ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

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Guido Ascari, Anna Florio

and Alessandro Gobbi

Number 788
March 2016

Manor Road Building, Oxford OX1 3UQ

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Guido Ascari† University of Oxford

Anna Florio‡ Politecnico di Milano

Alessandro Gobbi§ Università Cattolica del Sacro Cuore

March 2016

Abstract

A natural generalisation of the original Leeper (1991) taxonomy leads to the concepts of *globally active (or passive)* and *globally switching* policies to explain the determinacy properties of a model where both monetary and fiscal policies may switch according to a Markov process. Monetary and fiscal policies need to be globally balanced to guarantee a unique equilibrium: globally active monetary policies need to be coupled with globally passive fiscal policies, and switching monetary policies with switching fiscal policies. This new taxonomy also links the determinacy analysis to the model dynamics because it qualifies under which conditions expectations and wealth effects arise in the Markov-switching model.

Keywords: Monetary Policy and Fiscal Policy Interaction, Markov Switching, Non-linear models.

JEL classification: E5.

*We thank Luca Dedola, Martin Ellison, Andrea Ferrero, Eric Leeper, Bartosz Mackowiak, Magali Marx, Chris Sims, Jouko Vilmunen and Tao Zha for useful comments. We would also like to thank participants to the ECB-EABCN-FRB Atlanta conference on Nonlinearities in Macroeconomics and Finance in the Light of Crises in Frankfurt, 15/16 December 2014, to the first annual conference of the MACFINROBODS project in Paris, 15-16 June 2015, to the VI Workshop on Institutions, Individual Behavior and Economic Outcomes in Alghero, 20-25 June 2015, to the the Anglo-French-Italian Workshop on Macroeconomics in Milan, 16-17 October 2015, and seminar participants at the University of Bern, Oxford and the Bank of Finland.

†Department of Economics, University of Oxford, Manor Road, Oxford OX1 3UQ, United Kingdom. E-mail address: guido.ascari@economics.ox.ac.uk

‡Department of Management, Economics and Industrial Engineering, Politecnico di Milano, via Lambruschini 4/B, 20156 Milan, Italy. E-mail address: anna.florio@polimi.it

§Department of Economics and Finance, Università Cattolica del Sacro Cuore, via Necchi 5, 20123 Milan, Italy. E-mail address: alessandro.gobbi@unicatt.it

1 Introduction

Economists generally describe macroeconomic policy through the use of policy rules that are then embedded in dynamic general equilibrium models to analyse their stabilisation and welfare properties. There is a wealth of evidence, however, that monetary and fiscal policy rules change over time, so a growing body of literature has started using models with recurring regime changes to estimate and study monetary and fiscal policy interactions.[1] In such a setup, agents coherently form their expectations considering the possibility of a change in the policy regime. This affects the dynamics of the model and the nature of the rational expectations equilibrium. In particular, when agents' expectations incorporate the possibility that policy regimes may switch, the analysis of uniqueness or multiplicity of rational expectations equilibria is quite different from the standard literature on determinacy/indeterminacy (e.g., Lubik and Schorfheide, 2004), where the change in regime comes as a complete surprise and it is perceived to last forever. It is possible that policy combinations, that lead to an explosive equilibrium in a fixed regime model, do not violate the transversality condition in a Markov switching model because agents correctly impute a positive probability of returning to a stable regime. Similarly, policy mixes, that would lead to indeterminacy in a fixed-regime model, could result in a unique determinate equilibrium if agents again anticipate the probability of reverting to a different policy mix in the future.

The influential contribution by Davig and Leeper (2007b) shows how to generalise the Taylor principle - the proposition that the central bank can stabilise the macroeconomy by adjusting the policy rate more than one-to-one with inflation - to a Markov-switching context. They find that the Taylor principle can be satisfied in the long run "even while deviating from it substantially for brief periods or modestly for prolonged periods". Allowing for regime changes in the parameter that controls the response of the interest rate to inflation in the Taylor rule, a passive monetary policy - indeterminate in a static context - could return determinacy if monetary policy becomes sufficiently aggressive in the future regime. The authors conclude that this long-run Taylor principle dramatically expands the determinacy region when compared to the constant-parameter setup.[2]

This paper studies whether the long-run Taylor principle changes when fiscal policy is allowed to change regime too. Consistently with much of the literature, Davig and Leeper (2007b) place fiscal policy in the background by assuming a passive fiscal policy.[3] However, the seminal paper by

[1]The list is rather long so here we mention quite a few without the aim of being exhaustive: Davig and Leeper (2007a,b); Chung et al. (2007); Davig and Leeper (2008, 2011); Bianchi (2013); Foerster (2013); Bianchi and Ilut (2014); Leeper et al. (2015).

[2]See Barthelemy and Marx (2015) for a recent generalisation of the results in Davig and Leeper (2007b).

[3]We apply the terminology in Leeper (1991). Active monetary (AM) policy arises when the response of the nominal

Leeper (1991) analyses the determinacy properties of four different regimes in a fixed coefficient setting depending on fiscal and monetary policies being active or passive. We thus extend Davig and Leeper's paper to a model where both monetary *and* fiscal policies are allowed to switch between a passive and an active state. By allowing for regime switching in both policies, our contribution generalises the seminal paper of Leeper (1991) to a Markov-switching context.

While the literature features many papers that estimate and analyse models' dynamics allowing contemporaneous switching of both monetary and fiscal policy, we study the conditions for the existence of a unique stable solution in a simple New Keynesian model, using the algorithm based on the perturbation method developed by Foerster et al. (2015, henceforth FRWZ). Indeed, determinacy represents a desirable feature of monetary policy implementation since the existence of multiple stable equilibria exposes the economy to endogenous volatility. We show that coordination between policies across the two regimes is essential because with a simultaneous switching of monetary and fiscal policy almost anything can happen. In particular, the switching from a double active regime (explosive in fixed coefficients) to a double passive regime (indeterminate in fixed coefficients) can happen to return determinacy. Furthermore, switching between two determinate regimes does not assure determinacy. Our approach extends Davig and Leeper's (2007b) intuition and proposes a natural generalisation of Leeper (1991), introducing the concepts of *globally active* (or *passive*) and *globally switching* policies to explain the determinacy properties of the model under Markov switching. A globally active (or passive) monetary (or fiscal) policy admits a modest deviation into passive (or active) monetary (or fiscal) policy in one of the regimes, but this deviation needs to be modest. If the deviation is instead substantial we label the monetary (or fiscal) policy a globally switching one.

The paper offers two main results. The first main message is that monetary and fiscal policies need to be *globally balanced* to guarantee a unique equilibrium: a globally active monetary policy needs to be coupled with a globally passive fiscal policy, and globally switching monetary policies with globally switching fiscal policies. Despite the complexity of the algorithm, we are able to provide some analytical insights on our results. This allows us to establish the second main result of the paper: there is a direct link between the concept of balanced policies and the dynamics of the model in terms of the presence of expectation effects. Contrary to Davig and Leeper (2007a) and Chung et al. (2007), we do not find that the fiscal theory is always at work when agents attach a positive probability of moving

interest rate is more than one-to-one to inflation, otherwise there is passive monetary (PM) policy. Analogously, passive fiscal (PF) policy occurs when taxes respond sufficiently to debt to avoid its explosion, while it is named active fiscal (AF) otherwise. In many fixed coefficient models, a unique bounded equilibrium requires one active and one passive policy.

towards a regime with active fiscal policy. We find neither expectation effects nor wealth effects under a *globally AM/PF regime*, that is, a regime that admits only modest deviations into PM/AF in one of the two regimes. Our study solves the problem of establishing if a regime is Ricardian or not in a model where agents are aware of recurrent regime changes: a *globally AM/PF regime* is definitively Ricardian.

We (mainly) concentrate our analysis on the case where one of the two regimes is AM/PF, which is the benchmark case in the New Keynesian literature.[4] Moreover, many works in this literature study the consequence on inflation of the transition from the Great Inflation to the Great Moderation era, the latter commonly considered an AM/PF regime.[5] Finally, we think that the regime switching approach is particularly suited to understanding the economic consequences of returning to the benchmark AM/PF regime after a period where fiscal policy could be considered active and monetary policy has been constrained by the zero lower bound, as after the outbreak of the recent crisis.

The paper is structured as follows. Section 2 is devoted to present the model and the methodology. Section 3 analyses determinacy areas when one of the two regimes is AM/PF, it explains what we mean by a *globally balanced* regime and it shows the implications for policy coordination. Section 4 focuses on the dynamics of the model and on the expectation effects of regime shifts. Section 5 concludes.

2 Model and Methodology

2.1 The model

We consider a basic New Keynesian model with monetary and fiscal policy rules, as in Bhattarai et al. (2014). The model is well-known, so a more complete description is in the Appendix. In non-linear form, the equations of the model are the following

$$1 = \beta \mathbb{E}_t \left(\frac{Y_t - G}{Y_{t+1} - G} \frac{R_t}{\Pi_{t+1}} \right), \tag{1}$$

$$\phi_t \left(1 - \alpha \Pi_t^{\theta-1}\right)^{\frac{1}{1-\theta}} = \frac{\mu\theta\left(1-\alpha\right)^{\frac{1}{1-\theta}}}{\theta - 1} Y_t + \alpha\beta \mathbb{E}_t \left[\phi_{t+1} \Pi_{t+1}^{\theta} \left(1 - \alpha \Pi_{t+1}^{\theta-1}\right)^{\frac{1}{1-\theta}}\right], \tag{2}$$

$$\phi_t = \frac{Y_t}{Y_t - G} + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1}\right], \tag{3}$$

[4]Results, however, can be easily extended to any other regime combination.
[5]Note that some authors consider the Great Inflation an indeterminate regime, e.g., Bhattarai et al. (2012), while others consider it a PM/AF regime, e.g., Bianchi (2012).

$$\frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} + G - \tau_t, \tag{4}$$

$$\tau_t = \tau \left(\frac{b_{t-1}}{b} \right)^{\gamma_{\tau,t}} e^{u_{\tau,t}}, \tag{5}$$

$$R_t = R \left(\Pi_t \right)^{\gamma_{\pi,t}} e^{u_{m,t}}. \tag{6}$$

Equation (1) is a standard Euler equation for consumption, where Y is output, R the nominal interest rate, Π the gross inflation rate and G government spending, which is assumed to be exogenous and constant. Equations (2) and (3) describe the evolution of inflation in the non-linear model. ϕ_t is an auxiliary variable (equal to the present discounted value of expected future marginal revenues) that allows us to write the model recursively. Equation (4) is the flow budget constraint of the government, where $b_t = B_t/P_t$ is real government debt. We follow Leeper (1991) in using lump-sum taxes, i.e., τ, which are set according to the fiscal rule (5): taxes react to the deviation of lagged real debt from its steady state level (b) according to the parameter $\gamma_{\tau,t}$. Equation (6) describes monetary policy. It is a simple Taylor rule where the central bank reacts to current inflation according to the parameter $\gamma_{\pi,t}$. A variable without the time index (i.e., τ, b and R) indicates its steady state value. β is the intertemporal discount factor; θ is the Dixit-Stiglitz elasticity of substitution between goods; and α is the Calvo probability that a firm is not able to optimise its price. The key parameters of our analysis are $\gamma_{\tau,t}$ and $\gamma_{\pi,t}$, which describe the time-varying stance of fiscal and monetary policy, respectively. We assume that these parameters follow an underlying two-state Markov process and are equal to ($\gamma_{\tau,i}$, $\gamma_{\pi,i}$) when the economy is in regime i, for $i = 1, 2$. The transition probabilities of going from regime i to regime j are denoted by p_{ij}, so p_{ii} is the probability of remaining in regime i and $p_{ij} = 1 - p_{ii}$.

2.2 Solution method

As our model includes fiscal policy, we need to account for the dynamics of public debt, which is a state variable. We thus employ the perturbation method developed by FRWZ, which allows us to solve for the minimal state variable (MSV) solutions of a Markov-switching model in the presence of predetermined variables.[6] Following FRWZ, our model can be written as

$$\mathbb{E}_t \boldsymbol{f}\left(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, b_t, b_{t-1}, \boldsymbol{\varepsilon}_{t+1}, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta}_{t+1}, \boldsymbol{\theta}_t\right) = \mathbf{0}, \tag{7}$$

where b_t is the only predetermined variable, while the remaining non-predetermined variables are stacked in vector $\boldsymbol{y}_t' \equiv [Y_t, \Pi_t, \phi_t]$. The exogenous shocks appear in vector $\boldsymbol{\varepsilon}_t' \equiv [u_{m,t}, u_{\tau,t}]$, and $\boldsymbol{\theta}_t' \equiv [\gamma_{\pi,t}, \gamma_{\tau,t}]$ is the vector of Markov-switching parameters. The first-order Taylor expansions of the recursive solutions are

$$b_t \approx b + h_{i,b}(b_{t-1} - b) + \boldsymbol{h}_{i,\varepsilon}\boldsymbol{\varepsilon}_t + h_{i,\chi}\chi, \tag{8}$$

$$\boldsymbol{y}_t \approx \boldsymbol{y} + \boldsymbol{g}_{i,b}(b_{t-1} - b) + \boldsymbol{g}_{i,\varepsilon}\boldsymbol{\varepsilon}_t + \boldsymbol{g}_{i,\chi}\chi, \tag{9}$$

where χ is the perturbation parameter. Note that the slope coefficients of the solutions are regime-dependent, while the steady state is not. $h_{i,b}$ governs the stability properties of the solution and is therefore required for the analysis of determinacy. FRWZ show that $h_{i,b}$ and $\boldsymbol{g}_{i,b}$ can be jointly found after solving a system of quadratic equations. As this system cannot be solved using traditional approaches such as the generalised Schur decomposition, we follow FRWZ and adopt the Groebner basis algorithm to find all existing solutions.

Once all the admissible solutions have been found, a stability criterion needs to be imposed in order to select the stable ones. We use the concept of *mean square stability (MSS)* proposed by Costa et al. (2005) and Farmer et al. (2009).[7] The MSS condition constrains the values of the autoregressive roots in the state variable policy function in the two regimes. In the Appendix we show that the exact conditions for MSS are

$$\left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2 < 1, \tag{10}$$

$$p_{11} h_1^2 \left(1 - h_2^2\right) + p_{22} h_2^2 \left(1 - h_1^2\right) + h_1^2 h_2^2 < 1, \tag{11}$$

for $p_{11} + p_{22} > 1$. Therefore, any given parameter configuration could lead either to: (i) determinacy, when a unique stable solution exists; (ii) indeterminacy, when multiple stable solutions exist; (iii)

[6] While some other non-MSV solutions may still exist, this class of solutions is usually the one employed in the estimation of DSGE models. At the time of writing, the analysis of rational expectation solutions in a Markov-switching context is a very active research area. Beside FRWZ, see, among others, Farmer et al. (2009, 2011), Blake and Zampolli (2011), Cho (2015), Maih (2015), Barthelemy and Marx (2015).

