

Large spin entangled current from a passive device

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Abstract. We show that a large entangled current can be produced from a very simple passive device: a cluster of three resonant quantum dots, tunnel coupled to one input lead and two output leads. Through a rapid first-order resonant process within the cluster, entangled electron pairs are emitted into separate leads. We show that the process is remarkably robust to variations in systems parameters. The simplicity and robustness should permit experimental demonstration in the immediate future. Applications include quantum repeaters and unconditionally secure interfaces.

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1. Introduction

Entanglement lies at the heart of quantum information processing. It is an essential resource that must be generated and consumed in the execution of quantum algorithms [1]. The ability to generate entanglement between elements that can be well separated spatially would be particularly powerful. In the context of computation, this would allow the linking of small, nanoscale quantum registers [2], or the building of cluster states in a distributed architecture [3, 4]. Moreover, it would enable specific few-qubit functions related to quantum communication; this includes quantum repeaters [5] for sharing entanglement over arbitrarily long distances and certain approaches to quantum key distribution. The latter may even enable a secure local interface such as an ATM that is invulnerable to malicious devices interposed between the internal mechanism and the user's identification card [6].

There have been a number of recent proposals for generating such an entanglement between electron spins that can be subsequently separated by macroscopic distances. For example, two static spins can be entangled by a third passing spin [7, 8] or by a sea of conduction electrons [9]. Alternatively, a divided spin chain [10] can be used in which a single excitation splits into two entangled parts. A further idea is that a current carrying lead can somehow bifurcate, producing pairs of entangled spins that propagate down different leads [11, 12].

In this paper, we focus on the goal of creating a *large* current of spin-entangled electron pairs, from the *simplest possible* passive solid-state device. Our solution is illustrated in figure 1; it consists of a single input lead *C* and two output leads *A* and *B*, coupled to a core formed of three quantum dots (QDs). The whole device would typically be lithographically defined by gates that can deplete a two-dimensional electrons gas [13, 14]. Such a structure allows for exquisite control of both electron energies and tunnel couplings. Our device would require an initial configuration, but would then run passively. In what follows we will show that, under suitable conditions, two electrons enter the QDs and their spins become entangled with each other, and they then leave with this entanglement still intact. Moreover, the entanglement is channeled such that one electron leaves through one output lead, and the other electron leaves through the other lead. This is achieved through a resonant transfer of the entangled electrons, one each to dots *A* and *B*. We shall show that the entanglement generation and separation process within the QD system occurs on a timescale set directly by the coherent tunneling rate: this is faster than an alternative entangling device discussed in [11], which relies on a

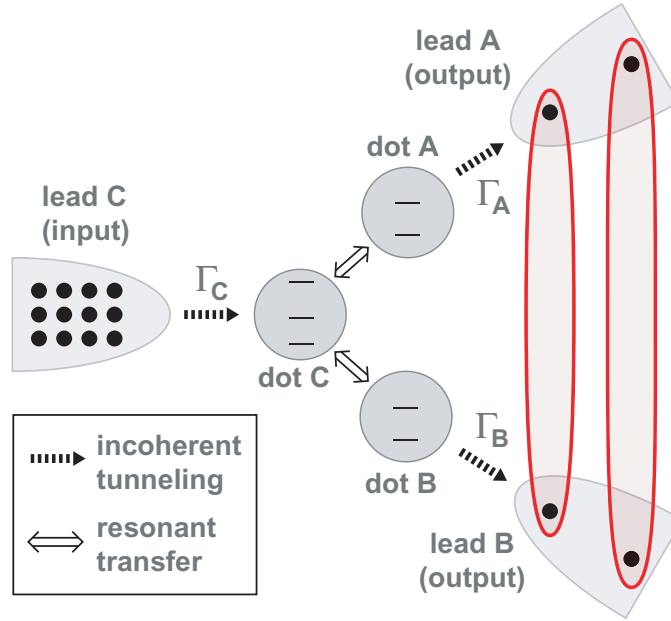


Figure 1. The entangler consists of a cluster of three QDs, each of the three being coupled to its own lead. Electrons tunnel incoherently from the input lead (left) onto dot C, populating a singlet state on that dot. This state is coherently coupled to two other states of the cluster; from these states the electrons can incoherently tunnel to the output leads. For certain parameter regimes, the electrons in each entangled pair will exit to *different* leads with high probability.

second-order tunneling process. Our device is fully resonant which allows us to generate a variety of entanglement qualities at different rates by changing system parameters.

