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Disposition Effect on Two Classical Expected Utility Models: Exponential and Power

by

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Abstract

A disposition effect is the observation that investors tend to sell winning stocks too early and hold losing stocks too long. In this paper, we investigate whether expected utility theory explains the disposition effect. We implement two models of expected utility theory: exponential and power. We show that for reasonable parameter values the disposition effect can be explained by expected utility theory.

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Contents

Chapter 1: Background **1**
1.1 Introduction 1
1.2 Utility and Expected Utility Theory 1
1.3 The Disposition Effect 3
1.4 Aim of the Dissertation 5
1.5 Organization of the Dissertation 6

Chapter 2: Two Classical Models that Apply Expected Utility Theory **7**
2.1 Optimal Trading Strategy of Expected Exponential Utility Model 7
2.1.1 The Single-Period Model 7
2.1.2 The Multi-Period Model 8
2.2 Optimal Trading Strategy of Expected Power Utility Model 12
2.2.1 The Single-Period Model 12
2.2.2 The Multi-Period Model 13
2.3 The Disposition Effect: Methodology 15

Chapter 3: Summary Of Empirical Results **17**
3.1 Expected Exponential Utility Model 17
3.2 Expected Power Utility Model 22

Chapter 4: Conclusion and Possible Improvement **28**
4.1 A Possible Improvement 28

Appendix A: MATLAB Code for Expected Exponential Utility Model **29**

Appendix B: MATLAB Code for Expected Power Utility Model **32**

List of Tables

3.1 Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.10$, $\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$ 18

3.2 Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.10$, $\sigma = 0.3$, $r = 0$, $N = 20$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$ 19

3.3 Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.10$, $\sigma = 0.3$, $r = 0$, $N = 30$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$ 19

3.4 Expected Exponential Utility Model List of values of PGR and PLR for $\mu = 0.06$, $\sigma = 0.3$, $r = 0$, $N = 20, 50, 100$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$ 20

3.5 Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.06$, $\sigma = 0.05, 0.10, ..., 0.30$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$ 21

3.6 Expected Exponential Utility Model List of values of PGR and PLR for $\mu = 0.08$, $\sigma = 0.3$, $r = 0.01, ...0.07$, $N = 12$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$ 21

3.7 Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.06$, $\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.1, 0.2, 0.3, 0.4$, $S_0 = 20$, $W_0 = 20$ 22

3.8 Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.06$, $\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$ 24

3.9 Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.06$, $\sigma = 0.3$, $r = 0$, $N = 20$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$ 24

3.10 Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.06$, $\sigma = 0.3$, $r = 0$, $N = 30$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$ 24

3.11 Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.05$, $\sigma = 0.3$, $r = 0$, $N = 20, 50, 100$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$. 25

3.12 Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.02$, $\sigma = 0.2, 0.25, ..., 0.4$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$ 26

3.13 Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.05$, $\sigma = 0.3$, $r = 0.01, ..., 0.04$, $N = 12$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$ 26

3.14 Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.05$,$\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.05, 0.10, ..., 0.40$, $S_0 = 20$, $W_0 = 20$ 27

1 Background

1.1 Introduction

Behavior economic has been studied by many disciplines. It incorporates psychology into economics and could be used to explain various things exist in the economy. An example of it can be found in the financial market in which many participants making decisions under risk and uncertainty. Individuals are recognized to have different tastes for time preference of consumption and different degrees of risk aversion. Economic theory recognizes these differences but cannot say much about why they exist and what cause them. This becomes particularly important when trying to predict outcomes in the market. The classical theory of preference represents people as expected utility maximizers. Expected utility theory is one of which been studied by many scholars[4]. We will discuss it in greater details next.

1.2 Utility and Expected Utility Theory

In economics, utility is defined as a measure of the relative satisfaction from, or desirability of, possession or consumption of various goods and services [4]. Very often, economists distinguish between cardinal utility and ordinal utility. Cardinal utility is a way to measure a particular good or service gained; hence its magnitude is important. While ordinal utility is a way to rank a particular good or service received, is a measure of preference [20].

Expected utility theory is one of which been studied by many scholars. In particular, Bernoulli [3], Cramer [5], Neumann and Morgenstern [15] are remembered for their common research interest in rational choice under uncertainty [8]. Cranmer was the first to introduce the notion of expected utility theory in 1728 [5]. But Daniel Bernoulli solved the expected utility model as the St. Petersburg paradox in 1738. He argues that individuals tend to maximize their wealth, i.e. the value of a logarithmic cardinal utility function, rather than to maximize their expected monetary payoff. Hence, the theory can describe realistic scenarios more accurately than expected value alone [3]. The first important use of the expected utility theory was to formulate game theory. Along with four axioms required in the formulation made the expected utility theory become a fundamental tool in the studies of behavior economics.

As stated Neumann and Morgenstern [15], in order to develop the theory of rational

decision making when facing uncertainty, it is necessary to make some precise assumptions about an individual's behavior. Known as the four axioms, which provide the minimum set of conditions for consistent and rational behavior. Once they are established, all the remaining theory must follow. The axioms are consisting of completeness, transitivity, continuity and independence.

1. The completeness axiom of preference states that for any two choice of P and Q, we mush have either $P \prec Q$ (Q is preferred to P), $P \sim Q$ (P is equally preferred to Q) or $P \succ Q$.

2. The transitivity axiom of preference requires that for any choice of P, Q and R, if $P \succeq Q$ (P is preferred at least as much as Q) and $Q \succeq R$, then $P \succeq R$.

3. The continuity of preference says that if $P \preceq Q \preceq R$, then there is a probability $s \in [0, 1]$ such that $sP + (1 - s)R \sim Q$. This axiom virtually tells us that all choices of intermediate preference are preferred the same as the combination of a better and worse choices.

4. The independence axiom claims that if $P \succeq Q$, then $sP + (1-s)R \succeq sQ + (1-s)R$ for any choice of R with probability s. This means that an individual prefers P at least as much as Q when making decision with uncertainty, he mush also prefer $sP + (1 - s)R$ at least as much as $sQ + (1 - s)R$ for any choice R with probability s.

The motivation behind the theory of expected utility is that individuals are assumed to evaluate their wealth in terms of the utility they gain from it. By definition, utility is described by a utility function which itself is just a mathematical representation of individuals' preferences over consumption goods and services. Let $X = \{x : x \text{ is an outcome}\}$ be a set of possible outcomes. An outcome, x_i, is defined as the result of an event, A_i. In general, expected utility theory applied to any type of outcomes, money being a special case. The probability of such event A_i occurring, with an outcome x_i, is given by p_i. A choice $P = (x_i, p_i)$ is a probability distribution defined on outcomes x_i associated with probabilities p_i. $U : X \longmapsto [-\infty, +\infty)$ denotes a utility function such that the value of $U(x)$ is a measure of preferences of individuals derive from the outcome x. x_i or x, y, z refer to the elements of X. Now,

$$x \succeq y \Leftrightarrow U(x) \geq U(y) \tag{1.1}$$

This represents an important property of a utility function, which is order preserving. Intuitively, equation (1.1) indicates that the utility function is a monotonic function of wealth. It should grow, with growth of wealth usefulness that individuals can extract from it, therefore ones may usually limit to consideration of continuous and increasing utility functions only. Moreover, people differ in how much they are willing to take risks. Economists often express one's willingness to take risk through a utility function of money. Expected utility theory takes account of it and assumes that people are risk

averse[17]. Mathematically, this implies that the utility function is concave. It can be described as for any $\alpha \in (0,1)$ and $x, y \in X$,

$$U(\alpha x + (1-\alpha)y) \geq \alpha U(x) + (1-\alpha)U(y). \tag{1.2}$$

where x is relatively large, and the small gradient of the concave utility function means the marginal utility of a unit increase of monetary gain diminishes as individuals become wealthier. To see why the utility functions are concave, consider a simple example of an extra marginal utility obtained by an acquisition of one additional pound. For someone who only has one pound to begin with, gaining one more pound is quite important. For someone who already has a million pounds, gaining one more pound is virtually meaningless. Concavity implies the decrease of the derivative of the utility function. The shape of the utility function represents the degree of risk aversion [4].

In this paper we consider two classical expected utility models with exponential utility function and power utility function, respectively. They are stated as follows:

$$U(x) = 1 - \exp(-\gamma x) \tag{1.3}$$

with $\gamma > 0$.

$$U(x) = \frac{1}{\gamma} x^{\gamma} \tag{1.4}$$

with $0 < \gamma < 1$. x denotes an investor's wealth.

By definition of pratt-arrow measure which is the attribution of a utility function, we can then calculate the absolute risk aversion (ARA) for the exponential utility function and relative risk aversion (RRA) for the power utility function. They are stated as follows:

$$ARA = -\frac{U'(x)}{U''(x)} \tag{1.5}$$

$$RRA = -\frac{xU'(x)}{U''(x)} \tag{1.6}$$

where $U(x)$ is the utility function of wealth x. Therefore, the ARA and RRA are all equal to γ in both cases.

1.3 The Disposition Effect

It is evident that individuals' behavior is largely different in finance. A great deal of efforts has been made to find out how behavior differs systematically from the normative models of standard finance theory. The disposition effect is one of the better documented

behavior patterns in many literatures [2]. In 1985, Shefrin and Statman [19] describe the disposition effect as the tendency to sell winners too early and ride losers too long, where the terms "winners" and "losers" refer to assets that have appreciated or depreciated since the time of purchase. Suppose an investor holds one share in his/her portfolio. If he/she believes that the price of the share will increase, we would think the share should be kept. If the investor believes that the price will decrease, we would think the share should be sold. This means that the point from which the investors make their decisions to sell is the current share price. The disposition effect arises from the fact that investors do not measure their expected gains and losses from the current price. Instead, actual gains and losses from some reference point in a finance setting are evaluated [19]. Many empirical studies show that this puzzling investor trading behavior is present across the markets.

