INCOME INEQUALITY AND CONSUMER MARKETS

BABAK SOMEKH
KEBLE COLLEGE
UNIVERSITY OF OXFORD

A thesis submitted for the degree of Doctor of Philosophy at the University of Oxford

For baba and mamman, you lit the candle.

Income Inequality and Consumer Markets

Babak Somekh
Keble College, University of Oxford
Doctor of Philosophy in Economics
Hillary Term 2012
Supervisor: Sir Tony Atkinson

Thesis Abstract

This thesis consists of three chapters that analyze theoretically the role of income inequality in consumer markets. Each chapter introduces distributional considerations into an economic model where previously inequality did not play a major role.

Chapter one uses a consumer search model to show under what conditions the distribution of income within a community is related to the type of firms that exist within that community, impacting the level of prices. We show that if time and money costs of search are high enough, only the middle class have incentive to search and therefore are the most aggressive shoppers. Using a supply side model, we argue that firms located in more informed communities are more likely to enter the market as large low-priced retailers. Connecting these two results, the model shows under what conditions the size of the middle class can have a negative relationship with the level of prices.

Chapter two demonstrates how firm pricing strategy and determinants of household location can interact to determine city structure. In this city, consumers and firms live on a continuous line interval. The model consists of two types of firms; many high-cost perfectly competitive firms located in the Central Business District, and one large low-cost "Superstore", choosing its price strategically. We show how the shopping habits of the consumer population, as determined by the relative price of the Superstore and the Corner Stores, can contribute to the various income segregation outcomes described in previous literature. In addition we consider the impact of city population structure on the pricing decision of a monopolist facing a competitive fringe.

Chapter three uses a simple model of banking services to consider how deposit-taking banks price for their services and choose the type of deposit customers that they target. This chapter goes beyond previous theoretical work on consumer banking, identifying the role of household income in the access to deposit services. We show that a higher rate of return on investments available to banks lowers financial exclusion, increasing the profitability of low-income consumers for deposit-taking institutions. This suggests that the possibility of financial exclusion increases in periods of recession. The chapter demonstrates how an increase in income dispersion can lead to a greater proportion of consumers excluded from mainstream banking.

ACKNOWLEDGMENTS

I owe an amazing amount of gratitude to my supervisor, Professor Tony Atkinson. His generosity and wisdom have been inspiring. I will carry his lessons with me throughout my professional career.

I would like to thank my examiners, Dr. John Thanassoulis and Professor Frank Cowell, for their challenging and clear-sighted feedback in regards to this thesis. I would also like to thank them for helpful guidance as I begin a new phase in my professional life.

I am grateful to John Quah and Mikhail Drugov for their insightful comments. A special thanks to Howard Smith for his help throughout my time at Oxford. I would also like to thank Antoni Chawluk for his friendship and academic guidance. Thank you to the moderators John Muellbauer and Meg Meyer as well as the participants in the Gorman workshop for their thoughtful feedback. Financial support from the Department of Economics and Keble College was very much appreciated.

Christina Wang from the Federal Reserve Bank of Boston provided helpful advice and feedback in relation to the third chapter of this thesis. Andrew Rhodes was very kind to provide comments and advice in relation to the first chapter. Rebecca Lane showed great patience and generosity by thoughtfully editing this entire thesis. I am indebted to my fellow students at the Department of Economics. Some of them have been on the journey with me from the start, others have joined along the way. In particular I would like to thank my good friends Mauro Caselli, Paolo Falco, Marta Troya Martinez, Gaston Yalonetzky, Martina Kirchberger, Salvatore Morelli, Ferdinando Giugliano and Charles Brendon.

Outside of the Department of Economics a wonderful group of friends in Oxford, London and New York have helped distract me in my time of need. A heartfelt thank you to my close friends Vincent, Mikey, Christian, Irene, Caterina, Igor, Julian, Shai, Fizzah, Jeev, Supriya, Martina and Dave.

My family, Bahareh, Aman, Sogol, Ariel and Mom, have been like a rock throughout my life. They have shown great patience with me, fully supporting my venture back into academia as well as my move away from New York. I would also like to thank my godparents, Doug and Nelda Lay. Doug inspired my decision to return to academia.

Lastly, I would like to thank Sian for being an amazingly supportive and loving girlfriend. Her kindness and wonderful sense of humor make life a great adventure.

Word Count

This thesis contains 156 pages (not including Bibliography while including Preface), with a typical page containing around 300 words, making the length of the document approximately 47,000 words (including footnotes).

CONTENTS

LIST OF FIGURES

PREFACE

The impact of income distribution on market outcomes has been a focus of economic research since the middle of the 20th century. Alonso (1964) considers the role of income distribution in outcomes in the real estate market. Alonso's study spurred an array of theoretical and empirical papers that look to explain the various agglomeration outcomes we observe in cities around the world. Similarly, in a seminal paper on the economics of information, Stigler (1961), proposes that consumers can have access to different levels of information, leading to price dispersion. This observation by Stigler inspired a whole field of literature focused on understanding the role of information in economics, and how uninformed consumers might struggle to compete in product markets. This thesis builds on this important tradition by connecting decision making by a consumer population heterogenous in income with the strategy of firms facing a diverse customer base. In addition, this thesis goes beyond previous work by more directly considering the impact of changes in the aggregate income distribution on outcomes in retail, real estate and financial services markets.

Each chapter connects different strands of economics when considering the role of income distribution in consumer markets. The first chapter brings together consumer search models inspired by Stigler (1961) with work on time allocation first developed in Becker (1965). The second chapter combines existing work in regional economics with the industrial organization literature. And the third chapter introduces both positive and normative considerations in income distributions into a model for financial services.

In the first chapter, we develop a model for a retail good when information is costly and consumers are heterogenous in income. Using some stylized assumptions about market structure, the model demonstrates two related results. First, that when there exists time and money costs of search, the middle class are the most aggressive shoppers. And second, that search intensity in a particular market determines the type of firms that choose to compete in that market. By connecting these two results, this model shows that a higher proportion of middle-income households leads to more competitive firms operating in the market, leading to lower prices. In this setup, the distribution of income can have a direct impact on the level of prices. Frankel and Gould (2001) have shown empirically that there exists a significant negative relationship between the size of the middle class and consumer prices. But their findings do not explain why such a relationship exists. This model looks to better understand the mechanisms that might lead to the middle class driving down prices. There is no consensus in the economic literature on whether or not the poor pay higher prices. This chapter demonstrate why it might just be the case that the middle class pay less.

Chapter two builds on the results in the first chapter by viewing the issue from a spatial perspective, rather than due to information asymmetries. This chapter considers how the pricing decision of firms interact with household location and shopping decisions, to determine city structure. The analysis adds to the literature on regional economics by considering shopping as a factor in a household's choice of location. How individuals choose where to live is a complicated question - driven by a variety of factors, some of which are rational and economic, while others are more personal. The rational set of incentives might include availability of quality education, access to job centers or to local amenities, as well as availability of public services and the neighborhood crime rate. Some of these factors can be interpreted as consequences of segregation, like the crime rate at a city center. While others can been used to explain segregation. This chapter contributes to the above discussion on the determinants of city structure in two ways. First the model looks to endogenize a cause of household location that goes beyond commuting costs by including access to affordable shopping in the consumers' decision process. And second, by allowing the large discount store to behave strategically, the analysis takes into account the firm's side of the problem. This more general framework helps better identify the cause and effect of city structure in relation to income. The results of the model show that a fully integrated city with the existence of a large discount store leads to the lowest costs of living for all consumers. The chapter also demonstrates that higher income consumers are better able to take advantage of the lower price offered by the discount store. The entry of a discount store in a city with spatial constraints leaves high-income consumers relatively better off compared to low-income households.

The third chapter introduces an income distribution into a model for financial services. To our knowledge this is the first instance of an economic model of retail banking services that takes into account the income distribution of the consumer population. Recent studies in the U.S. and U.K. have found that access to mainstream banking services, such as deposit accounts, is limited for a significant portion of the population. As these types of services play a greater role in the participation of consumers in the overall economy, it is becoming increasingly costly for those households that are left out of the financial services sector. Using an income distribution within a model for banking services helps focus specifically on what type of consumers are excluded from this market. The model in this chapter looks to identify how mainstream banks charge for deposit accounts and the customers they target. The chapter also looks at what happens to the consumers that are left out of the mainstream banking sector, and the costs they face when they are forced to turn to Alternative Financial Services. The results of the model show that exclusion from mainstream banking decreases with the rate of return that banks earn on deposits. This would suggest that exclusion increases during times of recession. This model does not consider specifically what drives the rate of return available to banks. The recent financial crisis has clearly demonstrated that high rates of return can be due to banks taking on a greater risk profile, exposing customer deposits to greater financial risk. This chapter demonstrates that it is important to consider both access as well as safeguarding deposits, in choosing to limit the returns available to banks.

CHAPTER 1

THE EFFECT OF INCOME INEQUALITY ON PRICE DISPERSION

Abstract

This chapter uses a consumer model with search to show under what conditions the distribution of income within a community is related to the types of firms that exist within that community, impacting the level of prices. Using a consumer model, we show that if time and money costs of search are high enough, only the middle class have the right cost and benefit balance and therefore, are the most aggressive shoppers. Using a supply side model, we argue that firms located in more informed communities are more likely to enter the market as large low-priced retailers. By connecting these two results, the model shows under what conditions the size of the middle class can have a negative relationship with the level of prices in a local market.

1.1 Introduction

Equal opportunity is one of the foundations of democratic economies. All individuals, no matter their race, gender, or economic background, are meant to be afforded the same opportunity to succeed. In general this promise is thought of in the context of access to education and the labor market. Also important is access to consumer markets. In this chapter we construct a theoretical model that describes why low-income families might have a hard time competing in consumer markets. Given certain assumptions about market structure, we are able to demonstrate two important and connected results. First we show that when there exists a time and money cost of searching for the lowest price, the middle class have the optimum balance of cost and benefit of search, and therefore search the most intensely. Second, we connect consumer search intensity in a particular market with the types of firms that choose to compete in that market. By connecting these two results, we show that a higher proportion of middle-income households leads to more competitive firms operating in that market, leading to lower prices. The main implication of our results is that separate is not necessarily equal, income segregation can result in less competition in the segregated market, leading to higher prices for low-income families.

Whether or not the poor face higher prices in consumer markets has been the subject of extensive debate. There have been various studies done at all levels and in many countries with inconsistent results. These studies sought to connect level of income with prices paid for consumer products such as food, transportation, housing and big ticket items. Though there are certain instances where the data supports the idea of the poor paying more, there are also examples of markets where there is no evidence of price dispersion due to income. An empirical paper by Frankel and Gould (2001) offers an alternative explanation for the instances where higher prices are observed for the poor. Using a search cost function U-shaped in income, they argue that the middle class search the most intensely for the lowest price, and therefore their presence in a given market leads to lower prices.

In this paper we construct a theoretical framework to explain the relationship between the size of the middle class and market prices. First we consider some of the relevant research that has gone into studying the idea of the poor paying more. Next we begin constructing our model by setting up the consumer's problem within a model that incorporates time usage and search behavior. Consumers have the option to pay a time and monetary cost to obtain the lowest price, or randomly choose a store in the market. By comparing the utility of consumption obtained from searching versus not searching across consumers with varying levels of income, we look to determine whether or not the middle class have the highest incentive to search.

Then we consider the idea of price dispersion from the point of the view of the firm. There have been a number of theoretical papers that try to explain the presence of price dispersion in consumer products. We use a version of a model originally developed by Salop and Stiglitz (1977), where price dispersion is a result of varying levels of informed consumers in a market. We show that when the proportion of the uninformed consumer population increases, smaller, higher priced firms enter the market, leading to higher average price.

Finally, we combine these two results to show that as the size of the middle class in a market increases, the portion of the perfectly informed consumers increases, and therefore prices decrease. Our results match previous findings in theoretical and empirical search models where even small amounts of search costs can lead to an outcome where no consumers search and all firms charge the maximum price (see for example Diamond (1971) and Baye et al. (2006)). We extend this result to show that as search costs become small enough relative to the extent of price dispersion only a portion of consumers search, and this portion consists of the middle class. We show that as long as search costs exist the very rich and very poor never search, therefore they depend on the middle class to keep markets competitive.

In our model there are some key assumptions about the time and money cost of search, as well as market structure, that lead to the results described above. Future research would need to consider characteristics of specific markets and households to determine if the assumptions we have made are consistent with empirical data. If it can be shown that the poor do not search as intensely as the middle class because of time and cost constraints, then policy should be directed at alleviating these costs and allowing more information to flow to the lower income households. On the other hand, if the problem is shown to be behavioral, that the poor choose not to search for the lowest price because of a lack of motivation or desire, then the problem becomes much more complicated, and would require a deeper understanding of how the poor interact in the economy.

Now we will motivate the assumptions of our theoretical model by discussing the relevant literature.

1.2 Empirical Background

When the clamor that "The Poor Pay More", Caplovitz (1963), was first heard in the U.S. in the midst of the racial riots in the 1960s it was greeted as one of paranoia. As economists began to study the issue it became clear that there was more to the assertion than first expected. What also became clear was that this question was a complicated one and would not be answered easily. In this section we survey research focused on determining if the poor do in fact pay more. As we would expect, the issue is one of both supply and demand. The makeup of a consumer population and the amount of information available to consumers determines the characteristics of consumer demand. What types of firms enter a market and the cost structure of these firms determine the price strategy of firms in a given market. These two forces working together can help explain the disparity in prices that triggered the original uproar. What started as a racial and sociological discussion has evolved into a more analytical and economic question.

There are several quantifiable parameters that are generally discussed when trying to determine why the poor face higher prices. First, there is the issue of the lack of availability of large discount stores in predominantly poor neighborhoods, or what is called the "store effect", Kunreuther (1973). It is proposed that prices at smaller groceries are higher than large supermarkets and discount stores. These larger stores are said to have lower per unit fixed costs and have higher purchasing power due to their ability to buy in bulk. Fixed costs are said to be even higher for stores in poor urban neighborhoods due to higher crime rates, which tend to push up insurance and maintenance costs. Next, it is proposed that goods sold in larger packages have lower per unit costs than the same goods in smaller sized packages, or what is called the "size effect". Smaller groceries, due to lack of shelf space, do not provide larger sized packages, adding to the higher per unit costs of shopping at these stores relative to supermarkets. A sub theory of the size effect is that lower income families have limited liquidity and limited storage space, especially freezer and refrigerator space, not allowing them to purchase bulk items [See Attanasio and Frayne (2006), and Rao (2000)]. Finally, it is argued that low-income families find it too expensive to search for and access the lowest priced firms, Clifton (2004). In general, low-income families are less likely to own a car or have access to the internet, forcing them to rely on public transportation to perform comparative shopping, which can be prohibitively time consuming and expensive.

The book by Caplovitz (1963) was one of the first attempts at linking income level to prices in a specific retail market. He found that the poor paid significantly more for major durables such as televisions and washing machines than the average consumer. In 1971, a study into food prices in New York City found no relation between food prices and neighborhood income level, Alcaly and Klevorick (1971). In 1974 an extensive study of consumer markets in the U.K. concluded that in general the poor seemed to face higher prices, and by all accounts never faced lower prices than other income classes, Piachaud (1974). Alcaly and Klevorick and Piachaud admit that their findings have weaknesses, mainly due to lack of detailed data. Fortunately it seems the information available for analysis has improved since Piachaud conducted his study. Through the advent of scanner data, computerized operations and the initiation of government sponsored surveys, researchers have access to better analytical tools. Nonetheless, the lack of consensus on the issue remains.

In 1991, New York's Consumer Affairs Department investigated grocery store price-fixing in several neighborhoods. Their survey of sixty stores and 140 interviews throughout New York found that the poor paid more for groceries in urban areas while receiving lower quality service, Freedman (1991). Further regional studies in Pittsburgh, Austin, and Minneapolis found that there was significant evidence that the poor paid higher retail food prices than the average consumer [See Dalton et al. (2003), Clifton (2004), and Chung and Myers (1999)]. Similarly a 2007 survey of the U.K. found that, in general, low-income families faced higher prices in financial services, utilities, telecommunications and durable goods purchases, Kober and Sterlitz (2007). These tests arrived at their conclusions from different perspectives. Some attributed the discrepancy in prices to the lack of supermarkets in poor neighborhoods, or the store effect. While others cited inability to buy in bulk due to either limited budgets or lack of choice. Another group found that poor households paid more because of higher absolute prices in stores located in low-income areas.

On the other hand, a 1991 study of ten regions using data on 322 retail supermarkets found no statistically significant evidence that consumers in low-income neighborhoods paid higher food prices than consumers in high-income areas, MacDonald and Nelson (1991). In 2000, a study using unpublished CPI data on prices paid by consumers across the United States found no evidence of higher prices faced by the poor, and in fact found that in some cases the poor faced prices up to 6% lower than the average, Hayes (2000).

There are also contradictory findings in developing economies. A study of two rural towns in India found the poor paid significantly higher prices for food products, due mainly to lack of storage space and access to credit, Rao (2000). A similar study of 122 medium sized towns in Colombia found that prices decreased with bulk purchases, leading to lower income families paying higher prices due to lack of capital, Attanasio and Frayne (2006). Both of these studies found some level of coping amongst poor families through communal purchases, though limited to very specific consumer goods. A study of 256 households in rural Rwanda, using detailed consumption data collected over the course of 2 years in the early 1980s, found a significant negative relationship between the standard of living of a household and the price index faced by the household, Muller (2002). In contrast, a study of consumer prices in Brazil found little evidence of a relationship between income and prices faced. In fact the study found some evidence of a positive relationship between household income and food prices, Musgrove and Galindo (1988).

There have also been studies performed to determine if it is true that the poor do not have access to discount firms. An extensive study into the shopping habits of consumers in the U.K. found that an increasing number of large retailers are moving from town centers to out-of-town locations, Piachaud and Webb (1996). Through a survey of consumer shopping habits, the study finds that low-income families with limited mobility are left with no choice other than to shop at high-priced local stores. In another study, the concept of "Food Deserts" in the UK, or lack of access to cheap and healthy food for low-income families, has been shown to be an increasingly common issue in cities across the country, Wrigley (2002). In the U.S., research in the late 1990s found that the number of supermarkets has dropped by 22% from 1966 to 1993, mainly due to a major consolidation in the industry[1]. The study found that supermarkets and food retailers are not as prevalent in low-income neighborhoods[2], low-income areas in nineteen U.S. cities had 30% fewer stores per capita compared to higher income areas. A regional study of Allegheny County, which contains the city of Pittsburgh, found similar results, Dalton et al. (2003). The study found that large supermarkets were on average a lot more accessible in middle to high-income suburban areas of the region. A Minneapolis study found that there was a significant positive relationship between income level and the number of supermarkets in a neighborhood, Chung and Myers (1999). The city of New York conducted a city wide study into the location of supermarkets. The study found that there was a significant shortage of supermarkets in the City. Lack of access to cheap and healthy food was found to be especially stark in low-income and minority neighborhoods, Gonzalez (2008).

If we accept that supermarkets are less likely to be located in poor areas, then it is important to determine if it is possible for poor families to commute to other neighborhoods to do their shopping. There are two conflicting factors affecting low-income families' ability to shop around and look beyond their own neighborhood stores. On the one hand, the poor are said to have a lower cost of time, which means it is more

[1] Economic Research Service, U.S. Department of Agriculture.

[2] Definition of what is considered a low-income neighborhood varies by research, but in general depends on data taken from the U.S. Census, and ranges from 20% to 30% of a neighborhood's population falling below the U.S. poverty line.

economic for them to comparison shop than wealthier consumers. At the same time, the poor on average tend to rely more heavily on public transportation, making it a lot harder for them to commute to the areas where lower prices might exist. Studies into the commuting habits of low-income families have found that most do not have direct access to their own cars, but are able to compensate through alternative methods of transportation, Dalton et al. (2003). A similar study in Austin, Texas, found that poor consumers are not necessarily confined to shopping within their own neighborhoods, and through the use of alternative methods of transportation are able to gain access to discount stores and supermarkets, Clifton (2004). But this access comes with a significant cost in time and money spent on transportation, which is not always economical. As a result, consumers rely heavily on local stores for their shopping needs.

Frankel and Gould (2001) attempt to overcome these conflicting findings by approaching the question from a new perspective. Using data on over 180 U.S. cities, they attempt to link the income distribution of a city with retail prices within that city. Assuming that search costs are U-shaped in income, that is, the rich have a high cost of time and the poor do not have the means, they argue that the presence of middle class consumers in a market leads to lower prices. They construct a basket of consumer goods in determining prices, including food, transportation, healthcare and other day-to-day products. After controlling for exogenous variables such as crime rate and real estate prices, they find a significant relationship between the size of the middle class and consumer prices. The study finds that with a transition of 1% from the middle-income group to the low-income group in the population, prices increased by about 0.7%, this effect increased to 1.1% when estimating using Instrumental Variables. The impact on prices was found to be slightly less, but still very significant, with transitions to the upper income groups. Either way these results would suggest that changes in the middle-income group have a significant impact on retail prices.

These finding have been supported by more recent empirical research. Blisard and Stewart (2008) use household expenditure data to measure who pays more for food. They find support for Frankel and Gould's findings that as households move from middle class into poor, prices rise. Myers (2011) find similar results using retail gasoline prices from three U.S. cities.

Since Frankel and Gould's 2001 paper there have been research focused on the relationship between income distribution and propensity to search. In an empirical paper considering the relationship between immigrant communities and prices, Lach (2007) argues that low income consumers have lower time costs therefore search more aggressively. Similarly, Fishman and Simhon (2005) demonstrate that lower income groups have more utility to gain from lower prices and are more likely to search. The latter paper considers the market for a particular good where consumers only buy one unit of the good and the utility function includes a continuous amount of a numeraire good. The former considers data for an immigrant community in Israel that initially had a higher rate of unemployment than the rest of the population, and therefore had more free time to conduct search. Both of these papers demonstrate alternative specifications and assumptions that impact the relationship between income and propensity to search.

Unfortunately, although there has been a significant amount of work done on competition and consumer search, there has not been much work done to quantify the time and monetary costs of search for consumers. Hong and Shum (2006) estimate a distribution of search costs for a population using only data on price dispersion for a particular good. This takes us in a positive direction in understanding search costs, but it does not say anything specific about the value of the time and monetary costs of search and how they vary with income. Finally, in a survey of the literature on consumer search and pricing, Ratchford (2009) concludes by admitting that not enough research has been done on measuring search costs.

In our model below we look to provide a theoretical framework for the results in Frankel and Gould (2001) and leave it to future work to determine if our assumptions on the nature of search costs are representative of the true relationship between income and the costs of search for consumers. Though Frankel and Gould clearly demonstrate that there is a negative relationship between the size of the middle class and consumer prices, their results do not tell us much about why this relationship exists. The purpose of our paper is to construct a theoretical model that will help understand why the poor cannot drive down prices without the help of the middle class.

We begin constructing our model by looking at the choice of search from the perspective of the consumer.

1.3 Demand Side

In this section we look to analyze how consumers decide whether or not to search for the lowest price. Taking the behavior of firms as given, we construct the consumer's problem using a model of time allocation, first formalized in Gary Becker's "A Theory of The Allocation of Time" (1965). Becker argues that consumers spend their time either working or consuming; therefore there is a trade-off between choosing to work and dedicating your time to any other activity. This seems a natural vehicle for our analysis, as consumers must decide if it is worth their time to search for the lowest price. In our model we assume a binary search rule, either consumers search or they do not. Clearly the real world is not so simple, most consumers perform some level of search when shopping for a good. We can interpret the binary rule used in our model in the same way as if we had used a search continuum with varying levels of search. If our analysis shows that only the middle class have the incentive to search, then we can say they search the most intensely. Then by making a link between search intensity in a market and lower prices we can argue that the presence of middle-income consumers in a market leads to firms charging a lower price, the results captured in Frankel and Gould's analysis.

In this section, using some intuitive assumptions on the costs and benefit of search we determine cutoff levels of income where search occurs with the following results:

(1) If price dispersion exists and is significant, and the cost of search is not too high, consumers falling in a middle range of income will search for the lowest price.

(2) Given any positive level of fixed cost associated with search, the very poor never have an incentive to search for the lowest price.

We consider a consumer model where the number of firms and consumers are assumed to be significantly large. Taking firm pricing behavior as given, consumers determine whether to search for the lowest priced firm or to randomly choose a firm. Consumers make the decision to search period by period; we do not consider a multiple period benefit to search in our model.

The Model

There are two types of **firms**, large low-priced superstores (ℓ-types) and small high-priced stores (h-types). A proportion of firms, $\beta \in [0,1]$, choose to be ℓ-types and charge the low price, $p_\ell = p_{\min}$, while $(1 - \beta)$ choose to be h-types and charge a high price $p_h = r$, where r is the consumers' **reservation price** above which they do not purchase the good. We can think of the reservation price as determined by their outside option, an option for the consumer away from the common market. This could be thought of as eating at home versus going out for dinner, growing vegetables in a garden versus purchasing from a grocery, or choosing to walk rather than buying a car. If a consumer encounters a firm that is charging a price above their reservation price they leave the market and consume their outside option. In this section consumers take the makeup of firms in the economy, β, as given and treat it as fixed. We will discuss the problem of the firms, and how β is determined, in more depth in our discussion of the supply side below.

All firms are assumed to be equally accessible by all consumers, therefore the average price across all firms in the market is given by[3]:

$$\bar{p} = \beta p_\ell + (1 - \beta) p_h \tag{1.1}$$

Consumers have varying levels of income, and face the choice of either purchasing the good from a randomly chosen firm, or searching for the lowest priced seller. There are two types of cost associated with searching. The first is a monetary cost that involves the cost of gathering information remotely or actually visiting each store to compare prices. We can think of this component as associated with the cost of a computer and a web connection in order to search the internet for price comparisons, the cost of a magazine subscription such as "Consumer Reports" in the U.S. or "Which" in the U.K.,

[3] We do not consider frictions related to spatial access in this model. We consider spatial access in a market for a retail good in Chapter Two of this thesis

or the cost of a car or other mode of transportation for going out to visit different stores. There is also a variable component such as the cost of gasoline or public transportation. We will treat all of these costs as fixed and refer to them as c.

The second type of cost is the opportunity cost of searching. Here we use a concept first formalized in Becker (1965). The idea is that the time spent searching by a consumer takes away from time available for working or consuming other goods. We refer to this time cost as s. As in the Becker model, the opportunity cost of the time spent searching is the foregone wage that the consumer could have earned working, w. Therefore, the time cost of searching for the lowest price is sw. The existence of a wage dependent time cost of search in effect creates a continuum of search costs across our consumer population as determined by the wage distribution. This is a different setup from previous work on price dispersion where search costs tend to be purely monetary and consumers fall into a discrete number of search cost levels. We will introduce an income distribution below so that we can more fully explore this additional dimension of our model.

Consumers will weigh the benefits and costs of search and will either search for the lowest priced firm or randomly choose a firm and purchase from that firm as long as price is below their reservation price.

Assumptions

In our model we make some simplifying assumptions that seem intuitive and help us focus on answering the question of how income affects search behavior.

(i) Consumers are risk neutral. This lets us focus on the tradeoff between the cost and benefit of search. Any degree of risk aversion would increase the incentive to search.

(ii) The monetary and time costs of search, c and s, are binary. Either a consumer searches and pays the minimum price, p_ℓ, or they do not search and randomly choose a firm. This is the "clearinghouse" approach used in Salop and Stiglitz (1977), Varian (1980) and Carlin (2009).

(iii) c and s are independent of income. We make this assumption to simplify our analysis, but in fact there is some evidence to suggest that the poor might face higher prices for transportation or big ticket items [See Clifton (2004) and Caplovitz (1963)]. This would suggest that the cost of search decreases with income, which would give the poor even greater disincentive to search. For now we will assume constant costs.

(iv) c and s are independent of the number of low and high-priced firms in the market. We can argue that these costs should go down as the number of low-priced firms increase or decrease, since in the first case there are more of them so it is more likely a consumer will find one, while in the second case they are more unique so more easily identifiable.

(v) Reservation price, r, is independent of income. In reality a consumer's outside option should vary with income. But the sign of variation is ambiguous, since the rich could have more outside options, while the poor need to do more with less. For the case of simplicity we will assume r to be constant.

(vi) Uninformed consumers do not have any information on prices, but their beliefs about the distribution and level of prices must be correct in equilibrium. This means that consumers will optimally choose whether or not to search taking the types of firms in equilibrium as given (in effect a simultaneous Nash equilibrium). This has become a standard assumption in consumer search models, and is a departure from Salop and Stiglitz (1977). In their model they assume that uninformed consumers know the distribution of prices and take into account possible firm deviations in their search strategy. The main difference between our assumptions about information and theirs is that in our model uninformed consumers are not able to observe when firms deviate from any proposed equilibrium strategy[4].

The Consumer's Problem

In this section we present the consumer side of our model and solve for the optimal choice of search depending on income. We solve for the consumer's expected consumption with search and without search and compare the utility achieved in each case.

Without search: If consumers do not search they will randomly choose a store and purchase the good at that store as long as the price is below their reservation price[5]. With probability β they will enter a low-priced store charging p_ℓ, and probability $1 - \beta$ they will choose a high-type charging p_h. We do not have any dynamics in the consumer's problem so once they have entered the store they do not leave and try another store (then

[4] We are not straying too far from S&S in our assumption here. Uninformed consumers knowing the distribution of prices in equilibrium is a strong assumption. Without some knowledge about prices, whether in equilibrium or outside of it, making the choice between searching or not searching would require a sequential search model. For a treatment of how consumers gather information when starting out at a point of zero information on prices see Rob (1985) or Rauh (1997).

[5] Here we are making an assumption about the shopping behavior of uninformed consumers by assuming that they randomly choose a store rather than some other strategy, such as simply going to stores frequented by middle income consumers. In essence we are assuming that consumers do not observe each others' shopping decisions, or alternatively, consumers do not know who is informed and who is uninformed. This assumption might be more appropriate for goods that can be bought online or big ticket items where consumers do not make frequent purchases. It is also possible that this type of uncertainty exists in markets where firms use intermittent sales to compete (see Varian (1980)). We will discuss this more in our discussion of firms' strategy below.

they would in effect be searching). Given this setup the consumer's problem without search can be written as the following probabilistic maximization problem:

$$\begin{aligned} \max_{x} \quad & \mathbb{E}[u(x)] \\ s.t. \quad & p_i x = Lw \quad \text{for } i \in \{\ell, h\} \quad \text{where } P[p_\ell] = \beta, P[p_h] = 1 - \beta \\ & tx = 1 - L \end{aligned} \tag{1.2}$$

$u(x)$ is a strictly monotonic well behaved utility function, dependent only on the consumption of one good, x, this assures us that both our budget constraints are binding. These assumptions on consumer preferences limit the consumer's problem to that of maximizing x, we will adopt this approach for the rest of this section.

The first constraint in (1.2) is the usual budget constraint except that consumers do not know prices and will face a low price with probability β. We do not include any non-wage income, since we are only interested in how changes in wage affects consumer choice. The second line in (1.2) is the time constraint from Becker's model. Consumers have 1 unit of time to allocate across working and all other activities. t is the amount of time needed to consume the good, and L is the amount of time a consumer spends working, earning wage w. The parameter t takes the place of leisure in our model of time allocation[6]. Different types of goods might have different levels of t, impacting the amount of time allocated to search, we will talk more about the impact of t on search below[7]. Note the inherent tradeoff between the different activities

[6]Note that there is no pure leisure in this model. Consumers either spend their time working, searching or consuming.

[7]Note that the number of firms in the market can have an impact on the time of consumption, t, since if there are more firms in a market consumers will have to travel a shorter distance to do their shopping, in effect spending less time consuming. But we believe this to be a second order effect in our results and choose to treat t as fixed for the purpose of our analysis.

in our time constraint. A consumer earning a wage w must choose between spending their time working, L, searching, s, or consuming, tx. Since the two constraints are binding, consumers indirectly choose L, by choosing their level of consumption, x, and by choosing whether or not to search.

We combine the two constraints and solve directly for the expected value of x purchased by uninformed consumers:

$$\mathbb{E}[x_U] \; = \; x_U(w) \; = \; \beta \left[\frac{w}{p_\ell + tw}\right] + (1-\beta)\left[\frac{w}{p_h + tw}\right] \quad \text{where} \quad \beta, t \in (0,1) \tag{1.3}$$

Next we will determine how introducing search will affect the consumer's problem.

