Triangulating ancient Egyptian mathematics

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Introduction

A long-standing tradition, stemming from the writings of the ancient Greek historian Herodotus in the fifth century BCE, held that the study of geometry had originated in ancient Egypt thanks to the needs of land-surveying [Can80, ch. 1]. For centuries, however, any details to back up this statement were lacking: Herodotus said little more than this basic assertion, and by the middle of the first millennium CE, when all knowledge of the Ancient Egyptian language was lost, he remained the only source.[1]

The following centuries saw much speculation about ancient Egyptian texts, but it was only in the nineteenth century that concrete progress was finally made in the reconstruction of the different phases of the Ancient Egyptian language, and in restoring an understanding of the various scripts that had been used [BJ20]. To begin with, however, knowledge of ancient Egyptian mathematics remained minimal, owing to a lack of relevant sources. This changed in the 1860s with the discovery of a new text that soon came to be known as the Rhind Mathematical Papyrus (RMP). Now housed in the British Museum, this papyrus consists of roughly 80 arithmetical and geometrical problems of types that would probably have been encountered during the course of a scribe's official career: the distribution of rations, for instance, and the calculation of some simple areas and volumes. Although the material covered is quite elementary to modern eyes, the RMP provided the very first detailed view of the nature and processes of ancient Egyptian mathematics.

The first comprehensive study of the content of the RMP took place in the 1870s, at a time when the renewed understanding of the relevant language (Middle Egyptian) was still evolving. Where knowledge of the language was lacking, scholars were able to follow the logic of the mathematics in order to extract meaning (or, as we shall see, to declare certain passages to be nonsense). Later on, as knowledge of the language, and also of its broader cultural context, improved, a two-pronged approach opened up for the study of the further ancient Egyptian mathematical texts that had emerged: simply put, one approach followed the philology, the other the mathematics. Only rarely were these two approaches mixed, however. Broadly speaking, the philological approach was (understandably) favored by Egyptologists, and the mathematical by those scholars who approached the texts from a more scientific or mathematical background. Each group naturally brought its own goals and biases to the study. For the most part, the two approaches resulted in complementary findings, and only occasionally clashed. In this article, we look at one such instance in which the mathematical and philological interpretations of a particular passage appeared to point in entirely different directions. This is Problem 51 in the RMP, concerning the calculation of the area of a triangle. After providing further background to the study of ancient Egyptian mathemat-

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[1] For a more detailed account of the themes explored in this introduction, as well as for fuller references for the content of the present paper more generally, see [HP22].

ics, we examine this problem in detail, starting from the original papyrus, and consider the competing interpretations, before discussing how this issue was eventually resolved, as a case-study in the issues that arise in reading such an ancient text.

The Rhind Mathematical Papyrus

The RMP takes its name from the pioneering Scottish archeologist Alexander Henry Rhind (1833–1863), who purchased it from an antiquities dealer in Luxor in 1863.[2] Following Rhind's early death later that year, it passed to the British Museum, where it remains to this day (though it is no longer on display, in order to ensure its conservation).[3] The papyrus was the first significant text on ancient Egyptian mathematics to emerge, and remains the most comprehensive such source within a rather small pool of extant materials [Imh16, §9.1].

The papyrus survives in the two large pieces into which it was evidently broken by its unknown discoverer. The total length of the two fragments is roughly 5m (a little over 16 feet), and the papyrus is 32cm (13 inches) in height. It was written by an individual who identified himself in an introductory title as "the scribe Ahmose". The same title dates the papyrus to Regnal year 33 of King Apepi (around 1537 BCE), and states that it is a copy of a text that was composed roughly 200 years earlier. The language is Middle Egyptian, written in the cursive hieratic script. As we see from the section of the papyrus shown in Figure 1, both black and red inks were used, with the latter employed to highlight certain parts of the text, such as the introductions of problems or their solutions. We also note the presence of images (we eschew the word "diagrams" for reasons that will be made clear below) within some of the geometrical problems, including Problem 51, which appears just to the left of the centre of Figure 1.

The acquisition of the RMP by the British Museum in the mid-1860s was accompanied by the recognition that here was a source that might finally reveal some of the details of ancient Egyptian mathematics. Early references to it in print focused on the images, referring to it as a "geometrical papyrus" [Bir68]. The first comprehensive survey of the content of the papyrus was carried out by the German Egyptologist August Eisenlohr (1832–1902), who published an edition at the end of the 1870s [Eis77]. Eisenlohr's publication of the papyrus caused a sensation, certainly among mathematicians, for here was the first detailed view of ancient Egyptian mathematics. The elementary nature of the mathematics (it is mainly arithmetic — even the so-called "geometrical" problems, as we shall see) and the fact that the papyrus contains nothing like the famous deductive structure of Euclid's *Elements* led to disappointment from some mathematical quarters [HP22, §8]. Nevertheless, the RMP had now pushed the detailed knowledge of the history of mathematics back an extra millennium, if not further, and this meant that it was quickly incorporated into the overarching narrative of the development of mathematics. This happened in particular in Moritz Cantor's *Vorlesungen über Geschichte der Mathematik* [Can80] and in many other works derived therefrom. The accounts of the RMP that had been digested for mathematical readers typically focused, understandably, on the details of the mathematics, and left aside the questions of language that were exercising the Egyptologists. More than that, they tended also to repackage the mathematics of the RMP in a modern form, and did not necessarily give their readers any indication of what this material was like in the original, so that mathematical readers became increasingly separated from the primary materials [HP22, §8]. In the longer term, this led to much speculation about ancient Egyptian mathematics that had little basis in the original sources.

While mathematical readers took on board the details of the RMP that they had gleaned from suitably digested summaries of Eisenlohr's findings, Egyptologists began refining the first translation as their

[2] The date is often given as 1858, and appears as such on sources such as Wikipedia (certainly as of 1st July 2024), but recent (unpublished) research by Margaret Maitland of National Museums Scotland (pers. comm.), using correspondence between Rhind and Samuel Birch of the British Museum, has shown that it was in fact early 1863. On Rhind, see [Bie12, p. 463].

[3] The two parts of the papyrus have the museum numbers P. BM EA 10057 and P. BM EA 10058.

