

# Synchronization in baroclinic systems

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**Abstract.** Synchronization of periodic and chaotic oscillations between two coupled rotating baroclinic fluid systems will be presented. The numerical part of the study involves a pair of coupled two-layer quasigeostrophic models, and the experimental part comprises two thermally coupled baroclinic fluid annuli, rotating one above the other on the same turntable. Phase synchronization and imperfect synchronization (phase slips) have been found in both model and experiments, and model simulations also exhibit chaos-destroying synchronization.

## 1. Introduction

The origin of oscillations and cyclic behavior in the weather and circulation of the atmosphere has long been of interest to scientists. These roughly cyclic oscillations vary on time scales from seasons to decades and even millennia. Commonly cited examples include the El Niño-Southern Oscillation, North Atlantic Oscillation/Arctic Oscillation and zonal index cycle [1], and others have been noted in regional events such as in records of precipitation in certain areas or the high-water level of rivers [4]. Some hypotheses propose that such oscillations are caused by periodic forcing, such as produced by solar or annual cycles, or teleconnections (phenomena in one region of the Earth that may originate in atmospheric features developing thousands of kilometers away) and natural internal climate variability [2, 3, 4]. Ideas of chaos synchronization have been used recently in attempts to better understand these kinds of behaviour [2, 4, 5]. The study of synchronization in simple experiments and simplified models may thus provide a useful source of insight for a better understanding of these atmospheric phenomena and in general in the study of climate variability.

Baroclinic waves have an important role in the atmospheric mid-latitude circulation, contributing strongly to the poleward heat transport in the form of waves and baroclinic eddies. These are some of the most energetic components of the atmospheric circulation and are responsible for major meteorological disturbances such as cyclones, anticyclones, and other weather phenomena on the Earth and other planets [6]. The evolution of these waves and, in general, the main aspects of the atmospheric circulation, can be investigated in laboratory analogues e.g. using the rotating, baroclinic annulus experiment [7]. Many different flow regimes are found in this experiment, including steady states, wave-like flows (baroclinic waves), and irregular flows (baroclinic chaos), some of which resemble flow regimes found in real atmospheres [7, 8, 9, 10]. Simplified two-layer quasigeostrophic models have also proved a useful tool in the theoretical investigation of baroclinic instability in rotating systems [11].

Some of the atmospheric phenomena mentioned above, such as due to diurnal and seasonal forcing, can be studied in the laboratory [12] and numerical models [13] by periodically

perturbing the thermal conditions of the system. In the same vein, teleconnection phenomena can be investigated using two coupled numerical and/or experimental baroclinic systems.

In the experimental part of the present study, we have thermally coupled two separate baroclinic systems in order to observe and analyze their resulting behaviour. The numerical part, on the other hand, consists of studies performed using a pair of five-dimensional versions of the two-layer quasigeostrophic model of baroclinic instability presented in [14, 15], that has been modified in order to obtain a system coupled through their zonal flows. Although it is clear that this simplified model is far removed from realistic atmospheric situations (at least on Earth), it can reproduce some of the basic dynamics found in rotating annulus experiments. With this model, we investigate synchronization between the systems in both periodic and chaotic regimes where the coupling can be either unidirectional or bidirectional.

## 2. The Model

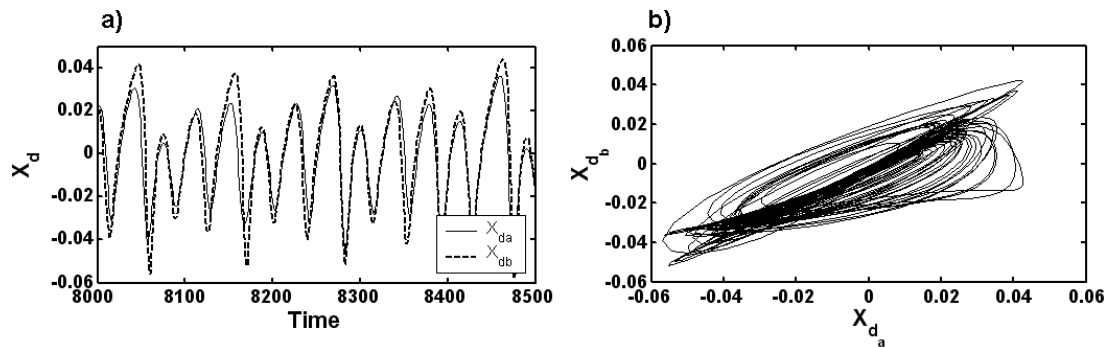
The model used here is an adaptation of the five-dimensional model developed and described in [14, 15], and the reader is referred to these papers for a detailed description and formulation of the model. Although this spectral model represents the simplest possible truncation, it has proved to provide valuable insight into some aspects of the behaviour of realistic systems and models. In this work we use a pair of these models, coupled via a linear *diffusive* term in the zonal flow correction,  $X_{d_i}$ , where  $i$  refers to the system  $a$  or  $b$ .  $A_s, B_s$ , and  $A_d, B_s$  represent the barotropic (real and imaginary) and baroclinic (real and imaginary) parts of the flow, respectively. The coupled non-identical system is then:

