On the Potential Future Effects of Population Structure on Financial Stability



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University of Oxford

A thesis submitted in partial fulfilment of the requirements for the M.Sc. in

Mathematical Finance

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Abstract

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In several advanced economies, the post World War II baby-boom was followed by a sustained drop in fertility rates to sub-replacement levels. In parallel, adult life expectancies have seen an unprecedented increase. The aim of this study is to better understand the potential future effects of these demographic dynamics on financial stability. Two major channels of influence are investigated. The first channel is through the housing market and the second is related to debt sustainability.

We present a probabilistic numerical model that simulates and projects future population dynamics. This model is used to project potential future growth of two major actors of the supply and demand in the housing market; old age market leavers and first time buyers. We also use this model to project actual income and consumption trends as well as their potential quantile distributions. The models parameters are estimated and calibrated to Canada, Germany, Spain, France, Italy, Japan, the United Kingdom and the United States. We run Monte Carlo simulations and sensitivity analysis to convey uncertainty.

Our results show an overall continuous and long lasting increase in old age generated supply. Potential first time buyers demand varies from country to country. However, in all countries in focus, it is increasingly sensitive to migration. Projections of actual income and consumption trends leads to decreasing support ratios. We discuss how these potential imbalances could impact the housing market, debt and to a larger extent financial stability.

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1 Introduction

1.1 Population structure, economic growth and debt

Population growth rate has long been the dominant demographic focus of economists and policy makers. However, significant changes in the age composition of populations have led to an increasing interest in studying the effects of population's age structure.

Over the past century, fertility and mortality rates undergone major changes. In the first half of the 20^{th} century, improvements in public health led to a sharp drop in infant and child mortality rates. In the second half of the last century, health improvements in middle aged and older adults led to a significant increase in life expectancy. These two developments have considerably increased age-specific survivorship rates. Fertility rates have also varied considerably over the past century. Following the second world war, many countries experienced a positive fertility shock that lasted for about two decades until socio-economic development in the late 1960s eventually led to a sharp drop in fertility rates (Figure B4).

The Demographic Transition Model (DTM) (Thompson 1929, (Kirk 1996)) summarizes the effects of these dynamics on birth and death rates, as well as on the total population and its age structure. The DTM initially had four stages, over which a population goes from a "primitive stability" to a "modern stability". However, most advanced economies are transitioning into a stage with sub-replacement fertility rates and high mortality rates (due to the delaying effect of increasing life expectancy). A fifth stage has, therefore, been proposed to describe this state of the population (Hall 1972). Extended periods with sub-replacement fertility rates tend to "inverse" the population pyramid (DTM: stage 5). Without compensating inbound migration, sub-replacement fertility ultimately leads to an increase in dependency ratios. Several studies have investigated the long-term implications of such population ageing on production, income, consumption and investment.

1.1.1 Demographic dividend and demographic burden concepts

In a properly functioning labour market, the decrease in fertility and mortality rates can lead to a population structure that is initially beneficial to the economy. During this pe-

riod, dependency ratios tend to fall, freeing up resources for education and economic development (Bloom, Canning, and Sevilla 2003). A drop in fertility can switch females' most productive years from child bearing and caring to education and economic activity, leading to higher consumption and economic growth (Bloom, Canning, Fink, et al. 2009). Williamson showed that a third of the east Asian economic growth in the 90s could be linked to the shift in their age distribution. On a larger scale, Kelley and Schmidt (Kelley et al. 2005) found that the combined effects of demographic shifts could account for about 20% of global per capita output growth between 1960 and 1995 (28% for Asia and 24% for Europe).

In a series of studies, Lee and Mason introduced the concept of 'second demographic dividend' due to wealth reinvestment and accumulation of the initial demographic investment described above (R. D. Lee et al. 2006). They see the first dividend as a window of opportunities that should be wisely used for asset and capital accumulation. Gonand (Gonand et al. 2015) links the amplitude and durations of the second demographic dividend to the population structure.

1.1.2 Productivity, economic growth and population structure

Over the past decade, the US labour participation rate decreased from around 66% to 62% (Taylor 2016). This trend is expected to persist, due to ongoing population ageing (Hurd et al. 2011). Workforce ageing is also expected to exert a negative pressure on productivity growth (Aiyar et al. 2016, Choudhry et al. 2016). This is mainly due to an age-specific productivity function with an inverted U shape, which peaks between 40 and 49 years of age (Feyrer 2007). Large variations across occupations mean that this productivity saturation effect is uneven across the different economic sectors (Bassanini et al. 2008).

As a result of the ageing population and the associated need for care and support, health care expenditures are expected to keep rising (in real terms and as a share of GDP) across many advanced economies (Orszag et al. 2007, Pammolli et al. 2012). With social and health care expenditure increasing at a faster rate than fiscal revenues, the fiscal imbalances of most OECD countries are projected to widen (Elmendorf et al. 2017).

Recent GDP growth forecasts tend to emphasize the extent of the continuous negative pressure due to population ageing in the Eurozone (Dieppe et al. 2015), the US (Maestas et al. 2016, (Fernald 2016)), China (Cai et al. 2013) and many South East Asian economies (Walmsley et al. 2017). In export-driven Japan, weak domestic demand has resulted in more than two decades of GDP stagnation (Muto et al. 2016). This situation has revived the secular stagnation hypothesis (Westelius et al. 2016). Some studies have also linked the persistence of low interest rates (Ferrero et al. 2017) and low inflation (Bullard et al. 2012 ,Yoon et al. 2014) to the recent demographic shifts. In the Figure 1.3 below we show the age specific labour income and consumption in the United-States as calculated by National

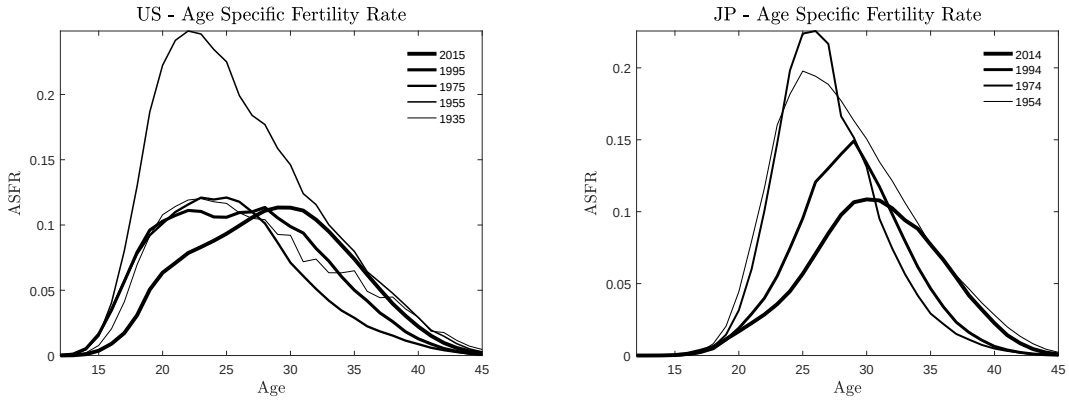


Figure 1.1: Age Specific Fertility rates in the U.S. and Japan

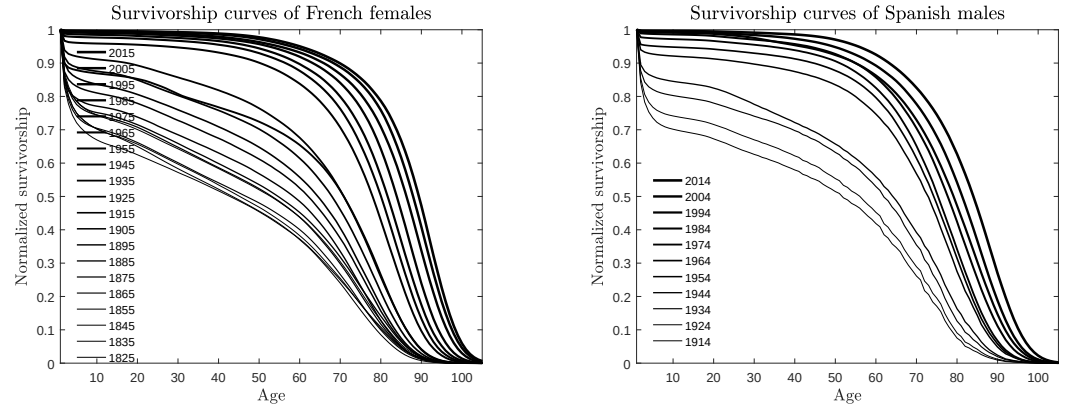


Figure 1.2: Age Specific Survivorship functions in France and Spain

Transfer Accounts economists.

1.1.3 Money, debt and financial stability

Advanced economies have accumulated large amounts of debt since the financial liberalization of the 1980s and 1990s (Schneider et al. 2004). Global OECD debt (public, household and corporate debt) is now more than three times the total global OECD GDP. Over the past decade, the low interest-rate policies of major central banks have kept debt-servicing expenses at a relatively low level. While persistent low interest rates have stimulated lending and risk taking, they have also increased borrowers' sensitivity to interest rate hikes. In the discussion below, we further elaborate on the effect of indebtedness on economic growth and financial instability in the context of an ageing population.

The Global Financial Crisis (GFC) and the European debt crisis exposed policy makers and the population in general to the impact and consequences of financial instabilities. A common subsequent criticism was that the orthodox paradigm and conventional models

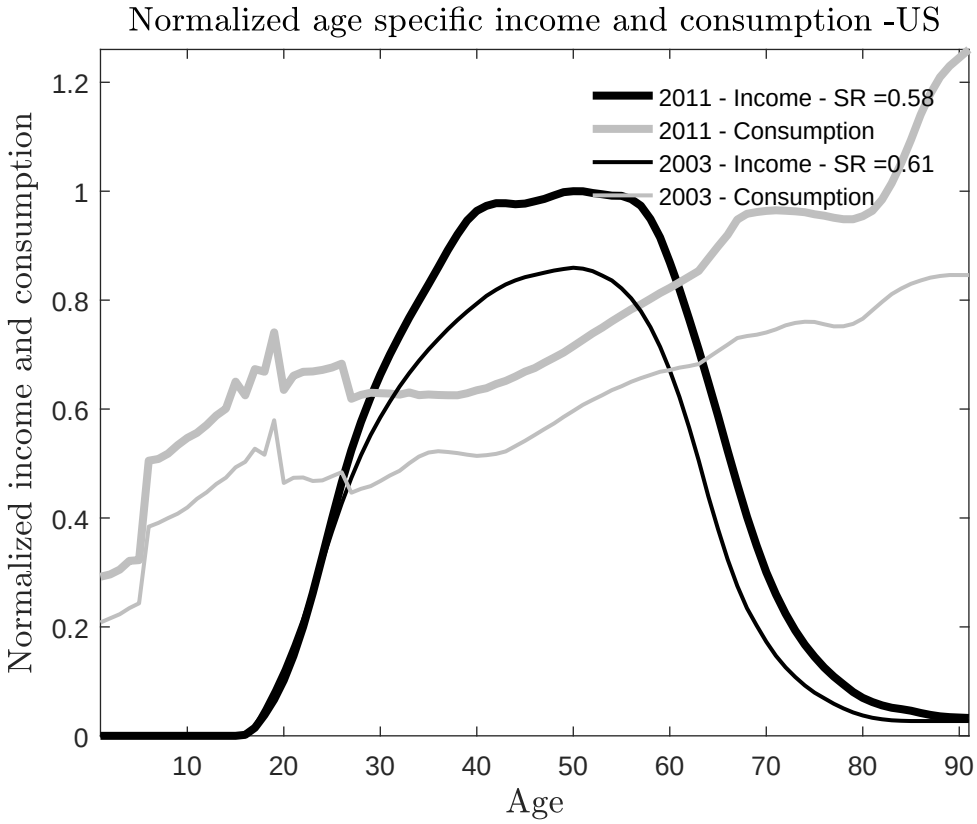


Figure 1.3: Normalized age specific labour income and consumption in the U.S.- 2003 and 2011. Source: National Transfer Accounts.

had major limitations. The neutrality of money - a common pre- crisis assumption - was highly questioned. The interaction between credit, money and macroeconomics emerged as one of the major subjects of research and discussion. This has also revived a fundamental debate on money creation (Werner 2014) and generated answers from major's central banks about money creation in modern economies (Deutsche Bundesbank 2017). Bank of England economists explain that (McLeay et al. 2014):

> *In the modern economy, most money takes the form of bank deposits. But how those bank deposits are created is often misunderstood: the principal way is through commercial banks making loans. Whenever a bank makes a loan, it simultaneously creates a matching deposit in the borrower's bank account, thereby creating new money. The reality of how money is created today differs from the description found in some economics textbooks:*
>
> - *Rather than banks receiving deposits when households save and then lending them out, bank lending creates deposits.*
> - *In normal times, the central bank does not fix the amount of money in circulation,*

nor is central bank money 'multiplied up' into more loans and deposits.

Both Central Banks' papers cite bank profitability, regulation, monetary policy and household behaviour as major factors in expanding and limiting credit growth. Since these variables are interdependent and tend to feed-back on each other, dynamic frameworks have been developed by heterodox and mainstream economists to explain their effects on leverage, economic growth and financial crises (Keen 1995, Godley et al. 2012, Kapeller et al. 2014). There is also a debate on whether this endogenous nature of money (created as a counterpart of an interest-bearing debt) implies a credit and economic growth imperative (M. J. Gordon 2003, Jackson et al. 2015, Strunz et al. 2015).

As discussed above, increasing dependency ratios are expected to put a negative pressure on growth rates. The implications of potential prolonged weak, or even negative, economic growth on financial stability, has resulted in many concerns among policy makers (International Monetary Fund 2016). This is exacerbated by ratios of debt over GDP that tend to drag down on growth (Cecchetti et al. 2011) and by ultra low interest rates (Summers 2014). Since these effects are shared and synchronized among many advanced economies, contagion risk further weakens global financial stability (Caccioli et al. 2015, Daly et al. 2017). In this context, one of the main purposes of this work is to better understand the implications of population ageing and shrinkage on potential credit growth.

1.2 Housing, credit growth and population structure

Housing is by far the largest expenditure and equity of the large majority of OECD households. In addition to its use as shelter, housing has been increasingly considered as a financial investment. This additional utility has led to an increase in the proportion of household expenditures that are allocated to it (Mian et al. 2010). Mortgages for housing represent more than two thirds of U.S. household debt and one of the main components of its money supply and economic growth (The Center for Microeconomic Data - Federal Reserve Bank of New York 2017). Housing also has a considerable financial impact on the other sectors of the economy.

Over the past decade, the age-specific U.S. debt balance stagnated for young borrowers and increased significantly for older ones (Brown et al. 2016). While ageing explains a large part of this increase for those over 50 years of age, a change of behaviour is mainly responsible for the changes for those who are over 65 years old. Over that age, home equity lines of credit account for the majority of this increase. For the younger age groups, there was a slight decrease in aggregate debt for those between 30 and 40 years old. However, debt origination showed significant changes. Student loans amounts per borrower more than doubled over this period, while mortgage origination more than halved over the same period. Similar trends have been reported in the UK, where the debt burden of student

loans is delaying home ownership and family formation. This is accentuated by property prices generally increasing at a faster rate than incomes for these age groups.

In the first study of this dissertation, we use a self-contained, custom-made demographics model to understand the dynamics between two major forces of the housing market. This involves and examination of the balance between First Time Buyers (FTB) as the main driver of housing demand and the supply generated by old age.

Our main hypothesis is that the sensitivity of the housing market to demographics could transition from an overall positive effect in recent decades to an overall negative one in the next few decades. In other words, many market segments with an actual high buying demand could saturate or even reverse into markets with a high selling pressure, due to ageing populations facing depleted younger cohorts. This widening balance should first offset the lack of housing stocks in many markets. However, if it persists for structural demographic reasons, this imbalance could lead to severe repercussions on credit and economic growth.

We weight the supply and demand imbalance through the ratio between old-age market-leavers and FTBs. While this benchmark doesn't explicitly include repeat buyers and rental investors, these two market forces are both sensitive to our benchmark. In a basic supply and demand framework, FTBs are similar to an upward force that dislodge repeat buyers in the "property ladder"(Paciorek 2013). One of the main components of this force is the equity gains proposed for repeat buyers to move (C. L. Lee et al. 2014), the quantity of FTBs has a significant influence on the number of potential repeat buyers (Anenberg et al. 2013). The remainder of this section provides a historical perspective on the effects of demographics on the housing market and reviews some relevant literature.

1.2.1 Housing and population structure: a literature review

During the late 19^{th} century and through the start of World War I, the US total fertility rate, adjusted for infant and child mortality, was around three children per woman. It started to drop after the first world war and reached a then all-time low of around two children per woman by the middle and late 1930s (Goss 2010). Hansen (Hansen 1939) and Barber (Barber 1978), have extensively discussed the influence of population growth on the Great Depression. Barber sees the excessive housing starts with regard to reduced family formation, as one of the main factors for the drop in housing demand and price in the mid-1920s.

Between 1940 and 1960, the U.S. home-ownership rate increased from 44% to 62%, its fastest recorded rise. Following various large-scale government interventions, the home ownership rate for men below 30 years old more than doubled (Chambers et al. 2014). Sternlieb et al. 1986 found that this favourable household-formation environment considerably helped the post-World War II baby boom. The U.S. mortality-adjusted TFR reached

its 20^{th} century's peak of 3.6 in the 1960s. Sternlieb also links the "Golden Housing Age" of the 1970s to the first wave of baby boomers entering the housing market.

In 1989, Mankiw and Weil (Mankiw et al. 1989) published the first study on the impact of baby boomers' ageing on house prices. They modelled housing demand as a function of age. Using 1970 U.S. Census data, they found a sharp decline in U.S. home prices over the following two decades. Pitkin and Myers (1994) suggested that this early and sharp drop in demand modelled by Mankiw and Weil was mainly due to an underestimation of the baby boomers' old age housing demand. The cohorts represented in the 1970 U.S. Census had a lower life expectancy and their housing demand was significantly declined after 45 years of age.

Green and Hendershott (R. Green et al. 1996) controlled for the quality and quantity of housing. They regressed the marginal prices due to these housing characteristics against several demographic variables. Using the 1980 U.S. Census data, they found that education and income were the main determinants of housing consumption and concluded that as long as these two variables were increasing, they should lead to a continuous increase in housing consumption (R. K. Green et al. 2005).

More recent studies show an increasing demographic impact on housing prices. Myers and Ryu (Myers et al. 2008) extrapolated age-specific buying and selling rates of every U.S. state (late 1990s data) on its own population forecast (2005 data). Their model shows a continuous increase in the number of states where housing sellers should exceed buyers, with most states crossing that point after 2025. Their model also shows the potential impact of internal migration; the states that attract young populations or retiring baby boomers, should suffer less from this imbalance than the states which are mainly experiencing out-migrations.

Eichholtz and Lindenthal (Eichholtz et al. 2014) use a hedonic framework to investigate the dependency of housing demand on age and other demographic characteristics. Using a cross-sectional survey of English households (data from 1991 to 2001), they found that housing demand increased with age and that changes in demographics led to heterogeneous demand responses.

