MS. D. Phil. d. 1175

Dep. 17 xii 52

EXPERIMENTS ON TRANSPORT PHENOMENA AT LOW TEMPERATURES

An account of some investigations on the phenomenon of film transfer in liquid helium II, and on the flow of liquid helium II in narrow channels.

A thesis submitted to the University of Oxford for the degree of Doctor of Philosophy

B. S. Chandrasekhar,
The Queen's College.

Trinity Term 1952

ABSTRACT.

This thesis deals mainly with some investigations on the helium II film in motion. Some experiments are also described on the flow of liquid helium II through a capillary packed with jewellers rouge. A summary, chapter by chapter, of the contents of the thesis follows.

Chapter 1.

This contains a brief survey of the state of our experimental knowledge about the helium II film when these investigations were started. The various theories of the film that have been proposed, are very briefly reviewed, with particular regard to their physical interpretation. This survey is relevant to an understanding of the experiments that are described in the succeeding chapters of the thesis. A brief description of the cryostat and auxiliary apparatus that were used in these experiments is given.

Chapter 2.

The isothermal transfer of the helium II film at sub-critical rates has been studied. The study of such transfer is of importance, for it can give information about frictional forces which may arise during the motion of the film. Sub-critical rates are produced in a manner analogous to that by which the current in a superconductor

is kept below its critical value by means of a limiting series resistance. A detailed analysis of the results shows that neither the hypothesis of vanishing viscosity for Helium II, nor the mechanism of siphon flow, can account for these sub-critical rates. The concept of the zero-point diffusion of the superfluid, suggested by Mendelssohn, is shown to provide a satisfactory explanation of the results.

Chapter 3.

An investigation of the transfer of the helium II film at sub-critical rates, produced by the thermomechanical effect, is described. This method is more versatile than the one described in chapter 2, for a wide range of sub-critical rates can be produced very much more simply. The apparatus consists essentially of a small glass dewar with a well-fitting lid, so that the inside of the dewar is thermally well isolated from the outside world. Flow of liquid helium II into the dewar through the film is produced by supplying heat electrically to the inside. The variation of the rate of flow with the heating rate, for different temperatures, has been studied. Some interesting results on the properties of the film follow:

(1) The flow in the film is frictionless for a wide range

of sub-critical velocities. This provides an independent and more complete corroboration of the conclusions obtained in chapter 2. It is pointed out that, for these sub-critical rates in the film, the Gorter-Mellink theory of mutual friction is inapplicable. On the other hand, it seems clear that a mechanism allowing a sharp onset of the critical rate, below which the flow is frictionless, is necessary. Such a mechanism is provided by all those theories of film transfer which ultimately appeal to the principle of indeterminacy for determining the minimum excitation energy of the moving liquid.

(2) The heat of transport, Q, of the helium II film has been determined as a function of the temperature. The difference between this quantity, and H, the heat content (or enthalpy) of the bulk liquid, is pointed out. This distinction has caused some confusion in the past. It is shown that the entropy carried by the moving film is vanishingly small (the Tisza-London hypothesis), by calculating the entropy S of the bulk liquid from the relation $Q = TS$, and comparing it with that obtained from specific heat measurements.

(3) The heat of transport Q has been determined in the present experiments directly; it is, in essence, an iso-

thermal, isobaric determination for the film. The experiments of Kapitza on the fountain effect also give values of Q, for bulk flow in narrow channels, deduced from H.London's equation. There is close agreement between the Q's obtained by the two methods, thus supporting the interpretation, on the two-fluid model, of these phenomena.

Chapter 4

Some observations are reported, which establish the conditions under which bulk liquid can be formed from the helium II film during isothermal transfer. It had been thought previously that bulk liquid could be so formed, at any point below the highest meniscus, in a given arrangement for film transfer. The present experiments show that this is a necessary, but not a sufficient condition. The film will only form bulk liquid if it has to flow at a super-critical rate in order to remain as the film. In other words, bulk liquid is formed if the film encounters a perimeter, smaller than the limiting perimeter above the highest meniscus. These considerations do not apply to the lowest point on the surface for film transfer, where bulk liquid can always be formed from the film. The experiments provide an indirect confirmation of the uniqueness of the critical rate of film transfer.

Chapter 5

The transfer rates of the helium II film on a variety

of surfaces have been studied, and the following results obtained:

(1) Experiments with specially polished stainless steel beakers show that the anomalous transfer rates previously observed on metal surfaces are to be associated with capillary flow in surface cracks and irregularities. If such a surface could be given a finish comparable to that of glass, the transfer rate would be the same as that on glass.

(2) A study of the transfer rates on perspex and lucite in dependence on their surface finish corroborates the hypothesis of capillary flow; it is suggested that this may also be responsible for the maximum in the curve of transfer rate versus temperature, observed in some cases.

(3) Surface flow of liquid helium just above the λ-point, simulating genuine film transfer, was predicted, and actually observed in a carefully shielded 'rough' perspex beaker. This provides further evidence for the hypothesis of capillary flow.

(4) Experiments are described, which show that the increase in the transfer rates observed when the higher meniscus is near the rim of the beaker, cannot be due to an increased thickness of the film at the rim. An alternative explanation is offered, that it is a manifestation of capillary

flow, possibly coupled with the effect of a random distribution of the (microscopic) available perimeters for film transfer.

(5) The transfer rates on glass have been measured under large gravitational pressure differences, up to 15 cms. of liquid helium. It appears likely that the increased potential difference has no influence on the transfer rate, the observed increase of which can be attributed to the surface structure of the beakers used.

All these experiments point indubitably to a uniquely defined critical rate of transfer of the Helium II film, independent of the nature of the substrate. Thus a complete theory of the critical rate should come out of the quantum mechanics of the helium film alone, independently of the nature of the forces between the atoms in the substrate and the helium atoms. This last factor would of course influence the thickness of the film. It thus appears that the theories of the critical rate based on considerations of zero-point energy are in the right direction.

Chapter 6.

The flow of helium II through a capillary tightly packed with jewellers rouge has been studied, under thermal

and gravitational potential gradients. The pressure at an intermediate point in the channel is measured by the method previously used by Bowers and Mendelssohn in their experiments with smooth narrow channels. It appears that superflow in such tortuous channels is of a turbulent nature under isothermal conditions. The possibility of turbulence in superflow, thus established, has an important bearing on the interpretation of a large number of experiments on the flow in capillaries, and on experiments with oscillating discs. The thermal flow measurements show a curious inconsistency with those on gravitational flow. For a wide range of velocities, thermal flow is frictionless, while gravitational flow is dissipative; this is deduced from the behaviour of the pressure at the intermediate point in the channel. Furthermore, thermal flow shows a more or less sharp critical rate, while gravitational flow does not do so. These observations indicate that the concept of 'pressure at a point' in a superflow channel may not have the same meaning as in classical flow; it is not simply dependent on the pressures at the two ends, but requires one or more other parameters to determine it.

TABLE OF CONTENTS.

Page.

Chapter 1. Introduction.

1.1 Previous Experimental Knowledge of the Helium II Film 1

1.2 Theories of the Helium II Film 10

1.3 The Apparatus 15

Chapter 2. Isothermal Sub-critical Flow in the Helium II Film.

2.1 Introduction 22

2.2 The Apparatus 30

2.3 The Results 33

2.4 Discussion 35

2.5 Some Miscellaneous Observations 38

Chapter 3. Subcritical Transfer of the Helium II Film under a Temperature Gradient.

3.1 Introduction 41

3.2 The Significance of Measurements of Sub-critical Rates 44

3.3 The Apparatus 50

3.4 The Experiments 53

3.5 The Equilibrium Position of the Inner Level 59

Page

3.6 Discussion of the Results 64

3.7 Frictional Processes in the Helium II Film 85

3.8 The Heat of Transport of the Helium II Film 91

3.9 Summary 106

Chapter 4. Isothermal Formation of Bulk Liquid from the Helium II Film 110

Chapter 5. Isothermal Transfer of the Helium II Film over Various Surfaces.

5.1 Introduction 119

5.2 Specially Polished Stainless Steel Beakers 128

5.3 Perspex and Lucite 147

5.4 Surface Flow of Liquid Helium above the λ-point 162

5.5 Film Transfer Rate near the Rim of a Beaker 172

5.6 Film Transfer in 'Long' Beakers 180

5.7 Summary 186

Chapter 6. Flow of Helium II through a Rouge-packed Tube.

6.1 Introduction 191

Page

6.2 Experiments and Results 199

Acknowledgements 218

CHAPTER 1

INTRODUCTION

1.1 Previous Experimental Knowledge of the Helium II Film.

A brief account of those properties of the helium II film which were already known when the present experiments were undertaken, and which are relevant to an understanding of the results obtained, is given here. Attention is confined to the film, so that the wealth of data that has been amassed regarding the flow of helium II through channels has not been considered here; reference will be made to these results, where necessary, in the body of the thesis (cf. chapters 3 and 6).

The earliest indication of the extraordinary flow properties of liquid helium II was contained in an observation of Kamerlingh Onnes's (1922), that the levels inside and outside a beaker dipping in helium II equalised with striking rapidity. Rollin (1936) suggested the existence of a film covering the walls above the helium II, in order to explain the anomalous change in the rate of evaporation as the temperature is lowered through the λ-point. Kikoin and Lasarew (1938, 1939) showed that such a film was probably responsible for equalising tem-

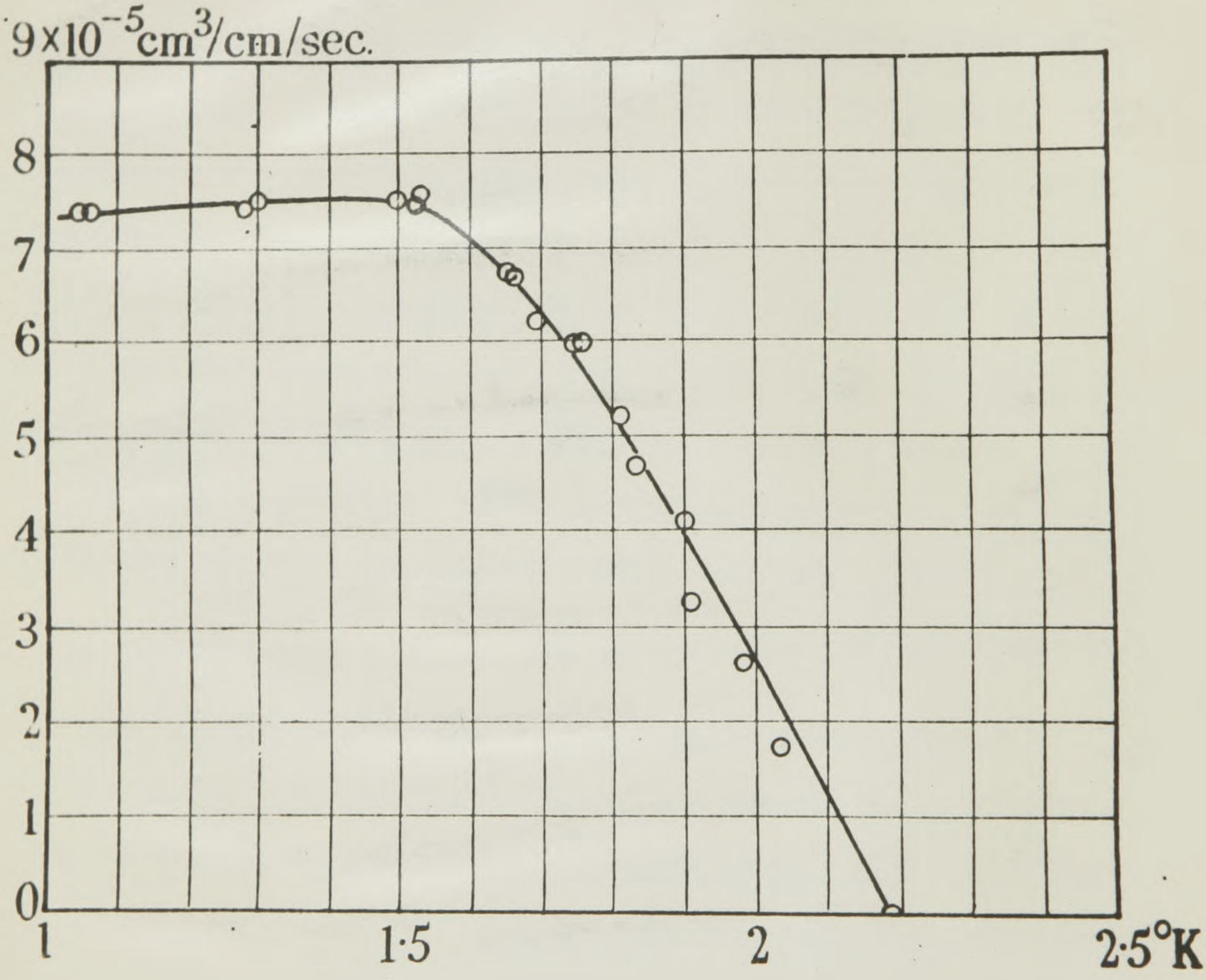


Figure 1-1 The dependence of the transfer rate of the helium II film on temperature, as determined by Daunt and Mendelssohn.

peratures along a tube dipping in helium II. By allowing the film to re-form on a large area connecting with the top of the tube from where it had been previously evaporated by heating, and measuring the corresponding loss of helium from the bath, they estimated the thickness of the film to be 2 to 3 x 10^{-6} cm.

The classic experiments of Daunt and Mendelssohn (1938, 1939a) established in detail many of the properties of the film. Their results may be summarised as follows:

(1) Liquid helium II always collects at the lowest available level.

(2) This 'transfer' takes place in a film along the solid surface of the containers.

(3) The rate of transfer is practically independent of the difference in height between the levels and of the underlying material.

(4) The rate of transfer is proportional to the smallest perimeter that the film has to pass through above the higher meniscus.

(5) The rate of transfer depends on temperature, rising from the value zero at the λ-point, and becoming nearly constant below about 1.5°K (figure 1-1).

All these results, with one exception, referred to experiments done with glass beakers. The exception was



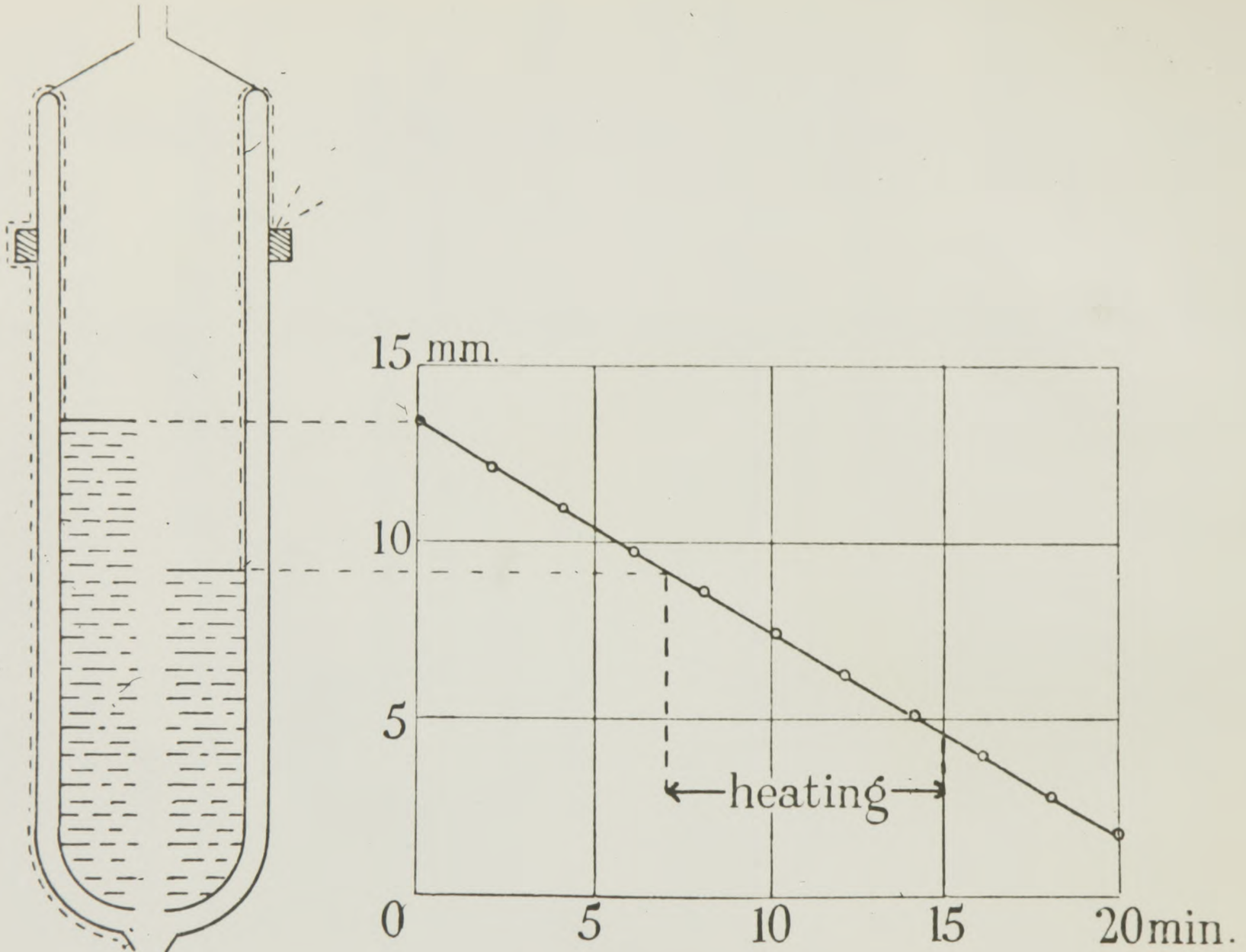


Fig 2.2 Heat flow in the helium II film (Daunt & Mendelssohn).

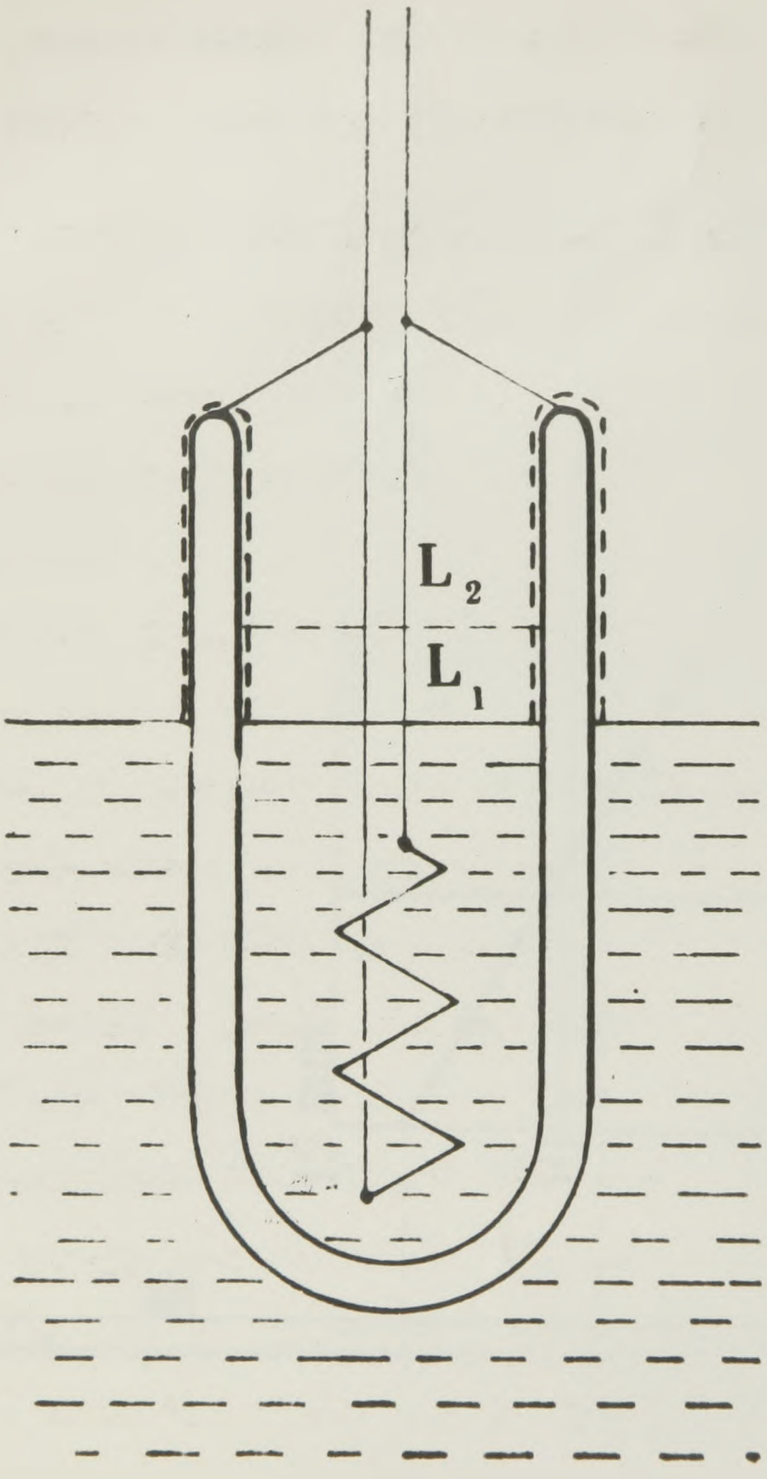


Fig. 1-3

a measurement on a smoothly polished copper beaker, which gave a transfer rate approximately the same as that on glass.

There were two more observations made by Daunt and Mendelssohn, which are of significance for our later discussions. They determined the heat conducted in the film when a temperature gradient was established along it, using the apparatus shown in figure 1-2. Heat was supplied to the film by inducing a current in the metallic ring R. If any heat had been conducted into the bath through the film, the evaporation of helium there would have caused an increase in the rate at which the bath level dropped. In fact no such increase was noticed, even when the heat supplied was enough to evaporate the film. It was found that less than 3% of the 'apparent heat flow' down the film was due to conduction, the rest being accounted for entirdy by the mass transport from the colder to the warmer place. It is to be remarked that this experiment showed not only the low thermal conductivity of the film, but also the fact that the critical rate of transport was unaltered by the creation of a small temperature gradient along the film.

The second observation of Daunt and Mendelssohn (1939b) which is of particular interest is the thermomechanical effect in the helium II film (figure 1-3) analogous to the

fountain effect discovered by Allen and Jones in fine channels. When the temperatures of the helium inside and outside the beaker were equal, the levels adjusted themselves to the same height. When a current was now passed through the heating coil, the inner level rose and reached a new equilibrium position a few centimetres above the outer level. This transfer of helium must take place through the connecting film, and thus shows that the thermo-mechanical pressure is transmitted through the film.

The remarkable properties of the flow of helium II through fine capillaries were studied in detail by Allen and Misener (1939), Kapitza (1941) and others. These experiments led to the notion of the superfluidity of helium II, i.e., its ability to flow under a vanishingly small potential gradient, with no measurable viscous dissipation, in fine capillaries (width about 1 μ or less). Particular mention must be made of the experiments of Kapitza, for he measured the thermal effects associated with the flow through narrow slits, which will be discussed in detail later (chapter 3).

The striking similarity between certain aspects of the superfluidity of helium II and superconductivity was first pointed out by Daunt and Mendelssohn (1942), and they suggested that film transfer in helium II takes place

under the zero-point momentum of the atoms in the film. This would lead to a critical velocity, below which the film transfer should be capable of occuring under even zero potential gradient. This conclusion was tested by them in their 'double beaker' experiment (1946) which will be discussed further in chapter 2.

Investigations on the helium film since 1945 have been mainly concerned with the critical rate under various conditions, and with measurements of the thickness of the film. Attempts have been made to study the mechanism of formation of the film by adsorption measurements, but they will not be gone into further here.

Atkins (1948) reported a series of observations of anomalously high flow rates on glass surfaces for the helium II film. He found also that, in the case of a beaker in the form of a bulb with a long narrow neck, the rate depended on the difference of levels between the inside and outside of the beaker. These anomalous rates in the case of isothermal film transfer were shown by Bowers and Mendelssohn (1950) to be due to deposits of solidified gases on the transfer surfaces. Atkins also found that the transfer rate was higher than that observed by Daunt and Mendelssohn, when the film flows under a temperature

difference. Brown and Mendelssohn (1950) showed that this increase in the transfer rate was only apparent; the particular experimental arrangement used by Atkins caused the outer perimeter of the beaker to be the limiting perimeter, instead of the inner one, as is the case in isothermal transfer. Their results will be considered further in chapter 3.

Ambler and Kurti (1952) have measured the transfer rate on glass below 1°K. They found that the rate, which is approximately constant from 1.5°K down to 1°K, starts to increase below the latter temperature, and appears to be still rising at 0.14°K, which was the lowest temperature they used. This remarkable behaviour is not predicted by any of the current theories of the film. It would be of great interest to repeat these experiments with some of the surfaces which have been used in the present experiments for isothermal film transfer (cf. chapter 5).

Apart from all these experiments on film transfer, there have been the measurements, by Burge and Jackson (1951), and later by Henshaw and Jackson (1951), of the thickness of the film by a direct optical method. Their results confirmed, in order of magnitude, the early measurements of Kikoin and Lasarew (1939) and Daunt and Mendelssohn (1939a). They were also able to measure

the variation of the thickness with the height, both for the stationary and the moving film. These measurements have opened up a new and fruitful technique for the study of the film, and have given remarkable visual proof of what had, at best, been indirectly deduced before (cf. chapter4). In an elegant series of experiments, Atkins (1950) has determined the thickness of the moving filml All these experiments agree in giving the order of magnitude of the thickness of the film as 10^{-6}cm.

1.2 Theories of the Helium II Film

The various theories of the helium II film that have been proposed will be briefly reviewed here, particularly in regard to their physical interpretation; this will be of use in considering their applicability to the experiments that are described in this thesis.

Bijl, de Boer and Michels (1941) made the first attempt to explain the existence of the film, and the critical rate of transfer. They pointed out that for an assembly of non-interacting particles, there was not only the ordinary zero-point energy determined by the density of the assembly, but also an additional zero-point energy depending on the dimensions of the container. A simple calculation from the de Broglie duality relations shows that this latter energy U is given by

$$U = \frac{\hbar^2}{8md^2}$$

$$U = \hbar^2/8md^2 \qquad (1.1)$$

where m is the mass of the helium atom, and d the width of the container. For a vessel of linear dimensions of 1cm. this comes out to be about 10^{-30} ergs, and may be neglected in comparison with the potential energy mgH of a helium atom at a height H=1 cm. (10^{-18} ergs). On the other hand, U becomes of the same order as mgH when d is about 10^{-6}cm., which was the order of thickness of the film as determined by Daunt and Mendelssohn by then. Using these ideas, Bijl et al showed that this zero-point energy has the effect of 'blowing up' the film, compared to the films that are deposited from ordinary liquids, which are about 10^{-7} cm. thick. They deduced the following relation between the thickness d and the height H of the film above the bulk liquid:

$$d = \frac{10^{-5}}{H^2} \qquad (1.2)$$

They also examined the results of Daunt and Mendelssohn on film transfer, and concluded that there existed a critical velocity v_c given by

$$v_c = \hbar/md \qquad (1.3)$$

They deduced this expression by examining the possibility of momentum exchange between an atom in the film, and the wall on which the film exists. The angular momentum of an atom, whose velocity is v, about the nearest point on the wall at a distance d, is mvd, and the lowest value it

can have is $\hbar$. Thus when its velocity is given by equation (1.3), it cannot go to a lower momentum state by losing momentum to the wall. It will be seen that this equation, which basically arises from dimensional considerations, is given by all the other theories which finally appeal to the indeterminacy principle.

Schiff (1941) examined the potential energy of a helium atom acted on by the van der Waals forces of the atoms of the atoms of the substrate, and of the surrounding helium atoms themselves, and balanced this against its gravitational potential energy. From this he deduced the following equation describing the variation of the thickness d of the film with height H from the bulk liquid:

$$d = \frac{k.10^{-6}}{H^{1/3}} \text{ cms.} \qquad (1\text{-}4)$$

where k = 4.3 for copper, 4.7 for silver and about 4 for glass. This theory, it should be pointed out, makes no explicit use of the concept of zero-point energy. The Bijl-de Boer-Michels theory, on the other hand, uses this concept, andit gives a variation as $H^{-\frac{1}{2}}$ for d.* Both

*This is easily seen by equating U in equation (1-1) to the gravitational potential energy mgH.

theories give the right order of magnitude for d, but the results of Burge and Jackson (1951) for the static film

appear to favour Schiff's theory. The measurements on the moving film (Henshaw and Jackson 1951, Atkins 1950) give very much smaller values (numerical) for the index of H, and have not been explained so far. Schiff's theory does not give any clear indication on the critical velocity of the film, particularly in its dependence on the chemical nature of the substrate.

Mendelssohn, in a series of papers (1945, 1947, 1951), has developed an empirical theory of liquid helium, the basis of which is a condensation of some of the atoms to the lowest level in momentum space. These condensed atoms may be called z-particles. It is necessary to introduce an energy gap between the z-states and the first excited state, to explain superfluidity and the shape of the specific heat curve. Mendelssohn suggests that the critical velocity of transfer is just the velocity with which the z-particles drift under their zero-point momentum, to equalise any disturbance in their statistical distribution through space. Thus superflow occurs even in the absence of an external field of force, as the experiments described in chapter 2 of this thesis will show. These ideas again give an inverse dependence of the velocity v_c on the thickness of the film, viz., equation (1-3).

Mott (1949) criticised the Bijl-de Boer-Michels theory of the film, for their assumptions led to a variation of density along the thickness of the film. He also gave a theory of the critical rate of flow, by considering the possibility of the formation of a stationary nucleus in flowing liquid helium. He assumed the expression for the surface energy between the nucleus and the moving liquid, and showed that the nucleus would grow and eventually bring the liquid to rest, only if the velocity v were greater than v_0, given by

$$v_c \approx \frac{\hbar}{m(ad)^{1/2}} \qquad (1\text{-}5)$$

where m=mass of the helium atom, a= interatomic distance in the liquid, d= width of the flow channel. This equation gives rather larger values for v_0 than equation (1-3), particularly for large d. It is seen that Mott's theory has no explicit reference to the dependence of the critical rate on the substrate.

Temperley (1949) worked out quantum-mechanically the ideas of Schiff already mentioned. He showed that the attraction of the wall alone on the helium atoms could account for a film only about 3.10^{-7} cm. thick, so that this length represented the range of the attractive forces of the wall. Taking account also of the forces between the helium atoms themselves and their gravitational energy,

he calculated the adsorption sites for the atoms on the solid surface, and showed that the formation of thick films could be explained. How the superflow in the film can be pictured on this model, is explained in chapter 5 of this thesis.

Ginsburg (1950) gave a simple and elegant derivation of the critical velocity, by considering the flow of helium through a slit of width d, and finding the minimum velocity v_c at which new excitations could be produced. He obtained the following expression:

$$v_c = \frac{\pi\hbar}{md} \qquad (1\text{-}6)$$

It is to be noted that this is half the value given by equation (1-3).

Finally, it must be emphasised that none of these theories would lead one to expect any marked dependence of the critical rate on the chemical nature of the substrate, with the possible exception of Temperley's. Even this theory, as will be discussed later, would at best predict only a slight dependence. This result is of importance in the experiments on film transfer on various surfaces, described in chapter 5 of this thesis.

1.3 The Apparatus

All the experiments described in this thesis were done



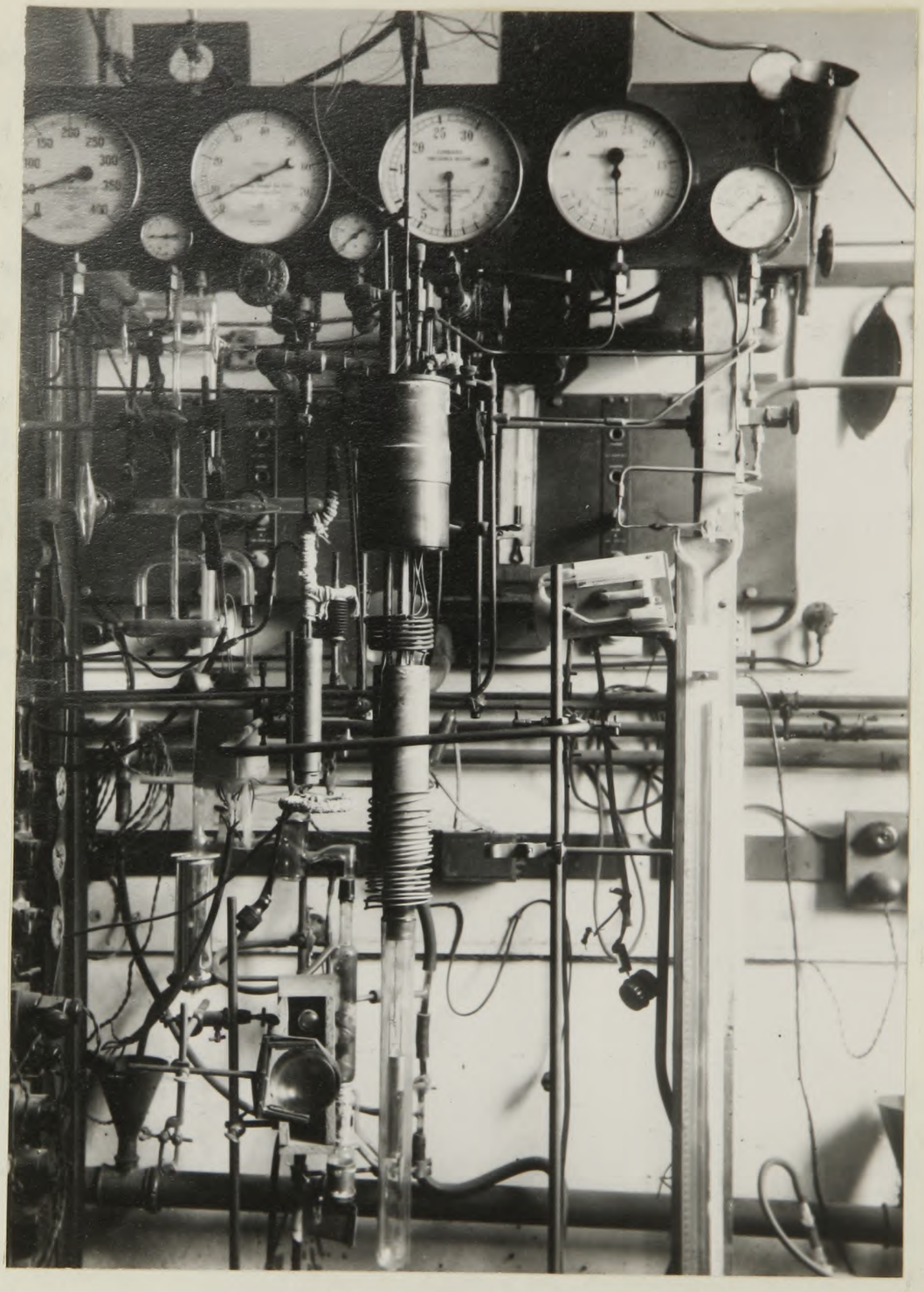


Figure 1-4 Photograph of the liquefier-Cryostat in which all the experiments were done.

with a combined liquefier-cryostat (figure 1-4) of the type originally used by Daunt and Mendelssohn (1948), and described in detail elsewhere (White 1950). The liquefier works on the Joule-Kelvin principle, and the double-walled glass vessel, in which the liquid helium collects, also serves as the experimental chamber. Some minor modifications were made in the liquefier described by White, so that a column of liquid helium I 36 cms. long (Which was reduced to 25 cms. of helium II by pumping) was available for experimenting, instead of about 18 cms. as in the earlier apparatus. This was of particular use in studying film transfer rates under gravitational potential differences much greater than had hitherto been used.

Temperatures were measured by measuring the vapour pressure of the liquid helium on a mercury manometer. Pressures of under one centimetre of mercury were measured on a 'Vacustat' gauge, which had been calibrated against an oil manometer. The temperature of the helium in the cryostat was held constant at any desired value by pumping the helium through a manostatic arrangement similar to the one described by Mendelssohn (1936).

The advantages of doing experiments on Helium II with a liquefier such as the one described above are numerous.

The very method by which liquid helium is obtained precludes the possibility of forming deposits of condensed gases on the surfaces used for film flow. This is a very important factor, as recent experiments have shown (Bowers and Mendelssohn 1950). Furthermore, the heat influx into the experimental space is remarkably low, as is shown by the fact that the bath loses, by evaporation, only about 2 cc. of liquid helium per hour. This is because the cryostat is surrounded by liquid hydrogen, and all the tubes leading to the cryostat from room temperature are long, made of bad conductors and taken through liquid hydrogen on their way down. These are advantages not easily obtained with a cryostat which is filled with liquid helium from a separate liquefier.

One of the main technical problems of visual work on liquid helium, is the one of suitable illumination. The low refractive index of the liquid (about 1.026 at 2.18°K) makes it difficult to distinguish the meniscus, particularly in narrow tubes. The low specific heat and latent heat of vaporisation make it necessary to ensure that not too much radiant energy is absorbed by the liquid.

A 12-watt electric filament lamp, with a glass tank filled with a solution of copper sulphate placed in front of it, was first used as the illuminant. While this was

quite satisfactory from the point of view of minimum heat influx into the cryostat, it was found unsuitable for making rapid observations over an extended length of the cryostat. So a vertical bank of neon lamps with a large condensing lens so placed as to illuminate uniformly that part of the cryostat over which observations were intended, was used and found to be an improvement. There was no appreciable heat influx from this lighting arrangement, as shown by the fact that the rate of loss of helium from the bath was independent of whether the lamps were switched on or not.

References to Chapter 1

Ambler, E. & Kurti, N., 1952, Phil.Mag., 43, 260.

Allen, J.F. & Jones,H., 1938, Nature, Lond., 141, 243.

Allen, J.F. & Misener, A.D., 1939, Proc.Roy.Soc.A, 172, 467.

Atkins, K.R., 1948, Nature, Lond., 161, 925; 1950, Proc. Roy.Soc.A, 203, 119, 240.

Bijl, A., De Boer, J.H. & Michels, A., 1941, Physica, 8, 655.

Bowers, R.& Mendelssohn, K., 1950, Proc.Phys.Soc. A, 63, 1318.

Brown, J.B. & Mendelssohn, K., 1950, Proc.Phys.Soc. A, 63, 1312.

Burge, E.J. & Jackson, L.C., 1951, Proc.Roy.Soc.A, 205, 270.

Dash, J.G. & Boorse, H.A., 1951, Phys.Rev., 82, 851.

Daunt, J.G. & Mendelssohn, K., 1938, Nature, Lond., 141, 911, 142, 475; 1939a, Proc.Roy.Soc.A, 170, 423, 439; 1939b, Nature, Lond., 143, 719; 1946, Nature, Lond., 157, 839; 1948, J.Sci.Inst., 25, 318.

Ginsburg, V.L., 1950, Dokl.Akad.Nauk.USSR, 69, 161.

Henshaw, D.G. & Jackson, L.C., 1951, Proc.Int.Conf.Low Temp. Phys.(Oxford), p.65.

Kapitza, P.L., 1941, J.Phys.U.S.S.R., 4,181; 5, 59.

Kikoin, A.K. & Lazarew, B.G., 1938, Nature, Lond.,141, 912; 1939, Ibid., 142, 289.

Mendelssohn, K., 1936, Proc.RoySoc. A, 155, 558; 1945, Proc.Phys.Soc., 57, 371; 1947, Rep.Int.Conf.(Cambridge) II, 35; 1951, Proc.Int.Conf. Low Temp.Phys.(Oxford), p.96.

Mendelssohn, K. & White, G.K., 1950, Proc.Phys.Soc. A, 63, 1328.

Mott, N.F., 1949, Phil.Mag., 40, 61.

Onnes, K., 1922, Trans.Far.Soc., 18, no.53.

Rollin, B.V., 1936, Actes du 7e Congr.Int.du Froid, 1, 187.

Schiff, L.I., 1941, Phys.Rev., 59, 838.

Temperley, H.N.V., 1949, Proc.Roy.Soc. A, 198, 438.

White, G.K., 1950, Thesis, Oxford.

CHAPTER 2.

ISOTHERMAL SUB-CRITICAL FLOW IN THE HELIUM II FILM.

2.1 Introduction.

Allen and Jones (1938) discovered that when a temperature difference was established between two volumes of helium II connected by a fine channel, a pressure difference resulted, causing a flow of the liquid from the colder to the warmer region. Under equilibrium conditions at any given temperature, the equation deduced by H.London (1939) gives the magnitude of this thermo-mechanical effect (Kapitza 1941, Meyer and Mellink 1947):

$$\frac{\Delta p}{\Delta T} = \rho \Delta S$$

where Δp and ΔT are the corresponding differences of pressure and temperature, ρ is the density of liquid helium, and ΔS is the difference of entropy between the bulk liquid and the liquid flowing through the channel. Now the first experiments of Daunt and Mendelssohn (1939a) on the mechano-caloric effect (the cooling produced in the liquid helium which flows out of a container through a fine channel) as well as the more accurate work of Kapitza, and Meyer and Mellink already referred to, showed that this entropy difference ΔS was equal to the entropy of the bulk liquid to a very close degree of approximation. This implied that the entropy of the moving part of the helium in the channel was sensibly zero.

Tisza (1938) examined the mechanism of the thermomechanical effect on the basis of his two-fluid model, and suggested that the temperature difference gave rise to an osmotic pressure difference of the normal fluid between the two volumes, and this caused the normal liquid to stream from the warmer to the cooler part. A compensating current of superfluid in the opposite direction gave rise to the observed thermal effects. This explanation is open to objections: it cannot satisfactorily explain the actual mass transfer which takes place in the fountain effect. It also fails to explain the fountain effect in the helium II film (Daunt and Mendelssohn, 1939d.

The reason Tisza proposed the picture of an osmotic pressure of the normal component is as follows. The superfluid is identified with the condensed particles of a degenerate boson gas. London (1939) showed that such particles cannot contribute to the total pressure in the fluid. Thus it was difficult to see how these particles, of zero momentum, derived the momentum necessary to allow them to participate in the flow under the thermo-mechanical effect.

The way out of this difficulty was shown by Mendelssohn (1945, 1947). Developing the remarkable analogy between the superfluidity of helium II and superconductivity (Daunt and Mendelssohn 1942), he pointed out that a particle of mass

m, confined to a space of linear dimension d, would have a zero-point momentum given by

$$mv \sim \frac{h}{d}$$

Its velocity v would therefore be of the order of h/md. The fact that v is proportional to 1/d would explain correctly the constancy of the film transfer rate along the length of a flow beaker, as also the observation of Allen and Misener (1939) that the flow of helium II through a fine capillary is proportional to its diameter. It should be remarked here that the above picture can only be an approximation, since the expression for the zero-point momentum given above implicitly assumes the wave function of a single particle in its lowest state confined to a length d, and this would give a non-uniform density distribution across a section of the film perpendicular to its surface (cf. Mott's criticism of the Bijl-de Boer-Michels theory of the helium film, 1949). On the other hand, there is no doubt that the superfluid particles would have a zero-point energy, and an associated zero-point momentum. Mendelssohn suggested that the momentum of superflow arose from this zero-point momentum.

A few words may be said here about the analogy between the superfluidity of heliumII, and superconductivity, in order to see the fruitfulness of Mendelssohn's concept

of zero-point momentum. The entropy of helium II, as also that of a superconductor, shows a steep decrease as the temperature is lowered through the transition point. The corresponding specific heat anomalies resemble the λ-anomalies observed in order-disorder transitions, and hence lead one to postulate a kind of ordering process to set in at the transition point.

Now both helium II and superconductors are characterised by a kind of frictionless transport, as shown by transport in the helium II film and by the current in a superconductor across which the potential drop is zero. Mendelssohn suggested that these effects were associated in each case with only some of the particles usually responsible for them in the normal state, and called them z-particles. These z-particles belonged to an energy state which was distinct from the states of all the other particles. In fact, as Mendelssohn clearly emphasized, it was not possible to label any given particle as a z-particle or not; the z-particles were distinguished solely by the fact that they all belonged to a single energy state, and hence one could only talk about a certain fraction of the total number of particles being in the z-state.

By considering the results of measurement of the

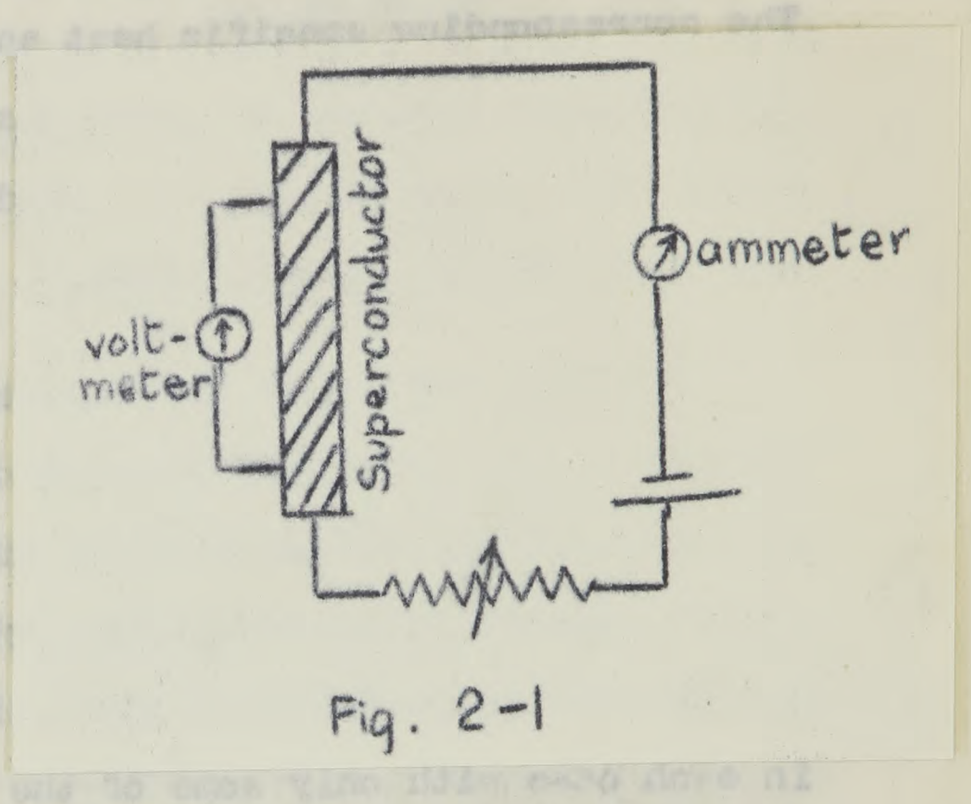


Fig. 2-1

Thomson heat in superconductors and the heat effects in the thermo-mechanical effect and its inverse, Mendelssohn concluded that the z-particles were energetically at the absolute zero of temperature, and possessed only a zero-point energy. Frictionless transfer with a critical velocity would be accounted for if there were a gap between this lowest energy state, and the spectrum of excited energy states.

On this hypothesis, the flow of a persistent current, for example, in a superconductor would be due to the zero-point momentum of the superconducting electrons. So also the flow of helium II in the film, or its flow under a temperature difference, would be caused by the zero-point momentum of the 'superfluid' helium atoms.

These considerations at once suggest an experiment with helium II, which should be in exact analogy to a similar experiment with a superconductor which is well known (figure 2.1). The flow of current in a superconductor can take place under zero potential difference between its ends, so long as the current is kept below a certain critical value. To limit the current to less than this critical value, an ordinary resistance has to be used in series with the superconductor. This indicates

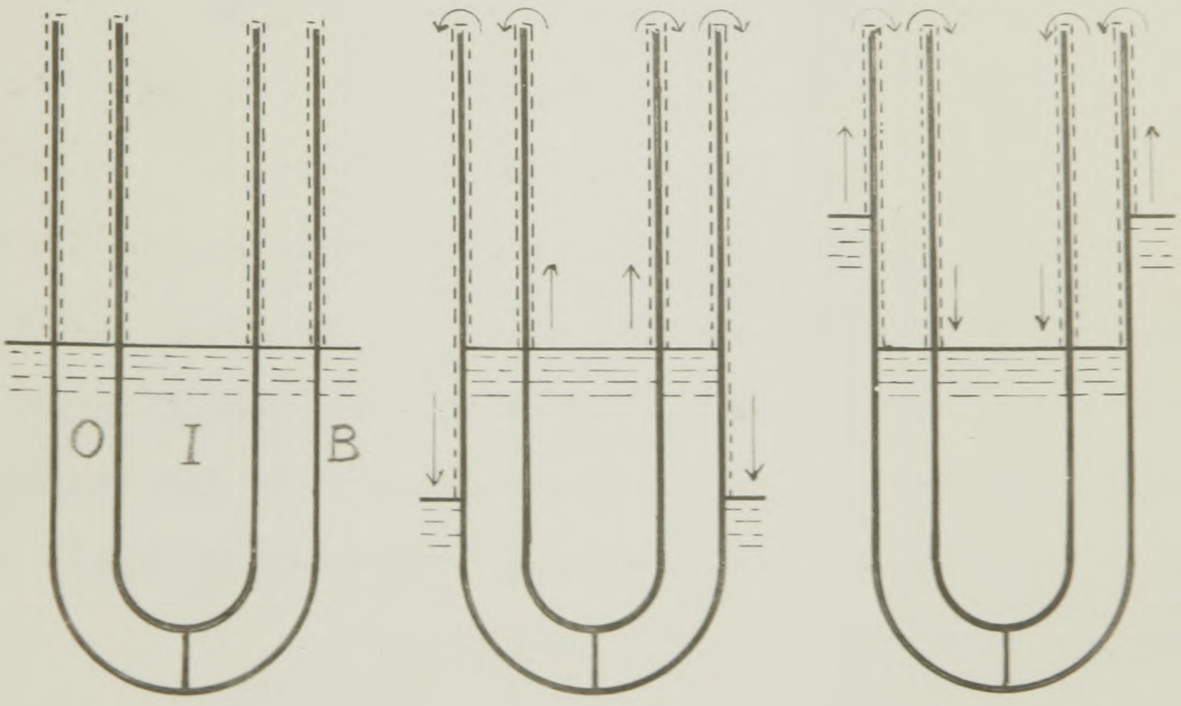


Figure 2-2 The double beaker experiment of Daunt & Mendelssohn.

that it should be possible to observe flow in helium II under zero potential gradient if the rate can be kept below the critical rate, by the insertion of a suitable flow resistance in series.

