On Moduli Stabilisation and Cosmology in Type IIB Flux Compactifications

Francisco Manuel Soares Veríssimo Gil Pedro
Exeter College
University of Oxford

A thesis submitted for the degree of
Doctor of Philosophy

Trinity 2012

To António, Maria, Joana and Giulia

Acknowledgments:

This Thesis was only made possible by the support I got from many people.

I am greatly indebted to my supervisor, Dr. Joseph P. Conlon, for all he taught me, for his constant encouragement and contagious enthusiasm. Over the last four years I have greatly benefitted from his knowledge and support while enjoying a lot of freedom to pursue my own interests.

It is a pleasure to thank my collaborators Gianmassimo Tasinato and Michele Cicoli from whom I have learned a great deal.

For introducing me to research in theoretical physics and for his continued support over the years I thank Prof. Orfeu Bertolami.

I would also like to thank Jay Armas, José Fonseca, Raquel Henriques and João Laia for the good times during Part III in Cambridge.

I thank the Portuguese and European citizens that through a studentship from Fundação para a Ciência e Tecnologia made my studies possible while asking for very little in return. I also acknowledge the financial support from the Physics Department and from the Exeter College of the University of Oxford.

For their love and support I thank my family, I would have not got this far without them.

Finally I thank Giulia for making my life exponentially better.

Summary:

This Thesis studies some aspects of string compactifications with particular emphasis on moduli stabilisation and cosmology.

In Chapter 1 I motivate the study of string compactifications as a way to build on the successes of the Standard Model of Particle Physics and of the theory of General Relativity.

Chapter 2 constitutes an overview of the technical background necessary for the study of flux compactifications. I sketch how the desire to obtain a supersymmetric theory in four dimensions constrains us to consider compactifications of the ten dimensional theory in six dimensional Calabi-Yau orientifolds. I argue that it is strictly necessary to stabilise the geometry of this compact space in order to have a phenomenologically viable four dimensional theory. I introduce the large volume scenario of type IIB compactifications that successfully incorporates fluxes and subleading corrections to yield a four dimensional theory with broken supersymmetry and all geometrical moduli stabilised.

The next four Chapters are devoted to the study of some phenomenological aspects of moduli stabilisation and constitute the original work developed for this Thesis.

In Chapter 3 I investigate the consequences of field redefinitions in the stabilisation of moduli and supersymmetry breaking, finding that redefinitions of the small blow-up moduli do not significantly alter the standard picture of moduli stabilisation in the large volume scenario and that the soft supersymmetry breaking terms are generated at the scale of the gravitino mass.

Chapter 4 deals with the putative destabilisation of the volume modulus by very dense objects. The analysis of the moduli potential shows that even the densest astrophysical objects cannot destabilise the moduli, and that destabilisation is only achievable in the context of black hole formation and cosmological singularities.

In Chapter 5 I present a model of inflation within the large volume scenario. The inflaton is identified with a geometric modulus, the fibre modulus, and its potential generated by poly-instanton effects. The model is shown to be robust and consistent with current observational constraints.

In Chapter 6 I introduce a model of quintessence, where the quintessence field and its potential share the same origin with the inflationary model of the previous Chapter. This model constitutes a stringy realisation of supersymmetric large extra dimensions, where supersymmetry, the low gravity scale and the scale of dark energy are intrinsically connected.

I conclude in Chapter 7 outlining the direction of future research.

The stubborn critic would say: "What is the benefit of these sciences?" He does not know the virtue that distinguishes mankind from all the animals: it is knowledge, in general, which is pursued solely by man, and which is pursued for the sake of knowledge itself, because its acquisition is truly delightful, and is unlike the pleasures desirable from other pursuits. For the good cannot be brought forth, and evil cannot be avoided, except by knowledge. What benefit then is more vivid? What use is more abundant?

Abū Rayhan Muhammad al-Bīrūni

Contents

Chapter 1

Introduction and Motivation

Our present understanding of the Universe around us is supported by the twin pillars of General Relativity and the Standard Model of Particle Physics. These two outstandingly successful theories give us insight on how the Universe works both at the very large and very small scales.

Born in the beginning of the 20th century, the theory of General Relativity (GR) [1] provides a beautiful geometric description of gravity that extends the previous ideas dating back to the works of Newton in the 17th century. The concept of gravitation as spacetime curvature is not only mathematically beautiful but also extremely successful in describing our observations. It has been extensively tested over almost one century on scales ranging from the solar system up to the radius of the observable Universe. It allows for a description of the orbits of planets, including small effects like the precession of Mercury's perihelion, gravitational lensing and redshift and emission of gravitational waves by binary systems.

Perhaps the most profound impact of the theory comes from its application to the study of the Cosmos where it has allowed us to build up a detailed picture of our past and changed our perceptions about our position in the Universe. The explanation of the expansion of the Universe, of the measured abundances of elements and the prediction of the cosmic microwave background (CMB) can all be counted as successes of GR applied to cosmology. When combined with a period of exponential expansion in the distant past, the standard cosmological model can explain the observed levels of isotropy and flatness of the Universe and provide an explanation for the origin of the observed anisotropies in the CMB.

Despite its successes there are still some loose ends in our theoretical description of the Universe based on GR. Hubble taught us that the Universe is expanding, so if we turn back time we see that it originated from what looks like an initial singularity. At this singularity the energy density approaches M_P^4, where $M_P \sim 2 \times 10^{18}$ GeV is the Plank mass, and GR ceases to be valid. An understanding of the nature of this singularity requires the knowledge of the ultraviolet completion of GR. Observations of supernovae and of the CMB made over the last decade indicate that 70% of the total energy in the Universe is in the form of a cosmological-constant-like fluid with an energy density around $\rho_\Lambda \sim 10^{-12} eV^4$ [2]. The physical origin of this fluid and an explanation of the hierarchy $\rho_\Lambda/M_P^4 \sim 10^{-120}$ are still unanswered questions. Despite the successes of inflation, the details of the inflationary mechanism are still very much a mystery, as questions about the origin of the inflaton and its potential remain unanswered. Furthermore, we have learned over the last century that the world is intrinsically quantum mechanical and that classical theories, such as GR, emerge as a limit of an underlying quantum theory. Identifying the quantum theory of gravity that extends GR is one of the great challenges in theoretical physics today. All these reasons should prompt us to look beyond GR and to identify and study the theory that builds on its successes and avoids its shortcomings. At the moment many avenues are being explored in search for an answer for one or more of the issues raised above. These include models with extra dimensions, scalar-tensor theories of gravity, modifications of the Einstein-Hilbert action among many others. Explaining just one of these issues would be a great step forward.

The Standard Model of Particle Physics (SM), like GR, is one of the great achievements of 20th century physics, providing us with a theory of strong and electro-weak interactions. Formulated within the framework of quantum field theory, the SM is a $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory with three families of chiral fermions. In order to spontaneously break the $SU(2)_L \times U(1)_Y$ symmetry down to the $U(1)_{EM}$ electromagnetic theory, the SM features a scalar degree of freedom, the Higgs field, that is also responsible for the origin of mass of elementary particles. Over the years the SM has been extensively tested to very high accuracy [3], a program that continues today with the experiments being undertaken at the Large Hadron Collider (LHC).

Even though it provides a well tested theory of three of the four known forces of Nature there are clearly hidden structures within the SM that prompt us to extend it. The SM by itself does not explain why there are exactly three families of chiral fermions or why their masses have a clear hierarchy. A deeper understanding of gauge coupling unification, of neutrino masses and of the origin of dark matter also motivate us to look beyond the SM. Perhaps the biggest mystery of the SM is tied to the Higgs sector as we cannot explain why the scale of electro-weak symmetry breaking is so much smaller than the Planck scale: $M_{EW} \ll M_P$. This is the well known hierarchy problem that has attracted so much attention over the last decades. The electro-weak scale, defined as the vacuum expectation value of the Higgs, is determined by experiment to be around 250 GeV. The structure of the Higgs potential makes this scale a function of the Higgs mass: $M_{EW}^2 \propto m_H^2$. Computations of loop corrections to the mass of the Higgs within the SM show that it is quadratically divergent with the integration cut-off $m_{loop}^2 \sim \Lambda_{UV}^2$, which in principle could be as high as the Planck scale. Fine-tuning of the bare mass to $m_0 \sim -\Lambda_{UV}$ would be required to keep the physical mass, $m_H^2 = m_0^2 + m_{loop}^2$, around the measured value of 125 GeV. For a Planck scale cut-off this corresponds to a tuning of the order of 10^{-32}. This solution is therefore highly unsatisfactory and the hierarchy problem should instead be interpreted as a sign that there is some form of new physics above the TeV scale.

Several ideas have been put forth to stabilise the electro-weak scale. Two of the most promising approaches, that are related to the work developed in this Thesis, are low energy supersymmetry and models with large extra dimensions.

Supersymmetry stabilises the Higgs mass by postulating that for each fermionic (bosonic) degree of freedom of the SM there is a corresponding boson (fermion). The problematic radiative corrections to the Higgs mass coming from fermionic loops are then exactly cancelled by the corresponding loops of superpartners. By doubling the spectrum of particles supersymmetry stabilises M_{EW} without the need for fine tuning parameters. Besides providing a solution to the hierarchy problem, low scale supersymmetry has interesting side effects. For instance when realised in the context of the Minimal Supersymmetric Standard Model (MSSM) it allows for gauge coupling unification and the lightest stable supersymmetric particle can be a candidate for dark matter. It is worth noting that if Nature employs some other mechanism to stabilise the weak scale, supersymmetry can still be a useful theoretical tool for constructing models of beyond the SM physics.

An attractive alternative to supersymmetry is provided by models with a low gravity scale [4]. If this scale is as low as a few TeV, then the radiative instability of the Higgs sector is tamed. This can be achieved by embedding the SM in a higher dimensional spacetime and demanding that only gravity propagates beyond the four visible dimensions. In these models M_P is related to the fundamental gravity scale M via the volume of the internal space $\mathcal{V}_n$: $M_P^2 \sim M^{2+n}\mathcal{V}_n$ where n denotes the number of extra dimensions. It then follows that the largeness of M_P is due to large $\mathcal{V}_n$, while the gravity scale M can be close to the TeV scale. We must note that for these models to provide a genuine solution to the hierarchy problem, rather than simply recasting it in a new language, a mechanism for generating naturally large $\mathcal{V}_n$ must be given.

A program trying to provide answers to some of the issues both of GR and of the SM, that has received a lot of attention over the last thirty years has been string theory [5–8]. Initially conceived as a theory of strong interactions, string theory was subsequently turned into a theory of quantum gravity and is at present the leading candidate for unifying and extending GR and the SM. Its spectrum includes a spin-2 field which we identify with the graviton and the gross features of the SM like non-Abelian gauge groups and chiral fermions. For consistency the theory must live in higher dimensional spacetimes and the description of fermions requires supersymmetry. If such a theory is to be of any use in the description of the real world, supersymmetry must be broken and all but four dimensions must be compactified, that is they must be wrapped into a size smaller than the smallest length scale we are able to probe with high energy experiments.

One important feature of string compactifications is the fact that the geometry of the compactification space determines the spectrum and the physics of the four dimensional theory, much like in the Kaluza-Klein theories that predate it [9, 10] . Even here the desire to solve the hierarchy problem has a profound impact as the requirement of a supersymmetric theory in four dimensions constrains the compact space to be Calabi-Yau as we will see in more detail in Chapter 2. This of course selects a general class of manifolds rather than pinning down a specific one. The hope is then that the building of realistic models of particle physics and cosmology will impose further constraints and narrow the space of possible choices. Assuming a particular topology is chosen and the particle spectrum is fixed, the resulting four dimensional physics is still dependent on the geometry of the compactification space as couplings and masses of particles are generically given by vacuum expectation values of moduli fields, scalar fields that parametrise the geometry of the compact space. To have any hope of making contact with reality one must first and foremost be able to stabilise the moduli, that is generate a potential for these fields. Furthermore moduli appear in the four dimensional theory as Planck coupled degrees of freedom and as such would mediate fifth forces between SM particles. These forces are severely constrained experimentally and as a result Planck coupled moduli must have masses outside the range $[10^{-17}, 10^{-2}]$ eV [11]. Once again the solution is to generate an adequate potential for these fields, so we observe that the stabilisation of the geometry of the extra-dimensional space is an essential requirement for any realistic string model trying to extend GR and the SM.

At present string theory is more a framework than a unique well defined theory. It is a term used to denote a set of different perturbative descriptions of the same underlying theory, each with different gauge symmetries, number of supersymmetries and particle content, connected by a web of dualities. In this Thesis we will deal exclusively with type IIB string theory and its phenomenology. In the context of IIB string theory, the standard model is realized locally on branes wrapping cycles in the internal space. This is an important feature of type IIB string theory that allows one to separately study the global and local aspects of the theory. This feature is not shared by all perturbative string theories, for instance in heterotic theories fermions are not confined to branes and live in the bulk so it is not possible to disentangle the SM physics from the physics that stabilises the extra dimensions. The possibility of studying separately local and global issues makes type IIB appealing for model building. Global/bulk physics encompasses issues like moduli stabilisation, the breaking of supersymmetry, the cosmological constant or inflation. Local/brane physics include the obtention of a chiral spectrum, the correct gauge group and Yukawa couplings to describe the SM, gauge coupling unification, reheating among others. The possibility of studying the global aspects neglecting the local issues and vice versa has yielded a number of advances over the last decade on both sides, however the final aim is to have a fully consistent and complete model. The first steps towards this goal are now being taken.

The physics of compactification mechanism and the breaking of supersymmetry in type IIB string theory will be the main focus of this Thesis with particular emphasis on the works developed in [12–15].

Chapter 2

Flux Compactifications of Type IIB String Theory

In this Chapter we introduce the framework of flux compactifications that forms the foundations in which we develop the work presented in Chapters 3-6. We start by presenting in Section 2.1 the Freund-Rubin compactification as a toy model that introduces some of the fundamental concepts in flux compactifications. In Section 2.2 we see how the requirement of supersymmetry constrains the geometry of the compact space to be Calabi-Yau and review some of the geometric concepts of these spaces that are relevant for the remainder of this Thesis. Section 2.3 is devoted to the review of the spectrum and action of type IIB string theory compactified in Calabi-Yau manifolds and their orientifolds. Section 2.4 introduces the stabilisation of the geometric moduli of the compact space through a combination of fluxes and subleading corrections to the effective field theory, with emphasis on the large volume scenario, while Section 2.5 deals with gravity mediated supersymmetry breaking.

2.1 Fluxes and extra dimensions: the Freund-Rubin compactification

In order to illustrate the role of fluxes in the stabilisation of the extra-dimensions we briefly analyse a 6D Freund-Rubin compactification [16]. This is a simplified scenario that captures the essence of the physics of the more complicated setups that we will introduce later. Let us then consider a 6 dimensional Einstein-Maxwell theory, with metric G and abelian field strength tensor F_2. The action for this theory is

$$\mathcal{S} = \int d^6X \sqrt{|G|} \left(M_6^4 \mathcal{R}_{(6)} - M_6^2 |F_2|^2 \right) , \tag{2.1}$$

where $X^M = (x^\mu, y^m)$ with $\mu = \{0,1,2,3\}$ and $m = \{4,5\}$. Assuming that the space-time is a direct product of a non-compact 4 dimensional space and a 2 dimensional compact manifold of genus g and volume $R^2\, l_6^2$ we can write the metric as

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + R^2(x)h_{mn}(y)dy^m dy^n, \tag{2.2}$$

where we assume that the radius of the compact space can be a function of the 4D coordinates. In this example $R(x)$ plays the role of the moduli in more complicated geometries and appears in the 4D theory as a scalar field. The point of this exercise is to show how by turning on the F_2 flux in the compact directions one can generate a potential for the "radial modulus" and stabilise the physical size of the extradimensional manifold. We must stress that F_2 can only have non-zero components in the compact directions due to the requirement of 4 dimensional Lorentz invariance.

Armed with the metric of Eq. (2.2) one can show that the 6D curvature scalar decomposes according to

$$\mathcal{R}_{(6)} \equiv G^{MN}\mathcal{R}^P{}_{MPN} = g^{\mu\nu}\mathcal{R}^\rho{}_{\mu\rho\nu} + g^{\mu\nu}\mathcal{R}^m{}_{\mu m\nu} + g^{mn}\mathcal{R}^\mu{}_{m\mu n} + g^{mn}\mathcal{R}^p{}_{mpn} \tag{2.3}$$

and that the determinant of the metric is $\sqrt{|G|} = \sqrt{|g|}\sqrt{h}R^2$. It then follows that the action will contain the following term

$$\mathcal{S} \supset M_6^4 \int d^4x\sqrt{|g|}(R\, l_6)^2 g^{\mu\nu}\mathcal{R}^\rho_{\mu\rho\nu}, \tag{2.4}$$

where we have assumed that $\int_\mathcal{M} d^2y\sqrt{h} \equiv 1$ in units of l_6^2. This allows us to establish the relation between the 4D and the 6D Planck masses typical of extra dimensional models

$$M_{(4)}^2 \equiv M_{(6)}^2 R^2, \tag{2.5}$$

and to note that a Weyl rescaling is needed to cast the action into the Einstein frame. We define the new metric f by

$$g_{\mu\nu} \equiv R^{-2}f_{\mu\nu}. \tag{2.6}$$

The integrated curvature of the 2 dimensional space is the topological invariant $\chi(M)$ related to the genus g of $\mathcal{M}$ via

$$\int_\mathcal{M} \sqrt{h}\mathcal{R}_{(2)}(h) \equiv \chi(M) = 2 - 2g. \tag{2.7}$$

In order to demonstrate the role of the gauge flux F_2 we thread $\mathcal{M}$ with some number of units of magnetic flux

$$\int_\mathcal{M} F_2 = N, \tag{2.8}$$

to find that the action then becomes [1]

$$\mathcal{S} = \int d^4x\sqrt{|f|}\left(\mathcal{R}_{(4)}(f) - 8\,(\partial R)^2 + \frac{\chi(\mathcal{M})}{R^4} - \frac{N^2}{R^6}\right). \tag{2.9}$$

So the integrated curvature term and the gauge flux generate a potential for the radius of $\mathcal{M}$:

$$V(R) = -\frac{\chi(\mathcal{M})}{R^4} + \frac{N^2}{R^6}. \tag{2.10}$$

Depending on the sign of $\chi(\mathcal{M})$, the interplay between the internal curvature and the flux potential can be sufficient to stabilise the radial mode. In the particular case when $\chi(\mathcal{M}) > 0$ one sees that these two contributions stabilise $R(x)$ at

$$\langle R\rangle = \sqrt{\frac{3}{2\chi(\mathcal{M})}}N. \tag{2.11}$$

So for relatively large flux, one can achieve large radii and small curvatures. For internal spaces with $\chi(\mathcal{M}) \leq 0$ the radial modulus potential will not have a minimum for R and will display run away behaviour towards infinity, showing the need to include further structure in this construction. In the context of string theory the combination of fluxes in the internal space with local sources like D-branes and orientifold planes allows for consistent realizations of moduli stabilisation that extend the basic example presented here. To understand how this can be achieved we now leave this illustrative example aside and turn our attention to the more realistic geometries that will lead us to considering orientifold compactifications of type IIB string theory.

2.2 Calabi-Yau manifolds

We have just seen how the geometry of the compactification manifold, together with non-trivial flux configurations can impact on the four dimensional theory one gets after compactification. In this Section we will continue to relate the geometry of the compact space with the physics of the four non-compact dimension. We will motivate the need for and explore some aspects of Calabi-Yau manifolds. These manifolds play a pivotal role in string phenomenology since they allow for the construction of quasi-realistic four dimensional theories starting from a higher dimensional string theory. The geometry of Calabi-Yau manifolds is an extremely rich subject that has attracted a lot of attention over the last twenty years and an extensive literature exists by now (for a review see e.g. [19, 20]). Therefore we do not aim to give a comprehensive

[1] The kinetic terms for the radial modulus can easily be computed using algebraic manipulation software like Cadabra [17, 18].

overview but rather pick our way through the subject, highlighting the features that are important for the work presented in later Chapters of this Thesis.

The crucial requirement that leads us to the study of Calabi-Yau manifolds is that of supersymmetry. Phenomenologically supersymmetry is rather desirable since it allows to solve the hierarchy problem, that is explaining why is $M_W \ll M_P$. If after compactification one wants to obtain a supersymmetric field theory in four dimensions then the geometry of the compact space is rather constrained. This was originally discovered in [21] in the context of heterotic string compactifications. Even though this Thesis will deal exclusively with the phenomenology of type IIB string theory, it is worth taking a brief historical detour and revisit the arguments of [21].

The requirement of unbroken supersymmetry is that of finding a supersymmetry transformation such that $\delta_\epsilon \Phi = 0$, where Φ denotes a generic (fermionic or bosonic) field of the theory and ϵ the infinitesimal supersymmetry parameter. For a bosonic Φ, $\delta_\epsilon \Phi \propto$ fermion. The requirement of Lorentz invariance forbids fermions from getting a vacuum expectation value (VEV) and so $\delta_\epsilon \Phi \sim 0$, we then see that this does not lead to any condition on the generator of the transformation ϵ. However if Φ is fermionic $\delta_\epsilon \Phi \propto$ boson which in general is non-zero and can therefore lead to a breaking of supersymmetry, so one must ensure that it equates to zero. With this in mind, we note that the field content of heterotic supergravity includes the graviton g_{MN}, the gravitino Ψ_M, a 2-form field B_{MN}, the string dilaton ϕ and its fermionic partner the dilatino λ in the gravity multiplet and the 2-form field strength F^a_{MN} and the gaugino χ^a in the Yang-Mills multiplet. To ensure that this theory admits a supersymmetric solution one must look carefully at the supersymmetry transformation of the fermionic fields Ψ_M, λ and χ.

Following [21] we restrict our attention to the case of vanishing H_3 and constant dilaton ϕ. In this regime $\delta_\epsilon \lambda = 0$ automatically, however the gravitino and gaugino transformations are in general non vanishing. It turns out that it is the gravitino transformation

$$\delta_\epsilon \Psi_M = \nabla_M \epsilon - \frac{1}{4} H_M \epsilon, \tag{2.12}$$

that constrains the geometry of the compact space. The requirement of unbroken supersymmetry in the limit of vanishing H flux maps to

$$\nabla_M \epsilon = 0. \tag{2.13}$$

This means that the ten dimensional spacetime $\mathcal{M}$ must admit a covariantly constant spinor ϵ. Assuming $\mathcal{M} = \mathcal{M}_4 \times \mathcal{M}_6$ with $\mathcal{M}_4$ a maximally symmetric 4 dimensional space (i.e Minkowski, deSitter or anti deSitter), the existence of a covariantly constant spinor restricts $\mathcal{M}_4$ to be Minkowski and $\mathcal{M}_6$ to be Ricci flat:

$$R_{mn} = 0. \tag{2.14}$$

The Ricci flatness of the metric of $\mathcal{M}_6$ can be shown to be equivalent to the requirement of SU(3) holonomy of the compact space. This follows from writing the $\mathcal{M}_6$ part of Eq. (2.13) as $\nabla_m \eta = 0$, where η is the 6 dimensional part of the spinor ϵ. Under parallel transport around a closed curve in $\mathcal{M}_6$ a spinor will in general be rotated by some $SO(6) = SU(4)$ matrix U: $\eta \to U\eta$. Requiring that the spinor be covariantly constant means that it must come back to itself, so U must be a member of a subgroup of $SU(4)$ for which $U\eta = \eta$, this turns out to constrain U to be in $SU(3)$.

Through a conjecture by Calabi, later proved by Yau, a Kähler manifold X with vanishing first Chern class admits a Ricci-flat metric of $SU(N)$ holonomy. This metric is unique up to scalings for a given complex structure of X and a given Kähler class. In these Calabi-Yau manifolds one can have a globally well defined closed (1,1) form J, which is related to the volume of the manifold, and also a closed holomorphic $(3, 0)$ form Ω. In fact there is exactly one such 3-form up to a constant.

Henceforth we will focus exclusively on manifolds with $SU(3)$ holomony. The topology of such Calabi-Yau manifolds plays an important role in the determination of the spectrum of the low energy effective field theory. This is due to the fact that in Kaluza-Klein reduction, the massless degrees of freedom in four dimensions are expanded in terms of harmonic forms on $\mathcal{M}_6$ and these are in turn in 1-to-1 correspondence with the elements of the Dolbeault cohomology $H^{(p,q)}$ (closed (p, q) forms modulo exact (p, q) forms). The dimension of the various Dolbeault cohomology groups $H^{(p,q)}$ are called the Hodge numbers of $\mathcal{M}_6$ and denoted by $h^{(p,q)}$. These are usually organised in a diagram called the Hodge diamond. For compactification of the 10 dimensional Type IIB theory we are interested in 6 dimensional Calabi-Yau manifolds for which the Hodge diamond generically looks like

$$\begin{array}{ccccccc} & & & h^{(3,3)} & & & \\ & & h^{(3,2)} & & h^{(2,3)} & & \\ & h^{(3,1)} & & h^{(2,2)} & & h^{(1,3)} & \\ h^{(3,0)} & & h^{(2,1)} & & h^{(1,2)} & & h^{(0,3)}. \\ & h^{(2,0)} & & h^{(1,1)} & & h^{(0,2)} & \\ & & h^{(1,0)} & & h^{(0,1)} & & \\ & & & h^{(0,0)} & & & \end{array} \tag{2.15}$$

There are a number of symmetries and constraints in the Hodge diamond of a Calabi-Yau 3-fold that follow from complex conjugation, Poincare duality, the uniqueness of the $(3,0)$ form and other properties. These imply that the Hodge diamond for a Calabi-Yau 3-fold has only two free parameters and looks like

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{(1,1)} & & 0 & \\ 1 & & h^{(1,2)} & & h^{(1,2)} & & 1. \\ & 0 & & h^{(1,1)} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{2.16}$$

Specification of the Hodge numbers selects a particular topology but does not pick a single Calabi-Yau: there is a continuum of manifolds with the same topology related by deformations of their size and shape. The parameters that describe these deformations are the famous geometric moduli fields. These arise from the fact that if g is a Ricci flat metric in $\mathcal{M}_6$ then so is $g + \delta g$ provided it satisfies the Lichnerowicz equation:

$$\nabla_k \nabla^k \delta g_{mn} + R_m{}^p{}_n{}^q \delta g_{pq} = 0. \tag{2.17}$$

The properties of the metric and curvature of a Kähler manifold imply that the pure and mixed components of the metric deformation decouple and can be studied separately. It also follows from Eq. (2.17) that δg are harmonic and so can be expanded in terms of elements of the Dolbeaut cohomology. The coefficients of this expansion are the Kähler and the complex structure moduli. As we will see in the next Section these appear in the compactified theory as Planck coupled massless scalar fields. From a phenomenological point of view it is imperative that one is able to generate masses for these fields as otherwise they will mediate fifth-forces that are severely constrained experimentally. Furthermore in the context of string compactifications the masses of the four dimensional particles are given as functions of the moduli vacuum expectation values and so one must be able to fix these moduli.

2.3 Dimensional reduction: action and spectrum

The arguments presented in the previous Section that lead to Calabi-Yau compactifications in the context of heterotic string theory can also be applied to type IIB string theory. These compactifications on Calabi-Yau manifolds constitute a simpler example of the more realistic compactifications with non-vanishing flux. We now follow [22, 23] and derive the action and spectrum of compactifications of type IIB in Calabi-Yau manifolds and their orientifolds.

2.3.1 $\mathcal{N} = 2$ compactifications on Calabi-Yau manifolds

The bosonic spectrum of type IIB string theory encompasses the string dilaton $\hat{\phi}$, the metric tensor $\hat{g}$ and the $\hat{B}_2$ two-form in the Neveu-Schwarz/Neveu-Schwarz (NS-NS) sector and the axion $\hat{l}$, a two form $\hat{C}_2$ and a 4 form gauge field $\hat{C}_4$ in the Ramond-Ramond (R-R) sector. These are the fundamental degrees of freedom of the theory that can be used to write the low energy effective action as [22, 23]

$$\begin{aligned} S_{IIB}^{(10)} = & -\int \Big(\frac{1}{2}\hat{R} * \mathbf{1} + \frac{1}{4}d\hat{\phi}\wedge *d\hat{\phi} + \frac{1}{4}e^{-\hat{\phi}}\hat{H}_3 \wedge *\hat{H}_3\Big) \\ & -\frac{1}{4}\int \Big(e^{2\hat{\phi}}d\hat{l}\wedge *d\hat{l} + e^{\hat{\phi}}\hat{F}_3\wedge *\hat{F}_3 + \frac{1}{2}\hat{F}_5\wedge *\hat{F}_5\Big) - \frac{1}{4}\int \hat{C}_4 \wedge \hat{H}_3 \wedge \hat{F}_3 \ . \end{aligned} \tag{2.18}$$

The field strength tensors $\hat{H}_3$, $\hat{F}_3$ and $\hat{F}_5$ are given in terms of the respective potentials as

$$\begin{aligned} \hat{H}_3 =& d\hat{B}_2 \ , \qquad \hat{F}_3 \ = \ d\hat{C}_2 - \hat{l}d\hat{B}_2 \\ \hat{F}_5 =& d\hat{C}_4 - \frac{1}{2}d\hat{B}_2\wedge\hat{C}_2 + \frac{1}{2}\hat{B}_2\wedge d\hat{C}_2 \ . \end{aligned} \tag{2.19}$$

In order to reflect the correct counting of the degrees of freedom that follows from the spectrum of the 10 dimensional type II string theory, the condition of self-duality must be imposed on $\hat{F}_5$: $\hat{F}_5 = *\hat{F}_5$.

The spacetime metric is chosen to have a direct product form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{i\bar{j}}dy^i d\bar{y}^{\bar{j}} \ , \tag{2.20}$$

where $g_{i\bar{j}}$ is the metric on a Ricci flat Calabi-Yau manifold $\mathcal{M}$. As reviewed in the previous section, the geometry of the internal space is characterized by the deformations of the Kähler form J and of the complex structure. The Kähler form J can be expanded in terms of ω_A, the basis of harmonic $(1,1)$ forms of $\mathcal{M}$ as:

$$J = v^A(x)\omega_A, \qquad A = 1, ..., h^{(1,1)}, \tag{2.21}$$

while the deformations of the complex structure are given in terms of the holomorphic 3-form Ω in the basis of harmonic $(1,2)$ forms

$$\delta g_{ij} = \frac{i}{|\Omega|^2}\overline{z}^K(x)(\overline{\chi}_K)_{i\bar{i}\bar{j}}\Omega^{\bar{i}\bar{j}}{}_j, \qquad K = 1, ..., h^{(1,2)}. \tag{2.22}$$

From the 4 dimensional point of view the $h^{(1,1)}$ coefficients $v^A(x)$ of Eq. (2.21) are scalar fields that go under the name of Kähler moduli. Similarly the $h^{(1,2)}$ $\overline{z}^K(x)$ of Eq. (2.22) are scalar fields usually called complex structure moduli. Together with the string dilaton ϕ these are the famous moduli fields of string compactifications that play a pivotal role in determining the physics in 4 dimensions much in the same way as the radial modulus $R(x)$ of the simpler Freund-Rubin compactification discussed in Section 2.1.

Following the standard Kaluza-Klein reduction program we expand the remaining degrees of freedom of the theory in terms of the harmonic forms on $\mathcal{M}$ and keep only the zero modes. Requiring $\mathcal{M}$ to be a Calabi-Yau manifold sets the non-trivial cohomology groups to be as described in Table 2.1. We then find:

Cohomology group	dimension	basis
$H^{(1,1)}$	$h^{(1,1)}$	ω_A
$H^{(2,2)}$	$h^{(1,1)}$	$\tilde{\omega}^A$
$H^{(3)}$	$2h^{(2,1)}+2$	$(\alpha_{\hat{K}}, \beta^{\hat{L}})$
$H^{(2,1)}$	$h^{(2,1)}$	χ_K

Table 2.1: Cohomology groups on $\mathcal{M}$.

$$\hat{B}_2 \;=\; B_2(x) + b^A(x)\,\omega_A\;, \qquad \hat{C}_2 \;=\; C_2(x) + c^A(x)\,\omega_A \tag{2.23}$$

$$\hat{C}_4 \;=\; D_2^A(x)\wedge\omega_A + V^{\hat{K}}(x)\wedge\alpha_{\hat{K}} - U_{\hat{K}}(x)\wedge\beta^{\hat{K}} + \rho_A(x)\,\tilde{\omega}^A\;, \tag{2.24}$$

where $A = 1, ..., h^{(1,1)}$ and $\hat{K} = 0, ..., h^{(1,2)}$. From this expansion we see that from the 4 dimensional point of view there will be the following scalar degrees of freedom: b^A, c^A ρ^A as well as the type IIB dilaton ϕ, the axion l and the geometric moduli. The spectrum of the 4D theory also features the $V^{\hat{K}}$, $U_{\hat{K}}$ one-forms and the B_2, C_2 and D_2^A 2-forms. The self-duality condition on $\hat{F}_5$ will eliminate half of the degrees of freedom in $\hat{C}_4$. Here we follow [23] and eliminate D_2^A and $U_{\hat{K}}$. The resulting spectrum is that of $\mathcal{N}=2$ supergravity with a gravity multiplet, $h^{(2,1)}$ vector multiplets, $h^{(1,1)}$ hypermultiplets and one universal double tensor multiplet with the field content summarized in Table 2.2.

