Sequestering of Kähler moduli in type IIB string theory

Lukas Witkowski

Oriel College

University of Oxford

A thesis submitted for the degree of

Doctor of Philosophy

Trinity 2013

Für meine Eltern
Leszek und Anna Witkowski
mit großer Dankbarkeit

Acknowledgements

First and foremost I want to thank my supervisor Joe Conlon for his continuous support, his immense knowledge, his careful guidance and his kind patience.

Further, I want to express my sincere gratitude to my collaborators Marcus Berg, M. C. David Marsh and Mathew Bullimore, from whom I have learnt many things. I am also indebted to Mark Goodsell, who so often pointed me in the right direction.

It has been a pleasure to work as a member of the Theoretical Particle Physics group in Oxford. In particular, I want to thank Michael Klaput for numerous discussions, invaluable comments on the manuscript and friendship. I am also grateful to Andrei Constantin, Georgios Giasemidis, Saso Grozdanov, Nikolaos Kaplis and Benjamin Niedner who made the time at Oxford very enjoyable.

I also need to mention Chris Wymant – my close friend throughout both undergraduate and graduate life.

Last but not least, I want to thank Youxin Kong for her continuous encouragement and moral support.

This work was supported by the Science and Technology Facilities Council.

Sequestering of Kähler moduli in type IIB string theory

Lukas Witkowski

Oriel College
University of Oxford

A thesis submitted for the degree of
Doctor of Philosophy

Trinity 2013

Abstract

In this thesis we employ string perturbation theory in toroidal orbifold models to study aspects of supersymmetry breaking in type IIB string theory.

First, we determine the dependence of physical Yukawa couplings on blow-up moduli in models with D3-branes at orbifold singularities. Blow-up moduli are scalar fields describing the size of small blow-up cycles in the compactification geometry. In models implementing moduli stabilisation these fields can acquire F-terms and break supersymmetry. We examine the moduli-dependence of physical Yukawa couplings at string tree-level by computing disk correlation functions involving a Yukawa interaction of visible sector fields and an arbitrary number of blow-up moduli. We perform the calculation for one blow-up insertion explicitly and find that the correlation function vanishes if the blow-up modulus is associated with a small cycle distant to the visible sector. For more than one blow-up insertion we show that all such correlation functions are exponentially suppressed by the compactification volume. We explain how these results are relevant to suppressing soft terms to scales parametrically below the gravitino mass.

Further, we determine corrections to holomorphic Yukawa couplings on D3-branes at an orbifold singularity due to non-perturbative effects such as gaugino condensation on a stack of D7-branes. This can be done by calculating a one-loop threshold correction to the gauge coupling on the D7-branes. We show that, if present, the new contributions to Yukawa couplings are not aligned with the tree-level couplings. As the new Yukawa couplings contribute to soft A-terms they are sources of flavour-changing neutral currents.

Last we discuss an effect unrelated to supersymmetry breaking. We show that orbifold models with D3-branes at orbifold singularities can exhibit kinetic mixing of different massless Abelian factors. For this to be possible, the relevant $U(1)$ factors have to be associated with more than one orbifold singularity.

Contents

Chapter 1

Introduction

As of July 4th 2012 the hierarchy problem has become real. On that day, both the ATLAS and CMS collaborations at CERN claimed the discovery of a new bosonic resonance with a mass of $m = 126$ GeV [1, 2], which we now consider as the Standard Model Higgs boson (for recent results see [3–5]). While the detection of a new particle is always a significant event in particle physics, this discovery is a landmark moment, as it has enriched the list of elementary particles by a new category. The Higgs boson is the first fundamental scalar particle observed, and it forces us to confront a theoretical challenge, which is inherent when coupling a scalar to the Standard Model: why is the Higgs so unnaturally light?

The question has its origin in the one-loop corrections to the mass of a scalar particle, which depend quadratically on the ultraviolet cutoff Λ of the theory. For example, the correction to the mass squared of a scalar due to a Dirac fermion loop contributes

$$\delta m^2 \supset -\frac{|\lambda_f|^2}{8\pi^2}\Lambda^2 , \tag{1.1}$$

where λ_f is the coupling of the scalar to the fermion. For the case of the Higgs boson such corrections arise from all Standard Model fermions, with the top quark giving the dominant contribution. The observation now is that if the cutoff squared Λ^2 for our theory is larger than the renormalised mass squared $m^2 = m^2_{bare} + \delta m^2$ of the scalar, we require a cancellation between the bare mass parameter m^2_{bare} and the loop correction. If we increase the cutoff Λ the cancellation has to occur to more and more decimal places and the bare mass parameter becomes increasingly fine-tuned. As the bare mass and the loop correction have different origins, excessive fine-tuning is theoretically poorly motivated and deemed unnatural. This leaves us with the problem that the discovered Higgs boson with its mass of $m_H = 126$ GeV is increasingly unnatural, if we require the Standard Model to hold beyond $\Lambda = 1$ TeV. However, as the Standard Model is a renormalisable theory, it could in principle remain valid up to the Planck scale. In this case we would need a fine-tuning to one part in $\sim 10^{30}$.

This is a manifestation of the technical hierarchy problem and it has sparked a vast amount of physics of the last decades. One of the solutions is supersymmetry, a new fundamental symmetry which relates bosons and fermions. In particular, it introduces a superpartner for each Standard Model particle whose spin differs from that of the latter by a half-integer. The presence of these new particles and new interactions then leads to a cancellation of corrections to the Higgs mass with quadratic dependence on Λ to all orders in perturbation theory. However, supersymmetry predicts that particles and their superpartners are degenerate in mass. This is not the case for the particle spectrum we observe in nature and correspondingly supersymmetry can only exist as a broken symmetry. If the breaking of supersymmetry is not to reintroduce a quadratic dependence of scalar masses on Λ, then the terms breaking supersymmetry should exclusively consist of operators with coupling constants with positive mass dimension, which are referred to as soft terms. These include gaugino masses $M_a \lambda_a \lambda_a$, scalar masses $\tilde{m}^2_{ij} \phi^*_i \phi_j$, bilinear B-terms $B_{ij} \phi_i \phi_j$ and trilinear A-terms $A_{ijk} \phi_i \phi_j \phi_k$, where the gauginos λ_a are superpartners of gauge bosons, and the scalars ϕ_i include Higgs fields and superpartners of Standard Model fermions. At most, supersymmetry breaking through soft terms contributes to the scalar masses at 1-loop as

$$\delta m^2 \sim m^2_{soft} \frac{\lambda}{16\pi^2} \ln \left(\frac{\Lambda}{m_{soft}} \right) \ , \tag{1.2}$$

where m_{soft} is the mass scale associated with the soft terms and λ is a dimensionless coupling constant. If supersymmetry is to constitute a solution to the hierarchy problem, soft terms should not be much larger than 1 TeV, which is referred to as TeV scale supersymmetry breaking.

Soft terms introduce a large number of parameters into our model of particle physics, which should be ultimately determined by experiment. However, if low-energy supersymmetry is true, these terms are far from random: there are already strong experimental constraints that imply a definite structure for soft terms. The most stringent bounds arise from requiring the absence of excessive flavour-changing neutral currents, for example in $K\bar{K}$ or $D\bar{D}$ mixing. These imply that – to a good approximation – soft terms must either be universal, or at least inherit their flavour structure directly from the Standard Model Yukawa couplings.

At low energies, soft terms are simply parameters that can be measured. However, we expect that in the correct theory of supersymmetry breaking, we would be able to calculate soft terms from first principle, and understand the physics that generates the structure exhibited by soft terms.

At the moment there is little experimental guidance to a mechanism of supersymmetry breaking. In phenomenologically viable models, supersymmetry is broken spontaneously in a hidden sector. This is a sector of the theory, which we have to add to the visible sector containing the supersymmetric extension of the Standard Model. Most importantly, the hidden sector fields are not charged under the Standard Model gauge group and there are no renormalisable tree-level interactions between the two sectors. Supersymmetry breaking is then mediated to the visible sector through messenger fields, which couple to both the visible and hidden sectors. Various mediation mechanisms have been proposed and studied in detail: the most popular include renormalisable gauge interactions, one-loop effects due to a conformal anomaly or contact terms of gravitational strength.

The latter is known as gravity-mediation, and we will focus on this mechanism in this thesis. In this case soft terms are generated by contact interactions between the visible and the hidden sector, which are suppressed by powers of the Planck mass M_P. At low energies, we can analyse the structure of soft terms by just studying the contact-interactions. However, these are non-renormalisable operators and a theory containing these terms has to be replaced by a more fundamental theory at the Planck scale. If we wish to arrive at a microscopic understanding of these operators, we have to determine them directly in the fundamental theory. For the case of gravity-mediated supersymmetry breaking, this is a theory which describes physics at the Planck scale. As this is the scale where quantum gravity effects become important, we are led to the following: if we wish to arrive at a microscopic understanding of gravity-mediated supersymmetry breaking, we have to consult a quantum theory of particle physics and gravity.

One of the most promising candidates for such a theory is string theory, which is the main subject of this thesis. String theory is a framework describing interactions of string-like objects at energies typically close to the Planck scale. At low energies the interactions of strings reproduce quantum field theory and Einstein gravity. Hence string theory provides a framework where a microscopic realisation of gravity-mediated supersymmetry breaking can be achieved. Consequently, in this thesis we assume that supersymmetry is the solution to the hierarchy problem, and then study TeV scale supersymmetry breaking in the context of string theory models. To identify the questions which we want to discuss, we first need to review how TeV scale supersymmetry breaking is achieved in string theory.

String theory and supersymmetry breaking

String theory is one of the most compelling frameworks to describe gravitational physics in a quantum theory, which is also able to incorporate interactions of matter. It has been an active area of research for decades and is still very much a work in progress, with consequences far beyond its application to particle physics. String theory has sparked new developments in mathematics, illuminated the physics of non-perturbative effects in field theory, led to a microscopic explanation of the Bekenstein-Hawking black hole entropy and is at the core of a class of dualities including the celebrated AdS/CFT correspondence.

According to our current understanding, string theory exists as a set of perturbative string theories, which are all related by dualities. It is expected that these distinct formulations may be obtained as limits of an underlying structure called M-theory. However, a compelling formulation of the latter is still unclear. As all perturbative superstring theories require ten spacetime dimensions for consistency, string phenomenology necessarily includes extra dimensions. To describe the four non-compact dimensions we observe, six of the space dimensions have to be compactified on a suitable manifold. If we wish to realise low energy supersymmetry breaking, the process of compactification should not break all of the supersymmetries. This can be achieved by compactifying on Calabi-Yau manifolds, which are thus widely studied as compactification manifolds. At energies below the string and compactification scales the four-dimensional physics is then described by an effective supergravity theory.

Geometrically different Calabi-Yau manifolds are connected by deformations which fall into two classes: modifications of the Kähler class can be understood as changes to the volumes of cycles while modifications of the complex structure affect the shape of the compactification manifold. If such deformations come at no energy cost to the theory, the corresponding fluctuations will appear as massless scalars in the low energy theory. Such fields, which correspond to continuous deformations of the vacuum, are called moduli. Beyond the Kähler and complex structure moduli from compactification, there is the dilaton describing the string coupling, and there can be further moduli which are related to positions of D-branes.

If these moduli remain massless or stay too light, they are highly problematic for both particle phenomenology and cosmology. Massless or light scalars coupling to the visible sector give rise to long range fifth forces, which are not observed. Further, the vacuum expectation values of these fields determine the coupling constants and Yukawa couplings of the effective four-dimensional theory. If these field values are not fixed, the theory will have no predictive power. Hence, if string phenomenology is to succeed, it is imperative that the string theory compactification generates a potential, which stabilises all moduli.

While the existence of moduli introduces problems, it also allows for interesting phenomenology. Moduli are not charged under the Standard Model gauge symmetries and thus they naturally realise hidden sectors. Most importantly, the process of stabilisation not only provides the moduli with vacuum expectation values and masses, but it can also generate F-terms, which indicate broken supersymmetry. As moduli couple gravitationally to visible matter, they can communicate supersymmetry breaking to the visible sector. This is a realisation of gravity-mediated supersymmetry breaking, which thus arises very naturally in the context of string theory.

Hence, in many string constructions moduli stabilisation and supersymmetry breaking are inherently linked. As both moduli stabilisation and supersymmetry breaking are formulated below the compactification scale, they can be analysed in the four-dimensional effective supergravity theory. Hence we are back to field theory, but the situation is better than when starting with field theory alone. The leading order string theory supergravity Lagrangian shows many remarkable properties, which would not have easily be written down by an effective field theorist. Nevertheless, we encounter the same problem as in any approach based on low energy effective field theory. To fully determine gravity-mediated soft terms, we need to understand the origin of non-renormalisable terms in the effective action. Thus we are again at a point where we cannot proceed with the effective low energy theory alone: we have to turn to string theory directly.

This thesis: sequestering and de-sequestering

Hence we move towards the subject of this thesis: in this work we will employ string perturbation theory to determine corrections to the four-dimensional effective theory, which are relevant for the calculation of soft terms. Corrections to the low energy effective theory are organised in a perturbative expansion in two parameters. There is one expansion in g_s, which is the string theory coupling constant. Further, there are corrections in α', which is related to the string length ℓ_s as $\ell_s = 2\pi\sqrt{\alpha'}$. By evaluating elements of the S-matrix in string perturbation theory, we can deduce terms in the effective action up to all orders in α', while staying at a particular order in g_s. Unfortunately, one cannot perform calculations directly for smooth Calabi-Yau backgrounds, and we will have to restrict ourselves to toy-models based on toroidal orbifolds. Nevertheless, we will examine to what extent our results should generalise to smooth compactifications.

The results in this thesis will be most relevant for one particular scheme of moduli stabilisation in string theory. The Large Volume Scenario (LVS) [6] allows for stabilisation of all moduli while breaking supersymmetry at a hierarchically low scale. Using only the leading terms in the supergravity Lagrangian, one finds a curious pattern of supersymmetry breaking. The setup exhibits no-scale supersymmetry breaking, such that gravity-mediated soft terms cancel at leading order. In addition, anomaly-mediated soft terms (as far as they have been calculated) vanish as well. To answer the question which scale soft terms arise at one then needs to consider higher order corrections to the supergravity Lagrangian.

This brings us to the aim of this thesis: using string perturbation theory we can address questions regarding both the scale, and the flavour structure of soft terms in string models of supersymmetry breaking. In particular, we will discuss the following problems:

1. *Sequestering.* The vanishing of soft terms in the LVS at leading order can be understood as follows: at leading order the source of supersymmetry breaking is decoupled (sequestered) from the visible sector. A direct consequence is that the physical Yukawa couplings do not depend on the supersymmetry breaking moduli at leading order. One finds that soft terms are generated to the extent that physical Yukawa couplings depend on supersymmetry breaking fields. By calculating string theory correlation functions involving Yukawa couplings and supersymmetry breaking moduli, we can examine whether sequestering survives beyond leading order. While this approach captures all orders in α', we stay at leading order in g_s.

2. *De-sequestering.* Superpotential de-sequestering refers to the generation of contributions to soft terms due to non-perturbative effects used for moduli stabilisation. These non-perturbative effects arise from Euclidean D-branes or D-branes exhibiting gaugino condensation. Focussing on the latter case, we can determine the presence of corrections to visible sector Yukawa couplings by calculating a threshold correction to the gauge coupling on the brane giving rise to the non-perturbative effect. The corrections to Yukawa couplings in turn give contributions to A-terms, which can source flavour-changing neutral currents. While these effects are typically subleading, they become dangerous if the model exhibits sequestering as in case 1 above.

This thesis is structured as follows. Chapters 2 and 3 introduce calculational methods we will use to derive our results. In chapter 4 we examine sequestering of supersymmetry breaking blow-up moduli. This chapter is based on paper [7] with Joseph Conlon. Chapter 5 examines de-sequestering due to non-perturbative effects and follows paper [8] with Marcus Berg, Joseph Conlon and David Marsh. Chapter 6 examines a topic outside supersymmetry breaking and contains results from the paper [9] with Mathew Bullimore and Joseph Conlon. In particular, we examine the possibility of kinetic mixing of various $U(1)$ factors in models with D3-branes at orbifold singularities. Chapter 7 summarises the results of this thesis and discusses possible future work.

In the rest of this chapter, we review supersymmetry breaking in string theory in more detail, to then precisely formulate the questions which we wish to answer in this thesis.

1.1 Motivation

1.1.1 Gravity mediated supersymmetry breaking

Low energy supersymmetry and its breaking are promising phenomenological applications for string theory. Below the Kaluza-Klein scale of the compactification, the low energy limit of string theory is given by four-dimensional $\mathcal{N} = 1$ supergravity. Thus, to get an understanding for low energy supersymmetry breaking in string theory, it is appropriate to begin with an analysis in supergravity. Here, we summarise the standard derivation of soft terms in $\mathcal{N} = 1$ supergravity from [10].

The first step is the determination of the supergravity Lagrangian, specified by the Kähler potential K, the superpotential W and the gauge kinetic functions f_a. These are functions of the chiral superfields Φ_i of the theory, which include moduli and visible sector matter. In the setups of interest, moduli are hidden sector fields, which are not charged under the SM gauge group, and which only couple with gravitational strength. Supersymmetry will be broken in the hidden sector and moduli will acquire large vacuum expectation values (VEVs). To then study the effects of this on the theory of visible sector fields, it is convenient to separate the chiral superfields into hidden sector fields h_m and visible sector fields C^α. The supergravity Lagrangian can then be expanded in powers of C^α:

$$K(\Phi_i, \bar{\Phi}_i) = \hat{K}(h_m, \bar{h}_m) + \tilde{K}_{\bar{\alpha}\beta}(h_m, \bar{h}_m)\ \bar{C}^{\bar{\alpha}} C^\beta + \left\{ \frac{1}{2} Z_{\alpha\beta}(h_m, \bar{h}_m)\ C^\alpha C^\beta + \text{c.c.} \right\} + \dots\ , \tag{1.3}$$

$$W(\Phi_i) = \hat{W}(h_m) + \frac{1}{2}\ \mu_{\alpha\beta}(h_m)\ C^\alpha C^\beta + \frac{1}{6} Y_{\alpha\beta\gamma}(h_m)\ C^\alpha C^\beta C^\gamma \dots\ , \tag{1.4}$$

$$f_a(\Phi_i) = f_a(h_m)\ , \tag{1.5}$$

To explain how the above Lagrangian parameters determine the soft terms, we need to first find the moduli vacuum expectation values (VEVs). This is done by minimising the supergravity scalar potential which only depends on the hidden sector fields:

$$\begin{aligned} V_F &= e^{\hat{K}/M_P^2} \left(\hat{K}^{i\bar{j}}\ D_i \hat{W}\ D_{\bar{j}} \bar{\hat{W}} - 3 \frac{|\hat{W}|^2}{M_P^2} \right) \\ &= \hat{K}_{i\bar{j}} F^i \bar{F}^{\bar{j}} - 3 m_{3/2}^2 M_P^2\ . \end{aligned} \tag{1.6}$$

A non-zero F-term $F^i = e^{\hat{K}/2M_P^2} \hat{K}^{i\bar{j}} D_{\bar{j}} \bar{\hat{W}}$ then indicates broken supersymmetry. The gravitino absorbs the goldstino of the broken supersymmetry and acquires a mass $m_{3/2} = e^{\hat{K}/2M_P^2} |\hat{W}|/M_P^2$. Visible sector soft terms are then found as follows: by expanding the scalar potential as

$$V_{soft} = \tilde{m}'^2_{\bar{\alpha}\beta} \bar{C}^{\bar{\alpha}} C^\beta + \left(\frac{1}{6} A'_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma + \frac{1}{2} B_{\alpha\beta} C^\alpha C^\beta + \text{c.c.} \right) \tag{1.7}$$

the soft terms are determined by replacing the scalars h_m and their auxiliary terms F^m by their VEVs. The primes indicate, that we have not normalised our fields yet to obtain canonical kinetic terms. As we are mainly interested in soft scalar masses and A-terms, we only present the results for those:[1]

$$\tilde{m}'^2_{\bar{\alpha}\beta} = (m_{3/2}^2 + V_0) K_{\bar{\alpha}\beta} - \bar{F}^{\bar{m}} F^n \left(\partial_{\bar{m}} \partial_n \tilde{K}_{\bar{\alpha}\beta} - \tilde{K}^{\gamma\bar{\delta}} \partial_{\bar{m}} \tilde{K}_{\bar{\alpha}\gamma} \partial_n \tilde{K}_{\bar{\delta}\beta} \right)\ , \tag{1.8}$$

$$\begin{aligned} A'_{\alpha\beta\gamma} = & \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} F^m \left[\frac{\hat{K}_m}{M_P^2} Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} \right. \\ & \left. - \left((\partial_m \tilde{K}_{\alpha\bar{\rho}}) \tilde{K}^{\bar{\rho}\delta} Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \right]\ , \end{aligned} \tag{1.9}$$

where V_0 is the vacuum energy at the potential minimum. These expressions can be seen as the starting point for the examinations in this thesis: they constitute a prescription for the derivation of soft terms given a supergravity Lagrangian. What remains to be done is to extract the coefficients $\hat{K}$, $\hat{W}$, $\tilde{K}_{\bar{\alpha}\beta}$ and $Y_{\alpha\beta\gamma}$ of (1.3) to (1.5) from high-scale string compactifications. The need for high scale physics can be understood from the

[1] The remaining soft terms can be found in e.g. [10].

above expressions for soft terms. Note that it is the moduli dependence of both $\tilde{K}_{\alpha\bar{\beta}}$ and $Y_{\alpha\beta\gamma}$ that also enters the above expressions. However, as both $\tilde{K}_{\alpha\bar{\beta}}$ and $Y_{\alpha\beta\gamma}$ are dimensionless functions, they depend on moduli as h_m/M_P. This implies that such terms correspond to non-renormalisable operators in the supergravity action, which arise from physics at the Planck scale.

In this thesis, we will be particularly interested in non-renormalisable operators contributing to the Kähler metric and the Yukawa couplings, which enter the expressions for soft terms. However, before we can explain what we need to calculate in detail, we have to first review what is already known. Hence, we now turn to moduli stabilisation and supersymmetry breaking in type IIB string theory and the resulting low energy effective actions.

1.1.2 Moduli stabilisation and supersymmetry breaking in type IIB string theory

Low energy actions have been extensively studied in string theory (e.g. see [11–17]). These studies have revealed a remarkable structure in the form of (1.3) to (1.5) that would not be written down easily by an effective field theorist. For example, the Kähler potential $\hat{K}(h_m, \bar{h}_m)$ factorises into parts depending on Kähler and complex structure moduli, with corrections suppressed by either the α' or g_s expansion. The dependence of the matter metric $\tilde{K}_{\bar{\alpha}\beta}(h_m, \bar{h}_m)$ is often universal at leading order and in some cases to all orders in α' [18, 19].

We will be mainly interested in the low energy effective actions in type IIB string theory, as moduli stabilisation with low energy supersymmetry breaking is most successfully implemented in this context (for recent advances in heterotic string theory see [20–22]). In practice, moduli stabilisation in type IIB string theory on Calabi-Yau manifolds proceeds in two distinct steps. First, the dilaton and the complex structure moduli are stabilised using fluxes at a high scale, while Kähler moduli remain unfixed [23].[2] As long as the volume of the compactification is large enough such that the fluxes are dilute, the compactification can be studied at the level of the four-dimensional effective supergravity theory. The residual theory of the Kähler moduli is of no-scale type with a moduli Lagrangian specified by

$$\hat{K} = -2\ln \mathcal{V}(T_i + \bar{T}_i) \ , \qquad \hat{W} = W_0 \ , \tag{1.10}$$

where $\mathcal{V}$ is the volume of the Calabi-Yau, which depends on the Kähler moduli T_i. The superpotential is given by the constant term W_0, which depends on the VEVs of the

[2] For reviews of flux compactifications see e.g. [24–27].

dilaton and complex structure moduli. The above no-scale Lagrangian is the starting point for the most celebrated schemes of moduli stabilisation in type IIB. By considering perturbative and non-perturbative corrections to (1.10) the no-scale structure is broken in the second step and all moduli can be fixed [6, 28–31].

The results of this thesis will be particularly useful for one scheme of moduli stabilisation, which breaks supersymmetry at a hierarchically low scale. As it stabilises the bulk volume at an exponentially large value, it is known as the Large Volume Scenario (LVS) [6, 32]. At leading order the moduli Lagrangian of the LVS is just given by the above no-scale model (1.10), which leads to interesting consequences. Supersymmetry is mainly broken by the Kähler moduli, with the modulus T_b describing the bulk volume acquiring the largest F-term [32]. The no-scale structure at tree-level will have far more drastic effects: as the leading order Kähler matter metric takes the form $\tilde{K}_\alpha \sim 1/(T_b + \bar{T}_b)$ [33], there are cancellations in the expressions for soft terms, such that both tree-level soft terms and loop level anomaly-mediated soft terms vanish [16, 34, 35]. This leads to a vanishing of soft terms at order both $m_{3/2}$ and $\alpha m_{3/2}/4\pi$. Soft terms have to be generated at some scale, and this opens the possibility of a spectrum with soft terms suppressed compared to the gravitino mass $m_{3/2}$ [36, 37] (for possibilities of avoiding this conclusion via 1-loop field redefinitions see [38–40]). However, determining exactly which scale soft terms will arise requires the consideration of higher order corrections both in α' and g_s.

1.1.3 Kähler metric in the Large Volume Scenario

To determine the scale and flavour structure of soft terms, we need to consider higher order corrections to $\hat{K}$, $\hat{W}$, $\tilde{K}_{\bar{\alpha}\beta}$ and $Y_{\alpha\beta\gamma}$. This is particularly difficult for the case of the Kähler matter metric, which is a great obstacle on the way to a precise computation of soft terms. Here we will ask a different question, which will nevertheless allow us to learn about the scale of soft terms. Instead of determining the corrections to $\hat{K}$ and $\tilde{K}_{\bar{\alpha}\beta}$ independently, one can check whether the corrections to these two quantities are correlated. To illustrate this point we will review the analysis which led to the leading order Kähler metric shown above. Later we will see how this can be extended to higher orders.

At leading order, the Kähler matter metric was derived using a scaling argument [33] (also see [41]). The argument is based on the fact that the visible sector in the LVS is to a certain extent 'local'. In concrete constructions the visible sector in LVS models can be modelled by D3-branes at orbifold singularities or D7-branes on a four-cycle, which is stabilised at string size by D-terms. In all cases the visible sector occupies a small volume of the compact space. In addition, it is spatially separated from other cycles which provide non-perturbative effects responsible for moduli stabilisation. Thus one would expect that bulk effects decouple from the visible sector at leading order.

The local nature of the visible sector then implies that the physical Yukawa couplings should not depend on the bulk modulus T_b at leading order. In other words, the Yukawa couplings should neither vanish nor diverge if we decouple gravity by letting $\mathcal{V} \sim \mathrm{Re}(T_b)^{3/2} \to \infty$. In addition, they should neither depend on the volumes of small distant cycles parameterised by T_s. To make a connection to the moduli-dependence of the Kähler metric, we express the physical Yukawa couplings $\hat{Y}_{\alpha\beta\gamma}$ in terms of quantities of the supergravity Lagrangian:

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma}} \ . \tag{1.11}$$

We note that the only dependence of the physical Yukawa couplings on Kähler moduli arises through $\hat{K}$ and the Kähler metrics $\tilde{K}_\alpha$, which are assumed to be diagonal. The reason is that the holomorphic Yukawa couplings originate from the superpotential, which does not depend on Kähler moduli in type IIB. Hence, if the physical Yukawa couplings are independent of the moduli T_b and T_s, we require a cancellation between $e^{\hat{K}/2}$ and the denominator. If we use the tree level Kähler potential $\hat{K} = -3 \ln T_b + \bar{T}_b$, independence of T_b and T_s at leading order gives

$$\tilde{K}_\alpha = \frac{k_\alpha(U,S)}{T_b + \bar{T}_b} \ , \tag{1.12}$$

where we have taken the Kähler metrics to be flavour-universal.[3] This is exactly the form of the Kähler metric which leads to a vanishing of soft terms at leading order.

This analysis allows for an obvious generalisation to higher order corrections: one should be allowed to ask why the independence of the physical Yukawa couplings of Kähler moduli should not hold to higher orders in α' and g_s? This would not be surprising as the Yukawa couplings are local renormalisable interactions. The implication would be that higher order corrections to $\hat{K}$ and $\tilde{K}_\alpha$ are correlated, such that the dependence of the physical Yukawa couplings on Kähler moduli vanishes. In particular, this leads to a Kähler metric of the form

$$\tilde{K}_\alpha \sim e^{\hat{K}/3} + \dots \ , \tag{1.13}$$

[3]For a visible sector based on D7-branes wrapped on a four-cycle with Kähler modulus T_a, the Kähler metric will typically depend on T_a. However, as this cycle is stabilised supersymmetrically, this does not lead to any contributions to the soft terms.

with corrections to this result appearing at the order where physical Yukawa couplings fail to be independent of Kähler moduli.[4] This form leads to further cancellations in the expressions for gravity-mediated soft terms: note that if (1.13) holds to all orders, gravity-mediated soft terms become [36]

$$\tilde{m}'_{\bar{\alpha}\beta} = \frac{2}{3} V_0 \tilde{K}_{\bar{\alpha}\beta} \approx 0 \ , \tag{1.14}$$

$$A'_{\alpha\beta\gamma} = e^{\hat{K}/2} F^m \partial_m Y_{\alpha\beta\gamma} \ , \tag{1.15}$$

where V_0 is the tree-level cosmological constant, which we assume to be negligibly small. This is referred to as 'sort-of sequestering' in the literature [42] and it opens the possibility that gravity-mediated soft terms vanish exactly: recall that in type IIB string theory the superpotential and hence the Yukawa couplings do not depend on the Kähler moduli up to non-perturbative terms. Unless Yukawa couplings receive non-perturbative corrections which depend on the Kähler moduli, the term $\partial_m Y_{\alpha\beta\gamma}$ is absent and gravity-mediated soft-terms vanish.

The implications for the LVS phenomenology are the following. Soft terms have to be generated at some scale, but this can now be suppressed w.r.t. to the gravitino mass by powers of $\mathcal{V} \gg 1$. Here $\mathcal{V}$ is a number, which measures the volume of the compact space in units of the string length ℓ_s^6, and which is exponentially large in realisations of the LVS. If 'sort-of sequestering' occurs, the soft terms can be suppressed w.r.t. $m_{3/2}$ as much as $m_{soft} \sim \frac{m_{3/2}}{\mathcal{V}}$, where $m_{3/2} \sim \frac{M_P}{\mathcal{V}}$. In this case soft terms are sourced by the dilaton F-term, which scales as $\mathcal{O}(\mathcal{V}^{-2})$. Then TeV soft terms are possible for a bulk volume of $\mathcal{V} \sim 3 \cdot 10^7$ in string units, which implies a string scale of $m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{15}$ GeV. This is an attractive scenario as it allows for field theory gauge coupling unification within string theory. If soft terms were less suppressed w.r.t. $m_{3/2}$, the requirement of TeV soft terms would then imply a lower string scale, which would come in conflict with the possibility of a GUT within string theory. Hence, detailed knowledge of the scale of soft terms is important for the phenomenology of this model beyond the realm of supersymmetry breaking.

In summary, the dependence of physical Yukawa couplings on Kähler moduli is thus a useful tool for examining contributions to soft terms in the LVS. In other words, the conjecture (1.13) states, that "moduli generate soft scalar masses to the extent to which the physical Yukawa couplings depend on the moduli", as it was put in [36].

Testing the validity of (1.13) is one of the main goals of this thesis. Phrased in terms of the moduli-dependence of the physical Yukawa couplings this question is ideally suited

[4] Note that expression (1.13) includes the leading order result (1.12)

for an analysis using string perturbation theory. By performing a relevant calculation in worldsheet conformal field theory one can determine the physical Yukawa couplings directly. As calculations in smooth Calabi-Yau spaces are not possible, the computation will be performed in a toroidal orbifold. By blowing up orbifold singularities, the result can be extended to a smooth geometry. This is the subject of chapter 4: by calculating a disk correlation function, we will examine the validity of (1.13) to all orders in α' while staying at leading order in g_s. The result gives information about the scale at which soft terms arise in string models of supersymmetry breaking.

1.1.4 Non-perturbative corrections to Yukawa couplings

While we addressed corrections to the Kähler potential above, contributions to soft terms also arise from the superpotential and in particular the holomorphic Yukawa couplings $Y_{\alpha\beta\gamma}$. So far we had a good reason to sideline the holomorphic Yukawa couplings: as they arise from the superpotential, they do not depend on the supersymmetry breaking Kähler moduli to all orders in perturbation theory. The only dependence on Kähler moduli can arise through non-perturbative corrections, which typically give subleading contributions to soft-terms. This situation changes if there are cancellations in the perturbative contributions to soft terms: if $\tilde{K}_\alpha \sim e^{\hat{K}/3}$ holds to higher orders, the non-perturbative contributions to Yukawa couplings can become significant.

The presence of non-perturbative corrections to Yukawa couplings is expected on general grounds. For one, non-perturbative effects are vital ingredients of any successful scheme of moduli stabilisation in type IIB string theory, including the LVS. Such corrections are sourced by Euclidean D3-branes (E3-branes) or gaugino condensation on D7-branes wrapping four-cycles in the compact geometry. In the low energy effective theory the resulting corrections to the superpotential take the form

$$W^{np} = \sum_i \mathcal{A}_i(\phi) e^{-a_i T_i} \,, \tag{1.16}$$

where a is a model-dependent constant. Here $\mathcal{A}(\phi)$ is a prefactor which in principle depends on other moduli except T_i. Most importantly, the non-perturbative contributions introduce a dependence of the superpotential on the Kähler moduli, which allows for their stabilisation.

The presence of non-perturbative corrections to visible sector operator can then be understood as follows. Both the visible sector and the non-perturbative effects are realised by D-branes wrapping various cycles in the compact geometry. The visible sector is given by D3-branes at an orbifold singularity or D7-branes on a small four-cycle.

Similarly, the non-perturbative effects arise from E3 or D7-branes on four-cycles. Even though the two sectors are spatially separated, there will be open strings which stretch across the compact space and which carry charges under both sectors. As pointed out in [42], integrating out these massive strings can then generate cross-couplings between these two sectors. For the above superpotential this implies, that the prefactor $\mathcal{A}(\phi)$ will not only be a function of the moduli (except T_i), but also depend on visible sector fields C^α as well. In particular, new contributions to Yukawa couplings arise if non-perturbative effects induce terms of the form

$$W^{np} \supset Y^{np}_{\alpha\beta\gamma}(\phi)\, C^\alpha C^\beta C^\gamma\, e^{-a_i T_i}\ . \tag{1.17}$$

Given such corrections to Yukawa couplings, the corresponding contributions to A-terms are:

$$\begin{aligned} \delta A'_{\alpha\beta\gamma} &= e^{\hat{K}/2} F^m \partial_m \left(Y^{np}_{\alpha\beta\gamma}(\phi) e^{-a_i T_i} \right) \\ &= -e^{\hat{K}/2} a_i F^{T_i} Y^{np}_{\alpha\beta\gamma}(\phi) e^{-a_i T_i} \end{aligned} \tag{1.18}$$

The important observation for supersymmetry breaking is that there is no reason why $Y^{np}_{\alpha\beta\gamma}$ should exhibit the same flavour structure as the tree-level Yukawa couplings. For example, it is known that D-brane instantons can give rise to Yukawa couplings in type IIA string theory, which are perturbatively forbidden [43,44]. In our case, this introduces potential problems for the model: if the soft A-terms are not proportional to the Yukawa couplings, they are sources of flavour changing neutral currents. Requiring the absence of excessive flavour violation then imposes constraints on the string construction.

The numerically most important contributions arise if one of the fields C^i in (1.17) is the Higgs. In this case masses will be induced for the remaining visible sector fields after electroweak symmetry breaking. By studying the effect of these masses on flavour-violating or CP-violating effects, one can exclude regions of parameter space of the string construction which break observational bounds. The constraints are most drastic for the sequestered LVS which exhibits 'sort-of sequestering': $\tilde{K}_\alpha \sim e^{\hat{K}/3}$. For this scenario, the absence of excessive flavour violation was used in [42] to derive a bound on the volume of the compactification. Unless the non-perturbative corrections exhibit the same flavour structure as the tree-level Yukawa couplings consistency with observational bounds requires $\mathcal{V} < 10^5$ in string units.

Hence the questions regarding the existence and structure of non-perturbative contributions to Yukawa couplings are important, especially in scenarios with sequestered supersymmetry breaking. The presence of such terms can be checked by an appropriate calculation in string perturbation theory. At leading order, the presence of superpotential terms of the form (1.17) can be checked by evaluating an appropriate cylinder amplitude stretching between the visible sector and the D-branes supporting the non-perturbative effect. This is the subject of chapter 5 of this thesis. In particular, we will want to demonstrate that cross-couplings of the form (1.17) exist and are not suppressed by the bulk volume. Further, we will compare the flavour structure of new contributions to Yukawa couplings to that of the tree-level couplings. Again, as CFT calculations are not possible in the Calabi-Yau geometry, we have to work in a toy-model based on a toroidal orbifold. Nevertheless, the conditions for the presence of non-perturbative corrections to Yukawa couplings in these backgrounds should provide some intuition for their behaviour in the more realistic geometry.

In this chapter we have laid out in detail the questions which we wish to answer in this thesis. In the next chapter, we will introduce the methods which we will use to derive the results. In particular, we will describe how one can use string perturbation theory to determine operators in the low energy effective action.

Chapter 2

String perturbation theory

In this section we review some aspects of string perturbation theory and conformal field theory. These are vast subjects and we do not attempt to give a complete picture. Instead, we focus on topics which will be useful for calculations in the following chapters. In particular, we will be interested in obtaining elements of the S-matrix in string theory and how these can be used to learn about the low energy effective action. References for this section include the textbooks by Polchinski [45, 46]. Pedagogical introductions to string theory and conformal field theory include [47, 48]. A standard reference for conformal field theory is [49].

2.1 The string S-matrix

String theory describes the quantum behaviour of vibrating one dimensional objects. The only dimensionful parameter of the theory is α', which is related to the length of the fundamental string $\ell_s = 2\pi\sqrt{\alpha'}$. If string theory is probed with energies $E\ell_s \ll 1$ the extended nature of the string cannot be resolved, and strings effectively behave as point particles. Thus it is not unexpected, that the quantum theory of strings should reproduce the interactions of quantum field theory in a low energy limit.

But how can one determine the low energy effective action of string theory? One way is to calculate an observable in a putative quantum field theory and compare it to the same observable in string theory at low energies. If all observables coincide, the quantum field theory captures the low energy behaviour of string theory. This is not easily implemented, as string theory is very poor in observables. The action is the integral over the worldsheet of a single string, which corresponds to a "first quantised" theory of the string. Correspondingly we cannot calculate correlation functions of strings in spacetime, as this would require a "second quantised" formulation where the action is an integral of a string field over spacetime. The "first quantised" version of string theory allows us to determine the spectrum of a free string. A well-defined problem is then to ask for the probability of scattering of asymptotic states of the free string. This is nothing but the S-matrix and this is the observable that will allow for comparison with field theory.

The prescription for determining the low energy effective action of string theory is then the following [50]. Identify the massless sates of the string and calculate the S-matrix. In the limit $\alpha' \to 0$ this should be identical to the S-matrix of a field theory just containing the massless degrees of freedom. We can find the low energy effective action by postulating Lagrangians and selecting the one that reproduces all elements of the S-matrix correctly.

2.1.1 Calculating the S-matrix

Given a Lagrangian the S-matrix of a field theory can be determined perturbatively with the use of Feynman diagrams. In string theory we have a different perturbation expansion. To see this we begin with the action for a string moving in background spacetime fields. Its bosonic part is an integral of the spacetime coordinate field $X(\sigma^1, \sigma^2)$ over the Euclidean worldsheet Σ parameterised by (σ^1, σ^2):

$$\begin{aligned} S = \; & \frac{1}{4\pi\alpha'} \int_\Sigma \mathrm{d}^2\sigma \sqrt{h} \left[h^{\alpha\beta} G_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu + i\epsilon^{\alpha\beta} B_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu \right] \\ & + \frac{1}{4\pi} \int_\Sigma \mathrm{d}^2 \sqrt{h} \Phi(X) R^{(2)} \; . \end{aligned} \tag{2.1}$$

The background fields are the spacetime metric $G_{\mu\nu}$, the antisymmetric Kalb-Ramond field $B_{\mu\nu}$ and the dilaton Φ. The worldsheet metric is $h_{\alpha\beta}$ and $R^{(2)}$ its Ricci scalar. The action on the Euclidean worldsheet is more suitable for the definition of a path integral and can be obtained from its Lorentzian equivalent by a Wick rotation. The worldsheet coordinate σ^2 describes Euclidean worldsheet time while σ^1 parameterises the direction along the string.

It is the dilaton-dependent term which is crucial for string perturbation theory. Without the dilaton, this term is purely topological and calculates the Euler characteristic χ of the worldsheet. Thus, if the dilaton acquires a vacuum expectation value Φ_0 that is constant (throughout spacetime or at infinity), we can isolate its contribution to the path integral

$$\exp\left(-\Phi_0 \int \mathrm{d}^2\sigma \, \sqrt{h} R^{(2)} \right) = e^{-\Phi_0 \chi} = g_s^{-\chi} \; . \tag{2.2}$$

Here we identified the string coupling $g_s \equiv e^{\Phi_0}$, which is the parameter of the string perturbation expansion. It is not a free parameter, but set dynamically by the value of the dilaton. We see that contributions from worldsheets with different Euler characteristic enter at different orders in the string coupling. Consequently, the perturbation series of string theory is an expansion in worldsheet topologies.

The Euler characteristic of a two-dimensional surface can be exactly specified by the number of handles g, boundaries b and cross-caps c present and is given by

$$\chi = 2 - 2g - b - c \ . \tag{2.3}$$

Open strings only interact via worldsheets with boundaries, while unoriented strings give rise to worldsheets with cross-caps. The terms at the lowest order in the perturbation expansion arise from the sphere for closed strings and the disk for open strings, with coupling constants g_s^{-2} and g_s^{-1} respectively.

We now present a prescription for calculating elements of the S-matrix in string theory. In the following, we will sketch how this procedure is performed in practice. Schematically, an element of the S-matrix is given by

$$S(\lambda_1, \lambda_2, \dots, \lambda_n) = \sum_{topologies} \int \frac{\mathcal{D}X\mathcal{D}h}{\mathrm{Vol}_{\ \mathrm{diff}\times\mathrm{Weyl}}} \left(V_1 V_2 \dots V_n\right) e^{-S(h,X)} \ , \tag{2.4}$$

where h is the worldsheet metric and X stands for all worldsheet fields (including fermions). The external string states λ_i enter as operators V_i to which we will return later. Most importantly, the factor $\mathrm{Vol}^{-1}_{\mathrm{diff}\times\mathrm{Weyl}}$ signifies, that in order to arrive at physical results we should only sum over configurations which are not related by a gauge symmetry. For the string, gauge symmetries correspond to diffeomorphisms and Weyl rescalings on the worldsheet and we will need to fix them to calculate S-matrix elements.

First, one can always employ a diffeomorphism to bring the worldsheet metric into the form

$$h_{\alpha\beta}(\sigma^i) \stackrel{\mathrm{diffeo.}}{\longrightarrow} e^{2\omega(\sigma^i)}\eta_{\alpha\beta} \ , \tag{2.5}$$

which is known as conformal gauge. Then, at least locally, one can turn the metric into the flat metric in two dimensions by a Weyl rescaling $e^{2\omega(\sigma)}\eta_{\alpha\beta} \stackrel{\mathrm{Weyl}}{\longrightarrow} \eta_{\alpha\beta}$. This, however, does not fix the gauge freedoms completely: the sets of diffeomorphisms and Weyl rescalings are not mutually exclusive. There are reparameterisations which can been undone by Weyl rescalings, and they remain as residual gauge transformations after having fixed the metric as in (2.5). These remnant gauge symmetries form a most fascinating class of transformations. They correspond to conformal transformations, and thus conformal field theory (CFT) in two dimensions plays a crucial role when studying string theory.

We can use the conformal symmetry to our advantage. As long as the worldsheets are related by a conformal map, the corresponding string amplitude will give the same result. Riemann surfaces are two (real) dimensional manifolds, which are equivalent under a conformal map. Correspondingly, the string theory perturbation expansion is a sum over Riemann surfaces of different topologies.

Now we turn to the insertions V_i in the above path integral. As we are calculating S-matrix elements these correspond to external string states which are inserted at asymptotic infinity. If the external states are closed strings, the worldsheet for a scattering of such states will be a surface with long cylinders stretching to the external states. If we include external open string states there will be long strips attached to the boundary of the worldsheet. The conformal invariance of the theory can help to simplify the situation enormously. By a conformal transformation we can map the setup onto a compact worldsheet with the insertions of external states as small punctures. For closed strings these punctures appear in the bulk of the worldsheet, while open string states will lead to disturbances on the boundary. The formalism of CFT can make this precise: there exists an equivalence between physical states and so-called vertex operators at a point. These are local operators in the conformal field theory which exhibit all quantum numbers of the external string states. As a result, we can identify external string states with CFT vertex operators V_i, which we need to insert into the path integral.

After having fixed the metric as in (2.5), the path integral (2.4) has to be evaluated over conformally inequivalent Riemann surfaces. This turns the functional integral over metrics into an integral of finite dimension over the moduli space $\mathcal{M}_{g,n}$ of a Riemann surface of genus g with n punctures, where vertex operators are inserted. We can get some understanding for this moduli space by considering transformations of the worldsheet which we have not succeeded in fixing by specifying a metric. There are two types:

1. For one, there can be conformal killing vectors (CKVs) which correspond to reparameterisations of the worldsheet which cannot be fixed by fixing the metric. We will then need to fix them explicitly.

2. On the other hand, there can be deformations of the metric, which cannot be compensated for by a Weyl rescaling and a reparameterisation. These are known as moduli (or Teichmüller parameters) and different values for them correspond to conformally inequivalent surfaces. Hence we will need to integrate over these parameters explicitly in the path integral.

Both the number of CKVs and moduli depend on the genus g of the worldsheet and are summarised in the following table:

g	no. of CKVs	no. of moduli
0	3	0
1	1	1
≥ 2	0	$3g-3$

At the level of the path integral, the gauge fixing described above is most conveniently performed using the Fadeev-Popov method. This necessitates the introduction of ghost fields with their corresponding action. In particular, we need to invoke anticommuting fields b and c and, for the superstring, there will also be the commuting ghosts β and γ. We do not elaborate on this any further and refer readers to [45, 46] for more details. While we will not ignore ghosts in the following, we will only state results and properties, which are necessary for calculations in this thesis.

The above discussion leaves us with the following strategy for calculating S-matrix elements in string perturbation theory at a fixed order in $g_s^{-\chi}$. First we choose vertex operators for the relevant external states. We insert these vertex operators into a Riemann surface with Euler characteristic χ. We fix the positions of as many vertex operators as there are CKVs. As it turns out, we need to introduce one c-ghost for every position we fix. In addition, every modulus of the surface requires the insertion of a b-ghost. The element of the S-matrix is then calculated as a correlation function in two-dimensional CFT. In the last step we need to integrate over the unfixed vertex operator positions and all moduli of the surface to arrive at the result.

