Developing Papua New Guinea teachers' mathematical knowledge for teaching: Prospective Teachers and Algebra Concept

Betty Bala

A Research & Development Project

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Abstract

Mathematics teachers must have Mathematical Knowledge for Teaching (MKfT) as this is the knowledge required to teach in a school. It is seen that teachers who do not have an adequate knowledge of the MKfT are less well equipped to help students learn. A school curriculum that is meaningful and useful for Papua New Guineans is the driving force behind the recent educational reformations. However, school children still lack the application of the mathematical knowledge (SMK) in their everyday life which makes the learning of mathematics meaningless problematic in Papua New Guinea (PNG) and which could be attributed to mathematics teachers' lack of MKfT. Hence, this study took an action reaction approach whereby cycle one was a base line survey that was conducted with beginning teachers and other stake holders in Goroka gauging their perceptions of the MKfT required by teachers to teach in secondary schools in PNG and the issues and challenges in developing this knowledge. Findings from cycle 1 showed that, some respondents perceived that trainee teachers' SMK was good but needed PCK (an aspect of MKfT) development through a workshop. Therefore in cycle two the findings from the base line data were used to develop a workshop designed to enhance student teachers' MKfT, and overall, the evaluation of the workshop found it successful in beginning this enhancement. It also found that more sustained professional development work with beginning teachers, mentors and teacher educators needed to be conducted in the PNG context.

Key words: Mathematical Knowledge for Teaching (MKfT), Subject Matter Knowledge (SMK), Pedagogical Content Knowledge (PCK)

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1.0 Introduction

Taking into consideration the impact of teacher knowledge on their practice in the classroom has been an interesting issue. Participating during teaching practice as a teacher mentor then, and, in the recent years whilst engaged as a teacher educator, this was an aspect I had considered. That is, prospective teachers' teaching is focused on worksheet/textbook, ie, teaches just the content prescribed in that text and they call it teaching. Indeed, research affirms that the issue teacher education institutions in Papua New Guinea (PNG) and worldwide are responsible for the prospective teachers' knowledge and are the major elements in the country's hope for educational progress (Palmer, 1987; Price and Valli, 2005). Therefore, it is a must that they develop and build prospective teachers' knowledge for teaching specific subjects (Palmer, 1987; Wu, 2005).

This paper begins with a brief review of the literature surrounding teacher knowledge about mathematics and its impact on teaching and learning of mathematical concepts in the classroom.

Secondly is a description of a mini-research project that follows which has this aim:

•The knowledge mathematics teachers need to develop in order to teach mathematics.

Finally I will examine the findings of the research project, relate them back to the literature and consider how they relate to my role as a Teacher Educator in PNG.

The concern that the prospective teachers must be trained with an in-depth mathematical knowledge for teaching (MKfT) during teacher training in Papua New Guinea is enduring (Matang, 1998). Thus, teacher education institutions in PNG and worldwide are responsible for the prospective teachers' knowledge and are the major elements in the country's hope for

educational progress (Palmer, 1987; Price and Valli, 2005). They can develop and build prospective teachers' content knowledge for teaching specific subjects (Palmer, 1987). Hence, is evident in research that a prospective teacher can acquire the form of mathematical knowledge for teaching from a university or a teacher training college (Wu, 2011).

However, my personal observation whilst a secondary school teacher for eight (8) years and currently a teacher educator in a teacher education institution is supported by research that teacher education institutions in the country continue to train teachers lacking this vital knowledge for teaching during teacher training (Kukari, 2004; Palmer, 1987). Hence, a result of this revealed by research is the obvious widespread traditional textbook based teaching approach that lacks suitable pedagogical content knowledge (PCK) of the subject matter knowledge (SMK) in that textbook. Henceforth, this paper entails a research and development project aimed at developing mathematical content knowledge for teaching, specifically on the algebra concept, for some secondary school prospective teachers at a teacher education institution in PNG.

The chapter firstly describes the educational structural and curriculum reforms that embedded reasoning and problem solving, particularly in mathematics that requires this knowledge for teaching. The second section describes the teacher context in this reformed structure and curriculum in Papua New Guinea. The third section of the chapter is an entailment of why this is the focus area of this research in PNG's educational reformations, and, the final section briefly provides information on the rest of the chapters of the dissertation.

1.1 Education in general in PNG.

An old but prominent educationist perspective of education is that by education a child can be developed completely in body, mind and spirit (Gandhi and Kumarappa, 1953). Matang (2002) claimed that PNG can develop and produce these aspects in a child by a more knowing teacher at any stage of its education system. Hence, Integral Human Development (IHD) is Papua New Guinea's first and foremost goal in its national goals and principles which had become the basis

of its philosophy of education. IHD in a quality education for all (EFA) Papua New Guineans is the driving force behind the educational structural and curriculum changes over the last twenty (20) years (Hahambu, 2011).

Thus, stated is its current structural reform aimed at complete development of a child at various stages. Elementary is the first stage that is community–based (Elementary Prep, Elementary 1 and Elementary 2) and uses community vernacular (changing to English) as the main language of instruction. The primary stage consists of grades 3, 4, 5, 6, 7 and 8 is the second in the formal education system gradually bridging to English as the language of instruction, while maintaining use of the students' vernacular. Hence, secondary is the final stage where student have two years of schooling in Grades 9 and 10 and then the number of years is increased to four for those students who are eligible to continue to Grades 11 and 12. Thus, only those pupils who pass the grade (8) examination proceed on to the secondary stage that consists of grades nine (9) and ten (10) forming the lower secondary. Henceforth, the grade ten (10) examination determines whether a pupil continues his/her secondary education at the upper end.

The above system is called the 'Out-come Based Education (OBE)' that implements an 'Out-comes Based Curriculum (OBC)' in all grades at the different stages. OBC was seen relevant curriculum for PNG because its emphasis is skills development and the use and maintenance of the local languages of the community. The curriculum was foreseen primarily to help provide the knowledge, skills, attitudes and values in a particular grade in a specific subject relevant to both Papua New Guinean and international societies. However, teachers' lack of content knowledge in the respective subjects at the specific grades affects students' learning in the classroom, hence the desired outcome (knowledge, skills, attitudes and values) of the curriculum not evident in the majority of the learners (Copeland, No date).

1.2 Teacher context in PNG

Teachers' lack of content knowledge, pedagogical content knowledge (PCK) in particular is partly contributed by the traditional perception of teaching in Papua New Guinea which is the passage of traditional knowledge and skills from an older generation (more knowing) to the younger orally and through imitation. The traditional perceptions about teaching still strongly influence teachers' teachings in the classroom today (Kember and Kwan, 2000; Prosser and Trigwell, 2004) in PNG. They are therefore perceived by both students and parents as the only source of all knowledge and information in the formal classroom environment (Matang, 2002: 28). Thus, learning for students in schools is still ineffective despite the establishment of many teacher education institutions for all stages in PNG (Copeland, No date). I believe that the teacher education institutions are wholly responsible for teachers' lack the knowledge for teaching as they continue to train teachers as providers of knowledge and not prepare teachers who can assist students build new knowledge onto their existing (Kukari, 2004).

1.3 Area of research

This research titled 'Developing Papua New Guinea teachers' mathematical content knowledge for teaching: Prospective Teachers and Algebra Concept' is my focus area. This is because the National Council of Teachers of Mathematics, NCTM (2000) emphasized that the students' gaining of mathematical knowledge and their capability to exercise this knowledge to work out mathematical problems is limited and is influenced by the teaching they encounter in schools by their teachers. In addition, my observations of mathematics lessons in Papua New Guinea suggest that the gaps in students' knowledge may also be a result of teachers' lack of mathematical knowledge for teaching, which results in teachers focusing on shallow skills rather than deep understanding. Thus, an in-depth understanding of mathematical content knowledge is required for effective teaching. Therefore, because the literature and my professional experiences suggest a need to develop student teachers' mathematical knowledge for teaching designed and delivered intervention that enabled prospective teachers that developed in-depth

mathematical knowledge for teaching (MKfT) was paramount for me in this research. The concepts, mathematical knowledge for teaching, SMK and PCK will be explained clearly in detail later in the literature review.

Henceforth, this research is intended to contribute to realising Papua New Guinea's vision 2050 through the mathematics curriculum to embed reasoning and problem solving in the next generation of students. Therefore, by increasing teachers' in-depth mathematical knowledge for teaching this study will contribute to changing mathematics teaching in PNG towards a greater emphasis on problem solving and reasoning. In summary personal observation and the literature suggested that prospective teachers lack knowledge and require interventions by a more knowing other to develop their professional knowledge and practice (Blanton et al.; 2015).

Thus are the following research questions which guided me throughout the conduction of the action research:

1. What do beginning teachers and other key stake holders (mentors and teacher educators) perceive are the issues and challenges related to mathematical content knowledge that trainee teachers are faced with?
2. What mathematical knowledge for teaching do beginning teachers and stake holders say is required by trainee teachers?
3. How can this knowledge be developed in a workshop on the teaching and learning of algebra for final year mathematics major students in UoG?

1.4 The outline of the write-up

Chapter 2 of the dissertation is a literature review synthesised from educationists worldwide, most of whom are mathematics educators in particular. The research sheds light on the mathematical content knowledge that is required by teachers for teaching. The chapter further discusses some implications faced by teachers lacking the knowledge. Finally, the chapter

in this research reveals how mathematical content knowledge can be developed for both prospective and practising teachers.

Chapter 3 discusses the methods applied to carry out the action research. In addition to discussing the appropriateness of the methods employed for answering the research questions, a rationale for each method used is stated. Additionally, this chapter addresses ethical issues faced and articulates how they were taken on board considerably with respect for Research.

Chapter 4 summarises the key findings bringing key literature back in and chapter 5 concludes and discusses implications of the research for my practice as a teacher educator.

2.0 Literature review

Research had shown that mathematical knowledge for teaching is the knowledge teachers need for teaching (Shulman, 1986; Morris, 2001; Ball et al., 2005; Ball et. al, 2008; Ell et al., 2008; Stylianides and Stylianides, 2010; Lannin et al., 2013; Nolan et al., 2015; Pournara, 2015; Chikiwa, 2017). Thus, mathematical knowledge for teaching is the form of knowledge that mathematics teachers require to make the mathematical content knowledge accessible and applicable in real life for students in schools (Shulman, 1986; Leikin and Waynberg, 2007; Chick and Stacey, 2013; Ball et al., 2008; Hill et al., 2008). The impact of mathematics teachers in schools who lack the mathematics knowledge for teaching is that the students lack in-depth knowledge of the mathematical concepts (Hill et al., 2005; Goe, 2007). Therefore, teacher education institutions must adequately develop mathematics teachers' mathematics knowledge for teaching during initial teacher training (Rowland et al., 2005; Pournara, 2009, Agyeia and Voogt, 2016; Subramanian, 2014). In addition, this form of knowledge can be continuously enriched through professional development programs in schools (Ball and Cohen, 1999; Clarke and Hollingsworth, 2002).

The literature review now goes on in more detail to examine this mathematical content knowledge for teaching through answering the following questions;

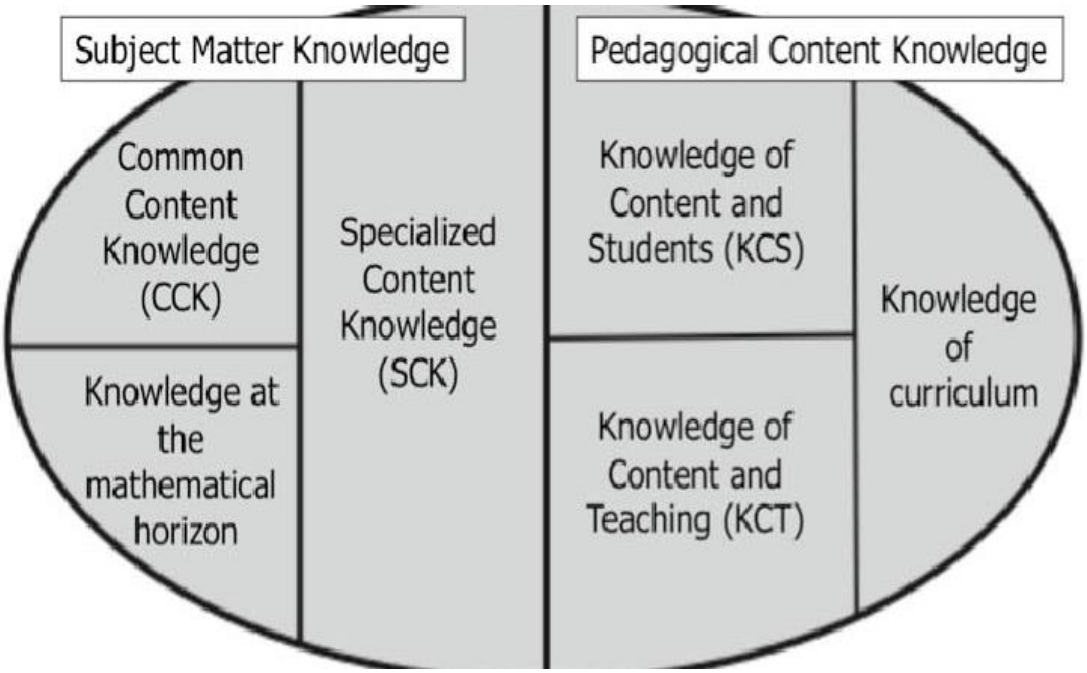
2.1 What does the literature say about the knowledge needed to teach mathematics?

2.2 What are some implications of teachers who lack in-depth mathematical content knowledge for teaching?

2.3 What does research say about how to develop teachers' in-depth mathematical knowledge for teaching?

2.1 What literature says about the mathematical knowledge needed to teach mathematics.

Most research (few examples - Morris, 2001; Ball et al., 2005; Ell et al., 2008; Krauss et al., 2008; Stylianides and Stylianides, 2010; Lannin et al., 2013; Nolan et al., 2015; Pournara, 2015; Chikiwa, 2017) which are related to the mathematical knowledge that is required by teachers for teaching have built on further from Shulman's (1986, 1987) work. According to literature, the knowledge needed by teachers to teach mathematics in schools is mathematical knowledge for teaching (MKfT) (Ball and Bass, 2000; Rowland et al., 2005; Ball et al., 2008; Krauss et al., 2008). The mathematics knowledge for teaching consists of Shulman's (1986) two categories, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). Ball et al. (2008: 395) claim this knowledge as the mathematical knowledge for teaching (MKfT) and is defined as "the mathematical knowledge needed to carry out the work of mathematics teaching". Ball and the team refined each category with the respective domains as shown in the diagram (2008: 403) below, which are used as the basis for what this literature say the mathematical knowledge needed for teaching mathematics.



2.1.1 Subject Matter Knowledge (SMK)

This section highlights subject matter knowledge (SMK) which is Shulman's (1986) first category of the mathematical content knowledge for teaching. In the section, SMK is defined in detail by employing Ball et al.'s (2008) three domains which are Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Knowledge at the Mathematical Horizon. SMK need emphasis as stated below because according to Wu (2005) and Ball et al. (2005) deeper understanding of the content can fundamentally alter a well prepared and planned pedagogical approach to a topic during instruction to suit the learning needs of the students.

Firstly, CCK is described by Ball and Bass (2000), Laren and Russo (2014), and, Venkat and Spaull (2015) as knowledge of mathematical concepts. This is the knowledge not required for teaching alone, but must be linked to teaching because teachers need it to teach the respective mathematical concepts during instruction with the students (Nolan et al., 2015). These mathematical concepts are unpacked in detail when required where necessary in teaching by teachers, for use in everyday life situations and work places (Sullivan, 2011). For example, four grade three students are asked to share a K10.00 equally amongst themselves but are stuck. After few minutes of thinking about how much each of them gets, they decided to unpack in detail their CCK of the mathematical concept 'division' to assist solve their sharing problem (Ball et al.,

2008). The mathematical concepts, how these mathematical concepts can be applied to solve problems in real life contexts, and, why these mathematical concepts arise in life must be known to everyone, including mathematics teacher (Ball and Bass, 2000; Pratt, 2012; Laren and Russo, 2014; Venkat and Spaull, 2015).

Secondly, in addition to teachers' CCK according to Wu (2011: 1), teachers cannot teach if they do not have the SCK. Ball et al. (2008) states that mathematics teachers' SCK is teachers knowing the mathematics they teach because their SCK must be their foundational knowledge that forms teachers' competency in which Adler et al. (2009) argue to be having full knowledge of the history of the mathematical concepts. Adler (2005) and Nolan et al. (2015) clarify that, unlike CCK, this knowledge is unique to teaching alone where teachers unpack mathematics work that teachers do to be able to do tasks such as to identify errors in students' assigned tasks and be able to identify students' misconceptions during instruction. Ball and Bass (2000), Llinares (2000), Krauss et al. (2008) and Watson and Harel (2013) affirm that in such situations, it is the teachers' SCK that enables clear understanding of respective mathematical concept(s) and be able to manage content related issues that arose during teaching. Thus, Laren and Russo (2014) argue that in addition to teachers' knowledge of algebra, they must successfully complete all algebraic problems. They must also know algebra deeply and flexibly so that they are able to assist students with their misconceptions. Therefore, mathematics teachers' knowledge for effective teaching in the classroom requires their SCK.