With search:

$$\max_x \quad u(x)$$

$$s.t. \quad p_\ell x = Lw - c$$

$$tx = 1 - L - s$$

Where p_ℓ is the minimum price across all firms, c is the fixed monetary cost of search, and s is the amount of time required to search for the lowest price. We solve directly for the level of x purchased by informed consumers:

$$x_I(w) = \frac{w - sw - c}{p_\ell + tw} \qquad \text{where} \quad s \in (0,1)$$

The restriction on the value of the time cost of search, s, assures us that consumers have enough time in a period to both search and consume, with some time left over to work[8]. We need to look at some comparative statics in order to determine at what levels of wage it is profitable for consumers to search. First we consider levels of x for wages close to zero and approaching infinity:

$$x_U(0) = 0 \qquad > \qquad x_I(0) = \frac{-c}{p_\ell} \tag{1.4}$$

$$\lim_{w \to \infty} x_U(w) = \frac{1}{t} \qquad > \qquad \lim_{w \to \infty} x_I(w) = \frac{1-s}{t} \tag{1.5}$$

From inequality (1.4) we can see that for any positive fixed cost of search, $c > 0$, low-income consumers with w close to zero would not choose to search[9]. Clearly if w is low enough a consumer would not have much money left to purchase the good after paying c to obtain the minimum price, therefore they would rather randomly choose a store. Similarly, from (1.5), we have that for any positive time cost of search, $s > 0$, high-income consumers would not search for the lowest price. For high levels of w the money cost of the good becomes irrelevant while time becomes increasingly valuable, so the consumer would not sacrifice any of their valuable time searching for a lower price. In fact, as wage approaches infinity the money cost of the good no longer matters, leaving consumers indifferent between the high and low price[10].

What is left to show is that there is a middle interval of income where it is optimal

[8]It is easy to show that the condition on s is sufficient to assure that $s + tx < 1$ for all consumers.

[9]Since a negative level of consumption is not very realistic, we can see the strict inequality in (1.4) as holding in the limit. That is, taking wage very close to zero, consumers who search would stop searching once their income runs out, leaving them nothing to consume, and consumers that do not search consume a small but positive expected quantity of the good. Alternatively we can see the negative level of consumption from search as the theoretical option for a consumer with no income that they would never choose to take.

[10]Note that since in our model consumption requires time, high-income consumers would never purchase more of the good than possible under the time constraint. Therefore, the ceiling for consumption of the good x when wage goes to infinity is given by $\frac{1}{t}$ where the amount of time spent working goes to zero as wage rises.

for consumers to search. We begin by looking at the first and second derivatives of our individual demand functions with respect to income:

$$x_U' = \frac{\beta p_\ell}{(p_\ell+tw)^2} + \frac{(1-\beta)p_h}{(p_h+tw)^2} \quad > \ 0 \qquad x_I' = \frac{(1-s)p_\ell+ct}{(p_\ell+tw)^2} \quad > \ 0 \tag{1.6}$$

$$x_U'' = \frac{-2t\beta p_\ell}{(p_\ell+tw)^3} + \frac{-2t(1-\beta)p_h}{(p_h+tw)^3} \ < \ 0 \qquad x_I'' = \frac{-2t[p_\ell(1-s)+ct]}{(p_\ell+tw)^3} \ < \ 0 \tag{1.7}$$

The first derivatives in equations (1.6) are positive, therefore both our bundles are increasing functions of income. The second derivatives in (1.7) are negative, therefore they are both concave in income, and since w is in the denominator of both functions in (1.7), the concavity of the two functions decreases with w, approaching zero concavity from below.

As we showed in inequalities (1.4) and (1.5) above the amount of x consumed from not searching $x_U(w)$, is higher for wages close to zero and wages approaching infinity. For search to be optimal for at least some consumers, there must exist a middle range of income, $w \in [w_1, w_2]$, where $x_I(w) > x_U(w)$.

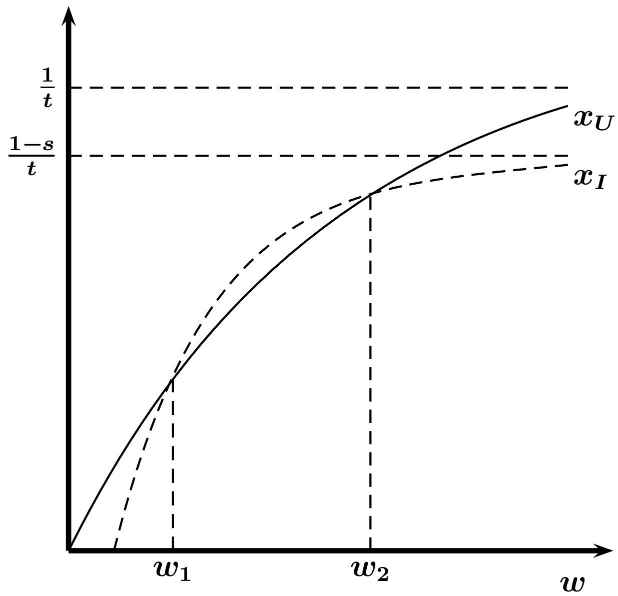

Figure 1.1: Demand as a Function of Income

We need to show under what conditions such a range of income exists. To calculate the values of w_1 and w_2, we solve for the roots of the following equation:

$$x_U(w) - x_I(w) = 0 \quad \Rightarrow \quad \beta\left[\frac{w}{p_\ell + tw}\right] + (1-\beta)\left[\frac{w}{p_h + tw}\right] - \frac{w - sw - c}{p_\ell + tw} = 0$$

$$\Rightarrow \qquad \frac{stw^2 + [sp_h + ct - (1-\beta)(p_h - p_\ell)]w + cp_h}{(p_\ell + tw)(p_h + tw)} = 0$$

The denominator of the above fraction is always positive, therefore we only need to consider the conditions under which the numerator takes negative values. Define:

$$f(w) = stw^2 + [sp_h - (1-\beta)(p_h - p_\ell) + ct]w + cp_h \tag{1.8}$$

Lemma 1.1: *The necessary and sufficient condition for there to exist a middle range* *of income,* $[w_1, w_2]$*, such that consumers within that range of income benefit from search, i.e.* $x_U(w) - x_I(w) < 0 \quad \forall \quad w \in [w_1, w_2]$*, is that the roots of the quadratic function* $f(w)$ *are positive and real.*

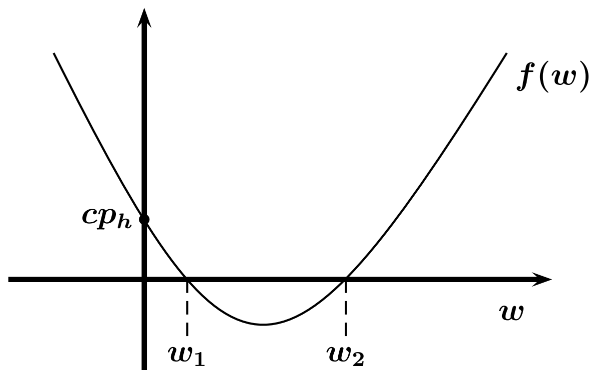

Figure 1.2: $x_U(w) - x_I(w)$

Proof: Given that $f''(w) = 2st$ and $2st > 0$, $f(w)$ is a strictly convex function of w. We also have that $f(w)$ crosses the vertical axis at $f(0) = cp_h > 0$, and that the limit of the function as w increases is positive, or more specifically: $\lim_{w\to\infty} f(w) \to \infty$. Therefore, if the roots of $f(w)$ are positive and real, then the function $f(w)$ crosses the x-axis at two points, $[w_1, w_2]$, to the right of the y-axis. Clearly $f(w)$ would be negative for all values of w between those two points, therefore we have that $x_I(w) > x_U(w) \quad \forall \quad w \in [w_1, w_2]$.

The roots for the quadratic equation $f(w)$ are given by:

$$w = \frac{(1-\beta)(p_h - p_\ell) - sp_h - ct}{2st} \pm \frac{\sqrt{[sp_h - (1-\beta)(p_h - p_\ell) + ct]^2 - 4sctp_h}}{2st} \tag{1.9}$$

We need the solutions to the above equation to be two **positive real roots**. A necessary, but not sufficient, condition for both roots to be positive is that the first term in equation (1.9) be positive. This gives us our first condition for the existence of a middle range of income where searching is optimal:

$$\Rightarrow \quad (1-\beta)\left(\frac{p_h - p_\ell}{p_h}\right) \; > \; s\left(1 + \frac{ct}{sp_h}\right) \tag{1.10}$$

This condition says that in order for there to exist a range of consumers that choose to search the gains from searching must be greater than the costs of search. The left side of (1.10) is the expected gains from search, $(1-\beta)$ is the proportion of high-priced firms and $\left[\frac{p_h - p_\ell}{p_h}\right]$ is the relative price difference between the two types of firms. While the right side is the costs of searching.

For the roots of $f(w)$ to be real we would require that the term underneath the square root in equation (1.9) be positive:

$$[sp_h - (1-\beta)(p_h - p_\ell) + ct]^2 - 4sctp_h > 0$$

$$\Rightarrow \quad (1-\beta)\left[\frac{p_h - p_\ell}{p_h}\right] > s + \frac{ct}{p_h} + 2\sqrt{\frac{sct}{p_h}}$$

Where the last step comes from the fact that the condition from (1.10) holds, therefore the argument inside the bracket on the left is negative. This gives us the second condition for existence. Rearranging the above inequality we have our necessary and sufficient condition for the existence of a middle income of consumers that search.

$$\Rightarrow \quad \textbf{Search Condition:} \quad (1-\beta)\left(\frac{p_h - p_\ell}{p_h}\right) \; > \; s\left(1 + \sqrt{\frac{ct}{sp_h}}\right)^2 \tag{1.11}$$

It is straightforward to show that if the requirement in equation (1.11) holds, then (1.10) will hold as well. To check the dimensionality of our condition we note that s and t are pure numbers and so are the ratio of prices and c to p_h. Therefore, our condition is valid and represents a relationship between values of pure numbers.

Since we have that the roots are real, then we know that w_1 and w_2 exist. To show that they are both positive it is sufficient to show that the smaller root is positive, which we can see is true from our demand curve in figure 1.1. Therefore, given our linear demand function, our requirement for positive roots as given by our "Search Condition" is the only condition necessary and sufficient for a middle range of income $[w_1, w_2]$ to exist where consumers search.

From the right hand side of the inequality we can see that s has to be quite small for the Search Condition to hold. This result is in line with existing literature on price dispersion and search costs. Diamond (1971) shows that as long as search costs exist, no matter how small, consumers will not be informed and firms will charge the monopoly price. Baye et al. (2006) show that Diamond's results seem to hold for a wide range of theoretical and empirical approaches, and that price dispersion continues to exist in both online and offline markets. Our result above differs from this previous work in the sense that even if the Search Condition holds, as long as search costs are positive, it will only hold for a middle income portion of the consumer population. Therefore, low-income consumers (as well as wealthy consumer) will always have to rely on the middle class in the market to keep prices competitive.

The extent of time costs, s, is a key feature of our model. The more readily consumers can access pricing information, the lower the time required to search for the lowest price. Improvements in information technology have gone a long way towards reducing the time cost of information for those consumers with access to the newest technology. Our model would suggest that as the speed of search increases it becomes more likely that consumers are informed about retail pricing. Here we have assumed that s is constant across the consumer population. This is very likely not to be the case. Poorer households are less likely to have access to the same technology and resources as higher income households, making it more difficult for them to search for prices. We will come back to this point below.

The left-hand side of of inequality (1.11) is decreasing with the proportion of firms charging the low price, β. This means that the incentive for middle-income consumers to search is decreasing with the proportion of low-priced firms. As the proportion of low-priced firms increases, the probability that an uninformed consumer happens upon a low-priced firm increases, therefore the benefit of paying the cost of search in order to find the lowest price decreases. Similarly the probability that there exists a middle-income group that searches is decreasing with the monetary cost of search relative to the monopoly price, $\frac{c}{p_h}$, which follows from the same intuition as above.

Interestingly as the time spent on consumption, t, increases, it becomes less likely that the Search Condition will hold. We can interpret this in two ways. Since we have defined t as our measure of time spent on leisure in our model, then it seems intuitive that as the time desired or required for leisure activities increases, it becomes more costly for consumers to dedicate their free time for searching for the lowest price. This in effect creates an inherent tradeoff between income and time spent on any other activity except consumption and labor. In addition, t encapsulates the time spent by the consumer taking part in activities other than working or searching. It is understandable that as these constraints on the consumer or household's time increases they will be less likely to spend time searching.

Finally, we have that the incentive for middle-income consumers to search is increasing with the discount offered by the low-priced firms, $(\frac{p_h - p_\ell}{p_h})$. As the percentage price difference increases the monetary benefit to search increases, giving more incentive for consumers to spend money and time to find the lowest price. This is also a result that we can directly observe in the real world: it is much more likely that consumers will take the time to shop around in a market where large discount superstores exist rather than a market with firms having similar economies of scale. This result has strong implications for households isolated in mainly poor communities. As we argued in the literature discussion above, one is much less likely to find large discount stores in low-income communities, making it more expensive for households located in these communities to access these low-priced stores.

These are all important considerations for the possibility of policy intervention. If we can show through more direct empirical analysis that the phenomenon described in our model does exist, then directed policy can target the relevant parameters above, increasing the incentive for the consumer population to search for the lowest price. As we will show below and as has been argued by Salop and Stiglitz (1977), a higher proportion of consumers that search will lead to a more competitive market environment.

Now we would like to consider the overall demand in our market, to do so we need to formally introduce an income distribution to our model.

Income Distribution

There are m consumers in our market. We consider values of X (total demand across the consumer population) as determined by the distribution of income in the market, continuing to take the firm side of the market as given. We begin by introducing a particular income distribution to our model, which will help us in determining aggregate demand as a function of the types of firms in the market, β.

In our analysis we consider the distribution of income in a community that lives within a larger economy. The larger economy is made up of many neighborhoods with varying number of middle-income individuals. We are interested in understanding how prices in a community vary as the size of the middle class in that community changes. We will work with a Uniform Income distribution, $G(w)$, defined over a range of income $[a, b]$, where the probability of a consumer earning wage $w \in [a, b]$ is given by $g(w) = \frac{1}{b-a}$. Though the Uniform Distribution is not necessarily an accurate representation of reality, it is mathematically more easy to deal with, and will allow us to more directly focus on the question at hand.

A possible extension of our results below would be to consider the impact on our analysis of introducing unbounded distribution functions with varying density across the income range (e.g. Normal or Pareto). We believe that these alternative distributions would not change our results below, but could potentially add greater depth to the analysis. Using an unbounded distribution would not impact our results directly, but would add justification for our assumption (vii) below (or even make it unnecessary in the case of distributions that are unbounded at both tails). Allowing for variation in the density function across the range of income would allow us to more carefully consider the impact of the shape of the distribution on prices, rather than being limited to using a mean preserving spread to represent inequality. Though we acknowledge the limitations imposed by using the Uniform, we find it more simple to work with, and we feel that our results below could be generalized to other types of distribution functions, including the Normal and Pareto.

We will focus our analysis in this section on mean preserving changes in the distribution of income centered around the mean of the overall economy (demonstrated by $\bar{w}$ in Figure 3.3 above), which is exogenously given. For the purpose of our theoretical model we are interested in focusing on the effect of changes in the size of the middle class on search. We do not lose any generality by centering our distribution about the mean and varying the variance of the distribution to signify changes in the concentration of the middle class.

We add the following assumption regarding our Income Distribution:

(vii) If there exists a range of income over which it is optimal for consumers to search, that range will be contained within the overall income range for the community, $[w_1, w_2] \subset [a, b]$. In other words, the range of income in any given community will never get too concentrated about the mean.

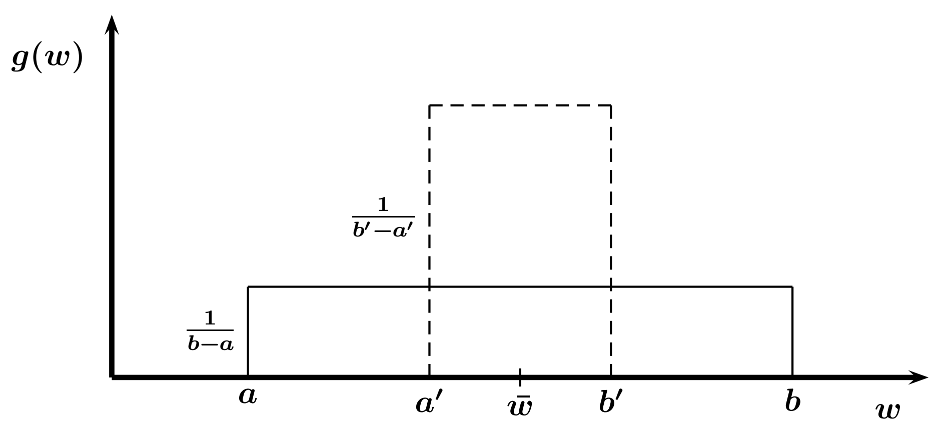

Figure 1.3: Uniform Distribution of Income

We lose very little in our analysis by using the "containment" assumption above. Our main question is whether or not increasing the size of the middle class will lead to lower prices. The income distribution we use is just a vehicle for representing changes in the size of the middle class in our model.

Aggregate Demand

In our model of Demand above we determined that as long as our Search Condition in (1.11) is satisfied with respect to our parameters there would exist a middle range of income $[w_1, w_2]$ where only consumers that fall within that range would choose to search. Now, using our income distribution $G(w)$ along with our Search Condition and our values of w_1 and w_2, we will look to calculate aggregate demand from informed and uninformed consumers as a function of our parameters and the proportion of low-priced firms, β. We begin by calculating the total demand in a given market as determined by the income distribution in that market.

As we can see in Figure 1.4 below where we have drawn consumer demand as a function of income[11], demand in a particular market is bounded by the range of income in that market, $[a, b]$. In addition, given our strictly monotonic utility function, the market demand curve is determined by the higher of the two demand curves calculated under search and no search.

[11]Note that in this case demand is upward sloping since higher income leads to greater demand.

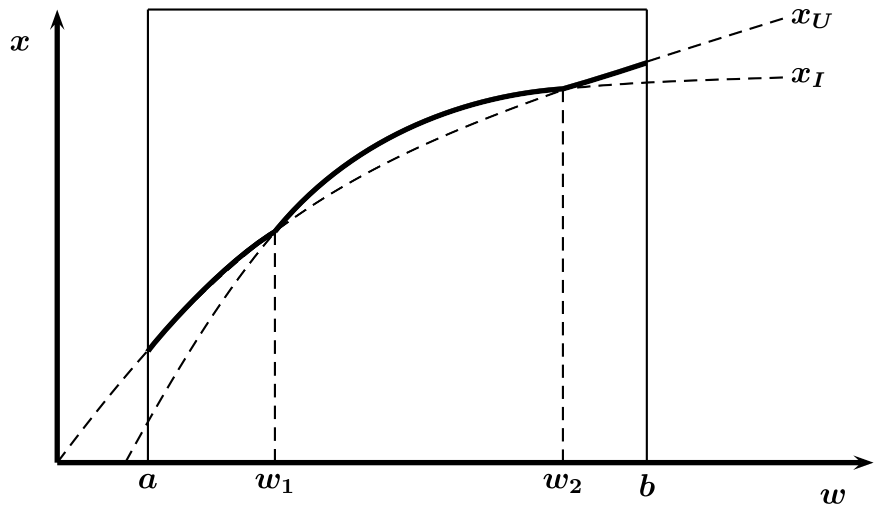

Figure 1.4: Market Demand Curve

Using our uniform distribution of income, we can calculate market demand by multiplying the total number of consumers per level of wage, $\frac{m}{b-a}$, by the integral under the curve. Total demand from consumers who search would be given by:

$$X_I \;=\; \tfrac{m}{b-a}\left(\int_{w_1}^{w_2} \frac{w - sw - c}{p_\ell + tw}\; dw\right) \tag{1.12}$$

Total demand from uninformed consumers is the sum of the two integrals of demand below and above our middle range of income. Rewriting equation (1.3) above we derive total demand from uninformed consumers:

$$X_U \;=\; \beta X_{U,\ell} \;+\; (1-\beta) X_{U,h} \tag{1.13}$$

Where: $$X_{U,i} = \frac{m}{b-a}\left(\int_a^{w_1} \frac{w}{p_i + tw}\ dw + \int_{w_2}^{b} \frac{w}{p_i + tw}\ dw\right)$$

The second line in equation (1.13) is aggregate uninformed demand if all uninformed consumers happened to walk into a type i firm. For a given distribution of firm types in the market, aggregate demand from uninformed consumers is given by the first line of equation (1.13).

Before moving on to the supply side of our model we consider the properties of the aggregate demand functions above. We would like to determine how the magnitude and composition of demand changes as we vary the proportion of low-priced firms, β. First we consider changes in demand from informed consumers with respect to the proportion of low-priced firms. Taking the derivative of equation (1.12) using Leibniz's Rule we have:

$$\frac{dX_I}{d\beta} = \frac{\partial w_2}{\partial \beta}\left(\frac{w_2 - sw_2 - c}{p_\ell + tw_2}\right) - \frac{\partial w_1}{\partial \beta}\left(\frac{w_1 - sw_1 - c}{p_\ell + tw_1}\right) < 0$$

We can show that $\frac{\partial w_2}{\partial \beta} < 0$ and $\frac{\partial w_1}{\partial \beta} > 0$, therefore we have that the above derivative is negative[12]. Demand from informed consumers is a decreasing function of the proportion of low-priced firms. This is as we would expect: a higher proportion of low-priced firms means that it is more likely that an uninformed consumer searching at random would happen across a low-priced firm, lowering the benefit of being an informed consumer.

Using the same results as well as that $X_{U,\ell} > X_{U,h}$ by construction, we can also

[12] See Appendix for proof.

show that the following results hold [12]:

$$\frac{dX_{U,\ell}}{d\beta} > \frac{dX_{U,h}}{d\beta} > 0 \Rightarrow \frac{dX_U}{d\beta} > 0 \quad \text{and} \quad \frac{dX}{d\beta} > 0 \tag{1.14}$$

Both demand from uninformed consumers and total demand are increasing functions of β. The first condition is the reverse of the result from the demand for informed consumers above, as the number of low-priced firms increases the incentive to search decreases, so there is more uninformed demand. Total demand increases with β, as more consumers do not pay the cost of search but still shop at the low-priced firms (since it is more likely they will randomly come across one) they have more money to spend on buying the good, leading to overall demand increasing. This result has interesting welfare implications. Imperfect price information costs consumers in terms of overall consumption, a portion of consumer income must be spent in gathering price information[13].

Next we look at the market from the perspective of firms. We look to determine how the presence of consumers who search and do not search effects the types of firms that enter a market.

1.4 Supply Side

In this section we consider how varying degrees of search by consumers can cause price dispersion in a market for a single good. Here we modify a version of a model of price dispersion first developed by Salop and Stiglitz (1977). We generalize away from most theoretical work on price dispersion by allowing consumers to buy a continuous amount

[13] By consumption we mean consumption of this particular good. The cost of search, at least the monetary cost, does go toward another form of consumption (Transportation or information goods).

of the good, $x \in \mathbb{R}_+$.

Firms

Our market consists of n firms that have identical production technology. There are different possible approaches to the cost structure of firms. Previous work on firm output and prices in a model with uninformed consumers have used various types of cost structures. Salop and Stiglitz (1977) and Braverman (1980) use a U-shaped average cost curve, Varian (1980) uses a strictly declining average cost curve, and several papers use constant marginal costs (See for example Carlin (2009)). All of these are justifiable approaches to the structure of firms in the market that we have described. Since we are interested in the choice of capacity as well as price, we use a U-shaped average cost curve characterized by a common fixed cost and increasing variable costs. This is analogous to a model of firms with soft capacity constraints, or decreasing returns to scale, allowing us to consider how firms choose their size (and in effect price) given the proportion of informed consumers that exist within that market.

Firms can choose over a continuum of prices between the consumers' reservation price r, and the minimum point on the average cost curve, $p_{\min}$. No firm would ever choose to price below $p_{\min}$, since it would be earning negative profits, and no consumer would ever buy the good above the price r. Given their choice of price, firms fall into two categories. If a firm is charging the lowest price in the market, then it is considered a Superstore, ℓ-type firm, characterized by high output, q_ℓ and low price, p_ℓ. These types of firms target both informed and uninformed consumers in the market. Otherwise the firm is considered a Corner store, h-type firm, with low output, q_h, and high price p_h. These types of firms only target uninformed consumers. Consumers are not able to recognize an ℓ-type or a h-type unless they are informed. This basically means that there is some level of uncertainty about what firm offers the lowest price. We can think of this as the case where a Superstore might not always charge the lowest price for all goods, and that a consumer would be able to pay a lower price for a small set of goods at a Corner store[14]. Firms compete based on price for demand from informed and uninformed consumers.

Given this setup and our assumption that all firms are equally accessible by all consumers, the firms charging the lowest price will share between them the demand from informed consumers, X_I. While both types of firms will share demand from uninformed consumers. The demand for each type of firm is given by:

$$q_\ell = \frac{X_I}{n_\ell} + \frac{X_{U,\ell}}{n_\ell + n_h} \qquad q_h = \frac{X_{U,h}}{n_\ell + n_h} \tag{1.15}$$

Where $X_{U,i}$ is as we defined in the previous section, n_h is the number of firms categorized as high-priced and n_ℓ is the number of firms categorized as low-priced. Firms must belong to only one of the two categories, so $n = n_h + n_\ell$. In this setup the proportion of low-priced firms β from the previous section is given by: $\beta = \frac{n_\ell}{n_h + n_\ell}$.

Each firm maximizes the following profit function taking the search decision of consumers and the pricing decision of other firms as given:

$$\max_{p_i} \quad \pi(p_i|p_{-i}) = p_i q_i(p_i|p_{-i}) - v[q_i(p_i|p_{-i})] - F \tag{1.16}$$

[14]Varian (1980) uses a model of sales to demonstrate that if firms intermittently switch from high to low price, consumers would have incentive to search and price dispersion would exist. This comes from the observation that large discount stores do not always offer the lowest price and that for a consumers to be fully informed they would have to compare prices between them and smaller retailers at the time of purchase.

$v(q)$ is variable cost and is increasing with quantity demanded ($v'(q) > 0$). $q_i(p_i|p_{-i})$ is the demand faced by firm i charging price p_i and taking the price of all other firms as given. We assume that there are fixed costs, F, but no other barriers to entry, therefore both types of firms earn zero profits in equilibrium. The average cost function is equal to the total cost function divided by quantity:

$$AC(q_i) = \frac{v(q_i)}{q_i} + \frac{F}{q_i}$$

Fixed costs plus increasing variable costs lead to a U-shaped average cost curve as drawn in figure 1.5 below.

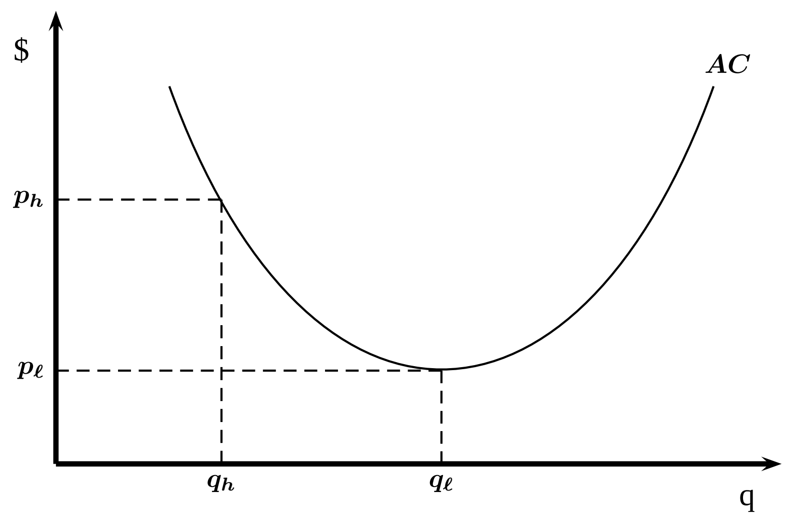

Figure 1.5: Average Cost Curve

From our zero profit condition we have that enough of both types of firms must enter to get both firms onto the average cost curve. Our assumption of profit maximization and the zero profit condition means that in equilibrium the demand curve faced by each type of firm lies below the average cost curve at each point other than that firm's choice of equilibrium price.

Using a similar setup, Salop and Stiglitz (1977) show that if search costs are high enough no consumers search and there exists a single price equilibrium (SPE) with all firms charging the reservation price. This is analogous to our Search Condition from above not holding due to high search costs. They also show that when search costs are low enough there exists a single price equilibrium with all firms charging the minimum price; we will consider this possibility in our discussion of equilibrium below. For intermediate values of search costs there exists a two price equilibrium (TPE) where some firms charge the minimum price and some charge a higher price less than or equal to the reservation price.

We differ from Salop and Stiglitz (1977) and other previous work on price dispersion in that we have consumers buying a continuous amount of the good and having a continuum of search costs. We also differ in our assumption that uninformed consumers do not know the distribution of prices. Salop and Stiglitz (1977) make the assumption that uninformed consumers know the distribution of prices in order not to stray too far from the standard competitive model, as well as to avoid the Diamond paradox, the outcome where no consumers search and all firms charge the highest price even when search costs are arbitrarily small, Diamond (1971).

As we will show below, our model still allows for a Diamond style outcome where if search costs exist, no matter how small, there exists an equilibrium where no consumers search and all firms charge the high price. We will also argue that even when there are search costs, there exists an alternative outcome where a portion of consumers search and some firms charge the low price. This is an alternative equilibrium to the Diamond no-search outcome, similar to the two price equilibrium suggested in Salop and Stiglitz (1977). Other papers have dealt with the possibility of moving away from the Diamond outcome, see for example Bagwell and Ramey (1992) and Rhodes (2011). We will focus the main part of our analysis on the possibility of the two-price outcome, and how price dispersion in such an outcome depends on the make up of the consumer population.

1.5 Equilibrium Analysis

As a final step in our theoretical model, we consider possible equilibria where firms and consumers act simultaneously (we are looking at Nash equilibria with respect to firm and consumer decisions). We would like to combine our results above to determine if and when there exists an "interior" equilibrium value for our consumer and firm populations that determines search intensity and firm type, given our model parameters and income distribution. Also, if this "interior" equilibrium does exists, we would like to determine how it changes with variations of inequality in our income distribution, as given by changes in $(b - a)$. In this final step we are able to answer the question initially set out by this chapter, "How do changes in income distribution affect consumer prices?"

An equilibrium in our model is characterized by the following:

1. Firms maximize profits, taking the prices of other firms and consumer search as given.
2. Firms earn zero profits.
3. Consumers search optimally, taking the pricing decision of firms as given.
4. All consumer demand is met, there is no excess demand in equilibrium.

The main question we are concerned with is: if there are two firms charging different prices, in what proportion will these firms enter the market, and how will that proportion depend on the makeup of the consumer population? We can show that there exists an equilibrium such that if a firm enters as a corner store it would price at $p_h = r$, while a Superstore would choose $p_\ell = p_{\min}$[15]. From now on we will limit our analysis to firms charging these two prices. The intuition for this result comes from the price elasticity of consumer demand. Informed consumers have perfectly elastic demand with respect to a price increase from the minimum price, therefore low-priced firms would not deviate from the minimum point on the average cost curve. Uninformed consumer have inelastic demand by construction, therefore firms targeting these types of consumers would be best off charging the maximum price as determined by the consumers' outside option, r. From our Search Condition above it is clear that the extent of the differential between the price of the two types of firms is a key factor in whether or not the middle class search. We will consider what drives the differential between the high and low price, below.

We now look for possible equilibria in our model.

Proposition 1.1: *If search costs are zero, $c = 0$ and $s = 0$, all firms charge the low price, $\beta = 1$.*

Proof: This is just the standard perfectly competitive outcome with perfect information.

Proposition 1.2: *For any positive cost of search, $c > 0$ and/or $s > 0$, there exists an equilibrium outcome where all firms charge the high price, $\beta = 0$, and no consumers search (this is the outcome described in the Diamond Paradox).*

Proof: If all firms charge the same high price the Search Condition does not hold and

[15] See Appendix for a more formal justification for $p_h = r$ and $p_\ell = p_{\min}$ as an equilibrium price outcome.

consumers do not have incentive to search. If consumers do not search then they will not be able to observe price deviations, therefore a deviating firm would not induce search. Given the inelastic demand from uninformed consumers, no firm has an incentive to choose a price below the reservation price[16].

Proposition 1.3: *For any positive cost of search, $c > 0$ or $s > 0$ there does not exist an equilibrium where all firms charge the low price, $\beta = 1$.*

Proof: We can see the proof for this result from inspection of our Search Condition in equation (1.11) above. For any positive value for s or c, and a non-infinite high price, p_h, as $\beta \longrightarrow 1$ our Search Condition does not hold and therefore no consumers search. Since demand from uninformed consumers is inelastic, if there are no consumers that search and all firms are charging the low price and earning zero profits, then one firm can deviate to the high price and earn positive profits.