Lakonishok and Smidt [12] test abnormal volume for NYSE and Amex stocks that increase or decrease in price over time length of 5, 11, 23 and 35 months. The results suggest more volume for winners. Ferris et al. [7] look at both price and volume data for thirty U.S. stocks. But the results cannot explain the disposition effect. Odean [16] has done the most convincing study using real data from U.S. stock markets. He studies the accounts of $10,000$ discount brokerage investors over the period of $1987 - 1993$. He finds a disposition effect in their stock market investments and all the effects are hugely significant due to large sample size and independence of individual investors. Each day when a sale takes place in a portfolio contains shares of stocks, each stock in that portfolio on that day falls into one of four categories: realized gains, paper gains, realized losses and paper losses. For every stock in that portfolio on the day that is sold, a "realized gain" is counted if the stock price exceeds the average price at which the shares were bought, and a "realized losses" is counted otherwise. For every stock in that portfolio on the day that is not sold, a "paper gain" is counted if the stock price exceeds the average price at which the shares were bought, and a "paper loss" is counted otherwise. Since the total number of realized gains and paper gains across all accounts for the whole sample set can be counted, the author computes the proportion of gains realized (PGR) as

$$PGR = \frac{\text{no. of realized gains}}{\text{no. of realized gains + no. of paper gains}} \tag{1.7}$$

Likewise, the proportion of losses realized (PLR)is calculated as

$$PLR = \frac{\text{no. of realized losses}}{\text{no. of realized losses + no. of paper losses}} \tag{1.8}$$

For the entire year, investors realize about 23% of the gains they could realize by selling. In contrast, they realize only 16% of their losses. The only exception is December in which investors realize losses at an even higher rate compare to realize gains. This suggests that the only time investors may not exhibit a disposition effect is at the year-end, when there is a tax advantage to selling. Moreover, the author examines the hypothesis

of investors are rational to keep losers and sell winners. The results suggest that the hypothesis is false: for winners that are sold, the average excess return is 3.4% higher than it is for losers that are not sold over the subsequent year.

In other empirical studies, the disposition effect has been documented in many other settings. Heisler [11] finds the disposition effect among a group of futures traders, Locke and Mann [13] among professional futures traders as well as those advised by brokers, Weber and Camerer [26] in an experimental setting and Genesove and Mayer [9] among individual home owners. Also, there are other facts relative to the disposition effect are discovered. For example, Ranguelova [18] finds that the effect is significant among large stocks. Dhar and Zhu [6] find evidence consistent with the idea of investors who are more capable of analytically processing information imply less disposition effect.

Moreover, Shefrin and Statman [19] explain how the disposition effect emerges in the prospect theory developed by Kahneman and Trversky [21]. Consider an investor who purchased stock at 70 pounds one week ago and who finds that the stock is now selling at 60 pounds. The investor must now decide whether he/she wants to hold the stock for one more week or realized the loss now. Note that it is assumed to be no taxes. In addition, let us suppose that one of the two following outcomes will occur during the coming week: either the stock price will increase by 10 pounds or decrease by 10 pounds. According to prospect theory, the investor would frame his/her choice as one of the following two:

A Sell the stock now, realizing a 10 pounds loss

B Hold the stock for one more week, given there is 50-50 chance between losing an extra 10 pounds or break-even.

Prospect theory implies that **B** will be selected over **A**, meaning the investor will hold his/her losing stock. This argument demonstrates why prospect theory gives rise to the disposition effect. Barberis and Xiong [2] show that, in simple two-period settings with a prospect theory functional form for utility, realized gains and losses can predict a disposition effect. they point out one extra ingredient, which is a prospect theory functional form for utility. This type of utility function is concave over gains and convex over losses. Such functional form explains the expediting of gains and the postponements of losses.

1.4 Aim of the Dissertation

In this paper, we study the trading behavior of an investor with expected utility theory. So far, no study has been done by this classical asset allocation model. We consider two implementations of the expected utility theory: exponential and power. In both models, it is not possible to obtain an analytical solution for the optimal trading strategy, I

generate an approximated solution via numerical simulation approach instead, and then to check, using Odean's [16] methodology, whether expected utility theory explains the disposition effect for reasonable parameter values. We pay particular attention to how the results depend on the appreciation rate μ, the volatility rate σ and the number of trading periods N.

1.5 Organization of the Dissertation

In Chapter 1, I review the elements of expected utility theory, as well as the evidences and interpretations on the disposition effect. Theoretical Framework is then set up in Chapter 2. The optimal trading strategy of expected exponential utility model and expected power utility model are obtained, for single- and multi-period of time. In Chapter 3, we analyze the two implementations: expected exponential utility model and expected power utility model. Chapter 4 concludes and gives a possible improvement for our study.

2 Two Classical Models that Apply Expected Utility Theory

2.1 Optimal Trading Strategy of Expected Exponential Utility Model

Consider a portfolio choice setting with $N+1$ step, where N is the number of trading periods whenever the investor trades his/her portfolio. Let T be the maturity date in years and $t = 0,T/N,2T/N,...,T$ be the date of trading with equal increment between 0 and T. Denote $h = T/N$ be the time interval. For simplicity, we use $h = 1$ to represent the time interval of one year when the investor manages his/her portfolio. For each investor, let us assume that he/she trades his/her portfolio for T years. We consider a market consisting out of two assets, a risk-free asset following a price process $B = \{B_t = \exp(rt), t \geq 0\}$, where r is the constant continuously compounded interest rate, and a risky asset, i.e. a stock, following a geometric Brownian motion price process $S = \{S_t, t > 0\}$ such that

$$dS_{t+1} = \mu S_t dt + \sigma S_t dw_t \tag{2.1}$$

where μ is the appreciation rate of the stock, σ is the volatility of the stock and w_t represents the Brownian motion with mean zero and variance t. Here we assume that the asset pays out no dividend.

$$S_{t+1} = S_t \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)h + \sigma\sqrt{h}\xi_t\right) \tag{2.2}$$

$$= S_t \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) + \sigma\xi_t\right) \tag{2.3}$$

where ξ_t are independent and identically normally distributed random variables with mean zero and variance one for all $t = 0,1,...,T$. Let us also assume that μ and σ are constant in this chapter. The risky asset has gross return $1 + R_{t,t+1} = S_{t+1}/S_t$ for time t to $t+1$. Finally let us define (F_t) is a filtration and F_t is σ-algebra generated by the stock with price $S_1, S_2, ..., S_t$ i.e. $F_t = \sigma(S_1, S_2, ..., S_t)$.

2.1.1 The Single-Period Model

Let us take $N = 1$ and $T = 1$, giving $h = 1$. Let W_1 be the investor's total wealth at time 1. The exponential utility model is of the following form:

$$U(W_1) = 1 - \exp(-\gamma W_1) \tag{2.4}$$

where γ is the Pratt-Arrow coefficient of absolute risk aversion and $\gamma > 0$.

At time 0, in order to maximize the expected utility function of final wealth the investor must decide how to split his/her wealth between the risk-free asset and the risky asset. Let the amount of money invested in stock market be π_0 at time 0 (at the beginning of the period). My goal for this asset allocation problem is to find the optimal π_0 so that it maximizes $E[U(W_1)]$ subject to the budget constraint:

$$W_1 = W_0 \exp(r) + \pi_0 (1 + R_{0,1} - \exp(r)) \tag{2.5}$$

Given the model is of form (2.4) we have:

$$\begin{aligned} \max_{\pi_0} E[U(W_1)] &= \max_{\pi_0} E[1 - \exp(\gamma W_1)] && (2.6) \\ &\Leftrightarrow 1 - \min_{\pi_0} E[\exp(-\gamma W_1)] && (2.7) \end{aligned}$$

Now the problem becomes to find π_0 which minimizes the $E[\exp(-\gamma W_1)]$. Applying the budget constraint to the equation $E[\exp(-\gamma W_1)]$ we have:

$$\min_{\pi_0} E[\exp(-\gamma W_1)] = \exp(-\gamma W_0 \exp(r)) \min_{\pi_0} E[\exp(-\gamma \pi_0 (1 + R_{0,1} - \exp(r)))]) \tag{2.8}$$

By definition, an optimal trading strategy is one that generates the optimal wealth allocations. In a complete market, such trading strategy always exists. Thus, we assume that there exists a π_0^* such that:

$$\min_{\pi_0} E[\exp(-\gamma W_1)] = \exp(-\gamma W_0 \exp(r)) E[\exp(-\gamma \pi_0^* (1 + R_{0,1} - \exp(r)))] \tag{2.9}$$

Since we know from Section 2.1 that $1 + R_{0,1} = \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) - \sigma\xi_t\right)$ is log-normal distributed, it follows from equation (2.9) that:

$$\min_{\pi_0} E[\exp(-\gamma W_1)] = \exp(-\gamma W_0 \exp(r)) E[\exp(-\gamma \pi_0^* (\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) - \sigma\xi_0\right) - \exp(r))] \tag{2.10}$$

Unfortunately, it is not possible to find the exact solution of π_0^* analytically. Instead, an approximated solution is generated via numerical method (see Appendix A).