[7] Davig and Leeper (2007b) employs a different concept of stability: bounded stability, which requires bounded paths and thus rules out temporarily explosive paths in one of the two regimes. See Farmer et al. (2009) and Barthelemy and Marx (2015) for a discussion in the context of Markov-switching DSGE models.

explosiveness, when no stable solutions exist. In what follows we want to explore the parameter space in order to identify the regions corresponding to these three cases.

2.3 Determinacy under fixed coefficients: Leeper (1991)

Assume for the moment that both $\gamma_{\pi,t}$ and $\gamma_{\tau,t}$ are constant over time and not subject to regime changes, as in Leeper (1991). The log-linearised model is a trivariate dynamic system in the two jump variables $\hat{y}_t$ and $\hat{\pi}_t$ and the predetermined variable $\hat{b}_t$

$$\frac{1}{\bar{c}}\hat{Y}_t = \frac{1}{\bar{c}}\mathbb{E}_t\hat{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t\hat{\Pi}_{t+1}\right), \tag{12}$$

$$\hat{\Pi}_t = \frac{\lambda}{\bar{c}}\hat{Y}_t + \beta\mathbb{E}_t\hat{\Pi}_{t+1}, \tag{13}$$

$$\hat{R}_t = \gamma_\pi\hat{\Pi}_t + u_{m,t}, \tag{14}$$

$$\hat{b}_t = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_\tau\right)\hat{b}_{t-1} - \frac{1}{\beta}\hat{\Pi}_t + \hat{R}_t - \frac{1}{\beta}\frac{\tau}{b}u_{\tau,t}, \tag{15}$$

where $\bar{c}$ is the steady state consumption-to-GDP ratio and $\lambda \equiv (1-\alpha)(1-\alpha\beta)/\alpha$ determines the slope of the Phillips curve. Hatted variables indicate log-deviations from steady state values. It is useful here to recall the necessary and sufficient conditions for determinacy of the rational expectations equilibrium (REE) in a fixed coefficient model. Using Leeper's (1991) renowned taxonomy, fiscal policy is said to be *passive* if the fiscal rule guarantees debt stabilisation in (15), that is if

$$\left|\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_\tau\right)\right| < 1. \tag{16}$$

In case of passive fiscal policy, it is easy to show that the following conditions have to hold to yield determinacy

$$\gamma_\pi > 1 \tag{17}$$

and

$$\gamma_\pi > \frac{\beta - 1}{\lambda}. \tag{18}$$

The first condition is the Taylor principle and it implies the second, which then becomes redundant. According to Leeper's taxonomy, monetary policy is labelled *active* if it satisfies the Taylor principle, otherwise is labelled as *passive*. Hence, the famous result in Leeper (1991) follows: when fiscal policy is passive, monetary policy needs to be active (i.e., $\gamma_\pi > 1$) to yield determinacy.

Conversely,in case of active fiscal policy (i.e., when (16) holds with the opposite sign), monetary policy should be passive to guarantee determinacy: $\gamma_\pi < 1$. In this case, the REE is non-Ricardian so a change in lump-sum taxation has real effects, and the so-called fiscal theory of the price level holds.[8] Summing up, in a fixed coefficient model as in Leeper (1991), the determinacy region is defined by the following conditions:

- Active monetary/passive fiscal (AM/PF)

$$\gamma_\pi > 1 \quad \text{and} \quad (1-\beta)\frac{b}{\tau} < \gamma_\tau < (1+\beta)\frac{b}{\tau}$$

- Passive monetary/active fiscal (PM/AF)

$$\gamma_\pi < 1 \quad \text{and either} \quad \gamma_\tau < (1-\beta)\frac{b}{\tau} \quad \text{or} \quad \gamma_\tau > (1+\beta)\frac{b}{\tau}$$

The REE equilibrium is indeterminate under PM/PF configurations and explosive under AM/AF ones.

2.4 Determinacy under regime switching

2.4.1 The general case

Applying the FRWZ method, the Appendix shows that solutions need to satisfy the following system of equations for the general case with $p_{11}, p_{22} < 1$

$$\begin{aligned} 0 = {} & g_{\pi,1}\left[1 + \lambda\gamma_{\pi,1} - p_{11}h_1(1+\beta+\lambda) + p_{11}^2\beta h_1^2\right] + (1-p_{11})(1-p_{22})\beta h_1 h_2 g_{\pi,1} \\ & + (1-p_{11})h_1 g_{\pi,2}\left[p_{11}\beta h_1 + p_{22}\beta h_2 - (1+\beta+\lambda)\right], \end{aligned} \tag{19}$$

$$\begin{aligned} 0 = {} & g_{\pi,2}\left[1 + \lambda\gamma_{\pi,2} - p_{22}h_2(1+\beta+\lambda) + p_{22}^2\beta h_2^2\right] + (1-p_{11})(1-p_{22})\beta h_1 h_2 g_{\pi,2} \\ & + (1-p_{22})h_2 g_{\pi,1}\left[p_{11}\beta h_1 + p_{22}\beta h_2 - (1+\beta+\lambda)\right], \end{aligned} \tag{20}$$

$$g_{\pi,1} = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)}, \tag{21}$$

$$g_{\pi,2} = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) - h_2}{b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)}, \tag{22}$$

where the 4 unknowns are h_1, h_2, $g_{\pi,1}$ and $g_{\pi,2}$. Recall that the debt, b_t, is the state variable of the system. h_i is the response of debt to its lag in regime i and $g_{\pi,i}$ is the response of inflation to the

[8] See Bhattarai et al. (2014) for a thorough analysis of the dynamics implied by such a parameter configuration.

lagged debt in regime i.

2.4.2 The absorbing case

In the subsequent analysis we will often refer to the case where regime 1 is absorbing, so that $p_{11} = 1$. This simplification allows us to derive analytical results on determinacy and, in turn, develop intuition about the numerical results in the general case. We refer the interested reader to the Appendix for the derivations of the analytical results in the text for this case. If $p_{11} = 1$, equations (19) and (21) reduce to

$$0 = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)}\left[1 + \lambda\gamma_{\pi,1} - h_1\left(1 + \beta + \lambda\right) + \beta h_1^2\right] \tag{23}$$

and the conditions for MSS, i.e., (10) and (11), simplify to: $|h_1| < 1$ and $|h_2| < \frac{1}{\sqrt{p_{22}}}$. Note that the Markov-switching nature of the economy only affects the condition in the non-absorbing regime. In particular, with respect to the fixed-coefficient case, the stability condition is less binding the lower is the probability of staying in the second state.

3 Determinacy Analysis

This section contains the main results of the determinacy analysis in our model. First, starting from an AM/PF regime, we want to understand how determinacy varies as the policy mix in the other regime changes. We will first analyse and provide analytical results for the case where regime 1 is absorbing (Section 3.1) and then numerical results for the more general case where the probability of staying in each regime is lower than one (Section 3.2). Since our main contribution is to extend the analysis of Davig and Leeper (2007b) to fiscal policy, Section 3.3 studies the conditions that fiscal policy needs to satisfy in the two regimes to yield a unique REE. An important insight is that determinacy depends on the existence of globally balanced policies, calling for coordination of fiscal and monetary authorities (Section 3.4).

3.1 The absorbing AM/PF case

Assume that regime 1 is absorbing, so that $p_{11} = 1$. If the economy is already in this regime, the conditions for determinacy evidently are the same as under fixed coefficients. Hence if fiscal policy is passive, condition (16) must hold, that is $\frac{b}{\tau}(1-\beta) < \gamma_{\tau,1} < \frac{b}{\tau}(1+\beta)$, and monetary policy must be active ($\gamma_{\pi,1} > 1$).[9] Figure 1a shows the combinations of the monetary ($\gamma_{\pi,2}$) and the fiscal ($\gamma_{\tau,2}$) coefficients for the second regime (setting $p_{22} = 0.95$) that return determinacy of the global equilibrium given an absorbing AM/PF regime 1 ($\gamma_{\pi,1} = 1.5$, $\gamma_{\tau,1} = 0.2$). Notably, there are two regions in the ($\gamma_{\pi,2}, \gamma_{\tau,2}$) space that return determinacy: an upper-right zone and a lower-left one.

First, let us analyse the upper-right zone. In this case there is MSS if for regime 2 the following conditions hold

$$\gamma_{\tau,2} \in \left(\bar{\gamma}_{\tau,2}, \frac{b}{\tau}\left(1 + \frac{\beta}{\sqrt{p_{22}}}\right)\right), \tag{24}$$

$$\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}, \tag{25}$$

with

$$\bar{\gamma}_{\tau,2} \equiv \frac{b}{\tau}\left(1 - \frac{\beta}{\sqrt{p_{22}}}\right), \quad \text{and} \quad \bar{\gamma}_{\pi,2} \equiv \sqrt{p_{22}} - \frac{\left(1 - \beta\sqrt{p_{22}}\right)\left(1 - \sqrt{p_{22}}\right)}{\lambda}$$

Determinacy clearly emerges when the second regime is AM/PF too. The threshold values $\bar{\gamma}_{\tau,2}$ and $\bar{\gamma}_{\pi,2}$\$, however, imply that both intervals for $\gamma_{\tau,2}$ and $\gamma_{\pi,2}$ widen, compared to the fixed coefficients result: there is determinacy even if the second regime deviates from Leeper's (1991) definition of the AM/PF mix. Looking carefully at the conditions above, one can realise that $\bar{\gamma}_{\tau,2}$ is negative, if $\sqrt{p_{22}} < \beta$, while $\bar{\gamma}_{\pi,2}$ is lower than one, because $\sqrt{p_{22}} < 1$. In other words, in order to have determinacy, fiscal and monetary policy in the second regime are not constrained to be, respectively, always passive and active. Rather, they can now vary "modestly" and be (to a certain extent), respectively, active and passive. This effect is more pronounced the lower is p_{22}. The modest changes for fiscal and for monetary policy are given by the following intervals (visualised, respectively, by the dotted and continuous arrows in Figure 1a)

$$\bar{\gamma}_{\tau,2} < \gamma_{\tau,2} < \frac{b}{\tau}(1 - \beta), \tag{26}$$

$$\bar{\gamma}_{\pi,2} < \gamma_{\pi,2} < 1. \tag{27}$$

Consider now what happens in the lower-left zone. There is global MSS if for regime 2 the following holds

$$\gamma_{\tau,2} < \bar{\gamma}_{\tau,2} \text{ and } \gamma_{\tau,2} > \frac{b}{\tau}\left(1 + \frac{\beta}{\sqrt{p_{22}}}\right), \tag{28}$$

$$\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}. \tag{29}$$

[9] Our calibration yields : $0.019 < \gamma_{\tau,1} < 3.892$. The calibration is described in Table 1 in the Appendix. We do not discuss it in the main text because it is very standard and our model is too stylised to make the case for a quantitative analysis. However, the logic of our analyses and results do not depend on the particular calibration chosen.

In this case, in order to have determinacy, fiscal and monetary policy in second regime are constrained to be, respectively, always "more than" active and "more than" passive with respect to Leeper's (1991) conditions. Hence, *both* monetary policy *and* fiscal policy must deviate "substantially" from the previous AM/PF regime.

The above analysis yields two key points. The first one relates to what we define as "modest" deviations; the second one to the nature of the solutions that characterise the different regions in Figure 1a.

First, the extent of the admissible deviations is given by the MSS conditions: $|h_2| < \frac{1}{\sqrt{p_{22}}}$, that allows a relaxation of Leeper's (1991) original conditions. With a certain abuse of terminology, we call a "fiscal solution" one that both depends on $\gamma_{\tau,i}$ and that implies $g_{\pi,i} = 0$ from (21) and (22), while a "monetary solution" is one that depends on $\gamma_{\pi,i}$, with $i = 1, 2$. Since the first absorbing regime is AM/PF, then we know from (23) that the only stable solution for this regime is the fiscal one: $h_1 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right)$, which implies $g_{\pi,1} = 0$. However, given that the value of $\gamma_{\pi,1}$ is bigger than 1, it does not exist a stable "monetary solution".[10] In order to have determinacy there should only be one corresponding stable solution, h_2. The threshold values $\bar{\gamma}_{\pi,2}$ and $\bar{\gamma}_{\tau,2}$ for the modest changes in monetary and fiscal policies above, define the conditions for the existence respectively of a monetary and a fiscal solution in the second regime. In particular, starting from an AM/PF absorbing regime, determinacy in regime 2 admits either only modest deviations from AM/PF (upper-right zone, $\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}$; $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$) or large deviations in both monetary and fiscal policy so that we are definitely in a PM/AF regime (lower-left zone, $\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}$; $\gamma_{\tau,2} < \bar{\gamma}_{\tau,2}$). Otherwise the model is either indeterminate or unstable.[11]

Second, the above analysis clarifies also the nature of these solutions. It follows immediately that for values of $\gamma_{\tau,2}$ larger than the threshold value, $\bar{\gamma}_{\tau,2}$, there exists a stable fiscal solution in both regimes, such that: $g_{\pi,1} = g_{\pi,2} = 0$. Notably, such a solution implies a *Ricardian regime* because the level of debt does not affect the dynamics of inflation. The fiscal solution is the unique solution in the upper-right zone, because the level of $\gamma_{\pi,2}$ is so high that there are no stable monetary solutions. Conversely, below the threshold value for the monetary policy coefficient, i.e., $\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}$ there exists a stable monetary solution, that is an admissible (stable) value of h_2 that is a function of $\gamma_{\pi,2}$. The

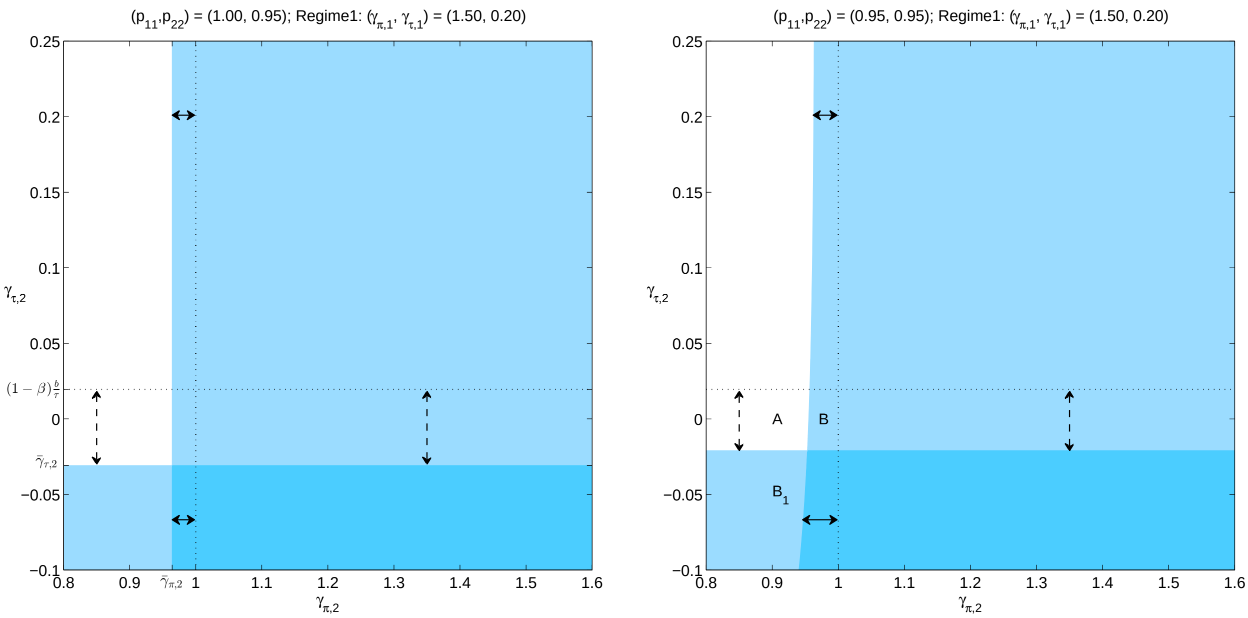
[10] More precisely, if $\gamma_{\pi,1} > 1$ in the absorbing regime, a stable solution does not exist in the first regime, i.e., a value of $h_{1,b} < 1$, such that the second square bracket in (23) is equal to zero.

