$, (4) is satisfied for all $a \in \big[\underline{a},\overline{a}\big]$ for each p given (2). Moreover, $u(p,a)$ is non-increasing in a, so (5) is satisfied for all $a \in \big[\underline{a},\overline{a}\big]$ if it is satisfied for $\overline{a}$. So from (2), with the bonuses specified, (5) is satisfied if

$$\frac{\delta}{1-\delta}\left(u-\underline{u}\right) \geq -b(p,0), \quad \text{for all } p. \tag{A.1}$$

(7) is satisfied if (A.1) is satisfied because $b(p,0) \leq 0$ and $b(p,0) \leq b(p,e(p,\overline{a}))$. Moreover, with $b\big(p,e\big(p,\underline{a}\big)\big) \geq 0$ the highest bonus given p, (6) can be replaced by

$$\frac{\delta}{1-\delta}\left(\pi-\underline{\pi}\right) \geq b(p,0) + c\big(e\big(p,\underline{a}\big),\underline{a}\big) + \int_{\underline{a}}^{\overline{a}} c_2(e(p,\tilde{a}),\tilde{a})\,d\tilde{a}, \quad \text{for all } p. \tag{A.2}$$

The left-hand and right-hand sides of (A.1) and (A.2) sum to the left-hand and right-hand sides of (3), respectively. So when (3) holds, there always exists a $b(p,0)$ satisfying the conditions specified that also satisfies both (A.1) and (A.2) for any non-negative $\left(u-\underline{u}\right)$ and $\left(\pi-\underline{\pi}\right)$ that sum to $\left(s-\underline{s}\right)$. A particular u is selected by setting

$$E_p\left[w(p)\right] = u - E_{p,a}\left[b(p,e(p,a)) - c(e(p,a),a)\right].$$

■

Proof of Proposition 2. First-best effort $e^*(p,a)$ is decreasing in a. Thus if it is unattainable, it must be because (10) is a binding constraint. Moreover, the left-hand side of (10) is independent of p and, with $c_1, c_{12} > 0$, its right-hand side is increasing in $e(p,a)$ for given (p,a). The rest of the proof uses this in parts that correspond to the parts of the proposition.

1. The first-best effort schedule $e^*(p,a)$ is increasing in p. Thus, if (10) is not satisfied by $e^*(p,a)$, it is certainly not satisfied by $e^*\left(p',a\right)$ with $p' > p$.

2. If the first-best effort schedule is not attainable for p and p', the right-hand side of (10) for p' must equal that for p. With that right-hand side increasing in $e(p,a)$ for given (p,a), to have $e(p,a) > e\left(p',a\right)$ for some interval requires $e\left(p,a'\right) < e\left(p',a'\right)$ for some a' outside that interval.

3. For any p for which $e(p,a)$ is the same for all a, the integral on the right-hand side of (10) equals $c(e(p,\overline{a}),\overline{a}) - c\left(e\left(p,\underline{a}\right),\underline{a}\right)$, so the right-hand side of (10) equals $c(e(p,\overline{a}),\overline{a})$ for all $p \in P'$. Pooling over all a for some p is optimal only if the first-best effort schedule is not attainable for that p, so (10) binds for all $p \in P'$. Its left-hand side is independent of p, so the right-hand side must take the same value for all $p \in P'$, which implies $c(e(p,\overline{a}),\overline{a})$, and hence $e(p,\overline{a})$, is the same for all $p \in P'$. But then, with $e(p,a)$ the same for all a for $p \in P'$, $e(p,a)$ must be the same for all $a \in \left[\underline{a},\overline{a}\right]$ for all $p \in P'$.

■

Proof of Proposition 4. From Proposition 3, if optimal $e(p,a)$ is strictly decreasing in a at $\hat{a}$ for given p, it is also strictly decreasing for all $a \in \left[\hat{a},\overline{a}\right]$ for that p. Moreover, for (p,a) for which optimal $e(p,a)$ is strictly decreasing in a, $e(p,a) = e^R(p,a)$ defined in (11). For the functional forms specified in the proposition, $s(e,p,a) = pe - a\hat{c}(e)$, so $s_1(e,p,a) = p - a\hat{c}'(e)$, and $c_{12}(e,a) = \hat{c}'(e)$. Defining

$$\hat{\mu}(p) \equiv \frac{\mu(p)}{\left(1 + \frac{\delta}{1-\delta}\sum_{\tilde{p}\in P}\mu(\tilde{p})\right)dG(p)}$$

for notational convenience in (11), these imply

$$\left[p - a\hat{c}'(e(p,a))\right]F'(a) - \hat{\mu}(p)\,\hat{c}'(e(p,a)) = 0, \quad \text{for } a \in \left[\hat{a},\overline{a}\right], p \in \left\{p',p''\right\}. \quad \text{(A.3)}$$

So, for $a'' \in \left(\hat{a},\overline{a}\right]$,

$$\left[\frac{p}{\hat{c}'(e(p,a'))} - a'\right]F'\left(a'\right) = \left[\frac{p}{\hat{c}'(e(p,a''))} - a''\right]F'\left(a''\right), \text{ for } a' \in \left[\hat{a},a''\right], p \in \left\{p',p''\right\},$$

and

$$p\left[\frac{F'\left(a'\right)}{\hat{c}'(e(p,a'))}-\frac{F'\left(a''\right)}{\hat{c}'(e(p,a''))}\right]=a'F'\left(a'\right)-a''F'\left(a''\right),\ \text{for } a'\in\left[\hat{a},a''\right),\ p\in\left\{p',p''\right\}.$$