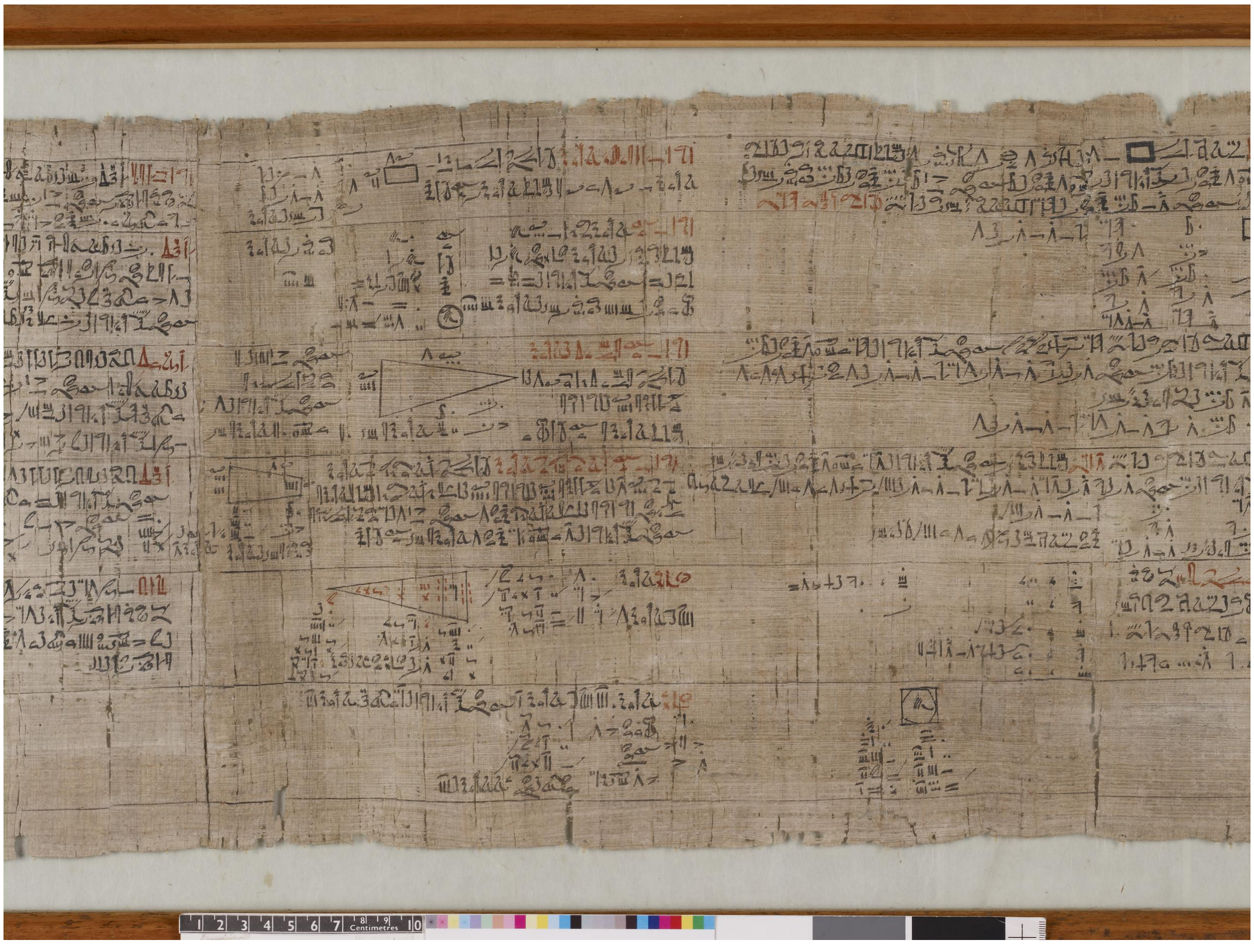

Figure 1: The Rhind Mathematical Papyrus. © The Trustees of the British Museum. Shared under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0) licence.

understanding of the underlying language increased thanks to fresh parallels from elsewhere. There was little in Eisenlohr's edition that was found to be wrong outright, but a richer knowledge of Middle Egyptian brought more nuanced interpretations of some of the problems — such as, eventually, Problem 51. It was an acknowledged fact that the study of ancient Egyptian languages was then so dynamic that an edition of a text became outdated almost as soon as it was published, and this is certainly what happened in the case of Eisenlohr's edition of the RMP. By the end of the century, a strong need for a new edition was felt.

Thomas Eric Peet

It is at this point that Thomas Eric Peet (1882–1934) enters the story. Although he graduated from Oxford University with a degree in classics in 1905 and went on to an Egyptological career, he had previously studied some mathematics, and a passing interest in the subject remained with him for the rest of his life (see [HP23] and [Bie12, pp. 420–421]). It was almost certainly this interest that attracted his attention to the RMP. Beginning in 1911, he set to work on a new edition. In the absence of any substantially new insights into ancient Egyptian mathematics in the decades after Eisenlohr's work on the papyrus, interest in the subject had declined. In 1923, however, when Peet's new edition finally appeared in print [Pee23], it caused a minor sensation among both Egyptologists and mathematicians, and served to reinvigorate the study of ancient Egyptian mathematics [HP22, §5]. Interest was also driven by rumors circulating in Western Europe and North America of a substantial Egyptian mathematical source in a collection in Russia, that was beginning to come to wider attention in the early 1920s. As was soon discovered, the content of this source, dubbed the Moscow Mathematical Papyrus (MMP), overlapped significantly with that of the RMP, but also contained new geometrical problems.[4] Although a full edition of the MMP was finally published only in 1930 [Str30], Peet had access to photographs of it while working on his edition of the RMP, and he used these to inform his general picture of the nature of ancient Egyptian mathematics. Indeed, his publication of the RMP went beyond the norms of such scholarly editions by offering commentary not only on the text itself, but also more broadly on the "general character" of ancient Egyptian mathematics [HP22, p. 10]. Peet's edition therefore appealed both to Egyptologists and to readers who were more interested in this overview, which often had more to say about the mathematical than the philological details; both types of reader gave the edition glowing reviews [HP22, §5].

During the rest of the 1920s, Peet continued to write on aspects of ancient Egyptian mathematics, including some of the seemingly new features that had been raised by the MMP.[5] This was also the decade in which a new type of scholar emerged in this connection. In the final decades of the nineteenth century, the mathematical readers who took an interest in the content of the RMP invariably did so via one of the digested versions of Eisenlohr's edition (Cantor's, for example), and certainly with no knowledge of Middle Egyptian. By the end of the 1920s, however, ancient Egyptian mathematics was being studied by scholars, such as Otto Neugebauer (1899–1990) and Kurt Vogel (1888–1985), who had strongly mathematical backgrounds, but who had learnt the languages necessary to read ancient Egyptian and ancient Mesopotamian sources. Although largely immune from some of the wilder speculations entered into by those mathematical readers who could not engage with the original materials, such "mathematician-philologists" were nevertheless inclined to follow mathematical over philological or cultural evidence, seeking the systematic aspects of ancient Egyptian mathematics that they felt ought to be there [HP22, §7]. For instance, much ink was spent (and is still being spent) on the question of the general rules that were supposed to underlie the table of fractions that takes up most of the recto of the RMP [Imh16, §10.3]. Since ancient Egyptian fractional notation employed only unit fractions (with

[4] The Moscow Mathematical Papyrus is now in the Moscow Museum of Fine Arts under museum number E4676.

[5] See the papers listed in the bibliography of [HP22].

one exception, namely $\frac{2}{3}$), any other fraction had to be expressed in terms of these and accordingly the recto of the RMP features a reference table for the duplication of unit fractions with odd denominators from 3 to 101 (for example, $\frac{2}{5}$ is represented there as $\frac{1}{3} + \frac{1}{15}$). Since no such representation is unique, this has opened up modern speculation about why the particular representations in the RMP were chosen. During the 1920s, both Neugebauer and Vogel wrote doctoral dissertations addressing precisely this question [HP22, pp. 211, 215]. Neither appears to have been swayed by the more prosaic, but perhaps more realistic, view taken by Peet that, in common with the construction of other ancient Egyptian texts, the table was probably compiled by gradual accretion over time, and had no special rules underlying its construction [Pee31a, p. 414]. As another example, Peet also found himself at odds both with mathematicians and with mathematician-philologists over the vexed issue of "ancient Egyptian algebra": some commentators insisted that certain problems in the RMP imply a knowledge of the solution of algebraic equations, whereas Peet, ever cautious about such matters, considered this term to be too anachronistic.[6] Very occasionally, Peet expressed his frustration at some of the suggestions made by the mathematician-philologists, as in May 1930 when he complained privately to his mentor Alan Gardiner (1879–1963) about the "crop of correspondence" that he had recently received from Neugebauer, Vogel, and others, "which get more abstruse each time".[7]

Two phases

Throughout this period, Peet was almost the only Egyptologist to be taking an interest in ancient Egyptian mathematics, the one significant exception being his sometime-coauthor, Battiscombe Gunn (1883–1950) [Bie12, p. 232], whom we shall meet again later on. Otherwise, the Egyptological community at large showed little interest, seemingly content with the fact that *someone* was doing such work, and that it was adding in a small way to the wider picture of ancient Egyptian culture and language — for example by the inclusion of sections on numeration, weights and measures in Gardiner's *Egyptian grammar*, completed with Peet's input [Gar27]. As the sole Egyptological commentator and authority on Egyptian mathematics, it was natural that in February 1931 Peet should be invited to deliver a lecture on this subject at the Rylands Library in Manchester. The lecture was subsequently published as a comprehensive overview of current knowledge of ancient Egyptian mathematics as it then was [Pee31a]. After twenty years of research, Peet also felt able to take a step back and comment on the changing nature of studies in this area. In particular, he identified two distinct phases through which he believed that the study of ancient Egyptian mathematics had passed [Pee31a, p. 409]:

1. "describing the actual processes used by the Egyptian mathematician in solving the problems which confronted him", and

2. "the attempt to analyse the mental processes which underlie the actual operations and in showing how far these agree with or differ from our own".

