

Brokered graph-state quantum computation

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Abstract. Using the *graph-state* approach to quantum computation, one can avoid the need for complex array nanostructures in which quantum bits (qubits) interact directly. Instead one can employ simple ‘atom-like’ nanostructures, coupled over macroscopic distances via optical emissions. Here, we describe a robust coupling procedure, which we call *brokering*, that is especially well suited to nanostructures bearing both nuclear and electron spins. We describe how this approach can be implemented with N–V centre materials.

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1. Introduction

The concept of *one-way* quantum computing offers a highly promising route towards a practical technology. Qubits are initially prepared in a particular form of entangled state, a graph state [1]–[5], also known as a cluster state. Computation then proceeds simply by measuring individual qubits, consuming their prior entanglement as a resource. Both the graph-state preparation and the subsequent measurements can be achieved by monitoring photon emissions from the qubits. Proposals for the generation of graph states have been made in a variety of physical systems [1], [6]–[10]. Optical graph states of up to four qubits have been generated in the laboratory [11]–[13].

A particularly attractive feature of one-way quantum computation is, that after the graph state has been created, the physical nanostructures representing its qubits can remain separated, effectively each in its own *dedicated apparatus* [8], [14]–[17]. This architecture appears very promising in its ease of scaling to many qubits. The use of graph states allows one to create the entanglement required by a quantum algorithm *before* that algorithm executes. Thus failures in creating entanglement, e.g. due to photons being lost, need not lead to failures in the algorithm. Unfortunately they can damage the nascent graph state, severely reducing the rate at which it can be ‘grown’.

Here, we describe a *broker–client* model which exploits the physics of atom-like nanostructures in order to provide risk-free graph-state growth. Leveraging ideas on distributed computation [18]–[20] together with work on probabilistic cluster state synthesis [8, 15], our model assigns to each multi-level nanostructure two logical qubits: the *broker* and the *client*. Groups of brokers negotiate new graph-state fragments via a risky optical protocol which may be prone to failure. Once complete, graph fragments are mapped from each broker to its client using safe local excitations. The clients, whose role is to store the nascent graph long term, remain insulated from failures during brokerage. We describe a solid state implementation using N–V centres in diamond, where brokers and clients are naturally embodied as electron and nuclear spins.

In mathematics, a graph is an entity consisting of a set of *vertices* (points), together with a set of *edges* (lines) connecting those vertices in some fashion. We can form a corresponding quantum *graph-state* by allocating a qubit in state $|+\rangle \equiv (|0\rangle + |1\rangle)/\sqrt{2}$ to each vertex, and performing a control-phase gate on each pair of qubits whose corresponding vertices are connected. Experimentally, we might employ qubits embedded in separate cavities, with some form of inter-cavity entanglement operation (EO) which can create the required phase relations. Suitable EOs have been derived; many of them are conceptually very elegant, involving the single photons being emitted from the qubits and entering a linear optics apparatus prior to detection. If a photon is detected but there is no information as to which qubit generated it, then those qubits typically become entangled (so-called ‘path erasure’ [21, 22]). However, these EOs fail if photons are lost from the apparatus—an unavoidable possibility for experiments in the foreseeable future. A crucial feature of all the EO schemes, we consider here is that such failures are always known, or *heralded*. In this way, failures can be tolerated in that systematic graph-state growth is always possible with a suitable strategy [8]–[10], [23], but for low probability of success p , the growth rate becomes extremely slow.