From this,

$$p'\left[\frac{F'\left(a'\right)}{\hat{c}'(e(p',a'))}-\frac{F'\left(a''\right)}{\hat{c}'(e(p',a''))}\right]=p''\left[\frac{F'\left(a'\right)}{\hat{c}'(e(p'',a'))}-\frac{F'\left(a''\right)}{\hat{c}'(e(p'',a''))}\right],\ \text{for } a'\in\left[\hat{a},a''\right),$$

so

$$\frac{\frac{F'(a')}{\hat{c}'(e(p',a'))}-\frac{F'(a'')}{\hat{c}'(e(p',a''))}}{\frac{F'(a')}{\hat{c}'(e(p'',a'))}-\frac{F'(a'')}{\hat{c}'(e(p'',a''))}}=\frac{p''}{p'},\quad \text{for } a'\in\left[\hat{a},a''\right).$$

For $p''>p'$, it follows that

$$\frac{F'\left(a'\right)}{\hat{c}'(e(p'',a'))}-\frac{F'\left(a''\right)}{\hat{c}'(e(p'',a''))}<\frac{F'\left(a'\right)}{\hat{c}'(e(p',a'))}-\frac{F'\left(a''\right)}{\hat{c}'(e(p',a''))},\quad \text{for } a'\in\left[\hat{a},a''\right),$$

so

$$F'\left(a'\right)\left[\frac{1}{\hat{c}'(e(p'',a'))}-\frac{1}{\hat{c}'(e(p',a'))}\right]<F'\left(a''\right)\left[\frac{1}{\hat{c}'(e(p'',a''))}-\frac{1}{\hat{c}'(e(p',a''))}\right],$$
$$\text{for } a'\in\left[\hat{a},a''\right).$$

Under Assumption 2, $F(a)$ is concave, so $F'\left(a'\right)\geq F'\left(a''\right)$ and thus

$$\frac{1}{\hat{c}'(e(p'',a'))}-\frac{1}{\hat{c}'(e(p',a'))}<\frac{1}{\hat{c}'(e(p'',a''))}-\frac{1}{\hat{c}'(e(p',a''))},\quad \text{for } a'\in\left[\hat{a},a''\right).$$

For $e\left(p',a''\right)\leq e\left(p'',a''\right)$, the right-hand side of this is non-positive, so $e\left(p',a'\right)<e\left(p'',a'\right)$, establishing (13). For $e\left(p',a'\right)\geq e\left(p'',a'\right)$, the left-hand side is non-negative, so $e\left(p',a''\right)>e\left(p'',a''\right)$, establishing (14).

From (A.3),

$$\hat{c}'(e(p,a))=\frac{p}{\hat{\mu}(p)+aF'(a)},\quad \text{for } a\in\left[\hat{a},\overline{a}\right],\ p=p',p''. \tag{A.4}$$

It follows from $\hat{c}$ strictly convex that $e(p,a)$ is strictly decreasing in a only if $aF'(a)$ is strictly increasing and that

$$e\left(p',a\right)\lesseqqgtr e\left(p'',a\right)\ \text{according as}$$
$$\frac{p'}{\hat{\mu}(p')+aF'(a)}\lesseqqgtr\frac{p''}{\hat{\mu}(p'')+aF'(a)},\quad \text{for } a\in\left(\hat{a},\overline{a}\right],$$

or, inverting the latter expression and re-arranging it,

$$e(p', a) \lesseqgtr e(p'', a) \text{ according as}$$
$$aF'(a)\left(\frac{1}{p'} - \frac{1}{p''}\right) \gtreqless \frac{\hat{\mu}(p'')}{p''} - \frac{\hat{\mu}(p')}{p'}, \quad \text{for } a \in (\hat{a}, \overline{a}].$$

So, given $a'F'(a') < a''F'(a'')$ for $a' \in [\hat{a}, a'')$, then $e(p', a') < e(p'', a')$ implies $e(p', a'') < e(p'', a'')$, establishing (15).

From (A.4),

$$e(p,a) = \hat{c}'^{-1}\left(\frac{p}{\hat{\mu}(p) + aF'(a)}\right), \quad \text{for } a \in [\hat{a}, \overline{a}], \, p = p', p''.$$

For notational convenience, define ϕ as $\hat{c}'^{-1}$. Then output is

$$pe(p,a) = p\phi\left(\frac{p}{\hat{\mu}(p) + aF'(a)}\right), \quad \text{for } a \in [\hat{a}, \overline{a}], \, p = p', p''.$$

For $a \in [\hat{a}, \overline{a}]$, $p = p', p''$,

$$\begin{aligned}\frac{\partial}{\partial a}(pe(p,a)) &= -p\phi'\left(\frac{p}{\hat{\mu}(p) + aF'(a)}\right)\frac{p}{[\hat{\mu}(p) + aF'(a)]^2}\frac{\partial}{\partial a}(aF'(a)) \\ &= -\phi'\left(\frac{p}{\hat{\mu}(p) + aF'(a)}\right)\left(\frac{p}{\hat{\mu}(p) + aF'(a)}\right)^2\frac{\partial}{\partial a}(aF'(a))\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial a}[p''e(p'', a) - p'e(p', a)] = \frac{\partial}{\partial a}(aF'(a))\Bigg[&\phi'\left(\frac{p'}{\hat{\mu}(p') + aF'(a)}\right)\left(\frac{p'}{\hat{\mu}(p') + aF'(a)}\right)^2 \\ &-\phi'\left(\frac{p''}{\hat{\mu}(p'') + aF'(a)}\right)\left(\frac{p''}{\hat{\mu}(p'') + aF'(a)}\right)^2\Bigg].\end{aligned}$$