Integrating over $\mathcal{M}$ and performing a further conformal transformation in the 4D metric we find that Eq. (2.18) reduces to

$$\begin{aligned} S_{IIB}^{(4)} = \int -\frac{1}{2}R*1 + \frac{1}{4}\mathrm{Re}\mathcal{M}_{\hat{K}\hat{L}}F^{\hat{K}}\wedge F^{\hat{L}} + \frac{1}{4}\mathrm{Im}\mathcal{M}_{\hat{K}\hat{L}}F^{\hat{K}}\wedge *F^{\hat{L}} - \\ - G_{KL}dz^K\wedge *d\bar{z}^L - h_{\hat{A}\hat{B}}dq^{\hat{A}}\wedge *dq^{\hat{B}}. \end{aligned} \tag{2.25}$$

gravity multiplet	1	$(g_{\mu\nu}, V^0)$
vector multiplets	$h^{(2,1)}$	(V^K, z^K)
hypermultiplets	$h^{(1,1)}$	(v^A, b^A, c^A, ρ_A)
double tensor multiplets	1	(B_2, C_2, ϕ, l)

Table 2.2: Bosonic components of the $\mathcal{N} = 2$ multiplets for IIB supergravity on a Calabi-Yau manifold.

Since in 4 dimensions B_2 and C_2 can be dualised to scalars the double tensor multiplet can be written as a hypermultiplet and so we denote the $h^{(1,1)} + 1$ hypermultiplets by $q^{\hat{A}}$. In Eq. (2.25) $F^{\hat{K}} = dV^{\hat{K}}$, $\mathcal{M}_{\hat{K}\hat{L}}$ is the gauge kinetic matrix, G_{KL} is the metric on the space of complex structure deformations and $h_{\hat{A}\hat{B}}$ is the metric on the moduli space spanned by the hypermultiplets $q^{\hat{A}}$.

There are two important conclusions to draw from this computation. Firstly that it is possible, once we choose the geometry of the compact space, to explicitly compute the spectrum of the 4 dimensional theory and to write down its action by following the standard Kaluza-Klein approach, this is the first step one must take to bring the 10 dimensional supergravity in contact with 4 dimensional physics. In doing so we find that the space spanned by the scalar bosons factorizes into a direct product form $\mathcal{M} = \mathcal{M}^{cs} \times \mathcal{M}^{q}$ that admits $\mathcal{M}^{cs} \times \mathcal{M}^{k}$ as a subspace. Secondly the spectrum is that of $\mathcal{N} = 2$ supergravity which is less appealing for phenomenology than the $\mathcal{N} = 1$ theory. In what follows we will show how the inclusion of extra string theory ingredients into the compactification geometry allows for the breaking of some of the supersymmetry leaving us with a $\mathcal{N} = 1$ theory.

2.3.2 $\mathcal{N} = 1$ compactifications on Calabi-Yau orientifolds

We now turn our attention to the problem of obtaining a $\mathcal{N} = 1$ theory having the $\mathcal{N} = 2$ supergravity described above as the starting point. A fundamental role in the breaking of part of the supersymmetry is played by orientifold planes, higher dimensional extended objects with negative tension that carry no physical degrees of freedom. The importance of these orientifold planes is twofold: they allow us to reduce the amount of supersymmetry and at the same time are crucial for the internal consistency of flux compactifications. We will leave the discussion of their role in the consistency of flux compactifications to Section 2.4.1 and focus for now on the truncation of the spectrum to that of $\mathcal{N} = 1$ supergravity.

Starting from type II supergravity compactified on a Calabi-Yau manifold $\mathcal{M}$ we can obtain a $\mathcal{N} = 1$ theory by modding out by one of the following two symmetry operations, usually called orientifold projections:

$$\mathcal{O}_{(1)} = (-1)^{F_L}\Omega_p\sigma^*, \qquad \sigma^*\Omega = -\Omega, \tag{2.26}$$

or

$$\mathcal{O}_{(2)} = \Omega_p\sigma^*, \qquad \sigma^*\Omega = \Omega. \tag{2.27}$$

Here Ω_p denotes an orientation reversal of the string world-sheet, σ is an internal symmetry that acts only on $\mathcal{M}$, leaving the 4D non-compact space intact and F_L is the left moving fermion number. σ must be a isometric and holomorphic involution ($\sigma^2 = 1$) of $\mathcal{M}$, it acts on the holomorphic 3-form Ω as can be seen in Eqs. (2.26), (2.27) and leaves the Kähler form unchanged: $\sigma^* J = J$. The orientifold projection $\mathcal{O}_{(1)}$ leads to compactifications with O3 and O7 planes while $\mathcal{O}_{(2)}$ leads to compactifications with O5 and O9 planes. The truncation of $\mathcal{N} = 2$ to $\mathcal{N} = 1$ is achieved by keeping in the physical spectrum only the degrees of freedom that are invariant under the orientifold projection. In what follows we will focus on orientifold compactifications with O3 and O7 planes.

The action of σ on cohomology groups $H^{(p,q)}$ is to split them in to two subgroups, one even $H_+^{(p,q)}$ and one odd $H_-^{(p,q)}$ under σ^*:

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}. \tag{2.28}$$

Consequently harmonic (p,q) forms which are the basis in which we expand the 10 dimensional degrees of freedom of the theory split into an even and an odd part under the action of σ. The action of σ on the non-trivial cohomology groups of $\mathcal{M}$, their respective dimensions and basis elements are given in Table 2.3.

cohomology group		dimension	basis
$H^{(1,1)}$	$H_+^{(1,1)}$	$h_+^{(1,1)}$	ω_α
	$H_-^{(1,1)}$	$h_-^{(1,1)}$	ω_a
$H^{(2,2)}$	$H_+^{(2,2)}$	$h_+^{(1,1)}$	$\tilde{\omega}^\alpha$
	$H_-^{(2,2)}$	$h_-^{(1,1)}$	$\tilde{\omega}^a$
$H^{(2,1)}$	$H_+^{(2,1)}$	$h_+^{(2,1)}$	χ_κ
	$H_-^{(2,1)}$	$h_-^{(2,1)}$	χ_k
$H^{(3)}$	$H_+^{(3)}$	$2h_+^{(2,1)}$	$(\alpha_\kappa, \beta^\lambda)$
	$H_-^{(3)}$	$2h_-^{(2,1)} + 2$	(α_k, β^l)

Table 2.3: Splitting of the cohomology groups under the action of σ.

The 10 dimensional fields of type IIB supergravity transform under world-sheet parity and left moving fermion number as described in Table 2.4.

	Ω_p	$(-1)^{F_L}$	$\Omega_p(-1)^{F_L}$
$\hat{\phi}$	+	+	+
$\hat{g}$	+	+	+
$\hat{B}_2$	-	+	-
$\hat{C}_2$	+	-	-
$\hat{l}$	-	-	+
$\hat{C}_4$	-	-	+

Table 2.4: Transformation properties of the 10 dimensional type IIB fields under $\Omega_p(-1)^{F_L}$.

From the transformation properties listed in Table 2.4 one sees that the invariant states must transform under σ^* as follows:

$$\begin{aligned} \sigma^*\hat{\phi} &= \hat{\phi}, & \sigma^*\hat{l} &= \hat{l}, \\ \sigma^*\hat{g} &= \hat{g}, & \sigma^*\hat{C}_2 &= -\hat{C}_2, \\ \sigma^*\hat{B}_2 &= -\hat{B}_2, & \sigma^*\hat{C}_4 &= \hat{C}_4. \end{aligned} \tag{2.29}$$

The 4 dimensional spectrum of the theory is then truncated by keeping only the terms invariant under the orientifold projection. These include the axion $\hat{l}$ and the dilaton $\hat{\phi}$, the following subset of the flux degrees of freedom:

$$\begin{aligned} \hat{B}_2 &= B_2(x) + b^a(x)\,\omega_a\ , \qquad \hat{C}_2 \;=\; C_2(x) + c^a(x)\,\omega_a \\ \hat{C}_4 &= D_2^\alpha(x)\wedge\omega_\alpha + V^\kappa(x)\wedge\alpha_\kappa - U_\kappa(x)\wedge\beta^\kappa + \rho_\alpha(x)\,\tilde{\omega}^\kappa\ . \end{aligned} \tag{2.30}$$

The invariance of the Kähler form under σ^* means that only the even terms in its expansion survive

$$J = v^\alpha\omega_\alpha, \qquad \alpha = 1, ..., h_+^{(1,1)}. \tag{2.31}$$

Furthermore we see that since $\hat{g}$ is invariant under σ^* and Ω is odd, the complex structure deformations that are kept in the spectrum are those corresponding to elements in $H_-^{(1,2)}$:

$$\delta g_{ij} = \frac{i}{|\Omega|^2}\bar{z}^k(\bar{\chi}_k)_{i\bar{i}\bar{j}}\Omega_j^{\bar{i}\bar{j}}, \qquad k = 1, ..., h_-^{(1,2)}. \tag{2.32}$$

It then follows that the bosonic spectrum of the $\mathcal{O}_1$ orientifold compactification, as described in Eqs. (2.30)-(2.32), is that of a $\mathcal{N}=1$ supergravity as summarised in table 2.5.

Comparing the $\mathcal{N}=2$ spectrum of Table 2.2 with the $\mathcal{N}=1$ spectrum of Table 2.5 one sees that V_0 left the gravity multiplet, that the $h^{(2,1)}$ vector multiplets of the

gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_+^{(2,1)}$	V^κ
	$h_-^{(2,1)}$	z^k
chiral multiplets	1	(ϕ, l)
	$h_-^{(1,1)}$	(b^a, c^a)
chiral/linear multiplet	1	(v^α, ρ_α)

Table 2.5: Bosonic components of the $\mathcal{N} = 1$ multiplets for IIB supergravity on a O3/O7 orientifold compactification.

$\mathcal{N} = 2$ compactification give rise to $h_+^{(2,1)}$ vector multiplets and $h_-^{(2,1)}$ chiral multiplets. Furthermore the $h^{(1,1)} + 1$ hypermultiplets lose half of their degrees of freedom and are reduced to $h^{(1,1)} + 1$ chiral multiplets.

To find the action governing the dynamics of the surviving degrees of freedom one must impose the truncation of the spectrum in the 4 dimensional action of Eq. (2.25). As shown in [23], the resulting action can be written in terms of chiral multiplets in the standard $\mathcal{N} = 1$ supergravity form, in terms of the Kähler potential K, the holomorphic superpotential W and the holomorphic gauge kinetic function f as

$$S_{IIB}^{(4)} = \int -\frac{1}{2}R{*}1 + K_{I\bar{J}} D\Phi^I \wedge {*}D\bar{\Phi}^{\bar{J}} + \frac{1}{2}\mathrm{Re} f_{\kappa\lambda} F^\kappa \wedge {*}F^\lambda + \frac{1}{2}\mathrm{Im} f_{\kappa\lambda} F^\kappa \wedge F^\lambda + V{*}1 \quad (2.33)$$

where Φ^I denotes all the complex scalars of the theory and $F^\kappa = dV^\kappa$. The scalar potential V is given by the sum of the F and D-term potentials

$$V = V_F + V_D \quad (2.34)$$

with

$$V_F = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2). \quad (2.35)$$

By definiton $K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K$, $K^{I\bar{J}} = (K^{-1})_{I\bar{J}}$ and the Kähler covariant derivative is $D_I W = \partial_I W + W \partial_I K$. The D-term potential is given by

$$V_D = \frac{1}{2}(Ref)^{-1\kappa\lambda} D_\kappa D_\lambda, \qquad \text{with} \qquad D_\kappa = (K_I + W_I/W)(T_\kappa)_{IJ}\Phi_J. \quad (2.36)$$

The field space coordinates that descend from the compactification of the 10 dimensional theory are not Kähler, that is to say that the kinetic terms do not take the form of $K_{I\bar{J}} D\Phi^I \wedge *D\bar{\Phi}^{\bar{J}}$ as in Eq. (2.33). In order to have the metric in field space taking a manifestly Kähler form one must therefore define the Kähler coordinates $\Phi^I \equiv (\tau, G^a, T_\alpha, z^k)$ in terms of the original fields descending from the dimensional reduction process. It turns out that the complex structure moduli z^k are good Kähler coordinates and that the remaining ones are defined by

$$\begin{aligned} \tau = l + ie^{-\phi}, \qquad G^a = c^a - \tau b^a, \\ T_\alpha = \frac{3i}{2}\rho_\alpha + \frac{3}{4}\mathcal{K}_\alpha(v) + \frac{3}{2}\zeta_\alpha(\tau'\bar{\tau}, G, \bar{G}), \end{aligned} \tag{2.37}$$

where

$$\mathcal{K}_\alpha = \mathcal{K}_{\alpha\beta\gamma}v^\alpha v^\gamma \qquad \text{and} \qquad \zeta_\alpha = -\frac{i}{2(\tau-\bar{\tau})}\mathcal{K}_{\alpha bc}G^b(G-\bar{C})^c. \tag{2.38}$$

In these coordinates the Kähler potential takes the form

$$K = K_{cs}(z,\bar{z}) + K_k(\tau, T, G), \tag{2.39}$$

where K_{cs} is the Kähler potential in the space of complex structure deformations

$$K_{cs}(z,\bar{z}) = -\log\left(-i\int \Omega(z)\wedge\bar{\Omega}(\bar{z})\right), \tag{2.40}$$

and K_k the Kähler potential for the axio-dilaton τ, Kähler moduli T^α and the 2-form moduli G^a, defined in terms of the volume of the compact space $\mathcal{K} \equiv \mathcal{K}_{\alpha\beta\gamma}v^\alpha v^\beta v^\gamma \equiv$ 6Vol(Y) as

$$K_k = -\log\left(-i(\tau-\bar{\tau})\right) - 2\log\left(\mathrm{Vol(Y)}\right). \tag{2.41}$$

For reasons that will become apparent in the next section, we are most interested in compactifications with O3 planes, for which the 2-form moduli vanish: $G^a = 0$. In this particular case we see that $T_\alpha = \frac{3i}{2}\rho_\alpha + \frac{3}{4}\mathcal{K}_\alpha$.

From the structure of the Kähler potential of Eq. (2.39) it follows that the metric in moduli space is block diagonal, with no mixing between the complex strucutre deformations and the remaining moduli, so the moduli space of the theory is given as a direct product: $\mathcal{M} = \mathcal{M}_{cs}^{h_-^{(1,2)}} \times \mathcal{M}_k^{h^{(1,1)}+1}$.

The F-term potential V_F one obtains from dimensional reduction can be derived from the Gukov-Vafa-Witten superpotential [24]:

$$W(\tau, z^k) = \int_Y \Omega\wedge G_3, \tag{2.42}$$

where G_3 is the combined 3-flux

$$G_3 = F_3 - \tau H_3, \tag{2.43}$$

as shown in [23]. Grimm and Louis also prove in [23] that the gauge kinetic functions $f_{\kappa\lambda}$ are holomorphic in the complex structure moduli, $f_{\kappa\lambda} = f_{\kappa\lambda}(z^k)$, and just like W, independent of the Kähler moduli T^α. This can be understood from the fact that the T^α possess exact Peccei-Quinn symmetries which prevent them from entering the holomorphic superpotential and gauge kinetic functions. This fact has far reaching consequences regarding the low energy phenomenology that follows from these models, in particular for the stabilisation of the Kähler moduli of the theory.

2.4 Flux compactifications and moduli stabilisation

Having looked at the spectrum and the action of orientifold compactifications of type IIB string theory we now focus on the flux generated potential and the issue of moduli stabilisation. We follow a chronological order, which is also an order of increasing sophistication and realism. We show how the IIB flux compactifications on Calabi-Yau orientifolds of [25] , whose spectrum was discussed in the previous Section, allow for the stabilisation of some but not all of the moduli of the theory. Crucially, the constructions of [25] are unable to stabilise the volume of the extradimensional manifold and so the inclusion of higher order corrections like those of [26–28] is needed to achieve this fundamental goal. We will see how these different corrections can by themselves or combined with each other give rise to more realistic models of particle physics and cosmology in string theory. Particular emphasis will be given to the large volume scenario developed in [29].

2.4.1 Orientifold compactifications

A very significant advance in string phenomenology came at the beginning of the century when Giddings, Kachru and Polchinski realised that the existence in string theory of extended objects like D-branes and orientifold planes allowed for a consistent higher dimensional theory with non-trivial fluxes. These flux compactifications on Calabi-Yau orientifolds form the foundations of the work presented in this Thesis and so I will now review the main arguments of [25].

The starting point of this analysis is the 10 dimensional action for type IIB supergravity. In the Einstein frame it is given by Eq. (2.18) which we rewrite here as

$$S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|g|}\left\{\mathcal{R} - \frac{\partial_M\tau\partial^M\tau}{2\,\mathrm{Im}(\tau)^2} - \frac{G_{(3)}.\overline{G}_{(3)}}{12\,\mathrm{Im}\tau} - \frac{\tilde{F}_{(5)}^2}{4.5!}\right\} + S_{CS} + S_{loc} \quad (2.44)$$

where

$$G_{(3)} = F_{(3)} - \tau H_{(3)} \tag{2.45}$$

is the combined 3-flux, τ is the axio-dilaton and

$$\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2} C_{(2)} \wedge H_3 + \frac{1}{2} B_{(2)} \wedge F_3 \tag{2.46}$$

is the self-dual 5-flux. S_{SC} is the Chern-Simons action, given by

$$S_{SC} = \int \frac{C_{(4)} \wedge G_{(3)} \wedge \overline{G}_{(3)}}{\mathrm{Im}\tau} \tag{2.47}$$

and S_{loc} is the action for the local sources which, neglecting fluxes, is given by

$$S_{loc} = -T_p \int_{R^4 \times \Sigma} d^{p+1}\xi \sqrt{-g} + \mu_p \int_{R^4 \times \Sigma} C_{p+1}. \tag{2.48}$$

Taking the spacetime metric to be given by the warped product of a 4D non-compact space and a 6D Calabi-Yau

$$ds^2_{10} = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn} dy^m dy^n, \tag{2.49}$$

allowing the axio-dilaton to vary in the compact space $\tau = \tau(y)$ and setting

$$\tilde{F}_{(5)} = (1 + *)(d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3), \tag{2.50}$$

one can show that the trace reversed Einstein equations in the non-compact directions reads

$$\mathcal{R}_{\mu\nu} = -g_{\mu\nu} \left(\frac{G_{mnp} \overline{G}^{mnp}}{48\, \mathrm{Im}\, \tau} + \frac{e^{-8A}}{4} \partial_m \alpha \partial^m \alpha \right) + \kappa^2_{10} \left(T^{\mathrm{loc}}_{\mu\nu} - \frac{1}{8} g_{\mu\nu} T^{\mathrm{loc}} \right). \tag{2.51}$$

Taking into account the warped metric of Eq. (2.49) this becomes

$$\tilde{\nabla}^2 e^{4A} = e^{2A} \frac{G_{mnp} \bar{G}^{mnp}}{12\, \mathrm{Im}\, \tau} + e^{-6A} \left[\partial_m \alpha \partial^m \alpha + \partial_m e^{4A} \partial^m e^{4A} \right] + \frac{\kappa^2_{10}}{2} e^{2A} (T^m_m - T^\mu_\mu)^{\mathrm{loc}}. \tag{2.52}$$

This equality severely constrains the brane/flux configurations that lead to warped solutions on compact manifolds. To see how, note that on a compact manifold the left hand side integrates to zero while on the right hand side we have a sum of positive definite terms. So in the absence of localised sources there is a no-go theorem that states that fluxes must vanish and the warp factor must be a constant [30, 31]. If however one allows for the presence of localised sources with negative tension like orientifold planes, warped metrics and non trivial fluxes can be made consistent with the integrated field equations (2.52).

The O3 planes that are required to circumvent the above no-go theorem carry D3 brane charge and so will source $\tilde{F}_{(5)}$. Therefore one must check that the addition of these localised sources is consistent with the equation of motion for the 5-form flux. To see this note that the equation of motion for the self dual $\tilde{F}_{(5)}$ is

$$d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} + 2\kappa_{10}^2 T_3 \rho_{loc}. \tag{2.53}$$

Integrating over the internal manifold yields

$$\frac{1}{2\kappa_{10}^2 T_3} \int_{\mathcal{M}} H_{(3)} \wedge F_{(3)} + Q_3^{loc} = 0, \tag{2.54}$$

where Q_3^{loc} is the total D3 brane charged of the localised objects such as D3 branes, O3 planes and fractional D7 branes. This is the so called tadpole cancellation condition and it states that the total D3 brane charge coming from localised objects and flux background must vanish.

In order to further constrain the configurations that might give rise to consistent warped flux compactifications note that the $\tilde{F}_{(5)}$ Bianchi identity can be written in terms of the potential α as

$$\tilde{\nabla}^2 \alpha = ie^{2A} \frac{G_{mnp}(*_6 \bar{G}^{mnp})}{12\,\mathrm{Im}\,\tau} + 2e^{-6A} \partial_m \alpha \partial^m e^{4A} + 2\kappa_{10}^2 e^{2A} T_3 \rho_3^{\mathrm{loc}} \ , \tag{2.55}$$

Subtracting Eqs. (2.52) and (2.55) we find

$$\begin{aligned} \tilde{\nabla}^2 (e^{4A} - \alpha) =& \frac{e^{2A}}{6\,\mathrm{Im}\,\tau} \left| iG_{(3)} - *_6 G_{(3)} \right|^2 + e^{-6A} |\partial(e^{4A} - \alpha)|^2 + \\ &+ 2\kappa_{10}^2 e^{2A} \left[\frac{1}{4} (T^m_m - T^\mu_\mu)^{\mathrm{loc}} - T_3 \rho_3^{\mathrm{loc}} \right]. \end{aligned} \tag{2.56}$$

The left-hand-side of the previous equation integrates to zero while in general the right hand side is non-negative. Therefore consistency requires that the warp factor and the potential α are related via

$$e^{4A} = \alpha^2, \tag{2.57}$$

the 3-form flux is imaginary self-dual (ISD)

$$iG_{(3)} = *_6 G_{(3)} \tag{2.58}$$

and the local sources satisfy

$$\frac{1}{4} (T^m_m - T^\mu_\mu)^{\mathrm{loc}} = T_3 \rho_3^{\mathrm{loc}}. \tag{2.59}$$

An example of sources that satisfy this condition are D3 branes and O3 planes, for which $T_0^0 = T_1^1 = T_2^2 = T_3^3 = -T_3\rho_3$ and $T_m^m = 0$.

The ISD condition implies that the only nonzero components of the flux are the $(2,1)$ and $(0,3)$. If one further requires unbroken supersymmetry then only the $(2,1)$ part survives. For a given choice of fluxes $F_{(3)}$ and $H_{(3)}$, the ISD condition fixes the complex structure moduli and the dilaton.

This setup then allows for consistent warped compactifications with flux, that stabilise the complex structure moduli and the axio-dilaton. They can accommodate both supersymmetric and non-supersymmetric solutions depending on the choice of internal fluxes.

2.4.2 Kähler moduli and the no-scale structure

Having introduced flux compactifications from a 10 dimensional point of view we now translate them into the language of 4 dimensional $\mathcal{N} = 1$ supergravity. Following the process of dimensional reduction outlined in Section 2.3 we find that the 4 dimensional theory for the scalar moduli is determined, to leading order, by the following Kähler potential and superpotential:

$$K = -2\log(\mathcal{V}) - \log(S + \overline{S}) - \log\left[-i\int_{\mathcal{M}} \Omega \wedge \overline{\Omega}\right] \equiv K_{tree}, \qquad (2.60)$$

$$W = \int_{\mathcal{M}} G_3 \wedge \Omega \equiv W_{tree}. \qquad (2.61)$$

These combine to generate the F-term potential for the moduli sector,

$$V = e^K \left(K^{A\overline{B}} D_A W \overline{D_B W} - 3|W|^2\right), \qquad (2.62)$$

which plays a pivotal role determining the vacuum structure of flux compactifications which we study in this Thesis. The sum in Eq. (2.62) runs over all the moduli of the theory: complex structure moduli (U_a), axio-dilaton (S) and Kähler moduli (T_a). The Kähler derivative is defined as

$$D_A W = \partial_A W + W \partial_A K, \qquad (2.63)$$

where $A = \{U_a, S, T_a\}$. Expanding the sum we find that V can be written as

$$V = e^K \left(K^{S\overline{S}} D_S W \overline{D_S W} + K^{U\overline{U}} D_U W \overline{D_U W} + K^{T\overline{T}} D_T W \overline{D_T W} - 3|W|^2\right). \qquad (2.64)$$

As explained in Section 2.4.1 one can choose the flux G_3 such that the axio-dilaton and the complex structure moduli are stabilised in a supersymmetric way, that is

$$D_S W = D_{U_a} W = 0. \qquad (2.65)$$

Note that the potential for these moduli is positive semi-definite, being given as the absolute value of a complex function, and so $V = 0$ corresponds to a minimum. We define the value of W for which Eq. (2.65) holds by W_0. With this choice of fluxes we find that the F-term potential becomes

$$\begin{aligned} V &= e^K \left(K^{T\overline{T}} D_T W_0 \overline{D_T W_0} - 3|W_0|^2 \right) \\ &= e^K \left(K^{T\overline{T}} (\partial_T W_0 \overline{\partial_T W_0} + \partial_T W_0 K \partial_{\overline{T}} K + \overline{\partial_T W_0} K \partial_T K) + (K^{T\overline{T}} \partial_T K \partial_{\overline{T}} K - 3)|W_0|^2 \right). \end{aligned} \tag{2.66}$$

Noting that the tree level superpotential, as defined in Eq. (2.61), depends on the axio-dilaton and on the complex structure moduli but not on the Kähler moduli and that for the leading order terms in the Kähler potential of Eq. (2.60) the following relation holds

$$K^{T_i\overline{T_j}} \partial_{T_i} K_{tree} \partial_{\overline{T_j}} K_{tree} = 3, \tag{2.67}$$

we see that there is complete cancellation of all terms in Eq. (2.66) at leading order in the expansion in g_s and α'. Consequently all the Kähler moduli are flat directions of the scalar potential. Phenomenologically this is a crucial hurdle that must be overcome if one wants to have any hope of relating the higher dimensional theory to our 4 dimensional world. This can be achieved if one manages to generate a potential for the Kähler moduli that has a minimum keeping the geometry of the extradimensional manifold fixed in a controlled regime.

Fortunately this no-scale behavior only holds at leading order and will break down once we consider subleading corrections to the Kähler potential and the superpotential of Eqs. (2.60) and (2.61). We will now review how this can be achieved.

2.4.3 Perturbative corrections to K

Generically one expects the tree level Kähler potential of Eq. (2.60) to receive both perturbative and non-perturbative corrections, such that the full Kähler potential can schematically be written as:

$$K = K_{tree} + K_{perturb} + K_{nonperturb}. \tag{2.68}$$

However the perturbative corrections are expected to dominate over the non-perturbative ones and in general will be the ones responsible for breaking the no-scale structure described above.

Recall that perturbative expansion of string theory has two independent parameters: the string coupling g_s, which is related to the VEV of the dilaton, and α', which is related to the string length via $l_s = 2\pi\sqrt{\alpha'}$. It then follows that there will be two types of contributions to $K_{perturb}$:

$$K_{perturb} = K_{\alpha'} + K_{g_s}. \tag{2.69}$$

We will now look at the effects of these two corrections separately.

α' corrections

The study of α' corrections to the 4 dimensional action was carried out in [26] following the work of [32]. This analysis is performed by including subleading corrections in α' in the 10 dimensional action for type II string theory and subsequently performing the Kaluza-Klein reduction as outlined in Section 2.3.

In principle there is an infinite series of such corrections and one should worry if it is consistent to truncate the series to a finite set of terms. We will show below that the expansion in α' is an expansion in inverse powers of the volume and therefore it can be controlled in the large volume limit. For a more detailed analysis of this issue see [33].

Explicit computations show that the first correction appears at order α'^3 and so the starting point is the following string frame 10D action [26] :

$$S = \frac{-M_{10}^2}{2} \int d^{10}x \sqrt{-g^{(10)}} e^{-2\phi} (\mathcal{R} + 4(\partial\phi)^2 + \alpha'^3 J_0), \tag{2.70}$$

where J_0 is proportional to $\mathcal{R}^4$ from which, after dimensional reduction one obtains the corrected Kähler potential

$$K = -2\log\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right), \tag{2.71}$$

where ξ is given in terms of the Euler number of the Calabi-Yau manifold X, $\chi(X) = 2(h_{1,1} - h_{2,1})$ as

$$\xi = -\frac{\chi\zeta(3)}{2(2\pi)^3}. \tag{2.72}$$

We have shown before that the no-scale structure descended from the fact that the holomorphic superpotential is independent of the Kähler moduli and from the particular functional form of the leading order Kähler potential K_{tree}. By including these subleading terms we are modifying K and so the no-scale cancellation will no longer happen. To see this note that in the limit of large volumes we can expand

$$K = -2\log\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) \sim -2\log\mathcal{V} - \frac{\xi}{g_s^{3/2}\mathcal{V}} + \mathcal{O}(1/\mathcal{V}^2) \tag{2.73}$$

and so the extra terms, which violate the condition of Eq. (2.67), generate the following correction to the F-term potential:

$$\delta V_{\alpha'} = 3e^{K_{tree}}(\xi/g_s^{3/2})\frac{(\xi/g_s^{3/2})^2 + 7(\xi/g_s^{3/2})\mathcal{V} + \mathcal{V}^2}{\left(\mathcal{V} - (\xi/g_s^{3/2})\right)\left(2\mathcal{V} + (\xi/g_s^{3/2})\right)^2} \sim \frac{3\xi|W|^2}{4g_s^{3/2}\mathcal{V}^3}. \tag{2.74}$$

Note that both in Eq. (2.73) and (2.74) the new terms generated by the α'^3 corrections appear as an inverse power of the volume $\mathcal{V}$. This justifies the claim that as long as the volume of the extradimensional space is sufficiently large, these corrections can be kept under control.

Before moving on to other possible corrections to K we must stress that while the α'^3 term breaks the no-scale structure it is insufficient to stabilise the Kähler moduli. Other ingredients are needed to achieve this goal.

g_s corrections

There is another kind of correction which will in general appear in the full Kähler potential for the moduli sector, these are the string loop corrections. String loop, or g_s, corrections can play an important role in the determination of the vacuum structure of the theory.

In order to estimate the effect of such correction to the 4 dimensional effective field theory, explicit string scattering amplitudes in Calabi-Yau backgrounds must be computed. Such computation is however beyond the reach of current techniques and so one must infer the general properties of the final result by looking at simpler cases. These explicit computations can be performed for a simpler $T^6/Z_2 \times Z_2$ background [34, 35], where it is found that they yield a correction to the Kähler potential of the form

$$K_{g_s} = K_{g_s}^{KK} + K_{g_s}^{W} \tag{2.75}$$

where $K_{g_s}^{KK}$ corresponds to exchange of closed strings carrying KK momentum between D3 and D7 branes and $K_{g_s}^{W}$ corresponds to exchange of winding strings between intersecting stacks of D7 branes. The functional forms of $K_{g_s}^{KK}$ and $K_{g_s}^{W}$ are known explicitly for the $T^6/Z_2 \times Z_2$ background and for the more complicated Calabi-Yau backgrounds are conjectured to be [27, 35]

$$K_{g_s}^{KK} \sim \sum_{i=1}^{h_{1,1}} \frac{\mathcal{C}_i^{KK}(U,\overline{U})a_{il}t^l}{Re(S)\mathcal{V}}, \tag{2.76}$$

where $a_{il}t^l$ is a linear combination of the 2-cycle volumes t_l that is transverse to the 4-cycle wrapped by the i-th D7-brane and

$$K_{g_s}^W \sim \sum_{i=1}^{3} \frac{\mathcal{C}_i^W(U,\overline{U})}{a_{il}t^l\mathcal{V}}, \tag{2.77}$$

where $a_{il}t^l$ is 2-cycle where the two D7-branes intersect. At this level $\mathcal{C}_i^{KK}$ and $\mathcal{C}_i^W$ are unknown functions of the complex structure moduli of the compactification.

Taking into account these perturbative corrections to K, we find that the leading g_s contributions to the scalar potential vanish exactly [27] . This cancellation goes under the name of extended no-scale structure and has been shown to have important phenomenological consequences. The largest g_s contributions to scalar potential take the form

$$V_{gs} = \frac{|W_0|^2}{\mathcal{V}^2}\left(\frac{(C_i^{KK})^2}{Re(S^2)}a_{ik}a_{ij}K^0_{k\bar{j}} - 2\sum_i K^W_{g_s,t_i}\right), \tag{2.78}$$

where $K^0_{k\bar{j}}$ is the Kähler metric computed with the tree level Kähler potential.