Daunt and Mendelssohn (1946) did perform such an experiment with a glass double beaker (figure 2.2). They observed that both in filling and emptying, the levels in the inner and outer beakers stayed together, the separation between them being certainly less than a millimetre. They showed that this pressure difference was inadequate to explain the velocity with which the helium film was moving over the surface of the inner beaker, and hence this flow was taking place under effectively zero potential gradient.

Now let the subscripts 'i' and 'o' refer to the inner and outer beakers respectively, both of them being cylindrical. And let

r_i and r_o = the radii,

$\frac{dh_i}{dt}$ and $\frac{dh_o}{dt}$ = the rates of change of the heights of the menisci,

R_i and R_o = the film transfer rates over the surfaces,

of the inner and outer beakers respectively. We know

from experiment that, under isothermal conditions, the film cannot flow against a potential gradient. We have, therefore, if the inner and outer levels start together,

$$\frac{dh_i}{dt} \leq \frac{dh_o}{dt}$$

What was actually observed, of course, was

$$\frac{dh_i}{dt} = \frac{dh_o}{dt}$$

we also have

$$\frac{dh_i}{dt} = \frac{2R_i}{r_i}$$

and

$$\frac{dh_o}{dt} = \frac{2R_o}{r_o}$$

so that

$$\frac{R_o}{R_i} = \frac{r_o}{r_i}$$

We thus have the result that, if the two levels move together, the rates over the two beakers are in the ratio of their radii. The rate R_o has necessarily got to be the critical rate of film transfer as ordinarily measured on a single glass beaker. Thus, by making the ratio $\frac{r_o}{r_i}$ as near to unity as is practical, we can arrange to have R_i correspondingly close to the normal critical rate. It is easy to see now the analogy between this flow system, and an electrical circuit containing a super-conductor carrying a current of less than the critical value. The outer beaker provides the external resistance which limits the rate of flow over the inner beaker

to less than the critical, and hence this latter flow takes place under zero potential gradient.

Soon after this experiment was done by Daunt and Mendelssohn, Atkins (1948) described observations he had made on transfer rates of the helium II film out of glass beakers of various shapes. He reported rates which were considerably greater than the ones which had been observed initially (Daunt and Mendelssohn 1939), and which also did not appear to be consistent within themselves. He attributed these high rates to temperature differences between the ends of the helium film. These apparent anomalies were resolved by the discovery that contaminated glass surfaces were responsible for the high rates, and that the transfer rate still retained the usual value when a temperature difference was created between the ends of the film (Bowers and Mendelssohn 1950, Brown and Mendelssohn 1950). Nevertheless, it was felt that the 'double beaker' experiment was of sufficient fundamental significance to require a fresh examination under more favourable conditions. Hence the following apparatus was devised.

2.2 The Apparatus

The double beaker that was used in the experiments

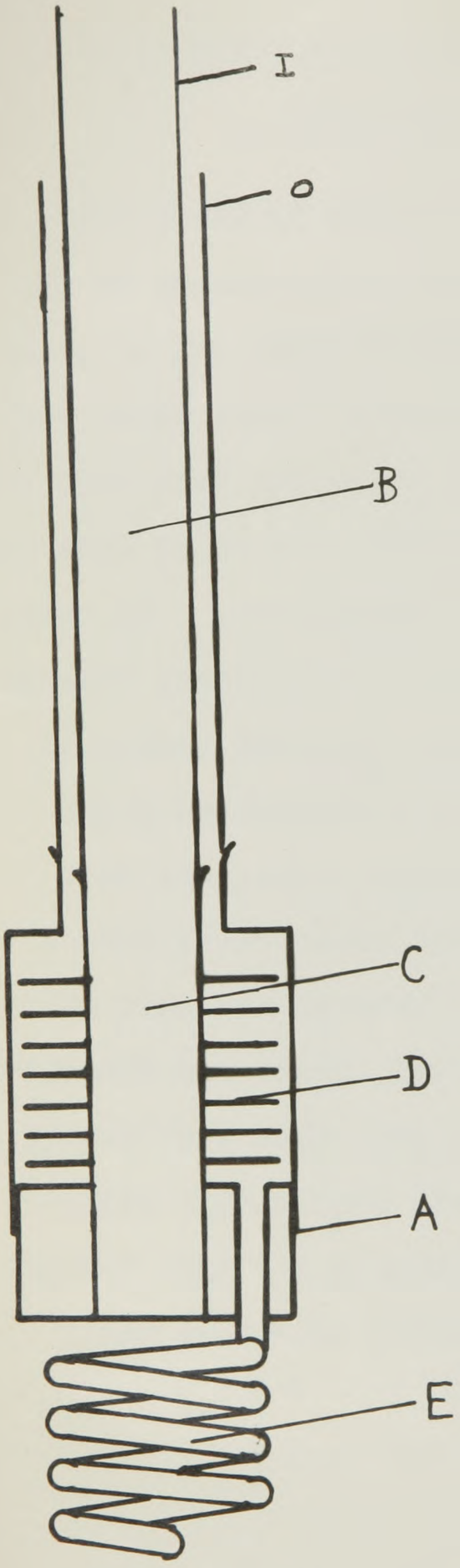


Figure 2-3.



Figure 2-4 Photograph of the double beaker.

to be described is shown in section in figure 2.3. The beaker was constructed in two parts, one fitting over the other, the two being soldered together with Wood's alloy. Each part had a copper-glass seal, so that the beaker was effectively of glass over most of its length, and of thin copper at the lower end (figure 2.4). During an experiment the copper part of the beaker was kept immersed in the liquid helium in the bath. The thinness of the copper walls, the vanes surrounding the inner tube, and the blocked-off spiral of copper tubing, all ensured a large area of thermal contact between the helium in the inner (I) and outer (O) beakers and the bath. The inside diameters of the two beakers were 0.84 and 1.26 cms respectively, and the wall thickness of each was 0.1cm.

2.3 The Results

The experiment consisted in measuring the separation between the levels of heliumII in I and O both when the beaker was filling and when it was emptying. A cathetometer was used for this purpose. All observations were made at a constant temperature which varied between 1.25°K and 1.3°K during the several runs. This temperature region was selected, for the characteristic of film rate versus temperature is almost flat here, so that a slight

fluctuation of temperature during a single experiment would not have any considerable effect on the transfer rate.

The transfer rate over the surface of the outer beaker agreed well with the ordinary transfer rate on glass (Daunt and Mendelssohn, 1939). The separation between the levels in the outer and inner beakers varied, in different experiments, between 0.05 cm and 0.02 cm, the outer level always being above the inner level because of surface tension. The calculated rise in the outer level, due to surface tension, assuming that the outer beaker was symmetrically placed round the inner, was 0.04 cm. The discrepancy between the calculated and observed values can be attributed to (a) slight asymmetry in the mounting of the outer tube round the inner, and (b) the experimental difficulty of determining the separation between two menisci which are so close to each other.

From the ratio of the radii of the two beakers, the transfer rate of the helium film over the inner beaker was calculated to be 0.67 times R_0, the rate over the outer beaker, which is itself the Daunt-Mendelssohn rate. From the measured values for the thickness of the helium film (Henshaw and Jackson, 1951) and the volume transfer rate of the film, the average linear velocity of the film

can be estimated to be about 50 cms./sec. Though the two-fluid model suggests that only the superfluid takes part in film transfer, the average velocity of the entire film can be assumed to be 50 cms/sec., as the superfluid concentration at the temperatures used is over 0.95. The average linear velocity of the film over the inner beaker would therefore be 33 cms./sec.

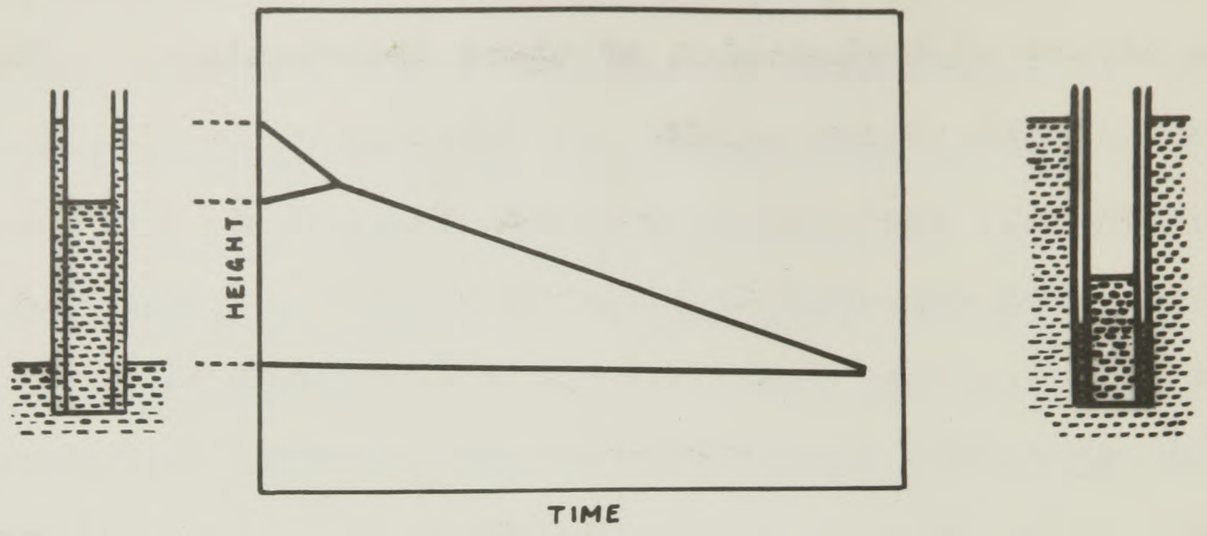
2.4 Discussion

There is one difficulty inherent in the experiment, and this is in determining exactly the separation Δh between the helium levels in the inner and outer beakers. It is only possible to say with certainty that this separation is less than a certain quantity. It was this upper limit to the separation which was quoted earlier. As has been mentioned earlier, this separation is of the same order of magnitude as the calculated rise due to surface tension of the meniscus in the outer beaker. It will be shown below that, in spite of this slight uncertainty in the difference of the two levels, the observations cannot be explained on a purely classical basis, while the concept of the motion of the superfluid atoms under their zero-point momentum does indeed throw light on the problem.

We shall first consider the possibility that Δh is

exactly equal to the calculated rise due to surface tension. Then the helium film is moving over the inner beaker under zero potential difference, and this, by itself, would be explained if the viscosity of the liquid were taken to be zero. But this does not fit in at all with the fact that the flow over the outer beaker takes place at a critical rate which is nearly independent of the potential difference between the ends of the film. For if the liquid obeyed the classical laws of flow with the viscosity $\eta = 0$, then the volume flow, being in the nature of siphon flow, should be proportional to $\Delta p^{1/2}$, where Δp is the pressure difference between the ends of the film. Thus for this case we see that it is not sufficient to assume merely that $\eta = 0$ to explain the flow over both beakers.

The other possibility is that Δh is slightly different from the calculated rise due to surface tension. From our observations it is obvious that the largest possible value for this difference is 0·05 cm., while in the majority of cases it was certainly less. The calculated average velocity of the film over the inner beaker was 33 cms./sec., and to produce this velocity would require a difference between the menisci in the inner and outer tubes of 2.2 cms., even assuming $\eta = 0$. This therefore is also incapable of explanation on purely classical grounds.



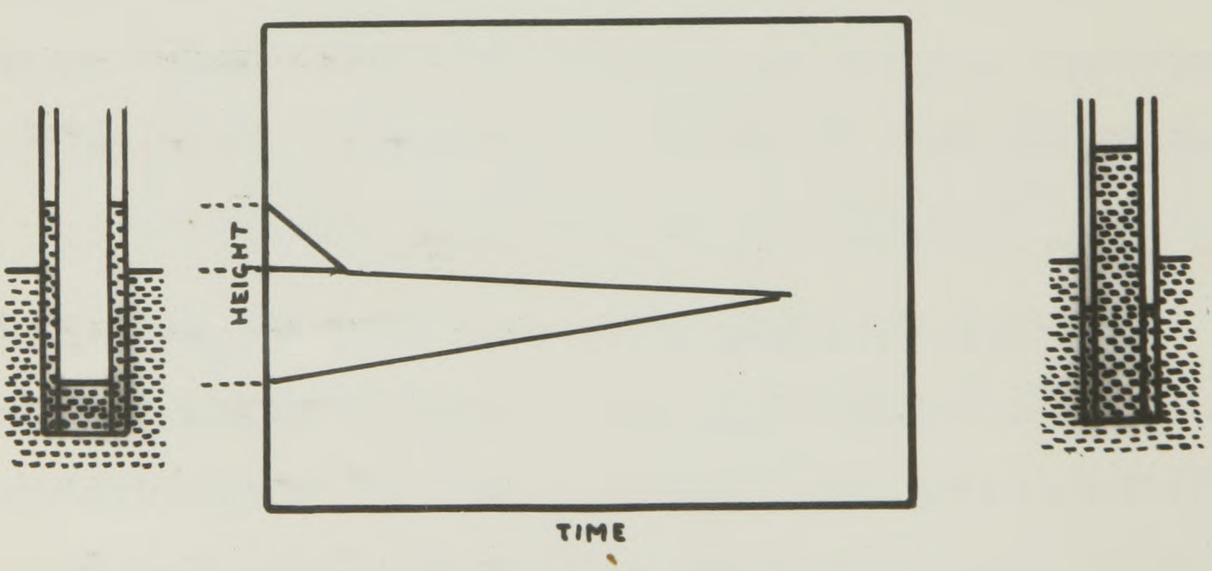


Figure 2-5

It is easy to see how the concept of zero-point momentum offers an explanation of these observations. The superfluid particles are capable of drifting with velocities up to a certain critical value which corresponds to this zero-point momentum, even in the absence of an external potential gradient. It is true that the above observations do not prove that flow at sub-critical velocities takes place under strictly zero potential gradient, but we can indeed say with confidence that most of the sub-critical flow is frictionless. A numerical estimate of the fraction of the total flow that is frictionless would not be reliable, because of the uncertainties in the measurements referred to above. We shall return to this topic in the next chapter.

2.5 Some Miscellaneous Observations.

The flow out of and into the beaker showed some interesting features when the initial conditions were so arranged that the liquid helium in the inner and outer beakers started at different heights. They are briefly described here, for they illustrate some aspects of what may be called the 'geometry' of film flow. The results are shown in figure 2-5. The graphs actually refer to the flow starting from the initial conditions pictured on their left. The conditions shown on the right give exactly similar results, except that now the liquid levels in the inner and outer

beakers move in a direction opposite to that on the left.

Consider the state of affairs pictured at the top left. The outer beaker would, to start with, empty into both the inner beaker and the bath, and its level would approach that in the inner beaker. If now one admits the possibility of the outer level getting very slightly below the inner level, so that I is now emptying into O, and O into the bath, then one might argue as follows: Since the volume flow rate is proportional to the perimeter for a cylindrical beaker, more helium can flow out of the outer beaker than is flowing into it from I. Hence there is a net loss of the liquid from O, and its level should continue to drop till it reaches the bath level. But this, it is obvious, is quite different from what actually happens, for the outer level stays with the inner level, and they both fall together.

The solution of this apparent paradox lies in the fact that the above argument takes account only of volume flows, and ignores the relative changes of the liquid levels. A simple geometrical consideration shows, in fact, that what happens is exactly what is to be expected.

The observations shown in the figure can be summarised by saying 'The outer level stays with the first level it encounters in its progress towards the bath level'.

Chapter 2: References.

Allen, J.F., And Misener, A.D., 1939, Proc.Roy.Soc.A, 127, 467.

Allen, J.F. & Jones, H., 1938, Nature, Lond., 141, 243.

Atkins, K.R., 1948, Nature, Lond., 161, 925; 1950, Proc. Phys.Soc.A, 203, 119, 240.

Bijl,A., De Boer, J.H., and Michels, A., 1941, Physica, 8, 655.

Bowers, R. and Mendelssohn, K., 1950, Proc.Phys.Soc. A, 63, 1318.

Brown, J.S. and Mendelssohn, K., 1950, Proc.Phys.Soc. A, 63, 1312.

Daunt, J.G. and Mendelssohn, K., 1939a, Nature, Lond., 143, 719; 1939b, Proc.Roy.Soc. A, 170, 423, 439; 1942, Nature, Lond., 150, 604; 1946, Ibid., 157, 839.

Henshaw, D.G. and Jackson, L.C., 1951, Proc.Int.Conf.Low Temp.(Oxford), p.65.

Kapitza, P.L., 1941, J.Phys.U.S.S.R., 4, 181; 5, 59.

London, H., 1939, Proc.Roy.Soc. A, 171, 484.

Mendelssohn, K., 1945, Proc.Phys.Soc., 57, 371; 1947, Phys.Soc.Cambr.Conf.(Low Temp) Report, p.35.

Meyer,L. and Mellink, J.H., 1947, Physica, 13, 197.

Mott, N.F., 1949, Phil.Mag., 40, 61.

Tisza, L., 1938, C.R. Acad.Sci., Paris, 207, 1035.

CHAPTER 3

SUB-CRITICAL TRANSFER OF THE HELIUM II FILM UNDER A TEMPERATURE GRADIENT.

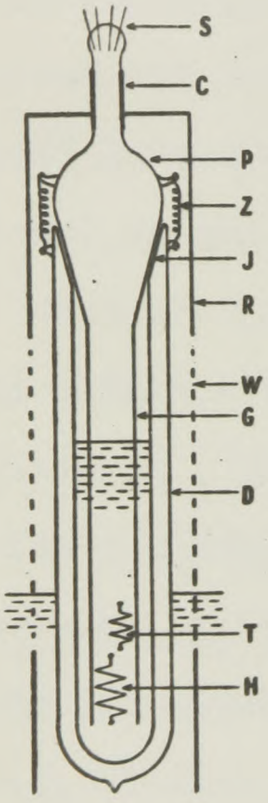
3.1 Introduction.

The flow properties of the helium II film under isothermal conditions have been the subject of detailed investigation since the original experiments of Daunt and Mendelssohn (1939a). But comparatively little work has been done on flow in the film connecting two volumes of helium II when heat is applied to one of them. The earliest experiment of this nature was that of Daunt and Mendelssohn (1939a), where they had a beaker filled with helium II emptying out through the film. A metallic ring attached round the beaker was heated by inducing a current in it. It was found that the rate at which the beaker emptied was independent of the rate at which heat was produced in the ring, even up to the point when the film was completely evaporated by the heat. There was a possibility, in this experiment, that the helium film was not continuous because of evaporation at the point where heat was applied, so that the flow of the film itself might be taking place

isothermally.

Atkins (1948) reported experiments on film transfer on glass under various conditions, all of which yielded rates higher than those observed before. It has however been shown since that the high rates in isothermal flow could be produced by deposits of condensed gas on the flow surface. Atkins's arrangement for studying the flow under a temperature difference consisted of a beaker in the form of a small glass dewar vessel, with a tight-fitting lid of ground glass. The vessel contained a heating coil and a resistance thermometer inside. Atkins found that the flow into the beaker produced by supplying heat electrically to the inside took place at a rate as great as 30.10^{-5} cm^3/cm.sec., as compared to the rate of $7{\cdot}5.10^{-5}$ cm^3/cm.sec. found by Daunt and Mendelssohn.

This observation was explained by the experiments of Brown and Mendelssohn (1950). They suggested that under the conditions of Atkins's experiments, helium could run down the inside surface of the beaker in the form of bulk liquid, because of the thermal energy which was being supplied to the inside. Thus the true critical perimeter for film transfer would be the outside perimeter of the beaker, and <u>not</u> the inside perimeter, as is the case for isothermal film transfer. To verify this suggestion,


S
C
P
Z
J
R
W
G
D
T
H

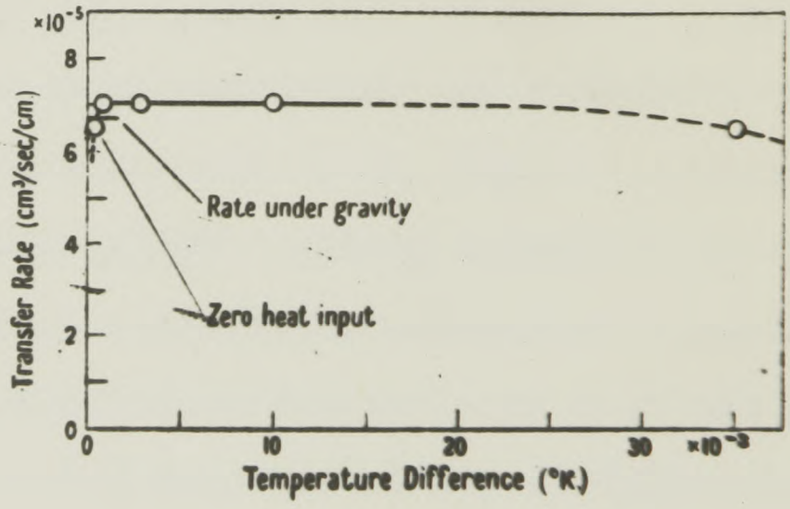

×10⁻⁵
8
6
4
2
0
Transfer Rate (cm³/sec/cm)
Rate under gravity
Zero heat input
0
10
20
30
×10⁻³
Temperature Difference (°K)

Fig. 3-1

they constructed a special transfer beaker (figure 3-1) which had the available inside perimeter for film transfer greater than the outer perimeter. The ambiguity which existed in the case of Atkins's apparatus regarding the critical perimeter was thus avoided, for in this case the outside perimeter was certainly the smallest perimeter the film had to go through. Using this apparatus, Brown and Mendelssohn showed that the transfer rate under a temperature difference was the same as that under isothermal conditions. It should be pointed out that even for the smallest temperature difference that they measured, the transfer took place at nearly the critical rate.

3.2 The Significance of Measurements of Sub-Critical Rates.

A large amount of experimental evidence has been accumulated (see chapter 5) to support the idea that there is a critical rate of transfer for the helium film when it flows from one container to another, under a difference of potential which may be either gravitational or thermal. Now, on the two-fluid theory of helium II, only the superfluid is supposed to move during film transfer, the normal fluid being immobilised owing to its viscosity and the narrowness of the channel (viz. $\sim 10^{-6}$ cm.) available

for motion. Thus there is relative motion between the superfluid and the normal fluid, and it is conceivable that this might give rise to a force of friction, tending to oppose any increase in the relative velocity.

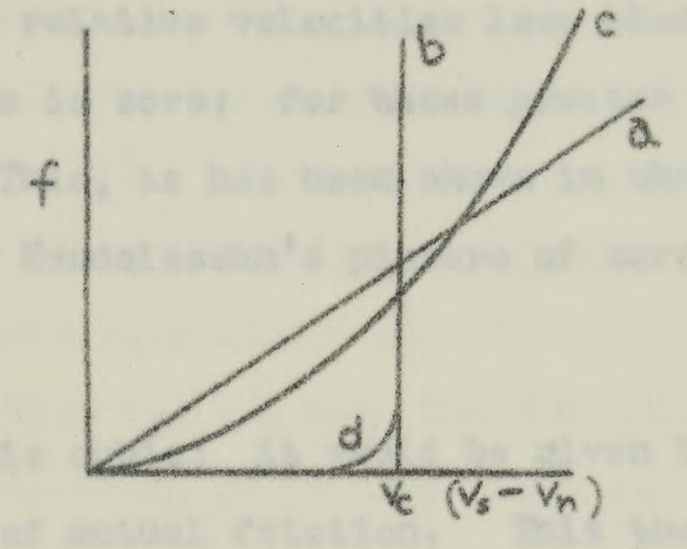


Fig. 3-2.(cf. Atkins 1950)

Figure 3-2 shows, quite in general, various possible ways in which this frictional force f can depend on the relative velocity $(v_s - v_n)$, where v_s and v_n are the velocities of the superfluid and the normal fluid respectively. Motion along a straight line is assumed for the sake of simplicity, so that v_s and v_n are the actual magnitudes of the two velocities. In the case of motion in the film, of course, $v_n = 0$. We shall discuss the different cases shown in the diagram in detail.

Curve (a) represents the case where the frictional force is proportional to the relative velocity. It is quite clear that such a force cannot be responsible for

a critical velocity such as is observed in films, and need not therefore be considered further here.

Curve (b) gives the case of the ideal critical velocity, v_c. For relative velocities less than v_c, the frictional force is zero; for those greater than v_c, it is infinite. This, as has been shown in chapter 2, would be explained by Mendelssohn's picture of zero-point diffusion.

Curve (c) is cubic; it would be given by the Gorter-Mellink theory of mutual friction. This theory was put forward (Gorter and Mellink 1949) to explain the conduction of heat by helium II in fine channels. It postulates a force of mutual friction between the normal fluid and the superfluid, of magnitude $A\rho_n\rho_s(v_s-v_n)^3$, where A is a constant and the other symbols have their usual meanings. While this has been very successful in explaining heat conduction in channels of about 10^{-2} to 10^{-4}cm. width filled with helium II, it has been inadequate to explain isothermal flow in capillaries, the Andronikashvili experiment at large amplitudes (Atkins 1952), and, as will be shown below, superflow in helium—both in the film and in very narrow channels.

Finally, curve (d) shows an intermediate case, where the possibility of a small frictional force for velocities

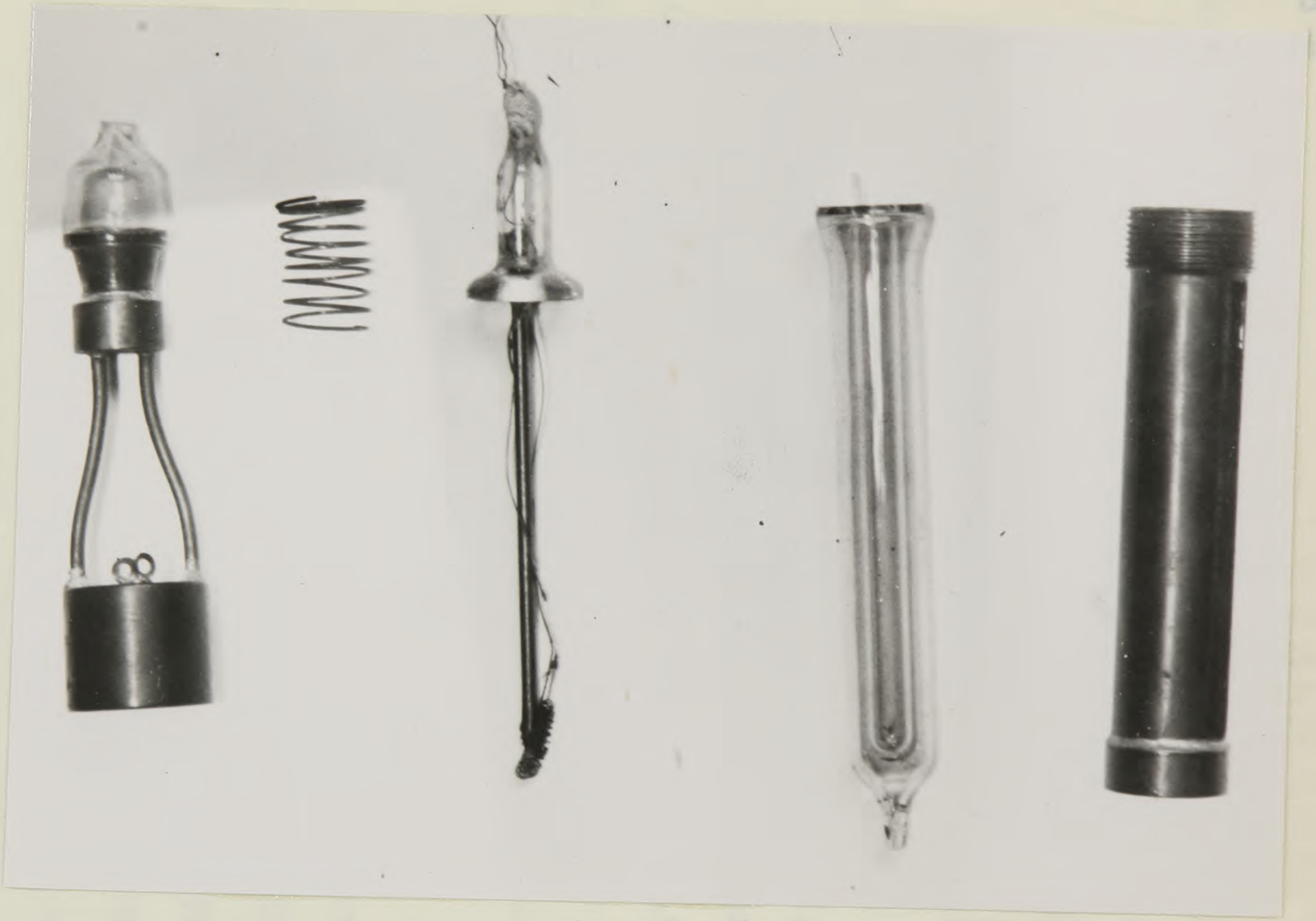
slightly less than v_c is admitted, and the critical velocity is provided for by a sudden and large increase in the friction at this point.

The experiments described in chapter 2 showed that the type of frictional force involved in film transfer was either (b) or (d). It would be obviously satisfying to approach the problem from a slightly different point of view, and investigate film transfer at velocities very much less than v_c, and look for the dependence of the frictional dissipation, if it does exist at all, on the velocity.

The experiments of Brown and Mendelssohn suggested a very convenient way of carrying out this idea. It was remarked before that the lowest flow rate they observed under a temperature difference was still very near the critical rate. They were primarily interested in producing a measurable temperature difference, of the order of 10^{-3}°K, and to do so would require a rate of heating considerably in excess of the critical rate. This indicates that, to measure sub-critical rates successfully, we should concentrate on measurement of small heating rates rather than temperature differences; The apparatus should therefore also provide the maximum possible thermal insulation, so that there is not much loss of heat from the inside



(a)



(b)

Figure 3-3 Apparatus for studying sub-critical film transfer under a temperature gradient: (a)as assembled, and (b) the parts, modified for the experiment described on page 81 .



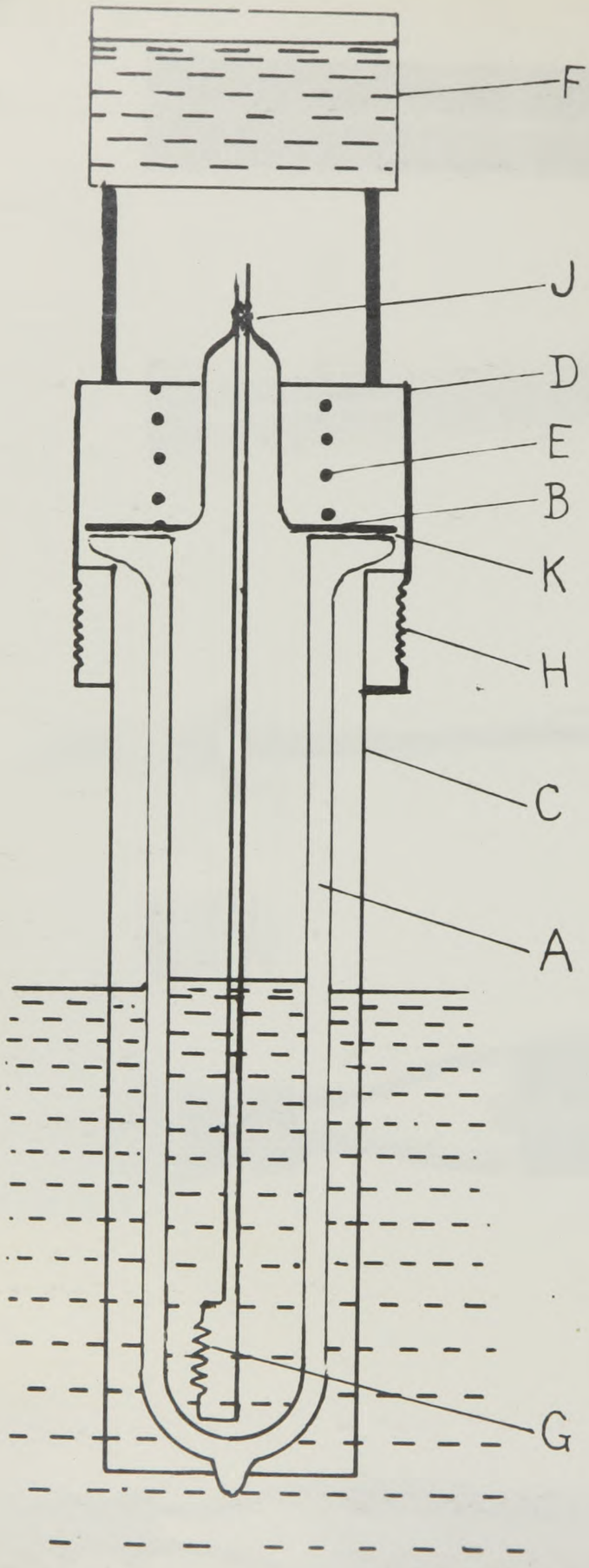


Figure 3-4

Sectional view of the beaker and radiation shield.

to the surroundings. With these aims in view the apparatus described in the next section was constructed.

3.3 The Apparatus.

The transfer beaker is shown in figures 3-3 and 3-4. A is a small unsilvered dewar vessel made of pyrex, of inside diameter 0.46 cm and depth 7 cms, and outside diameter 1.10 cm. The lip of the dewar was blown flat and optically polished. A glass cap B, with its lower side also optically polished, fitted on the dewar. This whole arrangement was surrounded by a cylindrical radiation shield made of thin copper. The shield was made in two parts C and D, which screwed together at H. By inserting the steel spring E between D and the glass cap, and then screwing D down, the cap was held firmly pressed against the dewar. C had two narrow slits to permit observation of the helium level inside the beaker.

Heat could be supplied to the inside of the beaker by passing a current through the resistance G, which was a small coil wound from S.W.G. 47 cotton-covered constantan wire. The leads from the heating coil were S.W.G.36 cotton-covered constantan wires which were led out through a small hole J in the glass cap, and then taken down to the helium bath surrounding the beaker before being taken out of the liquefier-cryostat to the electrical circuit

outside. This was to ensure that the heat being conducted down the leads from outside would enter the helium bath rather than the inside of the beaker. The resistance of the heating coil at room temperature was 64.1 ohms. Several specimens from the batch of constantan wire, from which this coil was wound, had been separately calibrated down to helium temperatures in the course of calorimetric determinations in this laboratory, and had shown identical and reproducible results. The resistance of the heating coil at liquid helium temperatures was therefore calculated from these calibrations.

The gap K between the optically polished surfaces was estimated by flow measurements, with air at room temperature and liquid helium at 4.2°K. The results by the two methods agreed, giving about 1μ for the width of this gap. The hole at J was blocked up with 'Fortafix' cement after inserting the leads through it. While this was almost certainly permeable to helium II, it still inhibited as flow of gas. The narrow gap between the dewar and the cap also prevented excessive flow of the gas, while the helium II film could easily pass through it. Thus, any heat introduced into the dewar could not leak out by evaporation of the helium inside.

The necessity of thus preventing a free flow of

helium gas between the inside of the beaker and the outside, for making measurements involving the thermomechanical effect, should be emphasised here. Such measurements may be of two kinds: a determination of the heat supplied to one of the two connected volumes of helium II and the rate of the resulting volume flow, or a measurement of the temperature difference between the two. The early experiments of Daunt and Mendelssohn (1950) on the thermomechanical effect in the film did indeed take account of the helium evaporated by the heat introduced into their open dewar vessel, but they measured the actual level difference between the inside and outside of the flow arrangement in the steady state. This, in effect, is a measurement of the temperature difference between the two regions. The highest level difference they were able to observe was about 0.25 cm. It is obvious that an accurate determination of the rate at which helium enters the beaker cannot be made under these conditions.

A small helium 'tank' was attached to the top of the radiation shield by means of two stout copper supports. The tank had a small hole at the top, through which it could be kept filled with liquid helium right through the course of an experiment by dipping it in the bath from time to time. This tank prevented radiation, from the warmer parts of the liquefier-cryostat, impinging on the top of the flow



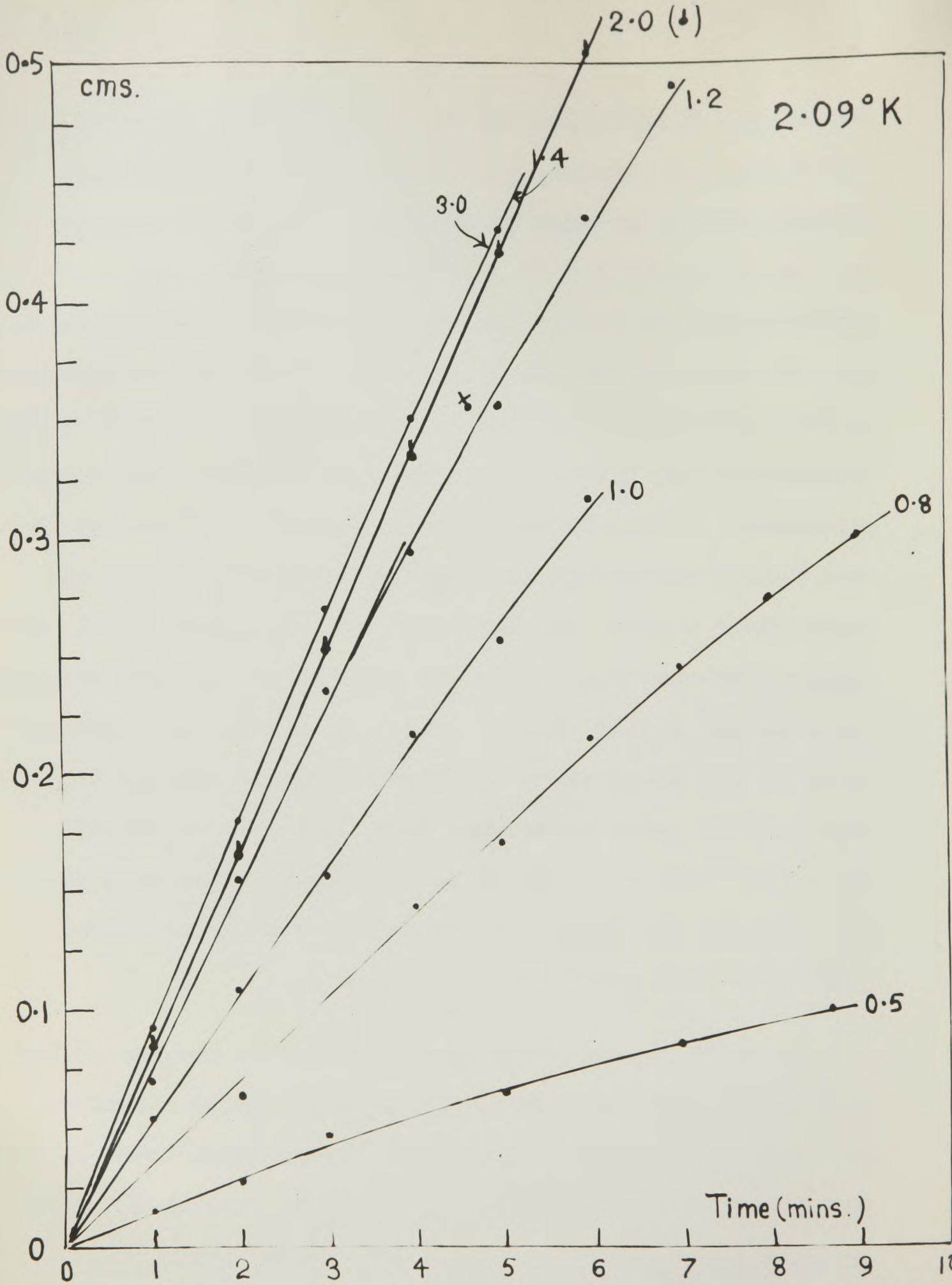


Figure 3-5 Thermal inflow at 2·09°K. The current in milliamperes is shown against each curve.

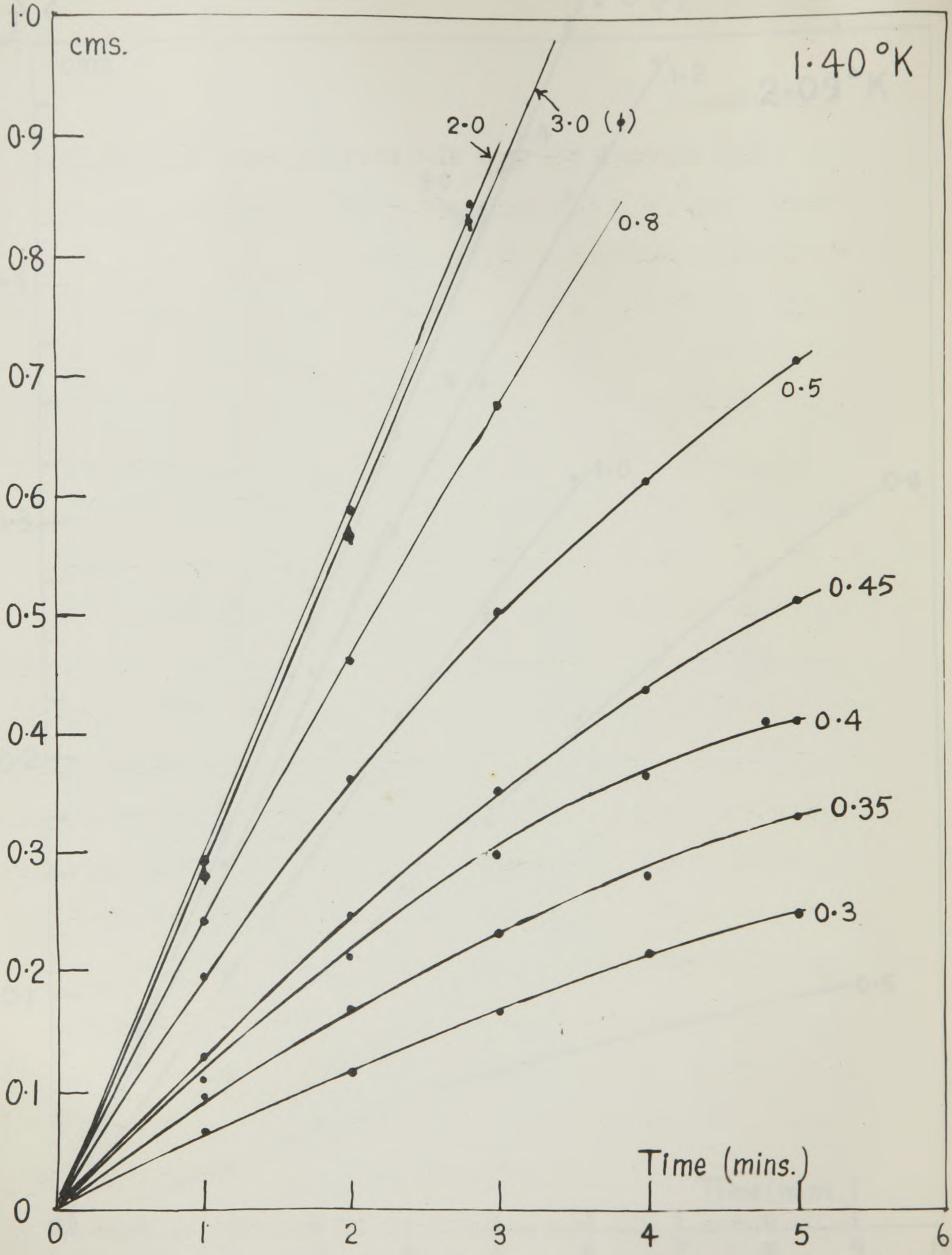


Figure 3-6 Thermal inflow at 1·40° K. The current in the heating coil is shown, in milliamperes, against the corresponding curve.

apparatus.

The current through the heating coil was obtained from a battery, and controlled by a resistance in series. It was measured by a multi-range ammeter, which had been compared with a calibrated ammeter.

3.4 The Experiments

The apparatus described above was suspended in liquid helium II in the cryostat, so that the bath level was a few centimetres below the polished rim of the beaker. The temperature of the bath was held constant, at any desired value between 1.2°K and the λ-point, by pumping the helium through the manostatic arrangement mentioned before (chapter 1). The level of helium inside the beaker was observed through a cathetometer with a vertical traverse. A predetermined heating current was switched on when the level inside the beaker was in its equilibrium position. The level rose, and was noted at suitable intervals of time and a graph plotted of the height against time. These observations were repeated for different heating rates; two typical sets of curves, for 2.09°K and 1.40°K, are shown in figures 3-5 and 3-6. The number against each curve gives the corresponding heating current in milliamperes.

Between successive determinations of the transfer

rate, the helium level in the beaker was allowed to return to its equilibrium position. This took a considerable length of time, and it was in fact used to estimate the heat transfer between the inside and outside of the beaker when there was a temperature difference between the two. This will be explained in detail later. But it may remarked here that the very small deviation from linearity which the curves in figures 3-5 and 3-6 show, particularly near the origin, itself indicates that the heat lost from the inside of the beaker by ordinary processes is very small. By 'ordinary processes' is meant radiation from the inside of the beaker, conduction through the glass, the leads and the residual gas in the dewar*,

*Small quantities of helium gas would probably diffuse into the vacuum space of the dewar in course of time, though all other gases would of course be condensed out at liquid helium temperatures.

and heat lost through the vapour over the liquid inside.

In a different sense, of course, almost the entire heat supplied to the inside of the dewar is finally transferred to the outside, for the temperature of the liquid helium inside does not rise by any appreciable amount. But the volume of the liquid inside increases because of inflow from the bath of helium whose entropy, as will be clear later, is vanishingly small. This causes an increase in the entropy per gram of the helium in the

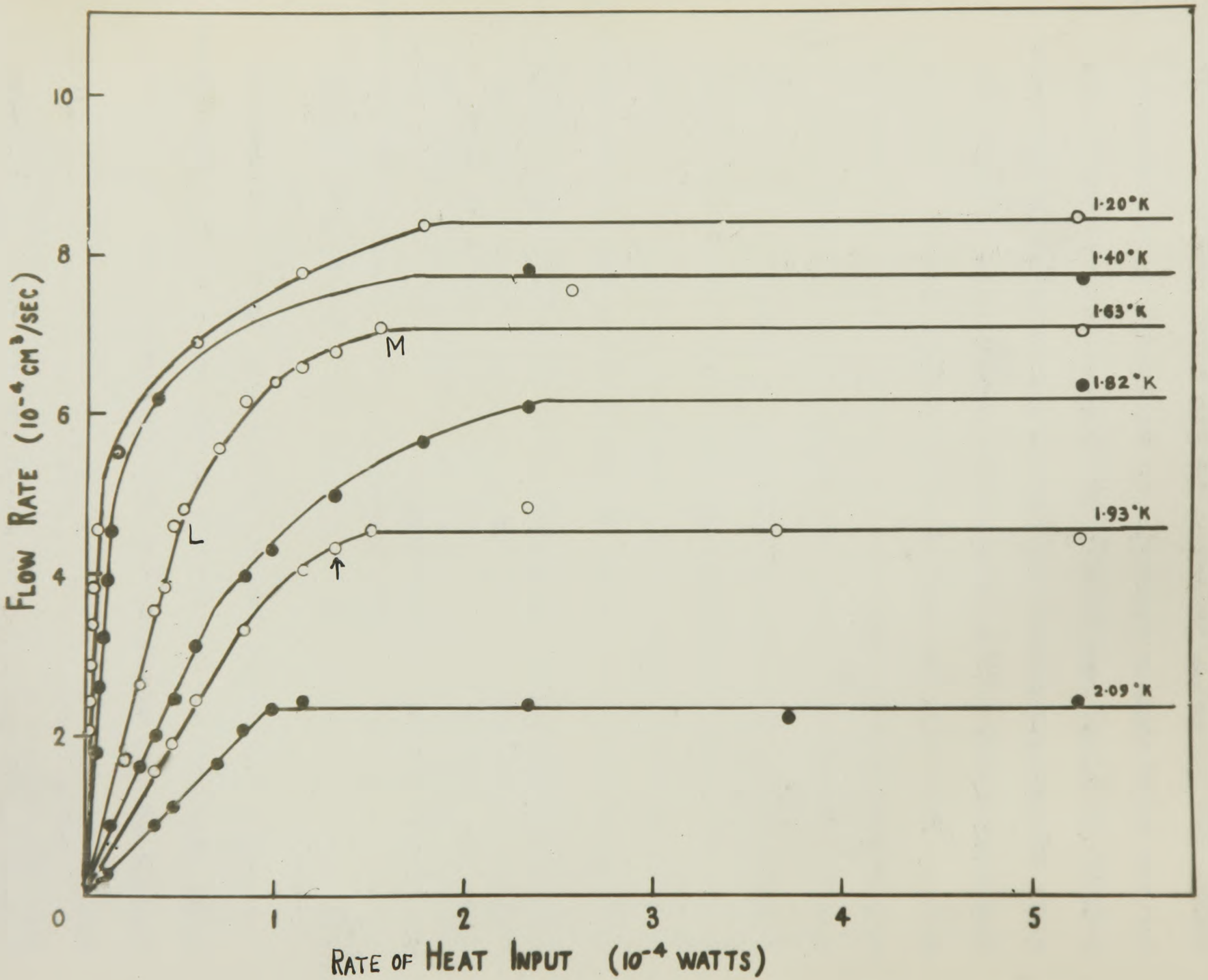

FLOW RATE (10^{-4} CM^3/SEC)
RATE OF HEAT INPUT (10^{-4} WATTS)
0
1
2
3
4
5
2
4
6
8
10
1·20°K
1·40°K
1·63°K
1·82°K
1·93°K
2·09°K
L
M

Figure 3-7.

bath, and hence an increase in its heat content. This increase is thus ultimately to be traced to the heat which has been supplied to the helium inside the beaker. This is the process which has been termed by Mendelssohn the 'isothermal flow of heat', corresponding to the flow of helium II under a zero potential gradient.