To the first phase belonged the strongly mathematically informed work of Eisenlohr, but Peet placed into the second phase his own more fully contextualized study of ancient Egyptian mathematics — a desire to understand the underlying thought processes was precisely what he had sought for in his attempt at a "general character" of Egyptian mathematics.

This lecture was to be Peet's last publication on ancient mathematics. In 1933, he left the professorship in Liverpool that he had held since 1920 in order to take up a similar post in Oxford, but died suddenly just a few months later. To a very large extent, any significant interest in mathematical topics among Egyptologists died with him. From an Egyptological point of view, it seemed that everything that could be said about ancient Egyptian mathematics strictly on the basis of the extant evidence had been said. Thus, over the following decades, the only substantial further studies were those of a mathematically speculative nature by non-Egyptologists, such as additional

[6] See [HP22, p. 215] and the references given there.

[7] Griffith Institute Archive, Oxford: AHG/42.230.39, Peet to Gardiner, 19th May 1930.

contributions to the discussion surrounding the recto table of the RMP. It was only towards the end of the twentieth century that a fully Egyptologically contextualized approach to ancient mathematics reasserted itself [Imh16].

1	10	100	1000	10,000	100,000	1,000,000
𓏺	𓎆	𓍢	𓆼	𓂭	𓆐	𓁨

Table 1: Ancient Egyptian hieroglyphic numerals.

RMP Problem 51

Having given some background to the study of ancient Egyptian mathematics in the late nineteenth and early twentieth centuries, we return now to Problem 51 of the RMP.[8] We have already seen the original text of the problem near the middle of Figure 1, but Figure 2 shows the problem itself. As noted previously, the RMP is written in the cursive hieratic script, which reads from right to left. The heading of the problem is picked out on the right-hand side of Figure 2 in red ink. The black text beneath the heading contains the statement of the problem; that at the top left of Figure 2 outlines the method of solution and states the answer without any working. The text underneath the central triangle is almost purely numerical and features a verification of the answer just given by feeding it back into the initial problem. This is a format common to many of the problems in the RMP: a problem is posed and an answer is given without justification, but this is then verified as satisfying the problem. Only in a few instances, such as here, is the method of solution outlined, if not actually performed [Pee23, p. 22].

Like any handwritten script, hieratic can be variable in its execution, though the RMP is a particularly neat example that is nowadays used in teaching the hieratic of that period. It is common practice in Egyptology to transcribe hieratic texts into hieroglyphs, since the two versions of the script are the same apart from their execution. Therefore, before proceeding further, let us do this for RMP Problem 51, with the result that appears in Figure 3. The handwritten hieratic signs are in fact derived from the corresponding more fully drawn hieroglyphs, though in most cases this is far from obvious, as a comparison of Figures 2 and 3 will show. Those readers who are already familiar with the Egyptian hieroglyphic numerals (as shown in Table 1) will notice several of these beneath the triangle, and also at other places throughout the text.

To start to read the problem, we begin with the heading in red. Transliterating this into Latin letters in the manner standard in Egyptology, we get *tp n ỉrt spdt m ꜣḥt*, which translates as "Example of reckoning a *sepedet* of land".[9] Like modern mathematical texts, the RMP makes use of standard formulations to set out problems and their solutions, and we see this from the opening words: *tp n* appears repeatedly throughout the papyrus as "Example of". Also like modern texts, ordinary words are co-opted as technical terms, and here the verb *ỉrt*, which has the broader meaning of "to do" or "to make", takes on the more specific sense of "to calculate" or "to reckon". The word *ꜣḥt* (𓅜𓐍𓏏𓈇𓏥),[10] "field, arable land", at the end of the sentence is translated as "land" because it ends with the so-called determinative (classifier sign) 𓈇, signifying irrigated land, and because the three strokes beneath 𓈇 indicate a sense of plurality, either as the plural "fields" or a collective singular noun "fieldage". There is therefore a sense here of quantity, so some authors have elected to translate the word as "acreage" or have gone even further and used "area" — the former is arguably the better, since, as we will see, this is not area in an abstract sense, but one that comes with units of measurement, and the word retains the determinative classifying it as a word to do with the land.

[8]The numbering of problems was introduced by Eisenlohr in his edition of the papyrus.

[9]The Ancient Egyptian script (largely) did not record vowels, and the modern convention for reading it aloud is to insert an *e* between consecutive consonants, as with *sepedet*, which is written *spdt*. In what follows, we do this explicitly only for key words such as *sepedet*. In the transliteration given above, the special characters *ỉ* and ꜣ represent a weak *y* sound and a glottal stop, respectively.

[10]Note that we have changed the direction of the signs because we are now writing from left to right. The character *ḥ* stands for an emphatic *h*.

Figure 2: Problem 51 in the Rhind Mathematical Papyrus. © The Trustees of the British Museum. Shared under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0) licence.

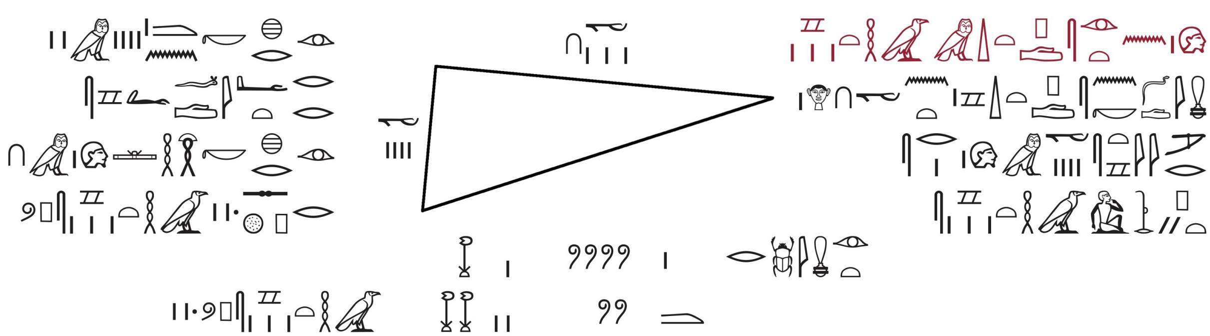

Figure 3: Hieroglyphic transcription of Problem 51 in the Rhind Mathematical Papyrus. Note that the text beneath the triangle begins in the bottom right-hand block in the original, but has been slightly displaced here for reasons of spacing.

Thus, as far as the heading is concerned, this leaves us with just one word that we have not yet translated: *sepedet* ( ). We take a little extra care over this word since the interpretation of technical terminology will be crucial in what follows, but in fact this one does not cause us any problems: it translates literally as "that which is pointed" and also ends with a determinative indicating pointed things.[11] Even without the evidence of the large triangular image in the middle of the problem, we would feel quite safe in translating this as "triangle". Problem 51 is therefore an "Example of reckoning a triangle of land".

We have gone through the translation of the heading of the problem in some detail in order to illustrate the systematic process that is needed to arrive at a secure translation, most particularly where technical terms may or may not be involved — one problem with Ancient Egyptian mathematical texts is that we often simply do not have a large enough sample to be able to decide definitively whether a particular word is being used in an established extended and/or technical sense in a given instance.

The text after the heading continues as follows: "As is said to you: A triangle of 10 *khet* upon its *meryt* and 4 *khet* as its *tep-ra*, what is its acreage?" Here again, we have a standard form of words (*mi ḏd nk*: "As is said to you") that is commonly used in the RMP to pose a specific problem. The word *ꜣḥt* that we translated as "land" in the heading, we now translate (slightly inconsistently!) as "acreage", since that seems more appropriate in this context. It may be obvious from the phrasing that *khet* is a unit of measure (equivalent to 100 cubits),[12] and so we see that the problem is asking us to find the area of the triangle, given two of its dimensions. The question is, which dimensions are they? The second, the *tep-ra* ( ), is the easier to deal with, and may be interpreted simply as the base of the triangle (allowing for the fact that the picture is drawn horizontally, and so the "base" is the left-hand edge). The word is a compound noun containing the word "mouth" (*ra*). The other part, *tep* ("head"), can often form compound nouns, and is used to mean "example", so this is literally the "head of the mouth". Here, we can perhaps, following a suggestion from Peet [Pee23, p. 91], think intuitively of the triangle as a mouth opening up from right to left. We are now left with just one word to interpret, *meryt* ( ), but since this one is somewhat problematic, we leave it untranslated for a moment and proceed to the next part of the problem.