We begin with an outline description of the process, then we proceed to an analytic treatment and finally a numerical model of the open dynamics.

2. The model system

Our protocol is schematically depicted in figure 2. Suppose the dot cluster is initially uncharged. Dot C receives an electron from its lead; this dot is less tightly confined than A or B so that the lone electron cannot resonantly transfer to those dots. However, the lowest two-electron state of dot C, i.e. the singlet state, is below the lead chemical potential and therefore a second electron can enter, populating this state. Resonant tunneling between dots is now possible. Double occupancy of dot A or of dot B will not occur because of their tighter confinement; therefore the potential resonant states are $\{|002\rangle, |101\rangle, |011\rangle, |110\rangle\}$ (using the notation $|n_A, n_B, n_C\rangle$). We find that asymmetry between A and B leads to exclusion of $|011\rangle$, and the remaining three-state dynamics leads to the desired emission to separate leads.

The three QDs have an internal Hamiltonian given by:

$$\mathcal{H} = \sum_{i,\sigma} \epsilon_i n_{i,\sigma} + \sum_{i,j,\sigma,\sigma'} U_{i,j} n_{i,\sigma} n_{j,\sigma'} + \sum_{i \neq C, \sigma} g_i (c_{C,\sigma}^\dagger c_{i,\sigma} + c_{i,\sigma}^\dagger c_{C,\sigma}), \quad (1)$$