By drawing tangents at the origin to the curves of height against time (figures 3-5 and 3-6) and measuring their slopes, the rates at which helium is entering the beaker are determined for the various rates of heating. Taking the tangents at the origin ensures that any error, due to heat losses to the surroundings, is minimised; for the corresponding temperature differences between the inside and outside of the beaker would be ideally zero, and in any case very small. These rates, expressed in terms of the rise of the inside meniscus, were converted to volume rates knowing the area of cross-section of the inside of the beaker. The observations were repeated at several temperatures between 1.2°K and the λ-point, and the results are shown in figure 3-7.

The rates are not expressed, as is usual for film rates, per centimetre of perimeter of the flow trrface. This is because of the difficulty of determining what the effective limiting perimeter is in a flow arrangement

such as the one used (cf. Brown and Mendelssohn 1950). It is likely that the inflow of helium takes place not only through the film on the outside of the beaker, but also through the film covering the radiation shield. But what can be asserted with certainty is that all the helium which has entered the beaker as a result of the heat supplied, must have passed through the film. This is so because the entire outside of the beaker is at the same temperature as the bath, as is surrounded by vapour which is in equilibrium with the helium in the bath. Hence no bulk liquid can form on the outside of the flow arrangement. And the amount of liquid helium that flows out of the tank at the top as film is negligible, as it has to pass through a hole less than one millimetre in diameter. These considerations will be of importance when we discuss later the shape of the curves of transfer rate versus heating rate.

3.5 The Equilibrium Position of the Inner Level

The inner level showed a remarkable behaviour under equilibrium conditions, i.e. when there was no current through the heating coil. It always stayed below the outer level; this level difference was different from experiment to experiment, but in no case was greater than one centimetre. This would seem, at first sight,

to be an anomalous behaviour, for, if there were any stray heat influx into the beaker, the temperature of the helium inside would rise, and this would cause a thermomechanical pressure difference to be established between the helium inside the beaker and that outside. The level inside would consequently rise above the level outside, by an amount given by the London equation.

But a closer examination of the situation suggested a possible explanation of the apparent anomaly. We refer back to figure 3-3, showing the apparatus. In spite of the precautions, described earlier, taken to avoid temperature gradients along the apparatus, it is very likely that the upper parts were slightly warmer than the lower parts. To start with, let the helium inside the beaker and in the bath be at the same temperature T, while the temperature at K, the gap in the flow apparatus, is $T+\Delta T$. Now this difference in temperature between the two ends of the film can be maintained, for K is a few centimetres above both the bath level and the inside level, and so a temperature difference corresponding approximately to this thermomechanical pressure difference can be maintained, without the film being completely evaporated at K.

Now, because K is at a slightly higher temperature

than either the bath or the helium inside the beaker, helium from these two parts moves to K through the connecting films, and evaporates. Thus the level inside falls, and at the same time the temperature rises slightly, owing to the loss of superfluid, until the temperature inside is just below that at K. Presumably the flow from the inside to K would stop when the gravitational potential difference between the two is exactly compensated by a thermomechanical pressure difference. It is obvious that the mechanism described above is made possible only because the inside of the beaker is thermally well insulated from the outside.

We must investigate another possibility that would also explain why the inner level comes to rest slightly below the outer level. This is that, for some reason, the heat influx into the beaker is so great that more helium is being evaporated from the inside than is being replaced by film flow from the bath. But this possibility can be discounted for the following reasons. Firstly, if such were the case, the inner level would continue to fall, and not reach a stationary position below the bath level, as has actually been observed. Secondly, if there were such a stray influx of heat, then an additional supply of heat to the inside of the beaker, e.g. by a current passing through the heating coil, would send the inner

level further down. This is exactly the opposite of what actually was observed. In fact, even the smallest heat inputs used in these experiments produced a rise of the inner level. We can therefore confidently rule out the possibility of stray heat influx being responsible for the position of the inner level below the bath level.

There is one more mechanism which must be examined, because it could also give the observed effect. Daunt, Probst, and Smith(1948) observed an osmotic pressure difference between pure He^4, and a solution of He^3 in He^4, the two liquids being connected by the film. Since He^3 does not take part in superflow, the film forms a semi-permeable link permitting the flow of only the heavier isotope. Taconis, Beenakker and Dokoupil (1950) showed that for dilute solutions (less than about 0.1% of He^3) the osmotic pressure P was given by van't Hoff's law:

$$P = \rho RT.C_L$$

where ρ = density of the liquid, C_L = concentration of He^3 in the solution, and the other symbols have their usual meanings.

The helium used in the present series of experiments was almost entirely derived from natural gas wells; such helium has a He^3 concentration of between 1 and 5 parts in

10^7. Since the helium in the beaker gets there only through the film, it may be assumed to be pure He^4. But the maximum osmotic pressure possible under these conditions comes out to be a fraction of 1 mm, and therefore this process may be ruled out of consideration as a factor in the present discussion. The equilibrium position of the inner level was taken as the 'zero point' for the calculations above.

There was one other remarkable observation, which lends support to our original explanation. This was the appearance of drops on the wires leading to the heating coil, when a current was passed through it, or when the temperature of the bath fluctuated slightly due to the action of the manostat. Due to the discontinuous action of the manostat, the temperature of the bath varied in a 'saw-tooth' manner, the amplitude being less than 0.01 ^{o}K. The drops appeared only when the temperature was dropping rapidly, and ceased when it began to rise slowly again. This is easily understood when it is realised that the temperature at K (figure 3-3) follows these variations closely, while that inside the beaker would be hardly affected at all. It is only the temperature gradient along the beaker which permits the formation of drops at points above both liquid levels; it is well known that it would not be possible under

isothermal conditions. During the momentary cooling at K, the thermomechanical equilibrium between K and the inside of the beaker is disturbed, and liquid flows from K to the inside; this accounts for the drops. As K gradually warms up, the equilibrium is re-established, with an upward flow of the liquid from the inside to K. Slight oscillations of the inner level corresponding to the above process were actually observed. The appearance of the drops makes it plausible that there is a temperature difference between K and the inside of the beaker of the right magnitude and sign to give thermomechanical equilibrium between the two. This would of course be after averaging out the fluctuation of temperature. The fact that drops appear as K is cooling implies that bulk liquid must have been present at K just before; and this is possible only with the temperature distribution suggested above, a distribution which is exactly the same as the one postulated earlier for explaining the position of the inner level at equilibrium.

We shall refer to the conclusions reached in the above discussion, when we consider the transfer rate in its variation with the heating rate.

3.6 Discussion of the Results

We shall first consider the shape of the heat input

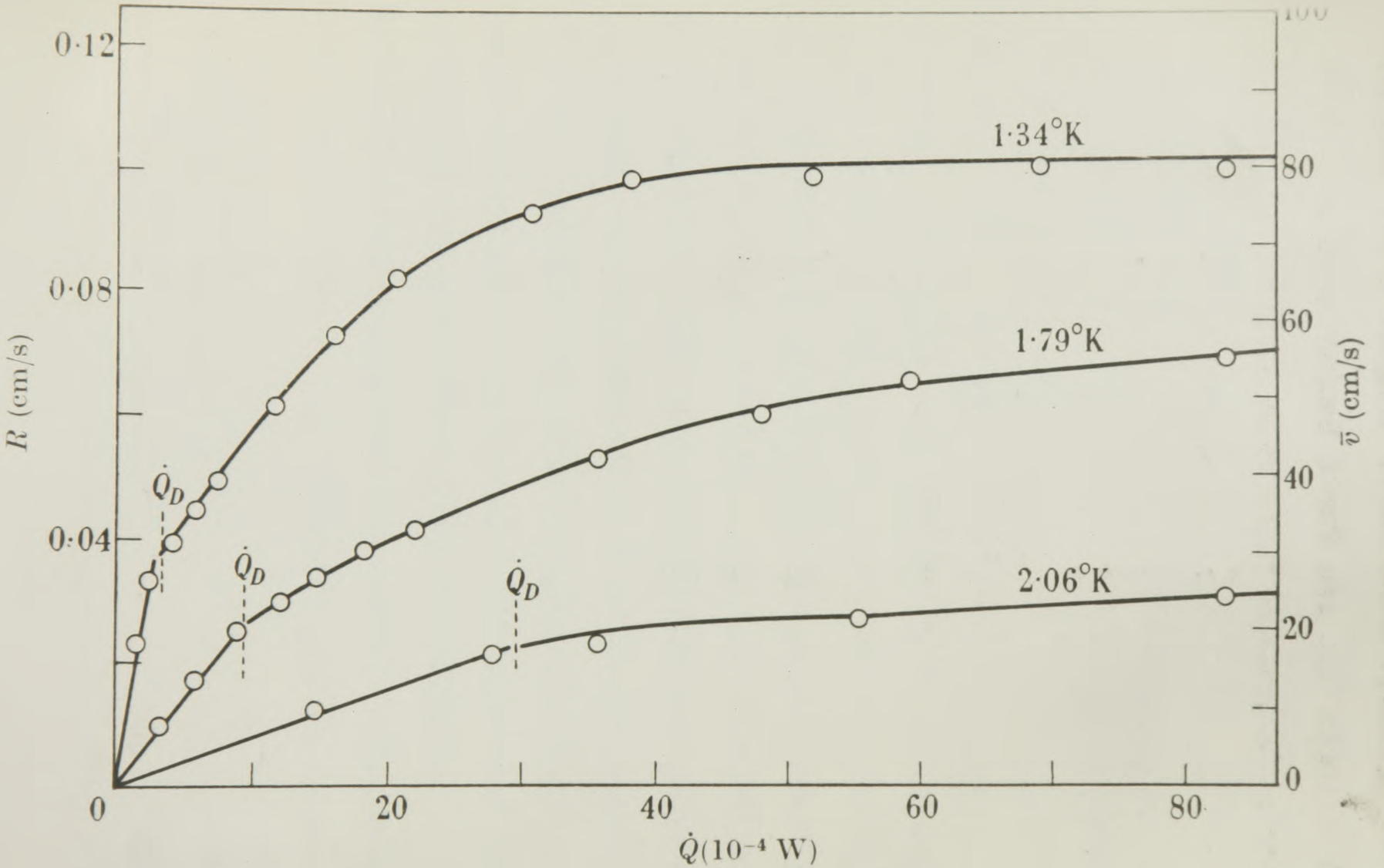


Figure 3-8. Variation of the flow rate through the 1·2 μ slit with varying heat input $\dot{Q}$ at three different temperatures. The flow rate is given as R, the directly observed rise of level in A in cm/s (1 cm rise = 0·238 cm³ of liquid passing through the slit); and as $\bar{v}$, the average velocity in cm/s of the liquid passing out of the slit into A. The broken lines indicate the values of $\dot{Q}_D$, the critical heat input of the depression effect for the three temperatures.

(Bowers and Mendelssohn 1952)

rate versus transfer rate curve (figure 3-7), taking as a typical example the one for a temperature of 1.63°K. It is natural to compare this with similar curves, for the case of flow of helium II through narrow channels, which have been previously obtained (Kapitza 1941, Bowers and Mendelssohn 1952). Figure 3-8 shows the curves obtained by the latter authors for flow through an annular slit of width 1.2 μ. Several striking differences between the two sets of curves are immediately apparent. In each case, the curves can be roughly divided into three regions: (1) low heating rates, where the transfer rate is proportional to the heat input rate, (2) high heating rates, where the flow rate is more or less independent of of the heating rate, and (3) an intermediate region, where a gradual transition from (1) to (3) occurs.

For low heating rates, the linearity of the curves in the case of the film is very much more pronounced than it is for flow in the 1.2 μ slit. And for high heating rates, the transfer rate in the film is independent of the heating rate, while for the slit it appears to be still increasing slightly. We shall discuss the three regions separately in greater detail.

3.61 Low Heating Rates.

The transfer rate in the film, at all temperatures,

is seen to be proportional to the heating rate when this is small. We shall examine the implications of this result on the basis of the two-fluid model for Helium II. Consider in general a system containing helium II, in which the pressure and temperature are, to start with, the same at every point. Then the superfluid is distributed uniformly throughout the system. If now the superfluid concentration is altered at any point in the system, then the superfluid and normal fluid move so as to restore the uniform distribution. The two motions would obviously be in opposite directions. The mechanisms underlying the motions for the two fluids are different: the superfluid particles drift under their zero-point momentum, while the normal fluid moves according to the classical ideas of flow under a temperature or pressure gradient, as the case may be. Applying these ideas to the present case, we find that the supply of heat to the inside of the beaker reduces the superfluid concentration there; therefore superfluid from the bath flows in through the film, till the concentration is the same as before (This being because the bath is very large compared to the volume inside.) The normal fluid, of course, is immobile in the film, and therefore does not contribute to this process.

There is another possible way of looking at the above process, which does not involve any assumption about the structure of heliumII. This idea will be briefly indicated here, and developed in section 3.8. In a given thermodynamical system, containing a fluid, if a mole of the fluid is transferred from one part (A) to another (B) by a definite process, then it is conceivable that an amount of heat Q is liberated at one part, say A, and absorbed at B. Q would, in general, depend on the process by which the transfer takes place. Then Q is defined as the heat of transport for this particular process. It is seen that Q is not uniquely defined for the fluid; it requires other parameters to determine it. In the case under consideration, there is an absorption of heat in the beaker, and a corresponding transfer of a proportionate amount of helium II from the bath, through the film, into the beaker. While there is not any direct experimental evidence yet that an equal amount of heat is liberated in the bath, our previous considerations show that this is a reasonable assumption to make. It would still be of interest to verify this. We can thus say that the heat supplied electrically provides the heat of transport of the helium transferred into the beaker from the bath through the film.

The helium that flows into the beaker causes the

level there to rise; hence the temperature there must also rise, to correspond to the resulting pressure difference. This follows from the observation that, in every case, for low heating rates, the inner level stopped rising as soon as the heating current was switched off, and started to fall very slowly. This means that at the instant of switching off the current, the temperature difference between the helium inside and that outside was just that given by the London equation for the corresponding pressure difference.

The quantity of heat used up in producing this rise of temperature can be shown to be a very small part of the total heat input. The pressure head, in centimetres of liquid helium, corresponding to a given temperature difference ΔT, can be calculated from the London equation. This is found to agree very well with experiment(cf. Meyer and Mellink 1947). Now our flow rates under a temperature difference were determined when the inner level was about 0.1 cm from its equilibrium position. The corresponding temperature difference, at 1.2°K, is 2.10^{-4}°K, and it is very much less at higher temperatures.

The enthalpy H of liquid helium in the temperature range from 1.2°K to the λ-point can be represented by the equation

$$H = kT^n \qquad (3.61)$$

where $n \approx 6.6$ (Keesom and Westermijze 1941). Hence the specific heat C is given by

$$C = nkT^{n-1} \qquad (3.62)$$

Let the initial mass of helium in the beaker be m_o, and let a mass m_1 be drawn in by applying heat to the inside, resulting also in an increase of temperature ΔT. As will be seen below, the heat of transport for the film is approximately equal to the enthalpy of the liquid; the two will be assumed to be the same for the purpose of this discussion without introducing any serious error. Hence we have:

Amount of heat Q_T used to provide the heat of transport of m_1 grams of helium

$$= Hm_1$$
$$= kT^n m_1 \qquad (3.63)$$

and amount of heat Q_c required to raise the temperature by ΔT of $m_o + m_1$ grams of helium

$$= C(m_o + m_1)\Delta T$$
$$= nkT^{n-1}(m_o + m_1)\Delta T \qquad (3.64)$$

Therefore

$$\frac{Q_T}{Q_C} = \frac{kT^n m_1}{nkT^{n-1}(m_o + m_1)\Delta T}$$

$$= \frac{T}{\Delta T} \cdot \frac{m_1}{n(m_0 + m_1)} \qquad (3.65)$$

Now $m_1/(m_0 + m_1)$ was about 0.1 under the conditions of the experiment, and, since T is of the order of $1^{\circ}K$, $T/\Delta T$ is about 10^4. Thus, with $n \approx 6.6$, we see that Q_0 is about 1% of Q_T, and may therefore be neglected.

There is another process occuring in the beaker while the helium is flowing in, which has to be examined. As the volume of helium inside increases, the amount of space available for the vapour above it decreases. As the gap between the optical flats at the top is narrow, the vapour cannot all escape, and some of it condenses on to the liquid, giving off its heat of condensation. The picture may be simplified by considering the extreme case, when <u>all</u> the vapour that is displaced by the incoming liquid is condensed. At $2.1^{\circ}K$, for example, when the inner level rises by 0.1 cm, the heat evolved by the condensing vapour is 4.5×10^{-4} calories per sq. cm. of the liquid surface*. The heat of transport involved in

---

*The estimation of this quantity involved a knowledge of the density of helium gas at the corresponding temperature and pressure. The density was calculated assuming the perfect gas laws, as the deviation due to the second virial coefficient is less than 6%.

---

causing the meniscus to rise through 0.1 cm by film transfer is 7.5×10^{-3} calories per sq. cm. of the liquid

surface. Thus, even in this extreme case, the heat evolved by the condensing vapour is about 6% of the total heat input. This fraction is nearly the same at the lowest temperatures studied, i.e. 1.2°K.

It should be emphasised that this figure of 6% represents an upper limit to the heat effect due to this particular mechanism. It is also seen that this represents an addition to the heat supplied electrically, while all the other effects are heat losses, so that taken together, they would tend to cancel each other out. Thus the overall error in neglecting them is very small.

In addition to all these, there is a heat loss from the inside of the beaker, when it is at a slightly higher temperature than the outside, due to the ordinary processes of conduction and convection. The conduction would take place through the glass, the film and the vapour which might have diffused into the vacuum space of the little dewar; convection could occur in the vapour above the liquid. Very little is known about the true thermal conductivity of the helium II film, except that it must be very small (Daunt and Mendelssohn 1939a. The difficulty is that the heat of transport of the film, being very large, would swamp the conductive

heat flow in any projected experiment. Moreover, it has not been possible to produce heat flow in the film without an accompanying net flow of matter, as was done by Meyer and Mellink in their experiments with narrow slits (1947). The calculation of these heat losses would be cumbrous, if not actually impossible. Therefore the following method, already indicated earlier, was used instead to estimate the heat loss due to these processes.

After the heating current was switched off, the rate at which the inner level dropped was determined. The increase in temperature of the inner liquid over its equilibrium value was proportional to the increase in height of the meniscus. The rate at which heat was being lost from the inside was calculated from the rate at which helium flowed out of the inside, assuming that this helium carried no entropy. Thus, if m grams of helium were flowing out per second, when the meniscus was at a height H from its equilibrium position, and Q its heat content at that temperature, then the rate of heat loss, in mechanical units, when the meniscus is 0.1 cm from the equilibrium position would be $(mQJ/H)\times 0.1$ watts. This implies the approximation that has been already used earlier, that the heat of transport of liquid helium II equals its heat content.

The above expression holds if H were not much greater Than 0.1 cm, and this condition was always satisfied in the experiments. This heat loss was of the order of 10^{-6} watt at a level difference of 0.1 cm when the heating rate was 10^{-4} watt. But the following consideration indicates that this correction need not, in any case, be applied. For, at any given temperature, this heat loss from the inside for a given level difference depends only on the corresponding temperature difference, and not on the heating rate. The level difference at which the transfer rates were measured was approximately the same in all cases, namely 0.1 cm. We have already seen that, for low heat inputs, the temperature inside follows the pressure difference, according to H. London's equation. Thus, for this pressure difference of 0.1 cm of liquid helium, the temperature difference would be the same for the different determinations of transfer rates. This means that, at any single temperature, the heat loss due to the processes considered in this discussion is the same for the different rates of heating. Thus the only effect of applying this correction would be to shift the linear parts of the curves in figure 3-7, parallel to themselves, by a small amount along the horizontal axis. The correction does not affect the linearity of the curves, or their slopes. As we shall

be concerned, in our further discussion, only with the slopes of the linear part of the curves, we are justified in neglecting this correction.

From the foregoing, it becomes apparent that the error introduced by the various factors considered is certainly less than a few per cent. The determination of the heating rate electrically, and the rate of inflow of the helium, could be made to a very much greater degree of accuracy. This margin of error, it should be emphasised, is likely to decrease when the slopes of the linear parts of the curves in figure 3-7 are considered.

Before discussing further aspects of the effects at low heating rates, such as the information that can be obtained about frictional processes in the moving film and the heats of transport involved, it will be necessary to examine the regions of intermediate and high heating rates. This is done in the following section.

3.62 Intermediate and High Heating Rates.

It will be noticed that, in figure 3-7, the curve for 2.09°K shows a fairly sharp break at the point where linear dependence of the flow rate on the heating rate changes to a constant transfer rate, while for lower temperatures this transition occurs gradually, over a range of heating rates. This raises the interesting

question, whether the critical velocity of the helium film is to be identified with that transfer rate at which non-linearity sets in, or with the transfer rate at very high heating rates. If the former is the case, then we have the critical velocity of the film, flowing under a temperature difference, depending to a certain extent on this difference, and becoming constant for larger temperature differences. If the latter were the case, then one would still have to explain the behaviour of the transfer rate for intermediate heating rates.

The first point to be decided was, whether the critical transfer rate for the film under isothermal conditions was to identified with the transfer rate, under a temperature difference, at the point where non-linearity sets in (L in figure 3-7 for 1.63°K), or with the constant rate at high heating rates (M in figure 3-7). The results of Brown and Mendelssohn (1950) already gave some information on this point, for they had observed the isothermal critical rate to correspond to the rate in the horizontal part of the transfer rate versus heating rate curve. In our apparatus, a direct verification was not possible, for the following reasons: (1) The limiting perimeter for film transfer under isothermal conditions would be the inside perimeter of the beaker,

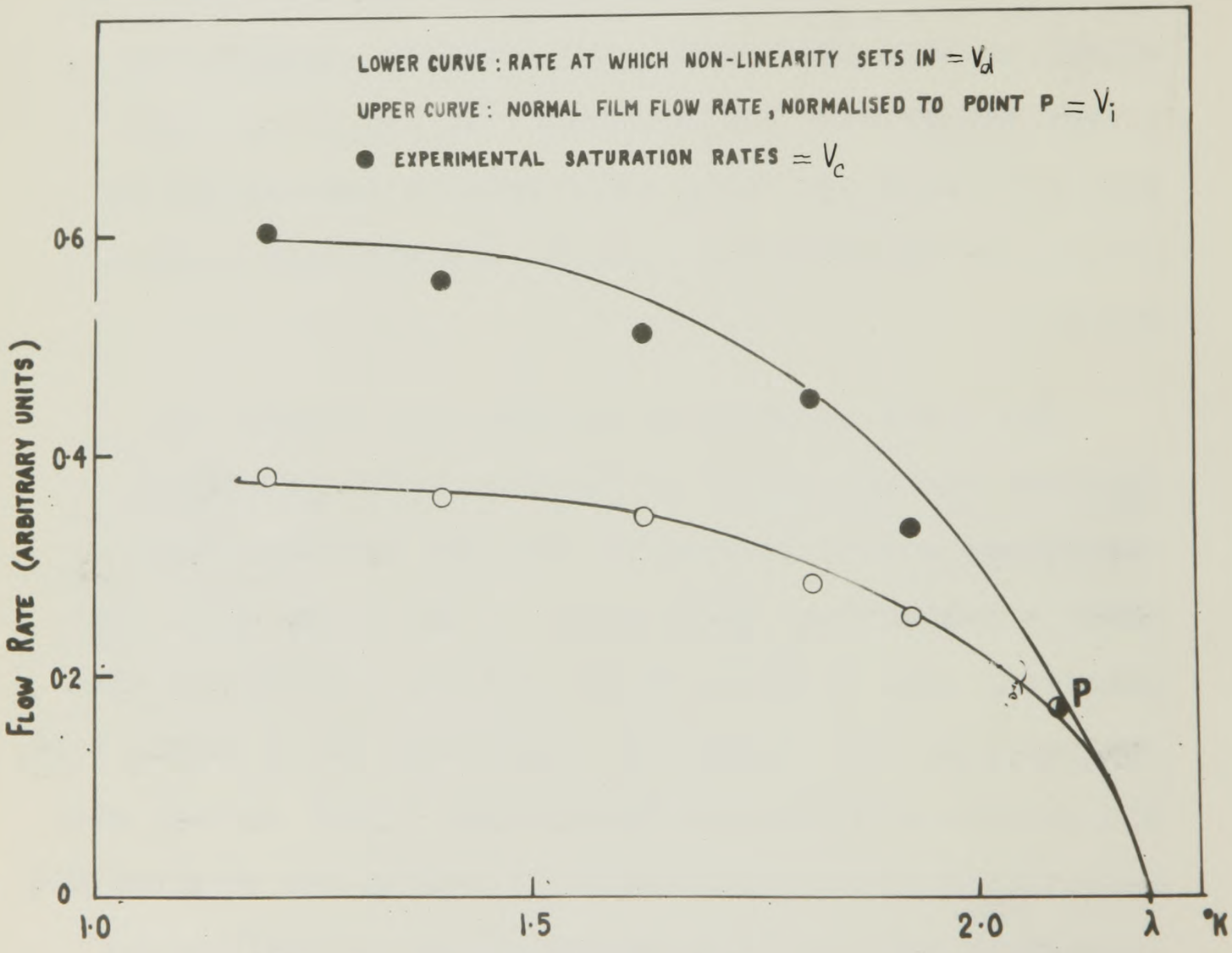

LOWER CURVE : RATE AT WHICH NON-LINEARITY SETS IN = V_d
UPPER CURVE : NORMAL FILM FLOW RATE, NORMALISED TO POINT P = V_i
● EXPERIMENTAL SATURATION RATES = V_c
FLOW RATE (ARBITRARY UNITS)
0·6
0·4
0·2
0
1·0
1·5
2·0
λ °K
P

Figure 3-9

while in the present experiments it was difficult to determine exactly, being certainly greater than the outside perimeter of the beaker. This has already been explained in detail earlier. (2) In any case, an attempt to measure isothermal flow in an arrangement such as the one used for these experiments is bound to produce results which are difficult to interpret, for the inside of the beaker is thermally isolated, and the temperature differences produced by the mechanocaloric effect (Maunt and Mendelssohn 1939b) cause rates which bear no simple relation to the isothermal transfer rate.

The rate at which non-linearity sets in will henceforth be denoted by V_d, and the constant rate for high heating rates V_c. It was felt that a comparison of the dependence on temperature of V_d and V_c, with that of V_i, the isothermal transfer rate (Mendelssohn and White 1950) would give some information about which of the two was nearer V_i. This comparison is made in figure 3-9. V_d and V_c were the same at 2.09°K, and so the isothermal rate V_i was normalised to this point P by multiplying by a suitable factor. The figure shows that V_c has the same variation with temperature as V_i, while V_d does not increase rapidly enough with decreasing temperatures. This confirms that the isothermal critical rate is in-

deed to be identified with the constant rate produced by high heating rates.

We now come to the shape of the curve of transfer rate versus heating rate, for intermediate heating rates. Several mechanisms can be suggested for explaining the non-linearity in the dependence of $\dot{V}$, the transfer rate, on $\dot{Q}$, the heating rate:

(1) The film does not have enough surface available for flow inside the beaker; that is, more helium is capable of entering the beaker in the form of film than can be carried as film on the available surface inside the beaker. The difference between the two would presumably decrease as the $\dot{Q}$ increases, and finally vanish for high $\dot{Q}$. This would be because, as the heating rate increases, a stage would be reached when the helium could run down the inside as bulk liquid (cf. Brown and Mendelssohn 1950).

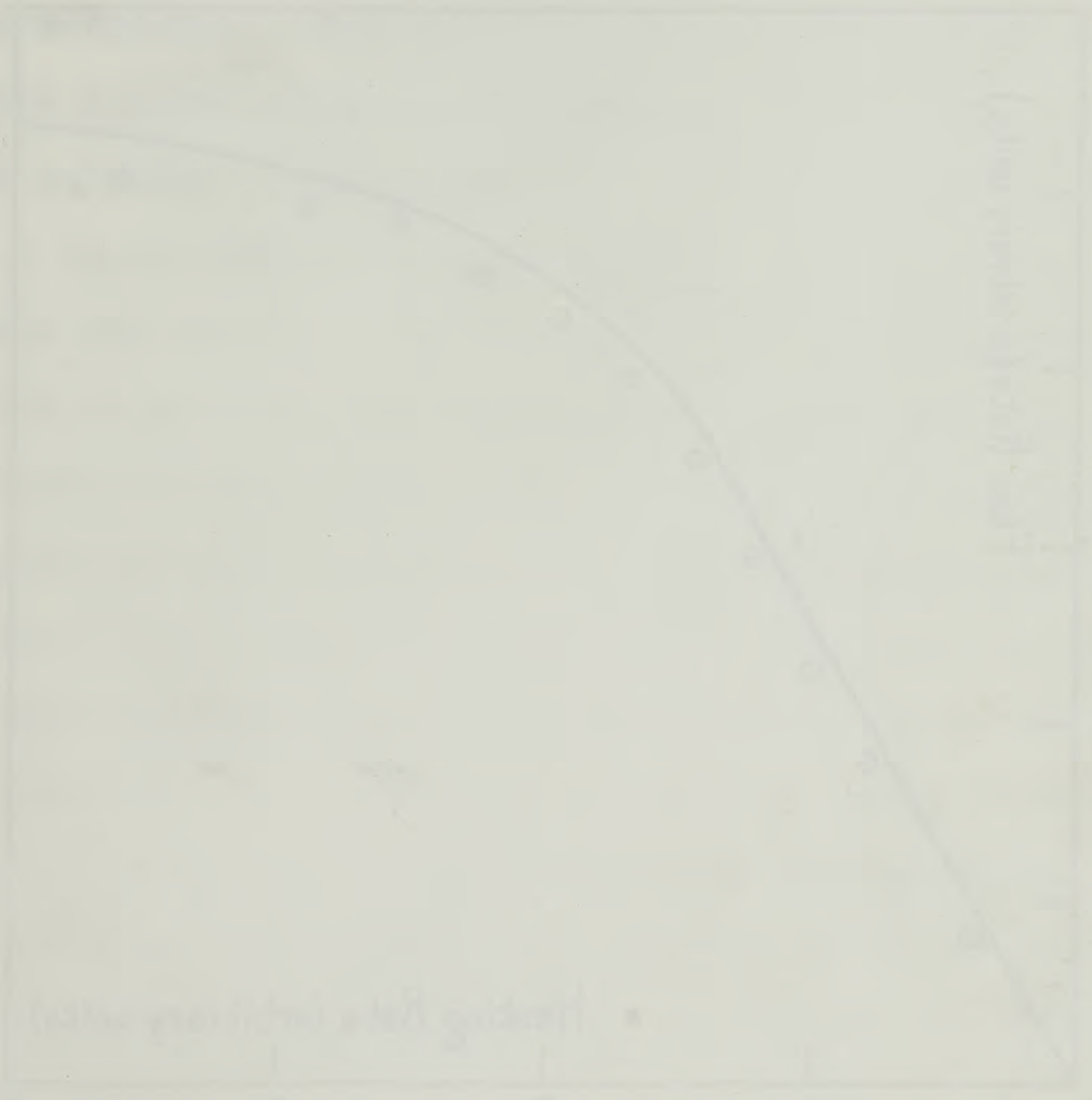
(2) When the transfer rate has reached a value V_d, so much helium is entering the beaker that the narrow gap between the polished surfaces at the top becomes filled with bulk helium, and beyond this point the transfer rate would be governed by the flow of bulk helium through this annular slit, with of course the ultimate limitation imposed by the film on the various surfaces outside

the beaker reaching its critical rate; this becoming manifest in the flat part of the curve.

(3) A third possibility, which may either account, by itself, entirely for the shape of the curve in this region, or may be effective simultaneously with one or both of the other two mechanisms, is that a frictional force between the normal fluid and the superfluid does become evident when the transfer rate approaches its critical value, so that more energy has to be supplied to the inside of the beaker than is necessary to make up the energy deficit of the superfluid flowing in.

The mechanisms (1) and (2) are basically similar: in (1) the controlling factor is film transfer on the inside of the beaker, while in (2) it is the flow of bulk liquid in the annular slit. Experiments were devised to test the applicability of each of the above explanations, and they gave the following results:

(1) To test the possibility that the inner surface of the beaker was insufficient to take the entire film transfer which would correspond to the heat supplied electrically, the flow apparatus was modified. A thin walled cupro-nickel tube, of outside diameter 0.2 cm, with 500 strands of S.W.G.46 bare constantan wire packed in it, was inserted inside the beaker. The wires projected



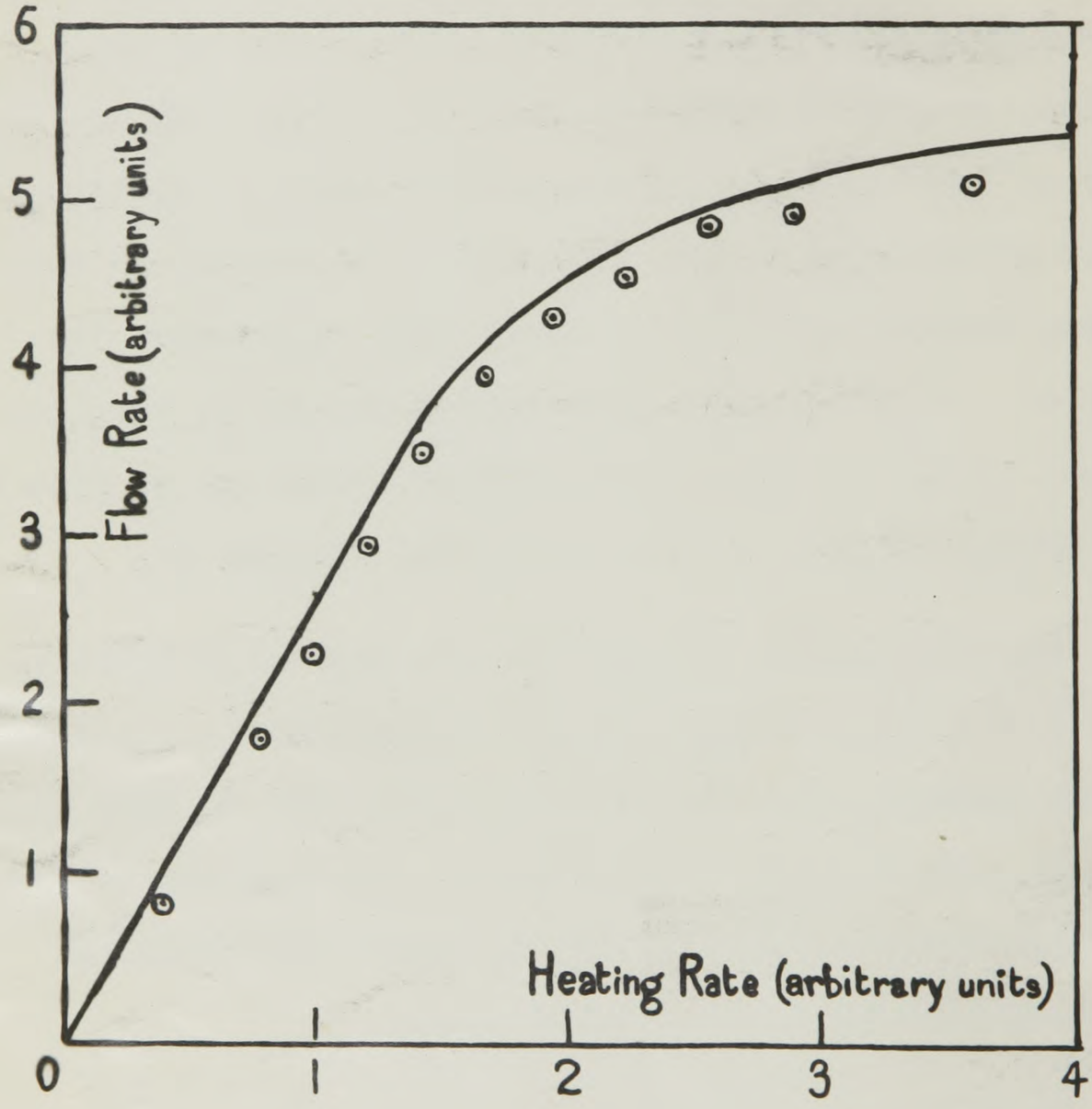


Figure 3-10 The encircled points show the rates at 1.78°K with the wire-packed tube in position. The curve is for 1.82°K without the tube.

outside the tube at both ends. At the top the wires were attached to the glass cap with a porous cement which extended up to the points of contact with the rim of the beaker. The lower end of the tube was immersed in the liquid helium in the beaker. This new arrangement provided an extra 9.5 cms of surface of constantan, and 1 cm of surface of cupro-nickel, for film transfer inside the beaker. It was expected that the porous cement at the top, and the increased perimeter made available by the wires, would together prove sufficient to take any film transfer with which the inner surface of the beaker alone could not cope. In fact, it will be seen from figure 3-7 that the thermal inflow observed corresponded to a limiting perimeter of about 10 cms of glass. Thus, even assuming the flow on constantan and cupro-nickel to be the same as that on glass, one would expect the total perimeter available inside the beaker to be adequate.

A transfer experiment, similar to the ones described before, was done with this new arrangement, at 1.78°K. The results are shown in figure 3-10, where the earlier results for 1.82°K (i.e. without the wire-packed tube inside the beaker) are also presented for comparison. The continuous curve is for 1.82°K, while the points are for 1.78 K. It is seen that the introduction of the

extra surface inside the beaker has had no material effect on the shape of the curve in the intermediate region.

(2) It is not really necessary that surface available inside for film transfer should influence the shape of the curve over the whole of the intermediate region, for, as was mentioned before, the helium sometimes formed drops inside the beaker at points which were above both menisci. This clearly indicates that helium can run down the inside of the beaker in the form of bulk liquid, and need therefore not be restricted by the available perimeter. This was the clue which suggested the second of the mechanisms mentioned above.

Now, as long as the transfer rate stays below a certain value, set by the geometry of the annular gap at the top of the beaker, and the velocity of flow through this gap, it will not offer any resistance to the transfer. But when the rate exceeds this value, the gap will fill up with liquid helium, and from this point onwards the transfer characteristics of bulk helium through an annular slit will affect the transfer into the beaker. An observation was made at 1.93°K which did support this possibility. The apparatus was lowered into the helium bath, so that the bath level was above the annular gap, but below the Fortafix seal J (see figure 3-4). Then

the flow into the beaker is almost entirely through the gap in the form of bulk liquid, any film flow through orifices in J being negligible. The flow rate observed under these conditions is indicated by the arrow in figure 3-7. This indicates that, for flow rates in the intermediate region, it is quite plausible to assume that the annular gap is filled with bulk liquid. This also links up with the observation of drops of liquid inside the beaker at points above both menisci.

While thus the above mechanism itself seems reasonable, it is rather more difficult to determine quantitatively its contribution to the non-linearity in the intermediate region. It should be similar in character to the flow through narrow slits observed by Kapitza(1941); this is indeed so. But the situation here is slightly more complicated, for the bulk liquid is connected by the film, certainly on one side, and possibly on the other, to volumes of liquid helium between which a temperature difference is maintained; while Kapitza had the bulk liquid in direct contact with the annular gap on both sides.

The obvious way of modifying the apparatus, so as to minimise this effect, would be to increase the width of the gap. But this, as has been explained already, would worsen the thermal isolation of the inside of the

beaker by allowing helium from inside to distil over to the bath, and is therefore not practicable. Another possible way would be to so reduce the outside perimeter of the apparatus available for film transfer, that the the critical rate is reached on the outside before the gap can be filled up with liquid. This unfortunately has not been practicable, because of the necessity of having a double-walled beaker, and of surrounding it with a metal radiation shield.

(3) We now come to the possibility of a true frictional dissipation of energy occuring in the film at rates near the critical, because of friction between the normal fluid and the superfluid, possibly of the Gorter-Mellink type. The sharpness with which the critical velocity sets in at 2.09°K makes it unlikely that, for that temperature in any case, there is such a mutual friction. At lower temperatures, no decision can be reached about the existence of such an effect, for it cannot be separated from the effect of bulk flow in the annular slit. But it will be seen later that certain conclusions can be drawn at least regarding the possibility of a partial application of a theory of friction to the present experiments. These matters are discussed in the next section.

3.7 Frictional Processes in the Helium II Film

As a background to the discussion that follows, it is necessary to survey briefly the theory of mutual friction between the normal fluid and the superfluid proposed by Gorter and Mellink (1949). It was observed experimentally (Allen and Reekie 1939, Duyckaerts and Keesom 1947, Mellink 1947) that the heat flow through helium II in fine channels was not linearly dependent on the temperature gradient. To explain this, Gorter and Mellink postulated a frictional force which comes into play when there is relative motion between the normal fluid and the superfluid, equal in magnitude for each of them but of opposite sign. Since heat flow in helium II is assumed to occur by a convective process (Kapitza 1941) it would be proportional to the relative velocity $(v_s - v_n)$, and the force is proportional to the temperature difference ΔT, where the symbols have their usual meanings. From the experimental observation that the heat flow varied as $(\Delta T)^{1/3}$, Gorter Mellink suggested that this mutual frictional force was proportional to $(v_s - v_n)^3$.

Now the theory of mutual friction was admittedly proposed originally only to explain heat flow in channels larger than 1 μ in width. But if it is to have a more

general physical significance, it should be applicable to flow under other conditions as well, such as isothermal flow in capillaries, or film transfer. This is particularly desirable, since attempts have been made (Zilsel 1950) to deduce the cube law from the hydrodynamical equations of the two-fluid theory, with the help of assumptions about irreversible processes. Furthermore, a crucial test that the theory has to pass is to describe the phenomenon of a critical rate of flow.

It is easy to see that, if the cube law holds between the frictional force and the velocity, and W is the rate of doing work which produces flow with a velocity v, then W should vary as v^4. Therefore, for not too small values of v, a small increment in v would require a considerable increase in W. This may be interpreted as a 'blurred' critical velocity (Dingle 1952), but not with complete justification; for, as many experiments have shown (Bowers and Mendelssohn 1952, chapter 6 of this thesis), the onset of the critical velocity is quite sharp, and flow at subcritical rates is frictionless. Such a behaviour cannot be reconciled with the Gorter-Mellink theory.

Ideally, if there were no friction and no critical rate, the extrapolated linear parts of the curves of

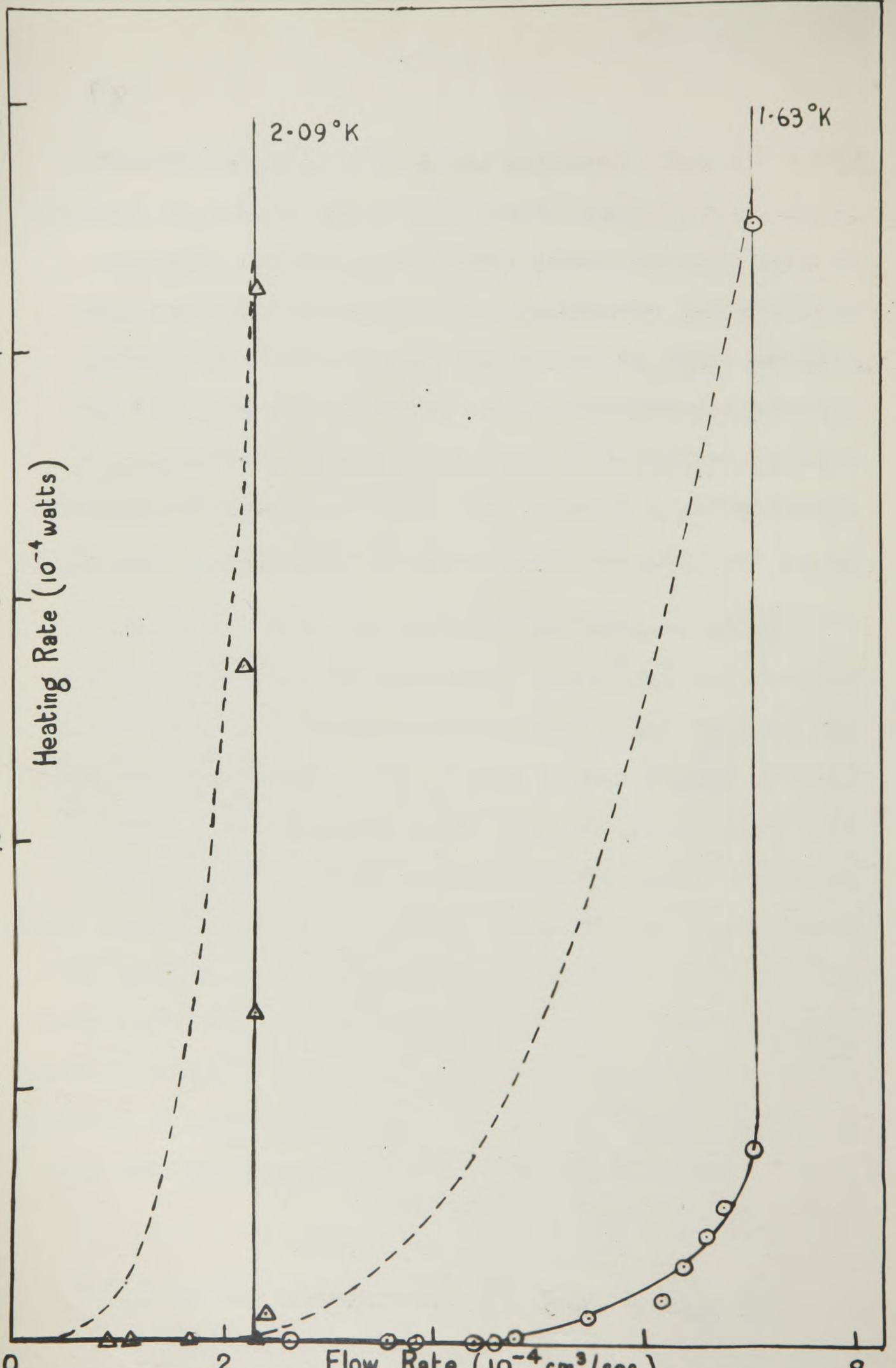


Figure 3-11 Comparison with the Gorter-Mellink theory.

figure 3-7 would describe the flow rate versus heating rate characteristics. The deviations from this straight line would be due to either frictional losses or the critical velocity appearing. In figure 3-11 are shown these deviations for temperatures of 2.09°K and 1.63°K, plotted against the transfer rate. The volume rates have been used, though the rate which enters directly in the theory of mutual friction is the linear relative velocity between the normal and super-fluids. But this substitution is justified, since the thickness of the film outside remains sensibly constant, independently of the velocity which it has. The volume rate is therefore proportional to the linear velocity in the film. Again, as the normal fluid is practically immobile in the film, the velocity of the superfluid is equal to the relative velocity between the two. On the same graph is also drawn curves showing the relation between the heating rate and the flow rate on the theory of mutual friction (broken curves). These latter were obtained by fitting a v^4 curve between the origin, and the observed flow rate for a heating rate of 5×10^{-4} watt. It is clearly seen that for a wide range of sub-critical rates, the transfer is frictionless, and cannot be accounted for at all on the Gorter-Mellink theory.

In the intermediate region, it is not possible to

say with certainty whether there is a gradual onset of friction, or the observed shape of the curve is entirely due to the mechanism (2) discussed in section 3.62 above. The curve for 2.09°K does indicate that, at this temperature at least, the critical rate appears suddenly. This makes it likely that the filling up of the annular gap with bulk liquid does contribute mainly to the non-linearity, without altogether ruling out the possibility of a gradual, but slight, appearance of friction at rates near the critical. A final decision on this point must be left to experiments on isothermal film flow under very small gravitational potential gradients.

It is nevertheless possible to reconcile these results with flow observations in wider channels ($\sim 1\mu$), if one assumes the existence of a critical velocity v_c which is inversely proportional to the width d of the channel. While all theoretical considerations regarding critical rates seem to agree in giving this result, experiments have not yielded consistent results. Kapitza (1941) did observe the critical velocity to decrease with increasing slit width, while on the other hand Meyer and Mellink (1947) and Bowers and Mendelssohn (1952) found v_c in channels of width about 1μ to be of the same order of magnitude as in the film ($d \sim 0.01\mu$). It must be pointed out that in all these experiments, there

was a considerable uncertainty in the determination of d, and there was also no assurance that the channel was uniform in width. It is clear that more measurements are needed on isothermal flow through narrow slits whose geometry is well defined.

If, however, we assume that the critical velocity is inversely proportional to the width of the slit, we shall then have to distinguish between two kinds of frictional processes for the superfluid. The first of them is against the wall, and gives rise to the observed critical velocity, below which frictionless flow can take place. The second is against the normal fluid, and becomes apparent only when the normal fluid also can move, as is the case in slits and capillaries. In these flow channels, which are wider than the film thickness, the critical velocity is reached at very low flow rates, and beyond this velocity the second type of friction, which might obey the cube law, sets in (cf. Bowers and Mendelssohn 1952).

The position may be summarised as follows: Using the two-fluid model, flow channels can be considered in two groups. (1) The very narrow ones, in which the normal fluid is completely immobilised, only the superfluid can move. The helium II film is the best, perhaps the

only, example of this. (2) Wider channels, where the normal fluid is also capable of motion.