As already noted, the text at the top left of Figure 3 continues with instructions (again repeatedly using a standard form of words: "You are to...") for how to solve the problem: "You are to take half of 4, namely 2, in order to give its rectangle. You are to multiply 10 by 2. This is its acreage." We will return to the phrase "to give its rectangle" later in the article,[13] but we first note what is going on here numerically: we are halving the base of the triangle and then multiplying this by the *meryt*, with an unstated answer of 20. Thus, if we were simply to follow the mathematics, then we would conclude that *meryt* indicates the height (or, in the picture, the horizontal length) of the triangle. Herein lies a problem, however: all other instances of the word *meryt* in surviving Egyptian texts indicate that its core meaning is a "riverbank" where boats moor [Fau62, p. 112], and so it is difficult in our case to interpret this word as anything other than the edge, i.e., the *side* of the triangle.[14] Even the labelling of the picture in Problem 51 seems to bear out this interpretation of *meryt*. The base of the triangle is labelled first with the sign for *khet* , beneath which there are four vertical strokes to indi-

[11]There is a small question mark over whether this sign is indeed the thorn determinative that indicates pointed things, or whether it represents a triangle directly. For our purposes, the practical upshot of each explanation is of course the same.

[12]Though not a standardized measure, the cubit was based on the distance from the elbow to the tip of the middle finger, and was therefore a little over 50cm (roughly 20 inches).

[13]But having taken care over the interpretation of *sepedet*, we ought also to comment on our use of the word "rectangle". The Egyptian word here is *ỉfd*, which is related to *ỉfdt*, meaning "quartet". Note that the hieroglyphic form of *ỉfd* ( ) ends with the land determinative.

[14]To the modern eye, there is perhaps a small quirk here in how we view Problem 51. We are clearly told at the beginning of the problem that this is a triangle of land. But if *meryt* is "riverbank", then it is possible to extend the above interpretation of *tep-ra* to mean "river mouth", in which case the picture in Problem 51 becomes an idealized view of a river delta. It is doubtful, however, that the scribe would have had such an aerial view of the shape of the Nile Delta.

cate the number four; the other label is quite clearly attached to the side of the triangle: the word for *khet* followed by the sign for ten ∩. The *khet* are plural, and so there are three vertical strokes to mark this as a plural noun (a grammatical feature required for ten of something but not for four).[15] In order to resolve this apparent impasse, let us consider how this difficulty was handled by the various scholars mentioned above.

Interpreting Problem 51: Eisenlohr

Eisenlohr's interpretation of Problem 51 appears in Figure 4. Certain differences from our analysis above are immediately apparent, not least the different (older, Germanic) conventions for transliteration into Latin letters. We note that in his translation into German, Eisenlohr retained the more literal translation of *i͗rt* as "to do" or "to make" (German "machen"); he commented on the extended usage "to reckon" in the notes immediately following his translation. Eisenlohr clearly had no qualms about translating *sepedet* as "Dreieck" ("triangle"), and we see that his translation contains the same small inconsistency as ours: *ꜣḥt* appears both as "Felde" ("field") and "Flächeninhalt" ("area"/"surface"). For Eisenlohr, *khet* was "Ruthe" ("rod"), a suitably biblical-sounding unit of measure; *tep-ra* was rendered as "Mündung", a German word for "mouth" that (in contrast to the simpler "Mund") captures the extended meaning of the mouth of a river — or here the "mouth" of a triangle, if we follow Peet's later interpretation.

Perhaps the most interesting feature of Eisenlohr's translation, however, is his rendering of *meryt* as "Schenkel" ("leg", i.e., "shank"). He chose here to follow the language over the mathematics, but it seems that he was influenced by the common mathematical use (in both English and German) of "leg" for the side of an angle or of a triangle. Leaving aside for the moment the objection that $\frac{1}{2}(\text{base}\times\text{side})$ does not give the correct area for a triangle, Eisenlohr's interpretation of *meryt* as "side" raises a more immediate problem: if "side" were indeed intended, then *which* side? Depending on how much weight we give to the labelling of the picture in the problem (a point that will be addressed below), we might argue that it was the upper side that was intended, but what of the lower? Given the lack of information about the lower side, Eisenlohr speculated from the shape of the picture that an isosceles triangle was in fact intended, although no comment to this effect appears in the text. Eisenlohr offered the tentative evidence that the area of an isosceles triangle was calculated in just such a manner in a considerably later Greek-language text, one from the first century BCE [Eis77, pp. 126–127].[16] The objection over the correctness of the calculation remains, however. As Eisenlohr noted, $\frac{1}{2}(\text{base} \times \text{side})$ gives only an approximation of the area of the triangle, and one that gets progressively worse as the length of the base increases. For the triangle in question, the error is not vast (19.6 square *khet* if *meryt* is interpreted as "side" vs 20 square *khet* if it means "height"), but for triangles of larger base, the error would surely have been noticeable to Egyptian scribes. Part of the problem lies in how much mathematical knowledge we are prepared to attribute to them; this is an irresolvable issue that has rumbled throughout the study of ancient Egyptian mathematics, thanks to the limitations of the surviving sources [HP22, §8]. It appeared therefore that attempts to reconcile the difficulties of Problem 51 on purely mathematical grounds not only required additional assumptions, but also quickly spiralled into more specialized notions (such as isosceles triangles) that are otherwise unattested in ancient Egyptian mathematics.

Faced with this, and perhaps not wanting to reject the philological evidence out of hand, Eisenlohr was left with little option but to conclude that there was something fundamentally wrong with Problem 51 — that the scribe had made a slip that had rendered

[15] Unfortunately these three vertical strokes indicating plurality look identical to the hieroglyphic sign for three, but the distinction is much clearer in hieratic, where the plural marker consists of three dots and the number as three longer strokes: see Figure 2.

[16] Peet later discussed Eisenlohr's suggestion in detail [Pee23, pp. 93–94].