Takats (Takáts 2012) used a panel regression model to study the relationship between population, old age dependency ratios and real housing prices. Using OECD data from 1970 to 2009, he found that an increase in the total population is associated with an increase in real housing prices, while an increase in the dependency ratio is associated with lower prices. By extrapolating the estimated coefficients on future population forecasts, he found a significant price decrease, due to increasing dependency ratios and decreasing population growth. Arestis and Martinez (Arestis et al. 2017) confirmed the historical demographic impact on housing by building a price equation based on demographic and macroeconomic indicators, with data from 1970 to 2013 for 17 OECD countries.

Hiller and Lerbs (Hiller et al. 2016) used a spatial panel model to study the effect of ageing on local housing prices in 87 German cities between 1994 and 2005. They found a substantially lower real price appreciation in urban areas with higher dependency ratios. They also found a heterogeneous effect across the different housing segments. Park (Park et al. 2017) used a panel regression on six metropolitan cities and seven provinces in South Korea. They found a negative effect of dependency ratios on home prices, with an amplitude more than two times larger than the effect of income growth.

1.3 Debt dynamics and population structure

In many advanced economies (AE), accommodative monetary policy contributed to the post GFC recovery in housing prices (Gabriel et al. 2017). This has led to an increase in home-owners and rental investors assets, to a lesser extent, it has also boosted consumption through the associated "wealth effect" (Domanski et al. 2016). With housing prices increasing at a faster rate than incomes in many AE markets, the home ownership rate sharply decreased among younger generations. Political policy makers have recently criticised the asymmetrical impacts of quantitative easing on inequality while monetary policy makers have oriented their critic on inefficient redistributive policies (International Monetary Fund 2017). Recent studies have linked the increase income and wealth inequality due to the asymmetrical effect of quantitative easing programs and low interest rate policies (Mumtaz et al. 2017, Montecino et al. 2015). Several transmission channels have been reported: low interest rates and asset repurchase programs have pushed investors toward higher yields and higher indebtedness. Since ownership of these assets was already concentrated among a relatively small portion of the population and highly skewed toward older ages groups, wealth inequality increased significantly following recent unconventional monetary policies. While some studies have found that home equity partially compensated for financial assets inequality, this effect has accentuated wealth inequality between age groups and geographic areas.

Wealth and income data from tax and survey reports have been used at different periods to construct empirical inequality distributions(Berman et al. 1953, Atkinson, J. P. F. Gordon, et al. 1989). Among the main recent findings was the evolution of the top tail of the distribution (Piketty et al. 2003). While advanced mathematical models have been developed to fit top incomes (Atkinson, Piketty, et al. 2011), recent studies on the scale of fiscal optimization and tax evasion have shed some light on the accuracy of this data and suggest that it might be significantly underestimating income and wealth inequality (Alstadsæter et al. 2017).

Literature on income inequality and economic growth suggests that inequality (especially at the bottom of the distribution) has a considerable negative impact on growth

(Cingano 2014). Several mechanisms have been identified and studied. High income inequality tends to limit human capital growth and negatively impact health and productivity. The Easterlin paradox - low population happiness and life satisfaction despite significant economic growth - has been associated with high income inequality (Oishi et al. 2015). A series of papers have found that excessive income inequality tends to make growth more unsustainable and leads to slower growth.

Studies on financial stability have long ignored the role of income heterogeneity. Kumhof and Rancière (Kumhof et al. 2015) looked at two pre-financial crisis periods where inequality and indebtedness rose considerably (1920-1929 and 1983-2008). They argue that the rise in US income inequality induced low-income individuals to borrow beyond sustainable means in order to maintain or improve consumption standards. Under their theoretical framework, the rising income on the upper side of the income distribution and increasing indebtedness on its lower side increases the demand for financial intermediation. As a consequence, the financial sector tends to increase as a proportion of the GDP and by feedback further increase income inequality. They conclude that the unsustainable over-leveraging at the lower tail of the income distribution considerably increases financial fragility.

Similarly, Rajan (Rajan n.d.) argues that rising income inequality tends to create political and economic pressure against the mean reversion of this inequality, by encouraging higher indebtedness at the low-income end of the spectrum, instead of more efficient and sustainable redistribution policies. Inequality has been cited as the main factor behind the divergence between median compensation and productivity growth since the late 1980s (Mishel 2012, Brill et al. 2017).

Income inequality is highly associated with a reduction in intergenerational income mobility, in what is known as "The Great Gatsby Curve" (Corak 2013, Jerrim et al. 2015). In a paper titled "The Fading American Dream", Chetty (Chetty et al. 2017) assessed intergenerational income mobility by estimating the fraction of children who earn more than their parents (absolute income mobility). They found a continuous decrease of this fraction from over 90% in the early 1940s cohorts to just over 50% for early 1980s cohorts.

2 Methodology

Support and Dependency Ratios are frequently employed in the academic and financial literature as a measure of the pressure of the inactive population on the active one. Common criticisms of these indicators are that they use static age groups with uniform weightings, whereas empirical data show that the income and consumption profiles of the active and inactive populations are highly dynamic.

The World Bank, and the OECD set the transition from inactivity to activity at 16 years of age. While this might have been a reasonable approximation many decades ago, this transition age has gradually increased. The transition age also tends to vary considerably between countries. Age-specific income and consumption data also show that there is a significant age dependency and time variability. By weighting all ages with the same constant contribution, dependency and support ratios introduce a considerable distortion to the population structure analysis.

Another source of error that we address in this work is population forecasts (Booth 2006). Long-term projections made by major institutions, such the UN, have continuously underestimated the persistence of sub-replacement fertility in advanced economies (United Nations 2013). This is mainly explained by their assumption that long-term fertility should converge to the replacement rate of about 2.1 children per woman. Similarly, some forecasting methodologies keep the age-specific fertility distribution constant over time and do not consider its temporal trend.

An additional source of error lies in the fact that many demographic models use age groups of five years, instead of the more granular one-year group. This study develops its own demographics model to overcome these limitations and gain full control and understanding of projection scenarios and accuracy. This study can also run Monte Carlo simulation to account for the high variability of some parameters. This allows us to present the results as a probabilistic distribution.

In this chapter, we describe the main models and techniques used in the study. The novelty of this work lies in its mix of low-level models and concepts in a "complex system" setting, with the aim of obtaining more representative high-level models. This chapter is divided into three sections. We first describe the population projection model, a simple

and self-contained model to forecast future populations with a good compromise between model simplicity and accuracy. We also describe how we calibrate its parameters to historical data. The accuracy of the model is assessed in the results section.

The second section of this chapter is related to the first study in our work. The study uses the demographic projection engine to forecast the potential evolution of two major supply and demand forces in the housing market. The projected population structure of several advanced economies is expected to negatively influence the growth in potential first-time buyers, while increasing the supply of homes sold due to old age-related conditions. In the second section, we show how we model these two market forces and how we calibrate the model to market data.

The final section of the chapter presents the models used in the second study of this dissertation. We use the demographic model presented in the first section to estimate future age-specific income and consumption, as well as their inequality. We calibrate the model to income, consumption and equality data. The projection of actual trends should provide an overview of potential future debt and its sustainability in the context of population ageing.

As a general consideration, individual models are kept as simple as possible to allow for a clear understanding of the mechanisms at work and to limit the overall number of parameters. We have made most of our model's parameters time dependent to account for their dynamic trend. These time variation parameters are calibrated by using datasets from at least two recent years. This study has been implemented on Matlab.

2.1 Demographics dynamics

2.2 An analytical formulation

The demographic structure of a population N(t) is mainly driven by three dynamics, namely: births, deaths and migration.

$$\frac{dN(t)}{dt} = births(t) - deaths(t) + migration(t) \tag{2.1}$$

The natural growth (excluding migration) of an age specific population $n(t \geq 0, a \geq 0)$ composed of females $f(t,a)$ and males $m(t,a)$ can be modelled as :

$$\frac{df(t,a)}{f(t,a)} = -h_f(t,a)dt$$
$$\frac{dm(t,a)}{m(t,a)} = -h_m(t,a)dt$$

where the hazard functions $h_f(t,a)$ and $h_m(t,a)$ are the age-specific mortality rates at time t and $n(t,a) = f(t,a) + m(t,a)$. Developing $df(t,a)$ gives:

$$\begin{aligned} df(t,a) &= f(t+dt, a+da) - f(t,a) \\ &= f(t+dt, a+da) - f(t+dt, a) + f(t+dt, a) - f(t,a) \\ &= \frac{\partial f(t,a)}{\partial a} da + \frac{\partial f(t,a)}{\partial t} dt \end{aligned}$$

With $da = dt$ both representing small time variations, we can rewrite the two previous equations in a differential form, also known as the Sharpe - Lotka - McKendrick differential equation :

$$\begin{aligned} \frac{\partial f(t,a)}{\partial a} da + \frac{\partial f(t,a)}{\partial t} dt &= -h_f(t,a) n(t,a) dt \\ \frac{\partial m(t,a)}{\partial a} da + \frac{\partial m(t,a)}{\partial t} dt &= -h_m(t,a) n(t,a) dt \end{aligned}$$

with the initial conditions set by the male and female age specific distributions at $t = 0$ and newborns as a boundary condition:

$$\begin{aligned} f(0,a) &= f_0(a) \\ m(0,a) &= m_0(a) \\ f(t,0) &= \int_0^\infty \frac{1}{s(t)} r(t) d(t,a) f(t,a) da \\ m(t,0) &= \int_0^\infty s(t) r(t) d(t,a) f(t,a) da \end{aligned}$$

where $r(t)$ is the total fertility rate, $d(t,a)$ is the normalized age specific fertility distribution and $s(t)$ is the population average sex ratio of male to female at birth. We will further discuss these distributions in the subsequent sections.

2.3 Numerical implementation and calibration

To solve the demographic growth model proposed in (1), we use a forward Euler algorithm with $da = dt = 1\,year$ to match the time periods of most of the calibration data. At every simulation step, the demographic model computes the variables in (1) for every age cohort as well as for both sexes. The subsections below explain how these variables and distributions are modelled, calibrated and projected over time.

2.3.1 From fertility to births

Total fertility rate

The total fertility rate (TFR) of a population is a synthetic index obtained by summing the weighted birth contribution of every women's age group during a given period, usually a

calendar year:

$$tfr = \sum_{a=0}^{maxage} asfr(a) = tfr \sum_{a=0}^{maxage} nasf(a)$$

Where $asfr(a)$ is the age specific fertility rate of women at age a and $nasf$ is the normalized age specific fertility distribution, $\sum_{a=0}^{maxage} nasf(a) = 1$. We can use the low and stable time variability of this distribution to approximate annual births. Under this model, total births $B(t)$ of a population is given by:

$$B(t) = tfr(t) \sum_{a=0}^{maxage} nasf(t,a) f(t,a) \tag{2.2}$$

where $f(t,a)$ is the female population of age a at time t. The NASF distribution reflects the age specific socio-economic behaviour of a population and tends to have an overall slow and stable variation over time. The TFR component of (2) tends to show larger variations and is one of the main sources of error in population forecasting (United Nations 2013). To account for this uncertainty, we have set different fertility scenarios and accordingly, have modelled the TFR as a random variable. We use Monte-Carlo sampling to simulate a large number of outcomes. The results are presented as a probability distribution to take into account this uncertainty.

Case 1 : The TFR follows a linear trend

In the late stage of the demographic transition model, fertility rates show little variation and might either slightly increase or continue to decrease (Alkema et al. 2011). Under this scenario, we model the TFR variation over time as an Arithmetic Brownian Motion (ABM):

$$dF_t = \mu_f dt + \sigma_f dW_t$$

Where μ_f is the linear trend and σ_f the variability of the Brownian Motion process W_t.

Parameters estimation of the Arithmetic Brownian Motion process

Drift μ_f and diffusion σ_f parameters are calibrated to TFR data from phase III of the fertility transition model. This correspond to the period after the 1980s for most advanced economies. Parameter estimation is done numerically by finding their maximum Likelihood estimator:

$$\mu_f = \frac{1}{N\Delta t} \sum_{n=0}^{N-1} F(t_{n+1}) - F(t_n)$$

$$\sigma_f^2 = \frac{1}{(N-1)} \sum_{n=0}^{N-1} \Big(F(t_{n+1}) - F(t_n) - \mu_f \Delta t \Big)^2$$

where N is the number of time steps Δt of the TFR time-series.

Case 2 : The TFR mean reverts around a stable value

In this case, we suppose that the TFR mean reverts around a long-term average. Demographers and major institutions tend to choose the replacement rate of 2.1 children per woman as the long-term rate. However, empirical data shows that total fertility rates are stabilizing at sub-fertility levels in most advanced economies. To simulate these potential outcomes, we model the total fertility rate as an Ornstein–Uhlenbeck process:

$$dF_t = \theta_f(\mu_f - F_t)dt + \sigma_f dW_t$$

Where θ_f is the speed of reversion, μ_f is the long term mean level and σ_f is the volatility factor. This model has the advantage of having several closed form formulas. The solution of the Ornstein–Uhlenbeck SDE is:

$$F_t = e^{-\theta_f t}F_0 + (1 - e^{-\theta_f t})\mu_f + \sigma_f \int_0^t e^{-\theta_f(t-s)}dW_s$$

The numerical simulation is implemented with (Gillespie 1996):

$$F_t = e^{-\theta_f \Delta t}F_{t-1} + (1 - e^{-\theta_f \Delta t})\mu_f + \sigma_f \sqrt{\frac{1 - e^{-2\theta_f \Delta t}}{2\theta_f}}dW_t$$

Parameter estimation of the Ornstein–Uhlenbeck process

Similarly to the ABM process, we estimate numerically the Ornstein–Uhlenbeck parameters from recent TFR time series. We use the maximum likelihood method, as described in (Valdivieso et al. 2009). By setting $\hat{\sigma}_f^2 = \sigma_f^2(1 - e^{-\theta_f \Delta t})/2\theta_f$, the log-likelihood function and its partial derivative are given by :

$$\mathcal{L}(\mu_f, \theta_f, \hat{\sigma}_f) = -\frac{n}{2}\ln(2\pi) - n\ln(\hat{\sigma}_f) - \frac{1}{2\hat{\sigma}_f^2}\sum_{i=1}^{N}\left[F_i - F_{i-1}e^{-\theta_f \Delta t} - \mu_f(1 - e^{-\theta_f \Delta t})\right]^2$$

$$\frac{\partial \mathcal{L}(\mu_f, \theta_f, \hat{\sigma}_f)}{\partial \mu_f} = \frac{1}{\hat{\sigma}_f}\sum_{i=1}^{N}\left[F_i - F_{i-1}e^{-\theta_f \Delta t} - \mu_f(1 - e^{-\theta_f \Delta t})\right]$$

$$\frac{\partial \mathcal{L}(\mu_f, \theta_f, \hat{\sigma}_f)}{\partial \theta_f} = -\frac{e^{-\theta_f \Delta t}\Delta t}{\hat{\sigma}_f^2}\sum_{i=1}^{N}\left[(F_i - \mu_f)(F_{i-1} - \mu_f) - e^{-\theta_f \Delta t} - e^{-\theta_f \Delta t}(F_{i-1} - \mu_f)^2\right]$$

$$\frac{\partial \mathcal{L}(\mu_f, \theta_f, \hat{\sigma}_f)}{\partial \sigma_f} = \frac{n}{\hat{\sigma}_f} - \frac{1}{\hat{\sigma}_f^3}\sum_{i=1}^{N}\left[F_i - \mu_f - e^{-\theta_f \Delta t}(F_{i-1} - \mu_f)\right]^2$$

Log-likelihood maximum is obtained by setting these partial derivatives to zero, with $F_x = \sum_i^N F_{i-1}$, $F_y = \sum_i^N F_i$, $F_{xx} = \sum_i^N F_{i-1}^2$, $F_{yy} = \sum_i^N F_i^2$ and $F_{xy} = \sum_i^N F_{i-1}F_i$:

$$\mu_f = \frac{\sum_{i=1}^N F_i - F_{i-1}e^{-\theta_f \Delta t}}{n(1 - e^{-\theta_f \Delta t})} = \frac{F_y - e^{-\theta_f \Delta t}F_x}{n(1 - e^{-\theta_f \Delta t})}$$

$$\theta_f = \frac{1}{\Delta t}\ln\frac{\sum_{i=1}^N (F_i - \mu_f)(F_{i-1} - \mu_f)}{\sum_{i=1}^N (F_{i-1} - \mu_f)^2} = -\frac{1}{\Delta t}\ln\left[\frac{-\mu_f(F_x + F_y) + F_{xy} + n\mu_f^2}{F_{xx} - 2\mu_f F_x + n\mu_f^2}\right]$$

After computing μ_f and then θ_f, we get σ_f with :

$$\hat{\sigma}_f^2 = \frac{1}{n}\sum_{i=1}^{N}\left[(F_i - \mu_f - e^{-\theta_f \Delta t}F_{i-1} - \mu_f)\right]^2$$

$$\sigma_f^2 = \frac{2\theta_f}{n(1-e^{-2\theta_f \Delta t})}\left[e^{-2\theta_f \Delta t}F_{xx} + F_{yy} - e^{-\theta_f \Delta t}F_{xy} - 2\mu_f(1-e^{-\theta_f \Delta t})(F_y - e^{-\theta_f \Delta t}F_x) + n\mu_f^2(1-e^{-\theta_f \Delta t})^2\right]$$

Age Specific Fertility Model

Advanced fertility models have been reported in the literature to account for highly asymmetrical Age Specific Fertility (ASF) distributions. These uneven distributions usually occur during a demographic transition or in populations with highly inhomogeneous fertility behaviour. However, since the ASF distribution of all countries in this study have smooth edges, we use the difference between two logistic functions to fit their distribution. The main advantages of using such a simple model are the relatively low number of parameters (two for each side) and the fact that all these parameters have a meaningful interpretation:

$$nasf(t,a,\sigma_l(t),\mu_l(t),\sigma_r(t),\mu_r(t)) = \frac{\frac{1}{1+e^{-\sigma_l(t)(a-\mu_l(t))}} - \frac{1}{1+e^{-\sigma_r(t)(a-\mu_r(t))}}}{\sum_{a=0}^{maxage}\left(\frac{1}{1+e^{-\sigma_l(t)(a-\mu_l(t))}} - \frac{1}{1+e^{-\sigma_r(t)(a-\mu_r(t))}}\right)}$$

$$\sigma_l(t) = \sigma_{l_0} + \sigma_{l_t}t$$

$$\mu_l(t) = \mu_{l_0} + \mu_{l_t}t$$

$$\sigma_r(t) = \sigma_{r_0} + \sigma_{r_t}t$$

$$\mu_r(t) = \mu_{r_0} + \mu_{r_t}t$$

Parameters $\mu_l(t)$ and $\sigma_l(t)$ set the midpoint and steepness of the left side of the distribution, while $\mu_r(t)$ and $\sigma_r(t)$ set the right side of the distribution. These parameters are time-dependent to account for variations in age specific fertility behaviour.