[11] More precisely: (i) if monetary policy is sufficiently passive (if $\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}$), there exists a stable monetary solution; (ii) if fiscal policy is not too active (if $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$), there exists a stable fiscal solution. Hence, to have only one solution either (i) or (ii) should be satisfied. When both are satisfied, we have two solutions as in the white region; when neither one is satisfied we have no stable solutions as in the dark blue region.

monetary solution is the unique solution in the lower-left zone, where no fiscal stable solution exists. This solution implies non-Ricardian dynamics because $g_{\pi,2} \neq 0$ and thus debt dynamics affect inflation: we are in a fiscal theory of the price level region of the parameter space.

3.2 The general case

Figure 1b shows that the general case where both regimes are non-absorbing ($p_{11}, p_{22} < 1$) exhibits the same qualitative results.[12] In particular, it is still true that the unique stable solution in the upper-right zone is the fiscal solution in both regimes, so the dynamics will be Ricardian in both regimes. In contrast, the unique stable solution in the lower-left zone is characterised by a monetary solution in both regimes, so the dynamics of the system will not be Ricardian in both regimes.



(a) Absorbing case (b) Non-absorbing case

Figure 1: Determinacy regions given an AM/PF regime 1.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

If the policy coefficients in the second regime are appropriately chosen one can get global determinacy across the two regimes under different policy mixes. Nothing ensures that switching among two determinate regimes in the fixed coefficient case always gives determinacy: it depends on the choice of policy coefficients. Note that point B in the upper-right zone and point A on its left, return, respectively, determinacy and indeterminacy. That is true even if both points correspond to an economy that switches between an AM/PF and a PM/AF mix: two regimes that, taken in isolation, are

[12] As the Appendix shows, for the general non-absorbing case with our calibration the threshold values for the fiscal policy coefficient are: $-0.02 < \gamma_{\tau,2} < 3.93$.

determinate.[13] The same happens if you compare point B_1 in the lower-left zone and point A. The next sections will be devoted to shedding light on these apparently puzzling findings.

3.3 Globally balanced policies

Davig and Leeper (2007b) indicate the conditions that a switching monetary policy needs to satisfy in the two regimes to yield a unique REE in the Markov-switching framework, assuming a passive fiscal policy. Since we want to extend the analysis to a switching fiscal policy, we look for the conditions that fiscal policy needs to satisfy to yield a unique rational expectation equilibrium assuming first a time-invariant active monetary policy (Section 3.3.1) and then a switching monetary policy (Section 3.3.2).

3.3.1 The fiscal and the monetary frontiers

The fiscal frontier. Consider again the absorbing case above and assume that monetary policy is always active ($\gamma_{\pi,1} = \gamma_{\pi,2} = 1.5$). This will be the symmetric case with respect to Davig and Leeper (2007b), who implicitly consider passive fiscal policy in both regimes. In this case we know that a stable monetary solution does not exist in both regimes because monetary policy is strongly active, so that $\gamma_{\pi,i} > \bar{\gamma}_{\pi,i}$ for $i = 1, 2$. Then, the only possible solution would be the fiscal one in both regimes: $h_i = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,i}\right); g_{\pi,i} = 0$, for $i = 1, 2$. Figure 2a displays what we label *the fiscal frontier* (henceforth FF) because it shows the combinations of fiscal policy rule coefficients ($\gamma_{\tau,1}$ and $\gamma_{\tau,2}$) in the two regimes that deliver determinate equilibria for the given monetary rule coefficients. As the figure shows, we have determinacy for the absorbing regime 1 when $\gamma_{\tau,1} > \frac{b}{\tau}(1 - \beta)$ and for the non-absorbing regime 2 when $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$. The fiscal policy combinations above the FF admit only modest deviations into AF in the second regime, so we name it a *globally PF* regime. As this definition suggests, this solution yields Ricardian behavior in both regimes. The other fiscal policy combinations in Figure 2a, instead, do not admit a mean square stable fiscal solution. In these cases, fiscal policy substantially deviates from PF (in the sense of being above the threshold values given by the MSS conditions) in at least one of the two regimes.[14]

Figure 2b plots the FF for the general case where both p_{11} and p_{22} are lower than one: the same intuition goes through. The only difference is that now the MSS condition (11) applies on both h_1

[13] The coordinates of the points on Figure 1b are: A: ($\gamma_{\pi,2} = 0.9$; $\gamma_{\tau,2} = 0$); B: ($\gamma_{\pi,2} = 0.97$; $\gamma_{\tau,2} = 0$); B_1: ($\gamma_{\pi,2} = 0.9$; $\gamma_{\tau,2} = -0.05$).

[14] Fiscal policy is no longer globally PF, but we label it as either globally AF (lower-left zone where both conditions for a globally PF are not satisfied), or globally switching between AF and PF (when only one of the two conditions for a globally PF is satisfied).

and h_2, intuitively smoothing the profile of the FF in Figure 2a and allowing for a modest deviation from PF also in regime 1. Following the same reasoning as in the absorbing case, we can define the analytical expression for the FF also for this general case.

Proposition 1. *Define the fiscal solution in both regimes as:* $h_i = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,i}\right) \equiv \bar{h}_i(\gamma_{\tau,i}); g_{\pi,i} = 0$ *for* $i = 1, 2$. *Then*

(i) For any policy parameter combination, this solution always exists;

(ii) For this solution to be MSS it must be true that

$$p_{11}\left[\bar{h}_1(\gamma_{\tau,1})\right]^2\left\{1 - \left[\bar{h}_2(\gamma_{\tau,2})\right]^2\right\} + p_{22}\left[\bar{h}_2(\gamma_{\tau,2})\right]^2\left\{1 - \left[\bar{h}_1(\gamma_{\tau,1})\right]^2\right\} + \left[\bar{h}_1(\gamma_{\tau,1})\bar{h}_2(\gamma_{\tau,2})\right]^2 < 1, \tag{30}$$

which defines the fiscal frontier in the space $(\gamma_{\tau,1}, \gamma_{\tau,2})$;

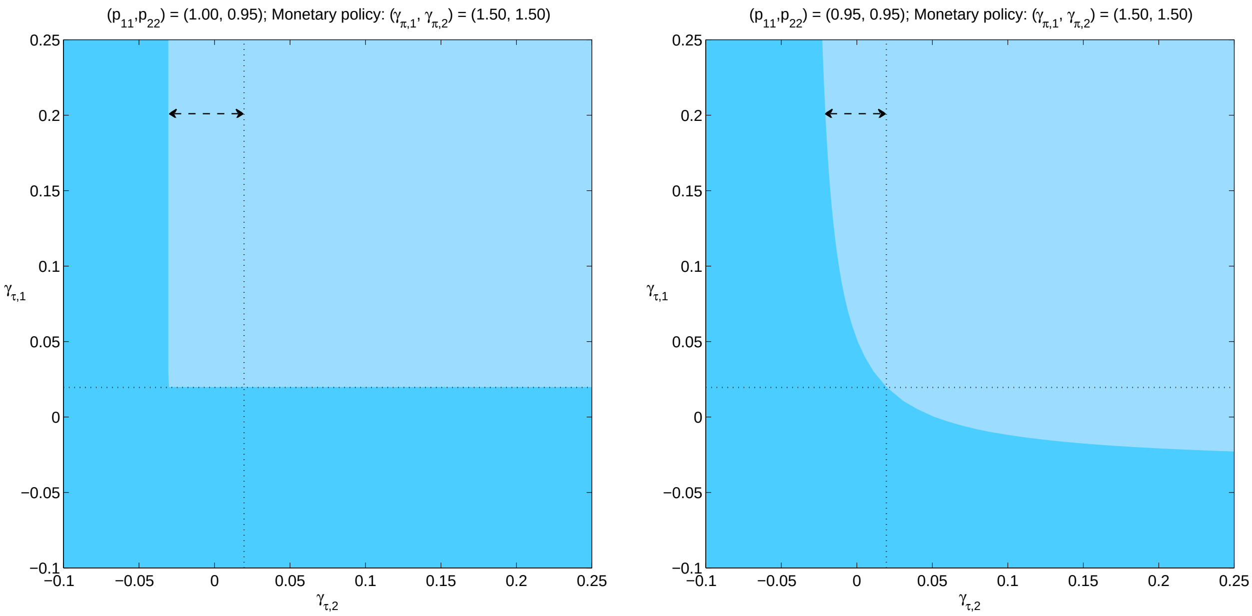
(iii) The fiscal frontier is independent from the monetary policy coefficients;

(iv) This solution yields Ricardian dynamics because $g_{\pi,i} = 0$.

Proposition 1 defines the FF and establishes two important results for the general case: (i) fiscal solutions are only stable for both regimes above the FF, and (ii) this solution yields Ricardian dynamics because $g_{\pi,i} = 0$.

The monetary frontier. We can map the insights of the FF into a specular graph where we vary the monetary policy coefficients while fixing the fiscal ones. When fiscal policy is PF, we know that a fiscal solution always exists in the two regimes, so determinacy requires no monetary solution to be stable. In the absorbing case, this happens when monetary policy is sufficiently active in the two regimes, that is, $\gamma_{\pi,1} > 1$ and $\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}$. These conditions are plotted in Figure 3a, which displays the monetary frontier (henceforth MF). The MF is the counterpart of the FF since it provides the combinations of monetary policy rule coefficients in the two regimes that deliver determinate equilibria, for given fiscal rule coefficients. Note that the monetary policy combinations above the MF admit only modest deviations into PM in the second regime, so we name them *globally AM* regimes. These solutions correspond to the ones above the FF and similarly yield Ricardian dynamics in both regimes. The other monetary policy combinations in Figure 3a, instead, admit a monetary solution that satisfies the MSS conditions in at least one of the regimes, yielding more than one stable solution.[15]

[15] In these cases, monetary policy substantially deviates from AM in at least one of the two regimes. Thus, we name monetary policy either globally PM or globally switching between AM and PM, according to the monetary policy coefficients being below the threshold values given by the MSS conditions in both regimes or in only one.



(a) Absorbing case (b) Non-absorbing case

Figure 2: The fiscal policy frontier.

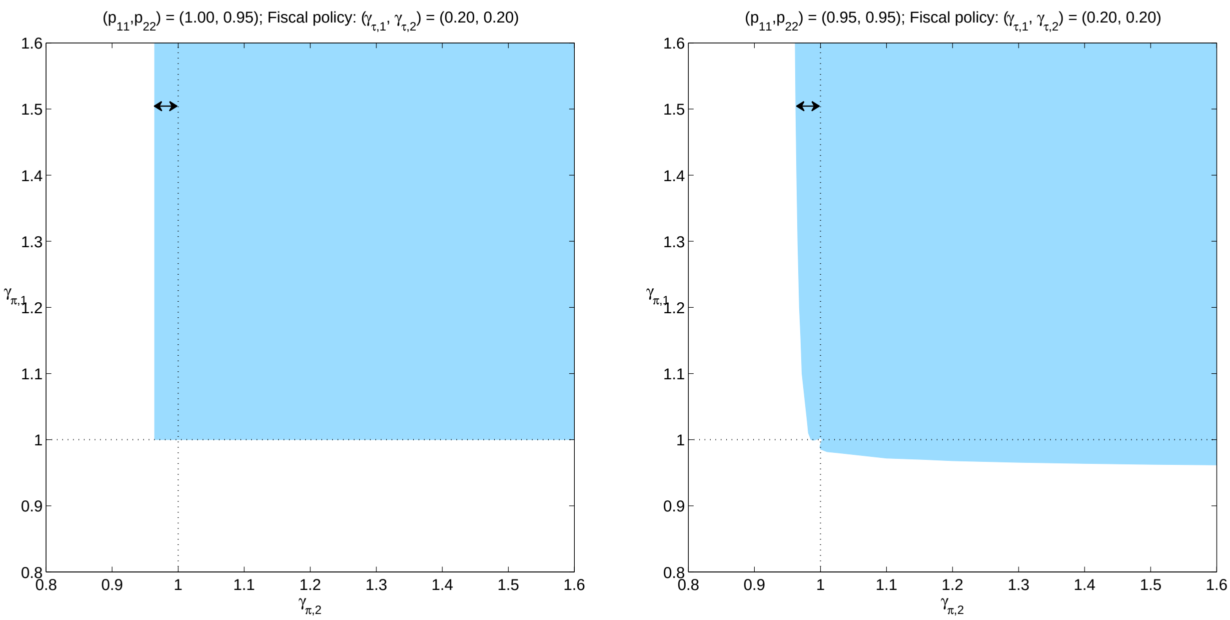
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

Figure 3b displays the MF for the general case where neither regimes are absorbing: the same intuition goes through and the only difference is that now the MSS conditions (10) and (11) apply on both the h_i's, creating the possibility of a modest deviation from AM in regime 1 as well. Figure 3b exactly reproduces in our framework the main result in Davig and Leeper (2007b): their long-run Taylor principle.[16] Two important new results stem from our analysis. First, consider the fiscal stance behind the long-run Taylor principle. It entails an always-passive fiscal policy: the central bank can stabilise the economy by following the Taylor principle or deviating from it, substantially for brief periods or modestly for longer periods, provided it is backed by a government that implements the fiscal adjustments necessary to stabilise the debt. As Figure 2b shows, we symmetrically find that even fiscal policy can deviate from a PF behaviour substantially for brief periods or modestly for longer periods and still return determinacy, provided that monetary policy is always active.[17] This establishes what we can analogously call the *long-run fiscal principle*, given by equation (30). Second, our analysis broadens the very concept of long-run Taylor principle. Consider a policy mix that lies to the right of the FF with passive fiscal policy in regime 1 and a modestly active fiscal policy in regime

[16] As in Davig and Leeper (2007b), asymmetric mean duration would expand the determinacy region in favour of the more transient regime both for the MF and the FF. Unfortunately, given the complexity of the system, a meaningful analytical expression for MF is not possible.

[17] See Section 4.1 for an example of determinacy after a substantial variation for a brief period.

2. The corresponding MF for this globally PF regime is very similar to the one in Figure 3b.[18] In other words, the MF is unaffected as long as the fiscal stance is globally passive, i.e. as long as the long-run fiscal principle is satisfied. It follows that the long-run Taylor principle assures determinacy not only when fiscal policy is passive all the time, as Davig and Leeper (2007b) maintain, but also when it deviates modestly into active fiscal territory for some time - provided that the long-run fiscal principle is satisfied.