For $e(p'', a) > e(p', a)$, it follows from (A.4) that

$$\frac{p''}{\hat{\mu}(p'') + aF'(a)} > \frac{p'}{\hat{\mu}(p') + aF'(a)}.$$

With $aF'(a)$ strictly increasing, $\partial(aF'(a))/\partial a > 0$. Thus $\partial[p''e(p'', a) - p'e(p', a)]/\partial a < 0$ if ϕ' is non-decreasing. But, by definition of ϕ as the inverse of $\hat{c}'$, $\phi(\hat{c}'(e)) = e$. Differentiation with respect to e gives

$$\phi'(\hat{c}'(e))\,\hat{c}''(e) = 1,$$

so $\phi' > 0$ given that $\hat{c}$ is strictly convex. Further differentiation gives

$$\phi''(\hat{c}'(e))\,\hat{c}''(e)^2 + \phi'(\hat{c}'(e))\,\hat{c}'''(e) = 0,$$

so

$$\phi''\big(\hat{c}'(e)\big) = -\frac{\phi'\big(\hat{c}'(e)\big)\,\hat{c}'''(e)}{\hat{c}''(e)^2},$$

which is non-negative, and hence ϕ' is non-decreasing, if $\hat{c}'''(e) \leq 0$, which establishes (16). ■

Proof of Proposition 5. From Proposition 3, for p with partial pooling of agent types, (12) applies for $e^R(p,a)$ given by (11). For the functional forms in the proposition, $s(e,p,a) = pe - a\hat{c}(e)$, so $s_1(e,p,a) = p - a\hat{c}'(e)$, and $c_{12}(e,a) = \hat{c}'(e)$. Thus (11) can be written

$$\left[p - a\hat{c}'\big(e^R(p,a)\big)\right]\frac{1}{\overline{a}-\underline{a}} = \frac{\mu(p)}{\Big(1+\frac{\delta}{1-\delta}\sum_{\tilde{p}\in P}\mu(\tilde{p})\Big)\,dG(p)}\hat{c}'\big(e^R(p,a)\big)\,. \tag{A.5}$$

Moreover, for $F(a)$ uniform,

$$\begin{aligned}\int_{\underline{a}}^{\hat{a}(p)} s_1\big(e^R\big(p,\hat{a}(p)\big),p,a\big)\,F'(a)\,da &= \int_{\underline{a}}^{\hat{a}(p)}\left[p - a\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)\right]\frac{1}{\overline{a}-\underline{a}}da \\ &= p\frac{\hat{a}(p)-\underline{a}}{\overline{a}-\underline{a}} - \frac{\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)}{2}\frac{\hat{a}(p)^2-\underline{a}^2}{\overline{a}-\underline{a}}.\end{aligned}$$

With this, (A.5) and $c_1(e,p) = a\hat{c}'(e)$, (12) yields

$$p\frac{\hat{a}(p)-\underline{a}}{\overline{a}-\underline{a}} - \frac{\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)}{2}\frac{\hat{a}(p)^2-\underline{a}^2}{\overline{a}-\underline{a}} = \left[p - \hat{a}(p)\,\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)\right]\frac{\hat{a}(p)}{\overline{a}-\underline{a}}.$$

This can be simplified to

$$\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)\left[\hat{a}(p)^2+\underline{a}^2\right] = 2\underline{a}p,$$

from which

$$\frac{1}{\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)} = \frac{\hat{a}(p)^2+\underline{a}^2}{2\underline{a}p}. \tag{A.6}$$

Because (A.5) applies to all a, it implies

$$\frac{p}{\hat{c}'\big(e^R(p,a)\big)} - a = \frac{p}{\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)} - \hat{a}(p)\,,$$

from which

$$\frac{1}{\hat{c}'\big(e^R(p,a)\big)} = \frac{1}{\hat{c}'\big(e^R\big(p,\hat{a}(p)\big)\big)} + \frac{1}{p}\left[a-\hat{a}(p)\right]$$

and, with (A.6),

$$\frac{1}{\hat{c}'\big(e^R(p,a)\big)} = \frac{\hat{a}(p)^2+\underline{a}^2}{2\underline{a}p} + \frac{1}{p}\left[a-\hat{a}(p)\right]. \tag{A.7}$$

From this and (A.6),

$$\begin{aligned}
&\frac{1}{\hat{c}'\left(e^R\left(p'', \hat{a}(p')\right)\right)} - \frac{1}{\hat{c}'\left(e^R\left(p', \hat{a}(p')\right)\right)} \\
&= \frac{\hat{a}\left(p''\right)^2 + \underline{a}^2}{2\underline{a}\,p''} + \frac{1}{p''}\left[\hat{a}\left(p'\right) - \hat{a}\left(p''\right)\right] - \frac{\hat{a}\left(p'\right)^2 + \underline{a}^2}{2\underline{a}\,p'} \\
&= \frac{1}{2\underline{a}\,p''}\left[\hat{a}\left(p''\right)^2 - 2\underline{a}\hat{a}\left(p''\right) + \underline{a}^2\right] + \frac{1}{p''}\hat{a}\left(p'\right) - \frac{1}{2\underline{a}\,p'}\left[\hat{a}\left(p'\right)^2 - 2\underline{a}\hat{a}\left(p'\right) + \underline{a}^2\right] - \frac{2\underline{a}\hat{a}\left(p'\right)}{2\underline{a}\,p'} \\
&= \frac{1}{2\underline{a}}\left\{\frac{1}{p''}\left[\hat{a}\left(p''\right) - \underline{a}\right]^2 - \frac{1}{p'}\left[\hat{a}\left(p'\right) - \underline{a}\right]^2\right\} + \left(\frac{1}{p''} - \frac{1}{p'}\right)\hat{a}\left(p'\right). \qquad \text{(A.8)}
\end{aligned}$$