2.4.4 Non-perturbative corrections to W

In theories with $\mathcal{N}=1$ supersymmetry, the perturbative holomorphic superpotential W is protected by non-renormalisation theorems. This fact that was originally established in [36] and more recently in [37, 38] has far reaching consequences for the physics of moduli stabilisation. Essentially it means that there will be no perturbative corrections to the tree level Wilsonian superpotential W_{tree} we have so far considered and so the leading corrections will come from non-perturbative effects which would otherwise be too small to play a role. The study of such non-perturbative corrections in conjunction with non-vanishing fluxes in the compact dimensions has allowed for a much closer contact than previously possible between the higher dimensional string theory and the four dimensional world [28].

For type IIB compactifications there are essentially two types of non-perturbative effects that are known to be relevant for the stabilisation of the Kähler moduli: Euclidean D3 branes [39] and gaugino condensation on D7 branes [40].

The action for an Euclidean D3 brane wrapping a 4-cycle Σ_i in the Calabi-Yau manifold is

$$S_{ED3} = \frac{1}{(2\pi)^3\alpha'^2}\int_{\Sigma_i} -\sqrt{g} + iC_4 \tag{2.79}$$

which will enter in the path integral as $e^{-S_{ED3}}$. Noting that $\int_{\Sigma_i}\sqrt{g} = vol(\Sigma_i)$ we see that such term introduces dependence on the Kähler modulus parametrising the volume of the cycle Σ_i. For this to yield a correction to the superpotential the instanton must have exactly two fermionic zero modes in order to generate a term of the form

$$\int d^4x d^2\theta(...),$$

which corresponds to imposing certain topological restrictions on the cycle Σ_i [39]. In such cases it generates an exponential contribution to the superpotential, that depends on the Kähler modulus T_i

$$W_{ED3} \sim e^{-T_i}. \tag{2.80}$$

D7 branes are also a key ingredient in type IIB flux compactifications. The action for a D7 brane wrapping a 4 cycle Σ_i in the Calabi-Yau contains the following term

$$S_{D7} \supset \int_{\mathcal{M}_4\times\Sigma_i} \sqrt{g} e^{-\phi} F_{\mu\nu}F^{\mu\nu} \tag{2.81}$$

and so the gauge coupling for the theory living in the world volume of the brane is

$$\frac{1}{g_{YM}^2} = Re\left(\frac{T_i}{2\pi}\right).$$

Considering an appropriate choice of fluxes is made, a stack of N_c branes gives rise to a pure $\mathcal{N}=1$ supersymmetric SU(N_c) Yang-Mills theory. This theory undergoes gaugino condensation at the scale Λ_{N_c}, generating a superpotential term of the form:

$$W_{D7} = \Lambda_{N_c}^3 = Ae^{2\pi T_i/N_c}. \tag{2.82}$$

This brings about a similar exponential dependence on the Kähler moduli as the ED3 instanton (albeit with different coefficients) and so the impact of both effects on moduli stabilisation can be studied by considering a superpotential of the form:

$$W = W_{tree} + \sum_i A_i e^{-a_i T_i}, \tag{2.83}$$

where the A_i are in general functions of the dilaton and complex structure moduli and $a_i = 2\pi$ for ED3 and $a_i = 2\pi/N_c$ for gaugino condensation.

The simplest illustration of the importance of these non-perturbative effects was put forward in [28] and became subsequently known as the KKLT scenario. We shall review it next.

Consider that the volume of the extradimensional manifold is given as a function of a single Kähler modulus as

$$\mathcal{V} = \left(\frac{T+\overline{T}}{2}\right)^{3/2}. \tag{2.84}$$

To leading order in α' and g_s, the Kähler potential in the Kähler moduli sector is then

$$K = -2\log\mathcal{V} = -3\log\left(\frac{T+\overline{T}}{2}\right). \tag{2.85}$$

Assuming that there is non-perturbative physics supported by the volume mode, the superpotential is

$$W = W_0 + Ae^{-aT}, \tag{2.86}$$

where

$$W_0 \equiv \left\langle \int_{\mathcal{M}} G_3 \wedge \Omega \right\rangle \tag{2.87}$$

is the flux induced superpotential that stabilises the axio-dilaton and the complex structure moduli in a supersymmetric minimum. The strategy of [28] is to look for a supersymmetric minimum for the volume modulus while assuming that the non-perturbative term in W will not displace the tree level minima for the dilaton and complex structure. Note that the condition for unbroken supersymmetry is

$$D_T W = 0 \tag{2.88}$$

which recalling that $D_T W = \partial_T W + W\partial_T K$ becomes

$$W_0 = -Ae^{-aT_0}(1 + \frac{2}{3}aT_0), \tag{2.89}$$

where T_0 denotes the position of the minimum of the volume modulus. So by playing the non-perturbative effects against the flux superpotential it is possible to find a supersymmetric minimum for the volume mode. This minimum is AdS, with

$$V_{AdS} = \langle -3e^K W^2 \rangle = -\frac{a^2A^2e^{-2aT_0}}{6T_0}. \tag{2.90}$$

For the effective field theory that derives from Eqs. (2.85) and (2.86) to be consistent, the volume must be stabilised at large values. This confers validity to the supergravity approximation and allows us to neglect further perturbative corrections to the Kähler potential. One must further require $aT > 1$ for the multi instantonic effects to be negligible and for the single instantonic contribution to be the leading order correction to W_{tree}.

In this setup control can be achieved by having a sufficiently low VEV for the flux superpotential as shown in Eq. (2.89). An illustrative example is provided in [28], where $W_0 = 10^{-4}$, $A = 1$, $a = 0.1$ results in $T_0 \sim 100$. At this level the vacuum is still supersymmetric and AdS, so if one seeks to make contact with the real world, supersymmetry must be broken and the minimum must be lifted. In the original KKLT proposal this is achieved by the addition of one $\overline{D3}$ brane, provided the fluxes are adjusted in order to satisfy the tadpole cancellation condition. This extra ingredient will simultaneously uplift the AdS vacuum to a more attractive dS by adding a term of the form

$$\delta V \sim \frac{D}{ReT^3} \tag{2.91}$$

and since it is non-supersymmetric the $\overline{D3}$ also breaks supersymmetry. The coefficient D depends on the number of $\overline{D3}$ and can be discretely tuned to yield a viable vacuum for the theory. The full potential is then given by

$$V = \frac{aAe^{-a\tau}}{2\tau^2}\left(\frac{1}{3}\tau aAe^{-a\tau} + W_0 + Ae^{-a\tau}\right) + \frac{D}{\tau^3}, \tag{2.92}$$

where we have assumed that all the parameters are real numbers, set the axion of the volume modulus to zero and defined $\tau = Re(T)$.

One crucial feature of the uplifting procedure using terms of the form of Eq. (2.91) is that the position of the minimum before and after uplifting is practically the same. This means that if the volume of the AdS minimum was large then the volume of the uplifted Minkowski or dS minimum will also be large and there will be no issues with the control over the perturbative expansion in the effective field theory.

The addition of the $\overline{D3}$ brane also introduces a barrier in the potential, separating the effective 4 dimensional dS minimum from a 10 dimensional Minkowski minimum at infinity. In this configuration the dS minimum is only meta-stable since it can decay through quantum mechanical tunneling. For this minimum to be phenomenologically viable its life time must be at least of the order of the age of the Universe. The issue of vacuum stability was also studied in [28] where the authors conclude that the life-time is greater than the cosmological timescales of 10^{10} years.

Despite being a huge step in the right direction, the proposal of [28] has also attracted some criticism, reviewed in [41]. In particular:

- It requires the tuning of the flux superpotential to small values, $W_0 \sim W_{np} \sim e^{-T} \ll 1$, while one would generally expect $W_0 \sim \mathcal{O}(1) \gg W_{np}$. In fact $W_0 \sim \mathcal{O}(10)$ are 10^{10} times more common in the flux landscape than $W_0 \sim \mathcal{O}(1)$ [29,42]. So while vacua with low W_0 exist in the landscape, they are statistically disfavoured.

- At least one non-perturbative effect is required on each cycle one wishes to stabilise. Given that generic Calabi-Yau manifolds have many cycles and that there are constraints on when such superpotential terms are generated this is a highly nontrivial requirement. For a possible way around this see [43].

- The addition of $\overline{D3}$ branes to uplift the original supersymmetric vacuum breaks supersymmetry explicitly and there are questions regarding the use of a supersymmetric setup in the first place.

Furthermore, recent considerations on the quantum nature of de Sitter space seem to suggest that the large values of N_c (or equivalently small values of a) required for control of the effective field theory are not consistent with the limits on the entropy of dS space in quantum gravity [44].

Fortunately all these issues can be solved or at least ameliorated if one considers the combined effects of perturbative corrections to K and of non perturbative corrections to W. This is the very core of the large volume scenario of [29] which we review next.

2.4.5 The large volume scenario

In this Section we will review the large volume scenario of type IIB compactifications following the seminal work of [29]. We start by giving the generic proof of the existence of such vacua before presenting an illustrative example.

General argument

In the previous Sections we have argued that the subleading corrections to the Kähler potential and superpotential play a very important role in breaking the no-scale structure and in generating minima for the Kähler moduli. Up to this point the phenomenological consequences of these small effects have been studied separately, first we looked at perturbative corrections to K and then to instantonic effects in W. In [29] it was shown that the combination of these two effects can generate minima of the scalar potential at exponentially large volumes. To study these we first stabilise the dilaton and the complex-structure moduli by solving

$$D_S W = D_{U_a} W = 0 \tag{2.93}$$

and then regard their values as fixed (this approximation is justified *post hoc* by the large value of the stabilised volume, which gives minimal cross-coupling between these sectors). The resulting theory is only dependent on the Kähler moduli and is specified by

$$\hat{K} = -2\ln[\mathcal{V} + \frac{\xi}{2g_s^{3/2}}] \tag{2.94}$$

and

$$\hat{W} = W_0 + \sum_i A_i e^{-a_i T_i}. \tag{2.95}$$

Recall ξ is related to the Euler number of $\mathcal{M}$ via Eq. (2.72). We require that $\xi > 0$ which is equivalent imposing the constraint $h^{(1,2)} > h^{(1,1)}$ on $\mathcal{M}$, so we will be looking at compactification manifolds with more complex structure than Kähler moduli. We further require that $h^{(1,1)} > 1$ so we will focus on manifolds with at least two Kähler moduli. The volume is given by

$$\mathcal{V} = \frac{1}{6}\int_{\mathcal{M}} J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k, \tag{2.96}$$

where J is the Kähler form of $\mathcal{M}$, k_{ijk} are triple intersection numbers and t^i are two-cycle volumes. The fields appearing in the 4D effective theory are the complexified Kähler moduli $T_i \equiv \tau_i + ib_i$ where b_i is the axion and the τ_i are the Einstein frame four-cycle volumes, given by:

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t^i} = \frac{1}{2} k_{ijk} t^j t^k. \tag{2.97}$$

Given Eqs. (2.94) and (2.95) we find that the scalar potential is given by

$$\begin{aligned} V =& e^K \left[G^{T_j \bar{T}_k} \left(a_j A_j a_k \bar{A}_k e^{-\left(a_j T_j + a_k \bar{T}_k\right)} + \left(a_j A_j e^{-a_j T_j} \bar{W} \partial_{\bar{T}_k} K + a_k \bar{A}_k e^{-a_k \bar{T}_k} W \partial_{T_j} K \right) \right) \right. \\ & \left. + 3\xi \frac{\left(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2\right)}{\left(\mathcal{V} - \xi\right)\left(2\mathcal{V} + \xi\right)^2} |W|^2 \right] \\ \equiv & V_{np1} + V_{np2} + V_{\alpha'}, \end{aligned} \tag{2.98}$$

where we have defined

$$V_{np1} \equiv e^K G^{T_j \bar{T}_k} \left(a_j A_j a_k \bar{A}_k e^{-\left(a_j T_j + a_k \bar{T}_k\right)} \right), \tag{2.99}$$

$$V_{np2} \equiv e^K \left(G^{T_j \bar{T}_k} a_j A_j e^{-a_j T_j} \bar{W} \partial_{\bar{T}_k} K + G^{T_k \bar{T}_j} a_k \bar{A}_k e^{-a_k \bar{T}_k} W \partial_{T_j} K \right), \tag{2.100}$$

and

$$V_{\alpha'} \equiv 3\xi \frac{\left(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2\right)}{\left(\mathcal{V} - \xi\right)\left(2\mathcal{V} + \xi\right)^2} e^K |W|^2 \sim \frac{3\xi}{16\mathcal{V}^3} e^{K_{cs}} |W|^2 + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \tag{2.101}$$

This scalar potential is a generalisation of KKLT, since $V_{KKLT} = V_{np1} + V_{np2}$ and so it admits a large volume supersymmetric minimum for small W_0. The argument of [29] is that this potential, taking into account the α'^3 correction, also admits a non-supersymmetric minimum at exponentially large volumes for $W_0 \sim \mathcal{O}(1)$.

The proof of the existence of the minimum relies on the analysis of the behaviour of the potential at small and large volumes: first one shows that at large volumes there is a decompactification limit in which the potential approaches zero from below and then that the potential is positive for small volumes, it then follows by continuity of V that there must be a local AdS minimum along the direction in Kähler moduli space where the volume changes.

Let us now see how this behaviour can be derived from Eq. (2.98). We start by defining the decompactification limit as the $\mathcal{V} \to \infty$ limit in moduli space where $\tau_i \to \infty$ for all moduli except one, which we call τ_s. There are two conditions in τ_s: first this limit should be well defined and second τ_s must appear non-perturbatively in W.

The leading order term in the volume expansion of Eq. (2.99) is

$$V_{np1} \sim e^K G^{T_s \overline{T_s}} a_s^2 |A_s|^2 e^{-2a_s\tau_s}, \tag{2.102}$$

since in the decompactification limit all other terms will be exponentially suppressed. Noting that in the large volume limit the inverse Kähler metric becomes

$$G^{T_s\overline{T_s}} = -\frac{4}{9}\mathcal{V} k_{ssk} t^k + \mathcal{O}(1) \tag{2.103}$$

we find

$$V_{np1} \sim \frac{-k_{ssk} t^k a_s^2 |A_s|^2 e^{-2a_s\tau_s} e^{K_{cs}}}{\mathcal{V}} + \mathcal{O}\left(\frac{e^{-2a_s\tau_s}}{\mathcal{V}^2}\right), \tag{2.104}$$

up to numerical prefactors. Despite the minus sign, this term is positive since $G^{T_i\overline{T_j}}$ is positive.

Performing the same expansion for V_{np2} we find that the leading order terms are those with τ_s in the exponential

$$V_{np2} \sim e^K \left(G^{T_s\bar{T}_k} a_s A_s e^{-a_s T_s} \bar{W} \partial_{\bar{T}_k} K + G^{T_s\bar{T}_j} a_s \bar{A}_s e^{-a_s \bar{T}_s} W \partial_{T_j} K \right). \tag{2.105}$$

The key to extracting the large volume behaviour of this term is to note that its overall sign is set by the axion of the T_s modulus. To see this note that the Kähler potential is a function of $T_i + \overline{T}_j$ and the Kähler metric and its inverse are symmetric. Keeping this in mind one can write

$$V_{np2} \sim X e^{-a_s T_s} + \overline{X} e^{-a_s \overline{T}_s}, \tag{2.106}$$

with $X = e^K G^{T_s\bar{T}_k} a_s A_s \bar{W} \partial_{\bar{T}_k} K$. Then explicitly writing the Kähler modulus in terms of its real and imaginary parts and defining $X \equiv |X| e^{i\phi_X}$ this becomes

$$V_{np2} \sim 2|X| e^{-a_s\tau_s} \cos(\phi_X - a_s b_s) \tag{2.107}$$

which is negative when the axion is at its minimum $\langle b_s \rangle = \frac{\phi_X - \pi}{a_s}$. Noting that $G^{T_s \bar{T}_k} \partial_{\bar{T}_k} K = -2\tau_s$ we write

$$V_{np2} \sim -\frac{4\tau_s a_s |A_s W| e^{-a_s \tau_s}}{\mathcal{V}^2}. \tag{2.108}$$

Finally we note that the large volume behaviour of Eq. (2.101) is given by

$$V_{\alpha'} \sim \frac{3\xi}{16\mathcal{V}^3} e^{K_{cs}} |W|^2 + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \tag{2.109}$$

Adding these three contributions together, we see that in the large volume limit the F-term potential becomes

$$V \sim \frac{-k_{ssk} t^k a_s^2 |A_s|^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{\tau_s a_s |A_s W| e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\xi}{\mathcal{V}^3}|W|^2 \tag{2.110}$$

up to overall normalisation factors. In the decompactification limit, when $\mathcal{V} \to \infty$ with $a_s \tau_s = \log \mathcal{V}$, this potential approaches zero form below, since the volume scaling of the three individual terms is

$$V \sim \underbrace{\frac{-k_{ssk} t^k a_s^2 |A_s|^2}{\mathcal{V}^3}}_{\mathcal{O}(\sqrt{\log \mathcal{V}}/\mathcal{V}^3)} - \underbrace{\frac{\log \mathcal{V} |A_s W|}{\mathcal{V}^3}}_{\mathcal{O}(\log \mathcal{V}/\mathcal{V}^3)} + \underbrace{\frac{\xi}{\mathcal{V}^3}|W|^2}_{\mathcal{O}(1/\mathcal{V}^3)}. \tag{2.111}$$

We then see that in the limit $\mathcal{V} \to \infty$ the second term, which is negative, will dominate. This justifies the claim that the potential will approach zero from below. On the other hand, in the limit of small volumes[2], the potential will be dominated by either the first or the third terms depending on the details of the model, since both are positive the potential will be positive. Continuity of the scalar potential then implies that somewhere between these two limits there will be an AdS minimum. Furthermore, since $\langle V \rangle \sim \mathcal{O}(1/\mathcal{V}^3)$ and $e^K |W|^2 \sim \mathcal{O}(1/\mathcal{V}^2)$ it is clear that the T_s modulus must develop a non-zero F-term and so the AdS minimum is non-supersymmetric.

Another interesting feature of this scenario is that the gravitino mass $m_{3/2} = \langle e^{K/2}|W| \rangle \sim |W_0|/\mathcal{V}$ is independent of the flux superpotential, since at the large volume minimum $\mathcal{V} = W_0 \times f(\tau_s, a_s, A_s)$. We will illustrate this in more detail below.

[2]Here the notion of small volume is a relative one as these are large in string units, in particular they will still be large enough for us to have control over the effective field theory. Further α' corrections to K, which will generate potential terms of the order $1/\mathcal{V}^n$ can still be safely neglected in this regime.

Example: Swiss cheese Calabi-Yau manifolds

Having presented the generic proof of existence of the large volume minimum we now give an explicit example following [29]. Discussions extending this to more general manifolds can be found in [45]. The simplest geometry that yields the desired phenomenology is a Calabi-Yau orientifold defined by a degree 18 hypersurface in $\mathbb{P}^4_{[1,1,1,6,9]}$:

$$z_1^{18} + z_2^{18} + z_3^{18} + z_4^3 + z_5^2 - 18\psi z_1 z_2 z_3 z_4 z_5 - 3\phi z_1^6 z_2^6 z_3^6 + ... = 0, \tag{2.112}$$

where z_i are the coordinates of the complex projective space and ψ and ϕ are complex structure moduli. This manifold has $h_{1,1} = 2$ and $h_{2,1} = 272$, so clearly it satisfies the requirement $\xi > 0$. The ellipsis denote other 270 terms that are not invariant under the orientifold projection.

For moduli stabilisation studies the most relevant geometrical quantity is the volume of the compact space, in particular the relation between the volume and the Kähler moduli of the theory. In this example the overall volume is given in terms of 2-cycle volumes as

$$\mathcal{V} = \frac{1}{6}(3t_1^2 t_5 + 18 t_1 t_5^2 + 36 t_5^3). \tag{2.113}$$

The 4-cycle volumes are therefore $\tau_4 = t_1^2/2 \equiv \tau_s$ and $\tau_5 = (t_1 + 6t_5)^2/2 \equiv \tau_b$ and we can rewrite the volume as

$$\mathcal{V} = \frac{1}{9\sqrt{2}}(\tau_b^{3/2} - \tau_s^{3/2}) \equiv \frac{1}{\lambda}(\tau_b^{3/2} - \tau_s^{3/2}). \tag{2.114}$$

Armed with (2.114) we can go ahead and compute the scalar potential for the Kähler sector. Assuming that there are non-perturbative effects in both 4-cycles and that the Kähler potential includes the α'^3 term we obtain

$$V = \frac{8}{3}\frac{\lambda a^2 |A|^2}{\mathcal{V}} e^{-2a\tau_s}\sqrt{\tau_s} - 4\frac{|AW|}{\mathcal{V}^2} a\tau_s e^{-a\tau_s} + \frac{3}{4}\frac{|W|^2 \xi}{\mathcal{V}^3 g_s^{3/2}}, \tag{2.115}$$

where we have neglected terms proportional to $e^{-a_b \tau_b} \ll 1$ since $\tau_b \sim \mathcal{V}^{2/3} \gg 1$.

The position of the minimum of this potential is found by solving $\frac{\partial V}{\partial \tau_b} = \frac{\partial V}{\partial \tau_s} = 0$. The first condition yields

$$\frac{\partial V}{\partial \mathcal{V}} = 0 \Leftrightarrow \mathcal{V} = \frac{3|W|}{2\lambda a A}\sqrt{\tau_s} e^{a\tau_s}\left(1 \pm \sqrt{1 - \frac{3\lambda\xi}{8 g_s^{3/2}\tau_s^{3/2}}}\right), \tag{2.116}$$

while the second

$$\frac{\partial V}{\partial \tau_s} = 0 \Leftrightarrow (\frac{1}{2} - 2a\tau_s)\left(1 \pm \sqrt{1 - \frac{3\lambda\xi}{8 g_s^{3/2}\tau_s^{3/2}}}\right) = (1 - a\tau_s). \tag{2.117}$$

In the limit of large $a\tau_s$ Eq. (2.117) becomes

$$\tau_s^{3/2} \approx \frac{\lambda\xi}{g_s^{3/2}} \left(\frac{1}{2} + \frac{1}{4a\tau_s} + \mathcal{O}(a\tau_s)^{-2} \right). \tag{2.118}$$

Equation (2.118) gives an implicit solution for τ_s, one can get an explicit expression by solving it iteratively. The first iteration will be

$$\tau_s^{3/2} \approx \tilde{\tau}_s^{3/2} \left(1 + \frac{1}{2a\tilde{\tau}_s} \right), \tag{2.119}$$

where $\tilde{\tau}_s^{3/2} = \frac{\lambda\xi}{2g_s^{3/2}}$. We will use Eq. (2.119) as our analytical expression for τ_s at the minimum, neglecting the small subleading $\mathcal{O}\left(a^{-2}\tau_s^{-2}\right)$ corrections. We can also expand Eq. (2.116) in powers of $\frac{1}{a_s\tau_s}$ to find:

$$\mathcal{V} = \frac{3|W_0|}{4\lambda a|A|} \sqrt{\tau_s} e^{a\tau_s} \left(1 - \frac{3}{4a\tau_s} + \mathcal{O}(a^{-2}\tau_s^{-2}) \right). \tag{2.120}$$

As an illustrative numerical example of the order of magnitude of the parameters of this vacuum, consider looking for minima where $m_{3/2} \sim$ TeV. This is motivated by the desire to have TeV-scale supersymmetry to address the hierarchy problem and stabilise the Higgs mass. Noting that $m_{3/2} \sim M_P/\mathcal{V}$ this maps into a constraint on the volume of the compactification manifold: $\mathcal{V} \sim 10^{15} l_s^6$. Considering a setup where $W_0 = 10$, $A = 1$, $a = 2\pi$, $g_s = 0.027$, $\xi = 1.31$, and $\lambda = 1/(9\sqrt{2})$ we get the desired minimum, with

$$\tau_s = 5.17 \qquad \text{and} \qquad \mathcal{V} = 2.15 \times 10^{15} \tag{2.121}$$

in string units, without needing to tune any of the parameters to unnatural values. In particular note that exponentially large volumes are attained without the need for very small flux superpotential, unlike in the KKLT scenario.

In conclusion, by combining perturbative corrections to K and non-perturbative corrections to W it is possible to show that the scalar potential for the moduli sector admits a non-supersymmetric AdS minimum at exponentially large volumes. This mechanism allows for the stabilisation of all geometric moduli that feature in the non-perturbative superpotential in a regime where the effective field theory is under control. The exponentially large volumes imply that the string scale will be parametrically below the Plank scale. It is possible to accommodate within this scenario TeV scale supersymmetry without the need for fine tuning the parameters.

Having derived the basic features of the large volume vacuum we now look more carefully at the physically relevant scales and at the spectrum of the theory.

The mass spectrum of large volume compactifications

The fundamental gravity scale in the large volume compactifications is the string scale, M_s, defined as the inverse of the fundamental unit of length $l_s \equiv 2\pi\sqrt{\alpha'}$. Through dimensional reduction, the string mass can be related to the 4 dimensional Planck mass

$$M_s = \frac{M_p}{\sqrt{4\pi\mathcal{V}}}, \tag{2.122}$$

where as usual $\mathcal{V}$ is the Einstein frame volume of the compact space.

Kaluza-Klein excitations of the massless fields in the compactification will have masses around $M_{KK} \sim M_s/R$ with R the radius (in string units) of a particular direction of the extradimensional space. Provided there is no large anisotropy in the compact space we can approximate $\mathcal{V} = (2\pi R)^6$ and write

$$M_{KK} = \frac{2\pi M_s}{\mathcal{V}^{1/6}}. \tag{2.123}$$

We must note that very anisotropic compactifications, like the ones considered in Chapters 5 and 6 will have more than one KK scale, as described in detail in [46].

The gravitino mass, that sets the scale for the supersymmetry breaking terms, as we will see below, is defined as $m_{3/2}^2 = e^{K/M_P^2}|W|^2/M_P^4$ and can be written as

$$m_{3/2} = \sqrt{\frac{g_s}{4\pi}}\frac{|W|}{\mathcal{V}}M_P. \tag{2.124}$$

The complex structure moduli and the axio-dilaton, being stabilised at tree level by fluxes on the compact space, get a mass at high scale when compared to the Kähler moduli sector masses that are generated by subleading effects. Denoting the typical number of fluxes by N one finds

$$m_{cs} \sim m_S \sim \sqrt{\frac{g_s}{4\pi}}\frac{M_P N}{\mathcal{V}}. \tag{2.125}$$

The mass spectrum for the Kähler moduli sector depends on the geometry of the compact space as well as on the details of the stabilisation mechanism. Assuming the setup described above, we find that the masses for the canonically normalised 4 cycle volumes (χ) and their respective axions (θ) are [3]:

$$m_{\chi_s} \sim m_{\theta_s} \sim \sqrt{\frac{g_s}{8\pi}}M_P\frac{\log\mathcal{V}}{\mathcal{V}}, \qquad m_{\chi_b} \sim \sqrt{\frac{g_s}{8\pi}}\frac{M_P}{\mathcal{V}^{3/2}\sqrt{\log\mathcal{V}}}, \qquad m_{\theta_b} \sim 0. \tag{2.126}$$

We see that in the limit of large volumes the volume mode χ_b is lighter than the small blow up mode and its axion. We also see that the T_b axion's mass is exponentially

[3]The factor of $\sqrt{g_s/8\pi}$ accounts for the correct normalisation of the scalar potential [47].

suppressed and so it remains effectively massless. A detailed derivation of the moduli and axion masses is given in the Appendix.

In Table 2.6 we present the numerical values for the mass spectrum for three distinct choices of volume of the internal manifold. Volumes are given in units of l_s^6 and masses in GeV. Internal volumes of 10^3 will be required for the inflationary model of Chapter 5. On the other hand $\mathcal{V} \sim 10^{15}$ yields a gravitino with TeV scale mass and so are attractive for building models of TeV supersymmetry. Volumes of the order of 10^{30}, when combined with highly anisotropic internal spaces, give rise to stringy models of supersymmetric large extra dimensions whose cosmology will be studied in Chapter 6.

$\mathcal{V}$	M_s	M_{KK}	$m_{3/2}$	m_{cs}, m_S	m_{χ_s}, m_{θ_s}	m_{χ_b}
10^3	10^{16}	3×10^{16}	10^{13}	$N \times 10^{14}$	7×10^{14}	10^{12}
10^{15}	10^{10}	3×10^8	8×10^2	$80N$	2×10^3	3×10^{-7}
10^{30}	5×10^2	3×10^{-2}	4×10^{-13}	$4N \times 10^{-14}$	2×10^{-12}	3×10^{-30}

Table 2.6: Mass spectrum (in GeV) for three different choices of $\mathcal{V}$.

It is important to note that for some choices of the internal volume, the compactification will exhibit one or more fields with problematic cosmological behaviour. This is the well known cosmological moduli problem [48] that afflicts theories with Planck coupled scalar degrees of freedom. Depending on their mass and couplings, these moduli will either overclose the Universe and/or decay during or after Big-Bang Nuclesinthesis (BBN), ruining its well tested predictions for the abundance of light elements. Possible solutions to the problem can involve dilution of these moduli by a period of low-energy inflation [49], a trapping mechanism keeping the moduli from getting large VEVs, among others.

2.5 Gravity mediated supersymmetry breaking in string compactifications

Up to this point we have focused on the physics that stabilises the extra dimensions. What we shall review now is how the very same physics is deeply connected to the particle spectrum of the low energy theory. With that in mind we review the generic mechanism of gravity mediated supersymmetry breaking following the original works of [50,51] and reviewed in [52] before applying it to the particular case of large volume compactifications [53, 54].

2.5.1 Framework and general results

Let us start by recalling that the Lagrangian for the scalar bosons of a $\mathcal{N} = 1$ supergravity theory is given by

$$\mathcal{L} = K_{M\bar{N}}(D_\mu Z_M)(D^\mu Z_N)^\dagger - V(Z, Z^\dagger), \tag{2.127}$$

where Z_M denote the scalar componets of the chiral superfields, $K_{M\overline{N}}$ is the Kähler metric, defined as the second derivative of the Kähler potential, $K_{M\overline{N}} \equiv \partial K/\partial Z_M \partial Z_N^*$. The scalar potential is given by the sum of the F-term and D-term potentials:

$$V(\phi_M, \phi_M^*) \; = e^{K/M_P^2}\left(\bar{F}_{\bar{N}} K^{\bar{N}M} F_M - 3W^\dagger W/M_P^2\right) + \frac{1}{2} D^a D^a, \tag{2.128}$$

with the D-terms defined as

$$D^a \equiv \left(\frac{\partial K}{\partial Z_M} + \frac{\partial W}{\partial Z_M}\frac{1}{W}\right) t^a_{MN} Z_N \tag{2.129}$$

and

$$F_M \equiv \frac{\partial W}{\partial Z_A} + M_P^{-2}\frac{\partial K}{\partial Z_A} W \tag{2.130}$$

are the F-terms which will play a pivotal role in the following discussion of SUSY breaking.

From the definitions above we see that the $\mathcal{N} = 1$ supergravity Lagrangian is determined by two functions: the Kähler potential K and the superpotential W. W is related to the Yukawa couplings and can include non-perturbative effects, while K determines the kinetic terms of the scalar degrees of freedom of the theory.

The space of scalar degrees of freedom can be split into two subspaces: the visible sector fields denoted by C^α and hidden sector fields h_m. The fields C^α are the scalar components of the Supersymmetric Standard Model (SSM) superfields and the hidden sector fields h_m will include the moduli fields that arise in compactifications of string theory.

One would like to find the low energy effective field theory of the visible sector fields by integrating out the hidden sector fields. This can be achieved by substituting the h_m and their F-terms by their VEVs and then taking the limit where $M_P \to \infty$ keeping the gravitino mass finite. This procedure eliminates the nonrenormalizable gravity corrections and one is left with a global SUSY Lagrangian for the visible sector plus a set of soft SUSY-breaking terms. This is usually designated by taking the flat limit. In order to take into account all the factors of M_P, crucial for taking the flat limit, it is useful to rewrite the hidden sector fields as $Z_m = h_m M_P$, where the h_m are dimensionless.