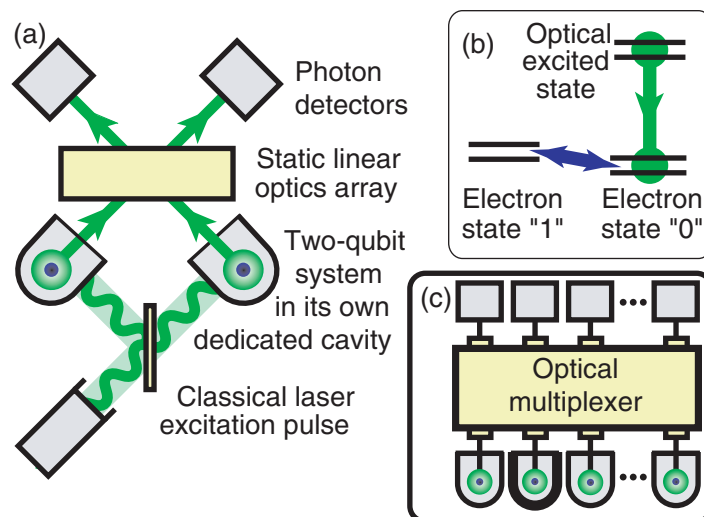


Figure 1. An overview of optical entanglement generation. (a) Schematic of the basic apparatus: individual atoms (or atom-like nanostructures such as N–V centres in diamond) are each isolated within a *separate* cavity. Each system has a multi-level eigenspectrum within which one can define two qubits: the *broker* and the *client*. Each cavity apparatus is capable of implementing deterministic local *broker–client* interactions by exciting level transitions. The broker qubits associated with different nanostructures can be entangled via the emission of a photon into a ‘path erasure’ optical apparatus. (b) A simplified view of one possible level structure (elaborated in figure 3). The broker state can be mapped into a photon via an optically-allowed transition. Doublets correspond to the client states. (c) In a mature form of the technology, multipartite brokering entangles sets of broker qubits in parallel, via an optical multiplexer. Related devices already exist [24].

2. Brokered graph-state formation

We suggest that each cavity should contain an atom (or atom-like nanostructure) that has a sufficiently complex level structure to constitute two logical qubits: the broker and the client (see figure 1). We assume that we can perform deterministic quantum gates between these two, in contrast to the inter-broker operations which may be both error prone and inherently indeterministic (see figure 2). For many systems, the natural embodiment would be an electron spin qubit for the broker, and a nuclear spin qubit for the client, as we discuss below. In this way, we effectively place a two-qubit quantum computer within each cavity, which has profound consequences for our strategy for creating entanglement *between* cavities. This scheme allows the generation of graph states with nodes of high degree (i.e. a high number of attached edges) without the risk of damage through entanglement failure, thus facilitating the synthesis of more complex graph structures, such as those required for fault tolerant computation [25]–[27] or robustness against qubit loss errors [28].