The experiments above have shown that in channels of group (1) there exists a critical rate which defines the maximum velocity possible for the liquid, and below which frictionless flow can occur. Experiments with channels belonging to group (2) (see chapter 6 of this thesis; Bowers and Mendelssohn 1952) show that frictionless flow is possible till a certain critical rate, beyond which flow can take place only with the appearance of friction, possibly of the Gorter-Mellink type. The two mechanisms suggested above would explain both types of flow. But the hydrodynamics of liquid helium for large velocities has not yet been fully developed, even where irreversible processes are ignored (Dingle 1952). There can be no doubt that such processes do play an important part in the flow phenomena discussed above. It may be expected that the final equations of motion, taking account of irreversible processes, will be capable of being interpreted in terms of these two mechanisms.

3.8 The Heat of Transport of the Helium II Film

The concept of the heat of transport has already been introduced ; it will now be developed in greater detail, with particular reference to the present experiments.

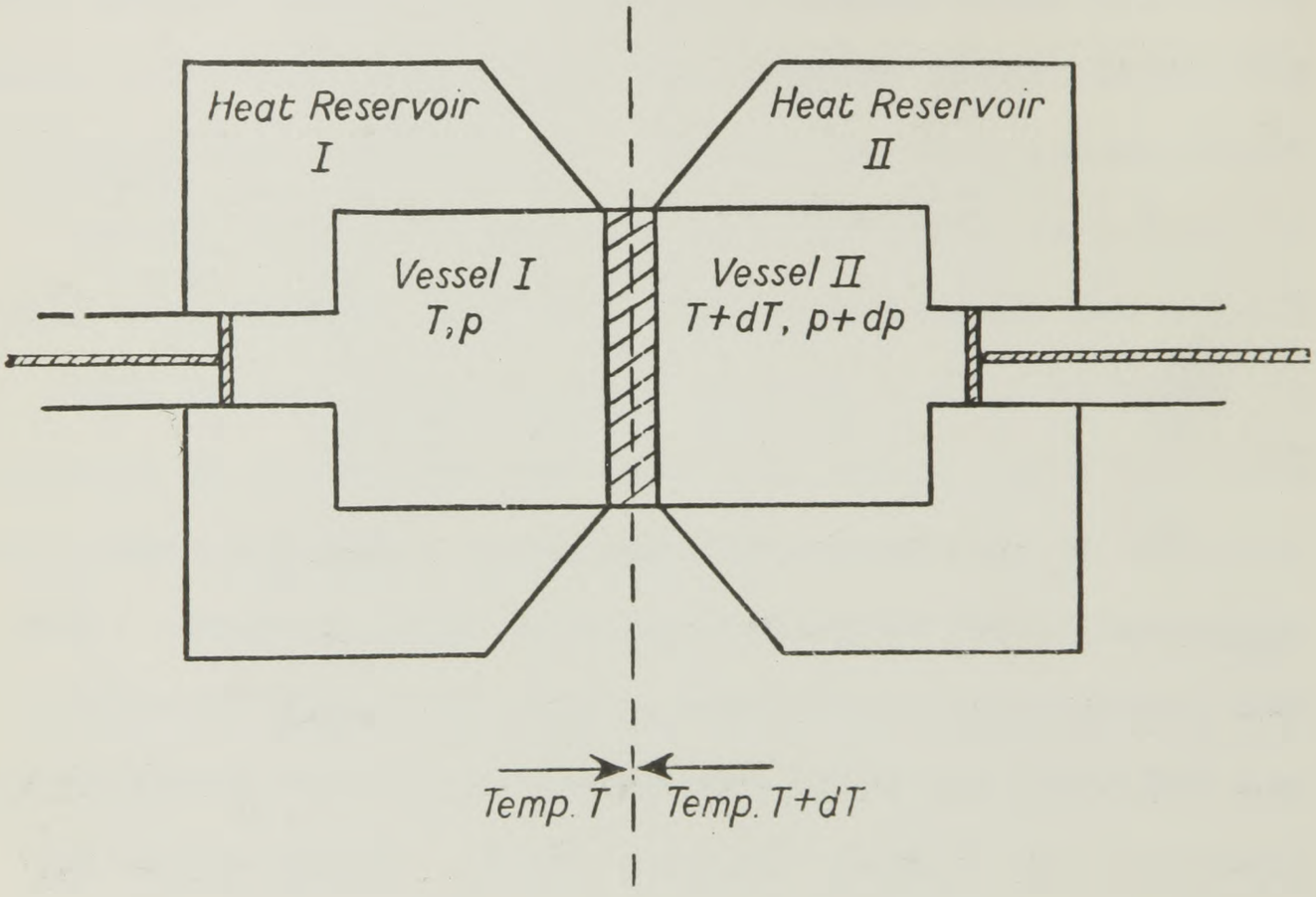

Heat Reservoir
I
Heat Reservoir
II
Vessel I
T, p
Vessel II
T+dT, p+dp
Temp. T
Temp. T+dT

Figure 3-12

Consider two vessels I and II (figure 3-12) connected by a superleak. The temperature and pressure in I are T and p, and in II, T+dT and p+dp. Now let a mole of the liquid be transferred from I to II, without altering the temperature or pressure in either vessel. This may be assumed to be done by suitable systems of heat reservoirs and pistons connected to the two vessels. If, during this process, a quantity of heat Q is liberated in chamber I and absorbed in II, then Q is the heat of transport for this particular arrangement.

In the course of the following discussion, the equation to the thermomechanical effect deduced by H.London (1939) will be frequently referred to, and so we shall first examine the thermodynamic derivation of the equation, which will assist us in appreciating the significance of the various quantities involved. The equation has been derived independently by several workers using different methods (Denbigh 1951). We shall consider here the one due to Eastman (1926), both because of its simplicity and its use of a mechanism which corresponds very closely to the actual arrangement used in these experiments.

Referring back to figure 3-12, consider the transport of one mole of liquid helium from chamber I to II through the link between them. We shall not, for the

present, specify the exact nature of this link. Let this transfer involve a heat of transport Q^*, and leave the temperatures and pressures unaltered, as already described. Thus the thermodynamic state in each vessel is unchanged, and we are dealing with a virtual displacement from a stationary state. We now calculate the various entropy changes caused by this displacement.

(a) Due to the constancy of pressures, the reservoirs lose heat to the liquid. Let U and U+dU be the internal energies of the liquid in vessels I and II, and V and V+dV the molar volumes. Then the heat absorbed during the displacement is

$$\begin{aligned} & dU + (p+dp)(V+dV) - pV \\ &= dU + pdV + Vdp \\ &= dH, \end{aligned}$$

where dH is the change in enthalpy of the liquid due to the change of state $(T, p) \rightarrow (T+dT, p+dp.)$ The corresponding change in the entropy of the reservoirs is denoted by dS_{r1} and is given by

$$dS_{r1} = -\frac{dH}{T} = -\frac{1}{T}\left(\frac{\partial H}{\partial p}dp + \frac{\partial H}{\partial T}dT\right) \quad (3.801)$$

(b) There is a further change in the entropy of the reservoirs, due to the heat of transport. This heat Q^* is evolved in I (at temperature T) and absorbed in II (at T+dT) by the liquid. The corresponding increase in

entropy dS_{r2} of the reservoirs is

$$dS_{r2} = \frac{Q^*}{T} - \frac{Q^*}{T+\Delta T} = \frac{Q^*}{T^2}\, dT \qquad (3.802)$$

(c) Finally, There is the change of entropy of the fluid due to the change of state from (p,T) to (p+dp,T+dT). This is denoted by dS_f and given by

$$dS_f = \left(\frac{\partial S}{\partial T}\right) dT + \left(\frac{\partial S}{\partial p}\right) dp \qquad (3.803)$$

We now apply Thomson's hypothesis, assuming that the sum of the above entropy terms, due only to the displacement at the stationary state, is zero:

$$dS_{r1} + dS_{r2} + dS_f = 0 \qquad (3.804)$$

or

$$-\frac{1}{T}\left[\left(\frac{\partial H}{\partial p}\right)_T dp + \left(\frac{\partial H}{\partial T}\right)_p dT\right] + \frac{Q^* dT}{T^2} + \left(\frac{\partial S}{\partial T}\right) dT + \left(\frac{\partial S}{\partial p}\right) dp = 0 \qquad (3.805)$$

We also have the following thermodynamical relations:

$$\begin{aligned} \left(\frac{\partial H}{\partial p}\right)_T &= V - T\left(\frac{\partial V}{\partial T}\right)_p \\ \left(\frac{\partial H}{\partial T}\right)_p &= C_p \\ \left(\frac{\partial S}{\partial T}\right)_p &= \frac{C_p}{T} \\ \left(\frac{\partial S}{\partial p}\right)_T &= -\left(\frac{\partial V}{\partial T}\right)_p \end{aligned} \qquad (3.806)$$

Substituting (3.806) into (3.804) we get

$$-\frac{V\, dp}{T} + \frac{Q^*}{T^2} dT = 0$$

or

$$\frac{dp}{dT} = \frac{Q^*}{VT}$$

or, expressed per gram of helium,

$$\frac{d\psi}{dT} = \frac{\rho Q}{T} \qquad (3.807)$$

where ρ = density of liquid helium, and Q is its heat of transport, per gram, for the arrangement considered above. This is identical with H.London's equation in its primary form.

It should be emphasised that in the above discussion, no account has been taken of the heat that may be conducted through the link. This being an irreversible process, would invalidate the application of the second law of thermodynamics, as has been done to get equation (3.807). But Thomson's hypothesis found a posteriori justification in Onsager's principle of microscopic reversibility, and hence equation (3.807) is correct, within the limitations imposed by the assumptions made in its derivation. This is as far as thermodynamics can take us.

H.London now gave a molecular interpretation to (3.807), based on Tisza's two-fluid model for helium II. If the capillary link between the two vessels I and II is fine enough to completely inhibit the flow of the normal component, than only the superfluid, whose entropy is zero, can flow through. This immediately enables us to specify Q, the heat of transport, more definitely. For

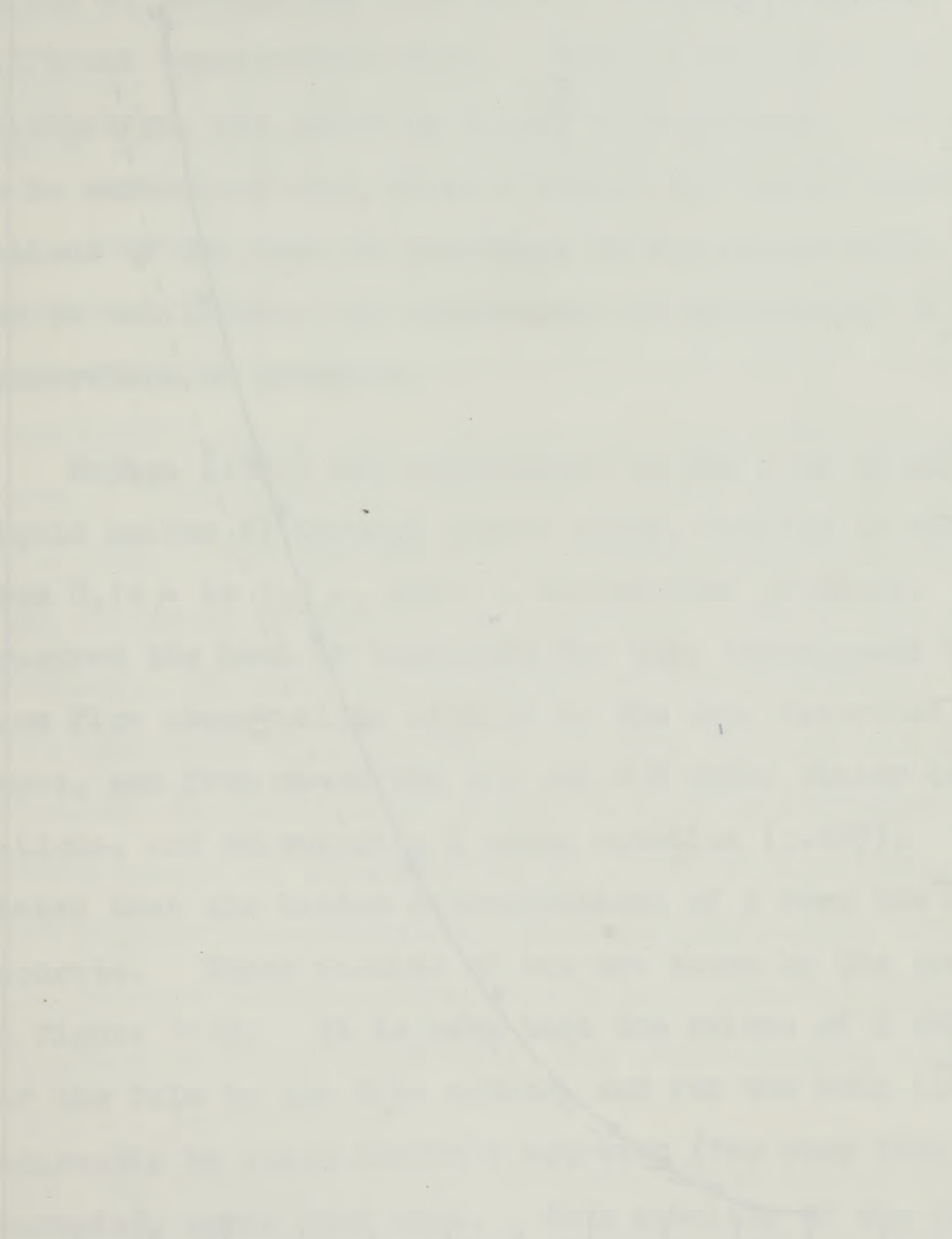
the removal of one gram of superfluid from reservoir I would require also the removal, at a temperature T, of a quantity of heat Q, or entropy Q/T. From a different point of view, the removal of one gram of the substance from I has left the pressure and temperature unaltered, and hence entropy removed must be equal to S, that of one gram of the substance. We thus have

$$S = Q/T \qquad (3.808)$$

where S is the entropy per gram of liquid helium at the temperature T. Inserting (3.808) into (3.807), we have H.London's equation in its familiar form:

$$\frac{dp}{dT} = \rho S \qquad (3.809)$$

We shall now discuss the application of these theoretical considerations to our experimental results. The experimental arrangement described above corresponds closely to the mechanism used by us to deduce the London equation. The temperature inside the beaker, of course, starts rising as soon as the heating current is switched on, as has already been explained. But as long as we confine our attention to the region of low heatings rates, the transfer rate can be regarded as being determined under isothermal conditions, for it is the extrapolated rate at time t=0, i.e. the instant at which the heating current is switched on. Thus, from the slope of the



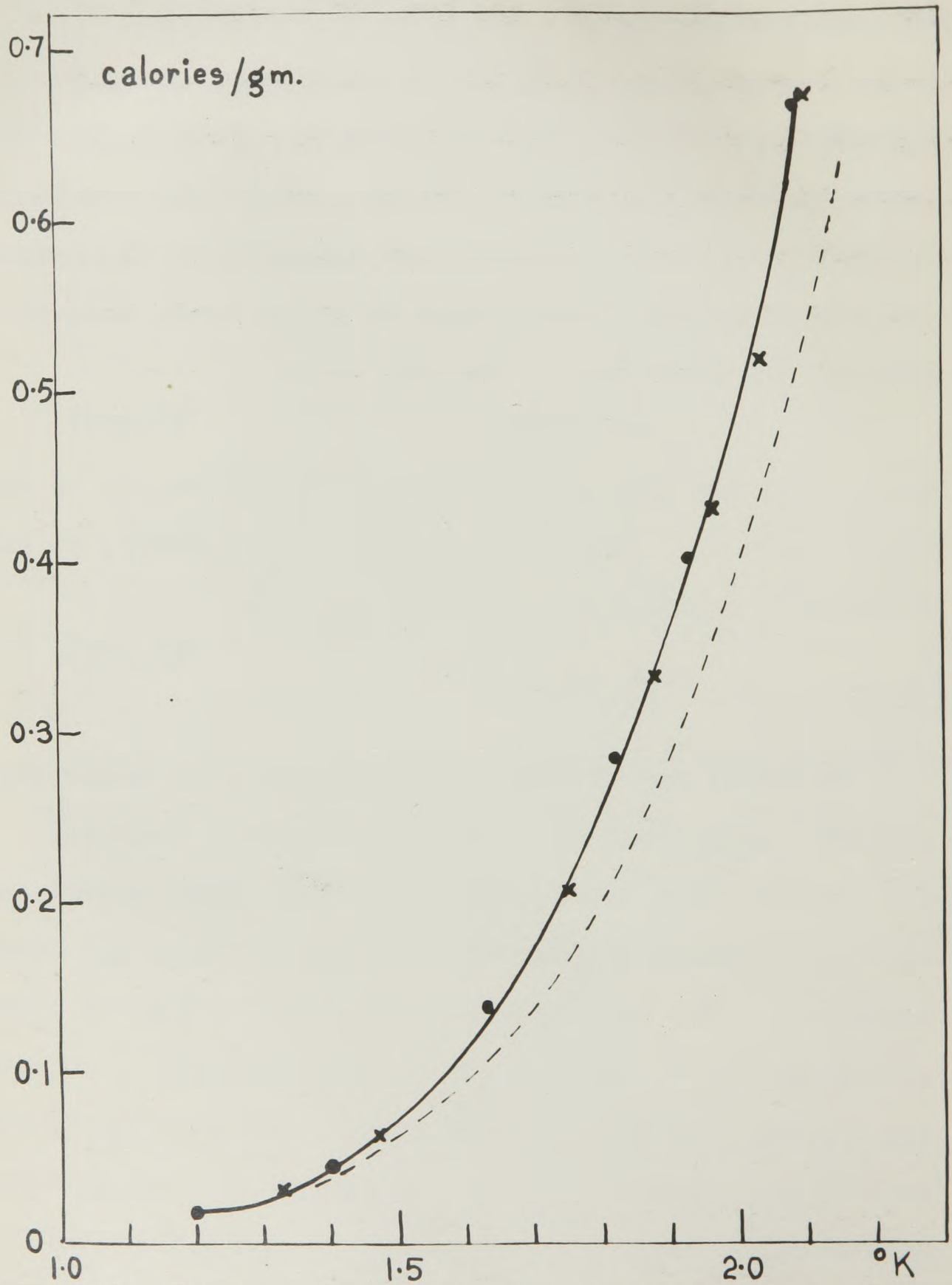


Figure 3-13. Heat of transport as a function of temperature. The dots represent the present results for the film, the crosses are Kapitza's results for channels about one micron in width. The broken curve is the enthalpy calculated from specific heat data.

linear part of the curve, the heat of transport Q of helium II through the film can be calculated for the different temperatures used. These values of Q, in calories/gm, are shown in figure 3-13 by dots. It is to be emphasised that these Q values are direct determinations of the heat of transport of the helium film, and do not involve any measurement of differences of temperature or pressure.

Kapitza (1941) did experiments on the flow of bulk liquid helium II through narrow slits, varying in width from 0.14 μ to 3.2 μ, under a temperature gradient. He measured the heat of transport for this arrangement both from flow observations similar to the ones described above, and from measuring Δp and ΔT under steady conditions, and calculating Q using equation (3.807). He states that the latter determinations of Q were the more accurate. These results of his are shown by the crosses in figure 3-13. It is seen that the values of Q obtained for the film by the flow method, and for the bulk liquid indirectly by using London's equation (for very fine channels), agree very well. This equality of the heats of transport for the two cases implies there is a basic physical similarity between the two phenomena, i.e. film transfer and flow in very fine channels of about 1 μ width. This is adequately explained by the two-fluid model, since

the normal component, which is immobile in the film, should be very nearly so in the narrow slits used by Kapitza, so that it is the superfluid in both cases which is responsible for the heat of transport.

Kapitza goes on to compare the experimentally determined values of Q with values calculated by Landau from the specific heat measurements of Keesom and Keesom(1935), and claims very good agreement between the two. It is not made clear how exactly the calculation was done. For the purposes of the ensuing discussion, the following thermodynamic functions must be clearly defined, and their mutual relationships clarified.

(1) The heat of transport Q for flow through narrow slits. In the general case, the heat of transport would depend on the geometry of the flow apparatus. But Kapitza's experiments, taken in conjunction with the present results, show that Q is a uniquely defined function of temperature only, for slits ranging in width from 1 μ to about 0.01 μ.

(2) The enthalpy, or heat content, H, of liquid helium, which can be calculated from specific heat measurements:

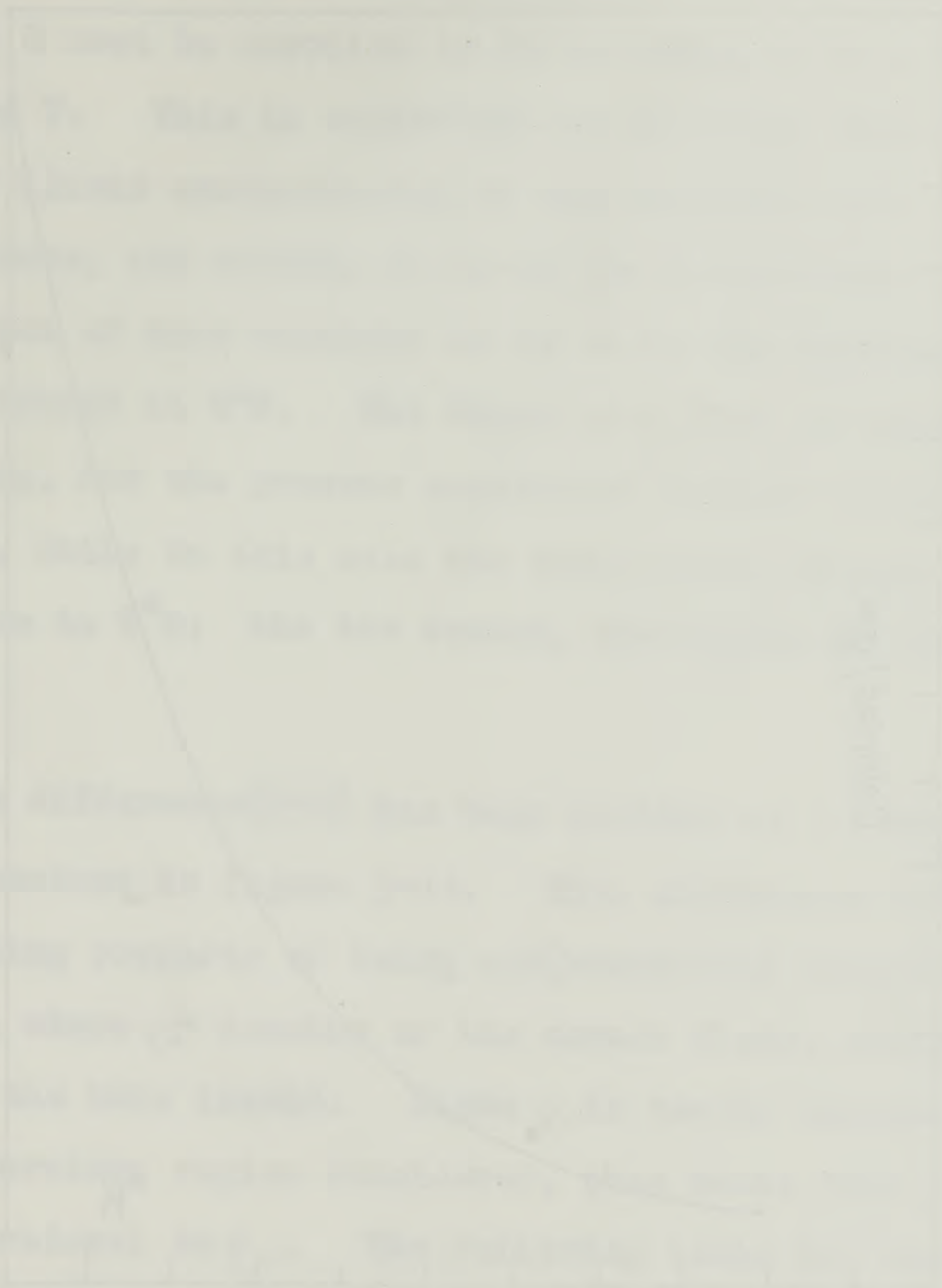
$$H = \int_0^T C_p \, dT \qquad (3.810)$$

(3) The entropy S of liquid helium, which may also be obtained from specific heat data:

$$S = \int_0^T \frac{C_p}{T} \, dT \qquad (3.811)$$

Now, if the Tisza-London hypothesis, that the entropy of that part of the liquid helium which actually moves in the capillary link is zero, were correct, then Q and S would be connected by equation (3.808). Kapitza's measurements give the quantity we have denoted by Q, but which he calls the heat content of liquid helium. It is possible therefore that Landau calculated the heat content H of liquid helium from Keesom and Keesom's specific heat data, for comparison with Kapitza's results. Band and Meyer (1948) have calculated both H and S, using the measurements of Keesom and Westermijze (1941). Their values for H are shown in figure 3-13 (broken curve), along with the present determinations of Q. While the two are of the same order of magnitude, Q is 15 to 20 per cent greater than H, and this is certainly outside the experimental error in determining Q. It is thus clear that neither Kapitza's results, nor the ones described in this chapter, for Q, agree with H from specific heat measurements.

Kapitza might have considered the interpretation of his results from a slightly different point of view, which would probably explain his attempt to compare the Q that he measured with the enthalpy of the bulk liquid. The process which was used above to deduce the London equation can be looked upon as follows: a gram of liquid,



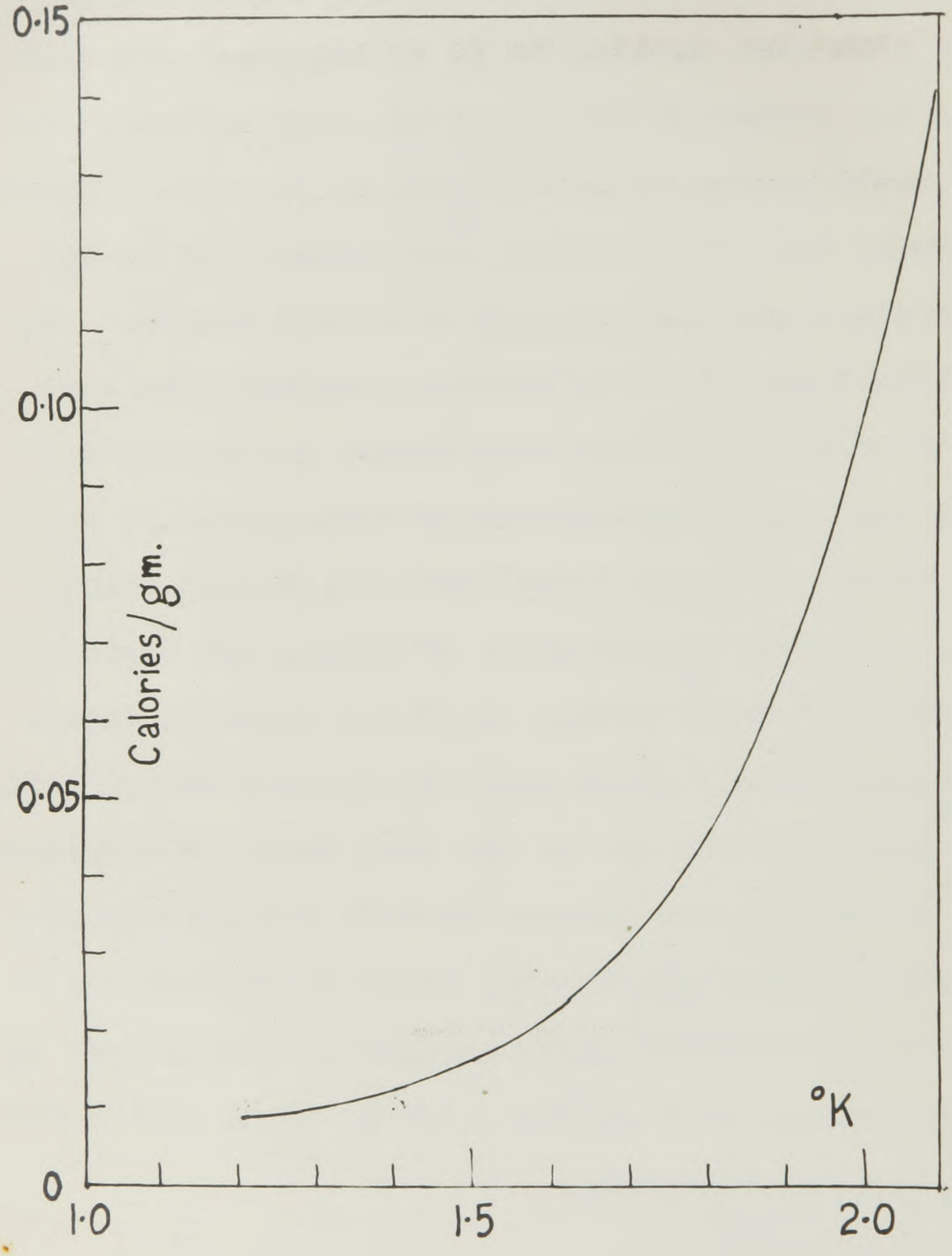


Figure 3-14. The difference (Q-H) as a function of temperature.

carrying no entropy, enters vessel II, and an amount of heat Q must be supplied to it to bring it to a temperature T. This is equivalent to starting with a gram of liquid energetically at the absolute zero of temperature, and warming it up to the temperature T. The amount of heat required to do so is the enthalpy H of the liquid at $T^{\circ}K$. But there is a flaw in this reasoning, for the process considered earlier is isothermal, while in this case the temperature changes from zero to $T^{\circ}K$; the two cannot, therefore, be equivalent.

The difference(Q-H) has been plotted as a function of temperature in figure 3-14. This difference has the interesting property of being approximately proportional to ρ_n/ρ, where ρ_n= density of the normal fluid, and ρ=density of the bulk liquid. Since ρ is nearly constant for the temperature region considered, this means that (Q-H) is proportional to ρ_n. The following table has been prepared using values of ρ_n/ρ given by Landau (1941), which have been verified by the experiments of Andronikashvili (1948).

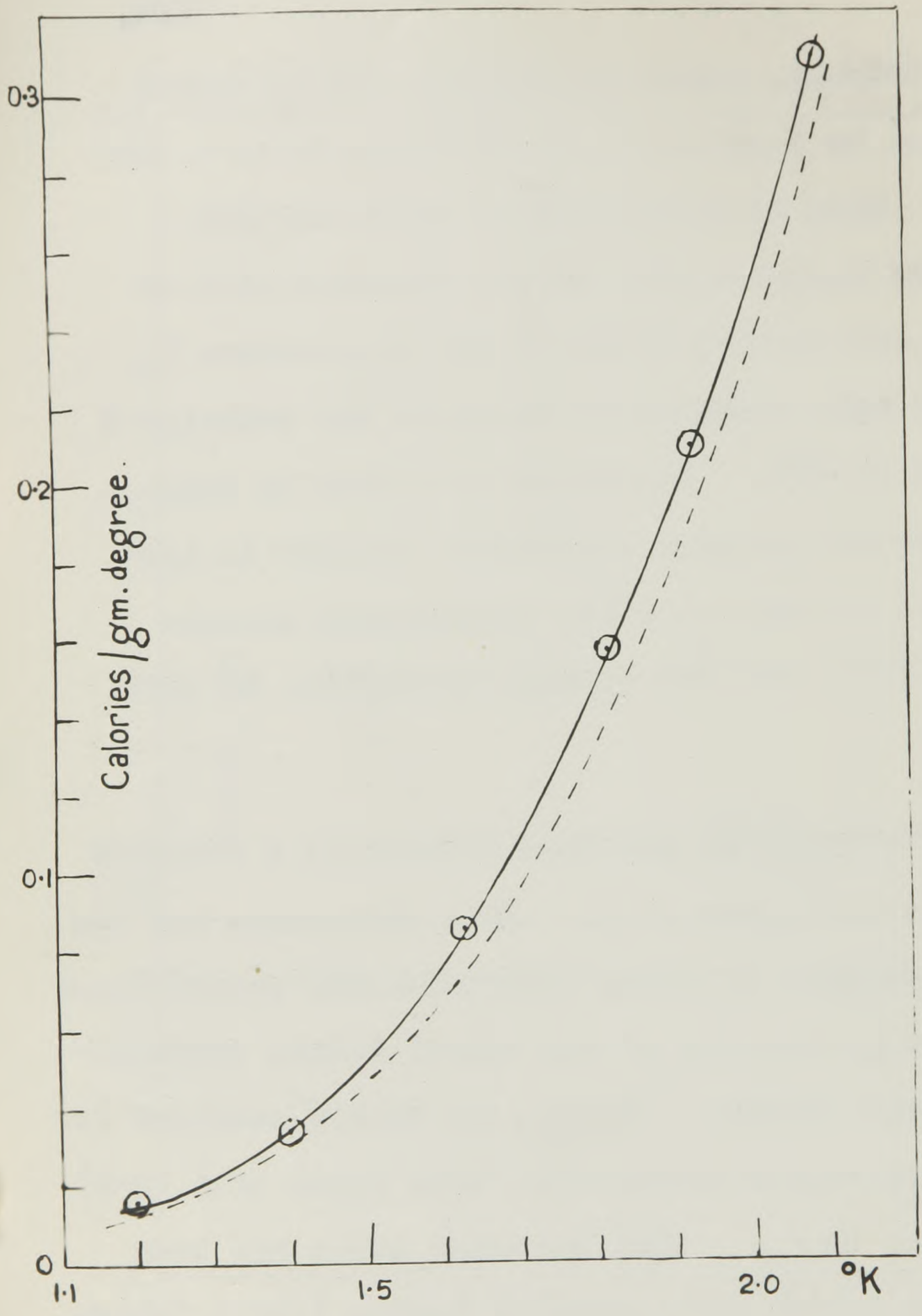


Figure 3-15. The entropy curve for helium II calculated from the present experiments (smooth curve). The broken curve gives the entropy obtained from specific heat data.

$T^\circ K$	(Q-H) cals/gm	ρ_n/ρ	$(Q-H)/(\rho_n/\rho)$ cals/gm
2.1	0.14	0.70	0.20
2.0	0.10	0.50	0.20
1.8	0.046	0.27	0.17
1.6	0.025	0.13	0.19
1.4	0.012	0.06	0.20
1.2	0.007	0.025	0.28

While it is reasonable that the figures in the last column should all be about the same†, it is not possible

---

†Q, H and ρ_n/ρ all have approximately the same power dependence on temperature.

---

to say whether the actual value has any significance. It does seem to represent a constant excess of heat which is associated with the normal fluid in this range of temperatures.

The hypothesis that the entropy of the fluid which moves in the film is zero, is capable of verification on the basis of the experimental results. The entropy S of liquid helium has been calculated from equation (3.808) using the experimentally determined values of Q. The results are shown in figure 3-15 (smooth curve). The broken curve is drawn using the values of S calcu-

lated from the specific heat data (Band-Meyer 1948). There is reasonable agreement between the two sets of values, the difference being nowhere greater than 10%, except at the lowest temperatures. This is not inconsistent with possible experimental error. In particular, specific heat measurements on liquid helium above 1.6°K have not given consistent results, because of experimental difficulties (cf. Hull, Wilkinson and Wilks, 1951). Thus one may conclude that the part of helium which takes part in superflow has effectively zero entropy.

3.9 Summary.

The results obtained in the above experiments are summarised in this section. It has been shown that frictionless transport can take place in the helium II film for a wide range of sub-critical rates. Thus the Gorter-Mellink theory of mutual friction is not valid for these rates. The critical transfer rate observed observed for thermal flow is the same as that obtained during transfer under isothermal conditions. A possible way is indicated of reconciling the differences between film transfer, and flow in fine channels.

The heat of transport, Q, of the helium II film

has determined as a function of the temperature. The difference between this quantity, and H, the heat content of the bulk liquid, is pointed out. This distinction has caused some confusion in the past. It is shown that the entropy carried by the moving film is vanishingly small (the Tisza-London hypothesis), by calculating the entropy S of the bulk liquid from the relation Q = TS, and comparing it with that obtained from specific heat measurements.

The heat of transport for the helium II film, obtained here, agrees with the heat of transport for bulk helium II in fine channels as measured by Kapitza. The significance of this on the two-fluid model is pointed out.

References to Chapter 3

Allen, J.F. & Reekie, J., 1939, Nature, Lond., 144, 475; Proc.Camb.Phil.Soc., 35, 414.

Andronikashvili, E.L., 1948, J.E.T.P., U.S.S.R., 18, 424.

Atkins, K.R., 1948, Nature, Lond., 161, 925; 1950, Proc. Roy.Soc.A, 203, 119, 240; 1952, Adv. in Phys., 1, 169.

Bowers, R. & Mendelssohn, K., 1952, Proc.Roy.Soc. A, 213, 158.

Brown, J.B. & Mendelssohn, K., 1950, Proc.Phys.Soc. A, 63, 1312.

Daunt, J.G. & Mendelssohn, K., 1939a, Proc.Roy.Soc. A, 170, 423, 439; 1939b, Nature, Lond., 143, 719; 1950, Proc.Phys.Soc. A, 63, 1305.

Daunt, J.G., Probst, R.E. & Smith, S.R., 1948, Phys.Rev., 73, 638.

Denbigh, K.G., 1951, Thermodynamics of the Steady State, (Methuen).

Dingle, R.B., 1952, Advances in Physics, 1, 111.

Eastman, 1926, J.Am.Chem.Soc., 48, 1482.

Gorter, C.J. & Mellink, J.H., 1949, Physica, 15, 285.

Hull, R.A., Wilkinson, K.R. & Wilks, J., 1951, Proc.Phys. Soc. A, 64, 379.

Kapitza, P.L., 1941, J.Phys.U.S.S.R., 4, 181; 5, 59.

Henshaw, D.G. & Jackson, L.C., 1951, Proc.Int.Conf. Low Temps.(Oxford), p.65.

Keesom, W.H. & Duyckaerts, G., 1947, Physica, 13, 153.

Keesom, W.H. & Keesom, A.P., 1935, Physica, 2, 557.

Keesom, W.H. & Westmijze, W.K., 1941, Physica, 8, 1044.

Landau, L.D., 1941, J.Phys.U.S.S.R., 5, 71.

London, H., 1939, Proc.Roy.Soc. A, 171, 484.

Mellink, J.H., 1947, Physica, 13, 180.

Mendelssohn, K. & White, 1950, Proc.Phys.Soc.A, 63, 1328.

Meyer, L. & Band, W., 1948, Phys.Rev., 74, 386.

Meyer, L. & Mellink, J.H., 1947, Physica, 13, 197.

Taconis, K.W., Beenakker, J.J.M. & Dokoupil, Z., 1950, Phys.Rev., 78, 171.

Zilsel, P.R., 1950, Phys.Rev., 79, 309.

CHAPTER 4

ISOTHERMAL FORMATION OF BULK LIQUID FROM THE HELIUM II FILM

4.1 Introduction

The observations described in chapter 3 showed that, under certain conditions of favourable temperature gradients, bulk liquid could be formed from the helium II film even at a point higher than the highest liquid meniscus. It was known that, under isothermal conditions, the film could form bulk liquid only at a point lower than the highest meniscus in any system containing liquid helium II. The explanation of the apparent anomaly mentioned above has already been given.

But the assumption that the film could form bulk liquid only below the highest meniscus, while it is necessary to ensure that energy is not created out of nothing, was felt to be an insufficient condition. For a host of other experiments indicated that the maximum velocity with which the film travelled was given by the critical rate. Therefore the helium, when it moved from one point to another in the form of the film, should



Figure 4-1 Special beaker to study formation of bulk liquid from the film.

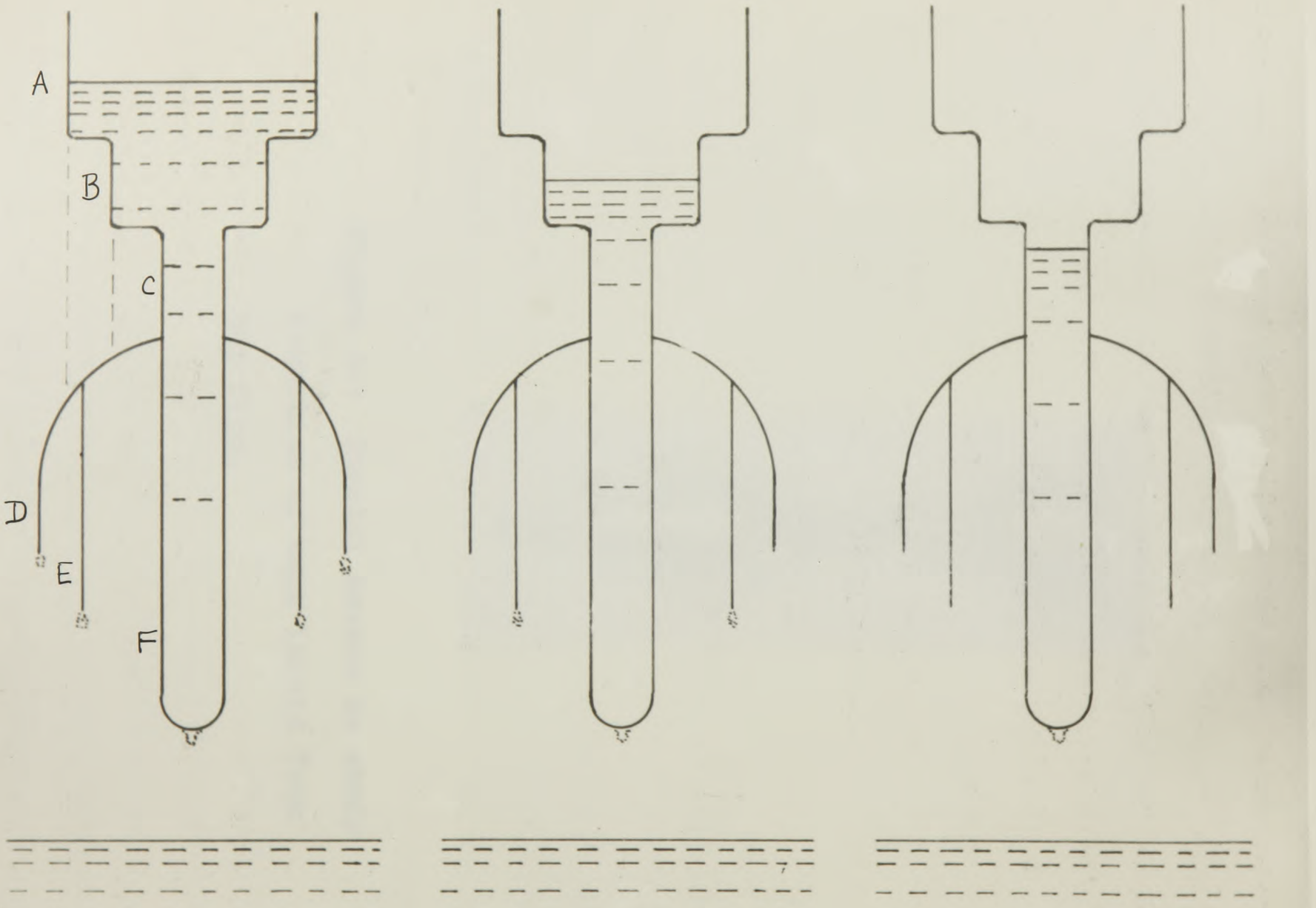


Fig. 4-2

in no case do so with a velocity greater than the critical one. And the critical volume rate is determined entirely, under isothermal conditions, by the smallest perimeter that the film has to pass through above the highest meniscus. It was of interest to see whether bulk liquid could be formed even from the film moving at a sub-critical rate. Therefore a special beaker was constructed, so designed as to make the film flow, below the higher meniscus, over perimeters which were both larger and smaller than the limiting perimeter.

4.2 The Apparatus

The beaker is shown in figures 4-1 and 4-2. It was made of glass, and its diameter decreased in steps from the top downwards, the widest section A being at the top. The glass 'skirt' D was attached to the 'waist' of the beaker, and inside D was another 'skirt' E. The maximum diameter of D was greater than the inside diameter of A, and that of E was between the inside diameters of A and B. D and E each had a glass 'drop' fused to the rim, to help collect any bulk liquid that might form. The actual dimensions of the apparatus, in centimetres, were as follows:

Inside diameters of: A = 1.8, B = 0.8, C = 0.35 ;

Maximum diameters of: D = 2.2, E = 1.4, F = 0.5 .

From these dimensions it is seen that (1) the maximum

perimeter in the flow system occurs in D, (2) the perimeter of E is between those of A and B.

4.3 The Experiments

In the first experiment, the beaker was suspended directly in the helium cryostat. The liquid helium was pumped down to 1.2°K, the beaker was filled by immersion in the bath, and then raised right out of the bath. The level in the beaker began to fall, at a rate which agreed with the known transfer rate on glass, but no drops were observed to form anywhere on the outside of the beaker. What was presumably happening was that, owing to the length of the beaker, the top was slightly warmer than the bath, and the film which crept out of the beaker evaporated entirely near the rim of the beaker. This observation emphasises the importance of providing adequate thermal shielding to apparatus used for film transfer experiments, so as to prevent temperature gradients.

For the next experiment, the beaker was enclosed in a thin-walled copper radiation shield, with two narrow longitudinal slits cut in it to permit observation. The top of the radiation shield was a cylindrical copper can, with a fine hole drilled through the top. The

can filled up with liquid helium through this hole during the liquifaction, and the size of the hole ($\sim$1 mm diameter) ensured that the loss of liquid helium by film transfer was small. The beaker was filled with liquid helium and raised from the bath, which was at a temperature of 1.2°K. The following phenomena were observed:

(1) With the inner level in A, drops were seen to form at all the three points D, E and F, detach themselves and fall into the bath. The rate of formation of these drops at each point remained constant as long as the inner level was in A.

(2) As the inner level passed through the narrowing part of the beaker from A to B, the intervals between successive drops from D increased, and when the inner level entered B, these drops ceased altogether. The drops from E and F continued.

(3) When the inner level entered C, drops ceased to form at E also, and continued to appear at F only.

4.4 Discussion

All these observations become coherent when the basic property of film transfer is applied, that it is possible only at a single critical rate, R say (expressed per unit perimeter). Let the perimeters of the different sections of the beaker be denoted by the corresponding

small letters. Then, when the inner level is in A, the volume of liquid helium flowing out is Ra. It forms bulk liquid on the outside of C, and thence travels to D. Now between D and E it has to move upwards, and it can do this only as a film; and the minimum perimeter it encounters here is e. So only a volume Re can reach E, and the difference R(a-e) drops off at D as bulk liquid. Similarly R(e-f) drops off at E, and the residue Rf at F.

When the level is in B, the helium again forms bulk liquid on the outside of C. But the fact that no drops form from D now, indicates that the liquid re-forms into the film, and travels over D and E. By reasoning as before, one sees that a volume R(b-f) drops off at E, and Rf at F. When the level is in B, the transfer over D is everywhere sub-critical. This is to be linked up with the inability of the film to form bulk liquid at D.

Thus the conditions for the formation of bulk liquid from the film under isothermal conditions are:

(1) The point of formation of the bulk liquid must be below the higher meniscus.

(2) The film must encounter during its motion a perimeter which is smaller than the limiting perimeter which is above the higher meniscus.

(3) Bulk liquid cannot be formed at a point where the

flow is sub-critical*.

---

*After this experiment was done, it was found that Burge (1949), in an elegant series of experiments, had reached identical conclusions, and in addition shown that bulk liquid can always be formed at the lowest point in the film. Therefore conditions (2) and (3) above do not apply at such a point.

It is easily seen that these conditions follow from a definition of the critical rate R as the maximum rate at which film transfer can take place, and from the assumption that under isothermal conditions the entire available surface of the beaker is covered with the helium film, any bulk liquid formed being in addition to this. This latter received a beautiful direct demonstration in the experiments of Henshaw and Jackson (1951). They used a beaker, the diameter of the lower half of which was less than that of the upper half. When the inner level was in the wider half, the lower half was covered with the film with rivulets of bulk liquid super-imposed. This bulk liquid disappeared when the inner level fell to the narrow part of the beaker.

Daunt and Mendelssohn (1939) reported a very interesting case of the formation of bulk liquid from the film.

They used the beaker shown in figure 5-3, for studying the influence of length of path on the transfer rate. With the levels as shown, they observed drops forming at A. But, if the innermost tube was of uniform bore, this would contradict condition (2) above. Their other observations preclude the possibility of temperature gradients. So the only explanation appears to be that the inner tube was actually slightly tapering, with the minimum diameter at its lower end.

References to Chapter 4

Burge, E.J., 1949, Thesis (Bristol).

Daunt, J.G. and Mendelssohn, K., 1939, Proc.Roy.Soc.A, 170, 423,439.

Henshaw, D.G. and Jackson, L.C., 1951, Proc.Int.Conf. Low Temp.Physics (Oxford), p.65.

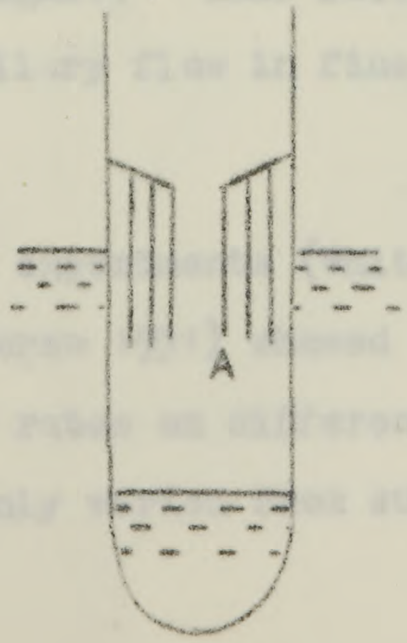


Fig. 5-3

CHAPTER 5

ISOTHERMAL TRANSFER OF THE HELIUM II FILM OVER VARIOUS SURFACES

5.1 Introduction

While the constancy of the transfer rate of the helium II film over glass surfaces may now be taken as a well-established experimental result, the position has not been so clear regarding the transfer over other surfaces, both dielectric and metallic. The earliest observations of transfer on a metallic surface were those of Daunt and Mendelssohn (1939). They found that the rate measured on a copper beaker was the same as that on glass, but that on a bundle of thin copper wires was higher. This latter, they suggested, was due to capillary flow in fine cracks on the surface of the wires.