One requires the low energy theory to be renormalisable and that the visible sector fields do not develop VEVs or couplings of order M_P. Using this as a guiding principle one can severely constrain the form of the Kähler potential and of the superpotenial. We can expand the superpotential in powers of M_P as

$$W(h_m, C^\alpha) = \sum_{n=0}^{\infty} M_P^n W_n(h_m, C^\alpha). \tag{2.131}$$

The requirement that in the scalar potential the visible sector fields are not multiplied by positive powers of M_P implies that $W_n = 0$ for $n \geq 3$. To see this note that

$$V \supset \frac{W^\dagger W C^\alpha C^{*\alpha}}{M_P^2} = \sum_{m,n=0}^{\infty} M_P^{m+n-2} W_n W_m^\dagger C^\alpha C^{*\alpha},$$

so that the most general superpotential that meets this requirement is

$$W(h_m, C^\alpha) = M_P^2 W_2(h_m, C^\alpha) + M_P W_1(h_m, C^\alpha) + W_0(h_m, C^\alpha). \tag{2.132}$$

Further noting that terms that violate the above requirement are in general present in the expansion of the $F_M \bar{F}_{\bar{N}}$ term in the scalar potential leads us to the conclusion that W_2 and W_1 can only depend on the hidden sector fields, so we find that

$$W(h_m, C^\alpha) = M_P^2 W_2(h_m) + M_P W_1(h_m) + W_0(h_m, C^\alpha). \tag{2.133}$$

Noting that the superpotential has mass dimension 3, one can expand $W_0(h_m, C^\alpha)$ in the visible sector fields which leads to

$$W = \hat{W}(h_m) + \frac{1}{2}\mu_{\alpha\beta}(h_m) C^\alpha C^\beta + \frac{1}{6} Y_{\alpha\beta\gamma}(h_m) C^\alpha C^\beta C^\gamma + \dots\,, \tag{2.134}$$

where we have defined $\hat{W}(h_m) \equiv M_P^2 W_2(h_m) + M_P W_1(h_m)$ and the ellipsis denote nonrenormalisable terms, suppressed by inverse powers of M_P.

The expansion of the Kähler potential can also be constrained using the same arguments as above. We start by noting that it can be written as

$$K(h_m, C^\alpha) = \sum_{n=0}^{\infty} M_P^n K_n(h_m, C^\alpha), \tag{2.135}$$

which implies that in general the scalar potential will include the following terms

$$F_M \overline{F_N} \supset M_P^{-4} \partial_M K \overline{\partial_N K} W \overline{W} + \partial_\alpha W M_P^{-2} \overline{\partial_\beta K} W.$$

Eliminating the undesired terms from the expansion of the first of the terms above leads to $\partial_\alpha K_m = 0$ for $m \geq 2$ while doing the same in the second yields $\partial_{\overline{\beta}} K_2^\dagger = \partial_{\overline{\beta}} K_1^\dagger = 0$. The expansion then becomes

$$K(h_m, h_m^*, C^\alpha, C^{*\alpha}) = M_P^2 K_2(h_m, h_m^*) + M_P K_1(h_m, h_m^*) + K_0(h_m, h_m^*, C^\alpha, C^{*\alpha}) + \\ + \sum_{n=3}^{\infty} M_P^n K_n(h_m, h_m^*, C^\alpha, C^{*\alpha}).$$

Noting that the terms with three or more powers of M_P would dominate the expansion of the exponential in front of the the scalar potential which would in turn lead to the generation of terms we are trying to suppress. Therefore one must further impose that $K_n = 0$ for $n \geq 3$. This leads to the final form for the Kähler potential:

$$K(h_m, h_m^*, C^\alpha, C^{*\alpha}) = M_P^2 K_2(h_m, h_m^*) + M_P K_1(h_m, h_m^*) + K_0(h_m, h_m^*, C^\alpha, C^{*\alpha}). \tag{2.136}$$

Given that the Kähler potential is a real, mass dimension 2 function, we can further rewrite it as

$$K = \hat{K}(h_m, h_m^*) + \tilde{K}_{\overline{\alpha}\beta}(h_m, h_m^*) C^{*\overline{\alpha}} C^\beta + \left[\frac{1}{2} Z_{\alpha\beta}(h_m, h_m^*) C^\alpha C^\beta + h.c. \right] + \ldots , \tag{2.137}$$

where we have defined $\hat{K}(h_m, h_m^*) \equiv M_P^2 K_2(h_m, h_m^*) + M_P K_1(h_m, h_m^*)$ and the ellipsis denote nonrenormalisable terms, suppressed by inverse powers of M_P.

Using the expansions of Eqs. (2.134) and (2.137) one can show that the potential for the visible sector fields can be written as:

$$V(C^\alpha, C^{*\alpha}) = \partial_a W_{eff} Z^{a\overline{b}} \overline{\partial_b W_{eff}} + V_{soft}(C^\alpha, C^{*\alpha}), \tag{2.138}$$

where the first term is the globally supersymmetric part, defined in terms of the effective superpotential W_{eff} and the second term is the supersymmetry breaking part. The effective superpotential is defined as

$$W_{eff} = \frac{\mu'_{\alpha\beta}}{2} C^\alpha C^\beta + \frac{Y'_{\alpha\beta\gamma}}{3} C^\alpha C^\beta C^\gamma \tag{2.139}$$

where the new un-normalised couplings $\mu'_{\alpha\beta}$ and $Y'_{\alpha\beta\gamma}$ are related to the original superpotential couplings $\mu_{\alpha\beta}$ and $Y_{\alpha\beta\gamma}$ via

$$Y'_{\alpha\beta\gamma} = \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} Y_{\alpha\beta\gamma} , \tag{2.140}$$

$$\mu'_{\alpha\beta} = \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} \mu_{\alpha\beta} + m_{3/2} Z_{\alpha\beta} - \overline{F}^{\overline{m}} \partial_{\overline{m}} Z_{\alpha\beta} . \tag{2.141}$$

The gravitino mass, defined as

$$m_{3/2} = e^{\hat{K}/M_P^2} |\hat{W}|/M_P^2 \tag{2.142}$$

is a crucial quantity in this analysis as it sets the overall scale of the soft supersymmetry breaking parameters as we will show below.

The information about supersymmetry breaking is encoded in $V_{soft}(C^\alpha, C^{*\alpha})$ which is given by

$$V_{soft}(C^\alpha, C^{*\alpha}) = M^2_{\tilde{Q}\overline{\alpha}\beta} C^{*\overline{\alpha}} C^\beta + \left(\frac{1}{6} A_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma + \frac{1}{2} B\mu_{\alpha\beta} C^\alpha C^\beta + h.c. \right), \tag{2.143}$$

where

$$\begin{aligned}
M^2_{\tilde{Q}\overline{\alpha}\beta} &= \left(m^2_{3/2} + V_0\right) \tilde{K}_{\overline{\alpha}\beta} \\
&- \overline{F}^{\overline{m}} \left(\partial_{\overline{m}} \partial_n \tilde{K}_{\overline{\alpha}\beta} - \partial_{\overline{m}} \tilde{K}_{\overline{\alpha}\gamma} \tilde{K}^{\gamma\overline{\delta}} \partial_n \tilde{K}_{\overline{\delta}\beta} \right) F^n \ , && (2.144) \\
A_{\alpha\beta\gamma} &= \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} F^m \Big[\hat{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} \\
&- \left(\tilde{K}^{\delta\overline{\rho}} \partial_m \tilde{K}_{\overline{\rho}\alpha} Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \Big] \ , && (2.145) \\
B\mu_{\alpha\beta} &= \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} \Big\{ F^m \Big[\hat{K}_m \mu_{\alpha\beta} + \partial_m \mu_{\alpha\beta} \\
&- \left(\tilde{K}^{\delta\overline{\rho}} \partial_m \tilde{K}_{\overline{\rho}\alpha} \mu_{\delta\beta} + (\alpha \leftrightarrow \beta) \right) \Big] - m_{3/2} \mu_{\alpha\beta} \Big\} \\
&+ \left(2m^2_{3/2} + V_0\right) Z_{\alpha\beta} - m_{3/2} \overline{F}^{\overline{m}} \partial_{\overline{m}} Z_{\alpha\beta} \\
&+ m_{3/2} F^m \Big[\partial_m Z_{\alpha\beta} - \left(\tilde{K}^{\delta\overline{\rho}} \partial_m \tilde{K}_{\overline{\rho}\alpha} Z_{\delta\beta} + (\alpha \leftrightarrow \beta) \right) \Big] \\
&- \overline{F}^{\overline{m}} F^n \Big[\partial_{\overline{m}} \partial_n Z_{\alpha\beta} - \left(\tilde{K}^{\delta\overline{\rho}} \partial_n \tilde{K}_{\overline{\rho}\alpha} \partial_{\overline{m}} Z_{\delta\beta} + (\alpha \leftrightarrow \beta) \right) \Big] && (2.146)
\end{aligned}$$

and V_0 is the vacuum expectation value of the F-term potential:

$$V_0 = \overline{F}^{\overline{m}} \hat{K}_{\overline{m}n} F^n - 3m^2_{3/2} \ . \tag{2.147}$$

From the fermionic part of the supergravity Lagrangian one finds that the canonically normalised gaugino masses are given as

$$M_a = \frac{1}{2} \left(Re f_a\right)^{-1} F^m \partial_m f_a \ . \tag{2.148}$$

Up to renormalisation group running effects, the spectrum of superpartners and their interactions are determined by Eqs. (2.144)-(2.148), which depend, among other things, on the hidden sector fields' F-terms, F^m and on the cosmological constant V_0.

Recalling that the hidden sector fields are to be identified with the moduli fields that describe the string coupling and the geometry of the extra dimensions, we are led to the conclusion that the the physics of moduli stabilisation has a direct effect on the spectrum of superpartners. This can ultimately be probed by accelerator searches for supersymmetry such as the ones being currently undertaken in the LHC thus providing a test for moduli stabilisation scenarios.

2.5.2 Supersymmetry breaking in the large volume scenario

The previous discussion outlined the generic method to relate the supersymmetry breaking terms to the hidden sector fields and their F-terms. We now apply these results to the case when the hidden sector fields are identified with the string theory moduli of type IIB, stabilised through the combined effects of non-perturbative terms in the superpotential and perturbative corrections to the Kähler potential. This work was originally developed in [53, 55] and further extended in [54].

We start by noting that in the cases when the spectrum of the theory is that of some version of the supersymmetric standard model, with two Higgs doublets, the expansion of the superpotential and Kähler potential of Eqs. (2.134) and (2.137) are

$$W = \hat{W}(h_m) + \frac{1}{2}\mu(h_m)H_1H_2 + \frac{1}{6}Y_{\alpha\beta\gamma}(h_m)C^\alpha C^\beta C^\gamma + \dots\ , \tag{2.149}$$

and

$$K = \hat{K}(h_m, h_m^*) + \tilde{K}_{\overline{\alpha}\beta}(h_m, h_m^*)C^{*\overline{\alpha}}C^\beta + \left[\frac{1}{2}Z(h_m, h_m^*)H_1H_2 \ +\ h.c.\right] + \dots\ . \tag{2.150}$$

respectively. We will use these expressions in the computation of the soft terms in the large volume scenario.

There are two crucial steps one must take in order to study the soft terms in the visible sector: first one must know how supersymmetry is broken, which depends on the vacuum structure of the theory and requires the knowledge of the moduli potential and second one needs to know how it is transmitted to the visible sector fields. We are interested in studying SUSY breaking in the large volume scenario and so the structure of the potential and the properties of its minimum were already analysed in Section 2.4.5. This addresses the first of the points mentioned above. Close inspection of the formulae for the soft terms, Eqs. (2.144)-(2.146), reveals that the knowledge of the the hidden sector F-terms, Kähler potential $\hat{K}$ and superpotential $\hat{W}$ is not enough to determine the soft SUSY breaking terms in the visible sector, explicit knowledge of the holomorphic gauge kinetic function, f_a, of the matter metric, $\tilde{K}_{\alpha\overline{\beta}}$, and also of $Z_{\alpha\beta}$ is required.

The gauge kinetic function

The gauge kinetic function, being constrained by holomorphy is relatively easy to compute. Furthermore, in the case where the SM is realised in a stack of D7 branes wrapping an internal cycle in the extradimensional manifold, it has a geometrical interpretation as the size of the cycle wrapped by the standard model stack of branes. This can be seen by reduction of the DBI action on the 4-cycle T_s which yields

$$f_s = T_s \tag{2.151}$$

for an unmagnetised brane and

$$f_s = T_s + \frac{1}{2} h_s(F) S \tag{2.152}$$

for a brane with magnetic flux F, where $h_s(F)$ is a flux dependent integral over the divisor wrapped by the D7's [56]. In the limit that the cycle is collapsed to a singularity the gauge kinetic function takes the form

$$f_i = \delta_i S + s_{ik} T_k, \tag{2.153}$$

where the factor δ_i depends on the geometry of the singularity and T_k are the blow-up modes that resolve the singularity.

The matter metric

The story is not so simple for the matter fields' metric, for which explicit computations could only be performed on flat toroidal backgrounds or for phenomenologically less interesting non-chiral D7 matter. It took the work of [55] to shed light on the modular dependence of the Kähler metric for bifundamental matter in Calabi-Yau backgrounds. It was show in [55] that the principles of locality in the extra dimensional manifold and of superpotential holomorphy can be used to provide the modular weights of the matter metrics without performing explicit string amplitude calculations in a IIB Calabi-Yau orientifold background. We will sketch how this works and then proceed to compute the soft supersymmetry breaking terms.

From Eq. (2.150) we see that the normalised standard model fields $\hat{C}^\alpha$ are defined as

$$\hat{C}^\alpha = \tilde{K}_\alpha^{1/2}(h_m, h_m^*) C^\alpha, \tag{2.154}$$

where C^α are the original unormalised fields that appear in the expansion of the Kähler potential and superpotential. It then follows that the physical (i.e. canonically normalised) Yukawa couplings, $\hat{Y}_{\alpha\beta\gamma}$, are given in terms of the superpotential Yukawas, $Y_{\alpha\beta\gamma}$, as

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{1/2}}, \tag{2.155}$$

where we have assumed that the moduli superpotential is real: $|\hat{W}| = \hat{W}$.

The key insight that allow us to use Eq. (2.155) to derive the modular dependence of the matter metric is that in the bottom-up approach that we are implicitly assuming, the local/brane physics which encompasses the choices of gauge group, chiral spectrum, Yukawa and gauge couplings is decoupled from the global/bulk physics which deals with the issues of moduli stabilisation, supersymmetry breaking, cosmological constant among others. This is what is meant by locality in the extra dimensions and it implies that the physical Yukawa couplings are determined by the local geometry and, crucially, are independent of the overall volume of the extra dimensional manifold.

One extra piece of information is required in order to extract the physics we want from Eq. (2.155) and that is the modular dependence of the superpotential Yukawas $Y_{\alpha\beta\gamma}$. To see why note that in the large volume scenario the dilaton and complex structure moduli do not break supersymmetry and therefore have vanishing F-terms: $F_{c.s.} = F_{dilaton} = 0$, only the Kähler moduli develop nonzero F-terms and so for soft term computation we are mostly interested in the Kähler moduli dependence of $Y_{\alpha\beta\gamma}$ and $\tilde{K}_{\alpha\beta}$. The Yukawa couplings $Y_{\alpha\beta\gamma}$ appear in the superpotential and are therefore protected by the holomorphy and non-renormalisation of W. The Kähler moduli of IIB flux compactifications on the other hand have an exact perturbative shift-symmetry $\text{Im}(T_i) \to \text{Im}(T_i)+\epsilon_i$. The holomorphy of W means that we can either have $W(T_i)$ or $W(\bar{T}_i)$ but not $W(T_i, \bar{T}_i)$. Considering that $W = W(T_i)$, then under the PQ shift-symmetry $W(T_i) \to W(T_i + i\epsilon_i) \neq W(T_i)$, violating the shift-symmetry. The Kähler moduli are then forbidden from entering in the tree level superpotential and the non renormalisation theorem of W then implies that they cannot enter at all in the perturbative W. One is then led to the conclusion that the perturbative superpotential is independent of the Kähler moduli and so are the superpotential Yukawas $Y_{\alpha\beta\gamma}$.

Combining the fact that $\hat{Y}_{\alpha\beta\gamma}$ is independent of the volume and that $Y_{\alpha\beta\gamma}$ is independent of all Kähler moduli we see that these have to enter in $\tilde{K}_\alpha$ in such a way as to cancel the factor of $e^{\hat{K}/2}$, that is:

$$\frac{e^{\hat{K}/2}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{1/2}} = \frac{1}{\mathcal{V}(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{1/2}} \to \tilde{K}_\alpha \sim \tilde{K}_\beta \sim \tilde{K}_\gamma \propto \mathcal{V}^{-2/3}.$$

For the computation of the soft terms we will follow [55] in using the following parametrisation for the matter metric

$$\tilde{K}_{\alpha\overline{\beta}} = \frac{k_{\alpha\overline{\beta}}(\tau_a, \phi)}{\tau_b^p}, \qquad \text{with} \qquad p = 1, \tag{2.156}$$

where τ_s denotes the volume of the 4-cycle that supports the standard models stack of branes and ϕ the complex structure moduli. We must stress that the value $p = 1$ is a very special one that leads to cancellation of the leading order contributions to the soft terms. This means that one will often find that the soft terms are no longer generated at the scale of the gravitino mass $m_{3/2}$ but rather at $m_{3/2}/\mathcal{V}^q$ for some $q \geq 0$.

Given that the subleading α'^3 corrections have been shown to play a crucial role in the determination of the vacuum structure of the potential for the Kähler moduli, consistency requires that we also include them in the matter metric. Knowing that they will constitute a small correction to Eq. (2.156), we assume they will take the form

$$\tilde{K}_{\alpha\overline{\beta}} = \frac{k_{\alpha\overline{\beta}}(\tau_a, \phi)}{\tau_b^p} \left(1 - \delta \left(\frac{Re(S)}{\tau_b}\right)^{n/2} + ...\right) \tag{2.157}$$

where n denotes the first order at which the α' correction enters the computation. Arguing once again that the physical Yukawa couplings are local quantities and therefore independent of the overall volume of the compactification space, we find that $n = 3$ and so the matter metric is given by [54]:

$$\tilde{K}_{\alpha\overline{\beta}} = \frac{k_{\alpha\overline{\beta}}(\tau_a, \phi)}{\tau_b^p} \left(1 - \delta \left(\frac{Re(S)}{\tau_b}\right)^{3/2}\right). \tag{2.158}$$

The above considerations about the modular dependence of the gauge kinetic function and of the matter metrics allow us to study how the supersymmetry breaking in the hidden/moduli sector can be transmitted to the visible sector. However before we proceed, a minimal extension of the simplest moduli stabilisation scenario discussed in Section 2.4.5 must be introduced.

The minimal geometry for constructing the standard model

It was originally assumed that if one wanted to introduce the standard model into the large volume framework it would suffice to consider the minimal geometry of Section 2.4.5 with only two Kähler moduli. In such setup the small cycle τ_s would simultaneously support the non-perturbative effects that stabilise the volume and be wrapped by the standard model stack of branes. However it was noted in [57] that instanton zero-mode counting forbids such scenario and therefore non-perturbative effects and the standard model stack of branes must be supported by different cycles in the Calabi-Yau geometry.

The volume of the simplest Swiss-cheese geometry that complies with this constraint is

$$\mathcal{V} = (\eta_b \tau_b)^{3/2} - (\eta_s \tau_s)^{3/2} - (\eta_a \tau_a)^{3/2}, \tag{2.159}$$

where τ_b determines the volume of the Calabi-Yau, τ_s is wrapped by the D3-brane instanton and the four-cycle of size τ_a supports the SM D7 branes. Examples of explicit Calabi-Yaus with these structures can be found in [58–60].

Since in this improved setup the standard model cycle τ_a no longer appears non perturbatively in the superpotential it has to be stabilised by D-terms, often close to the string scale, sequestering the SM physics from the bulk of the Calabi-Yau. The vanishing D-term condition reads

$$K_{T_a} = 0 \tag{2.160}$$

and shows a dynamical preference for a collapsed cycle $\tau_a \to 0$. The F-term for the standard model 4 cycle is as usual

$$F_a = e^{K/2}(W_{T_a} + W K_{T_a}). \tag{2.161}$$

Since the superpotential is independent of T_a, the vanishing of the D-term implies the vanishing of the F-term for the standard model cycle, and so τ_a does not break supersymmetry. Not only we see that $F_a = 0$ but also that $F^a = 0$ which implies that the standard model is sequestered from the supersymmetry breaking terms. Ultimately this means, as we will illustrate below following [54], that the soft terms will be generated at subleading order.

Depending on whether the standard model cycle is stabilised above or below the string scale, there will be two regimes in the effective field theory: the geometric regime and the singular cycle regime [54].

In the geometric regime all the four-cycles are larger than the string scale. In this case the effective field theory is determined by:

$$\begin{aligned}
\hat{K}(\Phi, \bar{\Phi}) &= -2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) - \ln(S + \bar{S}) + K_{CS}, && (2.162) \\
\hat{W}(\Phi) &= W_0 + A e^{-a T_s}, && (2.163) \\
f_i &= T_a - \frac{1}{2} \kappa_i S, && (2.164)
\end{aligned}$$

where the volume is given by Eq. (2.159) and the matter metric is

$$\tilde{K} = k\left(1 - \delta\left(\frac{Re(S)^{3/2}}{\mathcal{V}}\right)\right)/\mathcal{V}^{2/3} \tag{2.165}$$

with k being a function of the complex structure moduli and δ a function of $(Re(S)/\tau_b)^m$ with m determined by the the locality arguments discussed before to be $m = 3/2$.

In the singular cycle regime, the standard model four-cycle is smaller than the string scale. In this regime the theory is determined by:

$$\hat{K}(\Phi, \bar{\Phi}) = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) + c\frac{\tau_a^2}{\mathcal{V}} - \ln(S + \bar{S}), \tag{2.166}$$

$$\hat{W}(\Phi) = W_0 + Ae^{-aT_s}, \tag{2.167}$$

$$f_i = s_{ik}T_k + \delta_i S, \tag{2.168}$$

where the volume is given by

$$\mathcal{V} = (\eta_b\tau_b)^{3/2} - (\eta_s\tau_s)^{3/2} \tag{2.169}$$

and the matter metric is

$$\tilde{K} = (\beta - \delta/\mathcal{V} + \gamma\tau_a^m)/\mathcal{V}^{2/3}, \tag{2.170}$$

with β and γ being arbitrary functions of the complex structure moduli/dilaton. The quadratic term in τ_a in Eq. (2.166) is introduced to ensure that the τ_a kinetic term is well defined when $\tau_a = 0$.

The soft supersymmetry breaking terms

With the above information on how to build the standard model into the large volume setup one can go ahead and compute the soft supersymmetry breaking terms. Before doing that, let us note that we will be interested chiefly in finding the leading order behaviour of the soft terms and in particular, in the absence of explicit models from which to derive the fine details of the flavour structure, we assume the matter metric to be diagonal. This simplifying choice is further justified by the fact that the soft-terms are flavour universal to leading order in the volume expansion as found in [53].

With this in mind we define the canonically normalised scalars and gauginos as $\hat{C}^\alpha = \sqrt{K_\alpha}C^\alpha$ and $\hat{\lambda}^a = \sqrt{Re(f_a)}\lambda^a$ and define the physical Yukawas and μ term to be

$$\widehat{Y}_{\alpha\beta\gamma} = Y_{\alpha\beta\gamma}\,\frac{\hat{W}^*}{|\hat{W}|}\,e^{\hat{K}/2}\,(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)^{-1/2}\ , \tag{2.171}$$

$$\widehat{\mu} = \left(\frac{\hat{W}^*}{|\hat{W}|}e^{\hat{K}/2}\mu + m_{3/2}Z - \overline{F}^{\overline{m}}\partial_{\overline{m}}Z\right)(\tilde{K}_{H_1}\tilde{K}_{H_2})^{-1/2}\ , \tag{2.172}$$

We further simplify Eqs. (2.144)-(2.146) to:

$$M^2_{\tilde{Q}\alpha} = \left(m^2_{3/2} + V_0\right) - \overline{F}^{\overline{m}}F^n\partial_{\overline{m}}\partial_n\log\tilde{K}_\alpha\ , \tag{2.173}$$

$$A_{\alpha\beta\gamma} = F^m\left[\hat{K}_m + \partial_m\log Y_{\alpha\beta\gamma} - \partial_m\log(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)\right]\ , \tag{2.174}$$

$$\begin{aligned} B\hat{\mu} = {} & \widehat{\mu}^{-1}(\tilde{K}_{H_1}\tilde{K}_{H_2})^{-1/2}\Bigg\{\frac{\hat{W}^*}{|\hat{W}|}e^{\hat{K}/2}\mu\Big(F^m\Big[\hat{K}_m + \partial_m\log\mu \\ & - \partial_m\log(\tilde{K}_{H_1}\tilde{K}_{H_2})\Big] - m_{3/2}\Big) \\ & + \left(2m^2_{3/2} + V_0\right)Z - m_{3/2}\overline{F}^{\overline{m}}\partial_{\overline{m}}Z \\ & + m_{3/2}F^m\left[\partial_m Z - Z\partial_m\log(\tilde{K}_{H_1}\tilde{K}_{H_2})\right] \\ & - \overline{F}^{\overline{m}}F^n\left[\partial_{\overline{m}}\partial_n Z - \partial_{\overline{m}}Z\partial_n\log(\tilde{K}_{H_1}\tilde{K}_{H_2})\right]\Bigg\}\ . \end{aligned} \tag{2.175}$$

The F-term for the large modulus can be computed explicitly since we know the vacuum structure of the moduli potential:

$$F^b = e^{\hat{K}/2}\hat{K}^{b\bar{n}}\left(\partial_{\bar{n}}\overline{\hat{W}} + (\partial_{\bar{n}}\hat{K})\overline{\hat{W}}\right). \tag{2.176}$$

It was show in the appendix of [54], that taking into account the subleading terms, this becomes

$$F^b = -2\tau_b m_{3/2} - \frac{3}{8\sqrt{2}}\frac{\tau_b}{a\tau_s}\left(1 + \frac{3}{2a\tau_s}\right)\frac{W_0}{\mathcal{V}^2} + \mathcal{O}(\mathcal{V}^{-3}). \tag{2.177}$$

Therefore the soft breaking terms depending only on F^b are automatically of order the gravitino mass $m_{3/2}$. A similar computation shows that the F-term of the dilaton is:

$$F^S \sim \frac{3}{2\sqrt{2}}\gamma\frac{\xi}{g_s^2}\frac{W_0}{\mathcal{V}^2} \tag{2.178}$$

where γ is an order one constant introduced to allow for a small shift in the dilaton VEV due to the cross coupling generated by the α'^3 correction to K.

Before computing the soft SUSY breaking terms one needs to assume an explicit form for Z, we take it to be

$$Z = \tau_b^{-p_z}z(\tau_i). \tag{2.179}$$

We also assume for simplicity that the matter metric is diagonal and note that the fact that the perturbative superpotential is independent of the Kähler moduli implies that $\partial_m \log Y_{\alpha\beta\gamma} = 0$ which simplifies the A-terms and finally note that $\mu = 0$ for consistency [55] [4].

Armed with this knowledge we can go ahead and compute the soft terms, Eqs. (2.144)-(2.148). As reported in [54] we find that the only contribution to gaugino masses comes from F^S:

$$M_{\tilde{G}} = \frac{3}{4}\gamma \frac{\xi}{g_s^{3/2}} \frac{M_{3/2}}{\mathcal{V}}, \tag{2.180}$$

where it was assumed that the dynamical preference for small τ_a implies that the flux contribution will dominate the gauge kinetic function.

The soft scalar masses are given by

$$M^2_{\tilde{Q}} = m^2_{3/2} \left(-\frac{1}{4a\tau_s} \frac{\xi}{g_s^{3/2}\mathcal{V}} + \frac{15(\delta - \xi/3)}{4g_s^{3/2}\mathcal{V}} \right), \tag{2.181}$$

where only by the taking into account the leading α' correction (which gives rise to the second term in the bracket) is it possible to obtain non-tachyonic sfermion masses.

Considering that $\mu = 0$ one finds that the $\hat{\mu}$ and the $B\hat{\mu}$ terms scale as

$$\hat{\mu} \sim -\frac{M_{\tilde{G}}}{4a\tau_s} \qquad \text{and} \qquad B\hat{\mu} \sim -\frac{m_{3/2}}{2a\tau_s} M_{\tilde{G}}. \tag{2.182}$$

The leading contribution to the A-terms comes from the dilaton term in the expansion of Eq. (2.174) since the leading order F^b term cancels due to form of the modular dependence of the matter metric:

$$A_{\alpha\beta\gamma} = \frac{-3}{4\sqrt{2}} \frac{\xi}{g_s^{3/2}} \frac{|W_0|}{\mathcal{V}^2} M_P = -M_{\tilde{G}}. \tag{2.183}$$

We summarise the leading order contributions to the soft terms, both in the geometric and the singular cycle regimes, in the following table

Soft-term	Scale
$M_{\tilde{G}}$	$M_{3/2}\frac{\zeta^2}{4}$
$M^2_{\tilde{Q}}$	$M^2_{3/2}\frac{\zeta^2}{16\log\zeta}$
$\hat{\mu}$-term	$M_{3/2}\frac{\zeta^2}{32\log\zeta}$
$B\hat{\mu}$-term	$M^2_{3/2}\frac{\zeta^2}{16\log\zeta}$
A-term	$-M_{3/2}\frac{\zeta^2}{4}$

[4] If the superpotential μ is nonzero then μ' can be made arbitrarily larger than the string scale M_s in the limit $\mathcal{V} \to \infty$. Since M_s is the UV cut-off of the theory, consistency requires $\mu = 0$.

where we have defined $\zeta \equiv M_s/M_P \propto 1/\sqrt{\mathcal{V}}$. This then shows that the cancellation of the leading order terms lead to soft-terms that are volume suppressed relative to the naive expectation of $M_{3/2}^n$ (with $n = 1, 2$ depending on the mass dimension). This phenomenon is usually called sequestering and can have significant phenomenological consequences like alleviating the CMP and the tension between the scale of SUSY breaking and the inflationary scale.

Chapter 3

Moduli Redefinitions and Moduli Stabilisation

This Chapter is based on the paper [12].

In the study of string compactifications moduli stabilisation plays an essential role in attempts to generate a realistic phenomenology. Moduli stabilisation is necessary to give masses to the light moduli that would otherwise give fifth forces, and is also necessary to avoid runaway and decompactification. Furthermore the moduli potential plays a crucial role in setting both the scale of supersymmetry breaking and the value of the gravitino mass, and so represents a necessary ingredient of any ultraviolet-consistent model of supersymmetry breaking.

One set of promising models of moduli stabilisation are the large volume models originally developed in [29, 33]. In these models corrections to the Kähler potential [26] play a very important role as the minimum of the scalar potential comes from competition between a perturbative α' correction to the Kähler potential and non-perturbative corrections to the superpotential. While the form of the superpotential is restricted by non-renormalisation theorems, the Kähler potential is not constrained. To analyse the robustness of the large volume scenario it is therefore important to check the effect of all other possible corrections to the Kähler potential.

These corrections fall into several forms. The most obvious kind are those of other α' corrections. The large volume models involve the correction to the Kähler potential induced by the $\mathcal{R}^4$ term present at $\mathcal{O}(\alpha'^3)$ in the 10D IIB action. There are many other additional terms (for example flux terms such as $G^2\mathcal{R}^4$). Such α' terms were considered in [33]. At large volume such terms are all subleading compared to the $\mathcal{R}^4$ term, essentially because the $\mathcal{R}^4$ term is suppressed by six powers of length, while flux terms are suppressed by higher powers.

Another form of correction are those induced by string loop corrections to the Kähler potential. For some toroidal models these have been calculated in [34,35]. For more general models the Coleman-Weinberg potential has been used to constrain the form and magnitude of loop corrections [27, 45, 61]. One interesting property of such loop corrections is that they do in general induce corrections in the Kähler potential that dominate at large volume over the α'^3 corrections. However, due to an extended no-scale structure such loop corrections in fact cancel at leading order in the scalar potential [27], remaining sub-dominant to the α'^3 corrections. Other studies of the robustness of the large volume scenario include [62–64].

This Chapter will study another form of possible correction that does not seem to fall neatly into either of the classes above. This refers to the effect of field redefinitions that can occur at the loop level. In such redefinitions the relationship between the holomorphic moduli and the physical cycle volumes is modified at 1-loop, such that the holomorphic moduli no longer correspond directly to cycle sizes. As the Kähler potential involves the physical volume this should lead to a correction to the Kähler potential as the expression for the physical volumes in terms of the moduli changes.

The existence of such redefinitions has been established for various orbifold models where exact string computations are possible. It is certainly plausible that such redefinitions survive beyond the orbifold limit in which they were computed. We shall assume this applies, and then study the effect of redefinitions on moduli stabilisation and the computation of soft terms.

The structure of this Chapter is as follows: in Section 3.1 we review how field redefinitions occur in orbifolds and give a new argument for their occurrence in the geometric regime, in Section 3.2 we study the effect of such redefinitions on moduli stabilisation, and in Section 3.3 we extend this study to the effects on soft supersymmetry breaking. We conclude in Section 3.4.