(a) Absorbing case

(b) Non-absorbing case

Figure 3: The monetary policy frontier.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

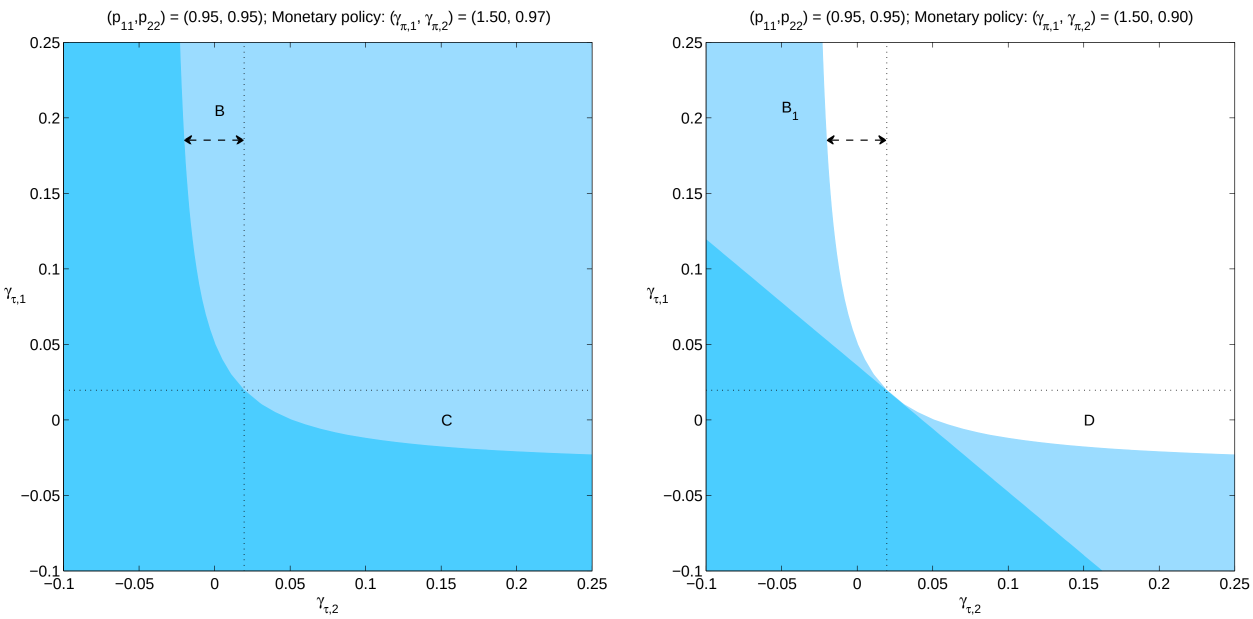
3.3.2 The fiscal frontier when monetary policy switches

Assuming AM/PF in the first regime, we now want to analyse the case in which there is a switch to PM in the second regime. Again, the aim is to study how fiscal policy should behave to yield determinacy. In order to define the FF for this general case, we need to distinguish two cases, according to whether $\gamma_{\pi,2}$ deviates from the Taylor principle to a lesser (case 1) or to a greater (case 2) extent.

Case 1: A modest $\gamma_{2,\pi}$ deviation. Assume monetary policy is active in the first regime and deviates only modestly in the second regime ($\gamma_{\pi,1} = 1.5$, $\gamma_{\pi,2} = 0.97$). If fiscal policy stays passive in both regimes we are above the MF in Figure 3 and, according to the long-run Taylor principle,

[18] For the sake of brevity we omit that figure, which is available from the authors upon request.

there would be determinacy.[19] Consider now Figure 4a, if fiscal policy in regime 2 becomes *active*, determinacy is preserved if $\gamma_{\tau,2}$ has only a modest deviation into the AF territory and thus if the fiscal mix is above the FF. In this case, when deviations from the current AM/PF regime are modest, we have a *globally AM regime* combined with a *globally PF regime*, which returns a *globally AM/PF regime*. Both the long-run Taylor principle and the long-run fiscal principle are satisfied.



(a) Modest deviations (b) Substantial deviations

Figure 4: The fiscal policy frontier for different monetary regimes.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

Case 2: A substantial $\gamma_{2,\pi}$ deviation. Now assume a substantial deviation of $\gamma_{\pi,2}$ from the AM case (e.g., $\gamma_{\pi,2} = 0.9$). Thus, determinacy generally calls for fiscal policy to deviate substantially from PF too. If fiscal policy stays largely passive in both regimes, according to the long-run Taylor principle, there would be indeterminacy (see Figure 3). Figure 4b shows that a modest deviation in regime 2 from PF policy and thus a fiscal policy combination above the FF, returns indeterminacy. If fiscal policy becomes active in regime 2, however, we get determinacy if it substantially deviates from the PF case. Fiscal policy combinations in Figure 4b need to be below the FF to yield determinacy, that is, the long-run fiscal principle should not be satisfied. Note that this is just a sufficient condition but not a necessary one because switching fiscal policies could also return instability. In Figure 4b, monetary policy is switching and there is only a limited set of fiscal policy combinations that yield

[19]The deviation is so modest that it does not admit a stable monetary policy solution and thus only the fiscal solutions are allowed. The Appendix provides the analytics for the absorbing case. See section 3.1 for an analytical condition of modest and substantial deviations in the absorbing case.

determinacy, and they imply a switching fiscal policy. When deviations from the current regime are substantial we refer to the global regime as "switching." Therefore, in the presence of substantial deviations from the AM/PF regime, we have both a *globally switching monetary regime* and a *globally switching fiscal regime* that return a *globally switching regime.*

It is easy to grasp the intuition about this result. In Figure 4b, a new condition appears and is represented by a straight line in the space $(\gamma_{\tau,1}, \gamma_{\tau,2})$. This line indicates the threshold for the existence of a stable monetary solution: above the line the parameter combinations (i.e., the monetary policy coefficients $\gamma_{\pi,i}$ and the probabilities of switching) are such that at least one stable monetary solution exists, while below the line it does not. We know from Proposition 1 that a fiscal solution always exists and is stable above the FF, and unstable below. In Figure 4b the threshold line is below the FF,[20] so above the FF there are at least two stable solutions: one fiscal and at least one monetary solution, so there is indeterminacy. Below the line no stable solution exists. Between the FF and the line, however, there are no stable fiscal solutions and only one stable monetary solution, so we have determinacy. Note that a globally switching policy solution such as this implies $g_{\pi,i} \neq 0$, so Ricardian equivalence does not hold and the dynamics would imply wealth effects.[21]

The general message from this analysis is that when monetary policy varies modestly, determinacy of the global equilibrium requires that fiscal policy varies modestly too. But when monetary policy varies substantially, determinacy generally requires fiscal policy to also vary substantially. To sum up, monetary and fiscal policy need to be *globally balanced* to guarantee the existence of a unique stable equilibrium: globally active monetary policies need to be coupled with globally passive fiscal policies; globally switching monetary policies with globally switching fiscal policies. Moreover, globally switching policies imply wealth effects, while globally AM/PF regimes do not.

3.4 The importance of coordination

According to the long-run Taylor principle in Davig and Leeper (2007b), for some parameters, switching from an indeterminate to a determinate regime (or vice versa) can still return determinacy. Further extensions of this literature (see Chung et al., 2007; Davig and Leeper, 2008) maintain that it is possi-

[20] To be clear, the line lies below the FF and it is not tangent to the FF, as it might appear from the Figure.

[21] The Appendix contains the full analytical characterization and an analytical expression for the threshold condition that defines the line in the absorbing case. Such an expression is not available in any meaningful way for the general case. In general, the slope and position of the line depend on the monetary policy coefficients $\gamma_{\pi,i}$ and the switching probabilities p_{ii}. For the parameters combinations in Figure 4b, the line lies below the FF. For larger switching into PM (i.e., lower $\gamma_{\pi,2}$) the line could also be cutting the FF. Though, our general message is still valid, because there will always be a *globally switching fiscal* regime that yields a unique determinate solution. However, in this case, particular combinations of AF in the two regimes (with both regimes deviating modestly from PF) could also return determinacy.

ble to visit indeterminate (double passive mixes) but even explosive (double active mixes) regimes and still have determinacy if agents expect a stable regime in the future. This means that coordination problems are irrelevant.

We showed above that with a simultaneous switching of monetary and fiscal policy anything can happen. In particular, as pointed out in the previous section, nothing ensures that switching among two determinate regimes gives determinacy. Take points A and B in Figure 1b. Since these two points entail the same modest deviation from the passive fiscal policy in regime 1, to have determinacy of the global equilibrium, monetary policy should also vary modestly. This does not happen at point A, since monetary policy is insufficiently active, while it does at point B (which indeed lies in the determinate area in Figure 4a). Compare now points A and B_1. Since these two points share the same substantial deviation from the active monetary policy in regime 1, to have determinacy of the global equilibrium, fiscal policy is also required to vary substantially. This is not the case for point A, since fiscal policy is insufficiently active, while it is as for point B_1 (which indeed lies in the determinate area in Figure 4b).

Furthermore, switching from a double active regime (AM/AF, explosive in fixed coefficients) to a double passive one (PM/PF, indeterminate in fixed coefficients) can happen to return determinacy. Consider, for example, point C in Figure 4a and point D in Figure 4b. They share the same fiscal policy coefficients: a PF policy in regime 2 and a modest deviation from it in regime 1. In both cases, monetary policy is active in the first regime and passive in the second. This would thus be a shift from a double active to a double passive regime that returns determinacy in Figure 4a but not in Figure 4b. According to our interpretation, this is due to the fact that at point C the global regime is balanced (since in Figure 4a there is also a modest change in monetary policy), while at point D it is not (since in Figure 4b there is a substantial change in monetary policy).

If policies should be balanced to get a determinate equilibrium then coordination is essential and the expectation of a stable regime in the future is not *per se* sufficient to achieve determinacy.

4 The expectation effect of regime shifts

The above analysis allows us to characterise both the number of admissible solutions in the different regions of the parameter space but also their nature, in terms of the implied dynamics. Recall that we identified two sets of possible solutions: a fiscal one that yields Ricardian dynamics because $g_{\pi,i} = 0$ and a monetary one that implies wealth effects because $g_{\pi,i} \neq 0$. These two solutions could be considered the counterpart in a Markov-switching context of the two original determinate solutions in Leeper (1991). This section considers the dynamics implied by the different solutions in more detail, to make clear its link with our definitions of global policies derived from the determinacy analysis.

Consider the case of a simultaneous monetary and fiscal switch and, in particular, the shifting from AM/PF to PM/AF. In a fixed coefficient model, these two regimes return determinacy. The first regime, the more common Ricardian case, assigns a passive role to fiscal policy, which has to behave promptly to satisfy the intertemporal government budget constraint. In this case, wealth effects are absent and the central bank, following the Taylor principle, manages to anchor agents' inflation expectations at a low level. The second regime, known as the non-Ricardian case, is the policy mix associated with the fiscal theory of the price level. In this regime monetary policy acts to stabilise debt and the failure of Ricardian leads to wealth effects. As Galí (2007) points out, however, once one allows policy regimes to change, the economy equilibrium properties are contaminated both by the characteristics of the other regimes and by the probability of shifting towards those alternative regimes. The so-called cross-regime spillovers are at work (see Davig and Leeper, 2007b).

According to the long-run Taylor principle, monetary policy could be passive in one regime, yet still have determinacy if this departure from an active monetary policy is substantial for brief periods, or modest for prolonged periods. It is agents' expectation of moving towards an inflation stabilising regime that allows for this result. Davig and Leeper (2008) define "expectation effects" as the difference between the equilibrium outcome from a model with fixed coefficients and from one that takes into account expected changes in regimes. Analysing a model of regime shifts in monetary policy in a context of an always passive fiscal policy, Liu et al. (2009) find that these expectation effects are asymmetric.[22] The shift from a dovish (or a less hawkish) monetary regime to a hawkish one lowers inflation volatility more than an inverse shift raises it: *inflation anchoring expectations* prevail. Moreover, Chung et al. (2007) find that the fiscal theory is always at work when agents attach a positive probability of moving towards active fiscal policy. As a consequence, in this case one has to also consider *wealth effect expectations*.

In what follows we want both to analyse how this asymmetry is affected by switching fiscal policy regimes and to check whether wealth effects are always in place in our framework.

Consider again point B and B_1 in Figure 1b. They are both characterised by the shift from an AM/PF regime to a PM/AF one with transition probabilities $p_{11} = p_{22} = 0.95$, and they both return a globally determined equilibrium. The AM/PF regime is the same for both points, while the PM/AF

[22] This result holds even if the two regimes have the same transition probabilities.

regime is different. As a consequence, point B was found to be a globally AM/PF regime (see Figure 4a), point B_1 was a globally switching regime (see Figure 4b).

Figure 5 shows the impulse responses to a lump-sum tax shock for the policy combinations implied by the points B (panel (a) on the left) and B_1 (panel (b) on the right).[23] The impulse response functions depend on the particular policy regime in place, so each panel displays two columns of Figures corresponding to one of the two regimes. Moreover, dashed lines in Figure 5 are the responses of the variables under a fixed-coefficients model; solid lines are the responses under a Markov-switching model. The difference between the solid and the dashed lines in each panel represents the expectation effects.

For the *globally AM/PF* regime (point B Figure 4(a) and left panel 5(a)):

1. The solid lines across the two regimes in Figure 5a are coincident except for the path of debt. In the PM/AF regime, the possibility of moving towards the Ricardian regime (with $p_{21} = 0.05$) makes the IRFs behave as in the Ricardian regime (i.e. inflation does not increase).

2. Look now at the differences between the solid and the dashed lines. The expectation effects are asymmetric (larger under PM/AF in the second column than under AM/PF). In the AM/PF regime there is actually no difference between these two lines, that is, there are no wealth effect expectations.