Thirdly, Ball et al. (2008) argues that teachers need the third domain of SMK which is the knowledge at the mathematical horizon. Knowledge at the mathematical horizon is the knowledge not directly related to instruction but comes either before or after a respective instruction (Llinares, 2000; Mosvold and Fauskanger, 2014). It is the knowledge of how different topics/mathematical concepts are related to each other in different grades/levels in whole of the school mathematics curriculum (Nolan et al., 2015). This knowledge helps the mathematics teachers where and what to build on from in a grade when teaching (Ma, 2010; Najike, 2004; Ball et al., 2005; Adler et al., 2009; Venkat and Spaull, 2015) because according to Adler et al. (2009) and Venkat and Spaull (2015), this knowledge makes teachers have rich knowledge in related topics/mathematical concepts. For example, Kashefi et al. (2010), Steel and Rogers (2012) and

Laren and Russo (2014) highlights the necessity of the knowledge at the mathematical horizon in the 'Calculus concept' that requires the knowledge of other concepts like algebra, trigonometry and geometry. Therefore, mathematics teachers must have the knowledge at the mathematical horizon so that they also can be rich with likely students' misconceptions before and after the grades at which they teach in order to appropriately fill in the necessary lacking knowledge (Hiebert, 1997; Steele and Rogers, 2012).

2.1.2 Pedagogical Content Knowledge (PCK)

PCK is Shulman's (1986) second category of the mathematical knowledge for teaching (MKfT). Again, Ball et al. (2008) defines PCK with the following domains which are Knowledge of Curriculum, Knowledge of Content and Students (KSC) and Knowledge of Content and Teaching (KCT).

Firstly, Knowledge of Curriculum is Shulman's (1986) second category of knowledge for teaching, however is a domain in PCK in Ball et al.'s (2008) work. According to Shulman (1986: 10), curriculum is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances. He classified the two as;

Vertical curriculum knowledge which is topics/concepts in school mathematics which will be taught later in the preceding years (Shulman, 1986). This is of course true in mathematics because Boley (1999) and Sullivan (2011) argue that mathematics unlike other school subjects begins with basic/fundamental mathematics. Thus, this fundamental mathematics necessitates the continuous growth of the mathematical knowledge deeper and wider with new knowledge (Klein, 1945; Baturo and Nason, 1996; Siegler and Booth, 2005; Sarama and Clements, 2009; Watson and Harel, 2013). Klein (1945) explains in detail the importance of the fundamentals of school mathematics which are required in the latter. In other words, Klein's perspective of school mathematics is more transmission orientated, which school mathematics is a set of discrete facts and methods that need to be learnt. That is, understanding deep ideas, such as that all the

operations in number and algebra can be built from repeated application of the filed laws, which happens to be a fundamental part of the teachers' knowledge. A dominant perspective is that mathematics grows like a tree, both upwards and downwards simultaneously, and, at the same time as mathematical knowledge branches out in new directions, it also grows deeper and more rigorous.

On the other hand, apart from knowing the whole of school mathematics curriculum in all levels, teachers are also required to know Lateral curriculum knowledge which is the relating of the taught mathematical curriculum to a different subject in the same/different grade(s) (Shulman, 1986). This is in line with Stylianides and Stylianides' (2010) argument that there is the usual association of mathematics to secondary school subjects such as business studies, economics and physics which include the application of mathematical knowledge. An example where mathematical knowledge was related to in real life was in Pournara's (2009) research work that involved a financial course that used spread sheets.

Furthermore, knowledge of curriculum also includes the use of curriculum materials/tools in certain circumstances (Shulman, 1986). Literature affirms that a deep understanding of mathematical knowledge is gained by students through the appropriate use of tools (Pratt, 2012: 23) that can make learning of mathematical knowledge dynamic and interactive (Fennema and Franke, 1992). In fact mathematics has some relationship with real life materials (Pimm, 2002) and so the experiences of mathematical content knowledge for teaching must include these real life materials (Davis and Simmt, 2006). The use of real life materials is able to convert the mathematical concepts to real situations (Chick, 2009; Dundara et al., 2012; Watson and Harel, 2013). Arcavi (2003) states that these real life materials can be used to model mathematical concepts, making it visible and enrich learners sense making of the mathematical concepts. In addition to enriching sense making of mathematical concepts, Julie (2002) in her study also showed that teachers perceived that their experience with modelling the mathematical concepts was motivational for students. Hence, the result of relating mathematics to appropriate materials during teaching in schools contextualizes mathematical concepts which has been shown to be able to develop deep understanding Matang, 2002; Najike, 2004; Kari, 2005; Pournara, 2009).

Secondly, in addition to mathematics teachers' knowledge of the curriculum for teaching in the PCK domain, teachers must also know their subject matter knowledge (SMK) and their students, which is teachers' Knowledge of Content and Students (KCS) (Ball et al., 2008). According to Nolan et al., (2015: 55), "KCS involves knowledge that combines knowing about students and knowing about mathematics in a way that enables teachers to (for example) anticipate what students may think and what they will find confusing, interesting and motivating, and to interpret students' (not fully coherent) spoken words and written work". Subramanian (2014: 179) defines this category of PCK as an interactive relationship between the teachers, their students and the mathematics content during instruction, and not just a process with fixed and pre-determined descriptions like beliefs. Shulman (1987), Llinares (2000) and Adler (2005) agree that KCS allows teachers in the classroom to make flexible decisions on students' evolving ideas during instructions encouraging their constructive participation in teaching. Moreover, this knowledge also allows teachers to identify possible students' difficulties beforehand, therefore allows teachers in their planning to simplify SMK possible enough for the learners and their difficulties (Ball and Bass, 2000). Thus, research has shown that in well planned lessons by teachers with KCS, every student of the class is a participant even for those who may think that mathematics is difficult and boring. They (teachers) engage students in constructive discussions (Stylianides and Stylianides, 2010) and are well prepared to assist them in different ways and clarify students' misconceptions when aroused (Ball et al., 2008; Krauss, 2008) through provision of appropriate guidance (Barwell, 2008). These lessons are characterized by prepared open ended questions which then prompt further teacher – student and student – student discussions that enable successful and deep understanding of mathematical concepts (Sullivan et al., 2006) which Ball et al. (2008) confirms that teachers' informed mathematical knowledge for teaching allows them to show students how to solve problems, answer students' questions and check students' work. An example of this is in the research by Watson and Harel (2013) that showed two teachers Linda and Paul whose in – flow pedagogic decisions on functions such as the questions they asked, the activities they designed and their flexible responses to students' suggestions were informed by their advanced level of the knowledge of the concept of function.

Sullivan (2011) affirms that it is KCS that aids teachers to interact with their students while they (students) are engaged and learn the mathematical concepts themselves.

Thirdly, further to mathematics teachers' knowledge of the curriculum and KCS also requires their Knowledge of Content and Teaching (KCT) (Ball et al., 2008). KCT is mathematics teachers knowing both their SMK and how to teach the required SMK (Ball et al., 2008), and, Nolan et al., (2015) says that it is the combination of the knowledge of mathematics and how to teach mathematics. Rowland et al. (2005) defines this domain of PCK as the clear setting of mathematical concepts for instruction for full understanding so that the intended knowledge is lead to carry on the prepared behavior in the work of teachers. An example in Llinares (2010: 51) work, that knowing her (Mrs M's) concept of function well, she explained her teaching goals based on two domains of function which assisted her in overcoming many of those possible learning difficulties she would have encountered during the teaching of the function concept. In addition to clear setting of mathematical concepts, Kari (2005) adds by pointing out that mathematical concepts can be made clearer during instruction by the teacher by integrating home cultures, which is mathematics utilized in life situations. Both Ball et al. (2005) and Ma (1999) also affirm that this KCT allows teachers to avail various suitable teaching strategies, for the easy understanding of mathematical concepts through appropriate representations (Bansilal et al., 2014; Chikiwa, 2017). The representation of mathematical concepts can enable better problem solving skills for children in the on-going changing environmental situations (Siegler and Booth, 2005; Sarama and Clement, 2009) which Adler (2005) confirms as mathematical teaching as a form of medium to solve mathematical problems en-counted in real situations.

It is clearly highlighted in detail in this section of the literature review that research confirms that Shulman's (1986) original notion, further fine-tuned by many others that in order to teach mathematics in schools and so the necessary knowledge for teachers' to teach mathematics are their SMK and PCK (Shulman, 1986; Ma, 1999; Llinares, 2000; Rowland et al., 2005; Ball et al., 2008; Subramanian, 2014) just to list a few. The mathematical knowledge for teaching is quite complex, however, had been simplified through its categorization so that it can be easily understood as it is critically reported in this section of the literature review.

2.2 Some implications of teachers who lack in-depth mathematical knowledge for teaching (MKfT)

The first section clearly outlined and discussed in detail what literature say the mathematical knowledge for teaching teachers need to teach mathematics in schools. This section discusses the key implications where teachers lack this mathematical knowledge to teach mathematics.

Firstly, much research had attributed students' low mathematics achievements to teachers' poor mathematical knowledge (Carnoy and Arends, 201; Taylor and Taylor, 2012 cited in Pournara et al., 2015). Pournara et al. (2015) and Chikiwa (2017) confirmed this that students who are taught by teachers lacking the mathematical content knowledge to teach mathematics results poor learning gains of mathematical concepts by students, that causes them to most often perform poorly in assessment tasks (Ball et al., 2005; Spaull, 2013; Graven, 2014; Venkat and Spaull, 2015; Spaull and Kotze, 2015), thus the low students' achievements (Mji and Makgato, 2006).

Secondly, mathematics being widely known as an abstract subject, the teachers' inadequacy of mathematical knowledge for teaching involving real life examples prompt learning difficulties for school children (Matang, 2002) which Morris (2001) claims has negative impacts on students' lives. According to Kahan et al. (2003), this seem to contribute to inadequate knowledge of mathematical concepts in a respective grade by students because Matang (1998) claims that their earlier school experiences, for example preschool which is within their own environment is most often ignored, thus produces knowledge gaps for them in the next grade/level (Adler, 2005; Pratt, 2012). This results in hindering students' learning of mathematical concepts in the next level (Kahan et al., 2003) that causes math anxiety for students (Uusimaki and Nason, 2004), and, Quinn (1997) and Bonner (2009) argue that this exacerbates the problems that students experience in learning mathematical concepts in the higher grades. This notion is affirmed with studies done in South African schools that mathematics teachers who lack the mathematical content knowledge for teaching created learning deficits in students' earlier schooling grades, which are the root cause of underperformance in their later years of schooling (Fleisch, 2008; Graven, 2016 cited in Chikiwa, 2017: 6).

The discontinuity in building of knowledge that writers have argued makes the subject difficult, boring (Osborne et al., 1997), pointless and is found to be perceived as not needed by many students in their real lives (Pratt, 2012; Sibuyi, 2012).

Thirdly, Huang and Kulm (2012: 425) argue that the teachers limited knowledge of mathematical content knowledge to teach mathematics, for example the skills in explaining and solving mathematical problems such as functions/equations/inequality, according to Huntley and Flores (2010) find it difficult during instruction just because they may not understand how such topics relate with each other (Kahan et al., 2003). Wu (2005) claims that when this happens, students do not learn mathematics because they are taught incorrect mathematics by teachers who lack in-depth knowledge of it. This lack even prevents teachers from understanding current mathematics that is based on more modern concepts which then can promote students' understanding of mathematical concepts (Even, 1993: 113). For example Bansilal's work (2012b) cited in Bansilal, Brijlall and Mkhwanazi (2014: 35-36) that focused on a teacher's poor SMK found that the teacher's explanations were often incoherent and illogical. The teacher's poor understanding of the concepts of ratio and number resulted in a teacher missing some key ideas and presenting convoluted explanations that involved circular reasoning, which made no sense to the learners. Boaler and Greeno (2000) claim that they resort to teacher centered lessons which are inefficient for most school children.

Finally, teachers who lack the mathematical knowledge for teaching focus their instruction on textbooks which often emphasize following rules and learning skills, rather than deep understanding of mathematical concepts (Watson and Harel, 2013). Teachers using only textbooks do not transform the static knowledge (Schon, 1983) well because they teach the content only in the way that the textbook presents it, rather than adapting to different learning situations (Swan, 2007). Simply following the contents of textbooks, teachers plan more traditional lessons that do not encourage, and at times do not leave room for anticipation of possible mistakes students might make, misconceptions, difficulties, possibilities for alternative explanations or examples, or consideration of interactions that might take place in a lesson

(Zazkis et al., 2009: 41). Thus, teachers are confronted with all these various learning situations created by students in the classroom during instruction (Mayson and Davis, 2013), hence, narrow wide range of knowledge during the supposed to be in those expected interactions between teacher and students (Kahan et al., 2003; Stylianides and Stylianides, 2010). Even worse is that it affects teachers' reactions to students' questions and ideas related to specific mathematical concepts (Even and Tirosh, 1995).

2.3 What research say about how to develop teachers' in-depth mathematical content knowledge for teaching.

Having outlined above what teacher knowledge is needed to teach mathematics and then in the last section having looked at the effects of lack of teacher knowledge on learning, this section looks at the ways in which the required knowledge to teach mathematics can be developed. Quality instruction for learners depends on knowledgeable teachers, and so their preparation during teacher training and continuing professional development is crucial to develop and enrich teachers' MKfT (Adler et al., 2005).

Firstly, a recommendation by Palmer (1987) is that the major focus of teacher training should be the development of teachers' SMK and PCK and is responsible to develop teachers' SMK and PCK (Lannin et al., 2013). Thus, Even (1993) argues that teacher education institutions must develop this two critical components, SMK and PCK (Shulman, 1986) teachers require to teach mathematics during teacher training. Stylianides and Stylianides (2010) points out that mathematics teacher education is to support any mathematical knowledge for teaching because it is evident in literature that a teacher must acquire this form of mathematical knowledge for teaching, in which Rowland et al. (2005) affirms as the trainees 'Foundational Knowledge' for teaching, from a teacher education institution during initial teacher training (Ball and Bass, 2000; Rowland et al., 2005; Ball et al., 2008; Krauss et al., 2008). Therefore, Chikiwa (2017) and Blanton et al. (2006) call for the teacher education programs, and, educational interventions respectively

to ensure that their courses are rich in both SMK and PCK and that these are presented in a way that correlates particular content with its pedagogy. This is because practicing teachers who are lacking the mathematical knowledge for teaching again calls for teacher educators to design and implement interventions which could be used to improve the poor situation (Bansilal et al., 2014).

In teacher education institutions, prospective teachers must be given the opportunity in their university course work to strengthen this subject content knowledge and pedagogical content knowledge (Manouchehri, 1996: 14) just like the elementary teachers who are prepared in their practice and content (Ball et al., 2009). Llinares (2000: 41) adds that in particular, that it is important that the secondary mathematics teacher education programs adopt approaches that take into account the central role played by teachers' knowledge about mathematics learning, and the dilemmas generated when teachers have to reconcile objectives that are apparently contradictory during their teaching practice (Llinares, 2000: 41). One way in which this knowledge can be developed is through the study of the history of each respective mathematical concepts because research reveals that prospective teachers whilst studying the history of mathematics in regards to the usefulness of each mathematical concepts gained deep understanding (Kolawole and Oluwatayo, 2005; Huntly and Flores, 2010).

Furthermore, prospective teachers require more constructivist beliefs (Muhtarom et al., 2017). Being constructivist during teacher training claimed by Stylianides and Stylianides (2010) is engaging prospective teachers with activities/tasks related to teaching in the domains of professional work such as economics, physics and statistics because, according to Subramanian (2014), these activities can influence and then develop prospective teachers' pedagogical knowledge for teaching. Examples of research in which prospective teachers developed in-depth knowledge of mathematical concepts through tasks done in the professional domains as part of teacher training are in Pournara (2009) where prospective teachers' engagement in a financial course that involved the use of spread sheets. Another in Bakker and Derry's (2011) whereby empirical illustrations of an inferential approach on learning how to draw statistical inference. In Agyeia and Voogt's (2013) research was through designing and enacting activity based

mathematical lessons with spread sheets as instructional tool, Turner et al. (2012) who developed problem solving mathematics lessons connected to mathematical practices in community settings. Timmerman (2004: 369) points out that when such teacher education courses are offered, mathematics teacher educators provide prospective teachers with opportunities to learn how to contribute to and implement different strategies of mathematical proficiency, prospective teachers are obliged to confront their existing beliefs, knowledge, and abilities about what it means to know and do mathematics, how children learn mathematics, and the strategies for teaching mathematics.

In addition to the teacher education programs that develops teachers' knowledge for teaching is research intertwined in the content teacher education courses where prospective teachers can be active and responsible for their own development of the knowledge for teaching (Palsdottir et al., 2008). For example, design research can involve provided proposed innovations that teachers adapt for their own local classrooms and build their own knowledge for mathematics and teaching (Gravemeijer and van Eerde, 2009). In another, teachers' pedagogical content knowledge with regard to planning of lesson, teaching of lesson, and assessing students' learning were developed during the course of a research (Susanta, 2012). Furthermore, information collected from problem solving journals, structural interviews and peer teaching were foundations for intervention that positively influenced prospective teachers' beliefs and knowledge about mathematics teaching and learning (Timmerman, 2004; Ell et al., 2007; Nolan et al., 2015). In particular, Nolan et al.'s (2015) research project designed and delivered components of a PCK course in a series of workshops developed and then built their own MKfT (Nolan et al., 2015).

On the other hand, research reveals that some teachers have learned 'on the job' to solve the mathematical problems they face during their practice (Adler, 2005: 10). Some of these have been through research that can act as catalyst and involve practicing teachers in research and experiment theoretical knowledge in their classrooms that may impact their practice (Gravemeijer and Eerde, 2009). For example, teachers who were involved as participants in Baird et al.'s (1987) research through collaboration amongst each other and outside consultants gained

knowledge on how to research their teaching and the students' learning, so that they became more able to make informed decisions based on perceptive evaluation of their practice. Chikiwa (2017) confirms that collaboration with other related stake holders can enrich and enhance teachers' MKT. There are teachers who used mathematical tasks (Swan, 2007) developed their MKT, and, those who made reflections throughout a research identified themselves as agents of change (Sales et al., 2011). Teachers' understanding of mathematics be impacted and changed by knowledge of mathematics teacher education research results (Johnson, 2000).

