Essays in Credence Goods and Repeated Games

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Abstract

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This thesis presents two chapters on credence goods and one on ongoing partnerships in an infinitely repeated game. The chapters on credence goods focus on the welfare and efficiency of equilibria in overcharging models of credence goods, something which has not been explicitly addressed before. The chapter on partnerships presents a theory explaining ongoing partnerships as solving a commitment problem for clients. There is a small literature on partnerships, and this chapter represents a novel but complimentary approach to that literature. At core, chapters 2, 3 and 4 of this thesis ask the following questions:

- Do competition and information increase welfare in credence goods markets?
- How do customers in credence goods markets discipline experts from committing fraud? Can these strategies be welfare ranked?
- Why do ongoing partnerships exist? What problem do they solve?

JEL Classifications: C72, C73, D40, D82, D83, L10

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If I've learned one thing about research over the past few years, it's that it's all about a willingness to embark off in strange new directions. Sometimes it works out, sometimes not. But to quote Albert Einstein "If we knew what it was we were doing, it would not be called research, would it?"

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Chapter 1

Introduction

Speaking loosely, credence goods can be thought of as being goods where a customer must first ask an expert for advice on what to buy before he buys it from him. Clearly, this can lead to misdirection and overcharging, and the most novel feature of these markets is that consumers may then search for a second opinion to improve on their initial advice. Chapter 2 presents a credence goods market where some customers require the advice of an expert to know what to purchase, while others know what they want and buy it direct from a wholesaler. Expert middlemen charge the same as wholesalers, but have incentives to claim that the customer needs a more costly good than is in fact the case, hoping to fraudulently raise their margin. This chapter explores welfare in credence goods markets like these when customers can get a second opinion. It finds that wholesale price competition may reduce welfare if customers face a greater probability of being fraudulently sold an expensive treatment than of genuinely requiring it. That is, although price competition amongst wholesalers decreases prices, it may also increase fraud, dissuading customers from entering the market and lowering welfare. And if some fraction of customers are also time-constrained and cannot seek a second opinion, then price competition becomes even less likely to increase welfare. Increasing the share of informed customers buying direct may increase welfare, but if that leads to an increase in price competition, again welfare may fall. Despite this, if the market is competitive enough, increasing the share of customers cutting out the middleman and buying direct must eventually be welfare improving.

Chapter 3 outlines a credence good market with competing private quotes where sellers may attempt to overcharge customers. In order to discipline sellers proposing a potentially fraudulent price here, customers may either seek a costly second opinion or may choose not to purchase at all. There are multiple equilibria in the market that rely on different combinations of these disciplining strategies, and these equilibria can be welfare ranked. Common wisdom suggests that fraud will be low and welfare high in equilibrium if customers always solicit a second opinion. But because search is costly such equilibria only exist when fraud is high and welfare low. So for low search costs welfare is higher when customers sometimes don't get a second opinion but purchase the repair regardless, so they sometimes search but always purchase. For high search costs that equilibrium ceases to exist and welfare is maximised if customers never seek a second opinion and sometimes don't purchase anything at all. This resembles a monopolistic model where customers cannot seek a second opinion. These results are found in both a model with 'quick' upfront quotes by sellers, where equilibrium prices are uniquely determined, and when the model is extended to 'slow' quotes where the seller takes some time to prepare his price offer, which permits a broader range of prices in equilibrium.

Chapter 4 presents a theory of ongoing partnerships where partnerships exist to solve a commitment problem for clients, creating incentives for juniors to exert unobservable effort. Because clients care about the age of agents working for them, partnerships also commit to hiring juniors by billing by the hour and then hiring cheap juniors to work those hours. As partners make a margin on each junior hired, this turns making partner into a prize for successful juniors in an up-or-out style career structure. And where ability matters partnerships only hire the most able juniors, raising profits per partner further. If valuable clients are scarce, these features emerge naturally from the partnership's role as a gatekeeper to partnership with limited competition. However if juniors are scarce, then partnerships may bid up junior wages. Partnerships with high effort and up-or-out careers for juniors may then cease to exist, replaced by low effort ordinary companies with agents paid their marginal product. Perhaps surprisingly, this can be avoided if juniors ignore high wage offers in a folk theorem which obtains if a deviation causes play to revert to a grim competitive equilibrium in the future. Even so, as the number of firms competing for juniors increases it becomes harder to sustain the continued existence of ongoing partnerships.

Chapter 2

Cut out the Middleman and Buy it Direct!

If someone's car engine is faulty they can either pay a mechanic to buy parts and fix it, or if they know something about cars, buy the parts direct and fix it themselves. Middlemen such as mechanics charge the same for a part as a wholesaler but may misdirect customers by claiming that they had to use more expensive parts than they did in fact use to fix the car, pocketing the difference. This chapter explores welfare in credence goods markets like these where customers can get a second opinion and finds that wholesale price competition may reduce welfare if customers face a greater probability of being fraudulently sold an expensive treatment than of genuinely requiring it. Similarly, if only some customers have the time to get a second opinion, a fall in price markups may end up lowering welfare in the market by raising the level of fraud. More information may also have ambiguous effects, possibly raising the level of fraud and reducing overall welfare. Despite this if the market is competitive enough, increasing the share of customers cutting out the middleman and buying direct must eventually be welfare improving.

2.1 Introduction

In markets with potentially fraudulent middlemen, does increasing wholesale price competition increase welfare? What about increasing the share of customers who can buy direct?

Here I find that increased price competition amongst wholesalers must be welfare improving if consumers can seek a second opinion and there is not too much fraud. If some customers are time-constrained and have to accept the first opinion offered to them, price competition is more likely to reduce welfare. Also, if prices are competitive enough more information and a greater share of customers buying direct is always eventually welfare improving as customers avoid any potential fraud, but if the share of customers buying direct increases a small amount it might be damaging to welfare as it induces middlemen to commit more fraud on those customers who still use them.

Car repairs are one example of a credence good market. In such markets consumers have a problem and at least some must rely on the advice of an expert to diagnose what they require to fix it. Fraud potentially arises because of this information asymmetry, as experts may attempt to overcharge customers by claiming that they need a more expensive good or service than is actually the case. And unlike standard models of moral hazard customers remain unaware of having been overcharged even after their problem is solved.

Consider how the market for car repairs operates. A customer who discovers his car is not functioning as it should goes to a mechanic who looks inside the car, observes what is wrong and then recommends a part be replaced to fix it. If the customer agrees to it, he then pays the going market rate for that part, which includes a small markup, and has his car returned working normally a few days later. Now while the mechanic may well have been truthful in his advice, he may also have claimed that the car needed a more expensive part then it did actually need, pocketing the difference. And as the quoted price was at the going rate from a wholesaler for that part, then as the car is returned working normally again, the consumer will never know whether he was defrauded or not.

There are many similar examples of markets for credence goods with potential overcharging. The literature suggests that examples of experts who can commit such fraud include doctors, mechanics, computer specialists (Dulleck and Kerschbamer, 2006), lawyers (Hyndman and Ozerturk, 2008), real estate agents (Fong, 2005), optometrists (Wolinsky, 1993), washing machine repairmen (Dulleck and Kerschbamer, 2005a), consultants (Pesendorfer and Wolinsky, 2003), taxi drivers (Darby and Karni, 1973), life insurance salesmen and undertakers (Emons, 2001). Empirical studies support the claim that such experts respond to incentives to commit fraud on their customers. Iizuka (2007) presents data from the market for prescription drugs in Japan where doctors both prescribe drugs and sell them to their patients. He shows that when markups go up on particular drugs that doctors start recommending those drugs more (if it isn't going to cause too much hardship for the customer) so overcharging their customers. Similarly, Bill Ross of Cumberland Law School finds in two surveys (1995-95 and 2005-06) that two-thirds of practising lawyers claim to personally know of bill-padding by colleagues. Indeed, overcharging seems endemic in some industries.

The contribution of this chapter is to investigate the role of price competition and information on welfare in credence markets where some customers can buy direct off wholesalers. To do so, it is assumed that some customers are informed about what their problem is (or can self-diagnose) and because they know exactly what good they need will purchase direct from a wholesaler. Uninformed customers instead do not know what their problem is and must visit a potentially fraudulent middlemen who can diagnose which good they require, and who then offers to fix their problem at the prevailing price found the wholesale market. Price competition enters the model via the competition among wholesalers for informed customers, while there is also competition in terms of honesty between experts diagnosing customers problems.

The chapter's first result is to show that if all uninformed consumers have the time to get a second opinion and an uninformed customers probability of genuinely requiring the potentially fraudulent treatment is greater than the probability that he will be fraudulently recommended it, then price competition increases welfare. Still, price competition is potentially welfare reducing if fraud is great enough because it may also increase incentives to commit further fraud. The chapter's second result shows that that the problem is worse when a fraction of consumers are time-constrained, and can only visit one diagnosing expert before purchasing treatment.

Three further results then outline the role of information. Proposition 3 shows that with fixed prices, changing customers from being uninformed to informed must increase welfare if prices are competitive enough. The reason for this is that when there is no fraud in the market, the wrong types of uninformed enter, with too many who need an expensive treatment and too few who need a cheap treatment. Switching uninformed to being informed then alters the make-up of entrants, which must be welfare improving if prices are close to costs. Proposition 4 shows that with flexible prices a marginal increase in the share of informed consumers may decrease welfare as (despite the potential positive effect from more informed customers) this intensifies price competition among wholesalers, which may lead to greater fraud among those consumers that remain uninformed. So overall there may be a decrease in welfare. The final proposition then shows that despite this, if prices are relatively competitive and the fraction of informed customers is increased enough, then welfare will eventually tend towards its maximum level. So as we tend toward full information (both in the sense that prices are close to costs and in the sense that customers know their own type) welfare in the model tends towards the first best outcome.

Dulleck and Kerschbamer (2006) review the literature on credence goods and following their terminology, this chapter investigates potential *overcharging* fraud. Customers cannot observe the actual repair made (denoted non-verifiability) but the repair must fix the customers problem (denoted liability). Two important earlier papers in this framework are those by Wolinsky (1993) and Fong (2005), which use a search and monopolistic framework respectively. Related papers include Liu (2011) and Hyndman and Ozertuk (2008) who follow Fong's (2005) overcharging setup, and introduce honest experts and heterogeneity in the customer's payoffs respectively. Papers by Sülzle and Wambach (2005) and Alger and Salanie (2006) instead follow Wolinsky's (1993) model but investigate insurance and a setting where experts set prices for both treatment and costly diagnosis. Of course other markets with information asymmetries, such lemons markets, are in a broader sense related to this chapter. And Muthoo and Mutuswami (2011) in a recent paper find that increased competition only sometimes improves welfare in lemons markets.

The chapter is organised as follows. Section 2 outlines the basic model, where consumers have time to get a second opinion and investigates the effect of an increase in price competition for the potentially fraudulent good. Section 3 then generalises this by assuming that a fraction of consumers are time-constrained and explores greater price competition in both goods. Section 4 examines the effect of increasing the share of informed consumers on welfare. Section 5 then concludes.

2.2 The Model

Consider a unit mass of consumers who each have a problem requiring treatment for a payoff of V. Consumers may have one of two different types of problem. A fraction $\alpha \in (0, 1)$ of consumers, denoted high types, have an expensive problem and require an expensive treatment costing c_h to fix it. The remaining $1-\alpha$ of consumers, denoted low types, have an inexpensive problem and require only a cheap treatment costing c_l to fix it. It is assumed that $V > c_h > c_l$.

Consumers also differ in their information and who they can buy from. A fraction $i \in (0, 1)$ of consumers is informed, and are aware of the type of treatment they need. These customers may buy direct from wholesalers, which are firms that sell goods but do not diagnose customers problems for them. The remaining fraction of $1 - i$ consumers are uninformed about which treatment they require. These consumers cannot buy treatment direct from a wholesaler but must visit a middleman who can diagnose them and then sell them treatment. Figure 2.1 displays the possible consumer types in the model. On the horizontal axis consumers up to the cutoff i are informed, with the remaining $1-i$ uninformed. While on the vertical axis, consumers up to the cutoff α need the expensive treatment, and the remaining $1 - \alpha$ only need the cheap treatment.

In a way standard to the credence goods literature, it is assumed that diagnosers may not provide customers the low cost treatment c_l, if the customer requires the more

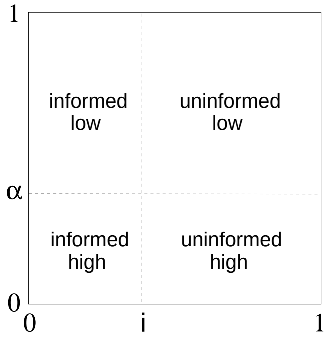

Figure 2.1: Different types of the unit mass of consumers

extensive treatment c_h (see Fong (2005), or Dulleck and Kerschbamer (2006) for example). This is referred to as the liability assumption and can be assumed to reflect malpractice laws. But while they cannot undertreat their customers, middlemen can attempt to overcharge them by recommending the more extensive treatment c_h when a customer in fact only requires c_l. The frequency with which middlemen fraudulently recommend an unnecessary expensive treatment is denoted F. Whichever type of treatment he recommends, the middleman may then offer to supply it at the prevailing wholesale market price, p_h or p_l. So notice here that a middleman can commit overcharging fraud by fraudulently diagnosing a customer who only requires c_l as requiring c_h, and then if that diagnosis is accepted, charging him p_h but only producing a treatment of cost c_l.

Customers pay different entry costs in the model. The entry cost of a type t customer is denoted k_t, and customers entry costs are uniformly distributed on $[0, V]$.

Uninformed customers who enter in some time period visit a middleman to have their problem diagnosed. But as the diagnosis may be fraudulent, they may either accept their first diagnosis or they may search again for a second opinion in the following time period. Search is assumed to be costless but for simplicity customers are restricted to only searching for a second opinion, and not a third or fourth opinion etc. A customer who has received two diagnoses may then choose which of them to accept. Once a customer has accepted his diagnosis, the middleman who offered it may also then offer to treat the customer himself at a price p_d. Customers then either

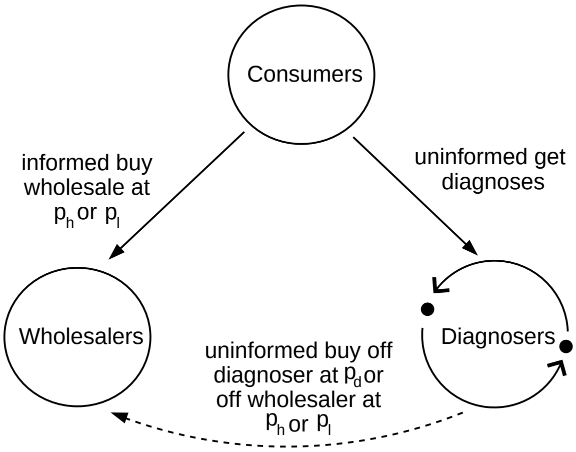

Figure 2.2: Consumers paths of play after entry depend on their types

purchase treatment off him, or use his diagnosis to purchase the diagnosed treatment off a wholesaler. As mentioned above, it is assumed that informed customers (who know the treatment they need) never visit a middleman and instead buy direct from wholesalers.

Firms may be wholesalers who compete for informed customers with publicly posted prices, but cannot diagnose a customer's problem, or they are middlemen who do not post public prices, but who can diagnose a customer. There are assumed to be 2 or more of both types of firms in the market, and wholesalers will in equilibrium all set the same price, $p_h(\theta)$ for the expensive treatment and $p_l(\theta)$ for the cheaper treatment, where θ represents the degree of price competition in the market. It is assumed that $\frac{dp_j}{d\theta} < 0$ for $j \in \{h, l\}$, $V > p_h > c_h$ and $V > p_l > c_l$.[1] The path of play for a consumer is shown in Figure 2.2. In each period, the timing in the model is as follows:

Timing

1. Wholesalers set prices.

2. Both informed and uninformed customers decide whether or not to enter the

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[1] This implies that standard models of competitive price setting, such as the Cournot model with $\theta = n$, could be used to solve for p_h and p_l in the wholesale market explicitly.

model at their individual entry cost k_t.

3. Uninformed customers then visit a middleman diagnoser, who offers them a diagnosis stating that either c_l or c_h is required.
4. If on their first visit, uninformed customers choose whether to accept the diagnosis or search again in the next period. If a customer has received two diagnoses, then he must choose one of the two to accept.
5. After a customer accepts a diagnosis, the middleman who's diagnosis was accepted then offers a price p_d to treat the customer.
6. Consumers purchase goods, with uninformed consumers choosing to purchase from either their middleman or from the wholesale market. Informed consumers purchase their treatment from the wholesale market.

That completes the discussion of the basic model. Proposition 1 below discusses equilibria in this model, and the relationship between welfare and price competition in p_h.

Proposition 1: Price competition in p_h may lower welfare if fraud is high. If $\alpha \geq (1-\alpha)F^2$ then price competition is welfare improving. However if $\alpha < (1-\alpha)F^2$ and (i) middlemen play a mixed strategy with $F \in [0,1]$ and (ii) $i < \hat{i}$ then welfare falls with greater price competition in p_h. If either (i) or (ii) does not hold, then price competition will again be welfare improving.

Proof: See appendix.

Put another way this proposition states that welfare may decline in p_h price competition only if there is a large enough fraction of customers buying through middlemen, those middlemen sometimes defraud their customers and customers face a greater probability of being fraudulently sold an expensive treatment than of genuinely requiring it. Otherwise, price competition in p_h will be welfare improving. This result is somewhat surprising, and as will be shown below it occurs because fraud increases as p_h decreases in the mixed strategy equilibrium. If fraud is relatively common (and F is high) and the number of consumers who must visit middlemen is also high (so i is low) then the negative effect of an increase in fraud more than offsets the usual positive welfare effects of lower prices.

To see this, note that there are three subgame perfect equilibria in the model. In the first equilibrium, middlemen always act honestly and tell the truth to agents genuinely requiring the c_l treatment, so $F = 0$. Here customers immediately accept a c_l diagnosis, but always search for a second opinion following a c_h diagnosis. In this equilibrium, fraudulently recommending c_h to a c_l type customer simply leads them to search for a second opinion, and that second opinion will be honest. So by attempting to defraud the customer the middleman misses out on charging him p_l for the cheap treatment and making a profit of $p_l - c_l > 0$. As such, there is no profitable deviation from telling the truth here. As agents anticipate being told the truth by middlemen, increased price competition here lowers prices, encouraging further entry and increases total welfare.

In the second equilibrium, middlemen always commit fraud by diagnosing all customers with c_h, so $F = 1$. Customers here do not bother to search for a second opinion (which they anticipate will again be c_h) and immediately accept a c_h diagnosis and then purchase. A middleman will not deviate and diagnose a customer as c_l here, as doing so only lowers his payoff. Clearly, the high level of fraud in the market dissuades many potential uninformed entrants from entering the market, but price competition here does increase entry, by lowering p_h the price every uninformed entrant expects to pay. So price competition increases welfare in this equilibrium as well (albeit from a lower starting point).

In the third equilibrium, middlemen mix between committing fraud and acting honestly, diagnosing a c_l type customer as c_h with probability $F \in [0, 1]$ where:

$$F = \frac{p_l - c_l}{p_h - p_l}$$

Price competition influences fraud in the market as fraud increases with p_l and decreases with p_h. The inverse relationship between p_h and F is especially interesting because it suggests that if price competition lowers p_h then while uninformed customers gain from lower prices they are hurt by more frequent fraud. If the net effect of a lower p_h and higher F is to increase the expected price an uninformed customer anticipates paying in the market, then that lowers entry by uninformed customers.

To see when this will be the case, first note that an uninformed customer with an entry cost k_t enters the market if $k_t \leq V - [\alpha + (1-\alpha)F^2]p_h - (1-\alpha)(1-F^2)p_l$. Recall that V is their payoff from being treated, and that their ex-ante probability of paying p_h is their probability of genuinely needing c_h, α plus their probability of not needing it $(1-\alpha)$ times the likelihood that they will visit two middlemen who both commit fraud and diagnose them as requiring c_h, which is F^2. So an uninformed customer expects with probability $[\alpha + (1-\alpha)F^2]$ to pay p_h. In a similar way, with probability $(1-\alpha)(1-F^2)$ uninformed customers expect to only require c_l and for it to be offered to them, so they will only have to pay p_l. As the entry costs of the $1-i$ uninformed are uniformly distributed on $[0, V]$, this implies that total uninformed entry denoted $Q_u^{F\in[0,1]}$ is given by:

$$Q_u^{F\in[0,1]} = \frac{(1-i)}{V}(V - \alpha p_h - (1-\alpha)F^2 p_h - (1-\alpha)(1-F^2)p_l)$$

As $F = \frac{p_l - c_l}{p_h - p_l}$ then as p_l falls F falls, so a fall in p_l increases entry by uninformed consumers. But as p_h falls, then F rises, and by differentiating $Q_u^{F\in(0,1)}$ with respect to p_h then $\frac{dQ_u^{F\in[0,1]}}{dp_h} > 0$ if:

$$\alpha < (1-\alpha)F^2$$

That is, a fall in p_h will deter entry by uninformed customers if their probability of genuinely needing the expensive treatment, α, is less than their probability of not needing it but having to pay for it anyway, $(1-\alpha)F^2$.

To determine the overall effect on welfare of a fall in p_h, note that although the net effect of a fall in p_h may discourage entry by uninformed consumers, which has a

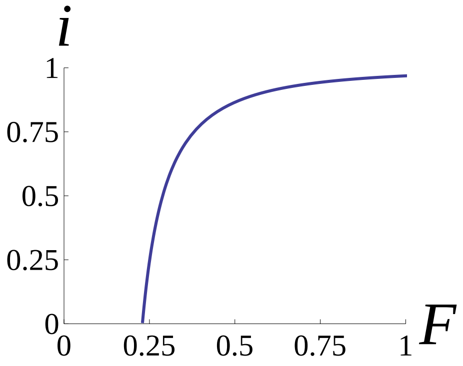

Figure 2.3: The bound $\hat{i}$ where $\alpha = 0.05$, $V = 7$, $p_h = 5$, $c_h = 4$, $p_l = 2.5$ and $c_l = 0$. Here $\hat{i} = 0$ at $F = \sqrt{\frac{\alpha}{1-\alpha}} \approx 0.23$.

negative effect on welfare, that it also encourages entry by informed consumers, which has a positive effect on welfare. By setting up an equation for total welfare, the proof shows that the overall effect on welfare of a fall in p_h is negative when $\alpha < (1-\alpha)F^2$ and the fraction of informed types i is below a bound denoted $\hat{i}$. Specifically if:

$$i < \hat{i} = \frac{(1-\alpha)F^2 - \alpha}{(1-\alpha)F^2 - \alpha + \frac{\alpha(p_h - c_h)}{\alpha(p_h - c_h) + (1-\alpha)(F^2 p_h + (1-F^2)p_l - c_l)}}$$

An example of $\hat{i}$ is shown in Figure 2.3. Here if $F \leq \sqrt{\frac{\alpha}{1-\alpha}} \approx 0.23$ then welfare is always increasing in price competition, but when $F > \sqrt{\frac{\alpha}{1-\alpha}} \approx 0.23$ then welfare is decreasing in price competition in the mixed strategy equilibrium if $i < \hat{i}$.

Proposition 1 indicates that price competition in p_h lowers welfare when fraud is high and there are few informed customers in the market buying direct (if firms are playing the mixed strategy equilibrium). But recall that fraud is endogenous and given by $F = \frac{p_l - c_l}{p_h - p_l}$. So fraud is low when $p_l - c_l$ is low. So another way to think of Proposition 1, is that price competition in p_h will raise welfare in the mixed strategy equilibrium only if the market for p_l is fairly competitive (implying F is low). This almost seems to suggest from a policy perspective that competition should be encouraged in cheaper, low price treatments in these markets before encouraging it in more expensive treatments, which could be fraudulently offered to consumers. Or at least that competition should not be encouraged in the more expensive treatments alone.

Below Proposition 2 will extend Proposition 1 to a more general setting, where some fraction of uninformed customers are time constrained and can only visit one middleman. There the effect of increased price competition lowering both p_h and p_l simultaneously will be considered as θ, the price competition parameter, rises.

2.3 Time Constrained Consumers

Consider the model presented above but now assume that a fraction $s \in (0, 1)$ of uninformed consumers only have the time to visit one middleman after entry. As such, they must accept the first diagnosis offered to them. Time constrained customers do not know that they are time constrained until they enter, and middlemen cannot distinguish them from non-time constrained customers. Proposition 2 below discusses the implications of this change to the model, and is followed by a numerical example of welfare lowering price competition using Cournot price setting. Again F is endogenous here, and as s rises from $s = 0$ here, the level of F in the mixed strategy equilibrium falls, implying that there will be less fraud than was the case in the mixed strategy equilibrium of Proposition 1.

Proposition 2: Price competition for both goods may lower welfare if fraud is high. If $\alpha \geq \frac{1-\alpha}{1-s}((1-s)F+s)^2$ then price competition is welfare improving. If $\alpha < \frac{1-\alpha}{1-s}((1-s)F+s)^2$ then price competition lowers welfare if (i) middlemen play a mixed strategy with $F \in [0, 1]$ (ii) $i < \hat{i}$ and (iii) the price of the potentially fraudulent good is relatively responsive to competition such that $\frac{dp_h}{dp_l} > \bar{p}$. If any of (i), (ii) or (iii) do not hold, welfare is again increasing in price competition.

Proof: See appendix.

As in the previous proposition there are three equilibria, and again in the two pure strategy equilibria (with $F = 0$ and $F = 1$) price competition is always welfare improving. In the third equilibrium, whereas previously it was sufficient that $\alpha \geq$

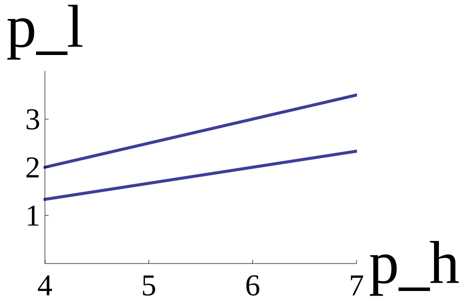

Figure 2.4: The critical p_l above which a fall in p_h may lower welfare. The lower line is for $s = 0$ as in Proposition 1, while the higher has $s = 0.75$. It is assumed that $\alpha = 0.5$ and $c_l = 0$.

$(1-\alpha)F^2$ for price competition in p_h to always be welfare improving, it is now only the case for $\alpha \geq \frac{1-\alpha}{1-s}((1-s)F+s)^2$. Notice that this expression nests the expression from Proposition 1 as a special case when $s = 0$. Here though, F in the mixed strategy equilibrium is now given by:

$$F = \frac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)}$$

Because $\frac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)} < \frac{p_l - c_l}{p_h - p_l}$ then given some p_h, p_l and c_l this is strictly less than the level of fraud found when $s = 0$ (as it was in the previous proposition). And differentiating with respect to s further reveals that it is falling monotonically in s.

Using this F, the condition $\alpha \geq \frac{1-\alpha}{1-s}((1-s)F+s)^2$ can be re-written as $\alpha \geq \frac{1-\alpha}{1-s}(\frac{p_l - c_l}{p_h - p_l})^2$. So the right hand side of this inequality is the same as that from the previous proposition (where $F = \frac{p_l - c_l}{p_h - p_l}$) divided by $1-s$. This condition tells us that price competition will no longer improve welfare in circumstances where it used to (given the same p_h, p_l and c_l). An example of this is shown in Figure 2.4 where $\alpha = \frac{1-\alpha}{1-s}(\frac{p_l - c_l}{p_h - p_l})^2$ is rearranged to solve for p_l as a function of p_h.[2]

But given that fraud is lower here, why is price competition (in the mixed equilibrium) more likely to lower welfare here than in Proposition 1? There are two

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[2] It is worth noting that this important fraction $\frac{p_l - c_l}{p_h - p_l}$ is equal to the constant $\frac{V - c_l}{c_h - c_l}$ if prices are set as $p_j = \frac{V + nc_j}{n+1}$ following Cournot competition with n firms supplying both types of informed customers. Similarly it becomes $\frac{t}{n^2(c_h - c_l)}$ using price setting from Salop's circle model with n firms supplying both types of informed customer, where the cost to a consumer of traveling x to purchase on the circle is given by tx^2. Of course many different price setting models could be 'plugged in' here. Below in the numerical example I use a fixed p_l and Cournot price setting for p_h.

reasons. First, the level of fraud may be lower but the change in the level of fraud $\frac{dF}{dp_h} = -\frac{p_l - c_l}{(p_h - p_l)^2}$ is exactly the same as it was in the earlier proposition. That is, a fall in p_h here will increase the level of fraud by exactly the same amount as earlier. And secondly, because $s > 0$ uninformed customers are relatively more wary of fraud because only some of them now get to seek a second opinion. Together, the overall effect is to make a fall in p_h less attractive that it was in the previous proposition (regardless of its level). Because of that (as shown in Figure 2.4) price competition is now more likely to decrease welfare than earlier.

As before, increased price competition here lowers welfare by causing a fall in p_h which increases F. This may reduce welfare by raising the expected price that an uninformed customer expects to pay (when the likelihood of greater fraud outweighs the benefit from a reduced p_h) and uninformed consumers are discouraged from entry by this if $\frac{dQ_u^{F\in[0,1]}}{dp_h} > 0$ or:

$$\alpha < \frac{1-\alpha}{1-s}((1-s)F + s)^2$$

Recall that this is now more likely than in Proposition 1. However unlike the earlier proposition, here I am considering a change in θ which effects both p_h and p_l. And a fall in p_l is unambiguously good for consumers as it lowers both p_l and F. That implies a further condition must be met for price competition to lower entry, and for $\frac{Q_u^{F\in[0,1]}}{d\theta} < 0$ (so price competition does lower entry) then:

$$\frac{dp_h}{dp_l} > \bar{p} = \frac{\frac{1-\alpha}{1-s} + (1-\alpha)(2F + (1-s)F^2)}{(1-\alpha)(2sF + (1-s)F^2 + \frac{s^2}{1-s}) - \alpha}$$

So as θ rises, if the fall in p_h is relatively great in comparison to the fall in p_l and $\alpha < \frac{1-\alpha}{1-s}((1-s)F + s)^2$ then entry by uninformed consumers will fall.

But that is still not a fall in welfare. A fall in prices that causes a fall in uninformed entry will still cause a rise in informed consumer entry, which may raise total welfare. In order for the net effect to be negative, again there must be relatively few informed types, so i must be low. First note that total welfare is given by:

$$W_T = Q_u^{F\in[0,1]}[V - \alpha c_h - (1-\alpha)c_l - \frac{\hat{k}_u^{F\in(0,1)}}{2}] + Q_h[V - c_h - \frac{\hat{k}_h}{2}] + Q_l[V - c_l - \frac{\hat{k}_l}{2}]$$

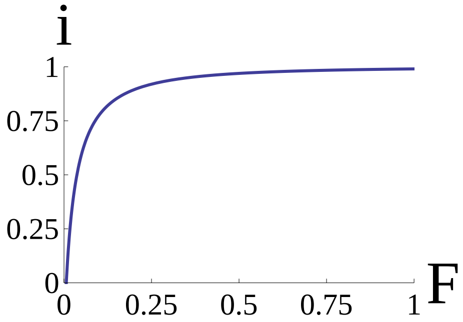

Figure 2.5: The bound $\hat{i}$ if $\frac{dp_l}{d\theta} = 0$, for $s = 0.5$, $V = 7$, $\alpha = 0.05$, $p_h = 5$, $c_h = 4$, $p_l = 2.5$ and $c_l = 0$.

The first term $Q_u^{F\in[0,1]}$ is the number of uninformed entrants, this is multiplied by the average welfare generated by each uninformed entrant, where the uninformed entrant with the highest entry cost who does enter has a cost $\hat{k}_u^{F\in(0,1)}$. Similarly, Q_h is the number of informed entrants with the c_h problem and is multiplied by average welfare per entrant of this type, where they enter up to some entry cost given by $\hat{k}_h$. And Q_l is the number of informed entrants with the cheap problem which is also multiplied by average welfare per entrant of this type where they enter up to some entry cost $\hat{k}_l$. So total welfare here reflects a fairly standard problem with linear downward sloping 'demand' curves but with three different types of consumer possessing different expectations of their final price, and different treatment costs.

The effect of increased price competition will be to reduce welfare then if $\frac{dW_T}{d\theta} < 0$. That is the case if and only if $i < \hat{i}$, so the number of informed customers who always benefit from lower prices is small. Here $\hat{i}$ is given by:

$$\frac{[(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha]\frac{-dp_h}{d\theta}-[\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)]\frac{-dp_l}{d\theta}}{[(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha]\frac{-dp_h}{d\theta}-[\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)]\frac{-dp_l}{d\theta}+\frac{(\alpha(p_h-c_h)\frac{-dp_h}{d\theta}+(1-\alpha)(p_l-c_l)\frac{-dp_l}{d\theta})}{\alpha(p_h-c_h)+(1-\alpha)(p_l-c_l)+(1-\alpha)(sF+(1-s)F^2)(p_h-p_l)}}$$

This is a complicated expression, but should resemble something like the $\hat{i}$ found for Proposition 1 and must be in $[0, 1]$ if $\frac{dp_h}{dp_l} > \bar{p}$. In Figure 2.5 an example of $\hat{i}$ is given, and clearly it does resemble the earlier bound. So again, as long as middlemen are playing the mixed-strategy equilibrium (and committing fraud some of the time) then if fraud is relatively common and there are few informed customers, and if the new condition that p_h changes enough relative to p_l as θ varies holds, then welfare falls. To illustrate this further, a numerical example is given below.

As F is a function of s, p_h and p_l in the mixed strategy equilibrium, solved examples of the mixed strategy equilibrium must be careful to ensure that $F \in [0,1]$. Bearing this in mind, in the numerical example below, I choose the parameters carefully so that this will be the case as θ varies using Cournot price setting where θ is the inverse of firms fixed costs, so as θ rises, entry and prices fall.

An Example with Cournot Competition In this example, I present a numerical solution with Cournot price setting, where (a) $\alpha < \frac{1-\alpha}{1-s}((1-s)F+s)^2$, (i) $F \in (0,1)$, (ii) $i < \hat{i}$ and (iii) $\frac{dp_h}{dp_l} > \bar{p}$. So (according to Proposition 2) an increase in price competition should decrease total welfare (further note that I am ignoring the welfare loss from fixed costs here, as I am not interested in the Cournot model per se, but wish to 'plug in' a model with tractable price setting in order to illustrate the welfare analysis of Proposition 2).

Begin by assuming that p_l is fixed at $p_l = 2.5$, this implies condition (iii) is met. There are n firms competing to set quantities for the expensive treatment in the wholesale market, and equilibrium prices are assumed to be given by the Cournot price $p_h = \frac{V+nc_h}{n+1}$. (Recall that here consumers all have a willingness to pay of V but in effect a linear downward sloping demand curve exists due to the uniform distribution of their entry costs on $[0,V]$. So here demand is $Q = \alpha i(V-p_h)$, leading to the Cournot prices above with n firms). The number of firms in the market is determined by the fixed cost K. The competition parameter θ is assumed to be the inverse of these fixed costs, so $\theta = \frac{1}{K}$. (And again using the usual Cournot reasoning the number of firms in equilibrium will be $n = \sqrt{\alpha i\theta}(V-c)$.) Thus as θ rises, the number of firms in the model rises.

In terms of the parameters I assume that $c_l = 0$, $c_h = 4$, $V = 7$, $s = \frac{1}{2}$, $i = \frac{1}{2}$ and $\alpha = 0.05$. Notice that these parameter imply that (a) is always met for any $F \in [0,1]$. As $\frac{dp_l}{d\theta} = 0$ the bound $\hat{i}$ starts at 0.974 and rises as θ increases. So as $i = 0.5 < 0.974 \leq \hat{i}$ so (ii) is also always met.

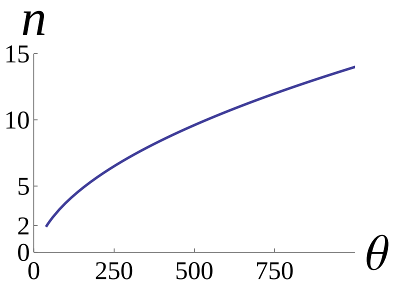

Figure 2.6: Firms

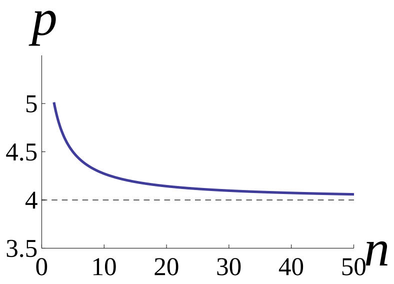

Figure 2.7: p_h

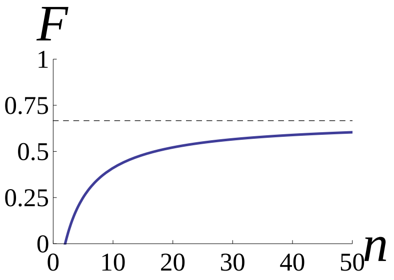

Figure 2.8: Fraud

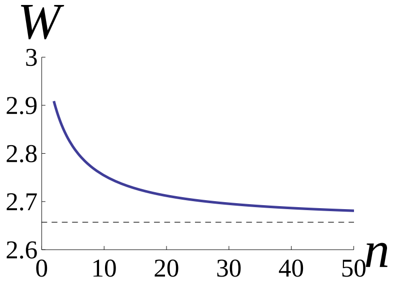

Figure 2.9: Welfare

To begin with I will set $K = \frac{1}{40}$ for firms in the c_h wholesaling market, which implies that exactly 2 firms enter and set p_h such that $F = 0$ in the mixed strategy equilibrium. Letting $\theta \to \infty$, $K \to 0$, so $n \to \infty$, $p_h \to c_h = 4$ and $F \to \frac{2}{3}$. The changes in the variables can be seen in the Figures presented. Notice that $F \in (0, 1)$ across these parameters so (i) is always met and as all the elements of the proposition are met in this case, then as θ rises then W_T should fall. This is shown in the final figure (noting again that W_T ignores the firms fixed costs).

2.4 Does more information help?

The previous section has shown that price competition may reduce total welfare in credence markets. But what is the effect of more information? Here I consider the effect of increasing the fraction of informed consumers i. That means more consumers are aware of exactly what treatment they require, and will enter the market to buy that treatment from a wholesaler if the price is low enough. In Proposition 3 I show that if prices are held constant and p_h is close enough to c_h then shifting consumers from being uninformed to informed must increase welfare regardless of F. Unfortunately, Proposition 4 then shows that if prices are not held constant (so an increase in the size of the informed market buying direct leads to greater price competition and decreased prices - as occurs in standard price setting models such as the Cournot or Salop circle models) then a marginal increase in information may cause welfare to fall for the same reasons as shown in Proposition 2. Despite this, Proposition 5 shows that if p_h is again close enough to c_h and $i \to 1$ then there is a U-shaped effect where despite causing welfare to initially fall, information then causes welfare to rise beyond its initial level. So in credence goods markets that are fairly competitive more information must eventually improve welfare.

Proposition 3: Information improves welfare with fixed prices if p_h is low For any $F \in [0, 1]$ where there is fixed prices $p_h = p_h^*$ and $p_l = p_l^*$ an increase in the share of customers buying direct is welfare improving if $p_h^* < \bar{p_h} = c_h + (c_h - c_l) + (p_l - c_l)$.

Proof: See appendix.

The intuition behind this result can be illustrated by assuming that $F = 0$, so there is no fraud. In that case, there is less entry by uninformed customers requiring the cheap c_l treatment and more entry by uninformed customers requiring the c_h treatment than there is by informed customers requiring those two treatments. To see this note that the fraction of the i informed customers entering is always given by:

$$\alpha \frac{V - p_h^*}{V} + (1 - \alpha) \frac{V - p_l^*}{V}$$

And if $F = 0$, the fraction of the $1 - i$ uninformed customers that enter is given by:

$$\frac{V - \alpha p_h^* - (1 - \alpha) p_h^*}{V}$$

This can be re-arranged as $\alpha\frac{V-p_h^*}{V}+(1-\alpha)\frac{V-p_l^*}{V}$ which means the total share of informed and uninformed customers entering is the same when $F=0$.

But a greater share of the fraction of uninformed types that do enter need the c_h treatment. To see this, notice that the share of uninformed entering that require the c_h treatment is given by α, their share of the population. Of the informed customers, the share of customers entering who require the c_h type treatment is given by

$$\begin{aligned} & \frac{\alpha\frac{V-p_h^*}{V}}{\frac{V-\alpha p_h^*-(1-\alpha)p_h^*}{V}} \\ = \quad & \alpha\frac{V-p_h^*}{\alpha(V-p_h^*)+(1-\alpha)(V-p_l^*)} < \alpha \end{aligned}$$

Where the inequality holds true as the denominator $\alpha(V-p_h^*)+(1-\alpha)(V-p_l^*) > V-p_h^*$ because $p_l^* < p_h^*$. So (although the total fraction of informed or uninformed entering is the same) a smaller share of informed entrants requires the expensive c_h treatment in comparison to the uninformed. Uninformed c_h types therefore overenter relative to informed ones.

It may appear that having more of the relatively expensive c_h types and less of the relatively cheap c_l types enter would imply that uninformed entry produced less welfare. But because customers make their entry decision based on the market prices p_h and p_l and not on the underlying costs of treatment c_h and c_l, this overentry may sometimes be welfare improving. When the margin on the expensive treatment is low and p_h is close enough to c_h then the overentry of uninformed c_h types is welfare reducing. But when p_h is relatively high, overentry can be welfare improving, so more information is unhelpful here. In contrast, there is always underentry of uninformed types requiring the c_l treatment, so information is always welfare improving for that type. But in general, more information may not improve overall welfare here.

So when does information improve welfare then? In order to state that moving some fraction of agents from being uninformed to informed (at fixed prices) increases welfare (still assuming no fraud), as shown in the proof, I require that $p_h < c_h + (c_h - c_l) + (p_l - c_l)$, which can be re-arranged as

$$V - c_l - \frac{\hat{k}_l}{2} > V - c_h - \frac{\hat{k}_h}{2}$$

This condition states that the average welfare of an informed c_l type who enters is greater than the average welfare of an informed c_h type who enters. This is the necessary condition for information to improve welfare, as information always induces relatively more c_l types and fewer c_h types to enter. As when this condition holds information improves welfare when $F = 0$, then when the condition holds, information must also improve welfare where $F > 0$ as not only are relatively more of the c_l types entering, leading to a welfare gain, but a greater share of informed types enter than uninformed when $F > 0$. So if $p_h < \bar{p}_h$ then increasing the share of informed customers is welfare improving for any level of fraud $F \in [0, 1]$.

Of course, this all assumes that prices are fixed. Below it is shown that if they are endogenous, then as a greater share of i types induces firms to set lower prices in the wholesale market, welfare may still decrease with more information, even if $p_h < \bar{p}_h$.

Proposition 4: But with endogenous prices, information may still cause welfare to fall. If (i) a mixed strategy equilibrium exists with $F \in [0, 1]$ and (ii) $i < \hat{\imath}$ then marginally increasing the share of informed customers buying direct reduces welfare.

Proof: See appendix.

Consider the case where $p_h < \bar{p}_h$ as shown in the preceding proposition. In that case, more information always improves welfare if prices are fixed because informed customers generate more welfare on average, and there is less overentry by c_h types. But even here if increasing the number of informed customers also leads to greater price competition amongst wholesalers then, as was shown in Proposition 2, information may reduce welfare. Here $\hat{\imath}$ is given by a rather involved expression that does not lend itself to economic interpretation, but can perhaps be thought of as the $\hat{\imath}$ from Proposition 2, adjusted for the welfare gains (or losses) of moving agents from being uninformed to informed. The purpose of this proposition is simply to illustrate that we cannot guarantee then that a marginal increase in information increases welfare even if $p_h < \bar{p}_h$ and to set the stage for Proposition 5, which does say something of interest.

Proposition 5: As all agents become informed, then welfare is maximised if p_h is low. If $p_h < \bar{p}_h$ then total welfare is maximised as the share of informed customers buying direct $i \to 1$.

Proof: First note that the effect of increasing the fraction of informed consumers is to increase price competition in the wholesale market, lowering prices there and increasing the average welfare of informed consumers. To see this recall that $W_i = \alpha(\frac{V+p_h}{2} - c_h)\frac{V-p_h}{V} + (1-\alpha)(\frac{V+p_l}{2} - c_l)\frac{V-p_l}{V}$ which is monotonically increasing in i as $\frac{dW_i}{di} = \frac{1}{V}(\alpha(p_h - c_h)\frac{-dp_h}{di} + (1-\alpha)(p_l - c_l)\frac{-dp_l}{di}) > 0$. So W_i is increasing in i and is maximised at $i = 1$.

Now note that total welfare W_T is a convex combination of W_i and W_u, such that $W_T = iW_i + (1-i)W_u$. As shown in Proposition 3, $W_i > W_u$ for all $F \in [0,1]$ if $p_h < \bar{p}_h$ which implies that $W_i > W_T$ as W_i must be greater than a strictly convex combination of itself and something smaller than itself.