2.1.2 The Multi-Period Model

For multi-period model, we assume that the investor's trading horizon is divided into $N > 1$ periods. At the end of each period the return of the held portfolio materializes and the investor can make a new decision on the allocation of his/her wealth. In order to obtain a simpler form of the result, let us set $T = N$.

Similar to the single-period case, we start from defining some notations. Let π_t be the amount of money invested in the risky asset at time t (at the beginning of $(t+1)$-th period), and W_t be the total wealth at time t.

There are two distinguishable ways to state the portfolio allocation problem. In one strategy each period is considered as it would be the last one, namely a myopic strategy. Under this strategy, the investor behaves in such a way that he/she makes the decision at each period only on the knowledge of the period initial wealth and the probability distribution of the risky asset return. He/she aims to maximize the expected utility of his/her final wealth in that period while disregarding the future completely. In another strategy the investor maximizes the expected utility of his/her future wealth as well as taking into account forecasts of future stock return for the entire time horizon. And he/she reallocates resources every next period according to the information. The objective is to find an optimal trading strategy $\pi_0, ..., \pi_{T-1}$ that maximizes the expected utility of terminal wealth, i.e. $\max_{\pi_t, 0\leq t\leq T-1} E[U(W_T)]$ for given initial wealth W_0. This strategy is often called non-myopic strategy. In this paper we would only consider the non-myopic strategy, and this problem is solved via the dynamic programming method [14].

At the beginning of the last period, the investor's problem is simply to divide the wealth W_{T-1} among the risky asset and risk-free asset so that $E_{T-1}[U(W_T)]$ is maximized. Let E_t denotes the expectation conditional on $S_0, ..., S_t$. In general, the optimal decision depends on W_{T-1}, i.e:

$$\max_{\pi_{T-1}} E_{T-1}[U(W_T)] = J_{T-1}(W_{T-1}) \tag{2.11}$$

Here the function J_{T-1} is called indirected utility function. The optimal decision at the beginning of the period $T-1$ is obtained by maximizing

$$E_{T-2}[J_{T-1}(W_{T-1})] = E_{T-2}[\max_{\pi_{T-1}} E_{T-1}(U(W_T))] \tag{2.12}$$

and the procedure is applied recursively. Define the indirected utility functions as follows:

$$J_t(W_t) = \max_{\pi_t} E_t[J_{t+1}(W_{t+1})] \tag{2.13}$$

for $t = 0, ..., T-1$ and $J_T(W_T) = U(W_T)$. Thus, by the Bellman principle of optimality we have

$$J_0(W_0) = \max_{\pi_0, ...\pi_{T-1}} E[U(W_T)] \tag{2.14}$$

Now the asset allocation problem for given exponential utility model becomes to find $\pi_0, ..., \pi_{T-1}$ which maximizes $E[U(W_T)]$ when $\gamma > 0$ subject to the budget constraint for $t = 1, ...T$

$$W_t = W_{t-1}\exp(r) + \pi_{t-1}(1 + R_{t-1,t} - \exp(r)) \tag{2.15}$$

To simplify this, at time T-1 we have:

$$\max_{\pi_{T-1}} E_{T-1}[U(W_T)] = \max_{\pi_{T-1}} E_{T-1}[1 - \exp(\gamma W_T)] \tag{2.16}$$

$$\Leftrightarrow \quad 1 - \min_{\pi_{T-1}} E_{T-1}[\exp(-\gamma W_T)] \tag{2.17}$$

Thus, at time $T-1$ the problem becomes to find π_{T-1} which minimizes $E_{T-1}[\exp(-\gamma W_T)]$. In order to apply the dynamic programming method stated above I define indirect utility function $J_{T-1}(W_{T-1})$ as follows:

$$J_{T-1}(W_{T-1}) = \min_{\pi_{T-1}} E_{T-1}[\exp(-\gamma W_T)] \tag{2.18}$$

At time T-2:

$$\max_{\pi_{T-2}} E_{T-2}[\max_{\pi_{T-1}} E_{T-1}(U(W_T))] \Leftrightarrow \max_{\pi_{T-2}} E_{T-2}[1 - \min_{\pi_{T-1}} E_{T-1}[\exp(-\gamma W_T)] \tag{2.19}$$

$$\Leftrightarrow \max_{\pi_{T-2}} E_{T-2}[1 - J_{T-1}(W_{T-1})] \tag{2.20}$$

$$\Leftrightarrow \quad 1 - \min_{\pi_{T-2}} E_{T-2}[J_{T-1}(W_{T-1})] \tag{2.21}$$

Hence the problem at time $T-2$ becomes to find π_{T-2} which minimizes the $E_{T-2}[J_{T-1}(W_{T-1})]$. Define J_{T-2} as:

$$J_{T-2}(W_{T-2}) = \min_{\pi_{T-2}} E_{T-2}[J_{T-1}(W_{T-1})] \tag{2.22}$$

Recursively, the problem now is to find $\pi_0, ..., \pi_{T-1}$ which minimizes $E[\exp(-\gamma W_T)]$
Thus, at time T-1 we have:

$$\min_{\pi_{T-1}} E_{T-1}[\exp(-\gamma W_T)] \tag{2.23}$$

Subject to the budget constraint for $t = 1, ..T$

$$W_t = W_{t-1} \exp(r) + \pi_{t-1}(1 + R_{t-1,t} - \exp(r)) \tag{2.24}$$

By applying the budget constraint(2.24) to (2.23), we get:

$$\min_{\pi_{T-1}} E_{T-1}[\exp(-\gamma W_T)] \tag{2.25}$$

$$= \min_{\pi_{T-1}} E_{T-1}[\exp(-\gamma(W_{T-1} \exp(r) + \pi_{T-1}(1 + R_{T-1,T} - \exp(r))))] \tag{2.26}$$

$$= \exp(-\gamma W_{T-1} \exp(r)) \min_{\pi_{T-1}} E_{T-1}[\exp(-\gamma \pi_{T-1}(1 + R_{T-1,T} - \exp(r)))] \tag{2.27}$$

The last equality is valid because $\exp(-\gamma W_{T-1} \exp(r))$ is measurable under the filtration at time $T-1$. Since it is always true that there exists an optimal trading strategy in a complete market, I assume that there is π^*_{T-1} follows (2.27) such as

$$(2.27) = \exp(-\gamma W_{T-1} \exp(r)) E_{T-1}[\exp(-\gamma \pi^*_{T-1}(1 + R_{T-1,T} - \exp(r)))] \tag{2.28}$$

Since $1 + R_{T-1,T} = \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) - \sigma\xi_{T-1}\right)$ is log-normal distributed and constant μ, σ and ξ_t is independent and identically distributed with mean 0 and variance 1, $\exp(\pi^*_{T-1}(1 + R_{T-1,T} - \exp(r)))$ is independent with the information available at time $T-1$ and

$$(2.28) = \exp(-\gamma W_{T-1}\exp(r))E[\exp(-\gamma\pi^*_{T-1}(\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) - \sigma\xi_{T-1}\right) - \exp(r)))] \tag{2.29}$$

Again it is not possible to find the analytical solution for π^*_{T-1}. Instead, we compute its approximation via numerical method (see Appendix A). In order to obtain a nice and clear recursively implicit formula for π^*_t, we denote that $K_{T-1} = E[\exp(-\gamma\pi^*_{T-1}(\exp((\mu - \frac{1}{2}\sigma^2) - \sigma\xi_{T-1}) - \exp(r)))]$ so that

$$J_{T-1}(W_{T-1}) = \exp(-\gamma W_{T-1}\exp(r))K_{T-1} \tag{2.30}$$

Then, at time T-2 we have

$$J_{T-2}(W_{T-2}) = \min_{\pi_{T-2}} E_{T-2}[\exp(-\gamma W_{T-1}\exp(r))K_{T-1}] \tag{2.31}$$

Subject to budget constraint

$$W_{T-1} = W_{T-2}\exp(r) + \pi_{T-2}(1 + R_{T-2,T-1} - \exp(r)) \tag{2.32}$$

Apply (2.32) on (2.31) we get

$$J_{T-2}(W_{T-2}) = \min_{\pi_{T-2}} E_{T-2}[\exp(-\gamma(W_{T-2}\exp(r) + \pi_{T-2}(1 + R_{T-2,T-1} - \exp(r))\exp(r))K_{T-1}] \tag{2.33}$$

$$= \exp(-\gamma W_{T-2}\exp(2r))K_{T-1}\min_{\pi_{T-2}} E_{T-2}[\exp(-\gamma\pi_{T-2}(1 + R_{T-2,T-1} - \exp(r))\exp(r)] \tag{2.34}$$

Let us assume that there exists a π^*_{T-2} such that

$$J_{T-2}(W_{T-2}) = \exp(-\gamma W_{T-2}\exp(2r))K_{T-1}E_{T-2}[\exp(-\gamma\pi^*_{T-2}(1 + R_{T-2,T-1} - \exp(r))\exp(r))] \tag{2.35}$$

Denote $\gamma_t = \gamma\exp((T-t)r)$ and we have

$$J_{T-2}(W_{T-2}) = \exp(-\gamma_{T-2}W_{T-2})K_{T-1}E_{T-2}[\exp(-\gamma_{T-1}\pi^*_{T-2}(1 + R_{T-2,T-1} - \exp(r)))] \tag{2.36}$$

Since $1 + R_{T-2,T-1} = \exp((\mu - \frac{1}{2}\sigma^2) - \sigma\xi_{T-2}$ is log-normal distributed and constant μ, σ and ξ_t is independent and identically distributed with mean 0 and variance 1,

$\exp(-\gamma_{T-1}\pi^*_{T-2}(1+R_{T-2,T-1}-\exp(r)))$ is independent of filtration at time $T-2$. Denote $K_{T-2} = E[\exp(-\gamma_{T-1}\pi^*_{T-2}(1 + R_{T-2,T-1} - \exp(r)))]$ and we have:

$$J_{T-2}(W_{T-2}) = \exp(-\gamma_{T-2}W_{T-2})K_{T-1}K_{T-2} \tag{2.37}$$

Recursively, we have:

$$J_0(W_0) = \exp(-\gamma_0 W_0) \prod_{i=0,...T-1} K_i \tag{2.38}$$

for $i = 0, ...T-1$

$$K_i = E[\exp(-\gamma_{i+1}\pi^*_i(1 + R_{i,i+1} - \exp(r)))] \tag{2.39}$$

Hence I obtain a series of implicit formula for optimal trading strategy $\pi^*_0, ..\pi^*_{T-1}$ which maximizes the expected utility of final wealth $E[U(W_T)]$. Since it is not possible to find the solution analytically, its approximation is generated via numerical method (see Appendix A).