Clarke and Hollingsworth (2002) emphasized a model of teacher professional growth based on their existing beliefs and knowledge through professional development programs. Sibuyi (2012) recommended workshops where teachers teaching strategies can be can exposed. For example, during a workshop, students' work was used that encouraged teachers' participation in developing practices on how to support teachers' involvement in constructing mathematical knowledge for teaching (Kazemi and Franke, 2004).

Another professional development program in the form of an intervention equipped mathematics teachers with advanced mathematical content knowledge for teaching in some Africa school. Teachers participated in a long year professional development course (Pournara et al., 2015). Project based used as professional development form which teachers implemented interactive and constructivist approach (Gunel, 2008: 209) because professional development programs can enhance and enrich teachers' mathematical content knowledge for teaching (Kanter and Konstantopoulos, 2010). DMI participants' (teachers) increased their MKfT during a nationally disseminated professional development program (Bell et al., 2010). In-service intervention that improved knowledge of mathematics for teaching (Ell and Lomas, 2007) and in-service training programs enhanced practicing teachers' practices (Khamis and Sammons, 2004).

It has been critically reported in this section of the literature review that it is the responsibility of teacher education institutions to develop the necessary mathematical knowledge which consists of SMK and PCK sufficiently for student teachers (Even, 1993; Ball and Bass, 2000; Blanton et al., 2001; Rowland et al., 2005; Krauss et al., 2008; Palsdottir et al., 2008;

Stylianides and Stylianides, 2010; Lannin et al., 2013; Bansilal et al., 2014; Chikiwa, 2017). Besides the teacher education institutions, empirical research involving both student and practicing teachers is also able to develop this knowledge (Johnson, 2000; Timmerman, 2004; Adler, 2005; Ell et al., 2007; Swan, 2007; Palsdottir et al., 2008; Gravemeijer and van Eerde, 2009; Sales et al., 2011; Susanta, 2012; Nolan et al., 2015; Chikiwa, 2017). Finally. Continuous professional development programmes can also develop and enrich teachers' SMK and PCK (Clarke and Hollingsworth, 2002; Kazemi and Franke, 2004; Khamis and Sammons, 2004; Ell and Lomas, 2007; Gunel, 2008; Bell et al., 2010; Kanter and Konstantopoulos, 2010; Sibuyi, 2012; Pournara et al., 2015).

3.0 Methodology

In this chapter of my dissertation, I describe the general philosophy that underpinned my chosen research method that carefully examined my research questions. Firstly, I begin with an explanation of the series of actions I undertook in the research design, by clarifying the method used most suitable to achieve the aims and the coverage of the research. Secondly, I continue the chapter with a description of my action research cycle one, selection of my research participants, data collection method and the ethical issues of my participants. Finally, the chapter ends with the description of my action research cycle two, selection of my research participants, data collection method and the ethical issues of my participants.

3.1 Action Research Methodology

I have chosen action research as my overarching methodological approach since I wanted to both research and develop my own practice as a teacher educator to develop the students' mathematical knowledge which I have identified as needing significant development (Edwards and Talbot, 2014).

Action research is an ongoing process of systematic inquiry, empirically based attempt to improve practice (Tripp, 2005). In educational context, "it is a step by step process that educators

can adapt to address their need for learning more about practice and successful interventions" (Ferrance, 2000, p: 29).

Thus, after a careful examination of these definitions, action research was used as my research design because it had the potential to enrich my knowledge and skills to correctly attend to my research questions and help develop my practice. Despite the various terms describing the steps, the concept is the same and that is action research. Shown below is a diagram of the various steps involved in an action research which are similar to my research (Lewin, 1948).



Firstly, in action research, educators work collaboratively to identify issues in their institutions and develop alternative practices for improvement (Stringer, 2008). This is exactly what had occurred in my action research cycle one that addressed its research questions by working collaboratively with key stake holders such as mathematics teacher educator, teacher mentors and beginning mathematics teachers. Secondly, educators conduct research that intersects with their practice in the institutions in which they work with their own students on problems that affect them directly, thus real change can occur (Ferrance, 2000; Elliot, 1978). Similarly, this characteristic of action research suitably fitted into the desire of my action research cycle two research questions with the prospective teachers. Thus, the action research created an opportunity for the research that began with a base-line survey that involved other key stake holders such as teacher educator, teacher mentors and beginning teachers. In addition to the useful information being provided by the participants (stake holders) which addressed my three research questions, the base – line survey provided specific information for the development of an intervention for my second group of participants. Finally, involving series of cycles is a feature of action research to indicate that there had been a change made in practice in the process of the research (Hine, 2013). In the same way, the action research I undertook provided the information from the first cycle to inform the intervention conducted in the second cycle.

Based on these two cycles, my attention is focussed on describing how each cycle was utilised to accomplish my research aims.

3.1.2 Action-Research Cycle One: Baseline investigation (fact-finding): Interviews with teacher educator, teacher mentors and beginning teachers.

The action research cycle one focused on answering the three (3) key research questions below;

1. What do beginning teachers and other stakeholders (mentors and teacher educators) perceive are the issues and challenges related to mathematical content knowledge that trainee teachers are faced with?
2. What mathematical knowledge for teaching do beginning teachers and stakeholders say is required by trainee teachers?
3. How can this knowledge be developed in a workshop on the teaching and learning of algebra for final year mathematics major students in UoG?

This is the first phase of the research that was a fact finding phase where key stake holders were asked the research questions stated. The fact finding phase was done almost over a period of three weeks through conducting interviews. An interview is a formal conversation between two or more people involving formal conversations in order to obtain quality data on a topic (Birmingham and Wilkinson, 2003). This allowed me to conduct the interviews with the key stakeholders and ask about the issues and challenges around mathematical content knowledge for teaching of prospective teachers in the Papua New Guinea context that addressed my key research questions for cycle one above.

3.1.2.1 Sample – Selection of the research participants

In the action research cycle one, the sampling strategy was purposive sampling because I selected my samples with the purpose that they have the best knowledge of the context to answer my research questions. I provided to the selected samples a leaflet that I constructed that contained the research information. Upon grasping the content of the leaflet by the participants listed below in the table, they agreed to participate in the research. Once they agreed to, they were given a letter to confirm their consent to be involved in the research.

Respondent	Role
	The mathematics teacher educator is the professional in UoG who trains trainees (pre-service year 3 and 4 students) on how to teach

Mathematics teacher educator x 1	mathematics in PNG secondary school classrooms. The teacher educator trains/teaches the trainees how to plan lessons based on PNG grades nine – twelve mathematics syllabus. The teacher educator works with students who teach about two to three grades nine to twelve topics during an assessment task called peer teaching. Whilst they teach, the teacher educator observes and assesses their teaching.
Teacher Mentors x 2	The teacher mentors are those teachers who are identified as well qualified, have more than five years of teaching experience, and either have been promoted from the base level to the next which is either senior subject master (SSM) or head of division (HoD) in teaching mathematics in secondary schools. Thus, when trainees are on teaching practice (professional experience), the teacher mentors mentor the trainees during pre-lesson discussions on lesson preparation and planning, and lesson presentation. The teacher mentors then observe the planned lesson during lesson presentation. After the lesson observation done by the mentor of the trainee, the mentor draws out the strengths and weaknesses during a post-conferencing. Before the trainees return to the respective teacher training institution, the mentor writes a report on the trainees' performance during the teaching practice period.
Beginning teachers x 2	The beginning teachers are those who recently graduated (one or two year(s) of teaching experience) from a teacher education institution and are teaching in secondary schools. They are new graduates and so their performance is based on the mathematical content knowledge for teaching acquired from the teacher education institution.

3.1.2.2 Data collection method: Semi-structured interviews

Qualitative data was collected in action research cycle one through, as said above, the use of individual interviews. The focus of the interview was to obtain facts about the issues and challenges around mathematical content knowledge of prospective mathematics teachers in PNG context from the participants which are related to those described in the literature. Interviews produce in-depth data of the researched issue (Birmingham and Wilkinson, 2003). The action research cycle one utilized semi-structured interviews in which, I as the interviewer directed the interview more closely (Birmingham and Wilkinson, 2003, p.45) so the conversations between me and the participants did not deviate from addressing the main research questions.

The interviews asked questions such as

- the challenges trainee teachers are faced with during their training,
- the levels of confidence in the subject knowledge and the knowledge of teaching of trainee teachers when they first start their training,
- the knowledge trainee teachers have about mathematics at the start of their teaching practice,
- what teacher training institutions might do differently to help develop trainee teachers' knowledge of mathematics and how to teach mathematics.

The semi-structured interview questions were pre-designed based on information in the existing literature, and to answer the research questions. The questions were given to the participants prior to the actual day of the interview.

3.1.2.3 Ethical issues

The action research cycle one was quite a challenge for me because I am a tutor, and is not in the mathematics department of UoG. For this reason, I was able to interview only one mathematics teacher educator. Before the interviews were conducted, as said above, I gave out a leaflet to the participants that contained all the information such as the purpose of the

information and how the data would be used. In addition to the leaflet, I assured the participants that all the information will be confidential by locking them away in a cabinet in my office, and where necessary, pseudo names will be used.

3.1.3 Action research cycle Two: Tailored Intervention for Prospective mathematics teachers

The intervention was primarily planned for two hours, however, I could only conduct for an hour due to circumstances beyond my control. The intervention was initially planned for the evening on the same day, but was changed a day before because some of the participants leave off campus and so asked for it to be held in the afternoon instead. The intervention began at three pm as expected to end at five pm. However, I did not know that there was a scheduled weekly workshop in the same room I was conducting the intervention. So, I ended the intervention at four pm instead and by that time, I thoroughly completed the first session but quite rushed through session two. Session one went on for about forty-five minutes and session two was only for about twenty minutes, which was still appropriate because it was a short session. The design of the intervention captured the information gathered from the action research cycle one, and intended to make a change in the prospective mathematics teachers' domain of practice (Clarke and Hollingsworth, 2002) in mathematics teaching. The intervention consisted of two sessions.

The key research questions for the action research cycle two are:

1. In what ways and to what extent did the prospective teachers perceive that the intervention was successful in addressing the issues expressed in cycle one?
2. In what ways and to what extent did the prospective teachers perceive that an intervention in the form of a workshop is necessary?

The first session of the intervention addressed mathematical content knowledge for teaching, in particular, pedagogical content knowledge (PCK) that is required by prospective mathematics teachers for the efficient delivery of the subject matter knowledge (SMK). The session outlined mathematical content knowledge, and, sub-divided it into SMK and PCK. The

session further focused into SMK, by looking into examples that mathematical knowledge for teaching (MKfT) is a form of applied mathematics.

The second session was about planning mathematics topics. The session offered a framework that can assist the prospective mathematics teachers construct the appropriate SMK of a topic through scaffolding.

After almost two weeks after the intervention, I interviewed the prospective teachers assessing the effectiveness of the intervention.

3.1.3.1 Samples – Selection of research participants

I was unable to conduct the interviews right after the intervention because I had to leave the school campus on business travel. By the time I returned after two weeks, the participants who are final year students were going to and from schools for their first practice teaching called micro-teaching. Besides, I do not know majority of them by name and face because as stated earlier, I did not teach them until this intervention. However, there are about less than five whom I got to know because they were in my tutorial classes in the general education courses. There were two of these students who assisted me gather the participants who are the final year major mathematics students for the workshop. Through these two students, they informed the others that I needed to interview about the intervention. Out of the twenty-one students who attended, I was able to interview eight students.

3.1.3.2 Data collection methods: Field notes on workshop and semi-structured interviews

The cycle utilized two data collection methods. Firstly, I wrote a personal field notes which is an extensive reflection of the intervention I conducted. Creation of the detailed field notes for interviews as well as for the study as a whole, ensured that the depth of my context is persistent with the data to allow robust research consistent with the qualitative interviews (Phillippi and Lauderdale, 2018, p.387). On this note, the field notes are my private extensive reflection stating my personal feelings and professional judgment of the success of the intervention drawing on

critical instances in the intervention which seemed to indicate a successful approach to develop the student teachers' mathematical knowledge for teaching. In addition, the field notes included an evaluation of the things that I could have done to make the intervention better.

Secondly, I conducted individual semi-structured interviews with the participants of the intervention. The questions asked during the interview were focused on answering the key research questions for this cycle which were assessing their perceptions of the workshop and their views of the workshop in general. The questions I asked were:

- did you enjoy the intervention,
- did you learn anything new from the intervention, if you did, what did you learn,
- do you want such interventions to be conducted for final year major mathematics students, if so, why.

I chose qualitative semi-structured interviews so I could have a core agenda but also follow up on some areas to give me some data I needed for action research cycle two (Birmingham and Wilkinson, 2003).

3.1.3.3 Ethics

The participants of the intervention were the final year major mathematics students. I know that I have power over them because I lectured a general education course to all the bachelor in education program taken by about four hundred final year students. The participants would know me well had they faithfully attended the lectures, but I did not due to the huge number of students population to know them all by name. However, those that I approached willingly agreed to participate in the interview. Before I began the interviews with each participant, I reminded them that the information they provided must be an honest perception of the intervention. In addition, they were informed that the information they provide will be used purposely for my research, and will not be shared with other staff and students of the institution, and even others elsewhere. I made sure that I did not influence their responses in any

way, or made them feel that I am in authority because am their tutor. Furthermore, they were reminded that I am not trying to make them compare their current mathematics education program, even though few of them tried to mention that in their responses.

4.0 Findings

4.1 Overview

The findings are presented according to each research question in the two cycles of the research.

4.2 Cycle 1: Fact finding

This section of the findings gives a detail account of what the respondents said, but are categorized in themes in each of the three research questions.

4.2.1 Issues and challenges trainee teachers are faced with

The analysis of the interview responses from the beginning teachers, mentors and teacher educator identified a number of common themes in the issues and challenges related to mathematical content knowledge that trainee teachers in Papua New Guinea are faced (see table W in appendix X for details of the analysis). The key themes are: lack of PCK, lack of confidence in teaching, lack of ability to practice and develop their PCK in school placements and lack of exposure to the school mathematics curriculum. Finally, a theme emerged to show that some of the teacher educators felt PCK and SMK did improve during teaching practice.

Firstly, a key theme that emerged was that respondents, both beginning teachers, teacher mentors and the teacher educator all perceived that a key challenge was trainees' lack of PCK, not necessarily SMK, in order to teach mathematics effectively. Both beginning teachers perceived that their SMK was 'good' but they struggled with how to transform it for pupils as they both said:

> "I have no problem with SMK, but how to deliver that SMK to the students is little bit a challenge to me". (Beginning teacher 1)

"My SMK is good but I need to adjust my teaching techniques. The challenge is how to teach/deliver the SMK thoroughly to the students". (Beginning teacher 2)

The two teacher mentors and the teacher educator also voiced these concerns as the Teacher mentor 1 said:

"The challenge that the training teachers face during their training period is delivering the content, they have the content but how to deliver the content". (Teacher mentor 1)

Therefore, for these respondents SMK does not seem to be a concern rather it is PCK that is the key focus of their concerns.

Off course, PCK is a vital knowledge base required for teaching because it allows teachers to identify possible students' difficulties beforehand, so that in their planning they simplify SMK enough for the learners and their difficulties (Ball and Bass, 2000). However, according to Wu (2005) and Ball et al. (2005), teachers' SMK can also influence their PCK because their deeper understanding of the SMK can alter even a well prepared and planned pedagogical approach to a topic during instruction to suit the learning needs of the students. This is because knowing their content (SMK) makes it easy to know their students (KCT) too which then allows them to make flexible decisions on students' evolving ideas during instruction that can encourage students' constructive participation in teaching in the classroom (Shulman, 1987; Llinares, 2000; Adler, 2005).

Secondly, and closely connected with lack of knowledge all the respondents also reported that trainees lacked confidence in delivering SMK. As beginning teacher 2 said:

"Lacking confidence when delivering content during lessons". (Beginning teacher 2)

The two teacher mentors and the teacher educator also voiced the same concern as both teacher mentors said:

"Level of confidence in content is disorganized and is low when they first start and so lacks confidence in teaching". (Teacher mentor 1)

"Level of confidence depend on individual trainee teachers but is generally very low with first contact". (Teacher mentor 2)

The teacher educator added more detail in perceiving that the trainees lacked confidence particularly in teaching difficult concepts such as 'Calculus'. This links to Ball et al.'s (2008) argument that teachers need the third domain of SMK which is knowledge at the mathematical horizon, a concept I will say more about below. For example, Kashefi et al. (2010), Steel and Rogers (2012) and Laren and Russo (2014) highlights the necessity of the knowledge at the mathematical horizon in the 'Calculus concept' that requires the knowledge of other concepts like algebra, trigonometry and geometry.

A third challenge was that, given lack of PCK was a concern, some respondents perceived that the time in school placement to develop strong PCK was limited. Both beginning teachers 1 and 2, for example, identified that there was a lack of practice/time in school to develop PCK and teacher mentor 2 talked about a lack of exposure of the trained to the classroom in order to develop their PCK.

Despite this concern voiced by some respondents, the major focus of teacher training should be the development of teachers' SMK and PCK (Palmer and, 1987) and so is responsible for developing teachers' SMK and PCK (Lannin et al., 2013). Even (1993) also adds that teacher education institutions must develop this two critical components, the SMK and PCK (Shulman, 1986) that teachers require to teach mathematics during teacher training and in their careers. Stylianides and Stylianides (2010) point out too that mathematics teacher education is there to support the development of mathematical knowledge for teaching (MKfT) because it is evident in literature that a teacher must acquire this form of mathematical content knowledge, in which Rowland et al. (2005) affirms as the trainees 'Foundational Knowledge' for teaching, from a teacher education institution during initial teacher training (Ball and Bass, 2000; Rowland et al., 2005; Ball et al., 2008; Krauss et al., 2008; Palsdottir et al., 2008).