Finally notice that as $i \to 1$ then $W_T \to W_i$. That implies that as $i \to 1$ then W_T tends towards the larger value in its support, a value that is also monotonically increasing as i increases. Therefore W_T must tend towards its maximum value as $i \to 1$. As this is true for all $F \in [0,1]$ then it is true for both the pure and mixed equilibria. ■

The implication of Proposition 5 is that as the fraction of informed consumers increases, even if total welfare decreases over some region, that if i is increased *enough* then welfare must eventually increase and tend towards its maximum value over i. So while a little information may be damaging to welfare, so long as the market is competitive enough, more information must eventually be welfare improving.

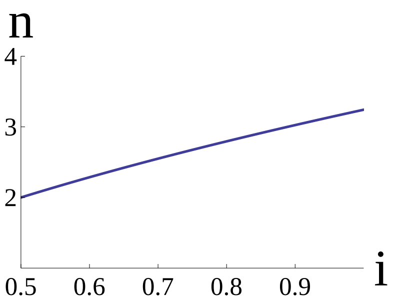

Figure 2.10: Firms

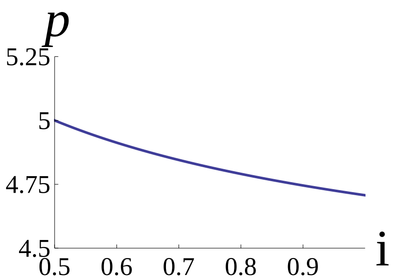

Figure 2.11: p_h

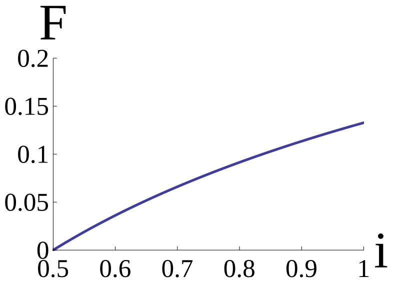

Figure 2.12: Fraud

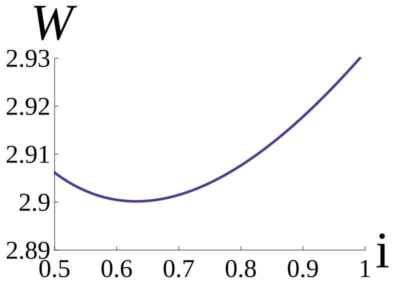

Figure 2.13: Welfare

Example with Cournot Competition Consider the numerical example with Cournot price competition given earlier. There it was assumed that $i = \frac{1}{2}$, $c_l = 0$, $p_l = 2.5$, $c_h = 4$, $V = 7$, $s = \frac{1}{2}$, $\alpha = 0.05$, and $K = \frac{1}{40}$ (where K is the fixed cost of entry for firms in the c_h wholesaling market). That initially leads to an equilibrium with $F = 0$, $n = 2$, and $p_h = 4$. Previously a decrease in entry costs K unambiguously lowered total welfare, but here it can be shown that if i increases (holding K constant) then while a similar effect persists for a while (with increased fraud and decreased welfare), as $i \to 1$ then welfare eventually begins increasing up towards its maximum. So there is a U-shaped relationship between information and welfare in this example which reflects that $p_h \leq 4$ and $\bar{p}_h = 10.5$. So as shown in Proposition 5, increased information must eventually increase welfare.

2.5 Conclusion

This chapter has studied the effect of price competition and information on entry and welfare in a credence goods market where some customers could get a second opinion. Greater price competition was shown in general to be welfare improving, but if middlemen play a mixed strategy, few customers are informed, and fraud is prevalent then it was shown that more price competition could be welfare reducing. Increasing the fraction of informed consumers with fixed prices was shown to be welfare improving if the market was relatively competitive, but if prices were flexible then again by intensifying price competition more information could also decrease welfare. Despite this, if the market is relatively competitive and the fraction of informed consumers rises enough then additional information is eventually welfare improving. So if we can aid customers in cutting out the middleman and buying direct, we should (when the market is competitive).

It's safe to say that most economists assume competition has a positive effect on welfare. The contribution of this chapter is not to disagree with that point *in general* but to point out that in credence markets where prices influence the level of fraud, price competition has a more complicated relationship with welfare than usual. Here it was shown that increased price competition may actually decrease welfare if there is enough deviation away from the standard assumptions of fully informed consumers, and if many of those uninformed consumers are then further constrained by not having the time to seek out competing advice.

Papers in the credence good literature have focused largely on the equilibrium strategies of the diagnoser-customer interaction (Pitchik and Schotter (1987), Fong (2005)), the decision to search for a second opinion (Wolinsky (1993, 1995)), or the correct prices for diagnosers to set in order to guarantee their honesty to customers (Emons (1997, 2001), Dulleck and Kerschbamer (2006, 2005b)). One paper which includes competition in terms of both the price and the honesty of expert advice is Pesendorfer and Wolinsky (2003), where experts offer customers a particular treatment at a price, and customers may make a costly search for further expert opinions before deciding on an offer of treatment. But again the focus is on qualitative features of the equilibrium strategy and not welfare.

The result that price competition has ambiguous effects on welfare in credence goods markets is I believe a general one. This is reminiscent of many results in information economics when the model is moved away from assumptions of full information. As shown by Dulleck and Kerschbamer (2006) there are also many ways to model credence goods, so welfare effects are likely to further depend on the assumptions made in terms of the way credence goods are modelled. In this chapter the potential problem is one of overcharging fraud and the equilibrium of interest is in mixed-strategies in a way similar to Wolinsky, Fong and Pitchik and Schotter. An alternative approach to overcharging, this time in pure strategies, is that of Dranove (1988) who models customers and experts choosing treatment cutoffs following a noisy signal. But despite the different framework, Dranove also finds an inverse relationship between fraud and price, although in his model there is only one price so it is difficult to draw too many comparisons. Nevertheless this inverse relationship suggests that the net effect of more price competition (lower prices and higher fraud) may again be to dissuade consumer entry and possibly reduce welfare in some circumstances. So while I expect the propositions in this chapter to be somewhat dependent on its formulation of the overcharging problem in a market for credence goods, I expect other formulations in the credence goods tradition will find similar results. Investigating this claim may be one avenue for future work.

2.6 Appendix

Proof of Proposition 1 In each period the game is solved backwards.

Step 6: The Purchase Decision As $V > p_h > c_h$ and $V > p_l > c_l$ by assumption, customers will always choose to purchase treatment. Informed customers will choose to purchase the treatment j at the prevailing price p_j from a wholesaler. Uninformed customers will purchase treatment from their diagnoser if his offered price p_d is at or below the prevailing wholesale price p_j, for the c_j type treatment. If $p_d > p_j$ then the customer will use the middleman's diagnosis to purchase the c_j type treatment off a wholesaler instead.

Step 5: Diagnoser's Price Offer A diagnoser who has had her diagnosis accepted by a customer may offer to treat the customer at some price denoted p_d. As she knows the customer will leave if she offers a price $p_d > p_j$ she will match this price so that $p_d = p_j$. As the customer accepts treatment at this price the diagnoser's profit will be p_j minus the true cost of treatment c_h or c_l.

Notice that if the customer purchases the cheap treatment from the diagnoser when he only actually requires the cheap treatment, then the diagnoser makes a profit of $p_l - c_l$. If the customer genuinely requires the expensive treatment and pays p_h to his diagnoser, then the diagnoser makes a profit of $p_h - c_h$. But if the diagnoser fraudulently recommended the expensive treatment when only the cheap treatment was necessary, then she only has to pay c_l to treat the customer, making a profit of $p_h - c_l$. In that case, the diagnoser has successfully committed fraud by overcharging the customer with a higher than market price for that treatment.

Step 4: Search? Search is costless but may only occur once. If $F = 1$ or $F = 0$ search will not change anything, as the second middleman will make the same diagnosis as the first. In the $F = 1$ equilibrium, I assume that consumers do not search, while in the $F = 0$ equilibrium I assume that they do, following a c_h diagnosis. If

$F \in (0,1)$ all consumers will search for a second opinion following a c_h recommendation because there is a positive probability the first expert has lied to them, and a positive probability that a second diagnosis will be c_l. So as $p_l < p_h$ and search is costless, agents will search for a second opinion following a c_h diagnosis. In all equilibria it is assumed that agents will immediately accept the diagnosis of a middleman if it is c_l as they cannot do better than this.

Step 3: Diagnosis First note that due to the liability assumption, if a customer genuinely requires the c_h treatment, then a middleman must recommend that treatment to him. However when a customer only requires the cheap treatment, the middleman must assess whether or not he should commit fraud. The expected equilibrium level of fraud committed in diagnosis is denoted F. To determine whether or not to commit fraud, note that a diagnoser knows that each period a number of c_l type customers enter and visit him first. This number is normalised to 1. Also, there is a fraction of c_l type customers who visited another expert but were told they need the c_h treatment by him, so are searching for a second opinion. This amounts to the fraction F of the 1 that visited another expert for their first diagnosis. Altogether then $1 + F$ c_l-type customers visit a middleman each period, with the fraction $\frac{F}{1+F}$ on their second visit (having been diagnosed c_h before) and the fraction $\frac{1}{1+F}$ just having entered and have never been diagnosed before.

A middleman knows that if he honestly recommends c_l all customers will stop and purchase from him at the price p_l. So the payoff is $p_l - c_l$. If the middleman defrauds his customer however, and falsely recommends the c_h treatment, then only the fraction that have to buy from him (those getting a second opinion) will do so. (I assume that customers offered c_h twice will always choose to accept the second middleman's c_h diagnosis. This does not effect the equilibrium in any way as whatever rule the customers use in equilibrium to decide which of two identical diagnoses to accept will still leave each middleman with the same share of the pool of twice-defrauded consumers.) That fraction is $\frac{F}{1+F}$ of the total number of c_l types. For each of those that do accept the expensive treatment he will make a profit of $p_h - c_l$ as he only provides the cheap treatment, despite charging for the expensive one. Therefore his expected profit per c_l type customer from recommending a fraudulent treatment is $\frac{F}{1+F}(p_h - c_l)$.

There are three symmetric equilibria. In the first, all experts expect all other experts to always commit fraud, so $F = 1$, and as mentioned in step 4, customers never search (as there is no point in doing so) and then purchase from the middleman at p_h. So there is no incentive to deviate to recommending c_l here. Second, if all diagnosers expect all other diagnosers to never commit fraud, so $F = 0$ then $\frac{F}{1+F}(p_h - c_l) = 0$ while $p_l - c_l > 0$ and no expert has any incentive to deviate from telling the truth here and this is an equilibrium. Finally, if diagnosers expect others to commit fraud with a frequency $F \in (0,1)$ such that $\frac{F}{1+F}(p_h - c_l) = p_l - c_l$, then diagnosers will be indifferent between committing fraud and acting honestly so will be happy to commit fraud with that frequency themselves. That is, there is a mixed strategy equilibrium with middlemen indifferent between committing fraud and acting honestly if:

$$\begin{aligned} & \tfrac{F}{1+F}(p_h - c_l) = p_l - c_l \\ \Rightarrow \qquad & F = \tfrac{p_l - c_l}{p_h - p_l} \end{aligned}$$

Notice that this equilibrium only exists if $F \in [0,1]$, and that $p_l \geq c_l$ implies $F \geq 0$ and $p_h \geq 2p_l - c_l$ implies $F \leq 1$.

Step 2: Customers decide whether to enter Informed customers enter if their entry cost is low enough to justify it at the equilibrium prices. That is an informed consumer with the c_h type problem and entry cost k_t will enter if:

$$V - p_h - k_t \geq 0$$

So denoting the highest type of informed customer with the c_h problem who enters as $\hat{k}_h$, then:

$$\hat{k}_h = V - p_h$$

Recalling that customers entry costs are distributed uniformly on $[0, V]$ and that of the unit mass of customers i are informed and α have the c_h problem, then denoting as Q_h the number of informed consumers with the expensive problem who enter, Q_h is given by:

$$Q_h = i\alpha\frac{V - p_h}{V}$$

In a similar way the number of informed types who enter with the inexpensive problem, denoted Q_l, can be shown to be:

$$Q_l = i(1 - \alpha)\frac{V - p_l}{V}$$

Uninformed customers must also weigh up if they should enter given the prevailing wholesaler prices, as well as the expected level of fraud that they will face and that they will get a second opinion. An uninformed customers expected payoff from entry is:

$$V - k_t - \alpha p_h - (1 - \alpha)F^2 p_h - (1 - \alpha)(1 - F^2)p_l$$

If $F = 1$ then recalling the assumption that entry costs are distributed uniformly on $[0, V]$, I denote the highest cost uninformed type who enters the model as $\hat{k}_u^{F=1}$ which is given by $\hat{k}_u^{F=1} = V - p_h$. As there are $1 - i$ uninformed consumers then the number of uninformed customers entering the model when $F = 1$ is given by:

$$Q_u^{F=1} = (1 - i)\frac{V - p_h}{V}$$

Similarly, if $F = 0$ then:

$$Q_u^{F=0} = (1 - i)\frac{V - \alpha p_h - (1 - \alpha)p_l}{V}$$

If $F \in (0, 1)$ then uninformed customers anticipate that in equilibrium the level of fraud will be that given in Step 3, $F = \frac{p_l - c_l}{p_h - p_l}$. Using this then an uninformed customers expected payoff from entry can be re-written as:

$$V - k_t - \alpha p_h - (1 - \alpha)p_l - (1 - \alpha)(p_l - c_l) - (1 - \alpha)(p_l - c_l - \frac{p_l - c_l}{p_h - p_l}(2p_l - p_h - c_l))$$

Denoting the highest cost uninformed type who enters the model when $F \in (0,1)$ as $\hat{k}_u^{F\in(0,1)}$ this is given by

$$\hat{k}_u^{F\in(0,1)} =$$

$$V - \alpha p_h - (1-\alpha)p_l - (1-\alpha)(p_l - c_l) - (1-\alpha)(p_l - c_l - \tfrac{p_l - c_l}{p_h - p_l}(2p_l - p_h - c_l))$$

So as there are $1-i$ uninformed and entry costs are uniformly distributed on $[0, V]$ then:

$$Q_u^{F\in(0,1)} = \frac{(1-i)}{V}(V - \alpha p_h - (1-\alpha)p_l - (1-\alpha)(p_l - c_l - \frac{p_l - c_l}{p_h - p_l}(2p_l - p_h - c_l)))$$

Step 1: Wholesalers set Prices Wholesalers anticipate that whatever the prevailing price is in the wholesale market that middlemen will match that price to ensure that no uninformed customers end up buying wholesale. So wholesalers set prices in order to compete solely for informed customers. Noting the intensity of price competition is given by θ then equilibrium prices are $p_h(\theta)$ and $p_l(\theta)$ where $p_h \in [c_h, V]$, $p_l \in [c_l, V]$ and $p_h > p_l$ for some θ.[3]

That completes the description of sub-game perfect equilibria in the model.

Entry and Welfare To show that the proposition is true, first note that there was only one equilibrium where middlemen played a mixed strategy with $F \in (0,1)$. In that equilibrium by differentiating $Q_u^{F\in(0,1)}$ with respect to p_h, the price of the expensive treatment:

$$\frac{dQ_u^{F\in(0,1)}}{dp_h} = \frac{(1-i)}{V}(-\alpha + (1-\alpha)(\frac{p_l - c_l}{p_h - p_l})^2) = \frac{(1-i)}{V}(-\alpha + (1-\alpha)F^2)$$

As $\frac{(1-i)}{V} > 0$ then $\frac{dQ_u^{F\in(0,1)}}{dp_h} > 0$ if $-\alpha + (1-\alpha)F^2 > 0$ or

$$\alpha < (1-\alpha)F^2$$

---

[3] Again note that an example of this is the Cournot model with $\theta = n$ so that in equilibrium $p_l = \frac{V+nc_l}{n+1}$ and $p_h = \frac{V+nc_h}{n+1}$.

So if the probability of genuinely needing the expensive treatment is less than the probability of not needing it but being defrauded twice, then price competition in p_h leads fewer uninformed customers to enter the market. The reason for this is the additional expected cost from more overcharging fraud outweighs the benefits of a lower price. Now let $\hat{F} = \sqrt{\frac{\alpha}{1-\alpha}}$ denote the level of fraud in this market for which a change in p_h will have no effect on uninformed entry as $\frac{dQ_u^{F\in(0,1)}}{dp_h} = \frac{(1-i)}{V}(-\alpha + (1-\alpha)\hat{F}^2) = 0$.

For total welfare denoted W_T to fall due to an increase in price competition and a fall in p_h we require that then $\frac{dW_T}{dp_h} > 0$. A lower p_h must encourage entry and improve the welfare of informed customers, so in order for the overall effect to be negative it must also lower entry and welfare of uninformed customers. Note that total welfare is given by the number of each type that enter multiplied by the expected welfare that their entry creates, $V - c_h - \frac{\hat{k}_h}{2}$ and $V - c_l - \frac{\hat{k}_l}{2}$ for informed high and low types respectively, and $V - \alpha c_h - (1-\alpha)c_l - \frac{\hat{k}_u^{F\in(0,1)}}{2}$ for uninformed types as they may require the high treatment with probability α and the low treatment with probability $1-\alpha$. So total welfare is given by

$$W_T = Q_u^{F\in(0,1)}[V - \alpha c_h - (1-\alpha)c_l - \frac{\hat{k}_u^{F\in(0,1)}}{2}] + Q_h[V - c_h - \frac{\hat{k}_h}{2}] + Q_l[V - c_l - \frac{\hat{k}_l}{2}]$$

As $F > \hat{F}$ then for $\frac{dW_T}{dp_h} > 0$:

$$\frac{dW_T}{dp_h} =$$

$$\frac{(1-i)}{V}((1-\alpha)F^2 - \alpha)[\alpha(p_h - c_h) + (1-\alpha)(F^2 p_h + (1-F^2)p_l - c_l)] - \frac{i\alpha}{V}[p_h - c_h] > 0$$

$$\Rightarrow \quad \frac{(1-\alpha)F^2-\alpha}{(1-\alpha)F^2-\alpha+\frac{\alpha[p_h-c_h]}{\alpha(p_h-c_h)+(1-\alpha)(F^2p_h+(1-F^2)p_l-c_l)}} > i$$

Let the left hand side of this inequality be denoted $\hat{i}$. Notice that because $F > \hat{F}$, $p_h > c_h > c_l$, $p_l > c_l$ and therefore $F^2 p_h + (1-F^2)p_l > c_l$ that implies $\hat{i} \in (0,1)$. So if $F > \hat{F}$ and $i < \hat{i}$ (or equivalently $1 - i > 1 - \hat{i}$) then price competition in the potentially fraudulent good that lowers p_h will decrease welfare as $\frac{dW_T}{dp_h} > 0$. If either

$F < \hat{F}$ or $i \leq \hat{i}$ or in either of the pure strategy equilibria, a fall in p_h will increase welfare. ■

Proof of Proposition 2 Solving for subgame perfect equilibrium again involves backward induction for some arbitrary period. This will be identical in steps 4, 5 and 6 to Proposition 1, except that a customer told they need c_h is now only able to search with probability $1 - s$. The rest must stop and purchase without a second opinion.

Step 3: Diagnosis In equilibrium, again normalise to 1 the number of c_l-types who have just entered the game and are visiting a particular middleman. A further $(1 - s)F$ c_l type customers will also be visiting the same middleman to get a second opinion. So of the c_l types getting a diagnosis, the fraction $\frac{1}{1+F(1-s)}$ will be getting their first opinion, while the fraction $\frac{F(1-s)}{1+F(1-s)}$ will be getting their a second opinion having been recommended c_h by the first middleman they visited.

As in the proof of Proposition 1, there are 3 equilibria. If $F = 1$ then customers do not search for a second opinion and purchase off the first middleman they visit, so middlemen have no incentive to deviate from committing fraud and this constiutes an equilibrium. If $F = 0$ then all customers search if they are recommended c_h, and middlemen know that by committing fraud a customer will purchase off another middleman, whereas by acting honestly they could make a profit of $p_l - c_l > 0$, so again here no middleman will deviate from telling the truth.

Finally, $F \in (0, 1)$ constitutes a mixed strategy equilibrium if a middleman's payoff from telling the truth $p_l - c_l$ is the same as recommending c_h. By committing fraud, a middleman expects a payoff of $\frac{s+F(1-s)}{1+F(1-s)}(p_h - c_l)$ as s of the $\frac{1}{1+F(1-s)}$ of new customers are time-constrained and must purchase while $\frac{F(1-s)}{1+F(1-s)}$ of the c_l-types are getting their second opinion and are assumed to accept the diagnosis of the second middleman who recommends c_h to them. (Again, it is irrelevant to the middleman's expected payoff how consumers decide which of two identical c_h diagnoses to accept, as in equilibrium all middlemen recommending c_h will receive the same share of the pool of twice defrauded customers.) Middlemen then are indifferent between truth and fraud when:

$$\begin{aligned} & \tfrac{s+F(1-s)}{1+F(1-s)}(p_h - c_l) = p_l - c_l \\ \Rightarrow \qquad & F = \tfrac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)} \end{aligned}$$

As long as $F = \frac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)} \in (0, 1)$ then this constitutes a mixed strategy equilibrium. Notice that $F \geq 0$ requires $s \leq \frac{p_l - c_l}{p_h - c_l}$ while $F \leq 1$ requires $1 - \frac{p_h - p_l}{p_l - c_l} \leq s$. If either of these conditions do not hold then this equilibrium ceases to exist.

Step 2: Customer Entry A consumer who is uninformed is also time constrained with probability s, but with probability $1 - s$ can visit two diagnosers. An uninformed customers expected payoff from entry is therefore:

$$V - k_t - \alpha p_h - (1-\alpha)(1-F)p_l - (1-\alpha)F(sp_h + (1-s)[Fp_h + (1-F)p_l])$$

Here every customer who enters expects to be a high type with probability α, to be a low type told the truth on their first diagnosis with probability $(1-\alpha)(1-F)$, and to be a low type offered a fraudulent diagnosis on their first visit with probability $(1-\alpha)F$. If they are a low type offered c_h on their first visit, then with probability s they have to stop and accept p_h, but with probability $1-s$ they can search for a second opinion, which will again be c_h with probability F but may be c_l with probability $1 - F$. Anticipating a level of fraud in the market $F = \frac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)}$ an uninformed customer will enter if his expected payoff is greater 0, taking his entry cost k_t into account so:

$$V - [\tfrac{\alpha-s}{1-s}p_h + \tfrac{1-\alpha}{1-s}p_l + (1-\alpha)(p_l - c_l)]$$
$$+(1-\alpha)(1 - \tfrac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)})(p_l - c_l - s(p_h - c_l)) - k_t \geq 0$$

The uninformed type with the highest entry cost who enters the model (for whom the above expression holds with an equality) is given by $\hat{k}_u^{F\in(0,1)}$ for some p_h and p_l assuming $F \in (0,1)$.

$$\hat{k}_u^{F\in(0,1)} = V - [\tfrac{\alpha-s}{1-s}p_h + \tfrac{1-\alpha}{1-s}p_l + (1-\alpha)(p_l - c_l)]$$
$$+(1-\alpha)(1 - \tfrac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)})(p_l - c_l - s(p_h - c_l))$$

As k_t is distributed uniformly on $[0, V]$ the number of uninformed entering is therefore given by $Q_u^{F\in(0,1)} = (1-i)\frac{\hat{k}_u^{F\in(0,1)}}{V}$ where:

$$Q_u^{F\in(0,1)} = \tfrac{(1-i)}{V}[V - [\tfrac{\alpha-s}{1-s}p_h + \tfrac{1-\alpha}{1-s}p_l + (1-\alpha)(p_l - c_l)]$$
$$+(1-\alpha)(1 - \tfrac{p_l - c_l - s(p_h - c_l)}{(1-s)(p_h - p_l)})(p_l - c_l - s(p_h - c_l))]$$

If it is the case that $F = 1$ or $F = 0$ in equilibrium, then regardless of whether he visits one or two diagnosers, a customer will receive the same expected payoff. If $F = 1$ then it will be:

$$V - k_t - p_h$$
$$\Rightarrow \quad Q_u^{F=1} = \tfrac{(1-i)}{V}(V - p_h)$$

If $F = 0$ then an uninformed customers expected payoff will be:

$$V - k_t - \alpha p_h - (1-\alpha)p_l$$
$$\Rightarrow \quad Q_u^{F=0} = \tfrac{(1-i)}{V}(V - \alpha p_h - (1-\alpha)p_l)$$

So in both of these cases, entry unambiguously rises as prices fall.

An informed customer's payoff from entry depends solely on the prices prevailing in the wholesale market, so as in the earlier model, the level of entry by informed customers with the c_h and c_l problems respectively will be given by:

$$\begin{aligned} Q_h &= \frac{i\alpha}{V}(V - p_h) \\ Q_l &= \frac{i(1-\alpha)}{V}(V - p_l) \end{aligned}$$

Step 1: Wholesalers set prices Play in step 1 is identical to that in Proposition 1. That completes the description of the sub-game perfect equilibria in the model.

Entry and welfare Again entry always increases when p_l falls, as a fall in p_l also lowers F so can only encourage entry. However, again if p_h falls then despite the fall in observed price the expected price of an uninformed entrant may rise as F rises. Taking the derivative of $Q_u^{F\in(0,1)}$ with respect to p_h implies that a fall in p_h will induce a decrease in entry by uninformed if:

$$\frac{dQ_u^{F\in(0,1)}}{dp_h} = (1-\alpha)(2sF + (1-s)F^2 + \frac{s^2}{1-s}) - \alpha > 0$$

So a decrease in the price of the potentially fraudulent good p_h lowers entry by uninformed when $\alpha < \frac{1-\alpha}{1-s}(s + (1-s)F)^2$. So when $s = 0$ this simplifies to the earlier condition $\alpha < (1-\alpha)F^2$. It also implies that as long as $F > \hat{F} = \frac{\sqrt{\frac{\alpha(1-s)}{1-\alpha}} - s}{1-s}$ that a decrease in p_h will lower entry by uninformed types.

In terms of welfare effects, unlike the previous proposition where I looked only at the effects from a change in p_h, here I will look at the effect of a more general increase in θ which causes a change in both p_h and p_l. First note that an increase in competition θ decreases entry if $\frac{dQ_u^{F\in(0,1)}}{d\theta} < 0$ or:

$$\begin{aligned} &[(1-\alpha)(2sF + (1-s)F^2 + \tfrac{s^2}{1-s}) - \alpha]\tfrac{dp_h}{d\theta} \\ &-[\tfrac{1-\alpha}{1-s} + (1-\alpha)(2F + (1-s)F^2)]\tfrac{dp_l}{d\theta} < 0 \\ \Rightarrow \qquad &\frac{dp_h}{dp_l} > \frac{\frac{1-\alpha}{1-s} + (1-\alpha)(2F + (1-s)F^2)}{(1-\alpha)(2sF + (1-s)F^2 + \frac{s^2}{1-s}) - \alpha} \end{aligned}$$

Notice that in the derivation I used $\frac{dp_l}{d\theta} < 0$ as an increase in price competition decreases prices. Further notice that the right hand side of the above inequality must be positive when $F > \hat{F}$ and a lower p_h discourages entry.

As $V > c_h > c_l$ and as every customer who enters the model will have their problem fixed, then a decrease in entry must be welfare decreasing. Noting that customers have heterogenous entry costs, the overall effect on welfare of a rise in θ must weigh the possible loss of welfare from uninformed customers dissuaded from entry against the increase in welfare from more informed customers entering.

If $F \in (0,1)$, then total welfare (denoted W_T) is given by:

$$W_T = Q_u^{F\in(0,1)}[V - \alpha c_h - (1-\alpha)c_l - \frac{\hat{k}_u^{F\in(0,1)}}{2}] + Q_h[V - c_h - \frac{\hat{k}_h}{2}] + Q_l[V - c_l - \frac{\hat{k}_l}{2}]$$

Using this a marginal increase in θ will decrease overall welfare if:

$$\begin{aligned}\frac{dW_T}{d\theta} &= \\ &\tfrac{(1-i)}{V}[[(1-\alpha)(2sF + (1-s)F^2 + \tfrac{s^2}{1-s}) - \alpha]\tfrac{dp_h}{d\theta} - [\tfrac{1-\alpha}{1-s} + (1-\alpha)(2F + (1-s)F^2)]\tfrac{dp_l}{d\theta}] \\ &\times[\alpha(p_h - c_h) + (1-\alpha)(p_l - c_l) + (1-\alpha)(sF + (1-s)F^2)(p_h - p_l)] \\ &-\tfrac{i\alpha}{V}[p_h - c_h]\tfrac{dp_h}{d\theta} - \tfrac{i(1-\alpha)}{V}[p_l - c_l]\tfrac{dp_l}{d\theta} < 0\end{aligned}$$

This can be re-arranged to solve for a bound on i:

$$i < \hat{i} = \frac{[(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha]\frac{-dp_h}{d\theta} - [\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)]\frac{-dp_l}{d\theta}}{[(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha]\frac{-dp_h}{d\theta} - [\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)]\frac{-dp_l}{d\theta} + \frac{(\alpha(p_h-c_h)\frac{-dp_h}{d\theta}+(1-\alpha)(p_l-c_l)\frac{-dp_l}{d\theta})}{\alpha(p_h-c_h)+(1-\alpha)(p_l-c_l)+(1-\alpha)(sF+(1-s)F^2)(p_h-p_l)}}$$

Note that the numerator is positive if $\frac{dp_h}{dp_l} > \frac{\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)}{(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha}$ and that $\hat{i} \leq 1$ as $\frac{A}{A+B} < 1$ when $A, B > 0$ which is also true here. So $\hat{i} \in (0,1)$. Further observe that multiplying both numerator and denominator by $1-s$ and taking the limit as $s \to 1$ implies that $\hat{i} \to 1$.

So if $F \in (0,1)$ in equilibrium and $\frac{dp_h}{dp_l} > \frac{\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)}{(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha}$, and $i < \hat{i}$ then welfare is decreasing as competition increases. As increased price competition will increase entry and welfare if $F = 0$ or $F = 1$ or if $F \in (0,1)$ and $i \geq \hat{i}$ then that completes the proof of the proposition. ■

Proof of Proposition 3 Assume $F = 0$ for now. In order for information to be welfare improving, then switching a customer (chosen at random) from uninformed to informed must be welfare improving. That is $E(w_i - w_u) > 0$ where w_i and w_u is the welfare of some customer as an informed and uninformed type. That implies $E(w_i) > E(w_u)$ which I will denote as $W_i > W_u$ where W_i and W_u are the expected (or average) welfare generated by an informed and uninformed agent respectively.

An informed customer who demands a type c_h good will enter the market and purchase if $k_t \leq V - p_h$, similarly an informed customer who demands the c_l good will enter the market and purchase if $k_t \leq V - p_l$. Given that an informed customer is with probability α a c_h type and with $1 - \alpha$ a c_l type then the expected welfare produced by an informed customer is given by:

$$
\begin{aligned}
& W_i = \alpha(V - c_h - \tfrac{\hat{k}_h}{2})\tfrac{V-p_h}{V} + (1-\alpha)(V - c_l - \tfrac{\hat{k}_l}{2})\tfrac{V-p_l}{V} \\
\Rightarrow \quad & W_i = \alpha(\tfrac{V+p_h}{2} - c_h)\tfrac{V-p_h}{V} + (1-\alpha)(\tfrac{V+p_l}{2} - c_l)\tfrac{V-p_l}{V}
\end{aligned}
$$

For an uninformed customer, assuming that no fraud is present in the market then they enter whenever $k_t \leq V - \alpha p_h - (1-\alpha)p_l$. That implies the probability of entry is $\frac{V-\alpha p_h-(1-\alpha)p_l}{V}$ and the expected welfare produced by an entrant is $(V - \alpha c_h - (1 - \alpha)c_l - \frac{\hat{k}_u}{2})$:

$$
\begin{aligned}
& W_u = (V - \alpha c_h - (1-\alpha)c_l - \tfrac{\hat{k}_u}{2})\tfrac{V-\alpha p_h-(1-\alpha)p_l}{V} \\
\Rightarrow \quad & W_u = (V - \alpha c_h - (1-\alpha)c_l - \tfrac{(V-\alpha p_h-(1-\alpha)p_l)}{2})\tfrac{V-\alpha p_h-(1-\alpha)p_l}{V} \\
\Rightarrow \quad & W_u = (\alpha(\tfrac{V+p_h}{2} - c_h) + (1-\alpha)(\tfrac{V+p_l}{2} - c_l))\tfrac{V-\alpha p_h-(1-\alpha)p_l}{V}
\end{aligned}
$$

If $W_i > W_u$ then

$$\begin{aligned}
\Leftrightarrow &\quad \alpha(\tfrac{V+p_h}{2} - c_h)(\tfrac{V-p_h}{V} - \tfrac{V-\alpha p_h-(1-\alpha)p_l}{V}) > (1-\alpha)(\tfrac{V+p_l}{2} - c_l)(\tfrac{V-\alpha p_h-(1-\alpha)p_l}{V} - \tfrac{V-p_l}{V}) \\
\Leftrightarrow &\quad \alpha(1-\alpha)(V+p_h-2c_h)(p_l-p_h) > \alpha(1-\alpha)(V+p_l-2c_l)(p_l-p_h) \\
\Leftrightarrow &\quad (V+p_h-2c_h) < (V+p_l-2c_l) \\
\Leftrightarrow &\quad p_h - c_h < (c_h - c_l) + (p_l - c_l)
\end{aligned}$$

So when $F = 0$ then switching a consumer from being uninformed to informed increases welfare if $p_h < \bar{p}_h = (2c_h - c_l) + (p_l - c_l)$. Because $W_i > W_u$ when $F = 0$ then it must also be true when $F > 0$ as W_i does not change and W_u falls because fraud causes fewer uninformed to enter. So increasing information at some arbitrary prices $p_h = p_h^*$ and $p_l = p_l^*$ where $p_h^* > p_l^*$ is welfare increasing if $p_h^* < \bar{p}_h$ for all $F \in [0,1]$. ■

Proof of Proposition 4 If $F \in (0,1)$, then total welfare (denoted W_T) is given by:

$$W_T = Q_u^{F\in(0,1)}[V - \alpha c_h - (1-\alpha)c_l - \frac{\hat{k}_u^{F\in(0,1)}}{2}] + Q_h[V - c_h - \frac{\hat{k}_h}{2}] + Q_l[V - c_l - \frac{\hat{k}_l}{2}]$$

As $\frac{d\theta}{di} > 0$ then denoting $W_i - W_u \equiv \triangle W$an increase in the fraction of informed decreases welfare if:

$$\begin{gathered}
\frac{dW_T}{di} = \\
\triangle W + \tfrac{(1-i)}{V}[[(1-\alpha)(2sF + (1-s)F^2 + \tfrac{s^2}{1-s}) - \alpha]\tfrac{dp_h}{di} - [\tfrac{1-\alpha}{1-s} + (1-\alpha)(2F + (1-s)F^2)]\tfrac{dp_l}{di}] \\
\times[\alpha(p_h - c_h) + (1-\alpha)(p_l - c_l) + (1-\alpha)(sF + (1-s)F^2)(p_h - p_l)] \\
-\tfrac{i\alpha}{V}[p_h - c_h]\tfrac{dp_h}{di} - \tfrac{i(1-\alpha)}{V}[p_l - c_l]\tfrac{dp_l}{di} < 0
\end{gathered}$$

This can be re-arranged to solve for a bound on i:

$$\begin{gathered}
i < \hat{i} = \\
\frac{[(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha]\frac{-dp_h}{di} - [\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)]\frac{-dp_l}{di} - \frac{\triangle W}{\alpha(p_h-c_h)+(1-\alpha)(p_l-c_l)+(1-\alpha)(sF+(1-s)F^2)(p_h-p_l)}}{[(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s})-\alpha]\frac{-dp_h}{di} - [\frac{1-\alpha}{1-s}+(1-\alpha)(2F+(1-s)F^2)]\frac{-dp_l}{di} + \frac{(\alpha(p_h-c_h)\frac{-dp_h}{di}+(1-\alpha)(p_l-c_l)\frac{-dp_l}{di})}{\alpha(p_h-c_h)+(1-\alpha)(p_l-c_l)+(1-\alpha)(sF+(1-s)F^2)(p_h-p_l)}}
\end{gathered}$$

So if $(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s}) > \alpha$ and $\frac{-dp_h}{di}$ is great enough, the numerator must be positive and $\hat{\imath} > 0$. If we further restrict consideration to $p_h < \bar{p}_h$ that ensures $\triangle W > 0$, so $\hat{\imath} < 1$ as it can be thought of as $\hat{\imath} = \frac{A-B}{A+C}$, where $A, B, C > 0$. Taken together that implies $\hat{\imath} \in (0,1)$.

So if $F \in (0,1)$ in equilibrium and $(1-\alpha)(2sF+(1-s)F^2+\frac{s^2}{1-s}) > \alpha$ with $\frac{-dp_h}{di}$ great enough, and if $p_h < \bar{p}_h$ then for some $i < \hat{\imath} \in (0,1)$ then $\frac{dW_T}{di} < 0$. Of course for $i > \hat{\imath} \in (0,1)$ then $\frac{dW_T}{di} > 0$. That completes the proof of the proposition. ■

Chapter 3

Competing Quotes for Car Repairs (and other Credence Goods)

This chapter outlines a market with private quotes for car repairs where sellers may attempt to overcharge customers. In markets like these, termed credence goods markets, it is assumed that customers remain uninformed about whether they have been overcharged even after a transaction has taken place. So in order to discipline sellers proposing a potentially fraudulent price, customers may either seek a second opinion (as in Wolinsky (1995)) or may choose not to purchase at all (as in Fong (2005)). Despite this, overcharging does sometimes occur. There are multiple equilibria in the market that rely on different disciplining strategies, and these equilibria can be welfare ranked. Common wisdom suggests that fraud will be low and welfare high in equilibrium if customers always solicit a second opinion. But because search is costly such equilibria only exist when fraud is high and welfare low. So for low search costs welfare is higher when customers sometimes don't get a second opinion but purchase the repair regardless, an outcome which resembles Wolinsky's model. For high search costs that equilibrium ceases to exist and welfare is maximised if customers never seek a second opinion and sometimes don't purchase anything at all. This resembles Fong's model of a credence good monopolist. These results are shown in both a model with 'quick' upfront quotes by sellers, where equilibrium prices are uniquely determined, and in a model with 'slow' quotes where the seller takes some time to prepare his price offer, which permits a broader range of prices in equilibrium.

3.1 Introduction

Consider someone who's car engine starts emitting smoke. If he visits a mechanic who diagnoses the problem and quotes a high price to fix it then it could either be an attempt to overcharge for a relatively minor repair or an honest assessment of the large amount of work the car needs. The customer might get a second opinion from another mechanic but there is still no guarantee that the second mechanic will truthfully report the extent of the problem. Eventually, if he wants to get his car fixed, the customer must put credence in some mechanic's quote and agree to it. And afterwards, so long as the car is returned working normally again, the customer will forever be unsure whether he was overcharged or not.

Car repairs are an example of a market for credence goods. These are goods or services where the customer must rely on the advice of an expert to identify what they require before purchasing treatment from them. And customers, even after they are treated, do not know whether they have been defrauded or not.[1] Credence goods were first identified by Darby and Karni (1973) who added them to Nelson's (1971) classifications of ordinary, search and experience goods. These categories of goods are distinguished by the timing of the customer's information about relative payoffs. For ordinary and search goods he knows his payoffs before he purchases, although he may need to see the good first. For experience goods such as tickets to the theater, he knows his payoff after purchasing the ticket and experiencing the show. But credence goods such as car repairs involve something like the choice between two experience goods. Customers may know their payoff afterwards but will never know what their payoff would have been from the alternative. So the distinction between credence and

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[1] Although it is usual to refer to credence goods, perhaps it would be better if they were termed credence services. After all it seems easier to overcharge for services (such as legal, medical, or advisory services) which leave no 'physical' trail. For example a lawyer might claim that he worked a certain number of billable hours to prepare a case and it is hard for a second lawyer to show that he didn't. But overcharging for goods (such as car or computer repairs) implies overcharging for physically removing or installing parts, which appears to leave less scope for excessive claims about the difficulty (or cost) of the job.

experience goods is blurry.[2] The main economic implication of these goods is that customers have no way to evaluate their outcome and then reward or punish sellers after a sale takes place. But the sellers advice about what to buy before the sale may contain useful information for a customer and gives customers an incentive to sometimes seek a second opinion before buying the good.

Credence goods markets are ubiquitous and examples of sellers in such markets include doctors, mechanics, computer specialists (Dulleck and Kerschbamer, 2006), lawyers (Hyndman and Ozerturk, 2008), dentists, real estate agents (Fong, 2005), optometrists (Wolinsky, 1993), washing machine repairmen (Dulleck and Kerschbamer, 2005a), consultants (Pesendorfer and Wolinsky, 2003), taxi drivers (Darby and Karni, 1973), life insurance salesmen and undertakers (Emons, 2001).

Empirical studies also confirm our intuition that experts do commit fraud in these markets. Emons (1997) cites a Swiss study showing that the probability of a patient receiving a major surgical intervention is one third higher if they are a member of the general public and not a physician themselves, or a member of a physicians family. Wolinsky (1993) cites a study by the United States Department for Transport which estimates that 53 per cent of all car repairs are unnecessary. While Iizuka (2007) finds that physicians in Japan who both prescribe and sell prescription drugs to their patients distort their recommendations towards drugs with a higher markup. Most recently, Dulleck, Kerschbamer and Sutter (2011) have also explored credence goods markets experimentally. As predicted they find that repeated interaction and reputation building do not discipline seller behaviour in these markets and that without strong additional assumptions (that either sellers are liable for undertreatment or that the costs they incur are verifiable to the buyer) then these markets break down in a similar way to lemons markets (the relevant literature starts with Akerlof (1970)).

This chapter explores outcomes in a credence good market where customers can

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[2] A theater-buff might view the ticket to a new play as an experience good, as he can easily compare his payoff to a known alternative, while a man dragged along to the theater once or twice in his entire life may view it as a credence good as even afterwards, he cannot compare his payoff with an alternative choice of show. This lack of a counterfactual separates credence and experience goods.

undertake costly sequential search to solicit *competing* quotes from sellers before choosing to purchase or exit. Each period consumers discover they have a problem and visit an expert who diagnoses them as suffering from either a cheap or costly problem and then offers a price for treatment. Consumers then decide whether to accept the offer or not, and may either make a costly search for a second opinion or exit the game without purchasing anything. Unlike some other models, after getting a second opinion customers may return to the first expert who diagnosed them to purchase. This model is similar to Wolinsky (1993) in that it has sequential search but differs in that it has non-public prices and allows customers to return and buy from their first diagnoser. And as in Fong (2005) customers payoffs are type dependent and give them an incentive to exit without purchasing if they believe sellers have committed fraud.

In this model with ‘quick quotes’ (where sellers offer prices straight away and customers use those prices to decide to search) prices suffer from Diamond’s paradox (see Diamond (1971)). That’s because price offers for the cheaper treatment will always be as high as possible and (knowing this) customers never search as the competing quote cannot do better. Meanwhile as the more expensive treatment is potentially fraudulent, customers have incentives to search, competing its price down to marginal cost.

In order to expand the set of equilibrium prices, the model is also extended to ‘slow quotes’. Now when customers are offered treatment, they can only be offered a type of treatment (cheap or expensive) up front, and a final price quote is only produced after they finish searching. This model reflects a professional making an informal assessment while the customer is present, and then mailing him the formal quote a few days later. With slow quotes, price offers between marginal cost and the consumers willingness to pay can now occur in equilibrium, avoiding Diamond’s paradox, and competition between firms is similar to that found in Burdett and Judd (1983) implying price dispersion in equilibrium. Here though, prices are influenced by the degree of fraud in the market.

There are four different equilibria (in both versions of the model) and each uses a different punishment to 'police' experts who would otherwise defraud their customers. The first equilibrium type has experts police themselves. Here customers always seek a costly second opinion and firms are prevented from always committing fraud by the relatively honest behaviour of their competitors. The remaining three equilibria are ones in which customers police firms. Two of these are similar to the equilibria found in the models of Wolinsky and Fong, with customers policing potentially fraudulent behaviour by either searching for a second opinion or exiting. The final equilibrium type has customers do both, punishing potential fraud by exiting immediately and otherwise searching for a second opinion before purchasing.

The main contribution of this chapter is to then welfare rank these equilibria. For low search costs where it exists Wolinsky's equilibrium (where customers sanction potential fraud by sometimes seeking a costly second opinion but always end up purchasing) creates the most welfare. For higher levels of search costs either Fong's equilibrium (where customers never search and sanction potential fraud by sometimes exiting without purchasing) or another equilibrium where customers only ever purchase treatment after getting a second opinion creates the most welfare. Once search costs are high enough however, then only Fong's equilibrium exists, so it welfare dominates by default.

3.1.1 Related Literature

There has been a broad range of approaches to modeling credence goods, and in general, the credence good literature is also related to the literature on moral hazard in product quality that follows Klein and Leffler (1981). Dulleck and Kershbamer (2006) categorise models according to four assumptions that may be made: homogeneity, liability, commitment and verifiability. Here it is assumed that customers are homogenous, as only sellers can observe information relevant to the client's problem, that a liability rule is in place so that experts may not undertreat customers and that customers are not committed to a particular seller as they may search for a second opinion. That makes this a model of overcharging fraud with private (competitive) price setting.