2.2 Optimal Trading Strategy of Expected Power Utility Model

Similar to the approach used in the exponential utility model, first of all, we define some notations. Consider a portfolio choice setting with $N+1$ step, where N is the number of trading periods. Let T be the maturity date in years and $t = 0,T/N,2T/N,...,T$ be the trading date with equal increment between 0 and T. Denote $h = T/N$ be the time interval with equal increment. For simplicity, we use $h = 1$ meaning the investor manages his/her portfolio at the time interval of one year. Again, we assume the market is complete, and it consists of two assets, a risk-free asset following a price process $B = \{B_t = \exp(rt), t \geq 0\}$ and a risky asset, i.e. a stock, following a geometric Brownian motion price process $S = \{S_t, t > 0\}$ such that $dS_{t+1} = \mu S_t dt + \sigma S_t dw_t$ where μ is the appreciation rate of a stock, σ is the volatility of a stock, and w_t represents a Brownian motion with mean 0 and variance t. Here I assume that the asset pays out no dividend. Again, $S_{t+1} = S_t \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) - \sigma\xi_t\right)$ where ξ_t is independent and identically normally distributed random variable with mean 0 and variance 1 for all $t = 0, 1, 2, ..., T$. Let us assume μ and σ are constant. The gross return of the risky asset is $1 + R_{t,t+1} = \frac{S_{t+1}}{S_t}$ for time t to $t+1$. Finally, let us define the (F_t) to be a filtration where F_t is σ-algebra generated by stock with price $S_0, ..., S_t$, i.e. $F_t = \sigma(S_1, S_2, ..., S_t)$. .

2.2.1 The Single-Period Model

Let $T = 1$ and W_1, be the investor's total wealth at time 1. The expected power utility model is

$$U(W_1) = \frac{1}{\gamma}W_1^\gamma \tag{2.40}$$

where γ is the Pratt-Arrow coefficient of relative risk aversion and $0 < \gamma < 1$.

At time 0, the investor must decide how to split his/her wealth between the risk-less asset and the risky asset. Define the proportion of total wealth invested in the stock market is θ_0 at time 0 (at the beginning of the period). The object of the asset allocation problem is to find the optimal trading strategy (in proportion) θ_0 so that it maximizes $E[U(W_1)]$ subject to the budget constraint:

$$\begin{aligned} W_1 &= W_0(1-\theta_0)\exp(r) + W_0\theta_0(1+R_{0,1}) && (2.41)\\ &= W_0(\exp(r) + \theta_0(1+R_{0,1}-\exp(r))) && (2.42)\end{aligned}$$

Apply this budget constraint to the model we get:

$$\begin{aligned} \max_{\theta_0} E[U(W_1)] &= \max_{\theta_0} E[\frac{1}{\gamma}(W_0(\exp(r)+\theta_0(1+R_{0,1}-\exp(r))))^\gamma] && (2.43)\\ &= \frac{1}{\gamma}W_0^\gamma \max_{\theta_0} E[(\exp(r)+\theta_0(1+R_{0,1}-\exp(r)))^\gamma] && (2.44)\end{aligned}$$

The last equality is valid because $\frac{1}{\gamma}W_0^\gamma$ is measurable under the filtration F_0. Since the market is assumed to be complete, we could always find an optimal θ_0^* such that

$$\max_{\theta_0} E[U(W_1)] = \frac{1}{\gamma}W_0^\gamma E[(\exp(r)+\theta_0^*(1+R_{0,1}-\exp(r)))^\gamma] \quad (2.45)$$

where $1+R_{0,1} = \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) - \sigma\xi_0\right)$ is log-normal distributed. Unfortunately, it is not possible to find such θ_0^* analytically. Instead, we generate its approximation via numerical method (see Appendix B).

2.2.2 The Multi-Period Model

Similar to the case of expected exponential utility model, we assume that the investor's trading horizon is divided into $N > 1$ periods. Other notations are defined as follows

- θ_t, the proportion of total wealth invested in the risky asset at time t (at the beginning of (t+1)-th period.
- W_t, the total wealth at time t.

The objective is to find the optimal non-myopic trading strategy $\theta_0, ..., \theta_{T-1}$(in proportion) which maximizes $E[U(W_T)]$, given the power utility model is:

$$U(W_T) = \frac{1}{\gamma}W_T^\gamma \quad (2.46)$$

and $0 < \gamma < 1$ subject to the budget constraint for $t = 0, ..., T$:

$$\begin{aligned} W_t &= W_{t-1}(1-\theta_{t-1})\exp(r) + W_{t-1}\theta_{t-1}(1+R_{t-1,t}) && (2.47)\\ &= W_{t-1}(\exp(r) + \theta_{t-1}(1+R_{t-1,t}-\exp(r))) && (2.48)\end{aligned}$$

Given indirected utility functions $J_t(W_t)$, I apply the dynamic programming method stated in Section 2.1 to get,

$$
\begin{aligned}
J_{T-1}(W_{T-1}) &= \max_{\theta_{T-1}} E_{T-1}[U(W_T)] && (2.49)\\
&= \max_{\theta_{T-1}} E_{T-1}[\frac{1}{\gamma}W_T^{\gamma}] && (2.50)\\
&= \max_{\theta_{T-1}} E_{T-1}[\frac{1}{\gamma}(W_{T-1}(\exp(r)+\theta_{T-1}(1+R_{T-1,T}-\exp(r))))^{\gamma}] && (2.51)\\
&= \frac{1}{\gamma}W_{T-1}^{\gamma}\max_{\theta_{T-1}} E_{T-1}[(\exp(r)+\theta_{T-1}(1+R_{T-1,T}-\exp(r)))^{\gamma}] && (2.52)\\
&= \frac{1}{\gamma}W_{T-1}^{\gamma}\max_{\theta_{T-1}} E[(\exp(r)+\theta_{T-1}(1+R_{T-1,T}-\exp(r)))^{\gamma}] && (2.53)\\
&= \frac{1}{\gamma}W_{T-1}^{\gamma}E[(\exp(r)+\theta^*_{T-1}(1+R_{T-1,T}-\exp(r)))^{\gamma}] && (2.54)\\
&= \frac{1}{\gamma}W_{T-1}^{\gamma}L_{T-1} && (2.55)
\end{aligned}
$$

Note that in (2.52) I take out $\frac{1}{\gamma}W_{T-1}^{\gamma}$ since it is measurable under filtration at time $T-1$. Despite $1+R_{T-1,T}=\exp\left(\left(\ \mu-\frac{1}{2}\sigma^2\right)-\sigma\xi_{T-1}\right)$ is independently log-normal distributed, taking expectation under filtration at time $T-1$ is equivalent to taking expectation under filtration at time 0. Thus, (2.53) follows. In a complete market, it is assumed that we can always find an optimal trading strategy θ^*_{T-1} which the solution is computed approximated via numerical method (see Appendix B). To find the implicit formula for θ^*_t for $t=1,...,T$, let us denote $L_{T-1}=E[(\exp(r)+\theta^*_{T-1}(1+R_{T-1,T}-\exp(r)))^{\gamma}]$. At time $T-2$ (at the beginning of period), the implicitly formula for the optimal trading decision θ^*_{T-2} is obtained

$$
\begin{aligned}
J_{T-2}(W_{T-2}) &= \max_{\theta_{T-2}} E_{T-2}[\frac{1}{\gamma}W_{T-1}^{\gamma}L_{T-1}] && (2.56)\\
&= \max_{\theta_{T-2}} E_{T-2}[\frac{1}{\gamma}(W_{T-2}(\exp(r)+\theta_{T-2}(1+R_{T-2,T-1}-\exp(r))))^{\gamma}L_{T-1}] && (2.57)\\
&= \frac{1}{\gamma}W_{T-2}^{\gamma}L_{T-1}\max_{\theta_{T-2}} E_{T-2}[((\exp(r)+\theta_{T-2}(1+R_{T-2,T-1}-\exp(r))))^{\gamma}] && (2.58)\\
&= \frac{1}{\gamma}W_{T-2}^{\gamma}L_{T-1}E_{T-2}[((\exp(r)+\theta^*_{T-2}(1+R_{T-2,T-1}-\exp(r))))^{\gamma}] && (2.59)\\
&= \frac{1}{\gamma}W_{T-2}^{\gamma}L_{T-1}E[((\exp(r)+\theta^*_{T-2}(1+R_{T-2,T-1}-\exp(r))))^{\gamma}] && (2.60)\\
&= \frac{1}{\gamma}W_{T-2}^{\gamma}L_{T-1}L_{T-2} && (2.61)
\end{aligned}
$$

Note that, for simplicity, we denote $E[((\exp(r) + \theta^*_{T-2}(1 + R_{T-2,T-1} - \exp(r))))^\gamma]$ by L_{T-2}. Recursively, we have:

$$J_0 W_0 = \max_{\theta_0 \ldots \theta T-1} E[U(W_T)] \tag{2.62}$$

$$= \frac{1}{\gamma} W_0^\gamma \prod_{i=0,\ldots,T-1} L_i \tag{2.63}$$

where $L_i = E[((\exp(r) + \theta^*_i(1 + R_{i,i+1} - \exp(r))))^\gamma]$ for $i = 0, ..., T-1$ and θ^*_i is implicitly stated. Since μ, σ are constant, and ξ_i is independent and identically distributed, we can obtain the same θ^*_i which maximizes the L_i for $i = 0, ..., T-1$ and we will denote it as θ^*. Since it is not possible to solve it analytically, numerical method is adopted in order to generate an approximated solution instead.