Recent experiments (White and Mendelssohn 1950, Dash and Boorse 1951) showed a bewildering variety of transfer rates on different metals and dielectrics, which not only varied from substance to substance, but

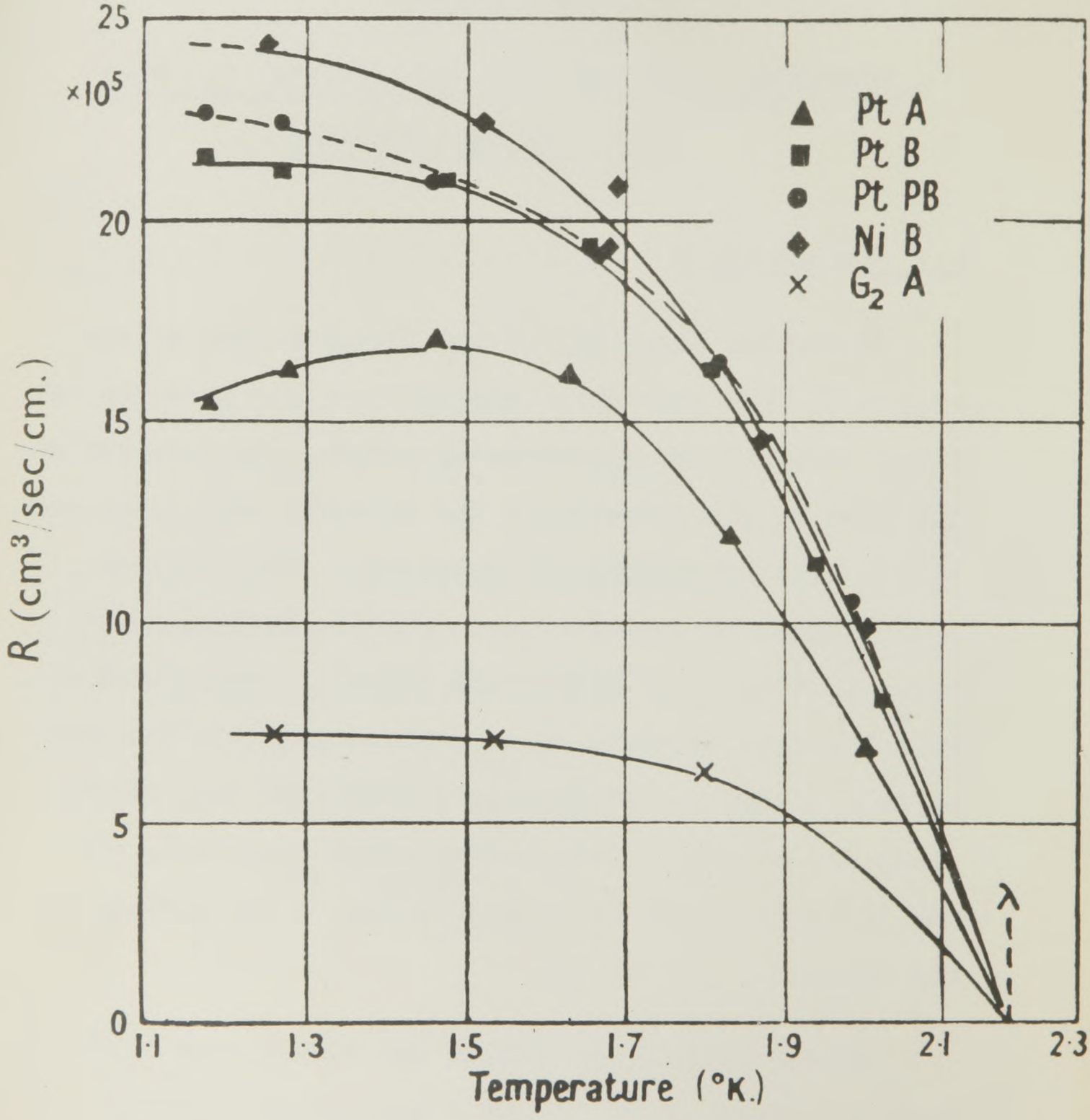

25
×10⁵
20
15
10
5
0
R (cm³/sec/cm.)
Pt A
Pt B
Pt PB
Ni B
G₂ A
λ
1·1
1·3
1·5
1·7
1·9
2·1
2·3
Temperature (°K.)

Fig. 5-1

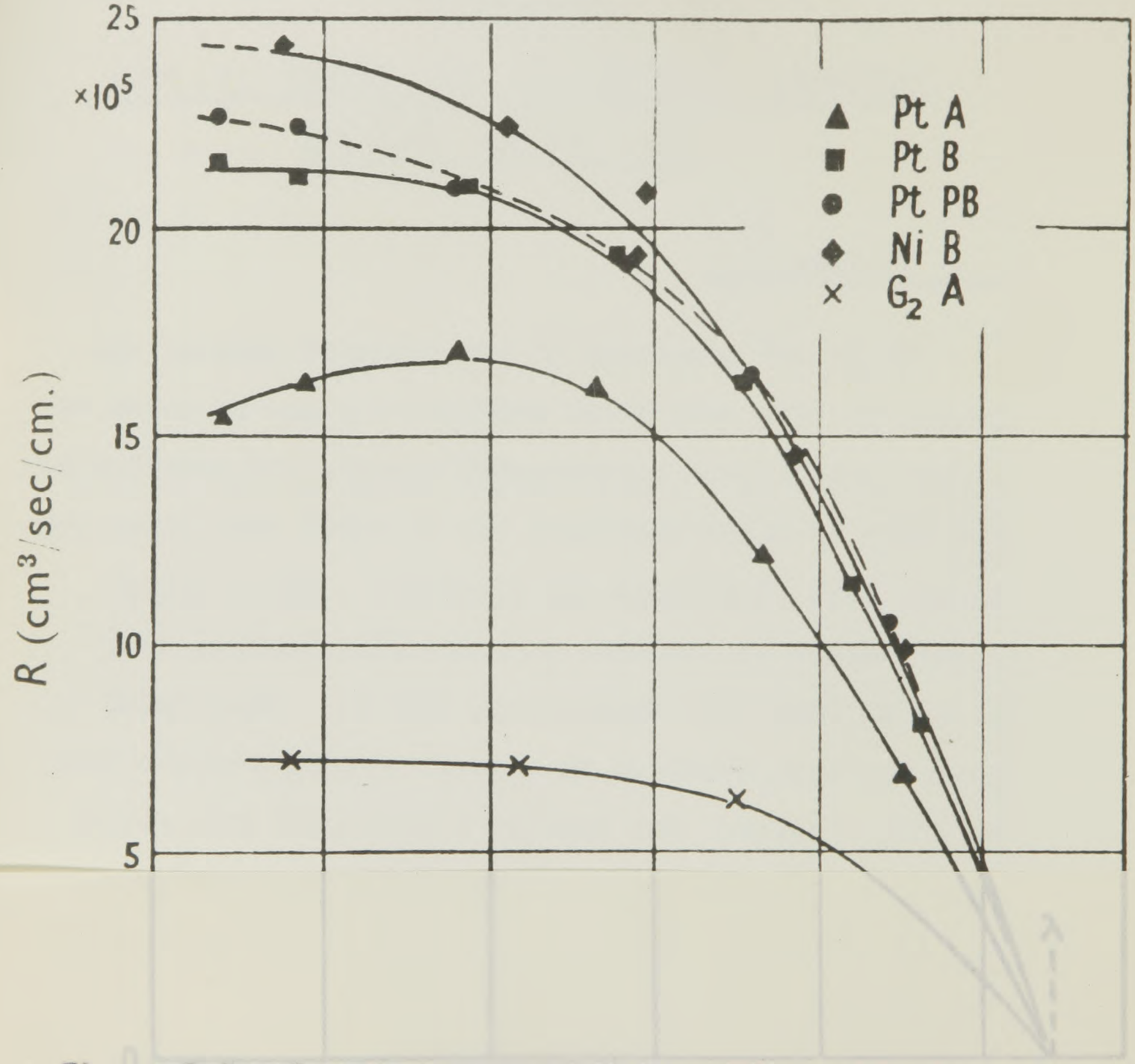
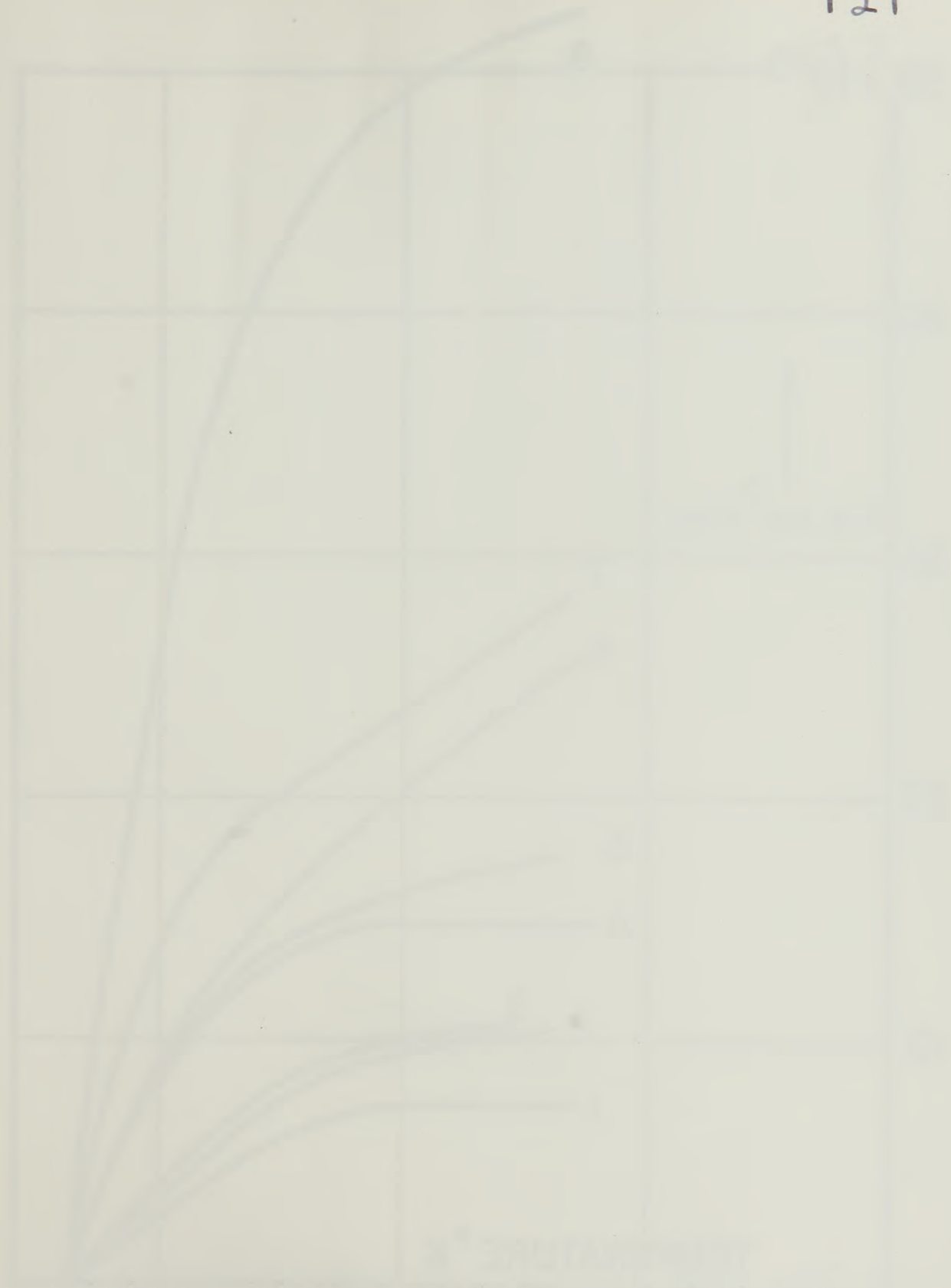


Figure 5-1. Film transfer rates on various surfaces.
PtA: ordinary platinum; PtB: same platinum baked out at 600°C; PtPB: same platinum polished and then baked out at 600°C; Ni B: Nickel baked out at 600°C; G_2A: glass (Mendelssohn & White)



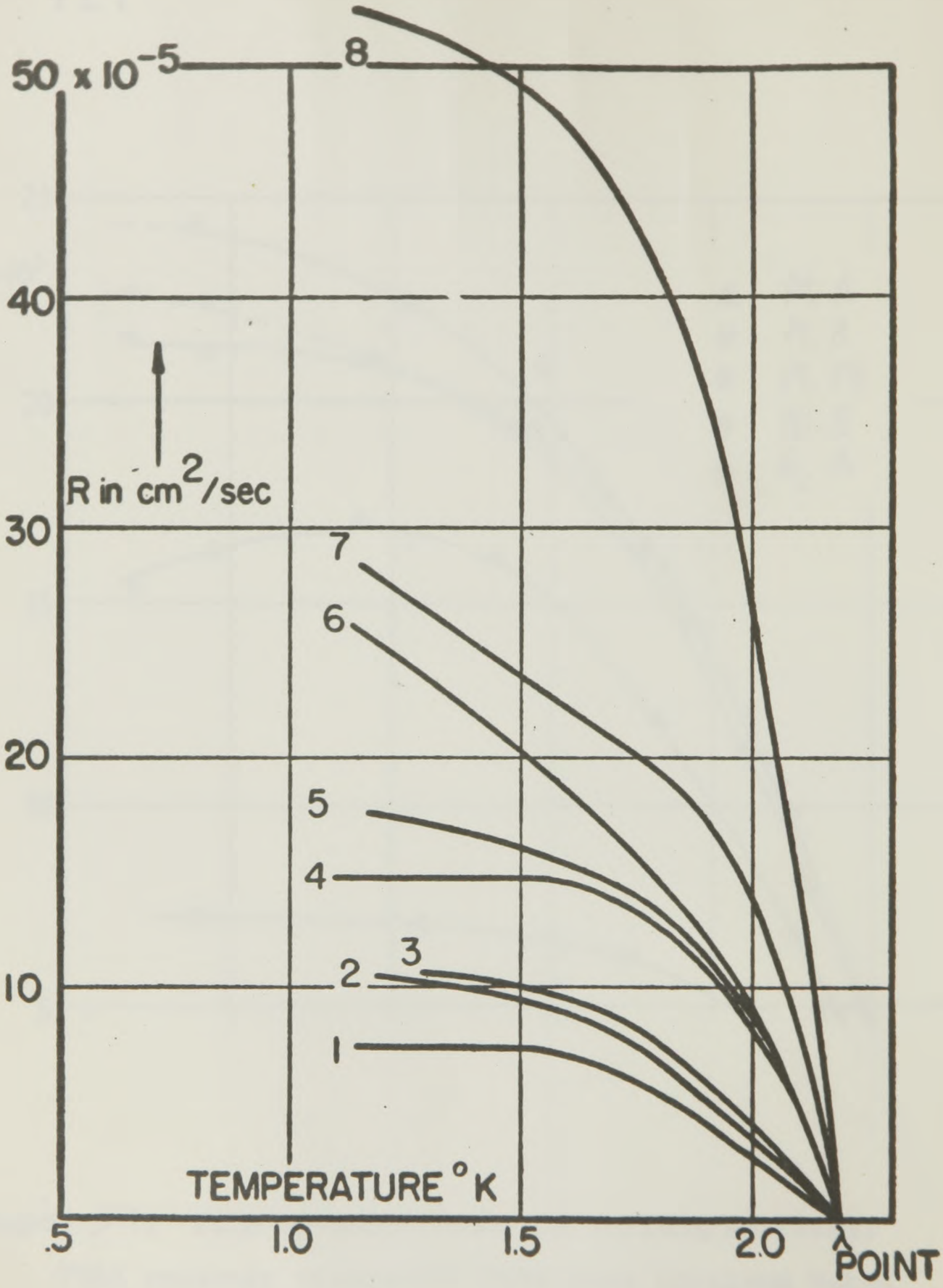

50 x 10⁻⁵
40
30
20
10
R in cm²/sec
8
7
6
5
4
3
2
1
TEMPERATURE °K
.5
1.0
1.5
2.0
λ POINT

Fig. 5-2.

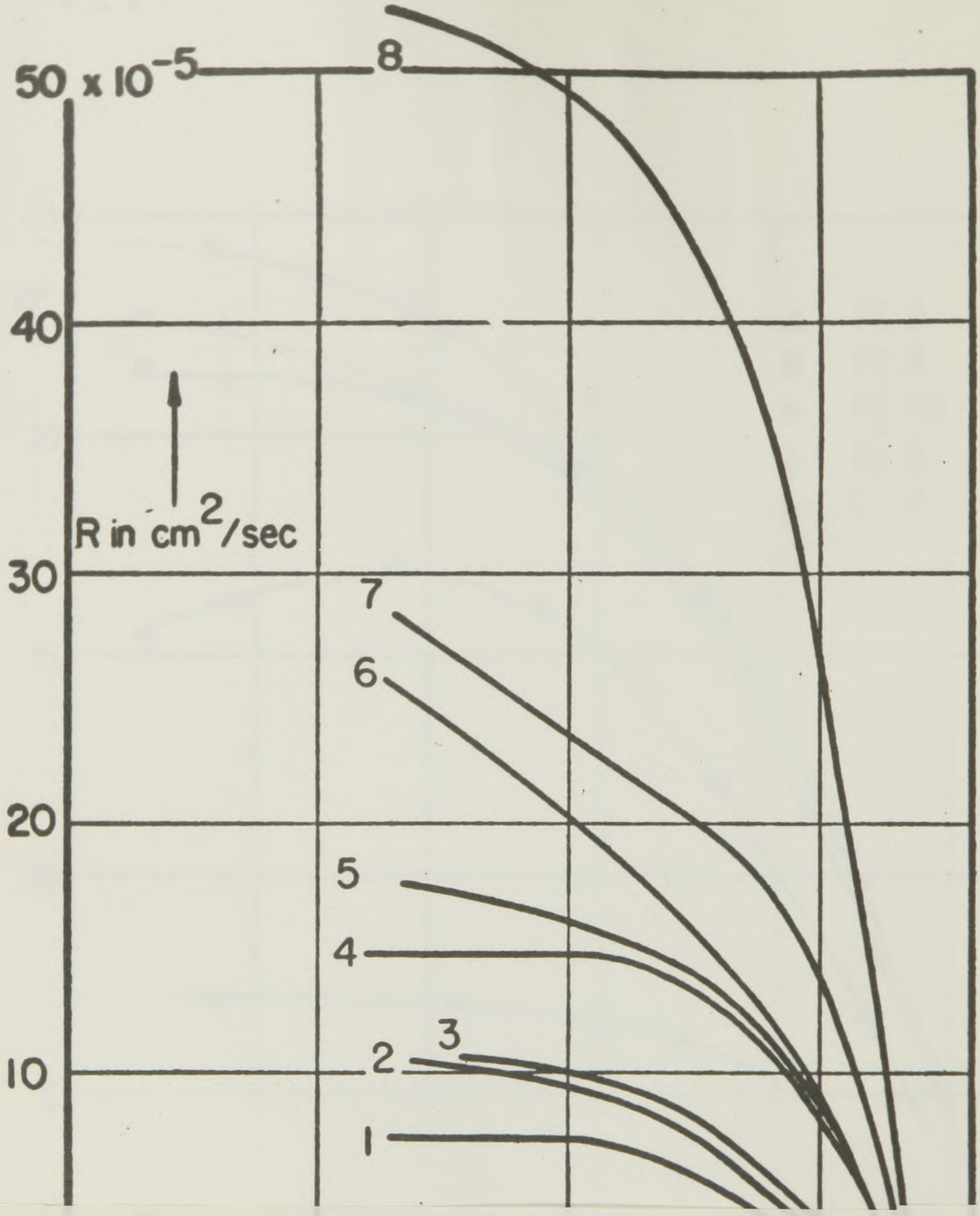


Figure 5-2. Film transfer rates on various surfaces.
(1) glass, (2) stainless steel, (3) iron, (4) machined copper, (5) lead, (6) burnished copper, (7) etched and oxidised copper, (8) etched copper.
(Dash & Boorse).

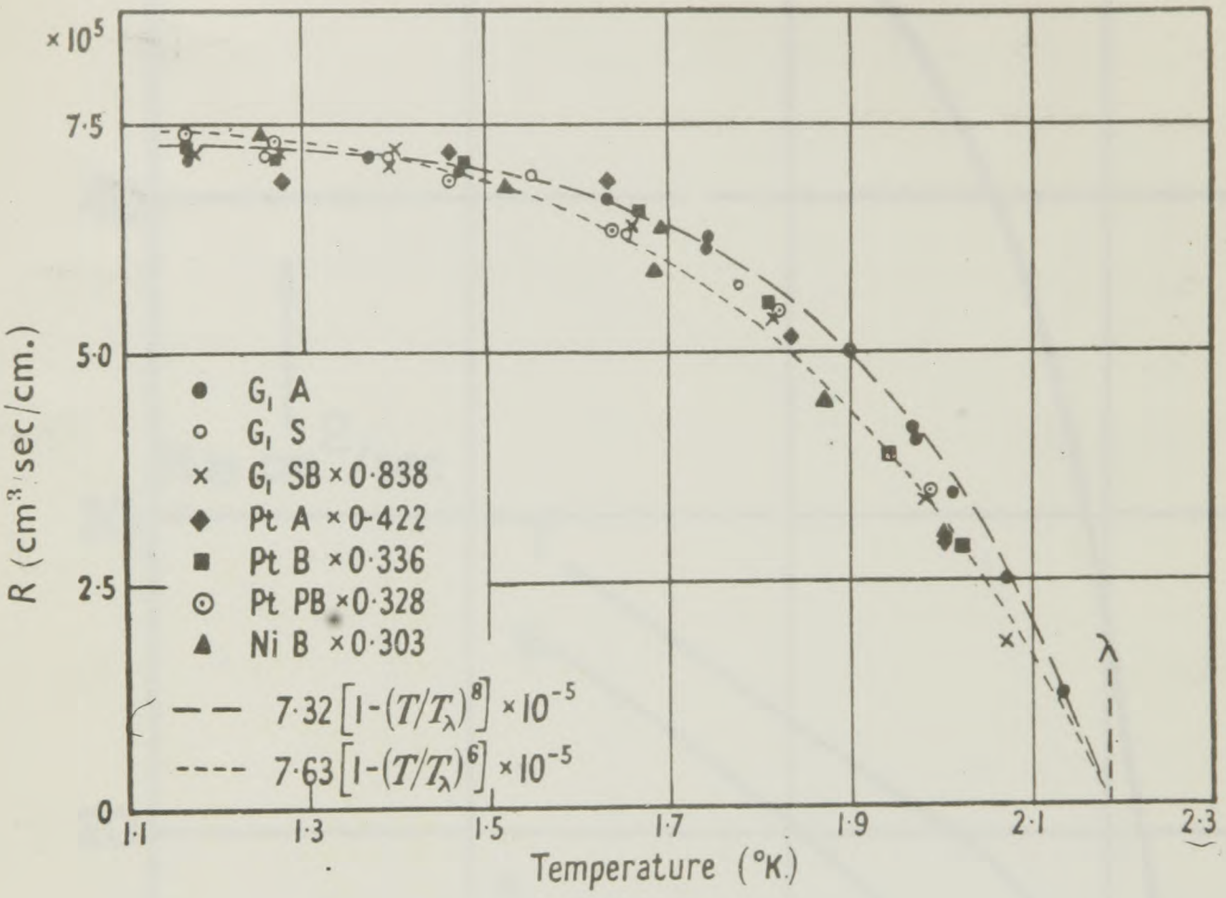

×10⁵
7·5
5·0
2·5
0
R (cm³ sec/cm.)
● G₁ A
○ G₁ S
× G₁ SB × 0·838
◆ Pt A × 0·422
■ Pt B × 0·336
⊙ Pt PB × 0·328
▲ Ni B × 0·303
— — 7·32 [1-(T/Tλ)⁸] ×10⁻⁵
---- 7·63 [1-(T/Tλ)⁶] ×10⁻⁵
λ
1·1
1·3
1·5
1·7
1·9
2·1
2·3
Temperature (°K.)

Fig. 5-3

also, for a single substance, depended on the method of preparation of the surface. Figures 5.1 and 5.2 show some of the results obtained. The following characteristics of these transfer rates deserve notice: (1) The rate on clean glass surfaces seems to be the lowest observed, with the exception of one case which is mentioned below. The curves of transfer rate versus temperature for all the other substances lie above that for glass. (2) Some of the curves, e.g. those for PtA (figure 5.1) and lucite (Dash and Boorse 1951), rise to a more or less pronounced maximum at about 1.5°K, and then fall at lower temperatures. Some again are continuing to rise at the lowest temperatures reached. But the vast majority of the curves seem to have a dependence on temperature which is the same as that of the transfer rate on glass. This point is clearly illustrated in figure 5.3 (White and Mendelssohn 1951), which shows the various rates normalised to agree with that on glass at 1.4°K. The exception to (1) mentioned above is the rate on solid hydrogen (van den Berg 1951), which was found to be 5.0×10^{-5} cm^3/cm.sec at the lowest temperature used, compared to 7.5×10^{-5} cm^3/cm.sec for glass at the same temperature. As the experimental details have not yet been published, it is not possible to comment on this result.

There are two possible explanations for the different transfer rates observed on these substances. The rate may depend on the chemical nature of the substrate, being influenced by the van der Waals forces between the helium atoms and the atoms in the substrate. Schiff (1941) gave a theory of the helium II film, considering the equilibrium of a helium atom acted on by van der Waals forces due to the other helium atoms, and the atoms in the substrate, as well as the gravitational field. He deduced the following formula for the variation of film thickness T with height H above the bulk liquid:

$$T = (K/H^3)^{\frac{1}{3}} \; 10^{-6} \text{ cm}$$

where H is in centimetres, and K is a constant for a given substrate. He gave some typical values for K: copper 4.3, silver 4.7 and glass about 4. It is interesting to note that this suggested variation of K with the chemical nature of the substrate is very much less than the observed variation of the transfer rate. But Schiff did not extend the above ideas to give a theory of the critical transfer rate, so that one cannot use the above formula for T to predict the transfer rate.

Temperley (1949) has calculated in detail the wave function of an atom in the film, taking account of the

various van der Waals interactions and the gravitational field. Discussing the critical rates, he says that most of the atoms which take part in the transfer must be in the outermost layers of the film, for the potential barriers between adjacent adsorption sites would decrease as the distance from the substrate increased; thus the transition probability for a helium atom to move from one site to a nearby vacant one would increase outwards from the substrate. On the other hand, he also shows that the van der Waals forces between the substrate and the helium atoms, by themselves, cannot support a film more than 0.1 times the thickness actually observed. Thus the distribution of the outer adsorption sites, and the energies associated with them, would be determined almost entirely by the forces between the helium atoms. This makes it unlikely, on this theory, that the critical rate of transport would be much influenced by the forces between the substrate and the atoms.

Thus neither of the existing theories of the film based on van der Waals interactions gives an indication that the substrate would influence the critical transfer rate. Thus, if the latter were actually the case, a new theory would be required, taking account not only of these interactions, but also other factors which are not immediately apparent.

There are, on the other hand, a group of theories (Bijl, de Boer and Michels 1941, Mendelssohn 1945,1947, Ginsburg 1950, London 1951) which, while differing in other details, have one common characteristic in dealing with the critical rate of transport. They all use a quantum mechanical argument, involving the principle of indeterminacy. From this a relation connecting v_c, the critical velocity, with d, the width of the flow channel (which would be the thickness of the film in the present case) of the following form is deduced:

$$v_c \sim \frac{h}{md}$$

But nowhere in the deduction is any assumption about the forces between the substrate and the atoms in the film introduced. We see therefore that none of the present theories of the film leads us to expect such forces to influence the critical rate.

We have, for this reason, and other detailed below preferred an alternative explanation of the anomalous transfer rates. This assumes that the true critical rate, expressed per unit perimeter, is a constant over all surfaces, provided the perimeter takes account of the microscopic structure of the substrate, at least

down to the order of magnitude of the thickness of the film. The micro-structure of the surface can affect the transfer rate by two different mechanisms, depending on the size and shape of the irregularities. Firstly, the actual perimeter for film transfer may be increased, for the film can 'see' the irregularities on the surface which escape macroscopic examination. Secondly, surface channels and cracks could give rise to ordinary capillary flow, which would be superimposed on the true film transfer, but which may, in general, have a different temperature difference (Allen and Misener 1939). If the first mechanism is effective on any given surface, then the rate would be larger than the rate on glass, but would have the same temperature dependence. But if bulk flow were possible because of cracks and narrow channels on the surface, the rate would not only be larger than the rate for glass, but would also, in general, have a dependence on temperature which need not be the same as for glass. These two possibilities therefore cover all the anomalous rates reported so far. It is likely that, in practice, both processes occur simultaneously.

5.2 Experiments with Specially Polished Stainless Steel Beakers

5.21 An opportunity to test our hypothesis arose when the Metrology Division of the National Physical Laboratory, Teddington, offered to prepare stainless steel beakers with a degree of surface polish better than any that had obtained on metallic surfaces used in earlier experiments. Previous determinations of the transfer rate on stainless steel had yielded varying results. Dash and Boorse (1951) reported a rate of about 10.5×10^{-5} $cm^3/cm.sec$*, while Henshaw and Jackson (1951) gave

*When the transfer rate is given as above, it refers to the rate at about 1.3°K, i.e. the value corresponding to the nearly horizontal part of the curve of transfer rate versus temperature.

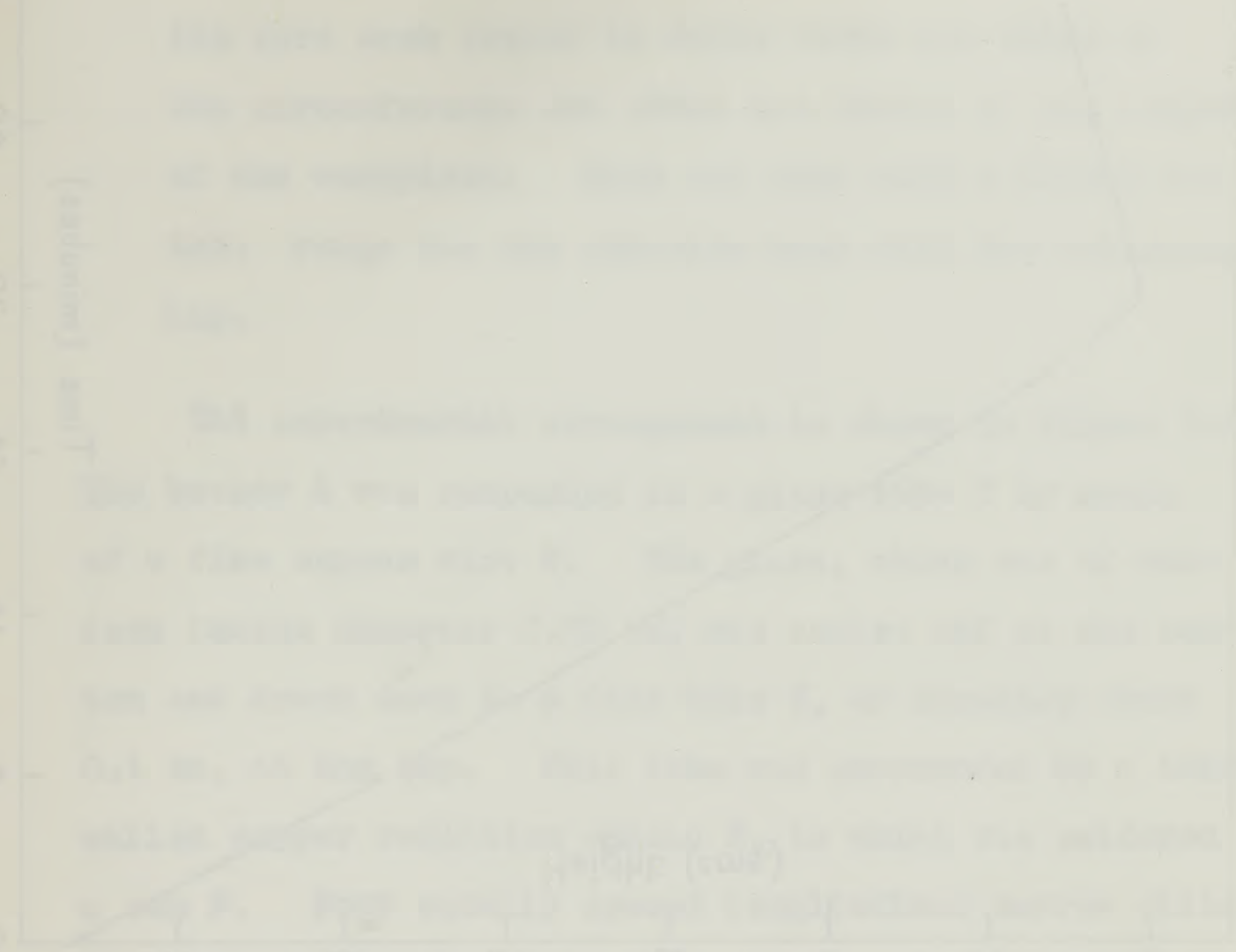
17.4×10^{-5} $cm^3/cm.sec$. The former result was obtained using an electrical method of measuring the transfer rate, out of a machined beaker of 18-8 stainless steel, while the latter was obtained by visual observation on a beaker details of which are not given. Both investigations were conducted with due precautions against the formation of condensed gas layers on the flow surfaces (Bowers and Mendelssohn 1950). It is thus difficult to see why there should be such a difference between the two sets of measurements, unless it were due to differences in the micro-

structure of the surfaces.

5.22 Experiments and Results

Two polished beakers of stainless steel were used in these experiments. They were identical in size, each being 6 cms long, and having inside and outside diameters of 0.58 and 0.63 cms respectively. Only the outside surface was specially polished, the inside being the usual turned finish. The National Physical Laboratory has kindly supplied the following description of the process of manufacture:

> Each beaker was made from 3/8 in. solid stainless steel rod (Staybrite F.S.T.) which was first drilled to produce the inside, then cut off to length and a centre formed at the closed end. A slightly tapered mandrel, centred on its outer end, was made to fit the bore, so making it possible to hold the workpiece, with the mandrel inserted, between centres. The outside diameter, after turning, was fine ground and then worked in a longitudinal direction with two natural stones (Dalmore yellow, and Water of Ayr) in succession, and finally polished with a pitch lap in in the same direction. The work was rotated slowly during each of these three stages. The stones and



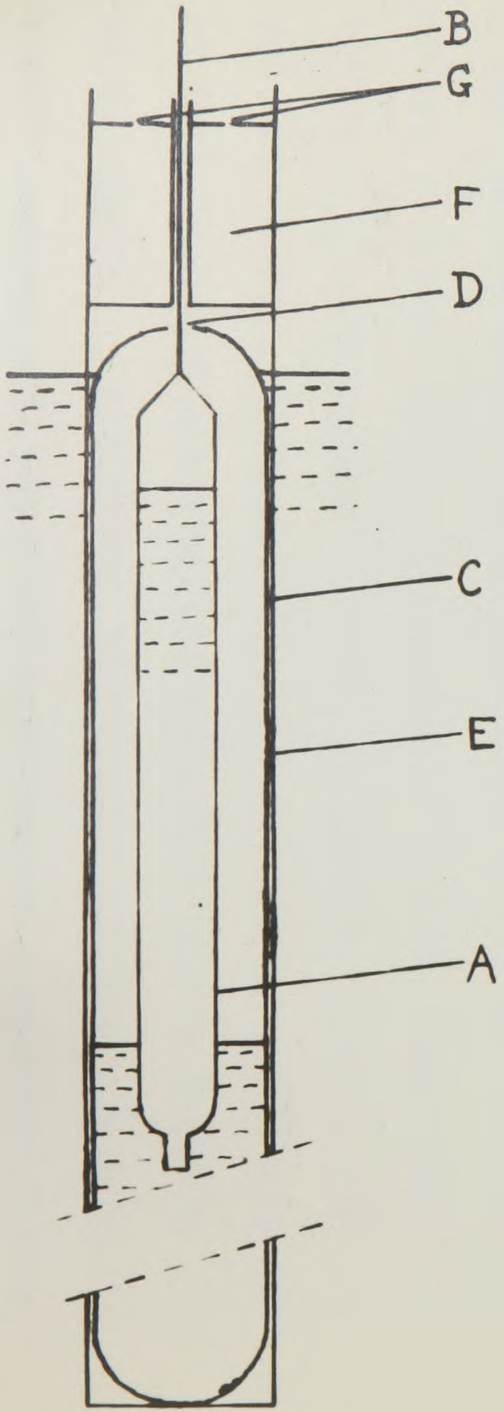


Fig. 5-4

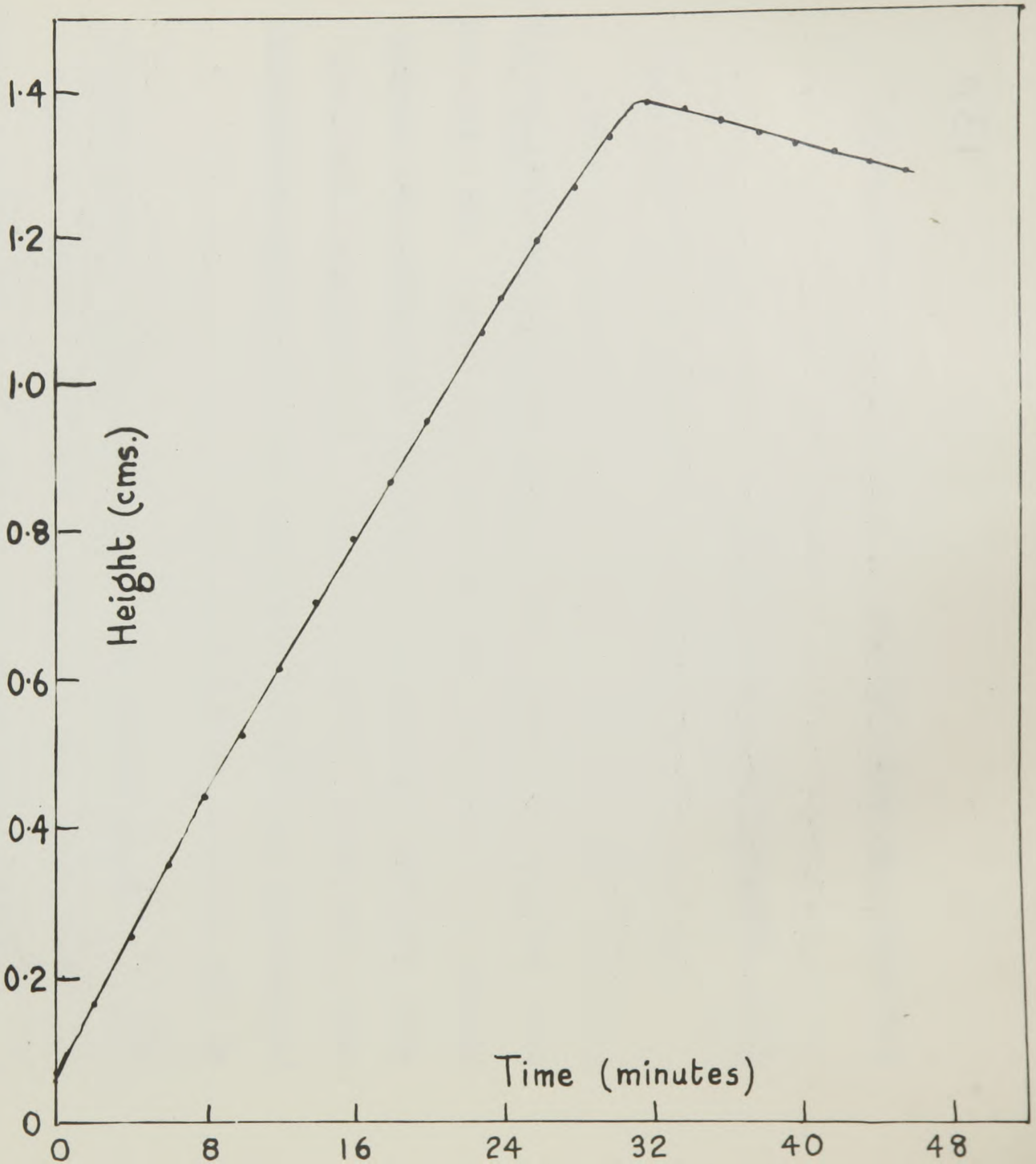


Fig. 5-5 Variation of level in C with time

lap were each shaped to cover about one third of the circumference and about two thirds of the length of the workpiece. Each was used with a little water; rouge was the abrasive used with the polishing lap.

The experimental arrangement is shown in figure 5.4. The beaker A was suspended in a glass tube C by means of a fine copper wire B. The glass, which was of uniform inside diameter 0.80 cm, was sealed off at the bottom and drawn down to a fine hole D, of diameter about 0.1 cm, at the top. This tube was surrounded by a thin-walled copper radiation shield E, to which was soldered a can F. Four equally spaced longitudinal narrow slits were cut in the shield, to permit observation of the liquid level in C on either side of the beaker. The can F was in the shape of a hollow annulus, and the wire supporting the beaker passed down its centre. Two fine holes G were drilled in the top of the can, so that it filled up with liquid helium during the liquefaction, and minimised the influx of heat from above into the beaker during the experiments.

The experiments consisted essentially in deducing the rate of transfer of the film from the beaker, by observing the rate at which the level in the glass tube C rose (or fell)

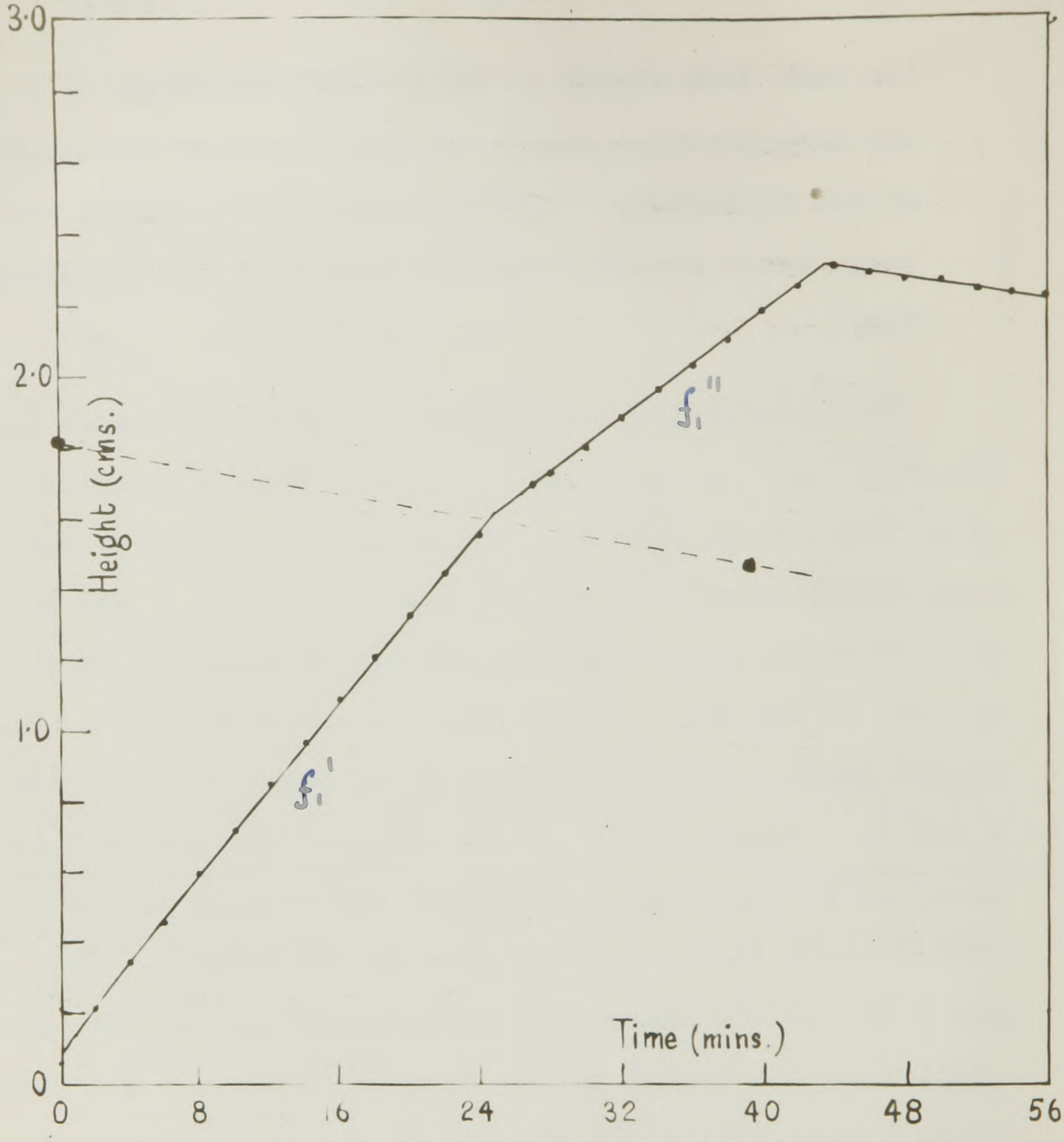


Fig. 5-6 : Smooth line : level in C.
Broken line : bath level

The relative dimensions of the beaker and the glass tube are critical in determining the accuracy with which the rates can be measured. For, if the annular area between the beaker and the tube were made small so as to give a large change of level for given flow, only a small length of the beaker would be actually available for the transfer, since the levels inside and outside the beaker would soon equalise. Also, any irregularities in the bore of the glass tube would influence the rate of change of the level in it considerably. But if, on the other hand, this annular area were large compared to the area of cross-section of the beaker, the advantage of having a large length of the beaker available is offset by the fact that the rate of rise of the level in C is small, and subject to greater error in determination. A simple geometrical consideration shows that the optimum condition is to have the annular area approximately equal to (preferably slightly larger than) the inside area of cross-section of the tube. The glass tube C was chosen to satisfy this criterion.

If the bath level and the level in C are not the same, there is film transfer between the two through the hole D which is not altogether negligible compared to that between C and the beaker. This is clearly evident in figures 5-5 and 5-6 which show the behaviour of the various

liquid levels in two typical cases. Figure 5-5 corresponds to a case where the bath level has been above the other two levels during the entire experiment. After these two levels come together, they continue to move up towards the bath level, because of film transfer through D. Similarly, if the bath level had been below, the two levels would have moved down towards it. In both cases the direction of transfer between C and the bath remains unchanged. In figure 5-6, the C level crosses the bath level on its way to the beaker level, and at this point the direction of transfer between the bath and C is reversed, so that the rate of change of the C level itself suffers an abrupt change. By observing the rate at which the C level moves after it has joined up with the level in the beaker, we can calculate the correction due to transfer between the beaker and the bath as follows:

The rate of evaporation of helium from the liquid surfaces inside C is negligible compared to the transfer rates, so that all observed changes of level can be considered to be entirely due to film transfer. Let f_1 be the rate of change of the C level before it meets the beaker level, and f_2 after. Let s_b and s_c be the cross-sectional areas inside the beaker and of the annular space between it and C. Then

Volume rate of transfer between C and the bath = $f_2(s_b+s_c)$



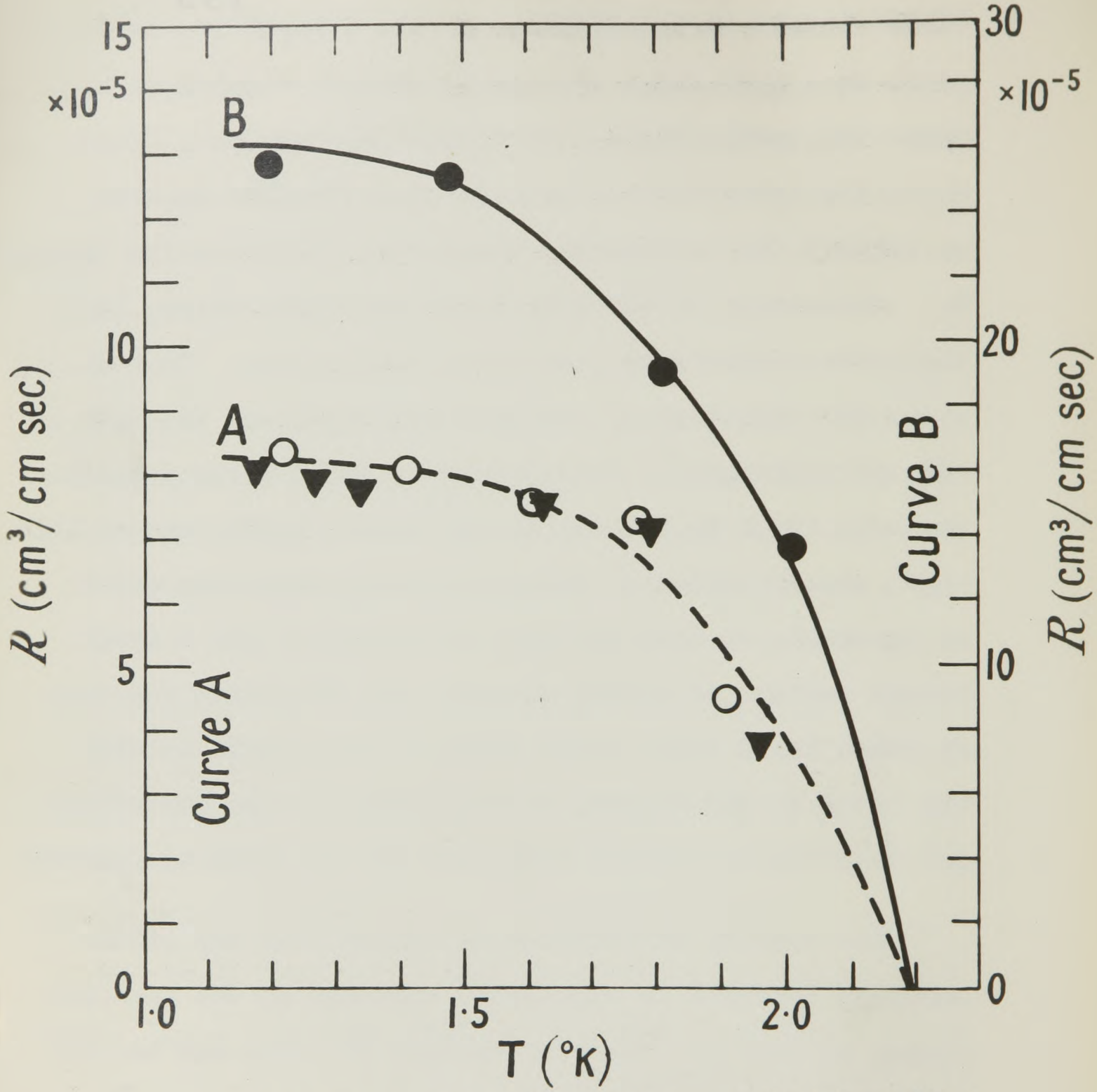


Fig. 5-7. Transfer Rates on Stainless Steel

Curve A : polished surface.