In general overcharging fraud only occurs in models with privately set prices, indeed Dulleck and Kershbamer (2006, pp. 27) state that “overcharging equilibria cease to exist if there is more than one expert and experts are free in choosing [public] prices.” One reason for this is the ‘undercutting’ argument given in Wolinsky (1993). That argument is simply that if sellers can specialise in one type of repair, then they will do so competing prices of both types of repair down to cost. That implies that no seller could ever honestly offer both types of repair in equilibrium as they have strict incentives to commit fraud. In other papers, sellers may commit to a *single* price at which they promise to repair any problem (see Dulleck and Kerschbamer’s Lemma 2 (2006)) or quite different assumptions are made on consumer information (in Emons (1997, 2000, 2001) for example customers can observe sellers total capacity while in Dulleck and Kerschbamer’s Lemma 1 (2006) costs incurred by the seller are verifiable). And in all of these models with flexible public price setting fraud is eliminated in equilibrium, so there is no overcharging.

Earlier work with overcharging in equilibrium includes Pitchik and Schotter (1987), Fong (2005) and Wolinsky (1993). Pitchik and Schotter’s (1987) model presents a simple mixed strategy with exogenously set prices and endogenously determined fraud. Fong’s model (2005) builds on this by letting a monopolist seller set public prices before playing a recommendation game similar to Pitchik and Schotter’s. Fong also investigates the role of heterogenous consumer characteristics that experts use to select whom to target for fraud and the provision of information from customers which could potentially be used to defraud them. Related papers in this framework include Liu (2011) and Hyndman and Ozertuk (2008) who follow Fong’s (2005) overcharging setup, and introduce honest experts and heterogeneity in the customer’s payoffs respectively.

Wolinsky’s initial contribution comes in two papers (1993, 1995) that use an overcharging framework with customers who may search for a second opinion. The earlier paper shows that in an equilibrium with public price setting, experts cannot credibly deliver both goods, so end up specialising in delivering only one of them. The second of Wolinsky's papers in this framework replaces public price setting with a private bargaining process between experts and customers. This permits experts to continue offering both goods, as prices don't get competed down to cost, and there is overcharging fraud in equilibrium. Papers by Marty (1999a, 1999b) and Sülzle and Wambach (2005) follow Wolinsky's (1993) setup, but simply assume exogenously fixed prices to avoid the undercutting argument. Those papers look at the effect on equilibrium of honest experts and co-insurance respectively. Alger and Salanie (2006) also modify Wolinsky's (1993) model but to a setting where experts set prices for both treatment and costly diagnosis. They further introduce the assumption that to commit fraud an expert must incur some additional verifiable cost.

Pesendorfer and Wolinsky (2003) introduce a model with private prices set by experts and fraud in equilibrium. But their paper looks at potential undertreatment, not overcharging, and incentives for costly diagnosis with flexible price setting. In their setup, a customer has an unknown problem and sequentially samples experts who offer the customer a contract with diagnosis and treatment prices. Similar to equilibria in this chapter, in equilibrium customers either exit, buy from the first expert they visit or search (and keep searching) for a second opinion until two opinions match, despite the different framework.

This article is organised in the following manner. Section 2 presents the model with quick quotes. Section 3 solves for equilibria in the model. Section 4 then welfare ranks the equilibria. The model is adapted to slow quotes in Sections 5 and 6. Section 7 then concludes. The appendix contains the proofs of the propositions.

3.2 Quick Quotes

Consider a mass of M consumers who are unsure of the amount of treatment they need to fix a problem so go to an expert for advice. There are $N \geq 2$ sellers (sometimes referred to as ‘experts’) who observe the type of treatment customers require and may use that informational advantage to commit fraud by claiming that a customer needs more treatment than is in fact necessary. A priori each customer requires a high (h) amount of treatment with probability $\alpha \in (0, 1)$ and low (l) amount of treatment with probability $1 - \alpha$. Experts offer to treat the customer’s problem at some price p, chosen after they have observed which treatment the customer requires. A liability rule is in place that ensures that experts cannot provide less treatment to a customer than they require, so if a customer accepts the offer of treatment, the expert must incur the cost of treating him. This is a standard assumption in the literature (see Fong (2005) and Dulleck and Kerschbamer (2006)). The cost to an expert is c_h for the h treatment and c_l for the l treatment. Customers do not benefit from more treatment than they require but their payoff is dependent on their type. Customers receive a payoff of V_h from treatment if they required h and V_l from treatment if they required l. The payoff from treatment of V_h for a h type is assumed to be greater then the V_l for an l type. This greater gain reflects either a better final outcome from more treatment (the car is like new again once the new parts from Tokyo are installed) or the avoidance of a worse outcome if left untreated (without the major overhaul the car would have stopped working entirely in a week). A customer who exits without purchasing any treatment receives a payoff of 0.[3] It is assumed that $V_h > c_h > V_l > c_l \geq 0$.

Experts must decide what price to quote customers when they visit them. Denote a high price as $p_h \in [c_h, V_h]$ and a low price as $p_l \in [c_l, V_l]$. An expert who offers a low price p_l would only do so for a customer requiring the cheap treatment c_l, and if this offer is accepted he makes a profit of $p_l - c_l$. However an expert who offers to treat a

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[3] Notice that a customer cannot condition his payment to the seller on the payoff he receives in a credence good market - this is similar to the moral hazard problem faced by customers buying a good of unknown quality in the product quality literature. But whereas models following Klein and Leffler (1981) often assume repeated interaction between a buyer and seller, giving cheated customers a way to punish deviating sellers in future periods, here the nature of credence goods implies that clients do not realise that they have been defrauded even after their payoffs are realised, so there is no mechanism to punish fraud by sellers in the future.

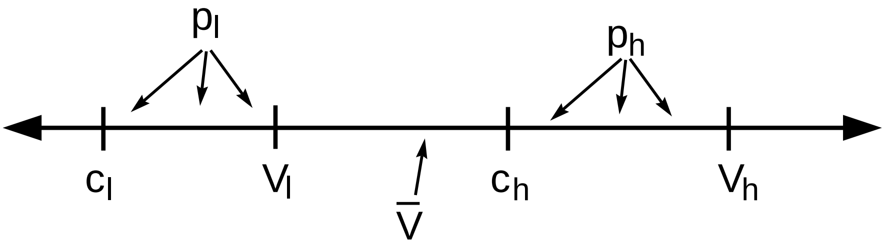

Figure 3.1: Costs, payoffs and prices for the two treatments

customers problem for p_h may be either genuinely offering a high price to a customer requiring the expensive c_h treatment, or he may be fraudulently trying to overcharge a customer who only needs the c_l treatment. If a customer genuinely requires c_h and accepts, then the expert makes a profit of $p_h - c_h$. But if the customer only needed c_l but accepts p_h, the expert makes a much greater profit of $p_h - c_l$. That is the nature of overcharging in this model. Notice that $p_h > V_l$ by assumption so a customer requiring l would never knowingly accept p_h. The two price offers for treatment, as well as costs and payoffs for the two types of customers are shown in Figure 3.1.

In this model, quotes are quick and customers are not committed to purchase from the first expert they visit. So after visiting a first seller who quotes them a price p_h or p_l in some period t customers can then choose to make a costly search and visit another expert in $t+1$ for a second opinion at a cost of $k \geq 0$. The second expert will then similarly offer the customer treatment at some p_h or p_l. It is assumed the customers may only search once. For their part, experts may encounter overlapping generations of customers as some customers will be on their first visit while others will be on their second. Experts cannot distinguish who is who. For convenience, I will also make four assumptions on parameter values to simplify analysis of the model:

Assumption 1 (No babbling equilibria exist as $\bar{V} < c_h$): This assumption is shown in Figure 3.1. In a babbling equilibrium the price p^* offered for treatment by experts contains no information on the customers type, so customers cannot update their beliefs on their type after visiting an expert, but they choose to purchase anyway.

Such an equilibrium exists if the expected payoff in a babbling equilibrium $\bar{V} = \alpha V_h + (1-\alpha)V_l$ is more than the cost of the high treatment c_h. Note that Assumption 1 could be re-written as $\alpha < \frac{c_h - V_l}{V_h - V_l}$. It also implies that $\frac{\alpha}{1-\alpha}(\frac{V_h - c_h}{c_h - V_l}) \in (0,1)$ as $V_h > c_h$ and $c_h > V_l$.

Assumption 2 (Normalise $c_l = 0$ and $V_l = 1$): To simplify the analysis of equilibrium in the following sections, a normalisation of $c_l = 0$ and $V_l = 1$ is made. This does not alter the qualitative results in any way.

Assumption 3 ($V_h - c_h > \frac{1-\alpha}{\alpha}$): This assumption is made simply to ensure that the equilibrium where 'experts police themselves' always exists in the quick quotes model. The assumption implies that $\frac{1}{c_h - 1} < \frac{\alpha}{1-\alpha}(\frac{V_h - c_h}{c_h - 1})$.

Assumption 4 ($c_h > 2$): This is a technical assumption used in the slow quotes model (Lemma 10) to simplify the analysis of the 'search benefit hill' ensuring that it has only one maxima. Strictly speaking it is not a necessary assumption.

Timing Recall that each period t a new group of M customers enters the model. Some either purchase or exit after their first diagnosis, but some may stay on and search for a second opinion in $t+1$. In equilibrium then, in any period some fraction of customers will be on their first diagnosis, and some will be getting a second opinion, where the timing of a customers decisions in the model is as follows:

1. After entering in some period t, a seller observes the customers problem and quotes a treatment price $p_l \in [c_l, V_l]$ or $p_h \in [c_h, V_h]$.
2. The customer observes the quoted price and decides whether to search for a second opinion at cost k in period $t+1$.
3. If the customer searches for a second opinion, a second seller observes the customers problem and quotes either $p_l \in [c_l, V_l]$ or $p_h \in [c_h, V_h]$.

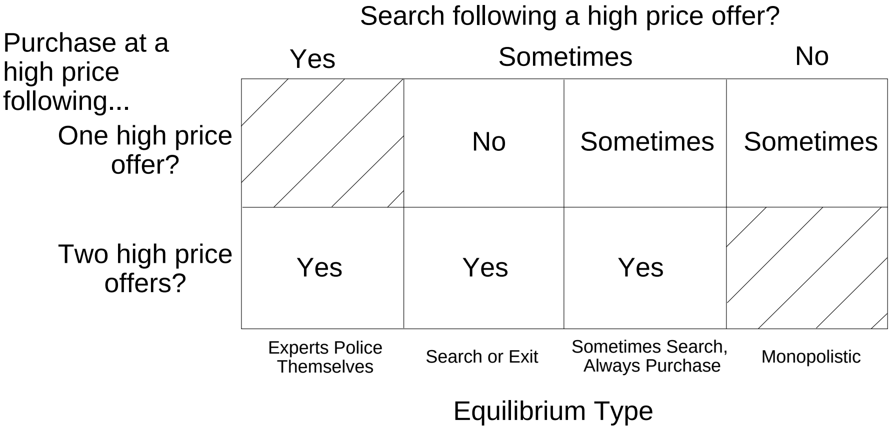

Figure 3.2: Consumer behaviour in different equilibria

4. After the final price quote, the customer then decides to accept treatment or exits without treatment.

3.3 Equilibria

There are four equilibrium types in this model. They can be distinguished by how experts are made indifferent in equilibrium between committing fraud and acting honestly towards l type customers. The equilibria are shown in Figure 3.2 and are labeled:

- experts police themselves
- search or exit
- sometimes search, always purchase
- monopolistic

In the first equilibrium type *experts police themselves*. In this equilibrium customers are concerned about the possibility of being defrauded so always search again if offered the potentially fraudulent high treatment. This creates competition between experts in terms of both their prices and their honesty, with dishonest (and high pricing) experts losing customers to honest (and lower pricing) ones. So if experts expect other experts to be relatively honest, that in turn forces them to act relatively honestly in order to stay competitive, and in that sense experts police other experts from committing fraud.

The three further equilibria are all ones in which customers police experts. They do so by either sometimes searching for a second opinion following a high price offer, or by simply exiting and not purchasing at all. In the *search or exit* equilibrium following a p_h offer consumers are indifferent between not searching then exiting, and searching and then purchasing treatment. In the *monopolistic* equilibrium (which resembles Fong's (2005) equilibrium) there is no search following a p_h offer but experts are again disciplined by customers exiting without purchasing. The final equilibrium type is one where customers *sometimes search, [but] always purchase* following a p_h offer. It resembles Wolinsky's (1995) equilibrium where the threat of search disciplines experts.

To establish these equilibria, below I present a series of propositions where F is denoted as the probability of an expert fraudulently recommending the h treatment to a customer who only needs the l treatment. I denote as s_h the probability of an agent stopping (and not searching) after a p_h offer from the first expert they visit and denote as s_l the probability of an agent stopping after a p_l offer from the first expert they visit. Recall that by assumption agents may only choose to search once. Agents choose to purchase a h treatment after one p_h offer with the probability b_h and after two p_h offers with a probability denoted b_{2h}. Finally note that an agent offered the l treatment will always purchase it at the lowest price offered, as the liability rule ensures that it cannot be a fraudulent offer.

Before moving on to the propositions, note that in general there are no pure strategy equilibria in this model where sellers always commit fraud. To see this, assume that $F = 1$, this then would constitute a babbling equilibrium, where the price offer by the seller does not contain any information on the customers type. As such, a customers expected payoff from accepting a price offer of $p_h \in [c_h, V_h]$ is given by his ex-ante expected payoff from treatment at that price $\alpha V_h + (1-\alpha)V_l - p_h$.

But by assumption 1, $\alpha V_h + (1-\alpha)V_l - c_h < 0$ so a customers expected payoff is less than 0 for any $p_h \in [c_h, V_h]$. In other words, no customer would purchase at p_h and sellers would have strict incentives to deviate to acting honestly and offering $p_l \in (0,1)$ as (due to the liability assumption) that signals that the customer must be an l type, and the customer will end up purchasing at that price. (This is true even if customers did search for a second opinion, as all other sellers offer a price $p_h > p_l$ in the proposed equilibrium). As sellers have an incentive to deviate from offering p_h this contradicts the assumption that $F = 1$, so this cannot be an equilibrium.

On the other hand, it will be shown below that there are two special cases where $F = 0$ can occur in equilibrium. The first of these is the monopolistic equilibrium, for the special case where $p_h = V_h$, and the second of these is the sometimes search, always purchase equilibrium when $k = 0$. Outside of these two special cases however, all equilibria have $F \in (0,1)$.

Proposition 1 (Experts Police Themselves): For a low enough k equilibria exist where experts police themselves. In these equilibria $p_l = V_l$, $p_h = c_h$, $s_l = 1$, $s_h = 0$, $b_{2h} = 1$, $F = \frac{1}{c_h - 1}$ and $(F, k) \in \Gamma^*$.

Proof: See Appendix.

In an experts police themselves equilibrium, following a potentially fraudulent offer of treatment at p_h customers strictly prefer to search for a second opinion because the search cost k is relatively low and they anticipate some level of fraud $F > 0$ in the market. Given that customers always get a second opinion, sellers are shown to be exactly indifferent between offering a fraudulent p_h price and an honest p_l price to l types given the anticipated frequency of fraud by other sellers. As they are indifferent between attempting to commit fraud and acting honestly, it is assumed that sellers commit fraud with the expected equilibrium probability.

The proof is broken into three lemmas. The first of these establish that:

- $p_l = V_l$
- $p_h = c_h$
- $s_l = 1$

This lemma resembles Diamond's paradox, but in a credence good market. It argues that given private price offers by sellers then just like the usual case of Diamond's paradox the price of the cheaper treatment p_l will be set at the consumer's maximum willingness to pay V_l and no customer will search following a p_l price offer. So $p_l = V_l$ and $s_l = 1$. But after a p_h price offer, if customers have enough incentive to search for a second opinion (because of the danger of potentially being defrauded), then p_h will be competed down to c_h. So $p_h = c_h$.

The second lemma asks: when will customers search for a second opinion following a p_h price offer? Consider the case of $k = 0$ (where it is costless to search). A customer is indifferent between searching or not if $F = 0$ because he expects the first p_h offer was truthful and that a second opinion would only reconfirm this. Or he is indifferent if $F = 1$ because while he anticipates that the first price offer was fraudulent with probability $1 - \alpha$ he expects that a second opinion will also be p_h. So in either case the customer does not expect a better price offer from the second expert, and as $k = 0$ he is indifferent between searching or not. For $F \in (0, 1)$ however, a customer will have strict incentives to search. He anticipates that the first p_h offer was potentially fraudulent and that a second opinion could potentially be honest, with a cheaper price offer of p_l. As it is costless to get a second opinion if $k = 0$ then customers strictly prefer to do so.

More generally for $k > 0$ a region in $F - k$ space denoted Γ^* is then defined, within which fraud is such that despite the higher search costs, customers have an incentive to search for a second opinion, but where fraud is low enough that customers would prefer to incur a search cost and get a second opinion rather than simply exiting for a payoff of 0. In this region it must also be that fraud is low enough that customers also

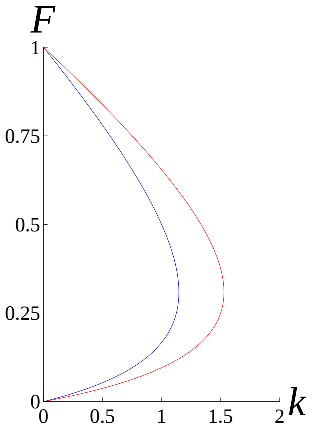

Figure 3.3: The Γ region with $p_l = 1$ and $p_l = 0$, assuming $p_h = 4$ and $\alpha = 0.2$

prefer to purchase at p_h after two p_h offers rather than exiting without purchasing. That is in Γ^*:

- $s_h = 0$
- $b_{2h} = 1$

To construct Γ^*, first the region where customers weakly prefer searching for a second opinion, rather than stopping and purchasing is found. Customers use Bayes theorem to update their beliefs about their own type following a p_h offer, taking the expected level of fraud and prices offered in the market as given. Setting their expected payoff from searching for a second opinion to be weakly greater than their expected payoff from simply stopping and purchasing after a p_h offer leads to the following quadratic equation:

$$F^2 - F(1 - \frac{k}{p_h - p_l}) + \frac{\alpha}{1-\alpha}\frac{k}{p_h - p_l} \leq 0$$

When $k \leq k^* = (p_h - p_l)\frac{(1+\alpha-2\sqrt{\alpha})}{1-\alpha}$ this quadratic can be solved for (real) roots between 0 and 1 which are the upper and lower cutoff levels of fraud between which a customer has strict incentives to search following a p_h offer. I denote the region in $F - k$ space where customers prefer searching to purchasing following a p_h offer as Γ. An example is given in Figure 3.3. Notice that the region expands outwards for

$p_l = 0$ because getting a truthful (cheaper) second opinion becomes more important to a customer for some given search cost. For $p_l = 1$ then $k^* = 1.14590$ while for $p_l = 0$ then $k^* = 1.52786$.

Recalling the story for $k = 0$ above, between the two roots customers have strict incentives to search following a p_h offer. But below the lower root or above the higher root, customers do not have incentives to search. The reason for this is that below the lower root, fraud is so uncommon that a customer believes that the offer of p_h was likely to be because he genuinely requires the expensive treatment, so he does not want to incur a search cost just to have the first opinion likely reconfirmed. Above the upper root, the customer will not search because fraud is so high that although the customer suspects that the first p_h offer was fraudulent, he also expects that getting a second opinion won't help matters, as the next seller he visits will also likely attempt to overcharge him. So again, he does not search. Within the Γ region however, for intermediate levels of fraud, a customer will search. He suspects that an initial p_h offer may have been fraudulent, and he thinks there's a good chance a second opinion could reveal that.

Within Γ customers prefer searching for a second opinion to simply stopping and purchasing. But it must also be shown that customers prefer to get a second opinion rather than simply exiting straightaway. To ensure this is the case a customers beliefs about his payoff from a second opinion must be greater than 0, his payoff from exiting without purchasing in equilibrium. That implies that the upper bound of the region in $F - k$ space where a customer would search for a second opinion is given by:

$$F_h = \frac{1}{2}(\frac{V_l - p_l - k}{p_h - p_l}) + \frac{1}{2}\sqrt{(\frac{V_l - p_l - k}{p_h - p_l})^2 + 4\frac{\alpha}{1-\alpha}\frac{(V_h - p_h - k)}{(p_h - p_l)}}$$

This upper bound is illustrated in Figure 3.4. It can be shown that (in this model) this level of fraud always lies below $\sqrt{\frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - V_l}}$, the level of fraud where customers would not purchase at p_h even following two p_h offers. That implies that if in equilibrium F and k lie in the Γ region and below F_h then customers would always get second opinion following a p_h offer, and that following a second p_h offer they would indeed

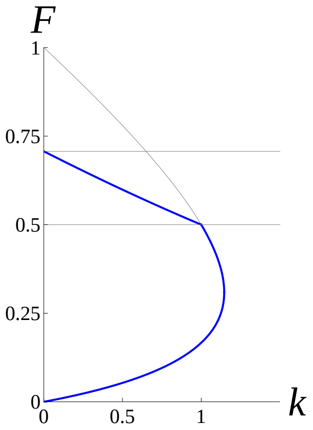

Figure 3.4: Region Γ^* assuming $\alpha = 0.2$, $p_l = 1$, $p_h = 4$ and $V_h = 10$. The upper horizontal line is $\sqrt{\frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - V_l}} = 0.70711$, the lower horizontal line is $\frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - V_l} = 0.5$.

purchase. I denote the region of Γ with $F \leq F_h$ is denoted Γ^* and within it both $s_h = 0$ and $b_{2h} = 1$.

The final lemma of the proof establishes that if $s_h = 0$ and customers always get a second opinion following an offer of treatment at c_h then sellers are indifferent between committing fraud and telling the truth if:

- $F = \frac{1}{c_h - 1}$

Assumption 3 assures us that this level of fraud will lie underneath the upper bound F_h, which can be shown to descend to intersect the right hand side of the Γ region at $F = \frac{\alpha}{1-\alpha}(\frac{V_h - c_h}{c_h - 1})$. Taken together, all of the above implies that experts police themselves equilibria exist for $F = \frac{1}{c_h - 1}$ if k is low enough, and in that equilibrium $s_h = 0$ and $b_{2h} = 1$ (as it lies inside Γ^*), and $p_h = c_h$, $p_l = V_l$ and $s_l = 1$ because customers always search following a p_h price offer but never search following a p_l price offer.

Proposition 2 (Search or Exit): For a low enough k equilibria exist where customers police experts with search or exit. In these equilibria $p_l = V_l$, $p_h = c_h$, $s_l = 1$, $s_h = 1 - \frac{V_l}{F(c_h - V_l)}$, $b_h = 0$, $b_{2h} = 1$, $F = F_h$ and $(F, k) \in \Gamma^*$.

Proof: See Appendix.

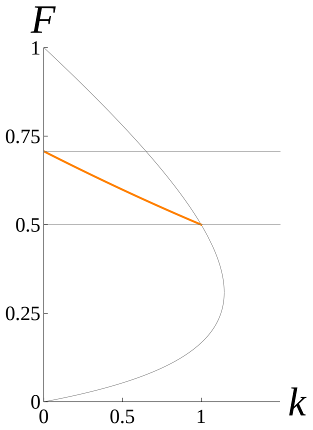

Figure 3.5: Search or exit equilibria are shown in yellow.

In a search or exit equilibrium expected fraud is higher than in the equilibrium above, so if all customers searched for a second opinion and then purchased, the threat of honest competition would not be enough to deter experts from strictly preferring to commit fraud. As such customers keep experts indifferent between fraud and honesty by sometimes exiting rather than merely searching for a second opinion (a stronger punishment for the seller as the seller has no chance at all of making the sale if the customer exits). As sellers are indifferent between acting truthfully and fraudulently in equilibrium they are assumed to commit fraud with a probability that just makes customers indifferent between searching and exit.

The proof of this proposition makes use of Lemmas 1 and 2 from Proposition 1. Lemma 1 established that $p_l = V_l$ and $s_l = 1$ and note that $p_h = c_h$ here because customers only ever purchase at p_h following two p_h offers. So the only high price that would be offered in equilibrium is $p_h = c_h$. Lemma 2 found that level of fraud F_h where customers were just indifferent between searching for a second opinion and exiting after a p_h offer. So at that level of fraud in equilibrium $b_h = 0$ as no one ever purchases after a single p_h offer. And as was observed above, in the quick quotes model F_h is always below $\sqrt{\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l}}$, so customers will always choose to purchase at p_h following two p_h offers, and $b_{2h} = 1$. It is also the case that for low enough k then F_h lies inside the Γ region, so those customers who do not exit have strict incentives

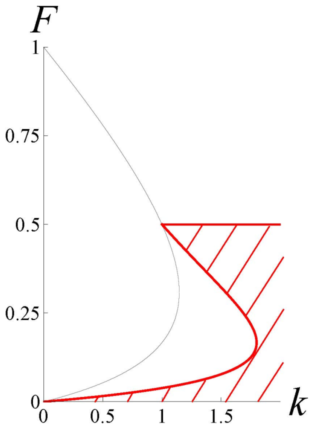

Figure 3.6: The region Γ^M with $p_h \in [c_h, V_h]$ where $p_l = 1$, $\alpha = 0.2$, $c_h = 4$ and $V_h = 10$.

to search. An example of the search or exit equilibrium is given in Figure 3.5.

As $F = F_h$ then customers are indifferent between exiting and searching, and can choose to search with any probability. In order for sellers to be just indifferent between telling the truth and fraud then customers must exit after being offered $p_h = c_h$ with probability:

- $s_h = 1 - \frac{1}{F(c_h - 1)}$

So as fraud rises, customers stop and exit more.

Proposition 3 (Monopolistic): For k great enough, equilibria exist where customers do not search and police experts by exiting. In these equilibria $p_l = V_l$, $p_h \in [c_h, V_h]$, $s_l = 1$, $s_h = 1$, $b_h = \frac{1}{p_h}$, $F = \frac{\alpha}{1-\alpha}(\frac{V_h - p_h}{p_h - V_l})$ and $(F, k) \in \Gamma^M$.

Proof: See Appendix.

In a monopolistic equilibrium, customers never choose to search following an offer of treatment at p_h and instead police sellers from always committing fraud by sometimes exiting. Sellers make customers indifferent between exiting and purchasing treatment at p_h by committing fraud in equilibrium with an appropriate probability, given the equilibrium p_h.

Again using Lemma 1, it can be shown that $p_l = V_l$ and $s_l = 1$, but here there is no search following a p_h price offer. As such, any $p_h \in [c_h, V_h]$ may be offered in equilibrium. In order for a seller to be indifferent between acting truthfully (for a payoff of $p_l = 1$) and acting fraudulently (for a payoff of $p_h b_h$) then customers only choose to purchase with probability:

- $b_h = \frac{1}{p_h}$.

Customers must also have no incentive to search in equilibrium. This is shown by finding the Γ^M region shown in Figure 3.6 where the expected payoff from search is less than 0, a customers expected payoff from accepting treatment at p_h in equilibrium. In this region $s_h = 1$ by construction. Customers must also be indifferent between purchasing and exiting in equilibrium, which implies a level of fraud:

- $F = \frac{\alpha}{1-\alpha}(\frac{V_h - p_h}{p_h - 1})$

Notice that at most this is $F = \frac{\alpha}{1-\alpha}(\frac{V_h - c_h}{c_h - 1})$ which is in $(0, 1)$ by Assumption 1. Further notice that $F = 0$ for $p_h = V_h$, and that higher prices imply lower equilibrium fraud. Finally notice that for a given level of fraud, that this equilibrium may exist if k is high enough, as beyond some k customers will not search.

Proposition 4 (Sometimes Search, Always Purchase): For a low enough k equilibria exist where customers always purchase and customers police experts by sometimes searching. In these equilibria $p_l = V_l$, $s_l = 1$, $p_h = c_h$, $s_h = \frac{1+F(1-c_h)}{c_h+F(1-c_h)}$, $b_h = 1$, $b_{2h} = 1$, $F = F_{1,2} \leq \frac{1}{c_h - 1}$ and $(F, k) \in \Gamma^*$.

Proof: See Appendix.

In the sometimes search, always purchase equilibrium customers are exactly indifferent between searching for a second opinion and simply stopping following an offer of treatment at p_h. Because fraud is low, regardless of whether they do or don't search customers have strict incentives to then purchase. This equilibrium is similar

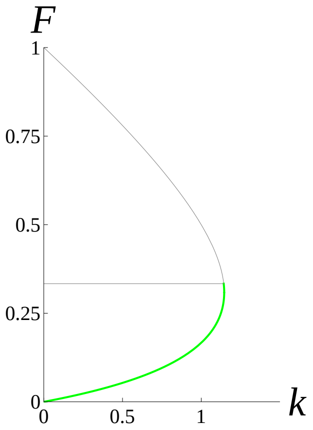

Figure 3.7: Sometimes search, always purchase equilibria are shown in green.

to the experts police themselves equilibrium, but because fraud is lower customers do not have to always search for a second opinion in order to make sellers indifferent between committing fraud and acting honestly. The customer sometimes searching, combined with other sellers more honest behaviour, is enough to keep sellers indifferent between attempting to commit fraud and acting honestly here. And because sellers are indifferent between the two, it is assumed that they will choose to commit fraud with the appropriate probability to keep customers exactly indifferent between searching and stopping in equilibrium.

Again, Lemma 1 ensures that $p_l = V_l$ and $s_l = 1$ here. And because when they are offered $p_h = c_h$ customers are indifferent between stopping and searching, then any seller offering a $p_h > c_h$ induces his customers to all search for a second opinion (which is at most $p_h = c_h$) so lose all of their customers. For this reason $p_h = c_h$ in equilibrium.

The proof shows that this equilibrium lies on the right hand side of the Γ region, where for that combination of k and F customers are indifferent between stopping and searching at $p_h = c_h$. So $F = F_{1,2}$, the roots of the quadratic that define the Γ region. The region where this equilibrium exists is shown in Figure 3.7. To keep sellers indifferent between fraud and honesty, customers must punish sellers for offering p_h by stopping with the probability:

- $s_h = \frac{1+F(1-c_h)}{c_h+F(1-c_h)}$

Here in contrast to the search or exit equilibrium as F grows then s_h falls. This occurs because in this equilibrium searching is a punishment meant to discipline experts who may have committed fraud, whereas in the search or exit equilibrium exiting is the punishment and searching is seen as a reward by sellers. This difference occurs because in this equilibrium the other seller visited by a searching customer is expected to behave relatively honestly, whereas in the search or exit equilibrium, a seller expects other sellers to behave relatively fraudulently. So search is a useful disciplining device in this equilibrium, whereas in the search or exit equilibrium it is far less useful.

In this equilibrium, notice that at $F = k = 0$, this implies that $s_h = \frac{1}{c_h}$, so customers sometimes do stop, but that as we move up the right hand side of Γ and F grows then s_h falls to eventually become $s_h = 0$ at $F = \frac{1}{c_h-1}$. At this point this equilibrium is identical to the experts police themselves equilibrium. And the sometimes search, always purchase equilibrium cannot extend further up the side of Γ as that would imply $s_h < 0$ which is impossible.

Finally as $F = \frac{1}{c_h-1} < \frac{\alpha}{1-\alpha}(\frac{V_h-c_h}{c_h-1})$ by Assumption 3, this guarantees that customers will always purchase in equilibrium, whether they have received one or two price offers at $p_h = c_h$. So $b_h = b_{2h} = 1$.

The four equilibrium types above are all shown in Figure 3.8 with the blue horizontal line the experts police themselves equilibria, the green line the sometimes search, always purchase equilibria, the orange line the search or exit equilibria, and the area in red is that area where the monopolistic equilibrium exists. The diagram is drawn for $\alpha = 0.2$ and $c_h = 4$.

3.4 Welfare

The four equilibrium types can be welfare ranked. In order to do so, notice that welfare losses occur either due to search costs or because customers sometimes exit

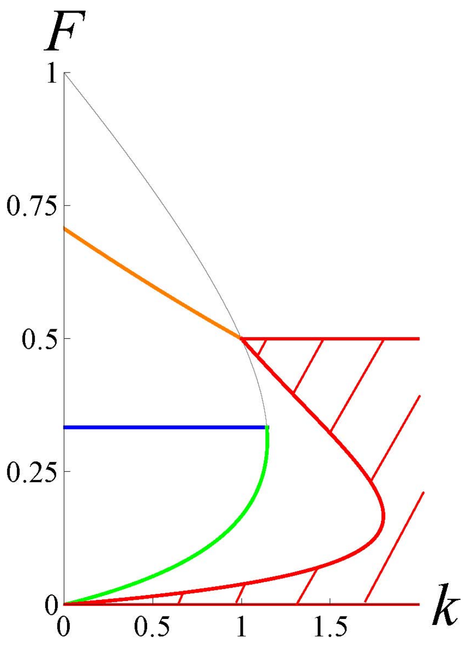

Figure 3.8: Equilibria in the Quick Quotes model

without purchasing treatment. As shown in Proposition 5, where k^{SP} is defined as the highest k at which the sometimes search, always purchase equilibrium exists and k^{SE} is defined as the highest k at which the search or exit equilibrium exists, welfare ranking the equilibria in the quick quotes model leads to a surprisingly simple result. Note that the sometimes search, always purchase equilibrium exists for all $k \in [0, k^{SP}]$, the search or exit equilibrium exists for $k \in [0, k^{SE}]$ and the monopolistic equilibrium exists for all k, if $p_h = V_h$ and $F = 0$ in equilibrium.

Proposition 5 (Welfare with Quick Quotes): For $k \leq k^{SP}$ the sometimes search, always purchase equilibrium welfare dominates. If $k^{SE} > k^{SP}$ then if $\alpha(V_h - c_h)(\frac{c_h-1}{V_h}) < 1-\alpha$ the search or exit equilibrium welfare dominates for $k \in [k^{SP}, k^{SE}]$, otherwise the monopolistic equilibrium welfare dominates in that range. For $k \geq \max[k^{SP}, k^{SE}]$ then only the monopolistic equilibrium exists and therefore welfare dominates.

Proof: See Appendix.

The sometimes search, always purchase (SP) equilibrium welfare dominates wherever it exists because fraud in equilibrium is low and customers offered p_h either accept immediately (so there is no welfare loss) or search for a second opinion (which is costly) but always end up purchasing treatment in the end. That equilibrium dominates the experts police themselves (EP) equilibrium because it involves less fraud and customers only sometimes seek a second opinion following a p_h offer (whereas they always search following a p_h offer in the experts police themselves equilibrium). The search or exit (SE) equilibrium is also welfare dominated because in SE fraud is higher than in SP and a customer offered p_h either exits (which is costly) or searches (also costly). So costs are incurred either way. Finally, the monopolistic (M) equilibrium is welfare dominated by SP because in M customers sometimes exit without treatment (which is costly) and the SP equilibrium only exists for low values of k (so search is not too costly).

For k beyond k^{SP} the SP equilibrium ceases to exist and either SE does not exist, in which case M welfare dominates, or SE does exist up to k^{SE} and either SE or M welfare dominate, depending on the parameter values. In the quick quotes model the condition required for SE to welfare dominate turns out to be the surprisingly simple rule that it does so if $\alpha(V_h - c_h)(\frac{c_h - 1}{V_h}) < 1 - \alpha$. In the slow quotes model below there is no easy rule involving the parameters such as this, and welfare dominance depends on equilibrium prices.

It is interesting to contrast the welfare loss in SP with the loss in Wolinsky's 1993 specialisation equilibrium, bearing in mind that payoffs are different in Wolinsky's model (as he assumes $V_l = V_h = V \geq p_h, p_l$) and he has public price setting, so the two models are somewhat different.[4] In his equilibrium, customers first visit an honest seller of only the cheap treatment and if they genuinely require the expensive

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[4] A further contrast between Wolinsky's model and this one is that he has a 'babbling' equilibrium where customers only visit one expert and they offer to treat all customers at a single price p, an offer which is always accepted. In that one-price-fixes-all equilibrium no search occurs and there is no welfare loss. Such equilibria are ruled out in this model by Assumption 1. This seems reasonable as they do not reflect the observed fraud and search behaviour found in equilibrium in credence goods markets, such as those for car repairs.

treatment they then visit a specialist in the expensive treatment. That implies that the average welfare loss is αk (as a customer always purchases but searches with probability α). In this model, the welfare loss in the SP equilibrium is given by $(\alpha + (1-\alpha)F)k(1-s_h)$ as upon entering the game an agents probability of being offered p_h is $(\alpha+(1-\alpha)F)$ and he will then decide to search with probability $(1-s_h)$ at a (welfare) cost of k. That means that for higher levels of k where SP exists (because F rises and s_h approaches 0) then the specialisation equilibrium must welfare dominate SP. Meanwhile for lower levels of k towards 0, then (as F falls towards 0 and $s_h > 0$) the SP equilibrium must welfare dominate the specialisation equilibrium. This suggests that a simple policy prescription to separate sellers vertically (thus ensuring honest treatment), may not necessarily deliver the highest welfare in equilibrium because of the costly additional search that must always be made for the more expensive treatment.

3.5 Slow Quotes

The model presented above had ‘quick’ quotes. Customers visited sellers who then immediately made a price offer for treatment, and using that information customers either searched for a second opinion or exited. In that model no one ever searches for a second opinion after a p_l offer, so $s_l = 1$, $p_l = V_l$ and (in all but the monopolistic equilibrium) $p_h = c_h$. It seems somewhat restrictive to assume that all credence goods markets have these characteristics, but without imposing exogenous prices on the market how could other prices arise? In what follows I consider instead a model with slow quotes. This is similar to the model above except that when customers visit sellers they can only informally advise them on the nature of their problem (c_l or c_h) and consumers must wait until they stop searching before a formal price offer arrives (and the price offer must be consistent with the informal advice). So customers condition their search behaviour on the expert’s advice, and only observe prices once they have finished searching. Under these assumptions the model can now sustain equilibria with $s_l < 1$, $p_l < V_l$ and (with appropriately chosen beliefs) $p_h \in [c_h, V_h]$ in equilibrium.

The timing in this model avoids Diamond's paradox as consumers do not condition search on prices themselves. So in a similar way to Burdett and Judd's (1983) model of price search, the decision to search is made independently of the actual price being offered (although unlike Burdett and Judd the decision to search is based on the type of offer, which is decided by the seller). The amount of potential fraud in the market also influences search for a second opinion and the amount of search influences the level of fraud. The major impact on the model is that now there may be high, low and (again) a no search equilibrium for s_l, following a low price offer p_l. As in the quick quotes model, the no search equilibrium implies $s_l = 1$ and $p_l = V_l$. In other s_l search equilibria by contrast (with $s_l < 1$) sellers now offer a p_l from an appropriate distribution and because customers know that there is price dispersion they have incentives to seek a second opinion to improve their chances of a low price.

In order to also expand the set of p_h prices offered in equilibrium, I further assume customers have 'fearful' off equilibrium path beliefs. That is, a customer observing any p_h aside from the equilibrium price now believes that the price offer is fraudulent for sure. Because of that he refuses to purchase at any non-equilibrium p_h. This expands the equilibria by ensuring any $p_h \in [c_h, V_h]$ can occur in equilibrium - even in the experts police equilibrium where customers always get two opinions.[5] There is one simplification that is useful to make with regard to p_h however. Recall that above in the quick quotes model Assumption 3 implies $\frac{1}{c_h-1} < \frac{\alpha}{1-\alpha}\frac{V_h-c_h}{c_h-1}$. That meant that the level of fraud in the experts police themselves equilibrium always lay below the upper bound F_h in the Γ^* region. It is useful (although not strictly necessary) in the slow quotes model to assume that $\frac{1}{p_h-1} < \frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1}$. This ensures that again the experts police equilibrium always lies below the upper bound F_h of the Γ^* region, simplifying the analysis. It further ensures that the sometimes search, always purchase

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[5] Notice that in a similar way Wolinsky (1993 pp. 385) assumes that a deviation by a seller to a lower p_h than $p_h = H + k$ in his model leads customers to believe that the expert will commit fraud. As customers do not visit a deviating expert no one ever deviates below $p_h = H + k$.

equilibrium exists up to $\frac{1}{p_h - 1}$, again simplifying the analysis. Note however that this assumption does imply $p_h < V_h - \frac{1-\alpha}{\alpha}$, so strictly speaking $p_h < V_h$ in what follows, although it can be made arbitrarily close to V_h with an appropriate choice of parameter values.

Timing Again, in any period t there will be some customers on their first diagnosis, and some getting a second opinion. The timing of a customers decision in the model is now:

1. After entering in some period t, a seller observes the customers problem and informally quotes a treatment type c_l or c_h.
2. The customer observes the type of offer (but not a price) and may choose to search for a second opinion at cost k in period $t + 1$.
3. If a customer chooses to search for a second opinion, a second seller observes the customers problem and quotes a type of treatment c_l or c_h.
4. Once the customer stops searching, the formal price quote(s) arrive(s). The customer then decides to accept treatment or exits without treatment.

Below, four propositions establish the equilibria in the slow quotes model. This largely involves accommodating the possibility of other equilibria in s_l with the most striking impact on the experts police equilibrium, where $F = \frac{s_l}{p_h - 2 + s_l}$ now. So in that equilibrium the level of fraud actually depends directly on s_l in equilibrium. The experts police equilibrium is discussed in Proposition 6. The other three equilibria are also impacted by s_l but not in such a dramatic way. Proposition 7 establishes the search or exit equilibrium and the monopolistic and sometimes search, always purchase equilibria are given in Propositions 8 and 9. The last two of these make use of results in Proposition 7 as for all three F is not directly a function of s_l.

Finally, again note that there is no equilibrium in the model with $F = 1$, again because a customers expected payoff from accepting p_h would be $\alpha V_h + (1-\alpha)V_l - p_h <$

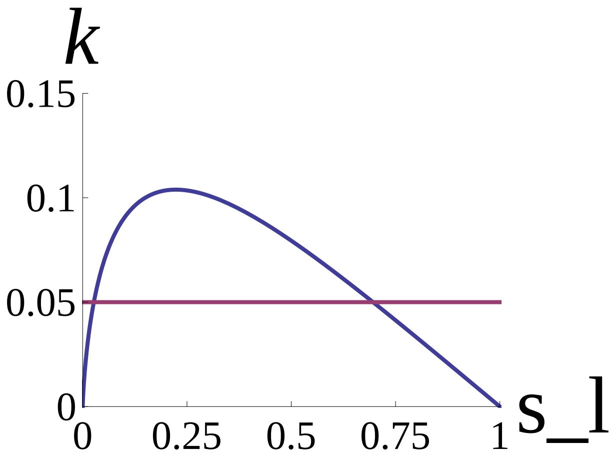

Figure 3.9: The benefit of search $[1-F][E(p_l)-E(p_l^{low})]$ and the cost of search $k=0.05$

0 by Assumption 1, so customers would never accept a p_h offer in this equilibrium. That implies that sellers would offer some p_l to l types instead (as there is always some probability of the offer being accepted, even when $s_l<1$) and that implies that $F=1$ cannot constitute an equilibrium. There are again the two special cases where $F=0$ in equilibrium, the monopolistic equilibrium when $p_h=V_h$ and the sometimes search, always purchase equilibrium when $k=0$, but in addition there is now also the special case of the experts police themselves equilibrium when $k=0$ and $s_l=0$.

Proposition 6 (Experts Police Themselves): For a low enough k equilibria exist where p_l is drawn from a distribution with cumulative density $G(p_l)$, $p_h \in [c_h, V_h]$, $s_l \in [0,1]$, $s_h=0$, $b_{2h}=1$, $F=\frac{s_l}{p_h-2+s_l}$ and $(F,k)\in\Gamma^*$.

Proof: See Appendix.

This equilibrium largely resembles its analogue in the quick quotes model, with customers always searching after a high price offer, but here the results have been generalised to any potential s_l, p_l is drawn from a distribution, and p_h may be greater than c_h in equilibrium. To begin with, recall that in the quick quotes model when experts policed themselves, $F=\frac{1}{c_h-1}$. Here that generalises to $F=\frac{s_l}{p_h-2+s_l}$ so the amount of search by customers following a c_l price offer $(1 - s_l)$ determines the level of fraud. Using this, the proof also establishes that there are up to three equilibria in s_l with high, low and no search respectively. Figure 3.9 illustrates the cost of search k (the horizontal line) and the benefit of search (the hill shape) following advice that the customer requires the c_l treatment.

The proof goes to some lengths to show that the benefit of search is 0 at $s_l = 0$ rises monotonically to a maxima at some $s_l \in (0, 1)$ and then falls monotonically to 0 again at $s_l = 1$. The intuition behind this shape is perhaps more useful than the technical details. At $s_l = 0$ everyone searches following a c_l price offer, and as $F = \frac{s_l}{p_h - 2 + s_l}$ then $F = 0$ so the p_l price must be Bertrand competed down to 0. But then there is zero benefit from searching following a c_l offer as the second opinion is also expected to be $p_l = 0$. Similarly, for $s_l = 1$ no one searches following a c_l offer, so firms all set $p_l = V_l$ and there is no expected benefit from search after a c_l offer. For intermediate levels of s_l however, firms offer a price from some distribution and $F \in (0, 1)$. That implies that there is some benefit to getting a second opinion, as that opinion may also be honest (p_l) and a second draw from the price distribution may yield the customer a lower final price in the end. It is less obvious that the 'search benefit hill' rises then falls monotonically, but this is shown to be true in the proof using Assumption 4. The points where the cost of search intersects the expected benefit of search yield two new equilibria here, the high and low search equilibria. The point where $s_l = 1$ and no one searches also remains an equilibrium because the benefit of search is 0 and is less than the cost of search $k > 0$. This also rules out $s_l = 0$ as an equilibrium as there the benefit of search is again 0 and $k > 0$ so it cannot be that in equilibrium everybody searches (and nobody stops).