2.3 The Disposition Effect: Methodology

In this paper, we follow the method proposed by Odean [16]. We say that the two models of expected utility theory explain the disposition effect if $PGR > PLR$. For each investor, we look at $T-1$ trading periods (since we use the time interval of one year, we have $N = T$). Let $t = 1,..., T-1$. On each date t belongs to $\{1, ..., T\}$ that a sale takes place in a portfolio contains shares of stocks, each stock in that portfolio on that day falls into one of four categories. Here we use the average purchase price as the reference point for the stock, whereas Odean [16] states that the highest purchase price, the first purchased or the most recent purchased price could also be reference point. For the stock in that portfolio on data t that is sold, a realized gain is counted if the stock price exceeds the average price at which the stock is purchased and a realized loss is counted otherwise. For the stock in that portfolio on date t that is not sold, we count this as a paper gain if the stock price exceeds the average price at which the stock is purchased and a paper loss is counted otherwise. We then count up the total number of realized gains and losses and paper gains and losses across of the investor for all trading periods and compute PGR and PLR as follows:

$$PGR = E\left(\frac{\sum_{t=1,\ldots,T-1} 1_{S_t > Saverage_t, x_t < x_{t-1}}}{\sum_{t=1,..,T-1} 1_{S_t > Saverage_t, x_t < x_{t-1}} + \sum_{t=1,..,T-1} 1_{S_t > Saverage_t, x_t > x_{t-1}}}\right) \tag{2.64}$$

$$PLR = E\left(\frac{\sum_{t=1,\ldots,T-1} 1_{S_t < Saverage_t, x_t < x_{t-1}}}{\sum_{t=1,..,T-1} 1_{S_t < Saverage_t, x_t < x_{t-1}} + \sum_{t=1,..,T-1} 1_{S_t < Saverage_t, x_t > x_{t-1}}}\right) \tag{2.65}$$

where x_t is the optimal number of shares held by the investor at time t, S_t is the price of the stock at time t, $Saverage_t$ is the average price of the stocks held by the investor at time t, and $1_{...}$ is the indicator function. For given price of the stock S_0 at time 0,

the average price, $Saverage_t$ is defined as follows:

$$Saverage_1 = \begin{cases} S_0 & if \quad x_1 < x_0 \\ \frac{x_0 S_0 + (x_1 - x_0) S_1}{x_1} & otherwise \end{cases} \tag{2.66}$$

$$Saverage_t = \begin{cases} Saverage_{t-1} & if \quad x_t < x_{t-1} \\ \frac{x_{t-1} Saverage_{t-1} + (x_t - x_{t-1}) S_t}{x_t} & otherwise \end{cases} \tag{2.67}$$

for $t = 2, ..., T-1$.
Unfortunately, given the value of parameters $\mu, \sigma, T, S_0, W_0, r, \gamma, N$ in expected exponential and power utility models, it is not possible to find an analytical solution of PGR and PLR. Instead, we are going to generate an approximated solution of the two via numerical method (see Appendix A,B).

3 Summary Of Empirical Results

In order to get more reliable results, we take M2 as large as possible (with limited CPU speed) when implementing the numerical method in MATLAB (see Appendix A,B). Simulate sample path of 100,000 and 10,000 of stock price which individual investor's trading strategy follows. The larger sample size is used in the simulation corresponding to the expected exponential utility model and the smaller sample size is used corresponding to the expected power utility model. Given the value of parameters S_0, W_0, μ, σ, r, γ, N, T, generate the average value of PGR and PLR. For reasonable parameter values, we say that the model of expected utility theory explains the disposition effect if $PGR > PLR$. It is also of interest to investigate how the results depend on each one of the parameters. Note that in this paper I refer the difference between the value of PGR and PLR as the degree of disposition effect.

3.1 Expected Exponential Utility Model

I wish to investigate how the disposition effect depends on the value of parameters S_0, W_0, μ, σ, r, γ, N, T in the expected exponential utility model. I start with the appreciation rate of the stock μ. For the value of absolute risk aversion of $\gamma = 0.1$, the utility function is fairly concave at around the investor's final wealth of 20. For this reason, I set the initial wealth of individual investor in his/her portfolio to be $W_0 = 20$ and the initial price of the stock to be $S_0 = 20$. Since we only investigate the trading behavior of individual investor within one year, the maturity time is set to be $T = 1$. In addition, set the risk-free interest rate $r = 0$. Since the price of the stock follows a geometric Brownian motion stated in (2.1), its gross return from t to $t+1$ is as follows:

$$1 + R_{t,t+1} = S_{t+1}/S_t = \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)h + \sigma\sqrt{h}\xi_t\right) \tag{3.1}$$

For time interval $h = 1$, the expected gross asset return is

$$\begin{aligned} E[1 + R_{t,t+1}] = &\quad E[\exp\left(\left(\mu - \tfrac{1}{2}\sigma^2\right) + \sigma\xi_t\right)] &(3.2)\\ = &\quad \exp(\mu) \simeq (1+\mu) &(3.3)\end{aligned}$$

where ξ_t is normally distributed with mean 0 and variance 1. It is also known that the gross return of risk-free asset is $1 + r$. Thus, I would only consider the case in which the appreciation rate $\mu > r$. The variance of gross return of the stock is

$$\begin{aligned} E[(1 + R_{t,t+1})^2] - (E[1 + R_{t,t+1}])^2 = &\quad E[\exp\left(\left(2\mu - \sigma^2\right) + 2\sigma\xi_t\right)] - \exp(2\mu) &(3.4)\\ = &\quad \exp(2\mu + \sigma^2) - \exp(2\mu) &(3.5)\end{aligned}$$

Thus, I set the volatility rate of the stock to be $\sigma = 0.3 > 0$. In order to get more reliable results, three sets of value of PGR and PLR for μ from 0.01 to 0.10 are generated when $N = 12$, 20, 30 (see Table 3.1-3.3). Clearly, the results indicate that there is a disposition effect for all values of μ and N due to $PGR > PLR$, which means that the investor has more propensity to sell the stock after a gain rather than a loss. Also, we find that the degree of disposition effect becomes slightly strong as μ increases. This is because the value of PGR increases slightly and the value of PLR decreases, as μ increases. Although there are a couple of outliers found, the approximation error is considered to be insignificant. The decreasing trend in PLR implicates that the investor tends to have more propensity to hold a stock with a paper loss rather than sell some shares of it with realized losses as μ increases. This is possibly because the probability of the investor ending up with a gain is larger for large value of appreciation rate of the stock. However, the slightly increase of PGR is probably due to larger value of μ implies that the investor's total wealth is more likely to exceed a transaction threshold over a short period of time. Thus, the investor tends to have more propensity to sell some shares of the stock with realized gains rather than hold them with paper gains as μ increases.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.01	0.3	0	12	1	0.1	20	20	0.6003	0.3132
0.02	0.3	0	12	1	0.1	20	20	0.6129	0.2406
0.03	0.3	0	12	1	0.1	20	20	0.6217	0.2394
0.04	0.3	0	12	1	0.1	20	20	0.6017	0.2244
0.05	0.3	0	12	1	0.1	20	20	0.6254	0.2125
0.06	0.3	0	12	1	0.1	20	20	0.6333	0.2156
0.07	0.3	0	12	1	0.1	20	20	0.6334	0.2117
0.08	0.3	0	12	1	0.1	20	20	0.6363	0.2085
0.09	0.3	0	12	1	0.1	20	20	0.6337	0.2027
0.10	0.3	0	12	1	0.1	20	20	0.6409	0.2014

Table 3.1: Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.10$, $\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.01	0.3	0	20	1	0.1	20	20	0.5771	0.3288
0.02	0.3	0	20	1	0.1	20	20	0.5961	0.2910
0.03	0.3	0	20	1	0.1	20	20	0.6081	0.3098
0.04	0.3	0	20	1	0.1	20	20	0.6151	0.2715
0.05	0.3	0	20	1	0.1	20	20	0.6011	0.2616
0.06	0.3	0	20	1	0.1	20	20	0.6232	0.2849
0.07	0.3	0	20	1	0.1	20	20	0.6123	0.2518
0.08	0.3	0	20	1	0.1	20	20	0.6231	0.2621
0.09	0.3	0	20	1	0.1	20	20	0.6262	0.2652
0.10	0.3	0	20	1	0.1	20	20	0.6332	0.2537