Now we consider the conditions for an equilibrium with an intermediate proportion of the two types of firms. From our zero profit condition we have that the quantity for each firm is fixed by the firms' choice of prices, $p_{\min}$ and r, and the shape of the average cost curve. Using $A(q)$ to represent the downward sloping portion of the average cost curve, this condition implies:

$$p_{\min} = A(q_\ell) = \frac{v(q_\ell)}{q_\ell} + \frac{F}{q_\ell} \qquad r = A(q_h) = \frac{v(q_h)}{q_h} + \frac{F}{q_h}$$

As we can see in the figure 1.6, for a given technology of production, only one value

[16] This results from our assumption that uninformed consumers do not know the distribution of prices except in equilibrium, and therefore do not observe deviations away from equilibrium. Salop and Stiglitz (1977) assume that uninformed consumers know the distribution of prices, therefore they are able to rule out the Diamond Paradox.

of $\bar{q}_\ell$ and $\bar{q}_h$ can satisfy the zero profit condition (intersect the two prices on the average cost curves). From now on we refer to these fixed quantities as $\bar{q}_\ell$ and $\bar{q}_h$.

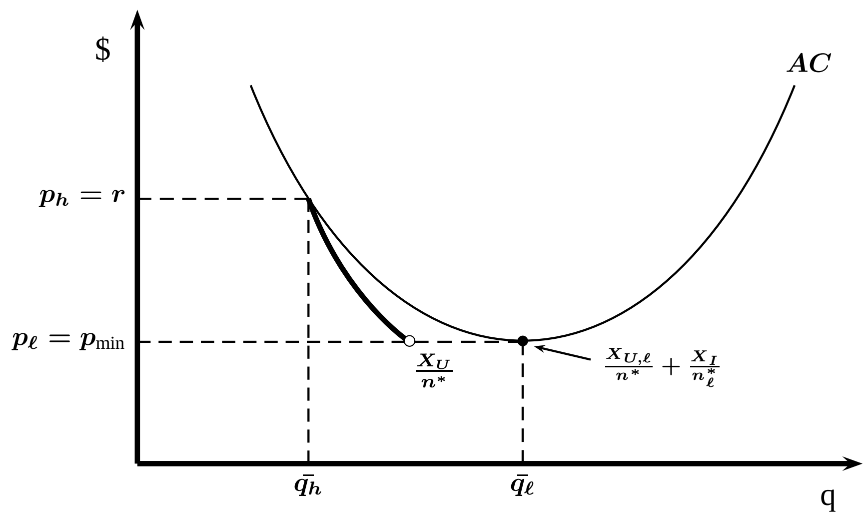

Figure 1.6: Two Price Equilibrium

Rearranging the formulas for the fixed quantities for each type of firm from (1.15) we have two equations that solve for the number of low and high-priced firms:

$$n_\ell + n_h = \frac{X_{U,h}}{\bar{q}_h} \qquad\qquad n_\ell + n_h = \frac{X_I(n_\ell + n_h)}{n_\ell \bar{q}_\ell} + \frac{X_{U,\ell}}{\bar{q}_\ell} \tag{1.17}$$

The levels of n_i (which in turn determine n and β) are determined by how many firms must be in the market at a given time for the two types of firms to be producing at their respective zero profit quantities. From Proposition 1.3 we know that the first equation must always be satisfied, while the second equation holds as long as there are some consumers that search, $X_I > 0$. Using these two equations we can solve for the number of low and high-priced firms in a two-priced equilibrium:

$$n_\ell^* = X_I \left(\frac{X_{U,h}}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}} \right) \qquad n_h^* = \frac{X_{U,h}}{\bar{q_h}} \left(\frac{\bar{q_\ell} X_{U,h} - \bar{q_h}(X_{U,\ell} + X_I)}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}} \right) \tag{1.18}$$

From examination of the two equations above we can see that as demand from informed consumers, X_I, decreases, n_ℓ^* approaches zero and n_h^* approaches $\frac{X_{U,h}}{\bar{q_h}}$. Combining the two equations we derive an expression for the proportion of low-priced firms in the market.

$$\beta^* \ = \ \frac{n_\ell^*}{n_\ell^* + n_h^*} \ = \ X_I \left(\frac{\bar{q_h}}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}} \right) \tag{1.19}$$

We can show that the denominator is positive as long as $X_I > 0$ (this is demonstrated in figure 1.6 above)[17].

Let us consider the equation for β^* in (1.19). The left and right side of the equation are both functions of β. For an equilibrium to exist we must have that there exists at least one fixed point such that $f(\beta^*) = \beta^*$, where $f(\beta)$ is the function on the right hand side of equation (1.19).

$$f(\beta) = X_I \left(\frac{\bar{q_h}}{\bar{q_\ell} X_{U,h} - \bar{q_h} X_{U,\ell}} \right)$$

Proposition 1.4: *If the Search Condition holds for some value of $\beta \in (0, 1)$ then there exists a unique equilibrium, $(w_1^*, w_2^*, \beta^*, n^*)$ such that n^* firms enter the market, β^* of firms choose to be ℓ-types and price at the low price, p_{min}, $1 - \beta^*$ of firms choose to be h-types and price at the reservation price, r, and consumers earning wage*

[17] See Appendix for proof.

$w \in [w_1^*, w_2^*]$ *search for the lowest price.* [This result is demonstrated in figure 1.7 below.]

Proof: First we consider the extreme points of the function, $f(\beta)$. From our Search Condition we have that as β approaches 1 the condition does not hold and $X_I = 0$, which in turn means $f(\beta) = 0$. In addition, as long as the Search Condition holds for some intermediate value of β, we have that as β approaches zero $f(\beta)$ approaches some positive constant, $k > 0$[18].

$$\beta \rightarrow 1 \quad \Rightarrow \quad f(\beta) = 0 \qquad \text{and} \qquad \beta \rightarrow 0 \quad \Rightarrow \quad f(\beta) \rightarrow k > 0 \tag{1.20}$$

In our analysis of demand in section 3 we show that demand from informed consumers is a decreasing function of the proportion of low-priced firms, $\frac{dX_I}{d\beta} < 0$. We also show that uninformed demand is an increasing function of β, that is $\frac{dX_{U,\ell}}{d\beta}, \frac{dX_{U,h}}{d\beta} > 0$. Using these results it is straightforward to show that $f(\beta)$ is monotonically decreasing over $\beta \in (0, 1)$[19].

Therefore, by Brouwer's Fixed Point Theorem, we must have at least one point where $f(\beta)$ crosses the $45°$ line, that is $\exists\ \beta^*$ s.t. $f(\beta^*) = \beta^*$. From monotonicity of the function $f(\beta)$, that β^* is unique.

[18] Note that when $\beta = 0$ we must have that $X_I = 0$ since no consumer will pay the cost of search if there are no low-priced firms to search for.

[19] See the Appendix for proof.

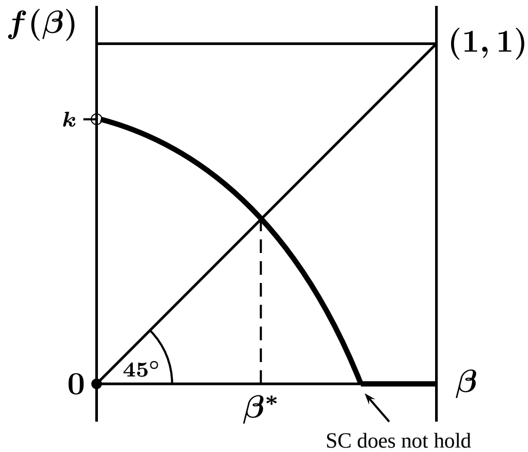

Figure 1.7: Equilibrium

We have now established that as long as the Search Condition holds for some intermediate value of β, there exists an equilibrium where some consumers search and some firms charge the low price.

Before we go on, it is interesting to note that the resulting equilibrium value of β^* depends both directly and indirectly on our Search Condition. It depends indirectly on the condition in that we need the condition to hold in order to allow for the intermediate equilibrium we have described above. The direct effect is demonstrated by the point on the graph in 1.7 where $f(\beta)$ hits the x-axis. This point is determined by the value of β where the Search Condition does not hold. A decrease in search costs, c and/or s, would cause this point to move out further on the x-axis, leading to a higher equilibrium proportion of low-priced firms. This is an intuitive result, as search costs go to zero, the intermediate outcome moves closer to the competitive outcome. Although as long as search costs are positive the alternative equilibrium with $\beta = 0$ continues to be a possibility.

Now we consider how this interior equilibrium changes with changes in our income distribution.

Variations Around Equilibrium

In this section we look to determine how our equilibrium proportion of low-priced firms, β^*, changes as we vary the range of our income distribution, $(b - a)$. Due to our use of a uniform distribution of income, an increase in the range of income is equivalent to a reduction in the size of the middle of the distribution. From our "containment" assumption in the section on income distribution, and examination of our Search Condition, we have that the numerator of the equation for $f(\beta)$ in (1.19) is not dependent on the range of income, $(b - a)$. Using the same argument we also have that the point where the function hits the x-axis, where the Search Condition does not hold and $X_I = 0$, does not depend on the range of income. What we need to determine is how the denominator in (1.19) changes as our distribution of income becomes more spread out. In our setup such a change is equivalent to a reduction in the size of the middle range of income. The denominator in the equation for β^* is:

$$\bar{q}_\ell \left(\int_a^{w_1} \frac{w}{p_h + tw} \ dw + \int_{w_2}^{b} \frac{w}{p_h + tw} \ dw \right) - \bar{q}_h \left(\int_a^{w_1} \frac{w}{p_\ell + tw} \ dw + \int_{w_2}^{b} \frac{w}{p_\ell + tw} \ dw \right)$$

We will focus on the impact of a mean preserving spread in the distribution. A mean preserving spread is demonstrated in figure 1.8 below, where $\bar{w}$ is mean income[20].

[20]In a uniform distribution $\bar{w} = \frac{a+b}{2}$.

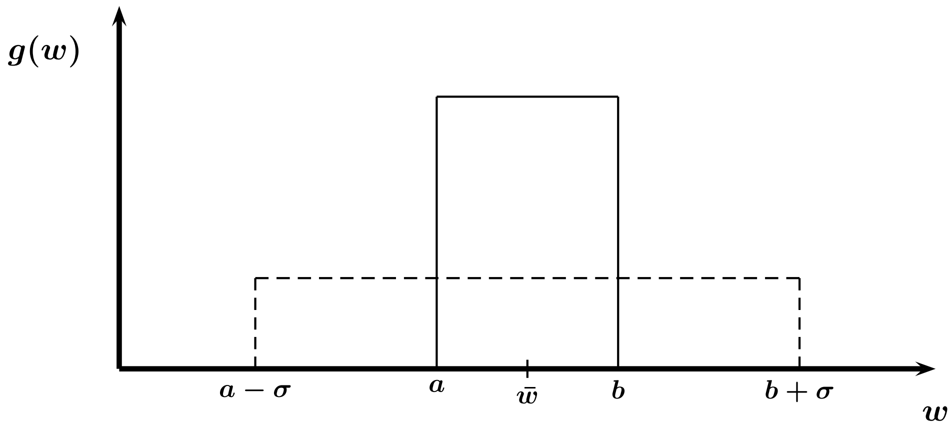

Figure 1.8: Mean Preserving Spread

Comparing the value of the denominator when the range of the distribution is $(b - a)$ with its value when the range increases to $[b + \sigma - (a - \sigma)]$, we can show that the denominator increases under the more spread out distribution[21]. Therefore, we have that $f(\beta)$ is a decreasing function of the spread of our distribution on the interior of the graph in 1.7.

[21] See Appendix for proof.

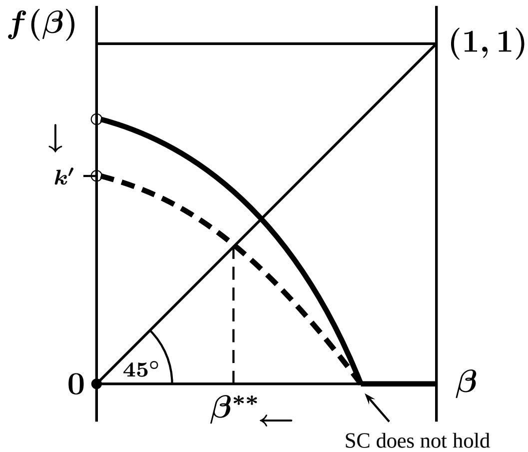

Figure 1.9: Effect of Decrease in Middle Class

As we can see in figure 1.9, when the range of income increases, $f(\beta)$ pivots down anchored at the point on the x-axis where the Search Condition is binding, leading to a lower equilibrium point, β^{**}. Therefore, a decrease in the size of the middle income group, represented by an increase in the variance in our model, leads to a lower number of firms choosing to be ℓ-types. This means that β^* decreases, which by examination of (1.1) leads to higher average prices in the market.

We have finally demonstrated that under certain market conditions we can have a negative relationship between the size of the middle class in a market and consumer prices in that market. Now we would like to explore more in-depth the conditions on our parameter values and market structure that allows this phenomenon to exist, and to discuss what types of markets would be more likely than others to allow for these conditions to hold.

Determinants Of Equilibrium

There are two main conditions on the model's parameter values that determine the intermediate equilibrium we have described. They are both represented in our Search Condition in (1.11) and the function $f(\beta)$ in (1.19). The first condition is that the costs of search, s and c, are not too high relative to consumer income. The second condition is that the difference between the two types of firms, represented by the difference between $p_{\min}$ and r, is significant enough to justify consumer search and therefore the existence of an ℓ-type firm in the market. In this final section we consider the significance of these conditions, and suggest possible consumer markets and types of cities, where these conditions are more likely to hold.

Search costs: This first condition is very intuitive, and seemingly the most relevant to policy initiatives. Clearly there are costs associated with searching for the lowest priced firm. For a household to feel that searching is optimal, they must believe that they can afford the time and money required. This is characterized by the left-hand side of the relation in our Search Condition. Lower search costs in our model would loosen the condition for search in (1.11). This would lead to a rightward shift of the $f(\beta)$ curve in figure 1.7 above, increasing the proportion of low-priced firms in the market. This result demonstrates how if we are able to lower the costs of search, we can increase the number of consumers that search, getting closer to the perfectly competitive outcome. Policy directed at lowering fixed costs through increasing availability of computers and journals, or lowering the time cost by making prices more transparent, would increase the incentive for all households to search. As we argued above, lowering search costs will bring the interior equilibrium above closer to the competitive outcome, reducing the loss to consumers of resources spent on searching.

Over the last several years, the internet has made it easier for consumers to locate the lowest priced firms in many markets, cutting down on the time cost of search. Various search engines such as "MySimon", "Google", "Kayak", and "esurance" provide highly efficient venues for consumers to compare prices in a wide ranging array of products, from electronics to car insurance. But these mediums are imperfect in two main aspects:

(i) Not all consumers have internet access readily available. 26% of the U.S. population do not have access to the internet, Nielsen (2008). Access is more varied in Europe, with those lacking internet acess ranging between 20-30% in most western European countries. These percentages are significantly higher for low-income households.

(ii) Not all consumer markets provide transparent pricing on the internet. The technologies for comparative shopping are still limited, with most households doing a majority of their shopping offline.

Lack of internet access for low-income families means that though the time cost of search is decreasing, the fixed cost of search still remains an issue for many households. Policy directed at increasing internet penetration in low-income consumers could be one way of bringing down these fixed costs.

Cities: The types of cities that would support the conditions we have presented above are those with significant structural frictions that make the gathering and dissemination of information difficult. Such frictions include:

(i) High levels of income segregation across neighborhoods.

(ii) Lack of readily available and affordable modes of transportation combined with a dispersed population.

(iii) Lack of community involvement and leadership.

(iv) Ineffective news media that do not reach the majority of the population.

These factors would require greater and more costly effort from the consumer to gather information on pricing on the firms available within the city. This would translate to higher values for s, c and possibly t in our model, making it more likely that low-income families would not be able to afford to search for the lowest price. The presence of these frictions is equivalent to a leftward shift in $f(\beta)$ in 1.7 above. Such a market environment would lead to a higher portion of high-priced firms in the economy.

An interesting consequence of the first item in the list of frictions above is the very famous concept of "Separate But Not Equal". In our example it might be the case that segregation of low-income families from the rest of the city might lead to higher prices in the poorer neighborhoods, making their separation from the rest of the city, in effect, unequal. We consider these types of spatial access frictions in Chapter Two of this thesis.

Difference Between $p_{\min}$ and r: The second main factor in our derived equilibrium is the difference between the two types of firms in the market, as represented by the price advantage of the low-priced firm. This structure represents a world where certain firms compete through high volume and small margins, ℓ-types, and others through low volume but higher margins, h-types. h-type firms choose to forgo selling to informed consumers so that they can sell at a higher price to uninformed consumers. An h-type firm is better positioned to compete in an industry with smaller fixed costs of entry, which results in a smaller difference between the consumer's reservation price relative to the minimum point on the average cost curve.

We can see the effect of this decreasing in "differentiation" in the left hand side of equation (1.11). As $p_{\min}$ approaches r the incentive to search decreases, reducing the demand from informed consumers and in turn reducing β^*. Therefore, we have that industries with higher fixed costs are more likely to support the type of equilibrium we have described above.

Product Markets: Product markets that are examples of industries that fit these characterisations include: Food Markets (Supermarkets versus groceries)[22], Household Goods (home product megastores versus local hardware stores), Healthcare (large hospitals versus local health centers), and Consumer Finance (large private banks versus small financial outlets).

All of the examples involve a high-fixed cost firm, such as a supermarket, that requires high volume of consumers to make up for their initial investment. If there exists a structural environment within a specific city where consumers are not perfectly informed about firm pricing, a firm can invest a small amount of fixed costs, for example a grocery, and look to attract a small volume of the uninformed consumers who will pay the higher price. These two tiers of firm types would not be sustainable in product markets with very little fixed costs, such as law firms and others in the services industry.

1.6 Conclusion

In our theoretical model above we have demonstrated that given a monetary and time cost of search, the middle class have the greatest incentive to search, therefore their presence in a market leads to greater competition among firms and lower prices. Frankel

[22] In the case of food markets one can argue that consumers are more aware of prices charged by different firms, since they tend to food shop more frequently relative to markets with one-off purchases. This might mitigate the impact of intermittent sales described in Varian (1980).

and Gould (2001) have shown empirically that there exists a significant relationship between the size of the middle class and consumer prices, but their findings do not tell us why such a relationship exists. Is it true that the middle class search the most intensely? And if so, what prevents the lower income class from searching, even though savings on price are a larger portion of their income relative to higher income consumers?

Through a closer analysis of specific cities and neighborhoods where these conditions are found to exist, we can attempt to see if the assumptions of our model hold. Do the poor face prohibitively high costs when shopping for the lowest priced firm? Is it a matter of search costs, or the costs of mobility that prevents low-income families from having access to the lowest price? By better understanding the link between the size of the middle class and consumer prices we can more effectively direct policy initiatives that will help the poor compete in consumer markets.

Whether or not income equality is ideal for a given society is an issue up for debate. But equal opportunity is inherent in the market ideas of free societies. Equal opportunity for low-income families to rise up out of poverty is as important a concept as that of equal opportunity based on race and gender. If we find that market frictions make it difficult for low-income families to search for the lowest price, then isolating low-income families in predominantly low-income neighborhoods can lead to them facing higher average prices. Higher prices would mean that money earned by a low-income family is worth less than that earned by the rest of society, making the climb out of poverty much steeper at the low-end of the income scale. This phenomenon, if found to exist, would be in line with an old concept usually associated with race, that "separate is inherently not equal".

1.A Mathematical Appendix

Proof That $\frac{\partial w_2}{\partial \beta} < 0$

Differentiating the equation for w_2 (the higher value for w in equation (1.9)) we have:

$$\frac{\partial w_2}{\partial \beta} = \frac{-(p_h - p_\ell)}{2st} + \frac{[sp_h + ct - (1-\beta)(p_h - p_\ell)](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}}$$

$$= \frac{-(p_h - p_\ell)}{2st} - \frac{[(1-\beta)(p_h - p_\ell) - sp_h - ct](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}} \quad < \quad 0$$

In the second line we have negated the term in brackets in the numerator of the second term, which was negative from the Search Condition holding. Therefore, we have shown that the derivative of the upper range of income that searches, w_2, with respect to the proportion of low-priced firms β is negative.

Proof That $\frac{\partial w_1}{\partial \beta} > 0$

Differentiating the equation for w_1 (the lower value for w in equation (1.9)) we want to show that:

$$\frac{\partial w_1}{\partial \beta} = \frac{-(p_h - p_\ell)}{2st} - \frac{[sp_h + ct - (1-\beta)(p_h - p_\ell)](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}} \quad > \quad 0$$

$$= \frac{-(p_h - p_\ell)}{2st} + \frac{[(1-\beta)(p_h - p_\ell) - sp_h - ct](p_h - p_\ell)}{2st\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h}} \quad > \quad 0$$

In the second line we have negated the term in brackets in the numerator of the second term, which was negative from the Search Condition holding. Multiplying both sides by the denominator of the second term and dividing through with $(p_h - p_\ell)$ we have:

$$-\sqrt{[sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h} \; + \; [(1-\beta)(p_h - p_\ell) - sp_h - ct] \;\; > \;\; 0$$

$$\Rightarrow \quad [(1-\beta)(p_h - p_\ell) - sp_h - ct]^2 \;\; > \;\; [sp_h + ct - (1-\beta)(p_h - p_\ell)]^2 - 4sctp_h$$

$$\Rightarrow \quad 0 \;\; > \;\; -4sctp_h$$

Therefore, we have shown that the derivative of the lower range of income that searches, w_1, with respect to the proportion of low-priced firms β is positive.

Proof That $\frac{dX_{U,i}}{d\beta} > 0, \quad \frac{dX_{U,\ell}}{d\beta} > \frac{dX_{U,h}}{d\beta},$ **and** $\frac{dX_U}{d\beta}, \frac{dX}{d\beta} > 0$

Differentiating demand from uninformed consumers from (1.13) we have:

$$\frac{dX_{U,i}}{d\beta} = \frac{m}{b-a}\left[\frac{\partial w_1}{\partial \beta}\left(\frac{w_1}{p_i + tw_1}\right) - \frac{\partial w_2}{\partial \beta}\left(\frac{w_2}{p_i + tw_2}\right)\right] \quad > \quad 0$$

Where we know that the inequality is true since $\frac{\partial w_1}{\partial \beta} > 0$ and $\frac{\partial w_2}{\partial \beta} < 0$. Therefore, demand from uninformed consumers is increasing with the fraction of low-priced firms. It is clear from the above inequality that:

$$\frac{dX_{U,\ell}}{d\beta} > \frac{dX_{U,h}}{d\beta} \qquad \text{since} \qquad \frac{w_j}{p_\ell + tw_1} > \frac{w_j}{p_h + tw_1} \quad \text{for} \quad j \in \{1, 2\}$$

We have that demand from uninformed consumers shopping at the low-priced store is increasing faster than demand from uninformed consumers shopping at high-priced stores.

To see how uninformed demand as a whole changes with respect to β we differentiate

X_U to get:

$$\frac{dX_U}{d\beta} = X_{U,\ell} - X_{U,h} + \frac{dX_{U,\ell}}{d\beta}\beta + \frac{dX_{U,h}}{d\beta}(1-\beta) \quad > \quad 0$$

Which we know is positive since $X_{U,\ell} > X_{U,h}$.

Finally in order to see how total demand changes with respect to β, we differentiate $X = X_I + X_U$ to get:

$$\frac{dX}{d\beta} = X_{U,\ell} - X_{U,h} + \frac{dX_{U,\ell}}{d\beta}\beta + \frac{dX_{U,h}}{d\beta}(1-\beta) + \frac{dX_I}{d\beta} \quad > \quad 0$$

The last three terms on the left of the inequality cancel out since at w_1 and w_2 we have:

$$\frac{w_j - sw_j - c}{p_\ell + tw_j} = \beta\left(\frac{w_j}{p_\ell + tw_j}\right) + (1-\beta)\left(\frac{w_j}{p_h + tw_j}\right) \quad \text{for} \quad j \in \{1, 2\}$$

As we argued above, $X_{U,\ell} > X_{U,h}$ by construction, so we have that total demand is increasing with the proportion of low-priced firms.

Proof that $p_h = r$ and $p_\ell = p_{\min}$ represent an equilibrium

In considering the choice of price by firms we consider the demand for the two types of firms and the profit function in (1.16). From examination of quantities for the two types of firms, q_ℓ and q_h, we can see that as long as the Search Condition is satisfied, consumer demand is not continuous at the point where price equals the minimum price available in the market, p_ℓ. If a firm raises its price above the minimum price they would lose their share of demand from informed consumers. Before we consider the implication of this discontinuity, we consider the decision of firms that only target uninformed consumers. Taking the derivative of the profit function above with respect to price we have:

$$\frac{\partial \pi_i}{\partial p_i} = q_i + \tfrac{\partial q_i}{\partial p_i} p_i - \tfrac{\partial v}{\partial q_i}\tfrac{\partial q_i}{\partial p_i}$$

When a firm only targets uninformed consumers, that is when they charge a price, p_i, above the minimum price, their demand from a consumer earning w is given by:

$$x = \frac{w}{p_i + tw} \qquad \text{where} \qquad \frac{\partial x}{\partial p_i} = -\frac{w}{(p_i + tw)^2}$$

Therefore, the price elasticity of demand for a consumer earning w and shopping at store charging p_i, is less than one.

$$\left|\frac{\partial x}{\partial p_i}\right| \frac{p}{x} \;=\; \frac{p_i}{p_i + tw} \;<\; 1 \qquad \forall\; w \in [a, b],\;\; p \in (p_\ell, r]$$

This gives us the following result regarding the pricing strategy of a firm that targets uninformed consumers.

Lemma 1.A.1: *If a firm does not target informed consumers, then it will charge a price equal to the consumers' reservation price,* $p_h = r$.

Proof: Let us assume the above statement to be true. Then the derivative of the profit function with respect to price from above is positive for any value of p below the reservation price. Setting the above differential to be greater than zero and dividing through by quantity we have:

$$1 + \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i} - \frac{1}{q_i}\frac{\partial v}{\partial q_i}\frac{\partial q_i}{\partial p_i} \quad = \quad 1 - |\epsilon| + \frac{1}{q_i}\frac{\partial v}{\partial q_i}\left|\frac{\partial q_i}{\partial p_i}\right| \quad > \quad 0 \qquad \forall \;\; p_i \leq r$$

Where ϵ is the price elasticity of demand. For the above relation to be true a sufficient but not necessary condition would be for the price elasticity of demand for a firm not charging the minimum price to be less than or equal to unity. Given that uninformed consumers do not know the distribution of prices, a change in the price of a high-priced firm does not impact consumer search decision, so a firm does not lose any customers by raising its price. From our examination of consumer demand we have shown that the demand for each individual uninformed consumer is inelastic for any price below the reservation price, therefore a high-priced firm faces inelastic demand up until the consumers' reservation price. Using these two results we have that if a firm chooses to price above the minimum available price, it would raise its price to the consumers' reservation price.

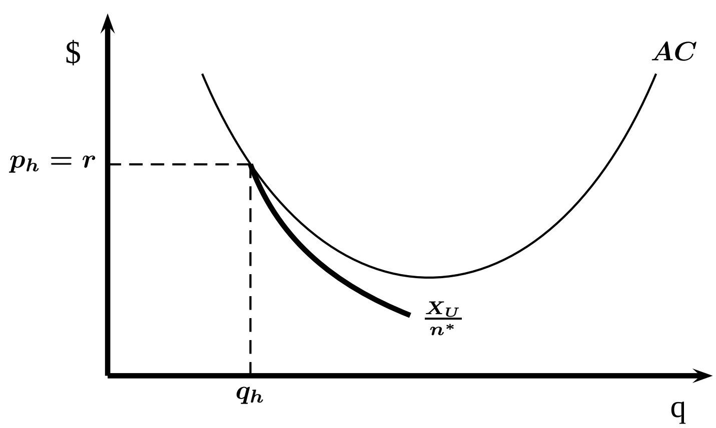

Figure 1.10: One Price Equilibrium

Lemma 1.A.2: *In a two-priced outcome where both types of firms are located on the average cost curve, the low-priced firms charging $p_\ell = p_{min}$ represents an equilibrium pricing strategy.*

Proof: As we argued above we must have that $p_h = r$. Now let us consider an equilibrium such that $r > p_\ell = p_{\min}$. For this to be an equilibrium it must be that there is some positive level of demand from informed consumers, $X_I > 0$. From our zero profit condition we have that both firms have to be on the downward sloping part of their average cost curve, with the low-types at the minimum point of the AC curve.

Let us consider a low-priced firm deviating to a price slightly above $p_\ell = p_{\min}$. Since we have that uninformed consumers can not observe this deviation, this change in price would not induce any additional search from consumers. In addition, since informed consumers know all prices in the market perfectly, they have perfectly elastic demand. Therefore, the deviating firm would lose all of its informed consumers and only face demand from uninformed consumers randomly choosing where to shop. As we argued above, uninformed demand for each firm is below the AC curve at all points except where $p = r$. Therefore, the deviating firm would earn negative profits.

Alternatively a deviating firm could choose to raise its price to $p = r$. Again, this does not induce any additional search by consumers. Since high-priced firms were operating on the AC curve, an additional high priced firm would shift the demand curve further to the left, earning all high priced firms negative profits, including the deviating firm.

Therefore, $p_h = r$ and $p_\ell = p_{\min}$ represent an equilibrium pricing strategy in a two-priced outcome.

Proof That $\bar{q}_\ell X_{U,h} - \bar{q}_h X_{U,\ell}$ Is Positive

When $X_I > 0$ we have that the following relations must hold for any value of n^* and n_ℓ^*:

$$\bar{q}_h \;=\; \frac{X_{U,h}}{n^*} \;<\; \frac{X_{U,\ell}}{n^*} \;<\; \frac{X_{U,\ell}}{n^*} + \frac{X_I}{n_\ell^*} \;=\; \bar{q}_\ell$$

Therefore we must have that the following hold:

$$\bar{q}_h \left(\frac{X_{U,\ell}}{n^*}\right) \;<\; \bar{q}_h \bar{q}_\ell \;=\; \left(\frac{X_{U,h}}{n^*}\right) \bar{q}_\ell \quad \Rightarrow \quad \bar{q}_h X_{U,\ell} \;<\; \bar{q}_\ell X_{U,h}$$

Which proves our relation.

Proof That $f(\beta)$ Is a Decreasing Function of β

Differentiating $f(\beta)$ from equation (1.19) we want to show that the following is negative:

$$\frac{df(\beta)}{d\beta} \;=\; \frac{\bar{q}_h \frac{dX_I}{d\beta}}{\bar{q}_\ell X_{U,h} - \bar{q}_h X_{U,\ell}}$$

$$-\;\frac{\bar{q}_h X_I \left[\bar{q}_\ell \left(\frac{\partial w_1}{\partial \beta} x_{U,h}(w_1) - \frac{\partial w_2}{\partial \beta} x_{U,h}(w_2)\right) - \bar{q}_h \left(\frac{\partial w_1}{\partial \beta} x_{U,\ell}(w_1) - \frac{\partial w_2}{\partial \beta} x_{U,\ell}(w_2)\right)\right]}{(\bar{q}_\ell X_{U,h} - \bar{q}_h X_{U,\ell})^2} \;<\; 0$$

Where $x_{U,i}(w_1)$ is demand from a consumer earning w_1 and shopping at firm i, and $x_{U,i}(w_2)$ is demand from a consumer earning w_2 and shopping at firm i. We know from our discussions above that the denominators in both terms are positive and that the numerator in the first term is negative. All that is left to show is that the argument inside the brackets in the numerator of the second term is positive. Rearranging the argument inside the brackets we need that:

$$\frac{\partial w_1}{\partial \beta}\left[\bar{q}_\ell x_{U,h}(w_1) - \bar{q}_h x_{U,\ell}(w_1)\right] - \frac{\partial w_2}{\partial \beta}\left[\bar{q}_\ell x_{U,h}(w_2) - \bar{q}_h x_{U,\ell}(w_2)\right] \quad > \quad 0$$

We know that the terms inside the brackets are positive from our inelastic demand. Coupled with our results above that have shown $\frac{\partial w_1}{\partial \beta} > 0$ and $\frac{\partial w_2}{\partial \beta} < 0$, we have that the above inequality is true.

Therefore, $f(\beta)$ is decreasing monotonically with β.