Starting with $k = 0$, then if k rises the high and low search equilibria will slowly converge, and for k greater than the highest point of the 'search benefit hill' those equilibria will cease to exist. This can be seen in Figure 3.10, where the old experts police themselves equilibrium (the horizontal line in blue where $s_l = 1$) is joined now by the high and low search experts police equilibria in purple below it. Notice how

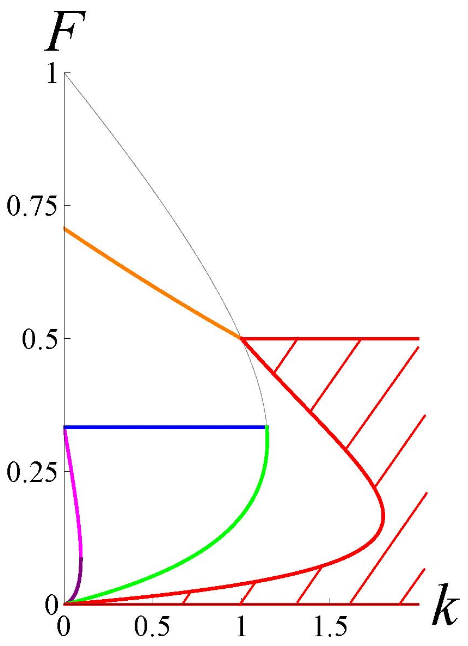

Figure 3.10: Equilibria in the Slow Quotes model

as k rises those two equilibria converge to the same F and then cease to exist.

Proposition 7 (Search or Exit): For low enough k equilibria exist where p_l is drawn from a distribution with cumulative density $G(p_l)$, $p_h \in [c_h, V_h]$, $s_l \in [0,1]$, $s_h = 1 - \frac{s_l+(1-s_l)F}{(p_h-1)F}$, $b_h = 0$, $b_{2h} = 1$, $F = F_h \leq F_{2h}$ and $(F,k) \in \Gamma^*$.

Proof: See Appendix.

This equilibrium is largely the same as it was in the quick quotes model, but again the results must be generalised to any s_l, p_l now follows a distribution in equilibrium, and p_h can be greater than c_h. Here these changes have less impact on the equilibrium then in the experts police case above. The reason for that is that F does not turn out to be a function of s_l. Here $F = F_h$, the upper bound of the Γ^* region, so if $s_l < 1$ that only influences F indirectly in that the expected low price p_l may change, slightly altering F_h. This is also true of p_h. Also in contrast to the experts police equilibrium, the 'search benefit hill' that existed for $s_l \in [0,1]$ following a c_l offer now has a slightly different shape. Rather than always being a hill shape starting at 0 for

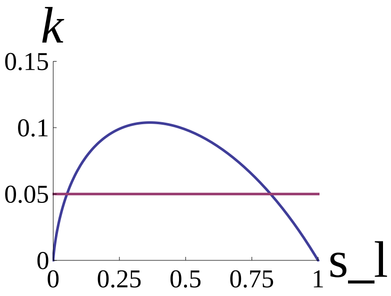

Figure 3.11: Search benefit with $F = 0$

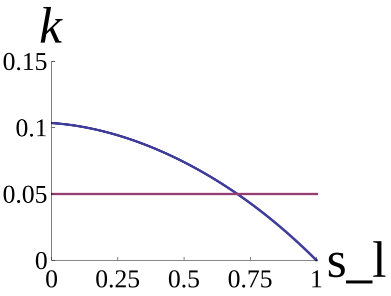

Figure 3.12: $F = 0.25$

$s_l = 0$ then rising and falling back to 0 at $s_l = 1$, now it may start at $s_l = 0$ above 0, but is still constrained to eventually fall to 0 at $s_l = 1$.

Observe in Figures 3.11 and 3.12 two examples. In the first figure where $F = 0$ this looks like the 'hill' seen when experts police themselves. This is because at $s_l = 0$ all customers get a second opinion following a c_l offer, and as $F = 0$ sellers Bertrand compete for l types, leading to an equilibrium price of $p_l = 0$. But in Figure 3.12 it is assumed that $F = 0.25$ so even if $s_l = 0$ and all customers get a second opinion following a c_l offer then sellers are only sometimes competing with a second c_l offer. Because of this p_l is drawn from a distribution and the expected benefit from search may be greater than 0. However it remains the case that at $s_l = 1$ where customers never search following a c_l offer that all sellers offer $p_l = V_l$ and there is no benefit from a second opinion. So the benefit of search must be 0 at $s_l = 1$ regardless of F.

The level of search following a c_h offer is also generalised in the slow quotes model. In the quick quotes model the probability of stopping was given by $s_h = 1 - \frac{1}{(c_h-1)F}$, whereas here it is generalised to:

$$s_h = 1 - \frac{s_l + (1 - s_l)F}{(p_h - 1)F}$$

So the equilibrium in the quick quotes model can be seen as a special case of this equilibrium.

Proposition 8 (Monopolistic): For a k great enough equilibria exist where customers do not search following a c_h offer, p_l is drawn from a distribution with cumulative density $G(p_l)$, $p_h \in [c_h, V_h]$, $s_l \in [0,1]$, $s_h = 1$, $b_h = (s_l + (1-s_l)F)\frac{V_l}{p_h}$, $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$ and $(F,k) \in \Gamma^M$.

Proof: See Appendix.

The analysis of search following a c_l offer is the same here as it was in the previous proposition. In this model again $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$, but whereas the probability of a customer purchasing the c_h treatment in the quick quotes model was $b_h = \frac{V_l}{p_h}$, here it has been generalised to $b_h = (s_l + (1-s_l)F)\frac{V_l}{p_h}$. So although there is no search following a c_h offer, s_l and search following a c_l offer may influence b_h.

Proposition 9 (Sometimes Search and Always Purchase): For low enough k equilibria exist with p_l drawn from a distribution with cumulative density $G(p_l)$, $p_h \in [c_h, V_h]$, $s_l \in [0,1]$, $s_h = \frac{s_l+(1-s_l)F+F(1-p_h)}{p_h+F(1-p_h)}$, $b_h = 1$, $b_{2h} = 1$, $F = F_{1,2} \leq \frac{1}{p_h-1}$ and $(F,k) \in \Gamma^*$.

Proof: See Appendix.

Again this proposition is generalised to any possible s_l, p_l follows a distribution now, and p_h may be greater than c_h. As in Proposition 7, again there may be a number of potential equilibria in s_l, depending on where the search benefit hill intersects the cost of search. Because of that the probability of stopping found in the quick quotes equilibrium $s_h = \frac{1+F(1-c_h)}{c_h+F(1-c_h)}$ is generalised to:

$$s_h = \frac{s_l + (1-s_l)F + F(1-p_h)}{p_h + F(1-p_h)}$$

This expression could be re-written as $F = \frac{s_l - s_h p_h}{p_h - 2 + s_l - s_h(p_h-1)}$ which can be compared to the expression $F = \frac{s_l}{p_h-2+s_l}$ in the experts police themselves equilibrium where $s_h = 0$. So again here fraud is always lower than in the experts police equilibrium, except for the single point where $s_h = 0$ where the two equilibria are identical.

3.5.1 Some Comparative Statics

Comparative statics are complicated in this model and the level of fraud (or indeed the level of search) in the above equilibria does not in and of itself say anything about the level of welfare in these markets. But it is nevertheless an interesting exercise to investigate some of the reactions of fraud and search to changes in prices now that a broader range of prices are admissible in equilibrium. Here I sketch out changes in the four equilibrium types as p_h rises or s_l (and therefore on average p_l) falls, thus giving sellers greater incentives to commit fraud on their customers. To begin with, notice that as p_h rises or the expected p_l falls then the Γ region will expand. This is because customers have a greater incentive to search if the potentially fraudulent p_h price is higher, and if the expected p_l price offered is lower. In what follows I will ignore the effect of this change in Γ (although I do not deny that it will impact on the equilibrium) because depending on exactly which equilibrium we are interested in, this change could either raise or reduce fraud, or may even imply that the old equilibrium no longer exists. Further changes in F may have an impact on the level of s_l in equilibrium, and again I will ignore those effects. So given those (large) caveats, what can we say? In the monopolistic equilibrium an increase in expected p_h and decrease in expected p_l have a first order impact on the equilibrium level of fraud $F = \frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - 1}$:

- Monopolistic: $\downarrow p_l \rightarrow$ no $\triangle F$; $\uparrow p_h \rightarrow \downarrow F$

So the monopolistic equilibrium has some comparative statics similar to the 'physician induced demand' hypothesis mentioned by Dranove (1988) among others. Here, as the potentially fraudulent price of the diagnosing expert (p_h) rises the amount of fraud decreases while changes in p_l have no impact on fraud. This suggests that less price competition is helpful in that it decreases fraud (or has no effect).

Recall that the key equation in the experts police themselves equilibrium was $F = \frac{s_l}{p_h - 2 + s_l}$. As s_l falls and p_h rises:

- Experts Police Themselves: $\downarrow p_l \rightarrow \downarrow F$; $\uparrow p_h \rightarrow \downarrow F$

So in contrast to the monopolistic equilibrium, these comparative statics imply that in the experts police themselves equilibrium a higher p_h may lead to lower fraud, but a lower p_l may reduce fraud. Indeed in equilibrium if prices are at cost (with $s_l = p_l = 0$) then $F = 0$. So although the 'physician induced demand' comparative statics are present for p_h, enough competition between experts ultimately drives p_l and fraud to 0.

The following two equilibria were ones on the upper and lower bounds of the Γ^* region respectively. Ignoring the effect of price changes on that region then in the search or exit equilibrium customers stopped with probability $s_h = 1 - \frac{s_l+(1-s_l)F}{(p_h-1)F}$ and in the sometimes search, always purchase equilibrium with probability $s_h = \frac{s_l+(1-s_l)F+F(1-p_h)}{p_h+F(1-p_h)}$. So the impact of a decrease in s_l or increase in p_h is:

- Search or Exit: $\downarrow p_l \rightarrow \uparrow s_h$; $\uparrow p_h \rightarrow \uparrow s_h$
- Search or Purchase: $\downarrow p_l \rightarrow \downarrow s_h$,; $\uparrow p_h \rightarrow \downarrow s_h$

So ignoring the effect of the change in Γ^* caused by a change in prices, then in the search or exit equilibrium, as an experts incentives to commit fraud increase (p_l falls, or p_h rises) customers stop more, and search less. In contrast, in the search or purchase equilibrium as an experts incentives to commit fraud rise, customers stop less, and search more. These reactions are naturally symmetric in these two equilibria, because in the search or exit equilibrium stopping is a punishment for experts, while in the search and purchase equilibrium, stopping is a reward. So the comparative statics naturally move in opposite directions in order to reduce sellers incentives to offer a fraudulent price.

3.6 Welfare in the Slow Quotes Model

In Proposition 5 it was shown that when $p_h = c_h$ and $p_l = V_l$ the sometimes search, always purchase equilibrium welfare dominates up to some $k = k^{SP}$ where it ceases to exist and that beyond that point either the search or exit or the monopolistic equilibrium welfare dominates. Proposition 10 observes that the proof of Proposition 5 also applies for any $p_h \in [c_h, V_h]$ if $s_l = 1$.

Proposition 10 (Welfare with Slow Quotes): Given some p_h, if $s_l = p_l = 1$ in equilibrium then for $k \leq k^{SP}$ the sometimes search, always purchase equilibrium welfare dominates. If the search or exit equilibrium exists for $k > k^{SP}$ then either it or the monopolistic equilibrium may welfare dominate. For $k \geq \max[k^{SP}, k^{SE}]$ then only the monopolistic equilibrium exists, so it welfare dominates.

Proof: See the proof of Proposition 5.

This proof establishes welfare rankings for some p_h and $s_l = p_l = 1$. As $s_l = 1$ this equilibrium is useful for welfare comparisons because a p_l price offer implies immediate acceptance and no welfare loss. But recall that due to the 'hill' shape of the benefit from search following a low price offer, there could be other potential equilibria for s_l. Equilibria with $s_l < 1$ are unfortunately not easy to deal with analytically, and while they can be welfare ranked they would have to be ranked on a case by case basis after solving for s_l numerically. This is further compounded by the problem that the search benefit hill itself differs among the four equilibrium types as they involve different levels of F, price dispersion and the hill itself is not even constrained to be a hill shape in three of the four equilibria. For all of these reasons it makes sense to focus on the case where welfare rankings can be produced in a fairly general way ($s_l = p_l = 1$) especially as that equilibrium in s_l is guaranteed to exist.

3.7 Conclusion

This chapter has proposed a theory of overcharging fraud in credence markets with competitive price setting by experts. In contrast to the extant credence goods literature, there are private quotes for treatment and customers may always purchase from the expert who offered them treatment at the lowest price. If treatment quotes are ‘quick’ then the market suffers from Diamond’s paradox in the cheap treatment, as customers cannot be induced to undertake costly search in equilibrium. But the expensive treatment does not suffer from this as customers always have some incentive to search as the price offer may be fraudulent. In order to expand the set of equilibrium prices, the model was also extended to ‘slow’ quotes, where customers choose to undertake search without knowing the actual price offer. Similarly, if customers hold the belief that unexpected price offers signal fraud, the set of equilibrium high prices can be expanded.

The model was found to have four potential equilibria which could be welfare ranked. Two of these resembled equilibria mentioned elsewhere in the literature (Wolinsky (1995) and Fong (2005)). And in contrast to the physician induced-demand hypothesis, presented for example in Dranove (1988), that argues experts induce more demand from their customers as prices fall, I found that was only necessarily the case in one of the four potential equilibria (the monopolistic one similar to Fong’s). In another equilibrium (where experts police themselves), lower prices may be accompanied by less demand inducement (and greater honesty) as customers seek a second opinion, changing experts incentives to commit fraud. Further, even if experts in the monopolistic equilibrium do set prices at customers maximum willingness to pay eliminating fraud and demand inducement, it was shown that welfare was always lower in the monopolistic equilibrium than in another equilibrium (resembling Wolinky’s (1995)) where customers sometimes searched for a second opinion. This implies that we should be careful not to simply equate fraud with low welfare in credence markets.

The different equilibria in the model also emphasise that in credence markets customers must have some way of disciplining experts for proposing potentially fraudulent treatment. Because in these markets customers do not know whether fraud was committed even after they have been treated, customers must either use a second opinion, or exit in order to punish experts for potential fraud. Reducing search costs may be useful then from a policy perspective, although not in every potential equilibrium. Perhaps more useful would be attempting to get consumers to move to an equilibrium where they sometimes get a second opinion. In this spirit I would suggest that customers in these markets should be encouraged to inform themselves about possible alternatives to their usual expert diagnoser and to seek a second opinion if they feel they may be being overcharged.

Further Work In Wolinsky's (1993) model publicly posted prices destroy the market with experts offering two types of goods, and he proposed a vertical specialisation equilibrium with each expert delivering only one type of treatment and customers being referred between them. To model competition in markets where firms might specialise more in one good than the other, it might be interesting to use a setup similar to Biglaiser and Ma (2003) who use a Hotelling model to investigate price setting in a market where customers demand two types of goods and firms set their quality levels. In their setup customers have the informational advantage, and they investigate equilibrium under bundling and separation of the two goods. It also seems natural to ask whether a more detailed model of referrals between firms such as that of Garicano and Santos (2004) could be adapted to look at referrals in a credence market to try and find regions where specialisation and the referral problem or non-specialisation and the credence problem exist. That could involve a trade off between the moral hazard problem faced by a customer visiting an expert and the adverse selection problem between referring experts. Finally, in a market for an experience good, Bouckaert and Degryse (2000) use a Hotelling model for competition between an expert and non-expert. Although the information asymmetry is not the same, an analogous model of competition between a full-service and specialist provider of a credence good might also be interesting and could perhaps be seen as a combination of Wolinsky's suggested equilibrium with full separation, and a market where all experts provide both services.

3.8 Appendix

Proof of Proposition 1

This proposition is established in three steps. The first of these is something like the credence goods version of Diamond's paradox. It assumes that $s_h = 0$, so consumers always search for a second opinion, and establishes that $p_l = V_l$, $s_l = 1$, and $p_h = c_h$. The next step in the proof defines a region Γ^* which is made up of pairs (F, k) of fraud and search costs where consumers will have an incentive to search following a price offer of p_h. In this region search costs are low and fraud is at an intermediate (not too high, or too low) level. Finally, I show that the equilibrium level of fraud when experts police themselves is $F = \frac{1}{c_h - 1}$ given $s_l = 1$, $s_h = 0$ and that given this level of fraud if search costs k are low enough, then this is in Γ^*. Together the three lemmas establish the proposition.

Lemma 1 (Diamond's Paradox in Credence Goods): In the quick quotes model $s_l = 1$, $p_l = V_l$, and if $s_h = 0$ then $p_h = c_h$.

Begin by noting that no price $p \in (V_l, c_h)$ will ever be offered to a customer, as a seller would only ever offer such a price to a customer requiring the l treatment, but that a customer will never pay more than V_l for such treatment. Similarly no seller will ever offer a price $p < c_l$ that does not cover his costs. And that no price $p > V_h$ will be offered as no customer would ever accept it. So the only prices ever offered to customers are either low prices denoted $p_l \in [c_l, V_l]$ or high prices denoted $p_h \in [c_h, V_h]$. Further, any p_l price offer signals to a customer that he must be an l type.

In equilibrium, all sellers will offer the same low price p_l when they choose to offer a price in $[c_l, V_l]$. To see this, first suppose that all sellers did set different p_l prices in equilibrium. That implies that some seller sets the lowest of these prices, denoted $\underline{p}_l$. Clearly any customer offered treatment at $\underline{p}_l$ will choose to stop and purchase at that price. But that cannot be an equilibrium because that seller could then raise his price by some small amount $\epsilon < k$ and his customers would still always stop and purchase from him, and the seller makes a greater profit. So no equilibrium exists with some seller offering a lower p_l than all the others.

Similarly, no group of two or more sellers could offer some lowest price $\underline{p}_l < V_l$ in equilibrium. To see this, assume that two or more sellers are all offering the lowest price $\underline{p}_l < V_l$. In that case, all customers of these sellers will stop and purchase once offered treatment at that price. But again, one of the sellers could deviate and raise his price by a small amount $\epsilon < k$ and his customers would still always stop and purchase at that price. So there cannot be an equilibrium with a group of sellers offering the same equilibrium price below V_l.[6]

The equilibrium low price offered by all sellers must therefore be $p_l = V_l$. At that price customers will not deviate from purchasing treatment, all sellers are offering the same price, and there is no profitable deviation to a higher price $p \in (V_l, c_h)$ as customers will refuse to purchase at such a price. (There are potentially profitable deviations to a fraudulent $p_h \in [c_h, V_h]$ however, and p_h is discussed below). There is also no profitable deviation downward from $p_l = V_l$ because although their customers continue to purchase from them, sellers make lower profits. As $p_l = V_l$ is the lowest price ever offered by any seller, then no customer offered $p_l = V_l$ will ever make a costly search for a second opinion. That is, as $p_l = V_l$ for all sellers then $s_l = 1$.

Customers may also be offered a high price $p_h \in [c_h, V_h]$, and it must be that $p_h = c_h$ in equilibrium. Assuming that $s_h = 0$ means that customers always search for a second opinion following an offer of p_h. By doing so, that second opinion must either be a second price offer of p_h or potentially a price offer of $p_l < p_h$. If the second opinion is $p_l < p_h$ then clearly the customer will choose p_l. If it is p_h then he will choose one of the two p_h prices. As the only case where he chooses p_h is following two offers of p_h, then following the logic of Bertrand competition $p_h = c_h$.

Lemma 2: A set Γ^* exists, where for $(F, k) \in \Gamma^*$ then $b_{2h} = 1$ and $s_h = 0$.

---

[6] Notice that I have ignored the case of $k = 0$ here. In that case, it is possible for $p_l < V_l$ to be an equilibrium price, but I will ignore that possibility as my focus is on equilibrium with costly search.

Lemma 1 found that $p_l = V_l = 1$ and $p_h = c_h$ if $s_h = 0$ in the quick quotes model. Here I will proceed more generally, for any equilibrium price $p_l \in [c_l, V_l]$ and $p_h \in [c_h, V_h]$ so that this proof can also be applied to the slow quotes model later. My strategy is to construct a set Γ^* in $F-k$ space by defining conditions where customers would always search for a second opinion following a p_h offer and where customers would always prefer purchasing at p_h rather than exiting without purchasing. To do so, first I will show that given some p_h and p_l that customers will always search for intermediate levels of $F \in (F_1, F_2)$ and a low enough search cost k. I will define this as the region Γ. I then show that if fraud is low enough and $F < \min\{F_h, F_{2h}\}$ that costumers prefer to purchase treatment at p_h rather than simply exiting for a payoff of 0. That allows me to define the region $\Gamma^* \subset \Gamma$ wherein customers will always search $s_h = 0$ and will be prepared to purchase at p_h after two recommendations so $b_{2h} = 1$.

Assume for now that $b_{2h} = 1$ so customers do in fact choose to purchase the h treatment after two offers of p_h. What is the region Γ in which customers will choose to search again after one price offer of p_h? To find this, note that in equilibrium there is an expected level of fraud in the market $F \in [0, 1]$ and that after being offered treatment at p_h a customer updates his belief about his own type. I will denote the customers belief that he requires the h type treatment as $P(h)$, and his belief about his type after having been offered p_h as $P(h \mid p_h)$. Applying Bayes' rule:

$$\begin{aligned} & P(h \mid p_h) = \tfrac{P(p_h|h)P(h)}{P(p_h|h)P(h)+P(p_h|l)P(l)} \\ \Rightarrow \qquad & P(h \mid p_h) = \tfrac{\alpha}{\alpha+F(1-\alpha)} \end{aligned}$$

A customers expected payoff from stopping and purchasing the high treatment at p_h is therefore:

$$\frac{\alpha(V_h - p_h) + F(1-\alpha)(V_l - p_h)}{\alpha + F(1-\alpha)} \tag{3.1}$$

A customer determines his expected payoff from searching again after being offered treatment at p_h by again applying Bayes' rule to account for the probability that he will again receive a p_h offer. With probability $P(h \mid p_h)$ the customer is a high type and will certainly receive a second p_h offer, and with probability $P(l \mid p_h)$ he is a low type, but will fraudulently receive a second p_h offer with probability F. So a customer's probability of seeking a second opinion and receiving a second h offer is:

$$\begin{aligned} & P(p_h \mid p_h) = 1 \times P(h \mid p_h) + F \times P(l \mid p_h) \\ \Rightarrow \quad & P(p_h \mid p_h) = \tfrac{\alpha}{\alpha+F(1-\alpha)} + F\tfrac{F(1-\alpha)}{\alpha+F(1-\alpha)} \end{aligned}$$

In a similar way, his probability of receiving a p_l offer from the second expert is given by:

$$\begin{aligned} & P(p_l \mid p_h) = 1 - P(p_h \mid p_h) \\ \Rightarrow \quad & P(p_l \mid p_h) = (1-F)\tfrac{F(1-\alpha)}{\alpha+F(1-\alpha)} \end{aligned}$$

Recalling that search incurs a cost k, the expected payoff from search is then given by:

$$\frac{\alpha(V_h - p_h) + (1-\alpha)F^2(V_l - p_h) + (1-\alpha)F(1-F)(V_l - p_l)}{\alpha + (1-\alpha)F} - k \tag{3.2}$$

For a customer to prefer to search after a p_h offer, it must be that (3.1)≤(3.2):

$$\tfrac{\alpha(V_h-p_h)+F(1-\alpha)(V_l-p_h)}{\alpha+F(1-\alpha)} \leq \tfrac{\alpha(V_h-p_h)+(1-\alpha)F^2(V_l-p_h)+(1-\alpha)F(1-F)(V_l-p_l)}{\alpha+(1-\alpha)F} - k$$

Rearranging this:

$$F^2 - F(1 - \frac{k}{p_h - p_l}) + \frac{\alpha}{1-\alpha}\frac{k}{p_h - p_l} \leq 0$$

Consider $k = 0$. Here search is costless and the above inequality becomes $F(F - 1) \leq 0$. This condition is met with equality for $F = 1$ and $F = 0$ because customers know that they will always be offered p_h again in the second opinion. When $F = 1$ this is because experts always commit fraud so only ever offer p_h, and when $F = 0$ experts are always honest so the initial offer of p_h must have been truthful, and the second expert will also offer it. Finally notice that for any $F \in (0, 1)$ that the inequality is strict. That implies that for intermediate levels of fraud customers strictly prefer to search for a second opinion. This is because with $F \in (0,1)$ the first p_h offer could have been fraudulent and the second opinion might then be honest. Clearly $(F,k) \in \Gamma$ for $k = 0$ and any $F \in [0,1]$. Below I will further delineate this region for $k > 0$.

Notice that at $F = 0$ the quadratic is positive and only falls in F if $k < p_h - p_l$. So there can only be positive (real) roots if $k < p_h - p_l$. Assuming this, then to find that region Γ within which customers prefer to search following a h offer, the above quadratic equation can be set to 0 and solved for its two roots. Real solutions for $F_{1,2}$ exist if k is small enough, and for those real solutions then if $F \in [F_1, F_2]$ customers will not deviate from searching for a second opinion:

$$F_{1,2} = \frac{1}{2}(1 - \frac{k}{p_h - p_l}) \pm \frac{1}{2}\sqrt{(1 - \frac{k}{p_h - p_l})^2 - 4\frac{\alpha}{1-\alpha}\frac{k}{p_h - p_l}}$$

For the solutions of the quadratic equation to be real, I require that $(1 - \frac{k}{p_h - p_l})^2 \geq 4\frac{\alpha}{1-\alpha}\frac{k}{p_h - p_l}$. Rearranging this leads us to another quadratic! (don't worry this is getting somewhere):

$$k^2 - k(2 + 4\frac{\alpha}{1-\alpha})(p_h - p_l) + (p_h - p_l)^2 \geq 0$$

Recalling that $p_h > V_l \geq p_l$, then this is another convex quadratic equation initially falling at $k = 0$, and as long as the quadratic is positive, which will occur for $k \in [0, k_1]$ and $k \in [k_2, \infty)$ then we get real solutions for $F_{1,2}$ out of the earlier quadratic. Solving for its roots:

$$k_{1,2} = (p_h - p_l)(1 + 2\tfrac{\alpha}{1-\alpha}) \pm \tfrac{1}{2}\sqrt{(2 + 4\tfrac{\alpha}{1-\alpha})^2(p_h - p_l)^2 - 4(p_h - p_l)^2}$$

$$\Rightarrow \qquad k_{1,2} = (p_h - p_l)\tfrac{(1+\alpha \pm 2\sqrt{\alpha})}{1-\alpha}$$

As $\alpha \in (0,1)$ then $\sqrt{\alpha} > \alpha$, this implies $k_1 < p_h - p_l < k_2$. Only the lower of these two values are of interest to us because as stated earlier, I need $k < p_h - p_l$ in order to get positive solutions for $F_{1,2}$. If $k > p_h - p_l$, then real solutions for $F_{1,2}$ can be found but they are negative, which cannot constitute part of an equilibrium as

$F \in [0,1]$. So these negative solutions for F when $k > p_h - p_l$, much like the complex solutions for F when $k_1 < k < k_2$ play no part in this model. Indeed these solutions simply indicate that the search cost is so high that customers do not search for any level of fraud. As there real positive solutions for F only for $k \in [0, k_1]$ I will denote $k_1 \equiv k^*$. As mentioned above at $k = 0$ the solutions are $F_{1,2} = 0, 1$ and as k rises these converge to a single root found at k^* which is $F^* = \frac{1}{2}(1 - \frac{k^*}{p_h - p_l})$. Note that $F^* \in (0,1)$ because $k^* < p_h - p_l$.

Assuming that $k \leq k^*$, then $F \in \{F_1, F_2\}$ are those levels of fraud at which the consumer would be indifferent between stopping after an offer of p_h and searching for a second opinion. Notice that as F_1, F_2 are 0 and 1 at $k = 0$ and that as k grows to k^* the roots monotonically converge to $F^* \in (0,1)$, so they are always between 0 and 1. Further note that for $F \in (F_1, F_2)$ customers strictly prefer to search.

I now denote the region with $k \leq k^*$ and $F \in [F_1, F_2]$ as Γ. Now I will argue that the Γ region always exists. This must be true as $k^* = (p_h - p_l)\frac{(1+\alpha-2\sqrt{\alpha})}{1-\alpha} > 0$. To see this note that $p_h - p_l > 0$ as by assumption $p_h \in [c_h, V_h]$ and $p_l \in [c_l, V_l]$ with $V_h > c_h > V_l > c_l$. Further:

$$\frac{1+\alpha-2\sqrt{\alpha}}{1-\alpha} = \frac{(1-\sqrt{\alpha})^2}{1-\alpha}$$

And $\frac{(1-\sqrt{\alpha})^2}{1-\alpha} > 0$ as $\alpha \in (0,1)$. As a $k^* > 0$ always exists in the model, so must the region Γ, with $k \leq k^*$.

The above shows that if $b_{2h} = 1$ and customers do purchase after a second p_h offer, then they will always prefer to search for a second opinion in the region Γ (with low search costs and intermediate levels of fraud) rather than purchase immediately. But there are two further requirements that must be met to ensure that after an initial p_h offer customers do go on to search again and purchase at p_h if the second expert recommends it. First customers must prefer to purchase after a second p_h opinion, rather than exiting without purchasing anything. Second customers expected payoff from a second search (and subsequent purchase) must be greater than exiting after the first p_h offer without purchasing.

In order to guarantee that customers will always purchase after their second p_h price offer, fraud must not be too high. To determine when this is the case, a customers expected probability of being the h type after having been made a second offer of p_h is given by again applying Bayes' rule:

$$
\begin{aligned}
& P(h \mid p_h, p_h) = \tfrac{P(p_h,p_h|h)P(h)}{P(p_h,p_h|h)P(h)+P(p_h,p_h|l)P(l)} \\
\Rightarrow \quad & P(h \mid p_h, p_h) = \tfrac{\alpha}{\alpha+F^2(1-\alpha)}
\end{aligned}
$$

So a customers expected payoff from purchasing the high treatment at p_h after two offers is therefore greater than exiting without purchasing and getting a payoff of 0 if:

$$
\begin{aligned}
& \tfrac{\alpha}{\alpha+F^2(1-\alpha)}(V_h - p_h) + \tfrac{F^2(1-\alpha)}{\alpha+F^2(1-\alpha)}(V_l - p_h) \geq 0 \\
\Rightarrow \quad & F \leq \sqrt{\tfrac{\alpha}{1-\alpha}\tfrac{V_h-p_h}{p_h-V_l}}
\end{aligned}
$$

Denote the right hand side of this inequality F_{2h} as an F below this levels guarantees that customers would purchase a h treatment after two h recommendations. So F_{2h} is potentially an upper bound on the level of fraud region where this equilibrium can exist.

When solving for $F_{1,2}$ above I used the fact that customers must prefer to search after their first p_h offer rather than simply purchasing treatment. That is the expected payoff from search had to be greater than or equal to the expected payoff from stopping and purchasing, or (3.1)$\leq$(3.2). But if F is high enough then $0 \geq$(3.1). That occurs when:

$$
\begin{aligned}
& \tfrac{\alpha(V_h-p_h)+F(1-\alpha)(V_l-p_h)}{\alpha+F(1-\alpha)} \leq 0 \\
\Rightarrow \quad & F \geq \tfrac{\alpha}{1-\alpha}\tfrac{V_h-p_h}{p_h-V_l}
\end{aligned}
$$

If F is that high, then the appropriate inequality to ensure that customers prefer to search rather than not searching is (3.2)≥ 0. That is:

$$\frac{\alpha(V_h-p_h)+(1-\alpha)F^2(V_l-p_h)+(1-\alpha)F(1-F)(V_l-p_l)}{\alpha+(1-\alpha)F}-k\geq 0$$

$$\Rightarrow \qquad F^2-F(\tfrac{V_l-p_l-k}{p_h-p_l})-\tfrac{\alpha}{1-\alpha}(\tfrac{V_h-p_h-k}{p_h-p_l})\leq 0$$

This quadratic will have two roots given by:

$$\tfrac{1}{2}(\tfrac{V_l-p_l-k}{p_h-p_l})\pm\tfrac{1}{2}\sqrt{(\tfrac{V_l-p_l-k}{p_h-p_l})^2+4\tfrac{\alpha}{1-\alpha}\tfrac{(V_h-p_h-k)}{(p_h-p_l)}}$$

Here, only the higher root is of interest to us. The reason for this is that the lower root will lie below $\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l}$, implying the original inequality (3.2)$\geq$(3.1) is appropriate, as (3.1)>0. So at the solution of the lower root to this quadratic then the inequality it solves cannot bind, and only the upper root of this quadratic has a meaningful interpretation here.

To see this is not obvious. First notice that (3.1)$=0$ at one unique $F=\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l}$ where customers receive a payoff of 0 from purchasing after the first p_h offer. This level of $F\in[0,1)$ by Assumption 1. Second, recall that the right hand side of Γ solve (3.2)=(3.1) and describes those pairs where a customer is exactly indifferent between searching for a second opinion and purchasing immediately after a p_h offer. As Γ always extends from $F=0$ up to $F=1$ then there must be a unique point on the right hand side of Γ at which $F=\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l}$. At that point, the value of a second search equals the value of purchasing straight away (as it lies on the right hand side of Γ) and the value of purchasing straight away equals 0 (as $F=\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l}$). So the value of search must equal 0 too at that point.

As at that point the value of search equals 0, then (3.2)$=0$ and that combination of F and k must satisfy the quadratic with equality. What happens when k now falls? $F^2-F(\frac{V_l-p_l-k}{p_h-p_l})-\frac{\alpha}{1-\alpha}(\frac{V_h-p_h-k}{p_h-p_l})$ is increasing in k so it will fall. Notice that its first derivative with respect to F is $2F-(\frac{V_l-p_l-k}{p_h-p_l})>0$ which is positive for $F>\frac{1}{2}\frac{V_l-p_l-k}{p_h-p_l}$, the case of the upper root of the quadratic, and negative for $F<\frac{1}{2}\frac{V_l-p_l-k}{p_h-p_l}$, the case of the lower root of the quadratic. So as k falls then the upper root of the quadratic must rise to ensure $F^2-F(\frac{V_l-p_l-k}{p_h-p_l})-\frac{\alpha}{1-\alpha}(\frac{V_h-p_h-k}{p_h-p_l})=0$. Alternatively, as k falls the lower root of the quadratic must fall to satisfy the quadratic. But that implies $F < \frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l}$, and the quadratic is no longer a binding constraint. So the lower root of the quadratic does not have a meaningful use to us here. Starting from the right hand side of Γ and reducing k then only the higher root of the quadratic is large enough to keep customers indifferent between searching and exit. Visually, in Figure 3.4 from the point where $\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l} = 0.5$ intersects the right hand side of Γ as k falls then the upper root rises, keeping it above 0.5.

So we can focus on the higher root as that will restrict the region $\Gamma^* \subset \Gamma$. Solving for this (higher) root which I will denote F_h:

$$F_h = \frac{1}{2}(\frac{V_l - p_l - k}{p_h - p_l}) + \frac{1}{2}\sqrt{(\frac{V_l - p_l - k}{p_h - p_l})^2 + 4\frac{\alpha}{1-\alpha}\frac{(V_h - p_h - k)}{(p_h - p_l)}}$$

The importance of F_h is that an F below this level in Γ implies that customers will prefer to search rather than exit after their first h offer. And notice that the above discussion implies the important result that $F_h \geq \frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1}$, as F_h is falling in k until it intersects the right hand side of the Γ region at $\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1}$.

For a customer to search following a p_h offer and then to purchase following a second p_h offer then $F \leq \min\{F_h, F_{2h}\}$. I define the region within Γ where this is true as $\Gamma^* \subset \Gamma$.

Note that so long as $p_l = V_l$ then at $k = 0$, $F_h = F_{2h}$. And as k rises then F_h falls, so F_h is always the upper bound of Γ^* in the quick quotes model where $p_l = V_l = 1$. This can be seen in Figure 3.4. However if $p_l < V_l$ then $F_{2h} < F_h$ at $k = 0$. So F_{2h} may be the upper bound of Γ^* for low levels of k. But as k increases F_h falls, and must eventually intersect the right-hand side of the Γ region at $\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l} < F_{2h}$. So F_h must always be the upper bound of Γ^* over some region of k even when $p_l < V_l$.

Finally notice that Γ^* always exists, although shrinks in size as $p_h \to V_h$. Indeed at $p_h = V_h$ the only element of the set is $k = 0$ and $F = 0$. As stated in the Lemma then, a set Γ^* exists where for $(F, k) \in \Gamma^*$ then $b_{2h} = 1$ and $s_h = 0$ by construction.

Lemma 3: If $s_h = 0$, $p_l = V_l$ and $p_h = c_h$ then $F = \frac{1}{c_h - 1}$.

In equilibrium for sellers to mix between attempting to commit fraud or not, then they must be indifferent between offering p_l and p_h to an l type customer. If an expert chooses to commit fraud, then as $s_h = 0$ customers always search again following p_h and a fraudulent expert's expected payoff depends on their expectations of customer behaviour. Note that normalising the number of new l type customers per expert to 1, then there are $1 + F$ of the l types visiting an expert each period. The fraction $\frac{F}{1+F}$ of these have been defrauded before while for $\frac{1}{1+F}$ this is their first diagnosis. I assume that customers offered the same price $p_h = c_h$ by two experts flip-a-coin to decide which expert to buy from. That implies of the l types getting a second opinion $\frac{\frac{1}{2}F}{1+F}$ will purchase given p_h, and as F of the new l types will also be defrauded by the second opinion they get, then similarly $\frac{\frac{1}{2}F}{1+F}$ will also eventually purchase given a p_h price offer. So a fraudulent expert's expected payoff in equilibrium is $\frac{\frac{1}{2}F+\frac{1}{2}F}{1+F}c_h$.[7]

As shown above, in equilibrium $p_l = V_l$ and all customers offered p_l immediately stop and purchase. So the expected payoff of offering p_l is simply V_l. Recall that $V_l = 1$ by Assumption 2. In equilibrium, for the fraudulent p_h offer and truthful p_l offer to have the same expected payoff:

$$\begin{aligned} \tfrac{F}{1+F}c_h &= 1 \\ \Rightarrow \quad F &= \tfrac{1}{c_h - 1} \end{aligned}$$

If in equilibrium all experts in the market are committing fraud with probability $F = \frac{1}{c_h - 1}$, so any individual expert is indifferent between committing fraud and honesty when they diagnose an l type customer, then no expert will deviate from committing fraud with probability $F = \frac{1}{c_h - 1}$. It is in this sense that experts police themselves in this equilibrium.

---

[7] Notice that in equilibrium this payoff would be the same regardless of how customers choose which of the two experts to purchase from. That is, if after being offered the same price p_h by two experts customers always chose to purchase from the first expert, or if they then all decided to purchase from the second expert then an expert's expected payoff from fraud in equilibrium would be $\frac{\frac{1}{2}F+\frac{1}{2}F}{1+F}c_h$. This is simply because the size of the pool of twice defrauded customers purchasing at p_h in equilibrium is unchanged, so all sellers will end up with the same equal share of it.

Together the three Lemmas above prove the proposition. To see this note that given Lemma 2, then the first lemma implies that $s_l = 1$, $p_l = V_l = 1$ and $p_h = c_h$. Given these prices, Lemma 3 states that $F = \frac{1}{c_h - 1}$. Recall that Assumption 3 is that $\frac{1}{c_h-1} < \frac{\alpha}{1-\alpha}\frac{V_h - c_h}{c_h - V_l}$. And Lemma 2 implied that the upper bound of Γ^* is F_h as $F_h \leq F_{2h}$ when $p_l = V_l$. Further F_h is at its lowest point when it intersects with the right hand side of the Γ region, and at that point $F = \frac{\alpha}{1-\alpha}\frac{V_h-c_h}{c_h-V_l}$. So $F = \frac{1}{c_h-1}$ is low enough to be in the Γ^* region. As that is the case then as long as k is low enough then in equilibrium $(F, k) \in \Gamma^*$. And in Γ^* then $s_h = 0$ and $b_{2h} = 1$. So experts police themselves equilibria as outlined in the proposition must exist. That proves the proposition. ■

Proof of Proposition 2

The proof of this proposition is in two steps. First I will show that Lemma 1 and 2 from Proposition 1 can be used to establish most of Proposition 2. I will then discuss the frequency of stopping, s_h, in equilibrium. Together that will establish the proposition.

To begin with notice that Lemma 1 continues to apply. That is, experts will only offer $p_l = V_l$ and as such $s_l = 1$ as customers will not search following the lowest possible price offer of $p_l = V_l$ for treatment. Experts will also only offer $p_h = c_h$ as in equilibrium customers only purchase treatment at p_h following two offers.

In equilibrium fraud is F_h, as defined in Lemma 2. That implies for a small enough k that fraud lies on upper bound of Γ^*. And as was discussed in Lemma 2, at $k = 0$ then $F_h = F_{2h}$ and as k increases then F_h falls to intersect the right hand side of Γ at $F = \frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l}$. And here as $\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-V_l} \leq F_h$ then $b_h = 0$ and as $F_h \leq F_{2h}$ then $b_{2h} = 1$. Further note that as Γ^* always exists then for for a small enough k that $(F_h, k) \in \Gamma^*$ also exists.

Lemma 4: If $F = F_h$ and $(F_h, k) \in \Gamma^*$ then $s_h = 1 - \frac{V_l}{F(c_h - V_l)}$.

By construction if $F = F_h$ and $(F_h, k) \in \Gamma^*$ then customers are indifferent between exiting and searching for a second opinion. That implies that customers may mix between the two strategies using any probability of stopping $s_h \in [0, 1]$ (and then exiting). In equilibrium they will mix between stopping (and then exiting) with the probability s_h that makes experts indifferent between honesty and committing fraud. This in turn implies that experts may mix, committing fraud with probability $F = F_h$.

Again normalise to 1 the number of new l types visiting an expert for their first diagnosis each period. That implies that an expert faces 1 unit of new l types and $F(1 - s_h)$ l types getting a second opinion, after having been defrauded by the first expert they visited and who then searched for a second opinion. So of the total number of l types he diagnoses each period, the fraction $\frac{1}{1+(1-s_h)F}$ are on their first visit and $\frac{(1-s_h)F}{1+(1-s_h)F}$ are getting a second opinion. If the expert decides to commit fraud, then of those on their first visit, $\frac{F(1-s_h)}{1+(1-s_h)F}$ will search for a second opinion which will turn out to also be fraudulent. Half of this number will purchase from the first expert they visited and half from the second. Similarly, by committing fraud half of the l types getting a second visit will end up buying from the expert, while the other half will return to their first expert and purchase. So the experts expected payoff from defrauding an l type customer is:

$$\frac{\frac{1}{2}F(1 - s_h) + \frac{1}{2}F(1 - s_h)}{1 + (1 - s_h)F} p_h$$

If instead the expert acts honestly and offers to treat an l type customer at $p_l = V_l$ then his expected payoff is simply V_l because all customers stop and purchase following a $p_l = V_l$ offer of treatment. So for an expert to be indifferent between honesty and fraud then:

$$\begin{aligned} &\tfrac{F(1-s_h)}{1+(1-s_h)F} p_h = V_l \\ \Rightarrow \quad &s_h = 1 - \tfrac{V_l}{F(p_h - V_l)} \end{aligned}$$

Noting that $V_l = 1$ and $p_h = c_h$ here, $s_h = 1 - \frac{1}{F(c_h-1)}$. Clearly $s_h < 1$. Note further that at the level of fraud where experts police themselves, $F = \frac{1}{c_h-1}$, then $s_h = 0$. As the lowest F_h is given by $F_h = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$ where F_h intersects the right hand side of the Γ region, then as by Assumption 3 $\frac{\alpha}{1-\alpha}(\frac{V_h-c_h}{c_h-V_l}) > \frac{1}{c_h-1}$ then $s_h > 0$ and lies between 0 and 1, so can be chosen in equilibrium. As a search or exit equilibrium with the features outlined in the proposition exists for a low enough k that $(F_h, k) \in \Gamma^*$ then that completes the proof of the proposition. ■

Proof of Proposition 3

This proposition is established in three steps. First Lemma 1 from Proposition 1 is used to establish that $p_l = V_l$ and $s_l = 1$ and p_h is discussed. Then the region Γ^M is defined by construction as that where customers will not choose to search following a price offer p_h, and are indifferent between purchasing and exit. Finally, b_h is found which makes experts indifferent between committing fraud and acting honestly. As in equilibrium experts are made indifferent between fraud and honesty they may choose $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$, and as customers are indifferent between purchasing and exit following a p_h offer they choose to purchase with probability b_h which establishes the proposition.