For the *globally switching* regime (point B_1 Figure 4(b) and right panel 5(b)):

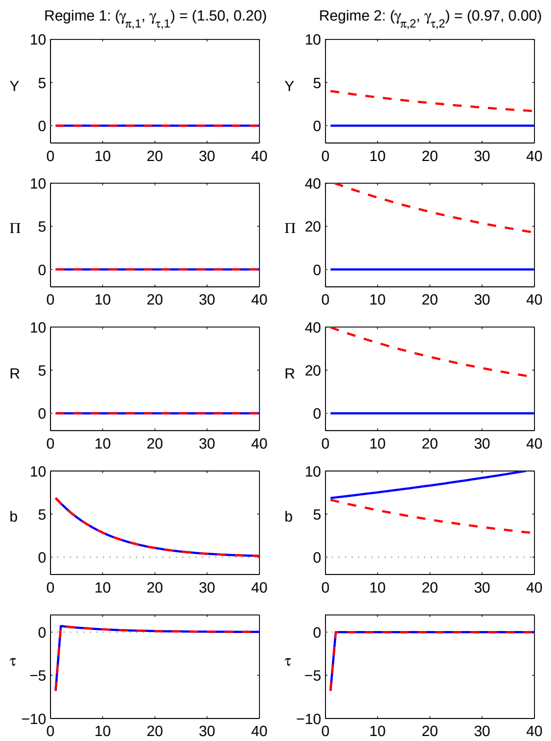
1. The solid lines are no longer coincident. Now, starting from a PM/AF regime, the possibility of switching to an AM/PF regime does not make the IRFs behave as in this last regime (inflation now increases under both regimes)

2. The expectation effects are again asymmetric and there are now wealth effects in the AM/PF regime.

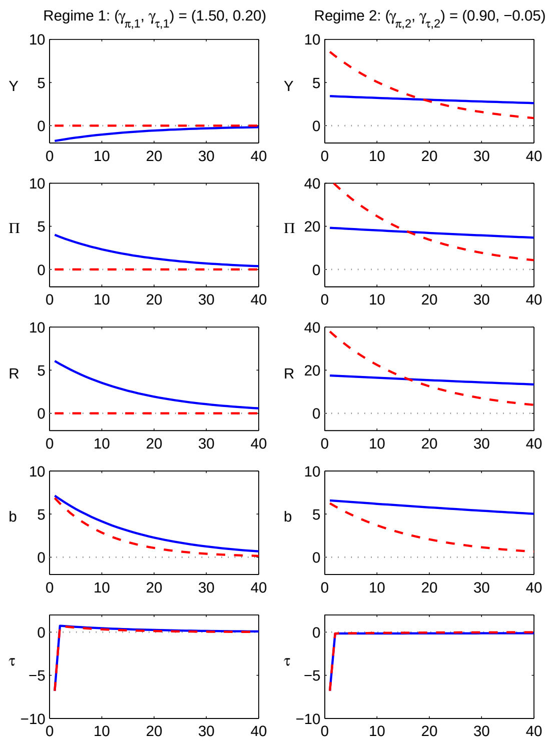
Why do impulse responses for these two points, that entail a switch from an AM/PF regime to a PM/AF one, return such strikingly different results?[24] Chung et al. (2007) find that the existence of a regime that would be termed as non-Ricardian under fixed coefficients is sufficient to generate wealth effects through an expectations channel: the fiscal theory of the price level is always at work if agents attach a positive probability of moving towards active fiscal policy in the future.

[23] Recall that the policy combinations are: (i) for point B, regime 1: ($\gamma_{\pi,1} = 1.5; \gamma_{\tau,1} = 0.2$) and regime 2: ($\gamma_{\pi,2} = 0.97; \gamma_{\tau,2} = 0$); (ii) for point B_1, regime 1: ($\gamma_{\pi,1} = 1.5; \gamma_{\tau,1} = 0.2$) and regime 2: ($\gamma_{\pi,2} = 0.9; \gamma_{\tau,2} = -0.05$).

[24] Both the results of asymmetric expectation effects and the coincidence of the solid lines under point B but not under point B1 hold even considering a monetary policy shock. Results are available from the authors.



(a) Globally AM/PF regime



(b) Globally switching regime

Figure 5: Impulse response function to a negative fiscal shock.
Notes: Blue solid lines: MS model; red dashed lines: fixed coefficients model.

We find this is not true if the policy mix is globally AM/PF, as it is the case for point B. Under the PM/AF regime, the possibility of switching to the AM/PF is now sufficient to stabilise inflation in both regimes. This is because fiscal policy is globally PF, such that the two regimes behave the same (except for the path of debt, as noted): there are no expectation wealth effects in the AM/PF regime and there are strong inflation anchoring expectation effects in the PM/AF regime. In contrast, when the global policy mix is switching, as in point B_1, the possibility to switch towards the AM/PF regime once in the PM/AF regime is not enough to stabilise inflation. Inflation actually increases under both regimes, though to a larger extent under the PM/AF one. Hence, when fiscal policy goes through an AF regime, there are wealth effects even in the AM/PF regime.

We do know the analytical reasons behind these results from our determinacy analysis. We labelled a globally AM/PF regime one in which only modest deviations from AM/PF are allowed. In this case, there is only one type of admissible stable solutions: the ones above the FF in Figure 4a. However, as implied by Proposition 1, we know that these solutions give a Ricardian dynamics because they are fiscal solutions where $g_{\pi,i} = 0$. On the contrary, in case of switching policies, Figure 4b shows that determinacy requires both policies to substantially switch across regimes, and the unique stable solutions in this case admits non-Ricardian dynamics in both regimes because they are monetary solutions where $g_{\pi,i} \neq 0$. Furthermore, at a point to the right of B1 in the white area in Figure 4b, monetary and fiscal policy are unbalanced so that there is indeterminacy. In this context, indeterminacy means that there exists at least a fiscal and a monetary solution, and both of them are admissible, because MSS stable. Then, depending on which solution one picks eventually in estimating the model, one might find the presence or absence of expectations and wealth effect in the different regimes. In that region of the parameter space, it might be possible to pick the monetary solution and then to conclude that the existence of a non-Ricardian regime is sufficient for wealth effects, without realizing that there is also another admissible solution where this is not true.

Do wealth effects disappear if agents are confident about a once-and-for-all switch towards an AM/PF regime? We find that under a globally AM/PF parametrisation impulse responses are identical to those in Figure 5a even if the AM/PF regime is absorbing, i.e. $p_{11} = 1$. Conversely, in the globally switching case with an absorbing AM/PF regime, the impulse responses of the PM/AF mix do not differ from the non-absorbing case. The responses of the (absorbing) AM/PF one are obviously identical to those under fixed coefficients. Then, under our new taxonomy, the expectation of an absorbing AM/PF regime for the future is neither a necessary nor a sufficient condition to avoid wealth effects in the PM/AF regime. It is not necessary because we do not find wealth effects in the global AM/PF case even when $p_{11} = p_{22} = 0.95$. It is not sufficient because we detect wealth effects in the PM/AF regime in the globally switching case even when $p_{11} = 1$.

4.1 Some theoretical and practical consequences

Our new taxonomy provides an answer to the problem of establishing if a regime is Ricardian or not in a model where agents are aware of recurrent regime changes. Usually one can refer to the AM/PF regime as Ricardian and to the PM/AF regime as non-Ricardian only where agents are assumed not to be aware of regime changes.[25] In a model with recurrent regime changes, as Bianchi and Melosi (2013) noted, the policy mix is not enough to establish if a regime is Ricardian or not. However, since we find neither expectation effects nor wealth effects under an AM/PF regime when agents expect a regime shift and the policy mix is *globally AM/PF*, we can conclude that the AM/PF regime of a *globally AM/PF* mix is definitively Ricardian.

The bulk of the literature[26] that estimates Markov-switching monetary-fiscal regimes and employs

[25]See note 4 in Bianchi and Ilut (2014).

[26]See Davig and Leeper (2008, 2011), Chung et al. (2007), Bianchi (2012) and Bianchi and Ilut (2014).

IRFs to study the impact of policy shocks finds results consistent with the ones we get under the *globallyswitching* regime case: inflation increases under both regimes. This is why that literature concludes that whenever agents believe it is possible for fiscal policy to become active, monetary and tax shocks always produce wealth effects. Without checking for determinacy, however, these results might arise from one of the multiple equilibria of an indeterminate parametrisation.

Our paper is consistent with results in Bianchi and Melosi (2013). They find that after a deficit shock under an AM/PF regime or under a short-lasting deviation towards a PM/AF one, there are no effects on inflation (and output). Inflation (and output), however, increase under a long-lasting deviation towards the same PM/AF. Since both the short and the long-lasting deviations are towards the same PM/AF, the authors consider the regime's persistence as the key ingredient to establish if a regime is Ricardian or not.[27] Our taxonomy is coherent with this finding because, although we have not focused on the role of transition probabilities so far, our definition of modest deviation depends on them (see Section 3.1).

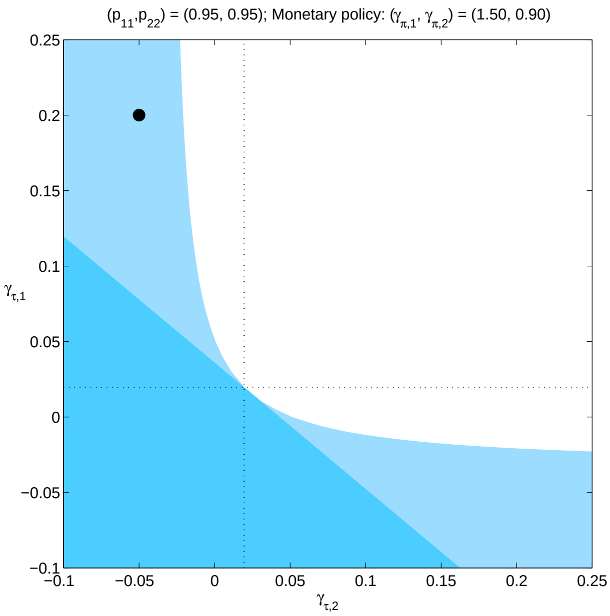
Consider a numerical example in the case of globally switching policies described by point B_1, reported as a black dot in Figure 6. Under this policy combination, if the second regime is long-lasting, say $p_{22} = 0.95$, a modest deviation is defined by $\gamma_{\tau,2} \in [-0.021; 0.02]$ and $\gamma_{\pi,2} \in [0.955; 1]$. Instead, if regime 2 is less permanent, say $p_{22} = 0.8$, a modest deviation is defined by: $\gamma_{\tau,2} \in [-0.16; 0.02]$ and $\gamma_{\pi,2} \in [0.67; 1]$.[28] Therefore, with a long lasting deviation, B_1 would correspond to a globally switching regime (see Figure 6a), while with a less permanent deviation we would have a globally AM/PF regime (see Figure 6b). In the first case the impulse responses to a negative fiscal shock would show up in rising inflation (see Figure 7a) because the unique stable solution is the monetary solution. In the second there would not be inflationary effects (see Figure 7b) because the unique stable solution is the fiscal one.

This analysis could have notable consequences for monetary policy: both for the timing of any exit strategy and for forward guidance. During the recent crisis the accumulated credibility of the Federal Reserve has permitted well-anchored inflation expectations, even though the US was potentially in a PM/AF regime. If we are ready to believe that during the crisis monetary policy deviated substantially from an AM regime, then the only way to avoid a future spike in inflation is to make this deviation short lasting.[29] A long-lasting deviation, on the contrary, could either de-anchor inflation expectations

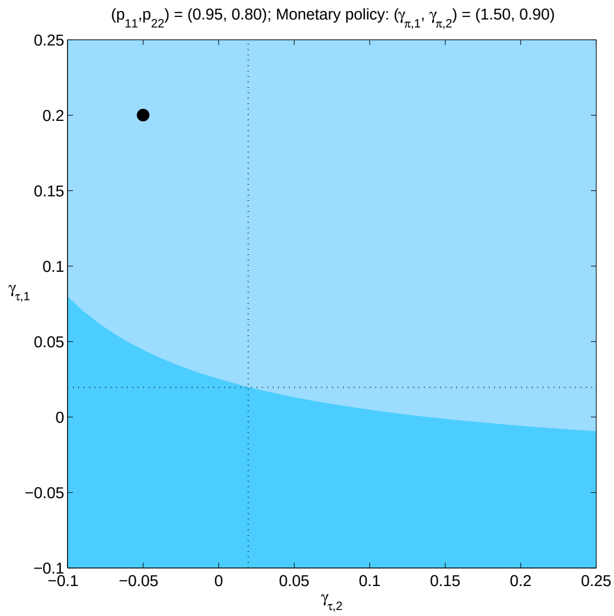
[27]See also Bianchi and Ilut (2014) on this point. They do not find any effect of a tax shock on inflation when the AM/PF regime is perceived to be fully credible (if agents expect to remain there forever) or if, being in a PM/AF regime, agents are confident about a return to the AM/PF regime.

[28]These intervals can be obtained following the procedure in Section 3.1 and Appendix A6.

[29]On the contrary, it would be useless to promise a more hawkish monetary policy in the future regime.



(a) $p_{22} = 0.95$



(b) $p_{22} = 0.80$

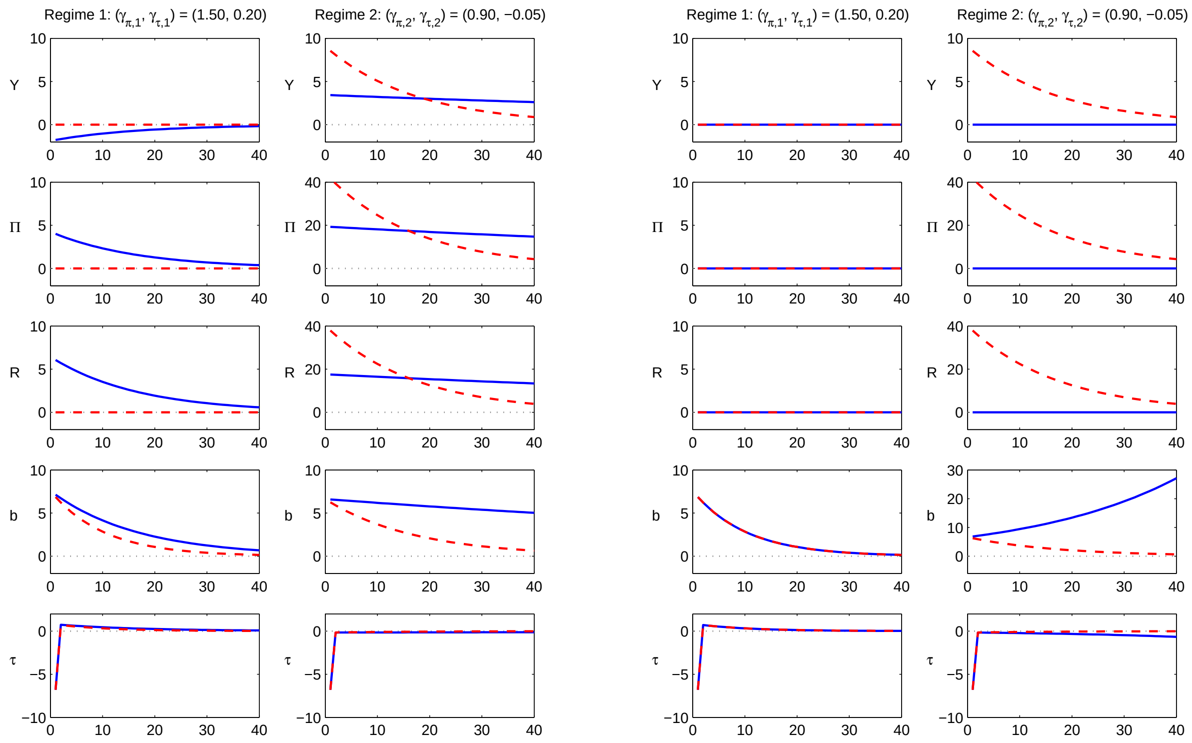
Figure 6: The monetary policy frontier for different levels of persistence of regime 2.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

and make inflation unavoidable, or generate multiple solutions, depending on the behaviour of fiscal policy. Indeed, if fiscal policy remains only modestly active, it may be difficult for policy makers to predict an inflationary surge because anything is possible.[30] If we believe such scenario as the relevant one, then, it might be that the observed subdued path of inflation is due to agents coordinating on a fiscal solution. However, these dynamics could abruptly revert into an inflation upswing if expectations were to suddenly switch to the other admissible rational expectations solution.

