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**COMPETITIVE MIXED BUNDLING
AND CONSUMER SURPLUS**

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Competitive Mixed Bundling and Consumer Surplus

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Abstract

Mixed bundling in imperfectly competitive industries causes some prices to rise and others to fall. This paper studies under what conditions mixed bundling works for or against the consumer interest. We find that if buyers incur firm specific costs or have shop specific tastes then competitive mixed bundling lowers consumer surplus overall and raises profits - the same is true of competitive volume discounts. Competition without these discounts causes all prices to be kept low as larger customers are targeted; with discounts the prices for heavy users drop, but more is extracted from small users. The consumer surplus result is reversed if the differentiation between components as opposed to firms is key.

Keywords bundling; loyalty rebates; volume discounts; competitive price discrimination

JEL Classification: L11, L41

1 Introduction

Mixed bundling is ubiquitous. It captures the marketing practice of charging a price dependent upon the total set of products bought. Examples include pay TV companies offering discounts on bundles of premium channels; credit card companies offering discounts which grow the more is spent on the card; supermarkets offering reductions on fuel; loyalty discounts for purchases across product lines and volume discounts. Much mixed bundling happens in competitive industries. By targeting lower prices at those that buy a bundle it is possible that prices rise for those small consumers that only buy one product. For example, car manufacturers provide lower prices to large fleet buyers as compared to small dealers. This has resulted in differences between fleet car prices and private sale prices that are so great that the UK parliament has urged an investigation.¹

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¹See "MPs urge probe into car prices", Financial Times, 10th September, 2004.

This paper seeks to explore when competitive mixed bundling acts in the consumers' interest and when not. The general context considered is of two imperfect competitors selling two differentiated products each. Mixed bundling here is a marketing/pricing decision² and so firms cannot credibly commit not to respond to a rival by mixed bundling. As a result the unconstrained Nash pricing equilibrium is one of mixed bundling: sellers use all of the tools they have available.³ The consumer surplus generated is compared to the counterfactual constrained equilibrium in which the competing firms only offer pure component prices. Such a constraint may arise through tacit collusion or as a result of the Competition Authorities taking an anti-bundling view. The effect of mixed bundling is found to hinge on the interplay of the two main contributions made in this work.

Firstly we introduce small buyers into the model who only desire one of the component products and not the bundle of both. These consumers are crucial as prices set to exploit large, possibly hybrid, bundle buyers will often cause profits to be lost on the smaller consumers. The small consumers therefore have a substantial impact on the overall equilibria. Secondly we make a distinction between firm specific preferences and product specific preferences. Firm specific preferences capture situations in which the disutility associated with buying from a firm does not increase in proportion to the products being bought. For example, consumers incur a hassle/time cost in contracting with a pay TV provider, but this cost does not increase as more premium channels are signed up for. Likewise the transport cost of going to a supermarket is not appreciably altered by the exact number of groceries bought. On the contrary, product specific preferences capture differences in the actual goods supplied at the competing firms - hence these taste costs grow as more products are bought.

This paper discovers that with firm specific preferences mixed bundling lowers consumer surplus and raises profits as compared to constrained pure component pricing. The firm specific preferences create an economy of scope for the large buyers who will decide to one-stop shop. As a result these large buyers have very high elasticity of demand. Under pure component pricing the firms compete to win these consumers and so lower the prices to keep the price of the bundle down. The large buyers are therefore shown to provide price protection for the small buyers. This protection is lost under mixed bundling and the price to the small buyers rises so much that consumer surplus falls.

Under product specific preferences these conclusions are reversed: mixed bundling is a prisoner's dilemma for firms and consumer surplus is higher than under constrained pure component pricing. As preferences are product specific, there is no economy of scope in demand under pure component pricing. Thus both large and small buyers have the same elasticity of demand for a given component; pure component pricing therefore involves no trade off between large and small buyers, and some large buyers form hybrid bundles from both firms. Mixed bundling however provides a means to target those who will buy both goods at only one firm and, at the optimal pure component prices, this subset of the large buyers have a high elasticity of demand. This pulls the bundle price down. As a result too many component sales as part of hybrid bundles are cannibalised and so the firms lower their component prices to try and stem the loss of profits. Hence all prices are lower under mixed bundling and consumer surplus rises.

²Stremersch and Tellis (2002) refer to this as price bundling as opposed to product bundling which requires a change in the production process.

³See McAfee et al. (1989).

There has been an upsurge of research on bundling generally though the two contributions discussed above have not been studied. Matutes and Regibeau (1992) is an important work in the area of bundling with product specific preferences. They consider a model in which all consumers must purchase both goods to derive any positive utility. There are not therefore any ‘small’ buyers and so the tradeoffs between serving the large and small buyers were not addressed; nor was the resultant effect on consumer surplus. Further Matutes and Regibeau consider product as opposed to price bundling so that firms could commit not to bundle. They therefore modeled competition as a multi-stage game in which firms first declared whether they would bundle or not, then based on this information they set prices. As a result their paper does not identify the intuition behind why consumer surplus should rise with product specific preferences, focusing instead on the motivations for declaring a bundle versus a pure component strategy prior to the price setting phase.

Gans and King (forthcoming) is an extension of Matutes and Regibeau (1992). Gans and King again make the assumption that all buyers will buy both goods so that there are no small consumers. The extension they study is that the firms set their bundle reduction in advance, commit to it, and then set their component prices observing their rival’s bundle reduction. They note that this form for price competition is suggested when independent firms combine to offer reductions across their combined product lines. In this setup competitive mixed bundling has no final effect on profits - a result very much at odds to that derived here. This difference can be traced to the fact that in the competitive context studied here, firms cannot commit in advance to a bundle reduction and so make pricing decisions simultaneously.

Armstrong and Vickers (2001) consider general competitive price discrimination in the context of firm specific preferences. The part of their paper most relevant here is the second model culminating in Proposition 5 which considers competing firms serving a population of consumers who differ in their tastes. This setup does allow for large and small buyers, and we find (Section 3) that the trade-off between these consumers is key. In this context the authors note that a symmetric mixed bundling equilibrium exists in which firms set their margins equal to the transport costs. The authors do not however turn to a discussion of profits and consumer surplus and their comparison with pure component pricing in this heterogeneous consumer case. The analysis presented here is complementary to Section 3 of Armstrong and Vickers (2001). Here the authors pursue a consumer surplus, profit comparison when all consumers are identical except for their taste between stores and the market is covered. This model is not applicable to bundling when consumers have unit demands as two-part pricing and per good pricing are equivalent here.⁴ The authors note that when multiple units of a component good are demanded and there are no fixed costs of service, then price discrimination through two-part tariffs can be damaging for consumer surplus (Proposition 3). However the authors note that a direct economic intuition is hard to provide because of the reliance on 2^{nd} order effects. We find that with mixed bundling and unit demands clear results and intuitions are available - and these stem from the analysis of the price rises to small buyers as against the price drops to large buyers.

It has been noted that mixed bundling, by allowing more pricing flexibility, will be the Nash equilibrium of a general simultaneous price setting game. This paper therefore offers an answer

⁴Under strong competition with consumers having identical unit demands all consumers will purchase the same set of component goods. The mixed bundling decision therefore collapses to setting one bundle price through the sum of the component prices.

to the question of when competitive mixed bundling is a prisoner's dilemma for firms. The fact that competitive price discrimination can lower firms' overall profits has been proposed by Thisse and Vives (1988) in the context of price discrimination by observable consumer location. However progress beyond this has been limited with Stole (2003, p26) noting that we do not currently have a general theory as to when competitive price discrimination will act for the firm or alternatively for the consumer.

The model considered here is one of imperfect competition in which rivals are able to match each other's mixed bundles. There has been a great deal of research into how a dominant firm with a larger product line might use bundling to foreclose entry. Whinston (1990) is perhaps the seminal reference here. Carlton and Waldman (2002) and Choi and Stefanadis (2001) build on this work to show that bundling can also inhibit research incentives by increasing the hurdle to successful market entry. These and other models of competitive bundling relevant to anti-trust considerations are discussed in the survey paper by Kobayashi (2005). Further references are available from the survey paper offered by Armstrong (forthcoming).

The rest of this paper proceeds as follows. The model is presented in Section 2. Consumer surplus and profit implications are considered with firm specific preferences in Section 3 and with product specific preferences in Section 4, and compared in Section 5. The paper's predictions are placed in the context of some actual industry examples in Section 6. Section 7 concludes with proofs gathered in the Appendices.