Begin by noticing that Lemma 1's analysis of p_l and s_l continues to apply, so $p_l = V_l$ and $s_l = 1$. But as in equilibrium here customers do not search following a price offer of p_h for treatment then it is not necessarily true that $p_h = c_h$. Indeed I am somewhat agnostic about equilibrium p_h here. If following Fong (2005) it is assumed that customers update their beliefs so that they remain indifferent between purchasing and exit for any offered p_h, then it can be shown (as Fong does in his paper) that profit maximising experts set $p_h = V_h$ (and in equilibrium there is no fraud). However other assumptions about beliefs may lead to other outcomes. For example if customers believe any deviation from an offer of the equilibrium price $p_h \in [c_h, V_h]$ to some other price in $[c_h, V_h]$ indicates that the diagnoser has definitely committed fraud, then they will not purchase. As such, a diagnoser will never deviate from offering the equilibrium p_h. So any price $p_h \in [c_h, V_h]$ can be set in equilibrium. Another possibility is that customers do not update their expectation of F given a non-equilibrium p_h offer. In that case an expert could undercut the prevailing equilibrium price by some small amount ϵ giving customers a strict incentive to purchase treatment at the offered price. By that assumption, only $p_h = c_h$ can be an equilibrium price. Because of all these possibilities, I present the equilibrium for any $p_h \in [c_h, V_h]$.

Lemma 5: A set Γ^M exists where for $(F, k) \in \Gamma^M$ then $F = \frac{\alpha}{1-\alpha}(\frac{V_h - p_h}{p_h - V_l})$ and k is great enough that $s_h = 1$.

The set Γ^M is defined as those pairs (F, k) where customers will not search for a second opinion following an offer of treatment at some $p_h \in [c_h, V_h]$ and where customers are indifferent between purchasing treatment at p_h and simply exiting.

To define Γ^M notice that if customers are indifferent between accepting treatment at some $p_h \in [c_h, V_h]$ and exiting after one price offer of p_h then their expected payoff from treatment at p_h must be equal to 0:

$$\begin{aligned} &\tfrac{\alpha(V_h - p_h) + (1-\alpha)F(V_l - p_h)}{\alpha + (1-\alpha)F} = 0 \\ \Rightarrow \qquad &F = \tfrac{\alpha}{1-\alpha}(\tfrac{V_h - p_h}{p_h - V_l}) \end{aligned}$$

This level of fraud is at its highest when $p_h = c_h$ and is an upper bound for F in Γ^M. Notice that the lower bound for F is 0, and occurs when $p_h = V_h$.

Secondly, in Γ^M customers don't search for a second opinion. So in the region Γ^M the expected payoff from search is 0 or less:

$$\frac{\alpha(V_h - p_h) + (1-\alpha)F^2(V_l - p_h) + (1-\alpha)F(1-F)(V_l - p_l)}{\alpha + (1-\alpha)F} - k \leq 0$$

$$\Rightarrow k \geq \frac{\alpha(V_h - p_h) + (1-\alpha)F^2(V_l - p_h) + (1-\alpha)F(1-F)(V_l - p_l)}{\alpha + (1-\alpha)F}$$

So given some equilibrium $p_h \in [c_h, V_h]$ that defines a unique $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$ (where customers are indifferent between purchasing and exiting) and that then defines a unique $k = \frac{\alpha(V_h-p_h)+(1-\alpha)F^2(V_l-p_h)+(1-\alpha)F(1-F)(V_l-p_l)}{\alpha+(1-\alpha)F}$ beyond which customers will not search for a second opinion. Taken together that F and k constitute the left hand side of Γ^M at p_h in $F-k$ space. So given p_h then Γ^M must exist for a great enough k. This region is shown in Figure 3.6.

Perhaps more intuitively, to find the lower bound for k in Γ^M then given some p_h observe where in $F-k$ space $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$ intersects the right hand side of the Γ region. Starting with $p_h = c_h$ then raise p_h towards V_h, and this intersection point will be at a lower F and the Γ region will expand to the right (as searching for a second opinion becomes more valuable to a customer). This process traces out the minimum level of k in Γ^M.

Lemma 6: If $(F, k) \in \Gamma^M$ then $b_h = \frac{V_l}{p_h}$.

In Γ^M then $s_h = 1$ so no one searches for a second opinion, and because $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$ customers are indifferent between choosing to purchase at p_h and exiting. As such, they may choose to purchase with some probability b_h. Recall also that $s_l = 1$ and all customers purchase at $p_l = V_l$ if it is offered to them. That implies an expert is indifferent between committing fraud and honesty if:

$$b_h p_h = V_l$$
$$\Rightarrow \quad b_h = \frac{V_l}{p_h}$$

As $V_l = 1$ and $p_h \in [c_h, V_h]$ then $b_h \in [\frac{1}{V_h}, \frac{1}{c_h}]$. So the purchase rate $b_h \in (0, 1)$ can always be chosen. Assuming that customers set $b_h = \frac{1}{p_h}$ then experts are indifferent between fraud and honesty, so they may set $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-V_l})$ in turn making consumers indifferent between purchasing and exit. That completes the proof of the proposition.
■

Proof of Proposition 4

This proposition is established in two steps. First I will use Lemma 1 and 2 to outline price, search and purchase behaviour. In particular that customers mix between searching and stopping. Then I will demonstrate that if customers search with the appropriate probability that experts are indifferent between honesty and fraud in equilibrium.

First notice that for a low enough k then if $F = F_{1,2}$ that $(F, k) \in \Gamma$ by the definition of Γ. And that as $F \leq \frac{1}{c_h - 1}$ then as $\frac{1}{c_h-1} \leq \frac{\alpha}{1-\alpha}(\frac{V_h - c_h}{c_h - V_l})$ by Assumption 3 that $(F, k) \in \Gamma^*$ as well. As $F = F_{1,2}$ this implies that customers are indifferent between stopping and searching after a single p_h offer, so any s_h may be chosen in equilibrium. And as $F \leq \frac{1}{c_h-1} \leq \frac{\alpha}{1-\alpha}(\frac{V_h - c_h}{c_h - V_l})$ then customers will prefer to purchase rather than exit, after one or two p_h offers. So $b_h = b_{2h} = 1$.

Using Lemma 1, note that again $p_l = V_l$ and $s_l = 1$. In a similar way to Lemma 1, $p_h = c_h$ here too. In equilibrium, customers offered the equilibrium p_h are indifferent between staying and purchasing and searching for a second opinion. If an expert deviated and offered a p_h above the equilibrium price, then the customer would strictly prefer to search, and receive a lower price offer from another seller (or he is on his second opinion and already has a lower price offer). So sellers cannot gain by deviating above the equilibrium p_h. A price $p_h > c_h$ also cannot be an equilibrium price as in equilibrium an expert expects to sell to all of his customers offered the p_h price who do not search and to some fraction of those customers who do search (as other experts are expected to offer the same price) so if $p_h > c_h$ an expert could deviate downwards by some small amount ϵ and sell to that fraction of customers who would buy from another expert if only offered p_h. As ϵ can be very small, this must be profitable, so experts would undercut any equilibrium price $p_h > c_h$. As such, the only equilibrium price is once again $p_h = c_h$.

Lemma 7: If $F = F_{1,2} \leq \frac{1}{c_h - 1}$ then $s_h = \frac{1+F(1-c_h)}{c_h+F(1-c_h)}$.

In equilibrium, an expert expects that $\frac{1}{1+(1-s_h)F}$ of the l type agents asking him for a diagnoses have just entered the game, and $\frac{(1-s_h)F}{1+(1-s_h)F}$ are on their second opinion having previously been offered p_h. Of the l types on their first diagnosis, $1 - s_h$ will search again, and with probability F be defrauded. Only half of this number will return to purchase from the first expert. And of the l types on their second opinion, only half will stay following a second p_h offer. So the expected payoff from offering to treat an l type at p_h is given by $\frac{s_h+\frac{1}{2}(1-s_h)F+\frac{1}{2}(1-s_h)F}{1+(1-s_h)F} p_h$. By offering to treat an l type at $p_l = V_l$ an expert knows in that the customer will accept and that his payoff will be $V_l = 1$. So in equilibrium, in order for an expert to be indifferent between committing fraud and acting honestly then it must be that:

$$\frac{s_h+(1-s_h)F}{1+(1-s_h)F} p_h = 1$$

$$\Rightarrow \quad s_h = \frac{1+F(1-p_h)}{p_h+F(1-p_h)}$$

Recall that $p_h = c_h$ so $s_h = \frac{1+F(1-c_h)}{c_h+F(1-c_h)}$. Notice that at $F = 0$ then $s_h = \frac{1}{c_h}$ and s_h falls as F increases until $s_h = 0$ at $F = \frac{1}{c_h-1}$ which was the level of fraud found in Proposition 1 where customers always searched following a p_h offer. So for $F \leq \frac{1}{c_h-1}$ then $s_h \in [0, \frac{1}{c_h}]$ and such a probability of stopping can be chosen.

As this level of search makes experts indifferent between truth and fraud in equilibrium, they may choose a probability of committing fraud $F = F_{1,2} \leq \frac{1}{c_h-1}$ that in turn makes customers indifferent between stopping and purchasing and searching for a second opinion. That completes the proposition. ■

Proof of Proposition 5

This proof is given generally for any $p_h \in [c_h, V_h]$ and as such these results apply to the quick quotes model where $p_h = c_h$ as well as the slow quotes model for the special case of $s_l = p_l = 1$. Further note that here I will denote the equilibria using the following shorthand:

- experts police themselves (EP)
- search or exit (SE)
- monopolistic (M)
- sometimes search, always purchase (SP)

Outline of the proof First, I will show that welfare in the SP equilibrium is always greater than welfare in EP, SE and M. But as the SP equilibrium lies on the right hand side of Γ then SP must cease to exist at some maximum k, which I will denote k^{SP}. Notice that the SE equilibrium cannot exist beyond this point. So for $k > k^{SP}$ then only the M or SE equilibrium may exist and either may welfare dominate. In the quick quotes model, where $p_h = c_h$, it is shown that the M equilibrium welfare dominates the SE equilibrium if and only if $\alpha(V_h - c_h)(\frac{c_h - 1}{V_h}) > 1 - \alpha$. Finally the proof notes that as SE lies inside Γ it too cannot extend beyond some maximum k denoted k^{SE} so for a large enough k the only equilibrium that continues to exist is the M equilibrium. As such, for a great enough k it must welfare dominate.

SP welfare dominates EP The EP equilibrium produces lower welfare than the SP equilibrium. To see this, notice that in EP welfare loss is given by the likelihood of an agent being offered p_h, multiplied by the cost of search k (as agents always search after a p_h offer) and that there is no welfare loss following a p_l offer as customers stop and purchase. The average welfare loss is therefore:

$$(\alpha + (1 - \alpha)F)k$$

Similarly, in the SP equilibrium the welfare loss is given by the probability of an agent being offered treatment at p_h, multiplied by their probability of search $1 - s_h$ and the cost of search k (recall in this equilibrium customers only sometimes search for a second opinion):

$$(1 - s_h)(\alpha + (1 - \alpha)F)k$$

If it can be shown that both equilibria exist at the same k and that F is lower in SP than in EP, then it must be that SP generates greater welfare. Recall that the EP equilibrium exists inside the region Γ (where customers always seek a second opinion) while the SP equilibrium lies on the right hand side of the region Γ (where customers are indifferent between purchasing and seeking a second opinion). As Γ always exists and its right hand side extends up from $F = 0$ to $F = 1$ (both at $k = 0$) then for any k where EP exists inside Γ there must be a SP equilibrium with the same k but a lower F (graphically in $F - k$ space, this equilibrium lies directly below the EP equilibrium). And for any EP equilibrium such an SP equilibrium must exist as SP equilibria exist for F up to $F = \frac{1}{p_h - 1}$ while EP equilibria exist where $F = \frac{1}{p_h - 1}$. So wherever an EP equilibrium exists, there exists a SP equilibrium with a lower F.

As $s_h \leq 1$ and F is always the same or lower, then the welfare loss in SP must be less than in EP (and the same at the point $F = \frac{1}{p_h - 1}$ where $s_h = 0$ as the two equilibria are identical at that point). So welfare in the SP equilibrium is always as great or greater than welfare in the EP equilibrium. That implies that SP welfare dominates EP for $k \in [0, k^{SP}]$.

SP welfare dominates SE It can be shown that for any k where they both exist that the SE equilibrium also has lower welfare than the EP equilibrium. That further implies that the SE equilibrium has lower welfare than the SP equilibrium.

In the SE equilibrium, fraud is F_h, customers offered treatment at p_h incur the search cost k with probability $1 - s_h$ and exit with probability s_h. As a customer is a h type with probability α and an l type with probability $1 - \alpha$ then the expected welfare loss of an agent here is given by:

$$\alpha(s_h(V_h - c_h) + (1 - s_h)k) + (1 - \alpha)F_h(s_h + (1 - s_h)k)$$

Notice that if a h type stops and exits the welfare loss is $V_h - c_h$, while if a defrauded l type stops and exits the welfare loss is $V_l - c_l = 1$ as $V_l = 1$ and $c_l = 0$ by assumption. Also that the welfare cost is k if either type decides to search. Rearranging the average welfare loss in SE:

$$s_h(\alpha(V_h - c_h) + (1-\alpha)F_hV_l) + (1-s_h)(\alpha + (1-\alpha)F_h)k$$

Replacing c_h with p_h (but keeping s_h fixed) this welfare loss is (weakly) greater than

$$s_h(\alpha(V_h - p_h) + (1-\alpha)F_hV_l) + (1-s_h)(\alpha + (1-\alpha)F_h)k$$

Notice that in SE that customers are indifferent between searching for a second opinion and exiting. That implies the following relationship between the variables:

$$\frac{\alpha(V_h-p_h)+(1-\alpha)F_h^2(V_l-p_h)+(1-\alpha)(1-F_h)F_h(V_l-p_l)}{\alpha+(1-\alpha)F_h} - k = 0$$

$$\Rightarrow \quad \alpha(V_h - p_h) + (1-\alpha)F_hV_l - (\alpha + (1-\alpha)F_h)k = (1-\alpha)F_h^2p_h + (1-\alpha)(F_h - F_h^2)p_l$$

As the right hand side of this expression is non-negative with $(1-\alpha)F_h^2p_h + (1-\alpha)(F_h - F_h^2)p_l \geq 0$ and as both $\alpha(V_h - p_h) + (1-\alpha)F_hV_l$ and $(\alpha + (1-\alpha)F_h)k$ are non-negative, then it must be that:

$$\alpha(V_h - p_h) + (1-\alpha)F_hV_l \geq (\alpha + (1-\alpha)F_h)k$$

So a convex combination of $\alpha(V_h - p_h) + (1-\alpha)F_hV_l$ and $(\alpha + (1-\alpha)F_h)k$ must be greater than $(\alpha + (1-\alpha)F_h)k$ at F_h. That is:

$$s_h(\alpha(V_h - p_h) + (1-\alpha)F_hV_l) + (1-s_h)(\alpha + (1-\alpha)F_h)k \geq (\alpha + (1-\alpha)F_h)k$$

Now recall that the welfare loss in the EP equilibrium is $(\alpha + (1-\alpha)F)k$. That implies that for the same k the welfare loss in SE is greater than in EP as $F_h > F_{EP} = \frac{1}{p_h}$ so $(\alpha+(1-\alpha)F_h)k > (\alpha+(1-\alpha)F_{EP})k$. So EP must welfare dominate SE. SP then welfare dominates SE by transitivity as SP dominates EP which dominates SE. To see this directly, note that SP lies on the right hand side of Γ and that SE lies inside

Γ. So at some k where both equilibria exist, then the SP equilibrium must lie directly below the SE equilibrium in $F-k$ space and $(\alpha+(1-\alpha)F_h)k > (\alpha+(1-\alpha)F_{SP})k$. Altogether that implies that SP welfare dominates SE for $k \in [0, k^{SP}]$.

SP welfare dominates M The final equilibrium to consider is the monopolistic one (M). In that equilibrium the fraction α of high type customers are offered treatment at p_h and exit with probability $1-b_h$ for a welfare loss of $V_h - c_h$. And the further fraction of $(1-\alpha)F$ of low type customers are also fraudulently offered treatment at p_h and exit with probability $1-b_h$ for a welfare loss of V_l. So the welfare loss in the M equilibrium is given by:

$$(\alpha(V_h - c_h) + (1-\alpha)FV_l)(1-b_h)$$

Noting that $b_h = \frac{1}{p_h}$ and $F = \frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1}$ then this welfare loss is smallest when $p_h = V_h$. In that case it becomes:

$$\alpha(V_h - c_h)\frac{V_h-1}{V_h}$$

It can be shown that SP always welfare dominates M as it produces a lower welfare loss than this. To see this, recall that the welfare loss in SP is $(\alpha+(1-\alpha)F)k(1-s_h)$. And a customer is always indifferent between stopping (and purchasing) and searching (and purchasing) in that equilibrium, so:

$$\begin{aligned} & \tfrac{\alpha(V_h-p_h)+(1-\alpha)F^2(V_l-p_h)+(1-\alpha)(F-F^2)(V_l-p_l)}{\alpha+(1-\alpha)F} - k = \tfrac{\alpha(V_h-p_h)+(1-\alpha)F(V_l-p_h)}{\alpha+(1-\alpha)F} \\ \Rightarrow \quad & (\alpha+(1-\alpha)F)k = (1-\alpha)(F-F^2)(p_h-p_l) \end{aligned}$$

So the welfare loss in SP can be written $(1-\alpha)(F-F^2)(p_h-p_l)(1-s_h)$. Substituting in $s_h = \frac{1+F(1-p_h)}{p_h+F(1-p_h)}$ then:

$$(1-\alpha)(F-F^2)(p_h-p_l)(\tfrac{p_h-1}{p_h+F(1-p_h)})$$

Noting that $F(p_h - p_l) \leq \frac{1}{p_h - 1}(p_h - p_l) = 1$ then this loss must be less than $(1-\alpha)(1-F)(\frac{(p_h-1)}{p_h+F(1-p_h)})$. Now notice that $(1-\alpha)(\frac{(1-F)(p_h-1)}{p_h+F(1-p_h)})$ is positive and declining in F. As it is decreasing in F then $(1-\alpha)(\frac{(1-F)(p_h-1)}{p_h+F(1-p_h)})$ must be less than $(1-\alpha)(\frac{p_h-1}{p_h})$ where $F=0$. By Assumption 3 this is then less than $\alpha(V_h - c_h)(\frac{p_h-1}{p_h})$. And as $V_h \geq p_h$ then this is less than $\alpha(V_h - c_h)(\frac{V_h-1}{V_h})$, the welfare loss in the monopolistic equilibrium. So where it exists SP produces a lower welfare loss, and thus always welfare dominates M for $k \in [0, k^{SP}]$.

SE and M equilibria Recall that the SP equilibrium lies on the right hand side of Γ and note that this equilibrium may only exist up to an $F = \frac{1}{p_h-1}$ as at this level of fraud all customers search following a p_h offer and $s_h = 0$. There are two possible scenarios. The first is that the SP equilibrium exists for every k where the SE equilibrium exists, in which case it welfare dominates up to k^{SP} and for k greater than this amount the only remaining equilibrium, the monopolistic one, welfare dominates by default. Alternatively, the SE equilibrium may exist for k greater than k^{SP}, in which case it may (or may not) welfare dominate M before it too ceases to exist at k^{SE}. We know that these equilibria only exist up to a certain k as they all lie in the set Γ which only contains k up to k^*.

There is no convenient expression to welfare rank SE and M in general beyond stating that M welfare dominates SE if:

$$s_h(\alpha(V_h - c_h) + (1-\alpha)F_h V_l) + (1-s_h)(\alpha + (1-\alpha)F_h)k > \alpha(V_h - c_h)\frac{V_h - 1}{V_h}$$

This expression can be evaluated and the equilibria welfare ranked on a case by case basis, but little more can be said in general. In the special case of the quick quotes model with $p_h = c_h$ however we can derive a simpler expression. Recall that in SE a customer is indifferent between exiting for a 0 payoff and searching for a second opinion (and then purchasing). So:

$$\frac{\alpha(V_h-p_h)+(1-\alpha)F_h^2(V_l-p_h)+(1-\alpha)(1-F_h)F_h(V_l-p_l)}{\alpha+(1-\alpha)F_h}-k=0$$

$$\Rightarrow \quad (\alpha+(1-\alpha)F_h)k=\alpha(V_h-p_h)+(1-\alpha)F_h^2(V_l-p_h)+(1-\alpha)(1-F_h)F_h(V_l-p_l)$$

Recall the welfare loss in SE is:

$$s_h(\alpha(V_h-c_h)+(1-\alpha)F_hV_l)+(1-s_h)(\alpha+(1-\alpha)F_h)k$$

Substituting out $(\alpha+(1-\alpha)F_h)k$ and setting $p_h=c_h$ then this becomes:

$$\begin{aligned}\Rightarrow \quad & \alpha(V_h-c_h)+s_h(1-\alpha)F_h+(1-s_h)(1-\alpha)F_h^2(V_l-c_h)\\ \Rightarrow \quad & \alpha(V_h-c_h)+(1-\alpha)(s_hF_h-(1-s_h)F_h^2(c_h-1))\end{aligned}$$

Substituting in $s_h=1-\frac{1}{F(c_h-1)}$.

$$\begin{aligned}\Rightarrow \quad & \alpha(V_h-c_h)+(1-\alpha)(F_h-\tfrac{1}{(c_h-1)}-F_h)\\ \Rightarrow \quad & \alpha(V_h-c_h)-\tfrac{1-\alpha}{c_h-1}\end{aligned}$$

This is the welfare loss in any SE equilibrium in the quick quotes model. So SE creates a lower welfare loss then M in that model if and only if:

$$\begin{aligned} & \alpha(V_h-c_h)-\tfrac{1-\alpha}{c_h-1}<\alpha(V_h-c_h)\tfrac{V_h-1}{V_h}\\ \Rightarrow \quad & \alpha(V_h-c_h)(\tfrac{c_h-1}{V_h})<1-\alpha\end{aligned}$$

This rule is surprisingly simple but only applies for the quick quotes model as it makes use of the fact that $p_h=c_h$. So if $p_h=c_h$ then if $k^{SP}<k^{SE}$ and if $\alpha(V_h-c_h)(\frac{c_h-1}{V_h})<1-\alpha$ then SE welfare dominates for $k\in[k^{SP},k^{SE}]$, otherwise M will welfare dominate in this range.

As both SE and SP lie in the Γ region (which only exists up to some k^*), then beyond some point only the M equilibrium continues to exist. Clearly, if it is the only candidate equilibrium remaining then the M equilibrium also welfare dominates. Putting all of the above together, then clearly as stated in proposition 5 then for $k \leq k^{SP}$ the sometimes search, always purchase equilibrium welfare dominates. If $k^{SE} > k^{SP}$ then if $\alpha(V_h - c_h)(\frac{c_h - 1}{V_h}) < 1 - \alpha$ SE welfare dominates for $k \in [k^{SP}, k^{SE}]$, otherwise M welfare dominates in that range. For $k \geq \max[k^{SP}, k^{SE}]$ then only M exists and therefore welfare dominates. That completes the proof. ■

Proof of Proposition 6:

There are two major differences between this equilibrium and its analogue in the quick quotes model. Here there are multiple equilibria in s_l implying p_l may now follow a distribution, and the level of fraud depends directly on s_l. Immediately below I start by discussing which out of Lemma's 1, 2 and 3 from Proposition 1 continue to apply. Then in three small lemmas I discuss first the equilibrium distribution of p_l, denoted $G(p_l)$, given some s_l. Then the level of fraud in equilibrium F. And finally equilibrium s_l. The discussion of s_l is quite involved as it can be shown that there are either three equilibria in s_l, a high, low and no search equilibrium, or if k is high there is only the no search equilibrium.

Begin by observing that Lemma 1 from the above model no longer applies here. In that model a customer offered treatment was offered a price upfront, but here a customer offered treatment is either offered a c_l or c_h type treatment with an appropriate price $p_l \in [c_l, V_l]$ or $p_h \in [c_h, V_h]$ to follow later. So customers now condition their search behaviour on the type of price, but not the price itself. A further major difference is that here a price $p_h > c_h$ may occur in any of the equilibria because customers hold 'fearful' off equilibrium path beliefs, and refuse to purchase at any p_h other than the expected one. As $p_h \geq c_h > V_l$ then customers offered p_h

and who believe that they have been defrauded for sure will never accept that price, so sellers will never deviate from offering the equilibrium p_h.

In Lemma 2 the region Γ and Γ^* were defined, and it was shown that they exist for any $p_l \in [c_l, V_l]$ or $p_h \in [c_h, V_h]$. This lemma continues to apply to the slow quotes model. The reason for this is that when a customer is advised that they need the c_h treatment, they make the decision whether to search or not based on the expected p_h and p_l offered in equilibrium. So in the derivation of Γ^* in Lemma 2 now, p_h and p_l are interpreted expected prices, following a c_h offer. Once customers stop searching, the actual price offers are revealed to them. With this in mind Lemma 2 continues to apply unchanged and again there exists a set Γ^* where for $(F, k) \in \Gamma^*$ then $b_{2h} = 1$ and $s_h = 0$.

Lemma 3 no longer applies. The level of fraud in equilibrium will be shown in Lemma 9 below to be $F = \frac{s_l}{p_h - 2 + s_l}$ now. This accommodates the other potential equilibria in s_l and in a sense generalises the earlier lemma.

Lemma 8 (Price Dispersion in p_l): Given $s_l \in (0,1)$ then p_l is drawn from a distribution with cumulative density $G(p_l) = 1 - \frac{(s_l + (2-s_l)F)(\frac{1}{p_l} - 1)}{2(1-s_l)(1-F)}$ with support $[\frac{s_l + (2-s_l)F}{s_l + (2-s_l)F + 2(1-s_l)(1-F)}, V_l]$. If $s_l = 1$ then $p_l = V_l$, and if $s_l = 0$ then $p_l = 0$.

Begin by noting that as in the previous model, if no one searches and $s_l = 1$ then all experts must offer $p_l = V_l$ and that constitutes an equilibrium. Also if $s_l = 0$ then as $F = 0$ in this equilibrium, firms Bertrand compete for l type customers and $p_l = 0$. The rest of this proof deals with $s_l \in (0, 1)$.

Here I will focus on symmetric equilibria, where all sellers set prices in the same way. Assuming $s_l \in (0,1)$ first observe that p_l cannot be set at a fixed price in equilibrium. The reason for this is that as $s_l \in (0,1)$ then some fraction of customers get a second opinion following a c_l offer of treatment, and this fraction of customer will split evenly between two sellers who both offer p_l. That implies that a seller could lower his price by ϵ and sell to all of this group of customers. And if ϵ is small enough this must be a profitable deviation. At $p_l = c_l$ there cannot be a downward deviation, but as sellers make no profit off any sale at that price, that also cannot be an equilibrium. Now sellers could raise their price to $p_l = V_l$ and make a positive profit off that group of customers who don't get a second opinion. So there cannot be a symmetric fixed price equilibrium in this model. Sellers can however offer p_l by drawing from the same distribution in equilibrium, this is discussed below.

In equilibrium with $s_l \in (0,1)$ three types of l customers visit an expert to ask for a treatment price. Normalising the number of l types entering the game each period per seller as 1, of the l types getting a diagnoses from a seller, the fraction $\frac{1}{1+F+(1-s_l)(1-F)}$ are new l types on their first opinion. The fraction $\frac{F}{1+F+(1-s_l)(1-F)}$ are previously defrauded l types now on their second opinion. And the fraction $\frac{(1-s_l)(1-F)}{1+F+(1-s_l)(1-F)}$ are l types on their second opinion following an honest offer of the c_l treatment by their first seller.

If for $s_l \in (0,1)$ sellers choose their price from some distribution with cumulative density $G(p_l)$ then they must be indifferent between choosing any price in support of that distribution. Note that at the highest price in the support sellers only sell to those customers who don't get a second opinion or who's second opinion is c_h, and as this group of customers would all purchase at any $p_l \in [c_l, V_l]$ then sellers will set V_l as the highest price in the support of $G(p_l)$. And at $p_l = V_l$ a seller expects a payoff of $\frac{s_l+(1-s_l)F+F}{1+F+(1-s_l)(1-F)}V_l$. The s_l in the numerator denotes those customers who don't get a second opinion, while the $(1-s_l)F+F$ denotes those who do get a second opinion but it turns out to be an offer of c_h.

A seller will offer a price $p_l < V_l$ only if he expects to get the same payoff as offering $p_l = V_l$. Again he will sell to the fraction $\frac{s_l+F+(1-s_l)F}{1+F+(1-s_l)(1-F)}$ of the l types found above, regardless of the price he sets. Now assuming other sellers also choose p_l following some distribution $G(p_l)$ then if a customer receives two offers of l treatment the probability of the second seller offering a lower price than the first is given by $1-G(p_l)$. So the additional fraction of customers the expert expects to win with a price $p_l < V_l$ is given by $\frac{2(1-s_l)(1-F)(1-G(p_l))}{1+F+(1-s_l)(1-F)}$. Note that the 2 is there because half this is the fraction of customers currently on their first opinion (but will get a second honest opinion) and the other half is getting a second opinion (after an honest first opinion).

On the support of $G(p_l)$ an expert must be indifferent to which p_l is chosen, so the expected payoffs from offering $p_l = V_l$ or $p_l < V_l$ must be the same. That is:

$$p_l(\tfrac{s_l+F+(1-s_l)F+2(1-s_l)(1-F)(1-G(p_l))}{1+F+(1-s_l)(1-F)}) = \tfrac{s_l+F+(1-s_l)F}{1+F+(1-s_l)(1-F)}V_l$$

Implicitly this defines our price distribution $G(p_l)$ as by rearranging this equation:

$$G(p_l) = 1 - \frac{(s_l + (2 - s_l)F)(\frac{1}{p_l} - 1)}{2(1 - s_l)(1 - F)}$$

Finally, to find the lowest price in support of this distribution, denoted $\underline{p}_l$, note that by definition $G(\underline{p}_l) = 0$ so:

$$\begin{aligned} & \underline{p}_l(\tfrac{s_l+F+(1-s_l)F+2(1-s_l)(1-F)}{1+F+(1-s_l)(1-F)}) = \tfrac{s_l+F+(1-s_l)F}{1+F+(1-s_l)(1-F)}V_l \\ \Rightarrow \quad & \underline{p}_l = \tfrac{s_l+(2-s_l)F}{s_l+(2-s_l)F+2(1-s_l)(1-F)} \end{aligned}$$

Notice that no seller would deviate to a price below $\underline{p}_l$ as they could sell to just as many customers at $p_l = \underline{p}_l$ and make a greater profit. That completes the derivation of the distribution $G(p_l)$.

So if all sellers choose p_l using $G(p_l)$ then all other sellers are indifferent among choosing prices $p_l \in [\underline{p}_l, V_l]$, so no seller has any incentive to deviate away from setting prices using $G(p_l)$. That completes the proof of Lemma 8.

Lemma 9 (Fraud): Given $s_l \in [0, 1]$ then $F = \frac{s_l}{p_h-2+s_l}$.

Given some s_l in equilibrium experts must be made indifferent between committing fraud or not. Recall to that $s_h = 0$ here, so customers offered c_h always get a second opinion. By offering c_h to an l type, all of the $\frac{1}{1+F+(1-s_l)(1-F)}$ new l types in the game will search, and of these, a fraction F will again be defrauded by the second expert they visit. As customers will split evenly between two sellers both offering the same p_h, a seller can expect to make a profit $\frac{\frac{1}{2}F}{1+F+(1-s_l)(1-F)}p_h$ off these new l types. Similarly, a seller will face a fraction $\frac{F}{1+F+(1-s_l)(1-F)}$ of l types who are currently on their second opinion, after having previously been offered c_h. By offering c_h then half of these will stay and purchase so a seller expects to make $\frac{\frac{1}{2}F}{1+F+(1-s_l)(1-F)}p_h$ from these l types. Taken together, by offering p_h a customer expects a profit of $\frac{F}{1+F+(1-s_l)(1-F)}p_h$.

From Lemma 8, we know that a seller who offers some p_l expects a profit of $\frac{s_l+(1-s_l)F+F}{1+F+(1-s_l)(1-F)}V_l$. For the seller to be indifferent between offering some p_l and p_h then:

$$\begin{aligned} & \frac{F}{1+F+(1-s_l)(1-F)}p_h = \frac{s_l+(1-s_l)F+F}{1+F+(1-s_l)(1-F)}V_l \\ \Rightarrow \quad & F = \frac{s_l}{p_h-2+s_l} \end{aligned}$$

So in equilibrium for all experts to be indifferent between truth and fraud towards the l types, it must be that $F = \frac{s_l}{p_h-2+s_l}$, and as sellers are indifferent between honesty and fraud given that F from other sellers, no individual seller has an incentive to deviate. That establishes the lemma.

Lemma 10: (Search Equilibria for s_l): There always exists a no search equilibrium with $s_l = 1$, and if k is low there exist one low and one high search equilibria with $s_l \in (0, 1)$.

An equilibrium with no search following a c_l offer and $s_l = 1$ always exists. To see this note that experts in such an equilibrium must set $p_l = V_l$. As such, for any cost of search $k \geq 0$ a customer receiving an l offer will not benefit from searching as at best they will receive another c_l offer at a price of $p_l = V_l$, so they will not search and $s_l = 1$. Notice that this is simply an application of Diamond's paradox and is equivalent to Theorem 1 in Burdett and Judd (1983).

I will now show that (exactly) two other potential equilibria in s_l exist if k is low enough. Notice that if offered a c_l treatment, customers know they are of the

l type, so their only incentive to search again comes from the increased probability of a low price, where p_l is draw from a distribution with the cdf $G(p_l)$. First note that a customer who stops after receiving one offer of treatment at c_l expects a price denoted $E(p_l)$ given by:

$$E(p_l) = \int_{\underline{p}_l}^{1} p_l g(p_l) dp_l$$
$$\Rightarrow \quad E(p_l) = \frac{s_l+(2-s_l)F}{2(1-s_l)(1-F)}(-\ln[\frac{s_l+(2-s_l)F}{s_l+(2-s_l)F+2(1-s_l)(1-F)}])$$

If a customer searches and receives a second l offer, then the customer expects that the lowest of the two p_l prices offered will be the expected value of the lowest order statistic, when drawing twice from the distribution $G(p_l)$ given above (see Casella and Berger (2001) for the generic derivation of the distribution of the the lowest of two draws from some distribution $G(x)$). The expected value of the lowest of two draws from $G(p_l)$ is denoted $E(p_l^{low})$ and:

$$E(p_l^{low}) = 2\int_{\underline{p}_l}^{1} p_l(1-G(p_l))g(p_l) dp_l$$
$$\Rightarrow \quad E(p_l^{low}) = \frac{(s_l+(2-s_l)F)^2}{2(1-s_l)^2(1-F)^2}(\frac{2(1-s_l)(1-F)}{s_l+(2-s_l)F} + \ln[\frac{s_l+(2-s_l)F}{s_l+(2-s_l)F+2(1-s_l)(1-F)}])$$

For s_l to be an equilibrium, a customer must be indifferent between searching and not following a c_l offer, here customers know that after their first diagnosis that they are an l type, but only with probability $1-F$ will the second expert they visit report l truthfully. In equilibrium then a customer will be indifferent between searching and not if:

$$[1-F][E(p_l) - E(p_l^{low})] = k$$

The benefit of search $[1-F][E(p_l) - E(p_l^{low})]$ is plotted against the cost of search k in Figure 3.9 and looks like a 'hill', starting at 0 for $s_l = 0$ rising towards a maximum, and then falling back towards 0 at $s_l = 1$. If it can be shown that $[1-F][E(p_l) - E(p_l^{low})]$ always rises monotonically from 0 at $s_l = 0$ to some maxima before falling again to 0 at $s_l = 1$ then there must be exactly two equilibria in s_l with $s_l \in (0,1)$ if k is low enough. Below I will go to some lengths to show that generically the benefit of search does always resemble a 'hill' of this nature.

The search benefit 'hill'

Using $F = \frac{s_l}{p_h-2+s_l}$ and $E(p_l)$ and $E(p_l^{low})$ from above, $[1-F][E(p_l)-E(p_l^{low})] = k$ can be stated as a function of only s_l, k and p_h:

$$-[\frac{p_h-2}{p_h-2+s_l}][\frac{s_lp_h}{2(1-s_l)(p_h-2)}\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})]$$
$$-[\frac{p_h-2}{p_h-2+s_l}][\frac{s_lp_h}{(1-s_l)(p_h-2)}+\frac{(s_lp_h)^2}{2(1-s_l)^2(p_h-2)^2}\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})] = k$$

In a way analogous to Burdett and Judd's (1983) Claim 3 I will now establish that the left hand-side of the above equation, which I will denote $V(s_l)$, limits to 0 as $s_l \to 0^+$ and $s_l \to 1^-$. That is, the value of search approaches 0 as the fraction searching approaches 1 and 0 respectively. Further it will be shown that for some point $s_l^* \in (0,1)$ then $V(s_l^*)$ is a maxima, such that for $s_l < s_l^*$ the function is increasing and for $s_l > s_l^*$ the function is decreasing. This establishes that the benefit of search resembles a 'hill' in turn implying that the cost of search k either cuts the hill twice so there are exactly two equilibria with $s_l \in (0,1)$, or k is above the hill's maxima, in which case the low and high search equilibria do not exist.

First I will show that the $\lim_{s_l\to 0^+} V(s_l) = 0$. To see this re-write $V(s_l)$ as:

$$-[\frac{p_h-2}{p_h-2+s_l}]\times$$
$$[\frac{s_lp_h}{(1-s_l)(p_h-2)}+\frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)(p_h-2)}{s_lp_h}}+\frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)^2(p_h-2)^2}{(s_lp_h)^2}}]$$

Clearly as $s_l \to 0^+$ the term in the first square brackets $\frac{p_h-2}{p_h-2+s_l} \to 1$. The first term in the second square brackets $\frac{s_lp_h}{(1-s_l)(p_h-2)} \to 0$, but for the next two terms it is not so clear, as $\frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)(p_h-2)}{s_lp_h}} \to \frac{-\infty}{\infty}$ and $\frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)^2(p_h-2)^2}{(s_lp_h)^2}} \to \frac{-\infty}{\infty}$.

To address this first apply L'Hopital's Rule to the first of the undefined terms:

$$\begin{aligned}
& \lim_{s_l\to 0^+} \frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)(p_h-2)}{s_lp_h}} \\
= & \lim_{s_l\to 0^+} \frac{\ln(s_lp_h)-\ln(s_lp_h+2(1-s_l)(p_h-2))}{\frac{2(p_h-2)}{p_h}\frac{(1-s_l)}{s_l}} \\
= & \lim_{s_l\to 0^+} \frac{\frac{1}{s_l}-\frac{4-p_h}{s_lp_h+2(1-s_l)(p_h-2)}}{-\frac{2(p_h-2)}{p_h}\frac{1}{s_l^2}} \qquad \text{by L'Hopital's} \\
= & \lim_{s_l\to 0^+} \frac{p_h}{2(p_h-2)}(\frac{s_l^2(4-p_h)}{s_lp_h+2(1-s_l)(p_h-2)} - s_l) = 0
\end{aligned}$$

Similarly to address the second of these undefined terms:

$$\begin{aligned}
& \lim_{s_l\to 0^+} \frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)^2(p_h-2)^2}{(s_lp_h)^2}} \\
= & \lim_{s_l\to 0^+} \frac{\ln(s_lp_h)-\ln(s_lp_h+2(1-s_l)(p_h-2))}{\frac{2(p_h-2)^2}{p_h^2}(\frac{1-s_l}{s_l})^2} \\
= & \lim_{s_l\to 0^+} \frac{\frac{1}{s_l}-\frac{4-p_h}{s_lp_h+2(1-s_l)(p_h-2)}}{-\frac{2(p_h-2)}{p_h}\frac{2(1-s_l)}{s_l^3}} \qquad \text{by L'Hopital's} \\
= & \lim_{s_l\to 0^+} \frac{p_h}{4(p_h-2)}(\frac{s_l^3(4-p_h)}{s_l(1-s_l)p_h+2(1-s_l)^2(p_h-2)} - \frac{s_l^2}{1-s_l}) = 0
\end{aligned}$$

So all three of the terms in the second square brackets tend toward 0 as $s_l \to 0^+$. So $\lim_{s_l\to 0^+} V(s_l) = -1(0+0+0) = 0$, which is what had to be shown.

Next I will show that the $\lim_{s_l\to 1^-} V(s_l) = 0$. Again note that $V(s_l)$ can be written as:

$$-[\frac{p_h-2}{p_h-2+s_l}] \times$$
$$[\frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)(p_h-2)}{s_lp_h}} + \frac{s_lp_h}{(1-s_l)(p_h-2)} + \frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)^2(p_h-2)^2}{(s_lp_h)^2}}]$$

Clearly the term in the first square brackets can be evaluated as: $\lim_{s_l\to 1^-} \frac{p_h-2}{c_h-2+s_l} = \frac{p_h-2}{p_h-1}$. The first term in the second square brackets however is undefined as:

$$\lim_{s_l\to 1^-} \frac{\ln(\frac{s_lp_h}{s_lp_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)(p_h-2)}{s_lp_h}} \to \frac{0}{0}$$

Again L'Hopital's rule can be applied, however before doing so it is useful to define $X = \frac{s_lp_h}{(1-s_l)(p_h-2)}$. The limit of the first term can then be re-written as:

$$\lim_{X\to\infty} \frac{\ln(\frac{X}{2+X})}{\frac{2}{X}}$$

$$\begin{aligned}
&= \lim_{X\to\infty} \frac{\ln(X)-\ln(2+X)}{\frac{2}{X}} \\
&= \lim_{X\to\infty} \frac{\frac{1}{X}-\frac{1}{2+X}}{-\frac{2}{X^2}} \qquad \text{by L'Hopital's} \\
&= \lim_{X\to\infty} \frac{X^2}{2(2+X)} - \frac{X^2}{2X} \\
&= \lim_{X\to\infty} \frac{-X^2}{2X+X^2} \\
&= \lim_{X\to\infty} \frac{-1}{\frac{2}{X}+1} = -1
\end{aligned}$$

Now note that the final two terms of $V(s_l)$ in the square brackets can be re-written jointly as:

$$\frac{\frac{2(1-s_l)(p_h-2)}{s_l p_h} + \ln(\frac{s_l p_h}{s_l p_h + 2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)^2(p_h-2)^2}{(s_l p_h)^2}}$$

So the limit of the second and third terms in the square brackets is also undefined as $\lim_{s_l\to 1^-} \frac{\frac{2(1-s_l)(p_h-2)}{s_l p_h}+\ln(\frac{s_l p_h}{s_l p_h+2(1-s_l)(p_h-2)})}{\frac{2(1-s_l)^2(p_h-2)^2}{(s_l p_h)^2}} \to \frac{0}{0}$. Again using $X = \frac{s_l p_h}{(1-s_l)(p_h-2)}$ this limit can be re-written as:

$$\begin{aligned}
& \lim_{X\to\infty} \frac{\frac{2}{X}+\ln(\frac{X}{2+X})}{\frac{2}{X^2}} \\
&= \lim_{X\to\infty} \frac{\frac{2}{X}+\ln(X)-\ln(2+X)}{\frac{2}{X^2}} \\
&= \lim_{X\to\infty} \frac{\frac{-2}{X^2}+\frac{1}{X}-\frac{1}{2+X}}{\frac{-4}{X^3}} \qquad \text{by L'Hopitals} \\
&= \lim_{X\to\infty} \frac{1}{-4}(-2X + X^2 - \frac{X^3}{2+X}) \\
&= \lim_{X\to\infty} \frac{1}{\frac{2}{X}+1} = 1
\end{aligned}$$

It can now be stated then that $\lim_{s_l\to 0^+} V(s_l) = -\frac{p_h-2}{p_h-1}(-1+1) = 0$, which is what had to be shown. So the function $V(s_l)$ limits to 0 as s_l approaches both 0 and 1.

Next I will demonstrate that for some unique $s_l^* \in (0,1)$ that $V(s_l^*)$ is a maxima, $V'(s_l) > 0$ for $s_l < s_l^*$ and $V'(s_l) < 0$ for $s_l > s_l^*$. Again, it will be useful to define $X = \frac{s_l p_h}{(1-s_l)(p_h-2)}$, and note that as $s_l \in [0,1]$ then $X \in [0,\infty)$. Re-writing $V(s_l)$ in terms of X:

$$V(X(s_l)) = [\frac{p_h + X(p_h-2)}{p_h + X(p_h-1)}][-X - \frac{X}{2}\ln(\frac{X}{2+X}) - \frac{X^2}{2}\ln(\frac{X}{2+X})]$$

Note that as $X = \frac{s_l p_h}{(1-s_l)(p_h-2)}$ then as X is monotonically increasing from 0 towards ∞ as s_l increases from 0 to 1, it is sufficient to show that the first order condition of the above equation with respect to X attains a unique maximum for some $X^* \in [0, \infty)$.