Curve B : after heat treatment.

This causes a rate of change of the C level of $f_2 \frac{(S_L+S_c)}{S_c}$
Therefore the corrected rate of change of the C level, caused only by film transfer with the beaker= $f_1 + f_2 \frac{S_L+S_c}{S_c}$
The sign is easily determined by considering the relative positions of the three levels during the period for which f_1 is measured. Thus, in figure 5-5 the negative sign is to be taken, while, in figure 5-6, the negative sign is taken with f_1', and the positive sign with f_1''. These corrections have been applied to all the results given below. If P is the outside perimeter of the beaker, then the transfer rate per unit perimeter for stainless steel is given by

$$\frac{S_c}{P}\left[f_1 + f_2\left(\frac{S_L+S_c}{S_c}\right)\right]$$

The validity of taking the outside perimeter as the limiting perimeter is discussed later.

The transfer rates for the two beakers (designated A and B)were determined at various temperatures below the λ-point, and the results are shown in figure 5-7. The dotted curve was obtained by Mendelssohn and White for baked-out glass.

After these experiments had been completed, the surface polish of beaker B was destroyed by heating it to just red heat, and immediately quenching it in water. The surface, which before had a mirror finish, was now

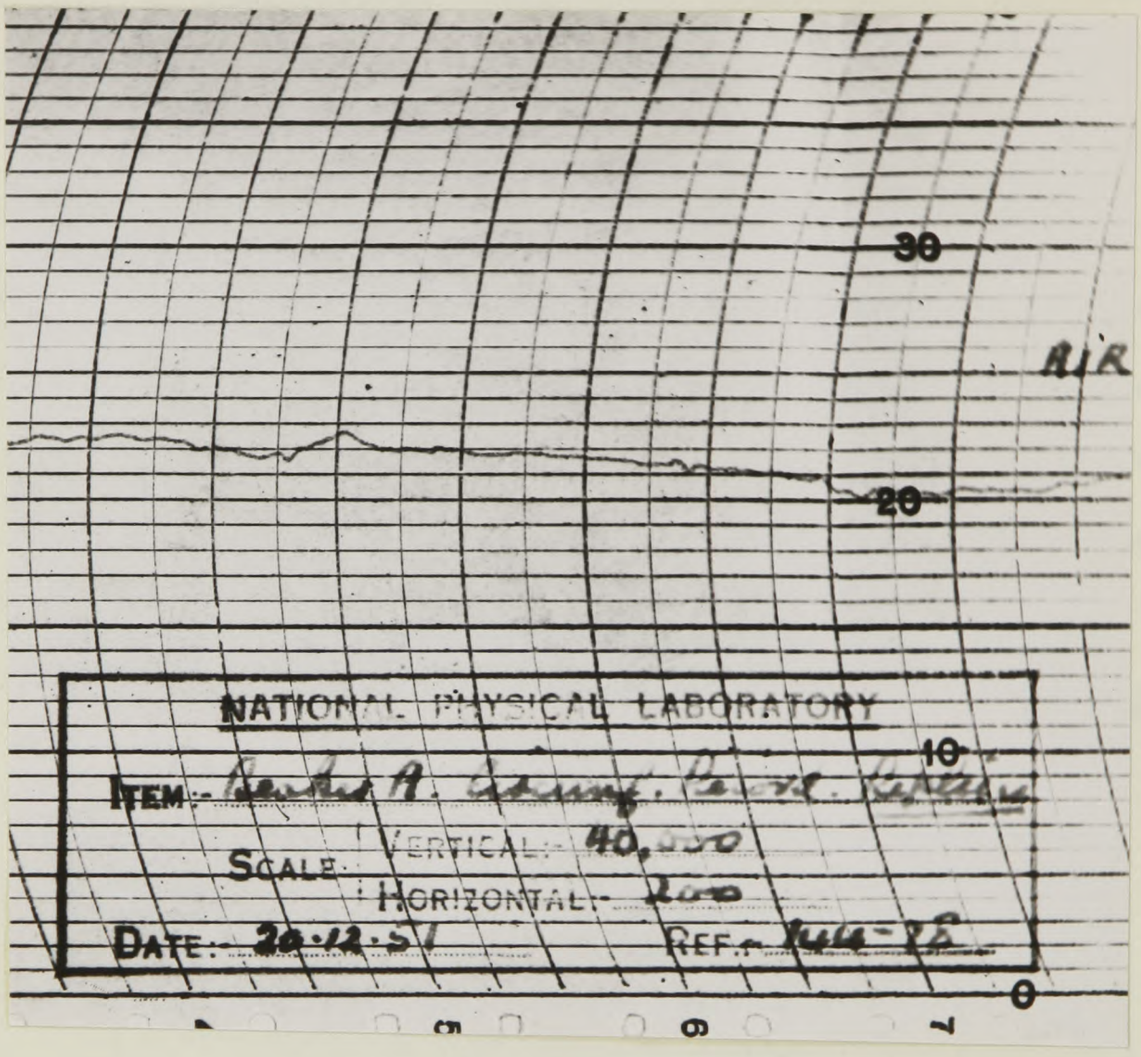


Fig.5-8. Talysurf chart of the polished surface of beaker A in a circumferential direction.

One horizontal grid division = 3.2×10^{-3} cm.

One vertical grid division = 9.05×10^{-6} cm.

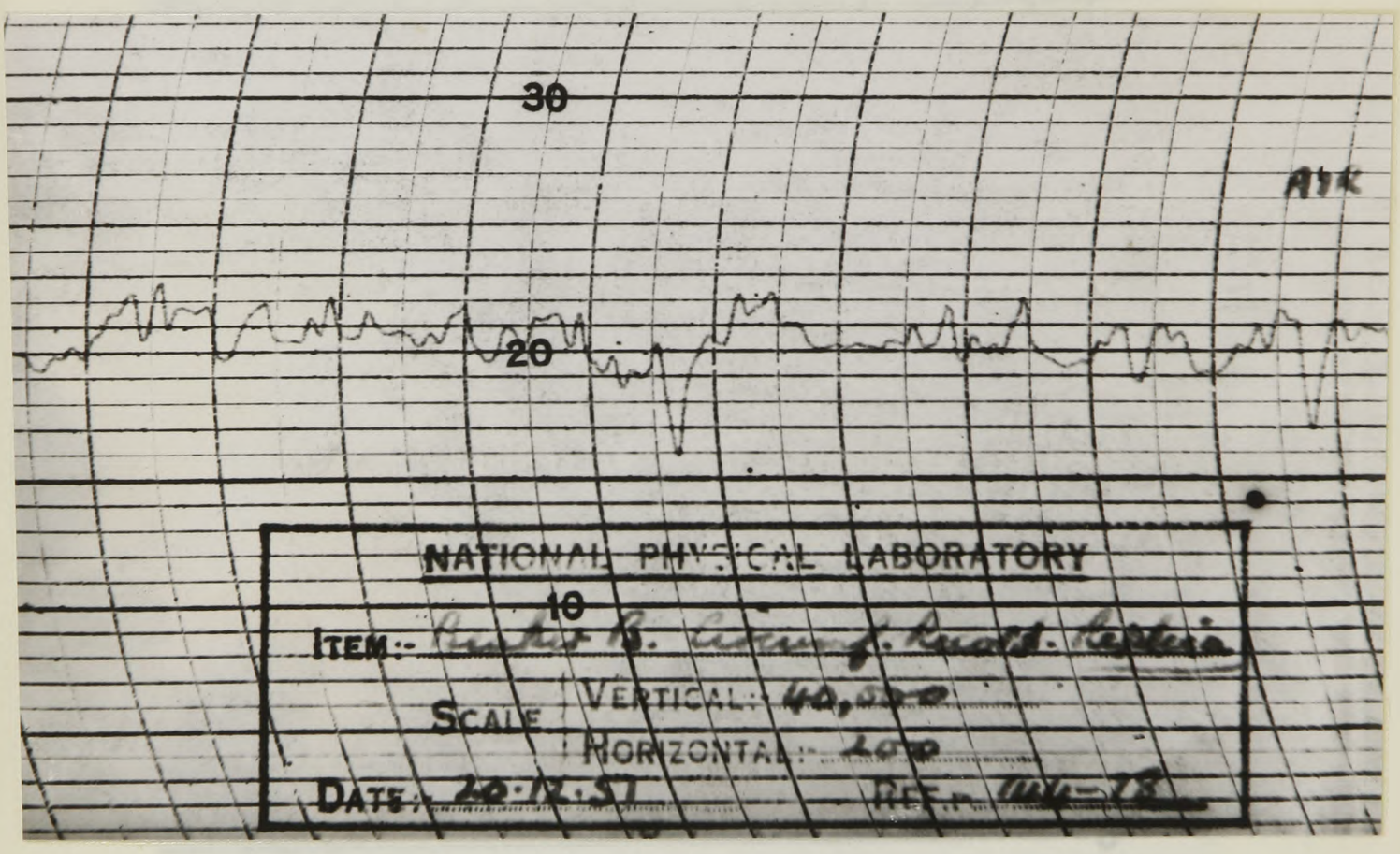


Fig. 5-9. Talysurf chart of the surface of beaker B, in a circumferential direction, after its surface finish had been destroyed by heating.

One horizontal grid division = 3.2×10^{-3} cm.

One vertical grid division = 9.05×10^{-6} cm.



Figure 5-10 Linnik photomicrograph of beaker A (polished surface). Magnification x600.





Figure 5-11 Linnik interference photograph of beaker A. Fringes in circumferential direction; magnification x600



Figure 5-12 Linnik photomicrograph of beaker B (after heat treatment) Magnification x600.

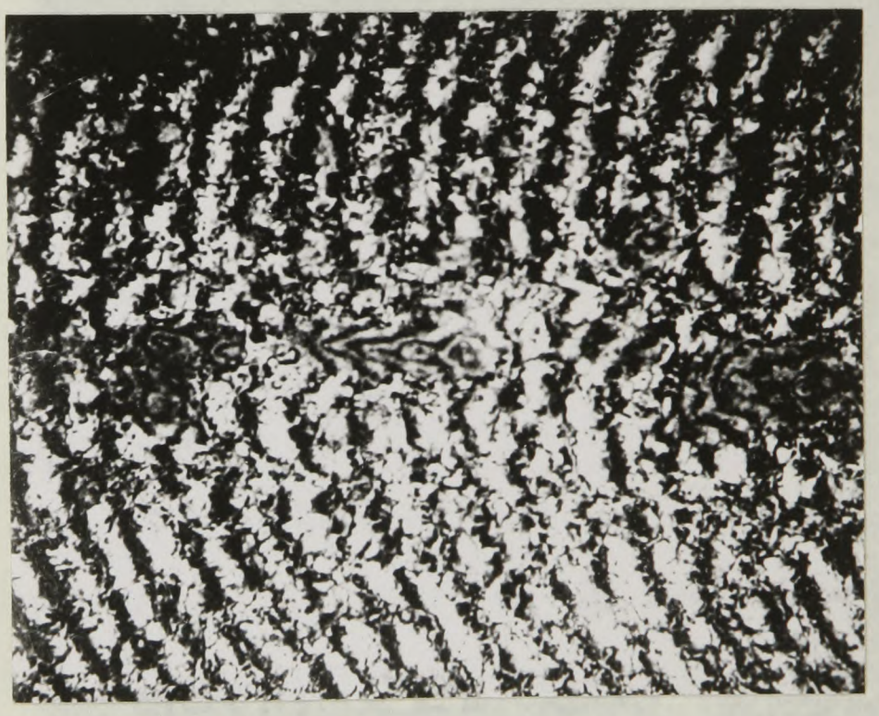


Figure 5-13 Linnik interference photomicrograph of beaker B (after heat treatment) Magn. x600 Fringes in circumferential direction.

matt. It was polished with cotton wool, though actually there was no visible formation of scale. It was then replaced in the apparatus, and the transfer rates again determined, giving curve B (figure 5-7). It is seen that the rate has increased by a factor of more than 3 over those for the polished beakers.

5.23 Discussion

We shall first consider the reasons for defining the outer perimeter of the polished beakers as the limiting perimeter for film transfer. The outside (polished) surface of beaker A, as well as the outside of beaker B after its finish had been destroyed, were examined by the Metrology Department of the National Physical Laboratory with the Talysurf instrument and the Linnik microscope. The results are shown in figures 5-8 to 5-13.

The Talysurf instrument traces the contour of a surface along a given direction by means of a fine diamond probe which is drawn along the surface in a controlled manner. It is not possible to measure the surface finish along a circumferential direction in the Talysurf instrument when the radius of curvature of the surface is as small as 0.15 in., and direct records of the finish were accordingly made only in an axial direction. The charts

for the circumferential direction were obtained from plastic replicas which were later opened out flat and the surface finish then recorded in the usual manner. Any major deviations from linearity in these latter are most probably due to this replicating process and should be disregarded*.

---

*This information was kindly supplied by the National Physical Laboratory.

A short section of each Talysurf chart has been reproduced in the diagrams. The points to notice about the charts are (1) the smoothness of the surface , to better than 10^{-5} cm, of the polished beaker, and (2) the remarkable change in the surface smoothness as a result of heating. A direct quantitative determination of the increase in perimeter due to heating is not feasible, because the magnification in the direction of the probe traverse is small compared to that in the direction at right angles. Also, such a determination, even if possible, might not be of great significance, for it is likely that capillary flow in cracks and interstices would be a major contribution to the increased transfer observed. The main conclusion to be drawn from these charts is the great increase in roughness on heating. The insides of the beakers, which had a turned finish, were certain to be very much rougher than the polished outside, so that

it seems reasonable to assume that the outside perimeter is the limiting one.

There is another argument in favour of this assumption which is, however, a negative one. For even if the inner perimeter were the limiting one, the rate would only be increased to 8.9×10^{-5} cm³/cm.sec., and this is still inexplicably less than the rates on stainless steel reported earlier—barring the one possible explanation which we have to fall back on, namely that the smoothness of the outer surface makes it the limiting perimeter.

The Linnik photomicrographs (figures 5-10 and 5-12) show that recrystallisation has occured in the stainless steel after heat treatment. It is easy to see how this would provide a large number of cracks for capillary flow. There are still visible, on the photomicrograph (5-10) of the polished surface, lines which may be minute draw-marks, made during the polishing. These may explain why the transfer rate even for the polished beaker is slightly higher than the lowest rate reported on glass ($\sim 7.5 \times 10^{-5}$ cm³/cm.sec.). It has already been suggested by Mendelssohn and White that the slight increase in the transfer rate on glass produced by baking it out at 600°C may be due to some surface cracks opening.

The Linnik interference photomicrographs show in an

even more striking manner the change in the surface produced by heat treatment. The interference is produced by reflections from the surface of the beaker and a flat reference surface, adjustments being provided in the interference microscope to enable the fringes to be orientated in any desired direction. These fringes effectively form height contours at about 2.5×10^{-5} cm. intervals. The photograph of the polished surface (5-11) clearly shows, when considered along with the Talysurf charts, that this surface had very few irregularities even as small as 10^{-5} cm. Its surface must have therefore approximated more closely to that of a glass surface than any other metallic surface investigated before.

From figure 5-7, we see that the rates on the two beakers (before heat treatment) agree with each other, and are approximately the same as that for glass which had been baked out at 600°C. Mendelssohn and White's suggestion that the latter rate is slightly higher than that obtained usually for glass because of the opening up of surface cracks due to heating, fits in with the possibility, supported by figure 5-10, that the stainless steel also had minute surface cracks or markings. After the surface finish of beaker B had been destroyed by heating, the rate went up to 26.2×10^{-5} cm /cm.sec. This agrees

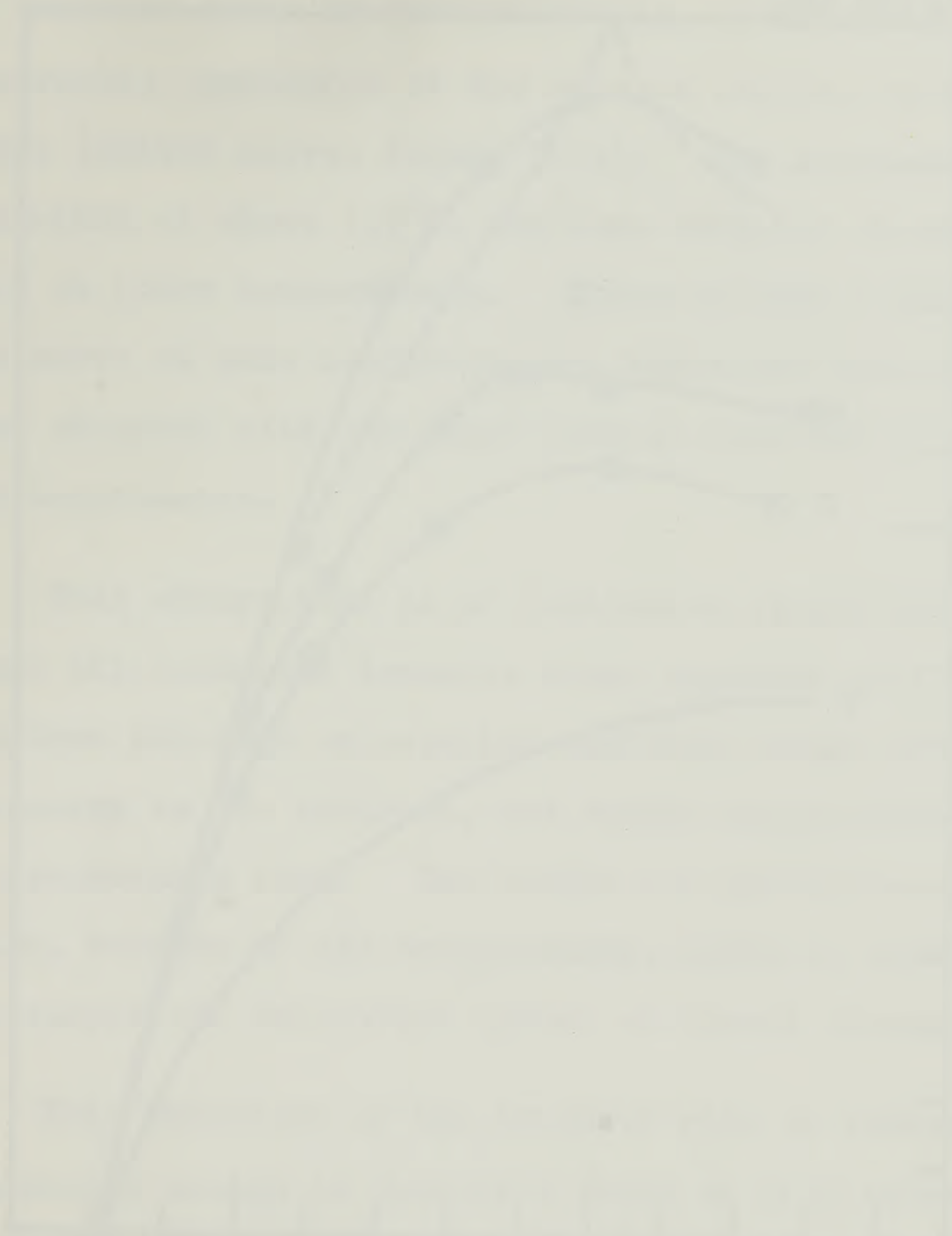
with the observation of Mendelssohn and White, that the transfer rate from a platinum beaker increased markedly after it was baked out at 600°C.

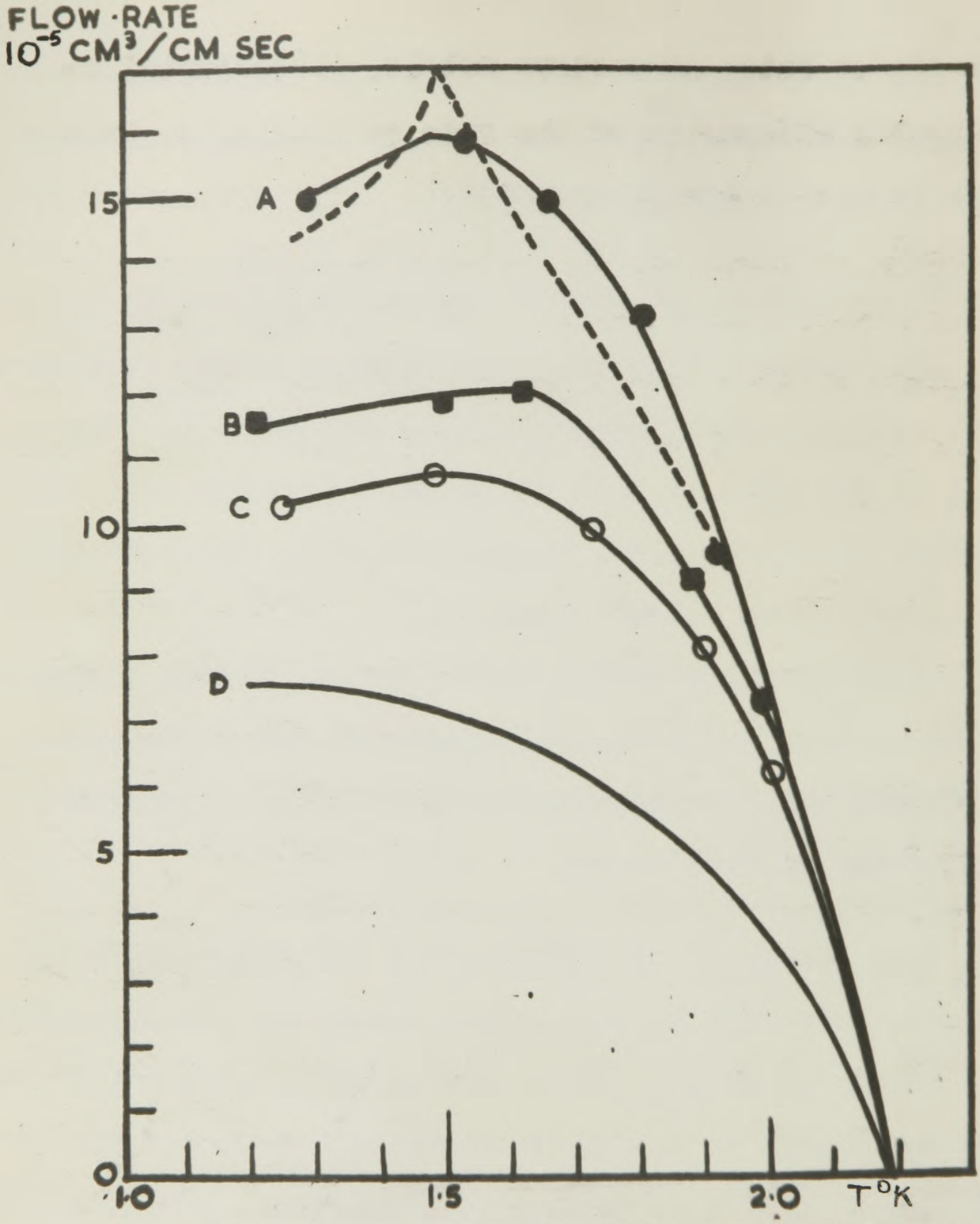
To summarise, we have obtained the following results:
(1) The transfer rate on a specially polished stainless steel surface, with irregularities less than 10^{-5} cm. deep, is the same as that on glass. Earlier work with machined stainless steel beakers, or other metal beakers, had always given larger rates.
(2) After heat treatment, when the only change produced in the surface was a recrystallisation and a consequent roughening and formation of cracks, the rate increased by a factor of over three.

From these, it is obvious that the increase in the transfer rate must be due to the change in the microstructure of the surface. It is not possible to say yet whether the change is due to an increased perimeter for the film, or to capillary flow in the cracks. This question will be taken up again after describing the next series of experiments.

5.3 Transfer Rates on Perspex and Lucite

Dash and Boorse(1951), in addition to their results





A perspex polished with commercial polish.

B lucite polished with rouge.

C perspex polished with rouge.

D glass.

The broken curve is for lucite by Dash and Boorse.

Fig. 5-14. Film transfer rates on perspex and lucite.

on transfer rates on various metals, also reported the remarkable dependence of the rate on lucite, on temperature (dotted curve, figure 5-14). The curve reaches a maximum at about 1.5°K, and then abruptly starts to fall at lower temperatures. There is thus a cusp in the curve at this temperature—a behaviour that had not been observed with any other surface used for film transfer experiments.

This observation is of particular significance, because all anomalous transfer rates reported till then had been obtained on metallic surfaces, which made it necessary to use indirect, and rather complicated, methods of determining them. But lucite now provided a substance which, because of its transparency, could be studied by the simple and well-tried method of visual observation.

This behaviour of the transfer rate on lucite seemed remarkable enough to justify a study of film transfer on perspex, which, like lucite, is a methyl methacrylate polymer (manufactured by I.C.I.Ltd.), to see if it had the same curious dependence on temperature.

All the experiments described in this section were done on machined beakers, of inside diameter 0.40 cm and wall thickness 0.1 cm, and polished by different methods as detailed below. The beaker was in each case mounted

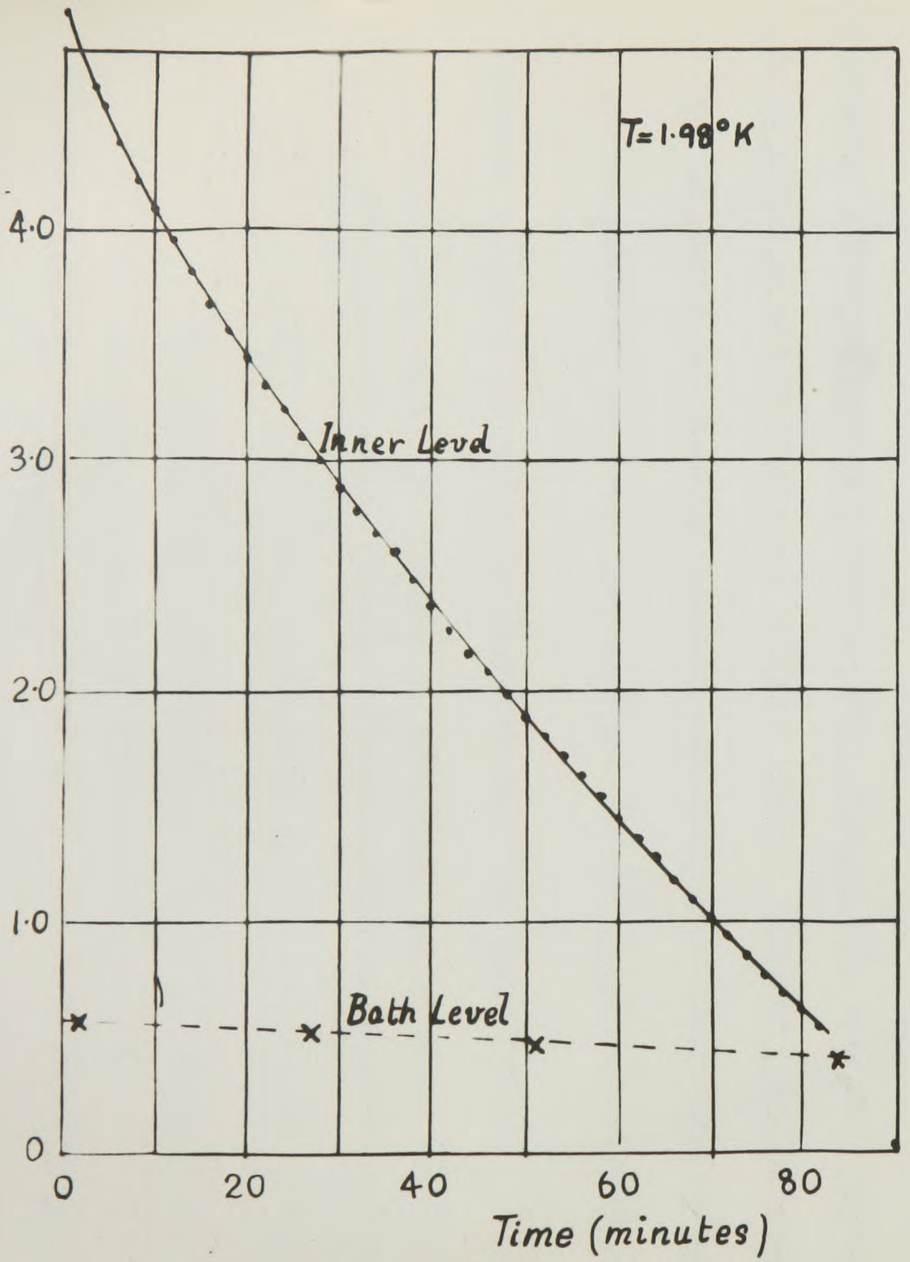


Fig. 5-15

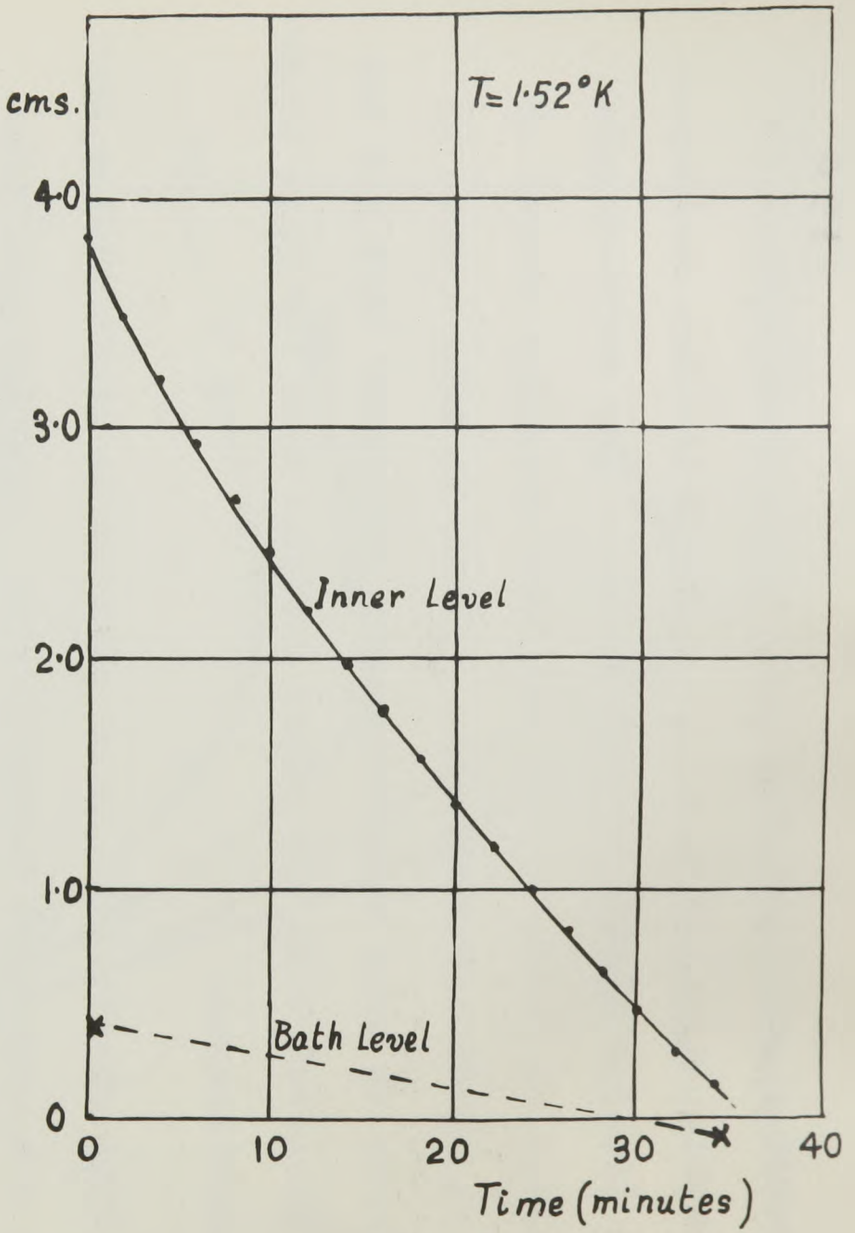


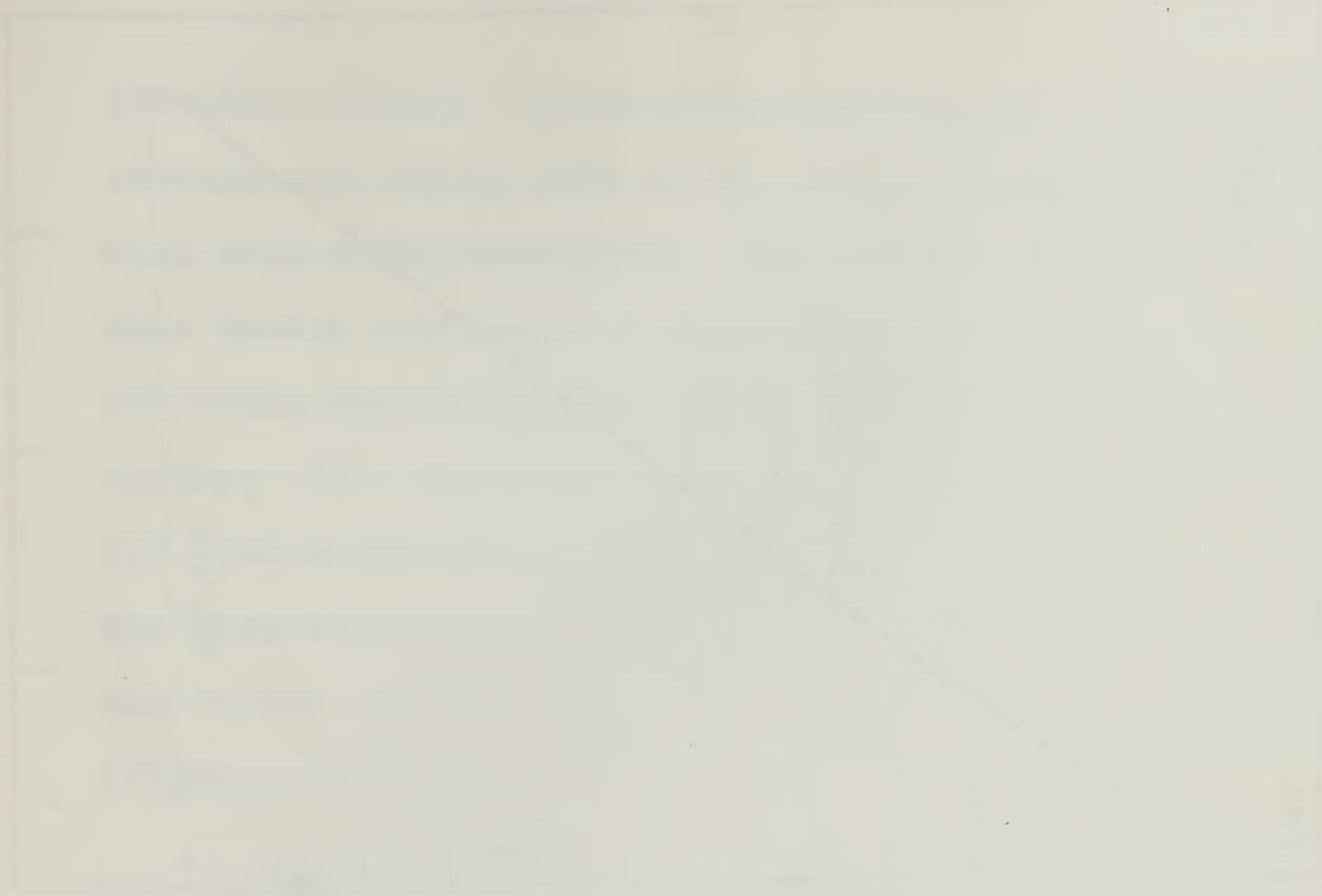
Fig. 5-16.

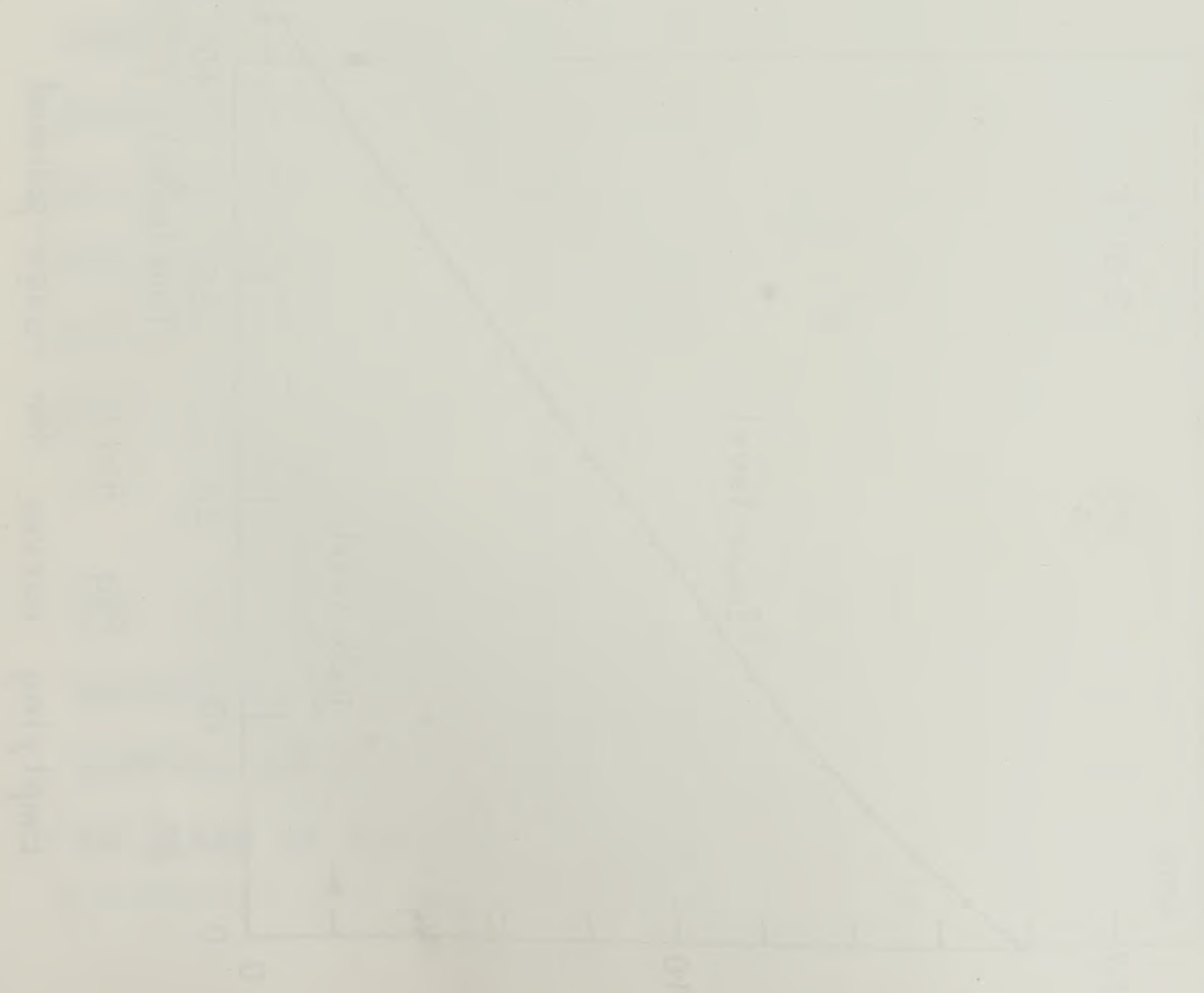
Emptying curves for 'rough' perspex beakers.

inside a copper radiation shield, with a helium can at the top, to reduce the heat influx into the beaker from above.

Curve A (figure 5-14) shows the transfer rate in dependence on temperature for a perspex beaker which, after machining, had been polished with a commercial polish. This polish consisted of a solvent, with an abrasive in fine suspension. The transfer out of this beaker was dependent on the position of the inner level, as is shown by the two typical emptying curves shown in figures 5-15 and 5-16. The rates given by curve A were determined when the inner level was close to the bath level, in every case at a depth of about 3 cms. from the rim. This particular aspect of the transfer will be discussed later, in comparison with the variation of transfer rate along the lengths of the other beakers. Curve A is seen to have the same shape as the one for lucite given by Dash and Boorse, though it cannot be said with certainty that it has a cusp at about 1.5°K. The inner surface of this beaker was subsequently examined under a microscope, and showed a streaky structure, with 'ridges' and 'valleys', about 10^{-3} cms wide, running axially (this would be explained by the method of polishing employed).

This perspex beaker was next polished by hand with





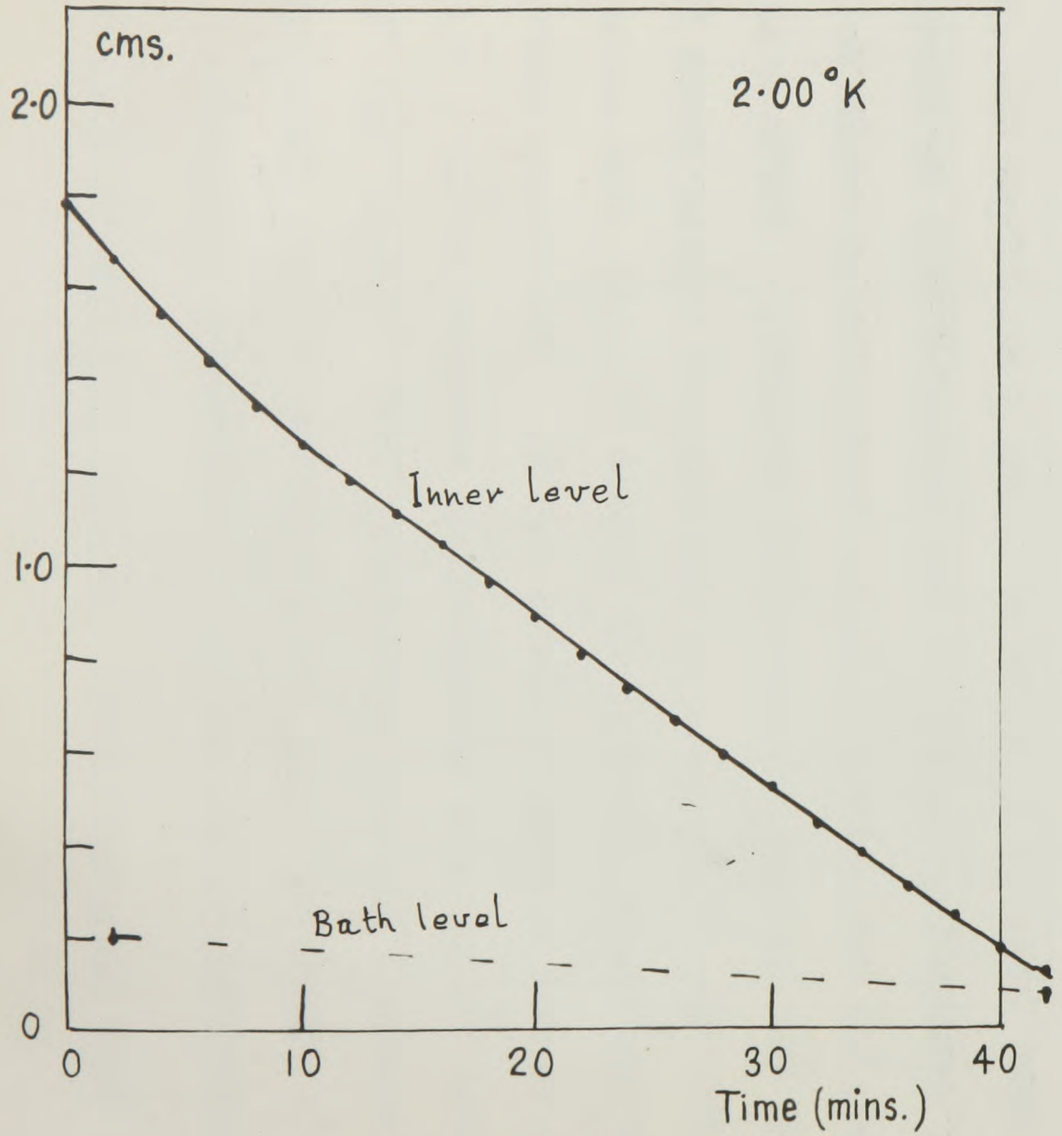


Fig. 5-17

Emptying curves for rouge-polished perspex beaker.

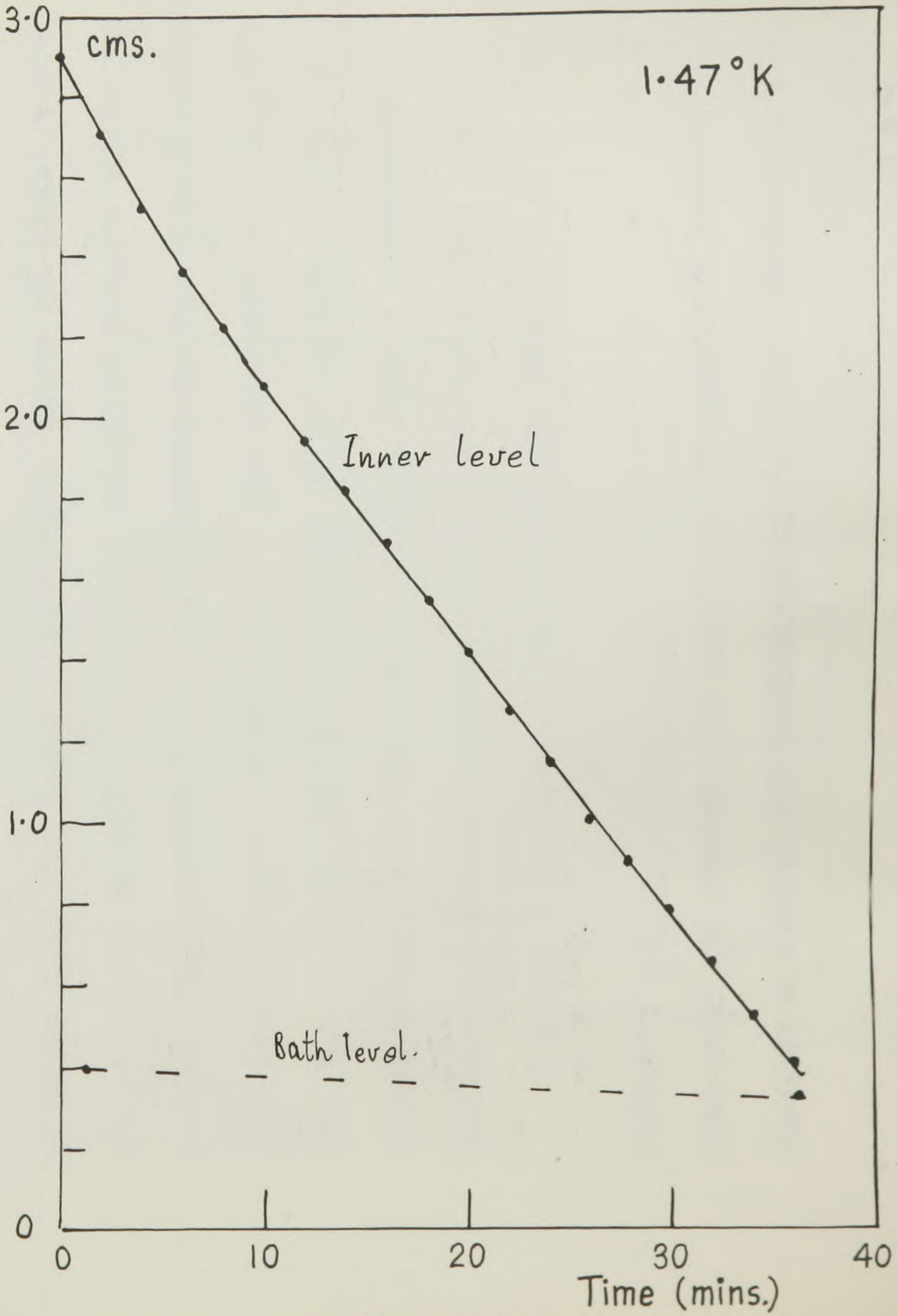


Fig. 5-18

jewellers rouge. Microscopic examination of the inner surface now showed that it was comparatively smooth, with only a few scratches. The transfer rates out of this beaker are shown in curve C, figure 5-14. The following characteristics of C, compared to A, are noteworthy: (1) the rates in C are smaller than those in A, (2) the maximum at 1.5°K appears to be less pronounced. The variation of the transfer rate along the length of the beaker was also less than that for the previous case (figures 5-17 and 5-18).

A length of lucite rod had in the meantime been obtained from America, and transfer experiments were done out of a beaker turned from this, and polished with jewellers rouge. The results are shown in curve B. It is seen that it is similar to C in magnitude and shape. The dependence of the transfer rate on the position of the inner level was also found to be about the same as for the rouge-polished perspex.