Suppose $\hat{a}(p') \geq \hat{a}(p'')$. Then $\hat{a}(p') - \underline{a} \geq \hat{a}(p'') - \underline{a}$ and, with $p'' > p'$, the term including the braces in (A.8) is strictly negative. With $p'' > p'$, the other term is also strictly negative. Moreover, $\hat{a}(p') \in [\hat{a}(p''), \overline{a}]$. By Part 2 of Proposition 3, $e(p, a) = e^R(p, a)$ for $a \in [\hat{a}(p), \overline{a}]$, so $e(p'', \hat{a}(p')) = e^R(p'', \hat{a}(p'))$ as well as $e(p', \hat{a}(p')) = e^R(p', \hat{a}(p'))$. Thus (A.8) implies

$$\frac{1}{\hat{c}'\left(e\left(p', \hat{a}(p')\right)\right)} > \frac{1}{\hat{c}'\left(e\left(p'', \hat{a}(p')\right)\right)},$$

so

$$\hat{c}'\left(e\left(p', \hat{a}(p')\right)\right) < \hat{c}'\left(e\left(p'', \hat{a}(p')\right)\right)$$

and, because $\hat{c}$ is strictly convex,

$$e\left(p', \hat{a}(p')\right) < e\left(p'', \hat{a}(p')\right).$$

For $\hat{a}(p') \geq \hat{a}(p'')$, that implies, by (15) in Proposition 4, $e(p', a) < e(p'', a)$ for all $a \in [\hat{a}(p'), \overline{a}]$. Moreover, with $e(p, a)$ equal to $e(p, \hat{a}(p))$ for $a \in [\underline{a}, \hat{a}(p)]$ and decreasing in a for $a \in [\hat{a}(p), \overline{a}]$, $\hat{a}(p') \geq \hat{a}(p'')$ then implies $e(p', a) < e(p'', a)$ for all $a \in [\underline{a}, \hat{a}(p')]$. But then $e(p', a) < e(p'', a)$ for all $a \in [\underline{a}, \overline{a}]$, which contradicts Part 2 of Proposition 2. Hence, it must be that $\hat{a}(p') < \hat{a}(p'')$, which establishes Part 1.

Suppose $e(p', \hat{a}(p')) \leq e(p'', \hat{a}(p''))$. With $\hat{a}(p') < \hat{a}(p'')$ and $e(p', a)$ strictly decreasing in a for $a > \hat{a}(p')$, (15) then implies $e(p', a) < e(p'', a)$ for all $a \in (\hat{a}(p'), \overline{a}]$. But then $e(p', a) \leq e(p'', a)$ for all $a \in [\underline{a}, \overline{a}]$, which contradicts Part 2 of Proposition 2. Hence, it must be that $e(p', \hat{a}(p')) > e(p'', \hat{a}(p''))$. Then, with $e(p, a) = e(p, \hat{a}(p))$ for $a \in [\underline{a}, \hat{a}(p)]$, $e(p', a) > e(p'', a)$ for all $a \in [\underline{a}, \hat{a}(p')]$, establishing Part 2.

Now suppose $e(p', \hat{a}(p'')) \geq e(p'', \hat{a}(p''))$. From $\hat{a}(p') < \hat{a}(p'')$ and $e(p', a)$ strictly decreasing in a for $a > \hat{a}(p')$, this implies $e(p', a) \geq e(p'', a)$ for all $a \in [\underline{a}, \hat{a}(p'')]$, with the inequality strict for some $a \in [\underline{a}, \hat{a}(p'')]$. Moreover, by (14) in Proposition 4, $e(p', a) > e(p'', a)$ for all $a \in [\hat{a}(p''), \overline{a}]$. That implies $e(p', a) \geq e(p'', a)$ for all $a \in [\underline{a}, \overline{a}]$, with the inequality strict for some $a \in [\underline{a}, \overline{a}]$, which contradicts Part 2 of Proposition 2. Hence, it must be that $e(p', \hat{a}(p'')) < e(p'', \hat{a}(p''))$. But then, by (15) in Proposition 4, $e(p', a) < e(p'', a)$ for all $a \in [\hat{a}(p''), \overline{a}]$, establishing Part 3. ■

Proof of Proposition 6. From the standard arguments in Laffont (1989, Chapter 10) and Milgrom and Segal (2002, Corollary 1), necessary and sufficient conditions for the principal to announce $\hat{p}_t = p_t$ are those involving (18) in the proposition together with the condition on the principal's one-period profit conditional on p given by

$$\pi(p) = -w(p) + \left[\int_{\underline{a}}^{\overline{a}} y(e(p,a), p) - b(p, e(p,a))\right] dF(a), \quad \text{for all } p \in P, \tag{A.9}$$

that

$$\pi(p) = \pi\left(\underline{p}\right) + \int_{\underline{p}}^{p} \left[\int_{\underline{a}}^{\overline{a}} y_2(e(\tilde{p},a), \tilde{p})\, dF(a)\right] d\tilde{p}, \quad \text{for all } p \in P. \tag{A.10}$$