Proof That $\bar{q}_\ell X_{U,h} - \bar{q}_h X_{U,\ell}$ Is Increasing with $(b-a)$

In showing this result we consider a change in the denominator defined above, which we call D, with a mean preserving spread of the range of income. Such a change is given by:

$$\Delta D = \bar{q}_\ell X_{U,h}(a-\sigma, b+\sigma) - \bar{q}_h X_{U,\ell}(a-\sigma, b+\sigma) - [\bar{q}_\ell X_{U,h}(a,b) - \bar{q}_h X_{U,\ell}(a,b)]$$

Rearranging the above equation and simplifying, we need the following to hold:

$$\bar{q}_\ell \left(\int_{a-\sigma}^{a} \frac{w}{p_h + tw} \; dw + \int_{b}^{b+\sigma} \frac{w}{p_h + tw} \; dw \right) - \bar{q}_h \left(\int_{a-\sigma}^{a} \frac{w}{p_\ell + tw} \; dw + \int_{b}^{b+\sigma} \frac{w}{p_\ell + tw} \; dw \right) \quad > \quad 0$$

Which we have shown to be true in our proof that $\bar{q}_h X_{U,\ell} < \bar{q}_\ell X_{U,h}$, above. Therefore, the denominator of $f(\beta)$ is increasing with a mean preserving spread of $(b-a)$.

CHAPTER 2

COMMUTING AND SHOPPING: DETERMINANTS OF CITY INCOME STRUCTURE

Abstract

This chapter demonstrates how firm pricing strategy and determinants of household location can interact to determine city structure. In this city, consumers and firms live on a continuous line interval. The model consists of two types of firms; many high-cost perfectly competitive "Corner Stores" located in the Central Business District, and one large low-cost "Superstore", choosing its price strategically. The chapter shows how the shopping habits of the consumer population, as determined by the relative price of the Superstore and the Corner Stores, can contribute to the various income segregation outcomes described in previous literature. In addition we consider the impact of city population structure on the pricing decision of a monopolist facing a competitive fringe.

2.1 Introduction

The purpose of this paper is to consider how the pricing decision of large retailers and Supermarkets (we group both together and refer to them as Superstores) interact with household location and shopping decisions as well as city structure. We look to build on the existing regional economics literature by extending the factors that impact the location choice of consumers to include their access to affordable shopping. Previous literature has focused on commuting costs and desire for space, access to neighborhood specific amenities, including high quality education, and the desire to locate close to job centers (see for example Guerrieri et al. (2010), Wasmer and Zenou (2002)). Though we acknowledge that all of these are very likely important factors in the decision of households on where to live, we argue that access to shopping is also an important factor, and we look to quantify the impact of shopping behavior on location while taking the impact of the above determinants as given. We go beyond previous spatial economics literature by endogenizing firms' role in the agglomeration forces within the city by considering the pricing strategy of firms when facing different city structures and a heterogeneous consumer population. Our recognition of the interdependence of firm strategy and consumer behavior in a spatial setting looks to connect the existing work in regional economics with the industrial organization literature.

One of the first attempts at understanding the economics of city structure is in the work of the pioneers of the mono-centric city model, Alonso (1964), Mills (1967) and Muth (1969), (collectively referred to as AMM). They argue that the two forces that determine the choice of location within a city are commuting costs, $\boldsymbol{t}$, and demand for space, $\boldsymbol{q}$. The ratio of these two factors, $\frac{\boldsymbol{t}}{\boldsymbol{q}}$, is said to be what determines city structure. The argument is that higher income households have a higher opportunity cost of time, and in order to cut down on commuting costs would prefer to live closer to the Central Business District (CBD), where presumably most economic and social activities are centered. At the same time higher income families are more likely to have a greater demand for space, drawing them out to the more spacious communities in the suburbs. These two opposing forces can result in different city structures depending on the extent of income inequality within the city, as well as the magnitude of demand for space across households with different levels of income. The authors also argue that the structure of the city would depend on the transportation infrastructure of the city, which can impact commuting costs and the availability of affordable space in the city center.

Though a very simple and intuitive model, the AMM explanation of city structure is not supported by the empirical data available. Wheaton (1977) tests the assumptions of the AMM model using U.S. household data and finds that the cost of transportation relative to demand for land does not vary across income, which would suggest that the AMM theory does not explain the various structures of income segregation that we observe. Wheaton's finding is corroborated by a more recent study, which finds that the elasticity of demand for land area is very low relative to the elasticity of demand for travel cost per mile, Glaeser et al. (2000), making it difficult for the AMM model to explain why in cities like Los Angeles and Detroit high-income families would choose to locate outside of the city center.

Brueckner et al. (1999) offer an alternative explanation using a model that links the location of different income groups to the spatial pattern of various types of amenities within a city. Using the assumption that the marginal valuation of amenities rises significantly with income, the authors argue that in cities like Paris where the amenities are concentrated in the city center, the rich tend to locate themselves closer to the central business district. While in cities like Detroit, where amenities are more spread out, the center of the city is mainly comprised of low-income families that cannot afford the commuting costs of living in the suburbs. In addition the paper looks to endogenize the amenities available within a city by including the neighborhood income, as well as a parameter for exogenous amenities, in the consumers' utility function. The argument is that certain types of amenities are directly related to the income of the population that live within that neighborhood, therefore wealthy consumers prefer to live in neighborhoods with high average income.

Guerrieri et al. (2010) expand on this argument through a model of "neighborhood effects". Using a continuous time model with no transportation costs they demonstrate under what conditions a segregated outcome can come about with high-income consumers located in the city center and poorer households in the periphery. Using the same argument as Brueckner et al. (1999) that all households prefer to live in richer neighborhoods, they show that poor neighborhoods located closer to the border of the high-income neighborhoods are more likely to experience price inflation during periods of economic boom. The price inflation in their model is partly driven by a process of gentrification, when higher income households expand into the low-income neighborhoods located at the border of the wealthy neighborhood in order to increase their housing consumption. Using data on intra-city housing prices in the U.S., the writers show that their results hold even after controlling for transportation costs and distance to natural amenities, factors independent of their "neighborhood effects".

This chapter adds to the above discussion by considering shopping as a factor in a household's choice of location. We approach this problem very much aware that the choice of where to live is a personal one, and is driven by a variety of factors, some of which are rational and economic, while others are more behavioral and based on personal experience. Even within the rational set of incentives there are a wide variety of important factors that can impact where families choose to live. Families with young children will very likely base their decision of location partly on availability of quality education, as is shown by Selod and Zenou (2003), and is one of the underlying concepts of the endogenous amenities of Brueckner et al. (1999). While young urban professionals will more likely be concerned with locating close to job centers, as is argued by Wasmer and Zenou (2002). Most households also consider access to local amenities, public services and the crime rate when choosing their neighborhoods. Some of these factors can be interpreted as consequences of city structure, like the crime rate at a city center, while others have been addressed in previous papers, such as Brueckner et al. (1999) use of amenities to explain segregation. The purpose of our paper is to isolate the impact of access to affordable shopping by taking these other variables as given. Clearly any real world consideration of our results would need to consider them within the context of all these other factors.

We look to contribute to the above discussion on the determinants of city structure in two ways. First we look to endogenize a cause of household location that goes beyond commuting costs by including access to affordable shopping in the consumer's decision process. Second we allow the Superstore to behave strategically, therefore we take into account the firm's side of the problem in our analysis. We attempt to address income segregation and city structure in a semi-general equilibrium framework[1]. In solving the consumer's problem, we look to determine to what extent shopping behavior drives household choice of location. We believe that since families tend to spend a significant portion of their income on household consumption, access to affordable shopping can be an important determinant of where families choose to live. We build our agglomeration model with the simplifying assumption that there are only two opposing factors that determine household choice of location, and in effect the rental rate across city neighborhoods. First is the desire to locate close to work, which in our model is the center of the city, or the Central Business District (CBD), in order to cut down on the cost of commuting. Second is the desire to live close to affordable shopping,

[1] Using the term "general equilibrium" is not fully accurate since we do not consider the labor market in our model, we take the hiring activities of firms as given. We are considering equilibria involving the other three aspects of the market, consumers, producers and the rental market within the city (we are taking agricultural rent at the end of the city as fixed).

where households will spend most of their earned income. We offer the consumer the option between convenient, but relatively expensive, shopping available at the Central Business District, and more affordable shopping in the outskirts of the city that would require additional money to be spent commuting in order to access. Households will simultaneously choose where to live and where to shop, and ultimately the tradeoff between the availability of affordable shopping with the convenience of access to the Central Business District will determine the possible equilibrium outcomes within our model.

We begin building our theoretical model with a base case that involves a homogenous consumer population living on a line interval. The Central Business District and the convenience stores (Corner Stores) are located at one end of the line, and affordable shopping (Superstore), is located at the other end. After we consider a base case with homogeneous consumers, we introduce a heterogeneous consumer population to the model and consider how are results differ. Within this framework we derive the conditions under which cities become segregated or fully integrated. We consider the impact these different outcomes have on the Superstore's pricing strategy and profits, as well as considering the impact on the welfare of the consumer population.

2.2 The Base Case With Homogenous Consumers

The main purpose of our theoretical model is to consider the connection between the location choice of households and the pricing decision of firms within a simple city framework. In each iteration of our model below we will solve for where households choose to live and shop, taking prices in the city as given. Then we consider how the decisions made by households impacts the demand faced by firms and ultimately their choice of price. For the base case of our City Model we consider a city containing a homogenous consumer population earning wage w. At first the only option for the consumer is to perform their shopping after work at the Corner Stores located in the Central Business District (CBD). We will then introduce a Superstore into the model. The timing of the model is as follows:

1. Corner Stores choose whether or not to enter the CBD and then choose their price.
2. The Superstore decides whether or not to enter, chooses its location and its price.
3. Consumers make their location and shopping decisions.

In regards to the timing above, one can argue that on a short term basis firms have more flexibility in choosing their price than consumers do in choosing their location, therefore we should have the Superstore choosing its price after consumers choose their location. We justify our timing above in two ways. First, we believe it is possible that in the long run consumers choose their location partially based on the prices offered by stores in various locations, therefore consumers would be reacting to the Superstore's choice of price. Second, although it might be seen as less realistic from a timing perspective, the Superstore choosing its price before the consumers choose location is more intuitive from a strategic perspective. It is more likely that the Superstore is aware of the impact of its price on consumers' choice of location rather than the reverse. In fact, as we will show in our analysis below, consumer welfare is monotonically decreasing with Superstore price, therefore if consumers moved first they would simply choose a city structure that would force the Superstore to charge the lowest price. We don't believe that this is a very realistic strategic setup and therefore choose our timing as above.

Throughout this paper we assume that zoning laws limit firms to the Farmland and CBD and that rents go to landlords, who are absentee.

The City

The spatial representation of our city is given by a straight line interval starting from the Central Business District (CBD) and going outwards[2]. The CBD is where all consumers work and where our Corner Stores are located. The other end of the city is Farmland, and represents the furthest out any consumer will choose to live. The city is closed such that the population size and composition are exogenously determined. We normalize the geographical and population size of the city to 1, therefore we will not include population density (population density being the ratio of population size to geographical space) in this model[3]. We use $z \in [0, 1]$ to represent the location of any given consumer or firm. The CBD will be used as the starting point of the line, $z = 0$, and the Farmland will be the outer point of the city, $z = 1$.

Figure 2.1: The City

The retail rental price at any given point in the city is determined competitively[4]. We

[2]Our choice of city shape is similar to Brueckner et al. (1999) and makes it easier to include rental lines in the model. For a treatment of spacial competition in a circular city or a bi-directional line please see Salop (1979).

[3]Population density within this framework does not impact the decision of firms and consumers.

[4]Note that this only refers to the rental price for housing, we will discuss commercial rental lines below.

will normalize the rental line by taking the rental rate at the Farmland as exogenously determined by the external land market, $r(1) = a$, and allowing the rest of the market to clear.

There are two types of transportation cost in our model, the money cost of commuting, t and shopping, g. These are both per-unit costs that need to be multiplied by distance from destination to determine the full cost to the consumer.

For the sake of focusing our question and keeping our analysis simple, we make the following assumptions about our model City:

(i) The distribution of the population across the city is uniform. In other words we assume that there are no high density points in the city.

(ii) Stores have to pay rent based on their location, but do not take up any actual space, therefore a consumer can live where a firm is located.

(iii) The money cost of commuting to work is higher than the the money cost of traveling to the store, $t > g$. This is an intuitive assumption as consumers commute to work daily, while shopping might happen on a weekly basis.

(iv) Zoning laws restrict the size of stores at the CBD to 1 unit of the good. There are no size constraints for stores located at the Farmland.

(v) There are no outside options in our model. Consumers must choose a place to live and a firm to shop at given what is available to them from our setup of the model.

(vi) Firms compete on price.

Based on the timing of our model and the assumptions above, an **equilibrium** in our model is characterized by the following:

1. No firm has an incentive to change their choice of entry, location or price, taking the strategy of other firms as given.

2. No consumer has an incentive to change their shopping and location choices, taking the decision of other consumers and the strategy of all firms as given.

As we stated above, we begin with a city where only Corner Stores exist, with a homogeneous consumer population, all earning wage w. All consumers work at the CBD and will have to commute from their homes to work, requiring them to pay a monetary cost of commuting equal to tz. Consumers shop after work at the Corner Stores located near the office, so do not have to pay any extra commuting costs related to shopping.

Consumers' Problem

Consumers spend all of their net earnings on one good, x, providing them with utility measured by $u(x)$, which we assume to be strictly increasing. They choose where to live based on the rental cost of the location and the costs of commuting to work and shopping. Therefore, the maximization problem for a representative consumer living at point z and shopping at the Corner Stores located at the CBD is given by:

$$\max_x \quad u(x)$$

$$s.t. \quad p_c x = w - tz - r(z) \tag{2.1}$$

Equation (2.1) is the usual budget constraint, we do not include any non-wage income, since we are only interested in how changes in wage affects consumer choice.

Here p_c is the price charged by the Corner Stores and $r(z)$ is the rent paid by the consumer located at z and shopping at the CBD.

Constraint (2.1) is binding by the monotonicity of our utility function. Given that we are only concerned with the consumption of one good and since there is no uncertainty in our model, we can solve directly for the consumers' optimal choice of x. This is analogous to using a linear utility function, which, without loss of generality, is what we will work with for the rest of this paper. Therefore, the indirect utility for our representative consumer is given by:

$$v_c = \frac{w - tz - r(z)}{p_c}$$

Consumers choose z to maximize v_c. Therefore, given our homogenous consumer population, $r(z)$ must be such that the utility level for consumers is constant across the city line. We can rearrange the equation above to solve for the rental rate at any point z on the city line interval while treating v_c as fixed with respect to z.

$$r(z) = w - tz - p_c v_c \tag{2.2}$$

We can think of this equation as the "rental indifference line". Since the utility of all consumers is independent of location in equilibrium, we can fix the utility for consumers living in our model city by the utility level of the consumer living at the Farmland, $z = 1$, paying a fixed rent of $r(1) = a$:

$$v_c(z = 1) = v_c = \frac{w - t - a}{p_c} \tag{2.3}$$

Combining equations (2.2) and (2.3) we can rewrite the rental formula along our city line:

$$r(z) = t(1 - z) + a \tag{2.4}$$

The rental cost in our city is a decreasing function of z, as consumers move out towards the Farmland they are able to save money on rent equivalent to the extra commuting costs they must pay to get to work.

Corner Stores

All Corner Stores are located in the CBD and behave competitively without any fixed costs or other barriers to entry. As we stated above, we assume zoning restrictions in the center of the city limit store size to one unit of the good, $x = 1$. Given this market structure, the higher the demand for the Corner Stores the higher the number of stores that will be able to operate at the CBD. Throughout this paper we will assume that the commercial rental market is independent of the housing rental market, therefore the cost structure of the firms in our city is independent of the consumers' rental lines. The Corner Stores face a constant marginal cost, $\bar{c}$, and compete on price, therefore their price is equal to their constant marginal cost, $p_c = \bar{c}$.

We can plug this price into the indirect utility from equation (2.3) to determine the resulting utility level in our base case model with the Corner Stores as the only option for our consumer population:

$$v_c = \frac{w - t - a}{\bar{c}} \tag{2.5}$$

Superstore

Now we will introduce the Superstore into our city model. Similarly to the Corner Stores, the Superstore faces a constant marginal cost, $\hat{c}$. The Superstore can choose to enter at the CBD without any fixed costs, but then its marginal cost will be equivalent to that of the Corner Stores (and it will be limited to unit capacity by zoning laws). Alternatively the Superstore can pay a positive fixed cost k and locate in the Farmland. If the Superstore locates in the Farmland it is able to take advantage of its more remote location and larger size to bring down marginal costs, therefore in the Farmland $\hat{c} < \bar{c}$. As we stated in our assumptions above, we do not assume any size restriction in the Farmland, therefore the Superstore is free to choose its capacity.

Proposition 2.1: *If a Superstore chooses to enter the market, only one will enter, and that store will choose to locate out in the Farmland.*

Proof: This is a direct result of our assumption on zoning restrictions as well as the cost structure described above. If the Superstore chose to locate at the CBD its marginal cost would be equivalent to that of the Corner Stores (in addition to not having fixed costs), therefore it would price at marginal cost and earn zero profits. Given the lower marginal costs associated with locating in the Farmland, if the Superstore locates at $z = 1$ it will be able to undercut the Corner Store price and attract positive demand. As long as fixed costs are small enough this would lead to positive profits[5]. Therefore, we have that if the Superstore chooses to enter it would locate out in the Farmland[6].

[5] In the analysis that follows, and our discussion of Superstore profits, we will inherently assume that if the Superstore attracts positive demand, the profits calculated will be enough to cover the Superstore's fixed costs.

[6] The strict assumptions we make on the laws of the city lead to the specific structure we have described here. Although there are clearly examples of large discount stores located in the center of cities, Superstores are much more likely to locate in the outer suburbs. Our specific model structure allows us to focus our question on the relative abundance of different stores at either ends of the city.

As for the number of Superstores, any level of fixed costs for entering as a Superstore, combined with our zoning restrictions, assures us that only one Superstore will ever enter in the Farmland. Otherwise we would have a duopoly under Bertrand competition, resulting in negative profits.

Therefore, the Superstore enters as a monopolist facing a competitive fringe (the Corner Stores), and chooses its price strategically.

If the consumer living at point z chooses to shop at the Superstore their maximization problem becomes:

$$\begin{aligned} &\max_{x} \quad u(x) \\ &s.t. \quad p_s x = w - tz - g(1-z) - r_s(z) \end{aligned}$$

Where p_s is the price charged by the Superstore and $g(1-z)$ is the commuting cost of living at point z and shopping at the Superstore. The resulting utility level is:

$$v_s = \frac{w - tz - g(1-z) - r_s(z)}{p_s}$$

Using the same argument as above we must have that the utility for consumers shopping at the Superstore is independent of location. Solving for $r_s(z)$ in the equation above we can determine the rental line faced by consumers that shop at the Superstore:

$$r_s(z) = w - tz - g(1-z) - p_s v_s \tag{2.6}$$

Now let us consider the consumer living at $z = 1$. Whether they shop at the Corner

Stores or the Superstore depends on the relative price of the two. But for the Superstore to exist in our city it must at least attract the consumer living in the Farmland, otherwise it will have zero sales and, due to its fixed costs of entry, would not choose to enter the market. The utility of the consumer living in the outer part of the city is given by:

$$v_s(z=1) = v_s = v = \frac{w-t-a}{p_s} \tag{2.7}$$

As before, equilibrium requires that the utility level is constant across our city in equilibrium, independent of where consumers shop, so we have that the utility for all consumers must be equal to that of the consumer at $z = 1$ as given by equation (2.7) above.

Combining equations (2.6) and (2.7) we can solve for the rental line for consumers in our city that shop at the Superstore:

$$r_s(z) = (t-g)(1-z) + a \tag{2.8}$$

The rental line for consumers shopping at the CBD is still the same as the general form we described in equation (2.2) above. Plugging in for utility from equation (2.7) we can solve for the rental line faced by consumers shopping at the Corner Stores in terms of the price charged by the Superstore:

$$r_c(z) = w - \tfrac{\bar{c}}{p_s}(w-t-a) - tz \tag{2.9}$$

If the Superstore enters the market there are two possible structural outcomes.

1. The Superstore captures the entire market, this would mean that all consumers

shop at the Farmland and the rental line for the entire city would be given by equations (2.8) above. In our analysis below we will consider under what condition on our parameters such a scenario would be an equilibrium in our model.

2. The Superstore will price in such a way as to capture a portion of the city's population as determined by a point in the city such that above that point all consumers shop at the Farmland. This scenario is depicted in figure 2.2 below.

Though we have not yet solved for the Superstore's price, we can clearly see that the slope for this rental line relative to z is equal to $-t$, which allows us to draw the graph for the new rental lines across our city after having introduced the Superstore.

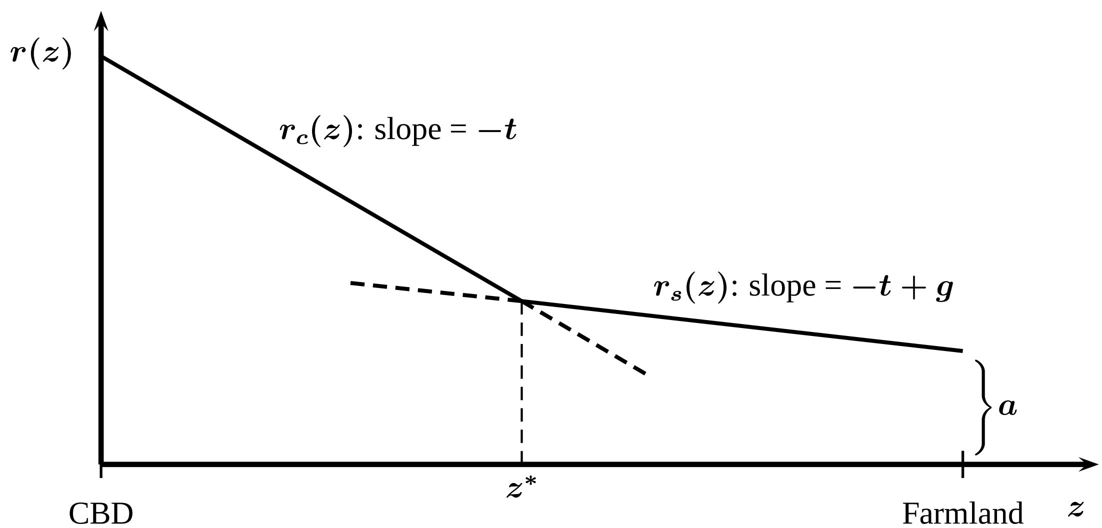

Figure 2.2: Rental Lines

Depending on the Superstore's choice of price, there is a point along the city line interval such that the consumers living to the right of that point will choose to shop at the Superstore. That point in the line, which we will call z^*, is determined by the value of z such that the two rental lines as given by equation (2.2) and (2.6) are equal, as shown in Figure 2.2 above. Setting the two equations equal to each other and solving for z we have:

$$1 - z^* = \frac{1}{\lambda}\left(\frac{\bar{c}}{p_s} - 1\right) \qquad \text{s.t.} \quad z^* \geq 0 \tag{2.10}$$

Where $\lambda = \frac{g}{w-t-a}$ is a transportation parameter that measures shopping cost as a percentage of the disposable income of the representative consumer living at $z = 1$. We define disposable income as earned wages net of the cost of commuting to work and rent. We can see from figure 2.2 and equation (2.10) that the demand faced by the Superstore is determined by the point z^*. As the Superstore increases its price, p_s, the point at which consumers switch from shopping at the Corner Stores to the Superstore increases, decreasing the number of consumers that pay the additional cost to shop at the Farmland.

Since we have assumed the Superstore has a cost advantage over the Corner Stores, $\hat{c} < \bar{c}$, if the Superstore enters the market it will always choose a price lower than $\bar{c}$ and capture a positive fraction of the market, that is $1 - z^* > 0$. If $1 - z^* = 1$ the Superstore would capture the entire market and would not have any incentive to lower its price, giving us the following lower bound for the Superstore's choice of price:

$$p_s = \frac{\bar{c}}{1 + \lambda} \tag{2.11}$$

The Superstore can drive out the Corner Stores from the city by providing a significant price discount relative to the transportation cost needed to commute out to the Farmland, represented here by λ. Interestingly, in low-income communities shopping and commuting costs represent a higher percentage of disposable income, high λ. A higher lambda would impose a tighter constraint on the Superstore's price in inequality (2.11) above, making it more difficult for the Superstore to drive out the Corner Stores[7]. Though not obvious, this result is somewhat intuitive. Low-income consumers find it relatively more costly to travel to the store with the lower price, forcing those that live too far away from the Farmland to shop at the more expensive, but more convenient Corner Stores. The role of income in the condition in equation (2.11) raises the question of what happens in communities where there are both rich and poor households. We will consider that question in detail in the next section when we introduce heterogeneous consumers into our model.

For now we turn to the choice of p_s by the Superstore in our current framework.

Superstore's Problem: Plugging in our values for v and z^* into the Superstore's profit function we have the Superstore's optimization problem in terms of its choice of price and our model parameters:

$$\max_{p_s} \quad \pi_s = (1 - z^*)v[p_s - \hat{c}] - k \;=\; \frac{g}{\lambda^2}\left(\frac{\bar{c}}{p_s} - 1\right)\left(1 - \frac{\hat{c}}{p_s}\right) - k \qquad (2.12)$$

$$s.t. \quad p_s \;\geq\; \frac{\bar{c}}{1+\lambda}$$

Before we go on to explicitly solve for the Superstore's price we can use our condition from equation (2.11) along with the objective function above to portray the relationship between the Superstore's choice of price and its profits. In the figure below, and in our analysis that follows, we take $k \rightarrow 0$ trivially, since any positive level of

[7] This observation is related to the results we present in the first chapter of this thesis, where now we are able to put the problem in a spatial setting and speak of the transportation costs of consumers rather than the costs of obtaining information.

fixed costs would deter an additional Superstore entering the market.

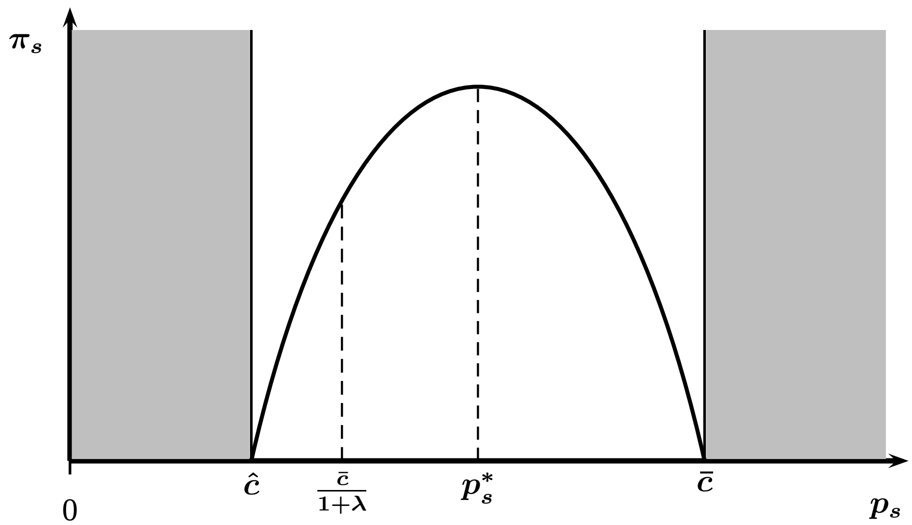

Figure 2.3: Superstore's Profit

The profit curve in figure 2.3 is drawn assuming an interior solution, the optimal price p_s^* is above the cutoff point identified in equation (2.11). If we had drawn the figure so that the cutoff point was above p_s^* then we would have had a corner solution: the Superstore would have chosen $p_s = \frac{\bar{c}}{1+\lambda}$ and captured the entire market.

In our figure above, the optimal price for the Superstore is p_s^*, which maximizes the Superstore's profit function in equation (2.12). Maximizing the objective function with respect to p_s we solve for the interior solution to the Superstore's problem. As we might expect the Superstore's choice of price is a function of the marginal costs of the two types of firms:

$$p_s^* = \bar{c}\left(\frac{2\hat{c}}{\bar{c}+\hat{c}}\right) \tag{2.13}$$

Since we have assumed that the marginal cost of the Superstore, $\hat{c}$, is lower than that of the Corner Stores, the term in the parenthesis is a positive fraction less than one. Therefore, we have that $p_s < p_c = \bar{c}$, as we expected. Interestingly, in this case of our model, consumer income, w, and the transportation costs in the city, t and g have a multiplicative effect on the Superstore's profits, and do not impact its choice of price. When we introduce consumers heterogeneous in income in the next section transportation costs as well as income impact the Superstore's choice of price as well as its profits.

Plugging in the above price into our equation for z^* above we can solve for the portion of the consumers that choose to shop at the Superstore:

$$1 - z^* = \begin{cases} \frac{1}{\lambda}\left(\frac{\bar{c}-\hat{c}}{2\hat{c}}\right) & \frac{\bar{c}}{\hat{c}} < 1 + 2\lambda \\ \\ 1 & \text{Otherwise} \end{cases} \tag{2.14}$$

Which is always positive, therefore the Superstore will enter the market and attract a proportion of the consumer population. As we argued above, whether or not it captures the entire market depends on our parameters. The condition for the existence of Corner Stores is $\frac{\bar{c}}{\hat{c}} < 1 + 2\lambda$. In order for a Corner Store to be able to survive in a market when a Superstore chooses to enter the difference between the costs of the two types of firms must be low relative to the extra commuting cost required to shop at the Superstore. This is in line with what we would expect in practice. In cities with large discount stores and low transportation costs it would be less likely that we would find a Corner Store. While in cities with more significant transportation costs and fewer large retailers, it is more likely that we would find small convenient Corner Stores.

Substituting in the resulting market price from equation (2.6) above into the Superstore's objective function in equation (2.12) we calculate the resulting level of profits for our Monopoly:

$$\pi_S = \frac{g(\bar{c} - \hat{c})^2}{4\bar{c}\hat{c}\lambda^2}$$

The profit of the Superstore is positive, and an increasing function of the marginal cost of the Corner Stores, $\frac{\partial \pi_s}{\partial \bar{c}} > 0$ (and decreasing with the marginal cost of the Superstore). As we would expect, profits for the Superstore decrease when the transportation costs in the city increase ($\frac{\partial \pi_s}{\partial t} < 0$, $\frac{\partial \pi_s}{\partial g} < 0$). An interesting point here, usually transportation costs would impact a business' profits directly through increasing their variable costs of bringing supplies to the stores. Here we have demonstrated that stores can also lose out from high transportation costs due to a decrease in demand, due to a decrease in purchasing power for the consumer. Big discount stores, or Superstores, can mitigate the effect of high transportation costs on their market share through helping reduce those costs for their customers. One example is when companies such as Tesco, Ikea and Walmart provide free buses that take customers from city centers out to their more remote store locations.

We determine the impact of the Superstore on the welfare of the consumers by comparing the indirect utility of the consumer population from equation (2.5) with the indirect utility after we introduce the Superstore in equation (2.7). Since we have shown that the price of the Superstore will be lower than that of the Corner Stores in equilibrium, we can clearly see that the indirect utility of consumers increases when the Superstore enters the market. Plugging in our calculated p_s^* into equation (2.7) we solve for the consumers' utility after the Superstore enters the market:

$$v = \frac{(\bar{c}+\hat{c})(w-t-a)}{2\bar{c}\hat{c}} = \frac{g(\bar{c}+\hat{c})}{2\bar{c}\hat{c}\lambda} \tag{2.15}$$

Subtracting the utility to consumers before and after introducing the Superstore we can quantify the change in consumer welfare due to the Superstore:

$$\Delta v = \frac{g(\bar{c}-\hat{c})}{2\bar{c}\hat{c}\lambda} \tag{2.16}$$

The positive impact of the Superstore on the consumers is an increasing function of the cost differential between the two types of firms and the purchasing power of the consumers themselves.

This positive change in consumer welfare brought about by the entry of the Superstore is in line with the empirical findings of Hausman and Leibtag (2007). In their paper they demonstrate that consumers benefit significantly from the entry of Superstores (or what they refer to as Supercenters) into a market. Though they do not consider a geographical model, they do argue that restrictive zoning laws that do not allow Superstores to enter certain markets can end up hurting consumers, an argument that our results above support.

Finally we consider the impact of the entry of the Superstore on the rental market. We found that the rental rate along the city line interval without the Superstore is given by equation (2.4). When we introduced the Superstore the rental line for consumers shopping at the Superstore is given by equation (2.8), which is clearly less than or equal to the old rental line for all values of z. We can determine the new rental line for consumers shopping at the Corner Stores by plugging in p_s from equation (2.6) and

$v_c = v$ from equation (2.7) into our general rental line for shopping at the corner store in equation (2.2).

$$r_c(z) = w - \frac{g}{\lambda}\left(\frac{\bar{c}+\hat{c}}{2\hat{c}}\right) - tz \tag{2.17}$$

Which we can show to be below our old rental line for all values of z. In figure 2.4 below the grey area represents the loss to the absentee landlords due to the Superstore's entry into the city.