2 The Model

The model is one of imperfect competition. There are two competing firms, I and II who each have a full product line of two goods denoted X and Y . The goods are manufactured by each firm at constant marginal cost, c_Y for good Y and similarly for good X . At any particular point in time there is a measure of consumers $B > 0$ who wish to buy the whole system of one unit of both X and Y . In addition there is a further measure of consumers $AX > 0$ who are only in the market for one unit of good X . Likewise measure AY . We assume that the valuations of consumers is sufficiently high that the market is fully served. That is all consumers of type AX will buy one unit of good X (only); all consumers of type B will buy the bundle of X and Y , though possibly not both goods from the same firm; and $AX + AY + B = 1$. In principal both the firms and the individual goods sold could be differentiated. We analyse each of these situations separately:

Firm Specific Preferences. The firms sell identical goods X and Y . However the firms themselves are differentiated. Thus consumers incur some utility cost in visiting either seller, but this cost does not grow proportionately with extra purchases. Formally, consumers are uniformly distributed along a Hotelling line of length 1 with the firms lying at either end. A consumer of type AX or AY located at point θ along the Hotelling line and purchasing from firm I incurs disutility of the price plus the transport cost of $\lambda\theta$, where λ is the parameter of differentiation. Consumers of type B , located at θ , who decide to purchase both of the goods from the same firm incur a reduced transport cost of $\lambda_B \times \text{distance}$ where $\lambda \leq \lambda_B \leq 2\lambda$. Should the type B consumers decide to split their purchases between firms and create a hybrid bundle then, as with the type AX, AY consumers, they will incur a transport cost of λ per each firm visited creating a total transport cost of $\lambda\theta + \lambda(1 - \theta) = \lambda$. We shall see that this will not happen

in equilibrium as each buyer travels at most distance $\frac{1}{2}$ and $\lambda_B \cdot \frac{1}{2} \leq \lambda$ by assumption. One motivating market for this setup is the pay-TV market: there is a learning and arrangement time cost of contracting with a pay TV provider, but this will not double if two channels are purchased as opposed to one.

Product Specific Preferences. The firms are not themselves differentiated, however the goods they sell are differentiated with a taste cost proportional to μ . The consumers of type AX are only in the market for good X and are modeled as uniformly distributed along a Hotelling line of length 1 with the firms at either end. Thus if the consumer is at location θ buying from firm II they incur a taste cost of $\mu(1 - \theta)$. For these single good consumers the two models are identical and μ is equivalent to λ . The consumers of type B however have separate tastes for X from I as opposed to II and similarly for Y . These consumers are modeled as uniformly distributed on a unit square with general location $(s, t) \in [0, 1]^2$, they incur a taste cost of μ times the (horizontal or vertical) distance traveled for each good. The firms are located at the extreme corners $(0, 0)$ and $(1, 1)$. Hence a type B consumer at location (s, t) purchasing the hybrid bundle of X from firm I and Y from firm II will incur a taste cost of $\mu s + \mu(1 - t)$.

Thus the formulation of demand makes two key additions to the previous literature. Firstly we include the consumers who are only in the market for one of the goods. These consumers will limit the flexibility of the competing firms as the component prices have a direct effect on them. Secondly we formally model the difference between firm specific and product specific utility costs. The interaction of these two parts will allow us to derive strong results concerning the effect of mixed bundling on consumer surplus.

We now turn to the nature of competition. In a further change to the previous literature on bundling we consider price bundling as opposed to product bundling (see Stremersch and Tellis (2002)). In the latter (product bundling), bundled goods are integrated in the design stage and so bundling has commitment value. Such bundling is well modeled as a multi-stage game in which first the firms observe whether their rivals have bundled and then decide their prices. We however turn to price bundling. This is a pricing decision and can be made at short notice. In particular one cannot give credible commitments not to bundle. This captures situations such as the price bundling of petrol and groceries, dinner and lodging and so on. Consequently our two firms simultaneously set their prices in an attempt to maximise their profits. If the firms are not restricted as to their price choices, then each good (in an economic sense) will have its own price, including an optimal price for the bundle. This is mixed bundling. In all that follows we restrict attention to the economically relevant case of sub-additive bundling: the price for the bundle must be weakly lower than the sum of the component prices. If, however, firms can coordinate (or are made) to avoid providing reductions across product lines, then we will have pure component pricing.⁵ The bundle can then be purchased at the additive sum of the component prices. This paper will compare the prices, profits and consumer surplus of the Nash equilibrium in prices under mixed bundling, as against the price equilibrium if the firms restrict themselves to pure component pricing. Thus we hope to answer when mixed bundling works in the consumers' favour.

⁵Such coordination on pure component pricing only might come from the application of Competition Law (see the Introduction) or through some form of collusion for example.

3 Firm Specific Preferences

We focus first on markets characterised by firm specific preferences: consumers thus incur a utility cost for each firm visited, but this cost does not double if both X and Y are purchased rather than just one. Recall from Section 2 that the firms sell identical products X and Y which they manufacture at marginal costs of c_X and c_Y . Due to the symmetry of the problem we seek symmetric pricing equilibria. We begin our analysis with the case of mixed bundling in which firms have full pricing freedom. Thus we search for a symmetric equilibrium in which the firms set prices (p_X, p_Y, p_B) with $p_B \leq p_X + p_Y$.⁶ It will be useful to define the margin $\rho_X = p_X - c_X$ for good X , and similarly for Y and the bundle, as the firms' choice variables. Note that in any symmetric equilibrium, as the goods are not differentiated, all consumers will decide to one stop shop - we do not assume this however in deriving the equilibria. All omitted proofs can be found in the appendix.

Lemma 1 *With full pricing freedom the unique symmetric equilibrium is given by margins*

$$\rho_X = \lambda = \rho_Y, \quad \rho_B = \lambda_B \quad (1)$$

Thus those who purchase only good X pay a price of $p_X = c_X + \lambda$. Lemma 1 is an extension of Armstrong and Vickers (2001, Proposition 5) for two reasons: firstly we show uniqueness of the equilibrium amongst the class of symmetric equilibria; secondly we do not assume that all consumers are only allowed to visit only one store and derive the equilibrium in this fuller behavioural setting.

This paper seeks to compare the general mixed bundling Nash pricing equilibrium to the equilibrium in which the firms are exogenously constrained to offer only pure component prices. To this end we have:

Lemma 2 *If the firms are exogenously constrained to pure component pricing, then the symmetric pricing equilibrium has margins:*

$$\begin{pmatrix} \rho_X \\ \rho_Y \end{pmatrix} = \frac{\lambda}{\frac{AX \cdot AY}{\lambda} + \frac{B}{\lambda_B} (AX + AY)} \begin{pmatrix} \frac{AX \cdot AY}{\lambda} + \frac{B}{\lambda_B} (AX + AY) - B \cdot AY \left(\frac{2}{\lambda_B} - \frac{1}{\lambda} \right) \\ \frac{AX \cdot AY}{\lambda} + \frac{B}{\lambda_B} (AX + AY) - B \cdot AX \left(\frac{2}{\lambda_B} - \frac{1}{\lambda} \right) \end{pmatrix} \quad (2)$$

We are now in a position to prove the central result of this section:

Theorem 1 *Suppose consumers have firm specific preferences and not product specific preferences. Comparing the symmetric mixed bundling equilibrium against the constrained pure component pricing equilibrium we see:*

1. *Mixed bundling strictly raises profits compared to pure component pricing*
2. *The component price is strictly higher under mixed bundling than under pure component pricing while the bundle price is lower under mixed bundling*

⁶It is conceivable that the firms might decide to set $p_B = p_X + p_Y$ which would correspond to pure component pricing. In fact they do not do this in equilibrium.

3. *Consumer surplus is strictly lower under mixed bundling as compared to pure component pricing*
4. *Welfare is unchanged.*
5. *The profit, welfare and consumer surplus effects shrink as λ_B grows towards 2λ .*

This is a no prisoners' dilemma result in a general setting; firms gain while consumers unambiguously lose out from mixed bundling. Note that mixed bundling here is equivalent to loyalty rebates; thus our results apply to such rebates also. This result will not hold with product specific preferences in the next section. Therefore here we first discuss what intuitions help to understand Theorem 1.

Part 2 of Theorem 1 says that the margins charged in a constrained pure component pricing equilibrium lie between those levied in a mixed bundling equilibrium. To understand this, note that the type B consumers all one-stop shop as a result of their economies of scope in going to only one firm (firm specific preferences with $\lambda_B < 2\lambda$). This implies that when the firms offer mixed bundled tariffs, there is one dedicated price aimed at each customer type. Competition in each consumer segment proceeds in a standard Hotelling way. Hence the mixed bundling equilibrium has the margins on each sale (X to AX types, the bundle to B types and so on) settle at transport cost. As $\lambda_B < 2\lambda$ we naturally have sub-additive bundling here. Note further however that the profit function each firm derives from serving one customer type, as is standard in Hotelling with uniformly distributed consumers, is concave in price. At a symmetric pricing equilibrium if margins $\rho_X > \lambda$ then the firms could gain profits on consumers of type AX by lowering the price. Vice versa if $\rho_X < \lambda$. Similarly for type B consumers around margin λ_B . Now suppose that the firms are constrained to charge only component prices. This implies that the bundle price is the sum of the component prices. It is still the case that firms have a profitable deviation, in respect of consumers AY , if the margin on good Y say differs from λ . Hence the only symmetric equilibria which can survive are if margins satisfy

$$\rho_X < \lambda, \rho_Y < \lambda \text{ and } \rho_X + \rho_Y > \lambda_B \quad (3)$$

For example, if the firms settled on some equilibrium with $\rho_Y < \lambda$ and $\rho_X + \rho_Y < \lambda_B$, but $\rho_X > \lambda$, then raising the margin on good Y would gain profits from type AY 's and type B 's and leave type AX 's unaffected. In the same way, if (3) is not satisfied there is always one margin which can be altered to weakly grow profits for all customer types. Thus the firms must trade off prices being too high for the bundle purchasers versus too low for the single component purchasers, explaining part 2 of Theorem 1.