Table 3.2: Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.10$, $\sigma = 0.3$, $r = 0$, $N = 20$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.01	0.3	0	30	1	0.1	20	20	0.5253	0.3851
0.02	0.3	0	30	1	0.1	20	20	0.5767	0.3058
0.03	0.3	0	30	1	0.1	20	20	0.5957	0.3131
0.04	0.3	0	30	1	0.1	20	20	0.6079	0.3335
0.05	0.3	0	30	1	0.1	20	20	0.6067	0.3137
0.06	0.3	0	30	1	0.1	20	20	0.6090	0.3034
0.07	0.3	0	30	1	0.1	20	20	0.6102	0.2908
0.08	0.3	0	30	1	0.1	20	20	0.6102	0.2841
0.09	0.3	0	30	1	0.1	20	20	0.6032	0.2913
0.10	0.3	0	30	1	0.1	20	20	0.6169	0.2859

Table 3.3: Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.10$, $\sigma = 0.3$, $r = 0$, $N = 30$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$

Secondly, I wish to find out how the disposition effect depends on the number of trading periods N within one year. Let us set $\sigma = 0.3$, $r = 0$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$. For $N = 20, 50, 100$, a random value of $\mu = 0.06$ is chosen to generate the value of PGR and PLR (see Table 3.4). The results report that there is also a disposition effect. However, the difference between the values of PGR and PLR becomes smaller, implying that the degree of the disposition effect is slightly weakened. The value of PGR decreases as N increases, meaning that the investor tends to have more propensity to hold the stock with a paper gain rather than selling some shares of it with realized gains. One possible reason for this is that the investor may believe the stock is potentially good to make a bigger gain after he/she sees a gain over a very short period of time. He/she would prefer to hold it for longer time rather than sell it now. In contrast, the value of PLR increases as N increases, showing that the investor tends to have more propensity to sell some shares of the stock with realized losses rather than hold them with paper losses. This is because the investor might believe the stock may be bad potentially to induce a larger losses after he/she discovered a loss just over a short period of time. And more trading periods means more opportunities to manage his/her portfolio in order to make a fortune. Hence, he/she would prefer to sell it now rather than hold it for longer time.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.06	0.3	0	20	1	0.1	20	20	0.6232	0.2849
0.06	0.3	0	50	1	0.1	20	20	0.5980	0.3390
0.06	0.3	0	100	1	0.1	20	20	0.5672	0.3940

Table 3.4: Expected Exponential Utility Model List of values of PGR and PLR for $\mu = 0.06$, $\sigma = 0.3$, $r = 0$, $N = 20, 50, 100$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$

Next, what would happen if we vary the value of volatility rate of the stock σ? For σ from 0.05 to 0.30, take $\mu = 0.06$, $r = 0$,$N = 12$ $T = 1$,$\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$, and generate the value of PGR and PLR (see Table 3.5). The results report that there is a disposition effect for all values of σ. We do not consider the results corresponding to $\sigma = 0.05$ since the approximation error is likely to be large. The values of PGR have almost little changes (less than 5% between the smallest and the largest value of PGR), whereas the value of PLR increases slightly, as σ increases. Since higher volatility means higher risk of the stock, the investor is more willing to sell some shares of the stock with realized losses rather than hold them with paper losses when increasing σ. However, if the investor hold a stock that induces a gain, he/she may not care too much about the risk of the stock. His/her willingness to continue hold the stock leads the values of PGR to have little changes.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.06	0.05	0	12	1	0.1	20	20	0.7074	0.1568
0.06	0.10	0	12	1	0.1	20	20	0.6554	0.1816
0.06	0.15	0	12	1	0.1	20	20	0.6308	0.2005
0.06	0.20	0	12	1	0.1	20	20	0.6376	0.2049
0.06	0.25	0	12	1	0.1	20	20	0.6213	0.2084
0.06	0.30	0	12	1	0.1	20	20	0.6308	0.2149

Table 3.5: Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.06$, $\sigma = 0.05, 0.10, ..., 0.30$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$

Moreover, I wish to investigate how the disposition effect depends on the value of risk-free interest rate r. Let us set $\sigma = 0.3$, $N = 12$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$. In order to get more values of PGR and PLR, I take μ as relatively large as possible such that $\mu = 0.08$ for $r = 0.01, 0.02, ..., 0.07$ (see Table 3.6). There is clearly a disposition effect as $PGR > PLR$, although the values of PGR and PLR seem not to change significantly. This is possibly because the risk-free interest rate has little effect on how the investor trades his/her risky asset in the portfolio.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.08	0.3	0.01	12	1	0.1	20	20	0.6306	0.2037
0.08	0.3	0.02	12	1	0.1	20	20	0.6279	0.2007
0.08	0.3	0.03	12	1	0.1	20	20	0.6191	0.1972
0.08	0.3	0.04	12	1	0.1	20	20	0.6134	0.2047
0.08	0.3	0.05	12	1	0.1	20	20	0.6185	0.2083
0.08	0.3	0.06	12	1	0.1	20	20	0.5775	0.2158
0.08	0.3	0.07	12	1	0.1	20	20	0.6058	0.2001

Table 3.6: Expected Exponential Utility Model List of values of PGR and PLR for $\mu = 0.08$, $\sigma = 0.3$, $r = 0.01, ...0.07$, $N = 12$, $T = 1$, $\gamma = 0.1$, $S_0 = 20$, $W_0 = 20$

Furthermore, what would happen if the value of absolute risk aversion γ is altered? Set parameters $\mu = 0.06$, $\sigma = 0.3$, $N = 12$, $T = 1$, $S_0 = 20$, $W_0 = 20$, $r = 0$, and generate the value of PGR and PLR for $\gamma = 0.1, ..., 0.4$ (see Table 3.7). The results show that there is a disposition effect for all values of γ. The values of PGR tend to be stable, which implicates that the investor keep the same trading strategy all the time when he/she makes a gain for different γ, whereas the value of PLR decreases slightly as γ increases, he/she has slightly more propensity to sell some shares of the stock with realized losses. Since an increasing γ means that the utility function of the model becomes more concave, the sensitivity to change of final wealth of the investor is decreased. Hence, to interpret my results, we say that the investor is little more sensitive to a gain

and more sensitive to a loss on his/her final wealth for smaller value of γ.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.06	0.3	0	12	1	0.1	20	20	0.6246	0.2322
0.06	0.3	0	12	1	0.2	20	20	0.6298	0.2175
0.06	0.3	0	12	1	0.3	20	20	0.6302	0.2176
0.06	0.3	0	12	1	0.4	20	20	0.6274	0.2128

Table 3.7: Expected Exponential Utility Model: List of values of PGR and PLR for $\mu = 0.06$, $\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.1, 0.2, 0.3, 0.4$, $S_0 = 20$, $W_0 = 20$

Finally, since the initial wealth W_0 and the initial stock price S_0 are independent of the trading strategy of an individual investor (explained in Section 2.1), we do not consider whether the disposition effect depends on their values.

3.2 Expected Power Utility Model

In the expected power utility model, I wish to investigate how the disposition effect depends on the value of parameters S_0, W_0, μ, σ, r, γ, N, T via a similar approach adopted in the expected exponential utility model. Firstly, in order to find how the disposition effect depends on the value of μ, I choose the relative risk aversion $\gamma = 0.3$. It is very clear that the utility function is fairly concave at around the investor's final wealth of 20. Thus, I take $S_0 = 20, W_0 = 20$. In addition, I choose the volatility rate of the stock to be $\sigma = 0.3$, the risk-free interest rate of the stock to be $r = 0$ and the maturity date to be $T = 1$. For similar reasons as in the expected exponential utility model, I consider a range of values of $\mu > r$ for different number of trading periods within one year, i.e. $N = 12$, 20, 30. But there is one extra condition in this case. The inequality of $r < \mu < 0.6$ is hold for given value of other parameters. Otherwise it is not guaranteed that there is an optimal trading strategy (in proportion) $0 \leq \theta < 1$ since the function of $f(\theta)$ is defined as

$$f(\theta) = E[(\exp(r) + \theta(1 + R - \exp(r)))^{\gamma}] \tag{3.6}$$

Then, the first derivative of $f(\theta)$ is

$$f'(\theta) = \gamma E[(\exp(r) + \theta(1 + R - \exp(r)))^{\gamma-1}(1 + R - \exp(r))] \tag{3.7}$$

And the second derivative of $f(\theta)$ for $0 \leq \theta < 1$ is

$$f''(\theta) = \gamma(\gamma - 1)E[(\exp(r) + \theta(1 + R - \exp(r)))^{\gamma-2}(1 + R - \exp(r))^2] \leq 0 \tag{3.8}$$

which implies that there exists a θ satisfying $0 \leq \theta < 1$ that maximize $f'(\theta)$. Since we got $f'(0) > 0$, and we want to find a θ satisfying $0 \leq \theta < 1$ such that $f'(\theta) = 0$, we need $f'(1) < 0$ such that

$$\gamma E[(1+R)^{\gamma-1}(1+R-\exp(r))] < 0 \tag{3.9}$$

$$\Leftrightarrow \gamma E[(1+R)^{\gamma} - \exp(r)(1+R)^{\gamma-1}] < 0 \tag{3.10}$$

$$\Leftrightarrow \gamma E[\exp((\mu - \frac{1}{2}\sigma^2)\gamma + \sigma\gamma\xi(t)) - \exp(r)\exp((\mu - \frac{1}{2}\sigma^2)(\gamma-1) + \sigma(\gamma-1)\xi(t))] < 0 \tag{3.11}$$