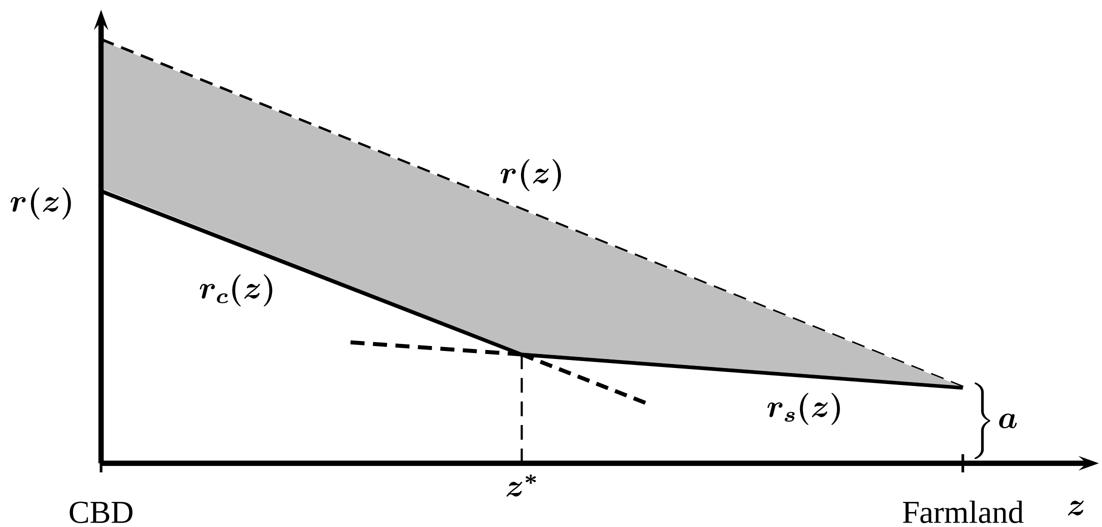

Figure 2.4: Superstore's Impact On Rental Lines

Introducing the Superstore into our simple model drives down rental costs all across our city except for at the Farmland, where the rental cost is determined by the world market. This is a very subtle but important result. Firstly, as we would expect, introducing the Superstore results in a relative increase in the rent at locations located closer to the Farmland, since the rent in the rest of the city has decreased while it has stayed the same at $z = 1$. This decrease in rent is weakly increasing as we move away from the

Farmland. Secondly, it is clear from above that the landlords are the only ones who lose out from the entry of the Superstore into the city. The only exception is the landlord who rents out to the Superstore, they continue to earn $r(z = 1) =$ a. This final result is necessary for the structure we just described to be an equilibrium, if the owner of $z = 1$ would have lost out by the Superstore's entry then she would probably not have rented out to the Superstore to begin with.

As we have shown, the entry of the Superstore benefits the consumers in the form of higher utility, leaves Corner Stores indifferent earning zero profits, and leads to the Superstore earning positive profits. The only "losers" in our simple scenario are the landlords who are now earning lower rents on their property.

Now we will continue our analysis by introducing a heterogeneous consumer population to our model.

2.3 Consumer Demand With Households Heterogeneous In Income

In this section we extend our base case analysis by introducing two exogenously determined levels of income for our consumer population. We continue to normalize our city size and population to 1, but now our city will be made up of high-income consumers (type h) and low-income consumers (type ℓ). A proportion, α, of the consumer population are type h consumers and earn high wages, w_h, and $(1 - \alpha)$ are type ℓ and earn low wages, w_ℓ. We take α as given.

All consumers work at the CBD and have to commute from their homes to work, requiring them to pay a money cost of commuting equal to tz. As before, we first consider the case where only Corner Stores exist in our city, then we introduce the Superstore and allow consumers the option of shopping after work at the grocery stores located near the office, or paying the additional transportation cost, $g(1 - z)$, and shopping at the Superstore.

As we discussed above, the timing of our model is in the form of a Stackelberg leadership model, where the Superstore sets its price and consumers choose housing and shopping location in response. Therefore we must first determine how consumers would respond to different price outcomes before considering the Superstore's choice of price in the next section. In our analysis below we use the following definition:

Definition 2.1: *An integrated portion of the city is a segment of the city line such that both types of consumers would choose to live in that section of the city at the prevailing level of rent and retail prices.*

Consumers' Problem: When consumers only have the option of shopping at the Corner Stores, the city rental lines will be similar to our base case above:

$$r_i(z) = w_i - tz - \bar{c}v_{i,c} \qquad \text{for } \; i = h, \ell \tag{2.18}$$

As in our base case the utility for consumers within the same income class must be constant with respect to location, $\frac{\partial v_i}{\partial z} = 0$, therefore the slope of the rental lines with respect to z is equal to $-t$ and does not depend on income. This results in rental lines for the two types of consumers that are parallel, with their relative position determined by the difference in their wages and their consumption of the good x (in effect the level of their rental line at the CBD given by: $r_i(0) = w_i - \bar{c}v_{i,c}$). Based on this structure the only possible equilibrium outcome where both consumers live within the city is where the rental lines for the two types of consumers overlap, a fully integrated city with high and low-income consumers living side by side (otherwise, if the rental line for one type of consumer is above the other, that consumer type would have incentive to lower their rental bid, consuming more of the good x, increasing their indirect utility without giving up their choice of location).

If a type i consumer lives at the outer limit of the city, $z = 1$, then the indirect utility for all type i consumers will be equal to that representative consumer. In an integrated city structure both types of consumers are indifferent between living at the Farmland or anywhere else in the city, therefore we can represent the indirect utility of the two types of consumers similar to our base case.

$$v_i = \frac{w_i - t - a}{\bar{c}} = \frac{g}{\bar{c}\lambda_i} \qquad \text{for } \; i = h, \ell \tag{2.19}$$

λ_i is a parameter representing transportation costs, as we defined above, but now varies with income[8].

Plugging equation (2.19) into our general rental line in equation (2.18) above, we can solve for the rental line in the city for the two types of consumers.

$$r(z) = t(1 - z) + a \tag{2.20}$$

Notice that this rental line is independent of income and is identical to the rental line in our base case model before we introduced a Superstore.

Before we move on it is important to note that this equilibrium structure does not mean we are suggesting that without a Superstore a city will be fully integrated, in our simple model we are ignoring some other important factors that contribute to segregation that has been noted by previous papers, Brueckner et al. (1999) and the AMM models among others. What we are looking to identify in this analysis is the kind of

[8]Recall that $\lambda_i = \frac{g}{w_i - t - a}$, therefore $\lambda_h < \lambda_\ell$ by construction. This means that $v_h > v_\ell$.

agglomeration pressures that introducing a Superstore into a city can create. What we see at first is that without a cheaper outside option shopping seems to have a neutral effect on household choice of location. Given how we have constructed our model this is as we would expect. Without a Superstore households shop after work at the CBD, so shopping has the same impact as commuting, so in effect no additional impact. Now we go on to consider what happens when we introduce a Superstore into our city with heterogeneous consumers.

Superstore: As before consumers can choose to shop after work at the Corner Stores located at the CBD, or pay the additional transportation costs and shop at the Superstore located at $z = 1$. The rental line for a type i consumer located at z and shopping at the Superstore would be given by:

$$r_{i,s}(z) = w_i - tz - g(1 - z) - p_s v_{i,s} \qquad \text{for } \; i = h, \ell \tag{2.21}$$

As with the rental line for shopping at the Corner Stores the gradient of the rental lines with respect to z for the two types of consumers are identical and equal to $-t + g$.

As before the utility for type i consumers from shopping at the Superstore with shopping at the Corner Stores must be equal, $v_{i,c} = v_{i,s} = v_i$. Setting the rent lines for shopping at the Corner Stores equal to that of shopping at the Superstore we can solve for the point in equilibrium where consumer of type i switches from shopping at the CBD to shopping at the Superstore[9]:

$$z_i^* = 1 - \frac{(\bar{c} - p_s)v_i}{g} \qquad \text{for } \; i = h, \ell \qquad s.t. \quad z_i^* \geq 0 \tag{2.22}$$

[9]Note that this is only true in equilibrium, where no consumers have incentive to change their location and shopping decisions.

From equation (2.22) we can clearly see that since v_h is larger than v_ℓ by construction, we must have that the switching point for the type h consumers falls below that of the type ℓ consumers. That is, $z_h^* < z_\ell^*$.

Based on our results above we can set a restriction on the equilibrium structure of our city model with heterogeneous consumers after the entry of a Superstore.

Proposition 2.2: *Given the order of switching for the two types of consumers we must have that in equilibrium $r_{\ell,j}(0) \geq r_{h,j}(0)$ and $r_{h,j}(1) \geq r_{\ell,j}(1)$ for $j \in \{c, s\}$. In other words, high-income households will always weakly prefer to live at $z = 1$, while low-income households will always weakly prefer to live at $z = 0$.*

Proof: Let us assume the opposite is true, that there exists an equilibrium such that the rich outbid the poor near the CBD, $r_{h,j}(0) > r_{\ell,j}(0)$ for all $j \in \{c, s\}$, only the rich would live in the city center. But given the fact that the rich switch to shopping at the Superstore earlier than the poor ($z_h^* < z_\ell^*$) and that both types of consumers have the same slopes for their rental lines, that would mean the rental line for the rich would be higher than that of the poor across the entire city line, the poor would be homeless. But this would not be an equilibrium. First of all, the rich have no incentive to strictly outbid the poor across the entire city line since we have not included housing size in the consumers' utility function [10]. Secondly, by assumption (v), the poor would have incentive to bid up their rental lines until they are able to live somewhere within the city. Given the order of switching for the two types of consumers, as the poor raise their offer the rental line for the poor living near the CBD would overlap with that of the rich at least as quickly as that of the poor and rich living out in the Farmland, giving us

[10]this is a simplifying assumption we make in our model in order to focus on shopping behavior, other papers dealing with city structure have considered the impact of space preference on structural outcome, see AMM models.

$r_{h,j}(0) = r_{\ell,j}(0)$, violating our assumption above.

In the same way, if we assume that the poor outbid the rich at the Farmland then their rental line would be above the rich across the entire city, giving the rich incentive to raise their rental lines until $r_{h,j}(1) = r_{\ell,j}(1)$.

One of the consequences of this result is that the utility of the type h consumers does not depend on the rental lines across the city. This is due to the equilibrium condition that the utility of each type must be constant with respect to their location and shopping choice. The utility of all type h consumers will be equal to the utility of the type h consumer living at the point $z = 1$, where rent is constant, regardless of the resulting city structure. We will come back to this point in the discussion of the equilibrium level of consumer utility.

The above argument means that our model does not allow for the structural outcome where high-income consumers live in the city center and low-income families are segregated out in the periphery of the city. This is an unrealistic limitation within our model. That type of structure is commonly observed in the real world, especially in major European cities, Brueckner et al. (1999). One way to allow for such a structure would be to add a time component to transportation costs, making it more costly for high-income families to live away from their job location at the CBD[11]. If time costs were high enough we could show that their exists a segregated outcome with type h consumers living near the CBD.

Alternatively we could add to our model a second good that does not have any

[11]This would only be true if the time cost of commuting to work was greater than the time cost of shopping.

spatial considerations[12]. The result in Proposition 2.2 is due to higher income consumers switching to shopping at the Superstore at an earlier point on the city line-interval than the low-income consumers. Wealthy consumers buy more of the good, and therefore in utility terms attribute greater value to the lower price offered by the Supestore. If there existed a non-geographical good that represented a greater source of utility for higher income consumers relative to the poor, then it would be possible that the poor would value the lower price of the Superstore more than the rich, and therefore would be able to outbid the rich in the outer part of the city (in such a scenario their switching point would be before that of the rich)[13]. We will leave the formal analysis of these extensions for future work.

Given Proposition 2.2 we must have that in equilibrium the utility for all type h consumers is equal to that of the high-income consumer living at $z = 1$ and shopping at the Superstore:

$$v_h = v_{h,s}(z = 1) = \frac{g}{p_s \lambda_h} \tag{2.23}$$

Therefore, the rental line for the higher income consumers shopping at the Corner Stores and the Superstore are respectively given by:

$$r_{h,c} = w_h - \tfrac{g\bar{c}}{p_s \lambda_h} - tz \qquad r_{h,s} = (t - g)(1 - z) + a \tag{2.24}$$

From the second equation in (2.24) we have that the rental line for the rich shopping at the Superstore does not depend on the price of the Superstore. That is because we have

[12]We can think of these as goods that could be ordered online or over the phone and will be delivered directly to the home. Alternatively we could think of these as goods that are available all along the city line.

[13]Brueckner et al. (1999) use historical amenities, such as the Seine and the Louvre in Paris, to draw the rich to the city center.

fixed the rent at $z = 1$ to a. Changes in the Superstore's price impacts the high-income consumers through their rental line from shopping at the CBD. As the Superstore's price, p_s, increases the rent the rich are willing to pay to live at the center increases, leading to fewer high-income consumers that choose to shop out in the Farmland.

The level of utility and rental lines of the low-income consumers are less straightforward, they depend on the structure of the city, which in turn depends on the choice of price by the Superstore.

Different Cases of Consumer Demand

As we have shown above, the location at which consumer i switches from shopping at the Corner Stores to shopping at the Superstore is determined by z_i^*. We can see from equation (2.22) that as the Superstore increases its price, p_s, z_i^* increases, consumer i has to be located further away from the city center in order for them to prefer to shop at the Superstore. We will demonstrate below that changes in z_i^* for the two types of consumers, as well as our the proportion of type-h consumers α, results in a non-continuously differentiable demand curve with two kinks.

(1) $D_s^{(1)} = (1 - z_h^*)v_h \quad \Longleftrightarrow \quad p_s \in \left[\frac{\bar{c}}{1+\alpha\lambda_h}, \bar{c}\right)$

(2) $D_s^{(2)} = \alpha v_h \quad \Longleftrightarrow \quad p_s \in \left[\frac{\bar{c}}{1+\alpha\lambda_\ell}, \frac{\bar{c}}{1+\alpha\lambda_h}\right]$

(3) $D_s^{(3)} = [\alpha v_h + (1 - \alpha - z_\ell^*)v_\ell] \quad \Longleftrightarrow \quad p_s \in \left(\hat{c}, \frac{\bar{c}}{1+\alpha\lambda_\ell}\right)$

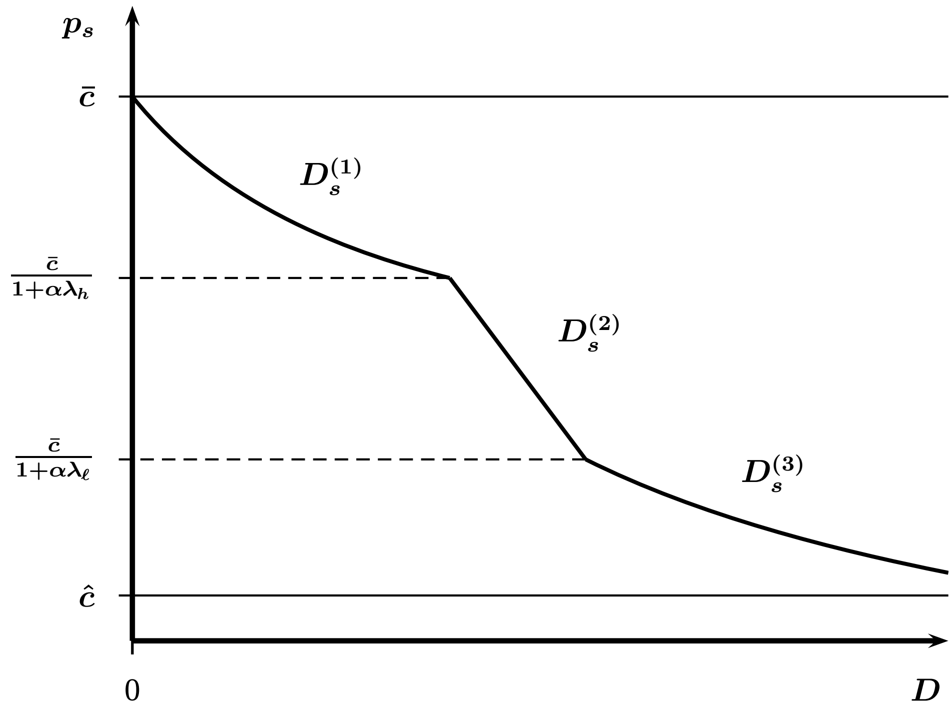

Figure 2.5: Demand Lines

In the above setup we have implicitly assumed that $\hat{c}$ is low enough relative to $\bar{c}$ to allow for the 2nd and 3rd cases of demand. In our analysis of demand below we will more formally define our assumption on the difference between the marginal cost of the two types of firms.

The demand line associated with $D_s^{(2)}$ is straight and drawn more steeply than the other two lines since in that price band demand is only increasing intensively as price decreases (no additional consumers are shopping at the Superstore, only the existing customers are buying more of the good), while in the other price ranges demand is increasing both intensively and extensively (the slopes of the lines are becoming less negative). In fact, we can show that the price elasticity of demand along $D_s^{(2)}$ is equal to -1, therefore because we assume constant marginal costs, along that section of the demand line the Superstore would increase its price until it reaches the kinked intersection between $D_s^{(1)}$ and $D_s^{(2)}$.

Since our demand line is non-continuously differentiable we will consider each of these demand lines separately. In each case of demand we will solve for the level of utility, which represents level of demand from each type of consumer, and rental bid lines in the city taking the price of the Superstore as given.

Case 1: $D_s^{(1)}$ $(\bar{c} > p_s \geq \frac{\bar{c}}{1+\alpha\lambda_h})$

In the first section of our demand line, the Superstore's price is high enough to push the switching points of the two types of consumers to a point that is further out in the city than the proportion of low-income consumers, $1-\alpha$. This means that the rental lines for the two types of consumers shopping at the CBD are overlapping, the city is integrated close to the CBD, while only high-income consumers live out in the Farmland.

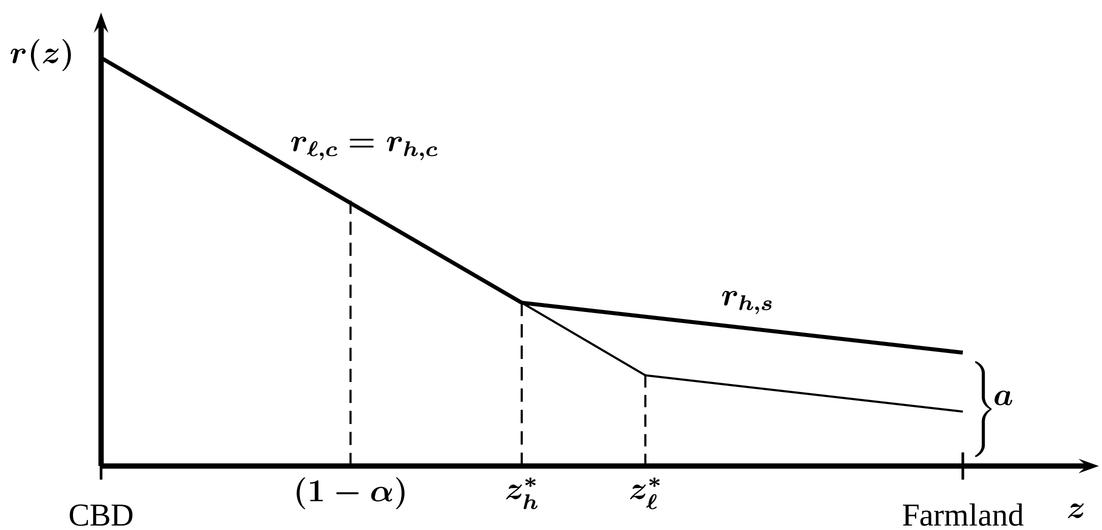

Figure 2.6: City Integrated Near the CBD ($D_s^{(1)}$)

The utility for the low-income consumers is determined by the point in the city where the rental line of the poor shopping at the CBD crosses that of the rich shopping at the Farmland, at point z_h^* in figure 2.6 above. Setting the two rental lines equal to each other we find the level of utility and the rental line for low-income consumers in terms of the Superstore's price:

$$v_\ell^{(1)} = \frac{g}{p_s\lambda_h} - \frac{w_h - w_\ell}{\bar{c}} \quad = \quad v_h - \frac{w_h - w_\ell}{\bar{c}} \tag{2.25}$$

$$r_{\ell,c}^{(1)} = w_h - \frac{g\bar{c}}{p_s\lambda_h} - tz = r_{h,c} \tag{2.26}$$

The utility of the poor is equal to the utility of the rich less the difference in the purchasing power of the two types of consumers. Interestingly, although low-income households do not shop at the Farmland, their utility is a decreasing function of the Superstore's price. In this configuration the rental lines of the poor and rich shopping at the Corner Stores are overlapping and must increase together as p_s increases, leading to a decrease in welfare for both types of consumers.

Case 2: $D_s^{(2)}$ $\left(\frac{\bar{c}}{1+\alpha\lambda_h} \geq p_s \geq \frac{\bar{c}}{1+\alpha\lambda_\ell}\right)$

On the second section of the demand line the two income classes in our city are completely segregated, with the rich living out in the Farmland and shopping at the Superstore and the poor living in the city center and shopping at the CBD. At the segregation point in our city, given by $1 - \alpha$, the rental line for the low-income households shopping at the CBD equals that of the high-income households shopping at the Superstore.

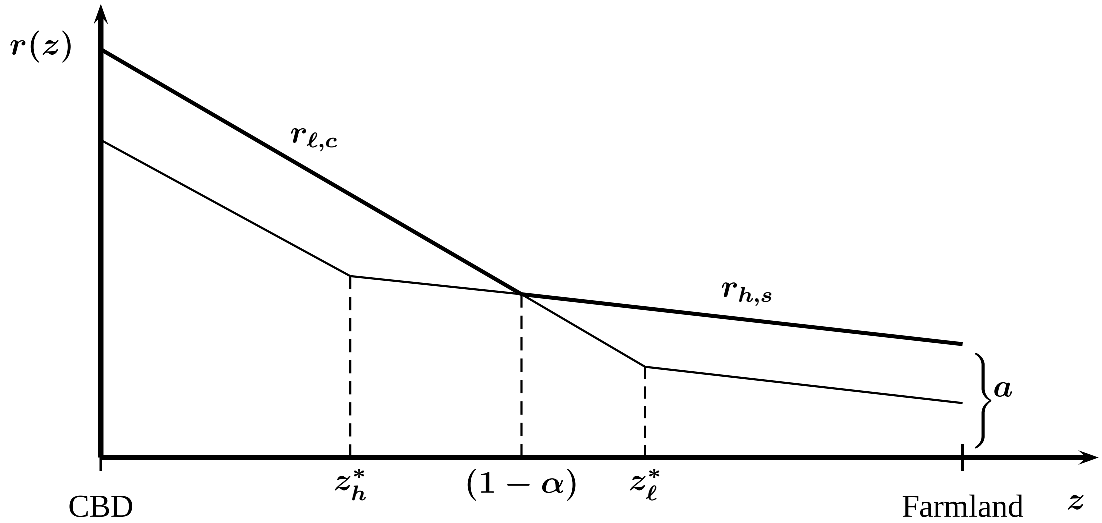

Figure 2.7: Segregated City Structure ($D_s^{(2)}$)

Evaluating the formula for $r_{h,s}$ in equation (2.24) at $z = 1 - \alpha$ and setting it equal to the equation for $r_{\ell,c}$ in (2.26) evaluated at the same point we can solve for the equilibrium level of utility for low-income households:

$$v_\ell^{*(2)} = \frac{\alpha g}{\bar{c}} + \frac{g}{\bar{c}\lambda_\ell} \tag{2.27}$$

We can now use our result above to get the rental line for low-income households in terms of our parameters:

$$r_{\ell,c}^{*(2)} = t(1-z) + a - \alpha g \tag{2.28}$$

From equations (2.23) and (2.27) we have that the utility of the high-income consumers depends only on transportation costs and the price of the Superstore, while the utility of the low-income households depends on the proportion of high-income households in the city, α, as well as transportation costs. In this case the utility of the poor only depends on the Superstore's price indirectly. p_s does not appear in equation (2.27) explicitly, but the Superstore's price does determine whether or not our city would be in this case of demand. The welfare of the poor increases with the percentage of the wealthy in the city because higher proportion of high-income consumers lowers the rent at the city center where the low-income households live, we can see this effect in the rental line for the low-income households in equation (2.26) above. This effect is limited in the sense that as α gets large enough it is less likely that pricing at this demand interval is optimal for the Superstore, we will get to this point below.

Case 3: $D_s^{(3)}: \quad \frac{\bar{c}}{1+\alpha\lambda_\ell} \geq p_s \geq \hat{c}$

In the final section of the demand line, the Superstore charges a low enough price that will push the switching points of the two types of consumers to a point that is closer to the city center than the segregation point, $1-\alpha$. This means that the rental lines of the rich and poor shopping at the Superstore are overlapping, the city is integrated out near the Farmland, while only low-income households live near the city center.

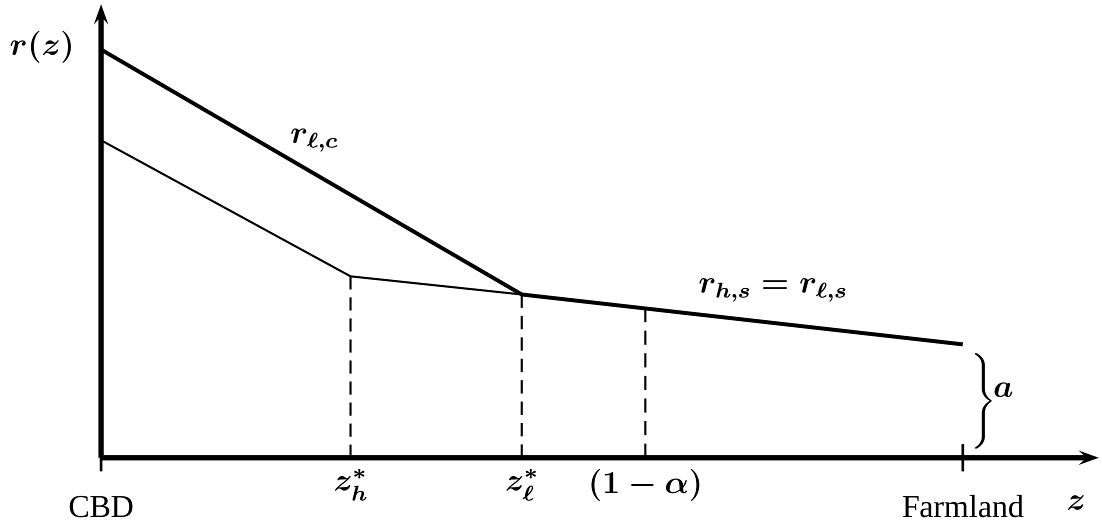

Figure 2.8: City Integrated Near the Farmland ($D_s^{(3)}$)

The Superstore captures all of the high-income consumers as well as a portion of the poor. Depending on the value of our parameters, there is also a scenario where the Superstore would choose to price low enough to capture the entire city population and push the Corner Stores out of the city. Under this scenario the city would be fully integrated with both consumer types shopping at the Superstore. We will consider under what condition such an outcome might occur below.

Similarly to equation (2.7) from our base case, since we have that the city is integrated out near the Farmland the utility for both types of consumers is given by the utility of the consumer living at $z = 1$ and shopping at the Superstore:

$$v_i = v_{i,s}(z = 1) = \frac{g}{p_s \lambda_i} \tag{2.29}$$

While the two rental lines for low-income households are given by:

$$r^{(3)}_{\ell,c} = w_\ell - \frac{g\bar{c}}{p_s\lambda_\ell} - tz \qquad\qquad r^{(3)}_{\ell,s} = (t-g)(1-z) + a = r_{h,s} \tag{2.30}$$

The rental line for the two types of consumers shopping at the Superstore are overlapping, therefore they are identical.

Finally, although technically the Superstore might choose a price as low as its own marginal cost, $\hat{c}$, and still earn non-negative profits, we would like to consider the price at which the Superstore would capture the entire market and push out the Corner stores out of our city. Under this scenario, the rental lines for the two types of consumers shopping at the Superstore would be overlapping over the entire city line. We can solve for this lower bound on the Superstore's price by setting the switching point for the lower income consumer, z^*_ℓ, to zero. Solving for p_s we get the following additional lower bound for the Superstore's price.

$$p_s = \frac{\bar{c}}{1 + \lambda_\ell} \tag{2.31}$$

Clearly this price could end up below the marginal cost of the Superstore, $\hat{c}$, but to allow for the possibility of a fully integrated city we will assume that $\hat{c} < \frac{\bar{c}}{1+\lambda_\ell}$.

2.4 Superstore's Problem With Households Heterogeneous In Income

Now that we have determined how the consumer population would choose their housing and shopping locations given the Superstore's price, we will look at how the Superstore will choose a price knowing the reaction of the consumer market. We will first set up the general form of the Superstore's problem, then we will solve for the optimal price within each case of demand as defined above. Finally we will identify under what conditions, if any, on our model parameters the proposed structures above constitute an equilibrium, and the resulting levels of utilities, rent and profit in each possible structure.

The Superstore's Problem: The general form of the Superstore's problem is given by:

$$\max_{p_s} \quad [\theta_h(p_s)v_h + \theta_\ell(p_s)v_\ell]\,[p_s - \hat{c}]$$

$$s.t. \quad \theta_h(p_s) \leq \alpha \quad \text{and} \quad \theta_\ell(p_s) \in [0, 1-\alpha]$$

$\theta_i(p_s)$ is the proportion of type i consumers that shop at the Superstore. We introduce this notation rather than using z_i^* since we would like to limit these proportions to correspond with the proportion of each type of consumer. We cannot do that using z_i^* since the latter can take any value between 0 and 1. The above set up is in effect a maximization problem with three inequality constraints (we know that θ_h is positive by Proposition 2.2 and the fact that it would always be optimal for the Superstore to charge a price below the Corner Stores and attract a positive number of consumers).

Whether or not the constraints in the maximization problem above are binding depends on which one of our cases of consumer demand the Superstore is facing. We will consider the Superstore's problem under each case of demand separately.

<u>The Superstore's profit in demand case $D_s^{(1)}$:</u>

$$\pi_s^{(1)} = \frac{g}{\lambda_h^2}\left(\frac{\bar{c}}{p_s} - 1\right)\left(1 - \frac{\hat{c}}{p_s}\right) \tag{2.32}$$

$$s.t. \quad p_s \geq \frac{\bar{c}}{1+\alpha\lambda_h}$$

Maximizing with respect to p_s we can solve for the Superstore's choice of price and resulting level of profit in our first case of demand:

$$p_s^{*(1)} = \frac{2\bar{c}\hat{c}}{\bar{c}+\hat{c}} \qquad \pi_s^{*(1)} = \frac{g(\bar{c}-\hat{c})^2}{4\bar{c}\hat{c}\lambda_h^2} \tag{2.33}$$

A similar result to what we calculated for the Superstore in our Base Case. This similarity is not surprising since, as in the Base Case, the Superstore is targeting a portion of one type of consumer.

Now that we have the Superstore's choice of price, we can substitute it into the levels of utility for the two types of consumers in equations (2.23) and (2.25) to determine the consumers' utility in terms of our parameters.

$$v_h^{*(1)} = \frac{g(\bar{c}+\hat{c})}{2\bar{c}\hat{c}\lambda_h} \qquad v_\ell^{*(1)} = v_h^{*(1)} - \frac{w_h - w_\ell}{\bar{c}} \tag{2.34}$$

We can do the same for our rental lines. We have already shown that the rental line for high-income consumers shopping at the Superstore does not depend on price, and is equal to the rental line from equation (2.24) above. The only binding rental line for the poor is from shopping at the CBD, and as we have previously shown, overlaps that of the rich.

$$r_{h,c}^{*(1)} = r_{\ell,c}^{*(1)} = w_h - \frac{g(\bar{c}+\hat{c})}{2\hat{c}\lambda_h} - tz \tag{2.35}$$

The Superstore's profit in demand case $D_s^{(2)}$:

This is the fully segregated outcome in our model, where the poor live in the city center and the rich live out near the Farmland. In this part of the demand graph all high-income consumers and none of the low-income consumers shop at the Superstore, therefore the Superstore's profit function is given by:

$$\pi_s^{(2)} = \frac{\alpha g(p_s - \hat{c})}{\lambda_h p_s} \tag{2.36}$$

$$s.t. \quad \frac{\bar{c}}{1+\alpha\lambda_h} \geq p_s \geq \frac{\bar{c}}{1+\alpha\lambda_\ell}$$

From figure 2.7 above, and examination of the profit function in equation (2.36), we can see that the demand faced by the Superstore is not dependent on z_i^* as long as $z_h^* \geq (1-\alpha)$, this means that any decrease in price by the Superstore does not lead to new customers, only existing customers purchasing more of the good. Since the Superstore faces constant marginal cost and demand is unit elastic as long as $p_s < \frac{\bar{c}}{1+\alpha\lambda_h}$, the Superstore would push its price up to the upper bound of the constraint above, decreasing costs without any loss to revenue. Therefore, in this case of demand we would have a corner solution with price given by:

$$p_s^{*(2)} = \frac{\bar{c}}{1+\alpha\lambda_h} \tag{2.37}$$

The price of the Superstore is an increasing function of w_h and does not depend on w_ℓ. As the income of the wealthier consumers increases, the price of the good at the Farmland approaches the price of the Corner Stores asymptotically. We can also see that p_s is a decreasing function of α, that is as the proportion of high-income families increases, it will be optimal for the Superstore to lower its price in order to prevent some wealthy consumers from switching to shopping at the Corner Stores. In the same way we can see that as the proportion of wealthy consumers decreases, the price of the Superstore approaches that of the Corner Stores, requiring higher prices from a smaller population of wealthy consumers being pushed further away from the CBD.