Consider the relational contract in the proof of sufficiency for Proposition 1, which specifies $b(p, e(p,a))$ and $E_p\left[w(p)\right]$ that ensure incentive compatibility when p_t is observed by both principal and agent but imposes no further requirement on $w(p)$ for each p. For the specified $b(p, e(p,a))$ and $E_p\left[w(p)\right]$, there certainly exists a $w(p)$ schedule that ensures $\pi(p)$ satisfies (A.10). Thus the only additional restrictions in addition to those in Proposition 1 required to implement the effort schedule $e(p,a)$ are those involving (18). ■

Proof of Proposition 7. The additional requirements over those in Proposition 6 to ensure both parties continue the relationship when either can quit without further payment after $\hat{p}_t$ is truthfully announced are

$$\pi(p) + \frac{\delta}{1-\delta}\left(\pi - \underline{\pi}\right) \geq 0, \text{ for all } p \in P; \tag{A.11}$$

$$\int_{\underline{a}}^{\overline{a}} u(p,a)\, dF(a) + \frac{\delta}{1-\delta}\left(u - \underline{u}\right) \geq 0, \text{ for all } p \in P. \tag{A.12}$$

From (2), (A.9) and $\pi - \underline{\pi} + u - \underline{u} = s - \underline{s}$,

$$\begin{aligned} \pi(p) + \frac{\delta}{1-\delta}\left(\pi - \underline{\pi}\right) + \int_{\underline{a}}^{\overline{a}} u(p,a)\, dF(a) + \frac{\delta}{1-\delta}\left(u - \underline{u}\right) \\ = \int_{\underline{a}}^{\overline{a}} \left[y(e(p,a), p) - c(e(p,a), a)\right] dF(a) + \frac{\delta}{1-\delta}\left(s - \underline{s}\right), \quad \text{for all } p \in P. \end{aligned}$$

Substitution for $u(p,a)$ and $\pi(p)$ from (4) and (A.10) allows this to be written

$$\begin{aligned} \pi\left(\underline{p}\right) + \frac{\delta}{1-\delta}\left(\pi - \underline{\pi}\right) + u(p, \overline{a}) + \frac{\delta}{1-\delta}\left(u - \underline{u}\right) \\ = -\int_{\underline{p}}^{p} \left[\int_{\underline{a}}^{\overline{a}} y_2(e(\tilde{p},a), \tilde{p})\, dF(a)\right] d\tilde{p} - \int_{\underline{a}}^{\overline{a}} \int_{a}^{\overline{a}} c_2(e(p,\tilde{a}), \tilde{a})\, d\tilde{a}\, dF(a) \\ + \int_{\underline{a}}^{\overline{a}} \left[y(e(p,a), p) - c(e(p,a), a)\right] dF(a) + \frac{\delta}{1-\delta}\left(s - \underline{s}\right), \quad \text{for all } p \in P. \end{aligned}$$

With $\pi(p)$ non-decreasing and $u(p, a)$ non-increasing in a, there certainly exist payments $w(p)$ that satisfy both (A.11) and (A.12) if the left-hand side of this is non-negative, which is the case if

$$\frac{\delta}{1-\delta}\left(s-\underline{s}\right) \geq \\ +\int_{\underline{p}}^{p}\left[\int_{\underline{a}}^{\overline{a}} y_2(e(\tilde{p}, a), \tilde{p})\, dF(a)\right] d\tilde{p} + \int_{\underline{a}}^{\overline{a}}\int_{a}^{\overline{a}} c_2(e(p, \tilde{a}), \tilde{a})\, d\tilde{a} dF(a) \\ -\int_{\underline{a}}^{\overline{a}} \left[y(e(p, a), p) - c(e(p, a), a)\right] dF(a), \quad \text{for all } p \in P,$$

or

$$\frac{\delta}{1-\delta}\left(s-\underline{s}\right) \geq \int_{\underline{a}}^{\overline{a}}\left[c(e(p, a), a) + \int_{a}^{\overline{a}} c_2(e(p, \tilde{a}), \tilde{a})\, d\tilde{a}\right] dF(a) \\ +\int_{\underline{p}}^{p}\left[\int_{\underline{a}}^{\overline{a}} y_2(e(\tilde{p}, a), \tilde{p})\, dF(a)\right] d\tilde{p} - \int_{\underline{a}}^{\overline{a}} y(e(p, a), p)\, dF(a), \quad \text{for all } p \in P. \quad \text{(A.13)}$$

It follows from the argument in the main text following (9) and (3) that this will certainly hold for any p for which the lower line is non-positive. That is the case for $p = \underline{p}$ because $y(e, p) \geq 0$. Moreover, the derivative of the lower line with respect to p is

$$-\int_{\underline{a}}^{\overline{a}} y_1(e(p, a), p)\, e_1(p, a)\, dF(a).$$

With the specification $y(e, p) = p\hat{y}(e)$ with $\underline{p} > 0$, (18) is sufficient to ensure this derivative is non-positive. Thus, the bottom line of (A.13) is non-positive for all p, which is sufficient to establish the result. ■