3.1 1-loop Kähler moduli redefinition

The purpose of this Chapter is to study the effects of field redefinitions on moduli stabilisation. It is a known feature of string compactifications that the definitions of moduli are altered at 1-loop level. Specifically, what is altered is the definition of the chiral superfields of the supergravity action in terms of the geometric quantities of the compactification, Eq. (2.37).

The large volume models involve a large overall volume together with smaller blow-up cycles. If such blow-up cycles were collapsed to a point, they would generate a singularity. Branes at singularities were recently studied in the context of threshold corrections in [65,66]. It was found there that in various cases consistency with the effective supergravity - specifically the Kaplunovsky-Louis formula describing the effect of anomalous contributions to gauge coupling running - required that the modulus controlling the blow-up cycle had to be redefined at 1-loop level. For other examples of 1-loop redefinitions in the context of theories with D-branes, see [67, 68].

The redefinition did not occur in all cases, where it did occur it took the form of a shift by a factor proportional to the logarithm of the overall volume of the CY:

$$\tau_{new} = \tau_{old} - \alpha \ln(\mathcal{V}), \tag{3.1}$$

where α is taken to be small. The physical cycle volume (which vanishes at the singularity) corresponds to τ_{old}. The holomorphic modulus τ_{new} is however non-zero at the singularity. We also note the types of cycles for which the redefinition occurs (blow-up cycles) are essentially the same as the blow-up moduli that play a crucial role in the large volume construction.

While such orbifold calculations show that field redefinitions occur, they are restricted to the singular limit. Let us give another argument for why field redefinitions should take place, this time in the geometric regime. We must stress that, unlike the explicit computation in the singular regime, the argument for redefinitions in the geometric regime is just an illustration. Consider an isolated collapsible cycle (e.g. a $\mathbb{P}^2$) taken to itself under an orientifold action, such that an O-plane wraps it and a stack of D-branes is on top of the O-planes giving a rigid $\mathcal{N} = 1$ SYM theory. This is the type of dynamics required on the small cycle in the large volume scenario. Such a theory will undergo gaugino condensation and generate a non-perturbative superpotential. The classical tree-level gauge coupling is given by

$$\frac{4\pi}{g^2} = \tau, \tag{3.2}$$

where τ is the Einstein frame physical volume of the 4-cycle measured in units of the string length $(e^{-\phi}\mathrm{Vol}_\Sigma/(2\pi\sqrt{\alpha'})^4)$. The running gauge coupling is then given by

$$\frac{1}{g^2}(\mu) = \frac{\tau}{4\pi} + \frac{\beta}{16\pi^2}\ln\left(\frac{\Lambda_{UV}^2}{\mu^2}\right), \tag{3.3}$$

and so $\Lambda_{strong} = \Lambda_{UV} e^{-\frac{2\pi\tau}{\beta}}$. The condensing gauge group also generates a holomorphic superpotential

$$W = M_P^3 e^{-\frac{3}{\beta}(2\pi T)}. \tag{3.4}$$

Here T is the chiral superfield corresponding to the 4-cycle wrapped by the branes, given classically by $T = \tau + i \int_\Sigma C_4$, but as we will see this does not have to hold at loop level. The prefactor of M_P^3 in (3.4) is enforced by supergravity: M_P is the only dimensionful quantity present in 4d supergravity and so is the only quantity that can appear here. In particular, quantities such as the string scale $M_s = M_P/\sqrt{\mathcal{V}}$ cannot appear in (3.4) as they violate holomorphy.

However, the normalised superpotential should also be identified with the strong coupling scale:

$$e^{K/2} W = \langle \bar{\lambda}\lambda \rangle = \Lambda^3_{strong}. \tag{3.5}$$

Using $\mathcal{K} = -2 \ln \mathcal{V}$, we therefore obtain

$$\Lambda_{strong} = \Lambda_{UV} e^{-\frac{2\pi\tau}{\beta}} = \frac{M_P}{\mathcal{V}^{1/3}} e^{-\frac{2\pi \mathrm{Re}(T)}{\beta}}. \tag{3.6}$$

Identifying the scale from which the coupling starts to run, Λ_{UV}, with M_{string}, consistency of equation (3.6) requires that

$$\mathrm{Re}(T) = \tau + \frac{\beta}{2\pi} \ln \mathcal{V}^{1/6}. \tag{3.7}$$

As τ involves a factor of $e^{-\phi}$ from the Einstein frame volume, this represents a 1-loop redefinition of T which appears to be required for consistency with the framework of 4d supergravity.

Both the above arguments suggest moduli redefinitions may be occurring in geometries relevant to moduli stabilisation. We therefore think it is important to study the possible effects on moduli stabilisation. While the form of the superpotential must be unaltered, the redefinitions can affect the Kähler potential. For the purpose of this Chapter we shall make a simple assumption about the nature of these effects. The classical Kähler potential is given by $K = -2 \ln \mathcal{V}$, where $\mathcal{V}$ is the physical (Einstein frame) volume of the Calabi-Yau. We shall assume that the dependence on the physical volume remains the same, and that the change in the Kähler potential as a function of the moduli comes simply from re-expressing the volume in terms of the redefined moduli. This assumption could be verified by a 1-loop computation of the Kähler potential, which is however beyond the scope of this work.

We note both arguments relate to redefinitions of the small blow-up modulus. The motivation for redefinitions of the large modulus is weaker. Indeed in this case the explicit loop calculation of [35] does not show any evidence for it (as this was a toroidal computation, it should incorporate the behaviour of the overall volume modulus). For completeness we shall however also study redefinitions of this large modulus in the following Section.

3.2 Moduli stabilisation

In this Section we investigate the consequences of 4-cycle volume redefinition in moduli stabilisation. We focus on the particular case of the $\mathbb{P}^4_{[1,1,1,6,9]}$ orientifold and study what happens to the position of the minimum when we redefine τ_b and τ_s. We study these two cases separately.

3.2.1 Redefining τ_s

Refedining τ_s according to Eq. (3.1) we find that the Kähler potential for moduli fields becomes:

$$K = -2\ln\left[\frac{1}{\lambda}(\tau_b^{3/2} - [\tau_s - \alpha\ln(\mathcal{V})]^{3/2}) + \frac{\xi}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2}\right] + K_{cs}. \tag{3.8}$$

We allow an arbitrary parameter α in the redefinition. Note the term proportional to α dominates at large volume over the term proportional to ξ. Computing the supergravity scalar potential we find

$$\begin{aligned} V = &\frac{1}{\tau_b^{9/2}}\left(-\frac{9}{2}|W_0|^2\alpha\sqrt{\tau_s}\lambda^2 + \frac{3}{4}|W_0|^2\frac{\lambda^3\xi}{g_s^{3/2}}\right) + \frac{2}{3}\frac{a^2|A|^2\lambda^2 e^{-2a\tau_s}}{\tau_b^{3/2}\sqrt{\tau_s}}(4\tau_s - 2\alpha\ln(\mathcal{V})) - \\ &- \frac{1}{\tau_b^3}a\left(\bar{A}e^{a\tau_s}W_0 + Ae^{a\tau_s}\bar{W}_0\right)e^{-2a\tau_s}\lambda^2(2\tau_s + 3\alpha - 2\alpha\ln(\mathcal{V})). \end{aligned} \tag{3.9}$$

We have dropped terms that are subleading by powers of $\mathcal{V}$. This can be further simplified to:

$$\begin{aligned} V = &\frac{3}{4}|W_0|^2\frac{\xi\lambda^3}{g_s^{3/2}}\left(1 - \frac{6\alpha\sqrt{\tau_s}}{\xi\lambda}\right)\frac{1}{\tau_b^{9/2}} + \frac{8}{3}\frac{a^2|A|^2\lambda^2 e^{-2a\tau_s}}{\tau_b^{3/2}}\sqrt{\tau_s}\left(1 - \frac{\alpha}{2\tau_s}\ln(\mathcal{V})\right) - \\ &- \frac{\lambda^2}{\tau_b^3}2a\tau_s\left(\bar{A}W_0 + A\bar{W}_0\right)e^{-a\tau_s}\left(1 - \frac{\alpha}{\tau_s}\ln(\mathcal{V})\right), \end{aligned} \tag{3.10}$$

by noting that at the minimum we expect $\ln\mathcal{V} = a\tau_s \gg 1$ and that the nonperturbative part of the superpotential is small compared to W_0. One can rewrite Eq. (3.10) as

$$V = \Lambda\left(1 - \frac{6\alpha g_s^{3/2}\sqrt{\tau_s}}{\xi\lambda}\right)\frac{1}{\tau_b^{9/2}} + \Omega\frac{\sqrt{\tau_s}e^{-2a\tau_s}}{\tau_b^{3/2}}\left(1 - \frac{\alpha}{2\tau_s}\ln\mathcal{V}\right) - \Psi\frac{\tau_s e^{-a\tau_s}}{\tau_b^3}\left(1 - \frac{\alpha}{\tau_s}\ln\mathcal{V}\right). \tag{3.11}$$

by defining

$$\Lambda = \frac{3}{4}|W_0|^2\lambda^3\xi/g_s^{3/2}, \qquad \Omega = \frac{8}{3}a^2|A|^2\lambda^2, \qquad \Psi = 2a\lambda^2(\bar{A}W_0 + \bar{W}_0 A). \tag{3.12}$$

At this point we again note that the contribution from the field redefinition is dominant over the α'^3 correction also in the scalar potential as well as the Kähler potential. This makes the study of this potential particularly interesting as the α'^3 correction played an important role in establishing the existence of the original large volume minimum.

To find the minimum of the potential one must solve $\frac{\partial V}{\partial \tau_s} = \frac{\partial V}{\partial \tau_b} = 0$. Keeping in mind that we expect $\ln \mathcal{V} \approx a\tau_s \gg 1$, from the first condition we find that at the minimum

$$\tau_b^{3/2} = \frac{\Psi}{2\Omega}\sqrt{\tau_s}e^{a\tau_s}\left(1 - \frac{3}{4a\tau_s} - \frac{\alpha}{2\tau_s}\ln \mathcal{V}\right), \tag{3.13}$$

while the second condition yields:

$$\tau_s = \left(\frac{4\Lambda\Omega}{\Psi^2}\right)^{2/3}\left(1 + \frac{1}{3a\tau_s} + \alpha\left(\frac{\ln \mathcal{V}}{\tau_s} - \frac{4g_s^{3/2}\sqrt{\tau_s}}{\lambda\xi}\right)\right). \tag{3.14}$$

Using the definitions in Eq. (3.12) we may rewrite these equations as:

$$\tau_s = \tilde{\tau}_s\left(1 + \frac{1}{3a\tau_s} + \alpha\left(\frac{\ln \mathcal{V}}{\tau_s} - \frac{2}{\tilde{\tau}_s}\right)\right), \qquad \tau_b = \tilde{\tau}_b\left(1 - \frac{1}{2a\tau_s} - \frac{\alpha}{3\tau_s}\ln \mathcal{V}\right), \tag{3.15}$$

where $\tilde{\tau}_s$ and $\tilde{\tau}_b$ are given by:

$$\tilde{\tau}_s = \left(\frac{\lambda\xi}{2g_s^{3/2}}\right)^{2/3}, \qquad \tilde{\tau}_b = \left(\frac{3|W_0|}{4|A|a}\sqrt{\tau_s}e^{a\tau_s}\right)^{2/3}. \tag{3.16}$$

These equations give the locus of the minimum as a perturbative expansion in α. As a check on the validity of these computations we also perform a numerical study of the minima of the potential for different values of α and compare the results with our analytical computation. The results are plotted in Fig. 3.1.

We can see that for small values of α ($0 < \alpha < 3 \times 10^{-3}$) there is good agreement between the numerical location of the minimum coordinates and the approximate analytic solution.

Observation of Fig. 3.1 reveals that the deviation between the numerical and the analytical grows with α, but remains small (order of a few percent) throughout the range of values considered. The deviation between the analytical and the numerical results for τ_b is one order of magnitude larger than the one for τ_s - this is due to

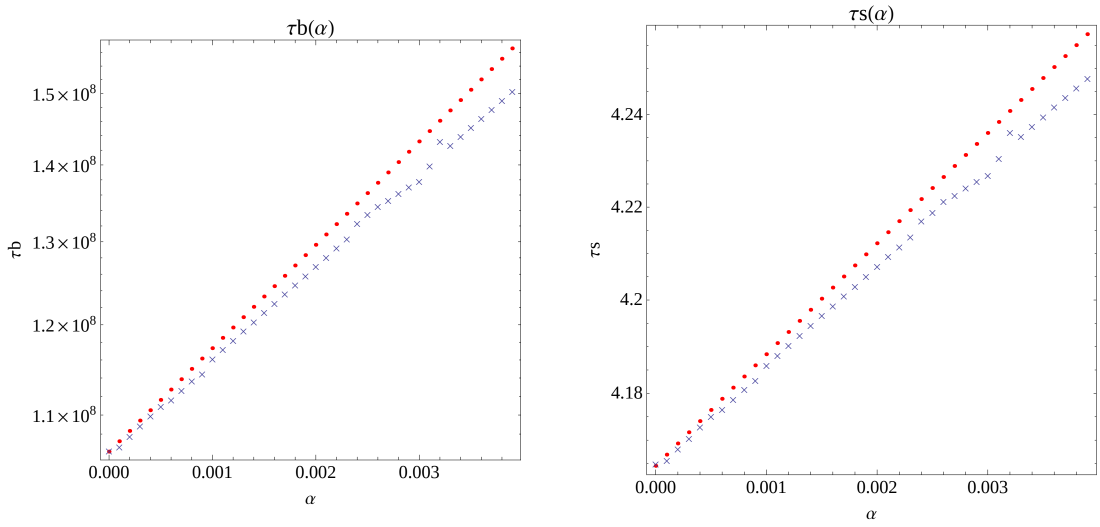

Figure 3.1: τ_b and τ_s at the minimum as functions of α. The dots are the analytical solutions while the crosses represent the numerical results.

the exponential dependence of τ_b on τ_s as seen in Eq. (3.16), which amplifies the deviation.

As we consider larger values of α the linear approximation breaks down, as is illustrated in Fig. 3.2. In this regime the effect of the α'^3 correction also becomes negligible in the large volume limit. However, we see that the structure of the potential (in particular the minimum at exponentially large values of the volume) remains unaltered. This is quite striking and shows that these two terms can play a similar role in the stabilisation of the Kähler moduli of the theory.

3.2.2 Redefining τ_b

We now consider the effects of redefining the large cycle τ_b. In this case the motivation for the redefinition is weaker as neither of the arguments presented in Section 3.1 directly apply to this cycle. However it is still worth considering for completeness. We redefine the τ_b 4-cycle of $\mathbb{P}^4_{[1,1,1,6,9]}$ according to Eq. (3.1) and follow the same procedure as in the previous section. The Kähler potential becomes:

$$K = -2\ln\left[\frac{1}{\lambda}([\tau_b - \beta\ln(\mathcal{V})]^{3/2} - \tau_s^{3/2}) + \frac{\xi}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2}\right] + K_{cs}. \tag{3.17}$$

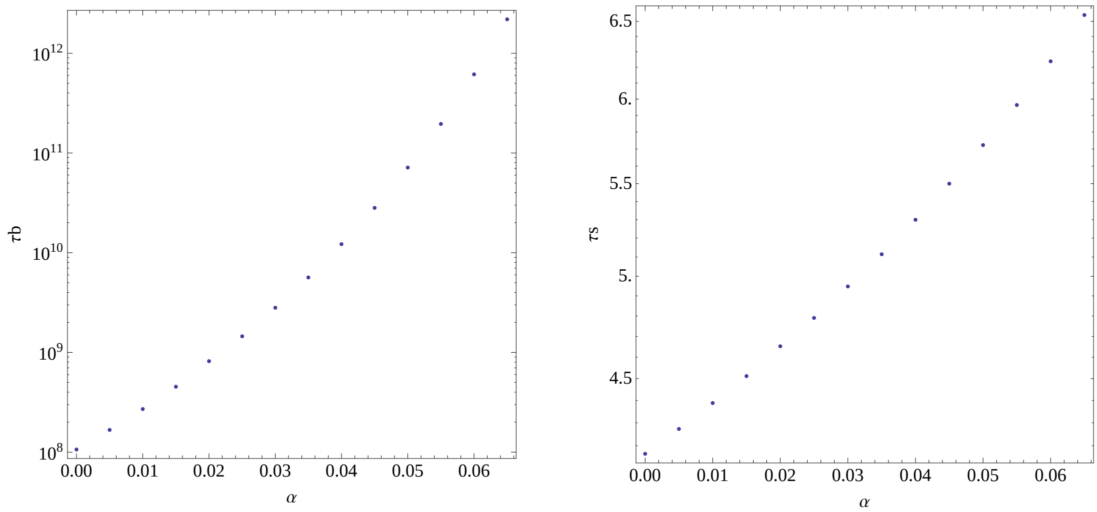

Figure 3.2: $\tau_b(\alpha)$ and $\tau_s(\alpha)$: The linear expansion breaks down for higher values of α.

The scalar potential is given by

$$\begin{aligned} V =& \frac{3|W_0|^2\lambda^3\xi}{4g_s^{3/2}\tau_b^{9/2}} + \frac{9|W_0|^2\beta\lambda^2}{2\tau_b^4} + \frac{8a^2|A|^2\lambda^2\sqrt{\tau_s}e^{-2a\tau_s}}{3\tau_b^{3/2}} \\ & - \frac{a\lambda^2\tau_s e^{-2a\tau_s}(6\beta\ln\mathcal{V} + 3\beta + 2\tau_b)(A(\bar{A} + e^{a\tau_s}\bar{W}_0) + c.c.)}{\tau_b^4}, \end{aligned} \tag{3.18}$$

which can be further simplified to

$$V = \frac{\Lambda}{\tau_b^{9/2}} + \frac{\Psi\beta}{\tau_b^4} + \Phi\frac{\sqrt{\tau_s}e^{-2a\tau_s}}{\tau_b^{3/2}} - \theta\frac{\tau_s e^{-a\tau_s}}{\tau_b^3}\left(1 + \frac{\beta}{\tau_b}(3\ln\mathcal{V} + \frac{3}{2})\right), \tag{3.19}$$

where

$$\Lambda = \frac{3|W_0|^2\lambda^3\xi}{4g_s^{3/2}}, \qquad \Psi = \frac{9|W_0|^2\lambda^2}{2}, \qquad \Phi = \frac{8a^2|A|^2\lambda^2}{3}, \qquad \theta = 2a\lambda^2(A\bar{W}_0 + c.c.). \tag{3.20}$$

In this case the effect of the redefinition in Eq. (3.19) is to generate a term that dominates in the large volume limit over any other term in the potential (as it scales as $\sim \tau_b^{-4}$ while all other terms scale as $\sim \tau_b^{-9/2}$). In the limit of small β we find that the minimum of Eq. (3.19) is located at

$$\tau_s = \tilde{\tau}_s\left(1 + \frac{1}{3a\tau_s} + \frac{\beta}{\tau_b}\left(\frac{-44}{9}\ln\mathcal{V} + \frac{16}{9}\frac{\tau_b^{3/2}}{\tilde{\tau}_s^{3/2}}\right)\right), \qquad \tau_b = \tilde{\tau}_b\left(1 - \frac{1}{2a\tau_s} + 2\beta\frac{a\tau_s}{\tau_b}\right), \tag{3.21}$$

where $\tilde{\tau}_s$ and $\tilde{\tau}_b$ are given by Eq. (3.16) .

In Fig. 3.3 we depict the numeric and analytic location of the minima as functions of β. The conclusions are essentially the same as for the previously studied case, the main difference being the fact that the allowed range for the loop factor β is much smaller.

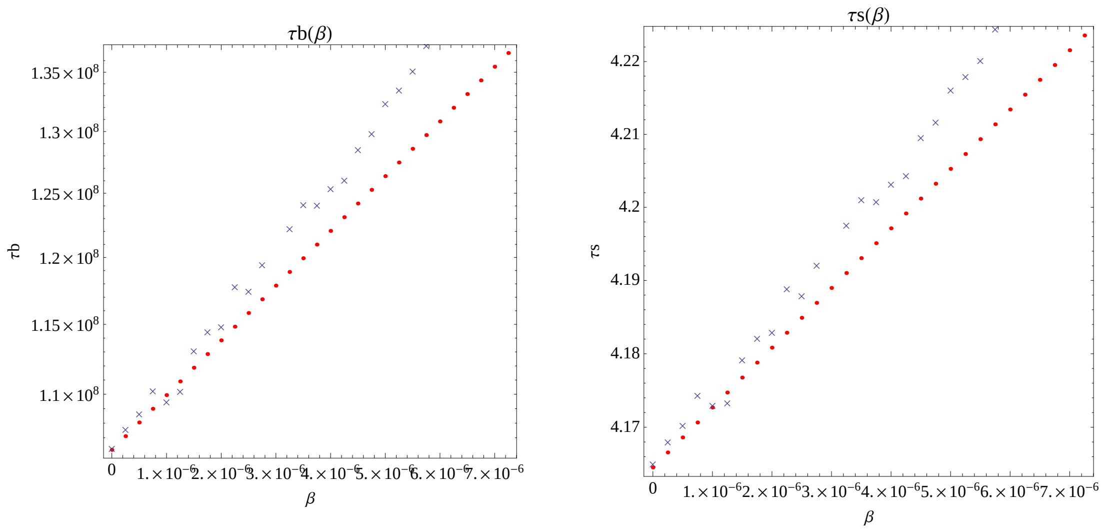

Figure 3.3: τ_b and τ_s at the minimum as a function of β. The dots are the analytical solution while the crosses represent the numerical result.

In contrast to the case with τ_s, in this case the large volume minimum does not survive for large values of β. This is because the correction to the scalar potential is dominant at large volumes over all other terms, and so gives a runaway behaviour. For sufficiently large β this runaway structure overwhelms all other features of the potential.

3.2.3 Summary

We have considered the effects of two possible field redefinitions on moduli stabilisation. In the better motivated case, that of the redefinition of τ_s, we find that the basic structure of the large volume minimum is unaltered even though the effects of the redefinition dominate at large volume the α'^3 effects that played a crucial role in establishing the original large volume minimum. For completeness we have also studied the case of the redefinition of τ_b. In this case the redefinition gives a term in the scalar potential that at sufficiently large volume dominates all other terms and gives runaway.

3.3 Soft supersymmetry breaking

Having studied the effects of field redefinitions on moduli stabilisation let us now see whether they can give any new effects in the study of supersymmetry breaking.

In the first formulation of the large volume model it was originally assumed that the D7-brane supporting the MSSM fields and the D3-brane instanton wrap the same four-cycle, namely τ_s. However it was pointed out in [57] that instanton zero mode counting forbids this scenario. It was proposed in [54] that the simplest way to avoid this issue is to assume a threefold with at least three four-cycles: one large cycle and two small four-cycles. The volume of this class of Calabi-Yau can be written as

$$\mathcal{V} = (\eta_b \tau_b)^{3/2} - (\eta_s \tau_s)^{3/2} - (\eta_a \tau_a)^{3/2}, \tag{3.22}$$

where τ_b determines the volume of the Calabi-Yau, τ_s is wrapped by the D3-brane instanton and the four-cycle of size τ_a supports the MSSM D7 brane. Examples of explicit Calabi-Yaus with these structures can be found in [58–60]. Since $\mathbb{P}^4_{[1,1,1,6,9]}$ has only 2 Kähler moduli clearly it cannot be an example of a manifold obeying Eq. (3.22). Nonetheless, the conclusions in the previous Section regarding moduli stabilisation will remain the same once we consider a geometrical setup obeying Eq. (3.22) as will be explained later.

We have reviewed in Section 2.5.1 that by performing an expansion of the Kähler potential and superpotential in powers of the matter fields one can compute the soft supersymmetry breaking terms. One must specify the Kähler potential and superpotential for moduli fields $\hat{K}(\Phi, \bar{\Phi})$ and $\hat{W}(\Phi)$ as well as the gauge kinetic function f_i and the matter metrics, $Z(\Phi)$. In doing so one must note note that there are two regimes in the effective field theory: the geometric regime and the singular cycle regime [54]. Our purpose here is to see whether moduli redefinitions can affect the results on supersymmetry breaking that were found in [54] and reviewed in Section 2.5.2.

In the geometric regime all the four-cycles are larger than the string scale. In this regime the effective field theory is determined by:

$$\hat{K}(\Phi, \bar{\Phi}) = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) - \ln(S + \bar{S}) + K_{CS}, \tag{3.23}$$

$$\hat{W}(\Phi) = W_0 + Ae^{-aT_s}, \tag{3.24}$$

$$f_i = T_i - \frac{1}{2}\kappa_i S, \tag{3.25}$$

where the volume is given by Eq. (3.22) and the matter metric is

$$Z = \frac{\tau_a^q f(\Phi)}{\tau_b^p}, \tag{3.26}$$

with $f(\Phi)$ being a function of the complex structure moduli. The form of the modular dependence of (3.26) comes from taking the leading powers of the moduli. Values of p and q can be either deduced or significantly constrained by examining the behaviour of the physical Yukawa couplings under rescaling of the several moduli in the theory, as described in [55]. Arguing that in local models the interactions should be independent of the overall volume it is found that $p_\alpha = 1, \forall\alpha$. The q is more delicate and can depend on whether a field originates from 'internal' or 'normal' modes of branes [69]. In what follows we leave p and q unspecified. This allows one to find more generic expressions for the soft terms and makes clear the nature of the cancellation of the leading order contributions.

In the singular cycle regime, the Standard Model four-cycle is much smaller than the string scale. In this regime the theory is determined by:

$$\hat{K}(\Phi,\bar{\Phi}) = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) + c\frac{\tau_a^2}{\mathcal{V}} - \ln(S+\bar{S}), \tag{3.27}$$

$$\hat{W}(\Phi) = W_0 + Ae^{-aT_s}, \tag{3.28}$$

$$f_i = s_{ik}T_k + \delta_i S, \tag{3.29}$$

where the volume is given by

$$\mathcal{V} = (\eta_b\tau_b)^{3/2} - (\eta_s\tau_s)^{3/2} \tag{3.30}$$

and the matter metric is

$$Z = \frac{g(\Phi) + \tau_a^q h(\Phi)}{\tau_b^p}, \tag{3.31}$$

with c being a constant and $g(\Phi)$ and $h(\Phi)$ being arbitrary functions of the complex structure moduli/dilaton. The quadratic term in τ_s in Eq. (3.27) is introduced to ensure that the τ_s kinetic term is well defined when $\tau_s = 0$.

3.3.1 Redefinition of T_a modulus and implications for soft SUSY breaking terms

Our aim is to investigate the effects of the T_a modulus redefinition in the scale of the soft SUSY breaking terms. In particular one seeks to compare the resulting terms to the ones found for the same geometrical configuration in [54]. One might also consider redefining the remaining Kähler moduli of the theory, but the effects will be subleading. We therefore neglect them and concentrate on the redefinition of T_a only.

A fully consistent analysis must investigate the effects of the T_a redefinition in the full Kähler potential, that is in the Kähler potential for moduli and in the matter metrics. We will proceed by steps, analysing first the case where we redefine T_a in the Kähler potential only and then studying the full case.

3.3.1.1 Redefining T_a in the Kähler potential

First we investigate the consequences of applying the redefinition in Eq. (3.1) to the Kähler potential for moduli fields, $\hat{K}(\Phi, \bar{\Phi})$. As the Standard Model cycle is taken to be stabilised by a D-term we must impose the following condition for the Kähler modulus T_a:

$$\partial_a K = 0 \Leftrightarrow \tau_a = \alpha \ln \mathcal{V}. \tag{3.32}$$

From this condition and since the superpotential is independent of T_a, it follows that $F_a = 0$. In [54] the vanishing of F_a led to the vanishing of F^a. However since the Kähler metric is non diagonal and $\tau_a \neq 0$ here we find

$$F^a = K^{a\bar{b}} F_{\bar{b}} + K^{a\bar{s}} F_{\bar{s}} = -3\alpha M_{3/2}. \tag{3.33}$$

This is a significant difference from [54]. For the computation of the soft terms one also needs the F-terms for the remaining Kähler moduli. These are found to be

$$F^b = -2M_{3/2}\tau_b \qquad \text{and} \qquad F^s = -2M_{3/2}\tau_s, \tag{3.34}$$

at leading order in the volume expansion. We note that even though the τ_a dependence of $\hat{K}(\Phi, \bar{\Phi})$ is different in the geometric and singular cycle regimes, the results in Eqs. (3.33)-(3.34) hold in both regimes.

In the computation of the soft SUSY breaking terms we will neglect the α'^3 contributions to the Kähler potential as well as the nonperturbative superpotential. This is because the SUSY breaking structure of the large volume models is essentially inherited from no-scale, and the contributions of the α' corrections to the soft terms is volume suppressed compared to the terms we will consider.

Geometric Regime

Redefining the τ_a in the Kähler potential, Eq. (3.23), we find the soft terms to be given by:

$$M_{\tilde{G}} = \frac{-3}{2}\alpha\frac{M_{3/2}}{\mathrm{Re}(f_i)}, \quad (3.35)$$

$$M_{\tilde{Q}}^2 = M_{3/2}^2(1-p) + \frac{9q}{4}\frac{M_{3/2}^2}{\ln^2\mathcal{V}}, \quad (3.36)$$

$$\hat{\mu} = \frac{Z}{\sqrt{Z_{H_1}Z_{H_2}}}M_{3/2}\left((1-p+\frac{3}{2}\frac{q}{\ln\mathcal{V}})\right), \quad (3.37)$$

where we have set the superpotential μ term to vanish.

The $B\hat{\mu}$ term is given in this regime by

$$B\hat{\mu} = \frac{M_{3/2}^2 Z}{\sqrt{Z_{H_1}Z_{H_2}}}\left((2-2p-p(p+1)+2p^2) + \frac{1}{\ln\mathcal{V}}3(1-p)(q_{H1}+q_{H2}) + \frac{1}{\ln^2\mathcal{V}}\frac{9q}{2}(q_{H_1}+q_{H_2}-\frac{q-1}{2})\right), \quad (3.38)$$

where we have assumed that $\mu = 0$ and that all the fields feel the overall volume of the compactification in the same way, i.e., $p = p_{H_1} = p_{H_2}$.

The A-term in the geometric regime is given by

$$A_{\alpha\beta\gamma} = M_{3/2}(3-3p) - 3M_{3/2}\frac{\tau_s^{3/2}}{\mathcal{V}} + \frac{3M_{3/2}}{2\ln\mathcal{V}}(q_\alpha + q_\beta + q_\gamma). \quad (3.39)$$

Following the rescaling arguments in [55], we set $p = 1$ and note that the leading order terms cancel in all soft terms, leaving only the volume suppressed contributions.

Neglecting $\mathcal{O}(1)$ factors we summarise the results in the following table

Soft-term	Scale
$M_{\tilde{G}}$	$\alpha M_{3/2}$
$M_{\tilde{Q}}^2$	$M_{3/2}^2/\ln^2\mathcal{V}$
$\hat{\mu}$-term	$M_{3/2}\ln\mathcal{V}$
$B\hat{\mu}$-term	$M_{3/2}^2/\ln^2\mathcal{V}$
A-term	$M_{3/2}\ln\mathcal{V}$

Singular Cycle Regime

We now compute the soft terms in the singular cycle regime. Taking into account the gauge kinetic function of Eq. (3.29), the gaugino masses are given by:

$$M_{\tilde{G}} = (-3\alpha s_{aa} - 2\tau_b s_{ab} - 2\tau_s s_{as})\frac{M_{3/2}}{2\mathrm{Re}(f_a)}. \quad (3.40)$$

We define

$$\Gamma \equiv \frac{h(\Phi)\tau_a^q}{g(\Phi) + h(\Phi)\tau_a^q}, \quad (3.41)$$

then the soft terms are:

$$M_{\tilde{Q}}^2 = M_{3/2}^2(1-p) + \frac{M_{3/2}^2}{\ln^2 \mathcal{V}} \frac{9q}{4} \Gamma(q-1-q\Gamma), \tag{3.42}$$

$$\hat{\mu} = \left(e^{K/2}\mu + M_{3/2}Z(1-p) + \frac{3q}{2}\frac{M_{3/2}}{\ln \mathcal{V}}\Gamma \right) \frac{1}{\sqrt{Z_{H_1}Z_{H_2}}}, \tag{3.43}$$

$$B\hat{\mu} = \frac{M_{3/2}^2}{\sqrt{Z_{H_1}Z_{H_2}}} \Big(Z(2-2p-p(p+1)+p(p+1)) + \frac{1-p}{\ln \mathcal{V}} \left(\frac{3}{2} Z\Psi + 3pq \frac{\tau_a^q h(\Phi)}{\tau_b^p} \right) + \frac{9q/4}{\ln \mathcal{V}} \frac{\tau_a^q h(\Phi)}{\tau_b^p} (\Psi - q - 1) \Big), \tag{3.44}$$

$$A_{\alpha\beta\gamma} = M_{3/2}(3 - p_\alpha - p_\beta - p_\gamma) - 3M_{3/2}\frac{\tau_s^{3/2}}{\mathcal{V}} + \frac{3M_{3/2}}{2\ln \mathcal{V}} \sum_{\xi=\alpha,\beta,\gamma} q_\xi \Gamma_\xi, \tag{3.45}$$

where

$$\Psi \equiv \frac{q_1 h_1(\Phi)\tau_a^{q_1}}{g_1 + h_1(\Phi)\tau_a^{q1}} + \frac{q_2 h_2(\Phi)\tau_a^{q_2}}{g_2 + h_2(\Phi)\tau_a^{q2}}.$$

In the singular cycle regime the rescaling arguments used to find the modular dependence of the matter metric no longer apply. Nonetheless locality implies that $p = 1$, then the leading order contributions to the soft terms vanish and the result found in this regime mirrors the one of the geometric regime up to factors of $\Gamma \approx \mathcal{O}(1)$.