Substituting the price above into equation (2.23) and (2.36) we derive the utility for high-income consumers and profit of the Superstore in terms of our parameters:

$$v_h^{*(2)} = \frac{\alpha g}{\bar{c}} + \frac{g}{\bar{c}\lambda_h} \qquad \pi_s^{*(2)} = \frac{\alpha g[\bar{c} - \hat{c}(1 + \alpha\lambda_h)]}{\bar{c}\lambda_h} \tag{2.38}$$

Since the utility of the poor as well as the effective rental line across the city are independent of the price of the Superstore, they are as we calculated in (2.27) and (2.28) above. We can see in this scenario of demand the utility of the rich and poor are almost exactly the same, except that the rich benefit from transportation costs being a lower percentage of their disposable income, $\lambda_h < \lambda_\ell$. It seems that the segregation of the poor into the city center does not cost the poor in terms of their level of indirect utility. That is, despite paying higher rents in the city center and being forced to shop at the more expensive Corner Stores, in our model the rental and goods markets adjust in a way to compensate the poor for their worse-off position. In real world situations there might be structural frictions that exist that would prevent such a compensation. One example would be if the poor had to pay higher commuting costs relative to wealthier households, allowing the Superstore or landlords to be less concerned with the impact of their choice of price on the less mobile poor population, LeRoy and Sonstelie (1983).

Comparing the equations above with our derived indirect utility without the Superstore in equation (2.19) we can see that under this demand scenario the utility for both types of consumers have increased by a factor of $\frac{\alpha g}{\bar{c}}$ due to the entrance of the Superstore. Clearly the positive impact of the entry of the Superstore increases as the cost of commuting to the store and the proportion of wealthy in the population increase. Therefore, from the consumers' perspective some level of transportation costs serves as a benefit by driving down rental costs.

Before we go on, it is important to note that the corner solution at the kink between $D_s^{(2)}$ and $D_s^{(3)}$ would never be an optimal outcome for the Superstore. As we argued above, once the Superstore reaches a price where all of the rich and none of the poor shop out in the Farmland, the elasticity of demand becomes -1 and the Superstore would choose to continue increasing its price until demand reaches the intersection of $D_s^{(1)}$ and $D_s^{(2)}$.

<u>The Superstore's profit in demand case $D_s^{(3)}$:</u>

$$\pi_s^{(3)} = g\left[\alpha(\frac{1}{\lambda_h} - \frac{1}{\lambda_\ell}) + \frac{1}{\lambda_\ell^2}(\frac{\bar{c}}{p_s} - 1)\right][1 - \frac{\hat{c}}{p_s}]$$

$$s.t. \quad \frac{\bar{c}}{1+\alpha\lambda_\ell} \; > \; p_s \; \geq \; \frac{\bar{c}}{1+\lambda_\ell}$$

We have a strict inequality on the upper constraint from our argument above ruling out the kink between $D_s^{(2)}$ and $D_s^{(3)}$. It is possible that the Superstore will drive down its price low enough to capture the entire market. This would be a corner outcome such that the switching point of the low-income consumer will be pushed all the way back to the center of the city, we will consider this corner out come below. Now we look for an interior solution in our third case of demand.

Maximizing with respect to p_s, we get the following for the Superstore's choice of price:

$$p_s^{*(3)} = \frac{2\bar{c}}{\frac{\bar{c}+\hat{c}}{\hat{c}} - \alpha\lambda_\ell\left(\frac{\lambda_\ell-\lambda_h}{\lambda_h}\right)} \tag{2.39}$$

Resulting in consumer utility and Superstore profit of:

$$v_i^{*(3)} = \frac{g}{2\bar{c}\lambda_i}\left[\frac{\bar{c}+\hat{c}}{\hat{c}} - \alpha\lambda_\ell\left(\frac{\lambda_\ell-\lambda_h}{\lambda_h}\right)\right] \qquad \text{for } i = h, \ell \tag{2.40}$$

$$\pi_s^{*(3)} = \frac{g\hat{c}}{4\bar{c}\lambda_\ell^2}\left[\frac{\bar{c}-\hat{c}}{\hat{c}} + \alpha\lambda_\ell\left(\frac{\lambda_\ell-\lambda_h}{\lambda_h}\right)\right]^2 \tag{2.41}$$

The welfare of the two types of consumers is increasing with the cost advantage of the Superstore, which is not surprising since in this segment of demand they are both shopping at the Superstore. Interestingly the welfare of both types of consumers decreases as the difference in income between the two increases. This is because the Superstore's price increases with the differential in income.

The rental line for the poor and the rich shopping at the Superstore is again not impacted by the Superstore's price and is given by the second equation from (2.24) above. We can derive the rental line of the poor shopping at the CBD buy plugging in our resulting price into the rental line from equation (2.30).

In the case of a corner outcome the Superstore drives down its price to the point where all consumers in the city shop at the Farmland, in other words the Superstore would price such that $z_\ell^* = 0$:

$$p_s^{*(4)} = \frac{\bar{c}}{1 + \lambda_\ell} \tag{2.42}$$

At this price the city would be fully integrated, the rental lines for the two types of consumers would be overlapping and cover the entire city line, with all consumers shopping at the Superstore. This scenario would result in consumer utility and Superstore profit equal to:

$$v_i^{*(4)} = \frac{g(1+\lambda_\ell)}{\bar{c}\lambda_i} \qquad \pi_s^{*(4)} = \frac{g[\bar{c} - \hat{c}(1+\lambda_\ell)]}{\bar{c}} \left(\frac{\alpha\lambda_\ell + (1-\alpha)\lambda_h}{\lambda_\ell\lambda_h}\right) \tag{2.43}$$

The city rental line would be determined by that of the poor and rich shopping at the Superstore and given by second equation in (2.24) above.

We will consider the implications for consumer utility for each of the outcomes described above.

2.5 Analysis of Equilibria

Now that we have fully described the choice of price by the Superstore under each demand scenario as well as the various city structures that result, we will consider under what conditions these different structural and price outcomes represent an equilibrium. By equilibrium we mean a case where the Superstore is maximizing profits under such a city structure, and where consumers have no incentive to move or change their shopping options. We begin by considering the constraints that we defined in the three cases of the Superstore's problem above. Comparing our derived price for the Superstore in each of the cases with the respective constraints, we obtain the following range of values for our parameters. The ranges on our parameters below signify the conditions for profit maximization for each structural outcome:

(i)	$\frac{\bar{c}-\hat{c}}{\hat{c}} < 2\alpha\lambda_h$	$\Rightarrow$	Integrated Center
(ii)	$2\alpha\lambda_h \leq \frac{\bar{c}-\hat{c}}{\hat{c}} \leq \alpha\lambda_\ell(1+\frac{\lambda_\ell}{\lambda_h})$	$\Rightarrow$	Segregated
(iii)	$\alpha\lambda_\ell(1+\frac{\lambda_\ell}{\lambda_h}) < \frac{\bar{c}-\hat{c}}{\hat{c}} < 2\lambda_\ell+\alpha\lambda_\ell(\frac{\lambda_\ell-\lambda_h}{\lambda_h})$	$\Rightarrow$	Integrated Periphery
(iv)	$2\lambda_\ell+\alpha\lambda_\ell(\frac{\lambda_\ell-\lambda_h}{\lambda_h}) \leq \frac{\bar{c}-\hat{c}}{\hat{c}}$	$\Rightarrow$	Fully Integrated

The above constraints follow the path we would expect. When income inequality in the city, measured in our model by α and $(\lambda_\ell - \lambda_h)$, is high relative to the Superstore's cost advantage over the Corner stores, $\frac{\bar{c}-\hat{c}}{\hat{c}}$, then the Superstore would be expected to charge a higher price and only target a portion of the wealthy consumers, as in our demand structure $D_s^{(1)}$. This leads to an equilibrium structure where only the rich live out in the Farmland, the area near the CBD is integrated, and only a portion of the high-income consumers shop at the Superstore.

As the cost advantage of the Superstore increases relative to the inequality within the city, we move through the different cases of city structure that we have described above. We can use the ranges of outcome on our parameters to draw figures depicting the ranges in which the four cases of demand obtain. The diagram below compares the cost advantage of the Superstore over the Corner stores, $\frac{\bar{c}-\hat{c}}{\hat{c}}$, relative to the proportion of high-income consumers in our city, α.

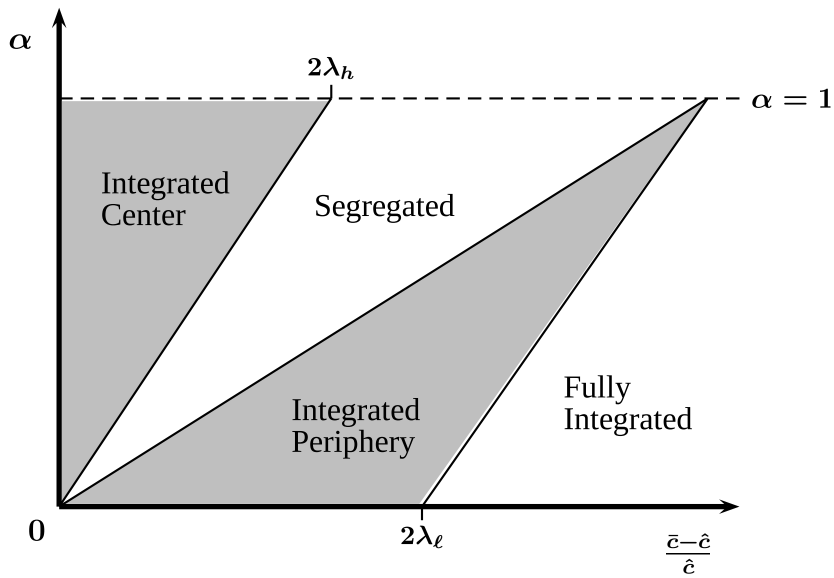

Figure 2.9: Regions of Equilibria: α

We can see that for low levels of α and a high cost advantage for the Superstore it becomes more likely that our city is fully integrated. In this structure both types of consumers are shopping at the Superstore and are indifferent as to where they live. This outcome is associated with the lowest price charged by the Superstore, and the lowest rental prices across the city. For middle-values of income inequality and Superstore cost advantage it becomes more likely that our city is either fully segregated, or integrated out near the Farmlands with only the poor living near the city center.

Similarly, for small income differentials (represented by a smaller difference between λ_h and λ_ℓ) and high cost advantage for the Superstore, the fully integrated outcome is more likely, while a middle range of the two factors leads to our segregated outcome. Interestingly, for low values of the Superstore's cost advantage, holding α constant, income differential does not impact the equilibrium outcome. Even for very small differences in income, the demand structure $D_s^{(1)}$ would be an equilibrium. We can interpret this result as a capacity requirement on stores in order for shopping options to impact consumer location decisions. For consumers to take the location of the Superstore into consideration when deciding where to live, there must be a significant difference between the price (and capacity) of the Superstore relative to the corner stores that are more readily available.

Equilibrium Utility

To see the impact of the Superstore on consumer welfare we look at the utility of each type of consumer in the different city structures. The following are the resulting utility levels from the various equilibrium outcomes described above. The first case is when we have heterogenous consumers without the Superstore. The next four outcomes are the various city structures that are possible when the Superstore enters the market.

Fully Integrated Without Superstore

$$v_i = \frac{g}{\bar{c}\lambda_i} \qquad \text{for } i = h, \ell$$

Integrated Center

$$v_h^{(1)} = \frac{g}{p_s\lambda_h} \qquad\qquad v_\ell^{(1)} = v_h^{(1)} - \frac{w_h - w_\ell}{\bar{c}}$$

Segregated

$$v_h^{(2)} = \frac{g}{p_s \lambda_h} \qquad v_\ell^{(2)} = \frac{g(1+\alpha\lambda_\ell)}{\bar{c}\lambda_\ell}$$

Integrated Periphery

$$v_h^{(3)} = \frac{g}{p_s \lambda_h} \qquad v_\ell^{(3)} = \frac{g}{p_s \lambda_\ell}$$

Fully Integrated With Superstore

$$v_i^{*(4)} = \frac{g(1+\lambda_\ell)}{\bar{c}\lambda_i} \qquad \text{for } i = h, \ell$$

The utility of type h consumers is always determined by the high-income consumer living at $z = 1$ and shopping at the Superstore. Therefore, the utility of the type h consumers is rising at an increasing rate as the Superstore lowers its price, irrespective of city structure or changes in the rental lines. The utility of the low-income consumers only increases in the semi-integrated outcomes, staying constant when the city is segregated. Figure 2.10 portrays how consumer utility changes as the Superstore's marginal cost, $\hat{c}$, changes relative to the Corner Stores', $\bar{c}$ (keeping α and λ_i constant), moving the city across the various income structures.

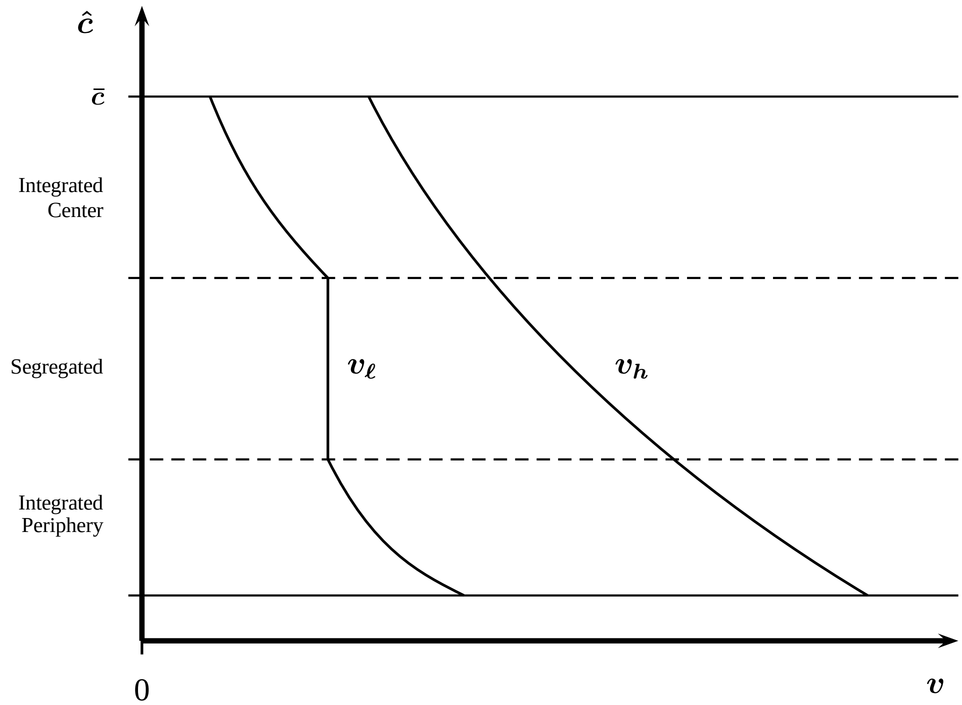

Figure 2.10: Equilibrium Utility

Interestingly, the utility of the type ℓ consumers increases at the same pace as the high types up until the price of the Superstore reaches the constraint for the segregated outcome. Although in the first case of demand low-income consumers do not shop at the Superstore, they benefit from the low price offered by the Superstore as if they were type h consumers. This is because high-income consumers live across the entire city. Therefore, the rent-savings in the city center must match the cost savings of shopping at the Superstore in order to keep the high-income consumers located at the city center indifferent to those living out near the Farmland.

In the segregated outcome the utility of the low-income consumers is fixed by the segregation point in the city. Their rent is not changing and they are continuing to pay the same price at the CBD. This is when the utility of the high-income consumers begins to increase relative to the ℓ types. In this segregated outcome h type consumers benefit from low rents and low prices in the outer part of the city. Since there are no h types living in the city center, the rent for low-income consumers does not change, leaving them relatively worse off. This is the period in the figure above where the utility of the two types of consumers diverges at the fastest pace.

Finally, when the city is integrated out in the periphery, the low-income consumers begin to benefit directly from the low price offered by the Superstore. Now the utility of both types of consumers is determined by the representative consumer living at $z = 1$ and shopping at the Superstore. Yet, the utilities of the two types of consumers continue to diverge. In this city structure rents are no longer changing across the city, only changes in Superstore's price affects consumer welfare. The greater buying power of high-income consumers allows them to benefit more from the decrease in the Superstore's price, their utility increases at a faster pace relative to that of the low-income consumers.

As we can see in the figure above, both consumers are better off from the entrance of the Superstore into the market. In addition, the welfare of both types of consumers is increasing monotonically with decreases in the price of the Superstore. These are not very controversial or surprising results. In most cases, consumer theory predicts that the entry of lower priced firms does not hurt, and in general benefits, consumers.

The interesting result in our equilibrium analysis above is that in certain forms of city income structure, the two types of consumers do not benefit equally from the Superstore's discount. When low-income consumers are segregated away from the Superstore they do not benefit from the low prices offered. This effect is mitigated by what we can call "Neighborhood Effects". When the city center is integrated, the rental lines across the city are interconnected, allowing low-income consumers to benefit from the Superstore's price through lower rent costs.

2.6 Conclusion

In this paper we have analyzed various income structures in cities and how they are related to firm pricing strategy. In our base case model with a homogeneous consumer population we showed that the Superstore's entry into the city leads to lower rental prices, allowing consumers to spend a higher portion of their income on consumption of our representative good, x. We also argued that without any income disparity, all consumers benefit equally from the entry of the Superstore into the market.

Then we introduced heterogeneous consumers into our model, with consumers earning two different exogenously determined levels of income. We showed that as the difference between the income of the rich and poor increases it becomes more likely that our city is segregated by income, with the rich consumers living in the outer part of the city. We also showed the conditions under which we would have more hybrid structural outcomes. More specifically, we showed that as the cost advantage of the Superstore increases relative to the Corner Stores, the city becomes more integrated, moving the resulting equilibrium towards the fully integrated outcome with all consumers shopping at the large discount store. At the same time we demonstrated that the two types of consumers do not benefit equally from the entry of the Superstore, especially when the city is completely segregated.

The determinants of city structure that we consider are similar to those considered in previous literature in that they are associated with the amenities available in a city. We go beyond previous literature on regional economics by focusing on the interplay of the rental market with firms' strategy, thereby looking to connect the existing work in regional economics with the industrial organization literature. We also introduce distributional considerations into the city model, demonstrating how the makeup of the consumer population can impact firm strategy and city structure. Of course households take into account many different factors when choosing where to live, and most of these issues have been dealt with in previous work. The purpose of our paper is to attempt to investigate the impact of firm pricing on household choice of location, taking the other factors analyzed in previous work as given.

The results we have presented above are an example of how spatial frictions can lead to divergence of welfare between consumers of different income levels. These results are similar to the information frictions that we discussed in the first chapter of this thesis. Whether lower income consumers are at a disadvantage because they do not have good information or physical access, the implications are similar. The existence of spatial and information frictions can lead to market outcomes that puts lower income households at an inherent disadvantage.

One natural question to ask, given our results, is what are the policy implications of such spatial frictions. Clearly the transportation costs present in our model are key factors in the outcomes we have described. But lowering transportation costs across the city line would not necessarily help remove these frictions. Our results depend mainly on $\boldsymbol{\lambda}$, transportation costs relative to consumers' disposable income. Lowering the cost of transportation would not lower the relative $\boldsymbol{\lambda}$ between the two types of consumers considered. An alternative would be to tax the high-income consumers, and/or the Superstore, and use the tax proceeds to subsidize the transportation costs of the lower income group. But these taxes might have a distortionary impact on Superstore pricing as well as our rental market. We hope to more carefully consider this question in a future paper.

Finally, one shortcoming of our model is that it does not explain a segregated city outcome with rich consumers living in the city center, a commonly observed city structure, especially in major European cities, Brueckner et al. (1999). One way to address this issue would be to include time costs of transportation into our model. The monetary costs of the time spent traveling from home to work is the loss of wages. Therefore, introducing time costs would increase the incentive for wealthier consumers to locate closer to their jobs in the city center (this is assuming that the time cost of commuting to work is greater than that of shopping at the Superstore, a reasonable assumption since most people commute to work everyday while they go shopping only one or two times a week). Adding time costs allows for a much wider set of structural outcomes in our city and would be a very interesting extension of our analysis.

CHAPTER 3

ACCESS TO BANKING AND INCOME INEQUALITY

Abstract

This chapter uses a simple model of banking services to consider how deposit-taking banks price for their services and choose the type of deposit customers that they target. This chapter goes beyond previous theoretical work on consumer banking, identifying the role of household income in the access to deposit services. We show that a higher rate of return on investments available to banks lowers financial exclusion, increasing the profitability of low-income consumers for deposit-taking institutions. This suggests that the possibility of financial exclusion increases in periods of recession. The model demonstrates how an increase in income dispersion can lead to a greater proportion of consumers excluded from mainstream banking.

3.1 Introduction

The importance of access to banking services for participation in the mainstream economy has made it increasingly costly for those households that are left out of the financial services sector. Over the last several decades financial services have become more sophisticated and prevalent in developed economies. Households rely on bank accounts to conduct basic financial transactions, build precautionary savings, and as a means for access to affordable credit. Most workers in advanced economies are no longer paid in cash, and require a way to cash checks or set up direct deposits in order "access" their wages. Households without a bank account not only end up paying more for basic financial services, but they may also be more vulnerable to loss or theft of their cash and asset holdings and often have difficulty building credit histories and achieving financial security.

The purpose of this paper is two fold. First, we go beyond previous theoretical work on consumer banking by considering a model of bank deposit services with a consumer population heterogeneous in income. This will allow us to focus specifically on what type of consumers are excluded from banking services. We look to identify how mainstream banks charge for deposit accounts and the customers they target. Second, we look at what happens to the consumers that are left out of the mainstream banking sector, and the costs they face when they are forced to turn to Alternative Financial Services. Finally we consider the role of AFS in the financial services market as well as what happens when we allow banks to participate in the AFS market. We show that the welfare impact of financial exclusion depends significantly on the extent to which consumers can participate in the economy without requiring the services of financial institutions.

A recent study in the U.S. by the Federal Deposit Insurance Corporation (FDIC) found that access to mainstream banking services such as deposit accounts, debit cards and checking services is lacking for a significant portion of the population, FDIC (2009). The 2009 survey found that over one quarter of households across the United States are either unbanked (7.7%, do not have a checking account) or underbanked (17.9%, have a checking account, but use Alternative Financial Services like check cashing services). Financial exclusion is also a problem in the United Kingdom where 5% of the population do not have access to a transaction account, FIT (2009)[1] , and an additional

[1]This is excluding households that did not respond. Including those who did not state the account status would raise this number to 7%.

$\mathbf{20\%}$ are considered underbanked, Kempson and Whyley (1998)[2]. Lack of access was especially stark amongst low-income households, where in the U.S. $\mathbf{20\%}$ were categorised as unbanked ($\mathbf{37\%}$ of low-income households did not have a current account in the UK, Devlin (2005)), with even higher levels of exclusion amongst minority groups (54% of black households and 43.3% of Hispanic households are either unbanked or underbanked in the U.S.). It is interesting to note the similarities between the exclusion numbers in the U.S. and UK. Studies into access to financial services in poorer countries have found financial exclusion to be much more widespread (See Beck et al. (2007) and Beck et al. (2008)).

Part of the reason for the prevalence of unbanked households is thought to be a lack of information on the services available to these households, a problem that banks attempt to alleviate though providing educational material and conducting community outreach. But it is acknowledged by the banks themselves that the lack of access is partially driven by the fact that very low-income households are not profitable customers for the banks. The latter is the main focus of our paper. We use our model of banking services to more formally consider the profitability of low-income customers of banks, hoping to better understand the economic causes of financial exclusion.

We begin by more formally describing what banking services entail.

Banking Services

Most mainstream bank accounts provide a variety of services for depositors. The most obvious benefit is the convenience of transaction services featured in the Baumol-Tobin

[2] This statistic might have decreased since the study by Kempson and Whyley (1998) and FSA (2000). The Financial Inclusion Taskforce (FIT) has observed a steady decline in the percentage of households without access to any transactional accounts, we would expect this decline to be reflected in the percentage of households considered on the margin of financial exclusion.

bank deposit model. Transaction services include internet/telephone banking, ATM access, direct debit and check cashing, automated payments and online and in person debit card transactions. Deposit accounts also provide security for account holders by providing theft and fraud protection. In addition, customers with deposit accounts are usually given preferential access to credit through overdraft services, credit cards and personal loans. Though we can take these perks for granted, they all play a significant role in our participation in the economy. Without a debit/credit account it is very difficult to participate in the e-retail market, cash checks, access cash locally or internationally, as well as rent accommodation or open mobile phone and utility accounts.

Banks charge for these services directly through fees and indirectly through foregone returns. Direct fees can be either in the form of periodic fees associated with holding a deposit account, or through charging fees for various bank services, like overdraft charges. Indirect fees are considered to be the difference between the consumer's so called outside option, the risk-free rate of return, and the interest paid on deposit accounts. These indirect fees make up a significant portion of revenues for deposit-taking institutions, and are a prominent aspect of the Baumol-Tobin model of transactions. From an accounting perspective, indirect fees are very difficult to measure, making it more difficult to quantify the contribution of the financial sector to GDP (See System of National Accounts 1993, 2008).

The direct fees tend to be a greater expense for low-income/low-balance account holders. Overdraft fees and fees associated with bounced checks only impact customers who have a low account balance and face the risk of triggering these charges. This seems to be less of an issue in the U.K., where most banks offer some basic banking services that have minimal to no fees. In the U.S. however, most commercial banks require a minimum balance and/or minimum periodic deposits in order for consumers to avoid fee payments, clearly a more difficult hurdle for low-income households. In addition, most banks have a tiered fee system where the higher the balance on customers accounts, the lower the fees. These penalties are cited as one reason why some households choose not to open up a bank account with a mainstream bank, resorting instead to Alternative Financial Services, FDIC (2009). High fees can also be seen as a way for banks to avoid less profitable customers. In times of financial distress, when bank profits and returns fall, banks tend to raise direct fees to replace lower revenues from indirect fees (see Dash (2011) and Son and Tighe (2011)).

In the global recession spurred by the financial crisis of 2007 banks were faced with declining returns on customer deposits as well as greater financial scrutiny and regulation on their investment portfolio. The CEO of Bank of America was quoted as saying that with the onset of the recession and greater regulation "We have 42 million retail customers, many of those don’t contribute or overcome their cost-to-serve.”, Son and Tighe (2011). The company responded to this decrease in profitability by raising fees for services and looking to target more profitable customers by providing lower prices for higher balances and more frequent deposits. Bank of America was not alone in looking to raise fees in response to a less profitable banking environment. This was a general trend in the U.S. retail banking sector (see Son and Tighe (2011) and Dash (2011)). As we will demonstrate in our theoretical model below, higher direct fees are in effect a regressive pricing mechanism and are usually a higher financial burden to low-income households. Therefore it is likely that in periods of recession low-income consumers are less likely to participate in mainstream banking.

Consumers that do not have a bank account turn to Alternative Financial Services for their banking needs. These include check cashing services, pre-paid direct debit cards, pawn brokerage, money orders and transfers as well as many forms of short term credit provisions. These services do not require a formal account but usually charge high fees. For example a recent product geared towards consumers without bank accounts are pre-paid debit cards, which allow consumers to put cash on debit cards not associated with a bank account. These types of cards have various forms of charges, including an application charge, transaction charges, an ATM withdrawal charge, a contribution charge as well as monthly fees. Considering the typically low balance on these cards for most consumers, these charges can add up to a high percentage of the volume of transactions for these customers, as well as a larger share of their disposable income.

In addition to being an issue of economic opportunity, financial exclusion is also a public policy concern. For example, social security, unemployment benefits and other benefits payments made by government institutions usually come in the form of checks. To the extent that those receiving these benefits have to pay high fees to cash them at AFS providers this is a transfer of public assets to these financial institutions. The U.K. government has taken steps to mitigate this effect by allowing for check cashing services through the country's postal service, FIT (2009). But these solutions are by no means universally available and do not address the transaction service needs of consumers.

The convenience of mainstream banks, the apparent need for a bank account for economic inclusion as well as the high fees associated with Alternative Financial Services are at odds with the widespread use of these services as well as the significant growth in the industry over the last decade. AFS providers have been growing steadily across the U.S. and are growing at a fast pace in Europe. Pre-paid debit cards are available on both sides of the atlantic and are provided by mainstream institutions such as Walmart in the

U.S. and Virgin in the U.K. In the U.S. $218 billion was loaded onto prepaid debit cards in 2007, representing a 100% increase in volume over four years, FDIC (2009).

We would like to better understand: why it is that these Alternative Financial Service providers exist and are becoming more prevalent; why consumers that have access to mainstream banks still choose to use these seemingly expensive services; and whether or not the prices charged for these services are determined by a well-functioning market or are a sign of the existence of market frictions.

In the following section we develop a theoretical model of the market for banking services. We look to use our model to better understand the importance of financial services for consumers and how banks choose the type of deposit customers that they target. In addition we consider the usage and pricing for Alternative Financial Services (AFS) by households left out of the mainstream banking sector and how this increases the prices they pay for financial transactions.

3.2 The Model

There has been extensive work done on modeling the business of commercial banks. Baumol (1952) and Tobin (1956) use an inventory style model to explain the economics of bank deposits, and the tradeoff consumers face when deciding how much cash to hold relative to keeping their money in less liquid assets. Other papers consider a bank's role as intermediary between lenders and borrowers, helping perform the role of choosing and monitoring the right investments for the funds provided by depositors (see Stiglitz and Weiss (1981), Diamond (1984), Holmstrom and Tirole (1997) and Shleifer and Vishny (2010)). Though all of these papers make important contributions towards understanding outcomes in the financial sector none of them have explicitly considered the role of income distribution in financial markets. The purpose of our paper is to begin thinking about how the distribution of income of consumers can impact bank decisions, focusing specifically on income distribution and the supply of deposits in the banking sector. By introducing consumers heterogeneous in income we are able to focus on the causes and extent of financial exclusion in bank deposit services. We consider how banks price for deposit services and how they determine the type of consumers that they accept deposits from. We abstract away from the monitoring problem, taking the return banks earn on deposits as given, and focus on the cost benefit tradeoff of the banks and deposit customers. The general framework of our model and our method of telling the story of the bank deposit market follows that of Shaked and Sutton (1982), who consider entry and the choice of quality in a monopolistically competitive market, and Atkinson (1995), who considers the exclusion of consumers from the market of a productive good. We have adjusted their assumptions about consumer preferences and firm strategy to reflect more closely the market for financial services. We begin with a simple model of consumers, Alternative Financial Service providers (AFS), and a mainstream bank.

Consumers

There is a unit mass of consumers that only differ in their income, w. Income is distributed according to a cumulative distribution function $G(w)$. The density, $g(w)$, is zero for values of w below the minimum wage, a and above the maximum wage, $a+h$, where h can take any positive value, $h > 0$. Consumers can choose to either keep their earnings at a mainstream bank providing all the deposit services described above, or to turn to an AFS that offers a minimum set of services (such as check cashing or pre-paid debit cards). More formally, banks provide consumers with full access to their earnings as well as an additional benefit of θw, where $\theta > 0$, to a customer earning w. Banks charge a fee, f_B, for these deposit services. AFS only provide consumers with access to their earnings (this is analogous to $\theta_A = 0$) and charge fee f_A.