Proof of Proposition 8. By the argument in the text, if the contract used in the proof of Proposition 5 satisfies (18) for all $(\hat{p}, p)$ when the set of possible $(\hat{p}, p)$ is treated as $P = \left[\underline{p}, \overline{p}\right]$, it is incentive compatible for the principal for $P \subset \left[\underline{p}, \overline{p}\right]$ consisting of a finite number of discrete points. With pooling of some agent types for all p, the inequality constraint in (10) holds with equality for all p. From Proposition 5 (see also Figure 2), $e\left(p, \underline{a}\right)$ is decreasing in p and, hence, so is $c\left(e\left(p, \underline{a}\right), \underline{a}\right)$. The left-hand side of the constraint in (10) is independent of p. Thus the integral in that constraint must be increasing in p. Under the conditions of Proposition 5 and with $\psi(a; p) = \hat{c}'(e(p, a)) \,/\, \int_{\underline{a}}^{\overline{a}} \hat{c}'(e(p, a))\, da$, that implies

$$\int_{\underline{a}}^{\overline{a}} e_1(p, a)\, \psi(a; p)\, da > 0, \text{ for all } p, \quad \text{(A.14)}$$

and (18) corresponds to

$$\int_{\underline{a}}^{\overline{a}} e_1(p, a)\, \frac{1}{\overline{a}-\underline{a}} da \geq 0, \text{ for all } p. \quad \text{(A.15)}$$

Comparing these two conditions corresponds to comparing the expected value of $e_1(p,a)$ over a for given p for two different distributions. From Proposition 5 (see also Figure 2), $e(p,a)$ is non-increasing for all a, and strictly decreasing for some a, so $\hat{c}'(e(p,a))$, and hence $\psi(a;\,p)$, have that same property. Thus the distribution in (A.15) stochastically dominates that in (A.14) in the second-order sense. So, by a standard result, the expression on the left-hand side of (A.15) is larger that on the left-hand side of (A.14), and hence condition (18) is satisfied, if $e_1(p,a)$ is non-decreasing for all a, and strictly increasing for some a.

From (A.7) in the proof of Proposition 5,

$$\frac{1}{\hat{c}'\left(e^R(p,a)\right)} = \frac{\hat{a}(p)^2 + \underline{a}^2}{2\underline{a}p} + \frac{1}{p}\left[a - \hat{a}(p)\right]$$

and, by definition of $\hat{a}(p)$, $e(p,a) = e^R(p,a)$ for $a \geq \hat{a}(p)$. Differentiation with respect to a thus yields

$$-\frac{\hat{c}''(e(p,a))\, e_2(p,a)}{\hat{c}'(e(p,a))^2} = \frac{1}{p}, \text{ for } a > \hat{a}(p)\,,$$

so

$$e_2(p,a) = -\frac{\hat{c}'(e(p,a))^2}{p\hat{c}''(e(p,a))}, \text{ for } a > \hat{a}(p)\,.$$

Differentiation of this with respect to p yields, for $a > \hat{a}(p)$,

$$\begin{aligned} e_{12}(p,a) = {} & \frac{1}{p^2}\frac{\hat{c}'(e(p,a))^2}{\hat{c}''(e(p,a))} \\ & - \frac{e_1(p,a)}{p\hat{c}''(e(p,a))^2}\left[\hat{c}''(e(p,a))\,2\hat{c}'(e(p,a)) - \hat{c}'(e(p,a))^2\,\hat{c}'''(e(p,a))\right] \\ = {} & \frac{1}{p^2}\frac{\hat{c}'(e(p,a))^2}{\hat{c}''(e(p,a))} - \frac{e_1(p,a)\,\hat{c}'(e(p,a))}{p\hat{c}''(e(p,a))}\left[2 - \frac{\hat{c}'(e(p,a))}{\hat{c}''(e(p,a))}\hat{c}'''(e(p,a))\right]. \end{aligned}$$

The first term on the right-hand side of this is positive given the specification. From Proposition 5 (see also Figure 2), the term multiplying the square bracket is also positive for $a > \hat{a}(p)$. So $e_{12}(p,a) > 0$ for $a > \hat{a}(p)$ if

$$\frac{\hat{c}'(e(p,a))}{\hat{c}''(e(p,a))}\hat{c}'''(e(p,a)) \geq 2.$$

This is certainly the case when (19) holds. For $a \leq \hat{a}(p)$, $e(p,a)$ is independent of a, so $e_{12}(p,a) = 0$. Thus, when (19) holds, $e_1(p,a)$ is non-decreasing in a for all a, and strictly increasing in a for some a, and hence the left-hand side of (A.15) is greater than the left-hand side of (A.14). So (18) holds for all $\hat{p}$, $p \in \left[\underline{p}, \overline{p}\right]$. ■

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Online Appendix

Appendix B Proof of Proposition 3

Define

$$S(e(.,a),a) = \int_{\underline{p}}^{\overline{p}} s(e(p,a),p,a)\,dG(p)\,. \tag{B.1}$$

In this notation, the optimization problem in Section 4 can be written

$$\max_{e(.,.)} \int_{\underline{a}}^{\overline{a}} S(e(.,a),a)\,F'(a)\,da \qquad \text{subject to} \tag{B.2}$$

$$\frac{\delta}{1-\delta}\int_{\underline{a}}^{\overline{a}} \left[S(e(.,a),a)\,F'(a)\,da - \underline{s}\right] \geq c\big(e\big(p,\underline{a}\big),\underline{a}\big) + \int_{\underline{a}}^{\overline{a}} c_2(e(p,a),a)\,da, \quad \forall p \in P, \tag{B.3}$$

$$e(p,a) \text{ non-increasing in } a, \quad \forall\,(p,a)\,. \tag{B.4}$$

This optimization problem has the same form as that in Levin (2003, p. 843) but with the finite number of additional variables $p \in P$ and corresponding constraints. Under Assumptions 1 and 2, it is a concave programming problem.