A fourth challenge, expressed only by the teacher educator was the lack of exposure to the school mathematics curriculum.

This is vital knowledge voiced by the teacher educator because according to Shulman (1986: 10), curriculum gives information about the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances. This is true in mathematics because confirm that in a mathematics curriculum unlike other school subjects, begins with basic/fundamental mathematics Boley (1999) and Sullivan (2011). The fundamental mathematics necessitates the continuous growth of the mathematical knowledge deeper and wider with new knowledge (Klein, 1945; Baturo and Nason, 1996; Siegler and Booth, 2005; Sarama and Clements, 2009; Watson and Harel, 2013). Klein (1945) explains in detail the importance of the fundamentals of mathematics which are required in the latter.

Finally, although concerns were expressed about levels of PCK, time in school to develop it and trainees' confidence, all the respondents did perceive that trainees' PCK does develop whilst on teaching practice and, the beginning teacher 2, teacher mentor 1 and the teacher educator, identified the mentor as key in helping trainees develop their PCK when they said:

> "Able to develop/gain teaching skills as time goes by during teaching practice under the mentorship". (Beginning teacher 2)
>
> "Knowledge changes during teaching practice as they are criticized meaningfully by their mentors". (Teacher mentor 1)
>
> "Once trainees are guided well by mentors and weaknesses are corrected at that moment during teaching, it helps them build a better understanding of the knowledge of subject and of teaching". (Teacher educator)

4.2.2 Key mathematical knowledge for teaching required by trainee teachers

The analysis of the interview responses from the beginning teachers, mentors and teacher educator identified two key mathematical knowledge for teaching required by trainee teachers

in PNG (see table X in appendix X for details of the analysis). The two key mathematical knowledge bases are: SMK and PCK.

Firstly, two beginning teachers, two teacher mentors and teacher educator perceived that good solid SMK is required by trainee teachers. Beginning teacher 1, teacher educator and teacher mentor said:

> "I taught well and was confident that reflected my good SMK and so develop SMK which is very important". (Beginning teacher 1)
>
> "A good SMK in mathematics has a great impact on the delivery of math and so theoretical concepts must be complemented with experiments and practical during teacher training". (Teacher educator)

This resonates with the knowledge indicated in the literature that is needed by teachers to teach mathematics in schools is 'mathematical knowledge for teaching (MKfT) (Ball and Bass, 2000; Rowland et al., 2005; Ball et al., 2008; Krauss et al., 2008). This is the knowledge required by teachers for teaching and is defined as "the mathematical knowledge needed to carry out the work of mathematics" (Ball et al., 2008: 395).

So some respondents spoke about the first of these knowledge bases, SMK. SMK defined in detail by Ball et al. (2008) in three domains are: Common Content Knowledge (CCK), Special Content Knowledge (SCK) and Knowledge at the Mathematical Horizon. SMK needs emphasis because, as said earlier, deeper understanding of the content can fundamentally alter a well prepared and planned pedagogical approach to a topic during instruction to suit the learning needs of the students (Wu, 2005 and Ball et al., 2005).

It (SMK) requires having thorough knowledge of the basic mathematical concepts, how they work in everyday life and why they exist is what the mathematics teachers are obliged to know (Ball and Bass, 2000; Laren and Russo, 2014; Venkat and Spaull, 2015). In addition it is also teachers knowing how and where to utilize the respective mathematical concepts in real life problems so that they are able to make these links during instruction (Nolan et al., 2015).

Furthermore, teachers cannot teach if they do not have the special SMK (Wu, 2011), which is teachers' foundational knowledge to add to their competency (Ball et al., 2008). This special SMK is unique to teaching alone and so is a requirement for teachers (Adler, 2005; Nolan et al., 2015). This is the knowledge in which teachers unpack mathematics work and 'identify errors in students' assigned tasks (Nolan et al., 2015). As teacher mentor 1 said teachers need more knowledge than their students so that:

> "The trainee teacher, before teaching the particular topic/unit must have enough knowledge of that topic because they were able to answer students' questions but to the level of that content". (Teacher mentor 1)

Teachers need the third domain of SMK which is the knowledge at the mathematical horizon, as mentioned above (Ball et al., 2008). Knowledge at the mathematical horizon is the knowledge not directly related to instruction but comes either before or after a respective instruction (Llinares, 2000; Mosvold and Fauskanger, 2014). It is the knowledge of how different topics/mathematical concepts are related to each other in different grades/levels in whole of the school mathematics curriculum (Nolan et al., 2015). This knowledge helps the mathematics teachers where and what to build on from in a grade when teaching (Ma, 1999; Najike, 2004; Ball et al., 2005; Adler et al., 2009; Venkat and Spaull, 2015) because according to Adler et al. (2009) and Venkat and Spaull (2015), this knowledge makes teachers have rich knowledge in related topics/mathematical concepts.

It is clearly indicated that the respondents identified SMK as important but, from their responses, it is not clear how aware they are about complexity of the nature of SMK, which is whether they are referring to beginning teachers' SMK of the school curriculum or their SMK beyond school curriculum. Therefore, this might suggest an area that needs to be developed in the teacher educators as they work with beginning teachers to develop their SMK particularly as RQ 1 for the teacher educators indicated that they didn't think SMK needed to be developed (neither did the Beginning teachers). This is because all respondents have quite a simple notion

of SMK and in mathematics education, SMK is split into Common Content Knowledge (CCK), Special Content Knowledge (SCK) and Knowledge at the Mathematical Horizon.

Secondly, the two beginning teachers, two teacher mentors and the teacher educator perceived PCK as the second key mathematical knowledge base required by trainee teachers. Beginning teacher 2, teacher mentor 1 and teacher mentor voiced the concerns below:

> "I think content knowledge of mathematics is okay to teach any level at secondary school. I am very confident. The only problem is knowledge of teaching". (Beginning teacher 2)
>
> "Trainees had difficulties explaining single question and to give alternative explanations". (Teacher mentor 1)
>
> "Yes at some degree when dealing with abstract concept that cannot be presented physically, eg., calculus and vectors". (Teacher mentor 2)

PCK is the second category of mathematical knowledge for teaching (MKfT). It is the form of mathematical knowledge for teaching with features of a most relevant content suitable for use in teaching that transforms SMK to make it accessible to students (Shulman, 1986). PCK is the clear setting of teaching ideas for full understanding so that knowledge is lead to carry on the prepared behavior in the work of a teacher (Rowland et al., 2005). PCK includes, for example, the use of real life materials to convert mathematical concepts and make them relevant to real situations (Dundara et al., 2012; Watson and Harel, 2013). These real life materials then can be used to model mathematical concepts, making these visible and then enrich learners' sense making of the mathematical concepts (Arcavi, 2003).

4.2.3 What respondents would like to see in a workshop to develop the student teachers' mathematical knowledge

The analysis of the interview responses from the beginning teachers, mentors and teacher educator identified three themes they would like to see in a workshop to develop the student teachers' mathematical knowledge (see table Y in appendix X for details of the analysis). The three themes are: SMK, PCK and more teaching practice and these are consistent with the findings from research questions 1 and 2.

Firstly, two beginning teachers, teacher mentor 1 and teacher educator perceived that the trainee teachers should be equipped well with SMK. Beginning teacher 2, teacher mentor 1 and teacher educator said:

> "As well, offer more content courses to overcome challenges or to be confident in answering questions." (Beginning teacher 2)
>
> "Equip them as much with content". (Teacher mentor 1)
>
> "Students must participate in experiments and write reports". (Teacher educator)

Examples from the literature of courses which can do this are present. For example, researcher report on a course, on the history of mathematics in which prospective teachers studied the usefulness of each mathematical concept and were shown to gain a deeper understanding (Kolawole and Oluwatayo, 2005; Huntly and Flores, 2010).

Also, prospective teachers can develop in-depth knowledge of mathematical concepts through tasks done in other professional courses they take as part of teacher training, some examples are; prospective teachers' engagement in a financial course that involved the use of spread sheets (Pournara, 2009), prospective teachers' empirical illustrations of an inferential approach on learning how to draw statistical inferences (Bakker and Derry, 2011), prospective teachers developed problem solving mathematics lessons connected to mathematical practices in community settings (Aguirre et al., 2012), and, prospective teachers designing and enacting activity based mathematical lessons with spread sheets as instructional tool (Agyeia and Voogt, 2013). So as well as the core teacher training programme it is important to look at other opportunities for trainee teachers to learn SMK.

Secondly, two beginning teachers and two teacher mentors voiced that teacher education institutions must also develop student teachers' PCK in addition to their SMK.

> "From experience, knowledge of how to teach mathematics is a problem so teacher training institutions should emphasize more on that". (Beginning teacher 2)
>
> "The institution should help the trainee teachers with teaching resources". (Teacher mentor 1)

"Since mathematics is abstract in nature, as much as possible practical application with a lot of examples (material use) should be taught/practice". (Teacher mentor 2)

The quotes are quite resonant with the need for prospective teachers to develop more constructivist beliefs (Muhtarom et al., 2017) during teacher training whereby they can be engaged with activities/tasks in other domains of professional work such as economics, physics and statistics but are related to mathematics teaching (Stylianides and Stylianides, 2010). These tasks then can influence and then develop prospective teachers' pedagogical content knowledge (PCK) for teaching (Subramanian, 2014) because their existing beliefs, knowledge, and abilities about what it means to know and do mathematics, how children learn mathematics, and the strategies for teaching mathematics are being confronted (Timmerman, 2004: 369).

Finally, two beginning teachers, teacher mentor 2 and teacher educator perceived that teacher education institutions should provide more time for student teachers' for practice teaching so that they can be able to develop their own PCK in collaboration with their peers or school-based mentor.

"Do thorough peer teaching with continuous lesson observations so that they (student teachers) can analyze how they would teach when they are out in the field (schools). BT 1

"More practice teaching must be emphasized and closely monitor and supervise these student teachers". TE

Examples of literature from which prospective teachers developed their PCK are: information collected from problem solving journals, structural interviews and peer teaching were foundations for intervention that positively influenced prospective teachers' beliefs and knowledge about mathematics teaching and learning (Timmerman, 2004; Ell et al., 2007; Nolan et al., 2015). In particular, Nolan et al.'s (2015) research project designed and delivered components of a PCK course in a series of workshops developed and then built their own PCK.

4.3 Cycle 2: Evaluation of the effectiveness of Intervention

4.3.1: What the student teachers and I said was successful in the workshop that helped them learn and why they thought it was successful

The intervention was designed to address the issues raised in cycle 1. It consisted of two parts: Part 1: introducing the student teachers to the different knowledge bases and showing them how complicated SMK and PCK are and what they are, and giving them strategies (including real life examples), ie, PCK. Part 2 of the intervention about planning a topic for a lesson, so this in other words, demonstrating what they (student teachers) learned in part 1.

The findings from research questions 1 and 2 show that the student teachers have quite a simple view of what SMK is, so I did present to them the knowledge bases and showed them how complicated these are to make it explicit to the student teachers what they have to learn. In part one of the intervention, I developed student teachers' mathematical knowledge for teaching (MKft). In here, MKfT was defined with examples, and then was categorized further into Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) which are the two very vital knowledge bases for teaching. These two knowledge bases were defined with examples so that the student teachers could differentiate between the two. Part 1 was concluded with an educational session which consisted of two approaches: a worksheet approach and matching cards activity approach which both were about 'algebraic expressions'. Part 2 was shorter compared to the part 1 in which students teachers were introduced to planning a topic for a lesson. (See attachment for details of the workshop)

The workshop was designed to address the issues identified in cycle 1 whereby both the student teachers and I perceive it to be successful. This section therefore now looks at what the student teachers and I perceived was successful about the intervention.

The analysis of the interview responses from the student teachers and my personal professional reflection identified a number of common themes that perceived to be successful in the workshop that helped them (student teachers) learn and why we thought they were effective (see table Z in appendix X for details of the analysis). The key areas that were perceived to be successful were: 1 the outlining and defining of the mathematical knowledge for teaching MKfT):

the categorization of the mathematical knowledge for teaching as SMK and PCK, and their definitions, 2 the information about mathematical knowledge being multi-dimensional, 5 helped developed PCK, 6 the information on the application of examples of mathematical concepts in real life, 7 how to plan and prepare concept(s)/topic for teaching, 8 suitable knowledge for teaching mathematics. Finally, a theme emerged to show that some of the student teachers suggested 9 more similar interventions to be conducted often for prospective mathematics teachers.

Firstly, a key theme that emerged was that the respondents: student teachers 1, 2, 7 and I perceived that the workshop was successful because they (student teachers) learned what is required to teach mathematics in a secondary school (and in all levels of schooling. In other words, in this area was the introduction and then the definition of the knowledge teachers required to teach mathematics. Student teacher 1 said:

> "I am now aware that what I require to be a mathematics teacher is mathematical knowledge for teaching (MKfT)". (Student teacher 1)

The literature supports these views and, although there are other qualities, the most important in order to be a mathematics teacher is his/her mathematical knowledge for teaching (MKfT) (see for example, Shulman, 1986; Morris, 2001; Ball et al., 2005; Ball et. al, 2008; Ell et al., 2008; Stylianides and Stylianides, 2010; Lannin et al., 2013; Nolan et al., 2015; Pournara, 2015; Chikiwa, 2017). So the Prospective teachers were informed in the intervention that they are required to know what this knowledge is because "this is the knowledge that is needed by mathematics teachers to carry out the work of mathematics teaching" (Ball et al., 2008: 395).

Secondly, having perceived that the mathematical knowledge for teaching (MKfT) they (student teachers) require to teach mathematics was successful, they were also quite surprised to be made aware that this knowledge (MKfT) was further categorized, ie, components of MKfT. The previous area introduced and then defined MKfT, in here, MKfT was categorized/divided into two components. So the student teachers 1, 2 and 7 perceived that the categorization of the

mathematical knowledge for teaching helped them to be aware of the specific knowledge they require to teach mathematics and student teacher 1 said:

> "You informed us of the strategies (categories) on what we need to equip ourselves before the actual implementation (teaching)". (Student 7)

Teachers require Shulman's (1986) two categories of MKfT which are Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). So it was a must that the intervention had to made these forms of knowledge known to the student teachers because they are the forms of knowledge "mathematics teachers need to carry out the work of mathematics teaching in schools" (Ball et al., 2008: 395). This was made known to the student teachers because quality instruction for learners in schools depends on teachers' adequate knowledge of both SMK and PCK, and so their (student teachers) preparation during teacher training and then continuing professional development must be crucial to develop and enrich their' SMK and PCK (Adler et al., 2005).

Thirdly, having categorized MKfT into SMK and PCK, or components of MKfT as SMK and PCK in the previous area, this area defined SMK and PCK. The student teachers 2, 7 and I raised that they have learned what each category (SMK and PCK) is.

> "I could see how amazed they (student teachers) seemed in the way they paid close attention when I presented to them the definitions of both SMK and PCK. I believe that the teachers must know what SMK and PCK are so that they are at least specific with their knowledge during their planning and preparation for instruction". (Personal reflection)

These are the two very vital knowledge mathematics teachers require in the field of teaching. Therefore, student teachers through this intervention were informed of what they are in teaching mathematics in schools. So SMK is having thorough knowledge of the basic mathematical concepts, how they work and why they arise in real life (Ball and Bass, 2000; Laren and Russo, 2014; Venkat and Spaull, 2015). Also, SMK is explicitly defined by the three domains: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Knowledge at the

Mathematical Horizon. On the other hand, PCK is defined as the clear setting of teaching ideas for full understanding so that knowledge is lead to carry on the prepared behavior in the work of a teacher (Rowland et al., 2005). Similarly, PCK is defined by the three domains which are Knowledge of Curriculum, Knowledge of Content and Students (KSC) and Knowledge of Content and Teaching (KCT) (Ball et al., 2008).

My purpose here was to demonstrate to the student teachers how complex a teacher's knowledge is and how important strong knowledge is and show them that teachers who have limited knowledge in these areas find it difficult during instruction to teach mathematics (Huntley and Flores, 2010). For example, the skills in explaining and solving mathematical problems such as functions/equations/inequality (Huang and Kulm, 2012: 425), just because they may not understand how such topics relate with each other (Kahan et al., 2003). When this happens, students do not learn mathematics because they are taught incorrect mathematics by teachers who lack in-depth knowledge of SMK and PCK (Wu, 2005). This lack even prevents teachers from understanding current mathematics that is based on more modern concepts which then can promote students' understanding of mathematical concepts (Even, 1993: 113). For example Bansilal's work (2012b) cited in Bansilal, Brijlall and Mkhwanazi (2014: 35-36) that focused on a teacher's poor SMK found that the teacher's explanations were often incoherent and illogical. The teacher's poor understanding of the concepts of ratio and number resulted in a teacher missing some key ideas and presenting convoluted explanations that involved circular reasoning, which made no sense to the learners. Lortie (1975) and Boaler and Greeno (2000) claim that they resort to teacher centered lessons which are inefficient for most school children.