Parameter estimation of the normalized ASF distribution

Fertility parameters $\mu_l(t)$, $\sigma_l(t)$, $\mu_r(t)$ and $\sigma_r(t)$ are modelled as linear over time with initial values μ_{l0}, σ_{l0}, μ_{r0}, σ_{r0} and rates of change μ_{lt}, σ_{lt}, μ_{rt} and σ_{rt}. We estimate these parameters by numerically minimizing the sum of squares:

$$C_{fertility}(\mu_{l0}(t),\sigma_{l0}(t),\mu_{r0}(t),\sigma_{r0}(t),\mu_{lt}(t),\sigma_{lt}(t),\mu_{rt}(t),\sigma_{rt}(t)) =$$
$$min\Big\{\sum_{t=0}^{maxcal}\sum_{a=0}^{maxage}\left(nasf_{data}(t,a) - nasf(a,\sigma_l(t),\mu_l(t),\sigma_r(t),\mu_r(t)\right)^2\Big\}$$

We use the period age specific fertility data compiled by the Human Fertility Database, a joint project of major public and academic research centres in demographics. Parameters

μ_{l0}, σ_{l0}, μ_{r0}, σ_{r0}, μ_{lt}, σ_{lt}, μ_{rt} and σ_{rt} are estimated for every country. These rates are used to project the normalized age specific fertility distribution.

The average age of mothers at birth has been increasing in most advanced economies (see Figure B2). This phenomenon has mainly been explained by increasing female education and a higher female labour participation rate. In our model, this is translated into a linear increase of $\mu_l(t)$ and $\mu_r(t)$ of about one to two months per year. With female fertility rapidly decreasing after the mid-thirties, there is a biological hard limit to how far the fertility distribution can shift to higher ages. Given this, we have introduced a saturation function to limit the maximum value of $\mu_l(t)$ and $\mu_r(t)$.

Birth projection

The total birth of a population at time t is given by (2). Once the fertility model parameters are extracted, we can use their linear trend parameters $\mu_{lt}(t)$, $\sigma_{lt}(t)$, $\mu_{rt}(t)$ and $\sigma_{rt}(t)$ for future projections. The normalized age specific fertility distribution (NASF) is constructed for every simulation time step and then multiplied by the projected TFR to get the age specific fertility rate. The total births of a given year is then given by (2). In all the economies studied in this work, sex ratio at birth $s(t)$ tends to be stable over time, between 1.03 to 1.05 male to female. At every simulation time step, female and male newborns are given by $f(t,0) = B(t)/(1+s(t))$ and $m(t,0) = B(t)s(t)/(1+s(t))$.

2.3.2 From hazard rates to deaths

First mortality model : Gompertz-Makeham

There is a large volume of literature on advanced mortality models. Since all the populations in this study have very low infant and child mortality rates, the Gompertz-Makeham model is a good compromise between accuracy, simplicity and the number of parameters. We model these parameters as a linear function over time to account for the increase in life expectancy. Under the Makeham model, the probability of dying within one year of an age a at time t is given by the hazard rate function:

$$
\begin{aligned}
h_{m,f}(t,a) &= \alpha_{m,f}(t)e^{\beta_{m,f}(t)a} + \gamma_{m,f}(t) \\
\alpha_{m,f}(t) &= \alpha_{t_{m,f}}t + \alpha_{0_{m,f}} \\
\beta_{m,f}(t) &= \beta_{t_{m,f}}t + \beta_{0_{m,f}} \\
\gamma_{m,f}(t) &= \gamma_{t_{m,f}}t + \gamma_{0_{m,f}}
\end{aligned}
$$

The first component (Gompertz component) of the hazard function is age dependent. The decrease in $\beta_{m,f}$ is the main contributor to the increase in life expectancy after the second World War. The second component (Makeham) is age independent and was the main contributor to the increase in life expectancy before the 1950s. Parameters $\alpha_{m,f}(t)$, $\beta_{m,f}(t)$

and$\gamma_{m,f}(t)$ are modelled as linear over time to account for the decrease in mortality hazard rates. Under this model, the survival function $v(a)$ at age a is:

$$v(t,a) = e^{-\gamma_{m,f}(t)a + \frac{\alpha_{m,f}(t)}{\beta_{m,f}(t)}\left(1-e^{\beta_{m,f}(t)a}\right)}$$

The models quantile function can be expressed in a closed form by using the (Lambert W_0) function:

$$Q_{m,f}(t,a) = \frac{1}{\gamma_{m,f}(t)}\left(\frac{\alpha_{m,f}(t)}{\beta_{m,f}(t)} - \ln(1-a)\right) - \frac{1}{\beta_{m,f}(t)} W_0\left(\frac{\alpha_{m,f}(t) e^{\frac{\alpha_{m,f}(t)}{\gamma_{m,f}(t)}} (1-a)^{\frac{\beta_{m,f}(t)}{\gamma_{m,f}(t)}}}{\gamma_{m,f}(t)}\right)$$

Parameter estimation of the exponential plus constant mortality model

For every calendar year t, the calibration function estimates the parameters α, β and γ by fitting the model to the age specific hazard rates qx of the life table. In actuarial life tables, qx is the probability of death between ages a and $a+t$, in our case $t = 1$ and a varies between 0 and 110 years of age. Parameter estimation is done for all counties in this study and for both sexes. We used the life table data compiled by the Human Mortality Database.

Curve fitting is done by least square regression. Nonetheless, the exponential plus constant function must first be linearised, integrating both sides of the hazard function $h(a) = \alpha e^{\beta a} + \gamma$ between a_1 and a_2 gives:

$$\begin{aligned}
\int_{a_1}^{a_k} h(a)da &= \frac{\alpha}{\beta}(e^{\beta a_k} - e^{\beta a_1}) + \gamma(a_2 - a_1) \\
&= \frac{1}{\beta}(h(a_k) - h(a_1)) + \gamma(a_k - a_1) \\
h(a_k) - h(a_1) &= -\beta\gamma(a_k - a_1) + \beta\int_{a_1}^{a_k} h(a)da \\
&= -\beta\gamma(a_k - a_1) + Q_{a_k} + \epsilon_a
\end{aligned}$$

where Q_{a_k} is a trapezoid approximation of the right hand side integral, we estimate it from the qx data to be fitted with $Q_{a_k} = Q_{a_{k-1}} + (h(a_k) - h(a_{k-1}))/(2(a_k - a_{k-1}))$. The simple linear regression is done by minimizing the sum of quadratic errors:

$$min\Big\{\sum_{a=1}^{a_{max}} \epsilon_a^2\Big\} = min\Big\{\sum_{a=1}^{a_{max}} \big(h(a_k) - h(a_1) + \beta\gamma(a_k - a_1) - \beta Q_{a_k}\big)^2\Big\}$$

Parameter α, β and γ are obtained by subsequently solving the two systems of equations as illustrated below:

$$\begin{bmatrix}\beta\gamma \\ \beta\end{bmatrix} = \begin{bmatrix}\sum_{a=1}^{a_{max}}(a_k - a_1)^2 & \sum_{a=1}^{a_{max}}(a_k - a_1)Q_k \\ \sum_{a=1}^{a_{max}}(a_k - a_1)Q_k & \sum_{a=1}^{a_{max}} Q_k^2\end{bmatrix}^{-1} \begin{bmatrix}\sum_{a=1}^{a_{max}}(h(a_k) - h(a_1))(a_k - a_1) \\ \sum_{a=1}^{a_{max}}(h(a_k) - h(a_1))Q_k\end{bmatrix}$$

$$\begin{bmatrix}\gamma \\ \alpha\end{bmatrix} = \begin{bmatrix}n & \sum_{a=1}^{a_{max}} e^{\beta a_k} \\ \sum_{a=1}^{a_{max}} e^{\beta a_k} & \sum_{a=1}^{a_{max}} e^{2\beta a_k}\end{bmatrix}^{-1} \begin{bmatrix}\sum_{a=1}^{a_{max}} h(a_k) \\ \sum_{a=1}^{a_{max}} e^{\beta a_k} h(a_k)\end{bmatrix}$$

The accuracy of the exponential plus constant regression is assessed in the results section. After obtaining the parameters α, β and γ for every year, we estimate the temporal rate of change of these parameters to get $\alpha_{t_{m,f}}$, $\alpha_{0_{m,f}}$, $\beta_{t_{m,f}}$, $\beta_{0_{m,f}}$, $\gamma_{t_{m,f}}$ and $\gamma_{0_{m,f}}$. These parameter are used for mortality projection. As we will discuss later, this model tends to over-estimate the mortality hazard rate at late ages in what is known as late-life mortality deceleration (Bebbington et al. 2014). This effect tends to push the strictly positive γ to negative values in the linear regression. To overcome this, we parametrized the algorithm to regress up to an age below the maximum age in the dataset (110 years of age).

Second mortality model : logistic increase

We have implemented a second mortality model. In this case the mortality hazard function is modelled by the left tail of a logistic function. This model allows us to account for late-life mortality deceleration. Above 80 to 90 years of age, hazard rate growth tends to decelerate from its log-linear growth, as modelled in the Gompertz-Makeham model. In this second model the mortality hazard function is as following:

$$\begin{aligned} h_{m,f}(t,a) &= \frac{1}{1+e^{-\alpha_{m,f}(t)(a-\beta_{m,f}(t))}} + \gamma_{m,f}(t) \\ \alpha_{m,f}(t) &= \alpha_{t_{m,f}} t + \alpha_{0_{m,f}} \\ \beta_{m,f}(t) &= \beta_{t_{m,f}} t + \beta_{0_{m,f}} \\ \gamma_{m,f}(t) &= \gamma_{t_{m,f}} t + \gamma_{0_{m,f}} \end{aligned}$$

In the results section we calibrate both models to the same life table data and compare their accuracy.

Parameters estimation of the logistic mortality model

Using the same data from the Human Mortality database, we estimate the parameters of this non-linear mortality model numerically, by minimizing the square error function as:

$$\begin{aligned} &C_{mortality}(\alpha_t(t), \alpha_0(t), \beta_t(t), \beta_0(t), \gamma_t(t), \gamma_0(t)) = \\ &\qquad min\Big\{ \sum_{t=ical}^{maxcal} \sum_{a=0}^{maxage} \big(h_{data}(t,a) - h(t,a,\alpha_t(t), \alpha_0(t), \beta_t(t), \beta_0(t), \gamma_t(t), \gamma_0(t))\big)^2 \Big\} \end{aligned}$$

Where h_{data} is the female or male hazard rate data (qx) and h the logistic model as outlined above. We calibrate the model to the latest available years to account for contemporary trends only.

Mortality projections

At every simulation time step t, new female and male hazard functions $h_{m,f}(t)$ are computed from $\alpha_{m,f}(t)$, $\beta_{m,f}(t)$ and $\gamma_{m,f}(t)$. In our discrete time Euler algorithm, the number

of male and females deaths $D(t,a)_{m,f}$ at time t for the age group a is given by:

$$d(t,a)_{m,f} = \{m,f\}(t,a)h_{m,f}(t,a)$$

where $t(t,a)$ and $m(t,a)$ are the female and male populations at time t and age a. The total number of deaths at time t is $D(t)_{m,f} = \sum_{i=a}^{a_{max}} d_{m,f}(t,a)$.

2.3.3 Immigration and emigration

Migration models

A visual observation of immigration and emigration age specific distributions in the studied countries shows three common regions. From birth to around 16 years of age there is a relatively small and slightly decreasing amount of positive net migration. These are primarily the children of migrant workers. From 16 years of age to the early thirties, net migration grows is rapidly and tends to include migrant workers and foreign students. After plateauing in the thirties, net migration figures tend to decrease slowly with age. This region is dominated by migrant workers (US: Johnson et al. 2013, EU: Raymer et al. 2011).

We model the first region with a constant level K_0. The second region is modelled by a rising logistic function (midpoint: μ_u, steepness: σ_u, amplitude: K_u) and we use a falling one for the third region (midpoint: μ_v, steepness: σ_v, amplitude: K_v). The age specific distribution is normalized to separate the total migration term $G(t)$ from the age specific distribution $g(t)$:

$$l(t,a) = G(t)g(t,a) / \sum_{a=0}^{agemax} g(t,a)$$

$$g(t,a) = G(t)\big(K_0 + K_u \frac{1}{1+e^{-\sigma_u(a-\mu_u)}} - K_v \frac{1}{1+e^{-\sigma_v(a-\mu_v)}}\big)$$

Calibration

Immigration and emigration flows are annually reported by most of demographics institutes. Table A summarizes the sources of calibration data. Parameter estimation is done by minimizing the square error between migration data and the model.

$$C_{migration}(a, K_0, K_u, \sigma_u, \mu_u, K_v, \sigma_v, \mu_v) = min\Big\{ \sum_{a=0}^{maxage} \big(g_{data}(a) - g(a, K_0, K_u, \sigma_u, \mu_u, K_v, \sigma_v, \mu_v)\big)^2 \Big\}$$

Since this distribution shows a significant volatility over time, we didn't include a time variation factor to its parameters. We will instead test several migration scenarios.

Migration projections

Under the migration model used in this work, the normalized age specific distribution is kept constant and only the total number of migrants changes over time. We assume immigration and emigration as random processes, ACF and PACF analysis shows inconclusive results, depending on the observed period and country. We thus implement two random process to represent the different migration policy. We use the Arithmetic Brownian Motion and the Ornstein–Uhlenbeck models described in the TFR section above to model a linearly increasing or a stabilizing migration policy. We estimate the total immigration and emigration parameters with the same methods described in the Total Fertility Rate section. Future projections are also based on the TFR numerical functions.

2.3.4 Population dynamics model

The main initial conditions of the demographics model are the initial male and female age specific population distributions. The total fertility rates and total net migration expectations are used as boundary conditions. Monte-Carlo simulations are used to account for the variability of these parameters.

At the initialization phase, we calibrate the fertility, mortality and migration models to the population in focus. For every simulation step representing one year, we use the calibrated parameters to compute the mortality rates, total immigration and emigration, as well as the number of newborns during that year. Finally, we simulate the effect of a one year forward timestep by shifting age groups by one year. Every age group is multiplied by its survival rate (1 - mortality rate) and the age specific migration is added. Newborns of the previous time step are split between males and females and set as a boundary condition at $age = 0$.

Algorithm 1 Algorithm 1: Demographics Dynamics Model

for t = 1 **to** tMax -1 **do**
  $newborns(t) = tfr(t) \sum_{a=1}^{aMax} nasf(t, a, \sigma_l, \mu_l, \sigma_r, \mu_r) f(t, a)$
  $mMortR = h(\alpha_{m_t}(t), \alpha_{m_0}(t), \beta_{m_t}(t), \beta_{m_0}(t), \gamma_{m_t}(t), \gamma_{m_0}(t))$
  $fMortR = h(\alpha_{f_t}(t), \alpha_{f_0}(t), \beta_{f_t}(t), \beta_{f_0}(t), \gamma_{f_t}(t), \gamma_{f_0}(t))$
  $netMig = h(\alpha_{f_t}(t), \alpha_{f_0}(t), \beta_{f_t}(t), \beta_{f_0}(t), \gamma_{f_t}(t), \gamma_{f_0}(t))$
  for a = ageMax **to** 2 **do**
    $m(t+1, a) = m(t, a-1)(1 - mMortR(a-1)) + mImm(a-1) - mEmi(a-1)$
    $f(t+1, a) = f(t, a-1)(1 - fMortR(a-1)) + fImm(a-1) - fEmi(a-1)$
  end for
  m(t+1,0) = $\frac{1}{(s+1)}$ newborns(t)
  f(t+1,0) = $\frac{s}{(s+1)}$ newborns(t)
end for

2.4 Housing Market and Population structure

We aim to understand the influence of the future population structure on the housing market, as one of the main vectors of money supply and economic growth. Instead of focusing on price dynamics, we look to the underlying supply and demand. Our focus is on the ratio between first time buyers and old age market leavers. In most advanced economies first-time buyers represent a considerable part of the total housing demand, while old age market leavers still represent a small but increasing fraction of the housing supply. The balance between these two markets forces is expected to change significantly. We hypothesise that an increasing number of old age market leavers will face a slowly increasing or even a decreasing number of potential first time buyers. In the following sections, we further explain how we model these two market forces.

2.4.1 First-time buyers

The age specific first-time buyers distribution tends to exhibit a rapid increase in the late twenties and a slower decrease through the thirties and early forties. We model this distribution with the difference of two logistic functions. The mid-point parameter μ_g and steepness σ_g set the left side of the distribution, while μ_h and σ_h set the right side of the first-time buyers distribution. These parameters have a linear time dependency to model its temporal variation.

$$
\begin{aligned}
ftb(t, a, \sigma_g, \mu_g, \sigma_h, \mu_h) &= FTB(t) nftb(t, a, \sigma_g, \mu_g, \sigma_h, \mu_h) \\
nftb(t, a, \sigma_g, \mu_g, \sigma_h, \mu_h) &= \frac{\frac{1}{1+e^{-\sigma_g(a-\mu_g)}} - \frac{1}{1+e^{-\sigma_h(a-\mu_h)}}}{\sum_{a=0}^{maxage} \frac{1}{1+e^{-\sigma_g(a-\mu_g)}} - \frac{1}{1+e^{-\sigma_h(a-\mu_h)}}} \\
\sigma_g &= \sigma_{g_t} t + \sigma_{g_0} \\
\mu_g &= \mu_{g_t} t + \mu_{g_0} \\
\sigma_h &= \sigma_{h_t} t + \sigma_{h_0} \\
\mu_h &= \mu_{h_t} t + \mu_{h_0}
\end{aligned}
$$

Where $FTB(t)$ is the number of first time buyers at time t and $ntbd$ is the normalised age specific first time buyers distribution. We define the potential first time buyers population $pftb(t)$ as the dot product between the age specific population distribution and the age specific FTB distribution.

$$
pftb(t) = \sum_{a=0}^{maxage} n(t, a) nftb\big(t, a, \sigma_g(t), \mu_g(t), \sigma_h(t), \mu_h(t)\big)
$$

Calibration

We were not able to compile age specific first-time buyer data across all ages and years in focus. To overcome this limitation, we linearly interpolated unavailable data or approximated it with data from countries with similar socio-economical conditions. Calibration is done by minimizing the error square function:

$$C_{ftbd}(t, a, \sigma_{g_t}, \sigma_{g_0}, \mu_{g_t}, \mu_{g_0}, \sigma_{h_t}, \sigma_{h_0}, \mu_{h_t}, \mu_{h_0} = min\Big\{ \sum_{t=0}^{maxage} \sum_{a=0}^{maxage} \big(ftbd_{data}(t,a) - ftbd(t, a, \sigma_{g_t}, \sigma_{g_0}, \mu_{g_t}, \mu_{g_0}, \sigma_{h_t}, \sigma_{h_0}, \mu_{h_t}, \mu_{h_0}\big)^2 \Big\}$$

First time buyer projections

Once calibration is completed, we use the linear variation time factors to approximate the potential future first-time buyer distribution. We expect positive μ_{g_t} and μ_{h_t} to reflect the overall increase in first-time buyer average age.