$$\begin{aligned}
 \dot{A}_{s_{a,b}} &= -\Delta_{s_{a,b}} A_{s_{a,b}} + \beta_{s_{a,b}} B_{s_{a,b}} - \left( \nu_{s_{a,b}} + \gamma_{s_{a,b}} X_{d_{a,b}} \right) B_{d_{a,b}}, \\
 \dot{B}_{s_{a,b}} &= -\Delta_{s_{a,b}} B_{s_{a,b}} - \beta_{s_{a,b}} A_{s_{a,b}} + \left( \nu_{s_{a,b}} + \gamma_{s_{a,b}} X_{d_{a,b}} \right) A_{d_{a,b}}, \\
 \dot{A}_{d_{a,b}} &= -\Delta_{d_{a,b}} A_{d_{a,b}} + \beta_{d_{a,b}} B_{d_{a,b}} - \left( \nu_{d_{a,b}} + \gamma_{d_{a,b}} X_{d_{a,b}} \right) B_{s_{a,b}}, \\
 \dot{B}_{d_{a,b}} &= -\Delta_{d_{a,b}} B_{d_{a,b}} - \beta_{d_{a,b}} A_{d_{a,b}} + \left( \nu_{d_{a,b}} + \gamma_{d_{a,b}} X_{d_{a,b}} \right) A_{s_{a,b}}, \\
 \dot{X}_{d_{a,b}} &= -\bar{\Delta}_{a,b} X_{d_{a,b}} + \bar{\gamma}_{a,b} \left[ A_{s_{a,b}} B_{d_{a,b}} - B_{s_{a,b}} A_{d_{a,b}} \right] + \underbrace{\eta_{a,b} (X_{d_{b,a}} - X_{d_{a,b}})}_{\text{Coupling}}
 \end{aligned} \tag{1}$$

where  $\eta_a$  and  $\eta_b$  represent the coupling strengths. If  $\eta_b = 0$  and  $\eta_a \neq 0$  we have *master-slave* coupling, and if  $\eta_a = \eta_b$  we have a symmetrical *mutual* coupling. So far, we have only used these two configurations. The de-tuning between the systems is produced by having different values for the Froude number,  $F_i$  which basically alters the parameters:  $\Delta_d, \bar{\Delta}, \nu_d, \bar{\gamma}$  and  $\gamma_d$ .

### 2.1. Numerical results

Depending on the chosen parameters, each single model is able to reproduce either periodic or chaotic dynamics. The dynamics of the coupled system was studied using nonlinear time series analysis diagnostics, such as the Lyapunov exponents, return maps, phase dynamics, frequency spectra, Lissajous plots, and bifurcation diagrams [16, 17]. For the *master-slave* configuration it is possible to observe imperfect synchronization (phase slips) and phase synchronization. Figure 1 shows an example of the synchronization found in the numerical model. The parameters are,  $F_A = 13$ ,  $F_B = 12$  and  $\eta = 0.14$  (coupled case). Using the auxiliary system approach [18], where an ‘extra’ *slave* is added and its dynamics is compared with the initial *slave*, generalized synchronization was also detected. Phase synchronization is even found when a chaotic slave system is driven by a master system that is in a periodic regime with relatively small coupling strength. Signs of chaos-destroying synchronization [17] is observed with a relatively large coupling strength.



**Figure 1.** Two chaotic signals originating from the coupled numerical systems are shown in **a)** and its corresponding Lissajous figure in **b)** (system *A* is the solid line and system *B* the dashed line). It is possible to observe that they are oscillating at the same frequency, but their amplitudes remain uncorrelated.