After being made aware of what SMK and PCK are in the previous area, the fourth area that was successful, perceived by two student teachers who both were interviewed at the same time, was that they learned how complex/difficult concepts can build on from the basics, and at the same time mathematical knowledge being multi-dimensional connected, ie, their Knowledge at the Mathematical Horizon, which is a domain of SMK was developed. Student teachers 6 said:

> "Helped understood mathematical concepts clearer, for example, connecting related concepts and mathematical knowledge being similar to building blocks". (Student teachers 6)

This knowledge they perceived was developed by the workshop seems to be resonant with teachers' knowledge of the mathematical horizon whereby different topics/mathematics concepts are related to each other in different grades/levels in whole of the school mathematics curriculum (Nolan et al., 2015). This knowledge helps the mathematics teachers know where and what to build on from in a grade when teaching (Ma, 1999; Najike, 2004; Ball et al., 2005; Adler et al., 2009; Venkat and Spaull, 2015). Mathematics teachers must have the knowledge at the mathematical horizon so that they also can be aware of with likely students' misconceptions before and after the grades at which they teach in order to appropriately fill in the necessary lacking knowledge (Hiebert, 1997; Steele and Rogers, 2012). The reason why prospective mathematics teachers must have the knowledge of the mathematical horizon is because mathematics begins with basic/fundamental mathematics concepts (Boley, 1999; Sullivan, 2011). This fundamental mathematics necessitates the continuous growth of the mathematical knowledge deepening and widening mathematical knowledge with new knowledge over time (Klein, 1945; Baturo and Nason, 1996; Siegler and Booth, 2005; Sarama and Clements, 2009; Watson and Harel, 2013). Klein (1945) explains in detail the importance of the fundamentals of school mathematics which are required in the latter. In other words, Klein's perspective of school mathematics is more transmission orientated, which school mathematics is a set of discrete facts and methods that need to be learnt. That is, understanding deep ideas, such as that all the operations in number and algebra can be built from repeated application of the field laws, which happens to be a fundamental part of the teachers' knowledge. A dominant perspective is that mathematics grows like a tree, both upwards and downwards simultaneously, and, at the same time as mathematical knowledge branches out in new directions, it also grows deeper and more rigorous.

As well as making student teachers much more aware of the knowledge they needed and why it was important the workshop also had a section that attempted to develop their SMK and

PCK. In the fifth theme, student teachers 5 and 7, and I perceived that the student teachers were showed some ways of PCK to deliver SMK. Student teachers 5 and I voiced that:

> "In-depth information about how to show and involve students". (Student teachers 5)
>
> "They tried to differentiate between textbook teaching and students' misconceptions developed during teaching, how to lead students out of their misconceptions which is not in textbooks in session 1". (Personal reflection)

Here both quotes are more focused on students and on student learning and misconceptions which forms part of PCK (Shulman, 1986). In mathematics education literature this might also be categorized as teachers' Knowledge of Content and Students (KCS) (Ball et al., 2008) which as Nolan et al. (2015: 55) say:

> "This knowledge is the combination of knowing about students and knowing about mathematics in a way that enables teachers to (for example) anticipate what students may think and what they will find confusing, interesting and motivating, and to interpret students' (not fully coherent) spoken words and written work".

It is an interactive relationship between the teachers, their students and the mathematics content during instruction, and not just a process with fixed and pre-determined descriptions like beliefs (Subramanian, 2014: 179). This knowledge allows teachers in the classroom to make flexible decisions on students' evolving ideas during instructions encouraging their constructive participation in teaching (Shulman, 1987; Llinares, 2000; Adler, 2005). It also allows teachers to identify possible students' difficulties beforehand, therefore allows teachers in their planning to simplify SMK possible enough for the learners and their difficulties (Ball and Bass, 2000).

The sixth theme where the workshop was perceived to be successful also involved my attempts to directly develop the student teachers' PCK in the area of the use of real life examples to make mathematical concepts clearer. This was perceived to be successful by student teachers

2, 3, 4 and I. Student teacher 3 gave an example of how she had subsequently used this strategy in his/her teaching:

> "During my micro-teaching, I implemented what I learnt from the intervention. That was, I had a packet of biscuit, and used it as an example to be shared between 2, 4 & 8 students to help explain the concept 'ratio'". (Student teacher 3)

This is a teacher's knowledge of curriculum which includes the use of curriculum materials/tools in certain circumstances (Shulman, 1986). A deep understanding of mathematical knowledge can be gained by students through the appropriate use of tools (Pratt, 2012: 23) that can make learning of mathematical knowledge dynamic and interactive (Fennema and Franke, 1992). This is because mathematics has some relationship with real life materials (Pimm, 1990) and so the experiences of mathematical content knowledge for teaching must include these real life materials (Davis and Simmt, 2006). The use of real life materials, as said before, allows abstract mathematical concepts to become more accessible in real life (Dundara et al., 2012; Watson and Harel, 2013). These real life materials can be used to model mathematical concepts, making it visible and enrich learners' sense making of the mathematical concepts (Arcavi, 2003). Relating mathematics to appropriate materials during teaching in schools contextualizes mathematical concepts which has been shown to be able to develop deep understanding (Bishop, 1993; Matang, 2002; Najike, 2004; Kari, 2005; Pournara, 2009).

The seventh theme that was identified by student teachers 1, 2 and by myself was that the planning and preparation of concepts/topic for teaching was considered as a successful part of the workshop. I evaluated in my reflection that:

> "I think that it is important that mathematics teachers need to plan and prepare concepts/topic well before teaching. That is, in their planning and preparation of a particular concept for teaching in a particular lesson, they must be aware of possible misconceptions and other related concepts students might experience during instruction". (Personal reflection)

Teachers must learn how to plan and prepare concept(s) well before instruction because when they do not, in my professional experience as a teacher educator and from the literature, they focus their instruction on textbooks which often emphasize following rules and learning skills, rather than deep understanding of mathematical concepts (Watson and Harel, 2013). Research by Zakis et al. (2009: 41) shows that teachers who do not plan and prepare well tend to follow the content of textbooks, plan traditional lessons, do not leave room for anticipation of possible mistakes students might make, deal with misconceptions and difficulties, and have space to explore possibilities for alternative explanations or examples. They (teachers) are then confronted with all these various learning situations created by students in the classroom during instruction (Mayson and Davis, 2013). Lack of planning has also been shown to affect teachers' reactions to students' questions and ideas related to specific mathematical concepts (Even and Tirosh, 1995).

Overall the programme and its success was summarized by student teacher 7:

> "So far the skills and techniques of how to deliver the content knowledge of mathematics had been a wonderful presentation at which you've showed us the strategies and approaches, on how to equip ourselves with the content before the actual implementation. You have done impressively with the outline of sessions and delivered well the methods/ways for us as prospective mathematics teachers to do the same". (Student teacher 7)

This emphasizes that the major focus of teacher training must be the development of teachers' SMK and PCK and so is responsible to develop teachers' SMK and PCK (Lannin et al., 2013).

Therefore, teacher education programs, and, educational interventions respectively must ensure that their courses are rich in both content and pedagogy and that these are presented in a way that correlates particular content with its pedagogy (Chikiwa, 2017; Blanton et al., 2001).

This is because practicing teachers who are lacking the mathematical knowledge again calls for teacher educators to design and implement interventions which could be used to improve the poor situation (Bansilal et al., 2014). In particular, it is important that the secondary mathematics teacher education programs adopt approaches that take into account the central role played by teachers' knowledge about mathematics learning, and the dilemmas generated when teachers have to reconcile objectives that are apparently contradictory during their teaching practice (Llinares, 2000: 41).

A final theme that was raised by student teachers 3, 4 and 6 was that more similar interventions and must be conducted often for prospective mathematics teachers. Student teachers 3 and 4 said:

> "Found it successful and so suggested that upcoming mathematics teachers should be provided this opportunity too". (Student teacher 3)
>
> "This intervention should be conducted with all prospective mathematics teachers because it helped me a lot as a teacher to be". (Student teacher 4)

Here are some examples of research on how teachers developed both their SMK and PCK. During a workshop, students' work was used that encouraged teachers' participation in developing practices on how to support teachers' involvement in constructing mathematical knowledge for teaching (Kazemi and Franke, 2004). A professional development program in the form of an intervention equipped mathematics teachers with advanced mathematical knowledge for teaching in some Africa school (Pournara et al., 2015). Project based used as professional development form which teachers implemented interactive and constructivist approach (Gunel, 2008: 209) because professional development programs can enhance and enrich teachers' mathematical content knowledge for teaching (Kanter and Konstantopoulos, 2010). DMI participants' (teachers) increased their MKfT during a nationally disseminated professional development program (Bell et al., 2010). In-service intervention that improved knowledge of

mathematics for teaching (Ell and Lomas, 2007) and in-service training programs enhanced practicing teachers' practices (Khamis and Sammons, 2004).

5.0 Conclusion

This chapter presents, firstly answers the research questions briefly, bringing back in how literature resonates with the study. Secondly, it discusses the key limitations of the study and finally, it considers the implications of the study for my own practice as a mathematics teacher educator.

5.1 Summary of findings

Firstly, the findings are helpful for professionals actively involved in the area of teacher education, particularly, mathematics teacher educators in teacher education institutions and teacher mentors in schools in PNG.

Firstly, the finding in research question one shows that the key issue and challenge the respondents perceive teacher trainees most pressingly face in schools is due to their lack of PCK and not so much their SMK. Literature resonates with this finding and supports the view that PCK is a vital knowledge base required for teaching because it allows teachers to identify possible students' difficulties beforehand, so that in their planning they simplify SMK enough for the learners and their difficulties (Ball and Bass, 2000). However, it is of concern that the respondents only identify PCK at this point and not SMK (although later they do) because in my professional experience SMK is also a challenge because, as Wu (2005) and Ball et al. (2005) say that teachers' SMK can also influence their PCK because their deeper understanding of the SMK can alter even a well prepared and planned pedagogical approach to a topic during instruction to suit the learning needs of the students. It is teachers' SMK that makes it easy to know their students (KCT) too and then allows them to make flexible decisions on students' evolving ideas during instruction that encourages students' constructive participation in teaching in the classroom (Shulman, 1987; Llinares, 2000; Adler, 2005). For example, teachers who do not know their

content well, let alone do not know how to transform it into something that could be learnt by the students (Bansilal's work (2012b) cited in Bansilal, Brijlall and Mkhwanazi, 2014). Therefore, in order for trainee teachers to overcome issues and challenges in schools, they need both their SMK and PCK. Even though the respondents said that the trainees' issues and challenges were due to their lack of PCK, the finding in research question two reveal that the key mathematical knowledge for teaching required in schools are SMK and PCK. Literature affirms that these are the two knowledge bases needed by teachers to teach mathematics (Ball and Bass, 2000; Rowland et al., 2005; Ball et al., 2008; Krauss et al., 2008) because they are knowledge bases needed to carry out the work of mathematics in schools (Ball et al., 2008). Therefore, the respondents said that they would want these knowledge bases for teaching to be addressed and developed in a workshop conducted for student teachers. Both student and practicing teachers are able to develop their SMK and PCK in such interventions as proven by literature (examples: Timmerman, 2004; Ell et al., 2007; Nolan et al., 2015). Similarly, this intervention was designed to help develop the student teachers' mathematical knowledge for teaching (MKfT), which are their SMK and PCK which responded to the findings from cycle. The student teachers during the intervention found that the information on MKft, and then its categorization simplified the complex knowledge. Their definitions and use of real life examples were informative that made the student teachers and I perceive the intervention successful.

5.2 Key limitations of the study

In the course of this project, I encountered numerous challenges which limited my research, particularly, in terms of data collection. The suitable participants for cycle one are teacher educators who are employed in teacher education institutions, teacher mentors who teach in secondary schools and beginning teachers who recently (at the time the data was collected) graduated from a teacher education institution. I initially asked through the lecturer in charge for mathematics education to interview at least about three, but only one agreed to participate in the data collection. The data collection with the teacher mentors was a also great challenge. I was turned down by the first secondary school I visited. This is the biggest state

secondary school in town and is the closest to where I work. I then resorted to asking a small private school within town and one outside which is about thirty minutes-drive from the main town. In addition, during the time of the data collection political influences within my institution and in the country meant it was challenging to gain access to participants particularly during the period where there was significant unrest within the student body of my institution.

In terms of working with student teachers in the intervention access was easier as I conducted lectures for a general education course to bachelor in education pre-service year four students in semester one in which all the mathematics prospective teachers were part of where they already knew me in person. The only challenge was for me to find a time and at which both me and the student teachers were free, and then find a venue so I could conduct the intervention because the students are in their final year of study and are busy with school work. We finally found a suitable time for both groups, however, the intervention did not go for two hours as scheduled because the room was going to be used by some senior lecturers for another workshop so I ended the intervention at the end of the first hour.

5.3 Key implications of the study

The key implications of my findings which are considered for my own practice as a teacher educator and the teacher institution at which am engaged are:

1. Although beginning teachers perceive that they need PCK they also need to understand the complexity of SMK similar what I have done in the the workshop.
2. Helping student teachers plan and then giving them suggestions for PCK (e.g. planning and giving them active strategies for teaching mathematics and the use of real life examples) was helpful but this was only a first step and needs to be done more systematically and for more topics.
3. More link between workshops and giving student teachers practice in schools with mentors in collaboration with teacher educators to develop their SMK and PCK.

4. Mentors and the other teacher educators are also, it seems not aware of the complexity of SMK and assume the beginning teachers' SMK was good – therefore there also seem a need for me to work with colleagues in the University of Goroka and schools which allow the student teachers do teaching practice to develop their understanding of the complexity of the knowledge base of teaching so they can be much more aware, as they work with teacher trainees, of the nuances of what needs to be developed and how they can work in a collaborative relationship to do so.

5. More similar interventions to be conducted for mathematics students teachers to their develop SMK and PCK.

6. A teacher education programme, particularly in mathematics to develop student teachers' SMK and PCK.
7. An efficient and effective collaboration between teacher educators working in teacher education institutions and other stake holders such as teacher mentors and other practicing teachers teaching in schools through workshops to inform and make them aware of the MKfT and help student teachers continue to build their SMK and PCK during their teaching practice.
8. Clarke and Hollingsworth (2002) emphasized a model of teacher professional growth based on their teachers' existing beliefs and knowledge being developed further through professional development programs. It is a recommendation that workshops must be conducted where teachers' teaching strategies can be can exposed (Sibuyi, 2012). For example, during a workshop, students' work was used that encouraged teachers' participation in developing practices on how to support teachers' involvement in constructing mathematical knowledge for teaching (Kazemi and Franke, 2004).

Students reported that the intervention was successful, however asked that more must be conducted most often for mathematics student teachers. The student teachers reported that all the aspects of the workshop was found to be appropriate for the teaching of mathematics, however, asked for time to put into practice what they have learnt. One of those very important

information I could have included was the domains of SMK and PCK, and, increased the time for the intervention.

Below are few areas to be researched by prospective researchers including myself:

1. Do student teachers require mathematics content beyond grade 12 mathematics curriculum or just the school curriculum as their SMK to teach in secondary schools?
2. Do student teachers need more practice time to develop both their SMK and PCK?
3. What sort of curriculum is required for teacher education programmes?
4. What sort of knowledge should teacher educators have to train student teachers who want to become secondary school teachers in teacher education institutions?

The notion that teachers are born and not made is misleading because teaching is complex so as the knowledge teachers require to teach. Therefore, student teachers may have their personal beliefs about teaching in which some may not be ideal for teaching. That is why it is the responsibility of any teacher education institution to change some of those beliefs such as textbook based teaching by developing their SMK and PCK which are the bases of teacher knowledge. Thus, student teachers enroll in teacher education institutions because their major focus of teacher training is the development of teachers' SMK and PCK and is responsible to develop teachers' SMK and PCK. Teacher education institutions must develop this two critical components, SMK and PCK teachers require to teach mathematics during teacher training because it is the responsibility of the mathematics teacher education institutions is to support any mathematical knowledge for teaching.

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APPENDIX I – Analysis of the data

Phase 1: Analysis

4. **What do Beginning teachers and other key stake holders (mentors and teacher educators) perceive are the issues and challenges related to mathematical content knowledge that trainee teachers are faced with?**

Summary of respondents in the table below.

Table W

Respondent	**Issues and challenges related to mathematical content knowledge that trainee teachers are faced with.**
Beginning Teacher 1 (TM 1)	-Explanation of concepts to students -Level of confidence is good in content -Lacks PCK on how to deliver/teach SMK -Minimum knowledge change in both content and teaching skills -Minimal teaching practice period. -"I have no problem with SMK, but how to deliver that SMK to the students is little bit a challenge to me, especially in explaining the mathematical concepts to the students".
Beginning Teacher 2 (BT 2)	-Lacking confidence when delivering content during lessons -Takes longer preparing lessons -Level of confidence is good in content -Lacks knowledge of teaching the content, ie, lacks teaching skills to deliver the content during lessons. -Able to develop/gain teaching skills as time goes by during teaching practice under the mentorship -"My SMK is good but I need to adjust my teaching techniques. The challenge is how to teach/deliver the SMK thoroughly to the students". -Lacks PCK on how to deliver/teach SMK

	-Minimal teaching practice period
Teacher Mentor 1 (TM 1)	-"The challenge that the training teachers face during their training period is delivering the content, they have the content but how to deliver the content". -Level of confidence in content is disorganised -Level of confidence is low when they first start -Knowledge changes during teaching practice as they are criticized meaningfully by their mentors. -Lacks PCK on how to deliver/teach SMK -Lacks confidence in teaching -Level of confidence increases during teaching practice whilst under the supervision of a mentor. -Student – teacher ratio
Teacher Mentor 2 (TM 2)	-Making adjustments to adapt to the new and challenging environment physically and psychologically -Level of confidence depend on individual trainee teachers but is generally very low with first contact -Very low in their level of PCK, yet to be explored -Level of content depends on individuals -Level of PCK picks up momentum as time goes by during teaching practice. -Lack of exposure to actual classroom setting -"The practical aspect/reality of actually teaching in a classroom setting and experiencing reality".
Teacher Educator	-Understanding school mathematics concepts -Presenting and teaching concepts -Lack of PCK on how to deliver/teach SMK

(TE)	-"Level of confidence is minimum for difficult topics/concepts" -The more students discuss topics during lessons, they improve -Trainees improve on their SMK and PCK during teaching practice whilst guided by mentors. -"Once trainees are guided well by mentors and weaknesses are corrected at that moment during teaching, it helps them build a better understanding of the knowledge of subject and of teaching". -Lacks exposure to school mathematics curriculum -Unable to solve all activity questions

Common themes from all respondents:

- Trainee teachers lacking PCK when they begin teaching practice.
- Trainees teachers lack confidence in teaching
- Lack of ability to practice and develop their PCK in school placements
- Lack of exposure to the school mathematics curriculum
- PCK and SMK did improve during teaching practice.