To use standard optimal control techniques, follow Levin (2003) in defining the control variable for each $p \in P$

$$\gamma\,(p,a) \equiv e_2(p,a)\,, \quad \forall a, \tag{B.5}$$

and the state variable for each $p \in P$

$$K(p,a) \equiv \int_{\underline{a}}^{a} \left[\frac{\delta}{1-\delta} S(e(.,\tilde{a}),\tilde{a})\,F'(\tilde{a}) - c_2(e(p,\tilde{a}),\tilde{a})\right] d\tilde{a}, \quad \forall a. \tag{B.6}$$

Constraints (B.4) can then be written

$$\gamma\,(p,a) \leq 0, \quad \forall\,(p,a)\,. \tag{B.7}$$

Constraints (B.3) can be written as boundary conditions

$$K\big(p,\underline{a}\big) = 0 \text{ and } K(p,\overline{a}) - \frac{\delta}{1-\delta}\underline{s} - c\big(e\big(p,\underline{a}\big),\underline{a}\big) \geq 0,\ \forall p \in P. \tag{B.8}$$

The optimization problem can then be written

$$\max_{e(.,.)} \int_{\underline{a}}^{\overline{a}} S(e(.,a),a)\,F'(a)\,da \qquad \text{subject to}$$

definitions: (B.5) and (B.6),

constraints: (B.7) and (B.8).

The Hamiltonian for this problem is

$$H = S(e(.,a),a)\,F'(a) + \sum_{p\in P}\eta(p,a)\,\gamma(p,a) \\ + \sum_{p\in P}\lambda(p,a)\left[\frac{\delta}{1-\delta}S(e(.,a),a)\,F'(a) - c_2(e(p,a),a)\right],$$

where $\eta(p,a)$ and $\lambda(p,a)$ are co-state variables assigned to $e_2(p,a)$ and to $K_2(p,a)$ respectively. The corresponding Pontryagin conditions (with $\nu(p,a)$ the multiplier assigned to (B.7) and $\mu(p)$ that assigned to the second constraint in (B.8)) are, for all (p,a),

$$\begin{aligned}
\nu(p,a) &= H_\gamma = \eta(p,a) \\
-\eta_2(p,a) &= H_e = s_1(e(p,a),p,a)\,dG(p)\,F'(a)\left(1 + \sum_{\tilde{p}\in P}\lambda(\tilde{p},a)\,\frac{\delta}{1-\delta}\right) \\
&\qquad - \lambda(p,a)\,c_{12}(e(p,a),a) \\
-\lambda_2(p,a) &= H_K = 0 \\
e_2(p,a) &= H_\eta = \gamma(p,a) \\
K_2(p,a) &= H_\lambda = \frac{\delta}{1-\delta}\int_{\underline{p}}^{\overline{p}} s(e(p,a),p,a)\,dG(p)\,F'(a) - c_2(e(p,a),a),
\end{aligned}$$

the boundary conditions

$$K(p,\underline{a}) = 0, \lambda(p,\overline{a}) = \mu(p), \eta(p,\underline{a}) = \mu(p)\,c_1(e(p,\underline{a}),\underline{a}), \eta(p,\overline{a}) = 0, \forall p,$$

and the complementary slackness conditions both on the monotonicity constraint

$$\nu(p,a) \geq 0, \gamma(p,a) \leq 0 \text{ and } \nu(p,a)\,\gamma(p,a) = 0, \forall(p,a),$$

and on the self-enforcement constraint

$$\begin{aligned}
\mu(p) \geq 0 \text{ and } K(p,\overline{a}) - \frac{\delta}{1-\delta}\underline{s} - c(e(p,\underline{a}),\underline{a}) \geq 0, \forall p, \\
\text{and } \mu(p)\left[K(p,\overline{a}) - \frac{\delta}{1-\delta}\underline{s} - c(e(p,\underline{a}),\underline{a})\right] = 0, \forall p.
\end{aligned} \quad \text{(B.9)}$$

Because $\lambda_2(p,a) = 0$ for all (p,a) and $\lambda(p,\overline{a}) = \mu(p)$, it must be that $\lambda(p,a) = \mu(p)$ for all a. In addition, because $\eta(p,a) = \nu(p,a)$ for all (p,a), then $\eta_2(p,a) = \nu_2(p,a)$ for all (p,a). Thus it is possible to substitute for $\eta(p,a)$ and $\lambda(p,a)$ in the Pontryagin conditions to reduce those conditions to

$$-\nu_2(p,a) = s_1(e(p,a)\,,\,p,a)\,dG(p)\,F'(a)\left(1+\frac{\delta}{1-\delta}\sum_{\tilde{p}\in P}\mu(\tilde{p})\right)$$
$$-\,\mu(p)\,c_{12}(e(p,a)\,,a)\,,\forall\,(p,a)\,, \tag{B.10}$$
$$\nu(p,a)\geq 0, e_2(p,a)\leq 0 \text{ and } \nu(p,a)\,e_2(p,a)=0, \forall\,(p,a)\,, \tag{B.11}$$
$$\nu\big(p,\underline{a}\big)=\mu(p)\,c_1\big(e\big(p,\underline{a}\big)\,,\underline{a}\big) \text{ and } \nu(p,\overline{a})=0, \forall p, \tag{B.12}$$

plus the complementary slackness conditions in (B.9). The constraints involving $K(p,\overline{a})$ are just the constraints (B.3), so the complementary slackness with $\mu(p)$ in (B.9) can be applied directly to (B.3). There are two cases to consider for each p, depending on whether the constraint (B.3) is binding for that p.