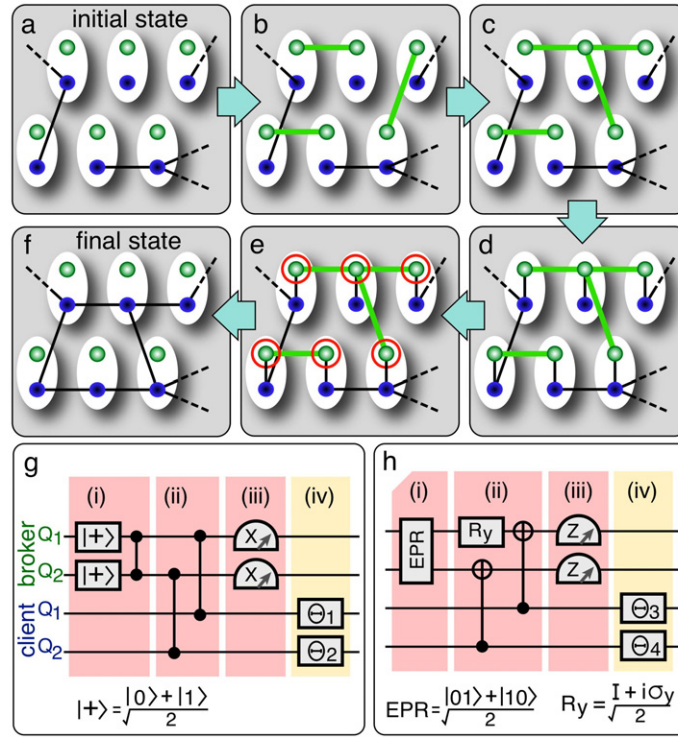


Figure 2. The procedure for brokering graph states. Ovals indicate the client (below, blue) and broker (above, green) qubits associated with a single nanostructure in a dedicated apparatus. (a) We can assume that an arbitrary set of graph edges have already been implemented as shown. (b, c) Where we wish to create new graph edges, we first create entanglement among the brokers. This may take many attempts, and may involve resetting the broker states. We depict two graph fragments brokered independently: a simple bipartite fragment and a more complex ‘3-node’ fragment formed in a two stage process involving four brokers. (d) We then deterministically create graph edges between the brokers and their client qubits *within* each pair. (e) Finally, we project the broker entanglement on to their client qubits via measurements on the brokers. (f) The clients thus acquire new entanglement relations with one another. (g) A naïve circuit diagram for the simplest bipartite brokering involves preparing state $|+\rangle$, phase gates and X measurement. (h) An optimized equivalent circuit can be constructed from realistic fast operations including control-NOT and Z basis measurements. The Z rotations on the client qubits, marked Θ , can be postponed and subsumed into the eventual measurement procedure.

3. Multi-partite brokerage

Brokerage is not restricted to the bipartite level, i.e. creating single graph edges prior to mapping those edges on to the clients. Broker qubits can arrange more complicated multipartite graph fragments, before mapping this on to the client qubits. The optimal strategy in this case depends on the experimental parameters, as we now discuss. In this regard, one should also consider the

utility of recruiting redundant brokers, i.e. broker qubits not presently being used by their clients, as an additional resource for rapidly creating sophisticated broker entanglement, regardless of whether their local clients require new entanglement.

Consider the example of figures 2(b) and (c) where a ‘3-node’ is established. If we only consider broker entangling operations which create entangled pairs of qubits, then three complete rounds of brokering and transfer to clients are needed to create the client entanglement seen in figure 2(f). The approximate time required will be

$$\tau_{\text{sequential}} = 3 \left(\frac{1}{p} (\tau_H + \tau_O + \tau_M) + \tau_{\text{CNOT}} + \tau_H \right),$$

where τ_H , τ_{CNOT} , τ_M are the times required to implement a broker Hadamard gate, a CNOT controlled by the client qubit, and a measurement in the broker computational basis, and where p is the success probability for the entangling operation. The average time spent on a single attempt to generate optical entangled is τ_O . If, however, the broker qubits are themselves used to create a 3-node before being entangled with the client qubit, as depicted in figure 2, then the time required will be

$$\tau_{3\text{-node}} = \frac{1}{p} \left(\left(\frac{1}{p} (\tau_H + \tau_O + \tau_M) + \tau_O \right) + \tau_{\text{CNOT}} + \tau_H \right).$$

For a typical physical implementation in terms of electron brokers and nuclear clients, we anticipate that $\tau_O \approx \tau_H$ (i.e. the electron rotations are the slow part of the optical entanglement) and $\tau_M \ll \tau_H$, since measurement is purely optical. It is then advantageous to use the latter scheme if the inequality, $\tau_{\text{CNOT}}/\tau_H \geq 1/(p^2) - 5/(2p) - 1$, holds. For experiments in the near future, it is reasonable to take probability $p = 0.25$. We may take ratio $\tau_{\text{CNOT}}/\tau_H = 10$ at least, due to the need to implement the CNOT with a frequency selective pulse, while unconditional electron rotations can correspond to faster ‘hard’ pulses. With these values there is already an advantage in brokering the larger fragment: $\tau_{\text{brokered}} \approx 2/3\tau_{\text{sequential}}$, and approaches $1/3\tau_{\text{sequential}}$ as this ratio grows. Higher values of p , e.g. from experimental refinements reducing photon loss, would increase this advantage. Thus, we conclude that it may be desirable to create mutual entanglement between four brokers. For more extreme values of the experimental parameters, it may even be desirable to further increase the degree of multipartite brokerage.