2.4.2 Old age market leavers and consequent selling rates

This supply market segment includes homeowners who leave the housing market due to old age conditions. This behaviour could be attributed to retirement in another country, entering specialised care, covering a home equity line of credit or ultimately death and probate. Age specific home selling rates in the US has been reported by Myers et al. 2008. The distribution exhibits a stable selling rate until about 55 years of age and increases exponentially after that. This behaviour is expected, since old age related home selling rates are expected to be proportional to old age healthcare needs and to mortality hazard rates. To model this distribution, we use a model akin to the one utilised to model mortality hazard rates, with a constant term and an age dependent term. The function $l_{m,f}(t, a)$ represents the conditional probability of leaving the housing market in the following year due to an old age condition.

$$\begin{aligned} l_{m,f}(t,a) &= \epsilon_{m,f}(t) e^{\theta_{m,f}(t)a} + \lambda_{m,f}(t) \\ \epsilon_{m,f}(t) &= \epsilon_{t_{m,f}} t + \epsilon_{0_{m,f}} \\ \theta_{m,f}(t) &= \theta_{t_{m,f}} t + \theta_{0_{m,f}} \\ \lambda_{m,f}(t) &= \lambda_{t_{m,f}} t + \lambda_{0_{m,f}} \end{aligned}$$

The number of market leavers at time t is given by:

$$\begin{aligned} L_m(t) &= \sum_{a=0}^{agemax} m(t,a) l_m(t,a) \\ L_f(t) &= \sum_{a=0}^{agemax} f(t,a) l_f(t,a) \end{aligned}$$

Calibration and Extrapolation

Home selling rates due to old age conditions and probate is not a commonly reported data. We approximated it from Myers et al. 2008, by minimizing the sum of square differences. The high uncertainty around this data and its corresponding parameters is compensated by running a Monte-Carlo simulation to cover the range of possible outcomes.

2.4.3 Housing Market benchmark

At every simulation time step, we approximate the number of potential FTB $pftb(t)$ and the old age market leavers $(L_m(t) + L_f(t))$. A sharp increase of the ratio between these two market forces gives us a hypothetical benchmark of potential supply and demand.

2.5 A demographics based model of income and consumption

In this study we use the demographics model to better understand potential future debt dynamics in several advanced economies. To achieve this, we model income and consumption distributions across all age cohorts and across time. We calibrate these distributions to the available data and run Monte-Carlo simulation on several test cases. We then obtain a range of potential outcomes and analyse the evolution of potential debt and its distribution across ages and income quantile. We define the age - income joint distribution $i(t, a, p)$ where p is the income percentile. The age consumption joint distribution is similarly $c(t, a, p)$.

Under this model, a population is represented by a matrix of "bins". At every time step t, the size of the male and female matrix is the number of quantiles times the number of age cohorts. In this study, we use one year age cohorts and percentiles. The total number of bins n_{bins} is thus 111 times 100. The number of persons per bin for each of the male and female population is :

$$N_{bins_{(m,f)}}(t) = \frac{popm, f}{n_{bins}} = \frac{\sum_{a=0}^{maxage}\{m, f\}(t, a)}{n_{quantile} maxage}$$

2.5.1 Age specific income and consumption models

National Transfer Accounts

National Transfer Accounts (NTA) framework has been introduced as a way to measure how populations produce, consume, save and distribute resources at every age. A common methodology has been promoted by the UN. Teams in more than 60 countries are actively constructing these accountsUnited Nations 2013. By disaggregating flows with regard to age, NTA are expected to give a better understanding and forecasting of these

flows. In this study, we mainly use the compiled NTA age specific income and consumption as a way to understand debt dynamics. In the figure 1.3 we show the normalized age specific labour income and consumption in the U.S.. The support ratio for a given year is defined as the average life cycle labour income over the average life cycle consumption. The difference between labour income and consumption for every age group is defined as the life cycle deficit (shown in the Figure 2.1 below). In many developing countries Support Ratios are close to unity and life cycle deficits tend to be balanced. In advanced economies, support ratios have decreased significantly over the past three decades.

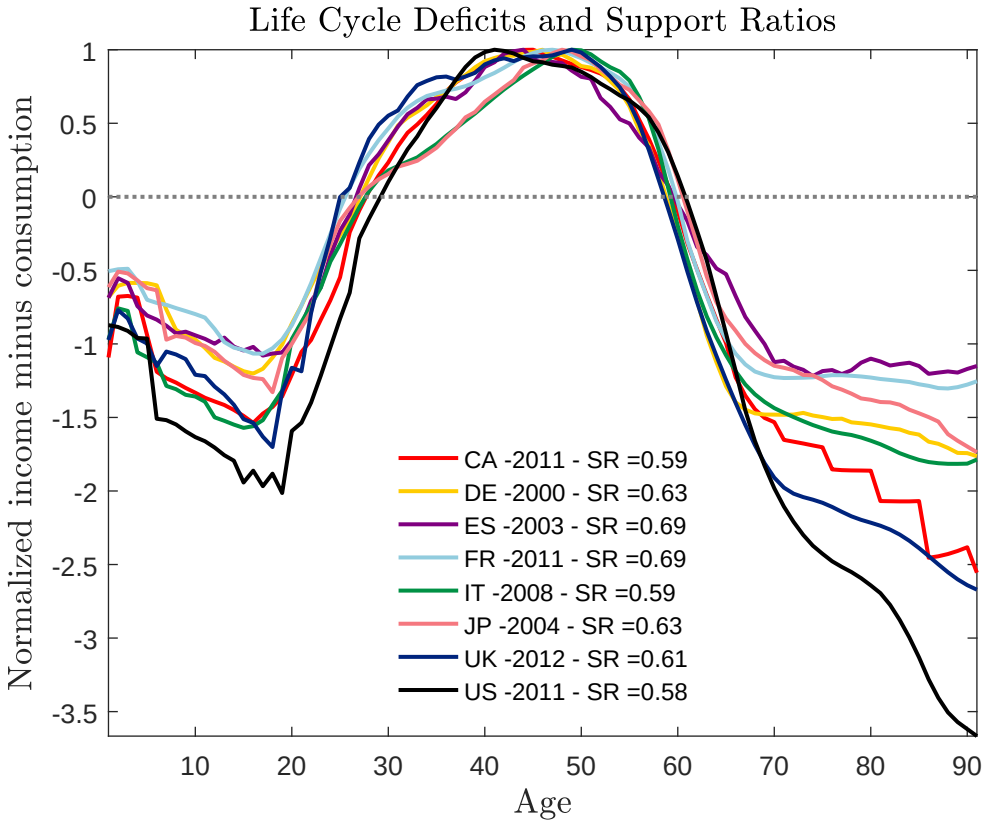


Figure 2.1: Life Cycle deficits in the studied countries.

Income model

The average age specific income distribution (Figure 1.3 and 3.12) increases across the twenties of age, saturates in the thirties and peaks in the mid forties. The income distribution decreases in the fifties and accelerates its fall in the sixties of age. We model the age specific rise and fall in income with two logistic functions of opposite polarity where

μ_i and μ_j are the left and right mid-points while σ_i and σ_j are the corresponding steepness:

$$
\begin{aligned}
i(t,a,\mu_i,\sigma_i,\mu_j,\sigma_j) &= I(t)i_n(a,\mu_i,\sigma_i,\mu_j,\sigma_j) \\
&= I(t)\frac{\frac{1}{1+e^{-\sigma_i(a-\mu_i)}} - \frac{1}{1+e^{-\sigma_j(a-\mu_j)}}}{\sum_{a=0}^{maxage}\frac{1}{1+e^{-\sigma_i(a-\mu_i)}} - \frac{1}{1+e^{-\sigma_j(a-\mu_j)}}} \\
\sigma_i &= \sigma_{i_t}t + \sigma_{i_0} \\
\mu_i &= \mu_{i_t}t + \mu_{i_0} \\
\sigma_j &= \sigma_{j_t}t + \sigma_{j_0} \\
\mu_j &= \mu_{j_t}t + \mu_{j_0}
\end{aligned}
$$

where $i_n(t,a,\mu_i,\sigma_i,\mu_j,\sigma_j)$ is the normalised age specific income at time t and $I(t)$ total income at t.

Consumption model

The average age specific consumption (Figure 1.3 and 3.12) increases slowly with age until pre-retirement (the fifties). After this point, it then tends to decrease with income in emerging economies and used to remain relatively constant in the majority of advanced economies. Yet, over the past few decades, old age consumption have seen a quasi-exponential growth with age in many advanced economies. This growth is mainly linked to healthcare and geriatric expenses. We model the early age specific consumption by its first order trend ($c_b + c_a a$).

$$
\begin{aligned}
c(t,a,c_{a_t},c_{b_t},\sigma_c,\mu_c) &= C(t)c_n(t,a,c_{a_t},c_{b_t},\sigma_c,\mu_c) \\
&= C(t)\Big(\frac{c_{b_t} + c_{a_t}a + 1/(1+e^{-\sigma_c(a-\mu_c)})}{\sum_{a=0}^{agemax} c_{b_t} + c_{a_t}a + 1/(1+e^{-\sigma_c(a-\mu_c)})}\Big) \\
c_{a_t} &= K_{c_a}c_{a_0}t \\
c_{b_t} &= K_{b_a}c_{b_0}t \\
\sigma_c &= \sigma_{c_t}t + \sigma_{c_0} \\
\mu_c &= \mu_{c_t}t + \mu_{c_0}
\end{aligned}
$$

where $c_n(t,a,c_a,c_{b_t},\sigma_c,\mu_c)$ is the normalized age specific consumption. Parameters c_{a_t}, c_{a_t}, σ_c and μ_c are all time dependent, to account for the time-variation of the age specific consumption.

Calibration and extrapolation

We calibrate our models to the age specific income and consumption data computed by NTA economists. Time variation factors (σ_{i_t}, μ_{i_t}, σ_{j_t}, μ_{j_t}, K_{c_a}, K_{b_a}, K_{σ_c} and K_{μ_c}) are computed, when the age specific data is available for at least two different years. Otherwise we use the time variation parameters of a country with similar socioeconomic conditions. Calibration is done by minimizing the error square function. Extrapolation of age specific income and consumption is completed by using the temporal rates of change obtained by the calibration function.

2.5.2 Income and consumption distribution models

One of the main novelties of this study is to combine age specific distributions with income and consumption inequality distributions. After finding the income $I(t,a)$ and consumption $C(t,a)$ of every age cohort we distribute it among a normalized percentile (frequency) distribution to obtain a joint income age distribution. Two quantile functions have been implemented in our inequality model. The first one is the simple Pareto distribution $P(\alpha)$. With only one parameter this distribution is simple and intuitive. In this case, income and consumption quantile functions for a given age group a are :

$$i(t,a,p,\alpha_{i_{t,a}}) = I(t,a)i_{p_n}(t,a,p,\alpha_{i_{t,a}})) = I(t,a)(1-p)^{-1/\alpha_{i_{t,a}}} \Big/ \sum_{p=q_i}^{q_f}(1-p)^{-1/\alpha_{i_{t,a}}}$$

$$c(t,a,p,\alpha_{c_{t,a}}) = C(t,a)c_{p_n}(t,a,p,\alpha_{c_{t,a}})) = C(t,a)(1-p)^{-1/\alpha_{c_{t,a}}} \Big/ \sum_{p=q_i}^{q_f}(1-p)^{-1/\alpha_{c_{t,a}}}$$

where $i_{p_n}(t,a,p,\alpha_{i_{t,a}}))$ and $c_{p_n}(t,a,p,\alpha_{c_{t,a}}))$ are the normalized income and consumption quantile functions. Parameters $\alpha_{i_{t,a}}$ and $\alpha_{c_{t,a}}$ are age and time dependent to account for the variability of income and consumption inequality across age and time.

The second quantile function implemented in the inequality model is the double Pareto-Lognormal distribution (dPLN) as presented in Reed et al. 2004. It has four parameters and allows better fittings at both tails. Its quantile function doesn't have a closed form, so we implemented it numerically. The cumulative distribution function of the dPLN distribution is :

$$F^{dPLN} = \Phi\left(\frac{\ln(a)-\nu}{\tau}\right) - \frac{1}{\alpha+\beta}\left[\beta a^{-\alpha}e^{\alpha\nu+\alpha^2\tau^2/2}\Phi\left(\frac{ln(a)-\nu-\alpha\tau^2}{\tau}\right) + \alpha x^{\beta}e^{-\beta\nu+\alpha^2\tau^2/2}\Phi^C\left(\frac{\ln(a)-\nu+\beta\tau^2}{\tau}\right)\right]$$

The dPLN distribution can be obtained as a geometric Brownian motion with log-normally distributed initial values and normally distributed final values.

Calibration

The dPLN parameters are estimated by minimizing the sum of square between the dataset and the dPLN model. Since these datasets tend to have larger age groups (5 to 10 years), one year age groups parameters are estimated by spline interpolation of the joint age-income distribution distribution. Similar interpolation is done for missing years in the calibration dataset.

2.5.3 Age specific income and consumption projection models

We combine the previously described demographics projection model with the age inequality distribution to model potential debt dynamics. Alike the housing model we use time steps and age cohorts of one year $\Delta a = \Delta t = 1$ year. We first extract calibration parameters and initialise population and age specific inequality distributions. For every time step of one year, we compute births, deaths, immigration and emigration for both sexes. We then compute the age specific income and consumption of this population and use the inequality model, to distribute income and consumption among every age group. The total fertility rate and net migration are modelled as random processes. Their parameters are calibrated to historical data and projected according to an ABM or OU process. We also include in the Monte-Carlo simulation some income, consumption and inequality variability to account for their potential future variation and the low quality of their calibration dataset.

3 Results

In this chapter, we further explain the models presented in the methodology chapter and assess their main performances. In the first section, we show the results related to parameter estimation and calibration of the fertility, mortality and migration sub-models.

Following this, we combine these three models to build a Bayesian population projection model. The accuracy of this model is assessed in Section Two. In Section Three, we use this demographic projection model to understand how projected changes in population structure could affect housing markets in many advanced economies. In Section Four we present the results related to the impact of population structure and income inequality on debt.

3.1 Demographics sub-models

3.1.1 Mortality

As detailedin the methodology section, our mortality hazard model implements two submodels. The one year conditional probability of death is modelled either as an exponential (Gompertz-Makeham model) or a logistic function. These two models mainly differ at late ages. where mortality hazard rates tend to decelerate. The exponential model is fitted with the closed form formulas as described in the methodology section. whereas the logistic model is fitted numerically by minimising the sum of square differences. We use life table data compiled by the Human Mortality Database for all the countries in this study (Canada, Germany, Spain, France, Italy, Japan, the United Kingdom and the United States). Parameter estimation is done for each sex separately. In the figure 3.1 below we show the mortality hazard rate data and its models for two separates years. Fitting residuals indicates that the logistic function fits better at late ages and considers a late age mortality deceleration. Residuals also illustrate that the logistic and exponential functions are unable to model infant and child mortality. However, the extremely low levels of infant and child mortality in the focused economies, significantly reduces the error related to early years mortality and increases the accuracy of these simple models.

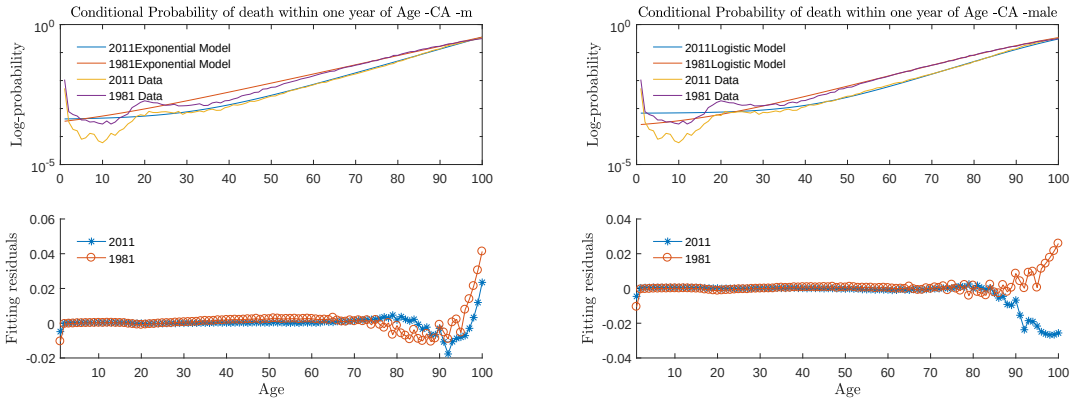


Figure 3.1: Mortality hazard rates and their fitted exponential model (left) and Logistic model (right) - Canadian Males

Projections of the mortality hazard rates is achieved by extrapolating the linear trend of these parameters. The variation over time of these parameters, $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ is shown in Figure 3.2 (males) and Figure B8 (females) for the exponential model $(h(t,a) = \alpha(t)e^{\beta(t)a} + \gamma(t))$ and logistic model $(h(t,a) = \frac{1}{1+e^{-\alpha(t)(a-\beta(t))}} + \gamma(t))$.

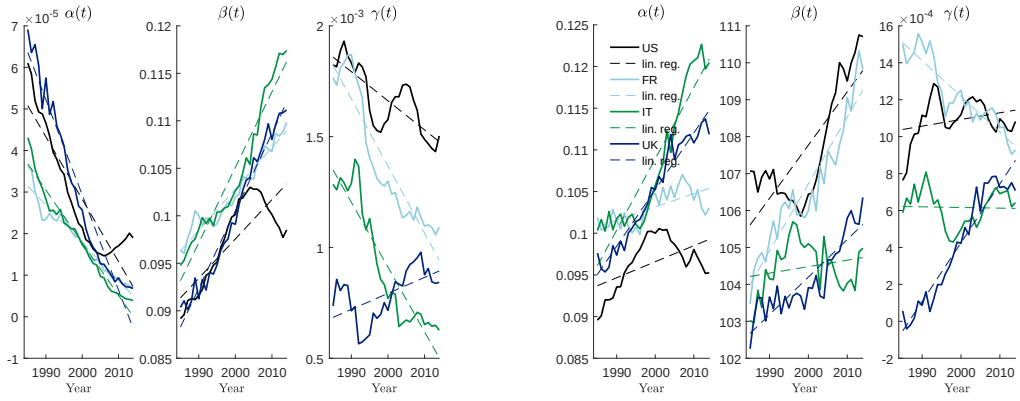


Figure 3.2: Exponential (left) and logistic (right) mortality hazard rate models parameters over time and their linear regression - Males

3.1.2 Fertility

Normalized age specific fertility rate

Fertility is one of the main components of population growth and is affected by many of its socioeconomic determinants. The normalized age specific component of the fertility rate tends to show slow and low variability. We modelled it as the difference of two logistic functions. Model parameters (midpoints and variability) are estimated by minimizing the sum of square difference. We fit the NASF model to the one year ASFR data compiled

by the Human Fertility Database. We show below the normalized age specific fertility of Germany and the Unites states, as well as the their fitted models. Figures for the other countries in this study are reported in the Figure B1.