1. Suppose that $\mu(p) = 0$. Then (B.3) for p is slack at the solution, so it suffices to consider the problem of maximizing the joint surplus subject to the constraint $e(p,a)$ non-increasing in a for that p. Because first-best effort $e^*(p,a)$ is strictly decreasing in a for given p, it both maximizes the joint surplus and satisfies $e(p,a)$ non-increasing in a. Hence $e(p,a) = e^*(p,a)$ at the optimum.

2. Suppose that $\mu(p) > 0$. Then (B.3) for p is binding at the solution. Then, with Assumptions 1 and 2, not only is $e^R(p,a)$ decreasing in a but also $e^R(p,a) < e^*(p,a)$ for all a for the given p.

Lemma 1 *On any interval for a where the solution $e(p,a)$ is decreasing for given p, $e(p,a) = e^R(p,a)$ given by (11).*

Proof. On any interval for a where $e_2(p,a) < 0$ for given p, it must be that $\nu(p,a) = 0$. Hence $\nu_2(p,a) = 0$ as well. Eliminating ν_2 in the Pontryagin condition (B.10) yields the result. ∎

Lemma 2 *If for some $\hat{a}(p)$, $e_2(p,a) < 0$ for given p, then $e(p,a)$ is decreasing on $\left[\hat{a}(p)\,,\overline{a}\right]$ for that p.*

Proof. The proof is by contradiction. Suppose that for p and some $a_0(p)$, $e(p,a)$ is decreasing in a below $a_0(p)$ and constant above it. By complementary slackness, $\nu\big(p,a_0(p)^-\big) = 0$ and $\nu\big(p,a_0(p)^+\big) > 0$. Hence $\nu_2(p,a_0(p)) > 0$. Moreover, it follows from the Pontryagin condition (B.10) that over any interval of a where $e(p,a)$ is constant, $\nu_2(p,a)$ must be increasing (using the assumptions that s_1 is decreasing in a, F is concave so F' is non-increasing in a, and c_{12} is non-decreasing in a). Thus $\nu_2(p,a)$ is both positive and increasing above $a_0(p)$. Consequently $\nu(p,a) > 0$ for all $a > a_0(p)$, contradicting the boundary condition in (B.12) that $\nu(p,\overline{a}) = 0$. ∎

Proof of Proposition 3. The proof uses the two lemmas above and the optimality conditions (B.10)–(B.12). First, observe from (B.12) that at the optimum $\nu(p, \underline{a}) = \mu(p)\, c_1\big(e(p, \underline{a}), \underline{a}\big)$ for each p. Thus, if (B.3) is binding for p, $\nu(p, \underline{a}) > 0$, which implies that $e_2(p, \underline{a}) = 0$ by (B.11). So there must be pooling of the lowest agent types. Combined with the two lemmas above, this yields two possibilities: either $e(p, a)$ is constant in a below some $\hat{a}(p) \in (\underline{a}, \overline{a})$ and decreasing in a above it, or $e(p, a)$ is constant on the entire interval $[\underline{a}, \overline{a}]$.

If there is partial pooling for given p, then for all $a \geq \hat{a}(p)$, $e(p, a) = e^R(p, a)$, while for all $a < \hat{a}(p)$, all types a are pooled at some $\hat{e}(p)$. By continuity, this means that $\hat{e}(p) = e^R\big(p, \hat{a}(p)\big)$. To identify the cut-off type $\hat{a}(p)$, observe that $\nu(p, \underline{a}) = \mu(p)\, c_1\big(e(p, \underline{a}), \underline{a}\big)$ and $\nu\big(p, \hat{a}(p)\big) = 0$. Integrating the Pontryagin condition (B.10) over a from $\underline{a}$ up to $\hat{a}(p)$, and substituting these boundary conditions yields (12). From Assumptions 1 and 2, it is easy to check that there is at most one value $\hat{a}(p) \in (\underline{a}, \overline{a})$ that satisfies (12) for given $\mu(\tilde{p})$ for all $\tilde{p} \in P$. If the $\mu(\tilde{p})$ are such that $\hat{a}(p) < \overline{a}$, the solution has $e(p, a) = e^R\big(p, \hat{a}(p)\big)$ for $a \leq \hat{a}(p)$ and $e(p, a) = e^R(p, a)$ for $a \geq \hat{a}(p)$. Furthermore, because the solution satisfies $e(p, a) \leq e^R(p, a)$ for all a and $e^R(p, a) < e^*(p, a)$ for all a, it follows that $e(p, a) < e^*(p, a)$ for all a given the specified p.

In the event that there is no cut-off type $\hat{a}(p) \in (\underline{a}, \overline{a})$ that solves (12), there is complete pooling at some $\hat{e}(p)$. To identify $\hat{e}(p)$, the Pontryagin condition (B.10) can again be integrated over a from $\underline{a}$ up to $\overline{a}$ and combined with (B.12) to yield

$$\int_{\underline{a}}^{\overline{a}} s_1\big(\hat{e}(p), p, a\big)\, F'(a)\, da = \frac{\mu(p)}{\Big(1 + \frac{\delta}{1-\delta} \sum_{\tilde{p} \in P} \mu(\tilde{p})\Big)\, dG(p)} c_1\big(\hat{e}(p), \overline{a}\big).$$

In this case, $\hat{e}(p) < e^*(p, \overline{a})$, so $e(p, a) = \hat{e}(p) < e^*(p, a)$ for all a given p. ■