The significance of this is that soft terms are of a similar order to the gravitino mass, in contrast to the behaviour in [54] where soft terms were significantly suppressed compared to the gravitino mass.

3.3.1.2 Redefining T_a in the Kähler potential and in the matter metrics

In this Section we study the effects on the scale of the soft terms of the Standard Model four-cycle redefinition in the matter metrics. Once we take the redefinition of τ_a into account, the geometric regime matter metric becomes

$$Z = \frac{(\tau_a - \alpha \ln \mathcal{V})^q f(\Phi)}{\tau_b^p}, \tag{3.46}$$

while in the singular cycle regime we should have

$$Z = \frac{g(\Phi) + (\tau_a - \alpha \ln \mathcal{V})^q h(\Phi)}{\tau_b^p}. \tag{3.47}$$

It is crucial to note that, in general, the metrics in Eqs. (3.46) and (3.47) are singular and/or have singular derivatives once we impose the vanishing D-term condition. This poses a problem for the computation of soft SUSY breaking terms. A way to avoid this singular behaviour is to argue that the Kähler potential will receive higher order α' corrections which will modify the vanishing D-term condition. We sketch this idea in the geometric regime of the effective field theory and argue that the same holds in the singular cycle regime. Let $\phi(T_i)$ denote an arbitrary function of the Kähler moduli of the theory, then the full moduli Kähler potential for the $\mathcal{N}=1$ SUGRA in the geometric regime can be written as

$$\hat{K}(\Phi,\bar{\Phi}) = -2\ln\left(\mathcal{V}+\frac{\hat{\xi}}{2}+\phi(T_i)\right) - \ln(S+\bar{S}) + K_{CS}, \tag{3.48}$$

where we expect $\phi(T_i) \ll \mathcal{V}$. Then the vanishing D-term condition becomes

$$\partial_a \hat{K} = 0 \Leftrightarrow \frac{\partial}{\partial \tau_a}\left(\mathcal{V}+\phi(T_i)\right) = 0. \tag{3.49}$$

After redefining τ_a in the volume this yields

$$\sqrt{\tau_a - \alpha \ln \mathcal{V}}\left(1+\frac{3}{2}\frac{\alpha\sqrt{\tau_a}}{\mathcal{V}}\right) = -\frac{2}{3}\frac{\partial}{\partial \tau_a}\phi(T_i). \tag{3.50}$$

Since we do not know the explicit form of $\phi(T_i)$ we cannot solve Eq. (3.50). Nonetheless one may write the solution as

$$\tau_a = \alpha \ln \mathcal{V} + \theta, \tag{3.51}$$

where $\theta \ll \alpha \ln \mathcal{V}$ is a function of the moduli of the theory, related to $\phi(T_i)$ by

$$\sqrt{\theta} \approx -\frac{2}{3}\frac{\partial}{\partial \tau_a}\phi(T_i). \tag{3.52}$$

Once these higher order corrections are taken into account, Eqs. (3.46) and (3.47) no longer have a singular behaviour and the soft terms will be given as functions of θ.

One must also point out that a full treatment of the problem does not involve a redefinition of τ_a in the gauge kinetic functions, Eqs. (3.25) and (3.29), since these are protected by holomorphy. As a consequence of this, gaugino masses in the geometric and singular cycle regimes will still be given by Eqs. (3.35) and (3.40) respectively in the full case.

Geometric Regime

Proceeding in the same way as before we compute the leading order contributions in the volume expansion of the soft terms:

$$M^2_{\tilde{Q}} = M^2_{3/2}\left(1 - p - q_\alpha \frac{3\alpha}{2\theta} + \mathcal{O}(\mathcal{V}^{-1})\right), \tag{3.53}$$

$$\hat{\mu} = \frac{M_{3/2} Z}{\sqrt{Z_{H_1} Z_{H_2}}} \left(1 - p + \frac{3}{2} \frac{\alpha q_\alpha}{\theta} + \mathcal{O}(\mathcal{V}^{-1}) \right), \tag{3.54}$$

$$B\hat{\mu} = \frac{Z}{\sqrt{Z_{H1} Z_{H2}}} M_{3/2}^2 \left((p^2 - 3p + 2) - \frac{3}{2} \frac{\alpha}{\theta} q_\alpha + \mathcal{O}(\mathcal{V}^{-1}) \right), \tag{3.55}$$

$$A_{\alpha\beta\gamma} = 3M_{3/2} \left(1 - p + \frac{1}{\mathcal{V}} \left(\theta^{3/2} + \frac{3}{4} \sqrt{\tau_a} \frac{\alpha^2}{\theta} (q_\alpha + q_\beta + q_\gamma) \right) + \mathcal{O}(\mathcal{V}^{-2}) \right). \tag{3.56}$$

Singular Cycle Regime

The soft terms in the singular cycle regime, once we redefine τ_a in the matter metric become

$$M_{\tilde{Q}}^2 = M_{3/2}^2 \left(1 - p - q_\alpha \frac{3\alpha}{2\theta} \tilde{\Gamma} + \mathcal{O}(\mathcal{V}^{-1}) \right), \tag{3.57}$$

$$\hat{\mu} = \frac{M_{3/2} Z}{\sqrt{Z_{H_1} Z_{H_2}}} \left(1 - p + \frac{3}{2} \frac{\alpha q_\alpha}{\theta} \tilde{\Gamma} + \mathcal{O}(\mathcal{V}^{-1}) \right), \tag{3.58}$$

$$B\hat{\mu} = \frac{Z}{\sqrt{Z_{H1} Z_{H2}}} M_{3/2}^2 \left((p^2 - 3p + 2) - \frac{3}{2} \frac{\alpha}{\theta} q_\alpha \tilde{\Gamma} + \mathcal{O}(\mathcal{V}^{-1}) \right), \tag{3.59}$$

$$A_{\alpha\beta\gamma} = 3M_{3/2} \left(1 - p + \frac{3}{4\mathcal{V}} \alpha^2 \frac{\sqrt{\tau_a}}{\theta} \sum_{\xi=\alpha,\beta,\gamma} q_\xi \tilde{\Gamma}_\xi + \frac{c\theta^2}{2\mathcal{V}} + \mathcal{O}(\mathcal{V}^{-1}) \right), \tag{3.60}$$

where $\tilde{\Gamma}_\alpha$ is defined as

$$\tilde{\Gamma}_\alpha \equiv \frac{h(\Phi)\theta^q}{g(\Phi) + h(\Phi)\theta^q}. \tag{3.61}$$

Even though an analysis with generic modular dependence in the matter metrics requires one to consider higher order α' corrections to the Kähler potential into account, there is a particular value of q for which this is not strictly necessary. If one sets $q = 2$ in the singular cycle regime, the matter metrics and its derivatives are well defined even if we set $\theta \to 0$. In this particular case we find, after setting $p = 1$:

$$\hat{\mu} = 0 = A_{\alpha\beta\gamma}, \tag{3.62}$$

$$M_{\tilde{Q}}^2 \propto \frac{\ln \mathcal{V}}{\mathcal{V}^3} \alpha^5, \tag{3.63}$$

$$B\hat{\mu} \propto \frac{\alpha^5 \ln \mathcal{V}}{\mathcal{V}^2}. \tag{3.64}$$

In this case one also finds that the cancellation of the leading order terms happens as before, leaving only highly volume suppressed terms. The contribution to the soft terms computed in [54] will then dominate over the ones considered here.

Going back to the generic case, one must note that in both regimes the usual cancelation of leading order terms is present if $p = 1$. The scale of the soft terms is then parametrised by the ratio $\frac{\alpha}{\theta}$, where θ is essentially a derivative of the higher order α' terms in the Kähler potential for moduli fields, $\phi(T_i)$.

A full computation of the soft terms must include at least the terms in Eqs. (3.53)-(3.60) and the ones computed in [54]. To understand the relative size of both contributions and the scale of resulting the soft terms it is necessary to know $\phi(T_i)$ explicitly. At the moment this is beyond our possibilities. Nonetheless it is interesting to point out that it might be possible that the scale of the soft terms is set by the subleading terms in the Kähler potential. The main point here is to note that if we redefine the MSSM four-cycle according to Eq. (3.1) the theory is still well behaved and we can get non vanishing soft terms.

3.4 Discussion

In this Chapter we have studied the effects of one-loop moduli redefinitions on moduli stabilisation in the large volume scenario. We have reviewed the origins of such redefinitions in orbifold models and also given a new argument for the existence of such redefinitions in the geometric regime. In our study of the effects on moduli stabilisation we have assumed that the form of the Kähler potential is only altered by re-expressing the geometric volume in terms of the redefined moduli variables.

We found that redefinitions of the small moduli do not alter the basic structure of the large volume minimum: the minimum remains in qualitatively the same location and at exponentially large volume. This is actually quite striking as the redefinition generates terms that in the scalar potential at large volumes dominate the α'^3 corrections. For redefinitions of the overall volume, which is less motivated, the modified Kähler potential gives a scalar potential that actually leads to runaway and removes the large volume minimum.

We also studied the possible effects of moduli redefinitions on supersymmetry breaking. There we found that redefinitions may have the ability to modify the results of [54] and induce soft terms of a similar order to the gravitino mass. However there were certain ambiguities which depend on the form of the matter metrics, and resolving these ambiguities depends on corrections to the Kähler potential. While the results of this section may be potentially interesting, what is really needed is a full CFT computation to see the effect of redefinitions on terms in the Kähler potential.

Chapter 4

Moduli-Induced Vacuum Destabilisation

This Chapter is based on the paper [13].

String theory has no free parameters. All coupling constants are instead determined as vacuum expectation values of scalar fields - moduli. The values these scalar fields take are determined by the moduli potential, and these values determine the parameters of the Standard Model and through them the masses, couplings and interactions of all known particles.

Moduli potentials have many ingredients, and much work has been done on constructing potentials that stabilise the moduli in phenomenologically attractive fashions. However moduli vevs are environmental and so there is no reason in principle why they should stabilise at the same values at all regions in space and time - indeed, we should expect the converse. In this article we therefore look at ways of destabilising moduli from their vacuum values.

The basic mechanism we investigate is simple: as moduli source the couplings of Standard Model fields, any form of energy density represents a source for the moduli potential. If the local energy density is sufficiently large - and large energy densities are realised both within neutron stars and in the context of cosmological singularities - then this gives a contribution to the moduli potential that can destabilise the modulus vev from its minimum.

Destabilised moduli vevs may be hard to achieve but the payoff if it can be done is large:

- Small changes in moduli vevs would give a continuous deformation away from the Standard Model, with associated small shifts in particle masses and couplings.

- Any region in which moduli vevs are altered can catalyse exotic processes that are forbidden or highly suppressed within the Standard Model like proton decay.

- Large shifts in moduli can potentially provide windows into entirely different vacua of the underlying theory.

For these reasons we think it worthwhile to investigate the possibility of destabilising moduli in various contexts. This discussion can be made concrete by recent developments in moduli stabilisation, as it is difficult to discuss the chances of destabilising moduli without concrete and well-motivated potentials that first stabilise moduli.

For other related work, see for example [70–74].

4.1 The effective field theory

We study the possibility and the consequences of moduli/matter interaction within the framework of the large volume models [29], which allow for the stabilisation of all the moduli while generating a non supersymmetric AdS minimum at exponentially large volumes.

As described in Section 2.4.5, the combined effects of α'^3 corrections to K and instantonic corrections to W allow for the stabilisation of the Kähler moduli describing the geometry of the compact extra-dimensional manifold. Recall that for the simplest geometries, the so called Swiss cheese manifolds, the volume is expressed in terms of the Kähler moduli as

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}. \tag{4.1}$$

Assuming that the 4-cycle τ_s supports an ED3 brane or a stack of D7-branes that undergo gaugino condensation, the scalar potential for the large volume models can be computed by taking the Kähler potential and the superpotential, Eqs. (2.94) and (2.95). One can choose to stabilise the axio-dilaton and the complex structure moduli in a SUSY preserving way, $DW = 0$, by turning on fluxes in the extra dimensions. This will cause their contribution to the scalar potential to vanish. For the two moduli model we are considering the scalar potential can be written as:

$$V = C_1 \frac{\sqrt{\tau_s} e^{-2a\tau_s}}{\mathcal{V}} - C_2 \frac{\tau_s e^{-a\tau_s}}{\mathcal{V}^2} + \frac{C_3}{\mathcal{V}^3}, \tag{4.2}$$

where

$$C_1 = \frac{8}{3}\lambda a^2|A|^2, \qquad C_2 = 4|AW|a, \qquad C_3 = \frac{3}{4}\frac{|W|^2\xi}{g_s^{3/2}}. \tag{4.3}$$

The large volume AdS minimum of the potential is located at

$$\tau_s^{3/2} = \frac{\lambda\xi}{2g_s^{3/2}}\left(1 - \frac{1}{2a\tau_s}\right), \tag{4.4}$$

$$\mathcal{V} = \frac{3|W|\sqrt{\tau_s}e^{a\tau_s}}{\lambda a|A|}\left(1 - \frac{3}{4a\tau_s}\right). \tag{4.5}$$

4.1.1 Moduli spectrum

We have shown in Section 2.4.5 (see also Appendix A.1) that in the large volume minimum the small modulus is much heavier than the large modulus:

$$m_s \approx \frac{M_P}{\mathcal{V}} \qquad \gg \qquad m_b \approx \frac{M_P}{\mathcal{V}^{3/2}}. \tag{4.6}$$

One can therefore consider that it decouples from the theory and take τ_b to be the only dynamical variable in the problem. This corresponds to using Eqs. (4.4) and (4.5) to eliminate τ_s dependence. We will later apply this to the scalar potential, Eq. (4.2), but first let us formulate the problem in terms of canonically normalised fields.

4.1.2 Canonical normalisation of the volume modulus

As discussed in the previous Section, the moduli associated with the volume of the 'small' four cycle gets, through moduli stabilisation, a very large mass and can therefore be integrated out. What remains is the theory of a single scalar field. The kinetic part of the Lagrangian is [1]

$$L_K = G_{b\bar{b}}\partial_\mu T_b \partial^\mu T_{\bar{b}} = \frac{3}{4\tau_b^2}\partial_\mu\tau_b\partial^\mu\tau_b. \tag{4.7}$$

For convenience we will work with the canonically normalised field Φ, defined by

$$\frac{3}{4\tau_b^2}\partial_\mu\tau_b\partial^\mu\tau_b \equiv \frac{1}{2}(\partial\Phi)^2, \tag{4.8}$$

so we find

$$\Phi \equiv \sqrt{\frac{3}{2}}\ln\tau_b = \sqrt{\frac{2}{3}}\ln\mathcal{V}. \tag{4.9}$$

Using Eqs. (4.5) and (4.9) it is possible to rewrite the potential, Eq. (4.2), as

$$V = (1 - \alpha\Phi^{3/2})e^{-\sqrt{\frac{27}{2}}\Phi}, \tag{4.10}$$

[1] recall that $G_{i\bar{j}} = \frac{\partial^2}{\partial T_i \partial T_{\bar{j}}}K$ and $T_i = \tau_i + ib_i$

where α is a constant and we have ignored factors of order unity[2]. In Fig. 4.1 we plot the potential for the canonically normalised volume modulus, Eq. (4.10), which exhibits the characteristic AdS minimum at exponentially large volume. In order to

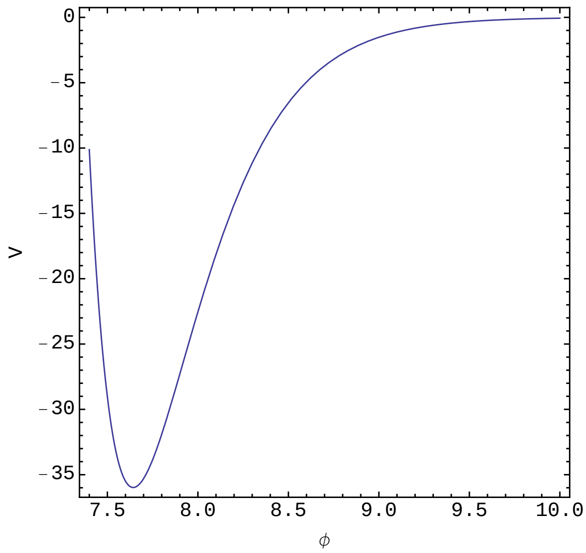

Figure 4.1: Potential (multiplied by 10^{15}) for the canonically normalised volume modulus for $\alpha = 0.05$.

obtain a phenomenologically viable model, it is necessary to lift the AdS minimum to dS or Minkowski, without spoiling the stabilisation of the Kähler moduli of the theory. The procedure proposed in [28] is to add a $\overline{D3}$ brane which generates a term in the potential of the form:

$$V_{\overline{D3}} \propto \frac{1}{\mathcal{V}^2} = e^{-\sqrt{6}\Phi}. \tag{4.11}$$

The full potential then takes the form

$$V = (1 - \alpha\Phi^{3/2})e^{-\sqrt{\frac{27}{2}}\Phi} + \epsilon e^{-\sqrt{6}\Phi}. \tag{4.12}$$

By tuning ϵ one can then find a dS or Minkowski minimum with all moduli stabilised. The minimum will lie in the same region as the initial AdS as illustrated in Fig. 4.2.

4.2 Moduli/matter interaction

Our goal is to investigate the interaction between matter and moduli fields. In particular, we want to know whether very large energy densities can destabilise the

[2]Note this requires including the subleading terms when integrating out the heavy modulus.

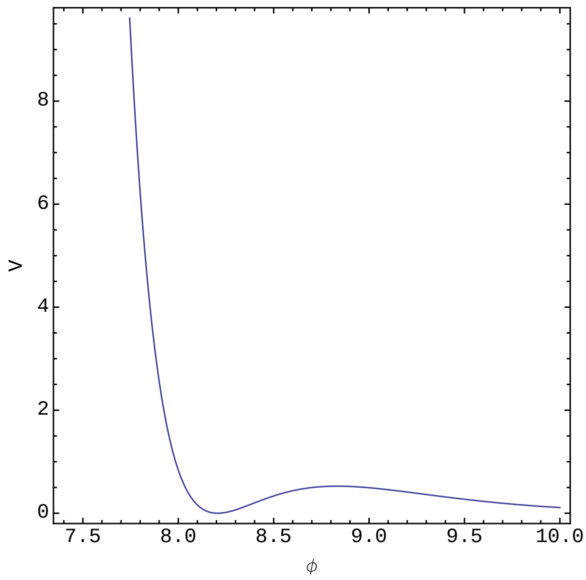

Figure 4.2: The full potential (multiplied by 10^{15}) for the canonically normalised volume modulus for $\alpha = 0.05$.

moduli or create bubbles of different vacua, and if so what will be the observable consequences.

Let us first enumerate some general features of this interaction. First, the volume modulus couples to everything. This is primarily because it sets overall scales and enters all dimensionful quantities. Secondly, the volume modulus is also the lightest modulus in these Swiss cheese geometries. This implies that it is the most appropriate field to consider the interactions of, as it should be easiest to destabilise. Finally, in order to try and destabilise the field we want the densest possible regions in order to have maximal effect on the potential.

Natural candidates to consider are neutron stars. These are very dense, gravitationally bound systems made up mostly of neutrons. We start by noting that the energy density of a neutron star is:

$$\rho_{NS} \approx \Lambda_{QCD}^4. \tag{4.13}$$

Due to gauge coupling running this term will be moduli dependent and therefore it will contribute to the moduli potential, Eq. (4.12). Let us briefly illustrate how this occurs. Recall that the QCD $\beta-$function is

$$\beta(g(\mu)) = -\frac{9}{16\pi^2} g(\mu)^3, \tag{4.14}$$

where as usual $\beta(g(\mu)) \equiv \mu\frac{dg}{d\mu}$. Substituting the definition of the $\beta-$function into Eq. (4.14) and integrating one finds that

$$\ln(\mu'/\mu'') = -\frac{8\pi^2}{9}\left(\frac{1}{g^2(\mu')} - \frac{1}{g^2(\mu'')}\right), \tag{4.15}$$

and setting $g(\mu') \to \infty$, $\mu' \equiv \Lambda_{QCD}$ yields

$$\Lambda_{QCD} = \mu'' e^{-\frac{8\pi^2}{9g^2(\mu'')}}. \tag{4.16}$$

Assuming that the coupling starts to run from the string scale M_s, we set

$$\mu'' = M_s = \frac{M_P}{\sqrt{\mathcal{V}}}, \tag{4.17}$$

$$g^2(\mu'') = g_{YM}^2, \tag{4.18}$$

Here g_{YM} is the QCD coupling at the string scale. In the context of the large volume models with Swiss-cheese geometry, the Standard Model comes from branes wrapping one of the small cycles, therefore g_{YM} evaluated at the string scale, is set by the size of these smaller cycles and so has no direct dependence on the overall volume $\mathcal{V}$. The moduli controlling the size of these smaller cycles are heavier than the overall volume modulus and were integrated out in obtaining the effective one-modulus potential (4.12).

It follows from Eqs. (4.17) and (4.18) that

$$\Lambda_{QCD} = \frac{M_P}{\sqrt{\mathcal{V}}} e^{-\frac{8\pi^2}{9g_{YM}^2}}. \tag{4.19}$$

The crucial fact to note here is that the volume dependence comes from assuming that the coupling starts to run from the string scale. The physics of this is simply that the QCD scale is a function of the scale from which the coupling starts running. However in string theory with a canonical gravitational action the string scale is itself a function of the moduli, and so the QCD scale itself is a function of the moduli.[3]

This allows us to add an extra term to the moduli potential, coming from the interaction with matter

$$\Lambda_{QCD}^4 \propto \left(\frac{1}{\sqrt{\mathcal{V}}}\right)^4 = e^{-\sqrt{6}\Phi}. \tag{4.20}$$

The full potential then becomes

$$V = (1 - \alpha\Phi^{3/2})e^{-\sqrt{\frac{27}{2}}\Phi} + \epsilon e^{-\sqrt{6}\Phi} + \rho_0 e^{-\sqrt{6}\Phi}, \tag{4.21}$$

[3] This is an advantage of focusing on the volume modulus, as this behaviour is model independent.

where we take

$$\rho_0 \propto \Lambda_{QCD}^4 e^{\sqrt{6}\Phi_0}. \tag{4.22}$$

At this point we must note that the assumption that the gauge coupling starts to run from the string scale can be relaxed. For example, in local models it is not the string scale but instead the winding scale from which couplings start running [65]. One can then take $\mu'' \propto \mathcal{V}^{-n}$, which implies $\Lambda_{QCD} \propto \exp(-\sqrt{\frac{-3n^2}{2}}\Phi)$, and therefore the term in the potential that parametrises the interaction with matter becomes

$$\Lambda_{QCD}^4 \propto e^{-\sqrt{24n^2}\Phi}. \tag{4.23}$$

Some potentially interesting cases include the Kaluza-Klein scale, $M_{KK} = M_P/\mathcal{V}^{2/3}$ and the Unification scale $M_{GUT} = M_P/\mathcal{V}^{1/3}$. Throughout the rest of this Chapter we will study the case where the couplings start to run from the string scale. The results from Kaluza-Klein our GUT scale running will be essentially the same with the feature that the larger the n, the smaller the modulus vev shift and the denser the environment required to destabilise the modulus form its vacuum vev.

The above computation explicitly shows the simplest possible way in which 4-dimensional energy densities depend on the moduli. However Eq. (4.23) is just an example of a more general underlying principle, namely that in a string theory context all masses, couplings and energy densities depend on vacuum expectation values of moduli fields and thus generate new terms in the moduli potential. This dependence may be complicated in general. However it must respect the property that as $\mathcal{V} \to \infty$, $M_s/M_P \to 0$ and so all dimensionful energy scales E in the 4d theory also satisfy $E/M_P \to 0$.

In the interest of having a well-defined model, we will assume throughout that the coupling to matter has the form derived above and the potential for the modulus field is given by Eq. (4.21).

4.3 Analysis of the volume modulus potential

In this Section we analyse the potential for the volume modulus, Eq. (4.21), investigating how the matter contribution can distort the potential and potentially lead to runaway. This will happen in the region of moduli space where the local energy density is comparable to the combination of the large volume potential plus uplifting term. For a fixed matter energy density, it is possible to achieve this by tuning the α

parameter in the potential, which is related to the Euler number of the extradimensional manifold.[4] For fixed α, the same effect occurs as the matter energy density is increased.

This potential tuning process is not completely free. One important constraint to this analysis comes from fifth force experiments. These limit the range of allowed masses for the volume modulus. For gravitational strength fifth force, the modulus mass must lie outside the range $[10^{-17}, 10^{-3}]$ eV (see e.g. [11]). We will first analyse the case where the local source is a neutron star and then perform a more generic analysis.

4.3.1 Neutron stars

We now examine the possibility of shifting the moduli vevs in a neutron star. We start by analysing the case where moduli physics is Planck coupled and then relax this assumption, treating the coupling as an extra free parameter.

We can approximate the potential in the vicinity of the minimum as

$$V(\Phi) = m^2(\Phi - \Phi_0)^2 + \Lambda_{QCD}^4 e^{-(\Phi-\Phi_0)/M_X}, \tag{4.24}$$

where m is the mass of the field Φ. Here $M_X = \lambda M_P$ gives the coupling strength of the modulus - if $\lambda \sim \mathcal{O}(1)$ then the modulus is Planck coupled, whereas if $\lambda \ll 1$ then the modulus-matter coupling is stronger than gravitational. Φ_0 is the vacuum expectation value of the modulus.

From this we see that the shift in the modulus away from its vacuum value Φ_0 is given by

$$(\Phi - \Phi_0) \simeq \frac{\Lambda_{QCD}^4}{2m^2 M_X}. \tag{4.25}$$

Recalling that in string compactifications the high energy couplings are directly related to the vevs of moduli fields, one observes that the fractional shift in a dimensionless coupling is of order

$$\frac{(\Phi - \Phi_0)}{M_X} \simeq \frac{\Lambda_{QCD}^4}{2m^2 M_X^2}. \tag{4.26}$$

These expressions are valid for small shifts in the modulus vev: for larger shifts, the global structure of the potential will become relevant.

[4]This modifies the gravitino mass and gravity-mediated SUSY breaking scale in the theory.

4.3.1.1 Planck coupled moduli

In general we expect moduli to be Planck-coupled, and this is true for the volume modulus in the large volume models. Since $\Lambda_{QCD}^4 \approx 10^{-80} M_P^4$, in order to get the minimum inside the compact object to differ from the one outside we are required to work at very large Φ where the local energy density effects become comparable to the background potential. This in itself does not pose any problem. However the mass of the modulus is given by Eq. (4.6) and is also determined by Φ. For $M_X \sim M_P$ we see from Eq. (4.26) that an $\mathcal{O}(1)$ shift in couplings (equivalently an $\mathcal{O}(M_P)$ shift in the modulus vev) requires a modulus mass of $m \sim 10^{-11}$eV (this corresponds to $\alpha \approx 3.1 \times 10^{-3}$ and $\Phi \approx 48$). However this mass value falls within the range that is excluded by fifth force experiments and in the context of the large volume models would also require a string scale of around 10keV, which is manifestly excluded.

4.3.1.2 Strongly coupled moduli

In a more phenomenological approach one might consider allowing the coupling strength M_X to be a free parameter rather than tying it to the Planck scale M_P, even though in the model considered the volume modulus with the specific potential, Eq. (4.12), is necessarily Planck coupled. Moduli in string models can certainly be coupled much more strongly than M_P (in the large volume models the blow-up moduli have matter couplings suppressed by M_s for example).

In this case we require the modulus mass not to be smaller than 10^{-2} eV and require an $\mathcal{O}(1)$ shift in a coupling. Depending on the precise value of the modulus mass, a numerical analysis now reveals that the interesting range for M_X is between 10^7 and 10^{10} GeV, consistent with the estimates of Eq. (4.26). Such strongly coupled moduli are in principle obtainable for models with low string scales. However this does not really help us, as we expect light moduli with masses $m \lesssim 1$eV and couplings only suppressed by $M_X \sim 10^9$GeV to be excluded by bounds on the cooling of SN1987A by emission of exotic light particles.[5]

4.3.2 Cosmic strings and other very dense objects

We now put neutron stars to one side and consider the typical density that would be required in order to cause Planckian displacements of moduli. We stress that we

[5] The cooling of SN1987A excludes axion decay constants $f_a \lesssim 10^{10}$ GeV - see for example [75]. The precise numbers entering the bound do depend on the pseudoscalar nature of the axion coupling, and so would be modified for emission of a modulus, but we do not expect the order of magnitude of the bound to change.

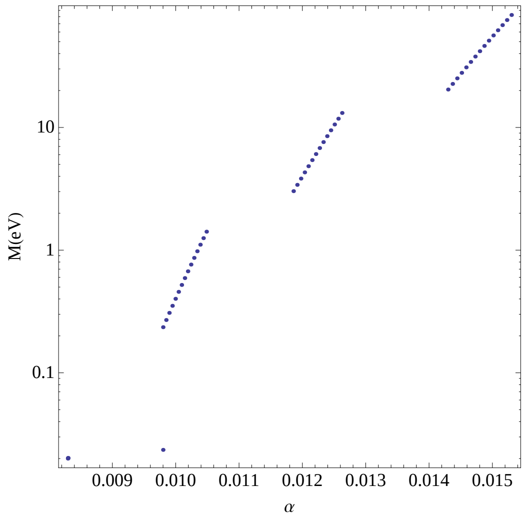

Figure 4.3: Modulus mass as a function of the α parameter for, from left to right, $M_x = 10^{10}, 10^9, 10^8, 10^7$GeV.

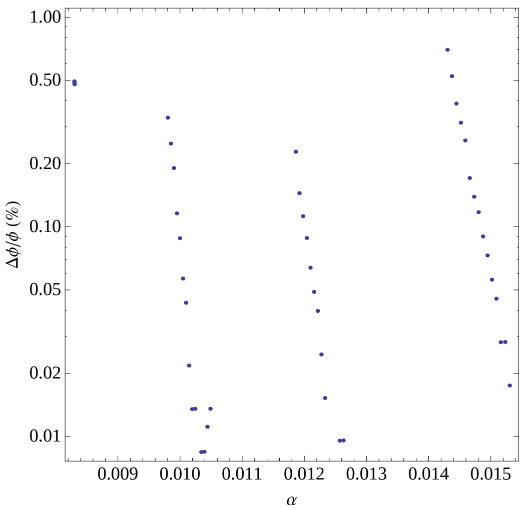

Figure 4.4: Normalised $\Delta\phi$ as a function of the α parameter for, from left to right, $M_x = 10^{10}, 10^9, 10^8, 10^7$GeV.

here keep the requirement that moduli physics is Planck coupled, as expected from string theory. We again aim to find the region of parameter space which allows for the shift of moduli vevs while remaining compatible with constraints from fifth-force experiments.

The fundamental reason why neutron star energy densities could not destabilize Planck coupled moduli is due to the hierarchy $\left(\frac{\Lambda_{QCD}}{M_P}\right)^2 \approx 10^{-40}$ in Eq. (4.26). It is therefore clear that we would need objects of higher energy density.

In what follows we perform a numerical scan for objects of energy densities $\rho \in [10^{-60}, 10^{-38}]\, M_P^4$. The results of the numerical study are shown in Figs. 4.5 and 4.6. We require an $\mathcal{O}(0.1)$ fractional displacement of the moduli from its vacuum value in a dense background. We plot in Fig. 4.5 the vacuum mass of the modulus for which this can be attained, for several different values of the background density. One sees that as the local perturbation becomes less dense, the region of parameter space where a deviation of order 1% in the modulus vev is attained corresponds to a region where the mass for this modulus is smaller. Keeping in mind that the fifth force lower limit for the mass is around 10^{-2} eV we conclude that the minimum energy density of an object capable of generating regions of different vacuum while still being compatible with fifth force constraints is $\rho \approx 10^{-60} M_P^4$. Figure 4.6 shows the fractional shift in the modulus vev for various energy densities.