While we sketched the formalism for calculating elements of the S-matrix in string theory, we have not collected all the ingredients. To perform calculations we need to know the relevant vertex operators and Riemann surfaces. First, we return to the string action.

2.1.2 Superstring action

We began the discussion of string perturbation theory by writing down the bosonic action for a single string in background fields (2.1). For non-trivial backgrounds this corresponds to an interacting two-dimensional field theory, which cannot be solved exactly in most cases. It can be studied using perturbation theory, with the expansion parameter given by $\sqrt{\alpha'}/R$, where R is a typical length scale over which spacetime fields vary. However, not all background fields give rise to a consistent string theory. In particular, consistency requires that the quantum theory remains Weyl invariant. A simple solution which satisfies this condition is given by a flat target space metric $\eta^{\mu\nu}$, vanishing Kalb-Ramond field and a constant dilaton. This is the background which we will use for the rest of this thesis: while this is a particular choice, it offers calculability as the worldsheet theory becomes free. As the dilaton is constant, it only contributes a factor $g_s^{-\chi}$.

Furthermore, we will now also introduce worldsheet fermions explicitly, which we ignored so far. These are crucial if we want to obtain a tachyon-free spectrum and space-time fermions. An elegant way of introducing fermions is by requiring two-dimensional supersymmetry on the worldsheet, which leads to the action for the appropriately named superstring. Fixing the worldsheet metric in conformal gauge (2.5) and using complex coordinates $z = \sigma^1 + i\sigma^2$ and $\bar{z} = \sigma^1 - i\sigma^2$ the superstring action in conformal gauge is given by [1]

$$S = \frac{1}{4\pi} \int d^2z \left(\frac{2}{\alpha'} \eta_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu + \eta_{\mu\nu} \psi^\mu \bar{\partial} \psi^\nu + \eta_{\mu\nu} \tilde{\psi}^\mu \partial \tilde{\psi}^\nu \right) . \tag{2.6}$$

The Grassmann-valued fields ψ^μ and $\tilde{\psi}^\mu$ can be understood as the components of a 2d Majorana-Weyl spinor. The equations of motion for the worldsheet fields are

$$\partial\bar{\partial} X^\mu = 0 \ , \quad \bar{\partial}\psi^\mu = 0 \ , \quad \partial\tilde{\psi}^\mu = 0 \ . \tag{2.7}$$

These imply that ψ^μ is a holomorphic field, $\tilde{\psi}(\bar{z})$ is an antiholomorphic field and the bosonic field X contains both holomorphic and antiholomorphic parts: $X^\mu(z, \bar{z}) = X^\mu(z) + \tilde{X}(\bar{z})$.

2.1.3 Vertex operators

We need to insert a vertex operator for each external state into the string path integral (2.4). These are primary operators of the conformal field theory on the worldsheet of conformal weight $(h, \bar{h}) = (1, 1)$. They are most rigorously constructed by analysing the physical states of the conformal field theory on the worldsheet and applying the state-operator correspondence. We refer readers to a text on CFT (e.g. [45, 49]) and, for brevity, only discuss some features of vertex operators.

For simplicity, we start with an example in bosonic string theory. Using complex coordinates $(z, \bar{z})$ on the conformally flat worldsheet, the vertex operator for a graviton is given by

$$V_g = \int \mathrm{d}^2z \ V_g(z, \bar{z}) = \frac{2g_c}{\alpha'} \ \epsilon_{\mu\nu} \int \mathrm{d}^2z \ \partial_z X^\mu \partial_{\bar{z}} X^\nu \ e^{ik\cdot X(z,\bar{z})} \ , \tag{2.8}$$

where we refer to $V_g(z, \bar{z})$ as the unintegrated vertex operator. Here, g_c is a normalisation for closed string operators, which can be fixed by unitarity. As the vertex operator should be allowed to be inserted anywhere on the worldsheet, we integrate over its position.

[1] We follow the conventions in the textbooks by Polchinski [45, 46].

We can also identify $\epsilon_{\mu\nu}$ as the polarisation of the graviton, while k is its spacetime momentum. Most importantly, the vertex operator shows the correct behaviour under conformal transformations: $\partial_z X^\mu$ is a primary field of conformal weight $(h,\bar{h}) = (1,0)$, while $\partial_{\bar{z}} X^\nu$ contributes $(h,\bar{h}) = (0,1)$. The momentum exponential is also primary with $(h,\bar{h}) = (\frac{\alpha' k^2}{4}, \frac{\alpha' k^2}{4})$. The conformal weight of the complete operator is the sum of all these contributions. To arrive at $(h,\bar{h}) = (1,1)$, we thus require that $k^2 = 0$. The graviton is massless as expected. Open string vertex operators V_{open} are inserted on the worldsheet boundary. Correspondingly, they are only integrated along the boundary $V_{open} = \int_{boundary} \mathrm{d}z\ V_{open}(z)$.

There is one more complication for the superstring: the vertex operators will also depend on superconformal ghosts β and γ, which are introduced when restricting to superconformal gauge [51]. The ghosts β and γ can be rewritten (bosonised) in terms of holomorphic and antiholomorphic scalar fields φ and $\tilde{\varphi}$, which enter the vertex operators explicitly in the form of fields $e^{a\varphi}$ and $e^{b\tilde{\varphi}}$ with conformal weights $h = -\frac{1}{2}a^2 - a$ and $\tilde{h} = -\frac{1}{2}b^2 - b$. One finds that the unintegrated closed and open string vertex operators are of the form

$$V^{a,b}_{closed}(z,\bar{z}) = e^{a\varphi(z)}\ e^{b\tilde{\varphi}(\bar{z})}\ \mathcal{O}_{closed}(z,\bar{z})\ , \tag{2.9}$$

$$V^{a}_{open}(z) = e^{a\varphi(z)}\ \mathcal{O}_{open}(z)\ . \tag{2.10}$$

The labels a and b are referred to as pictures, and there are constraints on their sums in consistent correlation functions. For combinations of vertex operators on worldsheets of genus $g = 0, 1$, the conditions are summarised in the table below.

Worldsheet	Vertex operators	Picture constraint	
no boundaries	$\prod_i V^{a_i,b_i}_{closed}$	$\sum_i a_i = \sum_i b_i =$	-2 sphere 0 torus
with boundaries	$\prod_i V^{a_i}_{open} \prod_j V^{a_j,b_j}_{closed}$	$\sum_i a_i + \sum_j a_j + \sum_j b_j =$	-2 disk 0 cylinder

Given an unintegrated vertex operator in one picture p, it is thus essential that we can modify its ghost charge to be able to write down consistent correlation functions. Starting with a vertex operator in picture p, it is easy to determine the appropriate form with $p+1$. The procedure is called picture-changing and is described in [51]. It amounts to evaluating the limit

$$V^{p+1}(w) = \lim_{z\to w} e^{\varphi(z)} T_F(z) V^p(w)\ , \tag{2.11}$$

where T_F is the worldsheet supercurrent.[2] Up to a normalisation, it is given by:

$$T_F(z) = \sum_{\mu=0}^{9} \partial X^\mu \psi_\mu(z) \ . \tag{2.12}$$

There is also an antiholomorphic equivalent. In appendix A.2.4 we list all vertex operators relevant for calculations performed in this thesis. Next, we discuss the surfaces where these vertex operators are inserted.

2.1.4 Riemann surfaces

The sum over worldsheets in string perturbation theory is a sum over Riemann surfaces, and here we will describe the surfaces which will be relevant for calculations in the following chapters. Again, it will be most useful to work on a Euclidean worldsheet described by complex coordinates $(z, \bar{z})$.

Sphere

To perform calculations on the sphere it is most useful to conformally map the sphere to the complex plane, including a point at infinity, where this map breaks down. The sphere has no moduli and three complex CKVs. The last statement is equivalent to the fact that the sphere exhibits an $\mathrm{SL}(2, \mathbb{C})$ isometry. This needs to be removed by fixing three complex vertex operator positions.

Disk

For practical purposes, the disk is most easily obtained from the sphere. Taking the sphere as the complex plane and identifying

$$z \sim \bar{z} \ , \tag{2.13}$$

we can take the upper half plane as the disk with the real line as the boundary. The isometry group is $\mathrm{SL}(2, \mathbb{R})$, which is the subgroup of the isometries on the sphere which are consistent with the identification $z \sim \bar{z}$. Hence on the disk, we have to fix three real parameters in the vertex operator positions.

Torus

The torus can be obtained from the complex plane by identifying under

$$z \sim z + 1 \qquad z \sim z + \tau \ , \tag{2.14}$$

[2] In practice, one needs to use the operator product expansions (OPEs) of the various fields to perform this limit (see appendix A.2.3). Strictly speaking, this operation should only be performed inside a correlation function. However, unless poles in $(z - w)$ appear when evaluating this limit, the naive procedure gives the correct result.

where τ is a complex modulus. Two tori with moduli τ and τ' are equivalent if the moduli are related by a $\mathrm{SL}(2,\mathbb{Z})$ transformation. Correspondingly, to only sum over inequivalent tori in the string path integral, we will have to integrate τ over the fundamental domain of $\mathrm{SL}(2,\mathbb{Z})$ with an invariant measure $\mathrm{d}\tau\mathrm{d}\bar{\tau}/(4\mathrm{Im}\tau)$.

Cylinder

To arrive at the cylinder (or annulus), we subject the torus to another identification. First, we restrict $\tau = it/2$ where $t \in \mathbb{R}$. Identifying points under

$$z \sim 1 - \bar{z} \tag{2.15}$$

the lines $\mathrm{Re}(z) = 0$ and $\mathrm{Re}(z) = \frac{1}{2}$ are identified with themselves and become boundaries. The fundamental domain of the cylinder is then given by $\mathrm{Re}(z) \in [0, \frac{1}{2}]$ and $\mathrm{Im}(z) \in [0, \frac{it}{2}]$. There is only one real modular parameter t which controls the circumference of the cylinder and we need to integrate over it in calculations with measure $\mathrm{d}t/(2t)$. The limit $t \to 0$ corresponds to the open string UV and closed string IR limit, whereas for $t \to \infty$ we replace IR $\leftrightarrow$ UV in the previous statement.

2.2 Correlation functions

In the first part of this chapter we saw that the calculation of elements of the S-matrix in string theory is equivalent to the evaluation of correlation functions of vertex operators in conformal field theory. Here we collect results which will be important for performing such calculations.

2.2.1 Worldsheets without boundaries

2.2.1.1 Correlation functions on the sphere

The starting point for this section will be the superstring action on the sphere, which we conformally map to the complex plane. With our choice of background fields the action is given by[3]

$$S = \frac{1}{4\pi}\int d^2z \left(\frac{2}{\alpha'}\eta_{\mu\nu}\partial X^\mu \bar{\partial} X^\nu + \eta_{\mu\nu}\psi^\mu\bar{\partial}\psi^\nu + \eta_{\mu\nu}\tilde{\psi}^\mu\partial\tilde{\psi}^\nu\right) \,. \tag{2.16}$$

This action describes a two-dimensional theory of free scalars and free fermions on the plane, and we will be able to calculate correlation functions using standard techniques of

[3]The superstring action (2.6) is defined on the Euclidean worldsheet of a single propagating string. For the closed string this is a cylinder, whose circumference is described by $\sigma^1 \in [0, 2\pi]$ and whose length is given by $\sigma^2 \in [-\infty, \infty]$. On this cylinder we use complex coordinates $z = \sigma^1 + i\sigma^2$ and $\bar{z} = \sigma^1 - i\sigma^2$. To arrive at the action on the complex plane, we can perform the conformal map $z \to e^{-iz}$, $\bar{z} \to e^{i\bar{z}}$. As the theory is conformally invariant, the form of the action remains unchanged.

quantum field theory (QFT). The all-important quantities will be the propagators (two-point functions) of the worldsheet fields: in a free quantum field theory all correlation functions can be expressed exactly in terms of the basic propagators. In particular, the procedure for calculating correlation functions is the path integral version of Wick's theorem: to arrive at the result we will have to sum over all pairwise contractions of the fields appearing in the correlation function. Each contraction then provides a factor of the basic propagator.

Hence, the starting point of any calculation are the propagators, which we list in the following. It will be convenient to split the bosonic field $X(z,\bar{z})$ into holomorphic and antiholomorphic parts $X(z,\bar{z}) = X(z) + \tilde{X}(\bar{z})$. Starting with the action (2.6) it is easy to show that the two-point functions are given by:

$$\langle X^\mu(z_1) X^\nu(z_2)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\ln(z_1 - z_2)\ , \tag{2.17}$$

$$\langle \tilde{X}^\mu(\bar{z}_1) \tilde{X}^\nu(\bar{z}_2)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\ln(\bar{z}_1 - \bar{z}_2)\ , \tag{2.18}$$

$$\langle \psi^\mu(z_1)\psi^\nu(z_2)\rangle = \frac{\eta_{\mu\nu}}{z_1 - z_2} \tag{2.19}$$

$$\langle \tilde{\psi}^\mu(\bar{z}_1)\tilde{\psi}^\nu(\bar{z}_2)\rangle = \frac{\eta_{\mu\nu}}{\bar{z}_1 - \bar{z}_2}\ . \tag{2.20}$$

Most importantly, there are no cross-couplings between holomorphic and antiholomorphic components. From the above results, we can also obtain the propagators involving ∂X and $\bar{\partial} X$ by differentiating (2.17) and (2.18).

Using these basic propagators, we can now evaluate correlation functions involving the above fields by the standard rules for a free 2d QFT. To give an example, the correlation function over two momentum exponentials appearing in vertex operators is

$$\langle e^{ik_1 X(z,\bar{z})} e^{ik_2 X(w,\bar{w})}\rangle = |z - w|^{\alpha' k_1 \cdot k_2}\ . \tag{2.21}$$

To perform practical calculations, it will be useful to rewrite the correlation functions over fermionic fields in a different form. In particular, the worldsheet spinors have non-trivial correlation functions with so-called spin fields, which appear in vertex operators of spacetime fermions (A.36). To treat both types of fields similarly, it will be convenient to define the following combinations of worldsheet spinors:

$$\Psi^{i=1} = \frac{1}{\sqrt{2}}\left(\psi^1 + \ \psi^0\right)\ , \qquad \bar{\Psi}^{i=1} = \frac{1}{\sqrt{2}}\left(\psi^1 - \ \psi^0\right)\ , \tag{2.22}$$

$$\Psi^{i=2} = \frac{1}{\sqrt{2}}\left(\psi^2 + i\psi^3\right)\ , \qquad \bar{\Psi}^{i=2} = \frac{1}{\sqrt{2}}\left(\psi^2 - i\psi^3\right)\ , \qquad \text{etc. for } i = 3,4,5\ , \tag{2.23}$$

with similar expressions for the antiholomorphic counterparts $\tilde{\Psi}$ and $\bar{\tilde{\Psi}}$. The basic correlation functions of these fields are

$$\langle \Psi^i(z_1)\bar{\Psi}^j(z_2)\rangle = \frac{\delta^{ij}}{z_1 - z_2}\ , \qquad \langle \Psi^i(z_1)\Psi^j(z_2)\rangle = \langle \bar{\Psi}^i(z_1)\bar{\Psi}^j(z_2)\rangle = 0\ . \tag{2.24}$$

In two dimensions, we can then rewrite the complex spinors in terms of free scalar fields $H(z)$ and $\tilde{H}(\bar{z})$ with a basic propagator $\langle H(z)H(0)\rangle = -\ln z$. The procedure is called bosonisation and amounts to locally replacing the complex spinors by

$$\Psi^i(z) \cong e^{iH^i(z)}, \qquad \bar{\Psi}^i(z) \cong e^{-iH^i(z)} \ . \tag{2.25}$$

One can confirm that the bosonised expressions exactly reproduce the correlators of the original spinors (2.24). In particular, correlation functions involving the bosonised fields become

$$\langle\prod_i e^{iq_i H(z_i)}\rangle = \begin{cases} \prod_{i<j}(z_i - z_j)^{q_i q_j} \ , & \sum_i q_i = 0 \ , \\ 0 \ , & \sum_i q_i \neq 0 \ , \end{cases} \tag{2.26}$$

where q_i are referred to as H-charges. The condition for a correlator over the fermionic fields to be non-zero can then be rephrased as a constraint on the H-charges: a correlator over spinors can only contribute a non-vanishing result if the sum of all H-charges vanishes.

Bosonisation can also be used on spin fields, which appear in vertex operators for fermions (A.36). These can be written as

$$S(z) = e^{i\sum_{i=1}^{5} q^i H^i(z)} \ , \qquad q = \left(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right) \ , \tag{2.27}$$

where we use the same scalar field $H(z)$, which was employed in the bosonisation of the worldsheet fermions. Then correlators involving both spin fields and worldsheet spinors can be determined using the expression (2.26).

In addition, superstring vertex operators will also contain the fields $\varphi(z)$ and $\tilde{\varphi}(\bar{z})$. These are free scalars which are introduced when bosonising the superconformal (β, γ) ghosts. As they are free fields, their propagators have the same qualitative form as (2.17) and (2.18). In particular, we have

$$\langle\varphi(z_1)\varphi(z_2)\rangle = -\ln(z_1 - z_2) \ , \qquad \langle\tilde{\varphi}(\bar{z}_1)\tilde{\varphi}(\bar{z}_2)\rangle = -\ln(\bar{z}_1 - \bar{z}_2) \ , \tag{2.28}$$

which completes the list of propagators on the sphere. Given the basic correlators from this section, all correlation functions on the sphere can be evaluated in principle.

2.2.1.2 Basic correlators on the torus

On the torus, the basic propagators have to satisfy the same local behaviour as on the sphere, but they also have definite periodicities under translations along the two cycles of the torus. Because of this Jacobi theta functions $\vartheta_{\alpha\beta}(z|\tau)$ (with $\alpha, \beta = 0, 1$) will feature prominently. These are quasi-doubly periodic under $z \to z + 1$ and $z \to z + \tau$ and can be used to construct doubly periodic functions under these shifts. Their definitions and some properties are listed in appendix A.1.

Bosonic propagator

We can use $\vartheta_1(z|\tau) = \vartheta_{11}(z|\tau)$ to write the propagator of two bosonic fields X on the torus. For small z the function $\vartheta_1(z|\tau)$ behaves as

$$\vartheta_1(z|\tau) = 2\pi\eta^3(\tau)\ z + \mathcal{O}(z^3)\ , \tag{2.29}$$

where $\eta(\tau)$ is the Dedekind eta function. This leads to the following ansatz for the propagator:

$$\langle X^\mu(z,\bar{z})X^\nu(w,\bar{w})\rangle_{\mathcal{T}} = -\frac{\alpha'}{2}\ln\left|\frac{\vartheta_1\left(z-w|\tau\right)}{\vartheta_1'(0)}\right|^2 + \frac{\pi\alpha'}{\tau_2}(\mathrm{Im}(z-w))^2\ . \tag{2.30}$$

Using the property (2.29), it clearly shows the same singular behaviour as its counterpart on the sphere (2.17). The second term is added to make the expression periodic on the torus and can be understood as a background charge.

Fermionic propagator

Correlators of fermionic fields on the torus exhibit one more complication compared to worldsheet bosons. The torus has two non-contractible loops and the worldsheet fermions can have either periodic (+) or antiperiodic (−) boundary conditions around them. This leads to four so-called spin structures on the torus, (−−) (+−) (−+) (++), and we will need to sum over them in the calculation. The first three are referred to as even spin structures, while (++) is called odd. The fermionic propagator can then be found by requiring

1. that it locally reproduces the result from the sphere (2.19),
2. that it satisfies the correct periodicities around the torus cycles.

Again, the result can be constructed using Jacobi theta functions. For the even spin structures (i.e. excluding $\alpha = \beta = 1$) such an expression is given by:

$$\langle\psi^\mu(z_1)\psi^\nu(z_2)\rangle_{\mathcal{T}}^{\alpha\beta} = \eta^{\mu\nu}\frac{\vartheta_{\alpha\beta}(z_1-z_2)\ \vartheta_1'(0)}{\vartheta_1(z_1-z_2)\vartheta_{\alpha\beta}(0)}\ , \tag{2.31}$$

which is also known as the Szegő kernel. Interestingly, by using properties of doubly periodic functions one can show that (2.31) is indeed the only possible expression consistent with the two conditions stated above (for details see e.g. [49]). The fermionic two-point function for the odd spin structure vanishes because of fermionic zero modes.[4]

[4] Correlation function of fermions in the odd spin structure are only non-zero if all fermionic zero modes are saturated. A good reference for fermionic one-loop correlators in both the even and odd spin structures is [52].

Correlators involving both fermionic fields and spin fields on the torus will also be sensitive to the spin structure. Their derivation is more technical and we refrain from showing it at this point. The same is true for correlation functions over superconformal ghosts. We refer readers to [53, 54] for the derivation.[5]

2.2.2 Worldsheets with boundaries

In this thesis we will be interested in performing CFT calculations on worldsheets with boundaries. In particular, we will analyse correlators on the disk and on the cylinder, which we defined in section 2.1.4. The presence of boundaries will introduce new conditions on the worldsheet fields, which will also have to be satisfied by the correlation functions. One important set of conditions will arise from extremising the action in the presence of a boundary. We will begin by deriving conditions on the bosonic spacetime fields. To this end we vary the action (2.16) w.r.t. the bosonic coordinate X^μ:

$$\delta S_b = -\frac{1}{\pi\alpha'}\int_\Sigma \mathrm{d}^2 z\ \delta X_\mu \partial\bar{\partial} X^\mu + \frac{i}{2\pi\alpha'}\int_{\partial\Sigma}\left(\mathrm{d}\bar{z}\ \delta X_\mu \bar{\partial} X^\mu - \mathrm{d}z\ \delta X_\mu \partial X^\mu\right) . \tag{2.32}$$

The first term gives the equation of motion $\partial\bar{\partial}X^\mu = 0$. The second term gives rise to boundary conditions and would be absent for closed string worldsheets. For open string worldsheets we have to ensure that these contributions vanish on each boundary explicitly. There are two possible solutions that lead to a vanishing boundary term:

$$\text{Neumann :} \qquad \mathrm{d}z\ \partial X^\mu\big|_{\text{boundary}} = \mathrm{d}\bar{z}\ \bar{\partial} X^\mu\big|_{\text{boundary}} , \tag{2.33}$$

$$\text{Dirichlet :} \qquad \delta X^\mu\big|_{\text{boundary}} = 0 . \tag{2.34}$$

To parallel the condition in the Neumann case, we can rewrite (2.34) as

$$\mathrm{d}z\ \partial X\big|_{\text{boundary}} = -\mathrm{d}\bar{z}\ \bar{\partial} X\big|_{\text{boundary}} . \tag{2.35}$$

The identification of these conditions with Neumann and Dirichlet boundary conditions arises as follows: using our definitions of the disk and cylinder from section 2.1.4, we can easily check that these expressions reproduce $\partial_n X|_{boundary} = 0$ for Neumann and $\partial_t X|_{boundary} = 0$ for Dirichlet boundary conditions, where n and t are directions normal and tangential to the boundary.

Similarly, there will also be conditions on the fermionic fields. Varying the fermionic part of the action gives

$$\delta S_f = \frac{1}{2\pi}\int_\Sigma \mathrm{d}^2 z\ \left(\delta\psi_\mu \bar{\partial}\psi^\mu + \delta\tilde{\psi}_\mu \partial\tilde{\psi}^\mu\right) - \frac{i}{4\pi}\int_{\partial\Sigma}\left(\mathrm{d}\bar{z}\ (\delta\tilde{\psi}_\mu)\tilde{\psi}^\mu - \mathrm{d}z\ (\delta\psi_\mu)\psi^\mu\right) . \tag{2.36}$$

[5] In [53, 54] the correlation functions of fermions, spin fields and superconformal ghosts were derived using the so-called stress-tensor method. In chapter 3 we will introduce this method to derive correlators involving twist fields.

Again, we recover the equations of motion from the first term. As before, the boundary terms have to vanish for each boundary separately. It follows that for both the disk and the cylinder we require:

$$\psi^\mu\big|_{\text{boundary}} = \pm\tilde{\psi}^\mu\big|_{\text{boundary}} \ . \tag{2.37}$$

The overall sign between ψ and $\tilde{\psi}$ is just a convention, but in the presence of more than one boundary we can choose a different sign on different boundaries.

There is one more condition on worldsheet fermions which we need to touch upon. The above shows that the holomorphic and antiholomorphic fermions are related on the worldsheet boundary. Worldsheet supersymmetry then requires, that they also become sensitive to Dirichlet and Neumann boundary conditions (see for example [55]). To summarise this section, the boundary conditions for the disk and the cylinder can be compactly written as

$$\partial X^\mu\big|_{\text{boundary}} = R^\mu{}_\nu\, \bar{\partial} X^\nu\big|_{\text{boundary}} \ , \qquad \psi^\mu\big|_{\text{boundary}} = \pm R^\mu{}_\nu \tilde{\psi}^\nu\big|_{\text{boundary}} \ , \tag{2.38}$$

where R is a diagonal matrix $R = \text{diag}(\pm1, \pm1, \ldots, \pm1)$ which encodes Dirichlet and Neumann boundary conditions. On the disk, Neumann directions require an entry $(+)$ in D, while an entry $(-)$ signifies Dirichlet boundary conditions. On the cylinder we need to exchange $(+) \leftrightarrow (-)$.

The significance of Dirichlet and Neumann boundary conditions is the following. In string theory, boundaries are associated with the presence of D-branes, which are higher dimensional surfaces on which open strings can end. While the ends of strings can move parallel to the brane, the string is fixed in the transverse direction. Hence, Neumann boundary conditions are required for directions parallel to the branes, while transverse directions exhibit Dirichlet boundary conditions. In the following, we will analyse how the boundary conditions are implemented on the level of correlation functions.

2.2.2.1 Basic correlators on the disk

To calculate correlation functions on the disk, we will be able to reuse many of the results on the sphere. Locally, correlators on the disk should exhibit the same behaviour as on the sphere. However, we also have to satisfy Dirichlet or Neumann boundary conditions which should also hold for the quantum theory. Taking the disk as the upper half plane, the boundary conditions can be written as follows:

$$\text{at } \text{Im}(z) = \text{Im}(\bar{z}) = 0: \quad \partial X^\mu(z) = \pm\bar{\partial} X^\mu(\bar{z}) \ , \quad \psi^\mu(z) = \pm\tilde{\psi}(\bar{z}) \ , \tag{2.39}$$

where $(+)$ encodes Neumann and $(-)$ Dirichlet boundary conditions. As mentioned before, worldsheet supersymmetry requires that the boundary conditions on bosons and fermions are correlated.

Method of images

In Electrodynamics problems with boundaries are typically solved by the method of images. We can also apply it here to derive Green's functions on the disk from those on the sphere. Recall that the disk can be obtained from the sphere by identifying points related by the involution $I_D : z \to \bar{z}$. Given the bosonic Green's function on the sphere

$$G_{S_2}(z_1, z_2; \bar{z}_1, \bar{z}_2) = \langle X(z_1, \bar{z}_1) X(z_2, \bar{z}_2) \rangle_{S_2} \tag{2.40}$$

we can find the corresponding result on the disk by (anti-) symmetrising under the involution I_D:

$$\begin{aligned} G_{D_2}(z_1, z_2; \bar{z}_1, \bar{z}_2) &= \frac{1}{2}\Big(G_{S_2}(z_1, z_2; \bar{z}_1, \bar{z}_2) + G_{S_2}(I_D(z_1), I_D(z_2); I_D(\bar{z}_1), I_D(\bar{z}_2)) \\ &\qquad \pm G_{S_2}(I_D(z_1), z_2; I_D(\bar{z}_1), \bar{z}_2) \pm G_{S_2}(z_1, I_D(z_2); \bar{z}_1, I_D(\bar{z}_2)) \Big) \\ &= -\eta^{\mu\nu} \, \frac{\alpha'}{4} \left(\ln|z_1 - z_2|^2 + \ln|\bar{z}_1 - \bar{z}_2|^2 \pm \ln|\bar{z}_1 - z_2|^2 \pm \ln|z_1 - \bar{z}_2|^2 \right) \\ &= -\eta^{\mu\nu} \, \frac{\alpha'}{2} \left(\ln|z_1 - z_2|^2 \pm \ln|z_1 - \bar{z}_2|^2 \right) \ , \end{aligned} \tag{2.41}$$

where the positive sign has to be used for Neumann and the negative for Dirichlet boundary conditions. The second term implies, that there are now non-zero cross-correlations between holomorphic and antiholomorphic parts of X. The method of images can also be applied to fermionic fields, but it is more involved since the boundary conditions (2.39) identify holomorphic and antiholomorphic fermions (see [56] for details).

The result for both bosons and fermions is that boundaries introduce cross-couplings between holomorphic and antiholomorphic fields. Hence, in addition to the correlation functions inherited from the sphere, the following correlators are non-zero on the disk [57–59]:

$$\langle X^\mu(z_1) \tilde{X}^\nu(\bar{z}_2) \rangle = \, - D^{\mu\nu} \, \frac{\alpha'}{2} \ln(z_1 - \bar{z}_2) \ , \tag{2.42}$$

$$\langle \psi^\mu(z_1) \tilde{\psi}^\nu(\bar{z}_2) \rangle = \frac{D^{\mu\nu}}{z_1 - \bar{z}_2} \ . \tag{2.43}$$

The matrix $D^{\mu\nu}$ encapsulates the conditions on the disk boundary:

$$D^{\mu\nu} = \begin{cases} \eta^{\mu\nu} & \text{Neumann} \\ -\eta^{\mu\nu} & \text{Dirichlet} \end{cases} \ . \tag{2.44}$$

While we do not give any more details, we note that we also require couplings between holomorphic and antiholomorphic parts of the superconformal ghosts [59]:

$$\langle \phi(z_1) \tilde{\phi}(\bar{z}_2) \rangle = \, - \ln(z_1 - \bar{z}_2) \ . \tag{2.45}$$

The main result for correlation functions on the disk is the following: in addition to the basic correlators on the sphere (2.17), (2.19) and (2.28) we also need to include cross-correlations (2.42), (2.43) and (2.45).

Fields on the boundary

Here, we wish to consider the special case where fields are located on the boundary. This is relevant when performing calculations involving vertex operators of open string states, which are inserted on worldsheet boundaries. Correlators of these fields can be obtained by taking a limit of the interior result. For example, the two-point function $\langle X(z_1,\bar{z}_2)X(z_2,\bar{z}_2)\rangle$ (2.41) for the Neumann case becomes:

$$\begin{aligned} \langle X(z_1,\bar{z}_1)X(z_2,\bar{z}_2)\rangle^{\mathrm{N}}_{D_2} &= -\eta^{\mu\nu}\,\frac{\alpha'}{2}\ln|z_1-z_2|^2 - \eta^{\mu\nu}\,\frac{\alpha'}{2}\ln|z_1-\bar{z}_2|^2 \\ \underset{z_{1,2}\to y_{1,2}}{\longrightarrow}\ \langle X(y_1)X(y_2)\rangle^{\mathrm{N}}_{D_2} &= -2\alpha'\eta^{\mu\nu}\ \ln|y_1-y_2|\ , \end{aligned} \tag{2.46}$$

where y_i is a position on the boundary. As $z_i=\bar{z}_i=y_i$ on the boundary we find that the basic correlator is effectively doubled compared to the sphere. For Dirichlet boundary conditions, the correlator vanishes.

2.2.2.2 Basic correlators on the cylinder

Before, we derived Green's functions on the disk from those on the sphere. Similarly, one can derive the propagators on the cylinder from those on the torus. For brevity, we will just show the results for the two-point functions of the bosonic fields. We can repeat the steps leading to (2.41), but use the appropriate involution for the cylinder $I_{\mathcal{A}}: z\to 1-\bar{z}$. We give the results for the bosonic propagator for Dirichlet and Neumann boundary conditions separately:

$$\begin{aligned} \langle X(z_1,\bar{z}_1)X(z_2,\bar{z}_2)\rangle^{\mathrm{N}}_{\mathcal{A}} = -\,\frac{\alpha'}{2}\left(\ln\left|\frac{\vartheta_1\left(z_1-z_2|\tau\right)}{\vartheta_1'(0)}\right|^2 + \ln\left|\frac{\vartheta_1\left(z_1+\bar{z}_2|\tau\right)}{\vartheta_1'(0)}\right|^2\right) \\ + \frac{2\pi\alpha'}{\tau_2}\left(\mathrm{Im}(z_1-z_2)\right)^2\ , \end{aligned} \tag{2.47}$$

$$\langle X(z_1,\bar{z}_1)X(z_2,\bar{z}_2)\rangle^{\mathrm{D}}_{\mathcal{A}} = -\,\frac{\alpha'}{2}\left(\ln\left|\frac{\vartheta_1\left(z_1-z_2|\tau\right)}{\vartheta_1'(0)}\right|^2 - \ln\left|\frac{\vartheta_1\left(z_1+\bar{z}_2|\tau\right)}{\vartheta_1'(0)}\right|^2\right) \tag{2.48}$$

If we considered fields X on the boundary, the Dirichlet correlator would vanish and the Neumann result would have to be doubled as before. The correlators for fermionic fields and superconformal ghosts can be found by similar methods (e.g. see [56,60]). The expressions which are relevant for calculations in this thesis are given in appendix A.2.6.

2.3 Toroidal orbifolds with D3-branes at orbifold singularities

In string perturbation theory we have to specify a background spacetime and this is the subject of this section. Before, we stated that we will restrict ourselves to flat backgrounds with vanishing Kalb-Ramond field and constant dilaton. Here we want to be more specific: to make contact with the observed universe, we take the background spacetime as a product of four-dimensional Minkowski space and a compact six-dimensional space. In this section, we will describe the compact space in more detail.

All calculations in the following chapters will be performed in toroidal orbifolds $(\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2)/\,\Gamma$, with Γ a $\mathbb{Z}_N$ subgroup of $SU(3)$. These spaces are simple enough to be able to perform calculations, but offer enough complexity to constitute semi-realistic models of particle physics [61, 62]. In particular, a toroidal orbifold is flat everywhere except at individual points, which nevertheless do not introduce any pathologies.

To construct the orbifold, we start with the three two-tori, which we parameterise by a complex coordinate X^i with $i = 1, 2, 3$. The geometry of each two-torus can be described by the geometric moduli

$$U = \frac{R_2}{R_1} e^{i\alpha} \ , \qquad T_2 = R_1 R_2 \sin\alpha \ , \tag{2.49}$$

where R_1 and R_2 are the radii of the two cycles of the torus. The orbifold point group $\mathbb{Z}_N$ acts on worldsheet fields through the orbifold twist vector $\theta = (\theta_1, \theta_2, \theta_3)$ with $\theta_i = k/N$ and k an integer:

$$X^i \to e^{2\pi i \theta_i} X^i \ , \tag{2.50}$$

$$\Psi^i \to e^{2\pi i \theta_i} \Psi^i \ , \tag{2.51}$$

where Ψ^i are complex spinors. The transformation of the spinor is needed to maintain worldsheet supersymmetry. The toroidal orbifold as a space is then obtained by identifying points related by the geometric action (2.50): $X^i \sim e^{2\pi i \theta_i} X^i$. For this to be consistent with the definition of the torus, only certain values for U are allowed.

Clearly, there can be fixed points under the geometric action (2.50), which are referred to as orbifold singularities. Interestingly, these do not lead to pathologies and string theory on this space is well-defined. The orbifold can be understood as a limiting case of a smooth space, where small cycles have shrunk to zero size to become singularities.

In addition to its geometric action, the orbifolding also affects further degrees of freedom. Open strings on a stack of D-branes or at a D-brane intersection can have endpoints on different branes. To keep track of strings ending on different branes one introduces a label λ, which is referred to as a Chan-Paton (CP) factor. The labels λ can be represented by matrices, and each vertex operator for an open string will contribute such a CP factor λ. In fact, the labels specify the representation of the string state under the gauge group on the stack of branes. When calculating elements of the S-matrix, we will need to include a trace over the CP factors of all vertex operators inserted on the boundary: $\mathrm{Tr}(\lambda_1\lambda_2\ldots)$. On these CP factors the orbifold acts as via a twist matrix γ_θ:

$$\theta: \ \lambda \to \gamma_\theta \lambda \gamma_\theta^{-1} \ . \tag{2.52}$$

On each stack of coincident branes we can choose an embedding of the form:

$$\gamma_\theta = \mathrm{diag}(\mathbb{1}_{n_0}, e^{\frac{2\pi i}{N}}\mathbb{1}_{n_1}, e^{\frac{4\pi i}{N}}\mathbb{1}_{n_2}, \ \ldots \ , e^{\frac{2\pi i(N-1)}{N}}\mathbb{1}_{n_{N-1}}) \tag{2.53}$$

where $n = \sum_{i=0}^{N-1} n_i$ is the total number of branes. By considering states which are invariant under the combined geometrical and CP action of the orbifold, the spectrum of massless states on D-branes at singularities can be derived [63–65].

D3-branes at orbifold singularities

As we will model visible sectors by D3-branes at an orbifold singularity, it will be useful to collect some features of the massless spectrum of these setups here. At a smooth point a stack of n D3-branes exhibits a $\mathcal{N} = 4$ Super-Yang-Mills theory with gauge group $U(n)$ on its worldvolume. At an orbifold singularity, only states which are invariant under the combined orbifold action will remain:

1. Gauge bosons arise from open string fluctuations in the external Minkowski directions. Correspondingly, they are not affected by the geometric action of the orbifold, which only affects internal directions. The CP factors then only have to be invariant under the CP action (2.52):

$$\lambda = \ \gamma_\theta \lambda \gamma_\theta^{-1} \ . \tag{2.54}$$

   Given the form of the twist matrix (2.53), the CP factor for a gauge boson has to be of block-diagonal form, with blocks of size $n_i \times n_i$. From this one can deduce that the gauge group on the stack of branes is now a product of multiple non-Abelian factors

$$\text{gauge group:} \ \ U(n_0) \times U(n_1) \times \ldots \times U(n_{N-1}) \ . \tag{2.55}$$

2. Chiral scalars arise from fluctuations (in the Neveu-Schwarz sector) in the internal directions. Correspondingly, they are also affected by the geometric action of the

orbifold. The CP factors have to obey

$$\lambda = e^{2\pi i\theta_i}\gamma_\theta\lambda\gamma_\theta^{-1} \tag{2.56}$$

In addition there will be chiral fermions (from the Ramond sector).[6] For $\theta_i \neq 0$ for all i we obtain a spectrum exhibiting $\mathcal{N} = 1$ supersymmetry. The scalars and fermions fill out chiral superfields which transform as bifundamentals under the gauge group on the stack of branes:

$$\text{Chiral multiplets: } \sum_{i=0}^{N-1}\sum_{r=1}^{3}(n_i, \bar{n}_{i-b_r}) \ , \tag{2.57}$$

for an orbifold twist $\theta = (\theta_1, \theta_2, \theta_3) = \frac{1}{N}(b_1, b_2, b_3)$. Consequently, we can label chiral multiplets as $C^r_{i,i-b_r}$. The subscripts, which we sometimes suppress, label the representations. The superscript denotes the complex internal direction. It is the appearance of chiral matter which makes these constructions interesting as semi-realistic models for particle physics.

3. The superpotential is inherited from the $\mathcal{N} = 4$ theory and takes the form

$$\begin{aligned} W &= \sum_{r,s,t=1}^{3} \epsilon_{rst} \ \mathrm{Tr}\ (C^r C^s C^t) \\ &= \sum_{i=0}^{N-1} \left(C^1_{i,i-b_r} C^2_{i-b_r,i-b_r-b_s} C^3_{i-b_r-b_s,i} - C^2_{i,i-b_r} C^1_{i-b_r,i-b_r-b_s} C^3_{i-b_r-b_s,i}\right) \end{aligned} \tag{2.58}$$

where ϵ_{rst} is the fully antisymmetric tensor. Consequently, only Yukawa couplings of the type $C^1C^2C^3$ or $C^2C^1C^3$ appear in the theory.

This concludes the analysis of the spectrum of D3-branes at an orbifold singularity.

In this chapter we reviewed the basics of string perturbation theory and sketched how to calculate CFT correlation functions involving the worldsheets fields X, ψ, $\tilde{\psi}$ and also spin fields S, $\tilde{S}$. In the setting of orbifolds, or when considering D-branes at angles, there are further fields, which implement non-trivial boundary conditions on the bosonic field X. Correlation functions involving these fields are technically more demanding, and we devote the next chapter to their study.

[6] The CP factors for fermions have to obey $\lambda = e^{2\pi i(\sum_i \theta_i s_i)}\gamma_\theta\lambda\gamma_\theta^{-1}$. The vector $\mathbf{s}$ represents the Ramond ground state and its entries take the values $s_i = \pm 1/2$. The GSO projection only allows states with $\sum_i s_i =$ odd.

Chapter 3

Twist fields

In this chapter we will discuss how to calculate CFT correlation functions involving twist fields, which introduce branch cuts into the map from the worldsheet to target space. In practical calculations we will also require expressions for Green's functions $\partial X(z_1)\partial \bar{X}(z_2)$ in a background of twist fields. The latter are a crucial starting point for the calculation of correlation functions of twist fields by the so-called stress-tensor method. Correspondingly, we will review this method for computing twist correlators on the sphere. As we are interested in worldsheets with boundaries, we finally discuss how the results can be carried over to the disk worldsheet. In this chapter we adopt the conventions of [66].

3.1 Twist fields on the sphere

A toroidal orbifold $\mathbb{T}^6/\mathbb{Z}_N$ is constructed as a quotient space of $\mathbb{T}^6$ under the linear action of the cyclic group $\mathbb{Z}_N$, yet the Hilbert space of string theory on this orbifold is much richer than the one on the toroidal space [61,62]. Take a closed string whose worldsheet is parameterised by a coordinate along the string $\sigma^1 \in [0, 2\pi]$ and a Euclidean time coordinate $\sigma^2 \in [-\infty, \infty]$. If the string endpoints are located at spacetime positions related by the action of $\mathbb{Z}_N$, this state is a closed string in a twisted sector:

$$X_{closed}(\sigma^1 + 2\pi, \sigma^2) = e^{2\pi i\theta} X_{closed}(\sigma^1, \sigma^2) \ . \tag{3.1}$$

Clearly, we need to work with complex spacetime coordinates $X = X^\mu + iX^{\mu+1}$ and $\bar{X} = X^\mu - iX^{\mu+1}$. At orbifold fixed points twisted closed strings do not have to stretch across spacetime and contribute massless states. Calculating scattering amplitudes involving these states is technically challenging, but it is certainly of phenomenological interest. For example, twisted closed string states are related to blow-up modes – Kähler moduli parameterising the sizes of cycles which have collapsed to zero size in the case of the orbifold. Consequently, by calculating correlation functions involving these blow-up modes one can in principle determine the effective field theory on the blown-up background [67,68]. Twisted closed string states are also important in heterotic string theory, where they appear in matter fields. Further, there are conditions similar to (3.1) in the context of open strings in models with intersecting D-branes. Twisted boundary conditions arise for open strings with two ends on different branes intersecting at angles. As massless twisted open string states are identified with matter excitations in these models, these states are of great phenomenological interest. Altogether, it is of great phenomenological importance to be able to perform calculations involving twisted states in string theory.

Upon a map $z = e^{i(\sigma^1 - i\sigma^2)}$ to the complex plane the condition (3.1) becomes:

$$X(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = e^{2\pi i\theta}X(z,z)\ . \tag{3.2}$$

At the level of the CFT we then enforce the above condition by the introduction of a twist field σ_θ, around which the bosonic field is multivalued [66]. It is these twist fields $\sigma_\theta(w,\bar{w})$ which then appear in vertex operators of twisted states and we need to be able to determine correlation functions involving them.

Monodromies and stress-tensor method

We will review the so-called stress-tensor method for deriving correlation functions involving twist fields, which was pioneered in [66]. While doing so, we collect results and expressions which will be important in the next chapter. The starting point is the monodromy of the bosonic spacetime field X around a twist field $\sigma_\theta(w,\bar{w})$ at the origin $w = \bar{w} = 0$:

$$X(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = e^{2\pi i\theta}X(z,\bar{z}) + v\ . \tag{3.3}$$

This differs from the expression in (3.2) as we also allow for displacements by v. These shifts will be members of a coset lattice which depends on both the twist angle and spacetime location of the twist field. We will separate the discussion into two parts by splitting the field X into a classical part and quantum fluctuations:

$$X = X_{cl} + X_{qu}\ . \tag{3.4}$$

The quantum fluctuation is ignorant of the embedding in spacetime and its monodromy condition is simply given by (3.2). In contrast, the full monodromy condition (3.3) applies for the classical part.

3.1.1 Quantum correlation functions

Beginning with the quantum part we note that the monodromy condition (3.2) leads to shifts in the Laurent mode expansion of X. We prefer to work with the holomorphic fields ∂X and $\partial \bar{X}$ which exhibit the same quantum monodromy conditions as X and $\bar{X}$ and whose Laurent expansions are

$$\begin{aligned} \partial X &= \sum_{n=-\infty}^{\infty} \alpha_{n-\theta}\ z^{-n-1+\theta}\ , \\ \partial \bar{X} &= \sum_{n=-\infty}^{\infty} \bar{\alpha}_{n+\theta}\ z^{-n-1-\theta}\ , \end{aligned} \tag{3.5}$$

with similar expressions for the antiholomorphic fields $\bar{\partial} X$ and $\bar{\partial}\bar{X}$. We can then derive the OPEs of the fields ∂X and $\partial \bar{X}$ with the twist field $\sigma_\theta(w, \bar{w})$ in the following way. The twist field is introduced to take us from the untwisted Hilbert space to the Hilbert space of twisted states. Correspondingly, the twisted ground state is given by $|\sigma_\theta\rangle = \sigma_\theta(0,0)|0\rangle$. Then we can find the relevant OPEs by requiring that all positive modes annihilate the twisted ground state:

$$\begin{aligned} \alpha_{n-\theta}\ |\sigma_\theta\rangle &= 0 \qquad n > 0\ , \\ \bar{\alpha}_{n+\theta}\ |\sigma_\theta\rangle &= 0 \qquad n \geq 0\ . \end{aligned} \tag{3.6}$$

This leads to the crucial result for the OPEs:

$$\begin{aligned} \partial X\ \sigma_\theta(w,\bar{w}) &\sim (z-w)^{-(1-\theta)}\ \tau_\theta(w,\bar{w}) + \cdots\ , \qquad (3.7) \\ \partial \bar{X}\ \sigma_\theta(w,\bar{w}) &\sim (z-w)^{-\theta}\ \tau'_\theta(w,\bar{w}) + \cdots\ , \\ \bar{\partial} X\ \sigma_\theta(w,\bar{w}) &\sim (\bar{z}-\bar{w})^{-\theta}\ \tilde{\tau}'_\theta(w,\bar{w}) + \cdots\ , \\ \bar{\partial} \bar{X}\ \sigma_\theta(w,\bar{w}) &\sim (\bar{z}-\bar{w})^{-(1-\theta)}\ \tilde{\tau}_\theta(w,\bar{w}) + \cdots\ , \end{aligned}$$

where τ denotes four different excited twist fields. The above result can be easily modified for the case of an antitwist $\sigma_{-\theta}$. Twisting by $-\theta$ is equivalent to a twist by $1-\theta$ in the monodromy condition (3.3). However, when writing down the mode expansion we assumed that $0 \leq \theta < 1$. Correspondingly, when dealing with antitwists we need to replace θ by $1-\theta$ everywhere, which lies in the allowed interval for the mode expansion.