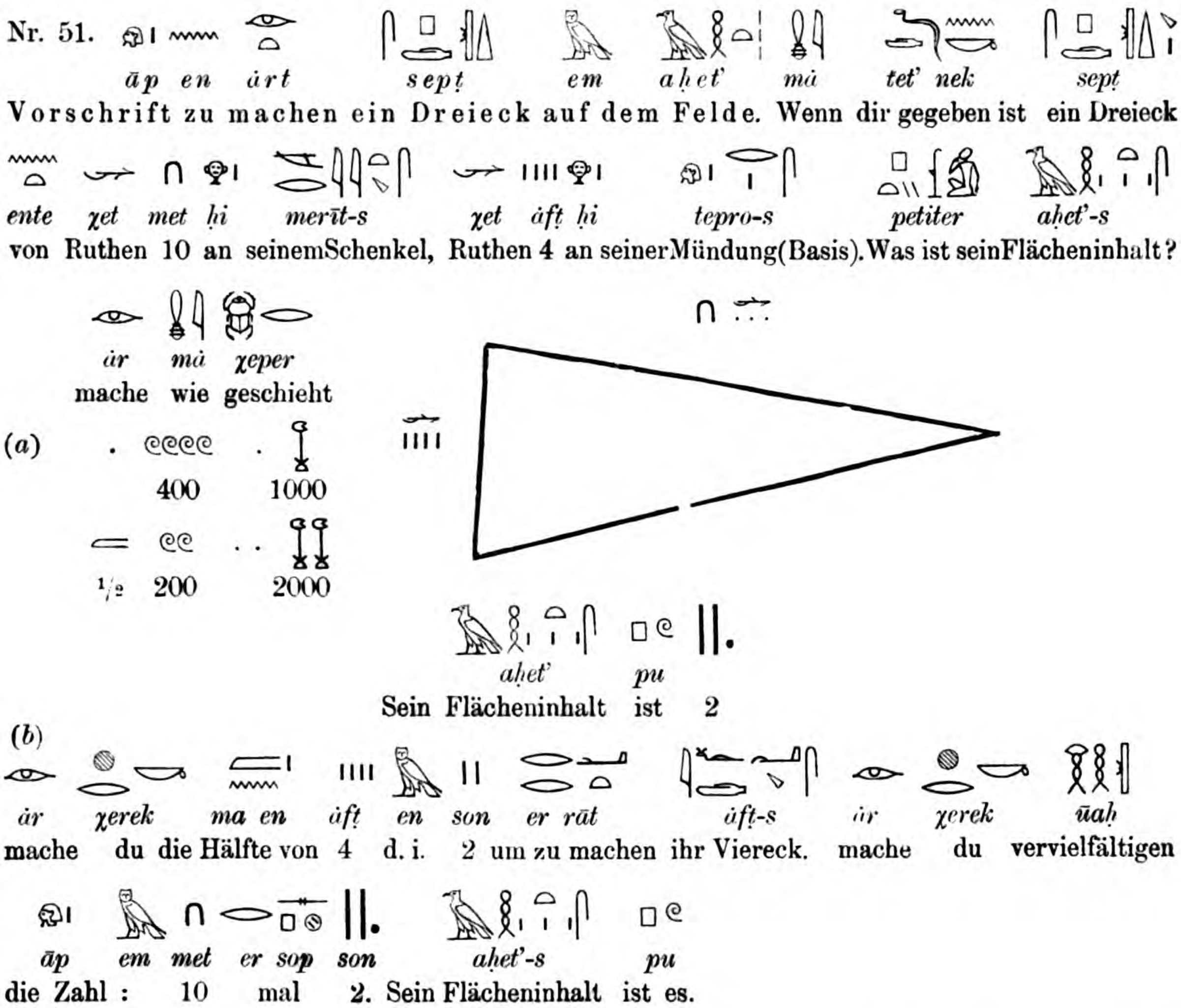

Figure 4: Eisenlohr's translation of Problem 51 in the Rhind Mathematical Papyrus [Eis77, p. 125].

\	1	15
	2	30
\	4	60
\	8	120
	13	195

Figure 5: An example of ancient Egyptian arithmetic: 15×13. Starting with 1 and 15, each row is repeatedly doubled until 13 is obtained as a sum of entries in the first column. The required product is then the sum of the corresponding entries in the second column. Although doubling and halving were the most commonly used operations in this context, a number of others were also permitted.

it nonsensical. Indeed, this was a possible conclusion, as the papyrus does contain some slips. As we have already noted, the RMP claims to be a copy of an older original, and there are several places where erroneous calculations may be easily explained away as slips made by the copyist. However, this urge to emend the text has represented something of a slippery slope in the study of the papyrus.

Eisenlohr's interpretation of the papyrus may also have been hindered by a further peculiarity of Problem 51 that we have not yet acknowledged. The numerical verification of the solution that appears beneath the triangle in Figure 3 (or to the left of that in Figure 4) may be translated as follows:

> The doing as it occurs:
>
> | | | | |
> |---|---|---|---|
> | 1 | 400 | 1 | 1000 |
> | $\frac{1}{2}$ | 200 | 2 | 2000 |
>
> Its acreage is 2.

The phrase "The doing like the happening", i.e., "the doing as it occurs" (*ỉrt mỉ ḫpr*) is another standard form of words,[17] used at various places in the RMP to introduce a calculation, particularly one that verifies a solution that has already been stated [Pee23, p. 24]. Taken in isolation, the numbers that appear in columns are easy to interpret: we have here two very elementary instances of the standard "two column" mode of presenting arithmetic in ancient Egyptian texts. (A slightly more complicated example is given in Figure 5.) The array of numbers on the left represents the calculation $\frac{1}{2} \times 400$, with result 200, while that on the right gives 2×1000, with result 2000. The calculations involved look very similar to those indicated earlier in the problem ($\frac{1}{2} \times 4$ and 2×10), but the orders of magnitude have seemingly gone awry. More puzzling still is the final comment "Its acreage is 2", considering not only that the number just arrived at is 2000, but also that we know from above that the answer to the problem should in fact be 20. Since these calculations are no longer explicitly in terms of *khet*, Eisenlohr speculated that perhaps other units were being employed [Eis77, p. 127]. The first calculation is in fact easily explained this way: it is the halving of the base of the triangle, but with the length of the base converted from *khet* into cubits. It would be another twenty years before a complete understanding of ancient Egyptian weights and measures was provided by Francis Llewellyn Griffith (1862–1934) in the 1890s (as summarized in [Pee23, pp. 24–26]), and this showed that Eisenlohr's speculations were indeed correct. The most commonly used unit of area measure in ancient Egypt was the *setjat* (*sṯꜣt*, also known by its later Greek name *aroura*),[18] equal to one square *khet*, often conceived as consisting of 100 strips, each 1 cubit × 100 cubits. However, two other units were sometimes preferred for calculations of a practical nature: the "cubit-of-land" (*mḥ-tꜣ*), equal to a strip of land 100 cubits × 1 cubit, and the "thousand-of-land" (*ḫꜣ-tꜣ*), a 1000-fold multiple of the cubit-of-land, equivalent to an area measuring 100 cubits × 1000 cubits (or 1 *khet* × 1000 cubits). With these conversion factors in mind, we can now make sense of the second calculation beneath the picture in Problem 51. Here, the 200 cubits that result from the first calculation have been implicitly converted back into 2 *khet*, which is then multiplied by the *meryt* given in cubits. The result is an area of

[17] The special character *ḫ* represents the sound of *ch* as in Scottish "loch".

[18] The special character *ṯ* represents the sound of *tj*.

2000, measured in the hybrid units of *khet* × cubits. By the comments above, we see that this is then equivalent to 2 thousands-of-land, hence the conclusion “Its acreage is 2”. Early commentators on the problem may be forgiven for thinking that these confusing calculations were simply a continuation of the perceived muddle of the first part.

Interpreting Problem 51: Peet

As the late nineteenth-century authority on the RMP, Eisenlohr’s conclusions regarding the erroneous nature of Problem 51 were adopted by most readers (see, for example, [Can80, vol. 1, p. 49]). One exception was Griffith in some notes that he published towards a new edition of the RMP. While criticizing the apparent copying errors that appear in some of the surrounding problems, Griffith asserted in passing that Problem 51 contains no mistakes [Gri94, p. 236]. He offered no explanation for his differing from Eisenlohr on this point, but he was evidently deciding to interpret *meryt* as “height”. Like Peet after him (see below), Griffith had perhaps developed a respect for the mathematical attainments of ancient Egyptian scribes, and it may have been anathema to him to credit them with a patently erroneous area calculation. He did note, however, in reference to the implied units in Problem 51 that “the language used is obscure to the last degree” [Gri94, pp. 236–237].

Twenty years later, however, Peet tackled these issues in his new edition of the RMP, and arrived at a rather more positive view of Problem 51 [Pee23, pp. 91–94] (see Figure 6). In reference to Eisenlohr’s interpretation, Peet noted that “the matter is hardly so simple as he supposed” [Pee23, p. 91], and proceeded to give a characteristically systematic and painstaking analysis of the problem. In Peet’s view, the difficulties surrounding Problem 51 consisted of two closely related issues that had arisen in Eisenlohr’s attempts at interpreting the problem: not only the question of whether the area calculation is correct, but also that of whether it was intended to apply to all triangles, or simply to a special class thereof. Peet arranged the available evidence under four broad headings [Pee23, p. 91]:

1. “The names of the triangle and its parts.”
2. “The position of the numbers marked in the figure.”
3. “The shape of the figure.”
4. “The striking phrase ‘This is its rectangle.’ ”

He then took each of these points in turn, and we follow his method here.