To explain part 1 of Theorem 1 we must understand why the firms focus more on setting the right component prices for type B buyers (low margin to sum to near λ_B) as opposed to type AX, AY buyers. The underlying reason for this follows from the fact that the reduced transport cost as a proportion of goods bought incurred by type B types makes them more elastic shoppers as compared to AX, AY buyers.⁷ To see this note that if a firm unilaterally lowers one of its component prices by ε it gains a measure of consumers equal to $\frac{\varepsilon}{2\lambda}$ from its rival. Thus the elasticity of demand of type AX consumers measured from symmetric prices of p is given by

⁷I am grateful to an anonymous referee for highlighting the importance of the relative elasticities of type B to AX, AY consumers in this context.

$\frac{p}{\lambda}$. Likewise, the elasticity of demand of the type B consumers around a bundle price of $2p$ is $\frac{2}{\lambda_B}p$. As $\frac{2}{\lambda_B} > \frac{1}{\lambda}$ type B consumers are more price sensitive and therefore punish price rises and reward price drops more; the pure component prices therefore give them more weight.⁸

A more technical intuition follows by noting that if profits were to be equal in both the constrained pure component pricing equilibrium and the mixed bundling equilibrium then we would require pure component pricing margins of $\rho_X^\dagger, \rho_Y^\dagger$ were

$$AX \cdot \rho_X^\dagger + AY \cdot \rho_Y^\dagger + B \cdot (\rho_X^\dagger + \rho_Y^\dagger) = AX \cdot \lambda + AY \cdot \lambda + B \cdot \lambda_B \quad (4)$$

Now suppose that the firms compete in pure components only and have set margins of $\rho_X^\dagger, \rho_Y^\dagger$ which deliver profits equal to the mixed bundling equilibrium. Suppose one firm contemplated lowering both of its component prices by $\varepsilon > 0$. This causes ε to be lost on existing AX and AY customers and 2ε on existing B type customers. However a measure of $\frac{\varepsilon}{2\lambda}$ AX consumers (and similarly for AY) are gained while a measure of $\frac{2\varepsilon}{2\lambda_B}$ type B customers are won. Combining we have the rate of change of the firm's profit with respect to ε given by

$$\begin{aligned} \left[\frac{\partial \Pi}{\partial \varepsilon} \right]_{\varepsilon=0} &= AX \underbrace{\left(\frac{\rho_X^\dagger - \lambda}{2\lambda} \right)}_{-ve} + AY \underbrace{\left(\frac{\rho_Y^\dagger - \lambda}{2\lambda} \right)}_{-ve} + B \underbrace{\left(\frac{\rho_X^\dagger + \rho_Y^\dagger - \lambda_B}{\lambda_B} \right)}_{+ve} \\ &> \frac{1}{2\lambda} \left[AX (\rho_X^\dagger - \lambda) + AY (\rho_Y^\dagger - \lambda) + B (\rho_X^\dagger + \rho_Y^\dagger - \lambda_B) \right] \quad \text{as } \lambda_B < 2\lambda \\ &= 0 \text{ by (4)} \end{aligned}$$

Hence the firm profits by lowering its margins below $\rho_X^\dagger, \rho_Y^\dagger$ exactly because the type B buyers incur economies of scope in bundle purchases and so have more elastic demand.

The welfare result in Theorem 1 follows as in both mixed bundling and constrained pure component pricing all consumers are served (by assumption) with their preferred product: that is symmetric prices for a given good result in consumers going to their nearest firms under both mixed bundling and pure component pricing. As welfare is given by the sum of profits and consumer surplus, a positive profit effect from mixed bundling translates into a negative consumer surplus effect. Thus when the firms are constrained to pure component pricing, the purchasers of both goods (large purchasers) have increased elasticity of demand due to their economies of scope in utility costs. The firms therefore price to win these consumers which forces the component prices down. The large buyers are therefore protecting prices for the small buyers. With mixed bundling this protection is lost. The firms can compete to lower prices to the large buyers, but the small buyers can now be targeted with higher prices as their elasticity of demand is less. On balance consumer surplus is lower when this price protection is lost.⁹ The effect of the loyalty rebate or mixed bundle therefore essentially acts as a transfer from the consumers to the firms.¹⁰

⁸The assumption of full market coverage suppresses any demand expansion for B types or contraction for AX, AY types. Thanassoulis (2005) showed that in the case that competition is sufficiently strong, the volume change effects did not alter the results.

⁹This consumer surplus change can be huge: taking the simplest model of $\lambda_B = \lambda$ and $AX = AY = B = \frac{1}{3}$ then introducing mixed bundling can raise component prices to AX, AY consumers by a factor of up to $\frac{3}{2}$.

¹⁰As all consumers one-stop shop due to type B 's economies of scope, mixed bundling becomes equivalent to third degree price discrimination. The conclusions of this model are therefore complementary to results from this

3.1 Generality of Theorem 1

The intuition behind Theorem 1 is based on the increased elasticity of demand the bundle purchasers have as a result of their economies of scope in utility costs. In this section we note that the results extend to the case of asymmetries between the firms. Specifically we suppose that firm I has a captive market of measure K with the firms competing for the custom of the undecideds, U ($= 1 - K$). To simplify and focus in on the firm asymmetry we assume that consumers are symmetrically distributed so that $AX = AY := \frac{1}{2}A$ and that the components have the same cost of production c . Finally we also assume that the large type B buyers have maximal economies of scope so that $\lambda_B = \lambda$. This does not imply symmetric pricing between firms I and II due to the captive consumers K .

Proposition 1 *Suppose that firm I has a captive market of measure K . The firms compete for the undecideds, U , who are uniformly distributed along the Hotelling line $\theta \in [0, 1]$. Theorem 1 parts (1) to (3) apply; further mixed bundling is now actively welfare reducing as compared to the constrained pure component pricing equilibrium.*

The margins and prices in this example now depend, in part, on the measure of captive customers, K : strong competition by firm I sees profits lost on these consumers. The actual market shares as well as prices will now be altered by the equilibrium (mixed bundling or not) and so the transport costs incurred change; the overall welfare is thus affected.

Irrespective of the asymmetry between the firms, and the fact that firm I now charges higher prices because of its captive customers, the intuition of Theorem 1 remains unaffected: without mixed bundling each firm competes vigorously for bundle sales and overweights these consumers in its component prices as large customers have more elastic demand due to their economies of scope in utility costs, small buyers are thus protected. Under the mixed bundling equilibrium, which we recall is the Nash pricing equilibrium, the price protection is lost for the smaller purchasers and so profits rise and consumer surplus falls, even though the price to the large purchasers declines.

3.1.1 An Aside on Competitive Volume Discounts

The results of Theorem 1 transfer directly to the case of competitive volume discounts by repeating the analysis with $AY = 0$ so that consumers either want one good or both goods. In such a situation it is natural to suppose that the transport costs do not grow proportionately with the volume bought ($\lambda_B < 2\lambda$). Theorem 1 thus implies that competitive volume discounts act for the firms and against the consumers (measured by overall consumer surplus). This result has immediate application to a wide range of situations, including the question of volume discounts for fleet car buyers raised by the UK parliament (footnote 1).¹¹ The results of this

field. Here however there are three different markets (for X , for Y , for the bundle) and a linear restriction on the margins ($\rho_X + \rho_Y = \rho_B$, or not in mixed bundling) rather than requiring margins are equal or not. The intuitions are therefore more intricate. Oligopolistic 3rd degree price discrimination for the standard case is discussed in Holmes (1989).

¹¹Stole (1995) considers a model of volume discounts (couched in terms of vertical differentiation over quality) in a competitive market in which firms are differentiated. However, in this case, the firms knew where each consumer was located in taste space, and no comparison was attempted with the equilibrium in which firms did not use volume discounts.

section do however depend on the firm specific preferences characteristic of consumer demand.

4 Product Specific Preferences

We next turn to study industries where consumers' taste differentiation is at the product level, between components, and not between stores or shopping experiences. Thus firms I and II manufacture differentiated versions of products X and Y at marginal costs of c_X and c_Y . The consumers incur utility taste costs for each component purchased. Recall from Section 2 that the type B consumers in the market for both goods are modeled as lying on a unit square, while the small type AX, AY consumers lie on a Hotelling line. For parsimony in the proofs that follow we assume that the measure $AX = AY := \frac{1}{2}A$. Note that as components are now differentiated, and bundle buyers do not have any economies of scope, some type B consumers will elect to purchase hybrid bundles of components (one component from each firm) as this will suit them best.