5 Conclusions

This paper studies the determinacy properties of the equilibrium in a simple New Keynesian model when both monetary and fiscal policies may switch according to a Markov process. In this context, nothing ensures that the switching among two regimes, which would be determinate under fixed-coefficient, returns determinacy. Davig and Leeper (2007b) define the long-run Taylor principle as the condition that the coefficients in the Markov-switching Taylor rule need to satisfy to guarantee a unique equilibrium, given a passive fiscal policy. This can be graphically visualised as a monetary frontier. Equivalently, we define a fiscal frontier that visualises the long-run fiscal principle as the condition

[30] Both the fiscal and the monetary solution are admissible: in the former case, there would be no inflation surge, in the latter, there would be.



(a) $p_{22} = 0.95$ (b) $p_{22} = 0.80$

Figure 7: Impulse response function to a negative fiscal shock for different levels of persistence of regime 2.

Notes: Blue solid lines: MS model; red dashed lines: fixed coefficients model.

that the coefficients in the Markov-switching government tax rule need to satisfy to guarantee a unique equilibrium, given an active monetary policy.

We propose a new taxonomy that generalises the seminal paper of Leeper (1991) to a Markov-switching context. We name a modest deviation from an active monetary policy into the passive monetary territory that respects determinacy - i.e. that satisfies the long-run Taylor principle - as "globally active monetary policy." Symmetrically, a modest deviation from a passive fiscal policy into the active fiscal territory - that satisfies the long-run fiscal principle - is named "globally passive fiscal policy." Substantial shifts in monetary and fiscal policies are termed "switching policies." Monetary and fiscal policies need to be globally balanced to guarantee a unique equilibrium. Globally active monetary policies need to be coupled with globally passive fiscal policies (i.e., a globally AM/PF regime), and switching monetary policies with switching fiscal policies (i.e., a globally switching regime).

Our new taxonomy also establishes an explicit link between the determinacy analysis and the dynamics of the Markov-switching model. If the policy mix is globally switching then the fiscal theory of the price level is always at work, though this is not true if the policy mix is globally AM/PF.

Under this latter case, there are no wealth effects because fiscal policy is globally PF. The taxonomy thus settles the problem of establishing if a regime is Ricardian or not in a model where agents are aware of recurrent regime changes. A globally AM/PF mix is definitively Ricardian because there are no wealth effects in both regimes. Moreover, the expectation of a fully credible (even absorbing) AM/PF regime for the future is neither a necessary nor a sufficient condition to avoid wealth effects in a PF/AM regime.

The analysis suggests some directions for future research. Our results are based on a very simple New Keynesian model. The advantage of such a model is to allow us to find some analytical results, to gain insightful intuitions on what drives determinacy and on the link between determinacy, dynamics, expectation effects and wealth effects. The natural next step forward in this research would be to see how much our new taxonomy and our results help to interpret the numerical results in a more structural, and possibly estimated, DSGE model. Finally, our definition of modest deviation has the same flavour of Leeper and Zha's (2003) definition of "modest policy interventions." However, our definition is based on the determinacy region of the parameter space and not on their modesty statistic. Checking empirically whether these definitions are somewhat consistent could be a possible fruitful point for future research.

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A Appendix

A.1 Parametrization

Table 1. Calibration

Parameter	Value	Description
β	0.99	Intertemporal discount factor
θ	11	Dixit-Stiglitz elasticity of substitution
α	0.75	Calvo probability not to optimise prices
N	0.333	Hours worked
$\bar{b}$	0.4	Debt-to-GDP ratio
$\bar{c}$	0.8	Consumption-to-GDP ratio

A.2 The model

In the paper we use a simple New Keynesian model with fiscal policy.

Households. The representative household maximises lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu N_t\right) \tag{31}$$

under a sequence of budget constraints given by

$$P_t C_t + (1+i_t)^{-1} B_t \leq P_t w_t N_t + F_t - \tau_t + B_{t-1}. \tag{32}$$

E_0 is the expectations operator conditional on time $t = 0$ information, C_t is real consumption, N_t is labour, w_t is the level of real wages, F_t are profits, τ_t are taxes, $R_t = 1 + i_t$ is the gross return on bonds purchased at date t (i.e. B_t). Maximization yields the first order conditions

$$1 = \beta \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} (1+i_t) \frac{C_t}{C_{t+1}} \right], \tag{33}$$

$$w_t = \mu C_t. \tag{34}$$

Final good producers. In each period, a final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{i,t}$ indexed by $i \in [0,1]$ and a standard CES production function $Y_t = \left(\int_0^1 Y_{i,t}^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)}$, with $\theta > 1$. Final good producers' demand schedules for intermediate good quantities are $Y_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$, where the aggregate price index is defined as

$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di\right)^{1/(1-\theta)}$.

Intermediate goods producers. There exists a continuum of intermediate goods produced by firms with constant returns to scale production function: $Y_{i,t} = N_{i,t}$. Intermediate goods producers compete monopolistically and set prices according to the usual Calvo mechanism. In each period each firm has a fixed probability $1-\alpha$ to re-optimise its nominal price $P_{i,t}^*$ in order to maximise profits. With probability α, instead, the firm keeps its nominal price unchanged. Using the stochastic discount factor $Q_{t,t+j} = \beta^j \frac{P_t C_t}{P_{t+j} C_{t+j}}$, the first order condition of the firm's problem gives the optimal relative price

$$p_{i,t}^* \equiv \frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t}\right)^{\theta} w_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t}\right)^{\theta-1}}. \tag{35}$$

As in Ascari and Ropele (2009), we introduce two auxiliary variables that allow to rewrite the last expression recursively

$$\psi_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t}\right)^{\theta} w_{t+j} = \frac{Y_t}{C_t} w_t + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta} \psi_{t+1}\right], \tag{36}$$

$$\phi_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t}\right)^{\theta-1} = \frac{Y_t}{C_t} + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1}\right], \tag{37}$$

where $\Pi_t \equiv P_t / P_{t-1}$ is the aggregate gross rate of inflation. As all re-optimizing firms face the same problem and pick the same relative price, aggregate inflation evolves according to

$$1 = \alpha \Pi_t^{\theta-1} + (1-\alpha) \left(p_{i,t}^*\right)^{1-\theta}. \tag{38}$$

Individual firms demand labour according to the relation $N_{i,t} = \left(P_{i,t}/P_t\right)^{-\theta} Y_t$. Aggregating this expression yields $N_t = Y_t s_t$, where $N_t \equiv \int_0^1 N_{i,t} di$ and $s_t \equiv \int_0^1 \left(P_{i,t}/P_t\right)^{-\theta} di$. The variable s_t measures the dispersion of relative prices across intermediate firms. s_t is bounded below at one and it represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism. s_t can be written recursively as

$$s_t = (1-\alpha) \left(p_{i,t}^*\right)^{-\theta} + \alpha \left(\Pi_t\right)^{\theta} s_{t-1}. \tag{39}$$

Fiscal and monetary policies. The government budget constraint is given by

$$(1+i_t)^{-1}\, b_t = \frac{b_{t-1}}{\Pi_t} + G - \tau_t, \tag{40}$$

where $b_t \equiv B_t/P_t$, G, and τ_t are the levels of government debt, expenditure, and taxes, all in real terms. Note that we assumed for simplicity that the government chooses a constant level of expenditure G. Taxes are set according to the fiscal policy rule

$$\tau_t = \tau \left(\frac{b_{t-1}}{b}\right)^{\gamma_{\tau,t}} e^{u_{\tau,t}}, \tag{41}$$

while the central bank sets the interest rate following the simple Taylor rule

$$R_t = R\Pi_t^{\gamma_{\pi,t}} e^{u_{m,t}}. \tag{42}$$

Note that the parameters $\gamma_{\tau,t}$ and $\gamma_{\pi,t}$ are indexed with time as they can take different values according to the underlying Markov switching process.

Complete nonlinear model. After imposing market clearing (with $Y_t = C_t + G$), the dynamics of aggregate variables is described by the following set of equations (here reproduced for convenience)

$$\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\frac{R_t}{\Pi_{t+1}} \frac{Y_t - G}{Y_{t+1} - G}\right] \\
w_t &= \mu \left(Y_t - G\right) \\
1 &= \alpha \Pi_t^{\theta-1} + (1-\alpha)\left(p_{i,t}^*\right)^{1-\theta} \\
p_{i,t}^* &= \frac{\theta}{\theta - 1} \frac{\psi_t}{\phi_t} \\
\psi_t &= \frac{Y_t}{Y_t - G} w_t + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta} \psi_{t+1}\right] \\
\phi_t &= \frac{Y_t}{Y_t - G} + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1}\right] \\
s_t &= (1-\alpha)\ \left(p_{i,t}^*\right)^{-\theta} + \alpha \Pi_t^{\theta} s_{t-1} \\
N_t &= s_t Y_t \\
\frac{b_t}{R_t} &= \frac{b_{t-1}}{\Pi_t} + G - \tau_t \\
\tau_t &= \tau \left(\frac{b_{t-1}}{b}\right)^{\gamma_{\tau,t}} e^{\sigma_\tau u_{\tau,t}} \\
R_t &= R\Pi_t^{\gamma_{\pi,t}} e^{\sigma_m u_{m,t}}
\end{aligned}$$

Zero-inflation steady state. If we switch off the exogenous processes $u_{\tau,t}$ and $u_{m,t}$, we can solve for the zero-inflation steady state

$$
\begin{aligned}
R &= \beta^{-1} \\
w &= \mu \bar{c} Y \\
p_i^* &= 1 \\
\psi &= \frac{1}{\bar{c}\left(1-\alpha\beta\right)} w \\
\phi &= \frac{1}{\bar{c}\left(1-\alpha\beta\right)} \\
s &= 1 \\
N &= sY = Y \\
\tau &= \left[\left(1-\bar{c}\right) + \bar{b}\left(1-\beta\right)\right] Y
\end{aligned}
$$

where we used the ratios $\bar{c} \equiv C/Y$ and $\bar{b} \equiv b/Y$. Further, note that the equation

$$
p_{i,t}^* = \frac{\theta}{\theta-1} \frac{\psi_t}{\phi_t}
$$

implies the following parameter restriction

$$
1 = \frac{\theta}{\theta-1} \frac{\psi}{\phi} = \frac{\theta}{\theta-1} \mu Y \bar{c}. \tag{43}
$$

Log-linearised model. The nonlinear model can be log-linearised around the non-stochastic zero-inflation steady state. Standard computations lead to

$$
\begin{aligned}
\frac{1}{\bar{c}} \hat{Y}_t &= \frac{1}{\bar{c}} \mathbb{E}_t \hat{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t \hat{\Pi}_{t+1}\right), \\
\hat{\Pi}_t &= \frac{\lambda}{\bar{c}} \hat{Y}_t + \beta \mathbb{E}_t \hat{\Pi}_{t+1}, \\
\hat{R}_t &= \gamma_{\pi,t} \hat{\Pi}_t + \sigma_m u_{m,t} \\
\hat{b}_t &= \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,t}\right) \hat{b}_{t-1} - \frac{1}{\beta}\hat{\Pi}_t + \hat{R}_t - \frac{1}{\beta}\frac{\tau}{b}\sigma_\tau u_{\tau,t},
\end{aligned}
$$

with $\lambda \equiv (1-\alpha)(1-\alpha\beta)/\alpha$. These equations correspond to equations (12)-(15) in the main text.

A.3 The FRWZ solution method

The analysis of determinacy under Markov-switching coefficients can be performed by checking the existence of a unique stable MSV solution. In order to find all the MSV solutions, we adopt the perturbation method proposed by FRWZ. The method can be applied directly on the nonlinear version of the model. However, instead of considering the complete nonlinear model outlined above, it is convenient to manipulate the equations and reduce the dimensionality of the system. The smaller system turns out to be:

$$
\begin{aligned}
1 &= E_t\left[\frac{\Pi_t^{\gamma_{\pi,t}} e^{u_{m,t}}}{\Pi_{t+1}} \frac{Y_t - G}{Y_{t+1} - G}\right], \\
\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} \phi_t &= \frac{\theta}{\theta-1}\mu Y_t + \alpha\beta E_t\left[\Pi_{t+1}^{\theta}\left(\frac{1-\alpha\Pi_{t+1}^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} \phi_{t+1}\right], \\
\phi_t &= \frac{Y_t}{Y_t - G} + \alpha\beta E_t\left[\Pi_{t+1}^{\theta-1}\phi_{t+1}\right], \\
\frac{b_t}{R\Pi_t^{\gamma_{\pi,t}} e^{u_{m,t}}} &= \frac{b_{t-1}}{\Pi_t} + G - \tau\left(\frac{b_{t-1}}{b}\right)^{\gamma_{\tau,t}} e^{u_{\tau,t}}.
\end{aligned}
$$

Using the notation of FRWZ, this system can be rewritten as

$$
\mathbb{E}_t \boldsymbol{f}\left(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, b_t, b_{t-1}, \boldsymbol{\varepsilon}_{t+1}, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta}_{t+1}, \boldsymbol{\theta}_t\right) = \mathbf{0},
$$

where b_t is the only predetermined variable and the remaining non-predetermined variables are stacked in vector $\boldsymbol{y}_t' \equiv [Y_t, \Pi_t, \phi_t]$. The exogenous shocks appear in vector $\boldsymbol{\varepsilon}_t' \equiv [u_{m,t}, u_{\tau,t}]$, and $\boldsymbol{\theta}_t' \equiv [\gamma_{\pi,t}, \gamma_{\tau,t}]$ is the vector of Markov-switching parameters. We look for recursive solutions such as

$$
\begin{aligned}
b_t &= h_t(b_{t-1}, \boldsymbol{\varepsilon}_t, \chi) \\
\boldsymbol{y}_t &= \boldsymbol{g}_t(b_{t-1}, \boldsymbol{\varepsilon}_t, \chi)
\end{aligned}
$$

perturbed around the non-stochastic zero-inflation steady state $[b, \boldsymbol{y}']'$, where χ represents the perturbation parameter. Note that in our model the solutions are regime-dependent (h_t and $\boldsymbol{g}_t$ follow the latent Markov process too), while the steady state is not. The stability properties of each solution is governed by parameters of the first order expansion of the solutions, which reads as follows under regime i

$$b_t \approx b + h_{i,b}(b_{t-1} - b) + \boldsymbol{h}_{i,\varepsilon}\boldsymbol{\varepsilon}_t + h_{i,\chi}\chi, \tag{44}$$

$$\boldsymbol{y}_t \approx \boldsymbol{y} + \boldsymbol{g}_{i,b}(b_{t-1} - b) + \boldsymbol{g}_{i,\varepsilon}\boldsymbol{\varepsilon}_t + \boldsymbol{g}_{i,\chi}\chi, \tag{45}$$

for $i = 1, 2$. In these expressions we used a matrix notation for the partial derivatives: for example, $\boldsymbol{g}_{i,\varepsilon}$ is a (3×2) matrix whose first column is given by the partial derivative of $\boldsymbol{g}_i$ with respect to $u_{m,t}$, and so forth.