We are inherently assuming that banks charge an indirect fee by not providing a deposit interest rate to customers, but we are not including these type of fees in the consumers' problem. This is based on the observation that most consumers that use AFS providers do not have access to a risk free rate, $r_f > 0$, as an outside option. In addition, most banks offer a zero interest rate on checking accounts. Therefore we believe that these fees are not a real consideration when choosing between an AFS and a transaction account with a bank.

In this section, the model assumes that consumers do not have full access to their cash without going through a financial service provider. Otherwise AFS customers would choose to keep their income w and not pay a fee. This is based on the observation that in the modern economy most workers are paid through checks or direct deposits. In addition, many consumer transactions, from online purchases to sending money to family members, usually require bank/AFS services. There is a role for cash payments within a developed economy, but we assume this to be sufficiently small so that even the poorest consumer would always prefer to pay the fee rather than solely rely on cash[3].

$$a - f_A \geq \lambda a \tag{3.1}$$

Where λ is the proportion of cash transactions in the economy. The above inequality requires that $\lambda < 1$. The condition above depends on the fee charged by AFS. We will

[3] Recall that a is the lowest level of income in our model

consider the choice of f_A in our discussion below[4].

The consumer's binary choice is between:

$$u_B = (1+\theta)w - f_B \quad \text{s.t.} \quad \theta > 0 \qquad \text{and} \qquad u_A = w - f_A \tag{3.2}$$

We can compare the two utility functions above to determine the income level, w^*, such that consumers earning an income below w^* choose to use an AFS over a mainstream bank.

$$w^* = \frac{f_B - f_A}{\theta} \tag{3.3}$$

Consumers earning below w^* are considered excluded from mainstream banking services. We are particularly interested in looking at how the proportion of consumers that are excluded, $G(w^*)$, is determined within our model, as well as the costs to consumers that are excluded from mainstream banking.

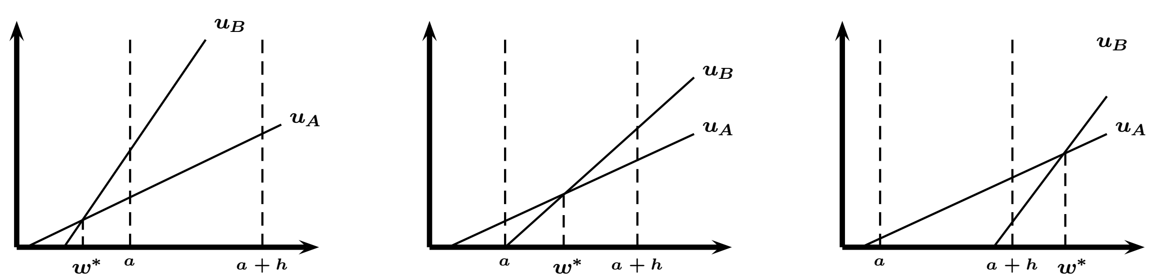

Figure 3.1: Cases of Financial Exclusion

[4]Note that λ is inherently included in our analysis of the consumers' problem. The bank and AFS provide access to the non-cash portion of consumer income, $(1-\lambda)w$. As we will show below, this means that λ, at least in our model, does not impact the level of financial exclusion. But as we would expect, λ does have an impact on overall consumer welfare. We will come back to the significance of λ to our results in the extension of our model below.

Figure 3.1 demonstrates the three possible cases for the market for banking services. The figure on the left represents the case when no consumer is excluded from mainstream banking. In this case the cutoff wage for bank customers, w^*, falls bellow the poorest consumer earning a. The figure in the middle represents an interior solution where a portion of consumers are excluded. In the third figure the mainstream bank would not enter the market, leaving all consumers to resort to using AFS for their transaction needs.

In the next section we will consider the bank side of the model to determine the conditions that would lead to each of the cases demonstrated above.

Banks

We use a basic deposit model where mainstream banks face fixed costs, k, such that only one bank enters, therefore we have a monopoly. Previous literature on the banking sector have used various levels of competition ranging from monopoly (see the Monti-Klein model described in Freixas and Rochet (2008)) to perfect competition. For the purpose of this paper we don't lose much generality by considering the strategy of a monopolist bank serving a consumer population with various forms of outside options. In fact the setup of our model is not too far away from the duopoly setup considered in Gabszewicz and Thisse (1979) and the monopolistically competitive model of Shaked and Sutton (1982)[5].

The bank takes in deposits and uses those deposits to invest in projects earning an assumed rate of return, r. The bank faces a fixed cost per deposit account associated

[5]We acknowledge that endogenous choice of entry and its consequences on the results that follow is an interesting extension to our model, but our initial findings suggest that entry of additional qualities of banking services do not significantly impact our results. We will leave a more detailed consideration of the impact of entry for future work.

with the administration and servicing of these account, c_B. Substituting for f_B from equation (3.3), the profit function for the bank is:

$$\pi_B = rD_B - (c_B - \theta w^* - f_A)N_B - k \tag{3.4}$$

$$\text{where} \quad D_B = \int_{w^*}^{a+h} wg(w)\,dw \quad \text{and} \quad N_B = 1 - G(w^*)$$

D_B is the total amount of deposits taken in by the bank and N_B is the number of bank accounts. For the sake of simplicity we are assuming that the bank can earn interest, r, instantaneously on the amount deposited by consumers. A more realistic setup would have consumers drawing down on their deposits continuously over the period, with the bank earning interest on an average deposit balance of $\frac{1}{2}D_B$. This adjustment would only add a $\frac{1}{2}$ in front of the interest component of the bank's marginal profit function in (3.5) below, but would not substantively change our analysis[6]. A potential extension of our model would be to consider the impact on the bank (as well as consumers) if consumers drew down a fixed amount of their deposits each period. In such a setup some low-income consumers would hit the "zero-bound" on their deposits, while higher income consumers would always be left with some positive balance in their accounts. This would make low-income consumers even less profitable for banks, and might lead them to charge penalty fees for consumers that reach the "zero-bound" (we observe these as overdraft fees in practice). In our analysis we bundle these potential fees in the general fee charged by the bank and leave this extension for future work.

In the integrals above, w^* is the level of income where consumers are indifferent between using the bank and an AFS provider, as defined in (3.3). In the general equi-

[6] In effect r would become $\frac{r}{2}$ throughout the rest of our analysis.

librium models of bank deposits, such as Basu and Wang (2007), the rate of return available to banks, r, is determined by a corporate market. For our purposes we take that return as given.

We assume no fixed costs in the Alternative Financial Services sector, therefore we treat AFS as a competitive fringe. Studies into the profitability of AFS providers have found that their high fees tend to be offset with high marginal costs. Both studies found that relatively low fixed costs of entry lead to high level of competition in the AFS industry (see Flannery and Samolyk (2005) and Skiba and Tobacman (2007)).

The bank takes the AFS fee, f_A, as given and equal to the constant marginal cost of providing AFS services, c_A[7]. The bank chooses its customers by choosing f_B, which in effect determines the cutoff level of income for bank customers, w^*. Differentiating (3.4) with respect to w^* we have:

$$\frac{1}{g(w^*)}\left[\frac{\partial \pi_B}{\partial w^*}\right] = -rw^* - (\theta w^* + c_A) + \theta\left[\frac{1-G(w^*)}{g(w^*)}\right] + c_B \tag{3.5}$$

The first and second terms on the right hand side are the loss in interest revenue and fees from the marginal consumer at w^*. The third term is the gain from higher fees charged to all remaining bank customers. The final term is the cost savings from not providing services to the marginal consumer. We can see from the cost and benefit terms that the interest available on the volume of deposits, r, makes it more costly for banks to raise their cutoff level of income. Therefore higher returns on bank assets makes it less

[7]Note that if we had assumed no fixed costs in mainstream banking services, in other words a competitive banking market, f_B would be less than c_B in order to satisfy the zero profit condition. In addition to fees, banks earn revenues by investing consumer deposits. Therefore the zero profit condition for banks is a bit more complicated and is not unique since it depends on the number of banks that choose to enter the market. If banks only played the role of financial warehouses then the investment return portion of the profit function would disappear and we would be in a more typical Shaked and Sutton setting.

likely that low-income consumers will be priced out of the mainstream banking market. Alternatively, the level of the bank's technology, or marginal cost of deposit services, c_B, increases the cost associated with low-income depositors and makes exclusion more likely.

From the equation above and the assumption that the density of our income distribution is zero below a, marginal profit is positive for any income below a. Therefore we have that the bank will not charge a fee below the point where the consumer earning the lowest wage will choose to use banking services[8]. Using equation (3.3) this gives us a lower bound for the fee charged by the bank:

$$f_B \geq \theta a + c_A \tag{3.6}$$

Lowering the fee below this level will not add any new consumers and will cost the monopolist revenues from existing customers. Raising fees above this level would only be profitable if the right side of equation (3.5) is positive for the lowest income level, a. In addition, whether or not a bank prices itself out of the market depends on the value of (3.5) at $w^* = a + h$. If the left hand partial derivative of the profit function is negative at that income level then the bank would have incentive to lower its price to at least attract the wealthiest consumer in the market. Evaluating the differential at these two points gives us conditions on our model parameters that would allow for a bank to operate in that market but only target a portion of the consumer population:

$$(r + \theta)a - \frac{\theta}{g(a)} \quad < \quad c_B - c_A \quad < \quad (r + \theta)(a + h) \tag{3.7}$$

[8]In fact this is also true for cases where $g(a) = 0$, that is when the probability of earning the subsistence level of income is zero then marginal profit is positive at that income level. This result is significant for our condition for an interior solution below.

The right hand side condition assures that it is worth it for a bank earning r and providing quality of service θ to enter a market where the wealthiest consumers earn $a + h$. The left hand side condition is when such a bank would not cater to the poorest consumers in the market, in other words it is the requirement for financial exclusion. Checking dimensions, all of the terms in the conditions above are in terms of income, we have monetary conditions as we would expect[9].

Interestingly, the condition for exclusion on the left hand side is a weaker condition on the level of a than the requirement for profitability of the poorest consumer, $(r + \theta)a + c_A > c_B$[10]. Therefore, it is not necessarily the profitability of the poorest consumer that might cause them to be left out of the mainstream banking sector, but rather the ability of the bank to price discriminate across consumers[11].

In practice it is very common for banks to price discriminate across consumers of different income levels. But the type of price discrimination we observe does not match the progressive form that the above results would predict. Price discrimination usually comes in two forms. The first form of price discrimination is through indirect fees, the theoretical foregone interest consumers could earn if they invested their funds in a risk free asset rather than depositing them in a bank. Banks tend to offer greater returns on savings accounts with higher average balances. Assuming that all consumers have the option to invest their funds in risk free assets, this is inherently a regressive cost to consumers. The extent to which poorer consumers do not have access to risk free returns mitigates this effect. It is possible that this form of price discrimination

[9] $\frac{1}{g(a)}$ is a monetary number.

[10] This condition says that the poorest consumer is profitable if the bank sets fees such that $w^* = a$, that is such that the poorest consumer's participation constraint is binding.

[11] If the bank could price discriminate then it would set fees such that the participation constraint for all consumers are binding and no one is excluded.

is directly related to the availability of a risk free rate of return to the consumer as an outside option. One can argue that higher income consumers tend to have access to higher rates of returns on their investments, requiring the bank to offer them a higher return on their savings in order to attract their business. The second form of price discrimination, which is much more common in mainstream banking in the U.S., is in the direct fees charged by banks. These fees tend to be waived for high-volume deposit accounts and tend to target lower volume accounts[12], making them a regressive price discrimination. In both of these cases price discrimination would exacerbate financial exclusion. We do not allow for price discrimination in our banking model.

The rate of return available to the bank, r, is an important factor in the inequality in (3.7). A higher r makes deposit resources more profitable for the bank, and less likely that poor consumers will be excluded. To the extent that exclusion from the financial sector negatively impacts low-income households, a higher return available for banks could be seen as a positive social outcome. This result is a bit misleading since in our model we do not consider what drives r. Higher returns for banks can be due to greater risk and uncertainty in the bank's investment portfolio, which can be a negative for the overall consumer population. This is a tradeoff that became more clear in the 2008 financial crisis and has spurred a debate about the role of banks as deposit-taking institutions. It is not clear to what extent banks should focus purely on safeguarding consumer deposits versus on their rate of return on investments. As the above inequality makes clear, there is a tradeoff for banks between making deposit services cheaper for their customers by offsetting high fees with high returns, and the extent that banks expose customer assets to financial risk. This result would be an

[12] In the sense that they tend to charge higher fees for accounts that have lower average balances and where customers do not regularly deposit funds into the account.

argument against the notion of limiting a bank to only serving as a money warehouse. If banks were not allowed to earn a return on customer deposits they would either respond by lowering the quality of deposit services, θ, or more likely by raising fees, and in effect increasing financial exclusion.

Another possible interpretation of r in the condition for exclusion above is in the context of economic recession. Zero or negative economic growth tend to coincide with periods of low returns on investments for financial institutions. To the extent that a low rate of return on the volume of deposits forces banks to increase their direct fees on deposit customers, as demonstrated in our results above, we can argue that financial exclusion is likely to increase in periods of slow to negative economic growth[13].

From the two conditions above we can see that as long as there is enough income in a community the bank will choose to enter the market. In addition, if there is significant difference between the technology of the two types of financial service providers, that is if $c_B - c_A$ is sufficiently large, relative to the income of the poorest consumer, then the left hand condition in (3.7) holds and we have an interior solution where the bank targets a portion of the consumer population. These preliminary results seem to match what we would expect. Banks that are targeting higher income consumers are more likely to provide better services in exchange for higher absolute fees, while financial companies targeting poorer neighborhoods are more likely to provide very basic services and charge lower fees. In the rest of this section we consider the pricing decision of a monopolist bank when facing a distribution of consumers that would result in a portion of the population being unbanked. We will also compare the fees paid by the

[13]In this instance, when considering the impact of a recession on exclusion we are ignoring any impact on the distribution of income. Clearly a recession might have redistributive effect or lead to a decrease in the standard of living, but here we are focusing only on the relation between periods of slow economic growth and the rate of return available to banks. We will consider the distributive impact on exclusion below.

"banked" and "unbanked" within this framework in order to determine if the unbanked end up paying a higher percentage of their transaction volume as financial fees.

The Distribution of Income

In the context of our banking model a and h are a measure of standard of living and are defined relative to the technology of the financial service providers, c_A and c_B. On its own h is not a sufficient summary statistic in our setting and only vaguely represents changes in dispersion. The impact of h on the dispersion of income will depend on the functional form of the distribution function $G(w)$ as well as the minimum level of income a. We will consider the impact of changes in these relative parameters, as well as changes in the income distribution of the overall population, when we introduce specific distribution functions below. Throughout our analysis, we are interested in how the relationship between the distributive and standard of living parameters from the consumer side and the technology parameters from the banking side interact to determine exclusion, as well as consumer welfare, within our model.

From (3.5) we have that if $g(a) = 0$ then $\frac{\partial \pi_B}{\partial w^*} = \theta > 0$ at a. Therefore, when the density at a is small enough our condition holds and a portion of the population will be excluded from banking services. If the overall standard of living, a, is sufficiently high, then the left hand side of the above inequality does not hold and everyone in the distribution uses mainstream banking. The right hand condition does not hold when the income of the richest consumer in the market is low relative to the difference in technology of the two types of firms. If this condition does not hold then the bank would not target any consumers and the entire population would have to resort to a lower θ (which in this case means an AFS, so they would be considered "unbanked").

This is not such an unrealistic possibility. There are neighborhoods in very poor urban and rural areas where branches of mainstream banks do not exist. These neighborhoods, depending on their income level, might be serviced by local banks that provide some level of deposit services, or they might rely fully on AFS providers.

All that is left to check is whether or not the second order condition for a maximum is satisfied. Similarly to Atkinson (1995), the second order condition for a local maximum is satisfied when the following condition holds:

$$\frac{(1-G)g'}{g^2} + \frac{r}{\theta} + 2 \geq 0 \tag{3.8}$$

A non-decreasing hazard rate ($\frac{g}{(1-G)}$) is sufficient but not necessary to assure that the condition in (3.8) holds[14]. We can show that the second order condition for a local maximum is satisfied for the specific income distributions considered below[15]. The global condition on the second derivative that will assure us a unique solution requires that:

$$g(w^*)\,(r+2\theta) \quad \geq \quad g'(w^*)\,[c_B - c_A - (r+\theta)w^*] \tag{3.9}$$

We can show that the condition for a global maximum in (3.9) holds for the two income distributions we consider below, therefore in both of these cases our solution for the profit maximising level of w^* is unique[15].

If the conditions for an interior solution are satisfied then there is a profit maximizing level of w^* such that:

[14] since $\frac{r}{\theta} + 2 > 1$.
[15] See Appendix.

$$r + \theta = \frac{\theta[1-G(w^*)]}{w^* g(w^*)} + \frac{c_B - c_A}{w^*} \tag{3.10}$$

Whether or not the cutoff wage for financial exclusion is decreasing with the rate of return available to the bank, r, depends on how the cumulative distribution function changes with w^*. If the first term on the right hand side of (3.10) is non-increasing with w^*, then we must have that an increase in the rate of return available to the bank leads to lower cutoff level of income. Our assumption of a non-decreasing hazard rate is again sufficient but not necessary for this condition to hold. As we argued above, this condition holds for the specific income distributions that we consider below. We also have that the cutoff level of exclusion, w^*, is increasing with the difference in technology between mainstream banks and Alternative Financial Service providers.

From equation (3.10) we can see that the proportion of consumers that get priced out of mainstream banking services, $G(w^*)$, depends on the distribution of income in a given market. We can illustrate possible outcomes by considering specific income distributions. For example, suppose that consumers are distributed uniformly from a to $a + h$, which gives us a density function $g(w) = 1/h$. Under this distribution the marginal profit of the mainstream bank is given by:

$$h\left[\frac{\partial \pi_B}{\partial w^*}\right] = -rw^* - (\theta w^* + c_A) + \theta(a + h - w^*) + c_B$$

Comparing the loss from raising the cutoff level of income (the first two terms) and the benefits (the last two terms) determines the choice of w^* for the bank facing a uniform distribution of depositors. This is illustrated in the figure below:

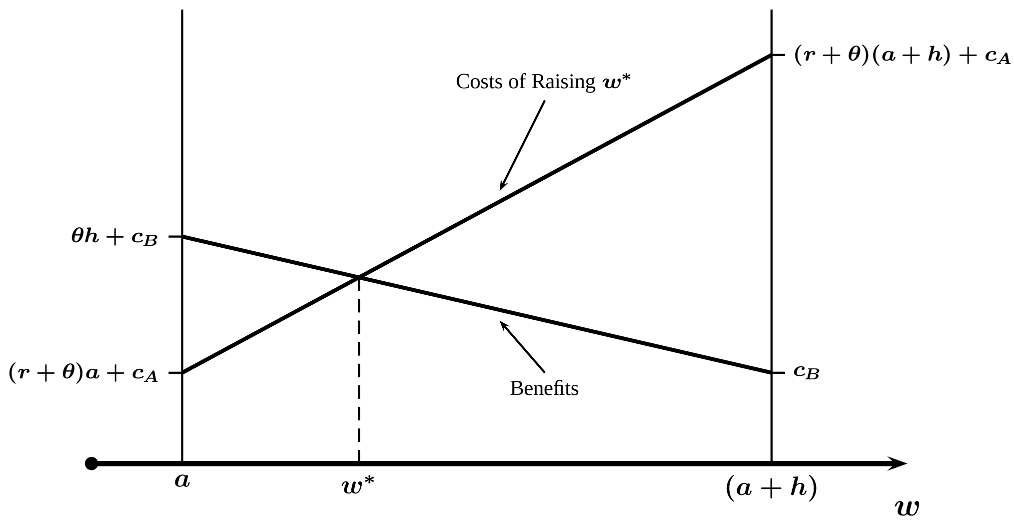

Figure 3.2: Costs and Benefits of Deposits

Evaluating the differential above at $w^* = a$ and $(a+h)$ we can derive the conditions for exclusion with a uniform distribution of income:

$$(r+\theta)a - \theta h \;\; < \;\; c_B - c_A \;\; < \;\; (r+\theta)(a+h) \tag{3.11}$$

As before the right condition is for a bank to enter this market, and the left is the condition for exclusion. The likelihood of a mainstream bank entering a market and there being consumers that are excluded increases with the difference between the richest and poorest consumers, h. We can see that from the left side of the inequality above, which is decreasing with h, and the right side, which is increasing with h. As we argued above, an increase in r reduces the possibility of exclusion and gives a bank greater incentive to enter a market.

The effect of θ, the quality of banking, on the possibility of exclusion is a little less clear. If the difference between the wealthiest and poorest consumers, h, is greater than the standard of living for the poorest consumer, a, then an increase in θ increases the possibility of exclusion. This result suggests that in populations with a high level of income dispersion and a low standard of living for the poor, an increase in the quality of banking would lead to greater exclusion.

Then w^* and the level of exclusion, $G(w^*)$, under a uniform distribution is:

$$w_U^* = \frac{\theta(a+h)+c_B-c_A}{r+2\theta} \quad \Rightarrow \quad G(w_U^*) = \frac{1}{r+2\theta}\left[\theta + \frac{c_B-c_A}{h} - a\left(\frac{r+\theta}{h}\right)\right] \qquad (3.12)$$

The level of financial exclusion is decreasing with the lowest income in the market, a. As we discussed above, h on its own is not a sufficient summary statistic. Increasing h increases the difference in income between the poorest and wealthiest consumers, but it would also lead to a higher average income when using a uniform distribution. To avoid this ambiguity in distributive effects we consider the impact of greater income dispersion by considering a mean preserving spread in the uniform distribution. A simultaneous and equal fall and rise in a and $a+h$ respectively[16].

A mean preserving spread of the distribution is demonstrated in figure 3.3 below, where $\bar{w}$ is mean income[17].

[16] Note that since we are lowering a by ϵ, in order to increase $a+h$ by ϵ we must increase h by 2ϵ.

[17] In a uniform distribution $\bar{w} = \frac{a+(a+h)}{2}$.

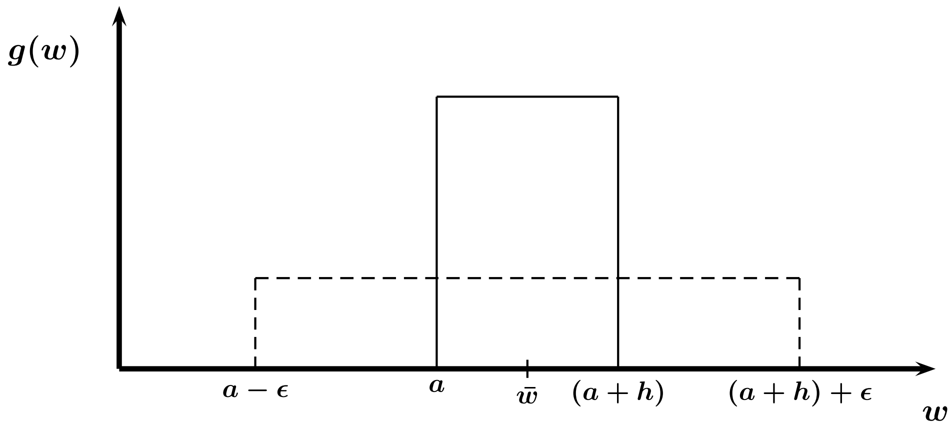

Figure 3.3: Uniform Distribution of Income

We can compare the proportion of consumers excluded from the mainstream financial sector by comparing the cumulative distribution function, $G(w)$, evaluated at the initial w^* with the extent of exclusion when our distribution is more spread out. Since $G(w)$ represents the proportion of consumers that fall below a particular level of income, an increase in $G(w^*)$ corresponds to a higher proportion of consumers excluded. The new w^{**} and proportion of exclusion, $G(w^{**})$, after a mean preserving spread becomes:

$$w_U^{**} = \frac{\theta(a+h)+\epsilon+c_B-c_A}{r+2\theta} \quad \Rightarrow \quad G(w_U^{**}) = \frac{1}{r+2\theta}\left[\theta + \frac{c_B-c_A}{h+2\epsilon} - (a-\epsilon)\left(\frac{r+\theta}{h+2\epsilon}\right)\right] \tag{3.13}$$

Comparing (3.12) with (3.13) we find that increasing the spread of our distribution leads to a greater proportion of consumer excluded if the following condition holds:

$$r\bar{w} + \theta\bar{w} + c_A \quad > \quad c_B \tag{3.14}$$

The left-hand side of the above inequality is the revenue to the bank from the average consumer if $w^* = \bar{w}$. This condition says that if the average person in the economy would be profitable for the bank then an increase in inequality would lead to greater exclusion. Based on the exclusion data discussed above, we would expect that the above condition tends to hold for the general population in the U.S. and U.K.[18]. The above condition might not hold in poorer regions within those countries where the average level of income is very low. In such areas an increase in inequality could lead to lower exclusion, but mainly in the upper tail of the income distribution.

Comparing the condition on exclusion from (3.7) with our condition for increasing exclusion in (3.14), it is clear that both conditions can hold for a range of values of $c_B - c_A$. As long as a is low enough relative to the cost differential between the bank and the AFS, exclusion can exist, and increase when income becomes more spread out.

Alternatively we could consider our results under a Pareto distribution, where income is greater than or equal to our lower bound a (this is equivalent to $h = \infty$). The cumulative distribution and density function are given by:

$$G(w) = 1 - \left(\frac{a}{w}\right)^{\alpha} \qquad g(w) = \frac{\alpha}{a}\left(\frac{a}{w}\right)^{\alpha+1} \qquad \text{s.t.} \quad \alpha > 1$$

Under this distribution $g(a) = \frac{\alpha}{a}$, where α is a shape parameter of the distribution. Therefore condition (3.7) becomes[19]:

$$(r+\theta)a - \frac{\theta a}{\alpha} \;<\; c_B - c_A \tag{3.15}$$

[18] This is based on the observation that total exclusion from transaction accounts for the entire U.K. and U.S. population tends to be below 10% and concentrated mainly in the poorer segment of the population, FDIC (2009) and Devlin (2005).

[19] In this case we do not have an upper condition since our income distribution does not have a finite upper limit.

As α decreases income becomes less concentrated in the lower part of the distribution, and it becomes more likely that consumers will be excluded from mainstream financial services. Alternatively, as the standard of living for the lowest income households, a, increases, the condition for exclusion is less likely to hold.

w_P^* and the level of exclusion, $G(w_P^*)$, under a Pareto distribution are given by:

$$w_P^* = \frac{c_B - c_A}{r + \theta\left(1 - \frac{1}{\alpha}\right)} \quad \Rightarrow \quad G(w_P^*) = 1 - \left(\frac{a\left[r + \theta\left(1 - \frac{1}{\alpha}\right)\right]}{c_B - c_A}\right)^{\alpha} \tag{3.16}$$

Where again we have that the percentage of the population excluded, $G(w_P^*)$, is decreasing with the rate of return, r, and the income of the poorest consumer (our standard of living parameter), a. We also have that both the cutoff level of exclusion and the proportion of those excluded are decreasing with α. The significance of α as a measure of inequality is not clear. Pareto himself referred to α as a measure of inequality. But if we measure inequality using the Gini coefficient, an increase in α leads to a decrease in inequality, Chipman (1974)[20]. An increase in α represents an increase in the density at the lower tail of the distribution, but it also represents a fall in the mean income[21]. In order to interpret the impact of changes in the Pareto distribution on exclusion in our model we need to consider both the standard of living parameter a and the shape of the curve, α.

In the case of the U.S., α has decreased over the last 30 years, leading to an increase in overall mean income. But as Atkinson et al. (2011) argue, this rise in mean real income has been driven mainly by an increase in the right tail of the income distribution, while the standard of living of the lowest income househods, a, has remained mostly

[20] Under a Pareto distribution the Gini coefficient is given by: $G = \frac{1}{2\alpha - 1}$.

[21] Mean income under a Pareto distribution is equal to: $a\left(\frac{\alpha}{\alpha - 1}\right)$.

unchanged[22]. This would suggest the opposite of how Pareto interprets α, meaning that a lower α can be associated with greater inequality (In fact, Atkinson et al. (2011) demonstrate that when using top-income data to measure inequality, the inverse of α is a measure of inequality). On the other hand, their study of real income in the U.K. found that although α has been decreasing, the standard of living for the lowest-income households, a, has increased[23].

Based on these results our model would predict that in the U.S. exclusion from mainstream banking must have increased over the last 30 years. From (3.16) we can see that, holding everything else constant, decreasing α without an increase in a would lead to greater exclusion. In the case of the U.K. the prediction of the model would be ambiguous. As we argued above, a decrease in α would lead to greater exclusion, while an increase in a would cause exclusion to decrease. Interestingly, the U.K. seems to have experienced a decrease in financial exclusion over the last decade. According to the Financial Inclusion Taskforce the proportion of the unbanked in the U.K. decreased steadily from 2000 to 2008, FIT (2009). This trend might suggest that in the U.K. the impact of a rise in a has outweighed a fall in α[24].

Before we go on we would like to briefly discuss how our results above would change under a unimodal distribution of income, which might be a more realistic representation of the types of income distributions we observe in practice. Examples of unimodal distributions with positive support include the Gamma distribution, Log-Normal

[22] Atkinson et al. (2011) show that although over the previous 30 years real income had grown at an average annual rate of 1.2%, the majority of that growth had been due to the growth in income of the top 1% of the population.

[23] Their study found this to be true for most English speaking countries, as well as to a smaller extent some Nordic countries.

[24] Note that these results might also be due to a variety of other factors, such as changes in the rate of return available to banks, r, as well as efforts by the U.K. government to increase access to banking. In addition, we currently do not have historical data on exclusion in the U.S., and the FIT study only provides data on exclusion in the U.K. for the last ten years.

distribution or more simply the Triangular distribution. As long as these distributions satisfy our non-decreasing hazard rate assumption (and our condition for a global maximum in equation (3.9)) then our general results in (3.7) and (3.10) would still hold, there would be a unique profit maximizing value of w^*[25]. Where our analysis above might change with a unimodal distribution would be when we consider the impact of a mean preserving spread of the distribution. Let us take the symmetric Triangular distribution as an example[26].

A mean preserving spread under a Uniform distribution unambiguously leads to a fall in the density at the initial choice of w^*. As we have shown in our analysis above, whether or not such a spread leads to greater exclusion depends on what happens to the inverse of the cumulative distribution at the initial w^*, that is what happens to $1 - G(w^*)$. We argued that if the condition in (3.14) is satisfied then the mean preserving spread would lead to greater exclusion. The main difference between the impact of a mean preserving spread under a symmetric Triangular distribution, as opposed to the Uniform distribution, is that under the former there is ambiguity as to what happens to the density at the initial cutoff level of income, $g(w^*)$. Depending on the initial cutoff level of income, a mean preserving spread of the Triangular distribution could lead to the density either increasing or decreasing. The ambiguity regarding the impact of such a spread on $1 - G(w^*)$ would be similar to the ambiguity under the Uniform distribution. Therefore the impact of a mean preserving spread under a Triangular distribution would depend on the initial level of w^*, which would determine which one of these effects would dominate.

[25]Note that under most unimodal distributions the probability of the lowest level of income, $g(a)$, is either zero or approaching zero, so the left-hand condition in (3.7) would always be satisfied and some consumers would be excluded.

[26]We assume that the condition for a global maximum is satisfied in our analysis that follows.

This is more easily demonstrated by looking at equation (3.10). As we can see from the first term on the right hand side of the equation, a decrease in the density, $g(w^*)$, combined with an increase in $1 - G(w^*)$, would require w^* to increase in order to maintain the equality (this is what happens under a mean preserving spread of the Uniform distribution when the condition in (3.14) is satisfied). Under a mean preserving spread of a Triangular distribution there is greater ambiguity regarding changes in $g(w^*)$ and $1 - G(w^*)$. Whether or not under such a distribution a mean preserving spread leads to greater exclusion depends on which of these effects dominates.

Financial Cost of Exclusion

Finally we would like to consider the costs of financial exclusion. Before we go into our analysis, there are two clarifications as to the purpose of this section. Firstly, in our model consumers choose their method of banking optimally. So from a utility perspective it is clear that excluded consumers would not prefer to use a mainstream bank. This result is driven partially from the fact that other than imposing a monopolist bank, we did not allow for any frictions. Our results would differ if something other than a bank's choice of fees led to the exclusion of the lower income class from banking services. These frictions can include information asymmetries, uncertain income flows coupled with risk aversion, or lack of spatial access, among others. Considering the welfare implications of these types of frictions is beyond the scope of our theoretical model. Alternatively, we can look at how the cost of transaction services as a percentage of income compares between the banked and unbanked, giving us a sense of the costs of being excluded from mainstream banking. This is the approach we will take here.