1. As we have already noted, *sepedet* is essentially a word that indicates a pointed thing, so, as Peet commented, we might be tempted to conclude that it refers only to a triangle with an acute angle at the “apex” (i.e., the right-hand end of the triangle in Figures 2–4). However, although he suggested that this might have been the original meaning of the word, Peet was content to take *sepedet* as a technical mathematical term for triangles in general. As to the parts of the triangle, Peet saw no difficulties with *tep-ra*, and held that it was “beyond all doubt” that this indicates the base of the triangle (and it was here that he gave the “river mouth” suggestion that we noted earlier [Pee23, p. 91]). He deemed the word *meryt*, on the other hand, to be “the crux of the problem” [*ibid*]. As we have already asserted, its core meaning was “riverbank”, and Peet noted this alongside the possible extended meanings of “harbor” or “quay”. The latter interpretations, though common at the time that Peet was writing, have since fallen out of favor, in part because of their implication of a man-made structure. Subsequent attempts to reconcile the use of the word *meryt* in this context focused on the possible shapes that a *constructed* quayside might take (see, for example, [Gun26]). *Meryt* has a land classifier sign at the end (𓈅), showing that the word was considered to relate to a landscape-feature, and that it had not taken on an entirely abstract meaning. The word’s signification of a natural riverbank could give it a sense of “edge”, which Peet suggested as a possible meaning in the context of a triangle, an interpretation that he saw as being particularly appropriate (by analogy with a river delta) in the case of a triangle with narrow base,[19] and it is easy to imagine that for a field bordering the Nile, the word

[19] But see the comment in footnote 14 above.

No. 51. (Pl. O.)

"Example of reckoning a triangle of land.[1] If it is said to thee, A triangle of 10 *khet* in its height and 4 *khet* in its base. What is its acreage?

The doing as it occurs:

You are to take half of 4, namely 2, in order to give its rectangle. You are to multiply 10 by 2. This is its acreage.

1	400	1	1000
$\frac{1}{2}$	200	2	2000

Its acreage is 2.[2]"

Figure 6: Peet's translation of Problem 51 in the Rhind Mathematical Papyrus [Pee23, p. 91].

"riverbank" could be used as a term for one side of an area of land. Peet, however, noted the naturally arising question of which side the *meryt* was meant to be in this case, before suggesting that we might alternatively reject the river analogy and take *meryt* to mean height. He concluded his discussion of this point with the remark that "This is just possibly the correct solution" [Pee23, p. 91].

2–3. In turning his attention to the picture at the centre of Problem 51, Peet moved beyond the largely philological considerations that we have outlined in the earlier parts of the present article, and indeed moved also beyond Problem 51 itself. His goal was to consider the status of such images in the context of the whole papyrus, and to assess just what information, precise or otherwise, they were intended to convey. We have already encountered the issue that in Problem 51 it is the upper edge of the triangle — and not the height — that appears to be labelled as the *meryt*. By looking at other more visual problems in the RMP (specifically, Problems 56–58), Peet was able to dismiss this difficulty immediately. These other problems concern the slope (*seked*) of a given pyramid, and each is accompanied by a two-dimensional representation of the pyramid. As Peet observed, the label for the dimension that is unequivocally the height of the pyramid in each case appears *outside* the depiction of the pyramid (Figure 7). The importance of being able to engage with the original manuscript is relevant here, for Peet noted that the reason for the images being labelled in this way was probably due to "the exigencies of arrangement in narrow horizontal bands" used to lay out the whole text [Pee23, p. 91]. His conclusion therefore was that we cannot necessarily take the labelling of the pictures at face value, and moreover that certain lengths (here, the height of the pyramid) that are central to a given calculation may not in fact be drawn into the corresponding image. Peet also found useful evidence elsewhere in the papyrus, in Problem 53, which is visible at the bottom of Figure 1 (the triangle with red writing inside and the black text immediately below it). This problem was probably intended as another area calculation, but some of the numbers within it are apparently muddled. It is largely numerical in its phrasing, and contains no special terminology, but it is possible to deduce that the height of the triangle in question is 7 *khet*, which is labelled outside the picture: it is the red mark, the hieratic sign for 7, which appears beneath the triangle's left-hand vertex. As Peet observed, this is, allowing for a 90° rotation, the same position in which the height was marked in Problems 56–58 (Figure 7). Overall, though, Peet used these deliberations to reach the conclusion that "nothing must be argued from the position of the 10 *khet* in No. 51" [Pee23, p. 92]. Following straight on from this, he dismissed his third point in just four lines: "It would be unwise to attribute too much importance to the shape of the triangles actually drawn in the papyrus" [*ibid*]. The triangles depicted in Problems 51–53 of the RMP are all of the same shape but of different dimensions, so we might speculate that this was simply the standard way in which ancient Egyptian scribes drew triangles

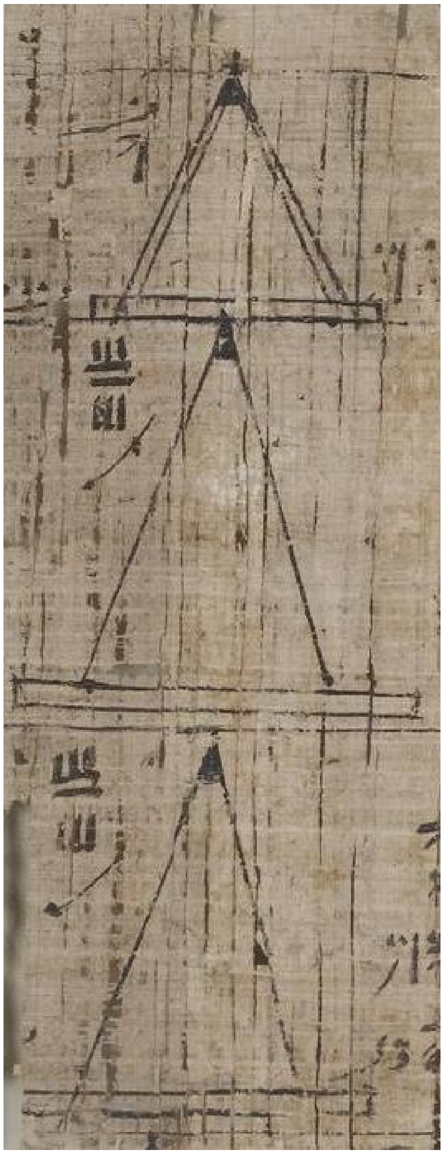

Figure 7: Two-dimensional representations of pyramids in Problems 56–58 of the RMP. In each case, the height of the pyramid is written at the top left of the picture. © The Trustees of the British Museum. Shared under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0) licence.

in such texts.[20] Either way, it appears to be inappropriate to try to extract precise information from the pictures — hence our preference for the term "picture" or "image" over "diagram".

4. Whenever he offered speculation as to how ancient Egyptian scribes might have arrived at a particular solution, Peet invariably opted for the empirical point of view, suggesting that trial-and-error and experimentation, possibly over generations, had ultimately led to usable methods and answers.[21] This is the approach that he took to the phrase "its rectangle". To begin with, there is little doubt here that the "it" in question really is the triangle with which we started, for the gender (feminine) of *sepedet* matches that of the suffix-pronoun that indicates possession of "its rectangle". Eisenlohr had tackled the interpretation of this particular phrase, and had suggested, in what seems like a rather Euclidean approach, that it calls for a rectangle whose height is half the base of the triangle to be erected on one of the long sides [Eis77, p. 126]. Peet dismissed this suggestion, however, by citing the outstanding issue of which side we are supposed to use. Peet appears implicitly to have regarded the idea of constructing the rectangle corresponding to a given triangle as being a general method; going beyond the immediate concerns of Problem 51, he asserted — albeit without much explanation — that Eisenlohr's approach could only be used for tall triangles, since it would become a "manifest absurdity" for short ones [Pee23, p. 92]. Instead, Peet suggested one of two possible graphical explanations of the phrase. In the left-hand diagram of our Figure 8, the triangle is assumed to be isosceles and the *meryt* is taken to be the vertical height, so that each of the smaller rectangles is enclosed by the

[20] Problem 52 is visible in Figure 1 between Problems 51 and 53: it consists of the text to the right of and immediately below the trapezium. Although seemingly a problem about the area of a trapezium, this was considered by Peet to be a problem about triangles, because the trapezium is introduced as a truncated triangle. The term used is *ḥȝkt*, where *ḥȝk* is attested elsewhere as meaning "to cut off the tail"; the meaning is reinforced by the use of the tail determinative ⟍ at the end of the word.

[21] See, for example, [GP29], where a particular volume calculation in the MMP is explained in terms of cutting up blocks of damp clay.