By the moment generating function of normal distribution we have

$$E[\exp((\mu - \frac{1}{2}\sigma^2)\gamma + \frac{1}{2}\sigma^2\gamma^2) - \exp(r)\exp((\mu - \frac{1}{2}\sigma^2)(\gamma-1) + \frac{1}{2}\sigma^2(\gamma-1)^2)] < 0 \tag{3.12}$$

Since exponential function is monotonic, the above condition is equivalent to the followings:

$$((\mu - \frac{1}{2}\sigma^2)\gamma + \frac{1}{2}\sigma^2\gamma^2) - (r + (\mu - \frac{1}{2}\sigma^2)(\gamma-1) + \frac{1}{2}\sigma^2(\gamma-1)^2)) < 0 \tag{3.13}$$

$$\Leftrightarrow (\mu - r) + \sigma^2(\gamma - 1) < 0 \tag{3.14}$$

Thus, we have $\mu < 0.63$ for $\sigma = 0.3$, $r = 0$ and $\gamma = 0.3$. If $\mu > 0.63$ we have a θ^* which maximizes $f(\theta)$ at 1. Let us take μ from 0.01 to 0.06 and generate the value of PGR and PLR (see Table 3.8-3.10). Again, a disposition effect is observed for all three sets of value of PGR and PLR as $PGR > PLR$. However, the results show that the degree of the disposition effect slightly decreases as μ increases. This is because the value of PGR decreases and the value of PLR increases slightly, as μ increases. The decreasing trend in PGR implicates that the investor tends to have more propensity to hold a stock with a paper gain rather than sell some shares of the stock with realized gains as μ increases. This is possibly because higher appreciation rate attracts the investor to have more willingness to hold stock longer even he/she currently make a gain. The slightly decrease in PLR is probably due to the optimal portfolio for power utility keeps the weight of risky investment in the total wealth to be a constant, so the transaction threshold will move in the same direction as the total wealth. For a investor who currently makes a loss, he/she is more likely to realized the loss for higher appreciation rate, since higher appreciation rate implies more probability to induce a smaller difference between the total wealth and the transaction threshold.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.01	0.3	0	12	1	0.3	20	20	0.6215	0.2190
0.02	0.3	0	12	1	0.3	20	20	0.6226	0.2180
0.03	0.3	0	12	1	0.3	20	20	0.6119	0.2229
0.04	0.3	0	12	1	0.3	20	20	0.6131	0.2228
0.05	0.3	0	12	1	0.3	20	20	0.6108	0.2308
0.06	0.3	0	12	1	0.3	20	20	0.5996	0.2304

Table 3.8: Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.06$, $\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.01	0.3	0	20	1	0.3	20	20	0.6232	0.2632
0.02	0.3	0	20	1	0.3	20	20	0.6219	0.2600
0.03	0.3	0	20	1	0.3	20	20	0.6255	0.2606
0.04	0.3	0	20	1	0.3	20	20	0.6173	0.2602
0.05	0.3	0	20	1	0.3	20	20	0.6122	0.2724
0.06	0.3	0	20	1	0.3	20	20	0.6105	0.2645

Table 3.9: Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.06$, $\sigma = 0.3$, $r = 0$, $N = 20$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.01	0.3	0	30	1	0.3	20	20	0.6135	0.2944
0.02	0.3	0	30	1	0.3	20	20	0.6194	0.2941
0.03	0.3	0	30	1	0.3	20	20	0.6096	0.2899
0.04	0.3	0	30	1	0.3	20	20	0.6130	0.2928
0.05	0.3	0	30	1	0.3	20	20	0.6078	0.2945
0.06	0.3	0	30	1	0.3	20	20	0.6080	0.2966

Table 3.10: Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.01, 0.02, ..., 0.06$, $\sigma = 0.3$, $r = 0$, $N = 30$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$

Then, I wish to investigate how the disposition effect depends on the number of trading periods N. Let us take $S_0 = 20$, $W_0 = 20$, $\gamma = 0.3$, $T = 1$,$r = 0$, $\sigma = 0.3$. For $N = 20$, 50, 200, a random variable $\mu = 0.05$ is chosen to compute the value of PGR and PLR (see Table 3.11). The results show that there is a disposition effect. However, the degree of the disposition effect is considered to be weakened as N increases. The value of PGR decreases, meaning that the investor have more propensity to hold the stock with a paper gain rather than sell some shares of the stock with realized gains for larger N. In comparison, the values of PLR increases, suggesting as N gets larger, the investor tends to have more propensity to realize losses rather than hold paper losses. In general, this result is similar to what we obtained in expected exponential utility model. Hence, the interpretation of such behavior could also be of similar reasons to the expected exponential utility model.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.05	0.3	0	20	1	0.3	20	20	0.6122	0.2724
0.05	0.3	0	50	1	0.3	20	20	0.5992	0.3266
0.05	0.3	0	100	1	0.3	20	20	0.5879	0.3680

Table 3.11: Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.05$, $\sigma = 0.3$, $r = 0$, $N = 20, 50, 100$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$

Next, would the disposition effect depend on the volatility rate of the stock change of volatility of the stock σ? In this case, I choose μ as relatively small as possible such that $\mu = 0.02$ to get more values of PGR and PLR, and satisfies the condition $(\mu - r) + \sigma^2(\gamma - 1) < 0$ (see Table 3.12). Set other parameters as $S_0 = 20$, $W_0 = 20$, $\gamma = 0.3$, $T = 1$, $N = 12$, $r = 0$, and generate the value of PGR and PLR for $\sigma = 0.2$, 0.25,..., 0.4. Even though in reality, 0.35 and 0.4 are relatively too large for the value of volatility rate of the stock, these two values are used so that a more reliable result may be found. In addition, I generate the value of PGR and PLR for $\sigma = 0.8$ and the result is similar to when $\sigma = 0.4$. The results show that the disposition effect exists, and the degree of the disposition effect gets slightly stronger as σ increases, due to slightly increase in PGR and slightly decrease in PLR. Since increasing σ implies increasing the risk of the stock. the increasing trend of PGR could be explained as the investor is risk-averse, and he/she might have more propensity to sell some shares of the stock rather than hold them when he/she makes a gain currently. The decreasing trend of PLR might not be explained in this way. It is possible that other financial reasons are behind this behavior. Nevertheless, we find a strong disposition effect in this case.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.02	0.20	0	12	1	0.3	20	20	0.6064	0.2315
0.02	0.25	0	12	1	0.3	20	20	0.6166	0.2256
0.02	0.30	0	12	1	0.3	20	20	0.6207	0.2217
0.02	0.35	0	12	1	0.3	20	20	0.6222	0.2159
0.02	0.40	0	12	1	0.3	20	20	0.6213	0.2174

Table 3.12: Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.02$, $\sigma = 0.2, 0.25, ..., 0.4$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$

Then, I wish to investigate how the disposition effect depends on the risk-free interest rate r. In order to generate the value of PGR and PLR for r =0.01,...,0.04, let us set parameters to be $S_0 = 20$, $W_0 = 20$, $\gamma = 0.3$, $T = 1$, $N = 12$, $\sigma = 0.3$, and choose the appreciation rate of the stock as relatively large as possible such that $\mu = 0.05$ (see Table 3.13). With consideration of the approximation error, the results indicate that there is a disposition effect. But, the degree of disposition effect is not affected much by the risk-free interest rate since the values of PGR and PLR tend to be unchanged, giving the difference of the two to be unchanged, as r increases. The reason might be similar to the expected exponential utility case, in which the risk free interest rate does not affect his/hers trading strategy in the portfolio.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.05	0.3	0.01	12	1	0.3	20	20	0.6124	0.2232
0.05	0.3	0.02	12	1	0.3	20	20	0.6167	0.2138
0.05	0.3	0.03	12	1	0.3	20	20	0.6196	0.2024
0.05	0.3	0.04	12	1	0.3	20	20	0.6193	0.2022

Table 3.13: Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.05$, $\sigma = 0.3$, $r = 0.01, ..., 0.04$, $N = 12$, $T = 1$, $\gamma = 0.3$, $S_0 = 20$, $W_0 = 20$

Moreover, it is of interest to check how the disposition effect depends on the relative risk aversion γ. Taking parameters $S_0 = 20$, $W_0 = 20$, $T = 1$, $N = 12$, $\sigma = 0.3$, $\mu = 0.05$, $r = 0$, when condition of $(\mu - r) + \sigma^2(\gamma - 1) < 0$, is satisfied and generating the value of PGR and PLR for γ = 0.05,0.1,... 0.4 (see Table 3.14). Here, the results show that there is a disposition effect for all values of γ. The values of PGR tend to be stable, whereas the value of PLR increases slightly, as γ increases. Since an increasing γ means that the utility function of the model becomes less concave, the sensitivity to change of final wealth of the investor is increased. Hence, to interpret my results, we say that the investor is more sensitive to a loss on his/her final wealth for larger value of γ. Thus, he/she might have more propensity to sell some shares of the stock with realized losses in order to hedge risk for higher value of relative risk aversion.