4. Broker entangling operations

A wide variety of options exist for entangling macroscopically separated qubits, suitable for a range of different physical implementations. Many proposals for the generation of entanglement between atoms in separate cavities combine the detection of emitted photons with the erasure of which path information to generate maximally entangled states (Bell states) in the atoms [21]. The starting point for these schemes is to prepare the atoms such that they emit a photon into a cavity mode conditional on their internal state. After leaking from the cavity, the photon(s) pass(es) through a beam splitter (which erases which path information) and arrives at detectors. Certain detection outcomes indicate the creation of a maximally entangled two qubit state

(Bell state) between the atoms. Although failure may occur, such failures are always known, or *heralded*—this is a crucial feature of all the schemes we consider here.

One versatile and robust-proposed implementation employs a Raman-type transition on a three-level Λ configuration of atom-like spectra [22, 29]. Raman-type transitions can be implemented in a variety of systems, including N–V centres when a suitably high magnetic field is applied [30]. Alternatively, if the broker qubit levels each possess a transition to an excited state via modes of different polarizations, methods using two-photon polarization interference can also be employed [31]–[33], with the advantage of being robust against path length fluctuations between cavities.

There are also schemes for non-deterministic entangling gates through which multipartite brokering may be achieved, building up fragments via successive pairwise entanglement of brokers [15, 17, 34]. Alternatively, more general projections may be used (for example, projections on to parity sub-spaces) which have the effect of ‘fusing together’ small graph states (for example, Bell states) so that larger graph states of arbitrary layout can be formed [16, 35]. In particular, the scheme given in [8] fully supports the preceding model together with our suggested N–V centre implementation.

5. A physical system: N–V centres in diamond

We have remarked that a natural implementation of the broker–client model could employ a pair of coupled electron and nuclear spin, such as can be found in atoms, or defects in solid state samples. Single electron spins can be detected optically [36], enabling remote entanglement as described above, while nuclear spins are ideal for storing entanglement during the primary graph’s growth, benefiting from coherence times as long as tens of seconds, even at room temperature in the solid state [37]. The electron–nuclear hyperfine interaction permits selective transitions [38] corresponding to, e.g. CNOT operations between client and broker.

While we stress that the requirements of our scheme permit many possible physical implementations, the discussion which follows illustrates a particular embodiment, using N–V defects in diamond, consisting of a substitutional nitrogen atom (^{14}N or ^{15}N) and a vacancy in an adjacent site [39]. This contains all the ingredients required by the brokered graph state generation scheme. The ground state of the defect is an electron $S = 1$ spin triplet (3A) which has a strong dipole-allowed optical transition to a first excited spin triplet state (3E), as shown in figure 3. The electron spin can be coupled to a range of different nuclei (^{13}C , ^{14}N or ^{15}N) via a hyperfine interaction or to nearby ‘dark’ electron spins by a dipolar interaction [40, 41]. Depending on which coordination shell the ($I = 1/2$) ^{13}C nucleus inhabits, a wide range of choices for the electron–nuclear interaction strength are available: up to 200 MHz (nearest neighbour), falling off rapidly for more distant sites [42, 43]. Alternatively, the coupling to the nitrogen nucleus may be exploited—this can vary from 2–100 MHz for ^{14}N ($I = 1$), depending on the precise species of N–V centre [43, 44] and crystal orientation [45], however the spectrum is complicated by the nuclear quadrupolar interaction. The less abundant ^{15}N avoids this additional splitting, being $I = 1/2$, and has a hyperfine constant approximately 40% greater than that of ^{14}N [43, 46].