Worldsheet supersymmetry requires that if the bosonic fields obey twisted boundary conditions, a compensating condition has to be imposed on the fermionic fields. Analogous to the bosonic case one introduces fermionic twist fields $s_\theta(w)$ and $\tilde{s}_\theta(\bar{w})$ such that the complex worldsheet spinors pick up a phase when moved around the insertion point of a twist. To get a consistent theory one defines the phase such that the worldsheet supercurrents $T_F(z) = \partial X \cdot \psi$ and $\tilde{T}_F(\bar{z}) = \bar{\partial} X \cdot \tilde{\psi}$ are single-valued around the insertion point of a twist $s_\theta(w)\tilde{s}_\theta(\bar{w})\sigma_\theta(w,\bar{w})$. From the OPEs, this is true if the fermionic twist fields are bosonised as

$$\begin{aligned} s_{\pm\theta}(w) &= e^{\pm i\theta H(w)}, \\ \tilde{s}_{\pm\theta}(\bar{w}) &= e^{\mp i\theta \tilde{H}(\bar{w})}, \end{aligned} \tag{3.8}$$

where H and $\tilde{H}$ are free scalar fields which we also used to bosonise the complex worldsheet fermions Ψ in (2.25).

The OPEs capture the local behaviour of the fields around twist insertions and allow us to determine the Green's functions in the presence of twists. These expressions will also be the starting point for the calculation of twist correlation functions via the stress-tensor method. We begin with the Green's function in a two-twist-background, which is completely determined by the OPEs. On the sphere we can thus write:

$$\begin{aligned} g_2(z_1,z_2)_{S_2} &= \frac{\langle -\frac{1}{2}\,\partial_{z_1}X\partial_{z_2}\bar{X}\ \sigma_\theta(w_1,\bar{w}_1)\ \sigma_{-\theta}(w_2,\bar{w}_2)\rangle}{\langle \sigma_\theta(w_1,\bar{w}_1)\ \sigma_{-\theta}(w_2,\bar{w}_2)\rangle} \\ &= (z_1-z_2)^{-2}\ (z_1-w_1)^{-1+\theta}\ (z_1-w_2)^{-\theta}\ (z_2-w_1)^{-\theta}\ (z_2-w_2)^{-1+\theta} \\ &\quad\ [(z_1-w_1)(z_2-w_2) - \theta(z_1-z_2)(w_1-w_2)]\ . \end{aligned} \tag{3.9}$$

The result is obtained by requiring that the fields ∂X and $\partial\bar{X}$ exhibit the correct local behaviour about all other fields: the double pole is typical of a $\partial X\partial\bar{X}$ propagator and the multivalued factors arise from the OPEs with the twist fields (3.7). The terms in the square bracket can be determined by requiring the absence of simple poles in $(z_1 - z_2)$, which we do not expect in a $\partial_{z_1}X\partial_{z_2}\bar{X}$ propagator. The dependence on the antiholomorphic coordinates $\bar{w}_1$ and $\bar{w}_2$ cancels between numerator and denominator.

This method can be extended to calculate Green's function in a background of more than two twists: However, these correlators cannot be fully determined by the OPEs alone. One can add to $g_2(z_1,z_2)_{S_2}$ classical solutions for $\partial X(z_1)\partial\bar{X}(z_2)$, which have the right monodromy around the twist insertions, but are not singular in the limit $z_1 \to z_2$. The Green's function can then be fixed by global monodromy conditions: we have to ensure that X is strictly periodic around all closed loops on the worldsheet encircling net zero twist.

We will now derive the conformal weight and the two-twist correlator. To do so we recall the OPE of a primary field $\phi(w,\bar{w})$ with the stress tensor $T(z)$:

$$T(z)\phi(w) = \frac{h\phi(w,\bar{w})}{(z-w)^2} + \frac{\partial_w\phi(w,\bar{w})}{z-w} + \dots\ , \tag{3.10}$$

where h is the conformal weight of ϕ. To make contact with (3.9) we note that the bosonic stress tensor $T(z)$ is the normal ordered product of $\partial X \partial \bar{X}(z)$ such that

$$-\frac{1}{2}\partial_{z_1} X \partial_{z_2} \bar{X} \sim \frac{1}{(z_1 - z_2)^2} + T(z_1) + \dots \ . \tag{3.11}$$

Thus, by subtracting the leading singularity we can turn the Green's function (3.9) into a correlator involving the stress-tensor:

$$\begin{aligned}\frac{\langle T(z_1)\ \sigma_\theta(w_1, \bar{w}_1)\ \sigma_{-\theta}(w_2, \bar{w}_2)\rangle}{\langle \sigma_\theta(w_1, \bar{w}_1)\ \sigma_{-\theta}(w_2, \bar{w}_2)\rangle} &= \lim_{z_2 \to z_1} \left[g_2(z_1, z_2) - \frac{1}{(z_1 - z_2)^2} \right] \\ &= \frac{1}{2}\theta(1-\theta) \left[\frac{1}{z_1 - w_1} - \frac{1}{z_1 - w_2} \right]^2 . \end{aligned} \tag{3.12}$$

By comparing (3.12) with (3.10) we can read off the conformal weight of a twist field and obtain a differential equation for the two-twist correlator. We find that the conformal weight of a twist (and antitwist) is $h = \frac{1}{2}\theta(1-\theta)$. Using $\bar{T}(\bar{z})$ we could also derive $\bar{h}$ which gives an identical result.

We now come to the main step of the stress-tensor method employed here: by comparing the simple poles of (3.12) with (3.10) one can derive a differential equation for the twist correlator. For our example we obtain

$$\partial_{w_1} \ln \langle \sigma_\theta(w_1, \bar{w}_1)\ \sigma_{-\theta}(w_2, \bar{w}_2)\rangle = -\theta(1-\theta)\ (w_1 - w_2)^{-1} \tag{3.13}$$

which can be integrated to give

$$\langle \sigma_\theta(w_1, \bar{w}_1)\ \sigma_{-\theta}(w_2, \bar{w}_2)\rangle = C(\bar{w}_1, \bar{w}_2)\ (w_1 - w_2)^{-\theta(1-\theta)} \tag{3.14}$$

where C is a "constant" of integration which depends on the antiholomorphic coordinates. The function C can be fixed by repeating the analysis starting with the antiholomorphic Green's function and relating it to a correlator involving the antiholomorphic stress tensor $\bar{T}(\bar{z})$. As the calculation is exactly parallel to the holomorphic case we can state the result for the two-twist correlator on the sphere directly:

$$\langle \sigma_\theta(w_1, \bar{w}_1)\ \sigma_{-\theta}(w_2, \bar{w}_2)\rangle_{S_2} = (w_1 - w_2)^{-\theta(1-\theta)}\ (\bar{w}_1 - \bar{w}_2)^{-\theta(1-\theta)} \tag{3.15}$$

The calculation of the two-twist-correlator could have also been achieved in a more direct Hilbert space approach by evaluating $\langle \sigma_\theta | T(z) | \sigma_\theta \rangle$. However, the stress-tensor method becomes particularly useful when analysing amplitudes with more than two twists. By comparing the correlation function of $T(z)$ in a background of n twist fields with the OPE (3.10) one can always derive a differential equation for the twist correlator.

3.1.2 Classical solutions

When calculating string scattering amplitudes one needs to sum over all classical backgrounds for the given process weighted by the classical action $e^{-S_{cl}}$ with

$$S_{cl} = \frac{1}{4\pi} \int \mathrm{d}^2 z \, \left(\partial X \bar{\partial} \bar{X} + \partial \bar{X} \bar{\partial} X \right) \, . \tag{3.16}$$

The classical solution has to satisfy the equation of motion $\partial\bar{\partial}X_{cl} = 0$, which implies that ∂X_{cl} and $\partial \bar{X}_{cl}$ have to be holomorphic while $\bar{\partial} X_{cl}$ and $\bar{\partial}\bar{X}_{cl}$ are antiholomorphic. Further, the classical solution has to display the correct monodromy around twist insertion points (3.3). This allows us to write down the classical solutions straight away up to a normalisation. For example, in a background of a twist $\sigma_\theta(w_1, \bar{w}_1)$ and antitwist $\sigma_{-\theta}(w_2, \bar{w}_2)$ they become

$$\begin{aligned} \partial X_{cl} &= a \, (z - w_1)^{-1+\theta} \, (z - w_2)^{-\theta} \\ \partial \bar{X}_{cl} &= \bar{a} \, (z - w_1)^{-\theta} \, (z - w_2)^{-1+\theta} \\ \bar{\partial} X_{cl} &= b \, (\bar{z} - \bar{w}_1)^{-\theta} \, (\bar{z} - \bar{w}_2)^{-1+\theta} \\ \bar{\partial} \bar{X}_{cl} &= \bar{b} \, (\bar{z} - \bar{w}_1)^{-1+\theta} \, (\bar{z} - \bar{w}_2)^{-\theta} \, . \end{aligned} \tag{3.17}$$

The normalisations a and b can be determined by requiring that

$$\Delta_{\mathcal{C}_i} X_{cl} = \oint_{\mathcal{C}_i} \mathrm{d}z \, \partial X_{cl} + \oint_{\mathcal{C}_i} \mathrm{d}\bar{z} \, \bar{\partial} X_{cl} = v_i \tag{3.18}$$

for all closed loops $\mathcal{C}_i$, where v_i is the displacement in spacetime associated with the loop $\mathcal{C}_i$. These shifts v_i describe the embedding of the worldsheet in spacetime, and depend on the twist locations. In the next paragraphs we will describe how they are determined.

Lattice shifts

Each twist field is associated with a space group element (θ, v) which implements the global monodromy conditions:

$$X(e^{2\pi i} z, e^{-2\pi i} \bar{z}) = \theta X(z, \bar{z}) + v \, . \tag{3.19}$$

The vector v is an element of a coset lattice that depends on the twist angle θ and the fixed point f of the twist field [62, 66]:

$$v \in (1 - \theta)(f + u) \qquad u \in \Lambda \, , \tag{3.20}$$

where Λ is the lattice of the orbifold space group.

The shift along a contour surrounding two twists will be associated with the product of the space group elements of the twists enclosed.[1] The product is performed as

$$(\theta, v)(\varphi, u) = (\theta\varphi, v + \theta u) \ . \tag{3.21}$$

Correspondingly, the inverse of a space group element (θ, v) is given by $(\theta, v)^{-1} = (\theta^{-1}, -\theta^{-1}v)$.

Consistent embeddings of the worldsheet into target space put restrictions on the shifts v_i associated with the vertex operator insertions (θ_i, v_i). In particular, consider a contour surrounding all twist insertions. We can pull this around the sphere to end up with a contour encircling no twists. To avoid inconsistencies we thus find the following space group selection rule: the ordered product of all space group elements (θ_i, v_i) on the sphere must be a set including the identity $(1, 0)$, otherwise the correlation function vanishes.

Classical action

We have now all necessary ingredients to calculate the classical action. The strategy is as follows:

1. Given the setup of twists identify a complete basis of loops C_i with net zero twist.
2. By calculating products of the space group elements of the individual twist insertions determine the displacements v_i associated with the loops C_i.
3. Normalise the classical solutions (3.17) by using the shifts v_i in (3.18).
4. Calculate the action (3.16) using the normalised solutions.

This concludes our review of twist fields on the sphere. In the following section, we will describe how the above results can be applied to study twist field correlation functions on the disk.

3.2 Twist fields on the disk

In this section we want to combine the cut-structure of twist correlation functions with the conditions enforced by worldsheet boundaries. The result will be the "doubling trick" for twist fields. On worldsheets with boundaries, twist fields can either be inserted in the bulk of the worldsheet, or they can be located on the boundary. The latter case occurs for open strings between branes intersecting at angles: in this context the twist

[1] To define products of space group elements consistently, we have to specify the ordering.

fields are also referred to as boundary changing operators and they have been explored in [14, 69–74]. We however will be interested in twist fields in the bulk of the disk. Such a setup has been considered before in [75].

3.2.1 Quantum correlation functions

The presence of D-branes requires strings to have Neumann boundary conditions parallel and Dirichlet boundary conditions transverse to the branes. Thus, the bosonic fields have to obey:

$$\text{disk:} \qquad \partial X = \begin{cases} \bar{\partial} X & \text{Neumann} \\ -\bar{\partial} X & \text{Dirichlet} \end{cases} \qquad \mathrm{Im}(z) = \mathrm{Im}(\bar{z}) = 0 \; . \tag{3.22}$$

On the sphere the presence of twist field insertions led to a cut-structure in the map from worldsheet to spacetime encoded in the OPEs (3.7) between the fields ∂X and $\bar{\partial} X$ and the twist fields $\sigma_\theta(w, \bar{w})$. These are local expressions and thus they must hold on the disk as well.

To see how to reconcile all these conditions let us return to the study of Green's functions. Using the OPEs (3.7) the $\partial X \partial \bar{X}$ propagator in the presence of one twist on the disk should thus behave as

$$g_1(z_1, z_2)_{D_2} = \frac{\langle -\frac{1}{2}\, \partial_{z_1} X \partial_{z_2} \bar{X} \;\sigma_\theta(w, \bar{w})\rangle}{\langle \sigma_\theta(w, \bar{w})\rangle} = \frac{(z_1 - w)^{-1+\theta}\; (z_2 - w)^{-\theta}}{(z_1 - z_2)^{-2}}\; f(z_1, z_2; w, \bar{w}) \; , \tag{3.23}$$

Next, we follow the same prescription to determine the antiholomorphic Green's function:

$$\tilde{g}_1(\bar{z}_1, \bar{z}_2)_{D_2} = \frac{\langle -\frac{1}{2}\, \bar{\partial}_{\bar{z}_1} X \bar{\partial}_{\bar{z}_2} \bar{X} \;\sigma_\theta(w, \bar{w})\rangle}{\langle \sigma_\theta(w, \bar{w})\rangle} = \frac{(\bar{z}_1 - \bar{w})^{-\theta}\; (\bar{z}_2 - \bar{w})^{-1+\theta}}{(z_1 - z_2)^{-2}}\; \tilde{f}(\bar{z}_1, \bar{z}_2; w, \bar{w}) \; . \tag{3.24}$$

The conditions (3.22) now require that the expressions for the two Green's functions coincide on the disk boundary when $z_1 = \bar{z}_1$ and $z_2 = \bar{z}_2$. The twist field is in the bulk of the disk such that $w \neq \bar{w}$. This condition is only satisfied if the holomorphic fields ∂X and $\partial \bar{X}$ also show a singular behaviour about the antiholomorphic coordinate $\bar{w}$. Similarly, the antiholomorphic fields $\bar{\partial} X$ and $\bar{\partial} \bar{X}$ must now also be branched about w. In particular, we require

$$\partial X(z, \bar{z}) \sigma_\theta(w, \bar{w}) \sim \begin{cases} (z - w)^{-(1-\theta)} & z \to w \\ (z - \bar{w})^{-\theta} & z \to \bar{w} \end{cases} \tag{3.25}$$

$$\partial \bar{X}(z, \bar{z}) \sigma_\theta(w, \bar{w}) \sim \begin{cases} (z - w)^{-\theta} & z \to w \\ (z - \bar{w})^{-(1-\theta)} & z \to \bar{w} \end{cases} \tag{3.26}$$

$$\bar{\partial} X(z, \bar{z}) \sigma_\theta(w, \bar{w}) \sim \begin{cases} (\bar{z} - \bar{w})^{-\theta} & \bar{z} \to \bar{w} \\ (\bar{z} - w)^{-(1-\theta)} & \bar{z} \to w \end{cases} \tag{3.27}$$

$$\bar{\partial} \bar{X}(z, \bar{z}) \sigma_\theta(w, \bar{w}) \sim \begin{cases} (\bar{z} - \bar{w})^{-(1-\theta)} & \bar{z} \to \bar{w} \\ (\bar{z} - w)^{-\theta} & \bar{z} \to w \end{cases} \tag{3.28}$$

Consequently, the Green's function on the disk satisfying all conditions including the absence of simple poles is

$$\begin{aligned} g_1(z_1,z_2)_{D_2} &= (z_1-z_2)^{-2}\,(z_1-w)^{-1+\theta}\,(z_1-\bar{w})^{-\theta}\,(z_2-w)^{-\theta}\,(z_2-\bar{w})^{-1+\theta} \\ &\quad [(z_1-w)(z_2-\bar{w})-\theta(z_1-z_2)(w-\bar{w})]\ . \end{aligned} \tag{3.29}$$

By comparing this result to the expression for the Green's function in a twist-antitwist background on the sphere (3.9) we find that the two expressions are identical if we replace $w \to w_1$ and $\bar{w} \to w_2$. This establishes the doubling trick for twist fields: we conclude that a single twist field $\sigma_\theta(w,\bar{w})$ in the bulk of the disk behaves like a twist at w and an antitwist at $w' = \bar{w}$ on the sphere.

This has direct consequences: as there is a non-zero correlation function between a twist and an antitwist on the sphere, we expect that a single twist field has a non-zero self-contraction on the disk. Starting with the Green's function (3.29) and using the stress-tensor method as detailed in the previous section we derive that

$$\langle \sigma_\theta(w,\bar{w})\rangle_{D_2} = (w-\bar{w})^{-\theta(1-\theta)}\ . \tag{3.30}$$

The correlation functions of the fermionic twist fields on the disk can again be easily determined via bosonisation in terms of free scalar fields $H(z)$ and $\tilde{H}(\bar{z})$. On worldsheets with boundaries we have non-zero correlation functions between the holomorphic and antiholomorphic fields $H(z)$ and $\tilde{H}(\bar{z})$. Consequently, there will be non-trivial correlation functions between the holomorphic and antiholomorphic fields $s_\theta(w)$ and $\tilde{s}_\theta(\bar{w})$:

$$\langle s_\theta(w)\ \tilde{s}_\theta(\bar{w})\rangle_{D_2} = \langle e^{i\theta H(w)}\ e^{i\theta \tilde{H}(\bar{w})}\rangle_{D_2} = (w-\bar{w})^{-\theta^2}\ . \tag{3.31}$$

Now we are in a position to collect a few results which will be important for later calculations. In particular, we will encounter

$$\langle \prod_{j=1}^{3} \sigma_{\theta_i}(w,\bar{w})\rangle_{D_2}\ , \qquad \langle \partial X^i(z_1)\partial \bar{X}^i(z_2) \prod_{j=1}^{3} \sigma_{\theta_i}(w,\bar{w})\rangle_{D_2}\ . \tag{3.32}$$

The first expression is just given by the product of three correlators of the form (3.30). The second correlation function is closely related to the Green's function in the presence of twists (3.29). In particular, we find

$$\begin{aligned} \langle \partial X^i(z_1)\partial \bar{X}^i(z_2) \prod_{j=1}^{3} \sigma_{\theta_i}(w,\bar{w})\rangle_{D_2} &= -2\ g_1(z_1,z_2)_{D_2} \langle \prod_{j=1}^{3} \sigma_{\theta_i}(w,\bar{w})\rangle_{D_2} \\ = \ -2\,(z_1-z_2)^{-2}\,(z_1-w)^{-1+\theta_i}&\,(z_1-\bar{w})^{-\theta_i}\,(z_2-w)^{-\theta_i}\,(z_2-\bar{w})^{-1+\theta_i} \\ [(z_1-w)(z_2-\bar{w})-\theta_i(z_1-z_2)(w-\bar{w})]&\,(w-\bar{w})^{-\sum_{j=1}^{3}\theta_j(1-\theta_j)}\ . \end{aligned} \tag{3.33}$$

3.2.2 Classical action

The doubling-trick also needs to be applied when considering the classical solutions. A priori, in the presence of one twist field $\sigma_\theta(w,\bar{w})$ we expect the same local behaviour on the disk as on the sphere:

$$\begin{aligned} \partial X_{cl} &\sim (z-w)^{-1+\theta} & \partial \bar{X}_{cl} &\sim (z-w)^{-\theta} \\ \bar{\partial} X_{cl} &\sim (\bar{z}-\bar{w})^{-\theta} & \bar{\partial} \bar{X}_{cl} &\sim (\bar{z}-\bar{w})^{-1+\theta} \end{aligned} \tag{3.34}$$

However, the classical solutions also have to satisfy Dirichlet or Neumann conditions (3.22) on the boundary. These conditions are only consistent with the above expressions if we modify the classical solutions on the disk as

$$\begin{aligned} \partial X_{cl} &\sim (z-w)^{-1+\theta}\,(z-\bar{w})^{-\theta} & \partial \bar{X}_{cl} &\sim (z-w)^{-\theta}\,(z-\bar{w})^{-1+\theta} \\ \bar{\partial} X_{cl} &\sim (\bar{z}-w)^{-1+\theta}\,(\bar{z}-\bar{w})^{-\theta} & \bar{\partial} \bar{X}_{cl} &\sim (\bar{z}-w)^{-\theta}\,(\bar{z}-\bar{w})^{-1+\theta}\,. \end{aligned} \tag{3.35}$$

The classical solutions on the disk with one twist insertion thus take the form of classical solutions on the sphere in the presence of a twist and an antitwist (3.17). This is what we expect from the doubling trick for twist fields: to summarise, we can obtain classical solutions on the disk in the presence of twist fields $\sigma_{\theta_i}(w_i,\bar{w}_i)$ by using the classical solutions on the sphere for twist fields at w_i and antitwists at $w_i' = \bar{w}_i$. Note that these solutions still need to be normalised by the global monodromy conditions.

3.2.3 Effect on Chan-Paton factors

There is one effect of twist fields on the disk which has no equivalent on the sphere. In particular, a twist field in the bulk of a disk will also affect the Chan-Paton factors. This can be demonstrated most easily by considering a single twist field on the disk. On the sphere such a setup is inconsistent according to the space group selection rule. This is not the case for the disk: employing the doubling trick a twist $\sigma_\theta(w,\bar{w})$ on the disk is equivalent to a twist and an antitwist on the sphere, which is permitted by the selection rule.

To illustrate the needed effect on the boundary, we will use a different definition of the disk than before. Starting with the sphere we identify points under the involution $I_D' : z \to 1/\bar{z}$. The fundamental region is then the unit disk $|z| \leq 1$. Using the doubling trick, a twist field placed at the origin of the disk will behave as a twist at the origin and an antitwist at infinity on the sphere. The insertion of the twist introduces a branch cut into the map from the worldsheet to target space. In this case the cut runs from the origin to infinity crossing the boundary at one point. Now imagine inserting a vertex operator at some point on the disk boundary, say $z_0 = e^{i\phi_0}$. To be specific, let this vertex operator be a D3 scalar:

$$V_\phi^{-1} = \lambda \; e^{-\varphi} \; \Psi(z_0) \; e^{ik\cdot X(z_0)} \; , \tag{3.36}$$

where λ is the Chan-Paton matrix. The only field which is sensitive to the twist is the complex spinor Ψ as the momentum exponential only contains external fields. Note that we return to the same configuration if we rotate the vertex operator once around the disk: $\phi_0 \to \phi_0 + 2\pi$. Hence the amplitude should be invariant under such operations. However, the spinor Ψ will have moved once around the twist insertion and will have picked up a phase: $\Psi \to e^{2\pi i\theta}\Psi$. For this phase to cancel we then need a compensating effect on the CP degrees of freedom. This can be achieved if we associate a twist matrix γ_θ with the point where the branch cut touches the boundary. When moving the vertex operator across the branch cut the Chan-Paton matrix has to be permuted with the twist matrix. The disk amplitude is left invariant if these matrices commute up to an appropriate phase factor:

$$\gamma_\theta \lambda = e^{2\pi i\theta} \; \lambda\gamma_\theta \qquad \text{for a D3 scalar.} \tag{3.37}$$

The fact that the twist matrix does not generally commute with Chan-Paton factors implies, that the ordering of vertex operators on the boundary with respect to the position of the branch cut matters. To see how the presence of a branch point on the boundary modifies the calculation, let us recall one aspect of the procedure for computing disk correlators without twist insertions.

The disk is invariant under reparameterisations of the worldsheet, which take the form of $\mathrm{SL}(2,\mathbb{R})$ transformations. To arrive at a physical result, we have to only consider configurations of vertex operators on the disk, which are not related by $\mathrm{SL}(2,\mathbb{R})$. For vertex operators on the boundary this implies the following. While a $\mathrm{SL}(2,\mathbb{R})$ transformation will shift the positions of these vertex operators along the boundary, it cannot change their ordering. Thus, when calculating disk amplitudes, we have to sum over all cyclically inequivalent orderings of vertex operators on the boundary.

In the presence of a twist field in the bulk of the disk, we have an additional object on the boundary in form of a branch point. While we have to choose a definite locus for the branch cut in specific calculations, the resulting amplitude should be independent of this choice. To ensure this, the procedure for calculating disk amplitudes from the previous paragraph has to be modified as follows [63]: to arrive at an amplitude which is independent of the position of the branch cut, the sum should now be over all cyclically inequivalent orderings of the vertex operators *and* the branch point. In effect, we should treat the branch point as an additional operator on the boundary, which introduces a factor γ_θ into the trace over Chan-Paton factors.

In the following chapter we turn to calculations, where we will employ the results and techniques explained in the previous pages. While we will have a clear phenomenological application in mind, the calculations will also be technically interesting: in particular, we will explain in detail how the sum over different orderings of vertex operators and the branch point can be performed in practice.

Chapter 4

Sequestering of blow-up moduli

In this chapter we will study the dependence of physical Yukawa on blow-up moduli at leading order in string perturbation theory. This is motivated by the observation that moduli generate soft terms to the extent to which physical Yukawa couplings depend on the moduli. In practice, this amounts to evaluating disk correlation functions involving a visible sector Yukawa interaction and an arbitrary number of blow-up moduli. While we calculate the amplitude with one blow-up modulus in detail, we will make general arguments about correlators with more than one blow-up insertion. This chapter is based on paper [7].

4.1 Motivation

Supersymmetry breaking in type IIB flux compactifications has remarkable properties, especially in realisations of the Large Volume Scenario (LVS). At leading order, the LVS exhibits no-scale supersymmetry breaking leading to a vanishing of both tree-level soft terms and loop level anomaly-mediated soft terms [16, 34–36]. To determine the scale and structure of soft terms we thus need to study higher order terms in the supergravity Lagrangian, and here we will focus on corrections to the moduli Kähler potential $\hat{K}$ and the Kähler matter metric $\tilde{K}_{\bar{\alpha}\beta}$.

In terms of the Kähler potential and the Kähler metric, the vanishing of soft terms can be understood as an interdependence of the form

$$\tilde{K}_{\bar{\alpha}\beta} \sim e^{\hat{K}/3} \,, \tag{4.1}$$

which holds at leading order [33]. The significance of expression (4.1) is, that if it continues to hold to higher orders, soft terms will continue to be suppressed. For example, if (4.1) is exact to all orders in in α' and g_s, gravity mediated soft scalar masses and A-terms will vanish exactly (up to non-perturbative corrections to A-terms).

While the form (4.1) was confirmed at leading order by a scaling argument, it is unclear whether it persists to higher orders in α' and g_s. In this chapter we will test this by a direct calculation in toroidal orbifold models and interpret our results for the case of smooth Calabi-Yau manifolds.

Instead of calculating corrections to $\hat{K}$ and $\tilde{K}_{\bar{\alpha}\beta}$ independently, we will study the validity of (4.1) by a different approach. The expression (4.1) was conjectured by analysing the physical Yukawa couplings. For diagonal Kähler metrics we have

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{K_\alpha K_\beta K_\gamma}} \ . \tag{4.2}$$

Supersymmetry is dominantly broken by Kähler moduli, which do not appear in the holomorphic Yukawa couplings. The statement that the Kähler metric takes the form (4.1) is then identical to requiring that the physical Yukawa couplings are independent of the supersymmetry breaking moduli. It is this statement which we will check using string perturbation theory by calculating CFT correlation functions involving a Yukawa coupling and supersymmetry breaking moduli. At zero momentum these correlators directly measure the dependence of the physical Yukawa couplings on the Kähler moduli and thus allow a direct test of (4.1).

This defines the strategy for this chapter. In particular, we will focus on the dependence of physical Yukawa couplings on blow-up moduli τ_s, which correspond to the Kähler moduli of small cycles. Thus, we will calculate correlation functions involving a physical Yukawa coupling and an arbitrary number of blow-up moduli. As the matter insertions arise from open strings, the leading order worldsheet is the disk. Hence, by calculating a disk correlator $\langle \tau_s \tau_s \dots \tau_s \ \psi\psi\phi \rangle$ we will examine the validity of (4.1) to all orders in α', although remaining at leading order in g_s.

Blow-up moduli and twist fields

To set up the calculation, we need to identify the states in the orbifold model which corresponds to the blow-up moduli. A blow-up modulus is a field whose VEV controls the size of a cycle which is shrinkable to zero size. The orbifold corresponds to the limit of a space, where all shrinkable cycles are collapsed into singularities. Thus, in the orbifold the blow-up moduli will correspond to massless closed string states, which are uniquely associated with singularities. The orbifold spectrum offers us such candidates in the form of twisted closed strings, which satisfy

$$X^i(\tau, \sigma + 2\pi) = e^{2\pi i \theta_i} X^i(\tau, \sigma) \ , \tag{4.3}$$

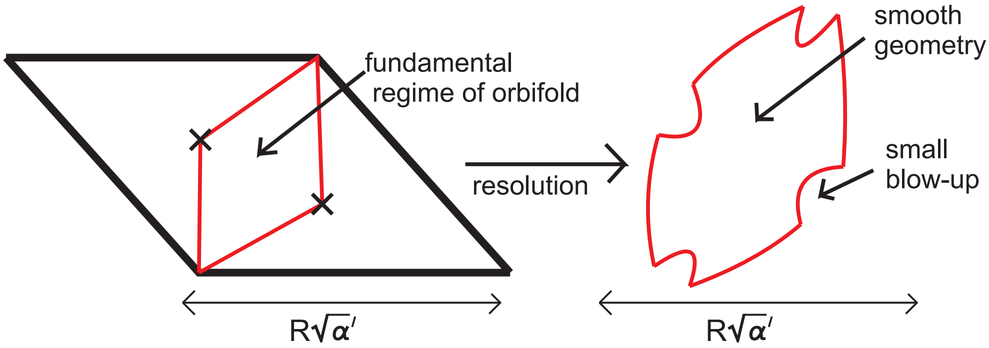

Figure 4.1: Resolving toroidal orbifold singularities gives the type of smooth geometry that appears in the Large Volume Scenario, with a large bulk attached to several small blow-ups.

in the vicinity of an orbifold singularity. This is the behaviour of X in the presence of twist fields, which will thus play a major part in the calculation. Blow-up moduli correspond to fully twisted modes, such that $\theta_1, \theta_2, \theta_3 \neq 0$ while $\theta_1 + \theta_2 + \theta_3 \in \mathbb{Z}$. In terms of cohomology the fully twisted modes increase the Hodge numbers of the compact space by a contribution $h^{1,1}_{tw}$ which is equal to the number of orbifold fixed points. Since the fully twisted modes modify $h^{1,1}$ of the manifold they correspond to deformations of the Kähler class. As a result the scalars corresponding to local blow-up modes can be identified as Kähler moduli. If the singularity is resolved into a smooth Calabi-Yau these modes map onto localised 2/4-cycles.

In contrast, if one $\theta_i = 0$, the mode arises from a partially twisted sector of the orbifold. In the orbifold these modes are tied to the singularity along two of the tori and are free to propagate in the third torus. Under the orbifold resolution to a smooth space, these correspond to metric modes that are visible locally, but are not normalised locally. They are not uniquely associated with one orbifold singularity and, correspondingly, will not be considered in this calculation.

In the end we will want to interpret our results for the case of a string compactification on a smooth Calabi-Yau. Luckily, toroidal orbifolds provide a good approximation for the Swiss-Cheese Calabi-Yaus present in the LVS. They consist of a large bulk (parameterised by the three two-tori) together with many small blow-up cycles, which in the orbifold limit are blown down to singularities. By increasing the radii of the tori it is possible to make the bulk arbitrarily large while retaining the exact worldsheet description. Placing D3-branes at certain singularities allows the incorporation of semi-realistic matter sectors such as considered in [65, 76–80]. If the blow-up cycles are resolved, we obtain the typical smooth Calabi-Yau geometry that appears in the LVS, with a very large bulk and small blow-up cycles. From a CFT perspective, these smooth Calabi-Yaus are obtained by giving a VEV to the marginal directions parameterised by the blow-up fields. Although working on an orbifold we can therefore probe the smooth limit – at least within the radius of convergence of this expansion – by computing correlators involving many blow-up fields. This is illustrated in figure 4.1.

In the models relevant to our discussion the dilaton and complex structure moduli are stabilised in a supersymmetric way and only the Kähler moduli break supersymmetry. In principle, the complex structure moduli and the dilaton will enter our results and appear explicitly in prefactors of the amplitudes we calculate. Here we are only interested in how our results are affected by the moduli breaking supersymmetry. As the dependence of our expressions on the complex structure moduli or the dilaton is not central to answering this question, we will ignore these prefactors in our calculations.

Thus, the main task of this chapter is to determine correlation functions of the form $\langle \tau_s \tau_s \dots \tau_s \; \psi\psi\phi \rangle$ on the disk with blow-up mode insertions τ_s in the bulk and matter fields on the boundary. Our investigation will proceed via the following steps.

1. To begin we will evaluate the amplitude $\langle \tau_s \; \phi_i \bar{\phi}_i \rangle$, where ϕ_i is a chiral scalar. While this does not give information about the moduli-dependence of the physical Yukawa couplings, it will be useful later on. This calculation will allow us to explain techniques involving twist fields, while the result will be used as a consistency check for the following computation.

2. Next we will examine the dependence of Yukawa couplings on a single blow-up mode in detail by determining both the quantum and classical contributions to the disk correlator $\langle \tau_s \; \psi\psi\phi \rangle$. We check our outcome by factorising the amplitude onto the previous result.

3. Amplitudes with more than one twist insertion will not be calculated explicitly, but we will be able to argue that terms of the form $\int \mathrm{d}^4x \; \tau_s \tau_s \dots \tau_s \; \psi\psi\phi$ are absent in the effective action up to contributions non-perturbative in α'.

4.2 Setting up the calculation

4.2.1 Vertex operator

For our model with D3-branes at orbifold singularities the matter states come from the open string sector while (twisted) moduli arise in the closed string sector. The open string vertex operators are untwisted and are inserted on the boundary of the disk. The relevant operators are summarised in section A.2.4.

The bosonic part of the blow-up mode superfield is given by a massless twisted closed string scalar whose vertex operator we introduce in this section. There are two conditions the operator has to satisfy for the consistency of the theory:

1. Unbroken worldsheet supersymmetry requires the worldsheet supercurrents $T_F(z)$ and $\bar{T}_F(\bar{z})$ to be single-valued on transport around the twist insertion point.
2. To keep conformal invariance intact the un-integrated vertex operator has to carry conformal weights $h = \tilde{h} = 1$.

Both conditions are satisfied by the following form for the vertex operator:

$$V_{\rm tw}^{(-1,-1)}(w,\bar{w}) = \; e^{-\varphi(w)} e^{-\tilde{\phi}(\bar{w})} \prod_{i=1}^{3} s_{\theta_i}(w) \tilde{s}_{\theta_i}(\bar{w}) \sigma_\theta(w,\bar{w}) \; e^{ik\cdot X(w,\bar{w})} \; . \tag{4.4}$$

Using the OPEs (3.7) and the bosonisation of the fermionic twist fields (3.8) one finds that worldsheet supersymmetry is indeed unbroken. The conformal weights are given by the sum of the ghost, twist and bosonic weights

$$h = \tilde{h} = \frac{1}{2} + \sum_{i=1}^{3} \left(\frac{1}{2}\theta_i^2 + \frac{1}{2}\theta_i(1-\theta_i) \right) = \frac{1}{2} + \frac{1}{2}\sum_{i=1}^{3} \theta_i \tag{4.5}$$

where we also used $k^2 = 0$ for massless states. The sum over twists obeys $\sum_{i=1}^3 \theta_i = 0 \bmod 1$, and thus a choice $\sum_{i=1}^3 \theta_i = 1$ is consistent with $h = \tilde{h} = 1$.

4.2.2 Kinematics

As string theory only exists in a "first quantised" formulation, we can only examine the scattering of on-shell states. This leads to ambiguities when calculating two- and three-point functions: for the scattering of two or three massless states, momentum conservation implies the vanishing of all kinematic invariants. Ratios of kinematic variables contribute finite parts to the overall amplitude, which cannot be unambiguously captured in this approach. To nevertheless determine the momentum-dependence, one can continue to use the on-shell formulation of string theory, but let the kinematic invariants become non-zero. Allowing k_i^2 to become non-zero would change the conformal weight of the vertex operators and break conformal invariance. For two-point functions this is circumvented by relaxing momentum conservation instead (see e.g. [81–83]):

$$s = 2k_1 \cdot k_2 = \delta \neq 0 \; . \tag{4.6}$$

After the calculation has been performed, momentum conservation is enforced again by setting $\delta \to 0$.

The problem also arises for three-point-functions of massless states. We will examine the scattering between two open strings with momenta p_1 and p_2 and a closed string of momentum q. All momenta lie parallel to the brane. The exponents of open string vertex operators involve $k_1 = 2p_1$ and $k_2 = 2p_2$ (as both holomorphic and anti-holomorphic parts contribute). It is useful to view the closed string vertex operator as a left-moving vertex operator located at w and a right-moving vertex operator at $\bar{w}$, with momenta $k_3 = q$ and $k_4 = q$, in effect turning the amplitude into a 4-point amplitude. We can write the equation of momentum conservation as

$$k_1 + k_2 + 2q = 0 \; . \tag{4.7}$$

We follow [14] to define Mandelstam variables as for a 4-point function:

$$s = k_1 k_2, \qquad t = k_1 k_3, \qquad u = k_1 k_4 \; . \tag{4.8}$$

However these kinematic variables are not independent. Momentum conservation leads to $s + t + u = 0$ and furthermore as $k_3 = k_4 = q$, we also have

$$t = u \; , \qquad s = -2t \; . \tag{4.9}$$

Note that for on-shell zero mass particles kinematics imply $s = t = u = 0$. Again, all kinematic invariants vanish once momentum conservation is enforced, but it is less clear how to unambiguously depart from this situation. We will deal with this situation as follows: we allow momenta to go off-shell despite the effect on the conformal weights of the vertex operators. In the end we justify the procedure by comparing our results to a factorisation limit of a four-point function.

The relevant four-point amplitude will involve three open strings (momenta p_1, p_2, p_3) and one closed string (momentum q). The open string vertex operators will carry momenta $k_i = 2p_i$. Again, we will only consider massless string states such that $k_i^2 = q^2 = 0$. Momentum conservation now requires that

$$k_1 + k_2 + k_3 + 2q = 0 \; . \tag{4.10}$$

Mandelstam variables are given by

$$s = k_1 k_2, \qquad t = k_1 k_3, \qquad u = 2k_1 q \; . \tag{4.11}$$

The ambiguity in the momentum-dependence is automatically resolved once we analyse the scattering of four states. We find two non-zero kinematic variables which are needed to fully describe the amplitude. The invariants are conveniently parameterised by the three Mandelstam variables together with the constraint $s + t + u = \sum_{i=1}^{4} \, m_i^2$.

4.3 Fayet-Iliopoulos term

In this section we will calculate a correlation function involving one blow-up mode and two visible D3 scalars. The result itself sheds light onto the D-term potential $\frac{g^2}{2}\left(|\phi|^2+\xi\right)^2$, but we will use it as a consistency check for a later calculation. To this end we calculate the disk amplitude $\langle \tau_s\, \phi^i \bar{\phi}^i \rangle$ with the two chiral scalars inserted at the boundary and the twisted closed string in the bulk.

The relevant vertex operators for a disk correlation function have to contribute a total ghost charge of (-2) on the disk worldsheet. To achieve this we take the twisted closed string in the $(-1,-1)$ picture and choose vertex operators with zero ghost charge for the matter fields:

$$V^0_{C^i}(z_1) = \lambda\ \left[\partial X^i + i(k_1\cdot\psi)\Psi^i\right] e^{ik_1\cdot X}(z_1)\ , \tag{4.12}$$

$$V^0_{\bar{C}^i}(z_2) = \lambda^\dagger \left[\partial \bar{X}^i + i(k_2\cdot\psi)\bar{\Psi}^i\right] e^{ik_2\cdot X}(z_2)\ , \tag{4.13}$$

$$\begin{aligned} V^{-1,-1}_{\rm tw}(w,\bar{w}) = e^{-\varphi(w)} e^{-\tilde{\varphi}(\bar{w})} \prod_{j=1}^{3} \sigma_{\theta^j}(w,\bar{w}) \\ e^{iq_3\cdot H(w)} e^{iq_4\cdot \tilde{H}(\bar{w})}\ e^{ik_3\cdot X(w)} e^{ik_4\cdot X(\bar{w})}\ . \end{aligned} \tag{4.14}$$

The H-charges are $q_3 = (\theta_1,\theta_2,\theta_3)$ and $q_4 = (-\theta_1,-\theta_2,-\theta_3)$. Chan-Paton factors are represented by λ and $\lambda^\dagger$. Our conventions are summarised in A.2.1, including the definitions of the complex fields X^i and Ψ^i appearing in the above expressions.

In the following we ignore the Chan-Paton factors λ at first and focus on the CFT calculation. Given the form of the vertex operators we can identify two individual contributions to the amplitude: we define $\mathcal{A}_1$ as the result of contracting the ∂X terms of both matter field vertex operators, while contracting the terms involving Ψ will lead to the expression $\mathcal{A}_2$. Cross-correlations will vanish due to uncancelled H-charge.

We begin our analysis by studying the amplitude $\mathcal{A}_1$ which consists of the following disk correlation functions:

$$\begin{aligned} \mathcal{A}_1 = \langle e^{-\varphi(w)} e^{-\tilde{\varphi}(\bar{w})}\rangle \langle e^{iq_3\cdot H(w)} e^{iq_4\cdot \tilde{H}(\bar{w})}\rangle \langle e^{ik_1\cdot X(z_1)} e^{ik_2\cdot X(z_2)} e^{ik_3\cdot X(w)} e^{ik_4\cdot X(\bar{w})}\rangle \\ \langle \partial X^i(z_1)\partial\bar{X}^i(z_2) \prod_{j=1}^{3} \sigma_{\theta^j}(w,\bar{w})\rangle\ . \end{aligned} \tag{4.15}$$

The correlators appearing in the first row can be evaluated using basic techniques. The last factor is the correlation function between the bosonic coordinate and twist fields which we evaluated before in (3.33). Here we will need:

$$\langle \partial X^i(z_1)\partial\bar{X}^i(z_2)\prod_{j=1}^{3}\sigma_{\theta_j}(w,\bar{w})\rangle_{D_2} = -\,(z_1-z_2)^{-2}\,(w-\bar{w})^{-\sum_{j=1}^{3}\theta_j(1-\theta_j)}$$
$$(z_1-w)^{-1+\theta_i}\,(z_1-\bar{w})^{-\theta_i}\,(z_2-w)^{-\theta_i}\,(z_2-\bar{w})^{-1+\theta_i}$$
$$[(z_1-w)(z_2-\bar{w})-\theta_i(z_1-z_2)(w-\bar{w})]\,. \qquad (4.16)$$

As the above correlator consists of two terms, this part of the amplitude splits into two contributions: we define $\mathcal{A}_{1b}$ as the part that is directly proportional to θ_i whereas $\mathcal{A}_{1a}$ is the remaining piece.

There is actually a subtlety in (4.16) that however does not affect the actual result. The vertex operators for fields with Dirichlet boundary conditions are not ∂X^i but instead $\partial_n X^i$. Writing $z = x + iy$ the derivative normal to the boundary is $\partial_n = i(\partial - \bar{\partial})$. So the correlator we actually require is not (4.16) but $\langle(\partial-\bar{\partial})X^i(\partial-\bar{\partial})\bar{X}^i\prod_j \sigma_{\theta_j}\rangle$. In fact, all four subcorrelators that enter this expression give identical results, and we recover (4.16). To see this, note that when $z = \bar{z}$, the local OPEs of $\partial X(z)\sigma_\theta(w,\bar{w})$ and $\bar{\partial}X(\bar{z})\sigma_\theta(w,\bar{w})$ are identical (using (3.25) and (3.27)). This ensures the singular behaviour is identical for all terms. Furthermore, the relative minus sign between $\langle\partial X\partial\bar{X}\rangle$ and $\langle\partial X\bar{\partial}\bar{X}\rangle$ is cancelled as $\partial X = -\bar{\partial}X$ on the boundary in the Dirichlet case.

We now turn to the amplitude $\mathcal{A}_2$ arising from the contraction of the terms involving spinors in the matter vertex operators. In terms of the individual worldsheet correlators we obtain:

$$\mathcal{A}_2 = -\,\langle e^{-\varphi(w)}e^{-\tilde{\varphi}(\bar{w})}\rangle\langle(k_1\psi)(z_1)(k_2\psi)(z_2)\rangle\langle e^{ik_1\cdot X(z_1)}e^{ik_2\cdot X(z_2)}e^{ik_3\cdot X(w)}e^{ik_4\cdot X(\bar{w})}\rangle$$
$$\langle e^{iq_1\cdot H(z_1)}e^{iq_2\cdot H(z_2)}e^{iq_3\cdot H(w)}e^{iq_4\cdot\tilde{H}(\bar{w})}\rangle\langle\prod_{j=1}^{3}\sigma_{\theta^j}(w,\bar{w})\rangle, \qquad (4.17)$$

where we bosonised the internal spinors $\Psi^i(z_1)$ and $\bar{\Psi}^i(z_2)$ with H-charges $q_1 = (\delta_{1i},\delta_{2i},\delta_{3i})$ and $q_2 = (-\delta_{1i},-\delta_{2i},-\delta_{3i})$. The bosonic twists only contribute their self-correlator (3.30) and the remaining pieces are elementary. One finds that $\mathcal{A}_2$ is proportional to the same worldsheet integral as $\mathcal{A}_{1a}$.

Next we use the SL$(2,\mathbb{R})$ invariance to fix three real parameters amongst the positions of the vertex operators. It is at this point where the ordering of vertex operators becomes important. The presence of the twist fields results in the presence of two branch points at w and $\bar{w}$ with a branch cut running between them. This branch cut has to run through the boundary of the disk, which we choose to do so at infinity. As explained in section 3.2.3 the position of the branch cut on the boundary has to be treated like a vertex operator when summing over all orderings of vertex operators on the boundary.

On the disk, configurations related by $\mathrm{SL}(2,\mathbb{R})$ give equivalent results and thus we only have to sum over orderings which are unrelated by this mapping. As $\mathrm{SL}(2,\mathbb{R})$ transformations amount to cyclic reorderings on the boundary there are only two different configurations for three boundary objects.