5. **What mathematical knowledge for teaching do Student teachers and stake holders say is required by trainee teachers?**

Summary of respondents

Table X

Respondents	**What key mathematical knowledge**
Beginning Teacher 1 (BT 1)	-SMK -Taught well that reflected good SMK -Very confident Example quote -"the feel of how to become a mathematics teacher" -Develop SMK which is very important
	-Enough mathematical knowledge

Beginning Teacher 2 (BT 2)	-Good SMK gave confidence to answer students' queries/questions -Had alternative explanations -Had confidence to respond to students' questions -PCK -More practice teaching to develop and build the PCK. Example quotes -"I think content knowledge of mathematics is okay to teach any level at secondary school. I am very confident. The only problem is knowledge of teaching".
Teacher mentor 1 (TM 1)	-Expanded university mathematics content knowledge -Some teaching methods -Knowledge of the topic/unit taught in a secondary school during teaching practice period -Lacks the PCK of the topics taught -"Trainees had difficulties explaining single question and to give alternative explanations" Example quote -"Trainees were able to answer students' questions but to the level of content". -They are confident if they had been faithful during teaching practice -SMK -PCK "The trainee teacher, before teaching the particular topic/unit must have enough knowledge of that topic"
	-University mathematics content -Knowledge of topics/content taught during teaching practice -Lacks confidence during delivery if one lacks SMK

Teacher mentor 2 (TM 2)	Example quote -"Yes at some degree when dealing with abstract concept that cannot be presented physically, eg., calculus and vectors". -Answered questions depending on the nature and degree of projected questions -Depended on the number of contact periods -SMK -PCK
Teacher educator (TE)	-Little of both SMK and PCK to begin teaching -Knowledge of the topic taught -Good SMK enables efficient delivery of math -Find explanation difficult at the beginning -Answer students' questions but not direct at most times -Just confident, not too much, not too little -Provide practical components of mathematical concepts -A good solid SMK Example quotes "A good SMK has a great impact on delivery on mathematical concepts". -"Trainee teachers initially find it difficult with explanations and make grammar and math errors". "Theoretical concepts must be complemented with experiments and practical"

Common themes from all respondents:

-SMK

-PCK

-Level of confidence

-Effects of mathematical for knowledge on teaching

6. **How can this knowledge be developed in a workshop on the teaching and learning of algebra for final year mathematics major students in UoG?**

Summary of respondents

Table Y

Respondent	**What the respondents said about what they'd like to see in a workshop to develop the student teachers**
Beginning Teacher 1 (BT 1)	- SMK -PCK -More practice teaching to develop and build the PCK. Example quote "Do thorough peer teaching with continuous lesson observations so that they (student teachers) can analyse how they would teach when they are out in the field (schools).
Beginning Teacher 2 (BT 2)	- SMK -PCK -More time for practice Example quotes -"From experience, knowledge of how to teach mathematics is a problem so teacher training institutions should emphasise more on that". "As well, offer more content courses to overcome challenges or to be confident in answering questions."
Teacher Mentor 1 (TM 1)	- SMK -PCK Example quotes

	-"The institution should help the trainee teachers with teaching resources". "Equip them as much with content".
Teacher Mentor 2 (TM 2)	-Use of material/tools to visualise concepts -More practice to build confidence Example quote -"Since mathematics is abstract in nature, as much as possible practical application with a lot of examples (material use) should be taught/practice".
Teacher Educator (TE)	-SMK Example quote - "Students must participate in experiments and write reports." -"More practice teaching must be emphasized and closely monitor and supervise these student teachers".

Common theme from all respondents:

-SMK

-PCK

-More practice teaching

Phase 2 Analysis of the success of the workshop

1. **In what ways and to what extent did the prospective teachers perceive that the intervention was successful in addressing the issues expressed in phase 1 of the research?**

Summary of respondents

Table Z

Respondent(s)	**What the student teachers said was successful in the workshop that helped them learn and why they thought it was successful**	**What the student teachers said was less successful in the workshop if anything and what might be done differently**

Student Teacher 1	-What Mathematical Knowledge for teaching is -Mathematical Knowledge for teaching categorised into; Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) -Preparation and planning of topics/concepts for lesson -Critically effective Example quote "I am now aware that what I require to be a mathematics teacher is mathematical knowledge for teaching."	-There should have been a time for practice to implement what was learnt -Should be done often
Student Teacher 2	-What Mathematical Knowledge for teaching is -Mathematical Knowledge for teaching categorised into; Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) -Clearly defined SMK and PCK -Examples and application of mathematical concepts in real life situations -Preparation and planning of topics/concepts for lesson	

Student Teacher 3	-Examples and application of mathematical concepts in real life situations Example quotes -"Found it successful and so suggested that upcoming mathematics teachers should be provided this opportunity too". "During my micro-teaching, I implemented what I learnt from the intervention. That was, I had a packet of biscuit, and used it as an example to be shared between 2, 4 & 8 students to help explain the concept 'ratio'".	
Student Teachers 4	-Use of real life situations to simplify mathematical concepts -"This intervention should be conducted with all prospective mathematics teachers because it helped me a lot as a teacher to be".	
Student Teacher 5	-Successful because its content was very suitable for teaching -"In-depth information about how to show and involve students".	-Required to do more
Student Teachers 6	-Helped understood mathematical concepts clearer, for example, connecting related concepts	-There should be another time for this information learned to be practised.

	-Mathematical knowledge being similar to building blocks. -Need more of similar interventions	
Student Teacher 7	-Defined mathematical knowledge for teaching -Categorised mathematical knowledge for teaching -Define SMK and PCK clearly -Developed our PCK on how to deliver SMK -How to equip ourselves with SMK before teaching Example quote "So far the skills and techniques of how to deliver the content knowledge of mathematics had been a wonderful presentation at which you've showed us the strategies and approaches, on how to equip ourselves with the content before the actual implementation. You have done impressively with the outline of sessions and delivered well the methods/ways for us as prospective mathematics teachers to do the same".	
	-Identified the 'Mathematical Knowledge for Teaching'	-Further categorisation of both SMK and PCK was not in detail

My own reflection notes	-Defined clearly mathematical knowledge for teaching -Categorised mathematical knowledge for teaching into 'Subject Matter Knowledge and Pedagogical Content Knowledge' -Defined SMK and PCK clearly -Outlined clearly PCK with related examples -Preparation and planning of concepts Example quote -"They tried to differentiate between textbook teaching and students' misconceptions developed during teaching, how to lead students out of their misconceptions which is not in textbooks in session 1". -Perceived the content of the intervention was very suitable for the prospective teachers. Example quotes "I could see how amazed they (student teachers) seemed in the way they paid close attention when I presented to them the definitions of both SMK and PCK. I believe that the teachers must know what SMK and PCK are so that they are at least	-Should have given some time during the intervention for student teachers to practice the preparation and planning section on given topics/concepts. -Amount of time for the intervention was not enough; just one hour -Did not have time to give an example of a mathematical concept, example, solve this **inequality** problem, and ask the student teachers what possible concepts their students would have in mind whilst solving this problem. -If I have to do this again, I will also arrange some time for students to do some practice teaching

	specific with their knowledge during their planning and preparation for instruction" "I think that it is important that mathematics teachers need to plan and prepare concepts/topic well before teaching. That is, in their planning and preparation of a particular concept for teaching in a particular lesson, they must be aware of possible misconceptions and other related concepts students might experience during instruction".	

Common themes:

-Outlined and defined mathematical knowledge for teaching

-Categorization of the mathematical knowledge for teaching

-Definition of SMK and PCK

-Mathematical knowledge being multi-dimensional

-Helped developed some ways of PCK

-Application of examples of mathematical concepts in real life

-Planning and preparing of concept(s)/topic for teaching

-Suitable knowledge for teaching mathematics

-More similar interventions often for prospective mathematics teachers

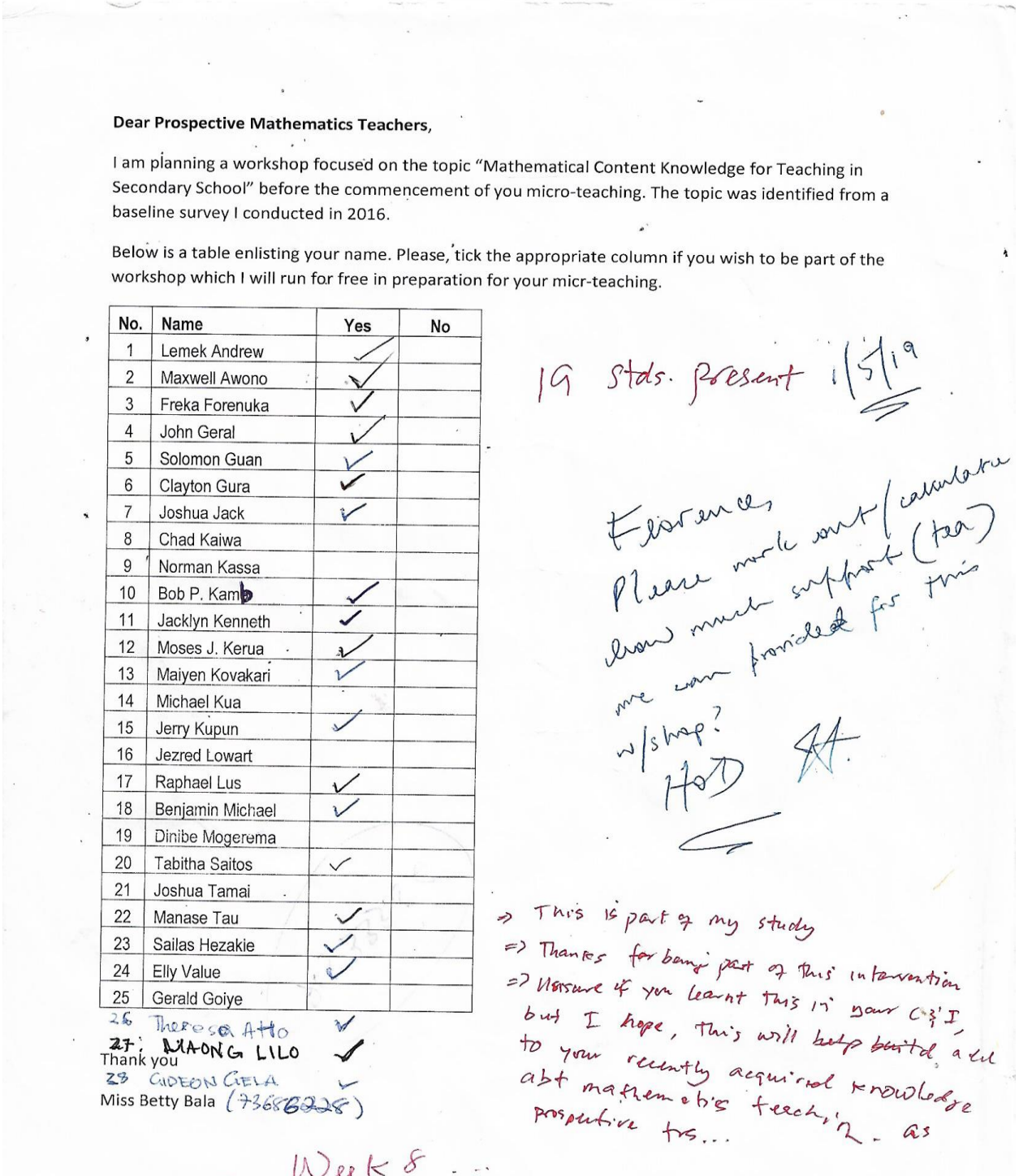
Common themes from all respondents: What Mathematical Content Knowledge for teaching is

-Mathematical Content Knowledge for teaching categorised into; Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK)

-Preparation and planning of topics/concepts for lesson

A quote from student teacher 4: "This intervention should be conducted with all mathematics prospective teachers because it helped me a lot as a teacher to be".

Appendix II – List of participants for intervention



Dear Prospective Mathematics Teachers,

I am planning a workshop focused on the topic "Mathematical Content Knowledge for Teaching in Secondary School" before the commencement of you micro-teaching. The topic was identified from a baseline survey I conducted in 2016.

Below is a table enlisting your name. Please, tick the appropriate column if you wish to be part of the workshop which I will run for free in preparation for your micr-teaching.

No.	Name	Yes	No
1	Lemek Andrew	✓	
2	Maxwell Awono	✓	
3	Freka Forenuka	✓	
4	John Geral	✓	
5	Solomon Guan	✓	
6	Clayton Gura	✓	
7	Joshua Jack	✓	
8	Chad Kaiwa		
9	Norman Kassa		
10	Bob P. Kamb	✓	
11	Jacklyn Kenneth	✓	
12	Moses J. Kerua	✓	
13	Maiyen Kovakari	✓	
14	Michael Kua		
15	Jerry Kupun	✓	
16	Jezred Lowart		
17	Raphael Lus	✓	
18	Benjamin Michael	✓	
19	Dinibe Mogerema		
20	Tabitha Saitos	✓	
21	Joshua Tamai		
22	Manase Tau	✓	
23	Sailas Hezakie	✓	
24	Elly Value	✓	
25	Gerald Goiye		

26 Theresa Atto ✓
27. MAONG LILO ✓
Thank you
28 GIDEON GELA ✓
Miss Betty Bala (73686228)

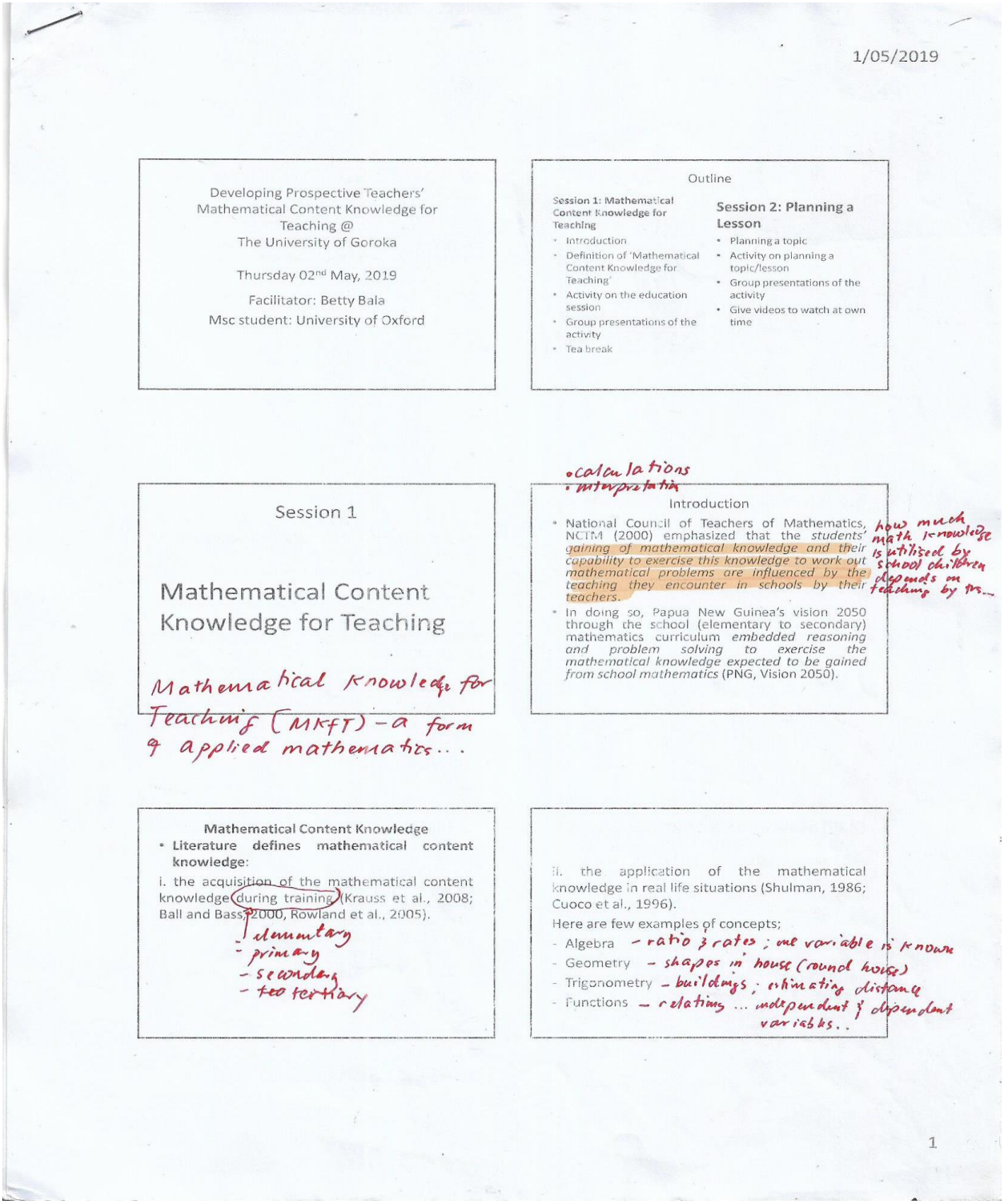
19 Stds. present 1/5/19

Florence,
Please work out/calculate how much support (fee) we can provided for this w/shop?
Hod

⇒ This is part of my study
⇒ Thanks for being part of this intervention
⇒ Unsure if you learnt this in your C.I, but I hope, this will help build a lil to your recently acquired knowledge abt mathematics teaching as prospective trs...

Week 8 ...