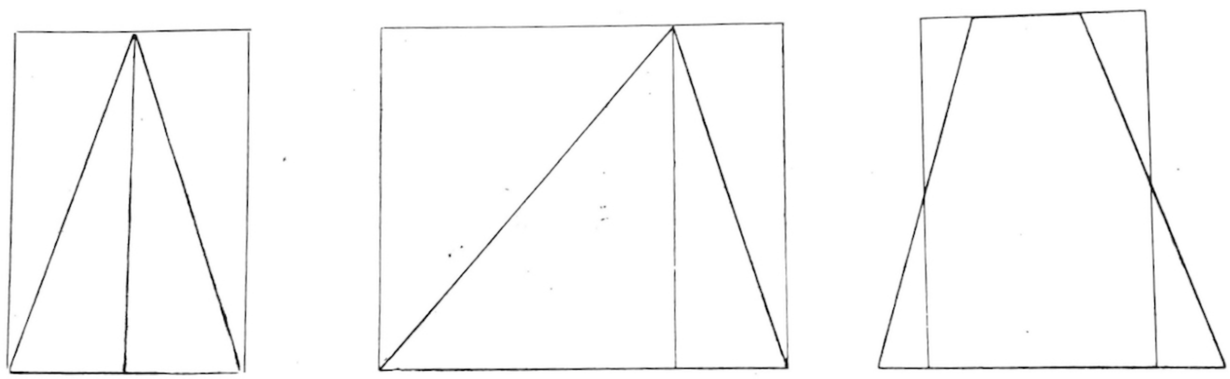

Figure 8: Diagrams used by Peet to explain the phrase "its rectangle" [Pee23, p. 92].

meryt and half the length of the base. The middle diagram covers the case of a non-isosceles triangle; "its rectangle" is not shown here, but Peet noted that it is easy to see from the symmetries of the smaller triangles involved that the large all-enclosing rectangle has area twice that of the original triangle. For someone who was so careful never to impose interpretations on a text, this last conclusion is arguably an unusual over-reaching statement for Peet, since such ideas of symmetry are not otherwise found in ancient Egyptian mathematical texts. But if we allow the speculation, this could perhaps have been the empirical route towards a method for finding the area of an arbitrary triangle, as Peet suggested. While he was considering the possible graphical background to Egyptian problem-solving, Peet also drew upon Problem 52, concerning the area of a trapezium: the right-hand diagram in Figure 8 suggests a graphical way in which knowledge of the area of a truncated triangle may have been arrived at. Problem 52, moreover, presents us with the same difficulty as Problem 51. Here, the area of the trapezium is calculated as half the sum of the parallel sides, multiplied by the *meryt*: if we interpret *meryt* as the height of the trapezium, then we get the correct answer, but not in general otherwise.

In beginning to sum up his treatment of the difficulties of Problem 51, Peet returned to the issue of the possible shapes of the triangle in question, and worked through these one by one. If, first of all, the triangle was intended to be scalene, then, Peet asserted, we must take *meryt* to mean the height of the triangle, and hence arrive at the correct calculation of the area, otherwise the solution is ambiguous. If the triangle is isosceles, then either interpretation of *meryt* can be taken, but if the *meryt* is the side length, then Problem 51 only works for triangles with small base compared to height (Eisenlohr's conclusion). Peet threw one final possibility into the mix, that the triangle is right-angled, but he immediately dismissed this option by noting that nothing about the problem points us in this direction, and that the notion of a right-angled triangle is dealt with in the MMP in its own special way, distinct from what appears in the RMP.[22] Having outlined these various possibilities, however, Peet did not commit himself to any one interpretation. Indeed, this is where we find Peet at his most cautious — and also his most frustrating. A century on, the impression that emerges from reading Peet's carefully considered analysis of Problem 51, which takes up several densely written pages of his edition of the RMP, is that it would be simpler overall, in terms of removing complications and contingencies, to reject the philological interpretation of *meryt* as "side" and instead follow the mathematical line. At the last moment, however, Peet backed away from making any firm conclusion [Pee23, p. 92]:

[22]See, for example, Peet's comments in [Pee31a, p. 433].

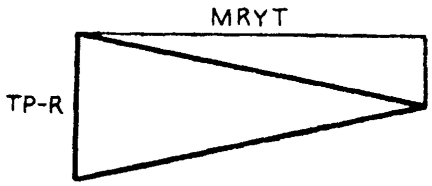

Figure 9: Gunn's suggested interpretation of *meryt* [Gun26, p. 133].

> The internal indications thus make it very difficult to draw any certain conclusion as to the exact meaning of *mryt* and the consequent correctness or otherwise of the Egyptian solution.

Although Peet was undoubtedly correct in asserting that we can never reach certainty over this issue, there was probably an element of professional positioning at play here as well. Peet had occasionally styled himself as a mathematician among Egyptologists [HP23], but here he nailed his philological colors to the mast. As someone who had trained initially as an archeologist and only later become a specialist in text-based studies, he may have felt that he had something to prove, and that he should align himself securely with the philologists.

In order to round off our discussion of Peet's treatment of RMP Problem 51, we consider briefly what other readers have made of his arguments. Among the first of these was the young Neugebauer, whose very first publication was a review of Peet's edition of the RMP. We would perhaps expect the more mathematically-led readers to gravitate naturally towards the interpretation of *meryt* that gives the correct solution to the problem, and this is indeed what we find with Neugebauer: despite Peet's extreme caution, there does not appear to have been any doubt in Neugebauer's mind that Peet had successfully shown that ancient Egyptian scribes did indeed have the correct formula for the area of an arbitrary triangle [Neu25, p. 68]. And it was not just mathematical readers who thought so. Another reviewer who cut straight across Peet's doubts was his later collaborator Gunn. Renowned for being exacting in his philological scholarship, Gunn appears to have had a technical turn of mind, stemming perhaps from his having briefly studied engineering before turning to Egyptology. His detailed review of Peet's edition of the RMP for the *Journal of Egyptian Archaeology* would later be described by Peet as an "invaluable" supplement to the main text [Pee31a, p. 409, n. 2]. In the review, Gunn was particularly firm in his opinion about Problem 51 [Gun26, p.133]:

> There can be no doubt that this [the *meryt*] is equivalent to what we call the "height" of a triangle. That the geometricians of the Middle Kingdom were stupid enough to calculate the area of a triangle by the formula $\frac{tp\text{-}r}{2} \times$ length of another side is hardly conceivable.

One aspect of Gunn's review that seems a little overelaborate in this connection, however, is his effort to find a new interpretation of the construction of the problem that fully reconciles the use of the word *meryt* in this context. He offered the suggestion that *meryt* might in fact denote a *pair* of perpendicular lines that can be added to the top right of the triangle in Problem 51 in such a way that the upper edge of the original triangle forms the hypotenuse of a new one (see our Figure 9). However, the diagrams that accompany Gunn's discussion look rather like cross-sections of possible man-made quaysides, and are probably somewhat wide of the mark.

It is interesting to observe that Peet's published views concerning the correctness of RMP Problem 51 shifted over time. By 1930, when he coauthored a paper with Gunn, *meryt* was taken immediately to mean "height" [GP29]. This paper, which dealt with triangle and pyramid problems in the MMP, added further fuel to the latter interpretation, for MMP Problem 17 features a triangle of the same dimensions as that in RMP Problem 51, though the question asked is a different one: given a triangle whose area is 2 thousands-of-land and whose base is $\frac{2}{5}$ of its height, find the base and the height.[23] Here, the solution of

[23]Elsewhere in the MMP, Problem 4 is a heavily damaged

the problem gives every indication that *meryt* must mean height, in part because this helps to make sense of a further technical term that appears, namely *ideb*, the ratio of the height to the base, literally meaning "bank". What is more, although Peet has dissuaded us from reading too much into the shape of the picture accompanying a problem, it is striking that the roughly drawn picture that appears in MMP Problem 17 is clearly scalene [GP29, Plate XXXVI]. In addition, the determinative that qualifies *sepedet* in the MMP is different from that in the RMP, and this time is a scalene triangle. The possibility of explaining discrepancies of language by reaching for the special case of an isosceles triangle does not appear to be available to us here, so we are left with little choice but to admit that *meryt* must mean "height".