We can take care of the two configurations by fixing the worldsheet coordinates as follows [14, 58]:

$$z_1 = x \quad , \quad z_2 = -x, \quad , \quad w = i \quad , \quad \bar{w} = -i. \tag{4.18}$$

Then $x > 0$ corresponds to the first ordering and $x < 0$ corresponds to the second ordering. In both cases we need to include the c-ghost contribution $\langle c(z_2)c(w)\tilde{c}(\bar{w})\rangle = (z_2 - w)(z_2 - \bar{w})(w - \bar{w}) = 2i(x^2 + 1)$.

First way of ordering

With the first way of ordering the resulting expressions are:

$$\mathcal{A}_{1a} + \mathcal{A}_2 = (1+2t)\,\frac{i}{4}\,2^{-1-4t} \quad e^{2\pi i\theta_i}\int_0^\infty \mathrm{d}x \frac{(x+i)^{2t+1-2\theta_i}(x-i)^{2t+1+2\theta_i}}{x^{2t+2}}, \tag{4.19}$$

$$\mathcal{A}_{1b} = \theta_i\,2^{-1-4t} \quad e^{2\pi i\theta_i}\int_0^\infty \mathrm{d}x \frac{(x+i)^{2t+1-2\theta_i}(x-i)^{2t-1+2\theta_i}}{x^{2t+1}}\,. \tag{4.20}$$

These can be evaluated using [14]

$$\begin{aligned} I(\delta,\alpha) &= \int_0^\infty \mathrm{d}x\; x^{\delta-1}(x-i)^{\alpha-\delta}(x+i)^{-\alpha-\delta} \\ &= \sqrt{\pi}2^{-\delta}e^{-\frac{1}{2}\pi i\alpha}\frac{\Gamma\left(\frac{\delta}{2}\right)\Gamma\left(\frac{1}{2}+\frac{\delta}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{\delta}{2}-\frac{\alpha}{2}\right)\Gamma\left(\frac{1}{2}+\frac{\delta}{2}+\frac{\alpha}{2}\right)}\,, \end{aligned} \tag{4.21}$$

in terms of which we obtain

$$\mathcal{A}_{z_1=x} = e^{2\pi i\theta_i}\,2^{-1-4t}\left((1+2t)\,\frac{i}{4}\,I(-1-2t,2\theta_i) + \theta_i\,I(-2t,2\theta_i-1)\right) \tag{4.22}$$

$$= i\pi e^{\pi i\theta_i}\Gamma\left(-2t\right)\left[\frac{\Gamma(1-\theta_i-t)+\theta_i\,\Gamma(-\theta^i-t)}{\Gamma(-\theta_i-t)\Gamma(\theta_i-t)\Gamma(1-\theta_i-t)}\right]\,. \tag{4.23}$$

We expand in powers of momentum to obtain

$$\mathcal{A}_{z_1=x} \propto \frac{i}{2}e^{\pi i\theta_i}\sin(\pi\theta_i)\left(1 + t\left[2\gamma_E + \psi(\theta_i) + \psi(1-\theta_i)\right] + \mathcal{O}(t^2)\right), \tag{4.24}$$

where $\psi(z) = \frac{\mathrm{d}}{\mathrm{d}x}\Gamma(z)$. This concludes the CFT calculation of this part of the amplitude.

Second way of ordering

We now repeat this analysis for the second way of ordering the vertex operators with respect to the branch point. We fix worldsheet coordinates as in (4.18), but now we integrate over $x < 0$. Performing the calculation with this choice we find:

$$\mathcal{A}_{z_1=-x} = e^{-2\pi i\theta_i}\, 2^{-1-4t} \left((1+2t)\, \frac{i}{4}\, I(-1-2t, -2\theta_i) - \theta_i\, I(-2t, -2\theta_i + 1) \right) \quad (4.25)$$

$$= i\pi e^{-\pi i\theta_i} \Gamma\left(-2t\right) \left[\frac{\Gamma(1-\theta_i - t) + \theta_i\, \Gamma(-\theta_i - t)}{\Gamma(-\theta_i - t)\Gamma(\theta_i - t)\Gamma(1-\theta_i - t)} \right] . \quad (4.26)$$

Having arrived at this expression the only difference to the result for the other ordering is in the overall phase factor, whereas the gamma functions are identical. For small momenta we thus find:

$$\mathcal{A}_{z_1=-x} = \frac{i}{2} e^{-\pi i\theta_i} \sin(\pi\theta_i) \left(1 + t\left[2\gamma_E + \psi(\theta_i) + \psi(1-\theta_i)\right] + \mathcal{O}(t^2)\right) . \quad (4.27)$$

Combination of results

When combing the results of the two previous sections we have to account for the fact that the vertex operators on the boundary are ordered differently with respect to the branch point. Different orderings contribute different traces over Chan-Paton factors which we reinstate at this point. The first way of fixing the vertex operators on the boundary provides a trace $\mathrm{Tr}(\lambda^\dagger\lambda\gamma_\theta)$ whereas the second analysis contains the factor $\mathrm{Tr}(\lambda\lambda^\dagger\gamma_\theta)$. Here γ_θ accounts for the branch point on the boundary as explained in section 3.2.3. We can write both our results using one common trace factor by noting that

$$\mathrm{Tr}(\lambda\lambda^\dagger\gamma_\theta) = e^{2\pi i\theta_i}\mathrm{Tr}(\lambda^\dagger\lambda\gamma_\theta) . \quad (4.28)$$

Here we used the commutator between the Chan-Paton factor for a complex scalar (3.36) and the twist matrix γ_θ. This allows us to combine the two partial expressions to arrive at the final result:

$$\mathcal{A}_{\mathrm{full}} = \mathrm{Tr}(\lambda^\dagger\lambda\gamma_\theta)\; i e^{\pi i\theta_i} \sin(\pi\theta_i) \left(1 + t\left[2\gamma_E + \psi(\theta_i) + \psi(1-\theta_i)\right] + \mathcal{O}(t^2)\right) . \quad (4.29)$$

As a side comment we note that the result is identical to the outcome of a calculation of the dependence of the twisted matter-metric on untwisted Kähler moduli in [14].

Although a warm-up, this calculation has required the use of the full set of calculational tools necessary for working with twist fields on the disk. Here, summing over the two orderings of the vertex operators with respect to the branch cut does not seem to be too important, as both configurations give the same result up to a phase. We will see that this changes if we consider three vertex operators on the boundary. There it will be crucial to sum over all orderings to arrive at the correct result.

The finite term in the expression (4.29) is the tree-level contribution to the FI-term ξ of the D-term potential $\frac{g^2}{2}(|\phi|^2 + \xi)^2$. To complete the result we still need to supplement the quantum correlator with the classical part. The classical action for one twist insertion is independent of the matter insertions and will be identical for the calculation that follows. We will discuss the relevant classical solution in section 4.4.2.

The term in (4.29) with a momentum prefactor is related to corrections to the matter metric and potentially interesting for our phenomenological applications.[1] However, we are mainly interested in correlations between corrections to the Kähler metric and the moduli Kähler potential, which can be determined more directly by studying the moduli dependence of Yukawa couplings. Correspondingly, we will not analyse the above result any further, but proceed with the calculation of Yukawa couplings in the presence of a twist field. Nevertheless, the expression (4.29) will be useful in what follows: we will employ the above answer as a consistency-check on the following calculation.

4.4 Yukawas with one twist insertion

4.4.1 Quantum correlator

In this section we begin our examination of the dependence of physical Yukawa couplings on blow-up moduli. Here, we will calculate the quantum contribution to the disk correlation function $\langle \tau_s \ \psi\psi\phi \rangle$, where τ_s is a blow-up mode and $\psi\psi\phi$ are visible sector fields from D3-branes at an orbifold singularity. To start we collect the relevant vertex operators in their canonical picture:

$$V_{\psi_1}^{-\frac{1}{2}}(z_1) = \lambda_1 \ e^{-\frac{1}{2}\varphi(z_1)} \ S^{\pm}(z_1) \ e^{iq_1 \cdot H}(z_1) \ e^{ik_1 \cdot X}(z_1), \tag{4.30}$$

$$V_{\psi_2}^{-\frac{1}{2}}(z_2) = \lambda_2 \ e^{-\frac{1}{2}\varphi(z_2)} \ S^{\mp}(z_2) \ e^{iq_2 \cdot H}(z_2) \ e^{ik_2 \cdot X}(z_2), \tag{4.31}$$

$$V_{\phi}^{-1}(z_3) = \lambda_3 \ e^{-\varphi(z_3)} \ e^{iq_3 \cdot H}(z_3) \ e^{ik_3 \cdot X}(z_3), \tag{4.32}$$

$$\begin{aligned} V_{\text{tw}}^{-1,-1}(w, \bar{w}) = \gamma_\theta \ e^{-\varphi(w)} \ e^{-\tilde{\varphi}(\bar{w})} \ &\prod_{j=1}^{3} \sigma_{\theta^j}(w, \bar{w}) \\ &\times e^{iq_4 \cdot H(w)} \ e^{iq_5 \cdot \tilde{H}(\bar{w})} \ e^{ik_4 \cdot X(w, \bar{w})}, \end{aligned} \tag{4.33}$$

where λ_i denote Chan-Paton factors. To complete the above information we list the H-charges of the above vertex operators. To this end we note that the external spinors $S^{\pm}$ can also be bosonised as $e^{\pm \frac{i}{2}(H^1 + H^2)}$ and we split the twist vertex operator into its

[1] Note that the above expression was obtained from a three-point function and the result is not conclusive as there are ambiguities in the off-shell extension of three-point amplitudes.

holomorphic and antiholomorphic parts:

$$V_{\psi_1}^{-\frac{1}{2}}(z_1) \sim |+\ +\rangle \otimes |\ \ +\ \ -\ \ -\rangle\ , \tag{4.34}$$

$$V_{\psi_2}^{-\frac{1}{2}}(z_2) \sim |-\ -\rangle \otimes |\ \ -\ \ +\ \ -\rangle\ , \tag{4.35}$$

$$V_{\phi}^{-1}(z_3) \sim |\ 0\ \ 0\ \rangle \otimes |\ \ 0\ \ \ 0\ \ (++)\rangle\ , \tag{4.36}$$

$$V_{\rm tw}^{-1}(w) \sim |\ 0\ \ 0\ \rangle \otimes |\ \ \theta_1,\ \ \theta_2,\ \ \theta_3\rangle\ , \tag{4.37}$$

$$V_{\rm tw}^{-1}(\bar{w}) \sim |\ 0\ \ 0\ \rangle \otimes |-\theta_1, -\theta_2, -\theta_3\rangle\ , \tag{4.38}$$

where $\pm$ represents an H-charge of $\pm 1/2$. The internal H-charges of the matter operators are consistent with a Yukawa coupling arising from a term $C^1C^2C^3$ in the superpotential. Then the external spinors $|s_0\ s_1\rangle$ are restricted by the GSO projection. Lorentz invariance allows us to impose without loss of generality $S^1 = |++\rangle$ and $S^2 = |--\rangle$. This restriction is equivalent to boosting to a frame with $k_1 = (k, k, 0, 0)$ and $k_2 = (k, -k, 0, 0)$. To see this, note that the physical state condition $(k \cdot \Gamma)|\psi\rangle = 0$ gives

$$(k \cdot \Gamma)|\psi\rangle = (k_0\Gamma^0 \pm k_1\Gamma^1)|\psi\rangle = -k_1\Gamma^0(\Gamma^0\Gamma^1 \mp 1)|\psi\rangle = -2k_1\Gamma^0(s_0 \mp 1/2)|\psi\rangle = 0,$$

and so $s_0 = \pm 1/2$ for $k_1 = \pm k_0$. The GSO conditions then imply $S^1 = |++\rangle$ and $S^2 = |--\rangle$. We also note that the physical state conditions imply $k_1^{1+} = k_1^{2+} = k_1^{2-} = 0$ and $k_2^{1-} = k_2^{2-} = k_2^{2+} = 0$, where we have introduced complex momenta $k^{1\pm} = \frac{1}{\sqrt{2}}(\pm k^0 + k^1)$ and $k^{2\pm} = \frac{1}{\sqrt{2}}(k^2 \pm ik^3)$.

The vertex operators (4.30) to (4.33) do not possess the correct overall ghost charge (-2) for a disk correlation function. To obtain the correct ghost charge we choose to picture-change two vertex operators on the boundary of the disk. An alternative would be to picture-change the bulk twist operator. However this would be technically more involved and require correlators involving excited twist fields. We thus picture-change the bosonic vertex operator $V_\phi(z_3)$ and the second fermionic vertex operator $V_{\psi_2}(z_2)$, modifying their ghost charges as $-1 \to 0$ and $-\frac{1}{2} \to \frac{1}{2}$ respectively.

Picture-changing of a vertex operator on the boundary is performed by evaluating the limit $\lim_{z\to w} e^{\phi(z)}T_F(z)V^{(c)}(w)$. The picture-changing operator involves the world-sheet supercurrent $T_F(z)$ which takes the following form on the boundary of the disk:

$$T_F(z) = \sum_{\mu=0}^{3} \partial X_\mu \psi^\mu(z) + \sum_{i=1}^{3} \left[\partial\bar{X}^i\Psi^i(z) + \partial X^i\bar{\Psi}^i(z)\right] . \tag{4.39}$$

The supercurrent consists of two terms: one consists of external fields only, while the other involves fields in the internal directions. Both parts contribute in the process of picture-changing and we will analyse the contributions separately.

Internal picture-changing

Here we consider the result of the picture-changing operator acting in the internal directions. Using the operator product expansions as detailed in (A.2.3) we find the following result:

$$\begin{aligned} V^{+\frac{1}{2}}_{\psi_2,int}(z_2) &= e^{+\frac{1}{2}\varphi}\ S^{\mp}\ e^{iq_2'\cdot H}\ \partial\bar{X}^3\ e^{ik_2\cdot X}(z_2), \\ V^{0}_{\phi,int}(z_3) &= \partial X^3\ e^{ik_3\cdot X}(z_3), \end{aligned} \tag{4.40}$$

where the internal H-charges are now $q_2' = (-\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and $q_3' = (0,0,0)$.

The full amplitude can now be written as a product of correlators over superconformal ghosts, external spinors, internal spinors, momentum exponentials and bosonic twists:

$$\begin{aligned} \mathcal{A}^{\text{int}} =& \langle e^{-\frac{1}{2}\varphi(z_1)} e^{+\frac{1}{2}\varphi(z_2)} e^{-\varphi(w)} e^{-\tilde{\varphi}(\bar{w})} \rangle \\ & \langle e^{+\frac{i}{2}(H^1+H^2)(z_1)} e^{-\frac{i}{2}(H^1+H^2)(z_2)} \rangle \\ & \langle e^{iq_1\cdot H(z_1)} e^{iq_2'\cdot H(z_2)} e^{iq_4\cdot H(w)} e^{iq_5\cdot H(\bar{w})} \rangle \\ & \langle e^{ik_1\cdot X(z_1)} e^{ik_2\cdot X(z_2)} e^{ik_3\cdot X(z_3)} e^{ik_4\cdot X(w)} e^{ik_4\cdot X(\bar{w})} \rangle \\ & \langle \partial\bar{X}^3(z_2) \partial X^3(z_3) \prod_{j=1}^{3} \sigma_{\theta^j}(w,\bar{w}) \rangle\ . \end{aligned} \tag{4.41}$$

The correlator involving twist fields is given by (4.16) while the remaining correlation functions can be calculated using standard CFT techniques on the disk. Reinstating the integration over vertex operator positions the result is:

$$\begin{aligned} \int \mathrm{d}z_1\ \mathrm{d}z_2\ \mathrm{d}z_3\ \mathrm{d}^2w\ & (z_1-z_2)^{-1+s}(z_2-z_3)^{-2+u}(w-\bar{w})^{-2}(z_1-w)^{-1+\theta^1+u/2}(z_1-\bar{w})^{-\theta^1+u/2} \\ & \times (z_2-w)^{\theta^2+t/2}(z_2-\bar{w})^{-\theta^2+t/2}(z_3-w)^{-1+\theta^3+s/2}(z_3-\bar{w})^{-\theta^3+s/2} \\ & \times \left[(z_3-w)(z_2-\bar{w}) - \theta_3(z_3-z_2)(w-\bar{w})\right]. \end{aligned} \tag{4.42}$$

The equation (4.42) is invariant under SL$(2,R)$ transformations which can be checked explicitly. The modding out of this symmetry is the subject of the next section after which we will return to this particular calculation.

Fixing SL$(2,\mathbb{R})$

Initially, the integral over worldsheet positions is of the form

$$\int_{\partial\mathcal{D}_2} \mathrm{d}z_1\ \mathrm{d}z_2\ \mathrm{d}z_3\ \int_{\mathcal{D}_2} \mathrm{d}^2w\ \ldots\ . \tag{4.43}$$

However, using the SL$(2,R)$ symmetry of the disk we can fix three real parameters in the worldsheet positions which thus drop out of the integration. This opens several choices

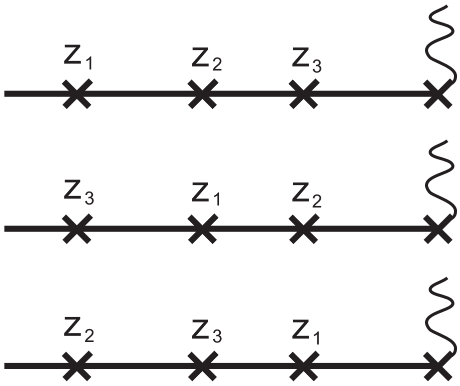

Figure 4.2: The three possible (and distinct) orderings of the vertex operator in relation to the location of the branch point on the boundary of the disk.

for us. For one, we could fix all three vertex operators on the boundary and integrate the twist position over the whole disk. We reject this possibility due to the technical difficulties it entails: it has to be carefully examined what happens if the twist operator collides with another vertex operator on the boundary. These issues can be avoided by fixing the twist positions away from the boundary:

$$w = i \ , \qquad \bar{w} = -i \ . \tag{4.44}$$

Then we are left with one more freedom to fix one vertex operator on the boundary.

Another consequence of the initial $\mathrm{SL}(2,\mathbb{R})$ invariance is, that certain configurations of vertex operators on the disk are equivalent to one another: in particular, as a $\mathrm{SL}(2,\mathbb{R})$ transformations amounts to cyclic reorderings of operators on the boundary, we need to sum over all cyclically inequivalent orderings of boundary operators. We thus need to know how many different orderings of vertex operators contribute to the amplitude.

It is at this point where the twist field makes another appearance. While we only have three vertex operators on the boundary, the position where the branch cut touches the boundary counts as an additional boundary object. In equation (4.42) we have always chosen this branch point to be at infinity (implicitly, by using the conventional definition of z^θ for $0 < \theta < 1$). The amplitude changes if a vertex operator is moved past the branch cut and hence different orderings will give different contributions. The four objects on the boundary thus allow for six different orderings, each endowed with its own trace over Chan-Paton factors:

$$\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\gamma_\theta) \quad \mathrm{Tr}(\lambda_2\lambda_3\lambda_1\gamma_\theta) \quad \mathrm{Tr}(\lambda_3\lambda_1\lambda_2\gamma_\theta) \tag{4.45}$$

$$\mathrm{Tr}(\lambda_1\lambda_3\lambda_2\gamma_\theta) \quad \mathrm{Tr}(\lambda_2\lambda_1\lambda_3\gamma_\theta) \quad \mathrm{Tr}(\lambda_3\lambda_2\lambda_1\gamma_\theta) \ . \tag{4.46}$$

As explained in section 3.2.3, the twist matrix commutes with the Chan-Paton factors

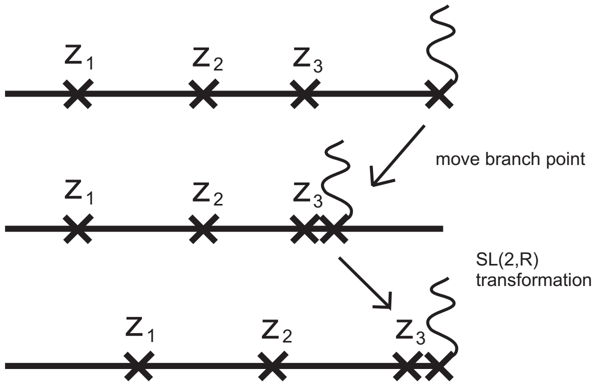

Figure 4.3: The procedure of fixing the $\mathrm{SL}(2,\mathbb{R})$ degeneracy while keeping the branch point at ∞.

up to a phase and thus the upper row is proportional to $\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\gamma_\theta)$ while the lower is proportional to $\mathrm{Tr}(\lambda_1\lambda_3\lambda_2\gamma_\theta)$. For a particular Yukawa coupling gauge invariance implies that only one of the rows contributes while the other traces vanish exactly. To be specific, we choose the upper row and learn that we need to consider three different orderings as shown in 4.2.

Now we are in a position to fix the remaining vertex operator position on the boundary. The mathematical expressions will be most tractable if we let one of the vertex operators go to infinity. To account for the three different orderings we have to set $z_1,\ z_2,\ z_3 \to \infty$ individually. To summarise, we fix $\mathrm{SL}(2,\mathbb{R})$ invariance by assigning

$$w = i \qquad \bar{w} = -i \qquad z_j \to \infty \quad \text{for} \quad j = 1,2,3\ , \tag{4.47}$$

where we pick up a factor of $\langle c(z_j)c(w)c(\bar{w})\rangle = 2i\ z_\infty^2$.

A little care is needed when fixing the last vertex operator. Having fixed $(w,\bar{w}) \to (i,-i)$ the residual transformation is of the form

$$z \to \frac{az+b}{-bz+a}, \qquad a,b \in \mathbb{R}, \quad a^2+b^2=1.$$

However in general such a transformation also moves the branch point location away from ∞, whereas in our fixed expressions we wish to keep the location of the branch point at infinity.

Amplitudes are continuous except when a vertex operator moves through a branch point location. Furthermore, the overall amplitude (summing over all possible orderings) must be insensitive to the location of the branch point. For each operator ordering the $\mathrm{SL}(2,\mathbb{R})$ degeneracy is then dealt with as follows.

1. For the ordering (z_1, z_2, z_3) the amplitude does not change unless the branch point

is moved through a vertex operator. We can use this freedom to bring the branch point from ∞ so that it resides next to z_3.

2. We can now use a $\mathrm{SL}(2,\mathbb{R})$ transformation to take $z_3 \to \infty$. As the branch point is next to z_3, the branch point is also moved to ∞, and so our mathematical expressions with the branch point at ∞ remain valid.

3. Repeat for the orderings (z_3, z_1, z_2) and (z_2, z_3, z_1). This is illustrated in figure 4.3.

As a direct consequence of our way of fixing worldsheet positions we will be able to write the results for our correlation functions in terms of the integral

$$I(a,b,c,d,e,f) = (2i)^f \int_{-\infty}^{\infty} \mathrm{d}x_1 \int_{x_1}^{\infty} \mathrm{d}x_2 (x_1-i)^a (x_1+i)^b (x_2-i)^c (x_2+i)^d (x_2-x_1)^e \ . \tag{4.48}$$

It was evaluated in [59] as

$$\begin{aligned} I(a,b,c,d,e,f) = & -(2i)^{3+a+b+c+d+e+f}\Gamma(-2-a-b-c-d-e)\times \\ & \Big[(-i)^{2(a+c)}\frac{\sin\pi(b+d+e)\Gamma(1+e)\Gamma(2+b+d+e)\Gamma(-1-d-e)}{\Gamma(-d)\Gamma(-a-c)}\times \\ & {}_3F_2(-c,1+e,2+b+d+e;2+d+e,-a-c;1) \\ & +(-i)^{2(a+c+d+e)}\frac{\sin(\pi b)\Gamma(-1-c-d-e)\Gamma(1+b)\Gamma(1+d+e)}{\Gamma(-c)\Gamma(-1-a-c-d-e)}\times \\ & {}_3F_2(-d,-1-c-d-e,1+b;-d-e,-1-a-c-d-e;1)\Big] \ , \end{aligned} \tag{4.49}$$

and we confirmed this result in [7].

We now continue with the calculation for the case of internal picture-changing. Fixing $(w,\bar{w}) \to (i,-i)$ and $z_3 \to \infty$ in (4.42) and including the c-ghost correlation function $\langle c(z_3)c(w)c(\bar{w})\rangle = (2i)\ (z_\infty)^2$, we obtain

$$\begin{aligned} \frac{-1}{2i}\int_{-\infty}^{+\infty}\mathrm{d}z_1 \int_{z_1}^{+\infty}\mathrm{d}z_2\ (z_1-i)^{-1+\theta_1+u/2}(z_1+i)^{-\theta_1+u/2}(z_2-i)^{\theta_2+t/2} \\ \times (z_2+i)^{-\theta_2+t/2}\ \left[(z_2+i)-2i\theta_3\right](z_2-z_1)^{-1+s} \ . \end{aligned} \tag{4.50}$$

We can evaluate this using the integral $I(a,b,c,d,e,f)$ given by (4.49), and in a similar vein we obtain the results for $z_2 \to \infty$ and $z_1 \to \infty$. We find:

$$\begin{aligned}
z_3 \to \infty :\ & -I\Big(-1+\theta_1+u/2, -\theta_1+u/2, \theta_2+t/2, 1-\theta_2+t/2, -1+s, -1\Big) && (4.51)\\
& + 2i\theta_3\ I\Big(-1+\theta_1+u/2, -\theta_1+u/2, \theta_2+t/2, -\theta_2+t/2, -1+s, -1\Big).\\
z_2 \to \infty :\ & -I\Big(\theta_3+s/2, -\theta_3+s/2, -1+\theta_1+u/2, -\theta_1+u/2, t, -1\Big) && (4.52)\\
& - 2i\theta_3\ I\Big(-1+\theta_3+s/2, -\theta_3+s/2, -1+\theta_1+u/2, -\theta_1+u/2, t, -1\Big).\\
z_1 \to \infty :\ & I\Big(\theta_2+t/2, 1-\theta_2+t/2, \theta_3+s/2, -\theta_3+s/2, -2+u, -1\Big) && (4.53)\\
& - 2i\theta_3\ I\Big(\theta_2+t/2, -\theta_2+t/2, -1+\theta_3+s/2, -\theta_3+s/2, -1+u, -1\Big).
\end{aligned}$$

External picture-changing

When picture-changing in the external directions we find two consistent possibilities. The picture-changed vertex operators can be characterised by their H-charges:

Case 1:

$$V_{\psi_2}^{+\frac{1}{2}}(z_2) \sim |(---)\ -\rangle \otimes |\ -\quad +\quad -\rangle\ , \qquad (4.54)$$

$$V_{\phi}^{0}(z_3) \sim |\ (++)\quad 0\,\rangle \otimes |\ 0\quad 0\quad (++)\,\rangle\ , \qquad (4.55)$$

Case 2:

$$V_{\psi_2}^{+\frac{1}{2}}(z_2) \sim |\ +\quad -\rangle \otimes |\ -\quad +\quad -\rangle\ , \qquad (4.56)$$

$$V_{\phi}^{0}(z_3) \sim |(--)\ \ 0\,\rangle \otimes |\ 0\quad 0\quad (++)\,\rangle\ , \qquad (4.57)$$

Both calculations are similar to the case of internal picture-changing and are displayed in appendix B.1. Case 2 is a bit more involved as one needs to consider higher terms in the OPE between picture-changing operator and vertex operator. The results are again conveniently written in terms of the integral (4.48). Instead of recording the individual results at this point, we will display the complete expressions in the next section.

Combination of results

The complete amplitude is a sum of all contributions arising from the various actions of the picture-changing operator. We thus have three different expressions for the three separate ways of fixing vertex operators, $z_3 \to \infty$, $z_2 \to \infty$ and $z_1 \to \infty$:

$$\begin{aligned}
\mathcal{A}_{z_3\to\infty} : &(u+t) \times I\Big(-1+\theta_1+u/2, -\theta_1+u/2, \theta_2+t/2, 1-\theta_2+t/2, -1+s, -1\Big) \\
&- \frac{t}{2} \times I\Big(-1+\theta_1+u/2, -\theta_1+u/2, -1+\theta_2+t/2, 1-\theta_2+t/2, s, -1\Big) \\
&- \frac{t}{2} \times I\Big(-1+\theta_1+u/2, -\theta_1+u/2, \theta_2+t/2, -\theta_2+t/2, s, -1\Big) \\
&- I\Big(-1+\theta_1+u/2, -\theta_1+u/2, \theta_2+t/2, 1-\theta_2+t/2, -1+s, -1\Big) \\
&+ 2i\theta_3 \times I\Big(-1+\theta_1+u/2, -\theta_1+u/2, \theta_2+t/2, -\theta_2+t/2, -1+s, -1\Big).
\end{aligned} \tag{4.58}$$

$$\begin{aligned}
\mathcal{A}_{z_2\to\infty} : &\frac{t}{2} \times I\Big(\theta_3+s/2, -\theta_3+s/2, \theta_1+u/2, -\theta_1+u/2, -1+t, -1\Big) \\
&+ \frac{t}{2} \times I\Big(\theta_3+s/2, -\theta_3+s/2, -1+\theta_1+u/2, 1-\theta_1+u/2, -1+t, -1\Big) \\
&+ u \times I\Big(\theta_3+s/2, -\theta_3+s/2, -1+\theta_1+u/2, -\theta_1+u/2, t, -1\Big) \\
&- I\Big(\theta_3+s/2, -\theta_3+s/2, -1+\theta_1+u/2, -\theta_1+u/2, t, -1\Big) \\
&- 2i\theta_3 I\Big(-1+\theta_3+s/2, -\theta_3+s/2, -1+\theta_1+u/2, -\theta_1+u/2, t, -1\Big).
\end{aligned} \tag{4.59}$$

$$\begin{aligned}
\mathcal{A}_{z_1\to\infty} : &\frac{t}{2} \times I\Big(-1+\theta_2+t/2, 1-\theta_2+t/2, \theta_3+s/2, -\theta_3+s/2, -1+u, -1\Big) \\
&+ \frac{t}{2} \times I\Big(\theta_2+t/2, -\theta_2+t/2, \theta_3+s/2, -\theta_3+s/2, -1+u, -1\Big) \\
&- u \times I(\theta_2+t/2, 1-\theta_2+t/2, \theta_3+s/2, -\theta_3+s/2, -2+u, -1) \\
&+ I\Big(\theta_2+t/2, 1-\theta_2+t/2, \theta_3+s/2, -\theta_3+s/2, -2+u, -1\Big) \\
&- 2i\theta_3 I\Big(\theta_2+t/2, -\theta_2+t/2, -1+\theta_3+s/2, -\theta_3+s/2, -1+u, -1\Big).
\end{aligned} \tag{4.60}$$

As we are interested in a contact term in the effective action we only need the part of the amplitude that is independent of the external momenta. Hence we will examine each of these expressions in the limit $s, t, u \to 0$ with $s+t+u = 0$.

We can also perform a powerful consistency check on the results at this point. The individual results from the various ways of picture-changing (4.51) to (4.53), (B.4) to (B.6), and (B.13) to (B.15) that constitute the above expressions have an unphysical pole of the form $(s+t+u)^{-1}$. These are incompatible with the known structure of either field theory or string theory. However on combination all such unphysical poles vanish: each of (4.58) to (4.60) is well-behaved in the limit $s+t+u \to 0$ and only has poles in $\frac{1}{s}$, $\frac{1}{t}$ or $\frac{1}{u}$. These checks are easiest to carry out numerically as it is cumbersome to treat the hypergeometric functions in $I(a, b, c, d, e, f)$ analytically.

Before combining the three results we reinstate the traces over Chan-Paton factors. We can write all traces in terms of $\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\gamma_\theta)$ as long as we recall that Chan-Paton

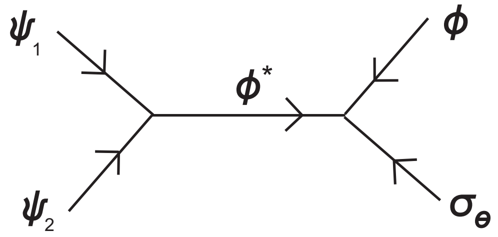

Figure 4.4: The origin of the $1/s$ pole as factorisation of the 4-point diagram onto a 3-pt Yukawa and an FI term.

factors commute with the twist matrix up to a phase. These phases are necessary to cancel the monodromy of a vertex operator under transport along the disk boundary. The relevant traces over Chan-Paton factors then are:

$$\mathcal{A}_{z_3\to\infty} : \mathrm{Tr}(\lambda_1\lambda_2\lambda_3\gamma_\theta) \ , \tag{4.61}$$

$$\mathcal{A}_{z_2\to\infty} : \mathrm{Tr}(\lambda_3\lambda_1\lambda_2\gamma_\theta) = \quad e^{-2\pi i(\theta_1+\theta_2)}\ \mathrm{Tr}(\lambda_1\lambda_2\lambda_3\gamma_\theta) \ , \tag{4.62}$$

$$\mathcal{A}_{z_1\to\infty} : \mathrm{Tr}(\lambda_2\lambda_3\lambda_1\gamma_\theta) = -e^{-2\pi i\theta_1} \qquad \mathrm{Tr}(\lambda_1\lambda_2\lambda_3\gamma_\theta) \ . \tag{4.63}$$

There is also an extra factor of (-1) that occurs as either $V(z_1)$ or $V(z_2)$ is moved through the branch point. This factor is due to our use of conventions with $\theta_1+\theta_2+\theta_3 = 1$, and comes from the phase $e^{\pi i(\theta_1+\theta_2+\theta_3)}$.[2]

The amplitudes $\mathcal{A}_{z_3\to\infty}$, $-e^{-2\pi i\theta_1}\mathcal{A}_{z_1\to\infty}$ and $e^{-2\pi i(\theta_1+\theta_2)}\mathcal{A}_{z_2\to\infty}$ have a remarkable pole structure, summarised below:

$$\begin{aligned}
\mathcal{A}_{z_3\to\infty} &= \frac{\alpha_s}{s} + \frac{\alpha_t}{t} + \frac{\alpha_u}{u}, \\
e^{-2\pi i(\theta_1+\theta_2)}\mathcal{A}_{z_2\to\infty} &= \frac{\beta_s}{s} \qquad\; + \frac{\beta_u}{u}, \\
-e^{-2\pi i\theta_1}\mathcal{A}_{z_1\to\infty} &= \frac{\gamma_s}{s} + \frac{\gamma_t}{t} + \frac{\gamma_u}{u},
\end{aligned} \tag{4.64}$$

where $\beta_s = -\gamma_s$, $\alpha_t + \gamma_t = 0$ and $\alpha_u - \beta_u + \gamma_u = 0$. The explicit expressions for the coefficients α, β and γ in terms of the angles $(\theta_1, \theta_2, \theta_3)$ are given in appendix B.2.

We see that the $\frac{1}{t}$ pole cancels and, up to an overall sign in $\mathcal{A}_{z_2\to\infty}$, so does the $\frac{1}{u}$ pole. The low-energy supergravity theory appears to require both the presence of a $\frac{1}{s}$ pole and the absence of poles in $\frac{1}{u}$ or $\frac{1}{t}$. The $\frac{1}{s}$ pole comes from the field theory diagram shown in figure 4.4, factorising onto the 3-point Yukawa interaction and the Fayet-Iliopoulos D-term $(\phi\phi^* + \xi)^2$, with $\xi \sim \tau_s$.

[2]This is easiest to see by considering the case of a gaugino vertex operator, with H-charges labelled by $(+,+,+,+,+)$ on a boundary with the twist field in the interior. The gaugino belongs to the untwisted sector and so should have no monodromy about the twist field. However it is easy to see from the H-charges that it does have a monodromy of $e^{i\pi(\theta_1+\theta_2+\theta_3)}$ on transport around the disk. In conventions where $\theta_1+\theta_2+\theta_3 = 1$ (rather than 0) we then need an additional minus sign appearing as spacetime spinors are moved through the branch cut.

The cancellations present strongly suggest that an overall sign is missing in the computation of $\mathcal{A}_{z_2\to\infty}$. This sign (which can be written as $\mathrm{sign}(z_3 - z_2)$) is presumably due to a cocycle factor that is present in the ordering of the vertex operators. We shall work on the supposition that the correct expression is indeed

$$\mathcal{A}_{\text{full}} = \mathcal{A}_{z_3\to\infty} - e^{-2\pi i\theta_1}\mathcal{A}_{z_1\to\infty} - e^{-2\pi i(\theta_1+\theta_2)}\mathcal{A}_{z_2\to\infty}. \tag{4.65}$$

In this case the surviving momentum pole of the full result is:

$$\mathcal{A}_{\text{full}} = \frac{ie^{\pi i\theta_3}\sin\pi\theta_3}{s} + \dots \tag{4.66}$$

and corresponds to the diagram in figure 4.4. The vertex on the left of the factorisation diagram is a tree-level Yukawa coupling which is a constant. The vertex on the right corresponds to an interaction $\tau_s\phi^3\bar{\phi}^3$ which we calculated before in section 4.3:

$$\lim_{k_i\to 0}\langle\tau_s\ \phi^3\bar{\phi}^3\rangle = ie^{\pi i\theta_3}\sin\pi\theta_3 \tag{4.67}$$

Putting these results together with a $\frac{1}{s}$ propagator we find perfect agreement. Our previous result is thus a strong confirmation of the analysis of this section including the additional sign in $\mathcal{A}_{z_2\to\infty}$ whose origin we leave for future work.

Our motivation for performing this calculation was to determine whether contact terms of the form $\int \mathrm{d}^4x\ \tau_s\psi\psi\phi$ exist in the effective action. To answer this question we have to go beyond the momentum pole: a non-zero contribution to our result for vanishing momenta beyond the pole is evidence for the presence of such a contact term. We do have an implicit analytic expression for this term; it is given by our result for the full amplitude (4.65) with the pole subtracted in the limit that s, t and u vanish:

$$\mathcal{A}_{\text{finite}} = \lim_{\substack{s\to 0\\ t\to 0\\ u\to 0}}\left(\mathcal{A}_{\text{full}} - \frac{ie^{\pi i\theta_3}\sin\pi\theta_3}{s}\right) \tag{4.68}$$

Although it would be very cumbersome to obtain the result in this limit analytically, one can nevertheless show numerically that a non-zero limit exists which also depends on the orbifold twist angles θ_i.

This concludes the study of the quantum contribution to the correlation function $\langle\tau_s\psi\psi\phi\rangle$. To complete the calculation, we also need to include an exponential weighted by the classical action. This will be the subject of the next section.

4.4.2 Classical action

In this section we complete the previous calculation by determining the classical contributions. The classical solutions are given by configurations of X^i which solve the equation of motion $\partial\bar{\partial}X^i_{cl} = 0$ and also satisfy the boundary conditions. In particular, the worldsheet boundary is to be identified with the locus of the visible sector D3-branes x_{D3}. Further, the insertion point of the twist field has to be mapped to the orbifold singularity of the blow-up mode x_{tw}. In addition, the classical solutions have to obey monodromy conditions about the twist insertion (3.35). Together, we find:

$$X^i_{cl}(z,\bar{z}) = x^i_{D3} \quad \text{for} \quad \mathrm{Im}(z) = \mathrm{Im}(\bar{z}) = 0 \ , \tag{4.69}$$

$$X^i_{cl}(w,\bar{w}) = x^i_{tw} \ , \tag{4.70}$$

$$\partial X^i_{cl}(z,\bar{z}) \sim (z-w)^{-1+\theta_i} \, (z-\bar{w})^{-\theta_i} \ , \tag{4.71}$$

$$\partial \bar{X}^i_{cl}(z,\bar{z}) \sim (z-w)^{-\theta_i} \, (z-\bar{w})^{-1+\theta_i} \ , \tag{4.72}$$

$$\bar{\partial} X^i_{cl}(z,\bar{z}) \sim (\bar{z}-w)^{-1+\theta_i} \, (\bar{z}-\bar{w})^{-\theta_i} \ , \tag{4.73}$$

$$\bar{\partial} \bar{X}^i_{cl}(z,\bar{z}) \sim (\bar{z}-w)^{-\theta_i} \, (\bar{z}-\bar{w})^{-1+\theta_i} \ . \tag{4.74}$$

These conditions can be implemented as follows. The Dirichlet conditions on the boundary imply that globally $X_{cl}(z,\bar{z})$ takes the form

$$X_{cl}(z,\bar{z}) = x_{D3} + f(z) - f(\bar{z}). \tag{4.75}$$

where, for simplicity, we suppress the label for the complex direction. The monodromy conditions around $(w,\bar{w})$ and holomorphy in z require $f(z)$ to be such that

$$\begin{aligned} \partial X_{cl}(z) &= \quad \alpha(z)(z-w)^{-1+\theta}(z-\bar{w})^{-\theta}, \\ \partial \bar{X}_{cl}(z) &= -\alpha^*(z)(z-w)^{-\theta}(z-\bar{w})^{-1+\theta}. \end{aligned} \tag{4.76}$$

The action is

$$S_{cl} = \frac{1}{4\pi\alpha'} \int \mathrm{d}^2 z (\partial X_{cl} \bar{\partial} \bar{X}_{cl} + \bar{\partial} X_{cl} \partial \bar{X}_{cl}). \tag{4.77}$$

and we have

$$\partial X_{cl} \bar{\partial} \bar{X}_{cl} = |\alpha|^2 |z-w|^{-2(1-\theta)} |z-\bar{w}|^{-2\theta}, \tag{4.78}$$

$$\bar{\partial} X_{cl} \partial \bar{X}_{cl} = |\alpha|^2 |z-w|^{-2\theta} |z-\bar{w}|^{-2(1-\theta)}. \tag{4.79}$$

As $z \to \infty$, we find that both $\partial X \bar{\partial} \bar{X}$ and $\bar{\partial} X \partial \bar{X} \longrightarrow |\alpha|^2 |z|^{-2}$. Now, if $\alpha(z)$ contains any non-negative powers of z the action is not normalisable as $z \to \infty$. Likewise, the presence of negative powers of z in $\alpha(z)$ makes the action non-normalisable as $z \to 0$. A finite result for the action can thus only be obtained if $\alpha(z) = \alpha(\bar{z}) = 0$. Hence the only possible solution is the trivial solution $\partial X = \bar{\partial} X = 0$ and $X = const.$

However, for a distant twist field we need $\Delta X = \int \mathrm{d}z \partial X + \mathrm{d}\bar{z} \bar{\partial} X = x_{\mathrm{tw}} - x_{\mathrm{D3}} \neq 0$ when integrating from the worldsheet boundary to the location of the twist. This non-zero result is incompatible with $\alpha(z) = \alpha(\bar{z}) = 0$ and hence the whole amplitude must vanish.

In contrast, for blow-up modes that share a singularity with the visible sector, the trivial solution $X = x_{D3} = x_{tw} = 0$ is allowed. In this case the classical part contributes a factor $e^{-S_{cl}} = 1$ to the amplitude and the quantum correlator (4.65) gives the full result.

This concludes the study of correlation functions of the form $\langle \tau_s \; \psi\psi\phi \rangle$ on the disk worldsheet. The result depends on the location of the blow-up mode w.r.t. the visible sector.

1. For blow-up modes that share the singularity with the visible sector we find that $\langle \tau_s \; \psi\psi\phi \rangle$ is non-zero and given by (4.65). In particular, the result implies the existence of a contact term $\tau_s \; \psi\psi\phi$, whose coefficient is given by (4.68). The presence of such a coupling is not surprising: visible sector quantities like the gauge coupling on the branes typically depend on this Kähler modulus. For our study it is important to state that the existence of this coupling is irrelevant for the study of supersymmetry breaking. In semi-realistic models the Kähler modulus of the cycle supporting the visible sector is stabilised supersymmetrically, and hence it does not contribute to soft terms.

2. We are mainly interested in distant blow-up modes, which correspond to supersymmetry breaking moduli in models of interest. Here we find that $\langle \tau_s \; \psi\psi\phi \rangle$ vanishes exactly as the classical action cannot be normalised. Hence, operators of the form $\tau_s\psi\psi\phi$ are absent at string tree-level for distant blow-up modes.

4.5 Yukawas with multiple twist insertions

In this section we will study the dependence of physical Yukawa couplings on more than one blow-up mode. Rather than calculating the individual amplitudes in detail we will argue that all such correlation functions vanish as long as the twist field location is geometrically separated from the locus of the visible sector.

The subject of this section are thus correlation functions of the form

$$\langle \tau_s^{(1)} \tau_s^{(2)} \; \ldots \; \tau_s^{(n)} \; \psi_1 \psi_2 \phi \rangle \; , \tag{4.80}$$

where

- all twist insertions are identical $\mathcal{N} = 1$ twisted closed strings or their corresponding antitwists, and

- the twist fields arise from orbifold singularities distant to the ones supporting the visible sector.

In particular, we want to answer, whether correlators of the above form give rise to contact terms $\int \mathrm{d}^4x\ \tau_s\tau_s\ldots\tau_s\psi\psi\phi$ in the low energy effective action.

Classical action

We analysed the amplitude with one distant twist insertion in the previous section, where we found that the correlator vanished as the classical action was not normalisable. It is tempting to check whether this is still the case for more than one twist field insertion. If we consider twist fields located at w_1, w_2, $\ldots$ w_n we find that the classical solutions behave as

$$\partial X_{cl}(z) \sim (z-w_1)^{-1+\theta}(z-w_2)^{-1+\theta}\ldots(z-w_n)^{-1+\theta}(z-\bar{w}_1)^{-\theta}(z-\bar{w}_2)^{-\theta}\ldots(z-\bar{w}_n)^{-\theta}\ . \tag{4.81}$$

It is now easy to check that for $n\geq 2$ the classical action is normalisable, as $\partial X\bar{\partial}\bar{X}$ does not diverge as $z\to\infty$.

Since we now know that the action is finite, we can continue to estimate its magnitude. As the classical action is the area of the worldsheet, we examine how the worldsheet is embedded in target space in the presence of twist fields. We have the following conditions:

$$\text{disk boundary: } X(z,\bar{z}) = x_{D3}\ , \tag{4.82}$$

$$\text{twist insertion: } X(w_i,\bar{w}_i) = x_{tw_i}\ , \tag{4.83}$$

where x_{D3} is the position of the visible sector in the compact space and x_{tw_i} is a distant orbifold singularity. The conditions thus force the worldsheet to stretch between the D3-brane stack on the boundary and each of the singularities for which a twist field is present. We can use the $\mathrm{SL}(2,\mathbb{R})$ symmetry of the disk to fix the location of one of the twist fields $(\sigma_\theta,\sigma_{-\theta})$ to $(i,-i)$ (this removes any subtleties with all twist operators approaching the boundary). For generic twist insertions, such a stretched worldsheet has a finite area which scales with the overall radius R of the compact space. As the classical action is simply the area of the worldsheet, this leads to a path integral suppression as $e^{-\lambda\frac{R^2}{2\pi\alpha'}}$, where λ is an $\mathcal{O}(1)$ number.