Appendix III – Intervention (Power point)



1/05/2019

Developing Prospective Teachers' Mathematical Content Knowledge for Teaching @ The University of Goroka

Thursday 02nd May, 2019

Facilitator: Betty Bala

Msc student: University of Oxford

Outline

Session 1: Mathematical Content Knowledge for Teaching

- Introduction
- Definition of 'Mathematical Content Knowledge for Teaching'
- Activity on the education session
- Group presentations of the activity
- Tea break

Session 2: Planning a Lesson

- Planning a topic
- Activity on planning a topic/lesson
- Group presentations of the activity
- Give videos to watch at own time

Session 1

Mathematical Content Knowledge for Teaching

Mathematical knowledge for Teaching (MKFT) - a form of applied mathematics...

Introduction

- National Council of Teachers of Mathematics, NCTM (2000) emphasized that the *students' gaining of mathematical knowledge and their capability to exercise this knowledge to work out mathematical problems are influenced by the teaching they encounter in schools by their teachers.*
- In doing so, Papua New Guinea's vision 2050 through the school (elementary to secondary) mathematics curriculum *embedded reasoning and problem solving to exercise the mathematical knowledge expected to be gained from school mathematics* (PNG, Vision 2050).

• calculations
• interpretation

how much math knowledge is utilized by school children depends on teaching by te...

Mathematical Content Knowledge

- **Literature defines mathematical content knowledge:**

i. the acquisition of the mathematical content knowledge during training (Krauss et al., 2008; Ball and Bass, 2000, Rowland et al., 2005).

- elementary
- primary
- secondary
- tec tertiary

ii. the application of the mathematical knowledge in real life situations (Shulman, 1986; Cuoco et al., 1996).

Here are few examples of concepts;

- Algebra – ratio & rates ; one variable is known
- Geometry – shapes in house (round house)
- Trigonometry – buildings ; estimating distance
- Functions – relating ... independent & dependent variables..

1

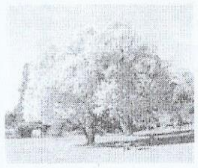
Example – Unit 10.2: *Functions and Graphs*

Topic: *Algebra and graphs*

Learning outcome: *calculating and using rates of quantities which change uniformly over time* (i.e. constant rates of change)

(Department of Education, 2006)

Functions



This tree grows 20 cm every year, so the height of the tree is ***related*** to its age using the function ***h***.

***h*(age) = age × 20**

So in 10 years
h(10) = 10 x 20
= 20 cm

iii. to be rich in and having multidimensional knowledge of the subject (Steele and Rogers, 2012; Ball et al., 2008; Hossain et al., 2009; Venkat and Spaull, 2015; Laren and Russo, 2014).

For example
- Algebra; (grades 6, 7 & 8; 10, 11 & 12)
- Calculus Concept' requires the knowledge of other concepts like algebra, trigonometry and geometry to find an equation of a line, an unknown side of a triangle formed and the area of shaded shape respectively.

Mathematical Content Knowledge for Teaching

1. Subject Matter Knowledge (SMK)
2. Pedagogical Content Knowledge (PCK)

Subject Matter Knowledge (SMK)

- The learning and knowing thoroughly the basic mathematical concepts, how they work and why they exist is what mathematics teachers are obliged to know (Ball and Bass, 2000; Laren and Russo, 2014; Venkat and Spaull, 2015).

For example- the algebra concept

Pedagogical Content Knowledge (PCK)

- The form of content knowledge with features of a most relevant content suitable for use in teaching that gives visibility to quality (Shulman, 1986). Refer to the example on function.
- Rowland et al (2005) defines it as the clear setting of teaching ideas for full understanding so that knowledge is lead to carry on the prepared behaviour in the work of a teacher.

Presentation of an Education session

- The session comprises two approaches

1. Worksheet approach to algebraic expressions
2. Matching cards activity approach to algebraic expressions

The detail of each approach is on the worksheet provided.

Have a look at 5-10 mins

Discussion

- Students continue working in groups and discuss the questions (on the worksheet) about the two approaches used in this session.
- This to be followed by a whole class discussion afterwards.

Session 2

Planning a topic/lesson

Planning a Topic

Preparing Yourself

⇓

Preparing for Learning

⇓

Planning for Learning

Preparing yourself–Foundation knowledge

- 'Splurge' diagram; An internal resource which you can call upon whenever you have to teach [a given topic] (from Preparing to Teach Framework).
- Prompts:
  - Key ideas, representations, misconceptions
  - Techniques, habits & behaviours, reasoning
  - Arising from, applying to, applying within

Preparing for learning – Topic planning

- What is to be addressed in the current series of lessons?
  - What concepts are to be worked on?
  - What skills are to be developed?
  - What contexts are relevant?
- In what order are these to be addressed (sequencing)?

Preparing for Learning

Conceptual	behavioural	contextual
relating to mental concepts	involving behaviour (observable)	relating to circumstances that form an idea ie.. the notion of 'equality' when working on "equations"...

Planning for learning – Lesson planning

- In a single lesson
- What learning outcomes are to be addressed?
- And in what order?
- For each learning outcome
- How are students to study? (Student activity)
- What is the teacher's role?(Teacher activity)
- How will the teacher/students know if the purpose is met? (Assessment)

Planning for Learning

Time	Outcomes	Student activity	Teacher activity	Assessment

Show UoG lesson planning format

Activity

In your groups, prepare and plan a topic ~~followed by~~ a ~~lesson on~~ (~~simultaneous equations~~) ≤

Preparing for learning..

=> first the conceptual ideas, behaviour, contexts...

Approach 1: A worksheet approach to algebraic expressions

I walk into the classroom and distribute the worksheet I prepared before the lesson. After all have a copy of the worksheet, I tell them to turn to page 91 and follow as I read through the notes and the examples up to page 93 which we would do this for about 20 – 25 minutes. In the next 6 – 8 minutes I would tell them to work on the 5 questions selected from Activity 10A on page 93: Write the following as algebraic expressions;

1. The sum of 9 and a
2. B decreased by 4
3. The product of 5 and c
4. x is increased by 4, then tripled
5. A number which is 3 less than twice the value of y

Now that the time is up for the exercise, I ask a volunteer at a time to show their answer on the white board for the 5 questions.

1. The 1st student walks up and writes 'the sum of 9 and a' as **9a**. Knowing that the answer is incorrect as the teacher, I ask another student to call out the correct answer which is '**9 + a'**.
2. I then move to question 2 "B decreased by 4" which a student writes '4 – b' which most students agree, but a student corrects it as 'b – 4' and approve it.

I take the same approach to questions 3 – 5. The 40 minutes of the lesson is over and so I tell them to do the rest of the questions on Activity 10A on page 93 for homework.

Approach 2: Matching cards activity approach to algebraic expressions. The activity card is extracted from the MARS assessment materials at http://map.mathshell.org/materials/download.php?fileid=694.

The lesson would begin with an interactive whole class introduction which should take 10 minutes. This is a short question and answer activity. If any incorrect answers are shown, further

discussions would be made till the correct answer would be reached, or ask leading on questions to lead students' out of the incorrect answer.

On a blank page, show me an algebraic expression that means;

Multiply n by 4, and then add 3 to your answer.	4n + 3
Add 3 to n, and then multiply your answer by 4.	4(3 + n)
Add 5 to n, and then divide your answer by 3.	$\frac{n+5}{3}$
Multiply n by n, and then multiply your answer by 5.	$5n^2$
Multiply n by 5, and then square your answer.	$(5n)^2$

The next part of the lesson is a Collaborative activity in which the expressions and words would be matched.

I tell the students to get into groups of two or three. I give each group two sets of cards, one with expressions written in algebra, and the other with words. I tell the students to choose an expression and find the words that match it.

But before they start, I would want them to pay attention while I model the activity with examples below.

4(n + 2) matches 'Add 2 to n then multiply by 4'; 2(n + 4) matches 'Add 4 to n then multiply by 2'.

Card Set A: Expressions **Card Set B: Words**

E1 $\frac{n+6}{2}$	E2 3n2	W1 Multiply *n* by two, then add six.	W2 Multiply *n* by three, then square the answer
E3 2n + 12	E4 2n + 6	W3 Add six to *n* then multiply by two.	W4 Add six to *n* then divide by two
E5 2(n + 3)	E6 $\frac{n}{6} + 6$	W5 Add three to *n* then multiply by two.	W6 Add six to *n* then square the answer
E7 (3n)2	E8 $(n + 6)^2$	W7 Multiply *n* by two then add twelve	W8 Divide *n* by two then add six.
E9 $n^2+12n+36$	E10 $3 + \frac{n}{2}$	W9 Square *n*, then add six	W10 Square *n*, then multiply by nine
E11 $n^2 + 6$	E12 $n^2 + 6^2$	W11	W12
E13	E14	W13	W14

If they find cards missing, they create these for themselves.

Now we have used two different approaches (the worksheet and the activity cards) so far in algebra to help transform the mathematics knowledge of algebraic expressions into conceptual understanding.

Students continue working in groups of two or three and discuss the questions below about the two approaches used in this session.

1. Critically describe the two approaches used in 100 words.
2. In which of the two have you been engaged lot more in discussions?
3. In which of the two do you think you learnt better? Give reason(s) for your answer.
4. What do you think each approach reflects on a teacher?
5. By working on the activities designed (worksheet and activity cards) in the two approaches, which of the two would you prefer the most for your students in the classroom? Give an explanation for your choose.
6. What do you need to know to successfully answer the questions?
7. What do the variables a, x or y mean?
8. What do these questions say about the purpose of learning algebra?
9. How much conceptual understanding do you need to answer the questions?
10. How would you know if your answers are correct?

Appendix IV – Personal reflection of intervention

Personal professional reflection of the intervention

- Planning of the intervention: It was initially planned for the 12th of May, 2019 which was a week before the student teachers were to go out for micro-teaching. However, I made an urgent change for it to be conducted on Thursday evening from 6 pm – 8 pm after all the rooms are freed up. The student teachers did not agree with this time since some of them leave off campus, and so was going to be difficult to go home to where they live which is not nearby to the university campus. I then resorted to a conference room and brought forward the time from 3 – 5 pm in the afternoon. It was unfortunate that the same room had been used for another workshop from 4 – 6 pm weekly which I was not aware of. I completed session one of the intervention

thoroughly, but rushed through session two, which is quite a short one. The intervention still was perceived well despite the cut in time.

- Content of the intervention: I believe that the contents of the intervention was very necessary and sufficient student teachers. They were informed of the knowledge they require to be mathematics teachers in schools. The intervention simplified this complex knowledge which they were able to understand the concept clearly. When I began the intervention, they were all paying close attention and I could see from their facial expressions how curious they were about what I was presenting. I believe within too that the contents in the intervention was very suitable for the student teachers, and this is what made me perceive that it was successful.
If I am to conduct another intervention which am planning for next year, I would include the domains of both SMK and PCK, and make sure that I will have about 2 hours of presentation so that I can be able to give some practical activity and then assist them, particularly in preparing and planning of at a topic.

Categorised mathematical knowledge for teaching into 'Subject Matter Knowledge and Pedagogical Content Knowledge'

-Defined SMK and PCK clearly

-Outlined clearly PCK with related examples

-Preparation and planning of concepts

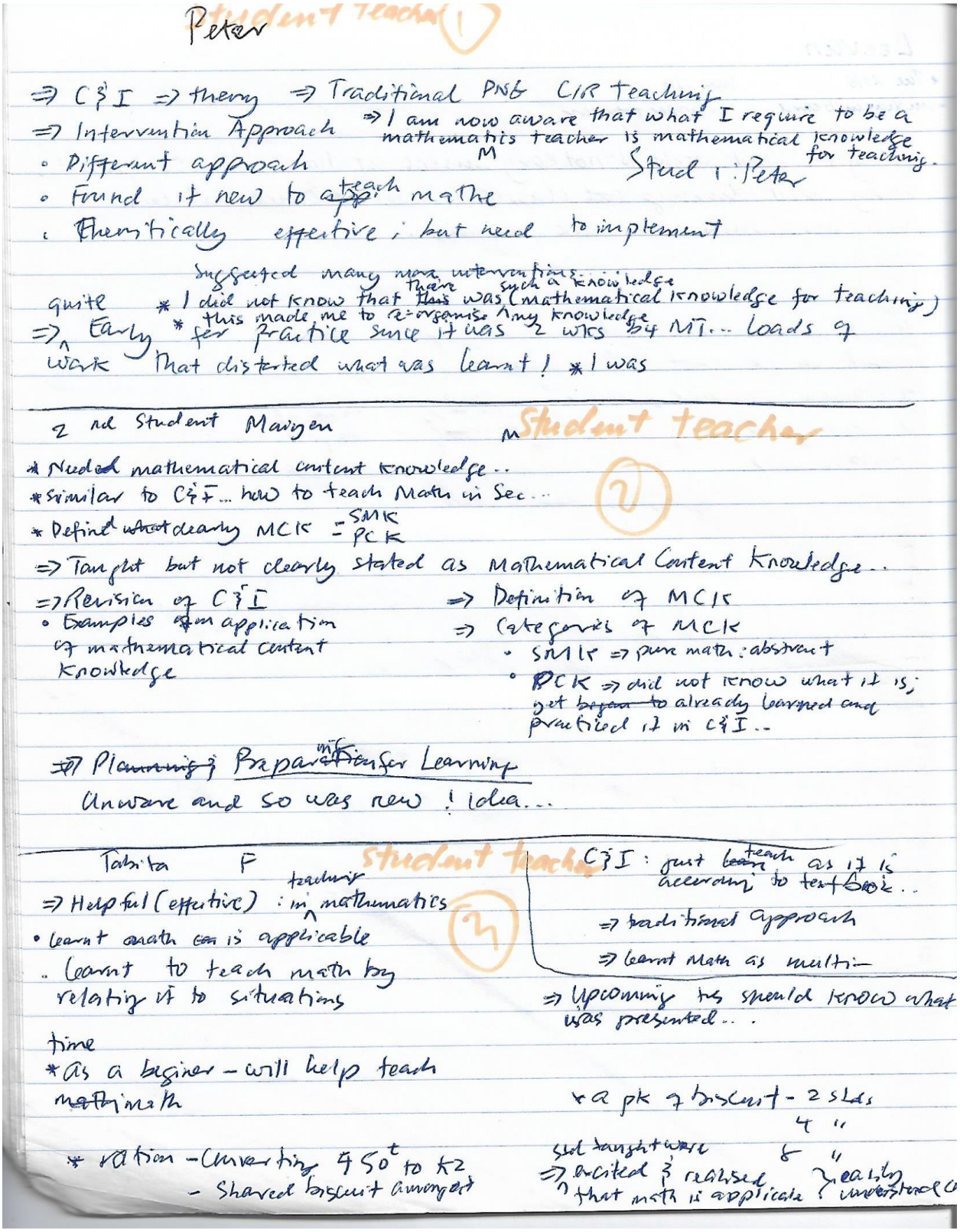
-"They tried to differentiate between textbook teaching and students' misconceptions developed during teaching, how to lead students out of their misconceptions which is not in textbooks in session 1".

-Perceived the content of the intervention was very suitable for the prospective teachers.

"I could see how amazed they (student teachers) seemed in the way they paid close attention when I presented to them the definitions of both SMK and PCK. I believe that the teachers must know what SMK and PCK are so that they are at least specific with their knowledge during their planning and preparation for instruction"

"I think that it is important that mathematics teachers need to plan and prepare concepts/topic well before teaching. That is, in their planning and preparation of a particular concept for teaching in a particular lesson, they must be aware of possible misconceptions and other related concepts students might experience during instruction".

Appendix V – Respondents of the evaluation of intervention (cycle 2)



Student teachers

Elly & Theresa

⇒ research topic by stds. then teach

C&I = Similar to Intervention

⇒ Must use real situation to simplify concepts... ⇒ learnt in C&I too

⇒ Find ~~not known~~ way to ~~Did not teach~~ write LP and then teach...

⇒ Shallow in C&I...
Intervention - indepth

*Write a reflection

Helpful

Should anytime with all stds.

- C&I brief
- help alot in math teaching

Student Teacher

Jacklyn (F)

Helpful - ~~was~~ was PCK approach was suitable...
- show stds
- involve stds
- use ↑ PCK

* Good timing ... MT
* Requested to do more.. in future
* ~~Shallow~~ Inadequate knowledge of PCK
*

⇒ 21/06/19 Student teachers

Joshua & Soloman

C&I: general info on how to deliver

- helps understand mathematical concepts
- eg.. making relationships with related concepts

⇒ new... found it effective & suggested more.. & be practised

* mathematics is building blocks..
* I/We think we are now able to integrate related concepts to build knowledge for stds.

Student 7

Interview questions to assess the Effectiveness of Intervention

- In addition to your mathematics education course that prepared you for mathematics teaching in secondary schools, is this intervention perceived effective in terms of its content presented and timing?
- Content of the intervention:

So far the skills and techniques of how to deliver the content knowledge of mathematics has been a wonderful presentation at which you've shared us the strategies and approaches on how to equip ourselves with the content before the actual implementation. You have done impressively with the outline of sessions and delivered well the methods/ ways for us as prospective mathematics teachers to do the same. Thank so much, may the Good Lord continue to bless you

- Timing of the intervention:

The timing was actually pertaining to the outline of your session. You have it all the control and management of it. You did well with an hour since disruption was not a problem you actually cover up and we covered all the sessions well with activities. The flow of timing was so good that you being the great deliverer. May God bless you!

Appendix VI – Respondents of cycle 1

BT (1)

Introductory questions to the beginning teacher.

I) Name: Josiah Francis

II) For how many years have you taken teacher education courses? 4 years

III) In which school did you do your teaching practice?
Asaroka Lutheran Secondary

IV) In which province/school did you complete your secondary education?
Goroka Secondary School. Goroka Eastern Highlands Province

V) What memories did you have of maths at school?
Challanging but also fun.