Triangles also arose in Peet's 1931 Manchester lecture, mentioned earlier [Pee31a]. In the lecture, he discussed questions concerning the area of a triangle in detail, and he was able to offer a new theory to his audience, advanced by the orientalist V. V. Struve (1889–1965) in his edition of the MMP [Str30]. Following Eisenlohr, Struve asserted that the triangle in RMP Problem 51 was isosceles, but differed from Eisenlohr in simultaneously interpreting *meryt* as "height". It is then possible to divide the triangle along its length into two equal parts, with the *meryt* forming the "limit" between the two. Peet, however, was not convinced: the *meryt* is not the "limit" between two things, but the "edge" of one thing [Pee31a, p. 431]. Recall that one of the reasons for declaring the triangle to be isosceles was the form of the picture in the papyrus. Struve had attempted to back up his claim by identifying isosceles triangles in the MMP, thus attempting to show that such triangles were an object of study by ancient Egyptian scribes. Again, Peet was unconvinced by some of these identifications, and felt that this pointed to an inconsistency in Struve's methodology [Pee31a, p. 431]:

> The evidence of these roughly drawn figures is of very uncertain value, but it must either be rejected altogether or respected when it tells against, just as much as when it tells in favour of, a theory.

Taking a step back from the technicalities of individual problems, Peet adopted a broader view of ancient Egyptian mathematics. Noting that Egyptian scribes had certainly been able to calculate the volume of a truncated pyramid (MMP Problem 14, as discussed in [GP29]), as well as the area of a circle to a reasonable degree of accuracy,[24] he suggested that it would be inconsistent to deny them a knowledge of the correct way of calculating the area of a triangle [Pee31a, p. 433]. In this last article, Peet's positive opinion of ancient Egyptian mathematics was clear: "those who have studied what Egypt did for mathematics before 2000 B.C. are moved to admiration rather than criticism" [Pee31a, pp. 440–441].

Conclusion

The reader who is familiar with the literature on ancient Egyptian mathematics will probably have noticed that we have not yet mentioned on other key modern text about the RMP: the further edition that was published by A. B. Chace (1845–1932) in two volumes in 1927 and 1929 [Cha27]. Although there had been questions of a collaboration between Peet and Chace early in the 1920s, this did not come about,[25] and the two editions evolved along different lines. Whereas Peet's edition is quite philologically focused, Chace concentrated much more on communicating the mathematics, with the result that while Peet's edition continues to be favored by Egyptologists, Chace's is better known to mathematicians. Chace's edition is also in many regards the more lavish one, containing photographs of the papyrus, as well as color facsimiles of the hieratic and hieroglyphic transliterations; in common with other purely

duplicate of RMP Problem 51 [Imh16, p. 122].

[24] This calculation (which consists of squaring eight-ninths of the diameter) appears in several places in the RMP, including in some problems visible in Figure 1. Immediately above Problem 51, for example, Problem 50 asks for the area of a circular field of diameter 9 *khet*, while Problem 48 (at the bottom right of the figure) compares the area of this same circle with that of its enclosing square.

[25] Mainly because Peet had a low opinion of Chace's scholarship: see [HP22, pp. 220–221].

Problem 51

Example of a triangle of land. Suppose it is said to thee, What is the area of a triangle of side[1] 10 khet *and of base* 4 khet?

Do it thus:

1	400
½	200
1	1,000
2	2,000.

Its area is 20 *setat.*

Take ½ of 4, in order to get its rectangle. Multiply 10 times 2; this is its area.

Figure 10: Chace's translation of Problem 51 in the Rhind Mathematical Papyrus [Cha27, vol. 1, p. 92]. The footnote attached to the word "side" refers the reader to Chace's earlier discussion of the translation, which we summarize in the main text here. Note that Chace chose to read the text strictly from right to left, thus placing the calculation beneath the triangle in the middle of the translation, rather than at the end, where it appears in Eisenlohr's and Peet's translations.

Egyptological editions, Peet's contains only the hieroglyphic transcription of the papyrus, without reproductions of the hieratic. Even Egyptologists admit that if one wants to identify a specific passage of text on the original papyrus, it is considerably easier to do so using Chace's version. The latter is, however, rather idiosyncratic in its layout, and does not always follow the conventions of Egyptological publishing, making it otherwise unsatisfactory for Egyptologists to use.

Given the comments in the preceding paragraph, we would therefore expect Chace to have taken the mathematical route and have interpreted *meryt* as "height". However, when we turn to his free translation of Problem 51 [Cha27, vol. 1, pp. 92–93], we see that he has taken it as "side" and makes curiously little comment on the inaccuracy that arises (Figure 10). Earlier in his first volume, however, Chace had provided a brief discussion of the types of area calculation present in the RMP, and he had stated the opinion that Problem 51 strictly concerns an isosceles triangle [Cha27, vol. 1, p. 36]. Chace offered little justification for this view, though footnotes to the relevant passage indicate that he was aware of some of the wider discussion of this issue. He attributed to Peet the view that *meryt* means "height", but was not convinced by Peet's arguments, which he found "rather inconclusive and not very clear" [Cha27, vol. 1, p. 37, n. 1] — a fair comment perhaps, given Peet's balanced caution. The only other clue that Chace offered about his point of view was the stark comment, influenced perhaps by the forms of the pictures in Problems 51–53, that "It does not seem probable that the author [of the RMP] had much conception of different types of triangles" [Cha27, vol. 1, p. 37]. If by the end of the 1920s, Peet's admiration for ancient Egyptian mathematics had grown to the point where he was inclined to give the scribe the benefit of the doubt over the area of a triangle, Chace adopted a more restrictive stance. His view may perhaps have stemmed from the same source as the mathematicians' disappointment over the elementary nature of ancient Egyptian mathematics that we noted earlier. By the 1920s, the idea that ancient Egyptian mathematics was "primitive" had become firmly established in mathematical writings [HP22, §8], and

Chace could have been influenced, consciously or otherwise, by this. In 1931, in interpreting a very fragmentary problem in the MMP, Peet commented that "the reasons which might incline us to the one solution or to the other are psychological rather than rational" [Pee31b, p. 106], and this observation might easily be extended to the study of ancient Egyptian mathematics more generally.

So where does this leave us? If we try to follow the philology literally, we end up in knots in trying to rationalize Problem 51 (and also 52) as written. The simplest approach is to follow the mathematics and arrive at an interpretation of the problem that is mathematically sound. Taken within the wider context of ancient Egyptian mathematics, this explanation is also the most sympathetic. It thus fits well with modern methodologies (which had also been advocated by Peet in the 1920s [HP22, §8]) that aim to consider Egyptian mathematics on its own terms, without any judgement stemming from a knowledge of later mathematics. The most recent general survey of ancient Egyptian mathematics has no hesitation in following the mathematically correct interpretation, with minimal commentary [Imh16, p. 121]. But we are still left with the problem of *meryt*. How can we interpret this as "height" when that is patently not what the word means elsewhere? A likely explanation, which appears to have been adopted implicitly in such texts as [Imh16], lies in the nature of technical terminology. The original meaning of *meryt* must be "riverbank", but the problems in the RMP suggest that the word might have gone the way of mathematical and other specialist terminology and have been actively co-opted away from its initial sense towards a technical meaning within mathematical texts. By the late Middle Kingdom, *meryt* may indeed have come to denote something specialized in mathematical texts, just as the verb "to reckon" was a technical adaptation of the more standard uses "to do" or "to make". The riparian terminology is not to be taken literally, but is a technical usage that derives from the "riverbank", and that is an especially appropriate extended usage when the problem being discussed in presented in terms of an area of land. Such specialized usages of the word *meryt* are known elsewhere, as when it is used as a collective noun for a "bankful of crocodiles" in a poem [Fau62, p. 112]. Context is everything, and here, a knowledge of Egyptian landscape can help: the riverbanks of the Nile can in fact be extremely high (see Figure 11), and the plot of one classic Egyptian narrative poem, *The Eloquent Peasant*, even depends on the unscalable height of the riverbank [Par97, pp. 58–59]. So the association of the word "riverbank" with "height" would not have been as counter-intuitive for an Egyptian scribe as it is to a modern reader. With only a few examples from the RMP and the MMP, this assertion can never be proven, but such a contextualizing approach can reconcile the apparently conflicting mathematical and philological perspectives.

Figure 11: *Meryt* in its original sense: a riverbank near Kom Ombo in 2003, showing the height above the Nile. Picture courtesy of Steve Lamb.

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