There is one important exception to this exponential suppression. Intuitively, the exponential suppression arises because the string is constrained to the orbifold fixed point near the twist insertion. This follows from the monodromy $X(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = e^{2\pi i\theta}X(z,\bar{z})$. However, if the twist insertions on the disk have zero net twist and come

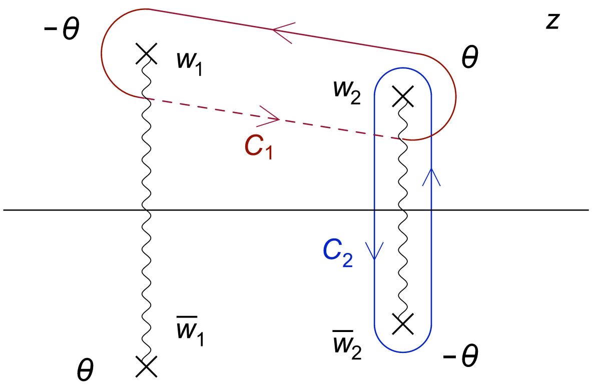

Figure 4.5: Setup of a twist and antitwist on the disk with the appropriate images. The two independent cycles $\mathcal{C}_1$ and $\mathcal{C}_2$ encircle zero net twist and form a basis for the homology of the cut worldsheet.

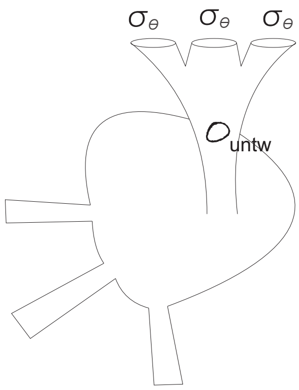

Figure 4.6: The structure of the contributions of distant twist operators to correlation functions. The only non-suppressed contributions come from regions where the twist operators factorise onto untwisted operators and the amplitude reduces to a correlation function involving untwisted operators.

together at one point on the worldsheet, the monodromy is lifted. In the OPE, bringing twist fields with net twist zero on top of each other factorises the amplitude onto an untwisted sector.

$$\lim_{w_1,w_2\ldots\to w_n}\sigma_\theta(w_1)\sigma_\theta(w_2)\ldots\sigma_\theta(w_n)=\sum(w-w_n)^{\lambda_i}\mathcal{O}^i_{untw}(w_n)\ . \tag{4.84}$$

Given that the boundary condition $X(w_i,\bar{w}_i)=x_{tw_i}$ still holds, it is not at first sight obvious that this eliminates the exponential suppression, as the worldsheet still has to stretch between the D3 location and the location of the twist fields.

It is easiest to convince oneself that the exponential suppression is absent by studying for example the explicit classical solutions for an antitwist $\sigma_{-\theta}(w_1,\bar{w}_1)$ and a twist $\sigma_\theta(w_2,\bar{w}_2)$ on the disk with Dirichlet boundary conditions. This setup is shown in figure

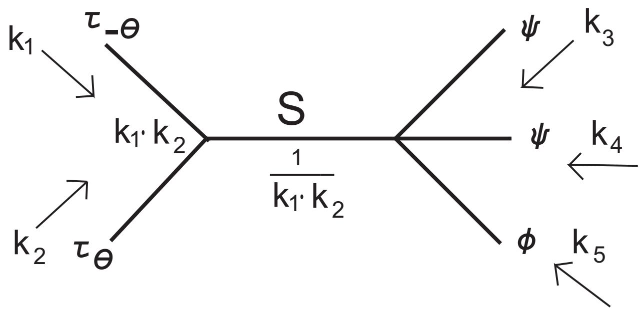

Figure 4.7: The generation of a finite contribution to the correlation function by factoring onto an intermediate massless state such as the dilaton.

4.5. The classical action was determined in [75] as

$$S_{cl,D} = \frac{4\pi}{\sin(\pi\theta)} \, |v_2|^2 \, \frac{F(x)\bar{F}(\bar{x})}{F(1-x)\bar{F}(\bar{x}) + \bar{F}(1-\bar{x})F(x)} \,, \tag{4.85}$$

where $F(x)$ is the hypergeometric function

$$F(x) = F(\theta, 1-\theta; 1; x) = \frac{1}{\pi}\sin(\pi\theta)\int_0^1 \mathrm{d}y \; y^{-\theta}(1-y)^{-1+\theta}(1-xy)^{-\theta} \,. \tag{4.86}$$

The result is obtained by fixing vertex operator positions as

$$w_1 = i \,, \quad \bar{w}_1 = -i \,, \quad w_2 = iy \,, \quad \bar{w}_2 = -iy \,, \tag{4.87}$$

where $y \in [0,1]$. The result is then most conveniently written in terms of $x = \left(\frac{1-y}{1+y}\right)^2$.[3]

Also, v_2 is the shift of X when transported around the loop $\mathcal{C}_2$ as shown in figure 4.5. It is given by an element of a coset lattice, which does not include a zero element for distant twist fields, and correspondingly $|v_2| \sim R$. The shift associated with $\mathcal{C}_1$ vanishes due to Dirichlet boundary conditions. Factorisation onto an untwisted state occurs in the limit $x \to 0$, where the twist and antitwist collide. Using the asymptotics of the hypergeometric function

$$F(x) \underset{x\to 0}{\rightarrow} 1 \,, \tag{4.88}$$

$$F(1-x) \underset{x\to 0}{\rightarrow} \frac{\sin(\pi\theta)}{\pi}\left(-\ln x + \ln(2\psi(1) - \psi(\theta) - \psi(1-\theta))\right) \,, \tag{4.89}$$

one can show that the classical action (4.85) is suppressed logarithmically in this limit, while it is $S_{cl} \sim R^2$ for generic twist locations. Thus, a correlation function involving a twist and an antitwist is typically suppressed by the classical action as $e^{-\lambda\frac{R^2}{2\pi\alpha'}}$. However, when factorising onto an untwisted state the suppression is lifted.

[3]The calculation was done in [75] by taking the worldsheet as the unit disk rather than the upper half plane, but here we transformed into our conventions by a conformal map.

In the large radius limit, we can therefore restrict our study of the multi-twist correlation function to the limit where the twist vertex operators come together and factorise onto states in the untwisted sector, as all other cases are exponentially suppressed at large radius. This is illustrated in figure 4.6. The factorisation can take two forms: it is either onto massless states in the untwisted sector (for example the graviton or the dilaton) or it is onto massive states (for example bulk KK modes).

Factorisation limits

Let us first consider the case of factorisation onto massless modes. While the amplitude can receive non-zero contributions in this limit, this does not constitute any evidence for a contact term $\int d^4x \; \tau_s \tau_s \dots \tau_s \psi\psi\phi$.

For example, we expect factorisation onto both the dilaton and the graviton to give non-zero results, as both have zero-momentum couplings to the Yukawas. The Yukawa couplings depend explicitly on the dilaton (which can be seen by noting that the D3-brane couplings are inherited from $\mathcal{N} = 4$ SYM)

$$\hat{Y}_{\alpha\beta\gamma} = g_s,$$

while the graviton will couple via the metric interaction

$$\int \sqrt{g} \; \psi\psi\phi.$$

We therefore expect – for example – the worldsheet correlation function $\langle S \; \psi\psi\phi \rangle$ to be non-zero at zero momentum.

Factorising onto the dilaton would generate the field theory diagram shown in figure 4.7. The vertex $\langle \tau_\theta \tau_{-\theta} \; S \rangle$ can only occur at finite momentum (a zero-momentum vertex would correspond to a tree level contribution to the moduli potential, which we know is absent). The diagram then has a positive power of momentum $k_i \cdot k_j$ from the $\tau_\theta \tau_{-\theta} \; S$ vertex and a negative power of momentum $\frac{1}{k_i \cdot k_j}$ from the dilaton propagator, giving overall no powers of momentum.[4]

However, by construction this interaction is formed simply by gluing together two lower-point interactions that are already present in the effective field theory. The diagram shown in figure 4.7 can be accounted for by a Lagrangian containing the terms

$$\mathcal{L} = S \partial_\mu \tau_\theta \partial^\mu \bar{\tau}_\theta + S \psi\psi\phi. \qquad (4.90)$$

This in no way implies the existence of an interaction $\tau_\theta \tau_{-\theta} \psi\psi\phi$ in the low energy effective action.

[4]We have not analysed whether the $\tau_\theta \tau_{-\theta} S$ vertex actually exists: the point here is to show that such contributions are reducible from the view of the low-energy field theory.

This argument applies for any factorisation of the twist fields onto massless modes in the untwisted sectors. In the language of Feynman diagrams this factorisation breaks the original diagrams into two sub-diagrams that are glued together by a propagator of the massless mode. By construction, the glued diagram involves a lower-point interaction between the twist fields and another massless field. As the massless fields are necessarily part of the low energy effective field theory, this limit is accounted for by lower-point interactions and there is no need to include a direct interaction $\tau_\theta \tau_\theta \ldots \psi\psi\phi$ in the Lagrangian.

The other case is where the factorisation is onto massive modes in the untwisted sector, for example bulk Kaluza-Klein modes. As the massive states are not in the low-energy theory, they have to be integrated out and we cannot use the above argument: the Feynman diagram does not manifestly factorise onto lower-point diagrams in the effective field theory.

In the bosonic string, twist operators have Yukawa-like interactions with such modes, coming from the OPE

$$\sigma_\theta(z,\bar{z})\sigma_{-\theta}(w,\bar{w}) \sim (z-w)^h (\bar{z}-\bar{w})^{\bar{h}} e^{ip_L \cdot X_L + ip_R \cdot X_R}, \qquad h, \bar{h} = \frac{1}{2}(\frac{m}{R} + \frac{1}{2}nR)^2, \quad (4.91)$$

where $p_L, p_R = \frac{m}{R} \pm \frac{nR}{2}$. These are direct Yukawa interactions with no momentum directly suppressing the three-point vertex.

For the superstring the situation is less clear and we do not attempt a direct calculation in this thesis. However, the twist operators correspond to Kähler moduli and so any Yukawa-like superpotential operator $\tau_s \tau_s \Phi_{heavy}$ would violate the perturbative shift symmetry of the Kähler moduli. This implies that, as with the massless modes, it is not possible to obtain a zero-momentum factorisation onto the heavy KK modes. This tells us that the field theory diagram involving factorisation onto a heavy KK modes involves a momentum prefactor coming from the factorisation vertex of twist fields onto KK modes. In principle this factor could be cancelled by an intermediate massless propagator, but as we factorise onto *massive* modes this cannot occur. As a result, the amplitude has an overall momentum prefactor and vanishes at zero momentum, and so cannot give rise to contact terms.

Sequestering from the worldsheet

We can also show that factorisation onto massive modes does not contribute to the amplitude by modifying an argument in [66] (see p60-63). We can then show that the amplitude $\langle \tau_s^{(1)} \tau_s^{(2)} \ \ldots \ \tau_s^{(n)} \ \psi_1 \psi_2 \phi \rangle$ will necessarily involve a momentum prefactor and thus vanish at zero momentum, except for in factorisation limits onto massless states.

We begin by analysing the quantum correlation function. First we rewrite the correlator (4.80) with the appropriate vertex operator insertions in their canonical picture:

$$\langle V_{\psi_1}^{-\frac{1}{2}}(z_1)V_{\psi_2}^{-\frac{1}{2}}(z_2)V_{\phi}^{-1}(z_3)\ V_{tw_1}^{-1,-1}(w_1\bar{w}_1)V_{tw_2}^{-1,-1}(w_2\bar{w}_2)\ \dots\ V_{tw_n}^{-1,-1}(w_n\bar{w}_n)\rangle\ . \tag{4.92}$$

This correlation function is not consistent as it stands: its ghost-charge is $(-2-2n)$ instead of (-2) as required for the disk. To correct for this we need to insert $2n$ picture-changing operators (PCO)

$$O_{PCO}^{+1} = e^{\varphi}\left(\partial X_\mu\psi^\mu + \left[\partial X^i\bar{\Psi}^i + \partial\bar{X}^i\Psi^i\right]\right) \tag{4.93}$$

into the amplitude. Picture-changing is performed by letting the PCO collide with a vertex operator[5]:

$$V^{p+1}(z) = \lim_{w\to z} O_{PCO}^{+1}(w)V^p(z)\ . \tag{4.94}$$

Note that the PCO consists of two qualitatively different parts, one involving external fields, and the other internal fields only. Both parts of the PCO give rise to qualitatively distinguishable terms:

- *External picture-changing:* In this case the active part of the PCO is $\partial X_\mu\psi^\mu$. OPEs between ∂X_μ and the momentum exponentials then lead to the appearance of explicit momentum factors k_μ.
- *Internal picture-changing:* Here picture-changing proceeds via $(\partial X^i\bar{\Psi}^i + \partial\bar{X}^i\Psi^i)$. As long as the momenta are purely along the external directions, no momentum-factors are generated in this case.

We now implement picture-changing for the amplitude of interest. On the disk it is only the sum of both holomorphic and antiholomorphic ghost charges which needs to yield (-2). A particularly useful way of picture-changing is the following one:

- Picture-change two of the matter vertex operators: $V_{\psi_2}^{-\frac{1}{2}} \to V_{\psi_2}^{+\frac{1}{2}}$ and $V_\phi^{-1}\to V_\phi^0$.
- For $n-2$ out of the n twist insertions we perform $V_{tw_j}^{-1,-1}\to V_{tw_j}^{0,0}$.
- Treat two of the twist fields differently: $V_{tw_1}^{-1,-1}\to V_{tw_1}^{0,-1}$ and $V_{tw_2}^{-1,-1}\to V_{tw_2}^{0,-1}$.[6]

[5]The procedure shown here can lead to ambiguities in the form of poles $(w-z)^{-1}$ Strictly speaking, the correlation function has to be evaluated with explicit insertions of $T_F(w)$ and the limit $w\to z$ taken at the end.

[6]If $V_{tw_1}^{-1,-1}$ is a twist and $V_{tw_2}^{-1,-1}$ is an antitwist the procedure should be modified as $V_{tw_1}^{-1,-1}\to V_{tw_1}^{0,-1}$ and $V_{tw_2}^{-1,-1}\to V_{tw_2}^{-1,0}$. The following argument then goes through unchanged.

Let us analyse the last step of this procedure. Picture-changing the twisted scalars in the internal directions can be performed by employing the following OPEs:

$$\begin{aligned}
&\lim_{z\to w} \quad e^{\varphi}\partial\bar{X}\cdot\Psi(z) \quad e^{-\phi}e^{-\tilde{\varphi}}\sigma_\theta s_\theta\tilde{s}_\theta(w,\bar{w}) \ \sim (z-w),\\
&\lim_{z\to w} \quad e^{\varphi}\partial X\cdot\bar{\Psi}(z) \quad e^{-\phi}e^{-\tilde{\varphi}}\sigma_\theta s_\theta\tilde{s}_\theta(w,\bar{w}) \ \sim 1\\
&\lim_{\bar{z}\to \bar{w}} \quad e^{\tilde{\varphi}}\bar{\partial}\bar{X}\cdot\tilde{\Psi}(\bar{z}) \quad e^{-\phi}e^{-\tilde{\varphi}}\sigma_\theta s_\theta\tilde{s}_\theta(w,\bar{w}) \ \sim 1,\\
&\lim_{\bar{z}\to \bar{w}} \quad e^{\tilde{\varphi}}\bar{\partial}X\cdot\bar{\tilde{\Psi}}(\bar{z}) \quad e^{-\phi}e^{-\tilde{\varphi}}\sigma_\theta s_\theta\tilde{s}_\theta(w,\bar{w}) \ \sim (\bar{z}-\bar{w}), \ .
\end{aligned}$$

According to the OPEs, internal picture-changing of two of the twists as $V_{tw}^{-1,-1} \to V_{tw}^{0,-1}$ can only be performed by two insertions of $e^{\varphi}\partial X\cdot\bar{\Psi}$, while $e^{\varphi}\partial\bar{X}\cdot\Psi$ does not contribute. Each of the $e^{\varphi}\partial X\cdot\bar{\Psi}$ insertions contributes two units of H-charge, such that four units of H-charge are necessarily introduced in this process. If these H-charges cannot be cancelled, the quantum correlation function is bound to vanish. However, it is easy to see that the H-charges are left uncancelled if we restrict ourselves to internal picture-changing. To transform $V_{tw_j}^{-1,-1} \to V_{tw_j}^{0,0}$ we need both $e^{\varphi}\partial\bar{X}\cdot\Psi$ and $e^{\varphi}\partial X\cdot\bar{\Psi}$ which are H-charge neutral. Similarly, picture-changing the matter fields does neither raise nor lower the overall H-charge. Thus internal picture-changing leaves us with uncancelled H-charge which forces the amplitude to vanishes. To obtain an expression with an allowed H-charge configuration we are forced to picture-change at least two of the vertex operators externally with $\partial X_\mu\psi^\mu$, leading to the appearance of a momentum-factor $k_i k_j$. The most general expression for our correlation function thus takes the form:

$$\langle\psi_1\psi_2\phi\ \tau_s^{(1)}\tau_s^{(2)}\ \ldots\ \tau_s^{(n)}\rangle \sim \sum_{i,j} k_i k_j\ \hat{\mathcal{A}}_{ij}(k_l; z_1,z_2,z_3; w_1,w_2,\ldots,w_n)\ . \tag{4.95}$$

The significance of this result is the following. We are interested in contact terms in the action and hence we wish to evaluate the amplitude for vanishing momenta. As the amplitude contains an explicit momentum-prefactor, the only finite contributions in this case arise from factorisation limits onto massless states. Previously, we argued that the only unsuppressed contributions to contact terms $\tau_s\tau_s\ldots\tau_s\psi\psi\phi$ can arise if the twist fields factorise onto a massive state. In this case the kinematic prefactor $k_i k_j$ remains uncancelled and the amplitude vanishes at zero momentum.

The conclusion of the argument presented in this section is the following: we find that at string tree level there is no term perturbative in α' in the (canonically normalised) effective action of the form

$$\int \mathrm{d}^4x\ \tau_s\tau_s\ldots\tau_s\psi\psi\phi, \tag{4.96}$$

where τ_s corresponds to a blow-up mode located at a singularity distant to that of the D3-brane. Although the argument has been developed in terms of twist fields located at a single singularity, it extends without modification to the case of many twist fields located at multiple singularities. In this case we equip the twist fields with a label indicating the orbifold singularity. The results of this section can then be directly applied to all twist fields carrying the same singularity label.

The argument had three steps:

1. For generic locations of twist operators, the classical worldsheet has to stretch. The classical contribution $e^{-S_{cl}}$ is then exponentially suppressed unless the twist operators coincide in a factorisation limit onto an untwisted sector. In this limit we can use the OPE to factor the twist operators onto either massless or massive states in the untwisted sector.

2. If the untwisted states are massless, we can decompose the diagram into sub-diagrams that come from lower point interactions in the effective field theory. These lower point interactions are sufficient for this limit and there is no need for a term of the form (4.96).

3. If the states are massive, the picture changing can be chosen to show that the amplitude has a kinematic prefactor that vanishes at zero momentum, excluding a term of the form (4.96).

The only possible loophole in the worldsheet argument would seem to be if there was a reason why the picture-changing as described above should be forbidden.

4.6 Result and discussion

In this chapter we examined the dependence of physical Yukawa couplings on blow-up moduli in a toroidal orbifold at string tree level. In models implementing the Large Volume Scenario (LVS) these blow-up modes correspond to the small cycles that are necessary for moduli stabilisation and supersymmetry breaking, and have non-zero F-terms.

The relevant calculations in this chapter are correlation functions of the form $\langle \tau_s \tau_s \dots \tau_s \ \psi\psi\phi \rangle$ on the disk, where τ_s are blow-up moduli and $\psi\psi\phi$ is a visible sector Yukawa coupling of matter fields from D3-branes at an orbifold singularity. At zero momentum, these correlators give information about the presence of contact terms $\int \mathrm{d}^4x \ \tau_s \tau_s \dots \tau_s \ \psi\psi\phi$ in the effective action.

For the case of one insertion τ_s we determined the full quantum and classical correlator. If τ_s is a distant blow-up mode, we find that the correlator vanishes, as the classical action cannot be normalised. Correspondingly, there are no contact terms $\tau_s \psi\psi\phi$ in the effective action in the case of a distant τ_s. For a blow-up mode τ_s sharing a singularity with the visible sector the classical correlator is trivial and the quantum correlator gives the full result. By analysing the correlation function at zero momentum, we determine the presence of a contact term $\tau_s \psi\psi\phi$. This shows explicitly that, as expected, blow-up moduli at the same singularity are not sequestered.

For the case of multiple blow-up mode insertions the full quantum correlator is beyond the scope of this thesis. By analysing the classical action we establish that such terms are suppressed exponentially as $e^{-R^2/\alpha'}$, where R is the overall radius of the compact space. The only exception to this suppression arises in factorisation limits onto untwisted states. However, any contributions to the amplitude $\langle \tau_s \tau_s \ldots \tau_s \; \psi\psi\phi \rangle$ are then accounted for by lower-point interactions or they vanish at zero momentum. In summary, contact terms of the form $\tau_s \tau_s \ldots \tau_s \psi\psi\phi$ are absent up to terms non-perturbative in α'.

While these calculations have been carried out in the orbifold limit, the importance of the multi-twist results lie in the fact that they allow us to extend our findings to a smooth space. If the blow-up modes acquire VEVs, then the orbifold is resolved to a smooth Calabi-Yau space. The absence of the Lagrangian terms $\int \mathrm{d}^4 x \; \tau_s \tau_s \ldots \tau_s \; \psi\psi\phi$ shows that even when we resolve onto the smooth space, there is still no dependence of the physical Yukawa couplings on distant blow-up modes (up to terms non-perturbative in α'). As the resolution of toroidal orbifold singularities leads to a space resembling the Swiss-Cheese structure frequently employed in the LVS, this provides a method to obtain CFT results for the geometries appropriate to the LVS.

This brings us back to the original motivation. We started by observing that the physical Yukawa couplings can only depend on supersymmetry breaking (Kähler) moduli via the moduli Kähler potential or the Kähler metric. The absence of any dependence of physical Yukawas on supersymmetry breaking moduli then implies that the moduli-dependence of the Kähler metric and the moduli Kähler potential has to be correlated: $\tilde{K}_\alpha \sim e^{\hat{K}/3}$. The interdependence between $\tilde{K}_\alpha$ and $\hat{K}$ is precisely of the form necessary for the cancellation of many contributions to soft terms. The main result from this chapter is that at leading order in g_s – as far as the dependence on supersymmetry breaking blow-up moduli is concerned – the structure $\tilde{K}_\alpha \sim e^{\hat{K}/3}$ survives to all orders in α' and is only broken by terms non-perturbative terms in α'.[7]

[7] The fact that the physical Yukawa couplings depend on Kähler moduli sharing the singularity with the visible sector does not invalidate this conclusion. As these local Kähler moduli are fixed at string size and do not acquire F-terms, they do not lead to contributions to soft terms. As long as we are interested in contributions to soft terms, our results imply that we can use $\tilde{K}_\alpha \sim e^{\hat{K}/3}$.

Chapter 5

Superpotential desequestering

In this chapter we will examine contributions to soft A-terms which arise from non-perturbative effects. While typically subleading, these contributions become important in string models with sequestered supersymmetry breaking. Here, we will calculate such contributions in a toy-model based on a toroidal orbifold. We will find that the flavour structure of these A-terms does not coincide with that of the tree-level Yukawa couplings. In consequence, the induced A-terms are sources of flavour-violating effects. This chapter is based on paper [8].

5.1 Motivation

Non-perturbative effects are essential ingredients in several celebrated moduli-stabilisation schemes in type IIB string theory [6, 28, 31]. These arise from Euclidean D3-branes or gaugino condensation on a stack of D7-branes wrapping some four-cycle in the internal dimensions. At the level of the four-dimensional effective theory, the non-perturbative effects induce a non-perturbative superpotential which depends on the volume of the corresponding four-cycle.

$$W \supset \mathcal{O}\ e^{-aT}\ . \tag{5.1}$$

Here T is a Kähler modulus of the cycle wrapped and a is a model-dependent parameter. The operator $\mathcal{O}$ can depend on visible sector fields, in which case the non-perturbative effects introduce cross-couplings between the visible sector and the supersymmetry breaking moduli. In this chapter, we will focus on such contributions to the superpotential which take the form of Yukawa couplings:

$$W \supset Y^{np}_{ijk}\ C^i C^j C^k\ e^{-aT}\,, \tag{5.2}$$

where Y^{np}_{ijk} are constants, C^i are various visible sector fields, a is a model-dependent parameter and T a Kähler modulus. After supersymmetry breaking in which the Kähler modulus T obtains a non-vanishing F-term, the above coupling gives rise to soft A-terms of the form

$$\delta A'_{ijk} = -e^{\mathcal{K}/2} a F^T Y^{np}_{ijk} e^{-aT} \ , \tag{5.3}$$

in natural units. Typically, such contributions to A-terms are subleading, but they can become relevant if other contributions to soft terms are suppressed. Such a suppression is indeed observed in the Large Volume Scenario, where soft terms vanish at leading order. Further suppression of soft scalar masses and A-terms is observed if the condition $\tilde{K}_\alpha \sim e^{\hat{K}/3}$ holds beyond leading order.

The importance of non-perturbative contributions to A-terms lies in the fact, that they can be sources of flavour-changing neutral currents: if one of the fields C^i in equation (5.2) is a Higgs field, soft masses for the remaining fields will be induced after electroweak symmetry breaking. Note that if the non-perturbative Yukawa couplings do not align with the tree-level Yukawa couplings, the soft masses induced by (5.3) will lead to flavour-violating processes. Consistency with observational bounds then restricts the parameter space of the string construction [42]. The severity of these constraints crucially depends on the size and structure of Y^{np}_{ijk}.

In this chapter, we study non-perturbative corrections to Yukawa couplings in some simple toroidal orbifolds which serve as computable toy models for more complicated compactifications. In our study we are motivated by previous results of flavour-violating effects due to Yukawa couplings sourced by string or D-brane instantons [43, 44, 84]. Similarly, non-perturbative effects in F-theory GUT models have been found to modify the flavour structure of the Yukawa couplings [85]. Here, we model the visible sector as arising from D3-branes at an orbifold singularity, while the D7-branes supporting the non-perturbative effects can either wrap two 2-tori of the compact space or a small 4-cycle instead. In particular, we want to check for the existence of non-perturbative contributions to Yukawa couplings in these setups and determine their flavour structure.

What to calculate?

In this chapter, we will focus on non-perturbative effects from gaugino condensation on a stack of D7-branes. In the four-dimensional field theory, this produces an effective superpotential which is proportional to

$$W^{np} = \mathcal{A} e^{-a f_{D7}} \ , \tag{5.4}$$

where f_{D7} is the gauge kinetic function on the D7-branes, and $a = 2\pi/c$, where c is the dual Coxeter number of the D7-brane gauge group, so that e.g. $c = N$ for $SU(N)$. At leading order f_{D7} is given by the Kähler modulus T of the cycle wrapped, but there will be corrections to the gauge kinetic function which will introduce a dependence on further fields. In particular, if f_{D7} depends on D3 matter fields, it is easy to see that operators of the form (5.2) are induced. Expanding the gauge kinetic function in gauge-invariant combinations of D3 matter fields C^i we find that new contributions to Yukawa couplings are present, if f_{D7} contains a trilinear term in visible sector fields C^i:

$$f_{D7}(\Phi^\alpha, C^i) = T + \ldots + y_{ijk}(\Phi^\alpha) C^i C^j C^k + \ldots \ , \tag{5.5}$$

where Φ^α are moduli. Plugging this expression into the non-perturbative superpotential (5.4) and expanding in matter fields C^i, we see that the trilinear term proportional to y_{ijk} indeed produces new contributions to Yukawa couplings of form (5.2).

Thus, to check whether new contributions to Yukawas are generated by gaugino condensation, we have to determine corrections to the D7 gauge coupling due to distant D3-matter fields. In the low-energy effective field theory the gauge kinetic function appears in the term:

$$\int \mathrm{d}^4x \int \mathrm{d}^2\theta \ f_{D7}(\Phi^i, C^j) W_\alpha W^\alpha \ , \tag{5.6}$$

where W_α is the D7 field strength superfield. Expanding the gauge kinetic function as above and integrating over the Grassmann coordinates, the trilinear terms produce the couplings

$$\begin{aligned} &\frac{1}{2} y_{ijk} \mathrm{Tr}(F^{\mu\nu} F_{\mu\nu}) \mathrm{Tr}(\phi^i \phi^j \phi^k) \qquad \text{and} \\ &2 y_{ijk} \mathrm{Tr}(\lambda\lambda) \mathrm{Tr}(\psi^i \psi^j \phi^k) \ , \end{aligned} \tag{5.7}$$

where λ are the gauginos corresponding to the D7 gauge bosons A^μ and ψ_i and ϕ_j are fermionic and scalar D3 matter fields. Hence, to generate non-perturbative corrections to Yukawa couplings, these operators must be present in the effective action.

The existence of such operators can then be checked using string perturbation theory – and this is the subject of this chapter. There are two kinds of string amplitude calculations we could perform to obtain y_{ijk}. First, one can compute the one-loop threshold correction to the D7-brane gauge kinetic function given by $\langle \mathrm{Tr}(A^\mu A^\nu) \mathrm{Tr}(\phi^i \phi^j \phi^k) \rangle$. Secondly one can calculate corrections to D3 Yukawa couplings due to gaugino condensation by evaluating $\langle \mathrm{Tr}(\lambda\lambda) \mathrm{Tr}(\psi^i \psi^j \phi^k) \rangle$. Both amplitudes are computed as a cylinder diagram with two D7-brane gauge boson or gaugino vertex operators inserted at one boundary, and three visible sector matter field operators inserted at the other boundary. In the following, we will perform the first calculation in detail and determine the size and flavour structure of y_{ijk}. We also briefly discuss the second calculation in appendix C. Most importantly, both approaches give the same result.

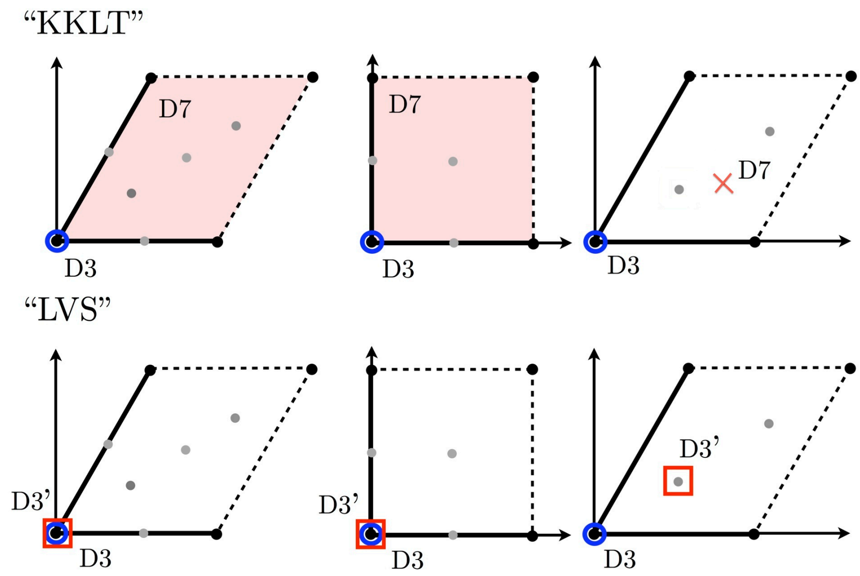

Figure 5.1: Brane configurations for the $\mathbb{T}^6/\mathbb{Z}_6'$ orbifold with $\theta = \frac{1}{6}(1, -3, 2)$. (This is the AaA lattice, in the classification of [86].) Dots indicate orbifold singularities. Stacks of D3-branes supporting visible matter are denoted by a blue circle at the origin. The branes responsible for non-perturbative effects are indicated by a red box and wrap the first two tori in the "KKLT" setup.

5.2 Setting up the calculation

Before we can begin with the calculation, we need to find the relevant setup in the toroidal orbifold background. In particular, we need to model the visible and the hidden sectors.

Visible sector

We will use D3-branes at an orbifold singularity to describe the visible sector. While they do not represent a fully realistic visible sector, they realise most features which we require. In particular, the spectrum (see section 2.3) contains chiral matter superfields $C^r_{a,b}$ in bifundamental representations $(n_a, \bar{n}_b)$. While we will often suppress the subscripts, we keep the superscript, which refers to the internal complex direction of the fluctuation giving rise to the matter field. Note that the superscript does not specify a chiral superfields uniquely, as for D3-branes at a $\mathbb{Z}_N$ orbifold singularity there are, in principle, N chiral superfields from each complex internal direction. As we are interested in new contributions to Yukawa couplings, it is important to review the Yukawa couplings present at tree-level. These arise from the superpotential:

$$W^{tree} = \sum_{r,s,t=1}^{3} \epsilon_{rst} \text{ Tr } (C^r C^s C^t) \, , \tag{5.8}$$

orbifold singularity	orbifold twist	allowed Yukawa couplings
$\mathbb{C}^3/\mathbb{Z}_3$	$\theta = \frac{1}{3}(1,1,-2)$	all $C^rC^sC^t$ for $r,s,t = 1,2,3$
$\mathbb{C}^3/\mathbb{Z}_4$	$\theta = \frac{1}{4}(1,1,-2)$	$C^1C^2C^3$ $C^1C^1C^3$ $C^2C^2C^3$
$\mathbb{C}^3/\mathbb{Z}_6$	$\theta = \frac{1}{6}(1,1,-2)$	$C^1C^2C^3$ $C^1C^1C^3$ $C^2C^2C^3$ $C^3C^3C^3$
$\mathbb{C}^3/\mathbb{Z}_6'$	$\theta = \frac{1}{6}(1,-3,2)$	$C^1C^2C^3$ $C^3C^3C^3$

Table 5.1: Gauge-invariant combinations of chiral superfields C^r arising on D3-branes at $\mathbb{C}^3/\mathbb{Z}_N$.

where ϵ_{rst} is the fully antisymmetric tensor. It follows that only Yukawa couplings involving the fields $C^1C^2C^3$ appear at tree-level.

However, there are in principle more combinations of three chiral superfields which are gauge invariant. Such combinations are given by

$$C^r_{i,i-b_r} C^s_{i-b_r,i-b_r-b_s} C^t_{i-b_r-b_s,i} \ , \tag{5.9}$$

and we list such combinations for several orbifold singularities in table 5.1. Thus there are in general more gauge-invariant Yukawa couplings than are realised in the superpotential.

Hidden sector

We have two possible choices for the D7-branes giving rise to the non-perturbative effects. They can either wrap a bulk cycle or a small (blow-up) cycle and we will analyse both situations. The first case is employed in scenarios of moduli stabilisation following the Kachru-Kallosh-Linde-Trivedi (KKLT) approach [28], while the second is relevant for the LVS [6]. While we are more interested in the latter, we nevertheless perform the calculation for both setups. For the case of bulk D7s, we let them wrap the first two tori of the compact space, while being located at a single smooth point on the third torus. If the D7-branes are located on a small four-cycle, we need to treat them differently. In the orbifold limit, all blow-up cycles collapse to zero size. In this case the D7s on a small cycle become fractional D3-branes at an orbifold singularity. To distinguish them from the visible D3-branes, we label this stack as D3'. In both cases, we separate the visible stack geometrically from the hidden stack. The relevant setups are shown in figure 5.1.

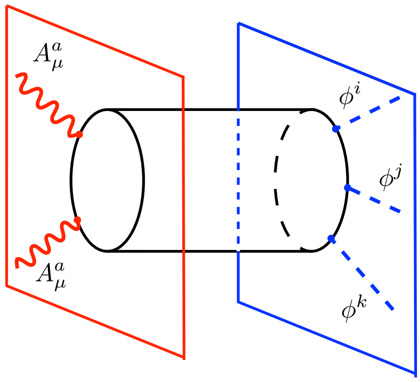

Figure 5.2: Cylinder amplitude that produces a threshold correction to a gauge coupling due to distant scalars. Vertex operators for gauge bosons are inserted on one boundary and the scalar operators are located on the other boundary.

CFT calculation

To confirm the presence of the relevant superpotential terms we need to calculate the correlation function $\langle \mathrm{Tr}(A^\mu A^\nu)\mathrm{Tr}(\phi^i\phi^j\phi^k)\rangle$. As this is a double-trace operator, we will need a worldsheet with two boundaries. Correspondingly, the leading contribution will be given by the cylinder diagram shown in 5.2. One could worry whether we should also consider unoriented worldsheets. Strictly speaking, the brane setups given above will in general be inconsistent as there are uncancelled tadpoles. However, one could cancel tadpoles by using the right number of branes with a combination of orientifold planes. The presence of orientifold planes then requires the consideration of unoriented worldsheets. Luckily, for the problem at hand, uncancelled tadpoles do not affect our result. Worldsheets with boundaries exhibit divergences due to tadpoles, if one boundary is unoccupied by vertex operators. The tadpole is then cancelled by a worldsheet where the empty boundary is replaced by a crosscap. As we are considering a cylinder diagram with insertions on both boundaries, tadpoles cannot contribute.

Orbifold sectors

All calculations in this chapter will be performed in compactifications based on toroidal orbifolds $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2\ /\mathbb{Z}_N$. To perform calculations on this space, we will insert a projection operator into the string path integral:

$$P = \frac{1 + \theta + \theta^2 \ldots + \theta^{N-1}}{N}\ . \tag{5.10}$$

This is needed to arrive at a result, which is invariant under the orbifolding. In consequence, the calculation splits into different sectors, where an orbifold twist θ^k is inserted explicitly. These sectors fall into three categories:

- *Fully twisted.* The orbifold action on all subtori is non-trivial: all $\theta_i^k \neq 0$.
- *Partially twisted.* The orbifold action leaves one $\mathbb{T}^2$ untwisted: $\theta_i^k = 0$ mod 1 for exactly one i.
- *Untwisted sectors.* The orbifold action leaves all tori untwisted.

In the following calculation, we will need to consider possible contributions from all these sectors.

5.3 Gaugino condensation: gauge threshold correction

The relevant term in the effective action is captured by a cylinder correlation function involving two D7 gauge bosons and three distant D3-brane scalars:

$$\langle V_A^0(z_1) V_A^0(z_2) V_{\phi_1}^0(z_3) V_{\phi_2}^0(z_4) V_{\phi_3}^0(z_5) \rangle \ . \tag{5.11}$$

The cylinder requires a background ghost charge of (0) and hence we take all vertex operators in the 0-picture. To start, we choose the three scalars to be consistent with a Yukawa coupling $C^1C^2C^3$. The vertex operators are then:

$$\begin{aligned}
V_A^0(z_1) &= [\partial X^\mu + i(k_1 \cdot \psi)\psi^\mu]\, e^{ik_1 \cdot X}(z_1) \\
V_A^0(z_2) &= [\partial X^\nu + i(k_2 \cdot \psi)\psi^\nu]\, e^{ik_2 \cdot X}(z_2) \\
V_{\phi_1}^0(z_3) &= \left[\partial X^1 + i(k_3 \cdot \psi)\Psi^1\right] e^{ik_3 \cdot X}(z_3) \\
V_{\phi_2}^0(z_4) &= \left[\partial X^2 + i(k_4 \cdot \psi)\Psi^2\right] e^{ik_4 \cdot X}(z_4) \\
V_{\phi_3}^0(z_5) &= \left[\partial X^3 + i(k_5 \cdot \psi)\Psi^3\right] e^{ik_5 \cdot X}(z_5) \ .
\end{aligned} \tag{5.12}$$

Here, Chan-Paton factors, polarisations as well as factors of the string coupling g_s are suppressed, and will be restored later. Further, X^μ and ψ^μ are external fields, while X^i and Ψ^i are complex internal fields as defined in appendix A.2.1. The fact that the scalars arise from a Yukawa coupling $C^1C^2C^3$ can be understood as follows: under an orbifold twist, the vertex operators acquire phases as $V_{\phi_1} \to e^{2\pi i\theta_1}V_{\phi_1}$, $V_{\phi_2} \to e^{2\pi i\theta_2}V_{\phi_2}$ and $V_{\phi_3} \to e^{2\pi i\theta_3}V_{\phi_3}$.

At this point we can already simplify the calculation considerably: as the correlation function only involves the complex internal fields ∂X^i and Ψ^i, but not the complex conjugates $\partial\bar{X}^i$ and $\bar{\Psi}^i$, the quantum correlator vanishes exactly! For terms containing the complex spinor Ψ^i, the vanishing is due to uncancelled H-charge. Similarly, one can check that the complex bosonic fields exhibit the following basic correlators:

$$\langle \partial X^i(z) \partial\bar{X}^j(0) \rangle \neq 0 \ , \quad \langle \partial X^i(z) \partial X^j(0) \rangle = \langle \partial\bar{X}^i(z) \partial\bar{X}(0) \rangle = 0 \ , \tag{5.13}$$

and correspondingly correlators only involving the fields ∂X^i give a vanishing result.

This does not imply that the whole amplitude is zero. While all quantum correlation functions of internal fields do not contribute to the result, there can be nontrivial classical solutions ∂X^i_{cl} which will be the only contribution from internal fields. Correspondingly, the spinorial fields can be ignored and the relevant terms of the scalar vertex operators are then:

$$\begin{aligned} V^0_{\phi_1}(z_3) &= \partial X^1 e^{ik_3\cdot X}(z_3) \\ V^0_{\phi_2}(z_4) &= \partial X^2 e^{ik_4\cdot X}(z_4) \\ V^0_{\phi_3}(z_5) &= \partial X^3 e^{ik_5\cdot X}(z_5)\ . \end{aligned} \tag{5.14}$$

Using this the correlation function 5.11 simplifies to:

$$\left\langle \left[\partial X^\mu + i(k_1\psi)\psi^\mu\right](z_1)\left[\partial X^\nu + i(k_2\psi)\psi^\nu\right](z_2)\ \prod_{i=1}^{5} e^{ik_i\cdot X}(z_i)\right\rangle \langle \partial X^1(z_3)\partial X^2(z_4)\partial X^3(z_5)\rangle$$

The first factor is essentially a well-known one-loop two-point function of gauge bosons (see e.g. [47,87,88]). The second factor only includes internal bosonic fields and contains all information about the field combinations appearing in the Yukawa coupling. If we analysed the case of a Yukawa couplings of fields $C^iC^jC^k$ the correlator would become $\langle \partial X^i\partial X^j\partial X^k\rangle$. We can thus state a necessary condition for the presence of new contributions to Yukawa couplings: as the second factor only gets contributions from classical solutions we note that a new Yukawa coupling of superfields $C^iC^jC^k$ can only be generated if a classical solution for the field combination $\partial X^i_{cl}\partial X^j_{cl}\partial X^k_{cl}$ exists. We will use this condition to scan through a variety of orbifold models. But before, we sketch the calculation of the first factor over external fields for completeness.

5.3.1 Gauge boson two-point function

There are in principle four terms coming from the correlator

$$\left\langle \left[\partial X^\mu + i(k_1\cdot\psi)\psi^\mu\right](z_1)\left[\partial X^\nu + i(k_2\cdot\psi)\psi^\nu\right](z_2)\ \prod_{i=1}^{5} e^{ik_i\cdot X}(z_i)\right\rangle\ , \tag{5.15}$$

however, the two cross-terms vanish as $\langle(\psi^\lambda\psi^\mu)(z)\rangle = 0$. Then there are two remaining contributions. One only consists of bosonic fields:

$$\left\langle \partial X^\mu(z_1)\partial X^\nu(z_2)\ \prod_{i=1}^{5} e^{ik_i\cdot X}(z_i)\right\rangle\ . \tag{5.16}$$

A purely bosonic correlator has to vanish at one-loop. It premultiplies the fermionic partition function which vanishes in a supersymmetric model

$$\mathcal{B}(z_1,z_2,z_3,z_4,z_5)\sum_\alpha \langle 1\rangle_\alpha = 0\ , \tag{5.17}$$

where the sum is over spin structures.

One is left with a correlator over spinorial fields and momentum exponentials.

$$\langle (k_1 \cdot \psi)\psi^\mu(z_1)(k_2 \cdot \psi)\psi^\nu(z_2)\rangle \left\langle \prod_{i=1}^{5} e^{ik_i\cdot X}(z_i) \right\rangle \, . \tag{5.18}$$

As there are no cross-correlations between these fields one can analyse them separately.

The fermionic amplitude involves four spinorial fields and depends on the spin structure. We only need to consider even spin structures: the amplitude vanishes in the odd spin structure as we can never saturate the fermionic zero modes. In the even spin structures we can contract the spinors using the Szegő kernel S_α:

$$\langle \psi^\mu(z_1)\psi^\nu(z_2)\rangle = \eta^{\mu\nu} S_\alpha(z_1 - z_2) = \eta^{\mu\nu} \frac{\vartheta_\alpha(z_1 - z_2)\ \vartheta_1'(0)}{\vartheta_1(z_1 - z_2)\vartheta_\alpha(0)} \, . \tag{5.19}$$

Introducing the polarisations ϵ_1 and ϵ_2, we thus find:

$$\epsilon_{1\mu}\epsilon_{2\nu} \langle (k_1 \cdot \psi)\psi^\mu(z_1)(k_2 \cdot \psi)\psi^\nu(z_2)\rangle = -\epsilon_{1\mu}\epsilon_{2\nu}(k_1 \cdot k_2 \eta^{\mu\nu} - k_1^\nu k_2^\mu) S_\alpha^2(z_1 - z_2)\langle 1\rangle_\alpha \, . \tag{5.20}$$

The momentum-prefactor captures the kinematics of two gauge bosons $F^{\mu\nu}F_{\mu\nu}$ which we formally extract. Continuing with the spinor amplitude we reexpress the Szegő kernel as

$$S_\alpha^2(z_1 - z_2) = \wp(z_1 - z_2) - e_{\alpha-1}, \tag{5.21}$$

where $e_{\alpha-1} = -4\pi i \partial_\tau \ln\left(\vartheta_\alpha(0|\tau)/\eta(\tau)\right)$, see e.g. [52]. Here $\wp$ is the Weierstrass function

$$\wp(z, \tau) = -\partial_z^2 \log \vartheta_1(z|\tau) + 4\pi i \partial_\tau \log \eta(\tau) \, . \tag{5.22}$$

As the Weierstrass $\wp$-function and the $\partial_\tau \ln \eta(\tau)$ piece of $e_{\alpha-1}$ are both independent of spin structure α, these terms will be proportional to the partition function and vanish upon summation over spin structures. Therefore this correlator only receives contributions from

$$-4\pi i \frac{\partial_\tau \vartheta_\alpha(0|\tau)}{\vartheta_\alpha(0|\tau)} \, . \tag{5.23}$$

Now as $\partial_\tau \vartheta_\alpha = 1/(4\pi i) \cdot \partial_z^2 \vartheta_\alpha$ we can write the contributions of the fermionic parts to the overall amplitude as

$$-\frac{\vartheta_\alpha''(0|\tau)}{\vartheta_\alpha(0|\tau)}\langle 1\rangle_\alpha, \tag{5.24}$$

where primes denote derivatives with respect to z. This concludes the analysis of the gauge-boson two-point function which gives a standard result. There is still the correlator involving momentum-exponentials which will contribute to the overall result. We move on to the remaining piece, the classical correlator.