Differentiating V with respect to X yields, after rearranging and simplifying:

$$V'(X) = -\frac{2(3p_h+2(p_h-1)X)(p_h(1+X)^2-X(3+2X))+(2+X)((2X^2+p_h^2(1+X)^2)(1+2X)-2p_hX(1+X)(2+3X))\ln(\frac{X}{2+X})}{2(2+X)(p_h+(p_h-1)X)^2}$$

Evaluating $V'(X)$ as $X \to 0^+$, then

$$\begin{aligned} &\lim_{X\to 0^+} V'(X) \\ = \quad & -\frac{6p_h^2+2p_h^2 \times \lim_{X\to 0^+} \ln(\frac{X}{2+X})}{4p_h^2} = \infty \end{aligned}$$

It is difficult to evaluate the limit of $V'(X)$ as $X \to \infty$,[8] but as $\lim_{X\to 0^+} V'(X) > 0$ we know that the function is initially increasing, so if it can be shown that $V'(X) = 0$ at only a single point X^*, then as $\lim_{X\to\infty} V(X) = 0$, then it must be that $\lim_{X\to\infty} V'(X) < 0$. It is shown below that there is indeed only one critical point X^* where $V'(X^*) = 0$.

To evaluate the shape of V further, note that it is extremely complicated to evaluate the second derivative of the function. Instead, it can be shown that this function only meets the first order condition at a single point, which is enough to show that there exists a single critical point, and that as the function initially increases from 0 before tending back to 0 as X grows very large, then that critical point must be a maxima. Setting the first order derivative above to 0 and rearranging:

$$\ln(1+\tfrac{2}{X}) + \frac{-2(3p_h+2(p_h-1)X)(p_h(1+X)^2-X(3+2X))}{(2+X)[(2X^2+p_h^2(1+X)^2)(1+2X)-2p_hX(1+X)(2+3X)]} = 0 \tag{3.3}$$

Clearly $\ln(1+\frac{2}{X})$ is positive and monotonically decreasing for $X \in [0, \infty)$. Next I will show that the second term in this equation is negative and monotonically increasing. To show that it is negative, note that for $p_h > 2$ the numerator must

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[8] At least 21 moderately complex terms would have to be evaluated using L'Hopital's rule multiple times for each.

be negative as $p_h(1+X)^2 - X(3+2X) = p_h + (2p_h - 3)X + (p_h - 2)X^2$. The denominator is also positive as the term in square brackets can be re-written as $2X^2 + p_h^2 + (4p_h^2 - 4p_h)X + (5p_h^2 - 10p_h)X^2 + (2p_h^2 + 4 - 6p_h)X^3$. Again, as $p_h > 2$, this must be positive. So the second term in Equation 3.3 is negative. To show that this term is monotonically increasing, note that the derivative is given by:

$$\frac{2((4X^4+p_h^4(1+X)^4)(11+4X(3+X))-4p_hX^2(-1+X(39+2X(38+X(25+6X)))))}{(2+X)^2(2X^2+p_h^2(1+X)^2(1+2X)-2p_hX(1+X)(2+3X))^2}$$
$$\frac{-2(2p_h^3(1+X)(1+X(1+X(39+2X(38+X(25+6X)))))+2p_h^2X^2(90+X(262+X(281+2X(68+13X)))))}{(2+X)^2(2X^2+p_h^2(1+X)^2(1+2X)-2p_hX(1+X)(2+3X))^2}$$

First note that the denominator must be positive as it consists of two squared terms. The numerator can be re-written with some effort as:

$$\begin{gathered}
2 \times [\\
(-2p_h^3 + 11p_h^4) + (-80p_h^3 + 56p_h^4)X \\
+(4p_h + 180p_h^2 - 308p_h^3 + 118p_h^4)X^2 \\
+(-156p_h + 524p_h^2 - 482p_h^3 + 132p_h^4)X^3 \\
+(44 - 304p_h + 562p_h^2 - 376p_h^3 + 83p_h^4)X^4 \\
+(48 - 200p_h + 272p_h^2 - 148p_h^3 + 28p_h^4)X^5 \\
+(16 - 48p_h + 52p_h^2 - 24p_h^3 + 4p_h^4)X^6]
\end{gathered}$$

As $p_h \geq c_h$ and $c_h > 2$ by Assumption 4, both the term $(-2p_h^3 + 11p_h^4)$ and each of the coefficients on the X terms must be positive. At $p_h = 2$ they can be shown to be 0 or greater, and it is then sufficient to note that each coefficient is increasing in p_h, for $p_h > 2$. As this is rather tedious, I won't go through it here, however this shows that the numerator is also positive. So the first term in Equation 3.3 is positive and monotonically decreasing, while the second term is negative and monotonically increasing. Therefore there can only be one point at which the first order condition is fulfilled, X^*. For $X < X^*$ then $V'(X)$ must be positive and for $X > X^*$ then $V'(X)$ must be negative. As that is true for some $X^* \in [0, \infty)$ it must also be true for some $s_l^* \in [0,1]$ at which $V'(s_l^*) = 0$ while for $s_l < s_l^*$ the function must be increasing and for $s_l > s_l^*$ the function must be decreasing.

The above has established that the search benefit 'hill' starts at 0 at $s_l = 0$ rises monotonically to a maxima then falls monotonically to 0 at $s_l = 1$. So if k is small enough then the function $V(s_l)$ will intersect k exactly twice, at some $s_{1,2} \in (0, 1)$. Note too that as k rises the intersection points $s_{1,2} \to s_l^*$ and there will be only one intersection point at $k = V(s_l^*)$. For $k > V(s_l^*)$ no equilibrium with customers searching following an l offer can exist, as the benefit from search is always less than the cost of search. That completes the proof of the Lemma.

Taken together the lemmas prove that p_l is drawn from a distribution with cdf $G(p_l)$ if $s_l \in (0, 1)$ or $p_l = V_l$ if $s_l = 1$. Further that $F = \frac{s_l}{p_h - 2 + s_l}$ in equilibrium and that s_l may be $s_l = 1$ in a no search equilibrium, $s_l = s_1$ in a low search equilibrium or $s_l = s_2$ in a high search equilibrium. The final point to note is that by assuming $\frac{1}{p_h - 1} < \frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - 1}$ then for a low enough k, $(F, k) \in \Gamma^*$, as the upper bound of Γ^* is weakly greater than $\frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - 1}$, so this equilibrium exists for $(F, k) \in \Gamma^*$. Taken together with the discussion at the start of the proposition, that completes the proof. ■

Proof of Proposition 7

Again note that Lemma 1 from the above model no longer applies as customers make the decision to search based on expected p_h and p_l not the actual price offered. Lemma 2 defined the region Γ and Γ^* and and again here $(F_h, k) \in \Gamma^*$. But now note that this equilibrium may not exist for very low values of k if $F_h > F_{2h}$. That occurs if the expected p_l offer in equilibrium is below 1 for low values of k, but as k rises then F_h must fall below F_{2h} and keep falling until F_h intersects the right hand side of the Γ region at $F = \frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - 1}$. As $F_h \geq \frac{\alpha}{1-\alpha}\frac{V_h - p_h}{p_h - 1}$, this equilibrium must exist for $F_h \leq F_{2h}$ where $(F_h, k) \in \Gamma^*$. Here customers do always purchase at p_h following two c_h offers but they refuse to purchase at p_h following only one offer. Further note that by definition for $F = F_h$ then customers are indifferent between exiting for a payoff of 0 and searching for a second opinion, after their first offer of c_h.

The next three lemmas discuss the equilibrium p_l distribution, s_h and s_l. They differ from those in the previous proposition as s_h does not necessarily equal 0 here, and the search benefit 'hill' from a second search following a c_l offer has a somewhat different shape.

Lemma 11 (Price Dispersion in p_l): if $s_l \in [0,1)$ then p_l is drawn from a distribution with cumulative density $G(p_l) = 1 - \frac{(s_l+(2-s_l-s_h)F)(\frac{1}{p_l}-1)}{2(1-s_l)(1-F)}$ with support on $[\frac{s_l+(2-s_l-s_h)F}{s_l+(2-s_l-s_h)F+2(1-s_l)(1-F)}, V_l]$. If $s_l = 1$ then $p_l = V_l$.

This Lemma is identical to Lemma 8, except that here $\underline{p}_l$ and $G(p_l)$ now reflect that s_h is not necessarily equal to 0. First though note that as before if customers do not search after a c_l offer then $s_l = 1$, $p_l = V_l$ and no expert would deviate from that price. Further, note that for the reasons given in Lemma 8 that in a symmetric equilibrium no fixed price p_l can be offered by all sellers.

Turning to the equilibrium distribution of p_l, again normalise the number of new customers with the l type problem entering each period per expert to 1, so now the fraction $\frac{1}{(1+(1-s_h)F+(1-s_l)(1-F))}$ of l types are on their first visit. The fraction $\frac{(1-s_h)F}{(1+(1-s_h)F+(1-s_l)(1-F))}$ are on their second opinion having previously been offered c_h, and the fraction $\frac{(1-s_l)(1-F)}{(1+(1-s_h)F+(1-s_l)(1-F))}$ of the l types are on their second visit, having been offered c_l previously. Given some s_l then p_l prices are set following some distribution with a cdf $G(p_l)$ so an expert must again be indifferent between recommending a price of V_l and a lower price on the support of $G(p_l)$ where $p_l < V_l$. That is:

$$p_l(\frac{s_l+(1-s_h)F+(1-s_l)F+2(1-s_l)(1-F)(1-G(p_l))}{1+(1-s_h)F+(1-s_l)(1-F)}) = \frac{s_l+(1-s_h)F+(1-s_l)F}{1+(1-s_h)F+(1-s_l)(1-F)}V_l$$

Rearranging this to find $G(p_l)$ yields:

$$G(p_l) = 1 - \frac{(s_l + (2 - s_l - s_h)F)(\frac{1}{p_l} - 1)}{2(1 - s_l)(1 - F)}$$

To find $\underline{p}_l$ again set $G(\underline{p}_l) = 0$ in the above equation for:

$$\underline{p}_l = \frac{s_l + (2 - s_l - s_h)F}{s_l + (2 - s_l - s_h)F + 2(1 - s_l)(1 - F)}$$

So in equilibrium here, p_l is drawn from a distribution with cdf $G(p_l) = 1 - \frac{(s_l+(2-s_l-s_h)F)(\frac{1}{p_l}-1)}{2(1-s_l)(1-F)}$ which has support on $[\frac{s_l+(2-s_l-s_h)F}{s_l+(2-s_l-s_h)F+2(1-s_l)(1-F)}, V_l]$. As all sellers choose p_l prices in this way, sellers will not deviate from doing so. That completes the Lemma.

Lemma 12 (Search s_h): Following a c_h offer customers do not search with probability $s_h = 1 - \frac{s_l+(1-s_l)F}{F(p_h-1)}$.

As fraud is $F = F_h$ then customers are indifferent between exiting for a payoff of 0 and searching (and then purchasing as $F_h \leq F_{2h}$). To also ensure sellers are indifferent between offering a p_l price and p_h then customers must choose to stop and exit with the right probability s_h. To find that s_h note that as shown above, if the fraction $\frac{1}{(1+(1-s_h)F+(1-s_l)(1-F))}$ of l agents are on their first visit then there are also $\frac{(1-s_h)F}{(1+(1-s_h)F+(1-s_l)(1-F))}$ that are visiting their second expert having been offered c_h previously and then having chosen to search, and there are $\frac{(1-s_l)(1-F)}{(1+(1-s_h)F+(1-s_l)(1-F))}$ of the l types on their second visit, having been offered c_l previously. By offering c_h then of the customers on their first visit, a fraction $1 - s_h$ will search again and F of them will be defrauded again. So the expert will end up with half of that pool of agents. An expert committing fraud will also end up with half of the previously defrauded l types. By offering c_h however an expert loses all those l type customers that get a second offer of p_l. So the experts expected payoff from defrauding an l type customer is:

$$\frac{1 \times (1 - s_h)F\frac{1}{2} + (1 - s_h)F\frac{1}{2}}{1 + (1 - s_h)F + (1 - s_l)(1 - F)} p_h$$

Again note that in equilibrium this payoff would be the same regardless of how customers choose which of the two experts to purchase from, as the number of twice defrauded types per expert is unchanged.

If the seller offers a price p_l, then his expected payoff is $\frac{s_l+(1-s_h)F+(1-s_l)F}{1+(1-s_h)F+(1-s_l)(1-F)}V_l$. To be indifferent between offering p_h and p_l then:

$$\begin{array}{rc} & \frac{s_l+(1-s_h)F+(1-s_l)F}{1+(1-s_h)F+(1-s_l)(1-F)}V_l = \frac{(1-s_h)F\frac{1}{2}+(1-s_h)F\frac{1}{2}}{1+(1-s_h)F+(1-s_l)(1-F)}p_h \\ \Rightarrow & s_h = 1 - \frac{s_l+(1-s_l)F}{F(p_h-1)} \end{array}$$

Clearly $s_h < 1$. Note further that at the level of fraud where experts police themselves, $F = \frac{s_l}{p_h-2+s_l}$, then $s_h = 0$. So as $F_h \geq \frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1}$ and $\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1} > \frac{1}{p_h-1}$ by assumption, then $s_h \in (0,1)$ so can be chosen. As customers are indifferent between stopping and exiting and searching, they are assumed to stop with probability s_h in equilibrium. That completes the lemma.

Lemma 13 (Search s_l): There always exists a no search equilibrium with $s_l = 1$, and if k is low there may also exist other equilibria with $s_l \in [0,1]$.

This lemma differs from Lemma 10 because here I cannot guarantee the search benefit 'hill' is in fact a 'hill'. That is because F does not necessarily fall to 0 as s_l falls to 0 here. Again though, begin by noting that if agents never search following an l offer then all experts will set $p_l = V_l$, and at that price a customer has no incentive to search as search is costly and would only lead to a second price offer of $p_l = V_l$. So $p_l = V_l$ and $s_l = 1$ is an equilibrium and always exists.

Of more interest is the equilibrium where customers sometimes do search following a p_l offer. Begin by noting that the expected price, given one draw from the distribution $G(p_l)$ is:

$$\begin{array}{rc} & E(p_l) = \int_{\underline{p}_l}^{1} p_l g(p_l) dp_l \\ \Rightarrow & E(p_l) = \frac{s_l+(2-s_h-s_l)F}{2(1-s_l)(1-F)}(-\ln[\frac{s_l+(2-s_h-s_l)F}{s_l+(2-s_h-s_l)F+2(1-s_l)(1-F)}]) \end{array}$$

The expected value of the lowest p_l when the customer draws twice from $G(p_l)$ is:

$$E(p_l^{low}) = 2\int_{\underline{p}_l}^{1} p_l(1-G(p_l))g(p_l)dp_l$$
$$\Rightarrow \quad E(p_l^{low}) = \tfrac{(s_l+(2-s_h-s_l)F)^2}{2(1-s_l)^2(1-F)^2}(\tfrac{2(1-s_l)(1-F)}{s_l+(2-s_h-s_l)F} + \ln[\tfrac{s_l+(2-s_h-s_l)F}{s_l+(2-s_h-s_l)F+2(1-s_l)(1-F)}])$$

In equilibrium if a customer is indifferent between searching and not then:

$$[1-F][E(p_l)-E(p_l^{low})] = k$$

If $s_h = 0$ then this is exactly the same expression as in the experts police themselves scenario. However in general s_h may be more than 0, and more importantly F is now set in a different way. This means that the benefit of search (the 'hill') may not look like a hill anymore. Below I will show that although as s_l tends towards one then the search benefit tends towards 0, for s_l at 0 the search benefit may be greater than 0.

The first thing to show is that as $s_l \to 1$, then $V(s_l) = [1-F][E(p_l)-E(p_l^{low})] \to 0$. To see this denote $X = \frac{s_l+(2-s_h-s_l)F}{2(1-s_l)(1-F)}$ in order to simplify things. Note that $\lim_{s_l\to 1^-} E(p_l) = \lim_{X\to\infty} \frac{-\ln[\frac{1}{1+\frac{1}{X}}]}{\frac{1}{X}}$ is undefined. So using L'Hopitals rule:

$$\begin{aligned}\lim_{s_l\to 1^-} E(p_l) = & \quad \lim_{X\to\infty} \frac{-\ln[\frac{1}{1+\frac{1}{X}}]}{\frac{1}{X}} \\ = & \quad \lim_{X\to\infty} \frac{\frac{1}{1+X}-\frac{1}{X}}{\frac{-1}{X^2}} \quad \text{by L'Hopital's} \\ = & \quad \lim_{X\to\infty} \frac{1}{1+\frac{1}{X}} = 1\end{aligned}$$

Further note that $\lim_{s_l\to 1^-} E(p_l^{low}) = \lim_{X\to\infty} \frac{2(\frac{1}{X}+\ln[\frac{1}{1+\frac{1}{X}}])}{\frac{1}{X^2}}$ is undefined. Using L'Hopital's Rule:

$$\begin{aligned}\lim_{s_l\to 1^-} E(p_l^{low}) = & \quad \lim_{X\to\infty} \frac{2(\frac{1}{X}+\ln[\frac{1}{1+\frac{1}{X}}])}{\frac{1}{X^2}} \\ = & \quad \lim_{X\to\infty} \frac{2(\frac{-1}{X^2}+\frac{1}{X}-\frac{1}{1+X})}{\frac{-2}{X^3}} \quad \text{by L'Hopital's} \\ = & \quad \lim_{X\to\infty} X - X^2 + \frac{X^3}{1+X} \\ = & \quad \lim_{X\to\infty} \frac{1}{1+\frac{1}{X}} = 1\end{aligned}$$

So the $\lim_{s_l \to 1^-}[1-F][E(p_l) - E(p_l^{low})] = [1-F](1-1) = 0$. This is similar to the case where experts police themselves in that the value of searching limits to 0 if nobody searches. Intuitively, this is because at $s_l = 1$ then $p_l = V_l$ and there is the no search equilibrium where the benefit of search is 0 because of the lack of price dispersion in p_l.

Unlike the experts police case, as $s_l \to 0^+$ then $V(s_l)$ does not necessarily go to 0. The reason is that F is no longer $F = \frac{s_l}{p_h - 2 + s_l}$ in equilibrium. To see what $V(s_l)$ does limit to as $s_l \to 0^+$ then (treating F as a constant) note that $\lim_{s_l \to 0^+} E(p_l) = \frac{(2-s_h)F}{2(1-F)}(-\ln[\frac{(2-s_h)F}{(2-s_h)F+2(1-F)}])$. While $\lim_{s_l \to 0^+} E(p_l^{low}) = 2(\frac{(2-s_h)F}{2(1-F)})^2(\frac{2(1-F)}{(2-s_h)F} + \ln[\frac{(2-s_h)F}{(2-s_h)F+2(1-F)}])$. So then:

$$\lim_{s_l \to 0^+} V(s_l) = [1-F][-\frac{(2-s_h)F}{2(1-F)} \ln[\frac{(2-s_h)F}{(2-s_h)F+2(1-F)}] - 2(\frac{(2-s_h)F}{2(1-F)})^2(\frac{2(1-F)}{(2-s_h)F} + \ln[\frac{(2-s_h)F}{(2-s_h)F+2(1-F)}])]$$

It is possible to show that this limit becomes 0 as F tends towards either 0 or 1, and that for $F \in (0,1)$ that it is positive. However I will leave out those details, as it does not add much to the discussion, although they are happily available upon request. Because the left-hand end of the hill does not limit to 0, and in general the benefit from search given any s_h must be non-negative (as a second opinion might improve upon the first opinion, but the first opinion is still available) there is also little point in trying to count the number of equilibria in s_l. It may be that the benefit hill starts at $V(s_l) > k$ at $s_l = 0$ then falls to $V(s_l) = 0$ at $s_l = 1$ so there is an equilibrium with $s_l = 0$ as the benefit of search is greater than the cost, and also equilibria at any points where $V(s_l) = k$, and again one at $s_l = 1$ as discussed in the introduction. But this is not the only shape that the hill could now take on, so it is best to simply argue that there could, depending on the parameters, be multiple equilibria with $s_l \in [0,1]$ if k is low enough and we know that there is always one equilibrium at $s_l = 1$. As I have shown that there may be multiple equilibria in s_l, with $s_l \in [0,1]$ that completes the proof of the lemma.

So it has been shown above that p_l is drawn from a distribution with cdf $G(p_l)$ above if $s_l \in [0,1)$ and $p_l = V_l = 1$ if $s_l = 1$. And that $s_l = 1$ is always an equilibrium, while depending on k and other parameters there may be other equilibria with $s_l \in [0,1]$. Further $s_h = 1 - \frac{s_l+(1-s_l)F}{F(p_h-1)}$ in equilibrium. As $p_h \in [c_h, V_h]$ due to customers fearful beliefs and as $F = F_h \leq F_{2h}$ then $b_h = 0$ and $b_{2h} = 1$. Noting that for some k that $(F_h, k) \in \Gamma^*$ exists, then that completes the proof of the proposition. ■

Proof of Proposition 8

Begin by noting a set Γ^M again exists for a great enough k as shown in Lemma 5, but where now p_l and p_h are interpreted as expected prices when a customer is deciding whether to search or not. So for $(F,k) \in \Gamma^M$ customers have no incentive to search for a second opinion following a treatment offer c_h, and $s_h = 1$ by construction. Further, in equilibrium as $F = \frac{\alpha}{1-\alpha}(\frac{V_h-p_h}{p_h-1})$ which implies that when they do observe the price offered to them customers are indifferent between exiting and purchasing treatment at p_h. Lemma's 11 and 13 apply here with $s_h = 1$, so again there may be multiple equilibria in s_l. In particular $s_l = 1$ always exists as an equilibrium and if $s_l < 1$ in equilibrium then p_l is drawn using the cdf $G(p_l)$ given in Lemma 11. Further note that any $p_h \in [c_h, V_h]$ may be the equilibrium p_h here.

What remains to be shown in this equilibrium is that customers purchase probability once they have observed the p_h offered to them, denoted b_h is given by $b_h = (s_l + (1-s_l)F)\frac{V_l}{p_h}$. To see this, note that in Γ^M as $s_h = 1$, then if an expert encounters an l type customer he is either on his first diagnosis or is on his second having been offered c_l on his first opinion and then having chosen to get a second opinion. Normalising the number of new l type customers per seller to 1, then of the l types visiting the seller, a fraction $\frac{1}{1+(1-s_l)(1-F)}$ are getting a first opinion, and a fraction $\frac{(1-s_l)(1-F)}{1+(1-s_l)(1-F)}$ are getting a second opinion after an initial c_l offer. Recalling from Lemma 11 that a sellers expected payoff from offering a p_l is $\frac{s_l+(1-s_l)F}{1+(1-s_l)(1-F)}V_l$ when $s_h = 1$ then for an expert to be indifferent between offering a fraudulent p_h and a truthful p_l:

$$\begin{aligned} b_h \times \tfrac{1}{1+(1-s_l)(1-F)} p_h &= \tfrac{s_l+(1-s_l)F}{1+(1-s_l)(1-F)} V_l \\ \Rightarrow \qquad b_h &= (s_l+(1-s_l)F)\tfrac{V_l}{p_h} \end{aligned}$$

As $V_l = 1$, $p_h \in [c_h, V_h]$ and $s_l + (1-s_l)F \in [0,1]$ then $b_h \in [0, \frac{1}{p_h}]$, where $\frac{1}{p_h} < 1$. So this acceptance rate b_h exists. Customers are indifferent between accepting and rejecting the c_h offer at p_h in Γ^M and are assumed to choose this b_h. That completes the proof of the proposition. ■

Proof of Proposition 9

Again note that Γ^* exists as defined in Lemma 2, but here again we interpret p_h and p_l as the prices a customer expects from a c_h and c_l offer when he decides whether to search or not. For $F = F_{1,2} \leq \frac{1}{p_h-1}$ then a customer offered the c_h treatment expects the price to be p_h and is indifferent between stopping and purchasing and searching for a second opinion and as $F \leq \frac{1}{p_h-1}$ then customers will purchase after they stop searching, whether they have received one or two offers of treatment at p_h as $\frac{1}{p_h-1} \leq \frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1}$ by assumption. So $b_h = b_{2h} = 1$. As was shown in Lemma's 11 and 13, there may also multiple equilibria for s_l and equilibrium may consist of some $s_l \in [0,1]$ if k is low enough. As was further shown in that proof p_l is drawn from a distribution in equilibrium with cdf $G(p_l)$. Again note the special case of $s_l = 1$ and $p_l = 1$ always exists. Further note that customers fearful beliefs imply that p_h may be any $p_h \in [c_h, V_h]$ in equilibrium.

What must be shown here is that customers stop with probability $s_h = \frac{s_l+(1-s_l)F+F(1-p_h)}{p_h+F(1-p_h)}$. To see this, and again normalising the number of new l types per seller each period to 1, then a seller expects that a fraction $\frac{1}{1+(1-s_h)F+(1-s_l)(1-F)}$ of the l type are getting their first opinion, and the fraction $\frac{(1-s_h)F}{1+(1-s_h)F+(1-s_l)(1-F)}$ are getting a second opinion having been previously offered c_h, also the fraction $\frac{(1-s_l)(1-F)}{1+(1-s_h)F+(1-s_l)(1-F)}$ are getting a second opinion having been previously offered c_l. The expected payoff from offering an l type p_h is therefore:

$$\frac{s_h + (1 - s_h)F}{1 + (1 - s_h)F + (1 - s_l)(1 - F)} p_h$$

In equilibrium this expected payoff must be equal to the expected payoff to a seller from offering a p_l price. This payoff was found in Lemma 11 as $\frac{(1-s_h)F+s_l+(1-s_l)F}{1+(1-s_h)F+(1-s_l)(1-F)}V_l$. So for a seller to be indifferent between the two:

$$\begin{array}{rc} & \frac{s_h+(1-s_h)F}{1+(1-s_h)F+(1-s_l)(1-F)}p_h = \frac{(1-s_h)F+s_l+(1-s_l)F}{1+(1-s_h)F+(1-s_l)(1-F)}V_l \\ \Rightarrow & s_h = \frac{s_l+(1-s_l)F+F(1-p_h)}{p_h+F(1-p_h)} \end{array}$$

Notice that at $F = 0$ then $s_h = \frac{s_l}{p_h} \geq 0$ and s_h falls as F increases until $s_h = 0$ at $F = \frac{s_l}{p_h-2+s_l}$. That is at $F = \frac{s_l}{p_h-2+s_l}$ all agents offered c_h search again, and this is simply the experts police themselves equilibrium. We know that customers can choose to stop with probability s_h here as $s_h \in [0, 1]$ and $F = F_{1,2}$ so they are indifferent between stopping and searching. Further we know that this equilibrium exists as $\frac{s_l}{p_h-2+s_l} \leq \frac{1}{p_h-1}$ which is less than $\frac{\alpha}{1-\alpha}\frac{V_h-p_h}{p_h-1}$ by assumption. So for k small enough this equilibrium must exist with $(F, k) \in \Gamma^*$. That completes the proof of the proposition. ■

Chapter 4

Ongoing Partnerships, Clients and Effort

Why do ongoing partnerships exist? What problem do they solve? This chapter presents a theory of ongoing partnerships where partnerships exist to solve a commitment problem for clients, creating incentives for juniors to exert unobservable effort. Because clients care about the age of agents working for them, partnerships also commit to hiring juniors by billing by the hour and then hiring cheap juniors to work those hours. As partners make a margin on each junior hired, this turns making partner into a prize for successful juniors in an up-or-out style career structure. And where ability matters partnerships only hire the most able juniors, raising profits per partner further. If valuable clients are scarce, these features emerge naturally from the partnership's role as a gatekeeper to partnership with limited competition. However if juniors are scarce, then partnerships may bid up junior wages. Partnerships with high effort and up-or-out careers for juniors may then cease to exist, replaced by low effort ordinary companies with agents paid their marginal product. Perhaps surprisingly, this can be avoided if juniors ignore high wage offers in a folk theorem which obtains if a deviation causes play to revert to a grim competitive equilibrium in the future. Even so, as the number of firms competing for juniors increases it becomes harder to sustain the continued existence of ongoing partnerships.

4.1 Introduction

Partnerships are in many ways unusual firms. Senior employees (the partners) own the firm while junior employees, despite their often considerable talents, usually work in a relatively low-pay, high-effort environment. Over time, occasionally one of those juniors becomes a partner and the firm is handed down from generation to generation. These *unusual* intergenerational transactions are fairly standard in partnerships but are not observed in ordinary companies where employees are paid their marginal product and senior employees are not given ownership of the firm.

In this paper I argue that agents have two simple choices to make in their careers. First, do they join a partnership (with its unusual features), or instead do they join an *ordinary* company (where wages are set at marginal product). Second, having worked as a junior in a partnership do they stay on and make partner, or instead do they leave. The catch being that only agents who have worked as juniors in a partnership can go on to make partner, juniors at ordinary companies must remain employees. Initially two results emerge. One, juniors are willing to accept wages below the value of their marginal product if they will make significantly more than that amount as an owner-partner. Two, for a partnership in some industry to offer such a career tradeoff to juniors, it must be more productive than an ordinary company in that industry.

Because partnerships are prevalent in what are referred to as the 'professional services industries,' where the key input into production is (often unobservable) effort, I argue that partnerships in these industries exist because the chance to make partner significantly increases the level of effort exerted by junior employees. As such, partnerships dominate these industries because after matching other incentives offered by competitors, they create further incentives for effort (and boost output) by offering their employees the prize of the company itself.

Ordinary companies cannot match these incentives. And juniors will join partnerships instead, even if there exists a 'rat-race' to make partner in an up-or-out career structure with low junior wages, for the prize of partnership if they succeed. It's not by accident that Clifford Chance prominently mentions the possibility of making partner on their recruitment page.[1] The implicit bargain in such firms is that juniors work hard to make the current partners wealthy, and in turn earn the chance to be the next generation of partners.

This paper makes use of an overlapping generations framework to emphasise the intergenerational aspects of partnerships. It begins by using a simplified version of the model to show that an ongoing partnership can only induce young agents to join it, for low junior wages now and high profits per partner in the future, if that partnership is somehow more productive than an ordinary company which pays all agents their marginal product. It then asks why partnerships might be more productive than ordinary companies. As is discussed below, Morrison and Wilhelm (2004) argue that partnerships give old agents incentives to exert effort and mentor young agents who without mentoring would not buy them out from the partnership when they retire. In a different paper, Heski Bar-Isaac (2007) argues that because partnerships require both old and young agents of high ability to exert effort to produce output, that by hiring a young agent, partners re-introduce reputational uncertainty into their considerations, and exert effort in order to produce output and then sell the firm to their juniors. These contributions explain how partnerships can create additional output by getting old agents to exert unobservable effort, and are complimentary to the one I provide.

I argue partnerships exist because they create incentives for juniors to exert effort (ignoring the partners possible role in exerting additional effort to help ensure that a junior can buy out his partnership stake). And partnerships charge by the hour, giving them an incentive to maximise their billable hours by hiring juniors. But for partnerships to exist with these features, partners must earn more than their marginal product by paying less for juniors than the revenue they bring in for the firm. That may occur naturally if partnerships with valuable clients are scarce so they can make

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[1] It states: "You are our next generation of associates - and partners. We want the best people to join us, and make every effort to offer excellent training and career opportunities in return." www.cliffordchance.com

a take it or leave it offer to juniors. But with multiple partnerships competing to hire the same juniors they may compete up junior wages, eliminating the high profits per partner that previously motivated juniors to work hard. This leads to an inefficient outcome where firms resemble ordinary companies paying employees their marginal product and no effort being exerted. In order to avoid this, it can be shown that a folk theorem obtains where juniors avoid accepting deviating wage offers in order to maintain high profits per partner, if the discount factor is high enough.

Juniors may also have different abilities. And a single partnership not facing competition will hire fewer, but on average higher ability types as the discount factor increases. This occurs because higher types are willing to accept a more extreme trade-off, with lower wages as juniors, because they have a better chance of making partner. So hiring lower ability types implies lifting the junior wage by relatively more when the discount factor is low. It can further be shown though, that partnerships hiring more than one type of junior do not generate as great a total profit as they would if split up into multiple firms each hiring a different type of junior. This equilibrium with different partnerships hiring different types of juniors again obtains using a folk theorem if the discount factor is high enough. Unsurprisingly, the minimum discount factor for the efficient equilibrium in both folk theorems rises in the number of partnerships competing for juniors.

4.1.1 Related Literature

Some recent contributions have attempted to explain the unusual features of partnerships by emphasising the intergenerational aspect of ongoing partnerships using an overlapping generations framework. The first of these by Morrison and Wilhelm (2004) shows that partnerships can create incentives for partners to mentor their juniors, as only skilled juniors will want to buy existing partners out. Because partnerships “deal primarily in [the] human-capital intensive production of experience goods” (Morrison and Wilhelm, 2004, pp. 1683) unskilled agents know that as partners they will end up disappointing clients, ruining the firm’s reputation and destroying the value of their partnership stake. As such, an unmentored (and unskilled) junior will not be interested in buying out an existing partner's stake in the firm. By placing mentoring at the core of their paper, the important contribution of Morrison and Wilhelm is to explain the existence of partnerships in industries such as "law, management consulting, investment banking, and even accounting" which "remain arts in spite of the best efforts of professional schools to codify practice." (Morrison and Wilhelm, 2004, pp. 1682).

Heski Bar-Isaac (2007) also uses an OLG framework to model an agent's career. In a similar vein to Morrison and Wilhelm (2004) he shows that partners have incentives to exert costly effort on behalf of their juniors. But in Bar-Isaac's model, effort is exerted not on mentoring, but to build a reputation. He shows that an old agent with an established reputation will form a partnership with a young agent without a reputation in order to intentionally introduce reputational uncertainty back into his decisions. By doing so an old agent ensures he still has 'something to prove' and will exert effort to help his junior build a good reputation amongst the firm's clients. If they are successful the junior then has an incentive to buy the old agent out so that he can continue to work with the same clients.

Rebitzer and Taylor (2007) study legal partnerships using the 'property-rights' approach to the theory of the firm. That suggests firm owners derive their authority to command employees from control over the non-labour factors of production (See Grossman and Hart (1986) and Hart and Moore (1990)). They argue that because in law firms "assets reside in the brains of employees" (Rebitzer and Taylor, 2007, pp. 227) then both the nature of authority and the nature of employment within these firms will be quite unusual and that both the up-or-out career path of juniors in large law firms (where an associate either makes partner or is fired) and the practice of having partners own the firm can be explained quite naturally from this perspective.

They note that the most valuable 'assets' possessed by a law firm are its relationships with important clients who provide it with work. And unlike an ordinary firm, over time the employees in a law firm come to 'own' these assets as they build relationships with the clients. Given enough time, a lawyer could build up a strong relationship with clients and threaten to 'grab and leave' with them - unlike an ordinary firm where employees could not threaten to leave with the physical plant and equipment. As such, a law firm must organise itself to the maximum benefit of lawyers with strong client relations (by making them partners) or it must fire juniors before they build those relations in the first place. This leads naturally to firms adopting an up-or-out promotion contest to partner because a profit-sharing partnership maximises profit *per partner* in order to ensure that none of the partners has any incentive to 'grab and leave' with their clients. This implies a partnership will 'run lean' with fewer partners per junior than a *total* profit maximising firm. This result is related to Ward's (1958) observation that employee owned firms will in general be smaller than ordinary firms because at the margin, if an additional employee raises total profit he will be hired by an ordinary firm, but an additional employee must raise both total profit *and* profit per employee for an employee owned firm to take him. In Rebitzer and Taylor's partnership result exactly the same logic applies, except it is the number of owner-partners that will run lean. They show that partners will be hired up to the point that average profit equals the marginal product of a partner, but juniors will be hired up to the point where their wage equals their marginal product. So partnerships have fewer partners than juniors in order to maximise profit per partner, which explains the up-or-out structure of careers in law firms.

Ward's observation that profit-sharing will cause partnerships to run lean has also sparked a literature examining profit sharing rules in partnerships. An early paper was Farrell and Scotchmer (1988) who explore equilibria in coalition-formation games where agents are of different ability. Levin and Tadelis (2005) argue that profit-sharing in partnerships helps them commit to remain small and high quality, which may explain why they exist in industries where product quality is difficult to observe. In their model, firms hire from a pool of different quality agents. As the quality of its agents determines the quality of the good a firm produces, firms always hire the best quality agents first, but partnerships again run lean in comparison to ordinary companies, implicitly committing to a higher quality product. This implicit commitment to quality implies that partnerships will exist as they are more profitable in industries where product quality is harder to observe. So this “explains the relative scarcity of partnerships outside of professional service industries such as law, accounting, medicine, investment banking, architecture, advertising, and consulting” (2005, pp. 131).

Garicano and Santos (2004) study partnerships with profit-sharing rules in a market for ‘referrals.’ They study the institutions that may promote efficient referrals of opportunities to talent. A market could create incentives to refer by using fixed price contracts or income sharing agreements. But fixed price contracts can create incentives for agents to refer every opportunity (and adverse selection) and income sharing agreements suffer from moral hazard (with agents trying to free ride on others work on an opportunity). So here, “the boundaries of partnerships reflect tradeoffs between facilitating the exchange of referrals and effort incentives” (Garicano and Santos, 2004, pp. 516).[2]

Inevitably, questions relating to partnerships also brush up against questions of reputation. This is natural as partnerships often operate in industries where firms and agents survive on their ‘name.’ There is quite an extensive literature on reputation and for a recent overview Bar-Isaac and Tadelis (2008) is excellent. But for the purposes of this paper, note that the interplay between who’s reputation is at stake (an agent’s or a firm’s) is neatly captured in an OLG environment. For example, Tadelis (1999) presents an OLG model with a ‘market for names’ in which agents can buy and sell a firm’s reputation (summarised by its name). This is a hidden information (adverse selection) model, with customers unable to observe changes in ownership. The model was later extended to incorporate moral hazard (Tadelis, 2002). In Bar-Isaac’s paper (2007) discussed above, his use of an OLG model also

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[2] It is sometimes argued that partnerships exist to share-risk and to give agents incentives to diversify into specialist areas (for example see Gilson and Mnookin, 1985). Garicano and Santos (2004) argue against this hypothesis, stating that empirically partnerships do not diversify as they should and that instead they cover areas among which referrals are likely to occur.

permits a firm's reputation to be a mixture of those of its partner and junior. In Morrison and Wilhelm (2004) reputation is also important in that only mentoring will permit a junior to keep the firm's reputation high in the eyes of its clients.

In this paper, the overlapping generations (OLG) aspect of partnerships is emphasised. This is not to dismiss Ward's (or others) observations on the effects or importance of profit-sharing rules, but is done to focus on the intergenerational transactions in partnerships. In this, it follows in the spirit of Morrison and Wilhelm (2004), Bar-Isaac (2007) and Rebitzer and Taylor (2007).

In terms of its folk theorem results, this paper follows the literature on the folk theorem in repeated games with overlapping generations of short-lived players. The key papers here are those by Kandori (1992), Salant (1991), and Smith (1992). They present general folk theorems for a generic repeated stage game where players possess full information on the history of play in the game. Here, I will also assume that new players observe the history of play, and that the stage game reflects a one-shot interaction with the partnership making price offers to clients and junior wage offers to young agents.

The paper is organised as follows. Section 2 presents a basic partnership model where young agents compete for a limited number of junior positions in a partnership. Section 3 then extends the model to include costly effort and clients. Section 4 assumes that there may be multiple partnerships competing for juniors and presents the first folk theorem. Section 5 then returns to a single partnership that can now hire different types of juniors, and shows that as the discount factor rises, partnerships will in equilibrium hire fewer, but higher ability juniors. Finally, section 6 then presents a folk theorem with multiple partnerships competing to hire different types of junior, and shows that in that equilibrium total industry profits are greater if partnerships can limit themselves to each hiring only one type in equilibrium. The paper then concludes.

4.2 Wages and Output

Consider an overlapping generations framework where agents live for two periods. Each generation consists of a single agent so that at any time period t there exist only two agents, a *young* agent in their first period and an *old* agent in their second period. In period t agents apply a discount factor $\delta \in (0, 1)$ to income received in period $t + 1$.

Young agents may either choose to work in a partnership which is owned by an old agent or for an ordinary company. In a partnership a young agent produces output worth a_j and is paid w_j, in the company a young agent produces output worth $\bar{w}$ and is paid $\bar{w}$. After working as a junior in a partnership, an old agent may either choose to stay on and become a *partner* who both owns and works for the firm, or the agent may leave and work for a company instead. An old agent who becomes a partner creates output worth a_p and pays himself w_p. Only old agents who worked in the partnership while young may become partners when old, so an old agent may work for a company either because they did so when young, or because they choose to leave the partnership. At the company an old agent produces output worth $\bar{w}$ and is paid $\bar{w}$. The possible career paths of an agent starting in some period t are shown in Figure 4.1.

Timing

It is assumed that each period events occur in the following order:

1. If he was a junior at the partnership, an agent chooses to become a partner or to leave for the company.
2. If he becomes a partner, then he makes a wage offer of w_j to the junior.
3. If he was made a wage offer, the junior chooses between the partnership or the company. Otherwise, he simply joins the company.

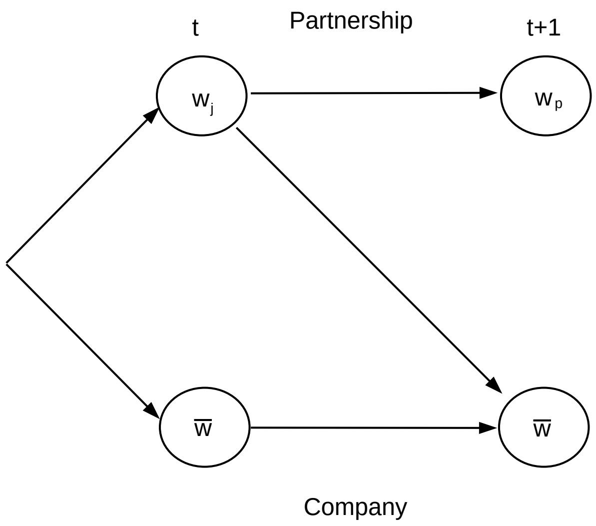

Figure 4.1: Possible career paths for a young agent in period t

4. If both agents are at the partnership, its total output is sold for $a_j + a_p$. If either agent did not join the partnership, then it ceases to exist, and they both work for the company where their total output is sold for $\bar{w} + \bar{w}$.

5. The wages of the agents are paid.

Before continuing, it's worthwhile briefly mentioning two restrictions that the model makes on agents actions. The first is that agents cannot join a partnership at the 'partner' level. There is an interesting discussion to be had on why this should be the case[3] but here it will simply be taken as a stylised fact - indeed that juniors from within the firm are promoted to owner-partner is perhaps *the* stylised fact about partnerships used in this paper. The effect of this assumption is to make the position of junior at a partnership into the only 'gateway' to possible future partner. If making partner is highly prized then this immediately puts the partnership in a strong position to dictate terms to juniors.

The second restriction in the model is that the partnership has all the bargaining power, and can make 'take-it-or-leave-it' wage offers. While this may not always be

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[3] Mentoring (Morrison and Wilhelm, 2004), reputation (Bar-Isaac, 2007), and property rights (Rebitzer and Taylor, 2007) have all been given as reasons. Speaking broadly, these are all claims that only juniors in a partnership build up sufficient *human capital* to be capable of running a partnership.

true, especially for the most talented juniors, there is often fierce competition for a limited number of places at well known professional partnerships.[4] This assumption is intended to reflect the strong bargaining position of partners who control a scarce resource (the 'gateway' to future partnership) and note that by allowing the partner to make a take-it-or-leave-it-offer, the same wage will be offered as when two juniors Bertrand compete their wages down to the lowest wage that a junior would possibly accept to work at a partnership. Of course a junior's wage must still make joining a partnership at least as attractive as the outside option, so the partnership cannot dictate terms completely.

4.2.1 Partnership Equilibrium

A *partnership equilibrium* is defined as a stationary, subgame perfect equilibrium where each period both old and young agents join the partnership. The equilibrium is stationary because each period the partner offers the junior the same wage w_j, the junior accepts and the partner makes a profit of w_p that is greater than his outside option. It is subgame perfect because it must be Nash in every subgame. That is, no agent can be made better off by deviating in any period t. In equilibrium, given w_j and w_p, two individual rationality constraints must be satisfied:

$$\begin{aligned} \text{For the old agent:} \quad & E(w_p) \geq \bar{w} \\ \text{For the young agent:} \quad & w_j + \delta E(w_p) \geq \bar{w} + \delta \bar{w} \end{aligned}$$

Notice that agents rely on their equilibrium expectations of w_p, given by $E(w_p)$, to make their decisions. Further notice that if the first of these constraints holds, it rules out the possibility of a young agent joining the partnership while young and then joining the company while old. So if both constraints hold then both young and old agents will choose to work in the partnership. Finally, note in equilibrium a partnership's budget constraint is given by $w_j + w_p = a_j + a_p$.

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[4] For example, Deloitte has around 10 applicants for every one of its graduate positions according to Jennifer Hughes of the Financial Times, 12 February 2008.

Proposition 1 If $a_j + a_p = 2\bar{w}$ then a partnership equilibrium exists with $w_j = w_p = \bar{w}$. So, if the value of output from a partnership is the same as the value of output from a company, then a partnership equilibrium exists with wages for juniors and partners equal to wages in the company.