This model is an extension of Matutes and Regibeau (1992) as we explicitly consider the small consumers (type A) who seek to purchase only one component. This is important as component prices aimed at bundle purchasers will have effects on the smaller purchasers and so the prices, profits and consumer surplus effects of mixed bundling will all change. Due to the symmetry of the margin setting problem we seek to compare the symmetric Nash equilibrium in mixed bundling against the equilibrium when the firms are exogenously constrained to set only pure component prices. We let ρ denote the margin on each component sale and ρ_B the margin on the bundle sale. We begin with the following result:

Lemma 3 *The unique unrestricted symmetric Nash equilibrium in prices is one of mixed bundling in which the margins satisfy:*

$$\rho \in \left[\frac{11}{12}\mu, \mu \right] \quad (5)$$

$$\rho_B = 2\rho - \mu + \mu \left\{ q + \sqrt{q^2 - 4q} \right\} \leq 2\rho - \frac{\mu}{2} \text{ where } q := \frac{\rho - \mu}{\mu} \quad (6)$$

Thus some type B consumers form hybrid bundles in equilibrium. Further, as the measure of type A consumers tends to 0 then the symmetric mixed bundling equilibrium has margins tending to $\rho = \frac{11}{12}\mu$ and $\rho_B = \frac{4}{3}\mu$.

Note that if there are no type A consumers at all then one further possible equilibrium is for the firms to only offer the bundle for sale (perhaps making the components available at very high prices) resulting in type B consumers one-stop shopping. In fact with no type A consumers the bundle margin in this case would have to be equal to the transport cost of μ . Lemma 3 notes that there is no such equilibrium if the measure $A > 0$. This is because competition for the type A consumers pushes component margins down to the transport cost of μ also. At this level those type B consumers with the strongest preference for the hybrid bundle are indifferent between the hybrid bundle and the pure bundle from either of the firms. Each firm then has a profitable deviation by lowering the component prices by an ε more and creating hybrid bundle purchasers. Turning to the case of the constrained pure component pricing equilibrium we have:

Lemma 4 *The constrained price equilibrium in pure component prices with product specific preferences is given by margins $\rho = \mu$.*

We can then compare the two equilibria of mixed bundling versus the firms being constrained to pure component pricing and have:

Theorem 2 *If consumers have taste costs between the products and no firm specific travel costs then:*

1. *Mixed bundling (loyalty rebates) strictly lowers profits compared to pure component pricing*
2. *All prices are strictly lower under mixed bundling than under pure component pricing*
3. *Consumer surplus is strictly higher under mixed bundling as compared to pure component pricing*
4. *Welfare is lowered under mixed bundling as compared to pure component pricing.*

In contrast to the situation when consumers have firm specific preferences, this is a prisoners' dilemma result - firms unambiguously lose out from mixed bundling. Further, unlike with firm specific preferences, here mixed bundling causes all prices to decline, both those for large buyers (type B 's) and those for small buyers (type AX, AY).

To understand this result we first turn to the constrained pure component pricing equilibrium. As there are no economies of scope in demand (unlike with the firm specific preferences of Section 3) the population of consumers in the market for good X (whether a type AX or type B consumer) have taste costs uniformly distributed along the Hotelling line of length one. This is clear for types AX and applies to types B as the valuations for the goods are independent. Therefore the elasticity of demand for good X from type AX 's and type B 's is equal, and further equals the Hotelling value.¹² As the elasticity of demand for good X is the same from all consumers there is no price conflict between populations and the equilibrium settles at the Hotelling value of margins equalling transport cost - a very different conclusion from Section 3.

Now consider the mixed bundling Nash equilibrium in prices. The first insight is that type B consumers buying the bundle from *one firm* only have a high elasticity of demand and these consumers can be targeted by the special price for bundle purchases. This will push the equilibrium price charged for the bundle down; yielding sub-additive bundling. To see this note that at the pure component pricing equilibrium, by definition, a unilateral reduction in the component price of X was not profitable. This is depicted in Figure 1, panel (a) in which the margin gained on $\gamma + \delta$ equaled the lost profits on $\alpha + \beta$. But now consider lowering the bundle price by ε keeping the margin charged on component sales of X and Y constant. This is depicted in Figure 1, panel (b). The margin gained from consumers γ' equals the margin gained on γ as a consequence of the symmetry assumption. The cost in lost profit is only α however as opposed to

¹² An ε price reduction in the the price of good X from symmetric prices grows demand by $\frac{\varepsilon}{2\mu}$ and so corresponds to an elasticity of $\frac{p}{\mu}$ where p is the component price of good X .

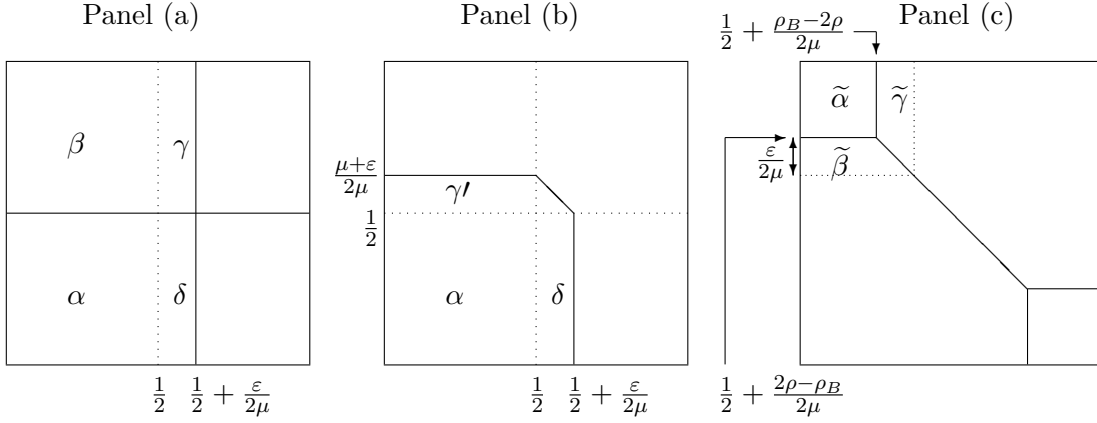


Figure 1: The demand choices of type B consumers

$\alpha + \beta$ with constrained pure component pricing. Therefore lowering the bundle price is certainly profitable.^{13,14}

It is therefore clear why the bundle price drops under mixed bundling as compared to pure component pricing. However, we have yet to explain the movement in the component prices. Particularly as nothing has changed for the type AX , AY consumers who are only in the market for one product. Any component price which charges a margin differing from μ will involve some profit loss on these consumers. Considering the type B consumers, panel (c) of Figure 1 shows the split of consumers between bundle purchases and hybrid sales. In particular the measure of consumers purchasing the hybrid bundle depends upon the bundle reduction compared to the component prices: specifically it depends on the value of $\frac{2\rho - \rho_B}{2\mu}$. If the gap between the overall price of the hybrid bundle versus the price of the bundle from one firm becomes too large then there will be few hybrid sales. Lowering the margin charged on a component will cannibalize some bundle sales costing margins of $(\rho_B - \rho)$. But it will also win some new hybrid customers who just buy good X at a margin of ρ . As $\rho > \rho_B - \rho$ due to sub-additive bundling this can be profitable if the component prices are too high. This is depicted in panel (c) of Figure 1. If Firm I were only serving type B 's then it would seek to alter its component price on good X so that an ε component price reduction which gained a margin of ρ on area $\tilde{\gamma}$ was exactly counterbalanced by the loss of margin $\rho_B - \rho$ on area $\tilde{\beta}$ and the loss of ε on area $\tilde{\alpha}$.

In the absence of type A consumers the above would give the optimal component margin to set as a function of the bundle margin. Specifically we would have:

$$\begin{aligned}
 \underbrace{\varepsilon \cdot \left[\frac{1}{2} + \frac{\rho_B - 2\rho}{2\mu} \right]^2}_{\tilde{\alpha}} &= \underbrace{\frac{\varepsilon}{2\mu} \cdot \rho \cdot \left[\frac{1}{2} + \frac{\rho_B - 2\rho}{2\mu} \right]}_{\tilde{\gamma}} - \underbrace{\frac{\varepsilon}{2\mu} (\rho_B - \rho) \cdot \left[\frac{1}{2} + \frac{\rho_B - 2\rho}{2\mu} \right]}_{\tilde{\beta}} \\
 &\Leftrightarrow 2\rho - \rho_B = \frac{\mu}{2}
 \end{aligned} \tag{7}$$

¹³In elasticity terms the pure bundle purchasers have a demand elasticity four times that of the type B consumers in a constrained pure component pricing equilibrium

¹⁴Without type A consumers Lemma 3 notes that this bundle margin would fall to $\frac{4}{3}\mu$ from 2μ .