5.3.2 Classical correlator — sums over winding modes

The part of the overall correlation function that is sensitive to the structure of the Yukawa coupling is the correlator of internal bosonic fields. A potential Yukawa coupling of the form $C^iC^jC^k$ is probed by a correlator

$$\left\langle \partial X^i(z_3)\partial X^j(z_4)\partial X^k(z_5)\right\rangle \ . \tag{5.25}$$

As mentioned before this correlation function only contributes its classical part. Summing over all classical backgrounds gives

$$\langle \partial X^i(z_3)\partial X^j(z_4)\partial X^k(z_5)\rangle = \sum_{\substack{\text{classical}\\ \text{solutions}}} \partial X^i_{cl}\partial X^j_{cl}\partial X^k_{cl}\ \langle 1\rangle\ e^{-S_{\rm cl}} \ . \tag{5.26}$$

The classical solution is defined by the embedding of the cylinder into the target space, and thus necessarily depends on the boundary conditions. As the two boundaries of our cylinder need to coincide with distant brane stacks, the cylinder has to stretch across the compact internal directions. Here, we parameterise the Euclidean worldsheet by $\sigma^1 \in [0, 1/2]$ and $\sigma^2 \in [0, t/2]$, such that the two boundaries are given by $\sigma^1 = 0, 1/2$. Then the required classical solutions are

$$X^i_{cl}(\sigma^1,\sigma^2) = x^i_0 + 4\pi\sqrt{\frac{T^i_2}{2U^i_2}}(m + U^i n + r_c)\sigma^1 \ . \tag{5.27}$$

Here U^i is the complex structure modulus of the torus wrapped, T^i_2 is its volume and m and n are winding numbers. We wrote the separation between brane stacks in terms of the dimensionless separation r_c, which is related to the physical distance ΔX^i as $\Delta X^i = 2\pi\sqrt{T_2/2U_2}\ r_c$.

Such a solution is allowed if the following conditions are satisfied:

- Both boundaries of the cylinder exhibit Dirichlet boundary conditions.
- The geometric action of the orbifold leaves the X^i direction untwisted.

The conditions are easily understood. Neumann boundary conditions are inconsistent with the winding part as $\partial_{\sigma^1}X^i \neq 0$ for finite r_c. Further, in a twisted sector the bosonic field X has to satisfy

$$X^i(\sigma^1, \sigma^2 + t/2) = e^{2\pi i\theta_i} X^i(\sigma^1,\sigma^2) \ . \tag{5.28}$$

Again, this is inconsistent with (5.27) for finite r_c.

We thus identified two conditions for the existence of a non-trivial classical correlator. We can now apply them to the D-brane constructions of interest. In particular, we will need to consider two cases. For one, the D7-branes are located on bulk cycles, while in the other setting they wrap small (blow-up) cycles.

D3-D7 models

In this case the gauge fields are located on a stack of bulk D7-branes whereas chiral superfields are supported on a stack of D3-branes at an orbifold singularity. All orbifold models have a singularity at the origin and we place the stack of visible D3-branes there. To be specific, let the D7-branes wrap the first two subtori and be located at a position $x^3_{D7} = 2\pi\sqrt{T^i_2/2U^i_2}\; r_c$ on the third 2-torus, which is not a singular locus. The boundary conditions on the two boundaries of the cylinder stretching between the two stacks are summarised below:

Complex direction	Ext 1	Ext 2	Int 1	Int2	Int 3
D3-D7	NN	NN	DN	DN	DD

where D stands for Dirichlet and N for Neumann boundary conditions. We conclude that the classical solution automatically vanishes for any combination of fields except for one, which is $\langle \partial X^3 \partial X^3 \partial X^3 \rangle$ for our choice of brane embedding.

For winding solutions to exist along this DD direction, it should not be twisted by the orbifold action. The geometric action of the orbifold can be classified into three different sectors. We examine them one by one for the D3-D7 system at hand:

1. **Fully twisted sectors** ($\mathcal{N} = 1$):
   Here the orbifold acts non-trivially on all three internal directions. As pointed out before, twisted directions do not allow for winding modes and this sector does not contribute.

2. **Untwisted sector** ($\mathcal{N} = 2$):
   Here the orbifold has no geometric effect as we insert the identity element of the orbifold group into the amplitude. Correlation functions are the same as for pure toroidal compactifications and winding modes are allowed along the single complex DD-direction.

3. **Partially twisted sector** ($\mathcal{N} = 2$): In partially twisted sectors the orbifold twists two complex directions while leaving exactly one direction untwisted. These instances only occur for $\mathbb{Z}_N$ orbifolds with N even. Which direction is left untwisted is model-dependent. If an untwisted direction coincides with the DD-direction the partially twisted sector adds to the result from the untwisted sector without changing it qualitatively.

As any orbifold compactification exhibits an untwisted sector the existence of winding solutions is thus automatic for D3-D7 models. Yet there is one more constraint which we need to take into account. So far we ignored the trace over Chan-Paton factors which, among other things, implements gauge invariance. The existence of a winding solutions for $\partial X^i \partial X^j \partial X^k$ is only a necessary condition for the generation of a new Yukawa coupling $C^i C^j C^k$. For this coupling to exist it has to be a gauge invariant combination of fields. As D3-D7 models only exhibit one DD-direction the new Yukawa coupling can only have the structure $C^r C^r C^r$. Such field combinations are gauge-invariant if the corresponding orbifold twist angle is $\theta_r = 1/3$. This occurs in $\mathbb{Z}_3$, $\mathbb{Z}_6$, $\mathbb{Z}_6'$, and $\mathbb{Z}_{12}$ orbifolds.

We now state the quantitative result for the classical contribution. Using the results from section A.2.2 (in particular (A.19) and (A.22)) we find:[1]

$$\langle \partial X^3 \partial X^3 \partial X^3 \rangle = \sum_{m,n} \left(2\pi \sqrt{\frac{T_2}{2U_2}} \right)^3 (m + Un + r_c)^3 e^{-\frac{\pi t}{\alpha'} \frac{T_2}{U_2} |m+Un+r_c|^2} , \tag{5.29}$$

where T_2 and U are volume and complex structure modulus of the third two-torus. In the following, it will be convenient to reexpress (5.29) as a triple derivative w.r.t. $\bar{r}_c$:

$$\langle \partial X^3 \partial X^3 \partial X^3 \rangle = \left(-\frac{\alpha'}{t} \sqrt{\frac{2U_2}{T_2}}\, \partial_{\bar{r}_c} \right)^3 \sum_{m,n} e^{-\frac{\pi t}{\alpha'} \frac{T_2}{U_2} |m+Un+r_c|^2} = \left(-\frac{\alpha'}{t} \sqrt{\frac{U_2}{2T_2}}\, \partial_{\bar{r}_c} \right)^3 \mathcal{Z}(t) , \tag{5.30}$$

where $\mathcal{Z}(t)$ is the sum over winding modes as defined in (A.23).

There is one subtlety for D3-D7 models on orbifolds which we ignored so far. To arrive at a setup that is invariant under orbifold twists we also need to include images of the D7-branes. For each stack of D7-branes we add $N-1$ identical stacks at the image loci of the original stack. Thus, for D7-branes located at r_c on the third 2-torus we include images at $r_c' = e^{2\pi i \theta_3 j} r_c$ for $j = 1, \dots N-1$. In our calculation we also need to include strings stretching between the D3-branes and all the image D7-branes. This leads to the following modification: each stack of D7-branes will give rise to an additional contribution of the form (5.29) with r_c replaced by the image brane locus r_c'. One can easily check that there is no cancellation between the various stacks. Instead, for $\theta_3 = 1/3$ all stacks give an identical result.

D3-D3' models

If the D7-branes carrying the gauge theory wrap a blow-up cycle they essentially become (fractional) D3-branes at an orbifold singularity. To distinguish this stack from the

[1] Strictly speaking, the vertex operators do not contain the field ∂X^i, but $\partial_n X^i = (\partial + \bar{\partial}) X^i$, where ∂_n is a derivative normal to the boundary. However, both ∂X and $\bar{\partial} X$ contribute the same classical result (A.20). Thus, by restricting our attention to ∂X^i only, we obtain the correct result up to an overall numerical factor.

visible D3s we call it D3'. In this case we have three complex directions with DD boundary conditions.

Complex direction	Ext 1	Ext 2	Int 1	Int2	Int 3
D3-D3'	NN	NN	DD	DD	DD

As D3-branes are pointlike in the internal dimensions model builders enjoy more freedom in arranging their spatial relations. The two stacks can be separated in one, two, or all three of the internal directions and these choices will affect the non-perturbative superpotential. Postponing this question we examine the various orbifold sectors for the possibility of winding solutions.

1. **Fully twisted sectors** ($\mathcal{N} = 1$): No contribution as before.

2. **Untwisted sector** ($\mathcal{N} = 4$):
   In principle winding modes are allowed for all internal directions. However, a D3-D3 system has $\mathcal{N} = 4$ supersymmetry which causes the quantum correlation function to vanish. While this is in stark contrast to the D3-D7 case considered above it is the typical behaviour for a D3-D3 setup.

3. **Partially twisted sector** ($\mathcal{N} = 2$): Partially twisted sectors have exactly one untwisted direction which allows for winding solutions. In other words, we are only free to stretch strings in at most one direction to connect the stacks. Consequently, a non-zero solution can only be found if the two stacks of branes are separated along one direction only which coincides with the complex dimension left untwisted.

To summarise, only partially twisted sectors can contribute, which only exist in $\mathbb{Z}_N$ orbifolds with even N. The result is more model dependent for the D3-D3' system than for the D3-D7 setup before. If the two stacks are separated in two or three directions the classical correlator vanishes. Most importantly, as winding solutions only exist for one direction only correlators of the form $\langle \partial X^r \partial X^r \partial X^r \rangle$ survive. This is gauge-invariant for $\mathbb{Z}_6$, $\mathbb{Z}_6'$, and $\mathbb{Z}_{12}$ orbifolds. The result is the same as before (5.29) and, as both stacks are at orbifold fixed points, no image branes are needed.

5.3.3 Results

Here we combine the results from the previous two sections with the appropriate partition functions. A list of cylinder partition functions is given in A.2.5. We also need to include the correlator over momentum exponentials which contributes

$$\left\langle \prod_{i=1}^{5} e^{ik_i \cdot X}(z_i) \right\rangle = \prod_{i<j} e^{-k_i \cdot k_j \mathcal{G}(z_i - z_j)} \tag{5.31}$$

where $\mathcal{G}(z_i - z_j)$ is the Green's function with NN boundary conditions.

D3-D7 models

In the case of D3-D7 models the result arises in the untwisted sector as explained above. Partially twisted sectors, if present, give an equal contribution. We find

$$\mathcal{A} = \int \frac{\mathrm{d}t}{t} \int \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}z_3 \mathrm{d}z_4 \mathrm{d}z_5 \frac{1}{(2\pi^2 t)^2} \left(\prod_{i<j} e^{-k_i \cdot k_j \mathcal{G}(z_i - z_j)} \right)$$
$$\sum_{\substack{\alpha,\beta \\ =0,1}} \frac{\eta_{\alpha\beta}}{2} \frac{\vartheta''_{\alpha\beta}(0)}{\eta^3} \frac{\vartheta_{\alpha\beta}(0)}{\eta^3} \left(\frac{\vartheta[{}^{1/2-\alpha/2}_{\beta/2}](0)}{\vartheta[{}^{0}_{1/2}](0)} \right)^2$$
$$\left(-\frac{\alpha'}{t} \sqrt{\frac{2U_2}{T_2}} \frac{\partial}{\partial \bar{r}_c} \right)^3 \mathcal{Z}(t) \ , \tag{5.32}$$

with $\mathcal{Z}(t)$ given in (A.23) and $\eta_{\alpha\beta} = (-1)^{\alpha+\beta}$.[2] We can apply the Riemann identity (A.12) to collapse the sum over spin structures to a constant.

$$\mathcal{A} = \int \frac{\mathrm{d}t}{t} \int \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}z_3 \mathrm{d}z_4 \mathrm{d}z_5 \frac{1}{(2\pi^2 t)^2} \left(\prod_{i<j} e^{-k_i \cdot k_j \mathcal{G}(z_i - z_j)} \right) \left(-\frac{\alpha'}{t} \sqrt{\frac{2U_2}{T_2}} \frac{\partial}{\partial \bar{r}_c} \right)^3 \mathcal{Z}(t) \ , \tag{5.33}$$

D3-D3 models

In the previous section we established that the only non-zero contribution arises in partially twisted sectors of the orbifold. To be specific we take $\theta_3 = 0$ mod 1 and $\theta_1 + \theta_2 = 0$ mod 1. Collating the previous results, the complete amplitude is

$$\mathcal{A} = \int \frac{\mathrm{d}t}{t} \int \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}z_3 \mathrm{d}z_4 \mathrm{d}z_5 \frac{1}{(2\pi^2 t)^2} \left(\prod_{i<j} e^{-k_i \cdot k_j \mathcal{G}(z_i - z_j)} \right)$$
$$\sum_{\substack{\alpha,\beta \\ =0,1}} \frac{\eta_{\alpha\beta}}{2} (-1)^\alpha \frac{\vartheta''_{\alpha\beta}(0)}{\eta^3} \frac{\vartheta_{\alpha\beta}(0)}{\eta^3} \left(\prod_{k=1}^{2} (-2 \sin \pi\theta_k) \frac{\vartheta_{\alpha\beta}(\theta_k)}{\vartheta_1(\theta_k)} \right)$$
$$\left(-\frac{\alpha'}{t} \sqrt{\frac{2U_2}{T_2}} \frac{\partial}{\partial \bar{r}_c} \right)^3 \mathcal{Z}(t) \ , \tag{5.34}$$

where $\mathcal{Z}(t)$ is given in (A.23) and $\eta_{\alpha\beta} = (-1)^{\alpha+\beta}$ as before. Again, we use the Riemann identity (A.11) to simplify the result.[3] Further, in the partially twisted sectors of the

[2]To calculate a correction to the gauge coupling, we have to sum over even spin structures only. Nevertheless, here we can perform the sum over all spin structures with $\eta_{\alpha\beta} = (-1)^{\alpha+\beta}$, as the expression (5.32) vanishes exactly for the odd spin structure $\alpha = \beta = 1$. The same is true for (5.34)

[3]If we repeated the analysis for the untwisted orbifold sector, the sum over spin-structures would be identically zero due to (A.10). Hence untwisted sectors do not contribute in the D3-D3 case.

relevant orbifolds (θ^3 sector of both $\mathbb{Z}_6$ and $\mathbb{Z}_6'$) we have $\theta_1 = \theta_2 = 1/2$. Thus we are left with:

$$\mathcal{A} = \int \frac{\mathrm{d}t}{t} \int \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}z_3 \mathrm{d}z_4 \mathrm{d}z_5 \frac{1}{(2\pi^2 t)^2} \left(\prod_{i<j} e^{-k_i \cdot k_j \mathcal{G}(z_i - z_j)} \right) \left(-\frac{\alpha'}{t} \sqrt{\frac{2U_2}{T_2}} \frac{\partial}{\partial \bar{r}_c} \right)^3 \mathcal{Z}(t) \tag{5.35}$$

which is the same as in the previous case. Hence we can discuss both results together.

Integral over t

We do not need to calculate the full amplitude to extract results of interest to us. On the contrary, as we only wish to determine corrections to the gauge coupling we only need to keep the pieces which exhibit the momentum dependence of a gauge kinetic term. To this end we recall that the above results (5.33) and (5.35) are premultiplied by a kinematic factor for gauge bosons which we extracted at an earlier stage of the calculation. Correspondingly, any additional momentum-dependence is undesirable and we let $k_i \to 0$. There are no momentum-poles in the amplitude and this limit can be taken without subtleties, eliminating the last term sensitive to the worldsheet operator positions. Subsequently, the integral over world-sheet positions can be performed without difficulties giving a contribution $\int \prod_{i=1}^{5} \mathrm{d}z_i \propto t^5$.

The last two remaining operations to be done are the sum over winding modes and the integration over t. Ignoring overall factors the amplitude becomes:

$$\mathcal{A} \sim \int_0^\infty \frac{\mathrm{d}t}{t} \frac{1}{t^2} t^5 \sum_{m,n} r_{mn}^3 \exp\left(-\frac{2\pi t}{\alpha'} |r_{mn}|^2 \right) , \tag{5.36}$$

where we define $r_{mn} = \sqrt{T_2^i / 2U_2^i}\, (m + Un + r_c)$. One can check that the integral does not diverge at either end of its range. The brane separation r_c acts as an IR cutoff. Upon Poisson resummation of the winding modes one can also check that the amplitude vanishes in the UV in virtue of the factor r_{mn}^3. We can press ahead with the integration over t which produces

$$\mathcal{A} \sim \sum_{m,n} \frac{\alpha'^3}{4\pi} \frac{r_{mn}^3}{|r_{mn}|^6} = \frac{\alpha'^3}{4\pi} \left(\frac{2U_2}{T_2} \right)^{\frac{3}{2}} \sum_{m,n} \frac{1}{(m + \bar{U} n + \bar{r}_c)^3} . \tag{5.37}$$

The double sum over m and n is of the form

$$\sum_{m,n} \frac{2}{(z + m + Un)^3} = -\wp'(z, U), \tag{5.38}$$

where we identified the first z-derivative $\wp'(z,U)$ of the Weierstrass $\wp$ function. Putting everything together we find

$$\mathcal{A} \sim \left(\frac{\alpha'}{2\pi} \sqrt{\frac{2U_2}{T_2}} \right)^3 \partial_{\bar{r}_c} \overline{\wp(r_c, U)} . \tag{5.39}$$

We can arrive at the result (5.39) from (5.36) by a different method. In the integrand, we restore the triple derivative from (5.35) above and recast the sum over winding modes as a genus-two theta function (A.8):

$$\mathcal{A} = \int_0^\infty \frac{dt}{t} \frac{1}{(2\pi^2 t)^2}\, t^5 \left(-\frac{\alpha'}{t} \sqrt{\frac{2U_2}{T_2}} \frac{\partial}{\partial \bar{r}_c} \right)^3 \vartheta[\begin{smallmatrix} \vec{R} \\ \vec{0} \end{smallmatrix}](\vec{0}, itG/2) \ , \tag{5.40}$$

where $\vec{R} = (r_1, r_2)$ is a real 2-component vector with $r_c = r_1 + U r_2$ and G is given by

$$G = \frac{T_2}{U_2} \begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix} \ . \tag{5.41}$$

We now move the derivatives with respect to the complex separation $\bar{r}_c$ outside of the integral over t. Unlike in the previous method, the integral now apparently becomes divergent, so we introduce a UV cutoff Λ in this intermediate step. Fortunately, we know from the previous method that the result is finite, so this is no cause for concern. The remaining integral is now the same as that which occurs in usual gauge coupling renormalisation (i.e. without the three scalar insertions we have here), so we can perform the integral as in [89]. We also collect all numerical constants in $\mathcal{C}$ which we leave undetermined. We obtain

$$\mathcal{A} = -\mathcal{C} \left(\frac{U_2}{T_2} \right)^{3/2} \frac{\partial^3}{\partial \bar{r}_c{}^3} \int_{1/\Lambda^2}^\infty \frac{dt}{t} \vartheta[\begin{smallmatrix} \vec{R} \\ \vec{0} \end{smallmatrix}](\vec{0}, itG/2) = -\mathcal{C} \left(\frac{U_2}{T_2} \right)^{3/2} \frac{\partial^3}{\partial \bar{r}_c{}^3} \left(\Lambda^2 \sqrt{G} - \mathcal{G}(0, r_c) \right) \ , \tag{5.42}$$

with the scalar boson propagator on the torus

$$\mathcal{G}(0, r_c) = -\ln \left| \frac{\vartheta_1(r_c|U)}{\eta(U)} \right|^2 + \frac{2\pi \mathrm{Im}(r_c)^2}{U_2} \ , \tag{5.43}$$

i.e. $\mathcal{G}(0, r_c)$ is the solution of the Laplace equation on the torus with a delta function source and a neutralising background charge. Happily, as expected from our previous calculation, the cutoff Λ disappears from the final result in virtue of the differentiation. Further, we see that the last term of the above expression is proportional to $\bar{r}_c^2$ and will be annihilated by the three derivatives. To make the connection to the previous result in terms of the Weierstrass $\wp$-function note that

$$\wp(z, \tau) = \partial_z^2 \ln \vartheta_1(z|\tau) + \text{const} \ . \tag{5.44}$$

5.3.4 Discussion

Let us begin by summarising the results. If present, the correction to the gauge coupling on D7-branes due to matter on distant D3-branes takes the form:

$$\mathcal{A} = -\mathcal{C} \left(\frac{U_2}{T_2} \right)^{3/2} \frac{\partial^3}{\partial \bar{r}_c{}^3} \ln \left| \frac{\vartheta_1(r_c|U)}{\eta(U)} \right|^2 = \mathcal{C} \left(\frac{U_2}{T_2} \right)^{3/2} \frac{\partial^3}{\partial \bar{r}_c{}^3} \mathcal{G}(0, \bar{r}_c) \ . \tag{5.45}$$

This result is valid for both D7-branes wrapped on bulk cycles and for D7s on a blow-up cycle.

Scaling

First, we will see if we can understand the scaling behaviour of (5.45). For this purpose, let us set the two torus radii equal: $T_2 = R_1 R_2 \sin\theta_U$ and $R_1 \sim R_2 \sim R$. We also understand R to be dimensionless and specify the radius in units of the string length. We find

$$\mathcal{A} \sim \left(\frac{U_2}{T_2}\right)^{3/2} \frac{\partial^3}{\partial \bar{r}_c{}^3} \ln \overline{\vartheta_1(r_c|U)} \sim T_2^{-3/2} \sim R^{-3} \, . \tag{5.46}$$

We can in fact understand the R^{-3} scaling from an analysis in four-dimensional effective supergravity. We have computed the superfield component coupling[4]

$$\frac{1}{2}\frac{\hat{y}_{ijk}}{M_s^3} \int \mathrm{d}^4x \, \sqrt{-g} \, \mathrm{Tr}\left(\phi_i \phi_j \phi_k\right) g^{\mu\nu} g^{\lambda\rho} \mathrm{Tr}\left(F_{\mu\lambda} F_{\nu\rho}\right) \tag{5.47}$$

in the string effective action for canonically normalised fields. Here $\hat{y}_{ijk}$ is a dimensionless parameter and the appearance of three factors of the string scale M_s can be understood for dimensional reasons: they should arise from the calculation through the factors of $\alpha' \sim \ell_s^2 \sim 1/M_s^2$ which we ignored.

The interaction (5.47) comes from the term

$$\int \mathrm{d}^4x \; \mathrm{d}^2\theta \; f_a(C^i) \mathrm{Tr}(W_\alpha W^\alpha) \tag{5.48}$$

in the superspace action. In particular, the Lagrangian term (5.47) arises from corrections to the gauge kinetic function of the form

$$f_a \supset \frac{y_{ijk}}{M_P^3} C^i C^j C^k \, . \tag{5.49}$$

However, we still need to normalise the matter fields. The Kähler metric for matter on D3-branes scales as $K_{i\bar{i}} \sim \mathcal{V}^{-2/3} \sim R^{-4}$ [33]. Performing the canonical normalisation and restricting to flat space we obtain

$$\frac{1}{2} y_{ijk} \frac{R^6}{M_P^3} \int \mathrm{d}^4x \; \mathrm{Tr}(\phi_i \phi_j \phi_k) \; \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) \, . \tag{5.50}$$

To make contact with (5.47) we note that $M_s \sim M_P/\sqrt{\mathcal{V}} \sim M_P/R^3$. We find

$$\frac{1}{2} \frac{y_{ijk}}{R^3 M_s^3} \int \mathrm{d}^4x \; \mathrm{Tr}(\phi_i \phi_j \phi_k) \; \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) \, . \tag{5.51}$$

The R^{-3} scaling of (5.46) is then consistent with supergravity expectations. Further, as the prefactor $T_2^{-3/2} \sim R^{-3}$ is the only part of our result (5.46) which depends on the volume of the compact space, we also learn that the Lagrangian parameter y_{ijk} entering the gauge kinetic function is independent of the volume.

[4] Note that this is Weyl invariant, so it is not affected by the Weyl rescaling we need to go to Einstein frame.

The last fact is also easily understood. The volume of the compact space is controlled by real part of a Kähler modulus T. However, the parameter y_{ijk} arises from a 1-loop correction to the gauge kinetic function f_a. As f_a is holomorphic, it cannot receive 1-loop corrections depending on Kähler moduli T_i: these fields exhibit axionic shift symmetries which are in conflict with the holomorphicity of f_a.

Another comment is in order: in this chapter we examined a cross-coupling between fields on branes which are separated in the compact geometry. The situation is similar to the one in chapter 4, where we examined cross-couplings between D3 matter fields and distant blow-up moduli. However, the outcome is very different: while such cross-couplings were suppressed as $e^{-R^2/\alpha'}$ in chapter 4, we do not observe this exponential suppression in the present case. In contrast, the Lagrangian parameter y_{ijk} is given by

$$y_{ijk} = \mathcal{C} \ (U_2)^{3/2} \frac{\partial^3}{\partial \bar{r}_c{}^3} \ \ln \left| \frac{\vartheta_1(r_c|U)}{\eta(U)} \right|^2 . \tag{5.52}$$

As long as the torus is not particularly degenerate, the above allows for $y_{ijk} \sim \mathcal{O}(1)$. To make a more precise statement, it would be useful to keep track of all numerical prefactors entering $\mathcal{C}$.

Model-dependence

One of the main results of the calculation is that only D3 Yukawa operators of the form

$$C^r_{a,b} C^r_{b,c} C^r_{c,a} \tag{5.53}$$

can correct D7 gauge couplings. If the gauge group on the stack of D7-branes condenses, new Yukawa couplings of the form (5.2) are induced. One of the primary questions of this chapter is whether the flavour structure of the new contributions to Yukawa couplings aligns with that of the tree-level Yukawas. To this end recall that the tree-level superpotential takes the form

$$W^{tree} = \epsilon_{rst} \ \mathrm{Tr} \ (C^r C^s C^t) \ , \tag{5.54}$$

and thus only operators of the form $C^1 C^2 C^3$ arise at tree-level. Here, the superscript refers to the complex internal direction and thus does not specify a single chiral superfield. In fact, for D3-branes at a $\mathbb{Z}_N$ orbifold singularity there are, in principle, N chiral superfields from each complex internal direction. Nevertheless, it is clear that Yukawa couplings of the form $C^r C^r C^r$ with $r = 1, 2, 3$ and operators of the type $C^1 C^2 C^3$ contain different superfields and thus never align. Thus, the flavour structure of new contributions to Yukawa couplings does not match the one of the tree-level couplings:

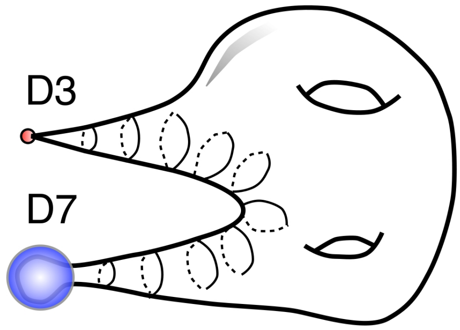

Figure 5.3: Calabi-Yau compactification with D3-branes at a singularity and D7-branes wrapping a small cycle. The singularity and the D7-branes share a homologous two-cycle. This setup appears in LVS constructions.

the result is that the non-perturbative corrections to Yukawa couplings can be sources of flavour-violating effects.

Whether such a correction to the D7-gauge coupling is indeed induced depends on the geometrical setup of D7 and D3-branes. Furthermore, the above combination $C^r C^r C^r$ has to be gauge invariant. In the orbifold model, the conditions for the generation of such couplings can be summarised as follows.

1. *Bulk D7-branes:* This setup is relevant for models based on the KKLT construction.[5] In the orbifold calculation we find, that if D7-branes wrap a bulk cycle, the generation of $C^r C^r C^r$ corrections to the gauge kinetic function is automatic. They arise from untwisted sectors, which are present in all orbifold models. Hence, such corrections are only absent if the coupling $C^r C^r C^r$ is forbidden by gauge invariance.

2. *D7-branes on small cycles:* In models implementing the LVS, the D7-branes wrap a small blow-up cycle. In the orbifold setting the D7-branes on blow-up cycles become D3-branes at an orbifold singularity. The necessary condition for non-zero cross-couplings is that the orbifold exhibits a partially twisted ($\mathcal{N} = 2$) sector. For corrections to the gauge coupling to then be induced, the two stacks can only be separated along an untwisted direction in a partially twisted sector. In homology, this statement is identical to the two stacks of branes sharing a homologous two-cycle. The advantage of rephrasing the condition for non-zero cross-couplings in terms of homology is, that it can be also applied to compactification spaces beyond toroidal orbifold models. Our result then suggests that cross-couplings between D3-branes and D7-branes on blow-up cycles are induced, if the two stacks are

[5] These effects are irrelevant in LVS as they are suppressed by a factor of $\exp(-\mathcal{V}^{2/3})$ in the superpotential.

connected by a homologous two-cycle. The situation is shown in figure 5.3. This generalisation to more general spaces should be checked by a direct calculation, which we leave for future work.

For completeness, we list all supersymmetric orbifolds and denote as "relevant A-term coupling" those operators that receive a non-vanishing contribution in our calculation.

Orbifold group	Twist vector θ	Partially twisted sectors	Relevant A-term coupling	
			D3-D3	D3-D7
$\mathbb{Z}_3$	$\frac{1}{3}(1,1,-2)$			$(C^3)^3$
$\mathbb{Z}_4$	$\frac{1}{4}(1,1,-2)$	Z^2 under θ^2		
$\mathbb{Z}_6$	$\frac{1}{6}(1,1,-2)$	Z^3 under θ^3	$(C^3)^3$	$(C^3)^3$
$\mathbb{Z}_6'$	$\frac{1}{6}(1,-3,2)$	Z^2 under θ^2 and θ^4, Z^3 under θ^3	$(C^3)^3$	$(C^3)^3$
$\mathbb{Z}_7$	$\frac{1}{7}(1,2,-3)$			
$\mathbb{Z}_8$	$\frac{1}{8}(1,3,-4)$	Z^3 under θ^2		
$\mathbb{Z}_8'$	$\frac{1}{8}(1,-3,2)$	Z^3 under θ^4		
$\mathbb{Z}_{12}$	$\frac{1}{12}(1,-5,4)$	Z^3 under θ^3	$(C^3)^3$	$(C^3)^3$
$\mathbb{Z}_{12}'$	$\frac{1}{12}(1,5,-6)$	Z^3 under θ^n for $n=2,4,6,8,10$.		

One important difference arises for $\mathbb{Z}_3$ orbifolds, which are popular in model-building. As there are no partially twisted sectors, no new Yukawa couplings can be generated for D7-branes on blow-up cycles in this case.

5.4 Summary

In this chapter we investigated certain non-perturbative corrections to Yukawa couplings in type IIB orbifold models in which the visible sector is realised as a stack of D3-branes on an orbifold singularity. For non-perturbative effects arising from gaugino condensation on a stack of distant D7-branes, these corrections to Yukawa couplings can be computed as one-loop threshold corrections to the gauge kinetic function of the D7-branes. We calculated the corresponding open string cylinder amplitude both in the case of D7-branes wrapping a large 'bulk' cycle in the internal dimensions, and in the case of D7-branes wrapping a small blow-up cycle, which in the orbifold limit corresponds to a stack of D3-branes at an orbifold singularity. If present, the result is the same independent of the embedding of the D7-brane. Despite arising from an interaction between distant brane stacks, we find that the new contributions to Yukawa couplings are not suppressed by the bulk volume.

It was shown in [42] that if the induced corrections to Yukawa couplings exhibit a non-trivial flavour structure, constraints on flavour-changing neutral currents may severely constrain the moduli stabilisation sector. In the models we consider in this chapter, we verify that the flavour structure of the corrections to Yukawa couplings does not align with the flavour structure of the tree-level Yukawa couplings. Correspondingly, the non-perturbative corrections to Yukawa couplings are potential sources for flavour-violating effects. We also identified a set of conditions under which such corrections are expected to be generated, independent of the finer details of the model. For new contributions to Yukawa couplings to be generated, the stack of D7-branes giving rise to the non-perturbative effects has to either wrap a bulk four-cycle or share a homologous two-cycle with the stack of visible sector D3-branes.

While we used conformal field theory to arrive at our results in this chapter, we expect that the calculation should also be possible by adopting a geometrical closed string approach. From this perspective corrections to the D7 gauge coupling are studied geometrically by analysing changes to the volume of the four-cycle wrapped by the D7-branes. Using this method the authors of [90] determined corrections to the D7 gauge coupling due to the presence of D3-branes. For our analysis it would be interesting to also allow for D3 matter fluctuations. The power of the closed string approach lies in the fact, that it can be applied to background geometries different from toroidal orbifolds. For example, the authors of [90] studied cross-couplings between D7 and D3-branes in warped throat geometries. Thus, by repeating our analysis using this closed string approach, we could extend our examination beyond orbifold constructions. This would be a first step towards a generalisation of our analysis to more realistic string compactifications.

Chapter 6

Aside — kinetic mixing of U(1)s in string models with D3-branes

Here we will consider another case, where $\mathcal{N} = 2$ sectors in an orbifold calculation give rise to non-negligible interactions between distant brane stacks. In particular, we will examine the possibility of kinetic mixing between two $U(1)$ factors in toroidal orbifolds, with one $U(1)$ realised in a visible and the other in a hidden sector. This chapter is based on paper [9].

6.1 Motivation

In (semi-)realistic string models there are often a multitude of $U(1)$ factors present. This is particularly true for models including D-branes. Although some of these $U(1)$s become massive and decouple, there are no theoretical obstructions to having multiple massless $U(1)$s present in a consistent string model. These $U(1)$s can inhabit both the visible and hidden sectors of the model. This is clearest in the context of models involving branes at singularities, where gauge groups can be geometrically separated in the compact space. Given the presence of several massless $U(1)$s in a theory, it is a natural question to ask whether kinetic mixing can occur in such a setup. This possibility can also be phenomenologically significant if one of the $U(1)$s participating in kinetic mixing is the weak hypercharge of the Standard Model.

In a theory with two $U(1)$ factors the low-energy effective Lagrangian density can contain the following terms:

$$\mathcal{L} \supset -\frac{1}{4g_a^2} F_{\mu\nu}^{(a)} F_{(a)}^{\mu\nu} - \frac{1}{4g_b^2} F_{\mu\nu}^{(b)} F_{(b)}^{\mu\nu} + \frac{\chi_{ab}}{2g_a g_b} F_{\mu\nu}^{(a)} F_{(b)}^{\mu\nu} + m_{ab}^2 A_\mu^{(a)} A_{(b)}^\mu. \tag{6.1}$$

The kinetic mixing term is a renormalisable operator and appears with the parameter χ_{ab} which can be generated at an arbitrarily high energy scale. We will be considering the effect of string scale physics on the terms in (6.1) and will only be interested in cases where string scale contributions to m^2_{ab} are vanishing. The main focus of this chapter will be to determine χ in a consistent string model.

The phenomenological interest in kinetic mixing is twofold. If hypercharge mixed kinetically with another massless $U(1)$ from a hidden sector we could expect the existence either of millicharged particles carrying small amounts of electric charge or alternatively Standard Model particles that are millicharged under exotic $U(1)$s [91–94]. $U(1)$s with weak couplings to Standard Model particles have also been of interest for models of dark matter explaining excess positron production in the galaxy [95]. The particle physics phenomenology of such weakly coupled $U(1)$s has been explored in [96, 97] and many subsequent works. Searching for kinetic mixing is also subject of experimental activity – for a recent review see [98].

In the past kinetic mixing in string theory has been studied in the context of type II and heterotic string theories and F-theory [99–111].[1] In previous work it was pointed out that there is tension between the possibility of kinetic mixing, the vanishing of gauge theory anomalies and the absence of mass mixing [103]. This observation provides the impetus for the work in this chapter: here we wish to determine whether the tension can be resolved in simple orbifold models in type IIB string theory with D3-branes at orbifold singularities. In particular, we will explore the possibility of kinetic mixing between visible and hidden $U(1)$s in these setups.

6.2 Kinetic mixing for D3-branes at singularities

6.2.1 Setting up the calculation

A priori, string models including D-branes give rise to a large number of $U(1)$ factors. This is also true for D3-branes at orbifold singularities, which support a gauge theory with gauge group

$$U(n_0) \times U(n_2) \times U(n_3) \times \ldots \times U(n_{N-1}) \; . \tag{6.2}$$

Each $U(n_i)$ gives rise to $SU(n_i) \times U(1)$ and there are potentially many $U(1)$s.[2] The gauge theory on the stacks of D3-branes will typically suffer from anomalies, unless certain conditions on the gauge groups are satisfied. For the case of both Abelian and non-Abelian factors in field theory, anomalies arise from triangle diagrams with external states $SU(n_i) \times SU(n_j)^2$, $U(1)_\alpha \times SU(n_j)^2$ and $U(1)_\alpha \times U(1)^2_\beta$. In practice, many of the $U(1)$ factors in D-brane constructions will turn out to be anomalous and not appear as

[1]See [98] for more references, in particular publications on phenomenological implications of kinetic mixing and the possibilities for detection.

[2]We reviewed the massless spectrum for D3-branes at orbifold singularities in section 2.3.

factors in the gauge group.

The three types of anomaly from above have corresponding interpretations in string models:

- Cubic non-Abelian anomalies should cancel as they correspond to inconsistencies. The consistency condition of string theory in the form of local tadpole cancellation automatically implies the vanishing of cubic non-Abelian anomalies [112, 113].

- Anomalies involving $U(1)$ factors are not fatal in string theory, as it automatically provides a mechanism for their cancellation. The field theory anomaly for $U(1)_\alpha \times G_j^2$ is generated in string theory by one-loop diagrams with all gauge bosons on the same boundary. At the same level in perturbation theory, string theory also allows for a cylinder diagram with the gauge boson for $U(1)_\alpha$ inserted on one boundary, and the two gauge bosons of G_j inserted on the other boundary. This is a four-dimensional version of the Green-Schwarz (GS) mechanism [114] and ensures the vanishing of these anomalies [115].

If a $U(1)$ is anomalous, the presence of the GS mechanisms implies the existence of a coupling between the $U(1)$ and a Ramond-Ramond (RR) two-form field:

$$\mathrm{Tr}(\gamma_k \lambda_i)\ C_2^{(k)} \wedge F_{U(1)_i}\ , \tag{6.3}$$

where λ_i is a Chan-Paton factor and γ_k is the Chan-Paton twist matrix in a twisted sector. Couplings of this form lead to the generation of a mass for the $U(1)$ gauge boson via the Stückelberg mechanism. Hence anomalous $U(1)$s automatically receive masses and drop out of the gauge group. While the anomaly of a $U(1)$ implies the presence of (6.3), non-anomalous $U(1)$ are not automatically massless. They can also couple to to RR two-form fields (in the $\mathcal{N} = 2$ sector) and thus become massive and leave the gauge group.

We will now proceed to study kinetic mixing in toroidal orbifolds with D3-branes at orbifold singularities. Visible and hidden sectors are modelled by stacks of branes which are separated in the internal directions. The above analysis of $U(1)$ factors in these setups suggests the following strategy

1. Ensure the consistency of the non-Abelian gauge theory on every stack of D3-branes by cancelling local tadpoles. Next, identify all non-anomalous $U(1)$s and select the massless ones. These will be candidates for $U(1)$s participating in kinetic mixing.

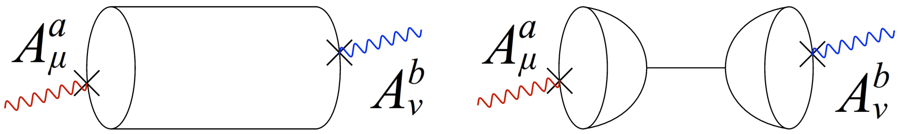

Figure 6.1: Relevant string diagrams for mixing between hidden $U(1)$s. The left diagram gives rise to both kinetic and mass mixing. The limit $\ell = 1/t \to \infty$ is shown on the right and gives the contribution to mass mixing only.

2. Cancel $\mathcal{N} = 2$ tadpoles, which are sourced non-locally. These have to be cancelled across branes sharing a homologous two-cycle, which will lead to further constraints on the allowed gauge groups.

3. Examine kinetic mixing by evaluating the corresponding string diagram connecting the visible and hidden sector.

Relevant string amplitude

Before we begin we will collect the relevant string amplitudes: here we need to evaluate a cylinder with two gauge boson insertions. As we are interested in mixing between $U(1)$s that do not have a tree-level coupling, we will consider diagrams with the two gauge bosons inserted on opposite boundaries. As both mass mixing as well as kinetic mixing arise from couplings to RR states, they have their origin in the same string diagram 6.1. Mass terms arise from couplings to massless RR-fields. At the level of the string amplitude this corresponds to the closed string IR limit, where the cylinder modular parameter approaches $\ell = 1/t \to \infty$. Kinetic mixing will arise from integrating over the whole range for t. In [9] we evaluated this amplitude via the background field method for all orbifold sectors. Here we will be able to use a result from the previous chapter. As it turns out, we will only need the result for the $\mathcal{N} = 2$ sectors explicitly. In this case we calculated the correlator of two gauge bosons in section 5.3.1 and the result can be extracted from (5.35). There the gauge bosons were inserted on the same boundary, whereas we insert them on opposite boundaries. This only affects the Chan-Paton traces, and so we can extract the relevant result from (5.35). For two gauge bosons from gauge groups $U(1)_a$ and $U(1)_b$ we find

$$\langle A^a A^b \rangle_{\mathcal{N}=2} = \mathrm{Tr}(\lambda^a \gamma_{\theta^k})\ \mathrm{Tr}(\lambda^b \gamma^*_{\theta^k})\ \epsilon^a_\mu \epsilon^b_\nu (\eta^{\mu\nu} k^2 - k^\mu k^\nu) \sum_{m,n} \int_0^\infty \frac{\mathrm{d}t}{t}\, e^{-\frac{\pi T_2}{\alpha' U_2}|m+Un+r_c|^2 t} \,. \tag{6.4}$$

The sum is over winding modes which wrap the two-torus which is left untwisted in the $\mathcal{N} = 2$ sector. Also, T_2 and U_2 are the imaginary parts of the Kähler and complex structure moduli of the two-torus wrapped and r_c is a dimensionless separation between the brane stacks supporting the two gauge theories. The amplitude is evaluated on-shell, such that $k^2 = 0$. Nevertheless, it is instructive to keep the kinematic prefactor, which in fact vanishes. The kinetic mixing parameter is then given by the part of the correlator multiplying the kinematic prefactor.

To check for the existence of mass mixing we need to study the limit $\ell = 1/t \to \infty$. To extract the dominant contribution, it will be useful to perform a Poisson resummation on the sum over winding modes. We find that the leading term diverges linearly and the correlator takes the form

$$\langle A^a A^b \rangle_{\mathcal{N}=2} = \text{Tr}(\lambda^a \gamma_{\theta^k})\ \text{Tr}(\lambda^b \gamma^*_{\theta^k})\ \epsilon^a_\mu \epsilon^b_\nu (\eta^{\mu\nu} k^2 - k^\mu k^\nu)\ \int_{\ell'}^{\infty} \mathrm{d}\ell\ \frac{\alpha'}{T_2} \left(1 + \mathcal{O}\left(e^{-\frac{\pi\alpha'\ell}{2T_2U_2}}\right)\right) \ . \tag{6.5}$$

This divergence is expected: in the limit $\ell = 1/t \to \infty$ the two gauge bosons couple via a massless closed string propagator, which contributes a momentum pole $1/k^2$. This poles cancels the momentum-dependence of the prefactor and the amplitude contributes to a mass term for the two gauge bosons. However, as we are evaluating the amplitude on shell, this propagator leads to a divergence. Hence the behaviour of (6.5) is consistent with the generation of a mass mixing parameter.

With the two expressions (6.4) and (6.5) at hand, we can begin our analysis of kinetic mixing of massless $U(1)$s in models with D3-branes at orbifold singularities. To proceed we need to identify consistent setups of branes and determine non-anomalous $U(1)$s. This is the subject of the next section.

6.2.2 Tadpole cancellation, anomalous and massless $U(1)$s

We begin by analysing the cancellation of local (twisted) tadpoles for D3-branes at $\mathbb{Z}_N$ orbifold singularities. Branes at such a singularity transform in the regular representation of the group. The appropriate transformation is encoded in the twist matrix

$$\gamma_\theta = \text{diag}\left(\mathbb{1}_{n_0}, e^{\frac{2\pi i}{N}} \mathbb{1}_{n_1}, \ldots, e^{\frac{2\pi i(N-1)}{N}} \mathbb{1}_{n_{N-1}}\right) \ , \tag{6.6}$$

where n_i are integers such that $\sum_{i=0}^{N-1} n_i = n$, where n is the total number of branes. This will then give rise to a gauge group on the branes as shown in (6.2). The cancellation of local tadpoles can then be written as a constraint on the allowed values for n_i. For every $\mathcal{N} = 1$ sector of the orbifold we require [65]:

$$\left(\prod_{r=1}^{3} 2\sin\pi\theta_r^{\ \mathcal{N}=1}\right)\ \text{Tr}(\gamma_{\theta,\ \mathcal{N}=1}) = 0 \ . \tag{6.7}$$

Now we move on and study the Abelian groups on the stacks of D3s. Using the $U(1)$s associated with the non-Abelian group factors as a basis, the generator of a general $U(1)$

on this brane stack can be written as

$$Q_c = \sum_{j=0}^{N-1} c_j \frac{Q_{n_j}}{n_j} \ . \tag{6.8}$$

The Chan-Paton factors λ for the $U(1)$ gauge bosons are simply diagonal matrices with the components of Q_c as their entries. The coefficients c_j for an anomaly-free $U(1)$ factor can be found as [65, 115]:

$$c_j = \frac{1}{N}\sum_{k=0}^{N-1} e^{-2\pi i jk} r_k \quad \text{where} \quad \left(\prod_{r=1}^{3} 2\sin \pi k\theta_r\right) r_k = 0 \ . \tag{6.9}$$

The above conditions allow for at least one solution in the form $c_j = \text{const}$. Thus, for all orbifolds there is a always an anomaly-free combination given by [65]

$$Q_{diag} = \frac{1}{N_{diag}} \sum_{j=0}^{N-1} \frac{Q_{n_j}}{n_j} \ , \tag{6.10}$$

where we introduced a normalisation such that $\text{Tr}(\lambda^2) = 1$. Further, if the orbifold has an $\mathcal{N} = 2$ sector, one direction is left untwisted and $\sin \pi k\theta = 0$ in this case. Correspondingly, the condition on one r_k is relaxed and we find one additional non-anomalous $U(1)$.