Proof By construction. Assume in equilibrium that each period both agents join the partnership and the partner offers the junior a wage $w_j = \bar{w}$. The partnerships budget constraint is $w_p + w_j = 2\bar{w}$. So $w_p = \bar{w}$. And as this occurs in every period in equilibrium, $E(w_p) = \bar{w}$.

In the proposed equilibrium both agents rationality constraints are satisfied, so they will not deviate from joining the partnership. But it must also be shown that the partner will not deviate in his wage offer $w_j = \bar{w}$. If a partner did deviate, he could increase his own payoff by reducing the offer to the junior to $w_j = \bar{w} - \epsilon$, where $\epsilon > 0$. But this violates the juniors second rationality constraint as $w_j + \delta E(w_p) = \bar{w} - \epsilon + \delta\bar{w} < \bar{w} + \delta\bar{w}$. A junior offered $\bar{w} - \epsilon$ would therefore choose to join the company, dissolving the partnership, and the partner would earn $\bar{w}$. The partner thus has no incentive to deviate down from $w_j = \bar{w}$. As a partner also has no incentive to deviate and lower his payoff by increasing w_j he has no incentive to deviate from $w_j = \bar{w}$. So no agent has any incentive to deviate from equilibrium in any period t, and this constitutes a partnership equilibrium. Thus if $a_j + a_p = 2\bar{w}$ then a partnership equilibrium exists with $w_j = w_p = \bar{w}$. ■

Proposition 2 If $a_j + a_p < 2\bar{w}$ a partnership equilibrium does not exist.

Proof By contradiction. Begin by assuming that a partnership equilibrium exists with $w_p = \bar{w}$. Because $a_j + a_p < 2\bar{w}$ then using the partnerships budget constraint $w_j = a_j + a_p - w_p < \bar{w}$. In stationary equilibrium it must be $E(w_p) = \bar{w}$, as the equilibrium is played each period. So $w_j + \delta E(w_p) = w_j + \delta\bar{w} < \bar{w} + \delta\bar{w}$, which violates the second rationality constraint, contradicting the assumption that a partnership equilibrium exists with $w_p = \bar{w}$. Greater values of w_p will continue to satisfy the first constraint but will reduce the value of $w_j + \delta E(w_p)$ continuing to violate the second rationality constraint. And smaller values of w_p would violate the first rationality constraint. Therefore a partnership equilibrium cannot exist if $a_j + a_p < 2\bar{w}$. ■

Proposition 3 If $a_j + a_p > 2\bar{w}$ then a partnership equilibrium exists with $w_j < \bar{w} < w_p$. Specifically:

$$\begin{aligned} w_j &= \frac{1+\delta}{1-\delta}\bar{w} - \frac{\delta}{1-\delta}(a_j + a_p) \\ w_p &= \frac{1}{1-\delta}(a_j + a_p) - \frac{1+\delta}{1-\delta}\bar{w} \end{aligned}$$

So, if the value of output from a partnership is greater than the value of output from a company, then a partnership equilibrium exists in which the junior's wage is strictly less than the company wage which is strictly less than the partner's wage.

Proof By construction. Assume in equilibrium that each period both agents join the partnership and the partner offers the junior a wage $w_j = \frac{1+\delta}{1-\delta}\bar{w} - \frac{\delta}{1-\delta}(a_j + a_p)$. The partnership's budget constraint is $w_p + w_j = a_j + a_p$. So $w_p = \frac{1}{1-\delta}(a_j + a_p) - \frac{1+\delta}{1-\delta}\bar{w}$. As this is an equilibrium played each period $E(w_p) = \frac{1}{1-\delta}(a_j + a_p) - \frac{1+\delta}{1-\delta}\bar{w}$ as well.

In this equilibrium, a young agent's rationality constraint is satisfied with an equality as $w_j + \delta E(w_p) = \frac{1+\delta}{1-\delta}\bar{w} - \frac{\delta}{1-\delta}(a_j + a_p) + \delta(\frac{1}{1-\delta}(a_j + a_p) - \frac{1+\delta}{1-\delta}\bar{w}) = \bar{w} + \delta\bar{w}$. As such, the junior will not deviate from joining the partnership. Now notice that if $E(w_p) - \bar{w} > 0$ then the old agent's rationality constraint is also satisfied. Here $E(w_p) - \bar{w} = \frac{1}{1-\delta}(a_j + a_p) - \frac{1+\delta}{1-\delta}\bar{w} - \frac{1-\delta}{1-\delta}\bar{w} = \frac{1}{1-\delta}(a_j + a_p - 2\bar{w}) > 0$ because $\frac{1}{1-\delta} > 1$ and $a_j + a_p > 2\bar{w}$. So a partner's rationality constraint is satisfied with an inequality.

As their rationality constraints are met, neither agent will deviate from joining the partnership in this equilibrium. A partner will also not deviate to offer a junior a lower wage. As the junior receives the same payoff from joining the partnership and the company, offering a lower w_j would cause a junior to choose to join the company, the partnership would dissolve and the partner would receive a payoff of $\bar{w}$. Clearly, this makes the partner worse off. A partner will also not deviate to offer a higher w_j as that must reduce w_p. As no agent will deviate in any period t, this constitutes a partnership equilibrium. So if $a_j + a_p > 2\bar{w}$ then a partnership equilibrium exists with $w_j < \bar{w} < w_p$. ■

Proposition 4 The only partnership equilibria are those outlined in proposition 1 and 3.

Proof Notice that the young agent's rationality constraint was satisfied with an equality in both proofs, so $w_j + \delta E(w_p) = \bar{w} + \delta\bar{w}$. Imagine an equilibrium with the junior wage $\epsilon > 0$ higher. Using the budget constraint, the equilibrium partner wage would be ϵ lower. And because w_p is the same each period in equilibrium, then $E(w_p) = w_p$ is also ϵ lower. In this equilibrium $w_j + \delta E(w_p) > \bar{w} + \delta\bar{w}$. But that means in some period a partner could deviate from the equilibrium junior wage to a lower junior wage and could still satisfy the juniors rationality constraint. As this would increase the partners payoff, he would do so. So in a partnership equilibrium the young agent's rationality constraint must be satisfied with an equality, or else it cannot be an equilibrium. ■

Figure 4.2 outlines the relationship between the wages in a partnership and the company wage. It has been drawn by varying $\bar{w}$ and assuming $a_j + a_p$ is constant. At $\bar{w} = 0$ partner wages are very high and junior wages are very low (below zero).[5] As $\bar{w}$, the wage offered by a company rises, then w_j increases as partnerships must offer juniors better conditions to retain them, and in turn the rise in w_j causes w_p to fall. Notice that as $\bar{w}$ rises, w_j rises at a steeper rate. The reason for this is that an increase in $\bar{w}$ improves juniors outside option, directly causing w_j to rise. But through the partnership's budget constraint, this causes w_p to fall. In equilibrium, $E(w_p)$ must

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[5] It might be argued that in equilibrium $w_j \geq 0$ should hold. Imposing that constraint, in Figure 4.2 it is easy to observe that if $\bar{w} < \frac{a_j + a_p}{2}$ then for $\bar{w}$ greater than some point $\bar{w}^*$, that $w_j > 0$. For $\bar{w} \leq \bar{w}^*$ a partnership equilibrium would exist with $w_j = 0$ and $w_p = a_j + a_p$.

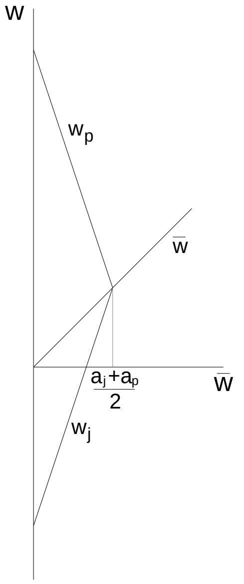

Figure 4.2: Wages in a partnership equilibrium with $\delta = 0.5$

also reflect this fall, making the partnership less attractive again to juniors. So w_j must rise to compensate for this as well.

At the point where $\bar{w} = \frac{a_j + a_p}{2}$, the wages offered by a company over an agents career, $2\bar{w}$, equal the value of output from an agent in a partnership over his career. At this point, as shown in Proposition 1, $w_j = w_p = \bar{w}$. If $\bar{w}$ increases beyond this point, using Proposition 2, no partnership will exist because $\bar{w} > \frac{a_j + a_p}{2}$, and both old and young agents will be paid the company wage, $\bar{w}$.

4.3 Clients and Effort

Above it was shown that ongoing partnerships exist where partners work with a junior if the partnership creates more output than an ordinary company. So why might partnerships create more output than ordinary companies? If partnerships are owned by partners who earn more then their marginal product, then this may give juniors incentives to exert effort for clients in order to keep them at the partnership when they themselves become partners. Clients for their part benefit from this additional effort as neither they nor an ordinary company can commit to rewarding juniors in the future, so cannot induce juniors to exert effort. But partnerships can do so, as partners are owners of the partnership, so may pay themselves ‘inefficiently’ high wages above marginal product.

The model can be extended in the following ways. Assume that a valuable client exists who wants effort exerted on his behalf at either the company or partnership. The client pays b to a firm each period, but if effort is not incurred in some period t then the client leaves in period $t+1$. Agents whether in a partnership or ordinary company produce output valued at $\bar{w}$ for no cost, and in addition may exert effort for the client. Exerting effort costs an agent c and it is assumed that $b > c$, so it is efficient to do so. It is further assumed that a partner does not have to hire a young agent in order for the partnership to exist. He can work alone if he chooses to. However before paying a firm b each period, a client observes whether the firm is an ordinary company, or a partnership and the type of agents working there.

If wages are private information, then ordinary companies will never be able to retain the client. An old agent at an ordinary company will never incur c as he does not benefit from it. And an ordinary company also cannot credibly promise a young agent in t a higher wage, or bonus, in $t+1$ if he exerts effort. Imagine if the company did promise a wage $w^* > \bar{w}$ if c is incurred in t. If the agent does incur c, then in $t+1$ the company knows the agent will produce $\bar{w}$ so only offers him this. As the wage offer is private information, future play cannot be influenced by it. As such, no agent, young or old will ever incur effort costs in an ordinary company.

The situation is very different in a partnership. Here again an old agent (the partner) has no incentive to exert effort, but a young agent does in order to keep the client and increase his profit as a partner. It is shown below that if the benefit from the client $\delta b > c$ then a partnership equilibrium exists with $w_p > w_j$ and juniors exerting effort before making partner. And in this equilibrium partners always hire juniors, as only juniors will exert effort. The equilibrium is given in the following lemma.

Proposition 5: Clients and Effort If $\delta b > c$ then a partnership equilibrium exists with $w_p > w_j$, and juniors at a partnership exerting effort for a client. Specifically:

$$w_j = \bar{w} + \frac{c - \delta b}{1 - \delta}$$
$$w_p = \bar{w} + \frac{b - c}{1 - \delta}$$

Notice that w_p is the same as that given in Proposition 3 with $a_j + a_p = 2\bar{w} + b - c$. But that w_j is the same as that given in Proposition 3 with $a_j + a_p = 2\bar{w} + b - c$ plus c in order to compensate the junior for his expected effort.

Proof By construction. In equilibrium both agents join the partnership each period, the partner offers the junior a wage of $w_j = \bar{w} + \frac{c-\delta b}{1-\delta}$ and pays himself $w_p = \bar{w} + \frac{b-c}{1-\delta}$, the junior exerts effort for the client and the client remains with the partnership. Begin by noting that a junior at a partnership who does not exert effort will lose the client, so in period $t+1$ the partnership will produce output of $2\bar{w}$ and following Proposition 1 will resemble an ordinary company with $w_p = \bar{w}$. If he does exert effort then the client stays and in equilibrium his payoff is $w_p = \bar{w} + \frac{b-c}{1-\delta}$. As such, a junior will exert effort at the partnership in some period t if $-c + \delta(\bar{w} + \frac{b-c}{1-\delta}) \geq \delta\bar{w}$, which is true as $\delta b > c$ by assumption. So juniors will exert effort.

As a young agent anticipates an effort cost of c at the partnership, and no effort at an ordinary company (as ordinary companies cannot induce agents to exert effort) where he will be paid $\bar{w}$ each period, then for a young agent to join the partnership $w_j - c + \delta E(w_p) \geq \bar{w} + \delta\bar{w}$. That condition is met with a strict equality here as $w_j - c + \delta E(w_p) = \bar{w} + \frac{c-\delta b}{1-\delta} - c + \delta(\bar{w} + \frac{b-c}{1-\delta}) = \bar{w} + \delta\bar{w}$. So a young agent will not deviate from joining the partnership at w_j in equilibrium, and no partner will deviate from offering w_j as a downward deviation implies the junior joins the company and the partner makes $\bar{w} < w_p$, while an upward deviation in w_j just reduces the partners wage w_p for no benefit.

Finally, notice that a partner always benefits from working with a junior here as because $\delta b > c$ then $w_j < \bar{w}$ and by hiring the junior a partner increases output by $\bar{w} + b$ but must pay w_j. So the benefit of having a junior outweighs his cost as $\bar{w} + b > w_j$, and partners will always choose to work with a junior.

The partnership in equilibrium produces output worth $2\bar{w} + b$ so a partner's wage must be $w_p = 2\bar{w} + b - w_j = 2\bar{w} + b - \bar{w} - \frac{c-\delta b}{1-\delta} = \bar{w} + \frac{b-c}{1-\delta}$. A partner will not deviate from remaining at the partnership as $w_p > \bar{w}$. Further $w_p > \bar{w} > w_j$ as $\delta b > c$ by assumption.

Finally note that because the partnership exists with juniors exerting effort each period, the client remains there and pays b. ■

So the stylised fact that many professional partnerships resemble a 'rat race' with juniors working very hard to make partner may simply reflect the unique ability of partnerships to create incentives for juniors to service valuable clients. This occurs as partners are owners of the partnership and capture the additional benefit b from having a client. As juniors at a partnership ensure effort will be exerted, clients are also happy to pay b and the value of a junior to a partner is $\bar{w} + b > w_j$. So partners choose to hire young agents even when they could work alone.

4.4 Competing Partnerships

How many juniors will a partnership hire? In this section the model is extended to N partnerships competing to hire a unit mass of juniors. Above it was shown that an ongoing partnership may create more output than an ordinary company because juniors have incentives to work hard for a client so the client doesn't leave when they make partner. But that commitment to work hard requires a partnership holding a junior's wages below his marginal product in order to increase the prize of making partner. If partnerships may only hire one junior per partner, then the number of junior positions at a partnership will naturally be quite scarce, and partnerships can dictate wages to juniors. But if partners may hire more than one junior each, then partnerships may compete up junior wages. If that occurs wages per partner fall and juniors may lose their incentive to exert effort.

Partnerships do not often have one partner working with one junior with the junior guaranteed to make partner in the future. Most large professional partnerships have an 'up or out' system with many juniors per partner, and only the highest ability juniors making partner. One good explanation for this is given by Rebitzer and Taylor (2007) who suggest that it may be because partnerships 'run lean' in the number of partners, maximising profit per partner, not total profit. Simplifying their argument, a firm hiring N employees for profit $\pi(N)$ will set $\pi'(N) = 0$. But an employee owned firm maximises $\frac{\pi(N)}{N}$, the profit per employee, so $\pi'(N) = \frac{\pi(N)}{N}$. Assuming profits follow a concave function in N, then the employee owned firm will always hire fewer employees than the ordinary firm.[6] Rebitzer and Taylor's partnership model (assuming a particular profit function for the joint output of both juniors and partners) finds that there will be fewer partners than juniors, as partners maximise profit per partner when deciding how large the partnership should be, but maximise the firms total profit when deciding how many juniors to hire. This is a very clear explanation for the up or out system, which leaves many juniors competing for relatively few positions as partner.[7] But in Rebitzer and Taylor's model there is no difference between juniors who make partner and those that do not, it's simply luck.[8]

In contrast to Rebitzer and Taylor I assume that the up-or-out system reflects the ability of juniors to produce output for the clients. Here each junior is assumed to have some ex-ante probability λ of producing output valued at b if they exert effort for a client at cost c. And clients choose to remain with agents who successfully produce output when they make partner, while agents that were unsuccessful choose to leave for their outside option. As partnerships may vary the number of juniors

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[6] This is Ward's key observation (1958), and is also the main assumption in Levin and Tadelis (2005).

[7] But note that this requires the profit function to be of a particular form.

[8] Further, as pointed out in Hart (2011) if an existing set of N^* partners charge new partners an entry fee of $f = \frac{\pi(N)}{N}$ where N is the total number of partners, then the existing partners maximise $\frac{\pi(N)}{N} + (\frac{N-N^*}{N^*})f = \frac{\pi(N)}{N^*}$. So by charging an entry fee, partnerships are able to act as though they are normal profit maximising firms where $\pi'(N) = 0$.

they hire, partnerships will also bill their clients by the hour. Although others have already given some explanations for this stylised fact in partnership, such as Emons (2000) who models it as a credence good problem revolving around getting the right treatment when the seller knows more than the buyer, here the problem is again commitment. Clients cannot observe the actions partners or juniors take once they pay them, but they can observe who is working at the partnership. As the client knows that partners don't exert effort, but charge by the hour, that gives partners incentives to hire cheap juniors and make a profit on the margin between what they bill per hour and what they pay the junior. Arguably this behaviour makes quite a lot of sense from the clients perspective. They know that nothing can motivate partners to exert effort, but they also know that juniors will exert effort, so if clients cannot command partners to hire juniors then they can still give partners incentives to do so if they are billed by the hour. So when partners can vary the number of juniors they hire, billing by the hour makes sense for the clients as it gives partners incentives to hire juniors who will then exert effort. Formally, the model is given below.

4.4.1 Partnerships Competing for Juniors

Consider an overlapping generations framework in an infinitely repeated game where each period a unit mass of new agents enter the game and live for two periods. At each time period t then there exist a unit mass of *young* agents in their first period and a unit mass of *old* agents in their second period. In period t agents apply a discount factor $\delta \in (0, 1)$ to income received in period $t + 1$. There are also N short lived clients who each arrive at a different location each period. All agents and clients are assumed to observe play before their arrival in the game.

Each period t a stage game is played. First old agents choose which of the N locations to go to, where together they form a partnership, or old agents may exit for their outside option. At both the partnership and outside option old agents are paid their marginal product of $\bar{w}$, and at the partnership each partner also receives an equal share of the partnership's profits. Every period one client arrives at each

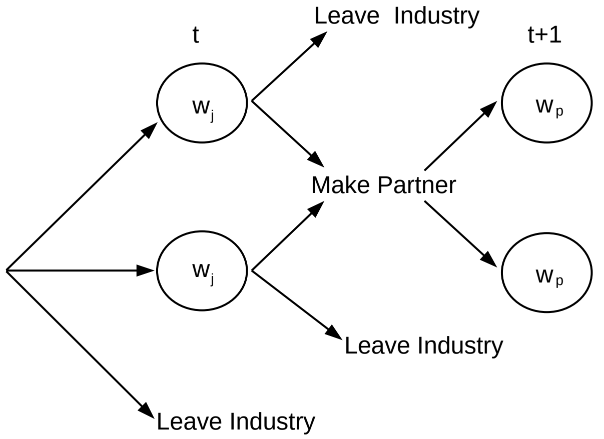

Figure 4.3: Possible careers for an agent at period t with two firms

location and once partnerships are formed the partners make a take it or leave it offer to the client. They do so by offering clients a price p_j per unit of juniors they hire. This can be thought of as offering the client a price per hour billed by the partnership. A partnership can bill more hours the more juniors it hires and here I simply assume that the total number of hours billed each period is fixed, so the price per hour is equivalent to charging a price per junior hired. Clients are assumed to value the expected output of a unit of juniors at $\bar{w} + \lambda b$ if they expect them to exert effort and $\bar{w}$ if they expect them not to.

After the client accepts or declines the partnerships price offer, a unit mass of young agents then enter the game. Partnerships then simultaneously offer a wage w_j to the juniors. Juniors have an outside option valued at $\bar{w}$ so they may either accept one of the partnerships wage offers, or leave for that outside option. Clients then pay partners based on the number of juniors hired, and juniors may exert unobservable effort for some cost c to produce output with probability λ. It is assumed that $\lambda b > c$ so that it is efficient for juniors to do so. Juniors who do not exert effort never produce output. The stage game then ends and it is observed which juniors successfully produced output. Figure 4.3 outlines a young agent's potential careers in a model with two partnerships.

The timing each period t in the model is as follows:

1. Old agents go to one of N locations and form partnerships, or exit for their outside option of $\bar{w}$.
2. Each partnership then makes their client a price offer p_j that they charge per unit of juniors hired.
3. Clients accept or reject this offer.
4. Partnerships offer juniors a wage w_j per unit of juniors to join them.
5. Juniors choose to go to a partnership and are paid or instead go to their outside option for $\bar{w}$.
6. Clients pay partnerships.
7. Juniors at partnerships then choose whether to exert effort or not, and output may be produced.

4.4.2 Partnership Equilibrium with Competition

A *partnership equilibrium* is defined as a stationary, perfect Bayesian equilibrium in the infinitely repeated game where each period young agents and partners join the N partnerships. The equilibrium is stationary because each period the partnerships offer clients the same price p_j, juniors the same wage w_j, clients and juniors accept, and partners make a profit w_p. It is subgame perfect because it is Nash in every subgame, so no agent can be made better off by deviating in any period t. The equilibrium is Bayesian because agents' beliefs in equilibrium must be consistent with the equilibrium actions of the different types of agent.

To ensure juniors join a partnership, the following rationality constraint must be satisfied:

$$w_j - c + \lambda\delta E(w_p) + (1-\lambda)\delta\bar{w} \geq \bar{w} + \delta\bar{w}$$

And for a junior to exert effort, the following compatibility constraint must hold:

$$-c + \lambda\delta E(w_p) + (1-\lambda)\delta\bar{w} \geq \delta\bar{w}$$

In Proposition 6 a competitive equilibrium is presented where each period juniors and partners join the N partnerships and the partnerships offer juniors a wage equal to their marginal product. Because of this partners and juniors earn their outside option $\bar{w}$ and as there is no incentive for juniors to exert effort. In effect the market resembles one populated by ordinary companies. In Proposition 7, reversion to this equilibrium will function as a grim trigger threat to juniors, allowing a folk theorem to obtain where juniors ignore above equilibrium wage offers.

Proposition 6: Competitive Equilibrium A competitive partnership equilibrium exists where all old agents join a partnership, offer the client a price of $p_j = \bar{w}$, clients accept, partners make a profit of $w_p = \bar{w}$, juniors are offered their marginal product $w_j = \bar{w}$, and juniors do not exert effort.

Proof Each period in equilibrium, $w_p = \bar{w}$. That implies that juniors must have consistent beliefs with $E(w_p) = \bar{w}$. So juniors will never exert effort as their effort compatibility constraint cannot be met. As juniors never produce output in equilibrium clients expect no effort so only ever accept $p_j = \bar{w}$ from the partnerships. Finally, as $p_j = \bar{w}$ partnerships earn no profit from hiring juniors and will not deviate from offering juniors $w_j = \bar{w}$. All old agents and juniors here have no incentive to deviate from joining the partnerships each period, so are assumed to do so, splitting evenly amongst them. ■

This equilibrium is clearly inefficient, but it does satisfy the usual requirement in competitive markets that price equals marginal cost. In the efficient equilibrium below, it is shown that clients can provide incentives for effort by only accepting price offers from partnerships if they are made up of agents who successfully produced output in $t-1$. In the equilibrium $w_j < \lambda b + \bar{w}$ a juniors marginal product, and $w_p > \bar{w}$ an old agents marginal product. This equilibrium is efficient because $b\lambda > c$, and juniors choose to exert effort in order to earn high partner profits in the future.

Proposition 7: Efficient Uncompetitive Partnerships If $\delta \geq \frac{\lambda b+\frac{c}{N}}{\lambda b+\frac{\lambda b}{N}}$ a partnership equilibrium is played every period t with each of the N partnerships made up of $\frac{\lambda}{N}$ successful old agents paid $w_p > \bar{w}$, and $\frac{1}{N}$ juniors who exert effort and are paid $w_j < \lambda b + \bar{w}$. Partnerships offer clients a price per unit of juniors $p_j = \lambda b + \bar{w}$ each period and that offer is accepted. Specifically w_j and w_p are given by:

$$w_j = \bar{w} + \frac{c - \delta\lambda b}{1-\delta}$$
$$w_p = \bar{w} + \frac{\lambda b - c}{\lambda(1-\delta)}$$

Proof The proof is by construction, first specifying an equilibrium strategy and then showing no agent has an incentive to deviate from that strategy.

Equilibrium Strategy On the equilibrium path, if there has been no deviation then each period t old agents who successfully produced output in $t-1$ split evenly among the N locations to form partnerships, so there are $\frac{\lambda}{N}$ partners at each firm. Partnerships then offer their client a price $p_j = \lambda b + \bar{w}$ per unit of juniors that they hire, and clients accept this price. Partnerships then all offer juniors the equilibrium wage $w_j = \bar{w} + \frac{c-\delta b\lambda}{1-\delta}$ and juniors also split evenly among the N partnerships. So there are $\frac{1}{N}$ juniors in each partnership. Partners are then paid $w_p = \bar{w} + \frac{\lambda b-c}{\lambda(1-\delta)}$ and juniors exert effort for the clients.

The equilibrium strategy must also specify play following potential deviations. The first possible deviation from equilibrium play is that an unsuccessful old agent does not exit for $\bar{w}$, but instead goes to one of the N locations and joins a partnership. In that case, play then immediately reverts to the competitive equilibrium, where it remains from then on.

A partnership may deviate to a non-equilibrium junior wage offer. If that occurs juniors are expected to not accept it and to split among the other $N-1$ partnerships. If they do so, then play continues as usual in $t+1$, with successful old agents splitting evenly among the N locations. If a junior does deviate however and accepts, then play reverts to the competitive equilibrium from $t+1$ onwards.

Clients observe whether a deviation from equilibrium play has occurred in the past and whether play is expected to revert to the competitive equilibrium. If no deviation has occurred they accept $p_j = \lambda b + \bar{w}$ but if a deviation has occurred then clients expect the competitive equilibrium will be played in $t+1$, so believe that juniors will exert no effort. As such, they only accept a price offer of $p_j = \bar{w}$ following a deviation.

Potential Deviations If no deviation has occurred then successful partners have no incentive to deviate from randomising among the N location as they are all ex-ante identical. No unsuccessful old agent will deviate from going to their outside option, as if they do then play reverts to the competitive equilibrium and they receive a payoff of $\bar{w}$ at the partnership, which is the same as their outside option. So old agents have no incentive to deviate.

Partnerships then have no incentive to deviate from the equilibrium p_j offer. If no deviation has occurred then clients expect each unit of juniors hired to produce output worth $\bar{w} + \lambda b$ so will not deviate from accepting that price offer. A partnership will not deviate from offering this price as a higher price will be rejected and a lower price lowers the partnerships profits. If a deviation has occurred then the client expects no effort to be exerted by juniors, so will only accept a price offer up to $\bar{w}$. Again, partnerships will not deviate from offering this.

If juniors are offered the equilibrium wage $w_j = \bar{w} + \frac{c-\delta\lambda b}{1-\delta}$ then they will not deviate from joining a partnership. To see this note that at this wage their rationality constraint is met with an equality as they have consistent beliefs. Note that there are $\frac{\lambda}{N}$ partners at each partnership and $\frac{1}{N}$ juniors at each partnership so $\frac{1}{\lambda}$ juniors per partner. So $E(w_p) = \bar{w} + \frac{\lambda b + \bar{w} - w_j}{\lambda}$. Using equilibrium w_j then $E(w_p) = \bar{w} + \frac{b\lambda + \bar{w} - \bar{w} - \frac{c-\delta\lambda b}{1-\delta}}{\lambda} = \bar{w} + \frac{b\lambda - c}{\lambda(1-\delta)}$. So the juniors rationality constraint is met as $w_j - c + \lambda\delta E(w_p) + (1-\lambda)\delta\bar{w} = \bar{w} + \frac{c-\delta\lambda b}{1-\delta} - c + \lambda\delta(\bar{w} + \frac{b\lambda - c}{\lambda(1-\delta)}) + (1-\lambda)\delta\bar{w} = \bar{w} + \delta\bar{w}$. Juniors will also exert if their effort constraint is met. That is $-c + \lambda\delta E(w_p) + (1-\lambda)\delta\bar{w} \geq \delta\bar{w}$. Again using equilibrium w_j and w_p this is met if $\delta \geq \frac{c}{\lambda b}$. That condition must be met in equilibrium also as by assumption $\delta \geq \frac{N\lambda b+c}{N\lambda b+\lambda b}$ and as $\lambda b > c$ then $\frac{N\lambda b+c}{N\lambda b+\lambda b} > \frac{c}{\lambda b}$. So juniors will not deviate from joining a partnership and exerting effort in equilibrium. Notice too that as $\delta\lambda b > c$ then $w_j < \bar{w}$ here.

Because in equilibrium $w_j < \lambda b + \bar{w}$ a partnership makes a profit off each junior it hires. So the partners have strict incentives to hire juniors. Below I discuss a partnership making a potential junior wage deviation, hoping to attract more than their $\frac{1}{N}$ juniors and boost profits.

Junior Wage Deviations If a partnership deviates from w_j to some other junior wage w_D hoping to attract juniors to work for it, then notice that following a deviation it expects to make a profit of $D^*(\lambda b + \bar{w} - w_D)$, where D^* is the number of juniors joining the partnership after a deviation. In order for this deviation to be profitable for the firm however, that deviation profit must be greater than the $\frac{1}{N}(\lambda b + \bar{w} - w_j)$ that the firm expects to earn if it doesn't deviate. That implies the highest possible deviation wage offer w_D is given by:

$$\begin{aligned} & D^*(\lambda b + \bar{w} - w_D) \geq \tfrac{1}{N}(\lambda b + \bar{w} - w_j) \\ \Rightarrow \quad & w_D \leq \lambda b + \bar{w} + \tfrac{1}{ND^*}(\lambda b + \bar{w} - w_j) \end{aligned}$$

So at most $w_D \leq \lambda b + \bar{w} + \frac{1}{N}(\lambda b + \bar{w} - w_j)$, when $D^* = 1$. In order to ensure that no partnership has a profitable deviation from w_j then juniors must reject this wage offer and choose to work at a non-deviating partnership. In equilibrium, all juniors expect all other juniors to do so, they therefore expect that if they avoid the w_D offer that play will continue on the equilibrium path, and they receive the expected payoff $w_j - c + \lambda\delta E(w_p) + (1-\lambda)\delta\bar{w} = \bar{w} + \delta\bar{w}$. Recall that if any junior does accept a deviating wage offer then play reverts to the competitive equilibrium in the next period. Therefore they won't exert effort if they accept w_D and receive a payoff of at most $w_D + \delta\bar{w} = \lambda b + \bar{w} + \frac{1}{N}(\lambda b + \bar{w} - w_j) + \delta\bar{w}$. Using $w_j = \bar{w} + \frac{c-\delta\lambda b}{1-\delta}$ then $\lambda b + \bar{w} - w_j = \frac{\lambda b - c}{1-\delta}$ and the non-deviation constraint is given by:

$$\lambda b + \bar{w} + \tfrac{1}{N}(\lambda b + \bar{w} - w_j) + \delta\bar{w} \leq \bar{w} + \delta\bar{w}$$

$$\Rightarrow \qquad \delta \geq \frac{\lambda b+\frac{c}{N}}{\lambda b+\frac{\lambda b}{N}}$$

Note that the right hand side of this inequality is positive and less than 1 as $\lambda b > c$ by assumption. If δ fulfils this condition, then partnerships know that any potentially profitable deviation from w_j will cause all juniors to join the other partnerships, reducing profits per partner to $\bar{w} < w_p$. So partnerships will not deviate from equilibrium w_j.

If $\delta \geq \frac{\lambda b+\frac{c}{N}}{\lambda b+\frac{\lambda b}{N}}$ then as no agent has any incentive to deviate from the equilibrium strategy in any period t then this constitutes a partnership equilibrium with agents playing a best response each period given their beliefs and beliefs consistent with play. ■

So partnership equilibria may continue to exist even when partnerships compete for juniors, if juniors ignore firms that try to raise wages. Such equilibria are more efficient than a competitive equilibrium (where no one exerts effort) if $\lambda b > c$, which is assumed to be true. That implies partnership equilibria exist where juniors are of a relatively high ability and the clients place a high enough value on their output. It becomes more difficult to sustain this equilibrium as N rises and the lower bound for δ grows because the payoff from deviating and hiring the $1-\frac{1}{N}$ juniors going to the other partnerships grows.

4.5 Types of Juniors

Juniors may not all be of the same type and the type of junior hired by a partnership influences the wages it pays, how much it bills clients and the number of juniors making partner. In this section the model is simply that given above, but it will be assumed that there is again only one partnership and that now there are J different types of junior. There is one unit mass of each type of junior, and they have different ability levels. The expected probability of each type of junior denoted $j = 1, 2, 3, ..., J$ producing output is given by $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_J$ and it is assumed that $0 < \lambda_1 < \lambda_2 <$

$... < \lambda_J < 1$. So higher types have a higher probability of successfully producing output. It is efficient for all types to exert effort here if $\lambda_j b > c$ for all j, which is assumed to be the case. In terms of notation, if a partnership hires each unit of $T = J - j + 1$ types from j to J then its average type is denoted $\bar{\lambda_j} = \frac{\lambda_j + \lambda_{j+1} + ... + \lambda_J}{T}$. It is also assumed that $\frac{\lambda_j}{\bar{\lambda_j}}$ is increasing at a decreasing rate in j. This is the case for example when types are evenly distributed on $[0, 1]$ with type j having ability $\frac{j}{J+1}$. The types of the juniors are known to both partners and clients, and each period the partnership makes its client a list of price offers denoted p_j, where p_j denotes the price the client will pay the partnership per unit of j type juniors hired.

Below I will show that partners in equilibrium maximise their profits per partner by only hiring one type of junior, but if they are free to vary the types of juniors they hire then they will hire more types when the discount factor is low. So as δ rises, partnerships hire fewer but on average higher ability juniors, make greater profits per partner, and because high ability agents make partner more often, the partnership will have more partners per junior. Proposition 8 establishes the partnership equilibrium with types of juniors, when clients do not allow partnerships to vary the types of junior they hire. Proposition 9 then shows that partnerships hiring fewer, but higher ability juniors make greater profits per partner. Finally Proposition 10 establishes that when partnerships may vary the juniors they hire, that large but low ability partnerships emerge when δ is low, and that as δ rises the partnership hires fewer, but higher ability juniors.

Proposition 8: Partnership Equilibrium with Types If $\lambda_j \delta b > c$ then a partnership equilibrium exists where every period t the partnership hires in total $T = J - j + 1$ juniors, the juniors are of types $j, j + 1, ..., J$, exert effort and are paid $w_j < \bar{w}$, there are also $\lambda_j + ... + \lambda_J$ successful partners paid $w_p > \bar{w}$ each period at the partnership. Partnerships offer clients a price list with $p_j = \lambda_j b + \bar{w}$ for types $j, j + 1, ..., J$ and $p_j = \bar{w}$ for types $1, 2, ..., j - 1$, and that price list offer is accepted by the client. Specifically w_j and w_p are given by:

$$w_j = \bar{w} + \frac{c - \lambda_j \delta b}{1 - \delta \frac{\lambda_j}{\bar{\lambda}_j}}$$
$$w_p = \bar{w} + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j (1 - \delta \frac{\lambda_j}{\bar{\lambda}_j})}$$

Proof By construction. In equilibrium, each period t play is as outlined in the proposition. If a deviation occurs and a junior that did not produce output in $t-1$ joins the partnership in t, then it is assumed that juniors in t and all future periods do not exert effort. As such, clients in t and all future clients only accept a price list offer from the partnership with $p_j = \bar{w}$ for all j. If only successful juniors from $t-1$ do join the partnership, but the partnership deviates in its price list offer, again it is assumed that no junior will exert effort this period, or in future periods. Finally, if the partnership deviates in its junior wage offer, then it is assumed that no junior will exert effort this period or future periods. Following any deviation all agents are expected to go to the outside option each period.

Begin by noting that if clients observe an unsuccessful agent at the partnership, then because they expect any juniors in $t+1$ not to exert effort, that they believe that clients from $t+1$ onwards will only accept a price offer with $p_j = \bar{w}$ for all j. As such, they expect $E(w_p) = \bar{w}$ in $t+1$ so juniors effort compatibility constraints cannot be met this period. That is juniors will not exert effort. As such, if an unsuccessful agent joins the partnership in t the client will only accept a price list with $p_j = \bar{w}$ for all j. So if an unsuccessful agent joins the partnership he anticipates $p_j = \bar{w}$ for all j will be offered and accepted, and the partnership will offer all juniors $\bar{w}$ to join it, as they cannot be induced to join for a wage below their outside option. As such $w_p = \bar{w}$ if an unsuccessful agents joins the partnerships and therefore they have no reason to deviate from going to their outside option.

If there has been no deviation, clients will not deviate from accepting a price list offer with $p_j = \bar{w} + \lambda_j b$ for $j = j, j+1, ..., J$ as they anticipate juniors will exert effort. Partnerships will also not deviate from offering such a price list, as they cannot charge more than the juniors marginal products, and a deviation will be rejected by a client that expects following a deviation no junior will exert effort. The only plausible deviation then is to $p_j = \bar{w}$ which a client would accept, but here juniors anticipate $E(w_p) = \bar{w}$ in $t+1$, so the partnership would have to offer $w_j = \bar{w}$ in order for them to join. As such, that deviation reduces w_p to $w_p = \bar{w}$ this period which as $\lambda_j b > c$ for all j is less than equilibrium profits per partner. So partnerships will not deviate from the equilibrium price list offer.

Once the client accepts the price offer, the partnership will not deviate from offering all juniors the equilibrium wage w_j. A deviation from w_j implies that in $t+1$ then $E(w_p) = \bar{w}$. So by joining the partnership juniors expect to exert no effort and receive a payoff of $w_j + \delta\bar{w}$. So the deviation wage offer must be $\bar{w}$ or above in order to hire any juniors. But that implies that the partnership makes no profit off the new juniors hired of types $1, 2, ..., j-1$ as the price list specifies $p_j = \bar{w}$ for juniors of these types. And now the partnership is also paying a higher wage to juniors of types $j, .., J$ as previously $w_j < \bar{w}$ as $\lambda_j b > c$ for all j. So there cannot be any profitable wage deviation up by the partnership. Partnerships will also not deviate down to a lower wage offer, as $w_j < \bar{w}$. So a downward deviation implies that no agents of any type will join the partnership, and the partners will make a profit of $\bar{w} < w_p$ this period.

Juniors of type j and higher will join the partnership at w_j in equilibrium because their rationality and effort constraints are met. To see this note that a junior of type j will not deviate from joining the partnership as his rationality constraint is satisfied with an equality. That is $w_j - c + \lambda_j \delta E(w_p) + (1-\lambda_j)\delta\bar{w} = \bar{w} + \frac{c-\lambda_j\delta b}{1-\delta\frac{\lambda_j}{\bar{\lambda}_j}} - c + \lambda_j\delta(\bar{w} + \frac{\bar{\lambda}_j - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}) + (1-\lambda_j)\delta\bar{w} = \bar{w} + \delta\bar{w}$. Also a higher type $j^* > j$ will have his rationality constraint satisfied strictly as $w_j - c + \lambda_{j^*}\delta E(w_p) + (1-\lambda_{j^*})\delta\bar{w} > \bar{w} + \delta\bar{w}$ so all higher types will also join. Notice that juniors of a lower type than j will not have their rationality constraint met however, so will go to the outside option instead. They will not deviate to join the partnership and then exert no effort either, as $w_j < \bar{w}$.

A junior of type j will however exert effort in equilibrium as his effort constraint is satisfied. If he doesn't exert effort he will not become a partner and expects his outside option in $t+1$ which is $\delta\bar{w}$. To see that he prefers to exert effort note that, $-c+\lambda_j\delta E(w_p)+(1-\lambda_j)\delta\bar{w} = -c+\lambda_j\delta(\bar{w}+\frac{\bar{\lambda}_j b-c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})})+(1-\lambda_j)\delta\bar{w} > \delta\bar{w}$ as $b\delta\lambda_j > c$ by assumption. Because this is true for a j type junior, then it is also true for any $j^* > j$ higher type junior as $-c+\lambda_{j^*}\delta E(w_p)+(1-\lambda_{j^*})\delta\bar{w} > \delta\bar{w}$. So juniors of type j and all higher types will exert effort.

Finally, notice that as there are $J-j+1$ juniors joining the partnership each period, then the number of partners in equilibrium must be $\bar{\lambda}_j(J-j+1) = \lambda_j+\lambda_{j+1}+...+\lambda_J$. So there are $\bar{\lambda}_j = \frac{\lambda_j+\lambda_{j+1}+...+\lambda_J}{J-j+1}$ partners per junior or $\frac{1}{\bar{\lambda}_j}$ juniors per partner. As all partners receive the same wage then it must be that $w_p = \bar{w}+\frac{1}{\bar{\lambda}_j}(\bar{\lambda}_j b+\bar{w}-w_j) = \bar{w}+\frac{\bar{\lambda}_j b-c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}$. So partners receive all of the firms profit.

As no agent has any incentive to deviate from this equilibrium in any period t and as agents beliefs are consistent with play, that concludes the proof. ■

In this equilibrium a partnership makes lower profits per partner the more (lower) types that it hires. Clearly for some w_j the partnership earns a greater profit the higher the type attracted to join it, so it would not make sense for a partnership to limit itself to just hiring low types at some w_j. But if a partnership can restrict itself to hiring only high types and not low types, then it will produce a greater profit per partner in equilibrium. This is shown below.

Proposition 9: Smaller Partnerships Make Higher Profits per Partner If a partnership hires fewer juniors so its lowest type is $j^* > j$, then $w_p^{j^*} > w_p^j$.

Proof To see this, notice that profits per partner when the lowest type is j are given by $w_p^j = \bar{w}+\frac{\bar{\lambda}_j b-c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}$ and when the lowest type is $j^* > j$ they are given by $w_p^{j^*} = \bar{w}+\frac{\bar{\lambda}_{j^*} b-c}{\bar{\lambda}_{j^*}(1-\delta\frac{\lambda_{j^*}}{\bar{\lambda}_{j^*}})}$. Begin by noting that:

$$\bar{\lambda}_{j^*} > \bar{\lambda}_j$$

$$\Rightarrow \qquad -\bar{\lambda}_j c > -\bar{\lambda}_{j^*} c$$

$$\Rightarrow \quad \bar{\lambda}_j(\bar{\lambda}_{j^*}b - c) > \bar{\lambda}_{j^*}(\bar{\lambda}_j b - c)$$

Both sides here are positive as $\bar{\lambda}_j \delta b - c > 0$ by assumption so $\bar{\lambda}_j b - c > 0$ and at $j^* > j$ then $\bar{\lambda}_{j^*}b - c > 0$ too. Now notice that $\frac{\lambda_{j^*}}{\bar{\lambda}_{j^*}} > \frac{\lambda_j}{\bar{\lambda}_j}$ by assumption. That implies that $\delta\frac{\lambda_j}{\bar{\lambda}_j} < \delta\frac{\lambda_{j^*}}{\bar{\lambda}_{j^*}}$ and that $1 - \delta\frac{\lambda_{j^*}}{\bar{\lambda}_{j^*}} < 1 - \delta\frac{\lambda_j}{\bar{\lambda}_j}$. So (as both sides of the derivation above are positive) then:

$$\frac{\bar{\lambda}_j(\bar{\lambda}_{j^*}b - c)}{1 - \delta\frac{\lambda_{j^*}}{\bar{\lambda}_{j^*}}} > \frac{\bar{\lambda}_{j^*}(\bar{\lambda}_j b - c)}{1 - \delta\frac{\lambda_j}{\bar{\lambda}_j}}$$

Which means that profits per partner are greater when the lowest agent type hired by the partnership is higher:

$$w_p^{j^*} = \bar{w} + \frac{\bar{\lambda}_{j^*}b - c}{\bar{\lambda}_{j^*}(1 - \delta\frac{\lambda_{j^*}}{\bar{\lambda}_{j^*}})} > \bar{w} + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1 - \delta\frac{\lambda_j}{\bar{\lambda}_j})} = w_p^j$$

That completes the proof. ■

The intuition here is that when a partnership expands to hire lower ability types, then the partners must raise the junior wage for all juniors. That implies the higher ability types have their rationality and effort constraints met strictly, and in effect earn rents from joining the partnership that they would not earn if only their type was employed. As the rents of juniors grow then the profits of each partner falls. Despite this, if a partnership can choose the types of juniors that it hires, it does not follow immediately that the partnership will only choose the highest types.