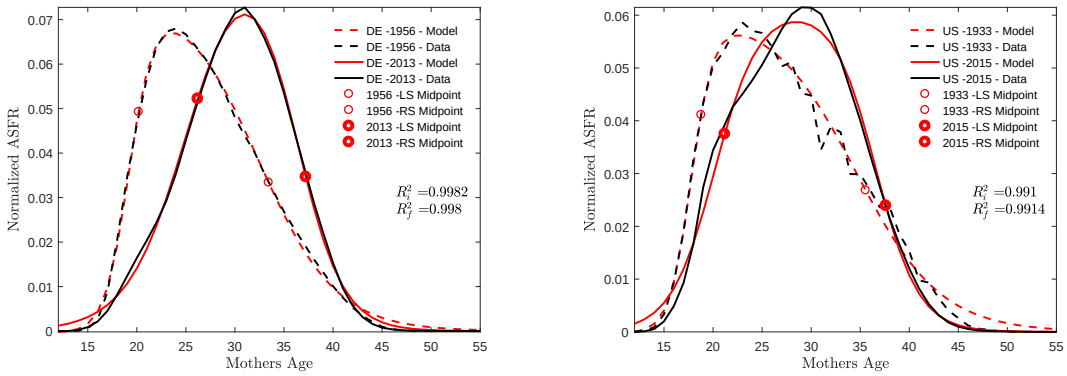


Figure 3.3: Normalized age specific fertility rates and their fitted models

The U.S. normalized age specific fertility model above has the lowest R-square among the countries in focus (See B3), this is attributed to the relatively high fertility in early twenties. Historical data shows that fitting accuracy for the U.S. increases with time, as the fertility curve gets more symmetrical.

Projections of the normalized age specific fertility distribution are obtained by extrapolating the linear trends of these parameters. Figure 3.4 below shows the fitted midpoint and variability parameters over the past 25 years. These parameters indicate a high level of linearity and reflect the increasing age of mothers at birth in advanced economies (Figure B2). A saturation function has been added to the mid-point parameters to reflect the biological limitations on fertility.

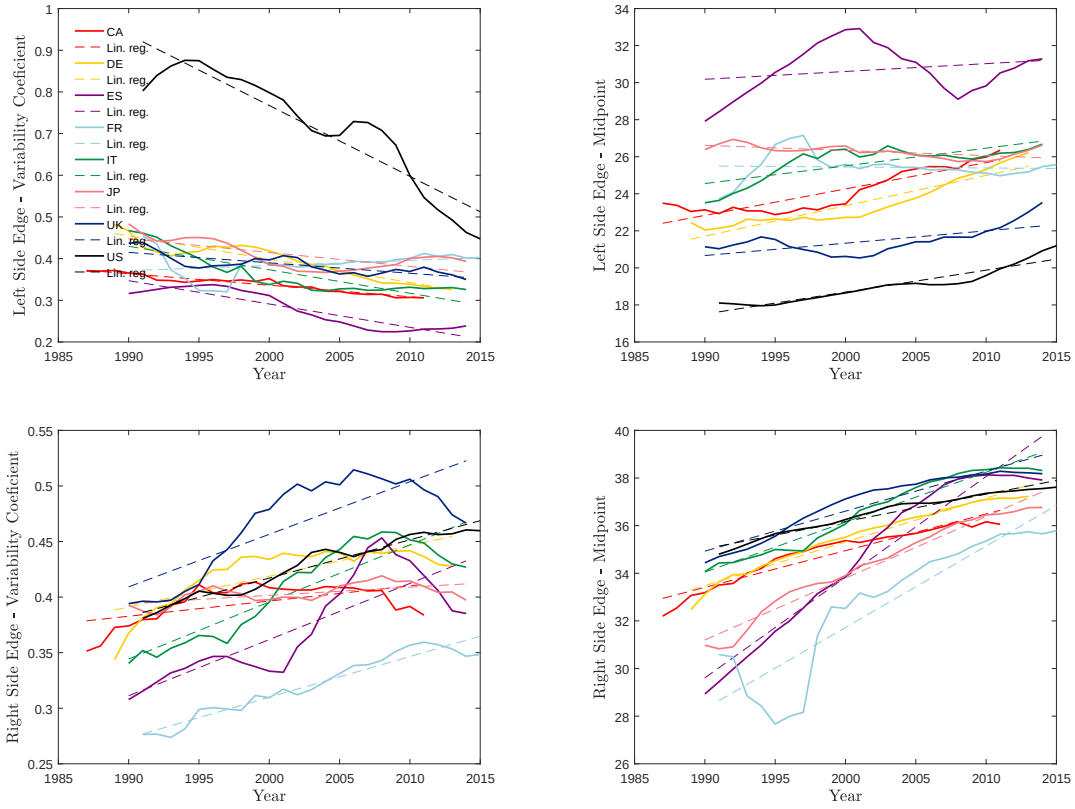


Figure 3.4: Normalized age specific fertility rates parameters over time and their linear regressions

Total Fertility Rates

Autocorrelation and partial autocorrelation analysis of Total Fertility Rates (TFR) in this study gives inconclusive results pertaining to their stationarity. To reflect this variation in fertility regimes, we implemented two random processes. The Arithmetic Brownian Motion (trend-stationary) can be used to project the past decrease in fertility. ABM parameters estimation is done with the technique presented in the methodology chapter. Figure 3.5 below shows past TFRs and some projected sample paths according to the ABM model. Parameter estimation is with data covering the last 30 years of the dataset. A key limitation of the ABM process to model fertility rates, is that it doesn't have a boundary at zero and thus allows negative values.

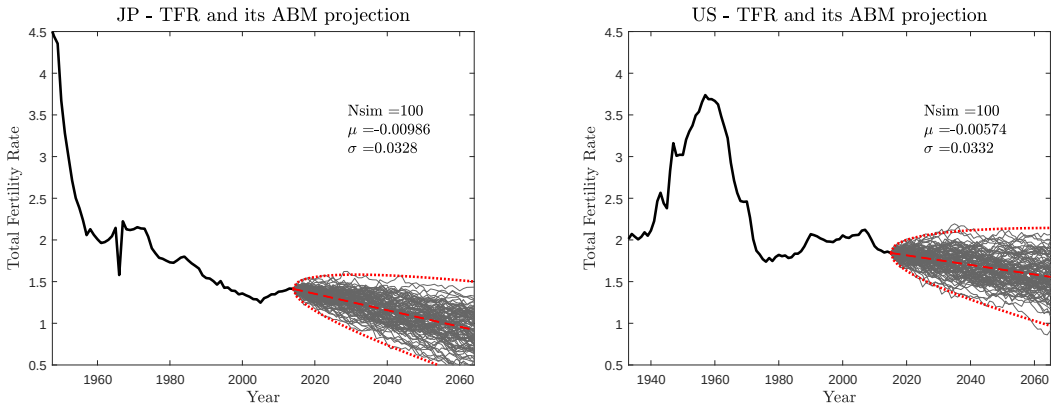


Figure 3.5: Japanese and U.S. Total Fertility Rates and their projections according to an ABM process

The Ornstein Uhlenbeck process is used to project a stationary TFR reverting to a mean value. Parameter estimation is done with the maximum likelihood method as described in the methodology chapter. In the Figure 3.6 we show past TFRs and some projected sample paths with the parameters estimated after calibration to the TFR data from the past 30 years.

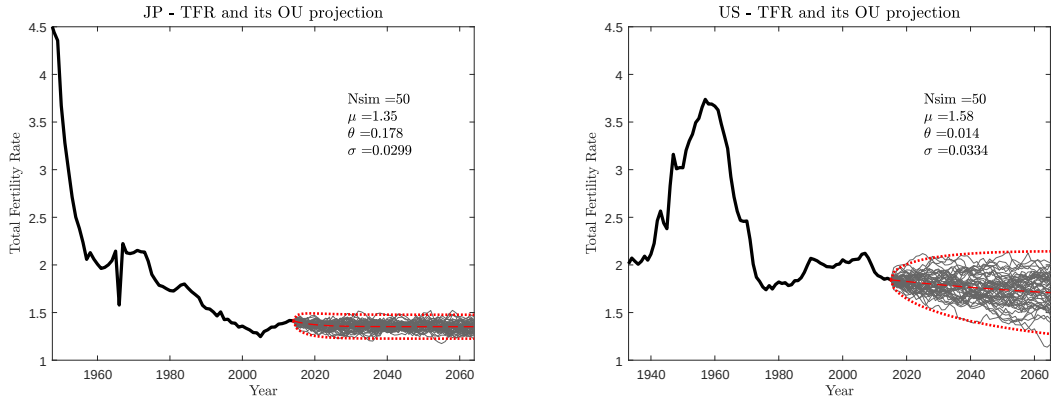


Figure 3.6: Japanese and U.S. Fertility Rates and their projection according to an Ornstein-Uhlenbeck process

Migration

Immigration and emigration models are divided into three components; normalized age specific distribution, total number and sex. We were able to compile Age Specific Migration (ASM) data for recent years only. Moreover, migration flow definitions tend to vary from one data source to another. High quality and homogeneous datasets on international migration are difficult to obtain for multiple reasons. The data compiled shows a non-significant trend in age specific migration variation over time. We thus used a constant

age distribution and computed it by averaging the recent years age specific migration distribution for every country, sex and migration flow direction. Migration model parameter estimation is done by minimising the sum of squares. The Figure 3.7 below shows the normalized age specific migration data of some of the countries in focus and their fitted models.

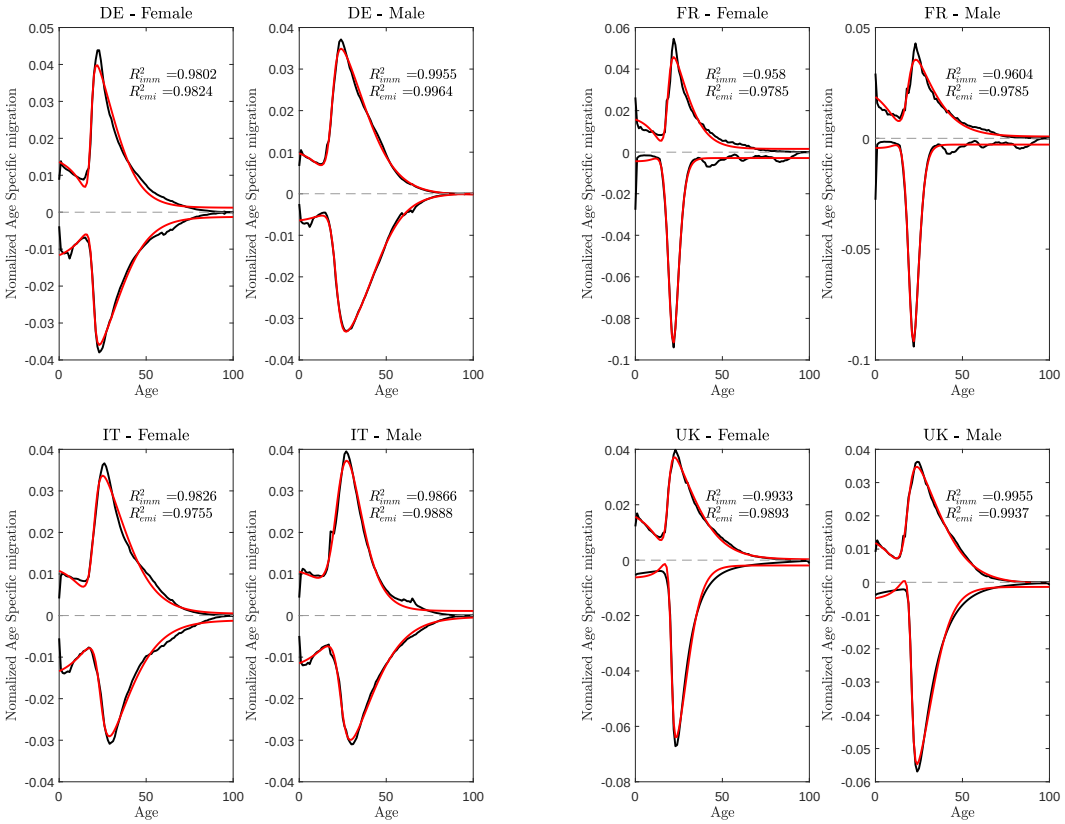


Figure 3.7: Average of age specific immigration (positive) and emigration (negative) annual flow data (black) and their fitted models (red).

The total immigration and emigration projections are done with Arithmetic Brownian Motion or the Ornstein-Uhlenbeck algorithms, similar to those used for the Total Fertility Rates projections. We first estimate their parameters and then simulate the sample paths, as described in the methodology chapter. The random processes parameters can also be modified to simulate the effect of migration policies. Figure 3.8 shows projected male migration in the UK after parameter estimation.

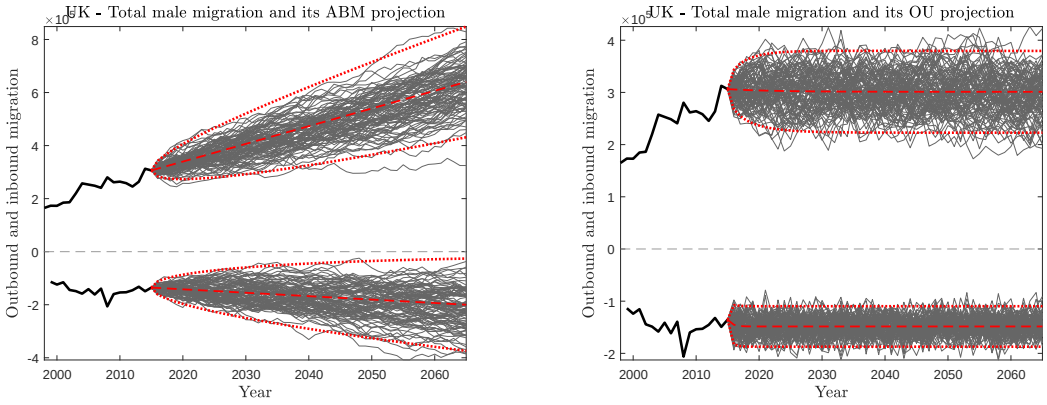


Figure 3.8: Projected male immigration (positive) and emigration (negative) in the UK according to an ABM (left) or OU (right) process.

3.2 Population projection model

This model projects future population dynamics by combining the fertility, mortality and immigration sub-models described above. These sub-models are calibrated to the latest data and considers first order time variations. Fertility (growth rate) is the main assumption and the primary source of inaccuracy in population projection models. To account for this variability we modelled it as a stochastic process. This results in population projections presented as a probability distribution rather than a median or an average. In this section, we present the main results related to our demographics model.

3.2.1 Accuracy assessment through Back-testing

We back-test the population projection model to assess its accuracy performance. In an initial stage, we estimate the sub-models parameters with historical data covering the period between 1980 to 1995. We then run Monte-Carlo simulations over the period between 1996 to 2010. Finally, we compare the population projection of these simulations with historical data.

In the Figure 3.9 we show the relative error of these projections at the end of the projection period (2010). These results were obtained by using the Ornstein-Uhlenbeck process for fertility and immigration. The projections tend to underestimate the total population. This can mainly be explained by the difference in projected migration and historical migration between 1996 and 2010. The United Kingdom, Italy and Spain show the largest errors, due to differences in migration behaviour between the calibration period and the simulation period. The Arithmetic Brownian Motion model for migration tends to give better results in these countries. Migration in Japan and the United States showed little variation between the calibration period and the test period with regard to the Ornstein-Uhlenbeck

model. Their population projections are very close to historical data.

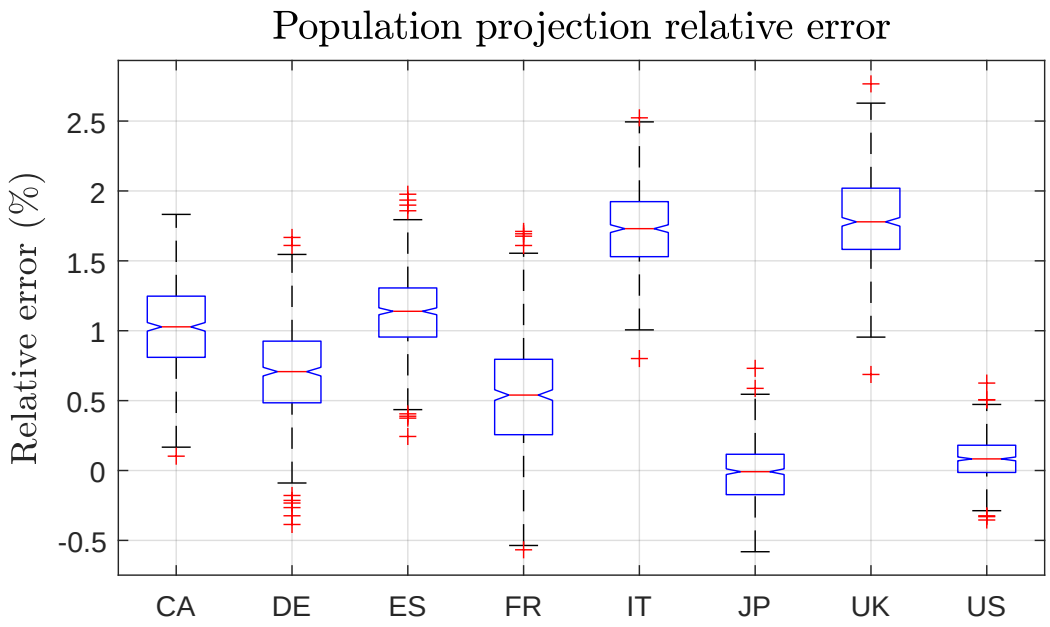


Figure 3.9: Population projection relative error. Calibration period: 1980 to 1995, simulation period: 1996 to 2010

With regard to fertility, the period between 1980 to 2010 shows a homogeneous behaviour in all the countries in focus. Simulations obtained with the Ornstein-Uhlenbeck model gives results comparable to historical fertility. Parameter estimation of fertility models in the period before the 1980, tend to have a larger error, due to the rapid decrease in fertility over that period.

The Figure 3.10 below shows the worst case error of the simulation set in Figure 3.8. Projection of the male population in the United Kingdom shows a larger error because of the underestimated migration (middle age region) and over estimated mortality (old age region). As discussed above, the UK witnessed a migration behaviour that is better modelled with an ABM process. The positive error at old age, means that survival rates of men in the UK has been higher than those expected from the parameter estimation between 1980 to 1995. These lower than expected mortality rates have been reported and explained by improvements in old age health care and changes in risky lifestyles.