We thus have the following results:

(1) The transfer out of a beaker of perspex depends markedly on its surface finish; it becomes less as the surface becomes smoother , and probably approaches that on glass as the limiting case.

(2) The variation of the transfer rate with the position of the inner level at which it is determined, also depends on the surface smoothness: being more for a rough surface than a smooth one of the same substance.
(3) The maximum in the curve of transfer rate versus temperature becomes flatter as the rates themselves decrease and probably approach those for glass.
It is to be noticed that the transfer rates were all determined when the beakers were emptying.

From these results, we conclude that the observed transfer rate on these surfaces is directly influenced by their comparative smoothness. The fact that with increasing smoothness the transfer rate versus temperature curve tends towards that for glass, leads one to expect that when the surface finish is comparable with that of glass, the rates on the two would be identical. This expectation has already been borne out in the case of the experiments with stainless steel beakers, as described earlier.

It would indeed seem that all the transfer rates observed on the perspex and lucite surfaces can be explained, if one postulates two different processes by which the transfer takes place, both of which act simultaneously: (1) pure film transfer, having a critical rate



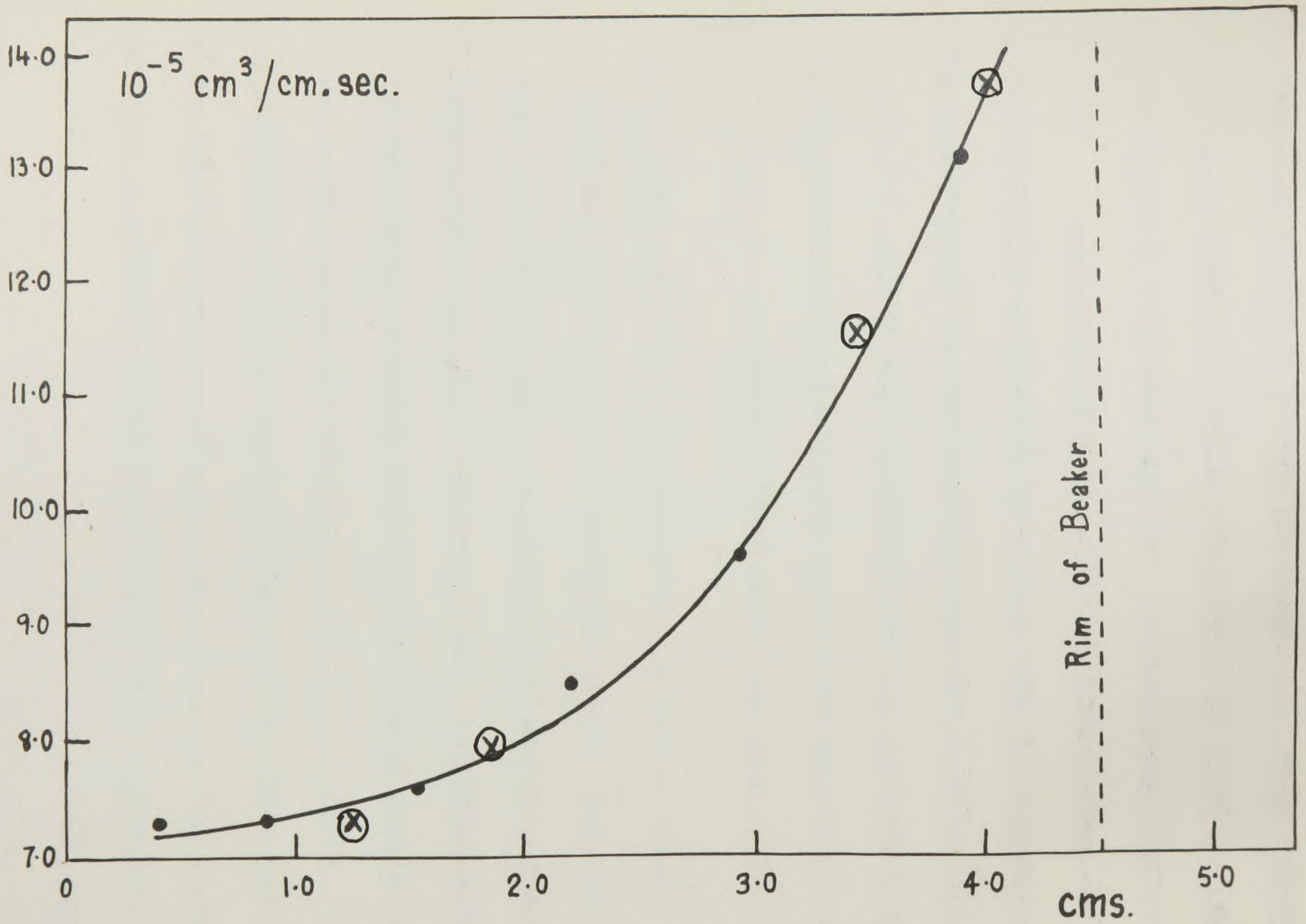


Fig. 5–19. Variation of rate with distance from rim of beaker. ('rough' perspex).

• 1·98° K

× 2·26° K (normalised).

approximately equal to that on glass; (2) capillary flow, which occurs in the irregularities on the flow surface. The latter kind of flow will obviously depend in a complicated manner on the size, shape and distribution of these irregularities. But the maxima in the curves at 1.5°K do indicate that this flow should decrease with temperature at least below 1.5°K. It is interesting that such a temperature dependence has been reported (Allen and Misener 1939) in the case of isothermal flow of the bulk liquid through capillaries of about 10^{-2} cm. diameter.

We shall now consider in greater detail the dependence of the transfer rate on the position of the inner level. Figure 5-15 shows a typical emptying curve, at a temperature of 1.98°K, for the perspex beaker polished with commercial polish. The rate varies continuously from a value of 13.1×10^{-5} cm.^{3}/cm.sec. when the inner level is near the rim of the beaker, to 7.3×10^{-5} cm.^{3}/cm.sec. when the two levels are close to each other. These rates are shown in figure 5-19 (•) as a function of the distance of the inner level from the rim. This latter has been chosen in preference to the level difference between the inside and the bath, because, on the mechanism postulated above for explaining the anomalous rates, the transfer rate will not depend in a simple man-

ner on the difference between the two levels. By expressing the transfer rate R as the sum of a pressure-independent term R_0 and one depending on pressure R_p, it was found that

$$R_p \propto (\Delta p)^3$$

where Δp is the difference between the inside and bath levels. It is obvious that such a dependence on the pressure difference cannot be obtained by any single simple mechanism of flow. For these reasons, it appears more justifiable to talk of the variation of the transfer rate with the distance of the inner level from the rim, rather than with the level difference between the inside and outside. The experiments to be described later, performed above the λ-point, lend support to this point of view.

Figure 5-17 shows a typical emptying curve for the same perspex beaker, at 2.00°K, after it had been polished with rouge. It shows a striking similarity to the well-known emptying curves for glass beakers. There is the same rather rapid drop of the level within a few millimetres of the rim, after which it settles down to a nearly constant rate.

A few words may be said here about the possible ways in which the roughness of the surface affects the

rate of transfer. The actual increase in the rate of transfer can be easily understood in terms of capillary flow in the surface irregularities. But the dependence of the rate on the position of the inner level is a slightly more complicated question. The rate would certainly be governed, to some extent, by the level difference, according to the usual laws of siphon flow. In addition to this, the rate would also be governed by the number of channels available for capillary flow for any given position of the inner level relative to the rim of the beaker. The number of such channels would decrease as the inner level moves further from the rim, for the capillary rise in some of them would reach a maximum, less than the distance of the rim from the inner level. In this connection, it may be pointed out that the rise in a capillary of circular cross-section, of 10^{-3} cm. radius, for liquid helium below the λ-point, is of the order of a centimetre.

One sees therefore that the plausible hypothesis of capillary flow on rough surfaces, explains all the observed features of helium II transfer on such surfaces. It also leaves intact the concept of a unique critical rate for pure film transfer. This latter is reinforced by the fact that the various curves of transfer rate versus temperature tend to flatten out as they approach

the curve for glass, and there is no instance yet reported of any such curve crossing the one for glass. It would be of great interest to extend the measurements of transfer rates on lucite and perspex to temperatures below 1°K, as has been done by Ambler and Kurti (1952) for glass. They reported an increase in the rate on glass below about 0.8°K, up to a value of 15×10^{-5} cm^3/cm.sec. at 0.2°K. Lucite and perspex have the great advantage, over all other surfaces which have hitherto been used in studying anomalous transfer rates, of permitting direct visual observations. This makes them particularly suitable for the method of Ambler and Kurti.

The hypothesis of capillary flow in surface irregularities, put forward above, has an important bearing on various properties of the mobile helium film which have been recently reported by other investigators. Mendelssohn and White (1950) state that the transfer rate out of a platinum beaker showed a dependence on the difference between the inner and outer levels, decreasing as this difference decreased. It is remarkable that this was the only beaker in their experiments which showed this variation of the transfer rate, and also the maximum in the rate versus temperature curve (PtA in figure 5-1). It is clear that the considerations applied to the lucite and perspex beakers above, are equally applicable to this case.

Brewer and Mendelssohn* have determined the trans-

*I am grateful to Mr.D.F.Brewer and Dr.K. Mendelssohn for permitting me to quote their results before publication.

fer rates of the helium film in equilibrium with the vapour at pressures P less than the saturation vapour pressure P_o, at several temperatures. They have done experiments on glass and german silver, and find that the transfer rates on the two surfaces are identical, up to a certain critical value of (P/P_o), which is itself a function of temperature. At 1.53°K, for example, they find $(P/P_o)_{critical} = 0.94$. For larger values of P the rate on german silver increases sharply over that on glass, till at $P/P_o=1$, it becomes more than twice that on glass. This can be explained by capillary condensation in narrow surface cracks setting in for P/P_o greater than 0.94 in the case of german silver. It also indicates that the actual increase in the perimeter available for pure film transfer is negligible, at least in this case. In the case of the lucite of and perspex beakers it is not very easy to separate out the contributions due to the two mechanisms, i.e. increase in the perimeter, and capillary flow.

Eselson and Lazarev (1951) have reported some new determinations of the film transfer rates on glass,

using radiation shields similar to the ones used in all the present series of experiments. In addition, they have also done experiments with beakers provided with copper bases to ensure equality of temperatures between the inside and the outside. Detailed results are not given, but they make the following points (all their experiments were done at 1.52°K):

(1) When the beaker is filled by immersion in the bath, and then the emptying rate is determined, it is found to decrease over a few millimetres from the rim, and then reaches a nearly constant value, identical with the one established by other workers.

(2) When the beaker, initially empty, is filled by film transfer from the bath, and then the emptying rate determined, it is slightly less than the rate found in (1).

(3) Detailed experiments show that the variation of rate along the length of the beaker depends on the distance of the inner level from the rim, and not on the difference of levels between the inside and the bath.

This third point, as has already been seen, also arises from our own considerations. Now it is conceivable that, when the beaker is filled by film transfer, some of the minute surface cracks on the inside surface do not get filled by the liquid, and hence do not contribute to the transfer out of the beaker subsequently

observed. This point will arise again in the experiments on flow above the λ-point, described below.

The increase in the transfer rate when the higher level is near the rim of the beaker is a curious phenomenon which has been observed in all film transfer experiments, since the earliest work of Daunt and Mendelssohn (1939). Some light on this problem will be thrown by experiments to be described later in this chapter.

5.4 Surface Flow of Liquid Helium above the λ-point

If the hypothesis of capillary flow in cracks to explain the anomalous rates on lucite and perspex were correct, it follows that, even above the λ-point, such a beaker filled with liquid helium should empty itself into the bath. The situation here is complicated by the fact that the viscosity and the surface tension of the helium have to be taken into account, as also the influence of radiation in evaporating the liquid from the surface. The surface tension limits the number of channels which are available for capillary flow, for any given depth of the higher liquid level below the rim. The viscosity of helium would influence directly the flow rate, if it exists, above the λ-point. Very careful shielding from radiation is required, as otherwise

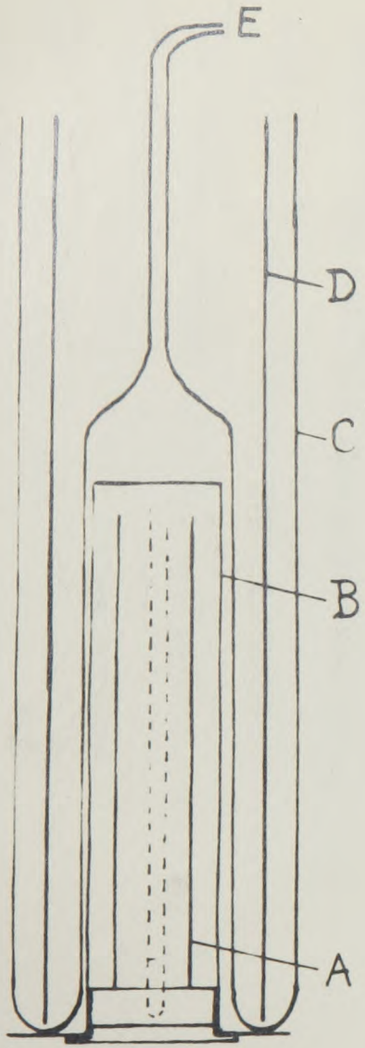


Fig 5-20.

Surface flow above the λ-point.

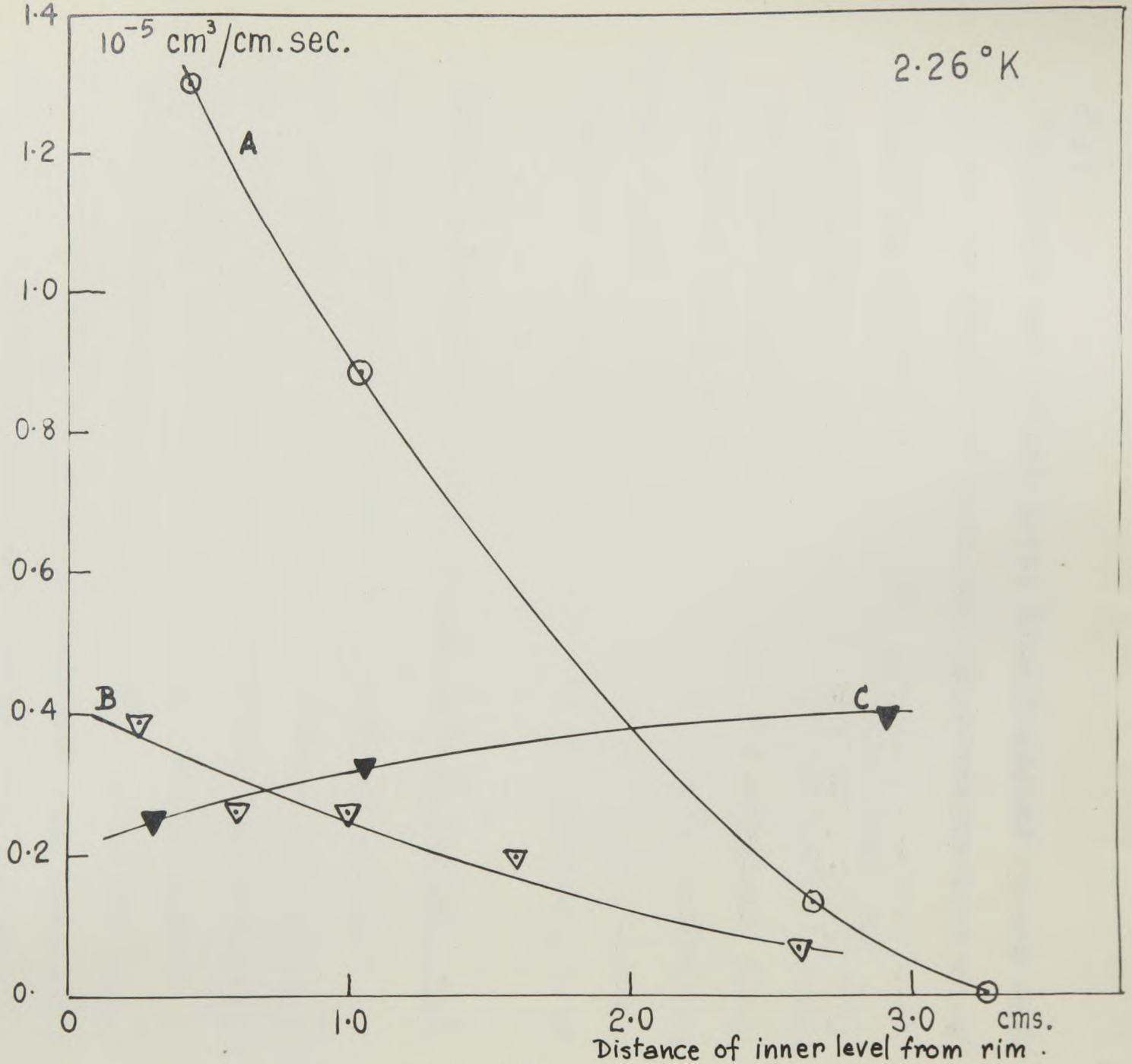


Fig. 5-21.

'rough' perspex beaker: A, outflow.
glass beaker: B, outflow; C, inflow.

any change in the liquid level inside the beaker may be due to evaporation, and also the liquid in the surface channels may evaporate faster than can be replenished from the bath.

5.41 Experiments with a 'Rough' Perspex Beaker.

With these considerations in view, the apparatus shown in figure 5.20 was constructed. A was the perspex beaker of inside diameter 0.40 cm. and wall thickness 0.10 cm., polished with a chemical polish, which had shown the anomalous rates below the λ-point described before. It was completely enclosed in the thin-walled copper radiation shield B, which had two narrow vertical slits, to permit observation of the liquid levels. This radiation shield itself was surrounded by the double-walled glass vessel C, whose inner tube was drawn out to a jet E well above the rim of the outer tube. The annular space of C contained a second cylindrical shield D, of polished aluminium, with two slits which were in line with the slits in B. The jet E permitted rapid equalisation of the levels of the liquid round the beaker, and that in the bath; this arrangement would of course not have been necessary for experiments below the λ-point. During the actual experiment C was kept filled with liquid helium, thereby ensuring a minimum influx of radiation to the beaker.

All the experiments were done at a temperature of 2.26°K, which was held constant to 0.01°K by the manostat mentioned in chapter 1.

It was found that, with the beaker initially empty, and the outer level about 1 cm. from the rim, no inflow could be detected; this appears to tie up with Eselson and Lasarev's observation (2) referred to above. It would mean that the surface channels have not been able to fill up with liquid and thus provide a flow link between the outside and inside of the beaker.

By carefully lowering the beaker into the bath, some helium was introduced into it; the beaker was then raised, so that the outside level was above the inside level. Observation through a cathetometer showed that the inside level was now creeping up slowly. To verify that this was not due to a distillation process, the beaker was raised further, so that the outside level was below the inside level. The latter was now observed to be falling slowly. These observations were alternated several times in succession, and in each case the inner level moved towards the outer level. This, taken in conjunction with the dependence of the creep rate on the position of the higher liquid level (discussed below), excludes the possibility of evapo-

ration being the only cause for the observed change of level. Also, the observation that the creep rate for approximately constant positions of the two levels remained the same for several hours makes it unlikely that it was due to the draining down of liquid sticking to the walls of the beaker.

The flow rate into the beaker, as a function of the distance of the outer level from the rim, is shown in figure 5-21 (curve A). The rate is expressed in cm.^{3} per cm.sec. only to facilitate comparison with the rates below the λ-point; the present data are not sufficient to say with certainty that the flow rate is proportional to the perimeter of the beaker. The actual distance through which the inner level moved during each determination was less than one millimetre, and it was at a distance of about 3 cms. from the rim in each case. There is a strong dependence of the flow rate on the relative positions of the rim and the two liquid levels--a dependence which is negligible in almost all cases of film transfer below the λ-point.

The liquid helium was then allowed to warm up to 4.2°K, and the beaker again dipped in the bath to allow the liquid to run over its surface. With the bath level now 4 mm. below the rim, no change in the inner level

could be detected during 30 minutes (the corresponding change at 2.26°K would have been about 0.25 cm.)

5.42 Experiments with a glass beaker.

For purposes of comparison, a glass beaker was substituted for the perspex beaker, and similar inflow experiments carried out at 2.26°K. The results are shown in figure 5-21 (curve B). It is remarkable that there should be a flow on glass also, though the magnitude is only a quarter of that on perspex. This flow is also seen to be dependent on the relative positions of the rim and the liquid levels.

An outflow experiment was done, with the inner level between 0.13 and 0.15 cm. from the rim, the position of the outer level being varied for the different sets of observations. These rates are shown ~~by the crosses~~ on figure 5-21 (curve C).

A curious effect was observed, when the beaker was filled to the rim with liquid helium and then raised a few centimetres . The meniscus in the beaker was quite flat. It remained stationary, and no movement could be detected even after 30 minutes. Then a little liquid was splashed out of the beaker by vigorously tapping the cryostat, so that the meniscus was about 1 mm. below the rim. It then started to creep down, at a

rate of about 0.006 cm/min. The only possible explanation of this strange behaviour seems to be that, when the meniscus is flat, surface tension effects prevent any liquid from flowing out. For, when the meniscus is flat, it has the minimum possible surface energy, and any change in its level would cause an increase in the area, thus requiring more energy. This effect was observed presumably because of the excellent thermal insulation of the beaker.

5.43 Discussion

To start with, we can rule out the possibility that any of the flows shown in figure 5-21 is caused by viscous flow in an adsorbed film of helium just above the λ-point. The evidence for such an adsorbed film is conflicting; Daunt and Mendelssohn (1939), in the course of their measurements of the thickness of the film, found no evidence for a film thicker than 10^{-7} cm. above the λ-point. The adsorption isotherm measurements of Kistemaker (1947) indicated that, even at 2.3°K, the thickness of the film on glass in equilibrium with the nearly saturated vapour was over 15 layers, compared to about 30 below the λ-point. The experiments of Burge and Jackson (1951) again seemed to show that the thick film disappeared immediately after warming up through the λ-point. It is clear that more experiments above the

λ-point, particularly optical measurements on films formed on a surface which is well insulated thermally, are needed. But even assuming a film thickness at 2.26°K which is the same as that observed below the λ-point, we find that the calculated viscous flow, under the conditions of the present experiments, is 10^{-4} times that observed, and therefore this type of flow cannot make a significant contribution to the observed flow.

It is easy to see how the hypothesis of capillary flow, in conjunction with the known differences between the surfaces of the perspex and glass beakers, explains the inflow curves for these beakers. The glass beaker has a very much smoother surface than the one of perspex, but it must nevertheless have a few surface cracks; these would explain the much smaller flow observed on glass, compared to that on perspex. This may also have a bearing on the observation, reported by various experimenters (van den Berg 1951, Eselson and Lasarev 1951) that even with extreme precautions against deposits of condensed gases on the flow surface, glass occasionally yields transfer rates slightly higher than 7.5×10^{-5} cm^3/cm.sec. This, on the basis of the above hypothesis, would be because a small fraction (which in general depends on the particular glass beaker employed

for the determination) of the observed transfer rate is due to capillary flow(cf. also Mendelssohn and White's observations on transfer out of baked-out glass beakers). The outflow from this particular beaker, at 1.27°K, was in fact observed to be 9.5×10^{-5} cm.^{3}/cm.sec.

The discussion, up to this point, has not indicated why the inflow above the λ-point varies with the relative positions of the rim of the beaker and the liquid levels. This variation can be due to (1) the gravitational potential difference between the two ends of the flow channels varying with the separation of the liquid levels, or (2) the number of channels available for flow being controlled by the depth of the higher meniscus from the rim (because of the influence of channel size, surface tension, viscosity and evaporation). The outflow from the glass beaker (curve C, figure 5-21) clearly indicates that the second explanation is to be preferred, at least as providing the greater part of the observed flow. For, during the outflow experiments, the inner level was at a distance of between 0.13 and 0.15 cm. from the rim, while the separation between the two levels varied between the values of 2.8 and 0.2 cm. It is seen that the outflow is approximately constant under these conditions. This shows that the separation

between the two levels has a very slight effect, if any, on the transfer. This, as has been mentioned before, explains one of the observations of Eselson and Lasarev (1951).

As the surface tension of liquid helium does not change much below 2.26°K, one might expect that there would be certain similarities between the transfer rate below the λ-point, and the surface flow above, for the same surface. Let the transfer rate, at a temperature below the λ-point, for the rough perspex surface, be expressed as the sum of a constant R_o and a variable R_p, where R_p depends on the position of the higher level; then R_p would be due to capillary flow, on the above assumptions. R_p would therefore have the same dependence on the depth of the higher level from the rim, as would the flow rate at 2.26°K. That this is indeed so, is seen from figure 5-19, where the encircled points represent the normalised flow R_p' above the λ-point, for perspex. These points were calculated from

$$R_p' = 7.3 + k.R_1$$

where R_1 is the flow above the λ-point, and k is a constant chosen to make $R_1 = R_p$ near the rim.

To summarise: the hypothesis of capillary flow

predicted that (1) there should be flow of liquid helium I out of, and into, a beaker just above the λ-point if the transfer below the λ-point had been anomalous, and (2) this flow should depend on the smoothness of the flow surface, decreasing with increasing smoothness. Both these predictions have been verified by the above experiments, and it has also been possible to interpret the variation of the rate with the position of the higher level.

5.5 The Helium II Film Transfer Rate near the Rim of a Beaker

5.1 Introduction.

In all experiments on film transfer on glass, it had been observed that, while the rate over the greater part of the beaker remained constant, it was slightly higher when the higher level was within a few millimetres of the rim. This is clearly brought out in figure 5-17, which, though it refers to perspex, has exactly the same characteristic.

Atkins (1950) gave the following explanation of this effect. It follows from quantum mechanical arguments that the critical velocity v_c is related to the thickness of the film d by the relation

$$v_c d = \text{constant}$$

This relation by itself would give a constant transfer rate over the entire length of the beaker. Atkins suggested that $v_c d$ was in fact slightly dependent on d, decreasing with decreasing d. Now the thickness d of of the film at a point at a height H above the liquid meniscus is given by

$$d = \text{const.}/H^{\frac{1}{n}}$$

where n is about 3 (Burge and Jackson 1951). Thus d decreases rapidly as H increases from zero, and more slowly for larger H. When the meniscus is near the rim, the large value of d at the rim gives a large $v_c d$ and a higher transfer rate. As the meniscus falls, the variation in d at the rim becomes less pronounced, giving a nearly constant transfer rate.

The above explanation appears untenable, as the following considerations show. Let the helium in region 1 (figure 5-22) flow to region 2 via the connecting film,

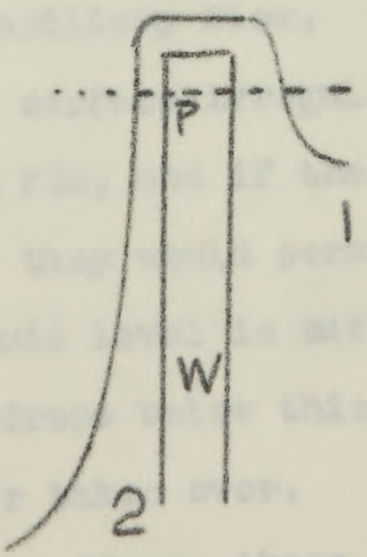


Figure 5-22.

over the thin dividing solid wall W. Assume that, during the transfer, the level in 2 remains unaltered. This would be very nearly true if 2 were the bath in any actual experiment. Consider the point P in the film in region 2, above both the liquid levels. The thickness of the film at P is constant, as it remains at a constant height above the liquid in 2. Furthermore, as it is above both liquid levels, no bulk liquid can be formed there, and the film has to move past P with a unique critical velocity v_{cp}. The volume rate of transfer past P is thus a constant, independent of the level in 1, so long as it is below P. By now taking P as close to the rim as is desired, so that it still remains above both liquid levels, we see that the variation of $v_c d$ with d, even if it exists, cannot explain the higher transfer rate near the rim.

A different explanation follows naturally from the hypothesis of capillary flow. During the construction of the beaker, surface irregularities are liable to appear near the rim, and if these are of a favourable shape and size, they would permit capillary flow as long as the liquid level is sufficiently close to them. When the level drops below this region, the normal type of film transfer takes over. It is possible that, in addition to this effect, there is also an increase in

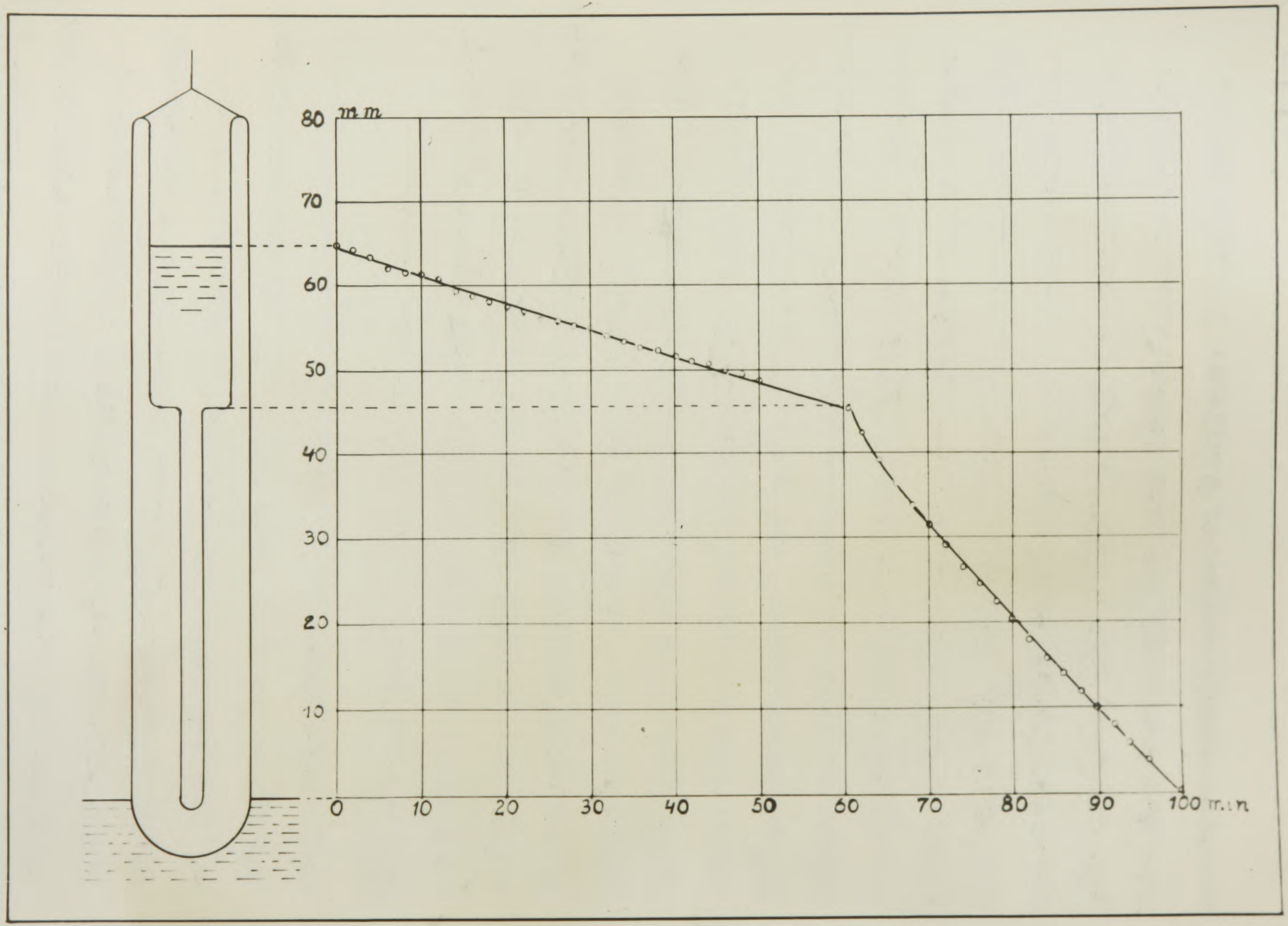


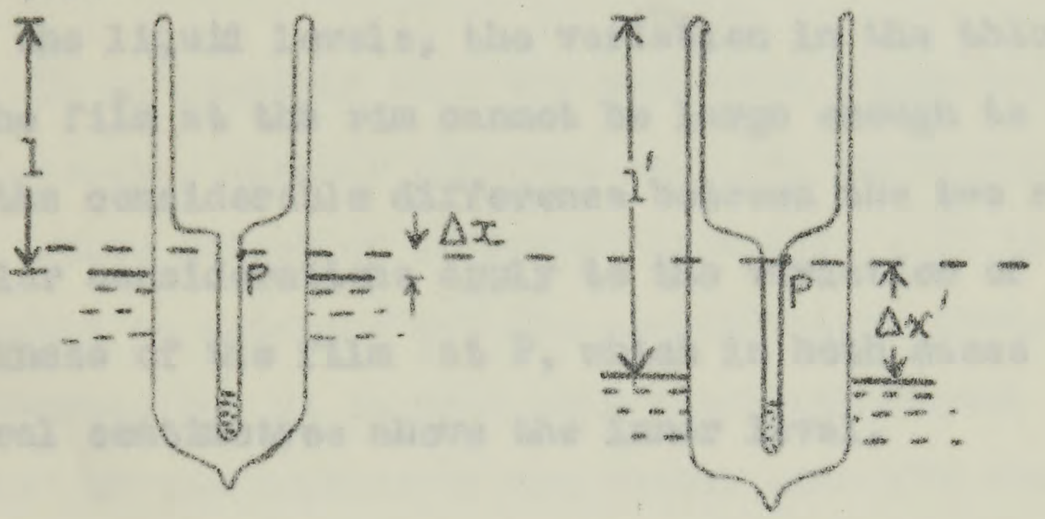
Fig. 5-23.

the microscopic perimeter available for film transfer near the rim. It has been suggested by Mendelssohn (unpublished) that if the microscopic perimeter varied along the length of the beaker in a random manner, then one would also expect an increase in the rate near the rim. A remarkable observation by Henshaw and Jackson (1950) seems to indicate that the first mechanism, i.e. capillary flow in surface irregularities, is the major contributory cause. They found that, when the inner level was near the rim during an outflow experiment, 'drops' of liquid helium, of an elongated shape, about 10^{-2} cm. long, appeared on the outside of the beaker at points <u>above</u> both liquid levels. These drops, which appeared in addition to the normal film, moved down at a speed of about 1 cm./sec. They disappeared when the transfer rate reached a constant value.

5.52 Experiment

To demonstrate that variation of thickness of the film at the rim could not explain these high rates, the following experiment was done with a glass beaker which had originally been used by Daunt and Mendelssohn (1939) to prove that the transfer rate is proportional to the minimum perimeter above the bulk liquid (figure 5-23). The outflow curve shows that, after the inner level enters the narrow part of the beaker, the transfer rate

decreases over a length of about 0.5 cm., and then becomes constant. In the present experiments, the inflow rates were determined, with the bath level successively occupying the positions shown below in (a) and (b).



(a) (b)

P is a point near the top of the narrow tube, just before it joins on to the wide tube. The inflow rates were determined in the two cases over the same range of positions of the inner level. All the experiments were done at a temperature of 1.27°K. We shall consider the two cases separately.

(a) In this position Δx was about 2 mm., and l was over 4 cms. The inflow rate was found to be 10×10^{-5} cm^3/cm.sec.

(b) $\Delta x'$ was now 1.7 cms., and l' was about 6 cms. The measured inflow rate was 7.8×10^{-5} cm.^{3}/cm.sec.

5.53 Discussion

The shape of the outflow curve (figure 5-23) could

by itself be accounted for if there were a slight constriction at P. But this would not explain at all the different rates observed in cases (a) and (b), and is therefore to be rejected. Again, as the rim of the beaker is, in both cases, at a considerable distance from the liquid levels, the variation in the thickness of the film at the rim cannot be large enough to account for the considerable difference between the two rates. Similar considerations apply to the variation of the thickness of the film at P, which in both cases is several centimetres above the inner level.

The explanation favoured by us, that the increased rate near the rim is to be associated with the increased roughness of the surface there, does indeed account for the above observations. For in case (a), the limiting perimeter is just below P, and the higher transfer rate implies that this is larger than the limiting perimeter in case (b)*. It must be emphasised that the rates in

---

*The term 'limiting perimeter' used here is taken to include the effect of capillary flow in addition to pure film transfer.

---

the two cases cannot be explained by postulating that the macroscopic perimeter at P is greater than at points further below. Firstly, such a difference as would be large enough to cause the observed transfer rate difference

could not be detected by visual examination, and secondly, if it existed, then during outflow, the inner level would move past P more slowly than when it is lower down. This is just the opposite of what was actually observed (figure 5-23).

It is possible that the mechanism suggested by Mendelssohn, already referred to, also contributes to the above effect of increased transfer. It is not possible to separate out the contributions from the two processes, but it is seen that they both depend on the nature of the surface of the beaker near the rim, and not on the variation of the thickness of the film with height.

Daunt and Mendelssohn (1939) suggested that the increased rate might be due surface tension effects. Since the capillary rise of helium II near a wall is less than 1 mm., such effects cannot be very noticeable.

To conclude: it is shown that the variation of the thickness of the film at the rim of a beaker cannot explain the high transfer rates observed in that region; on the other hand, the degree of smoothness of the flow surface near the rim, it is shown, provides mechanism for such rates.

5.6 Film Transfer in 'Long' Beakers

5.61 Introduction.

Kothari(private communication) made the interesting suggestion that, for large pressure heads, the critical rate of transfer in the helium film should be larger than what is normally observed. His argument was as follows: The normal critical rate can be obtained by calculating the ground state of a helium atom in the film, and equating its zero-point velocity to the critical velocity of film transfer. He suggested that, in the presence of a larger pressure head, the critical velocity would be given by the velocity of the atom in its first excited state, and perhaps even higher excited states. The dependence of the critical velocity on the pressure head that he suggested is shown in figure 5-24.

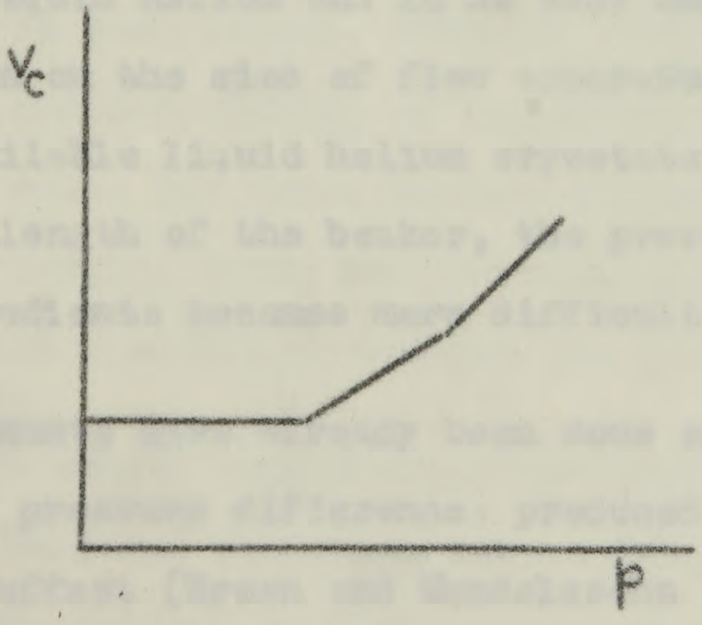


Figure 5-24. Variation of critical velocity v_c with pressure head p(Kothari)

There appears to be an objection to the above argument. On the two-fluid model, frictionless transport is possible only for those atoms which are in the ground state, all the excited atoms going into the normal fluid and therefore not participating in superflow. Thus only one critical velocity is possible for any given channel, determined by the corresponding zero-point energy.

But quite independently of these considerations, it seemed of interest to study film transfer under larger gravitational potential differences than had hitherto been used. It is remarkable that all film transfer experiments done since the earliest work of Daunt and Mendelssohn have been with pressure differences of about 5 cms. of liquid helium and in no case more than 10 cms. A limitation on the size of flow apparatus has been the size of available liquid helium cryostats. Also, with increasing length of the beaker, the prevention of temperature gradients becomes more difficult.

Experiments have already been done on film transfer under large pressure differences produced by the thermomechanical effect (Brown and Mendelssohn 1950, chapter 3 of the present thesis). These showed that the transfer rate was independent of the pressure difference used.

The corresponding problem in gravitational transfer is more complicated, because the pressure difference can be increased only by increasing the path length for the film (barring the possibility of using centrifugal force), and the transfer rate, as we have already seen, depends in a complex manner on the nature of the flow surface. This makes a complete interpretation of the results given below very difficult.

5.62 Experiments

All the experiments were done with glass beakers, which were surrounded by a copper radiation shield carrying a liquid helium reservoir at the top. The temperature was 1.27°K.

The preliminary experiments were done with Pyrex beakers, of 18 cms. length, prepared from lengths cut at random from ordinary tubing of inside diameter 0.26 cm. The outflow rates for these beakers varied irregularly along their length, owing to non-uniformity of the bore, and it was not possible to analyse the results.

The next set of experiments were performed with beakers of 'Veridia' precision bore tubing, whose inner diameters were uniform to better than 1%. The results are shown in figure 5-25.

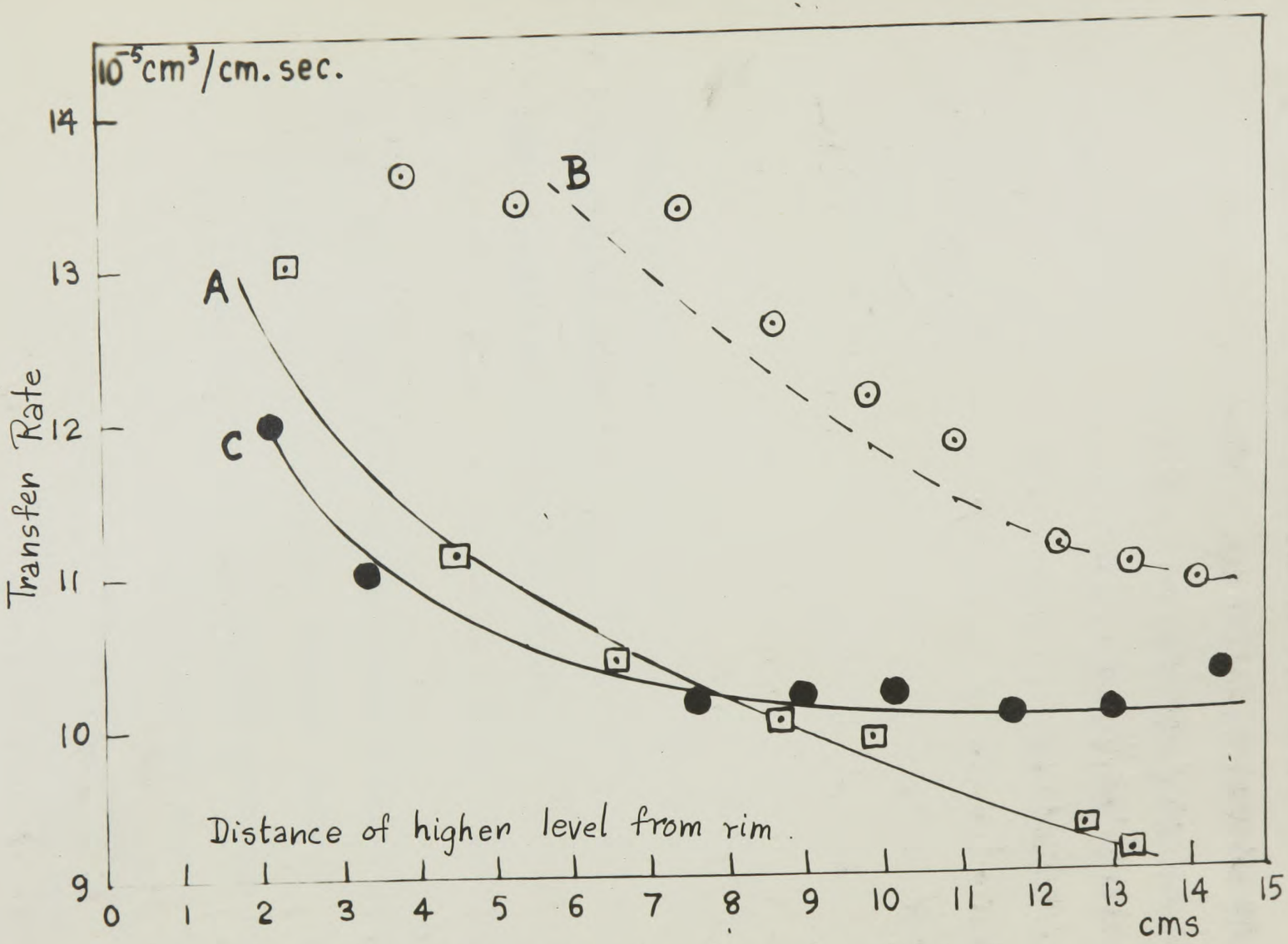


Fig. 5-25 Film Transfer from long beakers.

A & B, outflow.

C, inflow.

Curve A refers to outflow rates from such a beaker, of inside diameter 0.200 cm. These rates were determined by filling the beaker with liquid helium , and raising it by stages from the bath, and measuring the rates of transfer. This ensured that all these rates were determined for an approximately constant level difference between the inside and the bath, of about 3.0 cms. The other two curves refer to a 'Veridia' beaker of inside diameter 0.100 cm. Curve B is for outflow and curve C for inflow.

All these transfer rates are rather higher than that normally observed on glass surfaces at this temperature. This is probably connected with the nature of the inner surface of Veridia tubing; it is manufactured by being cooled on a copper mandrel, which is withdrawn when the glass is firm. This makes it likely that the bore of such tubing has a very much more uneven microscopic structure than that of glass tubing made by the usual methods. This was borne out by an examination of the two surfaces under a microscope. The inner surface showed fine closely-spaced streaks running right along the length of the beaker, while the outer surface was comparatively smooth. Thus uniformity of the bore, it seems, can be achieved only at the cost of surface finish.

The fact that the transfer rate decreases as the higher liquid level moves further from the rim, though the difference between the two levels remains the same (curve A), clearly indicates that this variation of the transfer rate cannot be connected with the gravitational potential difference between the ends of the film, but rather that it is yet another example of the influence of the microstructure of the flow surface.

A comparison of curves B and C shows that for outflow the rate is more strongly dependent on the position of the higher level vis a vis the rim, than it is for inflow. This can be understood on the basis of what has been said before about the possibility of capillary flow being present on rough surfaces, remembering the difference between the inside and outside surfaces in smoothness. It is conceivable that the contribution from capillary flow during inflow is less than during outflow, for in the former case not all the available channels on the inside may fill with liquid and take part in the flow.

One may conclude that these experiments with long beakers emphasise the influence of the surface finish on the transfer rate, but they give no indication that the hydrostatic head which determines the direction of the film transfer, has any effect on the rate.

2.7 Summary

A variety of experiments has been described and discussed in this chapter, and it may be well to summarise here the principal conclusions to be drawn from them. The experiments with the specially polished stainless steel beakers show that the anomalous transfer rates previously observed on metal surfaces are to be associated with capillary flow in surface cracks and irregularities. If such a surface could be given a finish comparable to that of glass, then the transfer rate would be the same as that on glass.

The transfer rates on perspex and lucite have been determined. It is again seen that the smoothness of the surface directly affects the transfer rate, and is also responsible for the maximum in the curves of transfer rate versus temperature observed in some cases.

On the hypothesis of capillary flow in surface irregularities, it was expected that, for a perspex beaker which had given anomalous rates below the λ-point, surface flow of liquid helium simulating genuine film transfer would be observed; such a flow is indeed observed, and is found to be consistent with the hypothesis.

When a beaker is filled with helium II and then

allowed to empty through the film, the rate is found to be slightly higher near the rim than over the rest of the beaker. It is shown that the explanation of this effect given by Atkins is untenable. Experiments with a special beaker are described, which show that this increase is due to the surface structure near the rim of the beaker.

Experiments have been done with long beakers to study the transfer under gravitational potential differences greater than any that have been previously used. It appears unlikely that the increased potential difference has any effect on the rate, the observed increase of which can be attributed to the surface structure of the beakers used.

From all these experiments, the main conclusion to be drawn is that there is a uniquely defined critical rate of transfer of the helium II film, which is a function of the temperature alone. The fact that this critical rate is independent of the nature of the substrate indicates that a complete theory of the critical rate should come out of the quantum mechanics of the helium II film alone, independently of the nature of the forces between the helium atoms and those of the substrate. These latter have of course to be considered

for understanding the formation of the film, or studying its thickness and its variation with height. It thus appears that the theories of the critical rate based on zero-point energy considerations (Bijl, de Boer & Michels 1941, Mendelssohn 1945,1947, Ginsburg 1950, London 1951) are in the right direction.

References to chapter 5.

Allen, J.F. & Misener, A.D., 1939, Proc.Roy.Soc. A, 172, 467.

Atkins, K.R., 1950, Proc.Roy.Soc. A, 203,119, 240.

Berg, G.J.van den, 1951, Proc.Int.Conf. Low Temp.Phys. (Oxford), p.69.

Bijl, A., de Boer, J.H. & Michels, A., 1941, Physica, 8, 655.

Bowers, R. & Mendelssohn, K., 1950, Proc.Phys.Soc. A, 63, 1318.

Burge, E.J. & Jackson, L.C., 1951, Proc.Roy.Soc. A, 205, 270.

Dash, J.G. & Boorse, H.A., 1951, Phys.Rev., 82, 851.

Daunt, J.G. & Mendelssohn, K., 1939, Proc.Roy.Soc. A, 170, 423, 439.

Eselson, B.N. & Lazarev, B.G., 1951, Dokl.Akad.Nauk. S.S.S.R., 81, 537.

Gânsburg, V.L., 1950, Dokl.Akad.Nauk.S.S.S.R., 69, 161.