Next, it needs to be checked whether the non-anomalous $U(1)$s remain massless. This amounts to checking whether couplings of the form $C_2 \wedge F$ are present. Fully twisted sectors do not contribute to such couplings for non-anomalous $U(1)s$. However, we need to check explicitly in partially twisted sectors. The existence of a coupling of a $U(1)$ gauge boson to a massless RR-form field in a partially twisted sector can be determined by evaluating a cylinder correlator $\langle A^a A^a \rangle_{\mathcal{N}=2}$ in the limit $t \to 0$. This is exactly of the form as (6.5). To ensure that the $U(1)$s remain massless, we thus require that $\lim_{t\to 0}\langle A^a A^a \rangle_{\mathcal{N}=2}$ gives a vanishing result. However, given the form of (6.5), the amplitude can only vanish through the Chan-Paton factors. For massless $U(1)$s we thus require

$$\text{Tr}(\lambda^a \gamma_{\theta,\ \mathcal{N}=2}) = 0 \ . \tag{6.11}$$

In summary, our candidate $U(1)$s for our study of kinetic mixing are thus given by non-anomalous U(1)s which also satisfy (6.11). It is at this point at the latest where we should notice a conflict between kinetic mixing and the absence of a mass for the gauge bosons. For the gauge bosons to remain massless, we require the vanishing of the trace $\text{Tr}(\lambda^a \gamma_{\theta,\ \mathcal{N}=2})$. However, as the expression responsible for kinetic mixing (6.4) is proportional to the same trace, non-zero kinetic mixing is incompatible with $\text{Tr}(\lambda^a \gamma_{\theta,\ \mathcal{N}=2}) = 0$. We are thus left with a "no-go theorem", which was also pointed out in [103]: either

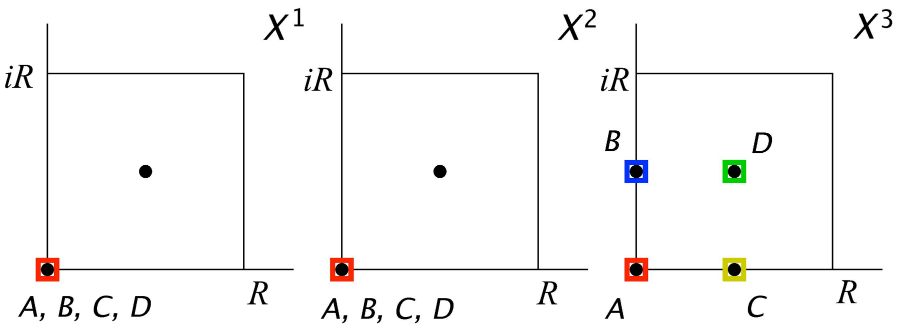

Figure 6.2: $\mathbb{T}^6/\mathbb{Z}_4$ orbifold setup with twist $\theta = \frac{1}{4}(1, 1, -2)$. We placed four stacks of D3-branes at orbifold fixed points A, B, C and D, which are only separated from one another along the third two-torus.

$U(1)$s are massless, but cannot participate in kinetic mixing, or, if they can mix, they will also become massive. For the non-anomalous $U(1)$s identified earlier, it is easy to show that $U(1)_{diag}$ remains massless. For all other non-anomalous $U(1)$ factors the trace (6.11) is finite and they will receive Stückelberg masses. The conclusion from this section thus is, that kinetic mixing of massless $U(1)$s from a single brane stack is impossible.

6.2.3 Resolving the puzzle

Our resolution follows the suggestion from [103], where kinetic mixing was studied in the context of D6-branes in type IIA string theory. If instead of considering $U(1)$s from individual brane stacks, we worked with $U(1)$s that have support on different separated stacks of branes, the mixing amplitude would consist of a sum over brane stacks of correlators of the form (6.4). In the limit relevant to mass mixing (6.5), the individual amplitudes are insensitive to the brane separations and only differ in the traces over Chan-Paton factors. A situation is then conceivable where the sum over different stacks cancels in the limit $t \to 0$, but becomes finite when brane separations become important for $t > 0$. Then kinetic mixing can occur while mass mixing is absent.

We demonstrate this explicitly by giving an example set in the toroidal orbifold $\mathbb{T}^6/\mathbb{Z}_4$ with twist $\theta = \frac{1}{4}(1, 1, -2)$. This space has 16 orbifold singularities. We place D3-branes at four fixed points A, B, C and D as shown in figure 6.2, which are separated from one another along the third two-torus. The twists θ and θ^3 define $\mathcal{N} = 1$ sectors, whereas θ^2 describes a $\mathcal{N} = 2$ sector. On every stack we define a twist matrix

$$\gamma_\theta = \text{diag}(\mathbb{1}_{n_0}, i\mathbb{1}_{n_1}, -\mathbb{1}_{n_2}, -i\mathbb{1}_{n_3}) \ . \qquad (6.12)$$

where the numbers n_i determine the gauge group as $U(n_0) \times U(n_1) \times U(n_2) \times U(n_3)$. Here, we choose the following setup, where M, N, K and L are integers:

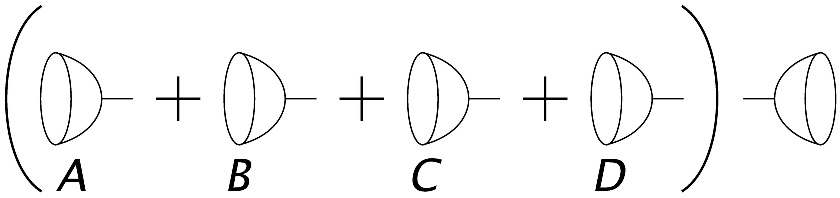

Figure 6.3: Contributions to partially twisted tadpoles from all brane stacks.

stack	n_0	n_1	n_2	n_3
A	M	N	M	N
B	N	M	N	M
C	K	L	K	L
D	L	K	L	K

We now show that this configuration satisfies all consistency conditions and indeed leads to kinetic mixing in the absence of mass mixing. We will go through all conditions successively.

Fully twisted tadpoles. The cancellation of fully twisted tadpoles (6.7) has to be satisfied locally at every individual stack. This is indeed the case due to $n_0 = n_2$ and $n_1 = n_3$. For example, on stack A we find $\mathrm{Tr}(\gamma_\theta^A) = M + iN - M - iN = 0$.

Partially twisted tadpoles. Partially twisted tadpoles contribute to the left-hand side of diagrams shown in figure 6.3. They arise in cylinder diagrams in partially twisted sectors in the limit $\ell = 1/t \to \infty$. Their cancellation has to occur over all brane stacks A, B, C and D. In fact, the right-hand side does not need to be empty in figure 6.3, so we insert two gauge boson vertex operators. Up to Chan-Paton factors the result is given by a sum of amplitudes of the form (6.5)

$$\mathcal{A}_{\mathcal{N}=2} \propto \sum_{I=A,B,C,D} \mathrm{Tr}(\gamma_{\theta^2}^I)\ \mathrm{Tr}(\lambda^2 \gamma_{\theta^2}^J)^* \int_{\ell'}^{\infty} \mathrm{d}\ell\ . \tag{6.13}$$

This then cancels in virtue of

$$\begin{aligned} & \mathrm{Tr}(\gamma_{\theta^2}^A) + \mathrm{Tr}(\gamma_{\theta^2}^B) + \mathrm{Tr}(\gamma_{\theta^2}^C) + \mathrm{Tr}(\gamma_{\theta^2}^D) \qquad (6.14)\\ =\ & (M - N + M - N) + (N - M + N - M) + (K - L + K - L) + (L - K + L - K) \\ =\ & 0\ . \end{aligned}$$

Massless U(1)s. Now we can construct massless $U(1)$s which have support over more than one stack. As the $\mathbb{Z}_4$ orbifold has one $\mathcal{N} = 2$ sector, there is one more non-anomalous $U(1)$ on each brane stack beyond the universal one in (6.10). Choosing it to be orthogonal to Q_{diag} gives:

$$Q_{tw} = \frac{1}{\mathcal{N}_{tw}} \left(\frac{Q_0 + Q_2}{n_1} - \frac{Q_1 + Q_3}{n_0} \right)\ , \tag{6.15}$$

where $\mathcal{N}_{tw}$ is the normalisation. The fact that Q_{tw} is anomaly-free implies that it does not couple as $C_2 \wedge F$ in $\mathcal{N} = 1$ sectors. However, it will get a mass from such couplings in the $\mathcal{N} = 2$ sector. In particular, for Q^A_{tw} defined on stack A the coupling is proportional to

$$\mathrm{Tr}\left(\lambda_{tw}\gamma_{\theta^2}\right) = \frac{2}{\mathcal{N}_{tw}}\left(\frac{M}{N} + \frac{N}{M}\right) \neq 0 \ . \tag{6.16}$$

To construct massless $U(1)$s, we define the following combinations, which have support over two brane stacks each:

$$Q_X = \frac{1}{2}\left(Q^A_{tw} - Q^B_{tw}\right) \tag{6.17}$$

$$Q_Y = \frac{1}{2}\left(Q^C_{tw} - Q^D_{tw}\right) \ . \tag{6.18}$$

These combinations are massless, as the couplings $C_2 \wedge F$ cancel between the two contributing $U(1)$s. This can be shown explicitly for Q_X. The contribution to mass mixing is then given by

$$\lim_{\ell\to\infty}\left(\mathcal{A}^{AA}_{\mathcal{N}=2} + \mathcal{A}^{BB}_{\mathcal{N}=2} - \mathcal{A}^{AB}_{\mathcal{N}=2} - \mathcal{A}^{AB}_{\mathcal{N}=2}\right) \ , \tag{6.19}$$

where each $\mathcal{A}^{IJ}_{\mathcal{N}=2}$ corresponds to an amplitude of the form (6.5). The superscripts labels the stacks involved. Here, we get a perfect cancellation, as Q^A_{tw} and Q^B_{tw} have the same coupling to $C_2 \wedge F$ given by the trace (6.16).

Kinetic mixing. Last, we can examine kinetic mixing between Q_X and Q_Y by evaluating the amplitude (6.4), with Q_X inserted on one boundary and Q_Y on the other. Again, the result will be a sum over different stacks. The kinetic mixing parameter is then given by:

$$\begin{aligned}\chi &\propto \left(\mathcal{A}^{AC}_{\mathcal{N}=2} + \mathcal{A}^{BD}_{\mathcal{N}=2} - \mathcal{A}^{AD}_{\mathcal{N}=2} - \mathcal{A}^{BC}_{\mathcal{N}=2}\right) \\ &\propto \left(\frac{M}{N} + \frac{N}{M}\right)\left(\frac{K}{L} + \frac{L}{K}\right)\sum_{m,n}\int_0^\infty \frac{\mathrm{d}t}{2t}\left(e^{-\frac{\pi R^2}{\alpha'}[m^2+(n+\frac{1}{2})^2]t} - e^{-\frac{\pi R^2}{\alpha'}[(m+\frac{1}{2})^2+(n+\frac{1}{2})^2]t}\right) \ .\end{aligned} \tag{6.20}$$

Here we wrote the winding sum for a square torus with radii $R_1 = R_2 = R$. We can see that mass mixing is absent by examining the limit $t \to 0$. As shown in (6.5), the sums over winding modes become insensitive to the separation of stacks and the two terms in the above result cancel. For finite t the two winding sums are different and we get non-zero contributions to the kinetic mixing parameter.

The above is a simple example to show that kinetic mixing is in principle possible in setups with D3-branes at orbifold singularities. Most importantly, we showed that it can arise for models that satisfy the relevant consistency conditions. However, with the inclusion only of D3-branes there still remains an uncancelled overall $\mathcal{N} = 4$ tadpole. As

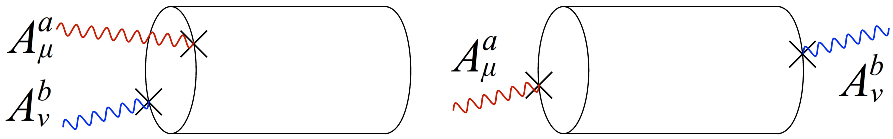

Figure 6.4: String diagrams contributing to kinetic mixing of U(1)s defined across multiple orbifold fixed points.

$\mathcal{N}=4$ sectors are conformal, they do not contribute to the running of gauge couplings and hence this tadpole does not affect our results. In the following, we want to leave the $\mathbb{Z}_4$ orbifold and discuss the possibility of kinetic mixing in setups with D3-branes at orbifold singularities more generally.

General considerations

A schematic calculation will enable us to establish conditions for kinetic mixing to occur in models with D3-branes at orbifold singularities. For simplicity we consider a single $\mathcal{N}=2$ sector leaving one complex plane invariant. We define two massless Abelian groups $U(1)_a$ and $U(1)_b$ that are each linear combinations of $U(1)$s from multiple orbifold fixed points. In general $U(1)_a$ and $U(1)_b$ will share fixed points and hence both diagrams in figure 6.4 can contribute to the amplitude. The contributions come solely from the $\mathcal{N}=2$ sector of the orbifold as the $\mathcal{N}=1$ sectors have to vanish to ensure tadpole cancellation and non-anomalous $U(1)$s.

There are two different contributions to the vacuum amplitude: one comes from strings that start and end on the same stack of D3-branes; the second from strings that start and end at different orbifold fixed points and whose winding sums depend on the complex separation of the singularities in the compact space. Schematically, the amplitude is:

$$\mathcal{A}_{\mathcal{N}=2} = \int \frac{\mathrm{d}t}{t} \sum_{n,m} \left[\sum_i C_i \, e^{-\frac{\pi T_2}{\alpha' U_2}|Un+m|^2 t} + \sum_{i\neq j} C_{ij} \, e^{-\frac{\pi T_2}{\alpha' U_2}|z_{ij}+Un+m|^2 t} \right] , \tag{6.21}$$

where the indices i and j run over orbifold fixed points contributing to the $U(1)$s and z_{ij} are their (dimensionless) complex separations. The coefficients C_i and C_{ij} are numerical prefactors that derive from the traces over the Chan-Paton states and are hence independent of t.

As we are interested in kinetic mixing between hidden $U(1)$s we need to ensure that there is no tree-level coupling between the two Abelian gauge bosons. In the string calculation this translates into the requirement that the above amplitude should vanish in the open string IR regime $t \to \infty$. In this limit only strings localised at single fixed points contribute:

$$\mathcal{A}_{\mathcal{N}=2} \underset{t\to\infty}{\to} \int \frac{\mathrm{d}t}{t} \sum_i C_i. \tag{6.22}$$

The divergence indicates a tree-level coupling between the $U(1)$s and to remove it we have to arrange the numbers of fractional branes across singularities accordingly so that the coefficient $\sum_i C_i$ vanishes. This conclusion here is the same whether the U(1)s share a fixed point or not.

Furthermore, we must analyse the string amplitude in the open string UV limit $t \to 0$. In this regime the diagram with both generators on the same boundary is proportional to tadpoles, whereas the diagram with the generators on opposite boundaries will depend on the Green-Schwarz couplings of the U(1)s with a partially twisted RR-field. Both diagrams then have to vanish: tadpole cancellation and the requirement of massless $U(1)$s prohibit any contributions. After Poisson resumming the string amplitude and re-expressing it using the modular parameter $\ell = 1/t$ we arrive at:

$$\mathcal{A}_{\mathcal{N}=2} \underset{l\to\infty}{\to} \int \mathrm{d}\ell \, \frac{\alpha'}{T_2} \left[\sum_i C_i + \sum_{I\neq J} C_{ij} \right]. \tag{6.23}$$

We already know that for hidden U(1)s we must have $\sum_i C_i = 0$. For the above amplitude to vanish for large ℓ we are thus forced to arrange our model such that $\sum_{i\neq j} C_{ij} = 0$ for strings starting and ending on different stacks. The vacuum amplitude must therefore take the form

$$\mathcal{A}_{\mathcal{N}=2} = \int \frac{\mathrm{d}t}{t} \sum_{n,m} \left[\sum_{i\neq j} C_{ij} \, e^{-\frac{\pi T_2}{\alpha' U_2}|z_{ij}+Un+m|^2 t} \right] \tag{6.24}$$

where $\sum_{i\neq j} C_{ij} = 0$. The construction of models with kinetic mixing between hidden and massless U(1)s now depends crucially on the magnitude of the complex separations z_{ij}.

If the complex separations z_{ij} all have the same magnitude then the winding sums are identical and can be factored out: the condition $\sum_{i\neq j} C_{ij} = 0$ then forces the amplitude to vanish identically. Only if some of the complex separations z_{ij} have different magnitudes then the winding sums do not cancel in the regime of intermediate t. This cancellation happens in the highly symmetrical orbifold setups based on $\mathbb{Z}_3$ and $\mathbb{Z}_6$, which fix the complex structure of all three two-tori. Otherwise, the integral gives a finite contribution to kinetic mixing despite vanishing in the IR and UV limits. Concluding, we see that to construct models with kinetic mixing the orbifold must have at least one $\mathcal{N} = 2$ sector and the presence of orbifold fixed points not all equidistant from one another.

6.3 Result and discussion

Above, we showed that kinetic mixing between massless $U(1)$s is in principle possible in setups with D3-branes at orbifold singularities, as long as the Abelian groups have support on more than one brane stack. To this end we constructed an explicit example based on the $\mathbb{Z}_4$ orbifold. Here, we summarise our results by giving a general expression for the kinetic mixing parameter in these setups. Hence, we perform the integral over the sum of winding modes in (6.4) as we did for (5.42) in chapter 5. Then the parameter for kinetic mixing between $U(1)_a = \sum_i U(1)_i$ and $U(1)_b = \sum_j U(1)_j$ takes the form

$$\chi_{ab} = \frac{g_a g_b}{4\pi^2} \sum_{\substack{i,j \\ i \neq j}} C_{ij} \left(\ln \left| \frac{\vartheta_1(z_{ij}, U)}{\eta(U)} \right|^2 - \frac{2\pi \mathrm{Im}(z_{ij})^2}{U_2} \right) . \tag{6.25}$$

Here C_{ij} encodes model-dependent contributions: these include traces over Chan-Paton factors and a factor $1/N$ coming from the orbifold projection. We note that the result does not depend on the Kähler modulus and thus the volume of the torus wrapped. This is consistent with type IIB expectations: holomorphy and the axionic shift symmetries prohibit the appearance of Kähler moduli in 1-loop corrections to the gauge coupling. We can also use the above to get a numerical estimate for the strength of kinetic mixing. As long as the torus wrapped by the winding strings is not particularly degenerate, the winding modes contribute a factor of $\mathcal{O}(1)$. The remaining parameters depend on the model, specifically, the numbers of D-branes and thus the gauge groups present. We choose phenomenologically reasonable gauge groups to arrive at a kinetic mixing parameter in the range $10^{-2} \lesssim \frac{\chi}{g_a g_b} \lesssim 10^{-1}$. This concludes our analysis of kinetic mixing in toy models based on D3-branes at orbifold singularities.

The phenomenologically most interesting case is kinetic mixing of the standard model hypercharge $U(1)_Y$ with another hidden Abelian gauge group. To this end one needs to construct a realistic visible sector, which is beyond the scope of this thesis. In a first step towards an examination of kinetic mixing in more realistic setups, one could generalise our analysis by also including D7-branes and orientifold planes.

Chapter 7

Summary and outlook: soft terms in the Large Volume Scenario

The structure of soft terms in type IIB string theory is much more intricate than one would expect from a field theory analysis alone. This thesis constitutes one further step towards a better understanding of both the scale and flavour structure of soft terms in the context of type IIB string theory. In this work, we focussed on soft scalar masses and soft A-terms which arise through gravity mediation in string models with low energy supersymmetry breaking. In particular, our results are most relevant for models implementing the scheme of moduli stabilisation known as the Large Volume Scenario (LVS). To summarise our results more succinctly, it is useful to recall that gravity-mediated soft scalar masses and A-terms are largely determined by the following set of functions in the supergravity Lagrangian: the moduli Kähler potential $\hat{K}$, the Kähler matter metric $\tilde{K}_{\bar{\alpha}\beta}$ and the holomorphic Yukawa couplings $Y_{\alpha\beta\gamma}$. Our results then concern corrections to these Lagrangian parameters which affect the soft terms.

Chapters 1 to 3 of this thesis were introductory. There we laid out the context of this work and introduced necessary calculational techniques.

In chapter 4 we examined correlations between the moduli-dependence of the moduli Kähler potential and Kähler matter metric. In particular, if the two functions are related as $\tilde{K}_{\alpha} \sim e^{\hat{K}/3}$, soft scalar masses and soft A-terms vanish exactly (up to non-perturbative contributions to the A-terms). The condition $\tilde{K}_{\alpha} \sim e^{\hat{K}/3}$ is equivalent to the physical Yukawa couplings being independent of supersymmetry breaking moduli, which allows for a direct test in string perturbation theory. In this work, we focussed on the dependence on blow-up moduli τ_s, which control the size of small cycles in the compactification geometry. In realisations of the LVS, blow-up moduli of cycles distant to the cycle supporting the visible sector acquire F-terms and thus break supersymmetry. We checked the dependence of physical Yukawa couplings on supersymmetry breaking blow-up moduli τ_s by calculating a correlation function $\langle \tau_s \tau_s \ldots \tau_s \; \psi\psi\phi \rangle$ in an orbifold setting. We modelled the visible sector by D3-branes at an orbifold singularity, while the relevant blow-up moduli can be identified with twisted closed string modes at distant orbifold singularities. We find that at leading order in g_s all correlation functions of the form $\langle \tau_s \tau_s \ldots \tau_s \; \psi\psi\phi \rangle$ vanish up to non-perturbative terms in α'. For large compactification volumes, as observed in the LVS, such terms are negligibly small. Our results imply that – as far as the dependence on supersymmetry breaking blow-up moduli is concerned – at leading order in g_s the structure $\tilde{K}_\alpha \sim e^{\hat{K}/3}$ survives to all orders in α' and is only broken by terms non-perturbative terms in α'. By giving a vacuum expectation value to the blow-up moduli, we can extend our result from the singular orbifold to a smooth space with a geometry not unlike the Swiss-Cheese Calabi-Yau manifolds employed in the LVS.

In the process of establishing this result, we also advanced calculational techniques involving twist fields σ_θ in the bulk of a worldsheet with boundaries. They give rise to branch cuts in the map from the worldsheet into target space which end on branch points on the boundary. In [63] it was suggested to sum over all inequivalent orderings of the branch points and vertex operators on the boundary to arrive at the correct result. Our calculation is the first detailed implementation of this procedure and we verify it explicitly by calculating the disk correlator $\langle \tau_s \psi\psi\phi \rangle$, where τ_s includes a twist field σ_θ. We find that unphysical poles in the amplitude only cancel when the sum is performed as suggested in [63]. Our result passes an additional test as the amplitude factorises correctly onto lower point interactions.

In chapter 5 we shifted our attention to holomorphic Yukawa couplings. In particular, we checked whether non-perturbative effects used for moduli stabilisation can give rise to corrections to Yukawa couplings. If present, these corrections depend on supersymmetry breaking Kähler moduli and thus enter expressions for soft A-terms. Specifically, we focus on corrections to the non-perturbative superpotential sourced by gaugino condensation on a stack of D7-branes. Calculations are performed in orbifold models with D3 and D7-branes, which contain the relevant features of more realistic constructions. By calculating a 1-loop threshold corrections to the gauge coupling of hidden D7-branes, we confirm that gaugino condensation on D7-branes can give rise to corrections to D3 Yukawa couplings in the visible sector. Despite the spatial separation between the hidden D7 and the visible D3-branes, we show that these effects are not suppressed by the bulk volume. In addition, we find that the flavour structure of the new Yukawa couplings does not align with that of tree-level Yukawa couplings, such that the non-perturbative contributions to A-terms can source flavour-changing neutral currents. We also derive conditions on the setups of branes which give rise to non-perturbative corrections to Yukawa couplings. We find that if the D7-branes wrap a bulk four-cycle these terms are typically present. For D7-branes on small (blow-up) cycles to give rise to corrections to D3 Yukawa couplings, both stacks of branes have to be connected by a homologous two-cycle.

Chapter 6 examines a different aspect of string phenomenology. There we checked for the possibility of kinetic mixing between different $U(1)$ factors in type IIB models with D3-branes at orbifold singularities. We find that if we restrict our attention to non-anomalous, massless $U(1)$s in setups which satisfy tadpole cancellation conditions, then kinetic mixing between $U(1)$s from the same brane stack is not possible. On the other hand, we showed that there can be kinetic mixing between massless $U(1)$s, which have support over more than one brane stack. In general, these non-local $U(1)$s can mix kinetically as long as all the brane stacks do not all have the same separation from one another.

Where does this leave the study of soft terms in type IIB flux compactifications, and in particular in the LVS? We motivated this work by asking at which scale soft terms arise compared to the gravitino mass. In particular, we pointed out that in the LVS soft terms are suppressed below the gravitino mass, and this suppression continues to the point until the structure $\tilde{K}_\alpha \sim e^{\hat{K}/3}$ is broken. While our results from chapter 4 suggest that at leading order in g_s this structure of the LVS remains intact (up to non-perturbative terms in α'), it is natural to ask whether it survives to higher orders in g_s. While our findings in chapter 4 show that all interactions between a visible sector and distant twist fields have to be strongly suppressed at tree-level, this need not hold at one-loop. For example, our calculation in chapter 5 shows that there can be unsuppressed interactions between distant brane stacks at 1-loop in string perturbation theory. An obvious extension of our work would be to repeat our analysis from chapter 4 at one-loop, and study the dependence of physical Yukawas on blow-up moduli by calculating the relevant annulus and Möbius strip diagrams.

The above constitutes an examination of correlations between the moduli-dependence of the Kähler matter metric and the moduli Kähler potential beyond leading order in g_s. However, having a detailed understanding of the explicit moduli-dependence of the above quantities is also highly relevant for any phenomenological investigation: corrections to the moduli Kähler potential can affect the procedure of moduli stabilisation which is a prerequisite of any phenomenologically viable construction. Note that the

LVS employs a perturbative α'^3 correction to $\hat{K}$ to successfully realise moduli stabilisation. If there are further corrections which enter the Kähler potential at similar strength, the whole vacuum could be destabilised. Such corrections exist, and their consequences for moduli stabilisation have to be examined. To give an example, using a calculation in a toroidal orbifold it was conjectured that string loops correct the Kähler potential at order $\alpha'^2 g_s^2$ [116]. While these can be the leading corrections in the Kähler potential, there are cancellations such that these corrections result in subleading terms in the scalar potential. Instead of destabilising the vacuum, they can then be used to stabilise flat directions in models of the LVS with many Kähler moduli [117]. This does not imply that all dangerous corrections to the Kähler potential are under control. Recently, a new α'^2 correction to the type IIB Kähler potential was found by considering higher derivative corrections to M-theory [118]. While still being subleading in the scalar potential, it is less suppressed than the corrections mentioned before. In consequence, it can spell danger for scenarios of moduli stabilisation in regions of parameter space, which were regarded as safe before. Interestingly, such corrections were only expected to arise at 1-loop in string perturbation theory, while this correction arises at string tree-level. Correspondingly, we take this as an example that there are still many open questions regarding perturbative corrections to the Kähler potential. String perturbation theory can be one of the methods that can help to give some answers.

Last, we need to address a point, which makes the study of string phenomenology in type IIB string theory unsatisfactory at the moment. It is the problem that model building in type IIB theory largely exhibits a modular structure: parts of the model are often studied in isolation, and one hopes that certain features carry over once all parts are integrated into a global construction. For example, the massless spectrum of a stack of D-branes can be analysed by studying the D-branes and their vicinity without reference to the whole compact space. However, not all local constructions can be carried over to a global model. There are global consistency conditions which constrain possible combinations of these local features. Recently, there has been increased effort to construct fully consistent models in type IIB string theory which contain visible sectors on D3/D7-branes and realise moduli stabilisation [80,119–121]. It would be interesting to construct models, which reproduce (a supersymmetric extension of) the Standard Model at low energies as realistically as possible, while still satisfying all consistency conditions. In these models soft terms could be studied in greater detail, and predictions could be made. All in all, a realistic model of particle physics from type IIB string theory would be a great feat in itself, and many other questions of physics beyond the Standard Models could be addressed.

The main subject of this thesis is low energy supersymmetry breaking and its implementations in string theory. Yet, at the time of writing no experimental support for low energy supersymmetry has been found. On the contrary, the allowed parameter space for many supersymmetric models is increasingly constrained by results from the Large Hadron Collider experiments. Many models are pushed to regions of parameter space, where the Higgs mass becomes increasingly fine-tuned. Nevertheless, due to the richness of supersymmetric models it is far too early to conclusively exclude low scale supersymmetry as the correct description of our world. While the LHC has confirmed the reality of the hierarchy problem it has not yet given any guidance to its solution.

Appendix A

Conventions and CFT tools

A.1 Theta functions and identities

The standard notation for the Jacobi theta functions with characteristics is:

$$\vartheta[{}^a_b](z,\tau) = \sum_{n=-\infty}^{\infty} \exp\left[\pi i(n+a)^2\tau + 2\pi i(n+a)(z+b)\right] . \tag{A.1}$$

Under shifts of the argument z they behave as

$$\vartheta[{}^a_b](z+1,\tau) \ = e^{2\pi i a}\vartheta[{}^a_b](z,\tau) \ , \tag{A.2}$$

$$\vartheta[{}^a_b](z+\tau,\tau) \ = e^{-\pi i\tau - 2\pi i(z+b)}\vartheta[{}^a_b](z,\tau) \ . \tag{A.3}$$

The following notation is also used: $\vartheta_{\alpha\beta}(z) \equiv \vartheta[{}^{\alpha/2}_{\beta/2}](z)$, and

$$\vartheta_1 \equiv \vartheta_{11}, \quad \vartheta_2 \equiv \vartheta_{10}, \quad \vartheta_3 \equiv \vartheta_{00}, \quad \vartheta_4 \equiv \vartheta_{01}. \tag{A.4}$$

Using $q = e^{\pi i\tau}$ the Dedekind η function is defined as

$$\eta(\tau) = q^{1/12}\prod_{m=1}^{\infty}(1-q^{2m}) = \left[\frac{\vartheta_1'(0,\tau)}{-2\pi}\right]^{1/3} . \tag{A.5}$$

Derivatives w.r.t. z give

$$\vartheta_1(z) = 2\pi\eta^3 z + \mathcal{O}(z^3), \tag{A.6}$$

$$\vartheta_i(z) = \vartheta_i(0) + \frac{z^2}{2}\vartheta_i''(0) + \mathcal{O}(z^4), \quad i=2,3,4 \tag{A.7}$$

where we left the argument τ implicit.

It will be useful to also introduce the genus-two ϑ-functions:

$$\vartheta[{}^{\vec\alpha}_{\vec\beta}](\vec\nu,G) = \sum_{\vec n\in\mathbb{Z}^N} e^{i\pi(\vec n+\vec\alpha)^{\mathrm{T}}G(\vec n+\vec\alpha)}e^{2\pi i(\vec\nu+\vec\beta)^{\mathrm{T}}(\vec n+\vec\alpha)} , \tag{A.8}$$

where G is an $N\times N$ matrix with $\mathrm{Im}(G)>0$. The case $N=1$ gives the usual Jacobi theta functions while the case $N=2$ will be used to reexpress the sum over winding modes on a two-torus.

Riemann summation formula

There is a multitude of Riemann identities for the four Jacobi theta functions defined above (see [122]). These can also be extended to theta functions with more general characteristics.

$$\sum_{\alpha,\beta} \frac{\eta_{\alpha\beta}}{2} \prod_{i=1}^{4} \vartheta \left[\begin{array}{c} \alpha/2 + g_i \\ \beta/2 + h_i \end{array} \right] (z_i, \tau) = - \prod_{i=1}^{4} \vartheta \left[\begin{array}{c} 1/2 + 1/2\ g_i' \\ 1/2 + 1/2\ h_i' \end{array} \right] \left(\frac{1}{2} z_i', \tau \right) \tag{A.9}$$

where $\eta_{\alpha\beta} = (-1)^{\alpha+\beta}$ and primed quantities are defined as

$$v_1' = v_1 + v_2 + v_3 + v_4 \ , \qquad v_2' = v_1 - v_2 - v_3 + v_4 \ ,$$
$$v_1' = v_1 - v_2 + v_3 - v_4 \ , \qquad v_2' = v_1 + v_2 - v_3 - v_4 \ .$$

Identities for D3-D3 models

In particular, we can use the above relation to derive the following identities that arise in the context of D3-D3 calculations. For untwisted sectors we find

$$\sum_{\alpha,\beta} \eta_{\alpha\beta}\ \vartheta''_{\alpha\beta}(0) \prod_{i=1}^{3} \vartheta_{\alpha\beta}(0) = 0 \tag{A.10}$$

where $\eta_{\alpha\beta} = (-1)^{\alpha+\beta}$ and derivatives are w.r.t. z. Further, we will rely on the following identity to simplify results for partially twisted sectors of the orbifold:

$$\sum_{\alpha,\beta} \eta_{\alpha\beta}(-1)^{\alpha} \frac{\vartheta''_{\alpha\beta}(0)}{\eta^3} \frac{\vartheta_{\alpha\beta}(0)}{\eta^3} \frac{\vartheta_{\alpha\beta}(\theta_1)}{\vartheta_1(\theta_1)} \frac{\vartheta_{\alpha\beta}(\theta_2)}{\vartheta_1(\theta_2)} = -4\pi^2 \tag{A.11}$$

where $\theta_1 + \theta_2 = 1$ mod 2.

Identities for D3-D7 models

Expressions arising in the context of D3-D7 models can be simplified as follows. For untwisted sectors we find:

$$\sum_{\alpha,\beta} \eta_{\alpha\beta} \frac{\vartheta''_{\alpha\beta}(0)}{\eta^3} \frac{\vartheta_{\alpha\beta}(0)}{\eta^3} \left(\frac{\vartheta[{}^{1/2-\alpha/2}_{\beta/2}](0)}{\vartheta[{}^{0}_{1/2}](0)} \right)^2 = 4\pi^2 \ . \tag{A.12}$$

Correspondingly, for partially twisted sectors with $\theta_3 = 0$ we can use:

$$\sum_{\alpha,\beta} \eta_{\alpha\beta} \frac{\vartheta''_{\alpha\beta}(0)}{\eta^3} \frac{\vartheta_{\alpha\beta}(0)}{\eta^3} \left(\frac{\vartheta[{}^{1/2-\alpha/2}_{\beta/2}](\theta_1)}{\vartheta[{}^{0}_{1/2}](\theta_1)} \right) \left(\frac{\vartheta[{}^{1/2-\alpha/2}_{\beta/2}](\theta_2)}{\vartheta[{}^{0}_{1/2}](\theta_2)} \right) = 4\pi^2 \ . \tag{A.13}$$

A.2 CFT building blocks

Here we summarise tools and results necessary for calculations in chapters 4, 5 and 6.

A.2.1 Geometry

The target space coordinates are the real worldsheet bosons $X^M(z,\bar{z})$ where $M = 0,\ldots,9$. We split them as $X^M = X^\mu, X^m$, where $\mu = 0,\ldots 3$ denotes external directions, while $m = 4,\ldots,9$ labels internal directions. It will be useful to group the internal directions into complex pairs

$$X^i = \frac{1}{\sqrt{2}}\left(X^{2i+2} + iX^{2i+3}\right) \qquad \bar{X}^i = \frac{1}{\sqrt{2}}\left(X^{2i+2} - iX^{2i+3}\right)\ , \tag{A.14}$$

where $i = 1,2,3$. While we label both real and complex variables with X, such coordinates carrying a latin index will always be understood as complex coordinates. In terms of these complex fields, correlation functions on the sphere become

$$\langle \partial X^i(z_1)\partial \bar{X}^j(z_2)\rangle = -\frac{\alpha'}{2}\frac{\delta^{ij}}{(z_1-z_2)^2} \tag{A.15}$$

$$\langle \partial X^i(z_1)\partial X^j(z_2)\rangle = \langle \partial \bar{X}^i(z_1)\partial \bar{X}^j(z_2)\rangle = 0\ . \tag{A.16}$$

Similarly, we define complex spinors as

$$\Psi^i = \frac{1}{\sqrt{2}}\left(\psi^{2i+2} + i\psi^{2i+3}\right)\ , \qquad \bar{\Psi}^i = \frac{1}{\sqrt{2}}\left(\psi^{2i+2} - i\psi^{2i+3}\right), \qquad i = 1,2,3\ . \tag{A.17}$$

In all calculations the internal space will be a toroidal orbifold $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2/\mathbb{Z}_N$. We can specify the geometry of a single two-torus $\mathbb{T}^2$ by its complex structure modulus U^i and its volume T_2^i. To use our coordinates X^i on the tori we identify under $X^i \sim X^i + 2\pi\sqrt{\frac{T_2^i}{2U_2^i}}\ (m + U^i n)$, where m, n are integers.

A.2.2 Classical winding solutions

In directions with Dirichlet boundary conditions, the boundary of a worldsheet is associated with a point in spacetime, where D-branes are placed. While the string endpoints might be fixed, the string can wrap the internal compact dimension leading to classical solutions in the form of winding modes. We will focus on cylinder worldsheets, which have two boundaries, which can be associated with different locations. Taking the Euclidean cylinder worldsheet to be parameterised by $\sigma^1 \in [0,1/2]$ and $\sigma^2 \in [0,t/2]$, we have the following classical solutions:

$$X^i_{cl} = x^i_0 + 4\pi\sqrt{\frac{T_2^i}{2U_2^i}}\ (m + U^i n + r^i_c)\ \sigma^1\ . \tag{A.18}$$

This is a winding solution stretching between two brane stacks and wrapping the two torus cycles m and n times. Here T^i and U^i describe the Kähler and complex structure moduli of the torus wrapped. One brane stack is at x^i_0 while the other is separated from the first one by a distance $\Delta X^i = 2\pi\sqrt{T_2^i/2U_2^i}\; r_c^i$. Then r_c is a dimensionless measure for the separation of the two branes. In terms of complex coordinates $z = \sigma^1 + i\sigma^2$ and $\bar{z} = \sigma^1 - i\sigma^2$ on the worldsheet this becomes:

$$X_{cl}^i = x_0^i + 2\pi\sqrt{\frac{T_2^i}{2U_2^i}}\,(m + U^i n + r_c^i)\,(z + \bar{z})\;. \tag{A.19}$$

From this we can find the classical solutions ∂X_{cl}^i and $\bar{\partial} X_{cl}^i$:

$$\partial X_{cl}^i = \bar{\partial} X_{cl}^i = 2\pi\sqrt{\frac{T_2^i}{2U_2^i}}\,(m + U^i n + r_c^i)\;. \tag{A.20}$$

Another useful quantity will be the classical action associated with a winding solution. For one complex direction, the classical action is given by

$$S = \frac{1}{2\pi\alpha'}\int \mathrm{d}^2 z\,\left(\partial X_{cl}^i \bar{\partial}\bar{X}_{cl}^i + \bar{\partial} X_{cl}^i \partial \bar{X}_{cl}^i\right)\;. \tag{A.21}$$

Using the classical solutions (A.20) and $\int \mathrm{d}^2 z = 2\int_0^{1/2}\mathrm{d}\sigma^1\int_0^{t/2}\mathrm{d}\sigma^2$ we arrive at

$$S = \frac{\pi t}{\alpha'}\frac{T_2^i}{U_2^i}\left|m + U^i n + r_c^i\right|^2\;. \tag{A.22}$$

We now also specify the contribution to the partition function due to winding strings on a torus. For each complex direction allowing for winding modes we obtain

$$\mathcal{Z}(t) = \sum_{m,n\in\mathbb{Z}} \exp\left[-\frac{\pi t}{\alpha'}\frac{T_2^i}{U_2^i}\left|m + U^i n + r_c^i\right|^2\right]\;. \tag{A.23}$$

If winding strings do not contribute, the classical partition function just provides a factor $\mathcal{Z}(t) = 1$.

A.2.3 Picture-changing and OPEs

To bring an amplitude into the appropriate ghost charge picture we can change pictures following the prescription of [51]:

$$V^{i+1}(z) = \lim_{z\to w} e^{\phi(z)} T_F(z)\, V^i(w)\;, \tag{A.24}$$

where we have the picture changing operator

$$T_F(z) = \sum_{\mu=0}^{3}\psi^\mu \partial X_\mu(z) + \sum_{i=1,2,3}\left(\Psi^i \partial\bar{X}^i(w) + \bar{\Psi}^i \partial X^i(z)\right)\;. \tag{A.25}$$

In practice the picture-changing is evaluated using operator product expansions (OPEs). We will find the following OPEs particularly useful

$$e^{ia\phi(w)}e^{ib\phi(z)} = (w-z)^{-ab}\,e^{i(a+b)\phi(z)} + \ldots\;, \tag{A.26}$$

$$e^{iaH(w)}e^{ibH(z)} = (w-z)^{ab}\,e^{i(a+b)H(z)} + \ldots\;, \tag{A.27}$$

$$\psi^\mu(w)\psi^\nu(z) = \eta^{\mu\nu}(w-z)^{-1} + \ldots\;, \tag{A.28}$$

$$\partial X^\mu(w)\, e^{ikX(z)} = -\frac{i\alpha'}{2}k^\mu (w-z)^{-1}\, e^{ikX(z)} + \ldots\;. \tag{A.29}$$

where the ellipses denote less divergent terms. Here H are free scalar fields which are used to bosonise complex fermions.

To perform picture-changing, it will sometimes be advantageous to complexify fields in the external directions. We then also define the momenta $k^{1\pm} = \frac{1}{\sqrt{2}}(\pm k^0 + k^1)$ and $k^{2\pm} = \frac{1}{\sqrt{2}}(k^2 \pm ik^3)$. The complexified external fields X^i and Ψ^i are then given by similar expressions. In terms of these fields we have

$$k \cdot X\,(z) = k^+ \cdot \bar{X} + k^- \cdot X \ . \tag{A.30}$$

Then we can also use the following OPEs to perform picture-changing:

$$\partial X^i\,(w)\, e^{ikX(z)} = -\frac{i\alpha'}{2} k^{i+}\,(w-z)^{-1}\, e^{ikX(z)} + \ldots \ , \tag{A.31}$$

$$\partial \bar{X}^i\,(w)\, e^{ikX(z)} = -\frac{i\alpha'}{2} k^{i-}\,(w-z)^{-1}\, e^{ikX(z)} + \ldots \ . \tag{A.32}$$

A.2.4 Vertex operators

Here we briefly summarise expressions of open string vertex operators used in calculations. We present operators for spacetime bosons both in their canonical and their picture changed form, with ghost charges (-1) and (0) respectively.
The operator for a four-dimensional gauge field is given by

$$V_A^{-1}(z) = \lambda e^{-\varphi} \epsilon_\mu \psi^\mu e^{ikX}(z) \ , \qquad V_A^0(z) = \lambda \epsilon_\mu \left[\partial_t X^\mu + 2i\alpha'(k\cdot\psi)\psi^\mu\right] e^{ikX}(z) \ , \tag{A.33}$$

where λ is a Chan-Paton factor, ϵ_μ is the polarisation and ∂_t is a derivative tangential to the boundary.
A chiral scalar (from a D3-brane at an orbifold singularity) corresponds to

$$V_{C^i}^{-1}(z) = \lambda e^{-\varphi} \Psi^i e^{ikX}(z) \ , \qquad V_{C^i}^0(z) = \lambda \left[\partial_n X^i + 2i\alpha'(k\cdot\psi)\Psi^i\right] e^{ikX}(z) \ , \tag{A.34}$$

where ∂_n is a derivative normal to the boundary. An antichiral scalar is given by

$$V_{\bar{C}^i}^{-1}(z) = \lambda e^{-\varphi} \bar{\Psi}^i e^{ikX}(z) \ , \qquad V_{\bar{C}^i}^0(z) = \lambda \left[\partial_n \bar{X}^i + 2i\alpha'(k\cdot\psi)\bar{\Psi}^i\right] e^{ikX}(z) \ . \tag{A.35}$$

For a spacetime fermion in the $-1/2$ picture we insert a vertex operator

$$V_\psi^{-\frac{1}{2}}(z) = \lambda e^{-\frac{\varphi}{2}} S_{10}\, e^{ikX}(z) \ , \tag{A.36}$$

where S_{10} is the ten-dimensional spin field. It can be locally bosonised as

$$S_{10} = \prod_{i=1}^{5} e^{iq_i H_i(z)} \ , \tag{A.37}$$

where $H(z)$ is a free scalar field. The H-charges q_i are given by the spin $\frac{1}{2}$ of the complex direction components of the spinor. The GSO projection fixes the number of negative spins to be even for a chiral spinor, while an antichiral spinor exhibits an odd number of $-\frac{1}{2}$ spins.

A.2.5 Cylinder partition functions

Here we list the relevant partition functions for cylinder worldsheets, which depend on the spin structure. In the external directions we have:

	even	odd
Bosonic:	$\frac{1}{\eta^4}\frac{1}{(4\pi^2\alpha' t)^2}$	$\frac{1}{\eta^4(it)}\frac{1}{(4\pi^2\alpha' t)^2}$
Fermionic:	$\left(\frac{\vartheta_{\alpha\beta}(0)}{\eta}\right)^2$	$(\eta^4)^2$
bc ghosts:	η^2	η^2
$\beta\gamma$ ghosts:	$\frac{\eta}{\vartheta_{\alpha\beta}(0)}$	$\frac{1}{\eta^2}$
Total:	$\frac{\vartheta_{\alpha\beta}(0)}{\eta^3}\frac{1}{(4\pi^2\alpha' t)^2}$	$\frac{1}{(4\pi^2\alpha' t)^2}$

Here η is the Dedekind eta function and we suppressed the argument τ. For the odd spin structure to contribute one assumes that the zero modes in the fermionic sector are saturated.

We will also need internal partition functions. These will depend on both the orbifold twist θ_I and the boundary conditions at the end of the cylinder. In D3-D3 models all internal directions exhibit Dirichlet-Dirichlet (DD) boundary conditions. In D3-D7 constructions we also have mixed Dirichlet-Neumann (DN) boundary conditions in two of the three complex dimensions. The combined bosonic and fermionic partition function for one compact torus I with twist $\theta_I \neq 0$ is[1]

$$\text{DD: } (-2\sin\pi\theta_I)\frac{\vartheta_{\alpha\beta}(\theta_I)}{\vartheta_1(\theta_I)}\ , \qquad \text{ND: } \frac{\vartheta_{(\alpha\pm1)\beta}(\theta_I)}{\vartheta_4(\theta_I)} \tag{A.38}$$

We will also need the partition function for an untwisted torus in the even spin-structures. There we have:

$$\text{DD: } \frac{\vartheta_{\alpha\beta}(0)}{\eta^3}\ , \qquad \text{ND: } \frac{\vartheta_{(\alpha\pm1)\beta}(0)}{\vartheta_4(0)}\ . \tag{A.39}$$

A.2.6 Fermionic and ghost correlators on the cylinder

We start with correlators involving both worldsheet spinors and spin fields. The correlation functions for the cylinder can be derived from those on the torus by the method of images. However, as long as we consider fields on the cylinder boundaries only, the result is very simple: in this case the correlators on the cylinder are identical to correlators on the torus [125]. The latter were derived in [53, 54] by considering the OPEs of spinors with the stress tensor in a background of spin fields. The resulting differential equations can then be solved for the correlation function. The expressions relevant to

[1] For the partition function derivation see for example [123, 124].

our calculation then are:

$$\langle\prod_i e^{iq_i\cdot H(z_i)}\rangle_{\mathcal{A}} = K_{\alpha\beta}\left[\prod_{i<j}\left(\frac{\vartheta_1(z_{ij})}{\vartheta_1'(0)}\right)^{q_iq_j}\right]\vartheta_{\alpha\beta}\left(\sum_i q_iz_i+\theta_I\right)\ , \tag{A.40}$$

where θ_I is an orbifold twist. Note that this expression is the full correlation function at 1-loop and contains the partition function, which is left after one has contracted all fields. The constants $K_{\alpha\beta}$ need to be determined for each amplitude by requiring that it factorises correctly. This amounts to taking the limit $z_i \to z_j$ for all i, j so that the amplitude factorises onto the field theory amplitude times the string partition function. The sum over spin structures is then matched to that of the partition function. Further, a correlator is only non-zero if all the H-charges cancel.