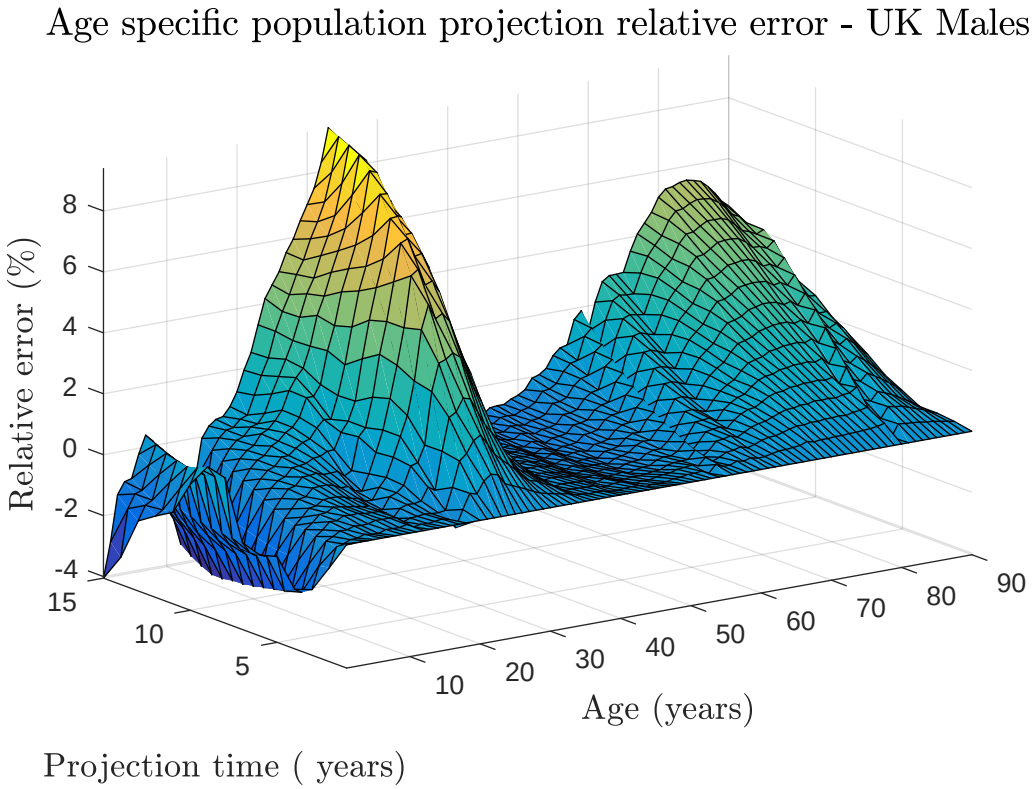


Figure 3.10: Age specific relative population projection error for UK Males. Calibration period: 1980 to 1995, simulation period: 1996 to 2010

3.3 Effects of population structure on the Housing market

3.3.1 Parameter estimation and projection

In this section, we use the above described population projection model and focus on two population groups representing major actors in the housing market. first-time buyers and old age market leavers. These population groups are expected to see significant changes over the next decades and could highly influence the housing supply and demand balance in many advanced economies.

The table in appendix A summarizes the dataset source used to calibrate the age specific first time buyers model. Parameter estimation is done for several years to extract the first order time variation parameters as described in the methodology chapter. Parameter estimation confirms several reports on the overall increase of first time buyer age.

In the context of this study, old age market leavers represent the home-owners leaving the housing market, due to old age conditions. Residential care, retirement expenses and ultimately death are the main reasons reported for definitely exiting the housing market

at old age. The hazard rate of these conditions is assumed proportional to the mortality hazard rate, as described in the mortality section. This statistic is not a standard one and is difficult to evaluate. In the case of inheritance or probate, the property is not necessarily reported as being sold by the old age market leaver. We use the findings of Myers et al. 2008 to calibrate the selling rate at old age in the U.S.. We modelled the hazard rate of quitting the housing market as a random variable to account for the high level of uncertainty regarding this statistic and to its potential change over time.

The Figure 3.11 shows the projected relative variation in first-time buyers and old age markets leavers over the next three decades. We have also estimated these populations over the past two decades to get a better overview of the past balance between these two market forces. Total Fertility Rates, immigration and emigration models are calibrated and projected according to an Ornstein-Uhlenbeck process. Parameter estimation is done with data covering the period between 2000 and 2015.

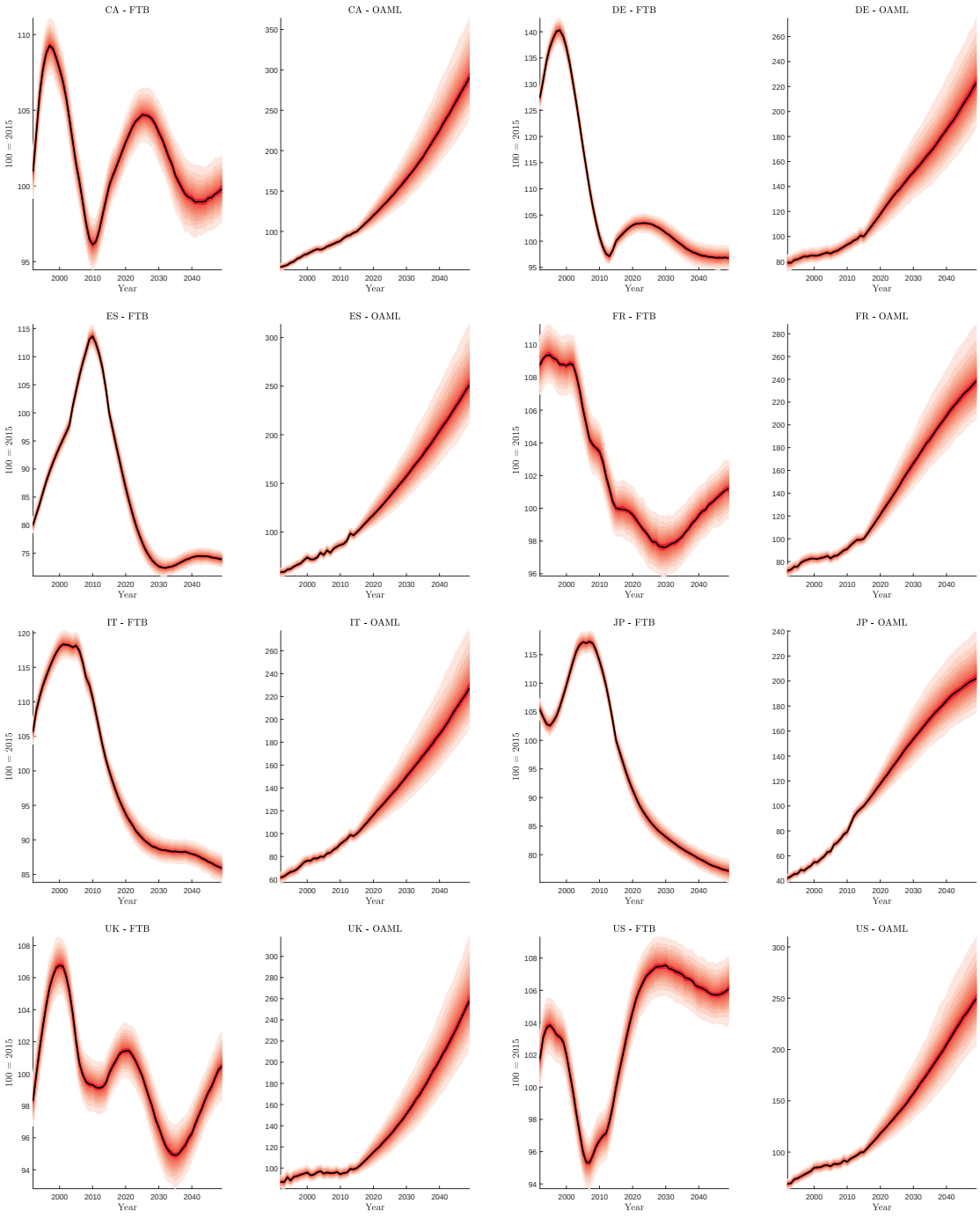


Figure 3.11: Past and Projected Potential First Time Buyers and Old Age Market Leavers

3.4 Income, consumption and support ratios

In this last study, we use the population projection model to simulate probabilistic projections of recent income and consumption trends. This will allow us to project potential Support Ratios, Life Cycle Deficits as well as their potential quantile distribution.

3.4.1 Parameter estimation and projection

We first estimate the income and consumption model parameters by minimizing the sum of squares between the models and the National Transfer Accounts data on income and consumption. Income is modelled as the difference of two logistic functions, while consumption is modelled by it's linear regression. In Figure 3.12 we show National Transfer Accounts historical data and the modelled curves obtained after parameter estimation. During the model calibration phase, we also extract the first order income and consumption rates of change for income and consumption projections.

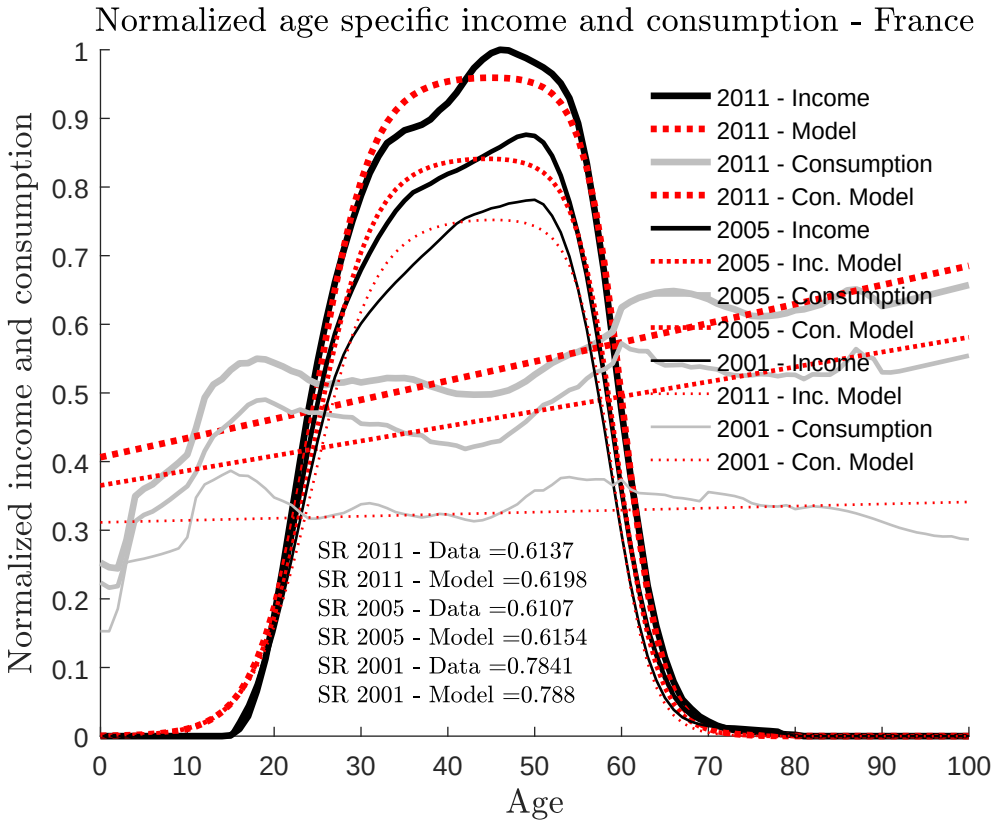


Figure 3.12: Age specific income and consumption in France - data and fitted models

In a second step, we estimate the parameters of the income and consumption quantile distribution. This is done by minimizing the sum of squares between the income and

consumption cumulative distribution functions and the double Pareto log-normal distribution. The four parameters of this distribution allows a very precise fit as shown in Figure 3.13 . This figure shows historical income quantile distribution data from the U.S. Census Current Population Survey. In black, we show the fitted dPLN distribution, up to incomes of 100,000$. In grey, we have estimated higher incomes with the dPLN parameters obtained by fitting lower incomes. We then generate an extrapolated joint age-income distribution and extract one year cohorts distributions as explained in the methodology section.

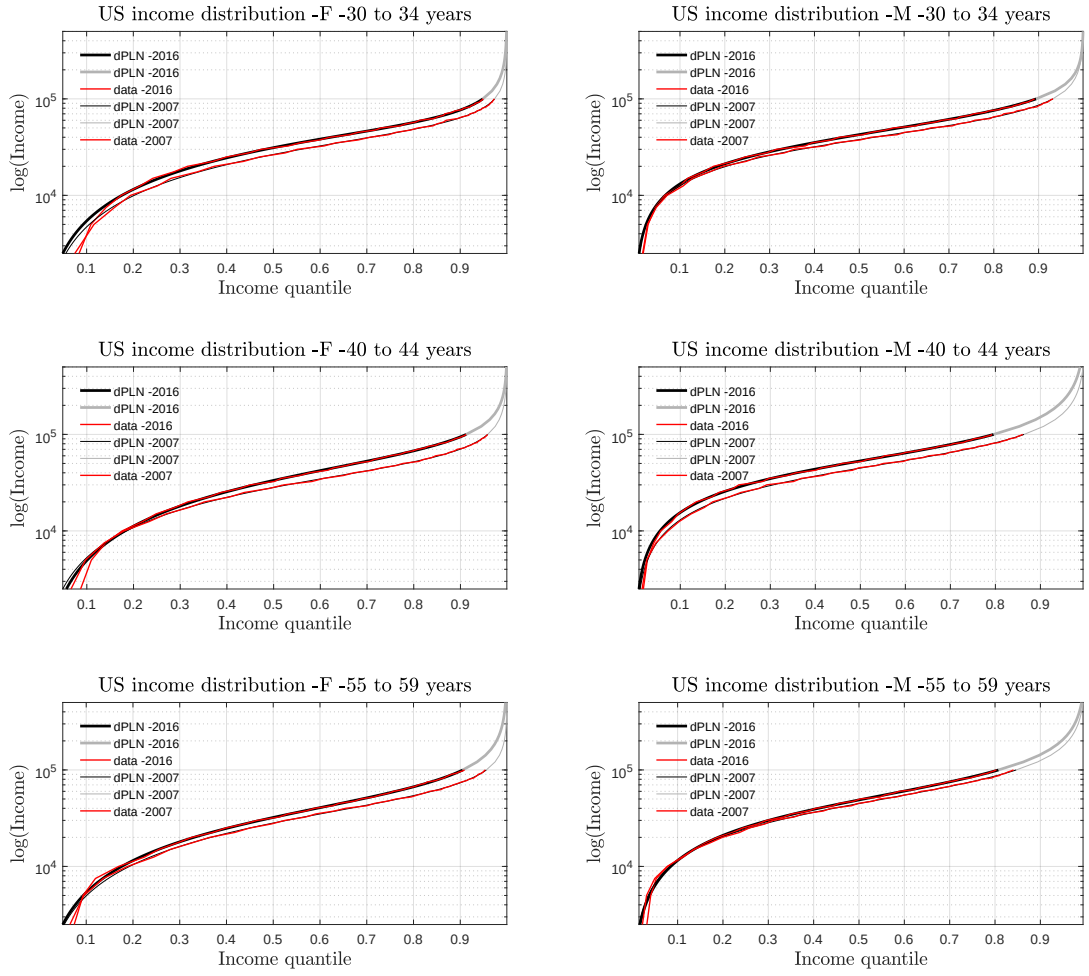


Figure 3.13: U.S. income distribution and their fitted dPLN distributions

Income inequality projections are obtained by projecting recent first order rates of change. The inequality rates of changes are normalized to the median. In the Figure 3.14 below, we show the median normalized average income variation between 2007 and 2016. To cover the whole income range, we only show the fitted dPLN curves. This Figure is discussed in the next chapter.

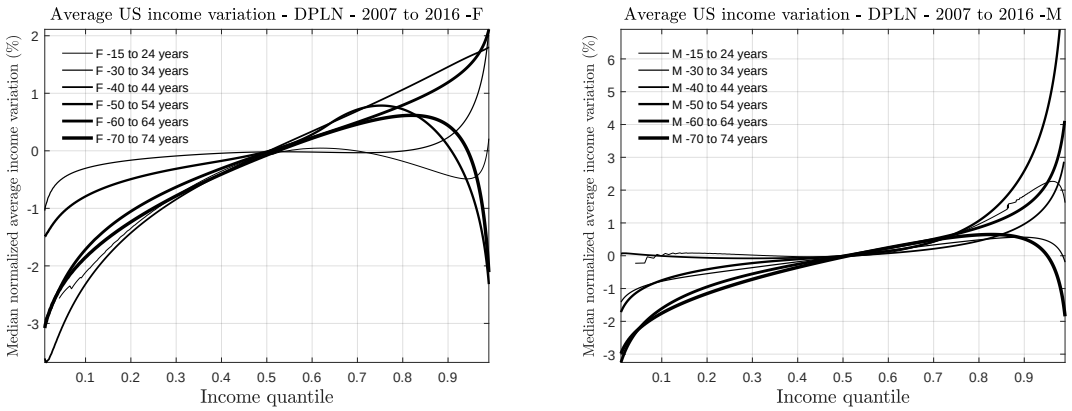


Figure 3.14: Median normalized average income variation in the U.S.- 2007 to 2016 - Fitted dPLN

Once sub models parameters are estimated, we project population, income, consumption and inequality distributions. For each of males and females we get at every simulation time step a joint age-income distribution and a joint age-consumption distribution. These joint distributions represent the total population of a country and are represented by two dimensional matrices of 111 years by 99 percentiles. These projections can be interpreted in many ways. In the Figure 3.15 below, we show the projected support ratios for the 10^{th}, 50^{th} and 90^{th} percentiles.