Having performed this generic analysis we should now consider where such densities could come from. We first note that any such object has to be *extremely* dense,

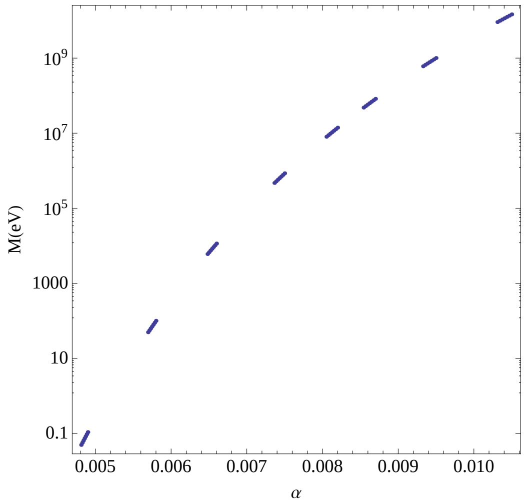

Figure 4.5: Modulus mass as a function of the α parameter for, from left to right, $\rho = 10^{-60}, 10^{-55}, 10^{-50}, 10^{-45}, 10^{-44}, 10^{-42}$, 10^{-40}, $10^{-38} M_{Pl}^4$.

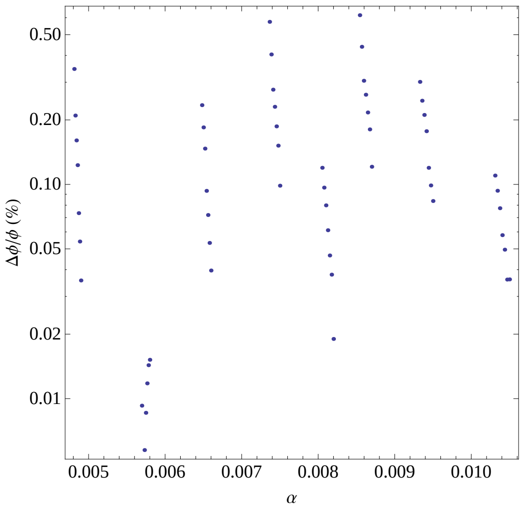

Figure 4.6: Normalised $\Delta\phi$ as a function of the α parameter for, from left to right, $\rho = 10^{-60}, 10^{-55}, 10^{-50}, 10^{-45}, 10^{-44}, 10^{-42}$, 10^{-40}, $10^{-38} M_{Pl}^4$.

about 20 orders of magnitude denser than neutron stars. For static objects a couple of possibilities present themselves. First, there are GUT cosmic strings. These topological defects are remnants of the GUT breaking which may have happened early in the history of the universe. Their mass per unit length μ is related to the scale of the breaking of the symmetry that generates them σ by $\mu \propto \sigma^2$. In the case of GUT strings, $\sigma \approx 10^{16}$ GeV and therefore $\mu \approx 10^{32}$ GeV^2. For cosmological purposes the strings are taken to be one dimensional objects, since they have to be either closed (unstable) or infinite. This is a simplification and from the theoretical point of view they are expected to have a finite radius of the order of the correlation length of the field that spontaneously breaks the GUT.

Secondly, one could imagine dark sector analogues of neutron stars - compact bound objects held together by degeneracy pressure in the same way as neutron stars are, in the case that there existed a dark analogue of QCD with a confinement scale $\Lambda_{dark} \gtrsim 1$ TeV. To avoid black hole formation such objects would need to be highly compact with rather small radii.

However these are rather speculative ideas and there is no strong reason to think either of these two objects exist. One case which does exist and where highly energy-dense regions are expected is that of singularities arising either cosmologically or through black hole formation. This will be analysed in the next Sections.

4.3.3 Cosmological singularities

The fact that near cosmological singularities energy densities may be arbitrarily high makes them good candidates for the study of a coupled modulus/matter system. This means that not only is it possible to destabilise the minimum for the moduli potential but also that decompactification might be possible.

The model we consider is that of a closed, matter dominated FRW universe. The overall spacetime structure is $\mathcal{R}^+ \times S^3 \times CY^6$. As before we assume that the matter is dusty and baryon-like, with a mass depending on the volume modulus as in Eq. (4.19) (where Λ_{QCD} can be replaced by any explicit mass scale). We take the modulus scalar field potential to be given as before by the large volume potential plus the uplifting term. We assume the volume modulus is initially at the minimum and unexcited, so that $\Omega_{\Phi,init} \equiv (\rho_\Phi/\rho_{tot})_{init} = 0$ while $\Omega_{matter,init} = 1$. The matter-moduli coupling follows from the assumption that the mass of the dust particles is moduli dependent, as in Eq. (4.19), and that their total number is conserved. The energy density is then moduli dependent and takes the form:

$$\rho(a, \Phi) = \frac{\rho_0}{a^3} e^{-\sqrt{6}(\Phi - \Phi_0)}. \tag{4.27}$$

The evolution of the system is determined, as usual, by the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\frac{\dot{\Phi}^2}{2} + V(\Phi) + \rho(a, \phi)\right) - \frac{\kappa}{a^2}, \tag{4.28}$$

and the Klein-Gordon equation for a homogeneous and isotropic field

$$\ddot{\Phi} = 3\frac{\dot{a}}{a}\dot{\Phi} + (V(a, \Phi) + \rho(a, \Phi))_{,\Phi}. \tag{4.29}$$

To study the coupled modulus-matter dynamics we solve the equations of motion numerically using a modified version of [76]. As expected, the initial evolution of this universe is the same as a regular closed matter dominated universe: the scale factor grows to a maximum before the universe starts collapsing, with the scale factor shrinking to zero. This behaviour is shown in Fig. 4.8. During most of the collapse, the system will not show any departure from the behaviour of a closed, matter dominated FRW universe. It is only when the scale factor becomes sufficiently small (let's call it $\overline{a}$), that the term in Eq. (4.27) will become large enough to modify the structure of the modulus potential, Eq. (4.21), and play a significant role in the evolution of the field. This happens, to first order, when the matter contribution to the potential at the minimum is comparable to the potential plus matter at the maximum, i.e.

$$V(\Phi) + \rho(a, \Phi)|_{min_0} = V(\Phi) + \rho(a, \Phi)|_{max_0}. \tag{4.30}$$

Note that min_0 and max_0 denote the minimum and maximum of the large volume plus uplifting potential (which differ from the extrema of the full potential). One can get an semi analytical estimate for $\overline{a}$, by solving Eq. (4.30), finding

$$\overline{a} = \left(\frac{\rho_0}{V(\Phi_{max_0})} (e^{-\sqrt{6}\Phi_{min_0}} - e^{-\sqrt{6}\Phi_{max_0}}) \right)^{1/3} . \tag{4.31}$$

This estimate can be refined by expanding $a_c = \overline{a} + \delta a$ and solving the condition

$$V(\Phi) + \rho(\overline{a} = \delta a, \Phi)|_{min} = V(\Phi) + \rho(\overline{a} + \delta a, \Phi)|_{max}, \tag{4.32}$$

where min and max denote the position of the minimum and the maximum of the full potential, i.e. including matter contribution, when $a = \overline{a}$. Solving Eq.(4.32) one finds

$$a_c \equiv \overline{a} + \delta a = \left(\frac{\rho_0}{V(\Phi_{max}) - V(\Phi_{min})} (e^{-\sqrt{6}\Phi_{min}} - e^{-\sqrt{6}\Phi_{max}}) \right)^{1/3} , \tag{4.33}$$

which is in good agreement with numerical estimates.

As the scale factor approaches a_c, the modulus expectation value begins to shift and the barrier to decompactification decreases. After a short period of evolution, the term (4.27) becomes the dominant term in the potential and there is no obstacle to prevent the field Φ from rolling towards infinity. The evolution of Φ is shown in Fig. 4.8 and we see the sharp increase in Φ beyond a critical time. This final stage of the evolution sees a rapid runaway of Φ to infinity while the scale factor continues to shrink to zero. The energy density of the universe will quickly be dominated by the kinetic energy of the volume modulus, with the dust and potential contributions becoming negligible. The evolution of the system is depicted in Figs. 4.7, 4.8.

In this last region it is possible to solve the equations of motion analytically. The equations of motion are

$$\frac{d}{dt}\left(a^3\dot{\Phi}\right) = -a^3\left(\frac{\partial V}{\partial \Phi}\right), \tag{4.34}$$

$$\frac{d}{dt}H = -\frac{\dot{\Phi}^2}{2} + \frac{\kappa}{a^2}. \tag{4.35}$$

Once Φ starts to runaway, the potential vanishes exponentially fast and ceases to be a significant contribution. In this asymptotic regime we can then solve these equations by

$$\begin{aligned} \Phi(t) &= \Phi_0 - \sqrt{\frac{2}{3}} \ln(t_0 - t), \\ a(t) &= \left(\frac{3}{2}\right)^{1/6} (t_0 - t)^{1/3}. \end{aligned} \tag{4.36}$$

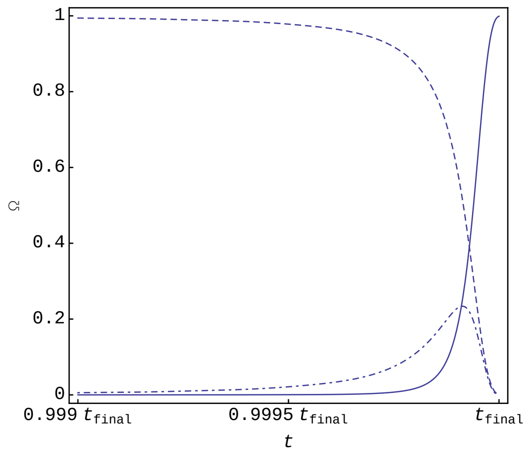

Figure 4.7: Late time evolution of the energy densities of matter and scalar field. Dashed line Ω_{dust}, dashed-dotted line $\Omega_{potential}$, full line $\Omega_{kinetic}$.

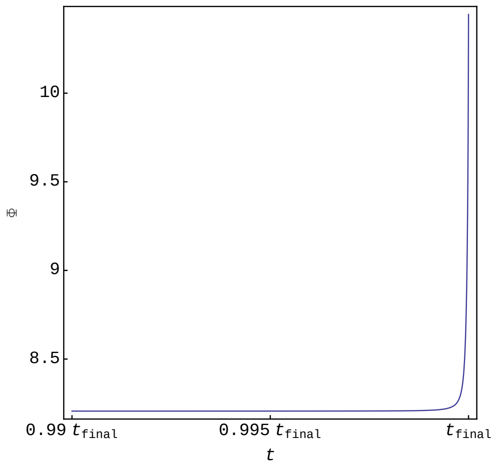

Figure 4.8: Late time evolution of the volume modulus in a closed, matter dominated FRW universe.

From the solution we see that we can self-consistently neglect the curvature term in Eq. (4.35) for $\frac{t_0-t}{t} \ll 1$. We can relate the canonically normalised field $\Phi(t)$ to the volume of the compact space using Eq. (4.9), to find that the compact volume $\mathcal{V}$ evolves as

$$\mathcal{V} = \frac{\mathcal{V}_0}{t_0 - t}, \tag{4.37}$$

and so diverges as $t \to t_0$.

In the limit as $t \to t_0$ this therefore describes a universe where

1. The 4-dimensional scale factor shrinks to zero size as $(t_0 - t)^{1/3}$.
2. The volume of the six compact dimensions is first destabilised before diverging as $(t_0 - t)^{-1}$.
3. There is a singularity at $t = t_0$, at which the 4-dimensional scale factor is formally zero and the 6-dimensional volume is formally infinite.

This represents a universe in which three spatial dimensions collapse and the extra six dimensions expand and reach infinite volume in finite time.

Let us consider the validity of a 4-dimensional effective field theory treatment. From the solution for $\Phi(t)$ in Eq. (4.36) it is clear that the (kinetic) energy density of the Φ field diverges in finite time,

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 = \frac{1}{3(t_0 - t)^2}. \tag{4.38}$$

Furthermore as decompactification occurs and the extradimensional volume increases, the 4-dimensional string scale decreases.

$$M_{s,4d}^4 = \frac{M_P^4}{\mathcal{V}^2} = M_P^4 (t_0 - t)^2 e^{-\sqrt{6}\Phi_0}. \tag{4.39}$$

Consequently, independent of initial conditions, as $t \to t_0$ the system will evolve to a state where the kinetic energy density in the field Φ is greater than the apparent cutoff of a 4d effective field theory.

Fortunately it is easy to understand what is happening from a higher dimensional perspective. Once the field Φ gets over its decompactification barrier, its potential soon becomes negligible - we can see in Fig. 4.7 how the kinetic energy of Φ is the dominant contributor to the energy density. Although we have written the problem in the language of 4-dimensional effective field theory, Φ is originally the volume modulus of the extra dimensions. Neglecting the potential energy, the system is then fundamentally that of 10-dimensional general relativity. The above solution then corresponds to a Kasner solution dimensionally reduced to 4 dimensions.

It is not immediately clear that the above numbers are consistent with a Kasner solution. Recall that the Kasner solution is

$$ds^2 = -dt^2 + \sum_i t^{2p_i} dx_i^2, \tag{4.40}$$

with $\sum p_i = 1$ and $\sum p_i^2 = 1$. For a $1 + 3 + 6$ dimensional Kasner solution, the allowed exponents are (using p_3 to denote the 3-dimensional growth and p_6 for the six-dimensional growth)

$$p_3 = -\frac{1}{3}, p_6 = \frac{1}{3}, \qquad p_3 = \frac{5}{9}, p_6 = -\frac{1}{9}. \tag{4.41}$$

It is clear that the evolution of the scale factor in (4.36) and the volume in (4.37) do not fit these conditions. However, note that the metrics are different: the metric used in the Kasner solution (4.40) is the 10-dimensional string frame metric, whereas the 4-dimensional metric for which Eqs. (4.36) and (4.37) applies is a dimensionally reduced metric that is related to the 10-dimensional metric by factors of the internal volume. It is then expected that the 4-dimensional scale factor does not have the Kasner exponent appropriate for a 10-dimensional (1+3+6) solution.

There is one striking feature about this behaviour. The initial conditions (a closed matter-dominated FRW universe) were unexceptional. However these conditions unavoidably evolve to give dynamic super-inflationary behaviour of the compact dimensions, which in theory reach infinite volume in finite time. Furthermore, this behaviour commences in the region controlled by effective field theory, where a 4d description is valid. In practice, the evolution of a Kasner solution should break down as the contracting dimensions approach the 10d string scale, regulating the infinity. This super-inflationary behaviour cannot be prevented - growth in the 4d energy density is a necessary consequence of a spacetime crunch - and this energy density must always eventually overcome the barrier to decompactification.

As formulated the dynamics have started with 3 large and 6 compact dimensions, ending with 6 large and 3 compact dimensions. However the physics is such that there is no reason not to reverse the process, and imagine starting with 6 large and 3 compact dimensions and ending with 6 compact and 3 large dimensions. In effect the universe bounces within effective field theory, which is achieved by the bounce occurring in different dimensions to the collapse: the collapse of certain dimensions triggers the expansion of others. This physics has some similarities to pre-Big Bang cosmology [77–79]. This was formulated using the heterotic dilaton and the $\mathcal{O}(d,d)$ symmetries of toroidal compactification. It would be interesting to make the connections more precise and see whether this super-inflationary growth of the compact dimensions is able to mimic some of the physics of conventional inflation.

4.3.4 A static solution

In this Section we study spherically symmetric configurations and analyse the resulting modulus profile. The models considered here are similar to the 'dark stars' mentioned in Section 4.3.2: we look for stable solutions of matter coupled to a modulus field, protected against gravitational collapse. In Section 4.3.5 we will allow for dynamical evolution of the profile and study gravitational collapse of dust balls.

Consider a spherical ball of dust of radius R. The dust has energy density ρ but is pressureless ($p = 0$). Let the line element inside the dust ball be curved FRW:

$$ds^2 = -dt^2 + a^2 \left(\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \tag{4.42}$$

where a is the scale factor and κ is the spacial curvature. By Birkoff's theorem, the spacetime for $r > R$ is the Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{4.43}$$

where M is the total mass in the region $r < R$.

For arbitrary values of ρ and p, the equations of motion for the scale factor $a(t)$ in Eq. (4.42) are:

$$\frac{\ddot{a}}{a} = -(\rho + 3p), \tag{4.44}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} - \frac{\kappa}{a^2}. \tag{4.45}$$

One can find a static solution ($\dot{a} = \ddot{a} = 0$) of Eqs. (4.44), (4.45) by taking

$$\kappa = \rho/3, \tag{4.46}$$

$$\rho = -3p, \tag{4.47}$$

where we have set $a = 1$. This is the Einstein Static Universe.

In the spirit of the previous Sections, let the mass of the dust particles be a function of the modulus field ϕ. The dust energy density is therefore given by:

$$\rho_{dust} = \rho_0 e^{-\sqrt{6}\phi}, \tag{4.48}$$

which represents a source for the volume modulus potential in the region $r < R$. The dust has zero pressure but there is a pressure contribution from the vacuum energy of the modulus field as it is displaced from its minimum.

The simplest modulus profile compatible with the equations of motion is obtained by considering three distinct regions. The first region is $r < R$, in which the field is at the minimum of the effective potential $V_{eff} = V_\phi + \rho_{dust}$. For $r > R + 1/m_\phi$ the field is at its vacuum minimum. In the transition region $R < r < R + 1/m_\phi$ the field interpolates smoothly between the two distinct minima.

The volume modulus profile described above implies that inside the dust ball,

$$\rho = \rho_{dust} + V_\phi, \tag{4.49}$$

$$p = -V_\phi, \tag{4.50}$$

where these are evaluated at the minimum of the combined potential $V_{dust} + V_\phi$. Although there is no pressure from the dust, the displacement of scalar fields from the minimum leads to a contribution to vacuum energy. One then finds that Eqs. (4.46) and (4.47) become

$$\kappa = \frac{\rho_{dust} + V_\phi}{3}, \tag{4.51}$$

$$\rho_{dust} - 2V_\phi = 0. \tag{4.52}$$

Since one can treat the spatial curvature as a free parameter, the condition for existence of a static solution for the region $r < R$ reduces to Eq. (4.52). As long as the combined potential $V = V_{dust} + V_\phi$ exhibits a minimum where $V_{dust} = 2V_\phi$, then a static solution will exist. There are two possibilities of tuning the system to generate a solution to Eq. (4.52): one may either tune the dust density ρ_0 or the parameter α in the large volume potential. There is then effectively a 1-parameter set of solutions parametrised by the density of the interior. There are various constraints on this set, for example by limits on the volume modulus mass coming from fifth force experiments, as discussed in Section 4.3.

In Table 4.1 we display four different static solutions. These were obtained by fixing α and then tuning the density ρ. It is interesting to study the properties of the dust distribution that sources the nontrivial volume modulus profile, in particular its mass and radius. Given that the density is uniform and fixed by Eq. (4.52), the mass will be given by $M = \frac{4\pi}{3}\rho R^3$, where R is the radius of the spherical dust distribution. One may write the radius in terms of the Schwarzschild radius as $R = \xi R_{sch}$, where $\xi > 1$. Since, by definition, $R_{sch} = 2GM$ one finds

$$R_{sch} = \sqrt{\frac{3}{8\pi G\rho\xi^3}}, \tag{4.53}$$

this implies that

$$R = \sqrt{\frac{3}{8\pi G\rho\xi}}. \tag{4.54}$$

One then concludes that the properties of the dust ball are completely determined by ξ since ρ is fixed by requiring a static solution. Note that the smaller ξ the larger the star's radius and mass. In Table 4.1 we display the radii and masses for the four cases under study considering $\xi = 1.1$.

α	$m_\Phi(M_p)$	$\rho(M_p^4)$	$R(M_p^{-1})$	$M(M_p)$
0.05	1.42×10^{-7}	5.39278×10^{-7}	449	204
0.01	4.45×10^{-19}	5.36521×10^{-15}	4.49×10^6	2×10^6
0.005	2.80×10^{-29}	6.288865×10^{-22}	1.3×10^{10}	6×10^9
0.001	5.16×10^{-82}	2.15993×10^{-57}	7×10^{27}	3×10^{27}

Table 4.1: Mass of the canonically normalised volume modulus, density, radius and total mass of the dust sphere, as functions of the α parameter in the large volume potential. Radius and total mass computed assuming $\xi = 1.1$.

The numerical results in Table 4.1 reveal that there is a very large hierarchy both in radius and mass between the various cases studied here. This exemplifies the issue raised in Section 4.3.2: for masses of Φ greater than that allowed by fifth force constrains, namely $m_\Phi \gtrsim 10^{-30} M_P$, the size of such objects is extremely small ($R \lesssim 10^{-23}$cm). The corresponding mass is $M \lesssim 1$kg. While they could in principle form part of dark matter, it is hard to see how interesting physics can be extracted from them. The local density of dark matter objects with kg masses is not larger than $10^{-21} m^{-3}$ and so objects would be both unobservable and undetectable.

4.3.5 A dynamic solution: black hole formation

In this Section we generalise the previous analysis to allow for dynamical evolution of the system. While Section 4.3.4 was restricted to static solutions, here we look for collapsing solutions of the coupled matter-modulus system. If one lets the gravitational collapse last for long enough, the final state will be a Schwarzschild black hole. However before one reaches the singularity, the local density becomes arbitrarily large, which could be sufficient to destabilise the modulus.

Here our initial conditions are a large, dilute dust ball which we allow to collapse towards a black hole. As before we assume that the spacetime is given by a curved FRW universe smoothly connected at the surface of the dust sphere ($r = R$) to a Schwarzschild solution. In contrast to Section 4.3.4, we now use the Friedmann equation to obtain dynamical solutions of the scale factor and of the energy density for $r < R$.

The profile for the volume modulus is obtained by considering:

$$\phi = \begin{cases} \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V_{,\phi} = 0 & , r \in [0, R] \\ (1 - 2M/r)\phi'' + (\frac{2}{r}(1 + 2M/r) + 2M/r^2)\phi' - V_{,\phi} = 0 & , r \in [R, R + m_\phi^{-1}] \\ \phi_\infty & , r \in [R + m_\phi^{-1}, \infty] \end{cases} \tag{4.55}$$

where $' \equiv \frac{\partial}{\partial r}$, $\dot{} \equiv \frac{\partial}{\partial t}$, M is the mass sourcing the Schwarzschild geometry and ϕ_∞ is the value of the field in the minimum of its vacuum potential.

The time dependence of the system arises in two different ways. Inside the dust distribution one has a homogeneous and isotropic positively curved universe, which is in general dynamic. This causes the scale factor and the volume modulus to be functions of time. For $r > R$ we assume that the time dependence comes only from the time variation of the radius of the dust ball (which can be traced to the time variation of the scale factor). In particular, we assume that the scalar field profile adjusts instantaneously to the change in dust density, i.e. that the characteristic time scale of the gravitational collapse is much larger than the typical timescale of the variation of the scalar field.

One must clarify what is meant by M in Eq. (4.55). Naively one would expect the mass of the dust ball to be given by the volume integral of the dust energy density. However there are also non negligible contributions from the potential and kinetic energies of the volume modulus. These are, for most of the evolution of the system, one order of magnitude smaller than the dust contribution but become more significant as the system evolves. Taking this effect into consideration, the mass sourcing the Schwarzschild geometry is

$$M = \int d^3x\sqrt{g}\left(\rho_{dust} + \rho_{V(\phi)} + \rho_{K(\phi)}\right). \tag{4.56}$$

In Fig. 4.9 we compare Eq. (4.56) with the naive estimate. The deviation between

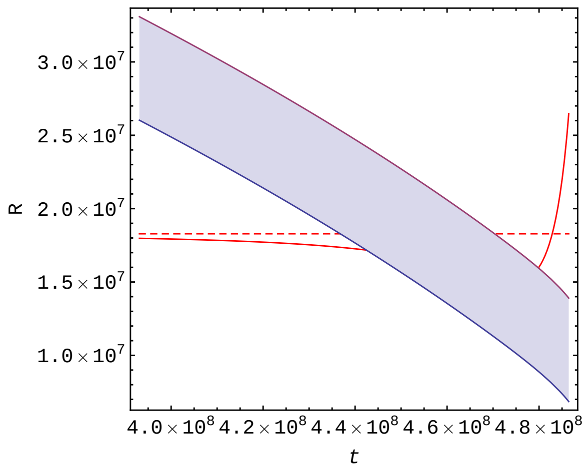

Figure 4.9: Comparison between the naive estimate for 2M (dashed line) and the result of Eq. (4.33) (solid line). The shaded area represents the region where the field varies between the value inside the dust ball and the vacuum minimum.

the two estimates is negligible throughout most of the evolution but increases with time, diverging at the end. This divergence is due to the fact that when the density reaches a critical value, the volume modulus potential no longer exhibits a minimum and decompactification happens. This means that the modulus starts to roll and its kinetic energy dominates the energy density inside the dust ball, in a similar way to the behaviour described in Section 4.3.3 for cosmological singularities.

The time evolution of the system is depicted in Fig. 4.10. We start with a spherical distribution of radius $r > 2M$, regime I. In this regime the whole transition region (shaded area) is outside the horizon and the deviation of the volume modulus from its vacuum minimum could in principle be observable. As the gravitational collapse evolves a black hole will form and subsequently the transition region will start to fall inside the horizon, this is regime II. In this regime, it is still in principle possible to observe the consequences of the matter modulus interaction since part of the transition region lies outside the horizon. When the whole of the transition region is veiled by the horizon, regime III, an observer sitting outside the black hole will not be able to measure the nontrivial profile of the volume modulus.

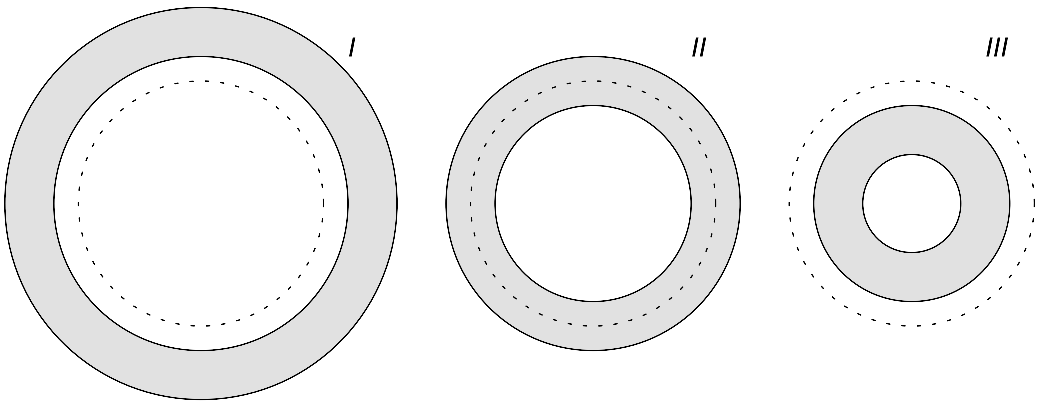

Figure 4.10: Sketch of the black hole formation process. The dashed line depicts the Schwarzschild horizon and the shaded area depicts the region $R < r < R + 1/m_\phi$ where the field rolls between its value inside and outside the dust ball.

The time evolution of the volume modulus profile in regime I is shown in Fig. 4.11. As the system collapses and the density increases, the volume modulus vev inside the dust ball increases and the transition region where the field is allowed to vary moves to the smaller radius region. The evolution of the field profile in regimes II and III is qualitatively similar to the one just described.[6]

4.4 Discussion

The purpose of this Chapter has been to consider the interaction of moduli and matter fields, and specifically to analyse the circumstances under which moduli fields can be destabilised from their vevs by dense concentrations of matter. We have described the origin of moduli/matter couplings and the assumed form of the moduli potential. We have tried to consider 'honest' values for parameters such as coupling strengths

[6]In regime II one must impose finiteness and continuity of the solution at the horizon

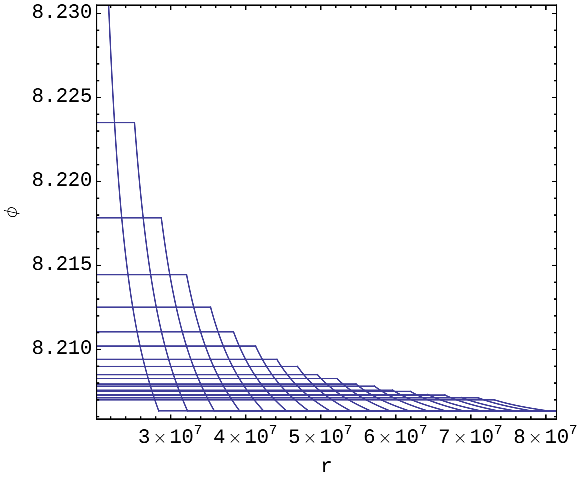

Figure 4.11: Time evolution of the volume modulus profile in regime I.

and moduli masses, enforcing the consistency constraints that emerge from string compactifications.

Our results have both negative and positive elements. On the negative side, it is not feasible to destabilise moduli through even the densest astrophysical environments. We considered neutron stars as the densest known form of matter, and found that Planck coupled moduli could only be destabilised if their masses were deep in the region excluded by fifth force constraints. If the coupling is relaxed from Planck strength, then the values of the coupling for which destabilisation is possible are excluded by the stellar cooling constraints that require $f_a < 10^9$GeV. This only leaves very exotic cases such as cosmic strings and hypothetical dark analogues of neutron stars.

On the positive side, our results show that the modulus/matter coupling will play a significant role in gravitational collapse. The cosmological collapse of a matter-dominated FRW universe was shown to lead to a super-inflationary decompactification of the internal dimensions as the volume modulus is destabilised. This super-inflationary decompactification leads to the internal dimensions reaching infinite volume at the same time as the external scale factor vanishes. It would be interesting to see whether this super-inflationary decompactification can mimic some of the features of conventional inflation. In a similar vein we also saw that the process of black hole formation will lead to decompactification during the period of collapse.

Chapter 5

A Model of Inflation

This Chapter is based on the paper [14].

It is now 30 years since cosmic inflation was proposed to solve outstanding problems of the standard cosmological model by postulating a period of exponential expansion in the early Universe [80–82].

Since the proposal of the original ideas, a plethora of inflationary models was put forward, among which we find the class of slow-roll inflationary models. At their heart is a slowly evolving field in a nearly flat potential that generates a quasi-de Sitter phase in the early history of the Universe. Due to its simplicity, the slow roll mechanism is among the preferred ways to generate an inflationary epoch. While as a class of models, slow roll provides a successful and elegant realisation of inflation, pinning down the details of the mechanism is a challenging question. In particular, identifying the inflaton field, and its connections with a fundamental theory of particle physics, are still issues open to debate. The best that can be done is to propose and analyse models that are compatible with current observational and experimental bounds, and compute their signatures so that they can be tested by forthcoming observations.

The very shallow inflaton potential yields a scalar mass lighter than the Hubble scale H, $m_\phi \ll H$. As in the Higgs case, it is notoriously hard to keep scalars light by preventing them from getting large contributions when integrating out heavy ultra-violet physics. This is the famous 'η-problem' [83] whose solution is crucial in order to trust any inflationary scenario. Due to this ultra-violet sensitivity of inflation, it is possible to find a robust solution to the η-problem only by embedding models in an ultra-violet complete theory.

For these reasons, a promising avenue to embed inflation into a model of particle physics is string theory (for reviews see [41, 84–87]). A key feature of string theory is the need for extra spacetime dimensions whose geometry is parameterised by scalar moduli fields. It is the vacuum structure of the moduli potential that determines the masses and couplings of the low-energy effective field theory. Over the past decade, significant progress has been made towards the understanding of the moduli potential [28, 33], allowing for a promising contact between string theory and particle phenomenology. The progress in moduli stabilisation also opened up the possibility of realising inflation in the moduli sector. String inflationary models based on single-field slow-roll can be broadly classified, based on the origin of the inflaton field [88,89], into open string models [90–101] and closed string models [102–115].

For closed string inflation there is a direct connection between the physics that stabilises the extra dimensions and inflation. Of particular interest for string phenomenology and inflationary applications is the Large Volume Scenario (LVS) of type IIB string theory [33] that through a combination of perturbative and non-perturbative effects allows for solutions with exponentially large volumes. In the context of the LVS, inflation can be driven by Kähler moduli rolling towards their minima.

As recently reviewed in [41], these models provide an interesting solution to the η-problem which does not rely on an axionic shift symmetry [116]. The two reasons why these models can evade the η-problem can be summarised in the following way:

1. The characteristic no-scale structure of the Kähler potential of type IIB supergravity is broken at leading order by α' effects which develop a potential *only* for the overall volume mode. Thus all the other $(h^{1,1} - 1)$ directions in the Kähler moduli space orthogonal to the volume are flat at leading order, and so constitute natural inflaton candidates. The 'extended no-scale structure' guarantees that string loop effects generate only a subleading potential lifting some of the remaining flat directions [27]. Hence any possible direction orthogonal to the volume is a good inflaton candidate.