The elements in $h_{i,b}$, $\boldsymbol{h}_{i,\varepsilon}$, $h_{i,\chi}$, $\boldsymbol{g}_{i,b}$, $\boldsymbol{g}_{i,\varepsilon}$, $\boldsymbol{g}_{i,\chi}$ are unknown and can be found by exploiting the fact that the derivatives of $\mathbb{E}_t \boldsymbol{f}$ are equal to zero. Proposition 1 in FRWZ uses the chain rule to state that the coefficients in $h_{i,b}$ and $\boldsymbol{g}_{i,b}$ can be obtained by solving a system of quadratic polynomial equations that corresponds to equation (A4) in FRWZ. To derive such system, we need to compute the partial derivatives of $\boldsymbol{f}$

$$\begin{bmatrix} \boldsymbol{f}_{ij,\boldsymbol{y}_{t+1}} & \boldsymbol{f}_{ij,\boldsymbol{y}_t} \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 & -\frac{1}{\bar{c}Y} & -\gamma_{\pi,i} & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta & -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\ 0 & \alpha\beta\left(1-\theta\right)\phi & -\alpha\beta & \frac{1-\bar{c}}{\bar{c}^2 Y} & 0 & 1 \\ 0 & 0 & 0 & 0 & \left(1-\beta\gamma_{\pi,i}\right) b & 0 \end{bmatrix},$$

$$\begin{bmatrix} \boldsymbol{f}_{ij,b_t} & \boldsymbol{f}_{ij,b_{t-1}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \beta & \frac{\tau}{b}\gamma_{\tau,i} - 1 \end{bmatrix}.$$

Note that the derivatives are indexed with ij to indicate that they must be evaluated at the steady state with $\boldsymbol{\theta}_t = i$ and $\boldsymbol{\theta}_{t+1} = j$ (refer to FRWZ for further details). With these derivatives in hand, we can apply formula (A4) of FRWZ and obtain a set of 8 equations in 8 unknowns

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\tau}{b}\gamma_{\tau,1} - 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{\bar{c}Y} & -\gamma_{\pi,1} & 0 \\ -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\ \frac{1-\bar{c}}{\bar{c}^2Y} & 0 & 1 \\ 0 & (1-\beta\gamma_{\pi,1})\, b & 0 \end{bmatrix} \begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta \end{bmatrix} h_1$$

$$+ p_{11} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} h_1 + p_{12} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} h_1,$$

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\tau}{b}\gamma_{\tau,2} - 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{\bar{c}Y} & -\gamma_{\pi,2} & 0 \\ -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\ \frac{1-\bar{c}}{\bar{c}^2Y} & 0 & 1 \\ 0 & (1-\beta\gamma_{\pi,2})\, b & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta \end{bmatrix} h_2$$

$$+ p_{21} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} h_2 + p_{22} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} h_2,$$

where, with a slight abuse of notation, the h_i and $g_{x,i}$ coefficients are the elements of $h_{i,b}$ and $\boldsymbol{g}_{i,b}$ defined above. This system can be simplified by subtracting the third equation from the second, and the seventh from the sixth, to eliminate $g_{\phi,1}$ and $g_{\phi,2}$. We then arrive at

$$0 = \frac{1}{\bar{c}Y}g_{y,1} + \gamma_{\pi,1}g_{\pi,1} - h_1\left[p_{11}\left(g_{\pi,1} + \frac{1}{\bar{c}Y}g_{y,1}\right) + p_{12}\left(g_{\pi,2} + \frac{1}{\bar{c}Y}g_{y,2}\right)\right], \tag{46}$$

$$0 = g_{\pi,1} - \frac{\lambda}{\bar{c}Y}g_{y,1} - \beta h_1\left(p_{11}g_{\pi,1} + p_{12}g_{\pi,2}\right), \tag{47}$$

$$0 = \beta h_1 + b\left(1 - \beta\gamma_{\pi,1}\right)g_{\pi,1} + \frac{\tau}{b}\gamma_{\tau,1} - 1, \tag{48}$$

$$0 = \frac{1}{\bar{c}Y}g_{y,2} + \gamma_{\pi,2}g_{\pi,2} - h_2\left[p_{21}\left(g_{\pi,1} + \frac{1}{\bar{c}Y}g_{y,1}\right) + p_{22}\left(g_{\pi,2} + \frac{1}{\bar{c}Y}g_{y,2}\right)\right], \tag{49}$$

$$0 = g_{\pi,2} - \frac{\lambda}{\bar{c}Y}g_{y,2} - \beta h_2\left(p_{21}g_{\pi,1} + p_{22}g_{\pi,2}\right), \tag{50}$$

$$0 = \beta h_2 + b\left(1 - \beta\gamma_{\pi,2}\right)g_{\pi,2} + \frac{\tau}{b}\gamma_{\tau,2} - 1. \tag{51}$$

Note that the term λ appears after exploiting the restriction (43). Finally, these equations can be further combined to obtain

$$
\begin{aligned}
0 &= g_{\pi,1}\left[1+\lambda\gamma_{\pi,1}-p_{11}h_1\left(1+\beta+\lambda\right)+p_{11}^2\beta h_1^2\right]+\left(1-p_{11}\right)\left(1-p_{22}\right)\beta h_1 h_2 g_{\pi,1}\\
&\quad+\left(1-p_{11}\right)h_1 g_{\pi,2}\left[p_{11}\beta h_1+p_{22}\beta h_2-\left(1+\beta+\lambda\right)\right],\\
0 &= g_{\pi,2}\left[1+\lambda\gamma_{\pi,2}-p_{22}h_2\left(1+\beta+\lambda\right)+p_{22}^2\beta h_2^2\right]+\left(1-p_{11}\right)\left(1-p_{22}\right)\beta h_1 h_2 g_{\pi,2}\\
&\quad+\left(1-p_{22}\right)h_2 g_{\pi,1}\left[p_{11}\beta h_1+p_{22}\beta h_2-\left(1+\beta+\lambda\right)\right],\\
g_{\pi,1} &= \frac{\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,1}\right)-h_1}{b\left(\frac{1}{\beta}-\gamma_{\pi,1}\right)},\\
g_{\pi,2} &= \frac{\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)-h_2}{b\left(\frac{1}{\beta}-\gamma_{\pi,2}\right)},
\end{aligned}
$$

which correspond to equations (19)-(22) in the main text.

As this system cannot be solved using traditional approaches such as the generalised Schur decomposition, we follow FRWZ and adopt the Groebner basis algorithm to find all existing solutions, i.e. all the possible 8-tuples made by coefficients h_1, $g_{y,1}$, $g_{\pi,1}$, $g_{\phi,1}$, h_2, $g_{y,2}$, $g_{\pi,2}$, $g_{\phi,2}$ that satisfy the system of equations.

Note that to characterise the first order expansion of the MSV solution we still have to determine the other coefficients $\boldsymbol{h}_{i,\varepsilon}$, $h_{i,\chi}$, $\boldsymbol{g}_{i,\varepsilon}$, $\boldsymbol{g}_{i,\chi}$ that appear in equations (44) and (45). Fortunately, doing so is an easy task. Proposition 1 in FRWZ shows that one has to solve two separate systems of linear equations corresponding to their equations (A5) and (A6).

A.4 Mean square stability under regime switching

To asses the stability of the MSV solutions when some parameters are allowed to switch, FRWZ use the notion of mean square stability (MSS), which is discussed by Costa et al. (2005) and Farmer et al. (2009).

MSS requires the existence of

$$
lim_{t\to\infty}\mathbb{E}_0\left(\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}\right), \quad \text{and} \quad lim_{t\to\infty}\mathbb{E}_0\left(\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}'\right)
$$

The MSS condition constrains the values of the autoregressive roots in the state variable policy function weighted by the probability of switching regimes. In our context with one state variable and two regimes, MSS formally states that one solution is stable if an only if all the eigenvalues of the matrix

$$\begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix} \begin{bmatrix} h_1^2 & 0 \\ 0 & h_2^2 \end{bmatrix} = \begin{bmatrix} p_{11}h_1^2 & (1-p_{22})h_2^2 \\ (1-p_{11})h_1^2 & p_{22}h_2^2 \end{bmatrix}$$

are inside the unit circle. The characteristic polynomial of this matrix is

$$z^2 + a_1 z + a_0 = z^2 - \left(p_{11}h_1^2 + p_{22}h_2^2\right) z + \left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2 = 0.$$

As discussed in LaSalle (1986, p. 28), both eigenvalues are inside the unit circle if and only if both the following conditions hold

$$\begin{aligned} |a_0| &< 1 \\ |a1| &< 1 + a_0, \end{aligned}$$

which in our case give

$$\begin{aligned} \left|\left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2\right| &< 1, \\ p_{11}h_1^2 + p_{22}h_2^2 &< 1 + \left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2. \end{aligned}$$

If we assume $p_{11} + p_{22} \geq 1$, the two conditions can be be rewritten as

$$\begin{aligned} \left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2 &< 1, \\ p_{11}h_1^2\left(1 - h_2^2\right) + p_{22}h_2^2\left(1 - h_1^2\right) + h_1^2 h_2^2 &< 1, \end{aligned}$$

which correspond to equations (10) and (11) in the main text.

A.5 The absorbing case

When regime 1 is absorbing ($p_{11} = 1$), MSS requires the eigenvalues of the following matrix to lie inside the unit circle:

$$\begin{bmatrix} h_1^2 & (1-p_{22})h_2^2 \\ 0 & p_{22}h_2^2 \end{bmatrix}$$

The two eigenvalues are equal to h_1^2 and $p_{22}h_2^2$, so that the conditions for MSS are

$$|h_1| < 1, \tag{52}$$

$$|h_2| < \frac{1}{\sqrt{p_{22}}}. \tag{53}$$

Moreover, by plugging $p_{11} = 1$ in equations (19) and (21) we obtain equation (23), that is

$$0 = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)} \left[1 + \lambda\gamma_{\pi,1} - h_1\left(1 + \beta + \lambda\right) + \beta h_1^2\right].$$

This equation has three solutions for h_1: one depends on the fiscal solution (first term), the other two on the monetary solution (square brackets).

When regime 1 is AM/PF then $g_{\pi,1} = g_{y,1} = 0$, since debt has no impact on inflation and output. Using these restrictions, equations (20) and (22) give

$$0 = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) - h_2}{b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)} \left[1 + \lambda\gamma_{\pi,2} - p_{22}h_2\left(1 + \beta + \lambda\right) + p_{22}^2\beta h_2^2\right], \tag{54}$$

which is itself a combination of a fiscal (first term) and two monetary (square brackets) solutions.

A.5.1 Figure 1a

Consider an AM/PF regime 1. The stability condition for the absorbing state (52) is the same as under fixed coefficients. As fiscal policy is passive, the fiscal solution in (23) is

$$h_1 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right),$$

so we have the condition

$$\left|\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right)\right| < 1,$$

that is $(1-\beta)\frac{b}{\tau} < \gamma_{\tau,1} < (1+\beta)\frac{b}{\tau}$. Employing our calibration, we have $\gamma_{\tau,1} \in (0.019, 3.892)$. The condition for not having another stable solution is that the two solutions of the monetary part of (23) should be outside the unit circle, that is, $\gamma_{\pi,1} > 1$: monetary policy needs to be active. We now analyse the MSS condition (53) for regime 2, given that regime 1 is AM/PF. To do so, we have to solve the third order equation (54) for h_2, and obtain one fiscal solution and two monetary ones. Let us distinguish two cases according to the stability of the fiscal solution in regime 2.

A stable regime 2 fiscal solution. In this case the fiscal solution must satisfy: $\left|\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)\right| < \frac{1}{\sqrt{p_{22}}}$, that gives equation (24)

$$\frac{b}{\tau}\left(1-\frac{\beta}{\sqrt{p_{22}}}\right) < \gamma_{\tau,2} < \frac{b}{\tau}\left(1+\frac{\beta}{\sqrt{p_{22}}}\right)$$

which, employing our calibration, returns: $\gamma_{\tau,2} \in (-0.032, 3.952)$.

To have a unique stable (fiscal) solution the other (two monetary) solutions must be both unstable, which translates into equation (25)

$$\gamma_{\pi,2} > \sqrt{p_{22}} - \frac{\left(1-\beta\sqrt{p_{22}}\right)\left(1-\sqrt{p_{22}}\right)}{\lambda},$$

that is, $\gamma_{\pi,2} > 0.964$. This first case describes the upper-right zone in Figure 1a.

An unstable regime 2 fiscal solution. Under this case $\left|\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)\right| > \frac{1}{\sqrt{p_{22}}}$ which corresponds to equation (28). Under our calibration, we have $\gamma_{\tau,2} < -0.032, \gamma_{\tau,2} > 3.952$.

In order to have only one stable solution, the two monetary solutions of (54) must be one inside and the other outside the unit circle, which yields equation (29)

$$\gamma_{\pi,2} < \sqrt{p_{22}} - \frac{\left(1-\beta\sqrt{p_{22}}\right)\left(1-\sqrt{p_{22}}\right)}{\lambda}.$$

Employing our calibration, we get $\gamma_{\pi,2} < 0.964$. Again, monetary policy can be passive, and the more so, the lower p_{22}. This second case describes the lower-left zone in Figure 1a.

A.6 Figure 1b: A numerical example for the modest fiscal deviations

Consider the case with $p_{11} = p_{22} = p < 1$ in Figure 1b. If regime 1 is AM/PF then $g_{\pi,1} = g_{y,1} = 0$ and from system (46)-(51) we can derive the equations

$$0 = \frac{\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,1}\right)-h_1}{b\left(\frac{1}{\beta}-\gamma_{\pi,1}\right)}\left[1+\lambda\gamma_{\pi,1}-p_{11}h_1\left(1+\beta+\lambda\right)+p_{11}^2\beta h_1^2\right], \tag{55}$$

$$0 = \frac{\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)-h_2}{b\left(\frac{1}{\beta}-\gamma_{\pi,2}\right)}\left[1+\lambda\gamma_{\pi,2}-p_{22}h_2\left(1+\beta+\lambda\right)+p_{22}^2\beta h_2^2\right]. \tag{56}$$

Take a passive fiscal policy in regime 1 with $\gamma_{\tau,1} = 0.2$ and $p = 0.95$. In this case the stable solution is the fiscal one and, in particular, $h_1 = \frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,1}\right) = 0.9068$. In order to have MSS, conditions (10) and (11) must hold. Under this case the most stringent one of the two turns out to be equation (11) that becomes $0.822p\left(1-h_2^2\right)+0.178ph_2^2-1+0.822h_2^2<0$. Then, in order to have MSS the stable solution in regime 2 must satisfy the following: $-1.0209<h_2<1.0209$.[31] If the stable solution in regime 2 is again the fiscal one then: $h_2=\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)\in(-1.0209,1.0209)$. In this case we get a "modest" fiscal deviation for:

$$-0.02<\gamma_{\tau,2}<3.93$$

Substituting $h_2=1.0209$ in the monetary solution for regime 2 in equation (56) we get $\gamma_{\pi,2}=0.955$ which is the lower bound of the correspondent "modest" monetary deviation:

$$0.955<\gamma_{\pi,2}<1$$

Note that these results hold for $\gamma_{\tau,1}=0.2$; obviously, for every given $\gamma_{\tau,1}$ we could obtain different $\gamma_{\tau,2}$ and $\gamma_{\pi,2}$ coefficients.