Therefore, considered alone, the mixed bundle prices to the type B consumers would have *all* margins pulled down as compared to constrained pure component pricing. As noted however, lowering the margins below μ for type A consumers lowers profits. Thus some convex combination results. Hence all prices come down in the mixed bundling equilibrium, but to a lesser extent the greater the number of type A customers.

This explains part 2 of Theorem 2. As all prices are reduced, consumers pay less and yet all type B 's receive both components while all type A 's receive the good they desire. Thus the total payments reduce and so firm profits must also reduce. This gives part 1 of the theorem. The lower prices raises consumer surplus by revealed preference.¹⁵ Finally welfare is lower with mixed bundling as some type B 's are bribed to buy the pure bundle when a hybrid bundle would be nearer to their true tastes.¹⁶

5 Firm versus Product Specific Preferences as Two Polar Cases

The results of this paper could be summarised by the following table:

Polar case 1: <i>Firm Specific Preferences</i> Utility economies of scope imply one-stop shopping [Section 3]	Polar case 2: <i>Product Specific Preferences</i> Hybrid purchases and no economies of scope [Section 4]
Mixed bundling causes: Profits \uparrow Consumer Surplus \downarrow Prices to small buyers \uparrow Prices to large buyers \downarrow	Mixed bundling causes: Profits \downarrow Consumer Surplus \uparrow Prices to small buyers \downarrow Prices to large buyers \downarrow

Consumers might have both firm specific and product specific preferences. Thanassoulis (2005) confirmed that as the relative strength of the preferences (λ versus μ here) changes, the profit and consumer surplus shrink then switch over as the model considered moves from one polar case to the other. Note that, in general, the price to small buyers effect and consumer surplus effect will not flip at the same point: prices to small buyers must first rise sufficiently before the consumer surplus falls as a result of mixed bundling. This paper shows that consumer surplus does indeed fall if firm specific preferences are dominant.

We conclude this section by noting that both economies of scope and hybrid purchases are needed to move between the consumer surplus and profit effects noted in the table above. To see this suppose that the product specific preferences model (Section 4) is adjusted by setting $A = 0$ and having proportion φ of the population uniformly distributed on the unit square with the remainder $(1 - \varphi)$ uniformly distributed along the diagonal from $(0, 0)$ up to $(1, 1)$.¹⁷ In

¹⁵If consumers all chose to remain purchasing the same set of goods in both regimes then they would be unambiguously better off under the mixed bundling regime. If they actually alter their purchase choice (buying pure bundles instead of hybrids) then by revealed preference they have found an option which is preferable to what they were consuming in the constrained pure component pricing equilibrium.

¹⁶Further permutations of this model, but with populations AX, AY set to zero removing small consumers, are available in the survey paper provided by Armstrong (forthcoming). In this work Armstrong notes the profit effect and that the literature has failed thus far to explain the result as we here try to do.

¹⁷I am grateful to an anonymous referee and to Mark Armstrong for recommending this analysis.

all cases the lack of economies of scope cause the component prices without mixed bundling to be set independently: component margins are given by μ and large buyers would offer no price protection to the small buyers if A rose from 0 as their elasticity of demand for each component good is no different. Under perfect positive correlation ($\varphi = 0$) all consumers lie along the diagonal, there are no hybrid bundle buyers, and the model is equivalent to that of Section 3 but with no economies of scope. As a result mixed bundling and pure component pricing are equivalent; economies of scope are needed to move us into the realm of Theorem 1. At anything less than perfect positive correlation ($\varphi > 0$), the profit, welfare and consumer surplus results of Theorem 2 all apply: mixed bundling works in the consumers' favour.¹⁸

6 Industry Insights

We have explored the theoretical effect of mixed bundling upon consumer surplus in imperfectly competitive markets. In this section I note that there exist industries which appear to exhibit predominantly either firm specific or product specific preferences. Consequently this paper makes (potentially) falsifiable predictions as to the effect of unconstrained competition in mixed bundling on consumer surplus.

- Premium TV Channel Bundles

The firm specific preferences model of Section 3 appears to fit the market for pay TV well. For example, in the UK many homes have two competing providers of Pay TV services (BSkyB via satellite and a cable operator). These platforms often deliver the same channels due to upstream content agreements. In such a setting consumers incur a time and hassle cost of contracting with any one platform; this utility cost does not grow as more premium channels are signed up for however. Therefore this paper would predict that if the TV platforms were unable to mixed bundle premium TV channels then all premium channels would be priced low to try and win the demand from the heavy users who require multiple channels. However, mixed bundling allows low prices to be targeted at the heavy users and high prices at those consumers who only want one channel to the detriment of consumer surplus overall.

This possibility was investigated by the Office of Fair Trading in the UK. However the legal view was taken that the appropriate test was whether mixed bundling either led to foreclosure or could be demonstrated to not be part of a profit maximising strategy.¹⁹ This paper notes that mixed bundling is the Nash equilibrium of standard competition, nevertheless it is possible that consumers would be better served by pure component pricing in this case.

- Electrical Appliance Stores

It is not uncommon to see TVs and DVD players sold at a reduction if purchased together. However, a Sony TV may be quite differentiated as compared to a JVC TV, or even another Sony

¹⁸The component prices only fall under mixed bundling if $\varphi > \frac{2}{3}$. Before that point the intuition leading to equation 7 noted that the optimal margins with no type A consumers satisfied $2\rho = \rho_B + \frac{\mu}{2}$; as most buyers lie on the positive diagonal the optimal bundle margin approaches 2μ from below and so the component prices are forced above μ .

¹⁹"BSkyB investigation: alleged infringement of the Chapter II prohibition", OFT, December 2002, Case CP 01916-00. In particular §591.

TV of a different specification. As stores cannot stock unlimited different numbers of models of all manufacturers, the particular set of electrical goods sold in any store will be differentiated from those of its rivals. This industry therefore displays product specific differentiation. On the other hand, shop specific differentiation is weak. For example, in the UK the Competition Commission accepts that consumers of electrical appliances search widely across different stores before selecting the products to purchase - a clear indication that transport costs between stores are small.²⁰ This paper therefore predicts that mixed bundling (reductions on groups of products) works for the consumer here. Mixed bundling allows the high elasticity pure bundle purchasers to be targeted with lower prices. These in turn pull the prices charged for the individual components down to prevent the low bundle prices cannibalising too large a proportion of the (now comparably profitable) hybrid bundle sales. I am not aware of any empirical study which has explicitly tested this proposition however.

7 Conclusions

This paper has sought to understand how competition with mixed bundles affects firms' pricing and consumers generally. The insight of the paper is that the structure of the buyers' tastes is key in determining whether mixed bundling is a prisoners' dilemma for firms or acts against consumer surplus. If there are economies of scope in buyer incurred shopping costs or investment costs, which exceed any product differentiation, then mixed bundling lowers consumer surplus as compared to pure component prices. The economies of scope manifest themselves as a lack of hybrid bundle purchases. The reverse profit and consumer surplus results hold when buyers see the product differentiation as key (so taste costs are incurred for every product bought) and greater than any shopping costs or economy of scope in purchases.

Thanassoulis (2005) extended this model in two ways. Firstly the market coverage assumption was relaxed in situations of strong competition. Secondly a model of combined firm specific and product specific consumer tastes was analysed. The general intuitions described in this paper were robust to these extended settings.

Finally the point has been made to me that I have analysed a one-shot model of competition, while in a repeated situation firms may be able to learn to avoid mixed bundling if this was in their interests. This is of course true, but it is also tacit collusion. Maintaining higher prices by threatening more competitive conduct otherwise is certainly possible and is certainly illegal. This model therefore provides a useful prediction: that moving to the Nash pricing equilibrium with mixed bundling is a threat in industries which exhibit predominantly product, and not firm specific, differentiation. Whether any firms do try and maintain collusion using this threat is an empirical question.

A Proofs from Section 3

Proof of Lemma 1. First let us assume one stop shopping so that all B consumers always go to only one store. The prices p_X are only paid by type AX consumers, p_B by type B and so on. Thus the model collapses to a standard Hotelling one with linear costs for each consumer type. This has a unique

²⁰See the UK Competition Commission Report on Extend Warranties, 2003, §2.81.

equilibrium as the profit functions are negative quadratics in price (a result of the uniform distribution) and so $\rho_X = \lambda = \rho_Y, \rho_B = \lambda_B$ is the Hotelling equilibrium.