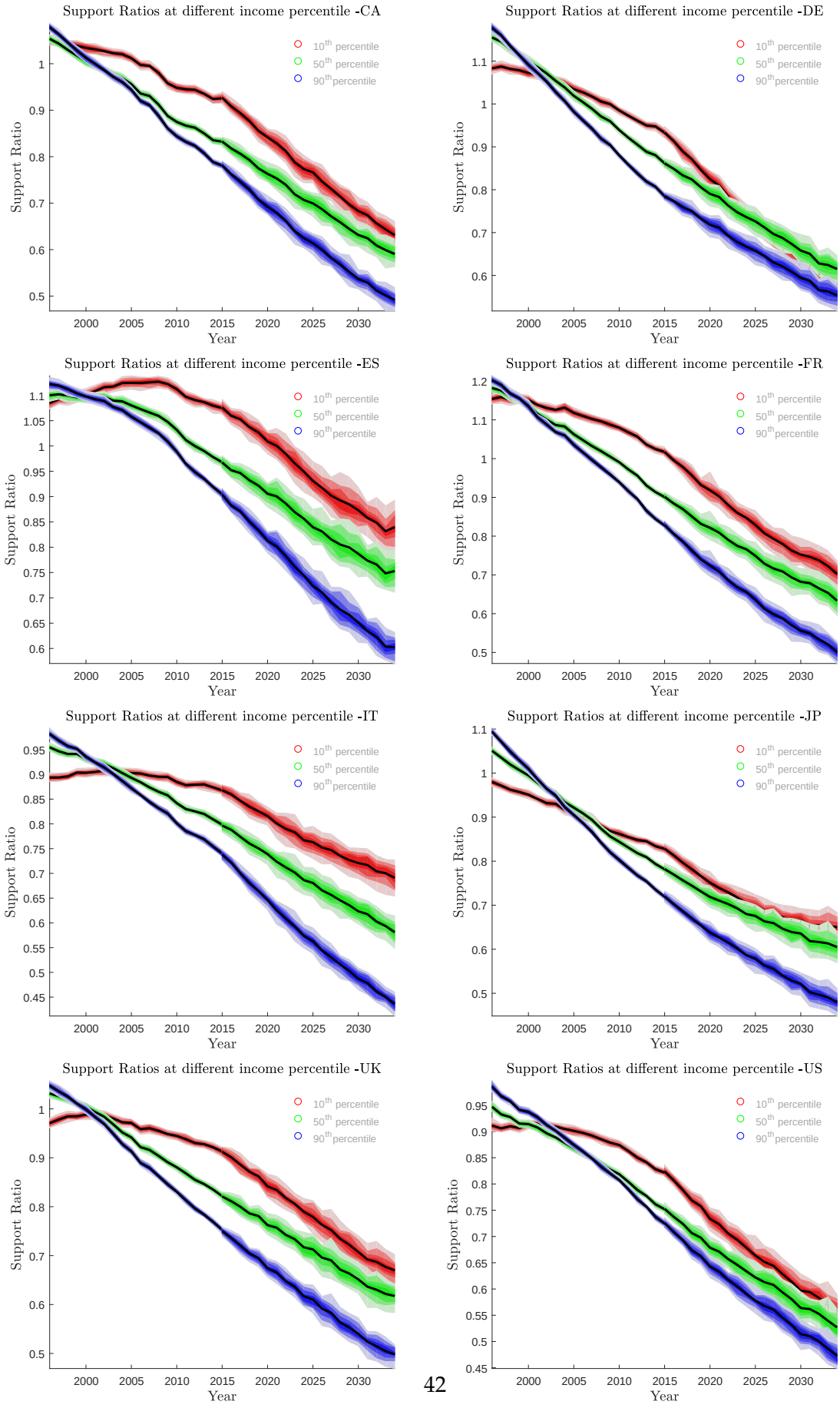


Figure 3.15: Projected Support Ratios across selected income quantiles

4 Discussion

4.1 Stochastic population projection

4.1.1 Fertility projection

Projections of Total Fertility Rates according to the ABM model gives accurate results over short-time projections. For longer ones, the actual decrease in total fertility rates pushes these fertility projections toward zero. Back-testing results show that the Ornstein-Uhlenbeck process fits well stabilized total fertility rates, however, it could still take negative values in case of low fertility levels and high volatility. The Cox–Ingersoll–Ross model overcome this Ornstein-Uhlenbeck limitation close to zero and could be considered as an alternative model for Total Fertility Rates. The Total fertility Rate of a country with populations with inhomogeneous fertility behaviour, could be improved by disaggregating the country TFR into a weighted sum of the TFR of these populations.

4.1.2 Mortality projection

Life expectancy deceleration has been recently reported in several populations, Figures 3.2 and B8 confirm these findings. The midpoint parameter of the logistic model increases at a slower rate for females than males. It also tend to saturate in countries with already high levels of life expectancy. The mortality model can be improved by substituting the linear time dependency of its parameters by saturating functions and by using more advanced models, like the Heligman-Pollard model. The latter has the advantage of modelling infant and child mortality.

4.1.3 Migration projection

Recent immigration surges in Italy and Germany pushes immigration projection to unreasonably high levels under the Arithmetic Brownian Motion assumption. This could be improved by adding a jump process to model immigration or emigration shocks. The ABM and OU process can also be modified to model the potential effect of immigration and emigration policies.

4.1.4 Population projection

Stochastic population projections offer the advantage of conveying uncertainty and its variation over time. For several types of financial and economic analysis this information has more value than the absolute levels. Similar models could be used by policy makers to improve long-term forecasting and planning. It may also be used to test the impact and sensitivity of policies. Spatial variations could be implemented to account for the variation in fertility rates and migration flows between regions, cities or even boroughs.

Population projection models could be improved by adding interdependencies and feedback loops between social and economic factors. For instance, internal and external migration tends to depend on the economic attractiveness of a region, its marginal carrying capacity and migration policies. There is also an abundant literature on the social and economic factors that affects fertility. Models of some of these factors could be added to the fertility model to test their sensitivity and potential effects.

4.2 Potential effects of population structure on the housing market

In most advanced economies, housing markets have a major effect on debt, money supply and economic growth as well as on the stability of their financial system. In a first step, the aim of this study was to identify the potential channels of influence of the population age structure on the housing market in the context of population ageing.

The largest identified demographic imbalance was that one side, large post World War Two cohorts - baby boomers - are reaching their end of life. This should liberate an increasing stock of housing units. On the other side, sub-replacement fertility rates since the late 1970s depleted the subsequent cohorts. Decreasing cohort size is expected to impact potential first time buyers (under the assumption that net migration is not high enough to counterbalance this effect).

4.2.1 First time buyers

In most studied economies, potential first time buyers increased during the 1990s and early 2000s (Figure 3.11). This could be explained by late baby boomers and early echo-boomers entering the housing markets. This population which was born between the late 1960s and late 1970s, corresponds with the period just before the drop of total fertility rates below replacement levels. Among the studied countries, Japan is one of the first to undergo such a fertility transition. According to the model, the number of potential first time buyers in Japan increased continuously over the 1970s and reached a first maximum in the late 1980s. A second peak in potential first time buyers is shown in Figure 3.11. It corresponds

with Japan's echo-boomers entering the housing market and it is the last cohort born to mothers with above replacement total fertility rates.

The evolution of potential first time buyers in Italy and Spain exhibit similar patterns to Japan. The late 2000s peaks corresponds with cohorts born just before the drop in total fertility rates below replacement levels in the late 1970s. This is coupled with high levels of net migration during the late 1990s and early 2000s. Potential first time buyers in Japan, Spain and Italy are expected to decrease significantly, due to the ongoing deep sub-replacement fertility and the relatively low net migration. In Japan, net migration rates have been historically low. In Spain and Italy, emigration has increased significantly over the past decade and tends to compensates for the effect of immigration.

Countries with total fertility rates close to replacement levels - the U.K., the U.S. and France - tend to show stable potential first time buyers over the next decades. With the Ornstein-Uhlenbeck migration projection calibrated to the latest 15 years of our dataset, projected net migration partially compensates for natural population shrinkage.

In this study, we have only computed the potential first time buyers distribution. The actual first time buyers number is a fraction of this population. This fraction depends on interest rates, housing prices, disposable income etc. The first time buyers model can be improved by incorporating the FTBs dependencies on these variables. Since disposable income and housing prices are highly dependent on location, this country level study could be disaggregated into more homogeneous regions for more representative models.

4.2.2 Old age market leavers

Middle and old age survivorship rates have increased considerably since the end of World War II (Figure 1.2). This has continuously increased the life expectancy of the baby boom generation and contributed to the decrease in mortality rates. This postponement of mortality has led to an accumulation effect. The post World War II generation will eventually reach peak mortality. This effect can also be observed by the increase in abruptness of the survivorship function (Figure 1.2). According to the population projection model, peak mortality growth - the derivative of mortality rates - should occur between 2025 and 2035. This corresponds with the largest baby boomer cohorts reaching their life expectancy. The mortality postponement effect described above, is the main factor behind the acceleration in old age market leavers in Figure 3.11. This effect is common to all countries in this study. Simulations over longer periods than those shown in figure 3.11 indicate that eventually, the quantity of old age market leavers will decrease (under the OU migration assumption with calibration from 2000-2015). This effect occurs around the period where sub-replacement fertility cohorts (those born around the 1970s) will reach their life expectancy. Japan, Germany and Italy are expected to be the first countries in this study to reach that maxima. Increasing net migration tends to delay this effect.

There are no standard statistics or models of the quantity of persons that leaves the housing market at old age. We associated a high level of uncertainty to this model, due to this lack of appropriate information. Microdata surveys could help considerably in improving the parameter estimation of the old age market leavers model.

A considerable proportion of the baby boomers housing legacy could be sold for liquidity. Two major arguments point toward this trend. The first one is related to the increasing use of home equity loans. This argument has been covered in section 1.2. The second argument is related to inheritance utility. Considering that baby boomers have a life expectancy of around 80 years and the average age of mothers at birth was around 25 years for that generation. We can expect echo boomers to inherit at an average age of around 55 years, which is well above the first time buyers age range. We can expect that increasing life expectancy, increases heirs age and thus could increase their liquidity preference with regard to inherited housing. These effects could further increase the proportion of inherited being sold and thus further increase the supply pressure.

4.2.3 Impact of demographics on financial stability : housing market channel

The dynamics described above points toward a potential long lasting supply pressure on the housing markets of the above studied economies. In a first phase, this could help offset the constrained housing supply in some of these markets. However, stagnating or even decreasing first time buyer demand may further depress the housing market supply and demand balance. It may also affect the expectations of repeat buyers and property investors. therefore, by positive feedback, it could affect the housing market as a whole. In this context, we can expect an increasing sensitivity of the housing market to internal and external migration. The high sensitivity of financial markets and many sectors of the economy to mortgages and the housing sector, increases the impact of population structure and ageing on financial stability.

4.3 Effects of population structure on Support ratios and Debt

The aim of this study is to use the population projection model to assess the sustainability of recent indebtedness trends in major advanced economies. The chosen methodology was to disaggregate total income and consumption by age and percentile level. For every simulation time step, age specific population, income and consumption growth are evaluated. Total income and consumption are aggregated to evaluate the overall support ratio.

4.3.1 Support ratios and life cycle deficit

Age specific consumption has been increasing at a faster rate than the related incomes in most advanced economies (Figures 1.3 and 3.12). The excess consumption has been

primarily financed by debt. We initially planned to project potential total debt requirement from support ratios, However we were not able to find the necessary data for parameter estimation and calibration. Under the above described conditions, the model shows that the population support ratio will continue to drop. This continuous decrease is mainly driven by increasing high life cycle deficits at over 60 years of age, coupled with increasing cohorts size. The increasing old age contribution to the lower support ratio is partially offset by decreasing early years cohorts.

National Transfer Accounts data covering the past two decades shows that the rise in life cycle deficit has mainly been driven by increasing public transfers, namely pensions and health care expenses. In most of the studied countries, these two major expenses are unfunded liabilities. They are, therefore mainly financed by taxation and deficits. With a projected slowly growing or even decreasing taxable base, we can expect an increase in taxation and deficit needs in most of the countries in this study.

4.3.2 Age specific inequality

In Figure 3.14 we showed the median normalized average income growth in the US between 2007 and 2016. The overall upward tilt of this figure confirms the increase in U.S. income inequality in the period following the Great Recession. It can also be noted that inequality has been higher among men than women and that it has as significant age dependency. Some high quantile groups at the extremes of the age spectrum have seen a significant drop in income. This could be explained by the drop in retirement income at old age and lower labour participation rates among the 15 to 24 years old age group.

Higher support ratios at lower income quantiles (red curves in Figure 3.15) has been partially explained in the literature by lower access to debt based consumption. However starting from the early 2000s we can see a decrease in support ratios at these lower income bands. This decrease has been explained by the increasing access to credit at low income bands and was a major factor leading to the 2007 Mortgage crisis.

4.3.3 Impact of demographics on financial stability : Debt channel

Above described mechanisms points toward an increasing impact of the projected population age structure on support ratios and potentially debt ratios and financial stability. This model could be complemented by introducing the effects of interest rates. This would allow to test the potentially stabilizing effect of negative interest rates. In Japan, the concept of grey democracy has been introduced to explain the feedback loop between the increase in population ageing and increasing deficits. This feedback system could also be introduced in our model to better understand its impact on financial stability.

5 Conclusion

Most advanced economies are facing an unprecedented and long-lasting period of population ageing. This study tries to understand the potential effects of changing population age structures on debt growth and on housing markets. We focus on these two factors due to their historical influence on financial instabilities. In a first stage, we introduce the concepts of demographic transition and demographic dividend. We review the major channels through which the age structure of a population can influence its economy. While there is no standard definition of financial stability, we introduced some elements and consequences of financial instabilities and their potential relation to "excessive debt". We also review some relevant economic and financial findings that followed the global financial crisis.

In a second stage we present the models used in this study. We also discuss how we estimate and calibrate their parameters. This study is based on a self-contained stochastic population projection model. This probabilistic model allows us to also project uncertainty. In the first study, we use the model to project the potential future growth of two important components of the supply and demand in the housing market, first time buyers and old age generated supply. We have calibrated our models to the historical data of Canada, Germany, Spain, France, Italy, Japan the United Kingdom and the United States. We found an overall gradual and long-lasting increase in old age generated supply until at least 2050. The highest growth rate was projected between around 2025 and 2035, depending on the country's age structure and the net migration projection scenario. We found a slow increase in first time buyer demand in countries with historical total fertility rates close to replacement level and relatively high positive net migration (France, The U.K. and the U.S.). In countries with historically deep sub-replacement fertility, we found a sharp decrease in first time buyer demand.

In the second study, we use the probabilistic population projection model to project recent labour income and consumption trends, as well as their quantile distributions. We found a continuous drop in support ratios, to levels that we described as a highly unsustainable. With excess public and private consumption, mainly financed through taxation and debt, we concluded that the ongoing synchronized population ageing should also

have a considerable effect on global financial fragility.

A decade after the Great Recession, the global economy remains in a weakened state. The crisis' impact is still reflected in major central banks' interest rates and balance sheets. With limited room for conventional monetary policy (including asset purchase programs), the next generation of unconventional monetary policies such as deep negative interest rate or helicopter money, will likely require models that take into consideration people's reactions to these policies and the distribution of these reactions. We have demonstrated a potential implementation of such a model. This model could be further detailed and improved by including the socio-economical dependencies and feedback loops between its different sub-models and the financial and economic variables. This would help identify and quantify the sensitive intervention points and could help in policy-making and planning.

A Appendix : Data Sources

	Canada CA	Germany DE	Spain ES	France FR	Italy IT	Japan JP	United Kingdom UK	United States US
Population	HMD	HMD	HMD	HMD	HMD	HMD	HMD	HMD
Mortality	HMD	HMD	HMD	HMD	HMD	HMD	HMD	HMD
TFR	HFD	HFD	HFD	HFD	HFD	HFD	HFD	HFD
ASFR	HFD	HFD	HFD	HFD	HFD	HFD	HFD	HFD
Migration	StatCan	Eurostat	Eurostat	Eurostat	Eurostat	OECD	Eurostat	OECD
FTB	CAAMP	Alik-Lagrange et al. 2015	Servihabitat	INSEE	Istat	StatJap	EHS	NAR
Income consumption	NTA	NTA	NTA	NTAt	NTA	NTA	NTA	NTA
Inequality	StatCan	Eurostat	Eurostat	Eurostat	Eurostat	OECD	Eurostat	US Cencus

CAAMP	Canadian Association of Accredited Mortgage Professionals
EHS	English Housing Survey
HFD	Human Fertility Database
HMD	Human Mortality Database
INSEE	Institut National de la Statistique et des Etudes Economiques
Statcan	Statistics Canada
Istat	Italian National Institute of Statistics
NAR	National Association of Realtors
NTA	National Transfer Accounts
OECD	Organisation for Economic Co-operation and Development
StatJap	Official Statistics of Japan