Henshaw, D.G. & Jackson, L.C., 1951, Proc.Int.Conf. Low Temp.Phys.(Oxford), p.65; 1950, Phil.Mag., 41, 1081.

Kistemaker, J., 1947, Physica, 13, 81.

London, F., 1951, Proc.Int.Conf. Low Temp.Phys.(Oxford), p.2.

Mendelssohn, K., 1945, Proc.Phys.Soc., 57, 371; 1947, Rep.Int.Conf.(Cambridge) II, 35.

Mendelssohn, K., & G.K.White, 1950, Proc.Phys.Soc. A, 63, 1328.

Schiff, L.I., 1941, Phys.Rev., 59, 838.

Temperley, H.N.V., 1949, Proc.Roy.Soc. A, 198, 438.

CHAPTER 6

FLOW OF HELIUM II THROUGH A ROUGE-PACKED TUBE

6.1 Introduction

Allen and Misener (1939), in the course of their measurements on the flow of helium II through fine channels, also made some observations on flow through a capillary tightly packed with jewellers rouge. They reported that the flow rate was proportional to $(\Delta p)^{\frac{1}{2}}$, where Δp was the pressure producing the flow, over the temperature range 1.1°- 2.17°K. They estimated the channels to be of the order of 10^{-6} cm in width. This seemed to indicate that the flow in tortuous channels of this size was quite different in character from that in slightly larger ones which were smooth; for their experiments on capillaries of about 10^{-4} cm. width gave a flow rate which was nearly independent of the pressure difference.

A variety of experiments done after the original investigations of Allen and Misener (cf.Kapitza 1941, Bowers & Mendelssohn 1952) had shown that it was possible

to define a critical rate of flow in slits of about 10^{-4} cm width, as the rate up to which no frictional dissipation appeared. The existence of a critical rate of transfer in the film, which is of thickness about 10^{-6} cm, had also been well established (chapter 3 of this thesis). The above observation on rouge-packed tubes naturally raised the question whether such a critical rate existed or not for such a flow.

The question of the existence of a critical rate of flow for helium II is one of basic theoretical importance. Simple quantum mechanical considerations suggest that a velocity v_c exists, below which no frictional dissipation of energy can take place, and that v_c is inversely proportional to d, the width of the channel (Bijl, de Boer & Michels 1941). For smooth channels less than about 10^{-3} cm wide, the existence of a critical rate has been, as pointed out above, well established. The experiments of Atkins on wide capillaries (1951) gave no clear indication of the existence of a critical rate, though they did not rule out the possibility of its existence. Even in the narrower capillaries, the dependence of the critical rate on channel size has not been conclusively determined. Meyer and Mellink (1947) and Bowers and Mendelssohn (1952) obtained a critical velocity for channels of width 10^{-4} cm

of the same order as that in the helium II film, of width about 10^{-6} cm. Kapitza (1941) and Allen and Misener (1939), on the other hand, found that the velocity of flow increased as the channel size was decreased. It is clearly necessary for more data on such flow to obtained, particularly through slits whose geometry is accurately known.

Gorter and Mellink (1949) introduced a new term in the hydrodynamical equations of the two-fluid model, which implied a force of mutual friction between the normal fluid and the superfluid varying as the cube of their relative velocity. They made this modification originally to explain the results of Keesom and Duyckaerts (1947) on heat flow through helium II in narrow capillaries. Efforts have been made (see review article by Atkins, 1952) to give, to what was essentially an ad hoc hypothesis, general validity in all flow phenomena, thermal as well as gravitational. Furthermore, efforts have been made to deduce this force theoretically (Zilsel 1950) taking account of irreversible processes. This makes it necessary to carefully examine the validity of mutual friction as applied to various types of flow experiments.

Now the theory of mutual friction does not contain

within it the concept of a critical transport rate. It can at best, as Dingle (1952) has pointed out, give only a 'blurred' critical rate, because of the rapid increase of the frictional force for large velocities. The crucial test between the two theories is therefore the existence of a critical rate, and the sharpness of its onset. The experiments on the helium II film described earlier (chapter 3) have already shown that for that type of flow the Gorter-Mellink theory is inapplicable. The experiments of Bowers and Mendelssohn (1952) have shown that flow through a channel 10^{-4} cms. wide is frictionless up to a well-defined critical rate, beyond which frictional dissipation, possibly obeying the cube law, sets in.

It is obvious that the complete solution of the hydrodynamical problem of liquid helium will have to retain the successes of the Gorter-Mellink theory, and at the same time account for the critical rate of transport. The theory should not only give frictionless flow for velocities less than the critical, but also take account of frictional processes (which themselves seem to be basically dependent on the channel width, see section 3.7 of this thesis) for supercritical rates. The beginnings of such a theory can be detected in the

recent work on the hydrodynamics of helium for large velocities (see Dingle 1952).

The existence of the critical rate in a given flow arrangement can be determined in several ways:

(1) A plot can be obtained of the flow rate against the pressure difference, and extrapolated back to zero pressure difference. If this gives a finite value for the flow rate, then this defines the critical rate. But this extrapolation is not always easy to make, particularly when there is an appreciable dependence of the flow on the pressure head (Atkins 1951). In such cases it can at best give an upper limit to the critical rate if it exists, without definitely indicating whether it does.

(2) Another method is to study the flow under the thermomechanical effect, and obtaining a curve of the flow rate against the heating rate. If a critical rate does exist, then there is a kink in the curve at this point, and it is linear for smaller rates. This method has been used successfully for narrow channels, of the order of 10^{-4} cm or less (chapter 3 of this thesis). It is not suitable for wider channels, where heat losses caused by mechanisms other than frictional dissipation as well, i.e. by return flow of the normal liquid, would introduce non-linearity in the curve.

It must be noticed that this method would not detect a frictional force which is independent of the velocity, directly.

(3) A very interesting method, suggested and used by Bowers and Mendelssohn (1950), can give a clear indication of the critical rate. This is to determine the pressure at an intermediate point in the flow channel. The method is based on the fact that flow at a sub-critical rate takes place under zero potential difference, so that the entire pressure drop must appear across that part of the channel where critical flow occurs. This is clearly seen, for the film, in the experiments described in chapter 2. Now in a perfectly uniform channel, the velocity is the same across any cross-section, so that the critical velocity is reached all along the channel at the same time. If, on the other hand, there is a constriction at any point in the channel, then the rate here would reach a critical value before elsewhere, and a pressure drop occurs here. Thus, by observing the pressure at a point in the channel in comparison with the pressures at its two ends, as the flow rate is gradually increased, the critical rate can be detected if it exists. The way this method works will become clear when the present experiments are described.

Bowers and Mendelssohn (1952), in their experiments

on flow through narrow annular slits of width about 10^{-4} cm., effectively used all the three methods described above. The isothermal flow under a gravitational potential difference was nearly independent of the difference, and hence the critical rate could be easily estimated by method (1). They also measured flow rates under the thremomechanical effect. These thermal flow measurements provide data for use with both methods (2) and (3): method (2) directly, and method (3) because the flow rate can be varied by varying the rate of heat input. They found that all three methods gave nearly the same critical rate.

The position is not so clear in the case of flow through rouge-packed tubes. According to the results of Allen and Misener, the flow was proportional to the square root of the pressure difference, and this indicated that it was turbulent. On the other hand, preliminary experiments on thermal flow in such a tube (Bowers, Chandrasekhar and Mendelssohn 1950) had shown that, up to a certain critical rate which depended on the temperature, the flow was frictionless. It seemed desirable to examine this curious contradiction between gravitational flow and thermal flow in greater detail, and so the present series of experiments were undertaken.

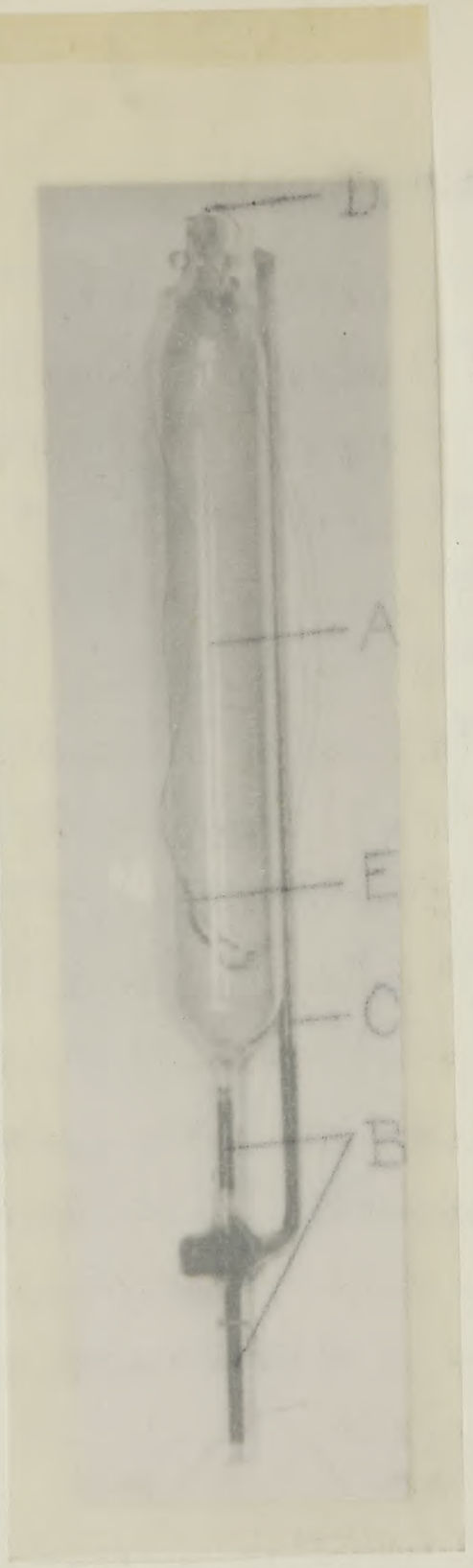


Figure 6-1 Rouge-packed tube.

6.2 Experiments and Results

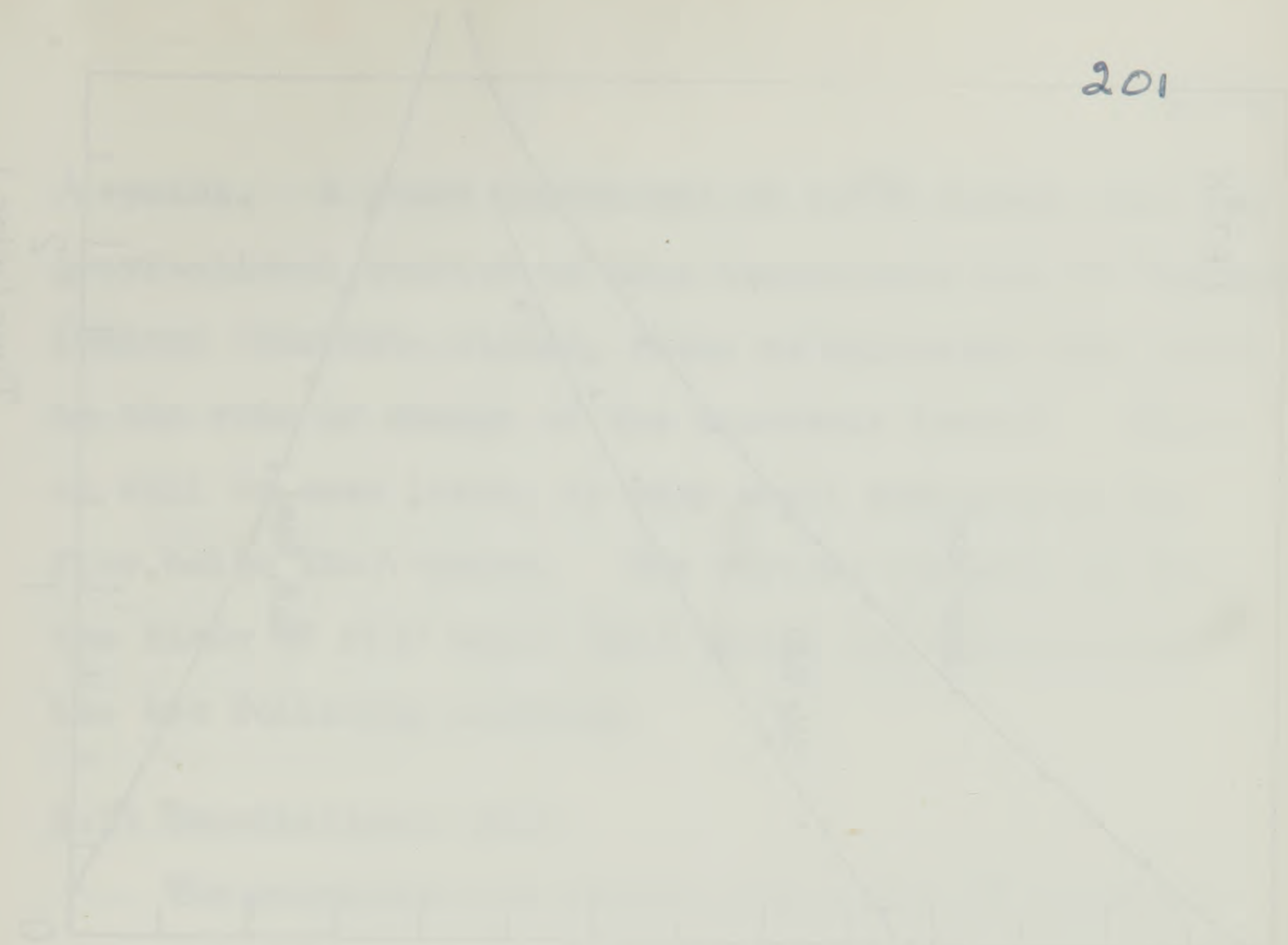
The flow apparatus is shown in figure 6-1. A is a graduated cylindrical glass reservoir, of inside diameter 1.7 cm. and length 10 cms. A thick-walled capillary B, 6 cms. long and 0.25 cm. in inside diameter, is attached to the lower end of the reservoir. B is blown out into a bulb at the middle, and the side tube C let into it. The top of A is brought to an orifice D of diameter 0.2 cm. The reservoir contains a heating coil E, wound from S.W.G. 47 constantan wire, with a resistance of 55.0 ohms at room temperature. Heat can be supplied to the helium in the reservoir by passing a current through this coil. The leads from the coil are taken out through D, and then through the bath and out of the cryostat. This ensures that heat conducted down the leads enters the bath, rather than the helium in the reservoir.

The two sections of the capillary are tightly packed with jewellers rouge whose grain diameter had been estimated to be about 10^{-4} cm. This was done by pouring in the rouge a little at a time, and stamping it down with a steel rod. The rouge is held in position by small plugs of cotton wool at each end. Silver foil

is wrapped round the entire capillary, as well as the lower end of the reservoir, to prevent spurious fountain effects caused by radiation incident on the rouge.

The orifice D and the top of the side tube were left open for all experiments on gravitational flow; for those on thermal flow, they were both sealed up with 'Fortafix' cement. It was found that, with D open, even the largest heating rates produced too small a fountain effect to permit an accurate determination of the flow rate. This would be because of the rapid equalisation of temperature between the helium in the reservoir, and the bath, by distillation. Correspondingly, the gravitational outflow rates with D blocked off were very low; the reason being that the helium in the reservoir warms up because of the mechanocaloric effect (Daunt and Mendelssohn 1939), and the temperature difference rapidly reaches a value large enough to exactly balance the pressure difference. After this the inner level merely follows the temperature, which itself drops very slowly because of the bad thermal contact with the bath.

The flow apparatus was suspended in the helium cryostat, and the gravitational and thermal rates of flow were determined at various temperatures below the





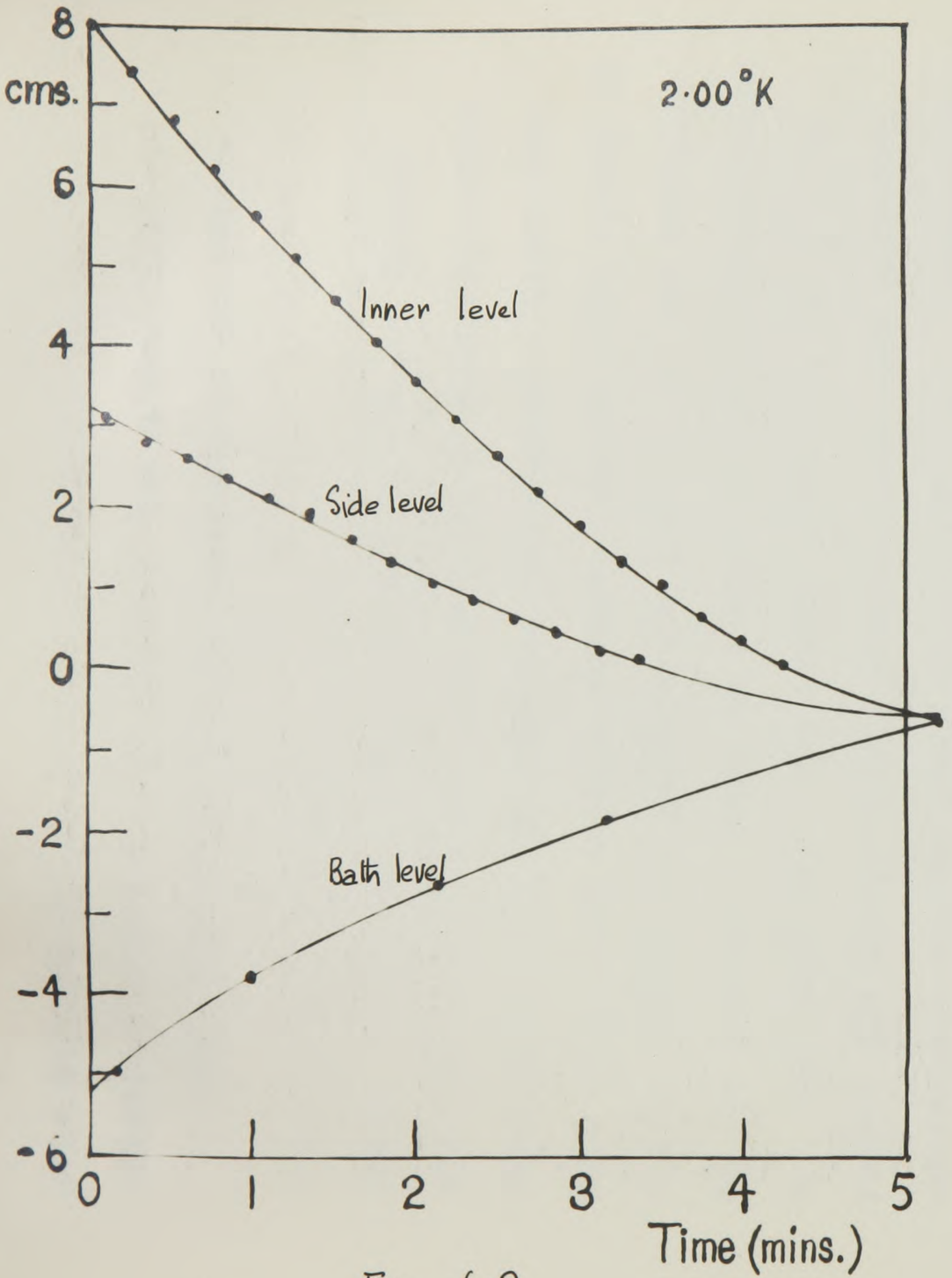

8
cms.
6
4
2
0
-2
-4
-6
2·00°K
Inner level
Side level
Bath level
0
1
2
3
4
5
Time (mins.)

Fig. 6-2

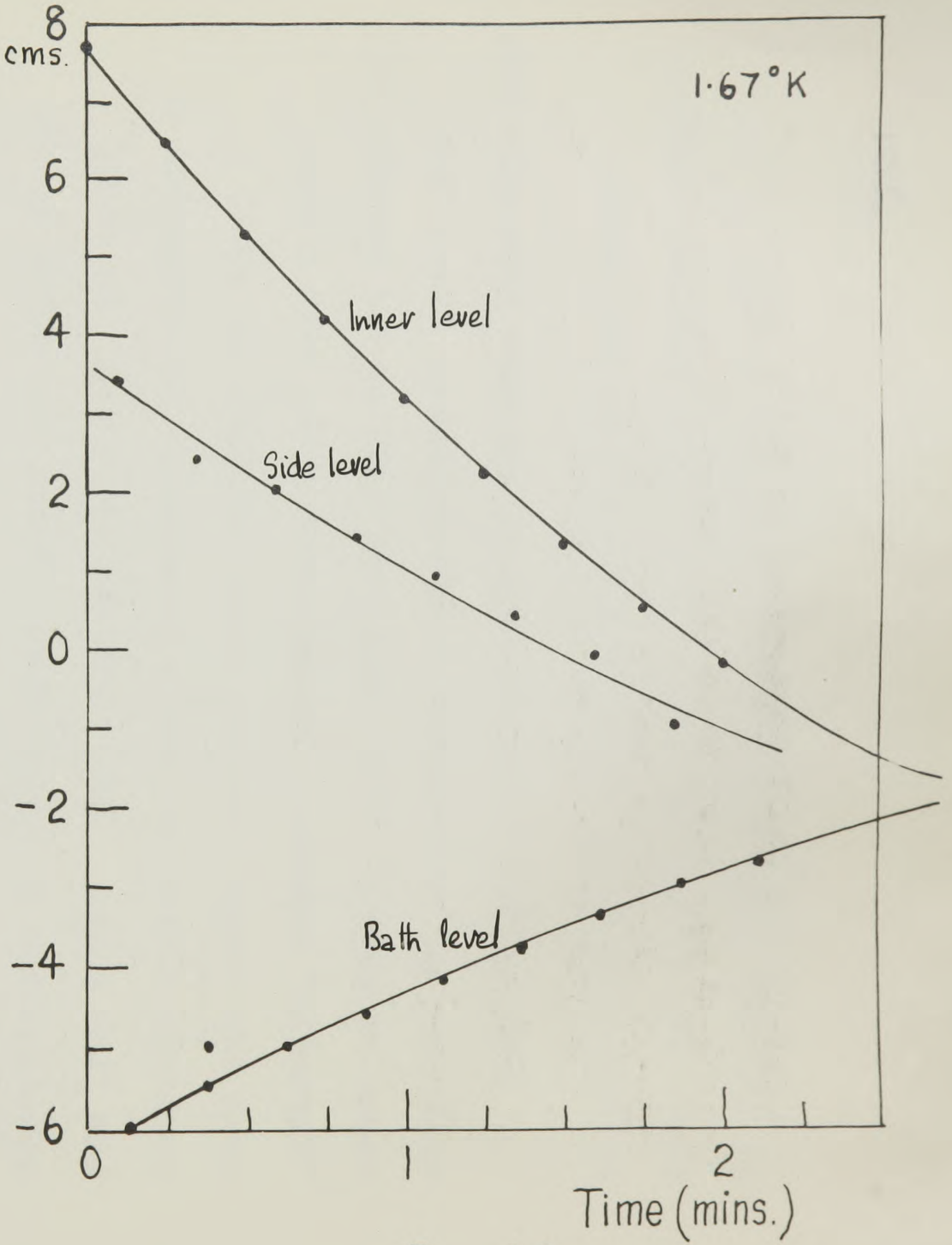

8
cms.
6
4
2
0
-2
-4
-6
1·67°K
Inner level
Side level
Bath level
0
1
2
Time (mins.)

Fig. 6-3.

λ-point. A check experiment at 2.6°K showed that the gravitational outflow at this temperature was 10^{-3}cm/min. (unless otherwise stated, rates so expressed will refer to the rate of change of the reservoir level). This, as will be seen later, is very small compared to the flow below the λ-point. The various features of the two kinds of flow below the λ-point are discussed in the two following sections.

6.21 Gravitational Flow

The reservoir was filled with helium II by immersion in the bath, and rapidly raised. Readings were taken of the levels in the reservoir, the side tube and the bath at suitable intervals of time. Figures 6-2 and 6-3 show two typical emptying curves. It is seen that the flow rate is pressure-dependent; a log-log analysis of the flow rate F as a function of the pressure difference showed that the dependence could be expressed as

$$F = k(\Delta p)^n \qquad (6.1)$$

where k was a constant for a given temperature, and n varied between 0.3 and 0.5. This variation may be due to the fact that the fall of the inner level was so rapid that it could not be timed with very great accuracy, and also to the difficulty of determining the position of the bath level exactly. But what was estab-

lished beyond doubt was that the flow rate was pressure-dependent, in sharp contrast to the flow through smooth channels of 10^{-4} cm width, where the flow was nearly pressure-independent.

The liquid level in the side tube indicates the pressure at a point between the two sections of rouge-packed capillary. It is seen from figures 6-2 and 6-3 that the position of the side level is between those of the inner level and the bath level during the entire flow. The ratio of the heights of the inner level and the side level above the bath was approximately constant, varying between 1.4 and 1.6 in the various experiments. This ratio is related to the ratio of the flow resistances in the two cases. This behaviour of the side level shows that the pressure drop occurs gradually along the length of the capillary, and hence the flow is not frictionless. The value of n in equation (6.1) indicates that the flow is of a turbulent nature.

6.22 Thermal Flow

For these experiments, the tops of the reservoir and side tube were sealed off with 'Fortafix' cement. The inflow rates were determined, at several temperatures, for different heating currents, by measuring the rate of rise of the inner level as soon as the current was

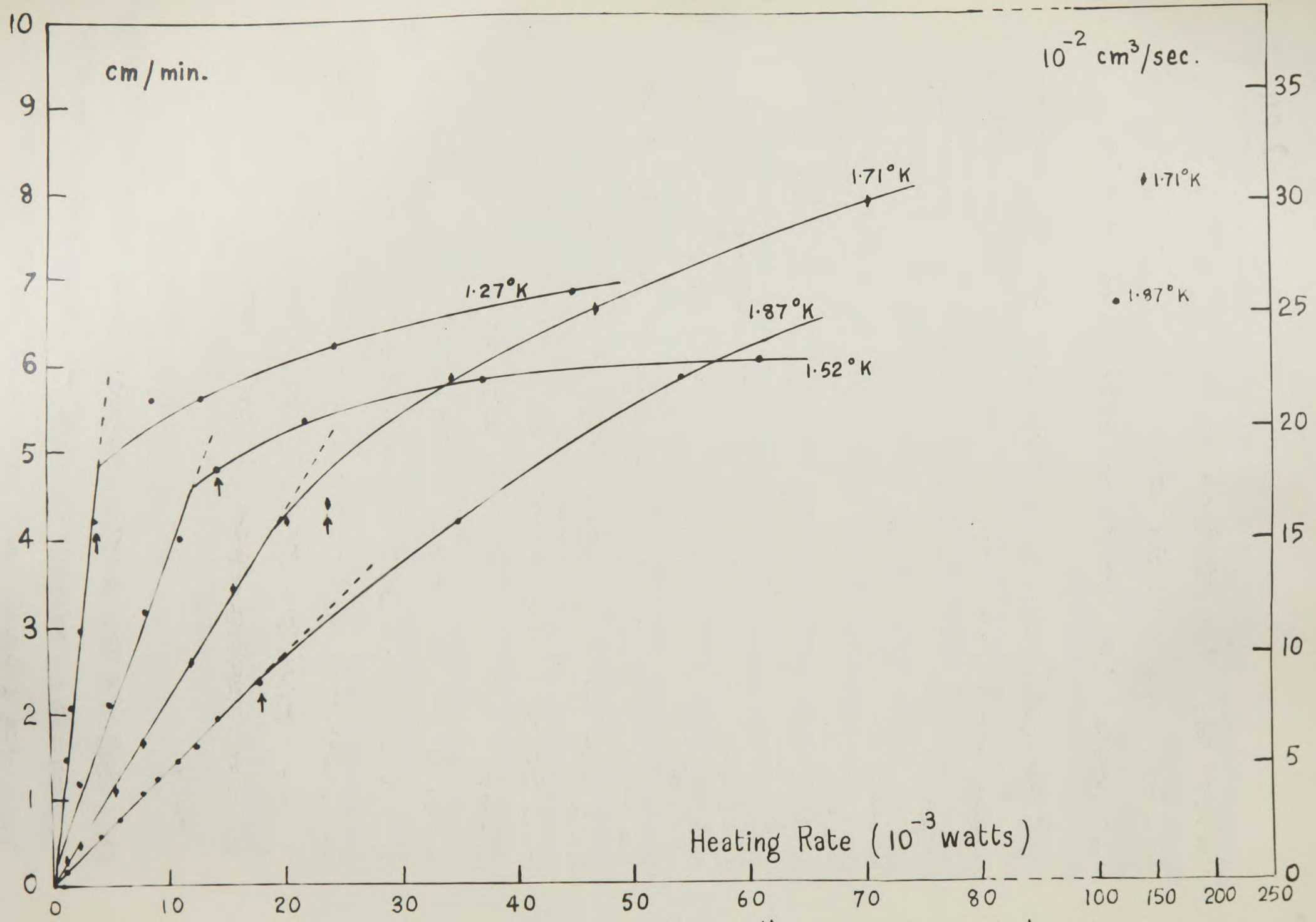


Fig. 6-4. Thermal flow through rouge-packed tube.

switched on. The behaviour of the side level, and of the inner level when the current was switched off, were also noted. The results are shown in figure 6-4.

For small rates of heating, the side level stayed steady, about 2 mm above the bath level (due to surface tension), while the inner level rose. As soon as the current was switched off, the inner level stopped, and started to fall slowly. This continued to happen till a critical rate of heating was reached, at which the side level started to go down, and continued falling as long as the current was on. When it was switched off, the side level rapidly returned to its equilibrium position near the bath, while the inner level continued to rise for some time, before coming to rest and falling slowly. This latter effect will henceforth be referred to as the 'overshooting' of the inner level. For heating rates larger than this critical value, the overshooting increased, as also the rate of depression of the side level. These two effects appeared simultaneously, and the corresponding points in figure 6-4 are indicated by arrows.

For the very largest heating rates used, the inflow became quite irregular; the inner level would rise, stop for a few seconds, and start rising again and so on.

When the heating rate was further increased, the helium inside was observed to boil. This suggests that the irregular behaviour was due to the pressure exerted by the vapour trapped in the reservoir, not being able to leak out fast enough through chance leaks in the Portafix seal. Hence the corresponding flow rates are not very reliable.

6.23 Discussion

A few words must be said, at the outset, about the practice, which is commonly adopted, of expressing the flow through fine channels by a simple power dependence on the pressure, as in equation (6.1). This equation implies that the flow vanishes as the pressure difference tends to zero, and therefore rules out the possibility of superflow. For, as has been proved for the film, and for very fine smooth channels, superflow is characterised by its ability to occur even as the pressure difference tends to zero. Thus the general equation describing the flow of helium II should be of the form

$$F = F_0 + F_1(p) \qquad (6.2)$$

where F_0 depends only on the geometry of the flow channel and the temperature, while F_1 describes the pressure dependence, in addition to being influenced by other

factors such as the temperature and size of channel. In the two limiting cases, i.e. the film and wide capillaries, F_1 and F_0 are respectively vanishingly small (chapter 3 of this thesis; Atkins 1951). But for intermediate sizes of channels, both terms should be taken into consideration. Thus a relation of the type of equation (6.1) is only approximate, and would not be applicable for small pressure differences in particular. The present results for rouge-packed tube have been expressed in this form, only to facilitate comparison with previous work.

The value (0.3 to 0.5) of the index n in equation (6.2) shows that the flow is pressure-dependent, and is probably in the nature of turbulent flow (for which the index n is 0.5).

There is, in fact, one other possible mechanism which can give a pressure-dependent flow. As the helium that flows out through the rouge-packed tube carries away very little entropy, the liquid left behind in the reservoir warms up. The temperature difference between the reservoir and the bath may eventually reach a value large enough to set up a thermomechanical pressure which opposes, and decreases the effect of, the hydrostatic pressure between them. The very small gravitational

flow observed with the top of the reservoir sealed off was an extreme example of this process, where heat exchange between the reservoir and the bath was almost completely inhibited. Even with the top of the reservoir open, it is conceivable that equality of temperature between it and the bath is not attained rapidly enough not to influence the rate of flow.

But the unsuccessful attempts which were made to produce thermal flow with the reservoir open, prove conclusively that the above cannot be the case. At 1.87°K, for example, a heating rate of 12.8×10^{-3} watt with the reservoir open, produced a fountain head of 2 mm. This is approximately the critical heating rate at this temperature, and is therefore also the order of the rate at which mechano-caloric heat is evolved during gravitational outflow. This proves that the heat evolved during gravitational outflow cannot have any significant effect on the dependence of the flow rate on the pressure difference, except when the latter is a few mm. of liquid helium or less. A simple calculation shows that the rate at which helium is lost by evaporation from the reservoir, due to the mechano-caloric heat, is H/L times the rate at which it is flowing out*. This introduces an error of 10% at 2.0°K,

---

*L = latent heat of vaporisation of liquid helium. This

assumes (1) the entropy of helium flowing out is zero, (2) the heat of transport for this particular arrangement is equal to H, the enthalpy of helium.

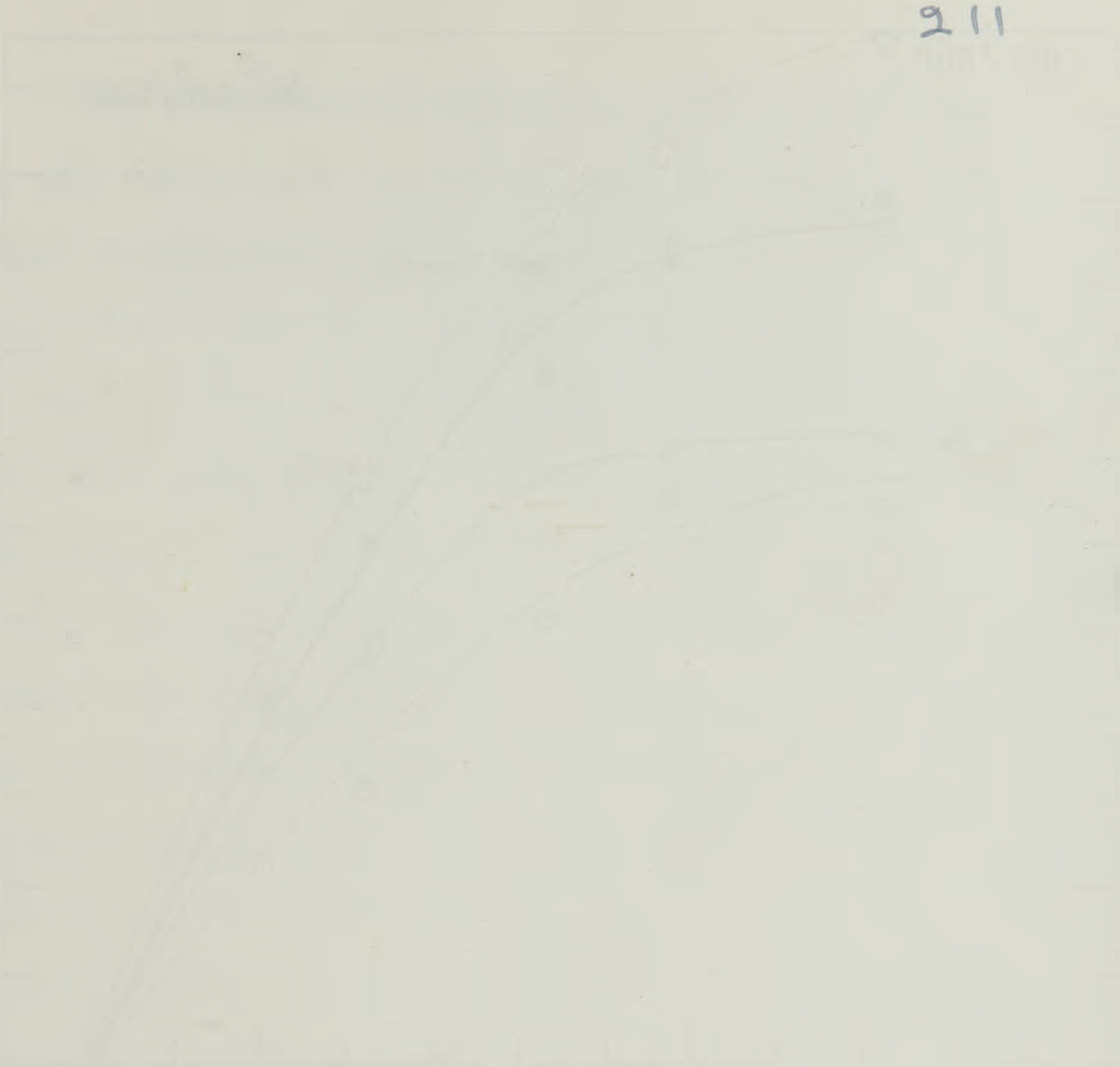
---

and very much less at lower temperatures. In any case, the loss by evaporation due to this mechanism, at any given temperature, forms a constant fraction of the true flow rate, and hence will not affect the relations between the flow rate and the pressure difference that are relevant to the present discussion.

The fact that there is a dissipation of energy along the tube is even more clearly brought out by the position of the side level (figures 6-2 and 6-3) during the flow. This behaviour of the side level cannot be attributed to the flow of the normal component, for the observed flow above the λ-point was less than 10^{-3} times the flow at, for example, 2.0°K, for similar pressure heads. The corresponding ratio of the viscosities of the normal fluid is about 0.3, and this makes it impossible to interpret the gradual drop of pressure in the tube as caused by normal flow. This presents therefore a remarkable instance of flow which is non-viscous, but still shows energy dissipation. This type of flow may be called 'turbulent superflow'. It will be seen later that this concept may have a profound significance in a large number of flow phenomena observed in helium II.

The flow under a temperature difference (figure 6-4) also shows very interesting features. At each temperature, the flow rate F is proportional to the heating rate $\dot{Q}$ up to a critical rate $\dot{Q}_c$ (corresponding to a flow rate F_c), beyond which the rate of increase of F with $\dot{Q}$ decreases gradually; there is thus a kink in the curve of F versus $\dot{Q}$. The similarity of this behaviour with that observed in the helium II film (figure 3-7) and smooth narrow channels (figure 3-8) should be noticed. We shall call the two regions $\dot{Q} < \dot{Q}_c$ and $\dot{Q} > \dot{Q}_c$ the linear and non-linear regions respectively, and consider each in detail.

The interpretation of the linear region is similar to that of the corresponding case for film transfer (chapter 3). The flow here is frictionless; this is substantiated by the side level staying close to the bath level during the flow. The height of the liquid helium in the reservoir at any instant is exactly enough to compensate the thermomechanical pressure. The heat of transport calculated for this flow was found to be slightly higher than for the film, but this is possibly because the apparatus was not specially designed to be adiabatic. The possibility of a constant frictional force must be ruled out, because of the behaviour of the side level.



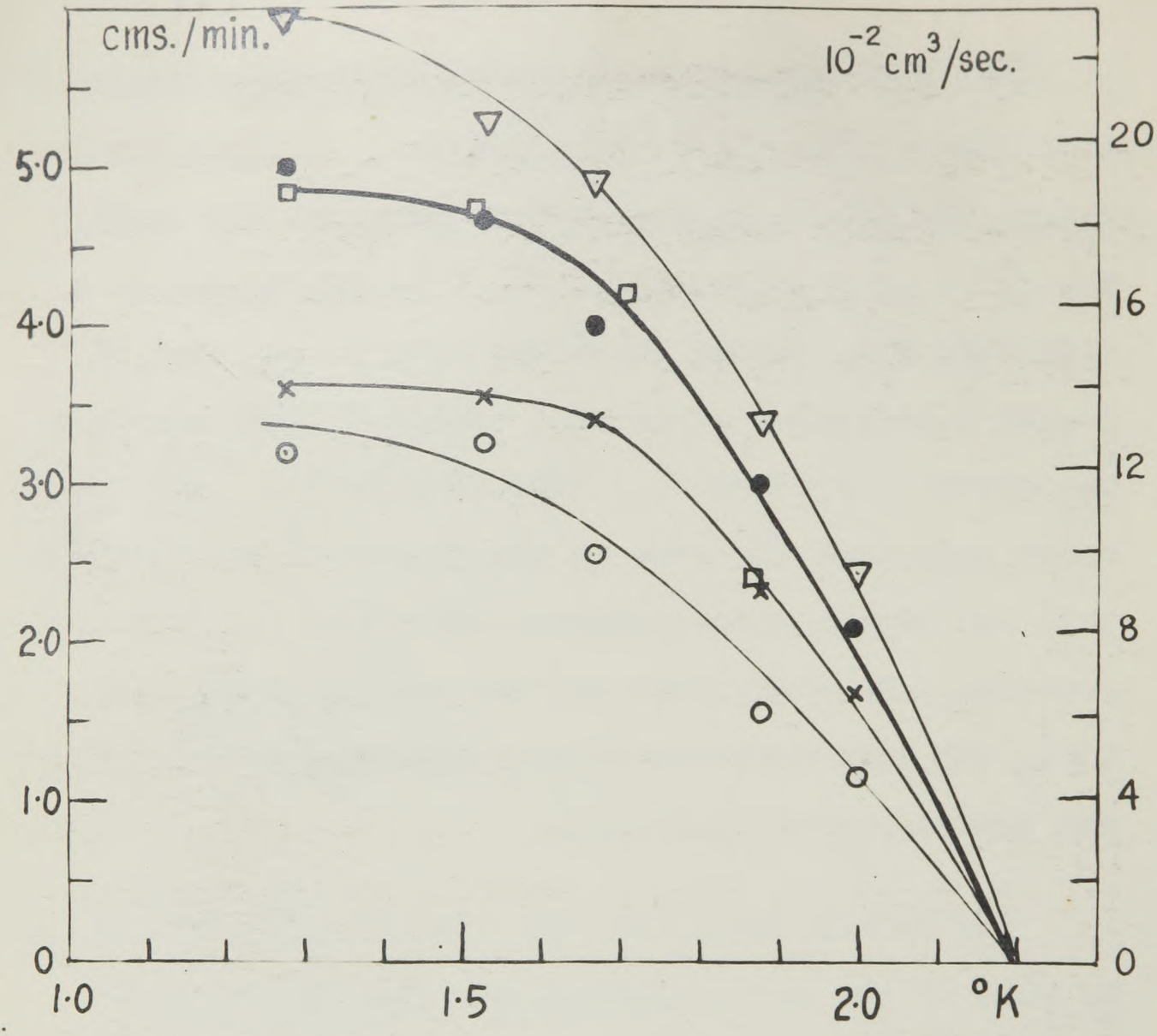


Fig. 6-5. Comparison of thermal and gravitational flows in rouge-packed tube.

□ thermal critical rate of flow.

Gravitational rates of flow under a pressure of Δp cms. of liquid helium:

⊙ $\Delta p = 2$ cms.

× $\Delta p = 4$ cms.

● $\Delta p = 8$ cms.

▽ $\Delta p = 12$ cms.

The inception of friction is indicated by the kink in the curve of F versus $\dot{Q}$, and this is supported by two other independent observations: the behaviour of the side level, and the 'overshooting' of the reservoir level. As long as $\dot{Q}$ is less than $\dot{Q}_c$, the thermomechanical pressure difference is instantaneously compensated by the flow of helium at a sub-critical rate. But when $\dot{Q} > \dot{Q}_c$, the helium cannot flow in fast enough, and this causes the side level to drop, and also the helium in the reservoir to overshoot, i.e. the temperature difference between the reservoir and the bath becomes larger than can be compensated by the level difference. Thus when the heating current is switched off, the helium continues to flow into the reservoir, till an equilibrium fountain head is attained. It is thus clear how the above observations on the reservoir and side levels give proof of the existence of a critical rate F_c.

While each of the two groups of phenomena, of gravitational flow and thermal flow, is consistent within itself, curious discrepancies appear when the two are compared. In figure 6-5, the critical rate F_c, deduced from the thermal flow measurements, is plotted as a function of temperature ($\square$). Also shown are the gravita-

tional flow rates for pressure heads of 2,4,8 and 12 cms of liquid helium. For every one of these gravitational rates, the flow showed frictional dissipation; while for the entire range of thermal rates less than the critical ones F_c, the flow was frictionless. We thus have the result that for a range of flow rates, thermal flow is frictionless, while gravitational flow is not. This is the only type of flow in which this remarkable result has been obtained. In the case of the helium II film, both gravitational and thermal flow at sub-critical rates are frictionless (chapters 2 and 3 of this thesis). Similar results have been obtained for flow through narrow smooth channels (Bowers & Mendelssohn 1952). It would thus seem that the flow properties of helium II through narrow tortuous channels, such as the rouge-packed tube offers, are significantly different from those in smooth channels of approximately the same width. In the latter case, the pressure at an intermediate point is uniquely determined by the velocity of flow. But for flow through a rouge-packed tube, even for the same rate of flow, the intermediate pressure depends on the mechanism by which the flow is maintained. It appears that the intermediate pressure in superflow is not simply a function of the end pressures, but requires a knowledge of other parameters to determine it.

In figure 6-5, it is seen that the gravitational rates for a pressure head of 12 cms of liquid helium are greater than the corresponding thermal critical rates. All the other gravitational rates are equal to, or less than, the thermal ones. It is tempting to suggest that the thermal rate sets a limit for the flow rate under isothermal conditions. This has certainly been verified for the film, and for smooth narrow channels, and therefore one might, a priori, expect the same to hold for the rouge-packed tube as well. It is certainly true that the determination of the flow rate, for large potential differences, is subject to some considerable error because of the steepness of the curves of height versus time in this region. On the other hand, it has already demonstrated that the flow through rouge-packed tubes has some characteristics which are not possessed by superflow in other kinds of channels. Further experiments are planned on flow under even larger pressure differences. In the present series, a limit was set on the greatest pressure difference that could be used, by the size of the cryostat.

Thus the two remarkable results of these experiments on flow through rouge-packed tubes are (1) the concept of 'pressure' in the flow of liquid helium II requires clarification, and (2) the superflow of helium II

can be sometimes turbulent. Prigogine (1951) suggests 'pressure' in helium II loses its simple classical meaning, and becomes dependent on the relative motion of the two fluids (see also Dingle 1952). This result, he claims, follows from the application of irreversible thermodynamics to the equations of motion.

The second result, on turbulent superflow, may have far-reaching consequences in the current interpretation of a large variety of flow phenomena. The only empirical theory which has tried to account for the phenomena of large velocities is the Gorter-Mellink theory of mutual friction. It is obvious that its application to experiments such as those with an oscillating disc (Hollis-Hallett 1950, 1952) or the flow of helium II through capillaries (Atkins 1951) will be considerably modified if the possibility of turbulence in the superfluid is admitted. On the other hand, F.London (1951) suggests that superfluidity is linked with the condition

$$\text{curl } v_s = 0$$

where v_s is the velocity of the superfluid, and this precludes the type of turbulence suggested above. The way out of this difficulty may lie in a re-examination of the basis of the two-fluid model, along the lines indicated by Dingle(1952).

In conclusion, it can be said that the present

experiments indicate the limitations of the simple two-fluid model. They emphasise the fact, which is sometimes overlooked, that the two-fluid model, as originally conceived by Tisza (1947), is applicable only to small velocities and reversible processes. But in a case like the present, where there is an irreversible dissipation of energy, the simple model breaks down, and perhaps the development of the hydrodynamical equations of liquid helium for large velocities may offer a solution.

References to Chapter 6

Allen, J.F. & Misener, A.D., 1939, Proc.Roy.Soc. A, 172, 467.

Atkins, K.R., 1951, Proc.Phys.Soc. A, 64, 833; 1952, Advances in Physics, 1, 169.

Bijl, A., de Boer, J.H. & Michels, A., 1941, Physica, 8, 655.

Bowers, R., Chandrasekhar, B.S. & Mendelssohn, K., 1950, Phys.Rev., 80, 856.

Bowers, R. & Mendelssohn, K., 1950, Proc.Phys.Soc. A, 63, 178; 1952, Proc.Roy.Soc. A, 213, 138.

Daunt, J.G. & Mendelssohn, K., 1939, Nature, Lond., 143, 719.

Dingle, R.B., 1952, Advances in Physics, 1, 111.

Gorter, C.J. & Mellink, J.H., 1949, Physica, 15, 285.

Hollis-Hallett, A.C., 1950, Proc.Phys.Soc. A, 63, 1367; 1952, Proc.Roy.Soc. A, 210, 404.

Kapitza, P.L., 1941, J.Phys.U.S.S.R., 4, 181; 5, 59.

Keesom, W.H. & Duyckaerts, G., 1947, Physica, 13, 153.

London, F., 1951, Proc.Int.Conf. Low Temp.Phys.(Oxford), p.2.

Meyer, L. & Mellink, J.H., 1947, Physica, 13, 197.

Prigogine, I., 1951, Proc.Int.Conf. Low Temp.Phys.(Oxford), p.100.

Tisza, L., 1947, Phys.Rev., 72, 838.

Zilsel, P.R., 1950, Phys.Rev., 79, 309.

Acknowledgements

I am grateful to Professor Lord Cherwell, F.R.S., for this opportunity to work in the Clarendon Laboratory, and to Professor F.E.Simon, F.R.S., for his continued interest in the research. I am deeply indebted to Dr.K.Mendelssohn, F.R.S., my supervisor, who has been a source of inspiration, and ever generous with his help and advice. Mr.F.Lenthall of the workshop and Mr.D.G. Saxton of the glass-blowing shop have earned my gratitude for the untiring skill with which they made the necessary apparatus.

I would also like to thank Mr.D.F.Brewer, Dr.R.Bowers, Mr.G.R.Heraus, Mr.N.G.McCrum, Dr.J.L.Olsen, Dr.J.Wilks, and the many others, who have, at various times, helped me with stimulating discussions, and experimental observations.

I am grateful to Messrs. H.Sarrell and F.H.Holt of the Metrology Division, National Physical Laboratory, for their most helpful co-operation with the experiments on the stainless steel beakers, described in chapter 5.

The munificence of the Rhodes Trust has made this stay in Oxford possible.