Interview questions based on the research questions for **beginningteachers.**

1. **What do beginning teachers and other key stake holders (mentors and teacher educators) perceive are the issues and challenges trainee teachers are faced with?**

1.1 What challenges did you as a trainee teacher face duringyour training period?

Does this mean you were not prepared well?

Communicate with student, esp: in explaining the concept to the student

1.2 What wereyour levels of confidence in the subject knowledge and the knowledge of teaching did you have when you first startedyour training?

My level of confidence in good and my subject knowledge (ie. content knowledge) is at a very good state.

1.3 Did this knowledge change during your training? If yes, how? Not enough time for TP

I mean your knowledge in the area of subject matter and knowledge of teaching, did it get any better?

No, the general idea is still the same. The knowledge is not change though the circulum do change.

BT (1)

1.4 What were you like at the end of your training? What strengths didyou have in your subject knowledge and knowledge of teaching? What challenges were you still faced with?

At the end of training am satisfied. My content knowledge is good but need to adjust on the technique of teaching. The challenge is how to drive the content in to student brain.

Would this have nothing to do with your content and knowledge of teaching?

1.5 What do you think influenced these challenges?

Lack in efficiency to communicate fluent to student in english. However, student cant properly understand english.

2 What mathematical knowledge for teaching do beginning teachers and stake holders say is required by trainee teachers?

You have learnt nothing from TP?

2.1 What knowledge did you have about mathematics at the start of teaching practice?Do you think this was enough knowledge to begin your training?

The knowledge of mathematics are good. Meaning very good with my content knowledge.

2.2 What knowledge did you have at the end of teaching practice?Do you think this is enough for you to be a teacher?

The knowledge is same nothing change, its only the the feel of how to become a mathematic teacher.

2.3 How did your knowledge of mathematics affectyour teaching experiences?

Very Good. I have the content knowledge. I taught in class well and ~~now~~ this reflect my teaching experience.

BT (1)

2.4 Did you encounter any difficulties in offering alternative explanations or representations in your teaching when students were struggling with the mathematics?

Yes; is because student don't understand the simple english.
Yes, I do speak pidgin moreder for them to fully understand.

2.5 How do you think your own personal knowledge of mathematics affected your ability to respond to students' questions?

My knowledge is ok, the student need to understand english Properly first, then they will absorb ~~engli~~ maths.

What made you to be very confident at the end?

2.6 How confident do you think were you to teach mathematics towards the end of teaching practice?

Very confidence. Role models / Supervisors during pre/post conferences.

2.7 As a beginning teacher, what do you think is the kind of knowledge you would want to be developed by a teacher training institution?

How about the knowledge for teaching?

The content knowledge is for most important. I prefer as a begining teacher content knowledge is very important.

Both SMK & PCK
60% 40%

3. How can this knowledge be developed in a workshop for final year mathematics major students in UoG?

3.1 As a beginning teacher, what do you think the teacher training institution should do differently to help develop trainee teachers' knowledge of mathematics and knowledge of how to teach mathematics?

Student must be thought with content. That content knowledge will be driven into student brain.
At a year of teaching the teaching aspect of a teacher, and rest must be the content knowledge.

RT (1)

3.2 What might the institution do to help develop trainee teachers' confidence to teach mathematics?

Do thorough peer teaching, with continuously do lesson observation so they can analyses how they would teach when they they are out in the field.

BT (2)

Introductory questions to the beginning teacher.

I) Name: YAME DICK (MR)

II) For how many years have you taken teacher education courses? 2 yrs

III) In which school did you do your teaching practice?
Kambubu Adventist S/S

IV) In which province/school did you complete your secondary education?
Goroka secondary school - E.H.P

V) What memories did you have of maths at school?
Learning maths was fun & interesting

Interview questions based on the research questions for **beginningteachers**.

1. **What do beginning teachers and other key stake holders (mentors and teacher educators) perceive are the issues and challenges trainee teachers are faced with?**

1.1 What challenges did you as a trainee teacher face duringyour training period?
- Laking confidence when devising lesson
- Lesson preparations takes more time

1.2 What wereyour levels of confidence in the subject knowledge and the knowledge of teaching did you have when you first startedyour training?
When it comes to subject knowledge / content knowledge, I am very confident, however I have no enough knowlege of teaching / teaching skills to deliver the content to the students.

1.3 Did this knowledge change during your training? If yes, how?
Yes. For teaching skills / technique I was able to develop / gain as time pass through school based supervision

BT (2)

1.4 What were you like at the end of your training? What strengths didyou have in your subject knowledge and knowledge of teaching? What challenges were you still faced with?

I have no problem with subject knowledge but how to deliver that knowledge to the students is little bit a challenge to me.

1.5 What do you think influenced these challenges?

I think the short period (6 weeks) of teaching practice (training) is the factor influences this challenge.

2 What mathematical knowledge for teaching do beginning teachers and stake holders say is required by trainee teachers?

2.1 What knowledge did you have about mathematics at the start of teaching practice?Do you think this was enough knowledge to begin your training?

I think content knowledge of mathematics is okay to teach any level at Secondary Sch.
lacks add in teaching !

2.2 What knowledge did you have at the end of teaching practice?Do you think this is enough for you to be a teacher?

Yes. I have enough mathematical knowledge and am confident to be a Teacher.
Still need more skills.

2.3 How did your knowledge of mathematics affectyour teaching experiences?

Give me confidence to answer any questions /queries from the students
content knowledge

BT (2)

2.4 Did you encounter any difficulties in offering alternative explanations or representations in your teaching when students were struggling with the mathematics?

Same concept explained differently

Yes (No). I always have alternative explanations to help struggling students.

2.5 How do you think your own personal knowledge of mathematics affected your ability to respond to students' questions?

Give me confidence to respond to students' question

2.6 How confident do you think were you to teach mathematics towards the end of teaching practice?

I am very confident. The only problem is knowledge of teaching.

2.7 As a beginning teacher, what do you think is the kind of knowledge you would want to be developed by a teacher training institution?

Teaching traning institution should taught teaching skills as well as allocate more teaching practise periods (6 weeks is not enough)!

3. How can this knowledge be developed in a workshop for final year mathematics major students in UoG?

3.1 As a beginning teacher, what do you think the teacher training institution should do differently to help develop trainee teachers' knowledge of mathematics and knowledge of how to teach mathematics?

For my experience, knowledge of how to teach mathematics is a problem so teacher training institutions should emphasis more on that as well as teachers' knowledge of mathematics should be develop by offering more content courses (maths)!

BT (2)

3.2 What might the institution do to help develop trainee teachers' confidence to teach mathematics?

Offer more content to overcome challenges or to be confident in answering questions as well as have more time for teaching practice

TM (1)

Introductory questionsto the teacher mentor.

I) In which province do you teach? Eastern Highlands.

II) In which institution and for how long did you do your teacher training?
I did my teacher training in Pacific Adventist University for 4 years.

III) For how long have you been teaching in a secondary school?
Eight years.

IV) What have you been teaching and for how long?
I've been teaching Advance Mathematics and General Maths and Chemistry for Grade 11 & 12 and also ~~a~~ grade 9 maths for 8 years.

Interview questions based on the research questions for **teacher mentor in secondary schools.**

1. **What do beginning teachers and other key stake holders (mentors and teacher educators) perceive are the issues and challenges trainee teachers are faced with?**

1.1 What challenges do trainee teachers face during their training period?
The challenge that the training teachers face during their training period is on delivering content; they have the knowledge but how to deliver.

1.2 What are the levels of confidence in the subject knowledge and the knowledge of teaching of trainee teachers when they first start their training?
The trainees level of confidence in ~~teaching~~ subject knowledge generally are all over the place. They are not very confident when they first start.

1.3 Does this knowledge change during their training? If yes, how?
Yes, the knowledge change during their training as they get critiques from their mentor.

1.4 What are they like at the end of their training? What strengths do they have in their subject knowledge and knowledge of teaching? What challenges do they still face?
They generally enjoyed the teaching career. They are confident to face the work force with their experience and knowledge content. The challenge would be the work load and the student teacher ratio imbalance.

TM (1)

1.5 What do you think influences these challenges?

The thing that influence these influx of student and student to teacher ratio imbalance is the Tuition fee free Education (TF

2. What mathematical knowledge for teaching do beginning teachers and stake holders say is required by trainee teachers?

2.1 What knowledge does the typical trainee teacher has about mathematics at the start of teaching practice?Do you think this is enough knowledge to begin their training?

The knowledge that typical trainee teacher have in University is just expending on Mathematic knowledge and some teaching methods to teach. But to teach in an institution, it needs time to gain experience and knows to apply the knowledge ~~learnt~~. and skills.

2.2 What knowledge does the typical trainee teacher has about mathematics at the end of teaching practice?Do you think this is enough knowledge for them to be a teacher?

I think the knowledge that the trainee teacher will have will be enough for that topic/unit taught. He/she need more time and experience on how to deliver the knowledge confidently.

2.3 In your experience, how does trainee teachers' knowledge of mathematics affect their teaching experiences?

The trainees have the knowledge of that particular topic, but to deliver them needs more time and confident.

2.4 Did the trainees encounter any difficulties in terms of having alternative explanations or representations when teaching students?

Trainees do have difficulties of explaining ~~say~~ single question and give alternative.

2.5 Were the traineesable to answer students' questions?

The trainees are able to answer students question but to their level of content.

TM (1)

2.6 How confident is do you think is a typical trainee teacher to teach mathematics towards the end of teaching practice?

I think ~~ate~~ by the time they reach the end of teaching, if they are faithful, they are ~~satisfy~~ satisfy and have confident to do the actual teaching.

2.7 As a teacher mentor, what do you think is the kind of knowledge you would want to be developed by a teacher training institution?

The trainee teacher, before teaching the particular topic/unit must have enough knowledge of that topic and do alot of research. peda 8087

3. How can this knowledge be developed in a workshop for final year mathematics major students in UoG?

3.1 As a teacher mentor, what do you think the teacher training institution should do differently to help develop trainee teachers' knowledge of mathematics and knowledge of how to teach mathematics?

The teachers training institution ~~to~~ should help the students to do alot of practice, starting from high school up and also they should teach different methods of teaching.

3.2 What might the institution do to help develop trainee teachers' confidence to teach mathematics?

The institution should help the ~~student~~ trainee teachers to get as much resource as they can and fill them with as much knowledge.

TM (2)

Introductory questions to the teacher mentor.

I) In which province do you teach? Eastern Highlands

II) In which institution and for how long did you do your teacher training?
6 weeks at Bena Bena Secondary (in 1999) Goroka Sec 6 weeks 2000.

III) For how long have you been teaching in (a) secondary school?
7 years at Goroka Sec, (in total 16 years in secondary schools).

IV) What have you been teaching and for how long?
ADVANCED MATH & PHYSICS, 16 years.

Interview questions based on the research questions for **teacher mentor in secondary schools**.

1. **What do beginning teachers and other key stake holders (mentors and teacher educators) perceive are the issues and challenges trainee teachers are faced with?**

1.1 What challenges do trainee teachers face during their training period? Is this an indication that they are not prepared well?
Adaptation - trying to adapt to the new challenging environment physically and psychologically.

1.2 What are the levels of confidence in the subject knowledge and the knowledge of teaching of trainee teachers when they first start their training?
Levels of confidence depend on the trainee teachers but generally very low, with respect to first contact
- Subject knowledge depends on individuals how much they have acquired. Knowledge of teaching will be very low yet to be explored

1.3 Does this knowledge change during their training? If (yes), how?
experience does plays a major role as Spring board.

1.4 What are they like at the end of their training? What strengths do they have in their subject knowledge and knowledge of teaching? What challenges do they still face?
End of training confidence builds up, subject knowledge triggers much concentration and review of teaching notes (i.e from UOG etc) Knowledge of teachings picks up

TM (2)

1.5 What do you think influences these challenges?

practical aspect/reality of actually teaching in a classroom setting and experiencing reality.

2. What mathematical knowledge for teaching do beginning teachers and stake holders say is required by trainee teachers?

Depends on the kind of stds. Is that enough knowledge??

2.1 What knowledge does the typical trainee teacher has about mathematics at the start of teaching practice?Do you think this is enough knowledge to begin their training? (Low)

The contents of respective mathematical syllabus that provide teacher acquired during his/her studies at University/college etc.

2.2 What knowledge does the typical trainee teacher has about mathematics at the end of teaching practice?Do you think this is enough knowledge for them to be a teacher?

Teaching practice is a parameter that measures & makes evaluation on one aspect of a teacher and that is how he/she mastered

2.3 In your experience, how does trainee teachers' knowledge of mathematics affect their teaching experiences?

contents based on the math syllabus or not

If one is not really equipped/ or mastered the contents of various maths topics in the syllabus, he/she lacks (DO) confidence in competence

2.4 Did the trainees encounter any difficulties in terms of having alternative explanations or representations when teaching students?

yes at some degree when dealing with abstract concept that cannot be presented physically eg- Calculus concept & vectors.

2.5 Were the traineesable to answer students' questions?

yes, depending on the nature and degree of projected questions.

TM (V)

2.6 How confident is do you think is a typical trainee teacher to teach mathematics towards the end of teaching practice?

That depends entirely on the trainee teacher him/her self and also on the number of contact periods/hours they have been teaching.

What about content and knowledge for teaching? ⇧ Representations of maths to make it real!

2.7 As a teacher mentor, what do you think is the kind of knowledge you would want to be developed by a teacher training institution?

Personal beliefs...

~~PCK~~ Philosophies of psychology SMK & PCK mainly focussing on cognitive development — child centered as well as teacher trainee personal development — strategic teaching.

3. How can this knowledge be developed in a workshop for final year mathematics major students in UoG?

3.1 As a teacher mentor, what do you think the teacher training institution should do differently to help develop trainee teachers' knowledge of mathematics and knowledge of how to teach mathematics?

Since mathematics is abstract in nature theoretically, as much as possible, practical application with alot of examples (i.e material wise) should be taught/practice to be implemented.

3.2 What might the institution do to help develop trainee teachers' confidence to teach mathematics?

Practise alot, as the saying goes, practise makes perfect. Confidence level only builds up when you do alot of presentation.

T Eductor...

Introductory questions to the teacher educator.

I) In which teacher training institution are you engaged?

University of Gorowa

II) What is your own experience as a teacher educator?

(i) Taught Gr 7 – Gr 12 with trainees

(ii) Taught Undergraduate Mathematics Courses.

III) What do you teach as a teacher educator?

Mathematics

/Edu.

IV) For how long have you worked as a teacher educator?

30 years.

Interview questions based on the research questions for **teacher educator in a teacher training institution.**

1. **What do beginning teachers and other key stake holders (mentors and teacher educators) perceive are the issues and challenges trainee teachers are faced with?**

1.1 What challenges do trainee teachers face during their training period?

(i) Understanding school mathematics Concepts

(ii) presenting & teaching Concepts.

1.2 What are the levels of confidence in the subject knowledge and the knowledge of teaching of trainee teachers when they first start their training?

level of Confidence is minimum for difficult Topics.

TE

1.3 Does this knowledge change during their training? If yes, how?

and effective supervision by school based supervisors.

Yes. the more students discuss these topics in class they Improve

1.4 What are they like at the end of their training? What strengths do they have in their subject knowledge and knowledge of teaching? What challenges do they still face?

once students are guided well and corrections are made on the spot during peer teaching it helps them to build a better understanding

new to class

* 1.5 What do you think influences these challenges? ← lack of exposure to sec. sch syllabus

(i) Exposure to all mathematics topics

(ii) solving every questions in the syllabus/ text book w/sheet

2. **What mathematical knowledge for teaching do beginning teachers and stake holders say is required by trainee teachers?** A good solid Content mathematics background.

* 2.1 What knowledge does the typical trainee teacher have about mathematics at the start of their teaching practice?Do you think this is enough knowledge to begin their training?

Knowledge not enough to begin teaching

students who begin their courses lack the Content in mathematics.

* 2.2 What knowledge does the typical trainee teacher has about mathematics at the end of their teaching practice?Do you think this is enough knowledge for them to be a teacher?

knowledge of school mathematics including that in the textbook (exercises)

A typical trainee teacher must solve all questions and exercises at the grade given to teach

2.3 In your experience, how does trainee teachers' knowledge of mathematics affect their teaching experiences?

A good Content in mathematics and being able to solve all mathematics questions has a great impact on delivering of maths

TE

2.4 Did the trainees encounter any difficulties in terms of constructing alternative explanations or representations when teaching students?

Trainee teachers initially find it difficult with explanations and make grammatical & maths error

2.5 Were the traineesable to answer students' questions?

Yes Some not direct at most times

*2.6 How confident do you think a typical trainee teacher is to teach mathematics towards the end of their teaching practice?

I Think trainee teachers must be confident at the end of TP experience

2.7 As a teacher educator, what do you think is the kind of knowledge you would want to be developed by a teacher training institution?

Theoretical Concepts must be Complemented with experiments and practical

3. **How can this knowledge be developed in a workshop for final year mathematics major students in UoG?** Display the concepts & theory on a cartridge paper or on Power point and have a concrete example displayed

3.1 As a teacher educator what do you think the teacher training institution should do differently to help develop trainee teachers' knowledge of mathematics and knowledge of how to teach mathematics?

Re-introduce practical mathematics. Students must participate in experiments and write reports. More peer teaching must emphasized

3.2 What might the institution do to help develop trainee teachers' confidence to teach mathematics?

Create a secondary ~~School~~ for this University. So lecturer can be done in the morning and practical teaching can be done every year. (2nd year to 4th year)

Eg. DFL here at UOG is now running classes for Matriculation.

Maths Trainee teachers can become tutors or assistant teachers from their second year till their final year. The clinical approach. These teachers will be closely monitored and supervised.