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Patterning through instabilities in complex media: Theory and applications

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Classical studies in engineering usually analyse the possibility of instability as a predictive tool that is used in order to avoid the onset of material failure in structural applications: how should one design a water tower so that it will not collapse when filled with water? Recently, a multidisciplinary research community has coalesced around the idea that such instabilities may also be used positively to guide the design and fabrication of new materials. For example, this guiding principle has recently been employed in elastic structures, where instabilities provide simple mechanisms through which to generate highly regular patterns or switchable morphologies, with applications ranging from self-folding machines to stretchable electronics and smart textiles. Such applications often generate extreme deformations in complex fluids and soft solids, opening the way to unexplored instability regimes that are associated with non-trivial phenomena. In fact, the combination of both geometrical and material nonlinearities often prompts the occurrence of intricate morphological changes following bifurcations and may lead to unconventional (or even counter-intuitive) behaviour.

Although instabilities in fluids and elastic solids have received much attention in their respective research fields over the last century, there remain many fundamental unanswered questions. This is particularly so where applications push instability into highly-developed regimes. The focus of this Special Issue is the wide class of instabilities that arise in complex media: from fluids to solids and those materials in between. Here we present a selection of some of the more recent developments in the analysis of such instabilities. The collected articles employ methods and techniques from a range of disciplines in Applied Mathematics, Physical Sciences and Engineering. These works focus on new fundamental scientific advances motivated by the wide

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range of applications in which these ideas can be used.

This Special Issue has 12 contributions, beginning with the review article by Gallaire and Brun [6]. This article provides a survey of some of the more common fluid instabilities, as well as highlighting some of their practical uses and laying some of the ground-work for other articles in the issue. The first group of papers concerns novel instabilities in complex media. Goehring and coworkers [7] investigate a drying colloidal suspension; this is an example of a phenomenon that combines fluid flow and transport with solid behaviours such as fracture and shear banding. Fracture is also a feature studied by Novak and Truskynovsky [10] in the context of fracture-induced segmentation in elastically constrained cohesive systems. Here, they focus on the presence of competing interactions that may lead to hierarchical self-assembly. Gourgiotis and Bigoni [2] then demonstrate how a spontaneous folding emerges in a generalized continuum material that is designed with extreme orthotropic properties, i.e. near an ellipticity failure. The dynamic properties of such an instability open the possibility to exploit Cosserat media for propagating waves in materials displaying origami-patterns of deformation. Riccobelli and Ciarletta [11] prove that a gravity-induced instability may occur when a dense elastic layer overlays a lighter layer. Crucially, the morphology of this instability is different to the classic Rayleigh–Taylor instability of fluids. The similarities and differences between viscous and elastic instabilities are highlighted again by Brun et al. [3], who study how a viscous fluid buckles when steadily dripped onto a moving belt. In particular, they show how this instability forms a series of regular stitches and, further, that when the flowing liquid is molten glass, permanent objects are formed that appear to have been 'sewn' together.

The second group of papers tackle a number of unresolved issues that arise when dealing with elastic instabilities in the well-developed regime. Hutchinson and Thompson [8] focus on the nonlinear buckling instability of a thin elastic shell subject to an external pressure. They provide new insights on the formation and development of multi-lobed dimples that occurs in the post-buckling regime. A different type of shell buckling instability, wrinkling, is studied by Taffetani and Vella [12]. They show how the properties of wrinkled shells can be understood well beyond the onset of instability by exploiting the large wavenumber of instability at threshold and, further, that the scale of instability observed in this highly-wrinkled state varies spatially. The spatial variation in wrinkle patterns caused by geometrical incompatibility is studied using rigorous mathematical methods by Bella and Kohn [1]. For their problem, Bella and Kohn show that the wrinkle wavelength is largely constant in space, but varies slightly to avoid the energetic cost of changing the number of wrinkles, which would be required to keep the wavelength precisely constant. Destrade and coworkers [5] then show that a soft solid subjected to an extreme deformation may form wrinkles in a direction other than normal to the direction of greatest compression; this wrinkled state may possibly become an initial bifurcation mode. Kuhl and coworkers [4] study the emergence of secondary bifurcations in a compressed elastic bilayer, providing useful guidelines for the design of smart surfaces with tunable morphology. Finally, Hazel and Mullin [9] investigate the peculiar buckling characteristics of thin elastic rings confined within containers of circular or regular polygonal cross section.

The collected articles unravel a number of urgent and fundamental questions by using a multi-disciplinary approach based on a combination of advanced theories, experiments and computations. Their results not only make advancements to our fundamental scientific understanding of such problems, but also push towards a paradigm shift in material design. We firmly believe that this reinvigorated interest in instabilities within mathematical and physical sciences will eventually open the path for guiding original engineering applications in many of the emerging fields at the core of modern technologies.

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