Now we relax the one stop shop assumption. At any symmetric price equilibrium firms I and II share the market. We wish to show that should firm I lower its component price by some small $\varepsilon > 0$ then no type B consumers would decide to create hybrid bundles of X from I and Y from II (denoted (I, II)) electing to one stop shop instead. Buying both goods from I gives a higher utility to (I, II) for all type B consumers with $\theta \leq \frac{\lambda}{\lambda_B} + \frac{p_X + p_Y - p_B}{\lambda_B} - \frac{\varepsilon}{\lambda_B}$. As $\lambda_B < 2\lambda$, $p_B \leq p_X + p_Y$ (by assumption of the model) and ε is small then (I, I) is preferred to (I, II) if $\theta \leq \frac{1}{2}$. Similarly the bundle from firm II is preferred to the hybrid bundle (I, II) for small ε if $\theta \geq \frac{1}{2}$. Thus in any small deviation from symmetric prices the type B consumers would one stop shop. Thus the price equilibrium found assuming one stop shopping is the only possible price equilibrium of the more general setting. ■

Proof of Lemma 2. At any symmetric pure component price equilibrium (p_X, p_Y) no firm must have an incentive to unilaterally lower its prices by some small ε . Suppose firm I did lower its good X component price by ε . We first note that in response to any such deviation all consumers will decide to make all their purchases from only one store. This is clearly true of type AX and AY consumers. Type B consumers will prefer to buy both goods from I rather than form the hybrid bundle (I, II) if $\theta < \frac{\lambda}{\lambda_B}$ and so this is true for all $\theta \leq \frac{1}{2}$. The bundle of both goods from II rather than the hybrid (I, II) will be preferred if $\theta \geq 1 - \frac{\lambda}{\lambda_B} + \frac{\varepsilon}{\lambda_B}$ and so for small ε given that $\lambda_B \leq 2\lambda$ this is true if $\theta \geq \frac{1}{2}$. Therefore type B consumers will always respond with purchases all from one store or another.

Taking one stop shopping as given therefore, we denote firm I 's profits from margins $(\rho_X - \varepsilon, \rho_Y)$ when facing firm II charging margins (ρ_X, ρ_Y) as $\Pi^I(\rho_X - \varepsilon, \rho_Y; \rho_X, \rho_Y)$ and we have

$$\Pi^I(\rho_X - \varepsilon, \rho_Y; \rho_X, \rho_Y) = AX \cdot (\rho_X - \varepsilon) \left[\frac{1}{2} + \frac{\varepsilon}{2\lambda} \right] + AY \cdot \frac{\rho_Y}{2} + B \cdot (\rho_X + \rho_Y - \varepsilon) \left(\frac{1}{2} + \frac{\varepsilon}{2\lambda_B} \right)$$

as in response to this price deviation firm I gains some new type B and type AX consumers. At a symmetric equilibrium we must have $\left[\frac{\partial}{\partial \varepsilon} \Pi^I(\rho_X - \varepsilon, \rho_Y; \rho_X, \rho_Y) \right]_{\varepsilon=0} = 0$ giving the first order condition

$$0 = AX \cdot \left(\frac{\rho_X}{\lambda} - 1 \right) + B \cdot \left(\frac{\rho_X + \rho_Y}{\lambda_B} - 1 \right)$$

Proceeding analogously for good Y gives the second first order condition. These can be written in matrix form as

$$\begin{bmatrix} \frac{AX}{\lambda} + \frac{B}{\lambda_B} & \frac{B}{\lambda_B} \\ \frac{B}{\lambda_B} & \frac{AY}{\lambda} + \frac{B}{\lambda_B} \end{bmatrix} \begin{pmatrix} \rho_X \\ \rho_Y \end{pmatrix} = \begin{pmatrix} AX + B \\ AY + B \end{pmatrix}$$

Inverting the matrix then gives the required result. ■

Proof of Theorem 1. Part 2: Comparing (1) to (2), and noting that $\lambda_B < 2\lambda$ we have that $\frac{2}{\lambda_B} - \frac{1}{\lambda} > 0$ and so the component prices are higher with mixed bundling than under constrained pure component pricing. The bundle price is lower with mixed bundling than pure component pricing if

$$\begin{aligned} \lambda_B &< \frac{\lambda}{\frac{AX \cdot AY}{\lambda} + \frac{B}{\lambda_B} (AX + AY)} \left(\frac{AX \cdot AY}{\lambda} + \frac{B}{\lambda_B} (AX + AY) - B \cdot AY \left(\frac{2}{\lambda_B} - \frac{1}{\lambda} \right) \right)^T \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &\Leftrightarrow \frac{\lambda_B}{\lambda} AX \cdot AY + B(AX + AY) < 2AX \cdot AY + B(AX + AY) \end{aligned}$$

which is true and so proves part 2.

Part 1: The profits accruing to each firm under mixed bundling are $\Pi_{mix\ bund}^I = \frac{1}{2} [\lambda (AX + AY) + \lambda_B B]$; under constrained pure component pricing the profits accruing to firm I are $\Pi_{no\ bund}^I = \frac{1}{2} [\rho_X (AX + B) + \rho_Y (AY + B)]$ where the margins are drawn from (2). Thus we have

$$\Pi_{mix\ bund}^I - \Pi_{no\ bund}^I = \frac{1}{2} [AX(\lambda - \rho_X) + AY(\lambda - \rho_Y) + B(\lambda_B - 2\lambda + (\lambda - \rho_X) + (\lambda - \rho_Y))]$$

Now note that $\lambda - \rho_X = \lambda \frac{B \cdot AY(2\lambda - \lambda_B)}{\lambda_B \cdot AX \cdot AY + \lambda \cdot B(AX + AY)}$ and so

$$\Pi_{mix\ bund}^I - \Pi_{no\ bund}^I = \frac{\lambda \cdot B \cdot AX \cdot AY \cdot (2\lambda - \lambda_B)}{2[\lambda_B \cdot AX \cdot AY + \lambda \cdot B(AX + AY)]} > 0 \quad (8)$$

as $\lambda_B < 2\lambda$ from the model.

Part 4: In both the mixed bundling Nash equilibrium and the constrained pure component pricing equilibrium all consumers are served (by assumption). The market is split in a symmetric equilibrium so all consumers get their preferred goods. Thus the outcome is equally efficient in either case.

Part 3: Welfare is the sum of consumer surplus plus profits. As welfare is unchanged and profits rise with mixed bundling, consumer surplus must fall.

Part 5: This now follows immediately from (8) as the profit difference between the two regimes shrinks as λ_B grows. In the limit of $\lambda_B = 2\lambda$ the profits are equal. The consumer surplus result follows as welfare is always unchanged. ■

Proof of Proposition 1 . Note that firms I and II will not set the same prices for a given good as firm I now has dedicated consumers.

Mixed Bundling Equilibrium

In any equilibrium we have, by symmetry, the margins charged by firm I for good X , good Y or indeed the bundle (as $\lambda_B = \lambda$ here) are equal at ρ^I . In this case the type B buyers strictly prefer the pure bundle from one or other firm to a hybrid due to the economies of scope in utility costs. But then any small reduction in the price of one component will cause no hybrid sales as there were no such marginal consumers. Thus we can assume wlog that all buyers one-stop shop.

For any undecided buyer, the point of indifference between the firms charging margins ρ^I and ρ^{II} is $\hat{\theta} = \frac{1}{2} + \frac{\rho^{II} - \rho^I}{2\lambda}$. Firm I must not gain by deviating from the competitive equilibrium by lowering the price of a good by ε . This gives rise to the first order condition

$$-\left[K + \frac{U}{2} + \frac{U}{2\lambda}(\rho^{II} - \rho^I)\right] + \frac{U}{2\lambda}\rho^I = 0$$

Repeating for firm II , we have:

$$\frac{1}{\lambda} \begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} \rho^I \\ \rho^{II} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \frac{K}{U} \\ \frac{1}{2} \end{pmatrix}$$

This can be solved to give the mixed bundling equilibrium margins as

$$\begin{pmatrix} \rho^I \\ \rho^{II} \end{pmatrix} = \lambda \begin{pmatrix} 1 + \frac{4K}{3U} \\ 1 + \frac{2K}{3U} \end{pmatrix} \Rightarrow \hat{\theta} = \frac{1}{2} - \frac{K}{3U} \text{ (the market share of the undecideds)} \quad (9)$$

Thus, recalling the captive market K , firm I makes a profit of $\frac{4\lambda}{3} \left(\frac{3}{4} + \frac{K}{U}\right) \left(\frac{U}{2} + \frac{2K}{3}\right)$ and firm II makes a profit of $\frac{2\lambda}{3} \left(\frac{3}{2} + \frac{K}{U}\right) \left(\frac{U}{2} + \frac{K}{3}\right)$.