A.7 Globally balanced policies: analytical results for an absorbing regime 1

A.7.1 Figure 2a: the fiscal frontier

Suppose that regime one is absorbing, and that monetary policy is active in both regimes. The equation to consider to solve for h_1 is (23). If $\gamma_{\pi,1}>1$, the two monetary solutions (i.e., the roots of the second order equation in the square brackets) are out of the unit circle. If the fiscal solution is outside too ($h_1=\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,1}\right)>1$), that is if there is an AF policy, then all solutions are explosive, independently from what happens in the second regime. On the other hand, if in the first regime there is a PF policy, the equation for the second regime reduces to equation (54). Given that we assumed an active monetary policy even in regime 2, we have global determinacy whenever fiscal policy is passive (or modestly active): $\frac{b}{\tau}\left(1-\frac{\beta}{\sqrt{p_{22}}}\right)<\gamma_{\tau,2}<\frac{b}{\tau}\left(1+\frac{\beta}{\sqrt{p_{22}}}\right)$, which corresponds to equation (25) in the text.

A.7.2 Fig. 3a: the monetary frontier

The condition for the absorbing state is (23). If in the absorbing regime 1 fiscal policy is passive, then the fiscal solution is inside the unit circle and $(1-\beta)\frac{b}{\tau}<\gamma_{1,\tau}<(1+\beta)\frac{b}{\tau}$. The condition for not having another stable solution is that the two solutions of the monetary part should be outside the unit circle, that boils down to the usual $\gamma_{\pi,1}>1$: monetary policy needs to be active. As for the non-absorbing regime 2, consider again equation (54). If fiscal policy is passive then to have a unique

[31] Condition (10) instead gives $-1.162<h_2<1.162$.

stable solution the other (two) monetary solutions must be both outside the unit circle, which gives (25): $\gamma_{\pi,2} > \sqrt{p_{22}} - \frac{(1-\beta\sqrt{p_{22}})(1-\sqrt{p_{22}})}{\lambda}$ that is, we have determinacy when $\gamma_{\pi,2} > 0.964$.

A.7.3 Figure 4b under an absorbing PM regime 1

Suppose now monetary policy is PM (with $\gamma_{\pi,1} = 0.9$) in the first (absorbing) regime and AM (with $\gamma_{\pi,2} = 1.5$) in the second regime. The condition for the absorbing state is, as usual, (23). That for the non-absorbing state is derived from (20) evaluated at $p_{11} = 1$ where, to simplify notation, we define $z = h_2\sqrt{p_{22}}$

$$\begin{aligned} 0 &= g_{\pi,2}\left\{1 + \lambda\gamma_{\pi,2} - z\sqrt{p_{22}}\,(1+\beta+\lambda) + \beta z^2 p_{22}\right\} \\ &\quad + (1-p_{22})g_{\pi,1}z\left[\beta z - \frac{1}{\sqrt{p_{22}}}\,(1+\beta+\lambda-\beta h_1)\right] \end{aligned} \tag{57}$$

where $g_{\pi,1}$ and $g_{\pi,2}$ are given, as usual, by equations (21) and (22) in the text. We can re-write the condition for the non-absorbing state as

$$z^3 + b_2 z^2 + b_1 z + b_0 = 0$$

where

$$\begin{aligned} b_2 &= -\frac{\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)p_{22} + (1+\beta+\lambda) + g_{\pi,1}(1-p_{22})b\left(\frac{1}{\beta}-\gamma_{\pi,2}\right)\beta}{\beta\sqrt{p_{22}}}, \\ b_1 &= \frac{\sqrt{p_{22}}\,(1+\beta+\lambda)\,\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right) + \frac{(1+\lambda\gamma_{\pi,2})}{\sqrt{p_{22}}} + g_{\pi,1}\frac{(1-p_{22})}{\sqrt{p_{22}}}\left(1+\beta+\lambda-\beta\bar{h}_1\right)b\left(\frac{1}{\beta}-\gamma_{\pi,2}\right)}{\beta\sqrt{p_{22}}}, \\ b_0 &= -\frac{\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)(1+\lambda\gamma_{\pi,2})}{\beta^2\sqrt{p_{22}}}. \end{aligned}$$

The necessary and sufficient condition for determinacy is that this cubic equation has exactly one solution inside the unit circle and the other two outside. By proposition C.2 in Woodford (2003), this is the case if and only if either of the following two cases is satisfied:

- Case I: $1 + b_2 + b_1 + b_0 < 0$ and $-1 + b_2 - b_1 + b_0 > 0$;
- Case II: $1 + b_2 + b_1 + b_0 > 0$, $-1 + b_2 - b_1 + b_0 < 0$, and $b_0^2 - b_0 b_2 + b_1 - 1 > 0$ or $|b_2| > 3$.

Let us study now how determinacy varies according to the fiscal policy undertaken in regime 1.

AF in the absorbing regime 1. Consider the case of an AF policy in the absorbing regime: regime 1 will have a PM/AF mix hence a unique stable solution. In this case the monetary solution in (23) should have a root inside and one outside the unit circle while the fiscal solution, being AF, is outside. The monetary coefficient $\gamma_{\pi,1} = 0.9$ generates a stable solution $h_1 = 0.94343$ (and an unstable one, that we discard, $h_1 = 1.1534$).

Studying the necessary and sufficient conditions for determinacy, we find that, under these parameters, Case I is never satisfied (since the second inequality never holds) while Case II is. In particular, the condition to have exactly one solution inside the unit circle for regime 2 (from the first condition in Case II) reads

$$\gamma_{\tau,1} > \frac{b}{\tau}(1-\beta h_1) - \frac{\left[1+\lambda\gamma_{\pi,2} - \sqrt{p_{22}}\,(1+\beta+\lambda) + \beta p_{22}\right]}{(1-p_{22})\left[\beta - \frac{1}{\sqrt{p_{22}}}\,(1+\beta+\lambda-\beta h_1)\right]\left(\frac{1}{\beta}-\gamma_{\pi,2}\right)}\left(\frac{1}{\beta}-\gamma_{\pi,1}\right)\frac{b}{\tau}\left[\beta\frac{1}{\sqrt{p_{22}}} - 1 + \frac{\tau}{b}\gamma_{\tau,2}\right], \tag{58}$$

that is represented by a negative sloped line in the space $(\gamma_{\tau,1}, \gamma_{\tau,2})$ and that depends, among other things, on h_1. Note that (58) corresponds to (57) for $z = 1$.

Hence there is a unique stable solution when $\gamma_{\tau,1}$ is above this line for $h_1 = 0.94343$. As a result, when the first regime is PM/AF we have a global determinate equilibrium in the hatched area of Figure A1.

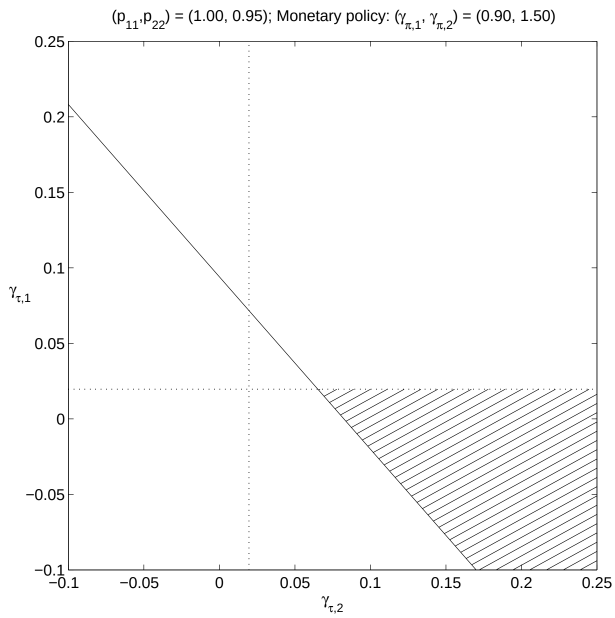

Figure A1: Stability for an absorbing PM/AF regime 1.

PF in the absorbing regime 1. When the first regime is PM/PF, the two monetary solutions are one inside ($h_1 = 0.94343$) and the other outside while the fiscal solution is inside with $g_{\pi,1} = 0$.

Now we have to consider two areas, one for each $h_1 < 1$ in regime 1. The solution $h_1 = 0.94343$, returns the same negative sloped straight line as before. So, again, there is a unique stable solution for regime 2 when $\gamma_{\tau,1}$ is above this line. Hence, when the first regime is PM/PF we have a global determinate equilibrium in the hatched area of Figure A2.

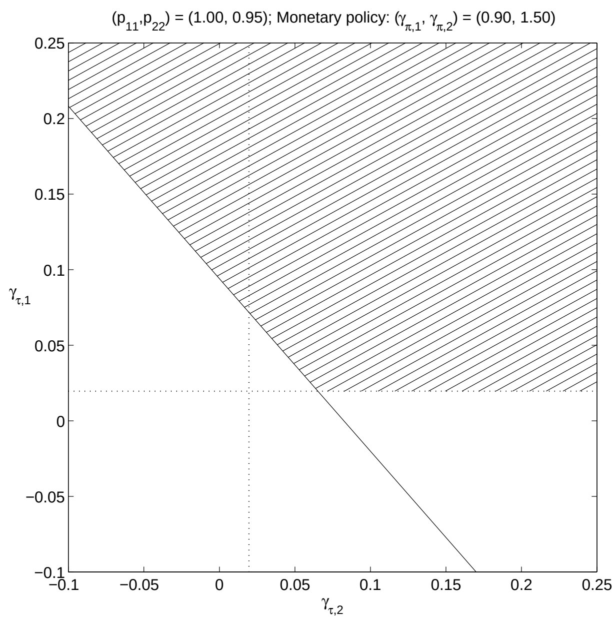

Figure A2: Stability for absorbing PM/PF regime 1: stable monetary solution case.

When we consider the other stable solution, with $g_{\pi,1} = 0$, equation (57) reduces to

$$0 = g_{2,\pi,b} \left[1 + \lambda\gamma_{\pi,2} - z\sqrt{p_{22}}\,(1 + \beta + \lambda) + \beta z^2 p_{22}\right].$$

Hence, we have just one stable solution if and only if the regime 2 is AM/PF (in this case there are 2 monetary solutions outside and 1 fiscal solution inside) and the area characterised by just one stable solution is the hatched area in Figure A3.

Overlapping these two figures (A2 and A3) we get the areas with just one stable solution when fiscal policy in regime 1 is passive: the two triangular areas in Figure A4.

Putting everything together. Putting together Figures A1 and A4, one obtains A5, which is the counterpart of Figure 4b for the case of absorbing PM regime 1.

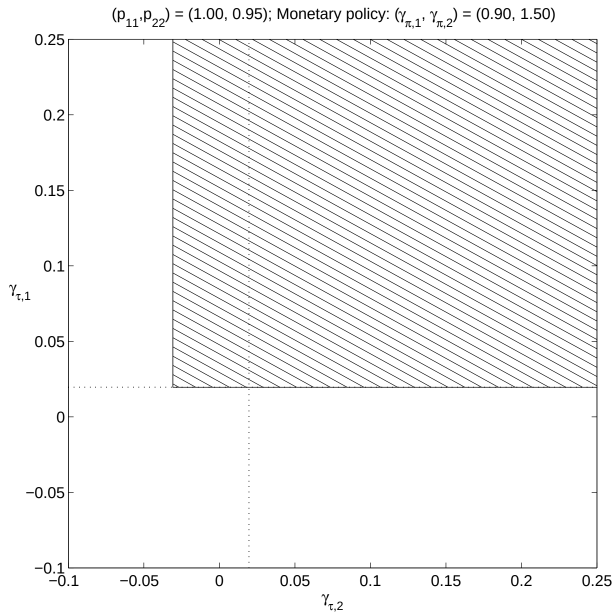

Figure A3: Stability for absorbing PM/PF regime 1: stable fiscal solution case.

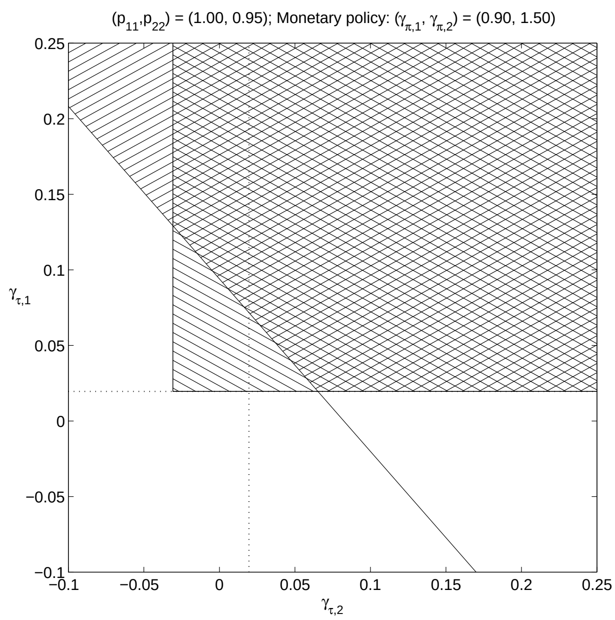

Figure A4: Stability for absorbing PM/PF regime 1: complete case.

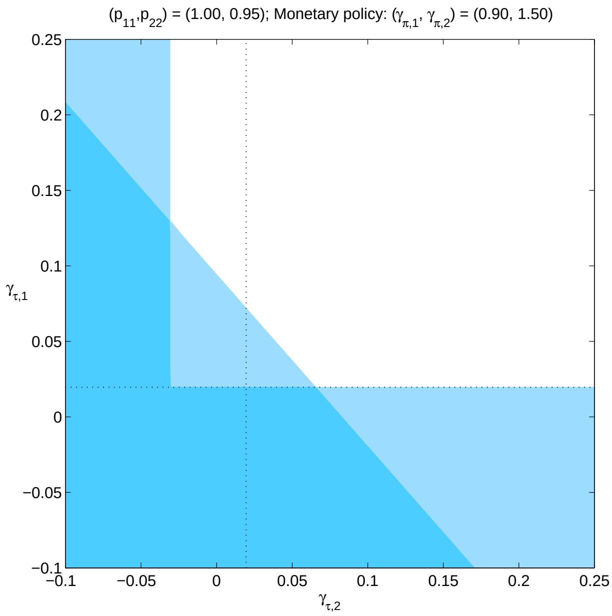

Figure A5: The fiscal policy frontier with substantial deviations in monetary policy and absorbing regime 1 with PM.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.