The second point of clarification is related to this approach. The direct fees charged by most deposit-taking institutions tend to be fixed fees, so by definition are regressive. That means that irrespective of the type of financial service provider households use (bank or AFS), low-income consumers pay a higher percentage of their transaction balances as fees than high-income consumers. We will discuss this issue in more depth below. In this section we look at the relative costs of the two types of service providers in order to determine the factors that can help mitigate or accentuate the costs of being unbanked.

Let us consider our results when using the Pareto distribution. We have from equations (3.3) and (3.16) that the fee charged by the bank is given by:

$$f_B = \theta w^* + c_A \quad \Rightarrow \quad f_{P,B} = \frac{c_B + c_A\left(\frac{r}{\theta} - \frac{1}{\alpha}\right)}{1 + \frac{r}{\theta} - \frac{1}{\alpha}}$$

When using a Pareto distribution, consumers with income greater than w^* pay $f_{P,B}$ for banking services, while consumer with income less than w^* pay c_A. In determining the relative costs to the two types of consumers (banked versus unbanked) we consider the outcome for a consumer δ below the cutoff level of income, $w^* - \delta$, relative to a consumer delta above, $w^* + \delta$. We would like to determine under what condition the unbanked pay a higher percentage of their income for financial services, $\frac{c_A}{w^* - \delta} > \frac{f_{P,B}}{w^* + \delta}$. Comparing these two ratios we have the following condition:

$$\delta\left[r + \frac{\theta(\alpha-1)}{\alpha}\right]\left[\left(\frac{r}{\theta} + \frac{\alpha-1}{\alpha}\right) + \frac{c_B - c_A}{c_A}\right] \quad > \quad \frac{(c_B - c_A)^2}{c_A} \tag{3.17}$$

If the condition in (3.17) holds, then the unbanked pay a higher percentage of their income for transaction services. It is clear that for income values in the neighborhood of w^* ($\delta \rightarrow 0$) this condition does not hold. This is because the fee charged by the bank is greater than the fee charged by AFS by construction. The bank provides a higher quality service than AFS and therefore can charge a higher price. Alternatively, for a high enough difference in income (higher δ) the condition above holds. This is in line with what we would expect. As we have stated above, the fixed costs of of banking is inherently regressive, and therefore it is not surprising that lower income consumers pay a higher percentage of their income for these fees.

More interestingly, a higher rate of return available to the bank, r, would make it more likely that those who are left out of the banking sector are worse off. Coupled with the result from above, where an increase in r makes exclusion less likely, this suggests that although the probability of being excluded goes down with the rate of return earned by the bank, the cost of being excluded increases. On the other hand, from the equations in (3.16) and the condition in (3.17), we have that both the probability of exclusion as well as the cost of being excluded are decreasing with Pareto's coefficient, α[27].

One factor that we have not considered so far is the low level of cash transactions in the modern economy. It is important to determine whether unbanked consumers pay higher fees because of being excluded from mainstream banking, or because they do not have a good outside option. There is evidence to suggest that the lack of an outside option is a major factor in the high cost of banking services to low-income households. Research into the fees charged by major banks has found that bank consumers with low deposit balances pay comparable fees to those charged by AFS, CRL (2011). In addition, there are many cases where mainstream banks either directly or indirectly participate in the AFS market, Epstein and Grow (2007).

In the next section we consider what happens to our results when we allow the bank to participate in the AFS market, as well as how our results depend on the consumers'

[27] Note that the condition in (3.17) does not depend on the standard of living parameter, a.

outside option, λ.

3.3 Alternative Financial Services

In the previous section we presented a very specific model of competition, a monopolist facing a competitive fringe. Now we consider some variations in the structure of our model to get a better sense of the policy implications of our results. We begin by allowing the bank to participate in the AFS market.

Bank as AFS Provider

Up to now we have assumed that the bank can only provide mainstream banking services and is not able to participate in the AFS market. But this does not have to be the case. Financial services involve some basic operations universal to banks and AFS. In most cases banks provide the same types of financial services to their deposit clients as AFS provide to their customers. It seems reasonable to expect that if banks face competition from AFS providers, they would consider the option of entering that market. In fact this observation seems to be true in practice. Mainstream banks have been shown to participate in the AFS market both directly, by providing AFS type services to clients with and without deposit accounts, and indirectly, by funding or owning AFS providers, Epstein and Grow (2007). We extend our model above to allow the bank to enter the AFS market. We maintain our assumption of a monopolist bank facing a competitive fringe.

The bank's role in the AFS market will depend on the marginal cost faced by the bank for transaction services, c_T. If $c_T \geq c_A$ then the bank would not be able to compete in the AFS market and therefore would not enter. A lower marginal cost of providing transaction services for a mainstream bank seems a reasonable assumption, therefore we consider the alternative case, and for simplicity choose $c_T = 0$[28]. Similarly to our model above we assume that the fixed cost of entering the AFS market is zero, but this is not essential to our results. Since we have that the bank's marginal cost of transaction is lower than that of the AFS providers, the bank will choose a price, f_T, less than c_A and drive the rest of the AFS providers out of the market. We consider the choice of fees, and in turn w^* by the bank in this setting. The bank's profit function when providing both deposit and transaction services is given by:

$$\pi'_B = rD_B - (c_B - f_B)N_B + f_T T_B - k \tag{3.18}$$

where $D_B = \int_{w^*}^{a+h} wg(w)\,dw$ and $N_B = 1 - G(w^*)$ and $T_B = G(w^*)$

We now have that $w^* = \frac{f_B - f_T}{\theta}$. D_B and N_B are as we defined them previously, and T_B are the banks transaction customers. These customer do not have deposit accounts, so their funds are not available to the bank to invest in the first term D_B. From w^* we have that the bank is competing with itself between deposit and transaction services. Changes in f_T impact w^* in an equal but opposite direction with changes in f_B. This property allows us to make the following proposition regarding the choice of fees by the bank.

Proposition 3.1: *If the bank chooses to enter the AFS market, it will fix f_T equal*

[28] The actual value of the marginal cost of transactions for the bank is not as important as the assumption that the bank has a cost advantage to provide AFS services over AFS providers. In fact the value of c_T does not impact the choice of f_B, and ultimately the level of w^*, as we will show below. Where actual value of c_T is relevant is in the profitability for the bank of entering the AFS market. Given the studies that argue that banks find this sector profitable, we are comfortable making this assumption.

to c_A (or more precisely a very small amount below c_A) and choose f_B as determined by the value of w^* that maximizes the profit function in (3.18)*. In other words, we only need to consider the bank's choice of the "excess price" for deposit accounts relative to a fixed level of transaction fees.*

Proof: It is clear that the bank must choose a fee for transaction services less than c_A, otherwise it would not be able to attract any AFS customers. In addition, given our assumption on the outside option for consumers from section 2 (where we have substituted in for f_A):

$$a - c_A \geq \lambda a$$

The bank has a captive AFS market for all choices of $f_T \leq c_A$. Finally, given that the absolute value of the changes in w^* are equal for changes in f_T and f_B, it is optimal for the bank to raise f_T to be the highest possible value (just below c_A), and then choose the value of w^* (by choosing f_B) that maximizes (3.18). Any lower value of f_T would lead to lower profits from fees for the bank, without increasing profits from deposit holdings.

Substituting in for $f_T = c_A$ in (3.18) and solving for the value of w^* that maximizes the monopolist's profits under a uniform income distribution, we have:

$$w_U'^* = \frac{\theta(a+h)+c_B}{r+2\theta}$$

Comparing this choice of w^* with the cutoff level of income when the bank did not participate in the AFS market, from equation (3.12), it is straightforward to show that the cutoff level of income has increased:

$$\Delta w_U^* = \frac{c_A}{r+2\theta}$$

If the bank chooses to enter the AFS market, the cost to the bank of losing non-banked customers decreases, leading it to price a higher proportion of consumers out of mainstream banking. From the difference in w^* above we can see that this impact is decreasing with the rate of return available to the bank, which is the cost of losing deposit customers, but increasing with the relative cost advantage of the bank in providing AFS services, $|c_A|$.

What remains to be checked is whether or not the bank would choose to enter the AFS market to begin with. By our inherent assumption of profit maximization, the bank would only choose to provide AFS sevices if the resulting level of profits, π'_B is greater than the level of profits in our original model π_B. Under a uniform distribution of income the difference in the two profit levels is given by:

$$(\pi'_B - \pi_B)h = -(r+2\theta)\left(\frac{w'^{*2}_U}{2} - \frac{w^{*2}_U}{2}\right) + \Delta w_U^* \left[c_B + \theta(a+h)\right] + c_A(w_U^* - a) \tag{3.19}$$

Which leads to the following condition for entry of the bank into the AFS market:

$$(r+\theta)a - \theta h < c_B - \frac{c_A}{2}$$

This is a weaker condition on the standard of living, a, than our condition for financial exclusion in (3.11) above. If a bank does not find low-income consumers profitable in its higher quality deposit-taking business, it would rather enter the AFS market and serve those excluded customers as a provider of lower quality transaction services. And as we have shown above, when a bank chooses to enter the AFS market, the cutoff level of income for customers choosing to open a deposit account increases.

From a policy perspective this would suggest that allowing banks to enter the AFS market might lead to greater exclusion from mainstream banking. Large financial institutions have a cost advantage over small pawnbroker type AFS. At least in the case of our model, a bank with such a cost advantage is able to profit from non-banked customers. This might lead the bank to increase its fees for deposit services, increasing the proportion of the unbanked without reducing the fees for transaction services. The policy response to this result is not very clear. Regulating banks to keep them out of transaction services might not be feasible. An alternative to increased regulation would be to focus on the outside options available to consumers of financial services.

Now we consider the role of consumers' outside option in our results.

Role of Outside Option

In the introduction to our original model we presented the concept "proportion of cash transactions in the economy", λ. This parameter represents to what extent consumers can rely solely on cash without ever having to resort to a bank or AFS. From the perspective of access to income, this would be the proportion of people (including public and private employees) who are paid their wages and other form of earning in cash, rather than through check or direct deposit. While from a perspective of using their earnings, λ represents the extent to which consumers can purchase goods and services in cash, rather than through online and in store debit/credit services.

Here we look to identify the role of this outside option in the results we have presented above. To this point λ has only played an indirect role in our results because we have assumed that the poorest consumer would always choose to use AFS rather than rely on cash alone, $\lambda a < a - c_A$. Now we relax this assumption by considering the choice of w^* by the bank if the AFS market did not exist, and so the consumers' only outside option is to rely on the existence of a cash economy. This extension will allow us to consider both the impact of a less cash dependent economy and the existence of an AFS market in our model. Using u_0 to represent the utility of a consumer that does not use financial services we can now represent the consumer's binary choice as between:

$$u_B = (1 + \theta)w - f_B \qquad \text{and} \qquad u_0 = \lambda w \tag{3.20}$$

As before we compare the two utility functions above to determine the level of income, w^*, below which consumers depends solely on a cash economy.

$$w_0^* = \frac{f_B}{1+\theta-\lambda} \tag{3.21}$$

Therefore consumers earning below w_0^* are excluded from mainstream banking services. Using a uniform distribution of income, we substitute the above cutoff into the bank's profit function from (3.4) and maximize with respect to w_0^*. Solving for the bank's choice for the cutoff level of income when AFS are not an option for consumers we have:

$$w_0^* = \frac{(1-\lambda+\theta)(a+h)+c_B}{r+2(1-\lambda+\theta)} \qquad f_{U,B} = \frac{(1-\lambda+\theta)(a+h)+c_B}{\frac{r}{1-\lambda+\theta}+2} \tag{3.22}$$

As we would expect f_B is a decreasing function of λ, as consumers' outside option improves, the bank is forced to lower fees to keep its customers. Differentiating w_0^* with respect to λ we have that if the following condition holds the level of exclusion is decreasing with the proportion of cash transactions, λ.

$$c_B < \frac{r}{2}(a + h)$$

If the level of income in a market is high enough relative to the technology of the bank, then as consumers' outside option increases the bank would choose to lower fees aggressively, resulting in less exclusion. The intuition behind this result is based on the rate of return available to the bank on customer deposits. As we have noted above, the bank earns revenues from charging customers direct fees and by earning a return on customer deposits. As λ increases the bank must lower its fees, resulting in lower direct revenues from customer accounts. If the level of income in a population is high enough, the lower direct fees puts greater emphasis on the return on deposits as a factor in bank profits. Therefore as long as the condition above is satisfied, when the percentage of the cash economy increases the bank would be willing to sacrifice the less profitable fees to attract more deposit customers.

The implication of this result is that an increase in consumers' outside option might not reduce financial exclusion if the bank is faced with a low-income consumer population. In such a case as the fees the bank charges decrease, the bank would not find returns on deposits high enough relative to the cost of administering accounts, therefore the bank would shrink its target market.

In order to see the impact of AFS on exclusion we can compare w_0^* with the cutoff level of income from our initial model. We can see by examination that (3.22) is greater than the w_U^* we calculated in our initial model, equation (3.12), regardless of the value of λ. This would suggest that despite their high fees, AFS do in some way improve the outcome for the consumer population by improving their outside option. The presence of AFS in the market forces the bank to lower its fees, lowering the costs of banking as well as financial exclusion.

Although it seems that λ does not impact the level of exclusion, it does have very important welfare implications. We can see by looking at the utility level of the two types of consumers in (3.20) the welfare of consumers left out of the financial services sector depends on λ. If we have a 100% cash economy the only difference between the banked and unbanked is the quality of service provided by the banking institution, θ.

This is a very important point. As we discussed above, the cost to consumers of being left out of the mainstream banking sector involves both a lack of access to the non-cash portion of the economy, as well as the inability to benefit from the security and convenience provided by deposit-taking institutions. Although both factors are very important issues facing households, they are very different from an overall welfare perspective. Security and convenience of consumer assets are similar to having a good security system or generous insurance on your home, they can be seen as goods bought in the market at a price. But access to your earnings should be considered more as a right. To the extent that some consumers are priced out of full access to their earnings is a much more fundamental problem. A problem that should interest public economists as well as policy makers, since it can impact redistribution measures that are rarely paid out in cash.

A Captive Audience

As a final extension, we consider the possibility of there existing a captive audience for AFS providers. By captive audience we mean the possibility that a portion of the consumer population does not have the bank as an option (or does not know the bank is an option). These could be illegal immigrants, financially uneducated consumers, or those who live in neighborhoods without banks, so are spatially constrained.

The impact of such a group within our model depends significantly on our choice of income distribution, as well as who the captive audience are, with respect to their level of income. To illustrate our point, consider a uniform distribution of income with density function $g(w) = 1/h$. If we assume that the captive audience is spread evenly across the income distribution, this is equivalent to a decrease in our density function, $g(w) = 1/h - \epsilon$. As we have shown in our analysis above, in a uniform distribution, h does not impact the choice of w^* by the bank, and therefore does not impact exclusion. Although a reduction in density would not impact the income level of inclusion, it would increase the percentage of consumers that resort to AFS providers.

Alternatively, if we assume that the captive audience is concentrated at the bottom of the income distribution, this is equivalent to cutting off a rectangle on the left hand side of the uniform cumulative distribution. In this case the captive audience would only impact our results if it extended beyond the bank's choice of w^*, meaning that the bank is not able to access as many customers as it would like. This seems to be a reasonable possibility in practice. According to the 2009 FDIC report on financial exclusion, some mainstream banks actively seek out unbanked consumers through community programs designed to improve understanding of the availability and benefits of deposit accounts. This would suggest that a portion of the exclusion data that we site in our introduction to this paper might be attributed to the lack of good information or understanding, rather than the kind of cost benefit analysis we have described in our model.

3.4 Conclusion

In this paper we have looked to more formally analyze the causes of exclusion from mainstream banking. We have used a stylized model of banking services to demonstrate how under certain circumstances it might be optimal for the bank to exclude the lower income portion of the population. In this setup, the existence of AFS in the market provides consumers with a better outside option relative to relying solely on cash for their day to day existence. In that sense the AFS market plays a positive role in our model, and forces the monopolist bank to price more competitively.

This result depends to some extent on our assumption of perfect competition in the AFS market, as well as our inherent assumption of perfect information and access for all consumers. Uninformed or segregated consumers may not have access to the mainstream banking sector, even if they would prefer to have a bank account rather than rely on AFS. To the extent that these frictions exist, consumers might be susceptible to predatory pricing. Skiba and Tobacman (2007) and others have shown that AFS is not a highly profitable business, therefore marginal cost pricing might not be very farfetched. On the other hand, the volume of transactions in the AFS market is growing very quickly, an indication that there are positive profits in this sector. More work needs to be done to understand the role of AFS and the consequences for consumers that are forced to rely on AFS for their financial needs.

We have also shown that the rate of return available to the bank, r, can play a positive role in reducing exclusion from mainstream banking. This result suggests that allowing banks to invest customer deposits has a positive impact on the consumer population by reducing the direct fees they have to pay for banking services. To the extent that consumers do not have access to a risk free rate of return for their assets, these direct fees make up a big chunk of the costs of banking. By allowing the bank to reduce direct fees, a higher rate of return on deposits reduces exclusion from banking services, as well as increasing consumer surplus. But the positive impact of r depends on what drives the increase in returns for the bank. If an increase in r is associated with economic growth and better investment opportunities, then it can be seen as a win win outcome for consumers and the overall economy. On the other hand, if increases in r are driven by higher risk in the bank's investment portfolio, the positive impact on consumers can be short lived; a phenomenon that we observed directly in the 2008 financial crisis. Future work on this topic should consider the tradeoff a bank faces when it chooses r, and how its choice of risk in its investment portfolio depends on the consumer population and the economic environment.

We believe that our results in this paper are a good demonstration of how introducing a heterogeneous consumer population adds greater depth to economic analysis. As far as we know, models of banking services have mainly ignored the role of income distribution in considering the strategic decisions of financial institutions. As we have shown above, how income is distributed can significantly impact firm strategy. In addition, changes in the income distribution can have important implications for outcomes for individual consumers. As we show in this paper, under certain circumstances, an increase in the dispersion of income can lead to the bank charging higher fees, and excluding a greater portion of consumers. Over the last several decades we have observed a trend towards greater income dispersion, our results would suggest a greater need for understanding the impact of this phenomenon on the workings of the modern economy, and the financial sector.

3.A Mathematical Appendix

Proof for local maximum

In case of a uniform distribution we have that $g(w) = 1/h$ and $1 - G(w) = \frac{a+h-w}{h}$. Therefore we have that:

$$\frac{g}{1-G} = \frac{1}{a+h-w}$$

Which is clearly an increasing function of w. Therefore we have a non-decreasing hazard rate, and the condition for a local maximum is satisfied.

In the case of a Pareto distribution we do not have a non-decreasing hazard rate. We need to show that the following condition for a local maximum holds:

$$\frac{(1-G)g'}{g^2} + \frac{r}{\theta} + 2 \geq 0$$

Using the Pareto distribution we can show that:

$$\frac{(1-G)g'}{g^2} = -\frac{\alpha+1}{\alpha} > -2$$

Where the relation at the end comes from the condition that $\alpha > 1$. Therefore our condition for a local maximum holds for any non-negative values of r and θ.

Proof for global maximum

The condition for a global maximum holds trivially for a Uniform distribution since the density function is constant with respect to w, that is $g'(w) = 0$.

To see that the local maximum under a Pareto distribution is also a global maximum we consider the derivative of our profit function.

$$\frac{\partial \pi_B}{\partial w^*} = -[\alpha r + (\alpha - 1)\theta] \left(\frac{a}{w^*}\right)^{\alpha} + \frac{\alpha}{a} (c_B - c_A) \left(\frac{a}{w^*}\right)^{\alpha+1}$$

We have already shown that the above function reaches a local peak at $w^* = w_P^*$ (where w_P^* is defined as in equation (3.16)). It is also straightforward to show that this function is positive for all values of w^* below w_P^* and negative for all values of w^* above w_P^*. Therefore the profit function is single peaked and we must have that w_P^* is the unique value of w^* that maximizes the bank's profit function.

Concluding Remarks and Direction for Future Research

Over the last several decades it has become increasingly clear that the distributions of income in most developed economies are becoming more and more unequal. The cause of this trend is not clear, although there is reasons to believe that the reasons are both policy related as well as due to structural changes in an increasing global economy. Whatever the causes of greater inequality, it is important for us to better understand how greater income inequality impacts the way markets operate. As the chapters above have demonstrated, it is possible that increasing inequality concentrates access to information and economic power in the hands of the few. This might make it more difficult for lower income households to participate in retail, real estate and financial markets on an equal footing with the rest of the consumer population.

The political battles that we observe taking place across the globe make it clear that there is no consensus within and between countries on an ideal level of inequality. But what most people who believe in market economies cannot deny, is that markets must allow equal access to all individuals regardless of their race, ethnicity, sex or economic background. The results of this thesis are not surprising in the sense that the story that the poor have a hard time competing in the economy has been told before. The purpose of this thesis is to put this story into a more rational and quantifiable framework.

These chapters look to identify specific economic issues and policy variables that are the reasons why low income consumers struggle to compete. We firmly believe that by removing these barriers to competition faced by a segment of the population, market economies can become more efficient and achieve greater overall prosperity.

The chapters in this thesis have pointed towards some specific policy steps that can be taken to alleviate the market frictions described. But the stylized nature of our models leave room for a significant amount of future work. Our hopes for future research involve two approaches. One approach is the extensions of some of the models above in order to better understand the mechanisms that drive the results we present. For example, in the second chapter we assumed only a monetary cost of transportation. As the results in our first chapter show, a time cost of access can have a significant impact on the decision of consumers. In future work we hope to introduce a time cost to commuting, either directly into the consumers utility function, or in the same style as our model in chapter one, as part of the household's time constraint. This type of extension will allow for a more rich set of city structural outcomes, helping us better understand the causes and costs of segregation.

In the third chapter the return on investments available to the bank is exogenously determined. As the model demonstrates, this rate of return is a key factor in the existence of financial exclusion. An obvious extension of this chapter would be to endogenize r by modeling the asset side of a bank's business. Better understanding the motivation for banks to increase or decrease their risk profile in relation to the rate of return available on assets, will help us understand what drives a bank's return on investments. Such an approach would make it easier to identify whether the cause of an increase in r is a better economic environment, or greater exposure to risk. By making this distinction, our approach to regulation of banks can become more nuanced, and in a sense we will be able to avoid throwing the baby out with the bathwater.

The second approach for future research would be an empirical study of the results and assumptions that we have discussed throughout this thesis. For example, in the first chapter the Search Condition was a significant precondition on whether or not our results are valid. In addition, we made some strong assumptions on the distribution of consumer income and the process of consumer search. Future work could focus on determining the types of markets where the search condition above holds. It would be interesting to build and update the work in Frankel and Gould (2001) in order to determine how their results would differ if the factors presented in this chapter were taken into account.

Finally, a recent study on financial exclusion in the U.S. by the federal deposit insurance corporation, FDIC (2009), provides a very rich set of data on households that are excluded from mainstream banking and resort to Alternative Financial Services. It might be possible to use this dataset in order to test the results presented in chapter three. Households might revert to Alternative Financial Services for rational or irrational reasons, the reasons could be spatial or a lack of information. All of these are factors that have strong implication for our results, as well as for any potential policy related approach.

Bibliography

Alcaly, R. E. and Klevorick, A. K. (1971). Food prices in relation to income levels in New York City. *Journal of Business*, 44(4):380–97.

Alonso, W. (1964). *Location and Land Use*. Harvard University Press, Cambridge, MA.

Atkinson, A. B. (1995). *Choice, Welfare, and Development*, chapter 2, pages 17–31. Oxford University Press, paperback, 2000 edition.

Atkinson, A. B., Piketty, T., and Saez, E. (2011). Top incomes in the long run of history. *Journal of Economic Literature*, 49(1):3–71.

Attanasio, O. P. and Frayne, C. (2006). Do the poor pay more? Working Paper.

Bagwell, K. and Ramey, G. (1992). The diamond paradox: A dynamic resolution. Discussion Papers 1013, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Basu, S. and Wang, J. C. (2007). Technological progress, "money" in the utility function, and the "user cost of money".

Baumol, W. J. (1952). The transactions demand for cash: An inventory theoretic approach. *Quarterly Journal of Economics*, 66:545 – 556.

Baye, M. R., Morgan, J., and Scholten, P. (2006). Information, search, and price dispersion. Working Papers 2006-11, Indiana University, Kelley School of Business, Department of Business Economics and Public Policy.

Beck, T., Demirguc-Kunt, A., and Peria, M. S. M. (2007). Reaching out: Access to and use of banking services across countries. *Journal of Financial Economics*, 85:234–266.

Beck, T., Demirguc-Kunt, A., and Peria, M. S. M. (2008). Banking services for everyone? barriers to bank access and use around the world. *The World Bank Economic Review*, 22(3):397–430.

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal*, 75(299):493–517.

Blisard, N. and Stewart, H. (2008). Who pays more for food? *Journal of Agricultural Economics*, 59(1):150–168.

Braverman, A. (1980). Consumer search and alternative market equilibria. *The Review of Economic Studies*, 47(3):487–502.

Brueckner, J. K., Thisse, J.-F., and Zenou, Y. (1999). Why is Central Paris rich and Downtown Detroit poor? an amenity-based theory. *European Economic Review*, 43(1):91–107.

Caplovitz, D. (1963). *The Poor Pay More: Consumer Practices of Low-Income Families*. The Free Press of Glenco, New York.

Carlin, B. I. (2009). Strategic price complexity in retail financial markets. *Journal of Financial Economics*, 91(3):278–287.

Chipman, J. S. (1974). The welfare ranking of pareto distributions. *Journal of Economic Theory*, 9(3):275–282.

Chung, C. and Myers, S. L. (1999). Do the poor pay more for food? an analysis of grocery store availability and food price disparities. *Journal of Consumer Affairs*, 33(2):276–296.

Clifton, K. J. (2004). Mobility strategies and food shopping for low-income families: A case study. *Journal of Planning Education and Research*, 23(4):402–413.

CRL (2011). Big bank payday loans: High-interest loans through checking accounts keep customers in long-term debt. Technical report, Center For Responsible Lending.

Dalton, E. S., Ehrlich, F. M., Heberlein, E., and Niemeyer, M. (2003). Food availability in Allegheny County, PA. *The Heinz School Review*, I(I).

Dash, E. (2011). Profits falling, banks confront a leaner future. *The New York Times*, page A1.

Devlin, J. F. (2005). A detailed study of financial exclusion in the UK. *Journal of Consumer Policy*, 28(1):75–108.

Diamond, D. (1984). Financial intermediation and delegated monitoring. *Review of Economic Studies*, 51(3):393–414.

Diamond, P. A. (1971). A model of price adjustment. *Journal of Economic Theory*, 3(2):156–168.

Epstein, K. and Grow, B. (2007). The poverty business. *Business Week*.

FDIC (2009). National survey of unbanked and underbanked households. Technical report, Federal Deposit Insurance Corporation.

Fishman, A. and Simhon, A. (2005). Can income inequality increase competitiveness. Working Paper.

FIT (2009). Fourth annual report on progress towards the shared goal for banking. Technical report, Financial Inclusion Task Force.

Flannery, M. and Samolyk, K. (2005). Payday lending: Do the costs justify the price? Working Paper 2005-09, FDIC Center for Financial Research.

Frankel, D. M. and Gould, E. D. (2001). The retail price of inequality. *Journal of Urban Economics*, 49(2):219–239.

Freedman, A. M. (1991). The poor pay more for food in New York, survey finds. *The Wall Street Journal*, pages B1, B5.

Freixas, X. and Rochet, J.-C. (2008). *Microeconomics of Banking*. The MIT Press, 2nd edition.

FSA (2000). In or out? financial exclusion: A literature and research review. Consumer Research Paper 3, Financial Services Authority.

Gabszewicz, J. J. and Thisse, J. F. (1979). Price competition, quality and income disparities. *Journal of Economic Theory*, 20(3):340–359.

Glaeser, E. L., Kahn, M. E., and Rappaport, J. (2000). Why do the poor live in cities? NBER Working Papers 7636, National Bureau of Economic Research, Inc.

Gonzalez, D. (2008). The lost supermarket: A breed in need of replenishment.

Guerrieri, V., Hartley, D., and Hurst, E. (2010). Endogenous gentrification and housing price dynamics.

Hausman, J. and Leibtag, E. (2007). Consumer benefits from increased competition in shopping outlets: Measuring the effect of Wal-Mart. *Journal of Applied Econometrics*, 22(7):1157–1177.

Hayes, L. R. (2000). Do the poor pay more? an empirical investigation of price dispersion in food retailing. Working Papers 825, Princeton University, Department of Economics, Industrial Relations Section.

Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics*, 112(3):663–691.

Hong, H. and Shum, M. (2006). Using price distributions to estimate search costs. *RAND Journal of Economics*, 37(2):257–275.

Kempson, E. and Whyley, C. (1998). *Access to current accounts*. London: British Bankers Association.

Kober, C. and Sterlitz, J. (2007). The poverty premium: How poor households pay more for essential goods and services. Non-profit, Family Welfare Association.

Kunreuther, H. (1973). Why the poor may pay more for food: Theoretical and empirical evidence. *Journal of Business*, 46(3):368–83.

Lach, S. (2007). Immigration and prices. *Journal of Political Economy*, 115(4):548–587.

LeRoy, S. F. and Sonstelie, J. (1983). Paradise lost and regained: Transportation innovation, income, and residential location. *Journal of Urban Economics*, 13(1):67 – 89.

MacDonald, J. M. and Nelson, P. J. (1991). Do the poor still pay more? food price variations in large metropolitan areas. *Journal of Urban Economics*, 30(3):344–359.

Mills, E. S. (1967). An aggregative model of resource allocation in a metropolitan area. *The American Economic Review*, 57(2):197–210.

Muller, C. (2002). Prices and living standards: evidence for rwanda. *Journal of Development Economics*, 68(1):187–203.

Musgrove, P. and Galindo, O. (1988). Do the poor pay more? retail food prices in Northeast Brazil. *Economic Development and Cultural Change*, 37(1):91–109.

Muth, R. (1969). *Cities and Housing*. University of Chicago Press, Chicago.

Myers, C. K. (2011). Retail redlining: Are gasoline prices higher in poor and minority neighborhoods? *Economic Inquiry*, 49(3):795–809.

Nielsen, N. R. (2008). www.internetworldstats.com.

Piachaud, D. (1974). *Do The Poor Pay More?* Child Poverty Action Group, London.

Piachaud, D. and Webb, J. (1996). *The Price of Food: Missing Out on Mass Consumption*. Suntory and Toyota International Centres for Economics and Related Disciplines.

Rao, V. (2000). Price heterogeneity and "real" inequality: A case study of prices and poverty in rural South India. *Review of Income and Wealth*, 46(2):201–11.

Ratchford, B. T. (2009). *Handbook of Pricing Research in Marketing*, chapter 5, pages 91–107. Edward Elgar Publishing.

Rauh, M. T. (1997). A model of temporary search market equilibrium. *Journal of Economic Theory*, 77(1):128–153.

Rhodes, A. (2011). Multiproduct pricing and the diamond paradox.

Rob, R. (1985). Equilibrium price distributions. *Review of Economic Studies*, 52(3):487–504.

Salop, S. and Stiglitz, J. (1977). Bargains and ripoffs: A model of monopolistically competitive price dispersion. *The Review of Economic Studies*, 44(3):493–510.

Salop, S. C. (1979). Monopolistic competition with outside goods. *The Bell Journal of Economics*, 10(1):141–156.

Selod, H. and Zenou, Y. (2003). Private versus public schools in post-apartheid south african cities: Theory and policy implications. *Journal of Development Economics*, 71(2):351–394.

Shaked, A. and Sutton, J. (1982). Relaxing price competition through product differentiation. *The Review of Economic Studies*, 49(1):3–13.

Shleifer, A. and Vishny, R. (2010). Unstable banking. *Journal of Financial Economics*, 97(3):306–318.

Skiba, P. and Tobacman, J. (2007). The profitability of payday loans.

Son, H. and Tighe, M. (2011). Bofa's moynihan says 'new normal' in banking not as profitable. *Bloomberg News*.

Stigler, G. J. (1961). The economics of information. *The Journal of Political Economy*, 69(3):213–225.

Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information. *American Economic Association*, 71(3):393–410.

Tobin, J. (1956). The interest elasticity of the transactions demand for cash. *Review of Economic Studies*, 38(3):241–247.

Varian, H. R. (1980). A model of sales. *The American Economic Review*, 70(4):651–659.

Wasmer, E. and Zenou, Y. (2002). Does city structure affect job search and welfare? *Journal of Urban Economics*, 51(3):515–541.

Wheaton, W. C. (1977). Income and urban residence: An analysis of consumer demand for location. *The American Economic Review*, 67(4):620–631.

Wrigley, N. (2002). "Food Deserts" in British cities: Policy context and research priorities. *Urban Studies*, 39(11):2029–2040.