Each period the number of partners is already fixed by the number of successful juniors in the previous period, as such the partners may have an incentive to either expand or contract the partnership in order to raise their current profit in period t. Below I present a proposition where partners are free to vary the types of the juniors hired by 1. That is, if in equilibrium they hire $T = J - j + 1$ types of juniors, $j, j+1, ..., J$ then they can deviate in some period t to hiring only $j+1, J+2, ..., J$ types or to hiring $j-1, j, j+1, ..., J$ types. This may occur if it is assumed that following a deviation to a different junior wage, play is then assumed to revert to equilibrium play

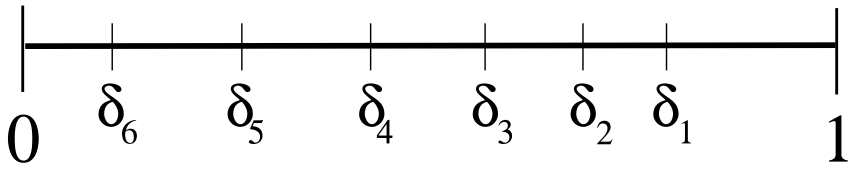

Figure 4.4: Example of δ_T with 7 types. Partnerships hire T types if $\delta_T < \delta < \delta_{T-1}$, all 7 types if $\delta < \delta_6$ and 1 type if $\delta_1 < \delta$.

in $t+1$ with the equilibrium types of juniors hired. Here it can be shown that when δ falls between two cutoff values δ_T and δ_{T-1} then a partnership will hire T types of junior in equilibrium. With J types in total then $0 < \delta_{J-1} < \delta_{J-2} < ... < \delta_1 < 1$, so as δ rises partnerships in equilibrium will hire fewer but higher ability types. In effect the partnership trades off the temptation of greater revenues through size in period t against the lower wage it can offer when juniors expect greater profits per partner in $t+1$. As with higher δ partnerships must be smaller in equilibrium, then equilibrium profits per partner are also higher.

Proposition 10: Partnerships hire fewer, but higher ability juniors as δ rises. If $\lambda_{j-1}\delta b > \frac{\lambda_{j-1}+\lambda_j+...+\lambda_J}{\lambda_j+...+\lambda_J}c$ for all j and $\frac{\lambda_j}{\bar{\lambda}_j}$ is increasing and concave in an agents type j (such when ability given by $\lambda_j = \frac{j}{J+1}$) then a partnership in equilibrium hires T types of junior $j, j+1, j+2, ..., J$ if $\delta_T \leq \delta \leq \delta_{T-1}$ where:

$$\delta_T = \frac{\lambda_{j-1}b - c}{\frac{\lambda_j}{\bar{\lambda}_j}(T+1)(\bar{\lambda}_{j-1}b - c) - \frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}}T(\bar{\lambda}_j b - c)}$$

$$\delta_{T-1} = \frac{\lambda_j b - c}{\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}}T(\bar{\lambda}_j b - c) - \frac{\lambda_j}{\bar{\lambda}_j}(T-1)(\bar{\lambda}_{j+1}b - c)}$$

Noting that $\delta_J = 0$ as for $\delta \in [0, \delta_{J-1}]$ a partnership already hires all possible types of junior, then $0 < \delta_{J-1} < \delta_{J-2} < ... < \delta_1 < 1$. So for $\delta \in [0, \delta_{J-1}]$ a partnership hires all types in equilibrium, and as δ rises it hires fewer and fewer types until $\delta \in [\delta_1, 1]$ where it hires only the highest type of junior. This also implies more partners per junior, lower junior wages, and greater profits per partner in equilibrium.

In Figure 4.4 an example of the δ_T cutoffs is given for a model with 7 junior types, $b = 2$, $c = 0.05$, and $\lambda_j = \frac{j}{8}$ for $j = 1, 2, 3, ..., 7$. This gives us $\delta_6 = 0.12, \delta_5 = 0.27, \delta_4 = 0.43, \delta_3 = 0.57, \delta_2 = 0.69, \delta_1 = 0.79$.

Proof By construction. Consider a partnership equilibrium such as that given in Proposition 8, but where partnerships may vary their wage offer w_j. Here, a deviating w_j is not assumed to have any effect on future play. It is instead assumed that the partnership will return to the equilibrium w_j offer in period $t + 1$ and clients will still expect effort to be exerted if successful juniors join the partnership. That implies that in some period t partners could deviate to either a higher w_j in order to attract more but on average lower types of agents, or they could deviate to a lower w_j in order to attract fewer but higher types of agents. If juniors accept this deviating wage, then they continue to exert effort, as they believe that play will revert to normal in $t+1$ but there will either be more or fewer agents making partner in $t + 1$ than in equilibrium. This changes their expectations of profits per partner and the wage needed to induce them to join the partnership. If in equilibrium a partnership is assumed to hire T types of juniors, and they do not have a profitable one-shot deviation to hiring either $T - 1$ or $T + 1$ types of junior then that is indeed an equilibrium. To further simplify the analysis, it is assumed that only deviations by one type need be checked, and that if a firm did deviate by more than one type in t then from period $t + 1$ onwards the client expects juniors to exert no effort and partnership profits will be $\bar{w}$. Using the proof of Proposition 8, that precludes deviations by more than one type. Below it will be shown that as stated in the proposition, as δ rises then the equilibrium number of types hired falls.

First consider a partnership that hires $T = J - j + 1$ types of junior in equilibrium $j, j + 1, j + 2, ..., J$. Note that as shown in Proposition 8 a w_j that induces type j agents to join the partnership must also induce higher types $j + 1, j + 2, ..., J$ to also join the partnership. As stated in that proposition, when j is the lowest type hired by the partnership then profit per partner in equilibrium is $w_p = \bar{w} + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}$. As there are T units of juniors each period of which $\bar{\lambda}_j T = \lambda_j + \lambda_{j+1} + ... + \lambda_J$ make partner each period then the partnership's total profit is given by:

$$\bar{w}\bar{\lambda}_j T + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\bar{\lambda}_j T$$

Notice that $\bar{w}\bar{\lambda}_j T$ of this total is produced by the partners themselves and $\frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\bar{\lambda}_j T$ is made off the margin between the junior wage and the amount charged to the client. If a partnership deviates to a higher w_j in some period t in order to hire $j-1$ type juniors, then those juniors expect in period $t+1$ that there will be $\bar{\lambda}_{j-1}(T+1) = \lambda_{j-1} + \lambda_j + \lambda_{j+1} + ... + \lambda_J$ partners in $t+1$. But the partnership in $t+1$ will revert to offering the equilibrium w_j to hire T units of juniors. So in $t+1$ it will make a total profit of $\bar{w}\bar{\lambda}_{j-1}(T+1) + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\bar{\lambda}_j T$. As such, profit per partner in $t+1$ following the deviation will be given by:

$$\bar{w} + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\left(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j-1}(T+1)}\right)$$

Using the juniors rationality constraint, a junior agent of type $j-1$ will join the deviating partnership if $w_{j-1} - c + \lambda_{j-1}\delta E(w_p) + (1-\lambda_{j-1})\delta\bar{w} \geq \bar{w} + \delta\bar{w}$ and this constraint is met with an equality here if:

$$w_{j-1} = \bar{w} + c - \lambda_{j-1}\delta\frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\left(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j-1}(T+1)}\right)$$

Recall that this wage will be offered to all juniors. Further notice that at this expected profit per partner all juniors of type $j-1$ and higher who join the partnership will exert effort. To see this, note that the effort constraint is that:

$$-c + \lambda_{j-1}\delta\left(\bar{w} + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\left(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j-1}(T+1)}\right)\right) + (1-\lambda_{j-1})\delta\bar{w} \geq \delta\bar{w}$$

$$\Rightarrow \quad \lambda_{j-1}\delta b \geq \frac{\lambda_{j-1}+\lambda_j+...+\lambda_J}{\lambda_j+...+\lambda_J}c\left(1-\delta\left[\frac{\lambda_j}{\bar{\lambda}_j} - \frac{\lambda_j+...+\lambda_J}{\lambda_{j-1}+\lambda_j+...+\lambda_J}\frac{\lambda_{j-1}}{\bar{\lambda}_j}\right]\right)$$

The term in square brackets is positive, so the right hand side is less than $\frac{\lambda_{j-1}+\lambda_j+...+\lambda_J}{\lambda_j+...+\lambda_J}c$ and $\lambda_{j-1}\delta b \geq \frac{\lambda_{j-1}+\lambda_j+...+\lambda_J}{\lambda_j+...+\lambda_J}c$ by assumption. So when the partnership deviates and offers a higher wage, $j-1$ type agents will exert effort, and that implies higher types with a better chance of making partner will as well.

A partnership deviating to a higher junior wage to attract the $j-1$ types will earn additional revenue of $p_{j-1} = \bar{w} + \lambda_{j-1}b$ (the $j-1$ types expected output when they are hired). And its total revenue is given by $(\bar{w} + \bar{\lambda}_{j-1}b)(T+1)$, the expected output per junior agent times the number of juniors plus $\bar{\lambda}_j T\bar{w}$, the output of the current partners. So a partnership deviating to hire the $j-1$ types will make a total profit given by:

$$(T+1)(\bar{w} + \bar{\lambda}_{j-1}b - \bar{w} - c + \lambda_{j-1}\delta\frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j-1}(T+1)})) + \bar{\lambda}_j T\bar{w}$$

For a partnership not to have an incentive to deviate to a higher junior wage then this profit must be less than the profit it expects to make in equilibrium. That is the non-deviation condition is:

$$(T+1)(\bar{\lambda}_{j-1}b - c + \lambda_{j-1}\delta\frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j-1}(T+1)})) + \bar{\lambda}_j T\bar{w} \leq \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\bar{\lambda}_j T + \bar{\lambda}_j T\bar{w}$$

Which implies:

$$\begin{aligned}
& (T+1)(\bar{\lambda}_{j-1}b - c)\bar{\lambda}_{j-1} + \lambda_{j-1}\delta\tfrac{\bar{\lambda}_j b - c}{1-\delta\frac{\lambda_j}{\bar{\lambda}_j}}T \leq \tfrac{\bar{\lambda}_j b - c}{1-\delta\frac{\lambda_j}{\bar{\lambda}_j}}\bar{\lambda}_{j-1}T \\
\Rightarrow\quad & (T+1)(\bar{\lambda}_{j-1}b - c)\bar{\lambda}_{j-1}(1-\delta\tfrac{\lambda_j}{\bar{\lambda}_j}) \leq (\bar{\lambda}_j b - c)T(\bar{\lambda}_{j-1} - \lambda_{j-1}\delta) \\
\Rightarrow\quad & (T+1)(\bar{\lambda}_{j-1}b - c)\bar{\lambda}_{j-1}(\bar{\lambda}_j - \delta\lambda_j) \leq T(\bar{\lambda}_j b - c)\bar{\lambda}_j(\bar{\lambda}_{j-1} - \lambda_{j-1}\delta) \\
\Rightarrow\quad & \bar{\lambda}_j\bar{\lambda}_{j-1}((T+1)(\bar{\lambda}_{j-1}b - c) - T(\bar{\lambda}_j b - c)) \leq \\
& \delta((T+1)(\bar{\lambda}_{j-1}b - c)\bar{\lambda}_{j-1}\lambda_j - T(\bar{\lambda}_j b - c)\bar{\lambda}_j\lambda_{j-1}) \\
\Rightarrow\quad & (T+1)(\bar{\lambda}_{j-1}b - c) - T(\bar{\lambda}_j b - c) \leq \delta((T+1)(\bar{\lambda}_{j-1}b - c)\tfrac{\lambda_j}{\bar{\lambda}_j} - T(\bar{\lambda}_j b - c)\tfrac{\lambda_{j-1}}{\bar{\lambda}_{j-1}})
\end{aligned}$$

The left hand side can be expanded out and simplified noting that $\bar{\lambda}_j = \frac{\lambda_j + \ldots + \lambda_J}{T}$ and similarly $\bar{\lambda}_{j-1} = \frac{\lambda_{j-1} + \lambda_j + \ldots + \lambda_J}{T+1}$. That implies that after expanding and simplifying the left hand side becomes $\lambda_{j-1}b - c$. Further as $\lambda_{j-1}b > c$ by assumption (as it assumed it is efficient for every type of junior to exert effort to create output) then $(T+1)(\bar{\lambda}_{j-1}b - c) > T(\bar{\lambda}_j b - c)$ as well. That implies that the right hand side of the above inequality is positive too as $\frac{\lambda_j}{\bar{\lambda}_j} > \frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}}$ by assumption.

So the non-deviation condition can be stated as:

$$\frac{\lambda_{j-1}b - c}{\frac{\lambda_j}{\bar{\lambda}_j}(T+1)(\bar{\lambda}_{j-1}b - c) - \frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}}T(\bar{\lambda}_j b - c)} \leq \delta$$

Notice that the left hand side of this expression is positive.

The other possible deviation a partnership that hires T types in equilibrium may make is to hire fewer juniors. That is, they reduce their junior wage offer so that only types $j+1$ and higher will join the firm. If that occurs then juniors know that in period $t+1$ there will only be $\bar{\lambda}_{j+1}(T-1)$ partners in period $t+1$, increasing profits per partner. Again using the total profit of the partnership in equilibrium, a junior of type $j+1$ will have his rationality constraint met if:

$$w_{j+1} = \bar{w} + c - \lambda_{j+1}\delta\frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j+1}(T-1)})$$

Notice that here the partnership is deviating to a lower junior wage, and there will be fewer partners than usual next period. This implies that agents of type $j+1$ will always exert effort, as they would do so for the usually lower expected profits per partner in equilibrium. So the effort constraint holds for all types $j+1$ and above.

The partnerships revenue is now $(\bar{w} + \bar{\lambda}_{j+1}b)(T-1)$ from its juniors. So a partnership deviating to hire only $j+1$ type juniors and higher, will make a profit this period of:

$$(T-1)(\bar{w} + \bar{\lambda}_{j+1}b - \bar{w} - c + \lambda_{j+1}\delta\frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j+1}(T-1)})) + \bar{\lambda}_j T\bar{w}$$

In order for the partnership to have no incentive to deviate for this profit, then it must be less than the usual total profit for the partnership. The non-deviation condition is therefore:

$$(T-1)(\bar{\lambda}_{j+1}b - c + \lambda_{j+1}\delta\frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}(\frac{\bar{\lambda}_j T}{\bar{\lambda}_{j+1}(T-1)})) + \bar{\lambda}_j T\bar{w} \leq \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1-\delta\frac{\lambda_j}{\bar{\lambda}_j})}\bar{\lambda}_j T + \bar{\lambda}_j T\bar{w}$$

After some rearranging that becomes:

$$\Rightarrow \quad T(\bar{\lambda}_j b - c) - (T-1)(\bar{\lambda}_{j+1}b - c) \geq \delta(T(\bar{\lambda}_j b - c)\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}} - (T-1)(\bar{\lambda}_{j+1}b - c)\frac{\lambda_j}{\bar{\lambda}_j})$$

As shown above the left hand side can be expanded and simplified to $\lambda_j b - c$, which is greater than 0 by assumption. This further implies that the right hand side is positive as $T(\bar{\lambda}_j b - c) > (T-1)(\bar{\lambda}_{j+1} b - c)$ and $\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}} > \frac{\lambda_j}{\bar{\lambda}_j}$ by assumption. So the non-deviation condition can be re-written as:

$$\delta \leq \frac{\lambda_j b - c}{\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}} T(\bar{\lambda}_j b - c) - \frac{\lambda_j}{\bar{\lambda}_j}(T-1)(\bar{\lambda}_{j+1} b - c)}$$

Note that the right had side is positive here. It remains to be shown that this upper bound within which partnerships do not deviate from employing T types of agents lies above the lower bound for δ found above. To show that this is indeed the case we require:

$$\frac{\lambda_{j-1} b - c}{\frac{\lambda_j}{\bar{\lambda}_j}(T+1)(\bar{\lambda}_{j-1} b - c) - \frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}} T(\bar{\lambda}_j b - c)} \leq \frac{\lambda_j b - c}{\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}} T(\bar{\lambda}_j b - c) - \frac{\lambda_j}{\bar{\lambda}_j}(T-1)(\bar{\lambda}_{j+1} b - c)}$$

Rearranging this the condition becomes:

$$\frac{\lambda_{j-1} b - c}{(\lambda_{j-1} b - c) + .. + (\lambda_J b - c)) - \frac{\bar{\lambda}_j}{\lambda_j}\frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}}((\lambda_j b - c) + .. + (\lambda_J b - c))} \leq$$
$$\frac{\lambda_j b - c}{\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}}\frac{\bar{\lambda}_j}{\lambda_j}((\lambda_j b - c) + ... + (\lambda_J b - c)) - ((\lambda_{j+1} b - c) + ... + (\lambda_J b - c))}$$

Note that for some $a, b, c, d > 0$ that $\frac{a}{b} \leq \frac{c}{d} \Leftrightarrow \frac{a}{b} \leq \frac{a+(c-a)}{b+(d-b)}$ so a sufficient condition here is to show $c - a > d - b$. The above condition is met here then, with $c - a = (\lambda_j - \lambda_{j-1})b$ if that is greater than $d - b$ which is given by:

$$\left(\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}}\frac{\bar{\lambda}_j}{\lambda_j} + \frac{\bar{\lambda}_j}{\lambda_j}\frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}}\right)((\lambda_j b - c) + ... + (\lambda_J b - c))$$
$$-(\lambda_{j-1} b - c) - (\lambda_j b - c) - 2((\lambda_{j+1} b - c) + .. + (\lambda_J b - c))$$

This is less than $(\lambda_j - \lambda_{j-1})b$ if:

$$2((\lambda_j b - c) + ... + (\lambda_J b - c)) > \frac{\bar{\lambda}_j}{\lambda_j}\left(\frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}} + \frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}}\right)((\lambda_j b - c) + ... + (\lambda_J b - c))$$
$$\Rightarrow \quad \frac{\lambda_j}{\bar{\lambda}_j} - \frac{\lambda_{j-1}}{\bar{\lambda}_{j-1}} > \frac{\lambda_{j+1}}{\bar{\lambda}_{j+1}} - \frac{\lambda_j}{\bar{\lambda}_j}$$

And this expression must be met as $\frac{\lambda_j}{\bar{\lambda}_j}$ is assumed to be increasing at a decreasing rate.[9]

One last noteworthy point is that the upper bound on δ for a partnership to hire T types of juniors is also the lower bound for a partnership to hire $T-1$ types in equilibrium. This can be seen by noting that the upper bound is simply the lower bound with $j+1$, and recalling that as the number of types T hired falls (by one) the lowest type j rises (by one). As stated in the proposition then I denote the lower bound of the region where a partnership may hire T types in equilibrium as δ_T, and the upper bound for that equilibrium is also the lower bound of the region where a partnership can hire $T-1$ junior types in equilibrium δ_{T-1}. We can ignore the lower bound δ_J of the equilibrium when J types are hired (as there is no $j=0$ type to deviate to hire in addition) and we can similarly ignore the upper bound δ_0 when $T=1$ as there are no higher types the partnership could restrict itself to.

Finally, notice that all of these bounds are positive and it can be shown that the highest of these bounds $\delta_1 < 1$. To see this recall that $\bar{\lambda}_J = \lambda_J$ and $\bar{\lambda}_{J-1} = \frac{\lambda_J + \lambda_{J-1}}{2}$. That implies:

$$\delta_1 = \frac{\lambda_{J-1} b - c}{2(\bar{\lambda}_{J-1} b - c) - \frac{\lambda_{J-1}}{\bar{\lambda}_{J-1}}(\lambda_J b - c)}$$

$$\Rightarrow \quad \delta_1 = \frac{(\lambda_{J-1} b - c)}{(\lambda_{J-1} b - c) + [(\lambda_J b - c) - \frac{\lambda_{J-1}}{\bar{\lambda}_{J-1}}(\lambda_J b - c)]}$$

Noting that $\frac{\lambda_{J-1}}{\bar{\lambda}_{J-1}} < 1$ then the denominator is greater than the numerator, and $\delta_1 < 1$. Further recall that $\delta_T > 0$ for some T, so as T rises then δ_T falls but remains positive. That is $\delta_T \in (0,1)$ for $T \in 1, 2, ..., J-1$ and $0 < \delta_{J-1} < ... < \delta_2 < \delta_1 < 1$. ■

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[9] A numerical example that easily fulfils these requirements is J types uniformly distributed on $[0,1]$ with the type $j = 1, 2, .., J$ having ability $\lambda_j = \frac{j}{J+1}$. Here the ratio $\frac{\lambda_j}{\bar{\lambda}_j} = \frac{2j}{(J+1)(j+J)}$, which is increasing in j as its first derivative is $\frac{2J}{(J+1)(j+J)^2}$ but it increases at a decreasing rate as its second derivative is $-\frac{4J}{(J+1)(j+1)^3}$. So this condition can be met quite easily.

4.6 Types of Partnerships

It might not just be agents that are differentiated by type, partnerships may also hold different reputations. And they don't always hire the best juniors, although that seemed to be the implication of the above section. Here partnerships are ex-ante identical but in equilibrium sort themselves by ability too, with high ability juniors going to one, and lower ability to another. While firms with higher quality juniors are more profitable, and generate more profits per partner, if partnerships stay small and hire only one type regardless of their ability, the partnership avoids having to pay rents to some juniors. That seems a good argument for each partnership to want to hire a single type. But as in the section on competing partnerships, an equilibrium strategy must be constructed to prevent firms from competing up junior wages, poaching one another's employees and destroying the efficient partnership equilibrium. In addition to their rationality and effort constraints, partnerships must now also ensure that agents sort themselves correctly into the expected partnerships. That imposes a further sorting constraint on the equilibrium.

In this section I consider J partnerships, each of which in equilibrium hires a different one of J types of junior. There is also a unit mass of each junior type $j = 1, 2...J$, so in total there are J units of juniors.

If a partnership that hires T types could split itself up into T individual firms by type, with each firm hiring a different type of agent in equilibrium, then the same number of agents would make partner but the overall profits of the partners would increase. In the previous section it was shown that a partnership that hired types $j, j+1, j+2, ..., J$ made a profit per partner of $w_p = \bar{w} + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j (1-\delta \frac{\lambda_j}{\bar{\lambda}_j})}$. And a partnership that hired fewer types of agent, starting at $j^* > j$ with $j^*, j^*+1, ..., J$ made a higher profit per partner of $w_p = \bar{w} + \frac{\bar{\lambda}_{j*} b - c}{\bar{\lambda}_{j*} (1-\delta \frac{\lambda_{j*}}{\bar{\lambda}_{j*}})}$. Profits per partner always increase the fewer (higher) types that a partnership hires. Ultimately they were maximised if the partnership only ever hired the highest type of agent J with $w_p = \bar{w} + \frac{\lambda_J b - c}{\lambda_J (1-\delta)}$.

But what if a firm could just hire type j agents and not those of a higher type? In that case the situation is analogous to that of the firm hiring only J types, because now the partnership only hires a single type, which in a sense is the highest available. So profits per partner are given by $w_p = \bar{w} + \frac{\lambda_j b - c}{\lambda_j(1-\delta)}$. While clearly this is less than when hiring only J types, as $\lambda_J c > \lambda_j c \Rightarrow \bar{w} + \frac{\lambda_J b - c}{\lambda_J(1-\delta)} > \bar{w} + \frac{\lambda_j b - c}{\lambda_j(1-\delta)}$, it is in a sense an improvement for the partners, because they no longer pay rents to higher types, who would have been prepared to work for a lower junior wage. To formalise this intuition the proposition below shows that if a firm hires different types $j, j+1, ..., J$ then it could increase total profits if it broke itself up into T firms, each of which only hires one type of junior.

Proposition 11: Industry Profit is Greater with Many Partnerships

Proof By construction. As shown in Proposition 8, a partnership hiring $T = J - j + 1$ types $j, j+1, ..., J$ makes a profit per partner of:

$$\bar{w} + \frac{\bar{\lambda}_j b - c}{\bar{\lambda}_j(1 - \delta\frac{\lambda_j}{\bar{\lambda}_j})}$$

The total number of partners in this firm is given by $\lambda_j + \lambda_{j+1} + ... + \lambda_J = \bar{\lambda}_j T$. Total profit is therefore:

$$\bar{w}\bar{\lambda}_j T + \frac{(\frac{\lambda_j + \lambda_{j+1} + ... + \lambda_J}{T})b - c}{\bar{\lambda}_j(1 - \delta\frac{\lambda_j}{\bar{\lambda}_j})}\bar{\lambda}_j T$$

Rearranging this:

$$\bar{w}\bar{\lambda}_j T + \frac{\lambda_j b - c}{1 - \delta\frac{\lambda_j}{\bar{\lambda}_j}} + \frac{\lambda_{j+1} b - c}{1 - \delta\frac{\lambda_j}{\bar{\lambda}_j}} + ... + \frac{\lambda_J b - c}{1 - \delta\frac{\lambda_j}{\bar{\lambda}_j}} \tag{4.1}$$

If instead we consider a firm that hires only one type j in equilibrium, then it makes a profit per partner of:

$$\bar{w} + \frac{\lambda_j b - c}{\lambda_j(1 - \delta)}$$

As in a firm with j type juniors they have a probability λ_j of making partner then the total profit of T firms each hiring a different type of agent $j, j+1, ..., J$ is given by:

$$(\bar{w} + \tfrac{\lambda_j b - c}{\lambda_j(1-\delta)})\lambda_j + (\bar{w} + \tfrac{\lambda_{j+1} b - c}{\lambda_{j+1}(1-\delta)})\lambda_{j+1} + ... + (\bar{w} + \tfrac{\lambda_J b - c}{\lambda_J(1-\delta)})\lambda_J$$

Which can be written as:

$$\bar{w}\bar{\lambda}_j T + \frac{\lambda_j b - c}{1-\delta} + \frac{\lambda_{j+1} b - c}{1-\delta} + ... + \frac{\lambda_J b - c}{1-\delta} \tag{4.2}$$

As $\frac{\lambda_j}{\bar{\lambda}_j} < 1$ then $1 - \delta\frac{\lambda_j}{\bar{\lambda}_j} > 1 - \delta$. Comparing Equation 4.1 to Equation 4.2 total industry profits are higher when T firms each hire a different type of agent, rather than a single firm hiring all types. ■

Can an industry with many partnerships hiring different types of junior exist? First I will show that again a competitive equilibrium obtains with partnerships resembling ordinary companies, no effort, and partners making no profit. This equilibrium again functions as a grim trigger threat to deviators in Proposition 13, which presents an equilibrium for the market with many partnerships each hiring different types, effort being exerted, and high profits per partner.

Proposition 12: Competitive Equilibrium (Redux) A competitive partnership equilibrium exists where all old agents join one of the J partnerships, partnerships offer the client a price of $p_1 = p_2 = ... = p_J = \bar{w}$ for each type of junior they hire, partners make a profit of $w_p = \bar{w}$, juniors are paid their marginal product $w_j = \bar{w}$, and juniors do not exert effort.

Proof: By construction. As in equilibrium each period both successful and unsuccessful agents make partner, then juniors will never exert effort, as effort is costly. As such, clients hold consistent beliefs that no junior will ever exert effort. They therefore only accept price offers of $p_j = \bar{w}$ for all j from partnerships. Old agents anticipate $E(w_p) = \bar{w}$ but have no incentive to deviate from joining the partnership. They expect clients will only accept $p_j = \bar{w}$ for all j so make that offer, and then offer a wage of $w_j = \bar{w}$ to juniors. Juniors accept this offer regardless of type as they anticipate $\bar{w}+\delta\bar{w}$ from joining a partnership which is the same as if they do not. In equilibrium, as $p_j=\bar{w}$ for all j and $w_j=\bar{w}$ then the firms budget constraint implies that $w_p=\bar{w}$. Finally, as all partnerships are identical, juniors and partners will split evenly amongst them. In this equilibrium no agent has an incentive to deviate from equilibrium play, and beliefs are consistent with play. ■

In the proposition below I denote the junior wage at the s type partnership as w_j^s and the profit per partner at the s type partnership as w_p^s, where $s \in \{1,2,...,J\}$ and the s type partnership hires only juniors of type s in equilibrium. Again it is assumed to be efficient for agents to exert effort, and $b\lambda_j > c$ for all $j=1,2,..,J$.

Proposition 13: The Partnership Folk Theorem with Types If $\delta \geq \frac{(\lambda_1+\lambda_2+...+\lambda_J)b+c}{(\lambda_1+\lambda_2+...+\lambda_J)b+\lambda_1 b}$ then a partnership equilibrium exists where each period t all type j juniors join the j type partnership, partners are paid $w_p^j > \bar{w}$ at the j partnership, juniors are paid $w_j^j < \bar{w}$ there, all juniors exert effort, and clients pay $p_j=\bar{w}+\lambda_j b$ for each unit of j type juniors a partnership hires. Specifically:

$$w_j^j = \bar{w} + \frac{c-\delta\lambda_j b}{(1-\delta)}$$
$$w_p^j = \bar{w} + \frac{\lambda_j b - c}{\lambda_j(1-\delta)}$$

Proof The equilibrium strategy again relies on a grim reversion to competitive equilibrium in $t+1$ should agents deviate from equilibrium play in t. Once more the only agents who have an ongoing stake in the game are the juniors, so are expected to punish partnerships deviating from their equilibrium w_j^j. Punishing deviating partnerships is a best response for juniors in equilibrium only if δ is high enough, and they value the loss of their future partnership profits highly enough.

Equilibrium Strategy On the equilibrium path, each period t all type j juniors join the j type partnership, partners are paid $w_p^j = \bar{w} + \frac{\lambda_j b-c}{\lambda_j(1-\delta)}$ at the j partnership, juniors are paid $w_j^j = \bar{w} + \frac{c-\delta\lambda_j b}{(1-\delta)}$, all juniors exert effort, and clients pay $p_j = \bar{w} + \lambda_j b$ for each unit of j type juniors a partnership hires.

Following a potential deviation, beginning with the juniors, in t if a junior of type j deviates by joining a partnership of a different type, then play reverts to the competitive equilibrium from $t+1$ on. If a partnership deviates from its expected junior w_j^j upwards, then all j type juniors continue to join it, and all other junior types are also expected to continue going to other partnerships. Play then continues as usual. If any non j type junior does join the deviating partnership, then play is expected to revert to the competitive equilibrium in $t+1$ and remain there from then on. Alternatively, if a j type partnership deviates its wage offer w_j^j downwards, juniors of other types again ignore it, but juniors of type j now do not join the partnership and instead go to their outside option for $\bar{w}$. In $t+1$ all of these agents then return from their outside option to join the partnership as partners. Play then continues as usual.

At the beginning of each period successful agents remain in their partnership location and make partner, while unsuccessful agents leave for their outside option of $\bar{w}$. If any agent deviates from this play in t, then play is assumed to immediately revert to the competitive equilibrium in t, and to remain there in future periods. The one exception to this is if a partnership deviated downwards in w_j^j last period, in that case if all j type agents left for their outside option and now turn up as partners at the j firm, then play is expected to continue as usual.

Clients observe whether a deviation from equilibrium play has occurred and whether play is expected to revert to the competitive equilibrium. If no deviation has occurred they accept $p_j = \bar{w} + \lambda_j b$ for all j and expect juniors to exert effort for them. But if a deviation has occurred then clients expect the competitive equilibrium will be played in $t+1$, so expect that juniors will not exert effort. As such, following a deviation they will only accept $p_j = \bar{w}$ for all j hired.

Potential Deviations Clients will not deviate from their equilibrium strategy as each period their beliefs are consistent with the effort that will be exerted by juniors. If effort is exerted then they expect a payoff of $\bar{w}+\lambda_j b$ per unit of j type agents hired, so they have no incentive to deviate from accepting the partnerships price offer list where $p_j = \bar{w} + \lambda_j b$ is charged per unit of j type juniors. Similarly, as following a deviation play reverts permanently to the competitive equilibrium, then juniors effort constraint cannot be met, and partnerships believe that they will exert no effort this period. As such, they have no reason to deviate from only accepting a price list of $p_j = \bar{w}$ for all j.

Old agents may also have incentives to deviate. Notice that if an old agent was successful then playing according to the equilibrium strategy he receives a payoff of $w_p^j > \bar{w}$ by staying at his current partnership (or returning from the outside option if the partnership deviated down from w_j^j last period). Old agents will thus not deviate to join a different partnership, as that causes play to be competitive this period with $p_j = \bar{w}$, $w_j = \bar{w}$ and $w_p = \bar{w}$, so partners make lower profits. Unsuccessful agents also know that if they join a partnership that play will be competitive and they will make $w_p = \bar{w}$. So they also have no incentive to deviate from going to their outside option for $\bar{w}$.

Partnerships (which attempt to maximise total profit) have no incentive to deviate from the equilibrium price offers, as if there has not been any deviations then they cannot charge a client more than $p_j = \bar{w} + \lambda_j b$ for j type juniors, and they will not deviate to a lower price offer as that would lower the partnerships profit. Also if there has been a deviation in the past then clients will not accept $p_j > \bar{w}$, so partnerships will not deviate from offering $p_j = \bar{w}$ for all j. Finally, partnerships will not deviate from hiring juniors as in equilibrium $w_j^j < \bar{w} + \lambda_j b$, their marginal product, so partnerships make a margin on each unit of juniors hired.

Equilibrium Rationality Constraints Juniors must have incentives to join a partnership, exert effort and to join the j type partnership. A junior of type j will not deviate for his outside option of $\bar{w} + \delta\bar{w}$ if $w_j^j - c + \lambda_j \delta E(w_p^j) + (1-\lambda_j)\delta\bar{w} \geq \bar{w} + \delta\bar{w}$. As in equilibrium there are $\frac{1}{\lambda_j}$ juniors per partner at the j partnership, then consistent beliefs imply $E(w_p^j) = \bar{w} + \frac{\bar{w}+\lambda_j b - w_j^j}{\lambda_j}$ so a junior of type j joins the j partnership if $w_j^j \geq \bar{w} + \frac{c-\lambda_j\delta b}{(1-\delta)}$. So juniors in equilibrium will not deviate for their outside option at this wage. In order to ensure that they also exert effort, then we require $-c + \lambda_j\delta E(w_p^j) + (1-\lambda_j)\delta\bar{w} \geq \delta\bar{w}$ in equilibrium. As $E(w_p^j) = \bar{w} + \frac{\lambda_j b - c}{\lambda_j(1-\delta)}$ then in equilibrium this constraint is met if $\delta \geq \frac{c}{\lambda_j b}$. This is true by assumption as $\delta > \frac{(\lambda_1+\lambda_2+...+\lambda_J)b+c}{(\lambda_1+\lambda_2+...+\lambda_J)b+\lambda_1 b} > \frac{c}{\lambda_j b}$, for all j. Notice that as $\delta\lambda_j b > c$ then it further implies that $w_j^j < \bar{w}$ (below an agents outside option) for all j and that $w_p^j > \bar{w}$.

Juniors of type j must also have no incentive to deviate and join a partnership of some other type denoted γ. If he does deviate then play reverts to the competitive equilibrium in $t+1$ onwards, so he has no incentive to exert effort. As such if he deviates to join some other partnership his expected payoff is that partnerships equilibrium wage $w_j^\gamma = \frac{c-\lambda_\gamma\delta b}{(1-\delta)}$, followed by $\delta\bar{w}$ in $t+1$. In order for him not to deviate for another partnership then this must be less than his equilibrium path payoff $w_j^j - c + \lambda_j\delta E(w_p^j) + (1-\lambda_j)\bar{w}\delta = \bar{w} + \delta\bar{w}$. That is we require $w_j^\gamma + \delta\bar{w} < \bar{w} + \delta\bar{w}$ for all γ. That is, $w_j^\gamma < \bar{w}$. This condition is met for all $\gamma = 1, 2, .., j, .., J$. So type j junior agents will not deviate from joining the j partnership and exerting effort.

Finally, partners will not deviate for their outside option of $\bar{w}$. To see this, note that in the j type partnership there is one unit of juniors and λ_j partners each period. That implies profit per partner is given by $w_p = \bar{w} + \frac{\bar{w}+\lambda_j b - w_j}{\lambda_j} = \bar{w} + \frac{\lambda_j b - c}{\lambda_j(1-\delta)}$ where all of the firm's profits are paid out to the partners in equilibrium. As $\lambda_j b > c$ by assumption then $w_p > \bar{w}$, so partners will not deviate from w_p to go to their outside option. Also following a downward wage deviation by a partnership in t, juniors go to their outside option but then all join the j partnership as partners. Again they have no incentive to deviate from doing so, as the deviating wage must be below $w_j^j < \bar{w}$ and it must be that wage per partner at j will be above $\bar{w}$ although lower than usual for that period.

Deviating Wage Offers A partnership of type γ may deviate to a higher junior wage offer w_D^γ hoping to hire more juniors and increase current profits. If it does so then it may hire D^* units of employees which generate an average marginal product of $\bar{w} + \bar{\lambda}^* b$. As such it expects to make a profit of $(\bar{w} + \bar{\lambda}^* b)D^* - w_D^\gamma D^*$. Now recall that in equilibrium by offering w_j^γ a partnership expects to make a profit of $\bar{w} + \lambda_\gamma b - w_j^\gamma$. A firm will only undertake a deviation if it expects its deviation profit to be greater than its equilibrium profit, that is:

$$\begin{array}{rc} & (\bar{w} + \bar{\lambda}^* b)D^* - w_D^\gamma D^* \geq \bar{w} + \lambda_\gamma b - w_j^\gamma \\ \Rightarrow & w_D^\gamma \leq \frac{(\bar{w} + \bar{\lambda}^* b)D^* + w_j^\gamma - \bar{w} - \lambda_\gamma b}{D^*} \end{array}$$

Now notice that:

$$\begin{array}{rc} & w_j^\gamma - \bar{w} - \lambda_\gamma b \\ = & \bar{w} + \frac{c - \delta\lambda_\gamma b}{1-\delta} - \bar{w} - \lambda_\gamma b \\ = & \frac{c - \lambda_\gamma b}{1-\delta} \end{array}$$

So the partnership with the highest potential deviation wage offer w_D^γ must be the firm where λ_γ is smallest, which is the partnership that hires only the lowest types $\gamma = 1$ for revenue of $\bar{w} + \lambda_1 b$ per junior hired in equilibrium. Denote $w_D = w_D^1$ as the highest deviation wage that can be rationally offered by a deviating partnership. Further notice that $(\bar{w} + \bar{\lambda}_D^* b)D^*$ is simply the total output of all juniors hired at the deviating wage. And further that $\bar{\lambda}_D^* D^* b$ is greatest if the partnership is successful in hiring all juniors at this wage so $\bar{\lambda}_D^* D^* b = (\lambda_1 + \lambda_2 + ... + \lambda_J)b$. That is w_D must be:

$$w_D \leq \frac{\bar{w}D^* + (\lambda_1 + \lambda_2 + ... + \lambda_J)b + \frac{c - \lambda_1 b}{1-\delta}}{D^*}$$

Following a deviating wage offer by some partnership γ, all juniors of other types are assumed to ignore it and to join their own type partnership as usual. Play then continues as usual next period. If they do not deviate then a j type agent expects a payoff of $w_j^j - \bar{w} - c + \lambda_j \delta E(w_p^j) + (1 - \lambda_j)\delta\bar{w} = \bar{w} + \delta\bar{w}$. If they do deviate play then reverts to the competitive equilibrium in $t+1$ so after joining the deviating firm juniors will not exert effort, and will receive $w_p = \bar{w}$ next period. Together, that implies that the non-deviation constraint preventing juniors ever joining a deviating partnership is:

$$\begin{aligned} & w_D + \delta\bar{w} \leq \bar{w} + \delta\bar{w} \\ \Rightarrow \quad & \tfrac{\bar{w}D^* + (\lambda_1+\lambda_2+...+\lambda_J)b + \frac{c-\lambda_1 b}{1-\delta}}{D^*} \leq \bar{w} \\ \Rightarrow \quad & (\lambda_1+\lambda_2+...+\lambda_J)b + \tfrac{c-\lambda_1 b}{1-\delta} \leq 0 \\ \Rightarrow \quad & \tfrac{(\lambda_1+\lambda_2+...+\lambda_J)b+c}{(\lambda_1+\lambda_2+...+\lambda_J)b+\lambda_1 b} \leq \delta \end{aligned}$$

As $\lambda_1 b > c$ by assumption the left hand side of this equality is positive and less than 1.

As stated in the proposition then if $\delta \geq \frac{(\lambda_1+\lambda_2+...+\lambda_J)b+c}{(\lambda_1+\lambda_2+...+\lambda_J)b+\lambda_1 b}$ then no profitable deviation by a partnership exists that will entice other types of juniors to join it. Further, no agent has any incentive to deviate from equilibrium play and agents beliefs are consistent with play, so this constitutes a partnership equilibrium. ■

Here I have considered J partnerships each of which each hired a different type of junior. It is possible to combine this result with Proposition 7's result by assuming that there are N partnerships of each type j. So in total NJ partnerships. Here if we follow through the derivation of w_D again, a partnerships maximum profit following a deviation is $\bar{w}D^* + (\lambda_1 + ... + \lambda_J)b - w_D D^*$ while its foregone equilibrium profits are $\frac{1}{N}(\bar{w} + \lambda_j b - w_j^j)$ so the maximum plausible $w_D = \frac{\bar{w}D^* + (\lambda_1+...+\lambda_J)b + \frac{1}{N}(\bar{w}+\lambda_j b - w_j^j)}{D^*}$. And as $w_j^j = \bar{w} + \frac{c-\lambda_j \delta b}{1-\delta}$ then w_D is highest for the N firms of type $j = 1$. So a partnership folk theorem exists with JN partnerships if:

$$\begin{aligned} & \tfrac{\bar{w}D^* + (\lambda_1+...+\lambda_J)b + \frac{1}{N}(\bar{w}+\lambda_j b - w_j^j)}{D^*} + \delta\bar{w} \leq \bar{w} + \delta\bar{w} \\ \Rightarrow \quad & \tfrac{(\lambda_1+...+\lambda_J)b + \frac{c}{N}}{(\lambda_1+...+\lambda_J)b + \frac{\lambda_1 b}{N}} \leq \delta \end{aligned}$$

Notice that the left hand side of this expression is less than one as $\lambda_1 b > c$ by assumption. And that just as in Proposition 7 the left hand side grows larger the greater is N. Dividing both numerator and denominator through by J is also revealing as it implies:

$$\frac{\bar{\lambda_1} b + \frac{c}{JN}}{\bar{\lambda_1} b + \frac{\lambda_1 b}{JN}} \leq \delta$$

So if both mean ability $\bar{\lambda_1}$ and the ability of the lowest type λ_1 stay the same, then an increase in J has exactly the same effect as an increase in N. If $\bar{\lambda_1}$ increases with J then J has a larger impact then N, and if λ_1 falls, then the left hand side grows as well. So increasing types and the number of firms hiring each type have similar but possibly distinct effects on the minimum discount factor necessary for an efficient equilibrium.

4.7 Conclusion

This paper presented a theory of partnerships where partnerships exist to give young agents incentives to exert unobservable effort and make partner, and partnerships bill by the hour in order to commit to hiring juniors. The unusual features observed in partnerships, with juniors working hard despite being paid below their marginal product and partners paid above their marginal product, emerge naturally when partnerships do not have to compete for juniors. And where multiple partnerships do compete for valuable juniors, the efficient equilibrium still obtains using a folk theorem, if the discount factor is high enough and there are not too many firms competing. It was further shown that partnerships will choose to hire fewer, but higher quality juniors as the discount factor rises. But that a more profitable equilibrium obtains for the industry if multiple partnerships each hire a different type of junior. It was again shown that this may occur when the discount factor is high enough using a folk theorem, and that the minimum discount factor necessary rises as the number of types and firms increases (assuming juniors average ability is unchanged).

The equilibrium strategies in the folk theorems are also interesting. It suggests that for traditional partnerships to exist where many firms could compete up junior wages, then juniors must somehow be induced to avoid accepting higher deviating wage offers. Here that was done through the threat of reversion to a competitive equilibrium in the future. But extending the model to include mentoring by partners of juniors, perhaps something similar could also occur if juniors expect partnerships paying high junior wages to invest less in their juniors, making it less likely for them to make partner in the future. That would entail extending the model to incorporate the insights of Morrison and Wilhelm (2004) or those of Bar-Isaac (2007). That is one avenue for future research.

But perhaps in industries where partnerships exist they simply do not have to compete fiercely for juniors. Ultimately, it is an empirical question, and anecdotally it seems likely that partnerships do have considerable power to set junior wages below their marginal product in the firm. Perhaps this is because partnerships with high value clients are relatively scarce. As stated earlier Deloitte, one of the big four accounting firms, had around 10 applicants for every one of its graduate positions.[10] And while these positions are not poorly paid in an objective sense, they are likely to pay less than what is billed to the client, helping to boost profits per partner. The Times for example reported that in each of the four largest UK law partnerships salary per partner was over one million pounds and there were around five junior lawyers per partner in these firms.[11] So while junior salaries may be high, they are dwarfed by profits per partner, and junior lawyers in these firms must be being paid less than their client billings.

A further extension to this paper could also be to agents that possess limited or no information of the history of play before their arrival in the game. In the literature on the folk theorem with overlapping generations of players who have limited information or memory of play before their entry, the major articles are those by Lagunoff and Matsui (2004), Anderlini and Lagunoff (2005), Kobayashi (2007) and originally Bhaskar (1998). In that literature some form of communication or information is required to sustain cooperative play between short-lived members in an

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[10] Jennifer Hughes, Financial Times, 12 February 2008

[11] 8 July 2011

ongoing infinitely repeated game. Perhaps then some form of costly communication with memoryless agents could also achieve a ‘cooperative’ partnership equilibrium in this model. That may be another avenue for future work.

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