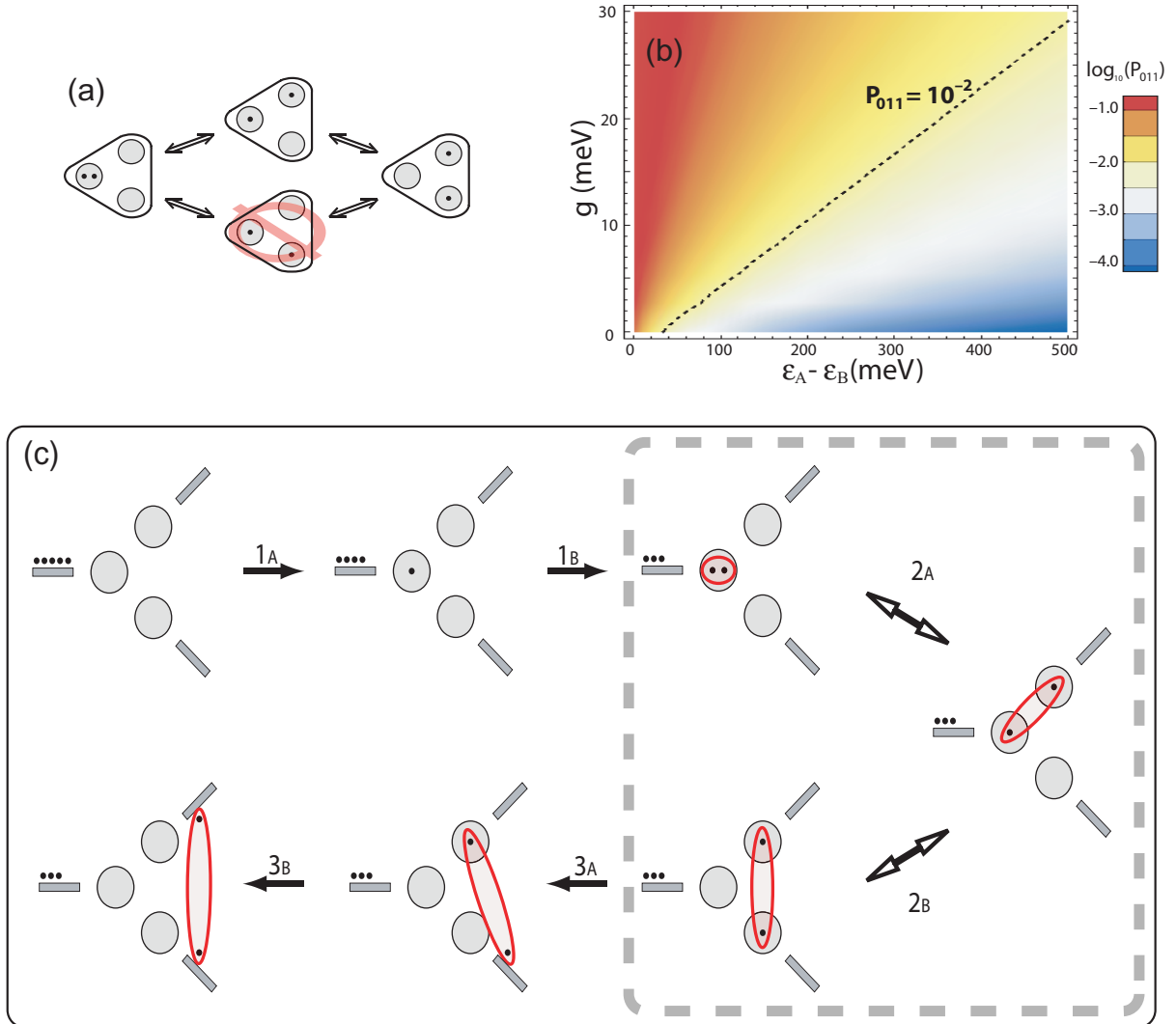


Figure 2. (a) The electrons may coherently tunnel from state $|002\rangle$ to $|110\rangle$ via two possible routes; we require that one route is suppressed. (b) Average population within the suppressed state $|011\rangle$ as a function of detuning between A and B , $\epsilon_A - \epsilon_B$, and resonant coupling strength g . We see that suppression of state $|011\rangle$ may be achieved for a wide range of the parameter space. (c) Resulting cycle of events; incoherent (and irreversible) tunneling events are denoted by single-headed arrows, while transitions between resonant states are shown by double-headed arrows. Steps 2A and 2B simply illustrate the route via which electrons tunnel through the device—the intermediate state $|101\rangle$ is never fully populated.

where $c_{i,\sigma}$ is the annihilation operator for an electron of spin σ on dot $i \in \{A, B, C\}$ and $n_{i,\sigma} \equiv c_{i,\sigma}^\dagger c_{i,\sigma}$. The first term represents the single particle energy ϵ_i for each dot, the second Coulomb repulsion $U_{i,j}$ between electrons on dots i and j . The dots are arranged such that tunneling is only significant between dot C and either A or B , given by tunneling strengths

$g_{A,B}$. Furthermore, it is assumed that U_{CC} dominates over all other repulsion terms and for simplicity we may set $U_{CC} = U$, $U_{AC} = U_{BC} = V$ and take $U_{AB} = 0$.

Each lead i is coupled to the neighboring QD i through incoherent tunneling of magnitude Γ_i . We bias the device and set the lead chemical potentials μ_i such that transport occurs from lead C to leads A and B . To ensure that two electrons may enter dot C and that their spin state is projected into the singlet state, we require that:

$$\mu_C > \epsilon_C; \quad \epsilon_C + U_{CC,T} > \mu_C > \epsilon_C + U_{CC,S}. \quad (2)$$

Meanwhile, to allow electrons to tunnel through dots, we require that $\mu_{A,B} < \epsilon_{A,B}$.

The incoherent tunneling processes can then be described by super-operators \mathcal{L}_i acting on the QD density matrix ρ :

$$\begin{aligned} \mathcal{L}_A \rho &= \Gamma_A \left[c_{A,\sigma} \rho c_{A,\sigma}^\dagger - \frac{1}{2} \left\{ c_{A,\sigma}^\dagger c_{A,\sigma}, \rho \right\} \right], \\ \mathcal{L}_B \rho &= \Gamma_B \left[c_{B,\sigma} \rho c_{B,\sigma}^\dagger - \frac{1}{2} \left\{ c_{B,\sigma}^\dagger c_{B,\sigma}, \rho \right\} \right], \\ \mathcal{L}_C \rho &= \Gamma_C \left[c_{C,\sigma}^\dagger \rho c_{C,\sigma} - \frac{1}{2} \left\{ c_{C,\sigma} c_{C,\sigma}^\dagger, \rho \right\} \right]. \end{aligned} \quad (3)$$

In general, the rates Γ_i will be functions of the density of states and the Fermi function evaluated at the energy difference between the chemical potentials μ_i and the respective on-site energies. However, in the present work the explicit values of the rates are unimportant, so long as they obey a strict hierarchy that we shall present shortly.