We propose using this optically detectable electron spin coupled to some $I = 1/2$ nuclear spin with an interaction strength between 5 to 200 MHz. The electron spin is initialized in the $M_S = 0$ ground state through laser cooling. Entanglement is achieved between the broker

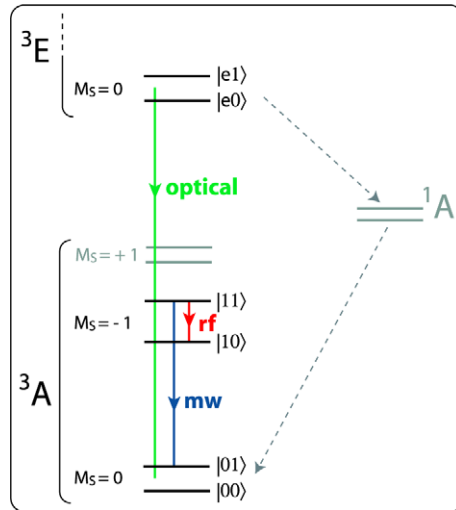


Figure 3. Energy level diagram for a typical N–V centre in diamond, including electron and nuclear spin states. Relevant qubit states are labelled in the $|broker, client\rangle$ basis. A small magnetic field is required to split the $M_S = \pm 1$ states; this also lifts the degeneracy of the nuclear spin in the $M_S = 0$ manifold, where the hyperfine interaction is 0. A selective π rotation applied to the transition labelled ‘mw’ corresponds to a CNOT gate targeting the electron spin, controlled by the nucleus (vice versa for the equivalent operation on the transition labelled ‘rf’). The zero-phonon-line energy of the optical transition is about 1.95 eV ($\lambda \approx 637$ nm).

(electron) spins by a suitable entangling operation, such as [8].⁵ Thereafter, a CNOT operation targeting the electron spin and controlled by the nuclear spin is achieved through a *selective* microwave π pulse, as shown in figure 3. This establishes entanglement with the client (nuclear) spins. Finally, the electron spin undergoes a rotation, achieved by a hard (non-selective) microwave pulse, and is then robustly measured by optical fluorescence—projecting the broker’s node in its graph fragment on to the client nucleus. Typical microwave pulse lengths are 50 ns, while optical measurement of the electron spin can be faster, given that the free space radiative lifetime of the 3E state is 13 ns [44].

When we wish to consume our graph state to perform computational steps, projective measurements of the client qubits are required. In order to achieve this in a nuclear–electron system such as the one we describe here, it will be necessary to map the nuclear state on to the electron for subsequent optical readout. We use a SWAP operation to exchange the client qubit from the nucleus to the electron; it is efficient to take this opportunity to also place a ‘fresh’ superposition on the nucleus, so that it can be reincorporated into the graph as a new client. Specifically, the electron spin is initialized into the $|+\rangle$ state, then SWAP’ed with the nuclear spin using three CNOT gates. This necessarily requires one CNOT targeting the nuclear spin,

⁵ Raman-like entangling operations [30] requiring a strong external magnetic field can also be employed here provided the external magnetic field is dynamically controlled. The magnetic field would be reduced during the broker–client CNOT and restored for subsequent entangling operations.

which must be performed using the a slower radiofrequency (RF) selective π pulse on the nucleus (shown in figure 3). A typical RF pulse duration is $10\ \mu\text{s}$. The electron spin can then be rotated and measured as described above.

Optical excitation occurs only within the $M_S = 0$ subspace, in which the electron–nuclear interaction is zero. Therefore the nuclear spin should remain unperturbed by the optical excitation to an excellent approximation. Note that only one frequency of each type of excitation in the hierarchy (optical, microwave and RF) is required, simplifying the experimental arrangement.

6. Conclusion

We have described a procedure for graph-state quantum computing that is tailored to fully exploit the physics of optically active multi-level nanostructures, and which allows arbitrary graph topologies to be grown efficiently, even when the brokerage failure rate is high. We have described an example implementation in terms of N–V centres in diamond, where brokers and clients are very naturally embodied as electron and nuclear spins. Demonstrator experiments appear to be feasible immediately.

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