B Appendix : Figures

B.1 Age specific fertility rates

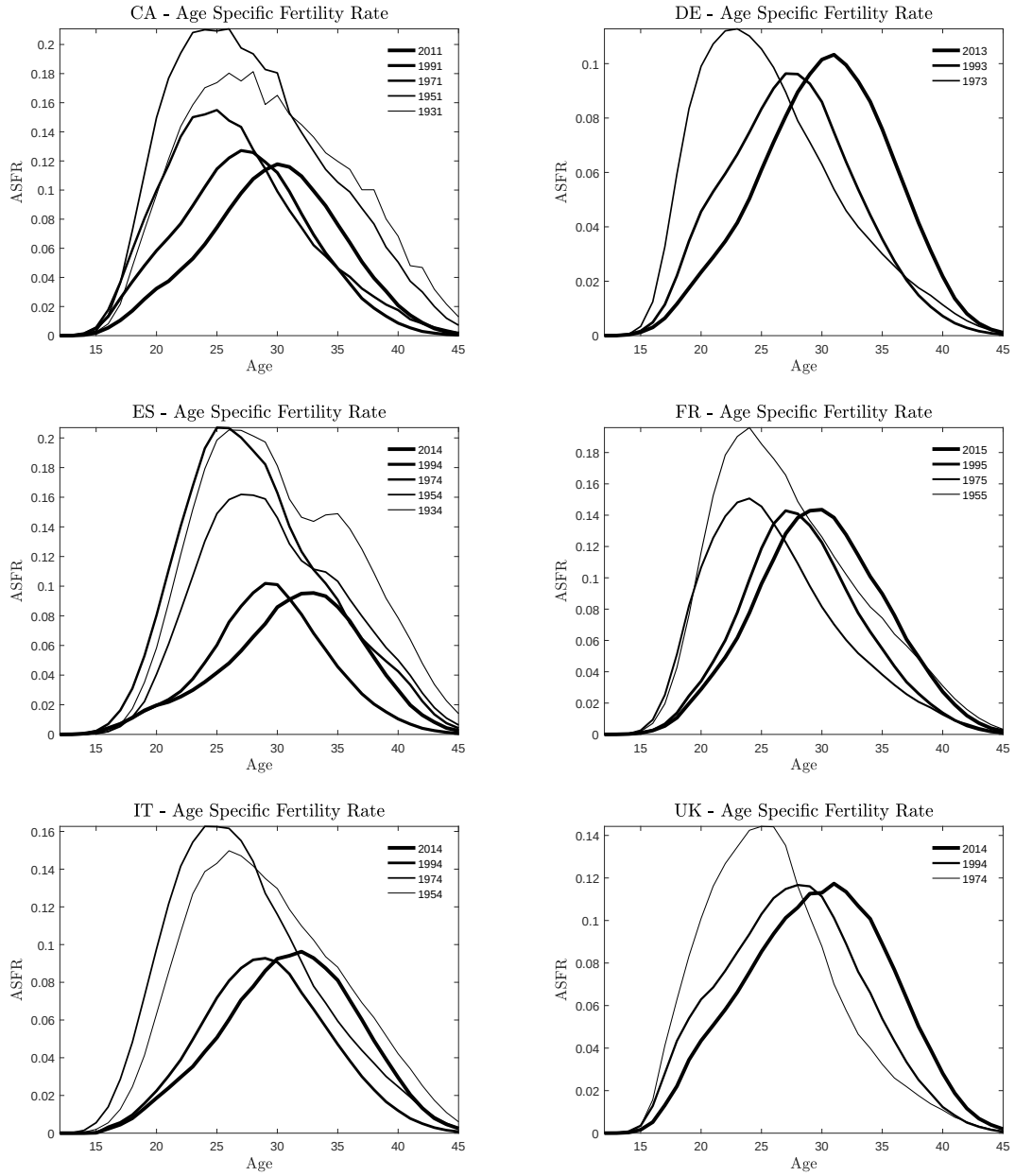


Figure B1: Age Specific Fertility rates in the studied countries

B.2 Average age of mothers at birth

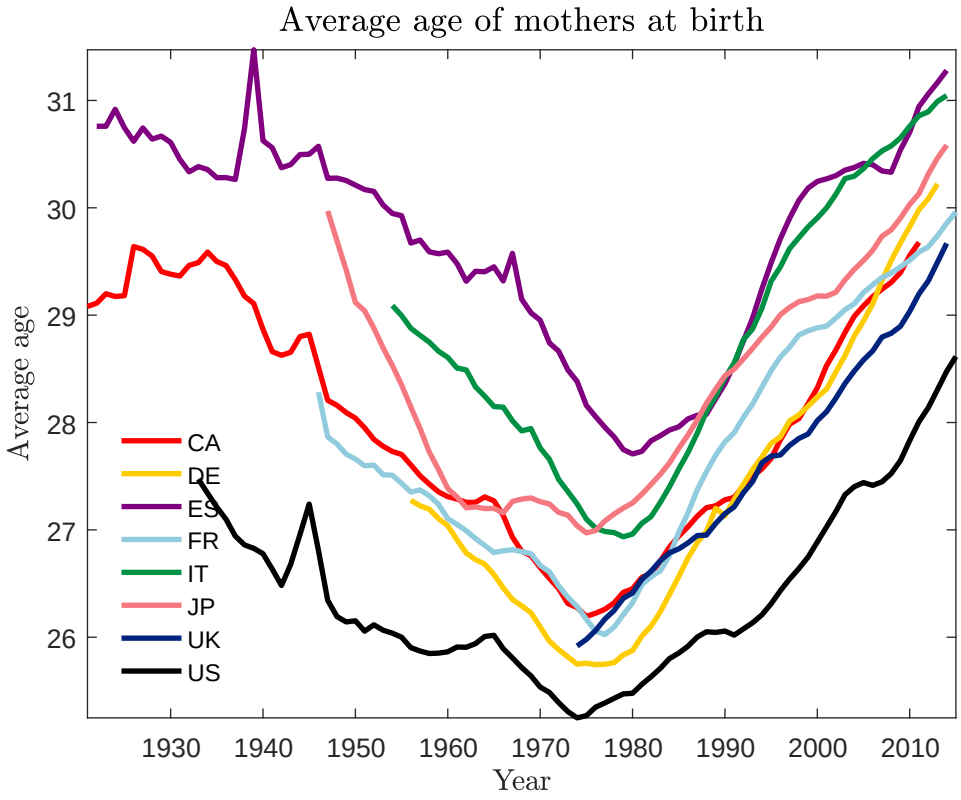


Figure B2: Average age of mothers at births

B.3 Normalized age specific fertility rate model

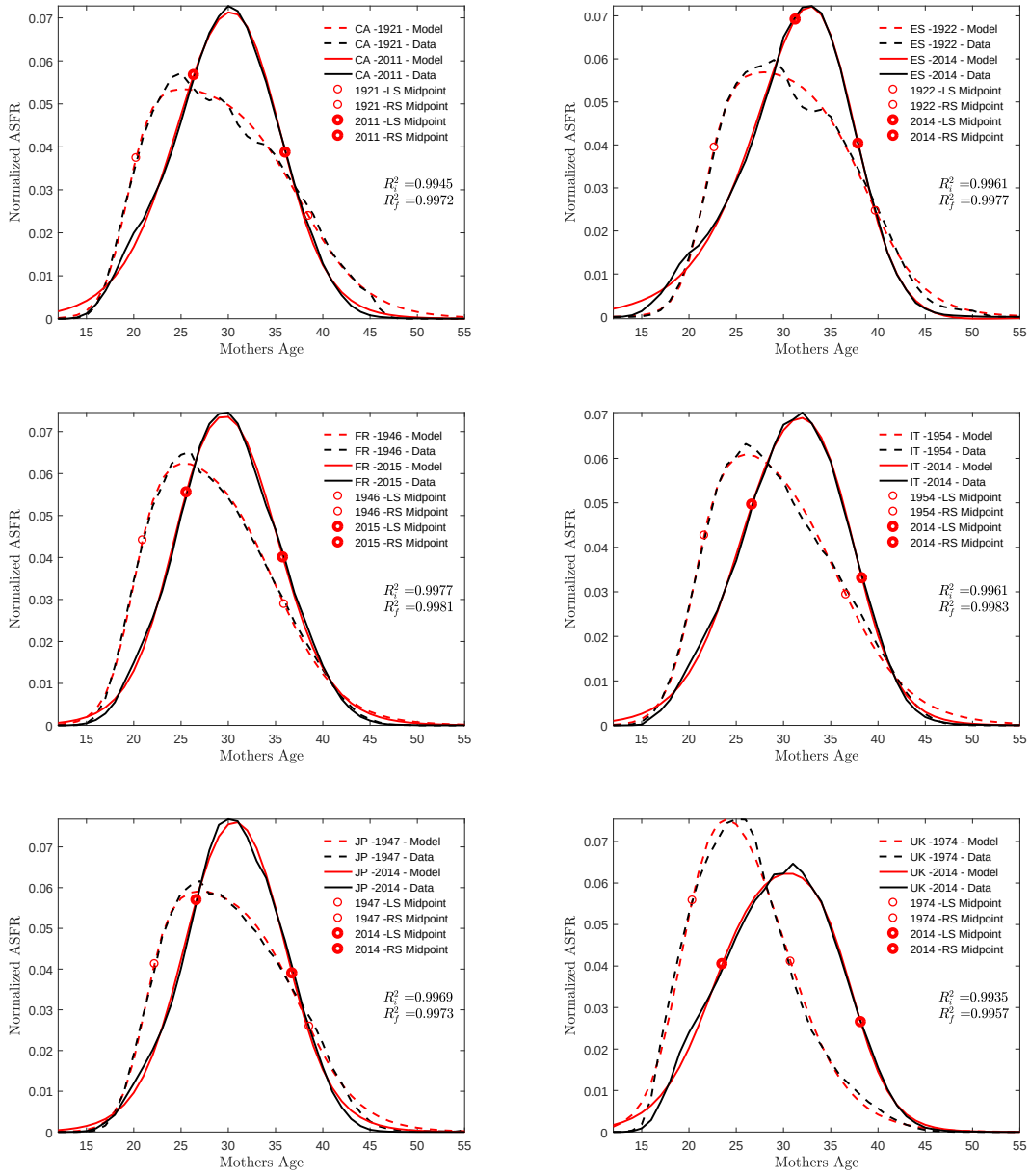


Figure B3: Normalized age Specific fertility rates and their fitted models

B.4 Total Fertility rates and their Arithmetic Brownian Motion projection

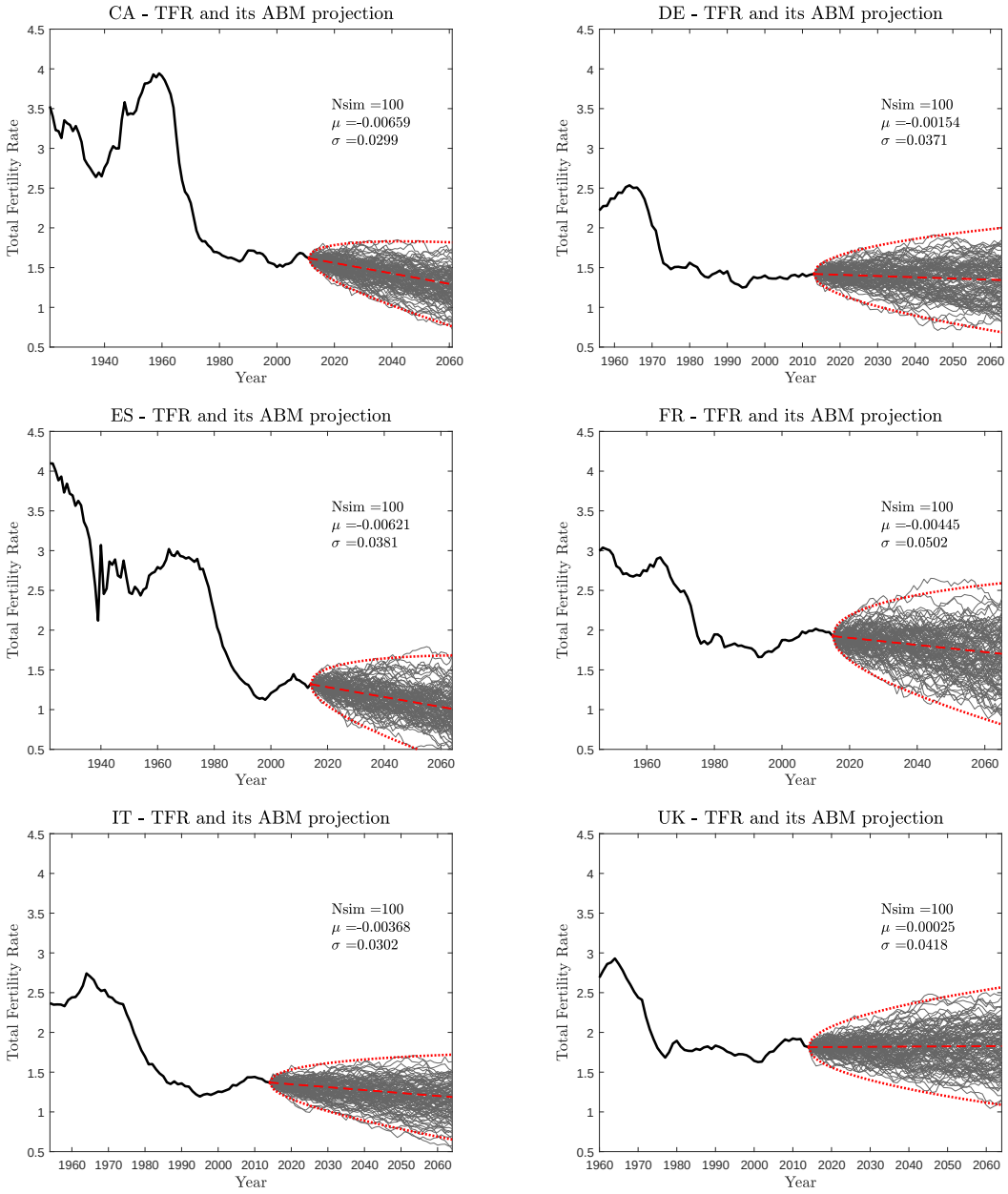


Figure B4: Total Fertility rates and their projection according to an ABM process

B.5 Total Fertility rates and their Ornstein–Uhlenbeck projection

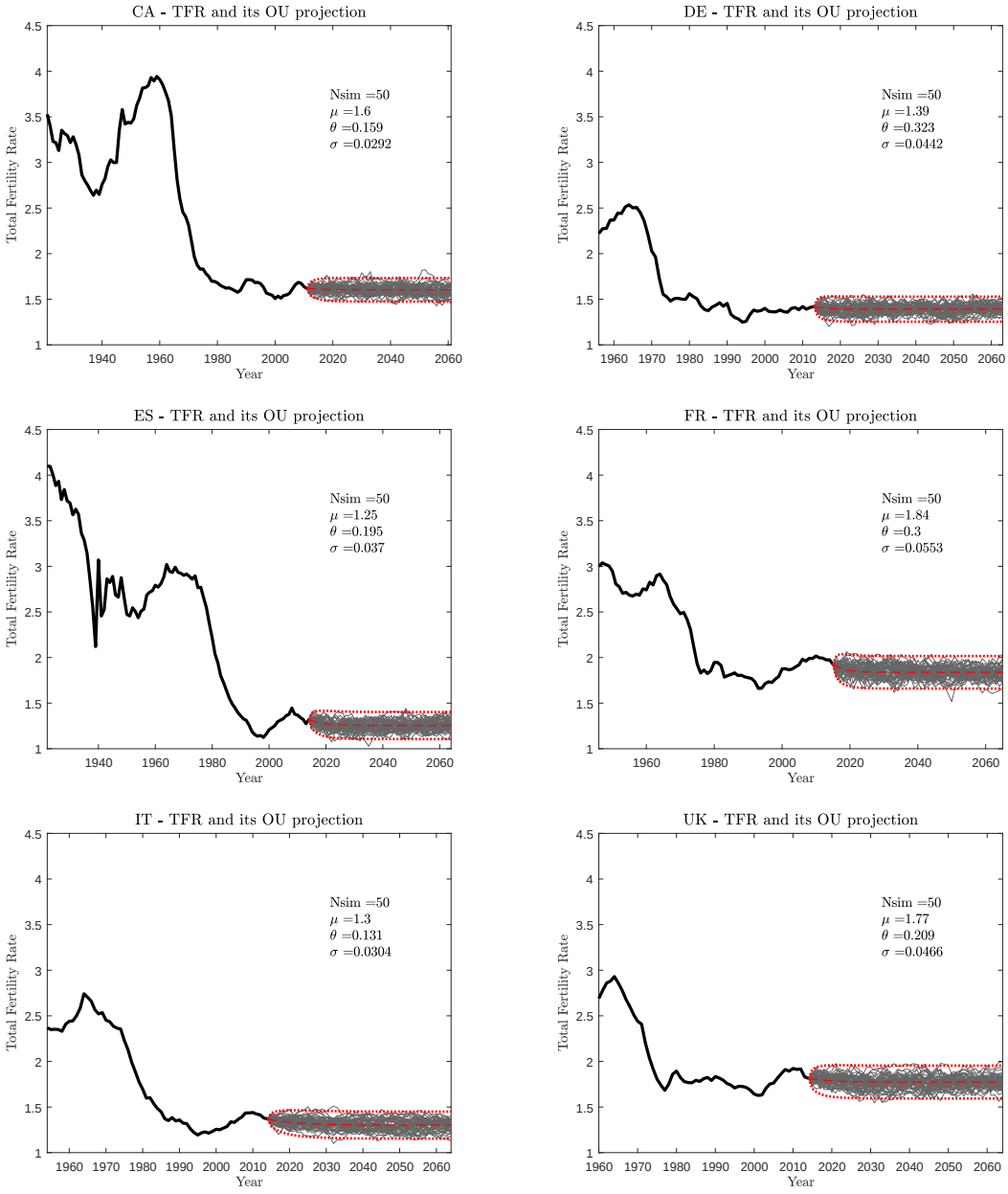


Figure B5: Total Fertility rates and their projection according to an OU process

B.6 Evolution of mortality hazard rates across different age groups

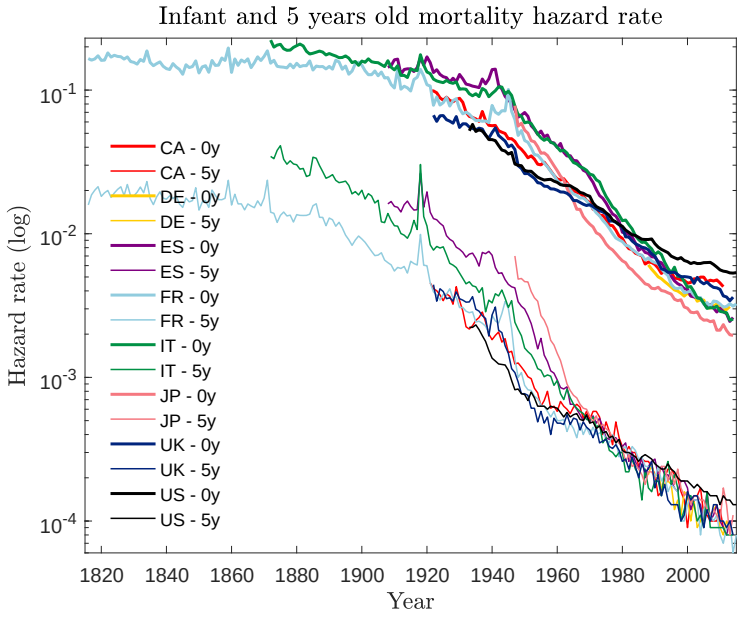


Figure B6: Evolution of the one year conditional probability of death within one year of birth and between five and six years of age

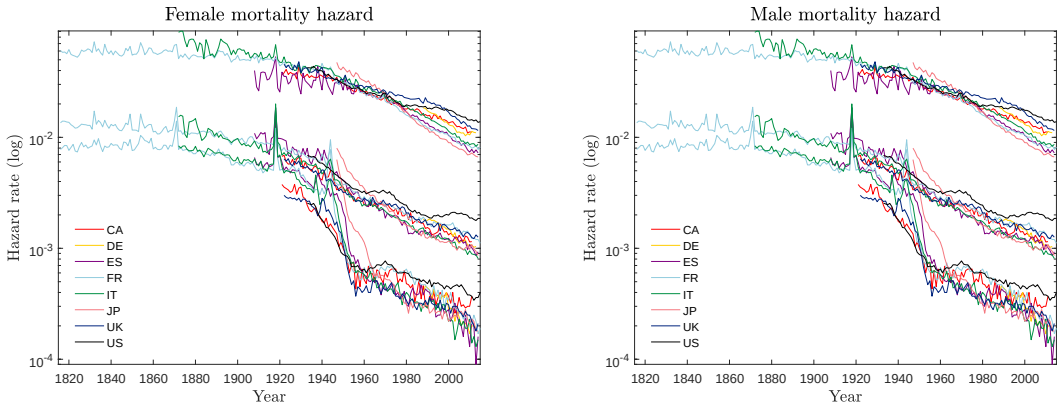


Figure B7: Evolution of the one year conditional probability of death within one year of 20 (bottom), 45 (middle) and 70 (top) years of age

B.7 Mortality hazard rate parameters over time

The figures below complement Figure 3.2.

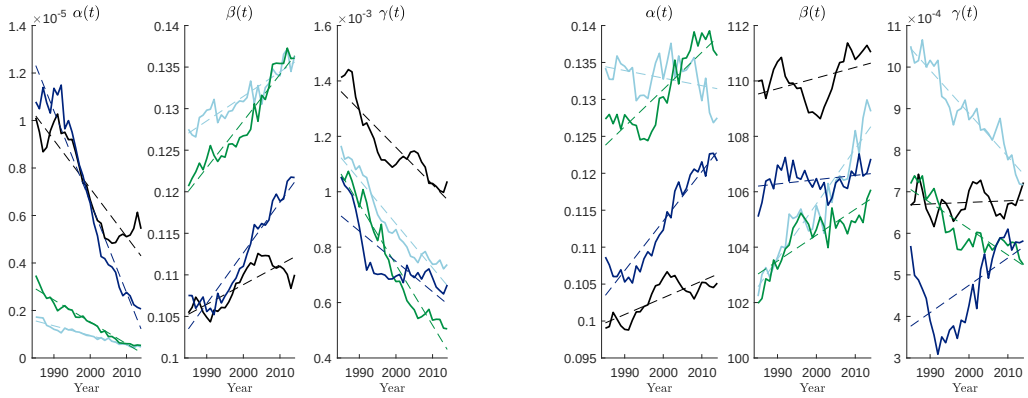


Figure B8: Exponential (left) and logistic (right) mortality hazard rate models parameters over time and their linear regression - Females

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