2.1. Resonant transfer

We require that the three states $|002\rangle$, $|101\rangle$ and $|110\rangle$ are on resonance. This may be achieved if the following conditions are met:

$$\epsilon_C - \epsilon_A = V - U, \quad \epsilon_C - \epsilon_B = -V. \quad (4)$$

Meanwhile, to suppress transfer via state $|011\rangle$, we require that:

$$\begin{aligned} |(\epsilon_C - \epsilon_B) + (U - V)| &\gg |g_B|, \\ |(\epsilon_C - \epsilon_A) + V| &\gg |g_A|. \end{aligned} \quad (5)$$

From conditions (4) and (5) we obtain a criterion relating the Coulomb terms: $(U - 2V) \gg g_{A,B}$. The resonance condition (4) can then be satisfied by careful tuning of the on-site energies such that $\epsilon_A - \epsilon_B = U - 2V$. On-site and Coulomb repulsion energies are typically an order of magnitude larger than coherent tunneling strengths: $U, \epsilon \approx 100 \mu\text{eV}$ and $g \approx 10 \mu\text{eV}$ [15]–[17]. Thus conditions (4) and (5) can indeed be met experimentally. Therefore, the essential coherent dynamics are governed by a simple Hamiltonian:

$$H' = \sqrt{2}g_A c_{2C}^\dagger c_A \left(1 - c_B^\dagger c_B\right) + g_B c_C^\dagger c_B c_A^\dagger c_A + \text{h.c.} \quad (6)$$

where we have introduced the new operators $c_C^\dagger = |1\rangle_C \langle 0|$ and $c_{2C}^\dagger = |2\rangle_C \langle 1|$. Note that these operators now simply describe the creation and annihilation of charge states. The first term corresponds to the tunneling $|002\rangle \leftrightarrow |101\rangle$ and has an enhanced coupling strength $\sqrt{2}g_A$ due to the fact that either electron on dot C may tunnel to dot A . Meanwhile, the second term corresponds to $|101\rangle \leftrightarrow |110\rangle$.

Now, the protocol is optimally efficient when the resonant dynamics lead to complete transfer of the electrons to dots A and B . However, if we assume that the inherent tunneling strengths are equal: $g_A = g_B = g$, then we never achieve complete transfer from dot C to dots A and B . To ensure complete transfer, we require that overall tunneling strengths are identical. We must therefore compensate for the enhancement, tuning the tunneling strengths such that $g_A = g_B/\sqrt{2} = g/\sqrt{2}$. This tuning may be achieved by applying a potential from a gate sitting between dots A and C . In this paper, we shall consider just the second case.

Electrons must tunnel off the device (step 3) on a timescale that is fast compared with the refill time Γ_C^{-1} of the system to avoid another electron entering the dot region before the first two leave, which may destroy the created entangled state. Thus $\Gamma_B, \Gamma_A \gg \Gamma_C$. Ideally, one of the pair of electrons should tunnel into lead B before lead A ; this prevents the remaining electron from being ‘stuck’ on dot C (i.e. $\Gamma_B \gg \Gamma_A$). However, in what follows we shall see that it is possible to relax this condition.

3. Monte Carlo simulations

In order to completely characterize the performance of the device for real parameters, we now exploit a quantum trajectories formalism [18] to describe the electron dynamics:

$$\begin{aligned} \rho(t+dt) = & -i[H', \rho(t)]dt + \sum_{i \in \{A, B\}} (\text{Tr}\{\mathcal{J}_i \rho\} \rho - \mathcal{A}_i \rho) dt \\ & + \sum_{i \in \{A, B\}} \left(\frac{\mathcal{J}_i \rho}{\text{Tr}\{\mathcal{J}_i \rho\}} \right) dN_i(t) + \mathcal{L}_C \rho dt + \rho(t), \end{aligned} \quad (7)$$

where $\mathcal{J}_i \rho = \Gamma_i c_i \rho c_i^\dagger$ represents the jump super-operator corresponding to an event where an electron hops off dot $i \in \{A, B\}$. $\mathcal{A}_i = \frac{\Gamma_i}{2} \{c_i^\dagger c_i, \rho\}$ and $dN_i(t)$ is the stochastic increment taking the values $\{0, 1\}$, which denotes the number of electrons that hop off dot i in the time interval $t, t+dt$.