μ	σ	r	N	T	γ	S_0	W_0	PGR	PLR
0.05	0.3	0	12	1	0.05	20	20	0.6169	0.2197
0.05	0.3	0	12	1	0.10	20	20	0.6160	0.2226
0.05	0.3	0	12	1	0.15	20	20	0.6164	0.2235
0.05	0.3	0	12	1	0.20	20	20	0.6129	0.2261
0.05	0.3	0	12	1	0.25	20	20	0.6133	0.2290
0.05	0.3	0	12	1	0.30	20	20	0.6120	0.2290
0.05	0.3	0	12	1	0.35	20	20	0.6019	0.2323
0.05	0.3	0	12	1	0.40	20	20	0.6105	0.2332

Table 3.14: Expected Power Utility Model: List of values of PGR and PLR for $\mu = 0.05$,$\sigma = 0.3$, $r = 0$, $N = 12$, $T = 1$, $\gamma = 0.05, 0.10, ..., 0.40$, $S_0 = 20$, $W_0 = 20$

Finally, We do not consider whether the disposition effect depends on the values of the initial wealth of the investor W_0 and the initial stock price S_0 because they are independent of the trading strategy of an individual investor.

So far, I have summarized and evaluated the empirical results for the expected exponential and power utility model, when varying the value of each parameter, respectively. I will make a conclusion and suggest a possible improvement in next chapter.

4 Conclusion and Possible Improvement

In context of expected utility theory, It is assumed that an individual investor follows a trading strategy which maximizes the two classical expected utility model: exponential and power. I evaluate whether these two explain the disposition effect. As a result, I find that both models of the expected utility theory explain a disposition effect: when the investor sells a stock in his/her portfolio, he/she has more propensity to sell a stock that has made a gain than one that has made a loss. We also pay attention on how changing any one of the parameters would affect the degree of the disposition effect in the two models. Although, the degree of disposition effect is weakened as the number of trading periods N increase, our results show that there is still a disposition effect in both cases. The degree of disposition effect becomes stronger as the appreciation rate of the stock μ increases in the expected exponential utility model. In contrast, the degree of disposition effect has little change in the expected power utility model. Changing one of other parameters each time has little or no effect on the degree of the disposition effect in both models.

4.1 A Possible Improvement

When computing the optimal trading strategy in both classical models, I assume that the market is consisting of two assets: one risk-free asset and one risky asset. To extend our analysis, one could further investigate the disposition effect based on trading strategy in a multi-stock setting. For this to be valid, there are two conditions that need to be satisfied. The first condition is that the investor is assume to be engaged in sometimes called "narrow framing" or "mental accounting", so that, even if the investor trades several stocks, he/she gets a utility on each one of them separately from the annual trading profit. The second condition is that W_0 needs to be reinterpreted as the maximum amount the investor is willing to lose by trading any one of his/her stocks. Under these conditions, the investor's trading strategy for each stock is independent of his/her other holdings in the portfolio. Therefore, it would be appropriate to predict whether the investor has a disposition effect in the multi-stock setting by the expected exponential and power utility models.

A MATLAB Code for Expected Exponential Utility Model

%%%%%%%%%%%%%%%%%%%%%%
Programming of Optimal Trading Strategy π_t^* of Expected Exponential Utility Model
N=T=1, for single period model; N>1, for multi-period model
%%%%%%%%%%%%%%%%%%%%%%

```
function nose(mu,sig,r,N,T,gamma,S0)
h= T/N;
if 1
M2 = 100000;
S=[];
S1 = S0 * ones(M2,1);
for n = 1:N
dW = sqrt(h) * randn(M2,1);
S1 = S1. * exp((mu-0.5 * sig * sig) * h+sig * dW);
S=[S S1];
end
S=[S0 * ones(M2,1),S];
end
X=[];
for i=1: N
Z=randn(1e+7,1);
f = @(P)sum((exp((mu - 0.5*sig*sig)*h+sig*sqrt(h)*Z)-exp(r*h)). *exp(((-gamma)*
exp(r * (N-i) * h)) * P * (exp((mu-0.5 * sig * sig) * h+sig * sqrt(h) * Z)-exp(r * h))));
Xe=fzero(f,1);
X=[X,Xe];
end
X
```

```
%%%%%%%%%%%%%%%%%%%%%
```
Programming of PGR and PLR of Expected Exponential Utility Function
```
%%%%%%%%%%%%%%%%%%%%%
function dissertation2(mu,sig,r,N,T,gamma,S0)
h= T/N;
M2 = 10000;
S=[];
S1 = S0 * ones(M2,1);
for n = 1:N
dW = sqrt(h) * randn(M2,1);
S1 = S1. * exp((mu-0.5 * sig * sig) * h+sig * dW);
S=[S S1];
end
S=[S0*ones(M2,1),S];
X=[];
for i=1: N
Z1 = randn(1,1e+7);
Z=Z1';
f = @(P)sum((exp((mu-0.5*sig*sig)*h+sig*sqrt(h)*Z)-exp(r*h)).*exp((-gamma)*
exp(r * (N-i) * h)) * P * (exp((mu-0.5 * sig * sig) * h+sig * sqrt(h) * Z)-exp(r * h))));
Xe=fzero(f,1);
Ye=Xe./S(:,i);
X=[X,Ye];
end
Saverage=[];
 for j=1:M2
Saverage1=[];
Saverage2=S0;
for i=2:N
if(X(j,i)>=X(j,i-1))
Saverage2=(X(j,i-1) * Saverage2+(X(j,i)-X(j,i-1)) * S(j,i))/X(j,i);
end
Saverage1=[Saverage1 Saverage2];
end
Saverage =[Saverage Saverage1'];
end
Saverage = Saverage';
G=0;
L=0;
for j=1:M2
C2=0;
C3=0;
C4=0;
for i=2:N
```

```
if(S(j,i)>Saverage(j,i-1)&&X(j,i)<X(j,i-1))
C1=C1+1;
elseif(S(j,i)>Saverage(j,i-1)&&X(j,i)>=X(j,i-1))
C2=C2+1;
elseif(S(j,i)<Saverage(j,i-1)&&X(j,i)<X(j,i-1))
C3=C3+1;
elseif(S(j,i)<Saverage(j,i-1)&&X(j,i)>=X(j,i-1))
C4=C4+1;
end
end
if((C1+C2)==0)
PGR = 0;
else
PGR = C1/(C1+C2);
end if((C3+C4)==0)
PLR = 0;
else
PLR= C3/(C3+C4);
end
G=G+PGR;
L=L+PLR;
end
G/M2
L/M2
```

B MATLAB Code for Expected Power Utility Model

```
%%%%%%%%%%%%%%%%%%%%%
Programming of Optimal Trading Strategy θ*_t of Expected Power Utility Model
N=T=1, for single period model; N>1, for multi-period model
%%%%%%%%%%%%%%%%%%%%%
function nos2(mu,sig,r,N,T,gamma)
h=T/N;
theta=[];
f0=[];
flag=(mu-r)+sig^2) * (gamma-1)
for rat=0:1e-3:1
Z =randn(1e+7,1);
f=sum((exp(r*h)+rat*(exp((mu-0.5*sig^2))*h+sig*sqrt(h)*Z)-exp(r*h))).^(gamma)));
theta=[theta rat];
f0=[f0 f];
end
plot(theta,f0)
[f0,I]=max(f0);
theta(I)
```

```
%%%%%%%%%%%%%%%%%%%%%
```
Programming of PGR and PLR of Expected Power Utility Function
```
%%%%%%%%%%%%%%%%%%%%%
function dissertation5(mu,sig,r,N,T,gamma,S0,W0)
h=T/N;
M2 = 10000;
S=[];
S1 = S0 * ones(M2,1);
for n = 1:N
dW = sqrt(h) * randn(M2,1);
S1 = S1. * exp((mu-0.5 * sig * sig) * h+sig * dW);
S=[S S1];
end
S=[S0 * ones(M2,1),S];
theta=[];
f0=[];
flag=mu-r+sig * sig * (gamma-1)
for rat=0:1e-3:1
Z =randn(1e+7,1);
f=sum((exp(r*h)+rat*(exp((mu-0.5*sig*sig)*h+sig*sqrt(h)*Z)-exp(r*h))).^ (gamma));
theta=[theta rat];
f0=[f0 f];
end
[f0,I]=max(f0);
theta=theta(I);
W=[];
for j=1:M2
W2 = W0;
for i = 1:N
W2 = W2 * (exp(r * h)+theta * (S(j,i+1)/S(j,i)-exp(r * h)));
W1 =[W1 W2];
end
W=[W W1'];
end
W=W';
for j=1:M2
for i=1:N
X(j,i)=W(j,i) * theta/S(j,i);
end
end
Saverage=[];
for j=1:M2
Saverage1=[];
Saverage2=S0;
```

```
for i=2:N
if(X(j,i)>=X(j,i-1))
Saverage2=(X(j,i-1) * Saverage2+(X(j,i)-X(j,i-1)) * S(j,i))/X(j,i);
end
Saverage1=[Saverage1 Saverage2];
end
Saverage =[Saverage Saverage1'];
end
Saverage = Saverage';
G=0;
L=0;
for j=1:M2
C1=0;
C2=0;
C3=0;
C4=0;
for i=2:N
if(S(j,i)>Saverage(j,i-1)&&X(j,i)<X(j,i-1))
C1=C1+1;
elseif(S(j,i)>Saverage(j,i-1)&&X(j,i)>=X(j,i-1))
C2=C2+1;
elseif(S(j,i)<Saverage(j,i-1)&&X(j,i)<X(j,i-1))
C3=C3+1;
elseif(S(j,i)<Saverage(j,i-1)&&X(j,i)>=X(j,i-1))
C4=C4+1;
end
end
if((C1+C2)==0)
PGR = 0;
else
PGR = C1/(C1+C2);
end
if((C3+C4)==0)
PLR = 0;
else
PLR= C3/(C3+C4);
end
G=G+PGR;
L=L+PLR;
end
G/M2
L/M2
```

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