Similarly, we can use the correlator of superconformal ghosts on the torus to determine correlation functions of superconformal ghosts on the cylinder boundary. The correlators also depend on the spin structure, but are unaffected by the orbifold twist. Using the same methods as above one finds [53, 54]:

$$\langle\prod_i e^{a_i\varphi(z_i)}\rangle_{\mathcal{A}} = K_{\alpha\beta}\left[\prod_{i<j}\left(\frac{\vartheta_1(z_{ij})}{\vartheta_1'(0)}\right)^{-a_ia_j}\right]\vartheta_{\alpha\beta}^{-1}\left(-\sum_i a_iz_i\right)\ . \tag{A.41}$$

As before, the factors $K_{\alpha\beta}$ are determined by factorisation onto the partition function.

Appendix B

Yukawas with a twist

B.1 Quantum results: external picture-changing

External picture-changing leads to two distinct contributions which must be computed separately. They differ by the structure of H-charges that arise.

Case 1

In this case the H-charges take the form

$$\begin{aligned} V_{\psi_1}^{-\frac{1}{2}}(z_1) &\sim e^{-\varphi/2}|++\rangle \otimes |+--\rangle \\ V_{\psi_2}^{+\frac{1}{2}}(z_2) &\sim e^{\varphi/2}|(---)-\rangle \otimes |-+-\rangle \\ V_{\phi}^{0}(z_3) &\sim |(++),\,0\rangle \otimes |0\,0\,(++)\rangle \\ V_{\rm tw}^{-1}(w) &\sim e^{-\varphi}|00\rangle \otimes |\theta_1,\ \theta_2,\ \theta_3\rangle \\ V_{\rm tw}^{-1}(\bar{w}) &\sim e^{-\tilde{\varphi}}|00\rangle \otimes |-\theta_1,\ -\theta_2,\ -\theta_3\rangle\ . \end{aligned} \tag{B.1}$$

As $e^{\varphi(z)}e^{-\varphi(w)/2} \sim (z-w)^{\frac{1}{2}}e^{\varphi/2}$, $e^{-iH(z)}e^{-iH(w)/2} \sim (z-w)^{\frac{1}{2}}e^{-3iH(w)/2}$ and $\partial X^I(z)e^{ik_2\cdot X(w)} \sim \frac{ik_2^I}{(z-w)}e^{ik_2\cdot X(w)}$, the picture changing of $V_{\psi_2}(z_2)$ is unambiguous and there is no need to consider subleading terms in the OPE. The momentum factors in this amplitude are given by $k_3^{1-}k_2^{1+}$. As k_2^{1+} is the only non-zero component of k_2, we can promote this to $k_2 \cdot k_3 = u$.

Prior to fixing the $\mathrm{SL}(2,\mathbb{R})$ symmetry, the amplitude is given by

$$\begin{aligned} -u\int \mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3\,\mathrm{d}^2w\,(w-\bar{w})^{-2}(z_1-z_2)^{-1+s}(z_2-z_3)^{-2+u}(z_1-w)^{-1+\theta_1+u/2}(z_1-\bar{w})^{-\theta_1+u/2} \\ \times (z_2-w)^{\theta_2+t/2}(z_2-\bar{w})^{1-\theta_2+t/2}(z_3-w)^{\theta_3+s/2}(z_3-\bar{w})^{-\theta_3+s/2} \end{aligned} \tag{B.2}$$

It is easy to check invariance of (B.2) under $\mathrm{SL}(2,\mathbb{R})$ transformations. There is an overall minus sign in this expression (compared to the analogous expressions (4.42) for internal picture changing and (B.9) for external picture changing case 2). This arises as the picture changing operator introduces negative H-charge for $V(z_2)$ and positive H-charge for $V(z_3)$. This is equivalent to introducing a correlator $\bar{\Psi}(z_2)\Psi(z_3)$, whereas the other two cases introduce a correlator $\Psi(z_2)\bar{\Psi}(z_3)$, leading to the minus sign differential. We then fix $(w,\bar{w}) \to (i,-i)$, giving

$$\frac{-u}{(2i)^2}\int \mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3\,(z_1-z_2)^{-1+s}(z_2-z_3)^{-2+u}(z_1-i)^{-1+\theta_1+u/2}(z_1+i)^{-\theta_1+u/2} \times (z_2-i)^{\theta_2+t/2}(z_2+i)^{1-\theta_2+t/2}(z_3-i)^{\theta_3+s/2}(z_3+i)^{-\theta_3+s/2} \quad \text{(B.3)}$$

Again, the result can be expressed in terms of the integral (4.48). We can then immediately write down the result for the three separate cases of $z_3 \to \infty$, $z_2 \to \infty$ and $z_1 \to \infty$. These are

$$z_3 \to \infty : u \times I(-1+\theta_1+u/2, -\theta_1+u/2, \theta-2+t/2, 1-\theta_2+t/2, -1+s, -1), \quad \text{(B.4)}$$

$$z_2 \to \infty : u \times I(\theta_3+s/2, -\theta_3+s/2, -1+\theta_1+u/2, -\theta_1+u/2, t, -1), \quad \text{(B.5)}$$

$$z_1 \to \infty : -u \times I(\theta_2+t/2, 1-\theta_2+t/2, \theta_3+s/2, -\theta_3+s/2, -2+u, -1). \quad \text{(B.6)}$$

Case 2

In this case the H-charges take the form

$$\begin{aligned}
V_{\psi_1}^{-\frac{1}{2}}(z_1) &\sim e^{-\varphi/2}|+\,+\rangle \otimes |+\,-\,-\rangle \\
V_{\psi_2}^{+\frac{1}{2}}(z_2) &\sim e^{\varphi/2}|+\,-\rangle \otimes |-\,+\,-\rangle \\
V_{\phi}^{0}(z_3) &\sim |(--),\,0\rangle \otimes |0\,0\,(++)\rangle \\
V_{\mathrm{tw}}^{-1}(w) &\sim e^{-\varphi}|0\,0\rangle \otimes |\theta_1,\ \theta_2,\ \theta_3\rangle \\
V_{\mathrm{tw}}^{-1}(\bar{w}) &\sim e^{-\tilde{\varphi}}|0\,0\rangle \otimes |-\theta_1,\ -\theta_2,\ -\theta_3\rangle\ .
\end{aligned} \quad \text{(B.7)}$$

The momentum prefactor is k_3^{1+}.

Here the picture-changing is more subtle as

$$\begin{aligned}
e^{\varphi(z)}e^{-\varphi(w)/2} &\sim (z-w)^{\frac{1}{2}}e^{\varphi(w)/2}, \\
e^{iH(z)}e^{-iH(w)/2} &\sim (z-w)^{-\frac{1}{2}}e^{iH(w)/2}, \\
\partial X^I(z)e^{ik_2\cdot X(w)} &\sim \frac{ik_2^I}{(z-w)}e^{ik_2\cdot X(w)} + \partial X^I(w)e^{ik_2\cdot X(w)}.
\end{aligned} \quad \text{(B.8)}$$

The leading term in the OPE is at $\mathcal{O}(z-w)^{-1}$, whereas we require the term at $\mathcal{O}(z-w)^0$. In principle we should expand the ghost, fermionic and bosonic OPEs to obtain the $\mathcal{O}(z-w)^0$ term. However in fact only the subleading bosonic term is relevant. The subleading ghost and fermionic terms necessarily involve the leading bosonic term, which has a factor of k_2^{1-} and so vanishes identically.

The ghost and fermion correlators give

$$(w-\bar{w})^{-2}(z_1-z_3)^{-1}(z_2-z_3)^{-1}(z_1-w)^{-1+\theta_1}(z_1-\bar{w})^{-\theta_1} \\ \times(z_2-w)^{\theta_2}(z_2-\bar{w})^{1-\theta_2}(z_3-w)^{\theta_3}(z_3-\bar{w})^{-\theta_3}. \tag{B.9}$$

The bosonic correlator is

$$k_3^{1+}\langle e^{ik_1\cdot X(z_1)}\partial_t X^{1-}(z_2)e^{ik_2\cdot X(z_2)}e^{ik_3\cdot X(z_3)}e^{ik_4\cdot X(w,\bar{w})}\rangle. \tag{B.10}$$

This gives

$$k_3^{1+}\left(\frac{k_1^{1-}}{(z_2-z_1)}+\frac{k_4^{1-}}{2(z_2-w)}+\frac{k_4^{1-}}{2(z_2-\bar{w})}\right)|z_1-z_2|^s|z_1-z_3|^t|z_1-w|^u \\ \times|z_2-z_3|^u|z_2-w|^t|z_3-w|^s. \tag{B.11}$$

We have dropped the $k_3^{1+}k_3^{1-}$ term as it Lorentz completes into $k_3^2=0$. Using $k_4^{1-}=-k_1^{1-}-k_2^{1-}-k_3^{1-}$, this effectively becomes

$$k_3^{1+}\left(\frac{k_1^{1-}}{(z_2-z_1)}-\frac{k_1^{1-}}{2(z_2-w)}-\frac{k_1^{1-}}{2(z_2-\bar{w})}\right)|z_1-z_2|^s|z_1-z_3|^t|z_1-w|^u|z_2-z_3|^u|z_2-w|^t|z_3-w|^s.$$

As k_1^{1-} is the only non-zero component of k_1, we can promote $k_3^{1+}k_1^{1-}$ to $k_3\cdot k_1=t$. The amplitude then becomes

$$t\int \mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3\,\mathrm{d}^2w\,(w-\bar{w})^{-2}(z_1-z_3)^{-1+t}(z_2-z_3)^{-1+u}(z_1-w)^{-1+\theta_1+u/2}(z_1-\bar{w})^{-\theta_1+u/2} \\ \times(z_2-w)^{\theta_2+t/2}(z_2-\bar{w})^{1-\theta_2+t/2}(z_3-w)^{\theta_3+s/2}(z_3-\bar{w})^{-\theta_3+s/2} \\ \times\left(\frac{1}{(z_2-z_1)}-\frac{1}{2}\left(\frac{1}{(z_2-w)}+\frac{1}{(z_2-\bar{w})}\right)\right). \tag{B.12}$$

One can again check that this is $\mathrm{SL}(2,\mathbb{R})$ invariant. We fix $(w,\bar{w})\to(i,-i)$ and consider the three separate cases $z_3\to\infty$, $z_2\to\infty$ and $z_1\to\infty$. The three results are then

$$z_3\to\infty: t\times I\Big(-1+\theta_1+u/2,-\theta_1+u/2,\theta_2+t/2,1-\theta_2+t/2,-1+s,-1\Big) \tag{B.13}$$
$$-\frac{t}{2}\times I\Big(-1+\theta_1+u/2,-\theta_1+u/2,-1+\theta_2+t/2,1-\theta_2+t/2,s,-1\Big)$$
$$-\frac{t}{2}\times I\Big(-1+\theta_1+u/2,-\theta_1+u/2,\theta_2+t/2,-\theta_2+t/2,s,-1\Big).$$
$$z_2\to\infty: \frac{t}{2}\times I\Big(\theta_3+s/2,-\theta_3+s/2,\theta_1+u/2,-\theta_1+u/2,-1+t,-1\Big) \tag{B.14}$$
$$+\frac{t}{2}\times I\Big(\theta_3+s/2,-\theta_3+s/2,-1+\theta_1+u/2,1-\theta_1+u/2,-1+t,-1\Big).$$
$$z_1\to\infty: \frac{t}{2}\times I\Big(-1+\theta_2+t/2,1-\theta_2+t/2,\theta_3+s/2,-\theta_3+s/2,-1+u,-1\Big) \tag{B.15}$$
$$+\frac{t}{2}\times I\Big(\theta_2+t/2,-\theta_2+t/2,\theta_3+s/2,-\theta_3+s/2,-1+u,-1\Big).$$

B.2 Pole structure of amplitudes

Here we collect the results for the poles in the partial amplitudes arising in the calculation of a Yukawa coupling with a twist insertion. The results were derived by expanding the full result for the amplitudes for small momenta. In some cases we were able to expand the generalized hypergeometric function ${}_3F_2$ appearing in the expression for the amplitude analytically; when this was not possible, the following results were obtained numerically and can be checked to hold for arbitrary angles θ_i for $\theta_1 + \theta_2 + \theta_3 = 1$.

$$\mathcal{A}_{z_3\to\infty} = \frac{ie^{\pi i\theta_3}(-1+4\theta_3)\sin(\pi\theta_3)}{s} + \frac{ie^{-\pi i\theta_1+\pi i\theta_3}\quad(1-2\theta_2)\sin(\pi\theta_2)}{t} + \frac{ie^{-\pi i\theta_1}(-1+2\theta_1-2\theta_3)\sin(\pi\theta_1)}{u} \tag{B.16}$$

$$e^{-2\pi i(\theta_1+\theta_2)}\mathcal{A}_{z_2\to\infty} = \frac{ie^{\pi i\theta_3}(-1+2\theta_3)\sin(\pi\theta_3)}{s} + \frac{ie^{-\pi i\theta_1}(-2\theta_3)\sin(\pi\theta_1)}{u} \tag{B.17}$$

$$-e^{-2\pi i\theta_1}\mathcal{A}_{z_1\to\infty} = \frac{ie^{\pi i\theta_3}\quad(1-2\theta_3)\sin(\pi\theta_3)}{s} + \frac{ie^{-\pi i\theta_1+\pi i\theta_3}(-1+2\theta_2)\sin(\pi\theta_2)}{t} + \frac{ie^{-\pi i\theta_1}(1-2\theta_1)\sin(\pi\theta_1)}{u}\ . \tag{B.18}$$

We need to sum these partial results to arrive at the full amplitude. The expressions with poles in t cancel when added. Once we modify the expression $e^{-2\pi i(\theta_1+\theta_2)}\mathcal{A}_{z_2\to\infty} \to -e^{-2\pi i(\theta_1+\theta_2)}\mathcal{A}_{z_2\to\infty}$ we see that the poles in u also sum to zero. The remaining pole in s is the result given in (4.66).

Appendix C

Yukawa couplings and gaugino condensation

In this section we perform a complementary calculation to the one in section (5.3), which gives information about new contributions to D3 Yukawa couplings. It involves computing the cylinder correlation function $\langle \mathrm{Tr}(\lambda\lambda)\mathrm{Tr}(\psi\psi\phi)\rangle$, where we insert gaugino and Yukawa vertex operators on opposite boundaries of the cylinder. Gauginos can arise from a stack of D3- or D7-branes, while the visible sector fields are located on D3-branes at an orbifold singularity. The relevant setups of branes are shown in figure 5.1.

C.1 Calculation

C.1.1 Setting up the calculation

We begin by specifying the H-charges of the relevant vertex operators:

$$\begin{aligned}
\lambda_1^{-\frac{1}{2}}(z_1) &= \frac{1}{2}(+,+,+,+,+) \\
\lambda_2^{-\frac{1}{2}}(z_2) &= \frac{1}{2}(-,-,+++) \\
\psi_1^{-\frac{1}{2}}(z_3) &= \frac{1}{2}(+,+,-,-,+) \\
\psi_2^{-\frac{1}{2}}(z_4) &= \frac{1}{2}(-,-,-,-,+) \\
\phi^{-1}(z_5) &= \frac{1}{2}(0,0,0,0,++)\ .
\end{aligned} \tag{C.1}$$

Here $(\pm)$ signify, that we can bosonise as $e^{\pm iH}$. Under an orbifold twist the vertex operators transform as $\psi_1 \to e^{2\pi i\theta_3}\psi_1$, $\psi_2 \to e^{2\pi i\theta_3}\psi_2$ and $\phi \to e^{2\pi i\theta_3}\phi$ and thus the Yukawa couplings above come from the superfield term $C^3C^3C^3$. The gaugino vertex operators are invariant under orbifold twists as expected: $\lambda \to e^{\pi i(\theta_1+\theta_2+\theta_3)}\lambda = \lambda$. We need to picture-change three vertex operators, which we choose to be the matter operators. To arrive at a setup with vanishing H-charge, we have to picture-change along the third internal direction. The result is

$$
\begin{aligned}
\lambda_1^{-\frac{1}{2}}(z_1) &= \frac{1}{2}(+,+,+,+,+) \\
\lambda_2^{-\frac{1}{2}}(z_2) &= \frac{1}{2}(-,-,+++) \\
\psi_1^{+\frac{1}{2}}(z_3) &= \frac{1}{2}(+,+,-,-,-) \\
\psi_2^{+\frac{1}{2}}(z_4) &= \frac{1}{2}(-,-,-,-,-) \\
\phi^{0}(z_5) &= \frac{1}{2}(0,0,0,0,0)\ .
\end{aligned}
\tag{C.2}
$$

and the vertex operators for the Yukawa coupling are given by

$$
V_{\psi_1}^{+\frac{1}{2}}(z_3) = \partial X^3 e^{iq_3\cdot H} e^{ik_3\cdot X}(z_3) \tag{C.3}
$$

$$
V_{\psi_2}^{+\frac{1}{2}}(z_4) = \partial X^3 e^{iq_4\cdot H} e^{ik_4\cdot X}(z_4) \tag{C.4}
$$

$$
V_{\phi}^{0}(z_5) = \partial X^3 e^{iq_5\cdot H} e^{ik_5\cdot X}(z_5) \tag{C.5}
$$

where q_3, q_4 and q_5 are the H-charges displayed above. We find that a Yukawa-coupling of the chiral superfields $C^3C^3C^3$ leads to the appearance of the internal bosonic fields $\partial X^3\partial X^3\partial X^3$. We could check explicitly that a Yukawa coupling arising from $C^rC^sC^t$ would lead to the appearance of $\partial X^r\partial X^s\partial X^t$.

C.1.2 Classical solution — winding modes

The correlation function over $\partial X^r\partial X^s\partial X^t$ only receives contributions from classical solutions in terms of winding modes. For D3-D3' setups the only contributions arise from partially twisted sectors, which allow winding solutions along one torus. The same is observed for D3-D7 setups as there is only one complex direction with DD boundary conditions. Hence only correlators of the form $\partial X^r\partial X^r\partial X^r$ can contribute. We find:

$$
\langle \partial X^r\partial X^r\partial X^r\rangle = \left(-\frac{\alpha'}{t}\sqrt{\frac{2U_2}{T_2}}\frac{\partial}{\partial\bar{r}_c}\right)^3\ \mathcal{Z}(t)
$$

where $\mathcal{Z}(t)$ is the partition function over winding modes (A.23).

C.1.3 Fermionic and ghost amplitude

We continue with the calculation for both D3-D3' and D3-D7 setups. The configuration of ghost and H-charges is summarised in (C.2).

D3-D3' models

In this case the spin-structure dependent terms of the correlators over fermionic and ghost fields contribute

$$\sum_{\alpha\beta} \frac{\eta_{\alpha\beta}}{2} \frac{\vartheta_{\alpha\beta}\left(\frac{z_1-z_2+z_3-z_4}{2}\right)\vartheta_{\alpha\beta}\left(\frac{z_1-z_2+z_3-z_4}{2}\right)}{\vartheta_{\alpha\beta}\left(\frac{z_1+z_2-z_3-z_4}{2}\right)} \vartheta_{\alpha\beta}\left(\frac{z_1+z_2-z_3-z_4}{2}+\theta_1\right) \tag{C.6}$$
$$\vartheta_{\alpha\beta}\left(\frac{z_1+z_2-z_3-z_4}{2}+\theta_2\right)\vartheta_{\alpha\beta}\left(\frac{z_1+z_2-z_3-z_4}{2}+\theta_3\right) .$$

The classical part from the previous section only contributes in partially twisted sectors, and thus we can set one of the θ_i to zero in the above. We choose $\theta_3 = 0$ and sum over spin-structures by employing a Riemann identity (A.9) with $\eta_{\alpha\beta} = (-1)^{\alpha+\beta}$:

$$\vartheta_1(z_1-z_4)\vartheta_1(z_2-z_3)\vartheta_1(\theta_1)\vartheta_1(\theta_2) \tag{C.7}$$

The spin-structure-independent terms for the fermions and ghosts evaluate to:

$$\frac{\vartheta_1'(0)}{\vartheta_1(z_1-z_4)}\frac{\vartheta_1'(0)}{\vartheta_1(z_2-z_3)} \tag{C.8}$$

Combining both results we arrive at the final expression for the correlator over fermionic and ghost fields in a partially twisted sector:

$$\vartheta_1'(0)\vartheta_1'(0)\vartheta_1(\theta_1)\vartheta_1(\theta_2) = 4\pi^2\eta^6\vartheta_1(\theta_1)\vartheta_1(\theta_2) , \tag{C.9}$$

where we have used $\vartheta_1'(0) = 2\pi\eta^3$. We find that the result does not depend on the worldsheet positions.

D3-D7 models

For D3-D7 models the presence of Dirichlet-Neumann boundary conditions for the directions along the worldvolume of the D7 brane implies that the theta functions appearing in correlation functions have to be modified as

$$\vartheta[{}^{\alpha/2}_{\beta/2}](z) \quad \to \quad \vartheta[{}^{(\alpha\pm1)/2}_{\beta/2}](z) . \tag{C.10}$$

Here we choose the D7-branes to wrap the first two 2-tori, which then exhibit Dirichlet-Neumann boundary conditions.

Using the above result we can revisit the fermionic and ghost correlators for D3-D7 models in a sector with $\theta_3 = 0$. The ghost correlators are unaffected while we have to modify the fermionic correlation functions on the first two sub-tori as described above.

By repeating the analysis of the previous section we thus find for the spin-structure-dependent terms:

$$\sum_{\alpha\beta} \frac{\eta_{\alpha\beta}}{2} \vartheta_{\alpha\beta}\left(\frac{z_1 - z_2 + z_3 - z_4}{2}\right) \vartheta_{\alpha\beta}\left(\frac{z_1 - z_2 + z_3 - z_4}{2}\right) \\ \times \vartheta_{(\alpha+1)\beta}\left(\frac{z_1 + z_2 - z_3 - z_4}{2} + \theta_1\right) \vartheta_{(\alpha-1)\beta}\left(\frac{z_1 + z_2 - z_3 - z_4}{2} + \theta_2\right) . \tag{C.11}$$

Using the Riemann identity (A.9) with $\eta_{\alpha\beta} = (-1)^{\alpha+\beta}$ this becomes

$$\vartheta_1(z_1 - z_4)\vartheta_1(z_2 - z_3)\vartheta_4(\theta_1)\ \vartheta_4(\theta_2) . \tag{C.12}$$

The spin-structure independent terms are unaffected and using the result from the D3-D3 case (C.8) we find the following result for the spinorial and ghost correlator:

$$4\pi^2\eta^6\vartheta_4(\theta_1)\vartheta_4(\theta_2) , \tag{C.13}$$

Again, we find the expression to be independent of the worldsheet positions.[1]

C.1.4 Completing the calculation — partition functions

To complete the results of the previous sections we need to combine them with the correlator over momentum exponentials and the appropriate partition functions. The fermionic partition function and the vacuum amplitude over ghosts have already been included implicitly in the result for the fermionic and ghost correlator.

D3-D3' models

For this setup we combine results (C.9) and (C.6) with the bosonic partition function. After having included the correlator over momentum exponential we find

$$\mathcal{A} \propto \int \frac{\mathrm{d}t}{t} \int \prod_{i=1}^{5} \mathrm{d}z_i \frac{\prod_{k=1}^{2}(-2\sin\pi\theta_k)}{(2\pi^2 t)^2} \left(\prod_{i<j} e^{-k_i\cdot k_j \mathcal{G}(z_i - z_j)}\right) \left(-\frac{\alpha'}{t}\sqrt{\frac{2U_2}{T_2}}\frac{\partial}{\partial \bar{r}_c}\right)^3 \mathcal{Z}(t) , \tag{C.14}$$

which is identical with the expression obtained earlier for corrections to the gauge kinetic function (5.35).

[1] As in the main text, we also need to introduce orbifold images of D7-branes at $r_c' = e^{2\pi i\theta_3 j} r_c$. All stacks contribute equally to the quantum correlator, but the sums over winding modes could be different. For the relevant value $\theta_3 = 1/3$ one can show that the contribution from winding modes (C.6) is the same for all stacks.

D3-D7 models

Bringing all results together for this setup we obtain in the untwisted sector:

$$\mathcal{A} \propto \int \frac{\mathrm{d}t}{t} \int \prod_{i=1}^{5} \mathrm{d}z_i \frac{1}{(2\pi^2 t)^2} \left(\prod_{i<j} e^{-k_i \cdot k_j \mathcal{G}(z_i - z_j)} \right) \left(-\frac{\alpha'}{t} \sqrt{\frac{2U_2}{T_2}} \frac{\partial}{\partial \bar{r}_c} \right)^3 \mathcal{Z}(t) \ . \qquad \text{(C.15)}$$

Again, the resulting expression coincides with the result for the calculation of corrections to the gauge kinetic function (5.33).

As the results for both the D3-D3' and the D3-D7 models match with the calculations for corrections to the gauge kinetic function we refer readers to the discussion of these previous results in section 5.3.4. The fact that the resulting expressions coincide is a welcome check of our previous calculation.

Bibliography

[1] **ATLAS Collaboration** Collaboration, G. Aad et al., *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys.Lett.* **B716** (2012) 1–29, [arXiv:1207.7214].

[2] **CMS Collaboration** Collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, *Phys.Lett.* **B716** (2012) 30–61, [arXiv:1207.7235].

[3] **CMS Collaboration** Collaboration, S. Chatrchyan et al., *Observation of a new boson with mass near 125 GeV in pp collisions at sqrt(s) = 7 and 8 TeV*, *JHEP* **06** (2013) 081, [arXiv:1303.4571].

[4] **ATLAS Collaboration** Collaboration, G. Aad et al., *Evidence for the spin-0 nature of the Higgs boson using ATLAS data*, arXiv:1307.1432.

[5] **ATLAS Collaboration** Collaboration, G. Aad et al., *Measurements of Higgs boson production and couplings in diboson final states with the ATLAS detector at the LHC*, arXiv:1307.1427.

[6] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **0503** (2005) 007, [hep-th/0502058].

[7] J. P. Conlon and L. T. Witkowski, *Scattering and Sequestering of Blow-Up Moduli in Local String Models*, *JHEP* **1112** (2011) 028, [arXiv:1109.4153].

[8] M. Berg, J. P. Conlon, D. Marsh, and L. T. Witkowski, *Superpotential de-sequestering in string models*, *JHEP* **1302** (2013) 018, [arXiv:1207.1103].

[9] M. Bullimore, J. P. Conlon, and L. T. Witkowski, *Kinetic mixing of U(1)s for local string models*, *JHEP* **1011** (2010) 142, [arXiv:1009.2380].

[10] A. Brignole, L. E. Ibanez, and C. Munoz, *Soft supersymmetry breaking terms from supergravity and superstring models*, hep-ph/9707209.

[11] L. J. Dixon, V. Kaplunovsky, and J. Louis, *On Effective Field Theories Describing (2,2) Vacua of the Heterotic String*, *Nucl.Phys.* **B329** (1990) 27–82.

[12] V. S. Kaplunovsky and J. Louis, *Model independent analysis of soft terms in effective supergravity and in string theory*, *Phys.Lett.* **B306** (1993) 269–275, [hep-th/9303040].

[13] T. W. Grimm and J. Louis, *The Effective action of N = 1 Calabi-Yau orientifolds*, *Nucl.Phys.* **B699** (2004) 387–426, [hep-th/0403067].

[14] D. Lust, P. Mayr, R. Richter, and S. Stieberger, *Scattering of gauge, matter, and moduli fields from intersecting branes*, *Nucl.Phys.* **B696** (2004) 205–250, [hep-th/0404134].

[15] R. Blumenhagen, B. Kors, D. Lust, and S. Stieberger, *Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes*, *Phys.Rept.* **445** (2007) 1–193, [hep-th/0610327].

[16] J. P. Conlon, S. S. Abdussalam, F. Quevedo, and K. Suruliz, *Soft SUSY Breaking Terms for Chiral Matter in IIB String Compactifications*, *JHEP* **0701** (2007) 032, [hep-th/0610129].

[17] G. Honecker, *Kaehler metrics and gauge kinetic functions for intersecting D6-branes on toroidal orbifolds - The complete perturbative story*, *Fortsch.Phys.* **60** (2012) 243–326, [arXiv:1109.3192].

[18] N. Akerblom, R. Blumenhagen, D. Lust, and M. Schmidt-Sommerfeld, *Instantons and Holomorphic Couplings in Intersecting D-brane Models*, *JHEP* **0708** (2007) 044, [arXiv:0705.2366].

[19] J. P. Conlon, *Mirror Mediation*, *JHEP* **0803** (2008) 025, [arXiv:0710.0873].

[20] M. Cicoli, S. de Alwis, and A. Westphal, *Heterotic Moduli Stabilization*, arXiv:1304.1809.

[21] L. B. Anderson, J. Gray, A. Lukas, and B. Ovrut, *Vacuum Varieties, Holomorphic Bundles and Complex Structure Stabilization in Heterotic Theories*, arXiv:1304.2704.

[22] L. B. Anderson, J. Gray, A. Lukas, and B. Ovrut, *The Atiyah Class and Complex Structure Stabilization in Heterotic Calabi-Yau Compactifications*, *JHEP* **1110** (2011) 032, [arXiv:1107.5076].

[23] S. B. Giddings, S. Kachru, and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys.Rev.* **D66** (2002) 106006, [hep-th/0105097].

[24] M. Grana, *Flux compactifications in string theory: A Comprehensive review*, *Phys.Rept.* **423** (2006) 91–158, [hep-th/0509003].

[25] M. R. Douglas and S. Kachru, *Flux compactification*, *Rev.Mod.Phys.* **79** (2007) 733–796, [hep-th/0610102].

[26] F. Denef, M. R. Douglas, and S. Kachru, *Physics of String Flux Compactifications*, *Ann.Rev.Nucl.Part.Sci.* **57** (2007) 119–144, [hep-th/0701050].

[27] F. Denef, *Les Houches Lectures on Constructing String Vacua*, arXiv:0803.1194.

[28] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, *De Sitter vacua in string theory*, *Phys.Rev.* **D68** (2003) 046005, [hep-th/0301240].

[29] A. Saltman and E. Silverstein, *A New handle on de Sitter compactifications*, *JHEP* **0601** (2006) 139, [hep-th/0411271].

[30] S. Parameswaran and A. Westphal, *de Sitter string vacua from perturbative Kahler corrections and consistent D-terms*, *JHEP* **0610** (2006) 079, [hep-th/0602253].

[31] M. Rummel and A. Westphal, *A sufficient condition for de Sitter vacua in type IIB string theory*, *JHEP* **1201** (2012) 020, [arXiv:1107.2115].

[32] J. P. Conlon, F. Quevedo, and K. Suruliz, *Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking*, *JHEP* **0508** (2005) 007, [hep-th/0505076].

[33] J. P. Conlon, D. Cremades, and F. Quevedo, *Kahler potentials of chiral matter fields for Calabi-Yau string compactifications*, *JHEP* **0701** (2007) 022, [hep-th/0609180].

[34] J. A. Bagger, T. Moroi, and E. Poppitz, *Anomaly mediation in supergravity theories*, *JHEP* **0004** (2000) 009, [hep-th/9911029].

[35] M. K. Gaillard and B. D. Nelson, *Quantum induced soft supersymmetry breaking in supergravity*, *Nucl.Phys.* **B588** (2000) 197–212, [hep-th/0004170].

[36] R. Blumenhagen, J. Conlon, S. Krippendorf, S. Moster, and F. Quevedo, *SUSY Breaking in Local String/F-Theory Models*, *JHEP* **0909** (2009) 007, [arXiv:0906.3297].

[37] S. de Alwis, *Classical and Quantum SUSY Breaking Effects in IIB Local Models*, *JHEP* **1003** (2010) 078, [arXiv:0912.2950].

[38] J. P. Conlon and F. G. Pedro, *Moduli Redefinitions and Moduli Stabilisation*, *JHEP* **1006** (2010) 082, [arXiv:1003.0388].

[39] K. Choi, H. P. Nilles, C. S. Shin, and M. Trapletti, *Sparticle Spectrum of Large Volume Compactification*, *JHEP* **1102** (2011) 047, [arXiv:1011.0999].

[40] C. S. Shin, *Anomalous U(1) Mediation in Large Volume Compactification*, *JHEP* **1201** (2012) 084, [arXiv:1108.5740].

[41] L. Aparicio, D. Cerdeno, and L. Ibanez, *Modulus-dominated SUSY-breaking soft terms in F-theory and their test at LHC*, *JHEP* **0807** (2008) 099, [arXiv:0805.2943].

[42] M. Berg, D. Marsh, L. McAllister, and E. Pajer, *Sequestering in String Compactifications*, *JHEP* **06** (2011) 134, [arXiv:1012.1858].

[43] M. Cvetic, J. Halverson, and R. Richter, *Realistic Yukawa structures from orientifold compactifications*, *JHEP* **0912** (2009) 063, [arXiv:0905.3379].

[44] M. Buican and S. Franco, *SUSY breaking mediation by D-brane instantons*, *JHEP* **0812** (2008) 030, [arXiv:0806.1964].

[45] J. Polchinski, *String theory. Vol. 1: An introduction to the bosonic string.* Cambridge, UK: Univ. Pr. (1998) 402 p.

[46] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond.* Cambridge, UK: Univ. Pr. (1998) 531 p.

[47] E. Kiritsis, *Introduction to superstring theory*, hep-th/9709062.

[48] D. Tong, *String Theory*, arXiv:0908.0333.

[49] P. D. Francesco, P. Mathieu, and D. Senechal, *Conformal Field Theory.* Graduate Texts in Contemporary Physics. Springer, 1997.

[50] D. J. Gross and J. H. Sloan, *The Quartic Effective Action for the Heterotic String*, *Nucl.Phys.* **B291** (1987) 41.

[51] D. Friedan, E. J. Martinec, and S. H. Shenker, *Conformal Invariance, Supersymmetry and String Theory*, *Nucl.Phys.* **B271** (1986) 93.

[52] M. Bianchi and A. V. Santini, *String predictions for near future colliders from one-loop scattering amplitudes around D-brane worlds*, *JHEP* **12** (2006) 010, [hep-th/0607224].

[53] J. J. Atick and A. Sen, *Correlation functions of spin operators on a torus*, *Nucl. Phys.* **B286** (1987) 189.

[54] J. J. Atick and A. Sen, *Covariant one loop fermion emission amplitudes in closed string theories*, *Nucl. Phys.* **B293** (1987) 317.

[55] C. Albertsson, U. Lindstrom, and M. Zabzine, *N=1 supersymmetric sigma model with boundaries. 1.*, *Commun.Math.Phys.* **233** (2003) 403–421, [hep-th/0111161].

[56] C. Burgess and T. Morris, *Open superstrings a la Polyakov*, *Nucl.Phys.* **B291** (1987) 285.

[57] A. Hashimoto and I. R. Klebanov, *Decay of excited D-branes*, *Phys.Lett.* **B381** (1996) 437–445, [hep-th/9604065].

[58] A. Hashimoto and I. R. Klebanov, *Scattering of strings from D-branes*, *Nucl.Phys.Proc.Suppl.* **55B** (1997) 118–133, [hep-th/9611214].

[59] M. R. Garousi and R. C. Myers, *Superstring scattering from D-branes*, *Nucl.Phys.* **B475** (1996) 193–224, [hep-th/9603194].

[60] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche, and T. Taylor, *Aspects of type I - type II - heterotic triality in four-dimensions*, *Nucl.Phys.* **B489** (1997) 160–178, [hep-th/9608012].

[61] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, *Strings on Orbifolds*, *Nucl.Phys.* **B261** (1985) 678–686.

[62] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, *Strings on Orbifolds. 2.*, *Nucl.Phys.* **B274** (1986) 285–314.

[63] M. R. Douglas and G. W. Moore, *D-branes, Quivers, and ALE Instantons*, hep-th/9603167.

[64] M. R. Douglas, B. R. Greene, and D. R. Morrison, *Orbifold resolution by D-branes*, *Nucl. Phys.* **B506** (1997) 84–106, [hep-th/9704151].

[65] G. Aldazabal, L. E. Ibanez, F. Quevedo, and A. M. Uranga, *D-branes at singularities: A bottom-up approach to the string embedding of the standard model*, *JHEP* **08** (2000) 002, [hep-th/0005067].

[66] L. J. Dixon, D. Friedan, E. J. Martinec, and S. H. Shenker, *The Conformal Field Theory of Orbifolds*, *Nucl.Phys.* **B282** (1987) 13–73.

[67] M. Cvetic, *Phenomenological Implications Of The Blown-Up Orbifolds, SLAC-PUB-4324 (1987)*, .

[68] S. Hamidi and C. Vafa, *Interactions on Orbifolds*, *Nucl.Phys.* **B279** (1987) 465.

[69] J. R. David, *Tachyon condensation in the D0 / D4 system*, *JHEP* **0010** (2000) 004, [hep-th/0007235].

[70] I. Antoniadis, K. Benakli, and A. Laugier, *Contact interactions in D-brane models*, *JHEP* **0105** (2001) 044, [hep-th/0011281].

[71] J. R. David, *Tachyon condensation using the disc partition function*, *JHEP* **0107** (2001) 009, [hep-th/0012089].

[72] M. Cvetic and I. Papadimitriou, *Conformal field theory couplings for intersecting D-branes on orientifolds*, *Phys.Rev.* **D68** (2003) 046001, [hep-th/0303083].

[73] S. Abel and A. Owen, *Interactions in intersecting brane models*, *Nucl.Phys.* **B663** (2003) 197–214, [hep-th/0303124].

[74] S. Abel and A. Owen, *N point amplitudes in intersecting brane models*, *Nucl.Phys.* **B682** (2004) 183–216, [hep-th/0310257].

[75] A. Lawrence and A. Sever, *Scattering of twist fields from D-branes and orientifolds*, *JHEP* **0709** (2007) 094, [arXiv:0706.3199].

[76] H. Verlinde and M. Wijnholt, *Building the standard model on a D3-brane*, *JHEP* **0701** (2007) 106, [hep-th/0508089].

[77] V. Balasubramanian, P. Berglund, and I. Garcia-Etxebarria, *Toric Lego: A Method for modular model building*, *JHEP* **1001** (2010) 076, [arXiv:0910.3616].

[78] S. Krippendorf, M. J. Dolan, A. Maharana, and F. Quevedo, *D-branes at Toric Singularities: Model Building, Yukawa Couplings and Flavour Physics*, *JHEP* **1006** (2010) 092, [arXiv:1002.1790].

[79] M. J. Dolan, S. Krippendorf, and F. Quevedo, *Towards a Systematic Construction of Realistic D-brane Models on a del Pezzo Singularity*, *JHEP* **1110** (2011) 024, [arXiv:1106.6039].

[80] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo, and R. Valandro, *D-Branes at del Pezzo Singularities: Global Embedding and Moduli Stabilisation*, *JHEP* **1209** (2012) 019, [arXiv:1206.5237].

[81] J. J. Atick, L. J. Dixon, and A. Sen, *String Calculation of Fayet-Iliopoulos d Terms in Arbitrary Supersymmetric Compactifications*, *Nucl. Phys.* **B292** (1987) 109–149.

[82] P. Bain and M. Berg, *Effective action of matter fields in four-dimensional string orientifolds*, *JHEP* **0004** (2000) 013, [hep-th/0003185].

[83] M. Berg, M. Haack, and B. Kors, *String loop corrections to Kaehler potentials in orientifolds*, *JHEP* **11** (2005) 030, [hep-th/0508043].

[84] S. A. Abel and M. D. Goodsell, *Realistic Yukawa Couplings through Instantons in Intersecting Brane Worlds*, *JHEP* **0710** (2007) 034, [hep-th/0612110].

[85] F. Marchesano and L. Martucci, *Non-perturbative effects on seven-brane Yukawa couplings*, *Phys.Rev.Lett.* **104** (2010) 231601, [arXiv:0910.5496].

[86] F. Gmeiner and G. Honecker, *Mapping an Island in the Landscape*, *JHEP* **0709** (2007) 128, [arXiv:0708.2285].

[87] I. Antoniadis, E. Kiritsis, and J. Rizos, *Anomalous U(1)s in type 1 superstring vacua*, *Nucl.Phys.* **B637** (2002) 92–118, [hep-th/0204153].

[88] M. Berg, M. Haack, and B. Kors, *On the moduli dependence of nonperturbative superpotentials in brane inflation*, hep-th/0409282.

[89] M. Berg, M. Haack, and B. Kors, *Loop corrections to volume moduli and inflation in string theory*, *Phys. Rev.* **D71** (2005) 026005, [hep-th/0404087].

[90] D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister, et al., *On D3-brane Potentials in Compactifications with Fluxes and Wrapped D-branes*, *JHEP* **0611** (2006) 031, [hep-th/0607050].

[91] B. Holdom, *Two U(1)'s and Epsilon Charge Shifts*, *Phys.Lett.* **B166** (1986) 196.

[92] B. Holdom, *Searching for epsilon charges and a new U(1)*, *Phys.Lett.* **B178** (1986) 65.

[93] S. A. Abel, J. Jaeckel, V. V. Khoze, and A. Ringwald, *Illuminating the Hidden Sector of String Theory by Shining Light through a Magnetic Field*, *Phys.Lett.* **B666** (2008) 66–70, [hep-ph/0608248].

[94] C. Burgess, J. Conlon, L.-Y. Hung, C. Kom, A. Maharana, et al., *Continuous Global Symmetries and Hyperweak Interactions in String Compactifications*, *JHEP* **0807** (2008) 073, [arXiv:0805.4037].

[95] C. Boehm, D. Hooper, J. Silk, M. Casse, and J. Paul, *MeV dark matter: Has it been detected?*, *Phys.Rev.Lett.* **92** (2004) 101301, [astro-ph/0309686].

[96] P. Fayet, *U-boson production in e+ e- annihilations, psi and Upsilon decays, and Light Dark Matter*, *Phys.Rev.* **D75** (2007) 115017, [hep-ph/0702176].

[97] P. Fayet, *Constraints on Light Dark Matter and U bosons, from psi, Upsilon, K+, pi0, eta and eta-prime decays*, *Phys.Rev.* **D74** (2006) 054034, [hep-ph/0607318].

[98] J. Jaeckel, *A force beyond the Standard Model - Status of the quest for hidden photons*, *Frascati Phys.Ser.* **56** (2013) 172–192, [arXiv:1303.1821].

[99] K. R. Dienes, C. F. Kolda, and J. March-Russell, *Kinetic mixing and the supersymmetric gauge hierarchy*, *Nucl.Phys.* **B492** (1997) 104–118, [hep-ph/9610479].

[100] A. Lukas and K. Stelle, *Heterotic anomaly cancellation in five-dimensions*, *JHEP* **0001** (2000) 010, [hep-th/9911156].

[101] S. Abel and B. Schofield, *Brane anti-brane kinetic mixing, millicharged particles and SUSY breaking*, *Nucl.Phys.* **B685** (2004) 150–170, [hep-th/0311051].

[102] R. Blumenhagen, G. Honecker, and T. Weigand, *Loop-corrected compactifications of the heterotic string with line bundles*, *JHEP* **0506** (2005) 020, [hep-th/0504232].

[103] S. Abel, M. Goodsell, J. Jaeckel, V. Khoze, and A. Ringwald, *Kinetic Mixing of the Photon with Hidden U(1)s in String Phenomenology*, *JHEP* **0807** (2008) 124, [arXiv:0803.1449].

[104] A. Arvanitaki, N. Craig, S. Dimopoulos, S. Dubovsky, and J. March-Russell, *String Photini at the LHC*, *Phys.Rev.* **D81** (2010) 075018, [arXiv:0909.5440].

[105] M. Goodsell, J. Jaeckel, J. Redondo, and A. Ringwald, *Naturally Light Hidden Photons in LARGE Volume String Compactifications*, *JHEP* **0911** (2009) 027, [arXiv:0909.0515].

[106] M. Goodsell, *Light Hidden U(1)s from String Theory*, arXiv:0912.4206.

[107] M. Goodsell and A. Ringwald, *Light Hidden-Sector U(1)s in String Compactifications*, *Fortsch.Phys.* **58** (2010) 716–720, [arXiv:1002.1840].

[108] J. J. Heckman and C. Vafa, *An Exceptional Sector for F-theory GUTs*, *Phys.Rev.* **D83** (2011) 026006, [arXiv:1006.5459].

[109] M. Cicoli, M. Goodsell, J. Jaeckel, and A. Ringwald, *Testing String Vacua in the Lab: From a Hidden CMB to Dark Forces in Flux Compactifications*, *JHEP* **1107** (2011) 114, [arXiv:1103.3705].

[110] M. Goodsell, S. Ramos-Sanchez, and A. Ringwald, *Kinetic Mixing of U(1)s in Heterotic Orbifolds*, *JHEP* **1201** (2012) 021, [arXiv:1110.6901].

[111] G. Shiu, P. Soler, and F. Ye, *Milli-Charged Dark Matter in Quantum Gravity and String Theory*, arXiv:1302.5471.

[112] R. G. Leigh and M. Rozali, *Brane boxes, anomalies, bending and tadpoles*, *Phys.Rev.* **D59** (1999) 026004, [hep-th/9807082].

[113] G. Aldazabal, D. Badagnani, L. E. Ibanez, and A. Uranga, *Tadpole versus anomaly cancellation in $D = 4$, $D = 6$ compact IIB orientifolds*, *JHEP* **9906** (1999) 031, [hep-th/9904071].

[114] M. B. Green and J. H. Schwarz, *Anomaly Cancellation in Supersymmetric D=10 Gauge Theory and Superstring Theory*, *Phys.Lett.* **B149** (1984) 117–122.

[115] L. E. Ibanez, R. Rabadan, and A. Uranga, *Anomalous U(1)'s in type I and type IIB $D = 4$, N=1 string vacua*, *Nucl.Phys.* **B542** (1999) 112–138, [hep-th/9808139].

[116] M. Berg, M. Haack, and E. Pajer, *Jumping Through Loops: On Soft Terms from Large Volume Compactifications*, *JHEP* **0709** (2007) 031, [arXiv:0704.0737].

[117] M. Cicoli, J. P. Conlon, and F. Quevedo, *General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation*, *JHEP* **0810** (2008) 105, [arXiv:0805.1029].

[118] T. W. Grimm, R. Savelli, and M. Weissenbacher, *On alpha' corrections in N=1 F-theory compactifications*, arXiv:1303.3317.

[119] M. Cicoli, C. Mayrhofer, and R. Valandro, *Moduli Stabilisation for Chiral Global Models*, *JHEP* **1202** (2012) 062, [arXiv:1110.3333].

[120] M. Cicoli, *Global D-brane models with stabilised moduli and light axions*, arXiv:1209.3740.

[121] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo, and R. Valandro, *D3/D7 Branes at Singularities: Constraints from Global Embedding and Moduli Stabilisation*, arXiv:1304.0022.

[122] D. Mumford, *Tata Lectures on Theta I.* Boston, USA: Birkhauser (1983).

[123] J. P. Conlon, *Gauge Threshold Corrections for Local String Models*, *JHEP* **04** (2009) 059, [arXiv:0901.4350].

[124] J. P. Conlon and E. Palti, *Gauge Threshold Corrections for Local Orientifolds*, *JHEP* **09** (2009) 019, [arXiv:0906.1920].

[125] S. Abel and B. Schofield, *One-loop Yukawas on intersecting branes*, *JHEP* **0506** (2005) 072, [hep-th/0412206].