We use a Monte Carlo method to generate values of the stochastic increment $dN_i(t)$: at every time step two random numbers r_i are generated, and if $r_i < \text{Tr}\{\mathcal{J}_i \rho\}dt$ then the stochastic increment takes the value $dN_i(t) = 1$. We then generate simulations of individual hopping events in two scenarios. The first, which we shall call *clean*, is depicted in figure 3(a) and corresponds to the regime where $\Gamma_B \gg \Gamma_A$. In this case, electrons hop onto the two leads in pairs, which from our previous argument are spin entangled with one another. The pairs are clearly grouped together, with typical pair–pair separation $2\Gamma_C^{-1}$. For the parameters considered here, this corresponds to a total current through the device of $I_{\text{total}} = 10$ pA. The second regime, which we shall call *dirty* is shown in figure 3(b) and corresponds to $\Gamma_B = \Gamma_A$. This simulation includes both pairs and single events because it is now possible for an electron to enter the system before both electrons from the previous cycle have left. However, the relaxed parameter constraints in this latter case may prove more experimentally accessible and so comparing the clean and dirty regimes allows us to explore the robustness of our protocol.

4. Post-selection

When two electrons leave the system that are closely separated in time they are almost always emitted into different leads, even in the *dirty* scenario. Such electrons have a high probability of

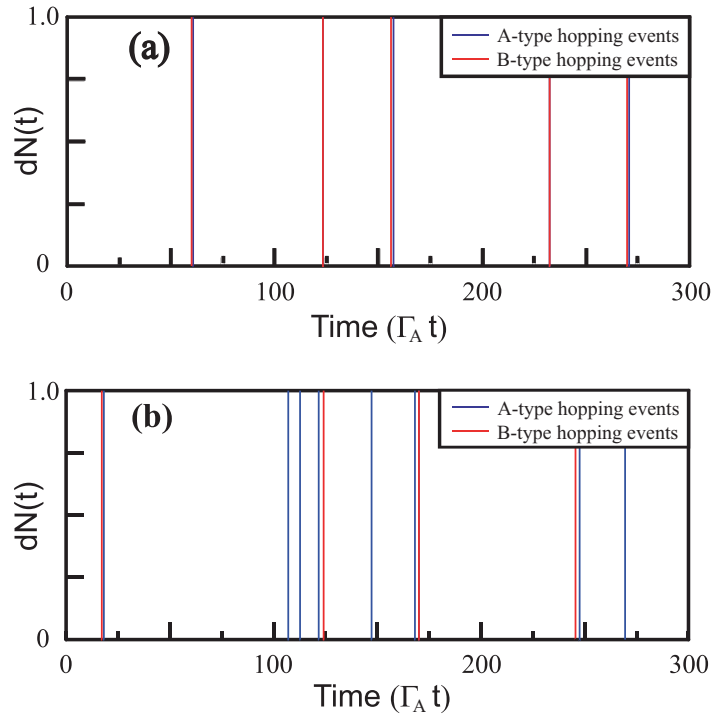


Figure 3. Plot hopping events from dot A/B to lead A/B as a function of time, from typical runs of the simulation. Two regimes are shown: (a) *clean*: $\Gamma_B = 10 \Gamma_A$, $\Gamma_C = \Gamma_A/25$ and $g = 10 \Gamma_A$; (b) *dirty*: $\Gamma_B = \Gamma_A$, $\Gamma_C = \Gamma_B/25$ and $g = 10 \Gamma_A$. Within the clean trace, the second and fourth lines correspond to a pair of hopping events, from B then A , whose separation cannot be resolved on this plot.

being entangled with one another, moreover one could further post-select such ‘good’ pairs by choosing only those electrons whose temporal separation is below some threshold. Any device of the kind we are describing emits electrons probabilistically and so must have some kind of electron detection system ‘down stream’ of the entanglement generator to collect the pairs prior to subsequent processing. Here, we can exploit that necessary detection system as a kind of filter, allowing us to identify those pairs that are most likely to be ‘good’ Bell pairs⁵. We emphasize that this does not involve any additional complexity beyond that which must be present in any case.