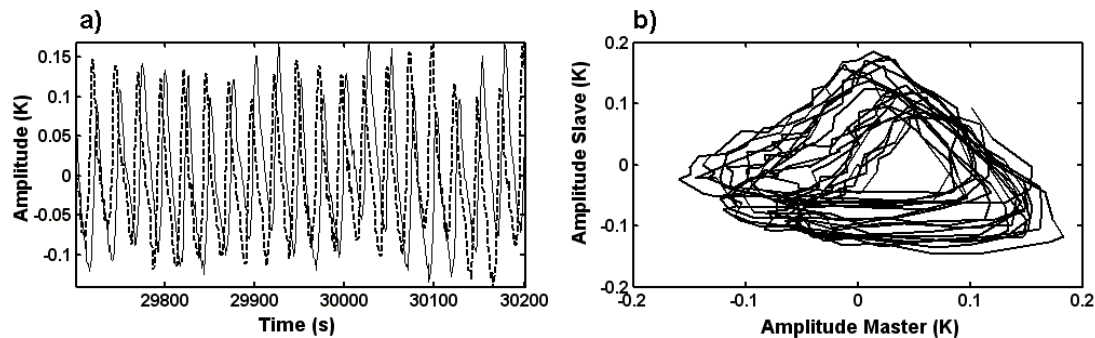
For the bidirectional coupling, imperfect and phase synchronization are also found and, when the de-tuning is large, the oscillation of both the master and slave disappears. This phenomenon is known as oscillation death (or quenching), and it has been found in other mutually coupled systems. However, we found a narrow region of imperfect synchronization between the regions of oscillation death and the region of no synchronization in the  $[\Delta F, \eta]$  parameter space.

### 3. The experiment

The experimental setup consists of two annular containers mounted on a turntable base, one above the other so they can both be rotated about a common vertical axis of symmetry. Each annulus consists of two concentric brass cylinders; the lid and the base are made respectively of perspex and tufnol. The inner and outer cylinders have radii of  $a = 25$  and  $b = 80$  mm respectively, and the depth of both annuli is  $d = 140$  mm. Inside the annular space, at mid depth and mid radius, there is a copper-constantan thermocouple ring, providing 32 measurement points, equally spaced in the azimuthal direction. The temperature of the inner and outer cylinders is regulated by circulating water through spiral grooves inside the inner cylinders and outside the outer cylinders. The coupling between the two annuli is performed by connecting the water output of the circulating water that maintains the temperature of the outer cylinder of one annulus (Annulus *A*) to the water input of the outer circuit of the second annulus (Annulus *B*). In this way, any temperature oscillation in the circulating water, due to the oscillatory heat transport caused by fluctuations in the amplitude of the baroclinic waves developing in Annulus *A*, will perturb the thermal boundary conditions of Annulus *B*.

The wavenumbers,  $m$ , drift frequencies,  $\omega_d$ , vacillation frequencies,  $\omega_\nu$  and modulated vacillation frequencies  $\omega_m$  are calculated by applying fast Fourier techniques (spatial and temporal FFTs) to the thermocouple data. A spatial Fourier transform is applied to the data provided by the thermocouple rings at each sample time in order to determine the dominant azimuthal wave number and its possible temporal amplitude variations. The time series of the dominant wavenumbers for each annulus are then compared and studied with the same tools used to analyze the numerical model results.

This work is ongoing, although preliminary experiments so far have found evidence for phase synchronization and phase slips (imperfect synchronization, probably due to unavoidable experimental noise). An example of the experimental results showing phase synchronization, visible here via the Lissajous plot, is shown in Figure 2. The parameters for the experiments are a rotation rate  $\Omega = 1.63 \text{ rad/s}$ ,  $\Delta T_A = 10$  K and  $\Delta T_B = 9.8$  K.



**Figure 2.** Two signals originating from the coupled system are shown in **a)** and its corresponding Lissajous figure in **b)** (system *A* is shown in a solid line and system *B* in a dashed line). Introduction of the coupling between the systems adjusts the frequencies and phases, although the amplitudes remain different. A circular structure in the coupled case in the Lissajous plot is typical with two signals at the same frequency and a constant phase shift.

#### 4. Conclusions and future work

Phase synchronization and imperfect synchronization have been found in a low-order model of baroclinic waves and in an experiment. For the numerical model these results also include chaos-destroying synchronization, oscillation quenching, and generalized synchronization. The experiment has shown both partial and complete phase synchronization so far. Further experimental work will map the synchronization regions in more detail, using different values of the coupling strength and de-tuning. There is also the possibility of modifying the experiment for bidirectional coupling by connecting not only the outer water circuits (warm walls), but also the inner circuits (cold walls).

#### Acknowledgments

AACP acknowledges the support of a Dorothy Hodgkin Award administrated via the UK Natural Environment Research Council and to the SEP-Mexico.

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