Pure Component Pricing Equilibrium

We can again assume one-stop shopping without loss of generality as at any equilibrium with margins ρ^I for firm I and margins ρ^{II} for II , there will be no consumers indifferent between buying the bundle from one shop versus forming a hybrid. But then a small ε reduction in price will not create any hybrid sales either. Proceeding, note that by symmetry, the price of the components will be the same for each component, though different across firms. The points of indifference will now differ for bundle purchasers. For component purchasers (type A) we have $\hat{\theta}_A = \frac{1}{2} + \frac{\rho^{II} - \rho^I}{2\lambda}$ while for bundle purchases (type B consumers) we have $\hat{\theta}_B = \frac{1}{2} + \frac{\rho^{II} - \rho^I}{\lambda}$. Firm I should not benefit by lowering both its component prices by ε . Such a move would change firm I 's profits to:

$$\Pi^I(\varepsilon) = A \cdot (\rho^I - \varepsilon) \left[K + U \left(\frac{1}{2} + \frac{\rho^{II} - \rho^I + \varepsilon}{2\lambda} \right) \right] + 2B \cdot (\rho^I - \varepsilon) \left[K + U \left(\frac{1}{2} + \frac{\rho^{II} - \rho^I + \varepsilon}{\lambda} \right) \right]$$

Taking the derivative of this expression with respect to ε , setting it to 0 gives the first order condition as $\rho^I - \frac{1}{2}\rho^{II} = \lambda \left(\frac{1}{2} + \frac{K}{U}\right) \left(\frac{A+2B}{A+4B}\right)$. Similar conditions for firm II allows the equilibrium margins and indifference points to be solved for:

$$\begin{pmatrix} \rho^I \\ \rho^{II} \end{pmatrix} = \frac{4\lambda}{3} \begin{pmatrix} A+2B \\ A+4B \end{pmatrix} \begin{pmatrix} \frac{3}{4} + \frac{K}{U} \\ \frac{3}{4} + \frac{K}{2U} \end{pmatrix} \Rightarrow \begin{pmatrix} \hat{\theta}_A \\ \hat{\theta}_B \end{pmatrix} = \begin{pmatrix} \frac{1}{2} - \frac{K}{3U} \left(\frac{A+2B}{A+4B}\right) \\ \frac{1}{2} - \frac{2K}{3U} \left(\frac{A+2B}{A+4B}\right) \end{pmatrix} \quad (10)$$

Comparison of mixed bundling to pure component pricing

Comparing (9) to (10) we see that part 2 of Theorem 1 holds: component prices are lower under pure component pricing while the bundle price is higher.

We now turn to the profit comparison. Recalling the captive market K for firm I and the contributions of groups A and B , the pure component pricing (no loyalty rebates) profits can be established as

$$\Pi_{PC}^I = \frac{4\lambda}{3} \left(\frac{3}{4} + \frac{K}{U}\right) \left(\frac{U}{2} + \frac{2K}{3}\right) \left(\frac{(A+2B)^2}{A+4B}\right) \quad \Pi_{PC}^{II} = \frac{2\lambda}{3} \left(\frac{3}{2} + \frac{K}{U}\right) \left(\frac{U}{2} + \frac{K}{3}\right) \left(\frac{(A+2B)^2}{A+4B}\right)$$

Therefore, for both firms, mixed bundling raises profits if $\frac{(A+2B)^2}{A+4B} < 1$ which is true as $A+B=1$ and $A, B > 0$. Thus loyalty discounts do indeed raise firm profits. This gives part 1 of Theorem 1.

We next turn to welfare: the consumers receive the same type of component(s) in both the mixed bundling (MB) equilibrium and the pure component pricing equilibrium (PC). However, the shop from which the consumer purchases may change. As the transport cost is wasteful (it is not profit for the producer) it contributes to welfare changes. If the indifference point moved from $\hat{\theta}_{MB}$ under mixed bundling to $\hat{\theta}_{PC}$ under pure component pricing, then the change in welfare would be given by:

$$W_{MB} - W_{PC} = \int_{\hat{\theta}_{PC}}^{\hat{\theta}_{MB}} -\lambda\theta + \lambda(1-\theta) d\theta = \lambda [\theta - \theta^2]_{\hat{\theta}_{PC}}^{\hat{\theta}_{MB}}$$

Inserting the actual expressions for the indifference points we have:

$$\begin{aligned} W_{MB} - W_{PC} &= A \cdot U \cdot \frac{\lambda K^2}{9 U^2} \left[\left(\frac{A+2B}{A+4B}\right)^2 - 1 \right] + B \cdot U \cdot \frac{\lambda K^2}{9 U^2} \left[4 \left(\frac{A+2B}{A+4B}\right)^2 - 1 \right] \\ &= \frac{\lambda K^2}{9 U} \left[\frac{(A+2B)^2}{A+4B} - (A+B) \right] < 0 \Leftrightarrow -AB < 0 \end{aligned}$$

Thus mixed bundling (competitive loyalty rebates) causes welfare to decline. As welfare is the sum of consumer surplus and profits, and mixed bundling causes profits to rise, we must have that consumer surplus falls under mixed bundling as opposed to pure component pricing. This completes the proof of Proposition 1. ■

B Proofs from Section 4

Proof of Lemma 3. We first consider only the type A consumers who seek to buy only one component. Though c_X may differ from c_Y , the production costs are assumed the same across firms and $AX = AY = \frac{1}{2}A$ implies that the margin setting decisions are symmetric. Thus suppose that both firms settle on charging margins ρ for X or Y purchased alone and ρ_B if the bundle is bought. Suppose that firm I lowered its good X margin by ε then the profit for the type A consumers only would be:

$$\Pi_I^{\text{Gp } A}(\rho - \varepsilon, \rho, \rho_B; \rho, \rho, \rho_B) = A \cdot \left(\underbrace{\frac{1}{2} \cdot \frac{1}{2} \rho}_{\text{Types } AY} + \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{\varepsilon}{2\mu}\right) (\rho - \varepsilon) \right)$$

where we use the convention that the first three arguments of the profit function give the margin levied for components X, Y and the bundle respectively from I . The next three arguments give the margins for these goods from firm II . The contribution to the first order condition is given by:

$$\left[\frac{\partial}{\partial \varepsilon} \Pi_I^{\text{Gp } A}(\rho - \varepsilon, \rho, \rho_B; \rho, \rho, \rho_B) \right]_{\varepsilon=0} = A \cdot \left(\frac{\rho - \mu}{4\mu} \right) \quad (11)$$

Turning now to the type B consumers, we first assume that some of these consumers weakly prefer the hybrid bundle to either of the pure bundles at equilibrium prices. At the end of the proof we will show that this must be the case. Suppose therefore that firm I decides to lower the price of component X by ε . The purchasing behaviour of all of the Group B consumers can be determined as a function of their tastes (s, t) , and is plotted in Figure 1, panel (c). For example, the hybrid bundle (I, II) is purchased if consumers lie in the top left of the Hotelling square with $t > \frac{1}{2} + \frac{2\rho - \rho_B}{2\mu} - \frac{\varepsilon}{2\mu}$ and $s < \frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} + \frac{\varepsilon}{2\mu}$. Overall the profit function is:

$$\begin{aligned} \frac{\Pi_I^{\text{Gp } B}(\rho - \varepsilon, \rho, \rho_B; \rho, \rho, \rho_B)}{B} &= \frac{\rho_B}{2} \left\{ 1 - \left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} + \frac{\varepsilon}{2\mu} \right]^2 - \left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} \right]^2 \right\} \\ &\quad + (\rho - \varepsilon) \left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} + \frac{\varepsilon}{2\mu} \right]^2 + \rho \left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} \right]^2 \end{aligned} \quad (12)$$

Proceeding analogously, if firm I were to lower the price of the bundle by ε then the profit function would be

$$\begin{aligned} \frac{\Pi_I^{\text{Gp } B}(\rho, \rho, \rho_B - \varepsilon; \rho, \rho, \rho_B)}{B} &= (\rho_B - \varepsilon) \left\{ 1 - \frac{1}{2} \left[1 - \frac{\varepsilon}{2\mu} \right]^2 \right. \\ &\quad \left. - 2 \left(\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} - \frac{\varepsilon}{2\mu} \right) \frac{1}{2} \left(\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} + \frac{\varepsilon}{2\mu} \right) \right\} \\ &\quad + 2\rho \left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} - \frac{\varepsilon}{2\mu} \right] \left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} \right] \end{aligned} \quad (13)$$

The first order conditions derived from the contribution of the Group B consumers can now be derived as

$$\left[\frac{\partial}{\partial \varepsilon} \Pi_I^{\text{Gp } B}(\rho - \varepsilon, \rho, \rho_B; \rho, \rho, \rho_B) \right]_{\varepsilon=0} = \frac{B}{2} \cdot \left[1 - \frac{2\rho - \rho_B}{\mu} \right] \left[\frac{2\rho - \rho_B}{\mu} - \frac{1}{2} \right] \quad (14)$$

$$\left[\frac{\partial}{\partial \varepsilon} \Pi_I^{\text{Gp } B}(\rho, \rho, \rho_B - \varepsilon; \rho, \rho, \rho_B) \right]_{\varepsilon=0} = B \cdot \left(\left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} \right]^2 - \frac{\rho}{\mu} \left[\frac{1}{2} - \frac{2\rho - \rho_B}{2\mu} \right] + \frac{1}{2\mu} (\rho_B - \mu) \right) \quad (15)$$

We first prove the final result concerning the equilibrium when $A \rightarrow 0$. When almost all consumers are drawn from Group B then the equilibrium margins are given by setting (14) = 0 = (15). From (14) we have that either $\frac{2\rho - \rho_B}{\mu} = 1$ or $\frac{1}{2}$ at the maximum. Setting $\frac{2\rho - \rho_B}{\mu} = \frac{1}{2}$ in (12) and taking the second derivative with respect to ε we find that we do have a local maximum at $\varepsilon = 0$. This is not the case if $\frac{2\rho - \rho_B}{\mu} = 1$ which would imply that a unilateral drop in the price of one component would be profitable. Substituting $\frac{2\rho - \rho_B}{\mu} = \frac{1}{2}$ in (15) we find that $\rho_B = \frac{4}{3}\mu$ and hence that $\rho = \frac{11}{12}\mu$ as required.