Post-selection will necessarily involve a decrease in the rate of production of acceptable pairs. We now proceed to calculate this effective rate. We define the rate of production in terms of the probability that the two electrons propagate down different leads. The probability that we see an electron hopping off dot j at time $t + \Delta$ given that an electron has hopped off dot i at some time t earlier, with no events in the interim, is given by $\langle \mathcal{J}_j(t + \Delta) \mathcal{J}_i(t) \rangle dt d\Delta$. Then, the

⁵ In practice, the post-selection would be accomplished through charge detection followed by a filtering process. In general, one might need to perform active gating downstream predicated on specific charge detection results. However, for specific applications e.g. communication protocols, we may simply move this filtering to the end, post-selecting on the bits within our final classical bit-string that are associated with good, entangled pairs of electrons.

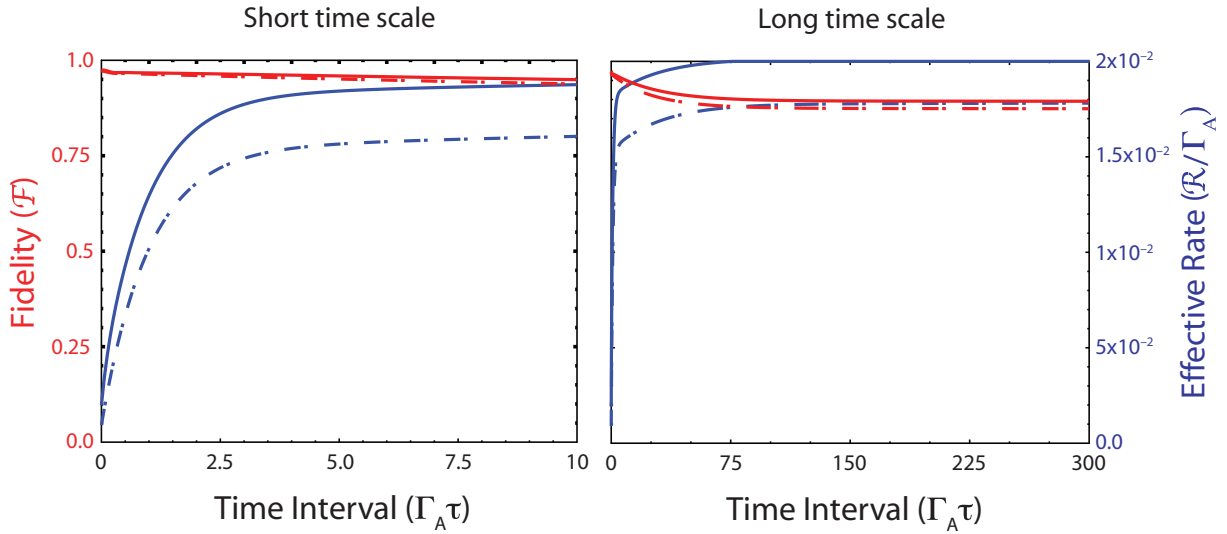


Figure 4. Effective rate of production of ‘good’ pairs of electrons (blue) and their entanglement fidelity (red) for short and long timescales in the two parameter regimes *clean* (solid) and *dirty* (dashed), as defined in the caption of figure 3.

probability that two electrons separated by a time interval Δ propagate down different leads is obtained by:

$$q(\Delta) = \int_0^\infty (\langle \mathcal{J}_A(t+\Delta) \mathcal{J}_B(t) \rangle + \langle \mathcal{J}_B(t+\Delta) \mathcal{J}_A(t) \rangle) dt. \quad (8)$$

If we post-select all pairs with a temporal separation of less than τ then the rate of production of pairs, $\mathcal{R}(\tau)$, is then given by

$$\mathcal{R}(\tau) = \frac{\Gamma}{2} \int_0^\tau q(\Delta) d\Delta. \quad (9)$$

The effective rates for the *clean* and *dirty* regimes are plotted in figure 4. As the selection time interval increases, the rate of ‘good’ pair production also increases, for both regimes. In both cases there is a short initial period where the effective rates increase rapidly: this is a signal from electrons already in the system which typically tunnel off at the rate Γ_A . There follows a longer period where the curves rise much more slowly: this is associated with the time needed for new electrons to enter the system, which is typically Γ_C^{-1} . As expected, we find that the maximum rate is larger for the *clean* regime than for the *dirty* regime.