Consider now the general mixed bundling equilibrium where consumers of type A and B are both present. For firm I to have no incentive to deviate on its bundle price we must have the first order condition (15) equal to zero. Hence we have the equilibrium bundle margin ρ_B in terms of ρ , the component margins. To make this explicit we use the notation $N := 1 - \frac{2\rho - \rho_B}{\mu}$ and $q := \frac{\rho - \mu}{\mu}$, (15) becomes $N^2 - 2qN + 4q = 0$ which implies that

$$N = q + \sqrt{q^2 - 4q} \quad (16)$$

The positive root is taken to maximise profit.²¹ Expanding N for ρ_B then gives the required

$$\rho_B = 2\rho - \mu + \mu \left\{ q + \sqrt{q^2 - 4q} \right\} \quad (17)$$

²¹If $A \rightarrow 0$ then the symmetric price equilibrium with mixed bundling has that $q = -\frac{1}{12}$ and $N = \frac{1}{2}$. This can only be the case if the positive root in (16) applies.

We now turn to the incentive to deviate on the component price. The condition for the bundle price, (17), gives us two key conditions. Firstly there must be a solution for ρ_B in equation (17). This implies that $q^2 - 4q \geq 0$ and so $q \leq 0$ or $q \geq 4$. In addition we restrict attention to the economically relevant sub-additive bundling so that $\rho_B \leq 2\rho$ and hence $q + \sqrt{q^2 - 4q} \leq 1$. But if $q \geq 4$ then the left hand side of this expression is greater than or equal 4. A contradiction. We therefore must have $q \leq 0$ and so $\rho \leq \mu$ for a sub-additive equilibrium. We now invoke the intermediate value theorem to show that the necessary condition for a Nash equilibrium can be satisfied sub-additively. From (11) and (14) we have

$$\left[\frac{\partial \Pi_I}{\partial \varepsilon} (\rho - \varepsilon, \rho, \rho_B; \rho, \rho, \rho_B) \right]_{\varepsilon=0} = A \cdot \frac{q}{4} + B \cdot \frac{1}{2} N \left(\frac{1}{2} - N \right) \quad (18)$$

We know that at $\rho = \frac{11}{12}\mu$ we have $q = -\frac{1}{12}$ and $N = \frac{1}{2}$ so that $\frac{\partial \Pi_I}{\partial \varepsilon} < 0$. We have already established that $q \leq 0$. Substituting the actual function $N(q)$ for N and considering q tending to zero from below gives

$$\begin{aligned} \lim_{q \nearrow 0} \frac{\partial \Pi_I}{\partial \varepsilon} &= \lim_{q \nearrow 0} \frac{1}{4} \left\{ Aq + B \left[9q - 4q^2 + (1 - 4q) \sqrt{q^2 - 4q} \right] \right\} \\ &= \lim_{q \nearrow 0} \left\{ O(q) + \frac{B}{4} (1 - 4q) \cdot \left(2 \sqrt{\underbrace{-q}_{+ve}} \right) \sqrt{1 - \frac{q}{4}} \right\} \\ &\rightarrow \frac{B}{2} \sqrt{-q} > 0 \text{ for } q \text{ small and negative} \end{aligned}$$

Thus the intermediate value theorem notes that there exists at least one value of $q \in (-\frac{1}{12}, 0)$ at which $\frac{\partial \Pi_I}{\partial \varepsilon} = 0$ and there will be no incentive to change the component prices.

To ascertain that there can be only one such solution with $q < 0$, note that $N(\frac{1}{2} - N)$ is concave and hence quasi-concave. In addition, $N(q)$ is a monotonic decreasing transformation of q on $q < 0$. Hence $\frac{\partial \Pi_I}{\partial \varepsilon}$ is quasiconcave and so can have at most two roots. One root is at $q = 0$ and we have found the second, label it q^* , lying in $q \in (-\frac{1}{12}, 0)$. This second root has $\frac{\partial}{\partial q} \frac{\partial \Pi_I}{\partial \varepsilon}(q^*) > 0$ by the quasi-concavity result, and so one can see that the associated margin ρ^* (where $q^* := \frac{\rho^* - \mu}{\mu}$) is such that profits would be increased by raising ρ from below ρ^* and lowering ρ from above ρ^* . As $q^* > -\frac{1}{12}$ we have $\rho_B \leq 2\rho - \frac{\mu}{2}$ as required.

We have thus shown the result as long as some type B buyers weakly prefer to purchase hybrid bundles: that is as long as $2\rho - \rho_B \leq \mu$.²² Therefore suppose, for a contradiction, that $2\rho - \rho_B > \mu$ so that no type B consumers purchased the hybrid bundle. Competition for the type A consumers would then be standard Hotelling and component margins would be given by transport cost, $\rho = \mu$. Turning to the type B consumers, suppose that the equilibrium margins are given by ρ_B . If firm I deviated by lowering its bundle margin by ε then it will make profits from B type consumers of $\Pi^{\text{Gp B}}(\varepsilon) = (\rho_B - \varepsilon) \left[1 - \frac{1}{2} \left(1 - \frac{\varepsilon}{2\mu} \right)^2 \right]$ for $\varepsilon \geq 0$.²³ This deviation must not be profitable and so at an equilibrium we require $\frac{\partial}{\partial \varepsilon} \Pi^{\text{Gp B}}(0) \leq 0$ which implies $\rho_B \leq \mu$. Proceeding similarly we can rule out any incentive to unilaterally raise the bundle margin if $\rho_B \geq \mu$. Thus the pure bundling equilibrium must have $\rho_B = \mu$. But in this case $2\rho - \rho_B = \mu$ which contradicts the original assumption. Hence the analysis following (12) and (13) above is valid confirming Lemma 3.²⁴

²²As then type B consumers located at $(0, 1)$ and $(1, 0)$ weakly prefer the hybrid bundles (I, II) and (II, I) to either of the pure bundles.

²³At equilibrium the boundary between the two firms' consumers is given by the leading diagonal of the unit square. The formula for the market share thus differs depending upon whether any price deviation makes firm I cheaper or more expensive than firm II .

²⁴We have consistently focused on sub-additive equilibria. In fact this is without loss of generality as there is no super-additive symmetric pricing equilibrium in this model. The firms would always rather deviate by lowering

■

Proof of Lemma 4. Suppose that the firms have arrived at an equilibrium in which the margin ρ is charged on components X and Y . There is no mixed bundling here by assumption. Neither firm must gain by altering either of these margins by some small amount. Consider first the effect on profits of the Group A consumers who buy only one component. These consumers are distributed along a Hotelling line between the firms. Thus, by a standard Hotelling analysis, ignoring type B 's, the equilibrium margins would be at transport cost. That is $\rho = \mu$.

Now consider the effect on profit from the Group B consumers (who buy both components). As there are no economies of scope in consumption, B consumers will decide where to buy good X and Y independently: and so a standard Hotelling analysis again applies. In particular, if firm I lowered the price of good X by ε then all type B buyers with $s < \frac{1}{2} + \frac{\varepsilon}{2\mu}$ will buy X from I . This results in an ε loss on those with $s \leq \frac{1}{2}$ and a margin gain on the new consumers with $s \in \left(\frac{1}{2}, \frac{1}{2} + \frac{\varepsilon}{2\mu}\right)$. The standard Hotelling margin of transport cost (μ) is the equilibrium result.

The equilibrium component margin for both populations A and B in isolation is thus μ and so this is the pure component (no loyalty rebate) price in equilibrium. ■

Proof of Theorem 2. Part 2 of the theorem, that all prices fall under mixed bundling as compared to pure component pricing, is immediate from Lemmas 3 and 4. As all margins decline under mixed bundling, profits from types AX, AY clearly fall. All type B consumers buy both goods X and Y at lower prices under mixed bundling. Industry profits are therefore lower with mixed bundling. By symmetry the firms share the industry profits equally and so firm profits are lower under mixed bundling also. This gives part 1 of the Theorem.

As all prices decline in mixed bundling as compared to pure component pricing, mixed bundling must have higher consumer surplus. This is because the consumers could stay purchasing the same goods as they do in the pure component pricing equilibrium and they would all be better off. If they change their purchase decision they are being even further rewarded in utility terms. Thus part 3 of the Theorem is shown.

Part 4 of the theorem is also immediate as in the mixed bundling equilibrium some type B consumers are induced to buy the sub-additively priced bundle and so do not receive their ideal hybrid bundle. ■

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the bundle margin if component margins are at or above μ ; while raising the component margins is profitable if these margins are below μ .

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