The rate of pair production is meaningless unless we have a measure of how entangled those pairs are. This can be determined by the probability that the sequence of events depicted in figure 2, which definitely produces an entangled state, has occurred. This probability is

$$p(\Delta) = \int_0^\infty (p_{A,B}(t, t+\Delta) + p_{B,A}(t, t+\Delta)) dt, \quad (10)$$

where

$$p_{i,j}(t, t+\Delta) = \langle (1 - n_i(t+\Delta)) \mathcal{J}_j(t+\Delta) n_j(t) \mathcal{J}_i(t) \rangle. \quad (11)$$

The initial state $\rho_0 = (1 - n_A)(1 - n_B)\rho_{SS}$ is the solution for the system in a steady state, projected onto the subspace with no electrons in either dot A or dot B . The probability of definitely obtaining an entangled pair whose separation in time is less than τ is then

$$\mathcal{P}(\tau) = \frac{\int_0^\tau p(\Delta) d\Delta}{\int_0^\tau q(\Delta) d\Delta}, \quad (12)$$

where we have normalized with the probability $q(\Delta)$ (as defined in equation (8)).

If the sequence in figure 2 does not occur, the electrons' state will be a mixture of all possible Bell states, one-quarter of which are the desired states. Thus, the fidelity \mathcal{F} of creating a perfect pair is $\mathcal{F} = (1 + 3\mathcal{P})/4$. Figure 4 shows this fidelity as a function of time in the clean and dirty regimes. As expected, the fidelity of pairs produced drops significantly once the cut-off time is of order Γ_C^{-1} .

A sensible strategy would be to choose a τ that is at the end of the fast initial rate rise. At this point, dirty pairs are emitted at 85% of the rate of clean pairs, while the fidelity of each dirty pair is only 0.94, compared with 0.96 for clean pairs. Both these values are within range where we may use entanglement distillation protocols to generate higher quality entanglement [19]. We therefore see that the relaxed, dirty regime performs comparably with the clean regime under these conditions.

We have also considered situations where the various intrinsic coherent coupling strengths within the tri-dot structure are no longer matched. Provided that the hierarchy of rates is respected, the operation of the device is found to be qualitatively identical and quantitatively very similar, for all such choices.

5. Decoherence

In this final section, we shall briefly address the effects of decoherence on the device. We must ensure that the operation time of the device is fast compared with the decoherence times of the spin and charge degrees of freedom. Spin decoherence is only an issue once the singlet has been formed on dot C . Therefore, the critical timescale for the spins is the time taken for both electrons to leave the tri-dot structure set by Γ_A . From our previous discussions of system parameters, we should expect a tunneling rate of $\Gamma_A = 1 \mu\text{eV}$ corresponding to tunneling time of 1.5 ns. Experiments have reported decoherence times of 1.2 μs with refocusing techniques [14]. Nuclear spin silent materials would eliminate the need for refocusing. Furthermore, spin coherence lengths in excess of 100 μm in GaAs quantum wires have been experimentally observed [20]. Hence, we can assume that the entanglement can be preserved for a considerable time after the electrons have left the device. Meanwhile for the charge degrees of freedom, coherence must be maintained on a timescale longer than the time taken for the resonant coherent processes g . This can be readily satisfied as $g \approx 10 \mu\text{eV}$ corresponding to a time of 150 ps, while charge decoherence times of 1 ns have commonly been observed [21, 22].

6. Summary

To summarize, we have proposed a passive device that can produce a stream of spin entangled electrons. We have shown that the rate and fidelity of these electrons depends on the choice of

system parameters and can be altered by a degree of post selection. This allows control over the characteristics of the generated entangled electrons that can be chosen to suit the application. This process is very fast: in practice the overall performance of the device will be limited only by rates of incoherent tunneling and the electronics used to subsequently handle the entangled pairs.

Acknowledgments

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