2. The tree-level Kähler potential K itself depends *only* on the overall volume. Therefore, if the inflaton is a combination of the Kähler moduli orthogonal to the volume mode (or equivalently if inflation takes place with the volume kept stable), no higher order inflaton-dependent operator gets generated by expanding the prefactor e^K of the F-terms scalar potential. Given that g_s corrections to K generically induce a dependence on all the Kähler moduli, inflaton-dependent higher order operators get indeed generated by expanding e^K once we consider the g_s-corrected Kähler potential. However, again due to

the 'extended no-scale structure', these operators are suppressed with respect to the leading inflationary dynamics.

Two possible effects to develop a potential for the inflaton field at subleading order have already been proposed. In the first case, Kähler moduli inflation [104, 105], the inflaton is a blow-up mode and its potential is generated by ordinary non-perturbative corrections to the superpotential, while in the second case, fibre inflation [106], the inflaton is a K3 divisor which develops a potential via perturbative string loop effects [1]. In fibre inflation a long enough period of inflation can be achieved rather naturally. On the other hand, in Kähler moduli inflation, a sufficiently long period of inflation can be driven by the non-perturbative potential only by fine-tuning the coefficients of the loop corrections, as pointed out in [106]. Notice that these g_s corrections would spoil inflation in the region rather close to the minimum where the non-perturbative potential would give rise to slow-roll. On the other hand, in regions further away from the minimum, due to the 'extended no-scale structure', these perturbative loop corrections would actually drive inflation in a way very similar to [106].

In this Chapter we shall provide the first model where the whole inflationary dynamics is driven entirely by non-perturbative effects since perturbative loop corrections can be shown to be negligible for natural values of the underlying parameters. The new key-ingredient is the use of poly-instanton corrections to the superpotential which are instanton corrections to the action of another instanton originally derived in type I compactifications in [118] and whose existence in type IIB compactifications was recently established in [119].

Here the inflaton is the volume of the fibre which supports the poly-instanton effects. The different topological origin of the inflaton between our model and the one developed in [104, 105] results in a different canonically normalised inflaton field. This leads to a potential which in our case gives rise to slow-roll in a region much closer to the minimum than in the model of [104, 105] even if in both cases the inflationary potential is generated by non-perturbative effects. This is the reason why in our model string loop corrections are less dangerous and can be shown to be negligible throughout all the inflationary dynamics for natural values of the underlying parameters.

The predictions of the model are rather independent on the choice of the microscopic parameters, as these influence mainly the scale of inflation but have little

[1]The authors of [112] tried to obtain inflation driven by the volume mode in order to solve the tension between inflation and TeV-scale SUSY [117]. However, for the reasons explained above, this model suffers from the η-problem, and so its realisation needs fine-tuning.

effect on the shape of the potential. Our model is characterised by a reheating temperature of the order $T_{\rm rh} \simeq 10^6$ GeV which requires $N_e \simeq 54$ e-foldings of inflation. The requirement of generating the correct amount of density perturbations fixes the Calabi-Yau volume of the order 10^3 in string units. This in turn leads to a high inflationary scale, $M_{\rm inf} \simeq 10^{15}$ GeV, corresponding to a small tensor-to-scalar ratio, $r \simeq 10^{-5}$, and a sub-Planckian motion of the inflaton field $\Delta\phi \simeq 0.5 M_p$. We also point out that the spectral index, $n_s \simeq 0.96$, is within the observationally allowed window.

This Chapter is organised as follows: in Section 5.1 we introduce fibered compactifications that will be fundamental for the remainder of the Thesis. In Section 5.2 we first show how the poly-instanton corrections to the superpotential can generate a potential for the fibre modulus, and then study its inflationary applications both analytically and numerically in Section 5.3. In the same Section we also present a detailed discussion of the η-problem stressing the similarities and the differences of our model with the previous ones developed in [104, 105] and [106], where the inflaton was also a Kähler modulus. We summarise our results in Section 5.4.

5.1 Fibered compactifications

As described in the previous Chapters, the original proposal of the large volume scenario was based on swiss-cheese manifolds, that is manifolds in which one single Kähler modulus parametrises the volume of the bulk of the compactification. However the procedure for moduli stabilisation through a combination of perturbative and non-perturbative effects also holds for geometries more general than Swiss-cheese as studied in [45]. Of particular interest to us are Calabi-Yau manifolds that admit a K3 or T^4 fibration over a $\mathbb{P}^1$ base. These manifolds are characterised by the fact that their volume is linear in the two-cycle giving the volume of the $\mathbb{P}^1$ base. Explicit examples of this kind of Calabi-Yau three-folds with additional del Pezzo divisors have been analysed in [120] using toric geometry. Here we shall just focus on the simplest of such manifolds whose volume takes the form:

$$\mathcal{V} = \lambda_1 t_1 t_2^2 - \lambda_2 t_3^3 \,, \tag{5.1}$$

where t_1 is the volume of the $\mathbb{P}^1$ base, $\tau_1 = \lambda_1 t_2^2$ is the size of the K3 or T^4 fibre, and $\tau_3 = 3\lambda_2 t_3^2$ controls the volume of a blow-up mode (the other four-cycle volume is given by $\tau_2 = 2\lambda_1 t_1 t_2$). Notice that the fibre is a K3 surface if its Euler characteristic is $\chi = 24$ whereas it is a T^4 divisor if $\chi = 0$ [121]. We shall not specify the value of χ

in order to be as generic as possible since our moduli stabilisation mechanism works in both cases. We can then rewrite the volume in terms of the correct Kähler moduli as:

$$\mathcal{V} = t_1\tau_1 - \alpha\gamma\tau_3^{3/2} = \alpha(\sqrt{\tau_1}\tau_2 - \gamma\tau_3^{3/2})\,, \tag{5.2}$$

where $\alpha = 1/(2\sqrt{\lambda_1})$ and $\gamma = \frac{2}{3}\sqrt{\lambda_1/(3\lambda_2)}$.

Due to the similar structure of the volume for a Swiss cheese type Calabi-Yau

$$\mathcal{V} = \tau_b^{3/2} - \sum_i^{h^{(1,1)}-1} \tau_i^{3/2} \tag{5.3}$$

and (5.2), an analysis of the scalar potential reveals that an AdS minimum at exponentially large volumes also exists for fibered geometries. There is however one crucial difference: while in the simpler geometries initially studied, with volumes described by Eq. (5.3), all moduli were stabilised, for K3 or T^4-fibered Calabi-Yau manifolds only the blow-up moduli and the volume $\mathcal{V} \sim \sqrt{\tau_1}\tau_2$ directions are fixed. Therefore the leading order stabilisation dynamics leaves one flat direction in moduli space which can be used to provide a stringy realisation of slow roll inflation provided that a suitable potential can be generated for this direction. We now turn our attention to this issue.

5.2 Fibre stabilisation via poly-instantons

In this Section we introduce poly-instanton corrections to the superpotential. These are the essential ingredient to lift the flat direction left after the study of the leading stabilisation dynamics in fibered Calaby-Yau's. We then show that the resulting potential for the fibre modulus can support slow-roll inflation. In Chapter 6 we will see how the same geometry can be pushed to a regime where the poly-instanton potential gives a rise to quintessential dynamics.

The existence of poly-instanton corrections was first established within the context of type I compactifications in [118]. These are instantons which do not give rise to a single contribution to the superpotential but, due to the presence of extra fermionic zero-modes, they correct the action of another instanton wrapping a different internal cycle. The zero-mode constraints for generating poly-instanton corrections in the type IIB T-dual version have been worked out in detail in [119].

Here we take a phenomenological approach and make the following assumptions:

1. The field theory living on a stack of D7-branes wrapping τ_3 can be broken into two gauge groups which separately undergo gaugino condensation, giving rise to a race-track superpotential of the form:

$$W = W_0 + A\, e^{-aT_3} - B\, e^{-bT_3}\,, \tag{5.4}$$

where A and B are threshold effects, while $a = 2\pi/n_a$ and $b = 2\pi/n_b$, with $n_a, n_b \in \mathbb{N}$.

2. On top of these effects, an Euclidean D3-instanton wrapping the fibre τ_1 yields non-perturbative corrections to the gauge kinetic functions of the two condensing gauge groups, resulting in a poly-instanton corrected superpotential which looks like:

$$W = W_0 + A\, e^{-a(T_3+C_1 e^{-2\pi T_1})} - B\, e^{-b(T_3+C_2 e^{-2\pi T_1})}\,, \tag{5.5}$$

where C_1 and C_2 are free constants.

Notice that we are following [46], where the same assumptions led to a very anisotropic compactification characterised by the presence of two micron-sized extra dimensions and strings around the TeV scale.

The scalar potential computed through Eq. (2.62) can be separated as:

$$V = V_{\mathcal{O}(\mathcal{V}^{-3})} + V_{\mathcal{O}(\mathcal{V}^{-3-p})}\,, \tag{5.6}$$

where the first piece scales with the volume as $\mathcal{V}^{-3}$, while the other as $\mathcal{V}^{-3-p}$, with p a positive parameter to be defined later. After minimising with respect to the two axions b_1 and b_3, these two pieces are:

$$\begin{aligned} V_{\mathcal{O}(\mathcal{V}^{-3})} =& \frac{8\sqrt{\tau_3}\left(A^2a^2e^{-2a\tau_3} + B^2b^2e^{-2b\tau_3} - 2AB\,ab\,e^{-(a+b)\tau_3}\right)}{3\mathcal{V}} \\ &+ \frac{4W_0\tau_3\left(Aae^{-a\tau_3} - Bbe^{-b\tau_3}\right)}{\mathcal{V}^2} + \frac{3\xi}{4\,g_s^{3/2}\,\mathcal{V}^3}\,, \end{aligned} \tag{5.7}$$

and:

$$\begin{aligned} V_{\mathcal{O}(\mathcal{V}^{-3-p})} =& -e^{-2\pi\tau_1}\Bigg\{-\frac{16C_1\sqrt{\tau_3}e^{-2b\tau_3}}{3\mathcal{V}}[Z^2a - ZB(a-b)] \\ &+\frac{4C_1W_0e^{-b\tau_3}}{\mathcal{V}^2}[2\pi\, Z\tau_1 + Za\tau_3 - Bb(a-b)\tau_3] \\ &+n\left[\frac{-16Bb^2\sqrt{\tau_3}e^{-2b\tau_3}Z}{3\mathcal{V}} + \frac{4W_0Bbe^{-b\tau_3}}{\mathcal{V}^2}(b\tau_3 + c\tau_1)\right]\Bigg\}\,. \end{aligned} \tag{5.8}$$

In Eq. (5.7) we kept only the leading order terms in the volume expansion, relegating terms proportional to $\mathcal{V}^{-n}$ for $n > 3$ and terms proportional to $e^{-2\pi\tau_1}$ to Eq. (5.8). In

Eq. (5.8) terms proportional to $e^{-2\pi n\tau_1}$ for $n > 1$ were left out as they are subdominant in the regime where the effective field theory is under control. Moreover, following [46], we also introduced new parameters to simplify the expressions:

$$Z \equiv B\,b - A\,a\,e^{-(a-b)\tau_3} \qquad \text{and} \qquad n \equiv C_2 - C_1\,. \tag{5.9}$$

The leading contribution, Eq. (5.7), is the standard LVS potential obtained from a racetrack superpotential. Notice that this term carries no information about the poly-instantons in W. Minimisation with respect to the volume and the blow-up modulus provides a minimum at:

$$\langle\tau_3\rangle^{3/2} = \frac{3\xi}{32\alpha\gamma f_1(1-f_1)g_s^{3/2}}\,, \qquad \langle\mathcal{V}\rangle = f_2\,e^{b\langle\tau_3\rangle}\,, \tag{5.10}$$

where we have defined:

$$f_1 \equiv \frac{(a-b)\,\tau_3 Bb + Z(1-a\,\tau_3)}{4\,(a-b)\,\tau_3 Bb + Z(1-4a\,\tau_3)} \qquad \text{and} \qquad f_2 \equiv \frac{3\alpha\gamma W_0\sqrt{\langle\tau_3\rangle}}{Z} f_1\,. \tag{5.11}$$

We stress that the result of Eq. (5.10) involves no approximations and is therefore exact. In particular the function f_1 generalises the definition of f_{corr} in [46]. The equation that determines the VEV of τ_3 can be solved iteratively, giving a more accurate result than the leading order estimate obtained by setting $f_1 = 1/4$. Notice that the volume is stabilised at exponentially large values, leading to a LVS.

As we have already mentioned, the leading order potential, Eq. (5.7), depends on τ_1 only through $\mathcal{V}$, therefore after stabilising τ_3 and $\mathcal{V}$ at their minima, Eq. (5.10), there is one flat direction left in the (τ_1, τ_2) plane. This flat direction is then lifted by the subleading effects due to poly-instantons encoded in Eq. (5.8). Notice that unlike $V_{\mathcal{O}(\mathcal{V}^{-3})}$ which depends only on τ_3 and $\mathcal{V}$, $V_{\mathcal{O}(\mathcal{V}^{-3-p})}$ depends on all the three directions in moduli space. The approach of using subleading non-perturbative corrections to the superpotential as in [46] provides an alternative for generating a minimum for the fibre modulus, with respect to using string loop contributions to the Kähler potential as originally explored in [45].

Setting the blow-up and volume moduli to their minima, Eq. (5.10), we can rewrite the subleading part of the resulting scalar potential in Eq. (5.8) as:

$$V_{\mathcal{O}(\mathcal{V}^{-3-p})} = -\frac{F_0}{\langle\mathcal{V}\rangle^3}\left(2\pi\,\tau_1 - p\,b\,\langle\tau_3\rangle\right)e^{-2\pi\,\tau_1}\,, \tag{5.12}$$

where $F_0 \equiv 4\,f_2\,W_0\,r_1$ and p is defined as:

$$p \equiv -\frac{r_1}{r_3}\,, \tag{5.13}$$

with:

$$r_1 \equiv C_1 Z + n\, b\, B\,, \tag{5.14}$$
$$r_3 \equiv (1-4f_1)\left[\frac{a}{b}\, r_1 - (a-b)\, B\, (C_1+n)\right]. \tag{5.15}$$

The minimum of $V_{\mathcal{O}(\mathcal{V}^{-3-p})}$ for fixed τ_3 and $\mathcal{V}$ can be shown to lie at:

$$2\pi\,\langle\tau_1\rangle = p\, b\,\langle\tau_3\rangle + 1. \tag{5.16}$$

By tuning the parameters in K and W, it is possible to obtain p positive and of order unity so that the minimum (5.16) is within the regime of validity of the effective field theory, $\langle\tau_1\rangle > 1$.

In [46] these effects were used to obtain a very anisotropic compactification with TeV-scale strings corresponding to volumes of the order $\mathcal{V} \sim 10^{30}$. We will leave the study of the late time cosmology of such models for Chapter 6, and focus here in models with $\mathcal{V} \sim 10^3$ which will allow us to derive a consistent model of inflation in the Kähler moduli sector of the compactification.

5.3 Poly-instanton inflation

Let us now study the inflationary dynamics by working in the single field approximation. We shall set the blow-up mode τ_3 and the volume $\mathcal{V}$ to their τ_1-independent vevs: $\tau_3 = \langle\tau_3\rangle$ and $\mathcal{V} = \langle\mathcal{V}\rangle$, and displace τ_1 far from its minimum. We shall then investigate if the fibre modulus can drive a long enough period of inflation while rolling down its potential. Notice that the approximation of stable τ_3 and $\mathcal{V}$ during inflation is justified by the mass hierarchy found in [46]:

$$m_{\tau_1} \sim \frac{M_p}{\mathcal{V}^{(3+p)/2}}, \qquad m_{\mathcal{V}} \sim \frac{M_p}{\mathcal{V}^{3/2}}, \qquad m_{\tau_3} \sim \frac{M_p}{\mathcal{V}}\,, \tag{5.17}$$

when all the moduli sit at their minima. In what follows we shall set $p \simeq \mathcal{O}(1)$, implying that for large volume $m_{\tau_3} \gg m_{\mathcal{V}} \gg m_{\tau_1}$. Moreover, as we shall see later on, the Hubble parameter during inflation scales with the volume as $H \sim m_{\tau_1}$. Hence both τ_3 and $\mathcal{V}$ are heavy during inflation since $m_{\tau_3} \gg m_{\mathcal{V}} \gg H$, justifying our single field approximation. We point out that the actual mass of τ_1 far from its minimum is much smaller than m_{τ_1} due to the rapid exponential suppression of its potential. Hence if we displace this field far from its minimum, it is naturally lighter than H, and can therefore drive inflation.

The effective potential for the inflaton field τ_1 can be written as:

$$V_{\rm inf} = V_{\rm up} + V_{\rm fib}\,, \tag{5.18}$$

where $V_{\rm fib}$ is the scalar potential (5.12) generated by poly-instanton effects, while $V_{\rm up}$ is the uplift term. Regardless of the nature of the uplifting, we shall assume that it gives rise to a τ_1-independent constant once the volume is fixed. This constant is obtained by requiring $V_{\rm up} = -V_{\rm fib}(\langle\tau_1\rangle)$ since we are tuning the uplifting in order to ensure the vanishing of the scalar potential at leading $\mathcal{V}^{-3}$ order. Notice that $V_{\rm up}$ will slightly modify the position of $\langle\mathcal{V}\rangle$ but we shall neglect this small effect since it only affects the overall inflationary scale.

Let us then rewrite the full inflationary potential as:

$$V_{\rm inf} = \frac{F_{\rm poly}}{\langle\mathcal{V}\rangle^{3+p}}\left[1-(1+2\pi\hat{\tau}_1)\,e^{-2\pi\hat{\tau}_1}\right]\,, \quad \text{with} \quad F_{\rm poly} = F_0 f_2^p\, e^{-1}\,, \tag{5.19}$$

where we defined $\hat{\tau}_1 \equiv \tau_1 - \langle\tau_1\rangle$ as the shift of the inflaton from its minimum. Notice that the inflationary potential scales with the volume as $1/\mathcal{V}^{3+p}$, and so we can get different values for the scale of inflation by varying the parameter p. This will be important in what follows.

Let us now canonically normalise the inflaton field by recalling that the kinetic Lagrangian is given by:

$$\mathcal{L}_{\rm kin} = K_{i\bar{j}}\partial_\mu T_i \partial^\mu \bar{T}_j = \frac{1}{4}\frac{\partial^2 K}{\partial\tau_i\partial\tau_j}(\partial_\mu\tau_i\partial^\mu\tau_j + \partial_\mu b_i\partial^\mu b_j)\,, \tag{5.20}$$

where in the second equality we rewrote the complexified Kähler moduli in terms of their real and imaginary components ($T_i = \tau_i + ib_i$) and used the fact that the Kähler potential is independent of the axionic fields. The Kähler metric turns out to be:

$$K^0_{i\bar{j}} = \begin{pmatrix} \frac{1}{4\tau_1^2} & \frac{\tau_3^{3/2}}{4\tau_1^{3/2}\tau_2^2} & -\frac{3\sqrt{\tau_3}}{8\tau_1^{3/2}\tau_2} \\ \frac{\tau_3^{3/2}}{4\tau_1^{3/2}\tau_2^2} & \frac{1}{2\tau_2^2} & -\frac{3\sqrt{\tau_3}}{4\sqrt{\tau_1}\tau_2^2} \\ -\frac{3\sqrt{\tau_3}}{8\tau_1^{3/2}\tau_2} & -\frac{3\sqrt{\tau_3}}{4\sqrt{\tau_1}\tau_2^2} & \frac{3}{8\sqrt{\tau_1}\tau_2\sqrt{\tau_3}} \end{pmatrix}\,, \tag{5.21}$$

where we kept only the leading order term in the volume expansion for each entry. Treating both the volume and the blow-up cycle as stable during inflation, the kinetic term for the fibre modulus simplifies to:

$$\mathcal{L}_{\rm kin} = \frac{3}{8\tau_1^2}\partial_\mu\tau_1\partial^\mu\tau_1\,. \tag{5.22}$$

It then follows that the canonically normalised field is defined as:

$$\phi \equiv \frac{\sqrt{3}}{2}\ln\tau_1 \qquad \text{or} \qquad \tau_1 \equiv e^{2\phi/\sqrt{3}}. \tag{5.23}$$

These definitions imply that the shift $\hat{\phi}$ of the canonically normalised inflaton from its minimum takes the form:

$$\hat{\tau}_1 = \langle\tau_1\rangle\left(e^{\frac{2}{\sqrt{3}}\hat{\phi}}-1\right), \tag{5.24}$$

and so the full inflationary potential (5.19) can be rewritten as:

$$V_{\rm inf} = \frac{F_{\rm poly}}{\langle\mathcal{V}\rangle^{3+p}}\left[1-e^{-c\left(e^{\frac{2}{\sqrt{3}}\hat{\phi}}-1\right)}\left(1+c\left(e^{\frac{2\hat{\phi}}{\sqrt{3}}}-1\right)\right)\right], \tag{5.25}$$

where $c = 2\pi\langle\tau_1\rangle$. This potential is sketched in Figure 5.1. The relation (5.24) suggests the following field redefinition:

$$\hat{\psi} \equiv \frac{\sqrt{3}}{2}\left(e^{\frac{2}{\sqrt{3}}\hat{\phi}}-1\right) \qquad \Leftrightarrow \qquad \hat{\tau}_1 = \frac{2}{\sqrt{3}}\langle\tau_1\rangle\,\hat{\psi}, \tag{5.26}$$

which allows us to write (5.25) in a very compact form as:

$$V_{\rm inf} \simeq \frac{F_{\rm poly}}{\langle\mathcal{V}\rangle^{3+p}}\left(1-\kappa\,\hat{\psi}\,e^{-\kappa\,\hat{\psi}}\right), \tag{5.27}$$

where $\kappa = \frac{2p}{\sqrt{3}}\ln\mathcal{V}$ and we have approximated $c = p\ln\mathcal{V}+1-p\ln f_2 \simeq p\ln\mathcal{V}$. We stress that we shall obtain a model of small field inflation where the inflaton travels a sub-Planckian distance in field space during inflation: $\Delta\hat{\phi}\simeq 0.5M_p$. Therefore we can Taylor expand the exponent in (5.26) finding that the field $\hat{\psi}$ gives the leading order approximation of the canonically normalised inflaton $\hat{\phi}$ since:

$$\hat{\psi} \equiv \frac{\sqrt{3}}{2}\left(e^{\frac{2}{\sqrt{3}}\hat{\phi}}-1\right) \simeq \hat{\phi}+\frac{1}{\sqrt{3}}\hat{\phi}^2+\dots. \tag{5.28}$$

Thus the compact expression (5.27) gives a rather accurate qualitative description of the inflationary dynamics when we just identify $\hat{\psi}$ with $\hat{\phi}$.

The aim of this work is to investigate whether the dynamics of the scalar field $\hat{\phi}$ rolling down the potential (5.25) is suitable for generating a prolonged period of inflation. With this in mind we compute the slow-roll parameters for the canonically normalised fibre modulus:

$$\epsilon = \frac{1}{2V_{\rm inf}^2}\left(\frac{\partial V_{\rm inf}}{\partial\phi}\right)^2 \qquad \text{and} \qquad \eta = \frac{1}{V_{\rm inf}}\frac{\partial^2 V_{\rm inf}}{\partial\phi^2}. \tag{5.29}$$

The slow-roll parameters turn out to be (for simplicity we express them in terms of $\hat{\psi}$):

$$\epsilon = \frac{2c^2}{3}\frac{\left(\frac{2c}{\sqrt{3}}\hat{\psi}\right)^2\left(\frac{2}{\sqrt{3}}\hat{\psi}+1\right)^2}{\left(e^{\frac{2c}{\sqrt{3}}\hat{\psi}}-1-\frac{2c}{\sqrt{3}}\hat{\psi}\right)^2} \simeq \kappa^4\, e^{-2\kappa\hat{\psi}}\,, \tag{5.30}$$

and:

$$\eta = -\frac{4c}{3}\frac{\left(\frac{2}{\sqrt{3}}\hat{\psi}+1\right)\left[\left(\frac{2c}{\sqrt{3}}\hat{\psi}\right)^2+(c-2)\frac{2c}{\sqrt{3}}\hat{\psi}-c\right]}{e^{\frac{2c}{\sqrt{3}}\hat{\psi}}-1-\frac{2c}{\sqrt{3}}\hat{\psi}} \simeq -\kappa^3\, e^{-\kappa\hat{\psi}}\,. \tag{5.31}$$

Both ϵ and η are exponentially small in the region $\kappa\hat{\psi} \gg 1$. Due to the large parameter $\kappa = \frac{2p}{\sqrt{3}}\ln\mathcal{V} \gg 1$, the slow-roll conditions are satisfied for small shifts $\hat{\psi} < 1$. This implies that we are dealing with a model of small field inflation and justifies our leading order approximation $\hat{\psi} \simeq \hat{\phi}$. Furthermore the model is characterised by the interesting relation:

$$\epsilon \simeq \left(\frac{\eta}{\kappa}\right)^2\,, \tag{5.32}$$

which implies the following hierarchy throughout all the inflationary region:

$$\epsilon \ll |\eta| \ll 1\,. \tag{5.33}$$

Requiring that $|\eta|$ is at most a few percent leads to values of ϵ within the interval $10^{-7} < \epsilon < 10^{-5}$. Notice that a negative η is a generic feature of our potential showing that the inflating region is tachyonic and therefore unstable. Hence the field $\hat{\phi}$ drives inflation as it slowly rolls towards its minimum.

5.3.1 Inflationary observables

In order to solve the basic problems of standard Big-Bang cosmology, the inflationary trajectory must also give rise to a sufficient number of e-foldings:

$$N_e \equiv \int_{\hat{\phi}_{\rm end}}^{\hat{\phi}_*}\frac{1}{\sqrt{2\epsilon}}d\hat{\phi} \simeq \int_{\hat{\phi}_{\rm end}}^{\hat{\phi}_*}\frac{\kappa}{|\eta|}d\hat{\phi} \simeq \frac{1}{\kappa^2}\int_{\hat{\phi}_{\rm end}}^{\hat{\phi}_*}e^{\kappa\hat{\phi}}d\hat{\phi} \simeq \frac{1}{\kappa^3}\left(e^{\kappa\hat{\phi}_*}-e^{\kappa\hat{\phi}_{\rm end}}\right)\,. \tag{5.34}$$

In our numerical analysis we shall consider the end of inflation as taking place at the point $\hat{\phi}_{\rm end}$ where $\epsilon \simeq 1$. We stress that there is nothing special in this choice since our final results for the cosmological observables are not sensitive to the exact point where inflation ends. From the expression (5.30) we can then see that $\hat{\phi}_{\rm end}$ is, in practice, just a function of κ since it is rather insensitive to the other underlying parameters.

In turn, from (5.34), we see that the number of e-foldings N_e is a function of κ and the point of horizon exit $\hat{\phi}_*$.

However the exact number of e-foldings depends on the inflationary scale $M_{\rm inf} = V_{\rm inf}^{1/4}(\hat{\phi}_{\rm end})$ and the reheating temperature $T_{\rm rh}$. In fact, under the general assumption that inflation is followed first by a matter-dominated reheating epoch and then by a radiation-dominated epoch with initial temperature $T_{\rm rh}$, we have [122]:

$$N_e \simeq 62 + \ln\left(\frac{M_{\rm inf}}{10^{16}\,{\rm GeV}}\right) - \frac{1}{3}\ln\left(\frac{M_{\rm inf}}{T_{\rm rh}}\right) . \tag{5.35}$$

As can be seen from (5.25), $M_{\rm inf}$ depends in general on $\mathcal{V}$, p and other parameters via the combination $F_{\rm poly}$ of Eq. (5.19). For fixed $F_{\rm poly}$, we can consider $M_{\rm inf}$ as depending just on κ given that the inflationary scale is rather insensitive to the actual value of p which we shall always set $p \simeq \mathcal{O}(1)$. As we will argue later on, $T_{\rm rh}$ is also a function of κ. Hence by equating (5.34) with (5.35), we obtain an equation in two unknowns: $\hat{\phi}_*$ and κ (or $\mathcal{V}$).

We can find a unique solution for each value of $F_{\rm poly}$ by noticing that the requirement of generating the correct amount of density perturbations gives a second equation in $\hat{\phi}_*$ and κ. In fact, the COBE normalisation can be written in terms of the inflaton potential as [2]:

$$A_{\rm COBE} \equiv \frac{g_s}{8\pi}\left(\frac{V_{\rm inf}^{3/2}}{V'_{\rm inf}}\right)^2\Bigg|_{\hat{\phi}=\hat{\phi}_*} \simeq 2.7\cdot 10^{-7} . \tag{5.36}$$

We solved numerically these two equations for $p \simeq \mathcal{O}(1)$ and different values of $F_{\rm poly}$. We found solutions with an internal volume large enough to trust the effective field theory, $\mathcal{V} \sim 10^3$, corresponding to $\kappa \simeq 8$, for $F_{\rm poly} \sim \mathcal{O}(10)$ which gives also $\hat{\phi}_* \simeq 0.9$. In turn, these results lead to $\hat{\phi}_{\rm end} \simeq 0.35$, $N_e \simeq 54$ and $M_{\rm inf} \simeq M_p/\mathcal{V} \simeq 10^{15}$ GeV. Notice that these numerical results confirm our initial statement that we are dealing with a model of small field inflation since $\Delta\hat{\phi} = \hat{\phi}_* - \hat{\phi}_{\rm end} \simeq 0.45$ in Planck units.

Since our model has a preference for moderately small volumes it seems hard to simultaneously accommodate GUT-scale inflation and TeV-scale SUSY which would require $\mathcal{V} \sim 10^{15}$. This is the infamous tension between the scale of inflation and the scale of SUSY breaking [117] which afflicts most of the models of string inflation. Given that in this work we are interested only in inflation, we shall not attempt to address this issue (see [112, 123, 124] for possible solutions).

[2]We included a prefactor of $g_s/8\pi$ for the correct normalisation of the potential in Einstein frame [47].

Let us now turn to the study of the observational footprints of our model. The spectral index and the tensor-to-scalar ratio are defined by:

$$n_s = 1 + 2\eta_* - 6\epsilon_* \qquad \text{and} \qquad r = 16\epsilon_* \,, \tag{5.37}$$

and they take the values:

$$n_s \simeq 0.96 \qquad \text{and} \qquad r \simeq 10^{-5} \,. \tag{5.38}$$

Notice that since $\eta < 0$ this inflationary model necessarily leads to the observationally preferred value $n_s < 1$. The smallness of ϵ implies the absence of observable primordial tensor modes in agreement with the fact that the inflaton range during inflation is sub-Planckian [125]. In Section 5.3.3, after discussing the effect of loop corrections on the inflationary potential, we shall present three different numerical examples for the underlying parameters which lead to these signatures in agreement with current data.

5.3.2 Reheating

The study of reheating for models of closed string inflation has been performed in [126–128]. After the end of inflation, the inflaton behaves as a classical condensate which oscillates coherently around its minimum. Due to the steepness of the potential, these oscillations end very rapidly and are followed by reheating which takes place via the perturbative decay of inflaton particles into visible sector degrees of freedom localised on D7-branes wrapping an additional blow-up mode [127, 128].

The gravitational coupling of the inflaton to all the other particles in the model can be read off from the moduli dependence of the kinetic and mass terms of open string modes by expanding the moduli around their VEVs and then expressing them in terms of the canonically normalised fields [74, 129]. Following this procedure, the inflaton coupling to visible sector gauge bosons scales with the overall volume as [46]:

$$g \simeq \frac{1}{M_p \mathcal{V}^p} \,. \tag{5.39}$$

Notice that this coupling is much weaker than gravitational, reflecting the fact that the direction τ_1 is not lifted by the leading order stabilising dynamics. Thus the width of the inflaton decay into visible sector degrees of freedom behaves as:

$$\Gamma \simeq g^2 m_{\tau_1}^3 \simeq \frac{M_p}{\mathcal{V}^{(9+7p)/2}} \,, \tag{5.40}$$

and so the corresponding reheating temperature is given by:

$$T_{\rm rh} \simeq \sqrt{\Gamma M_p} \simeq \frac{M_p}{\mathcal{V}^{(9+7p)/4}} \,. \tag{5.41}$$

For $p \simeq \mathcal{O}(1)$ and $\mathcal{V} \sim 10^3$ this expression gives $T_{\rm rh} \simeq 10^6$ GeV which is much higher than the Big-Bang Nucleosynthesis temperature $T_{\rm BBN} \simeq 1$ MeV. Notice that the inflaton dumps all its energy into visible degrees of freedom since its decay to hidden sector particles is kinematically forbidden. In fact, the condensing field theory on τ_3 develops a mass gap and all the particles acquire a mass of the order the scale of strong dynamics $\Lambda \sim M_p/\mathcal{V}^{5/6} \gg m_{\tau_1}$ [127, 128].

5.3.3 Loop corrections

In the discussion of the structure of the scalar potential for the fibre modulus we have so far neglected the effect of g_s corrections to the Kähler potential. In this Section we shall study their behaviour showing that they are naturally subleading with respect to the poly-instanton corrections both around the minimum and, more importantly, in all the inflationary region. Thus our model features a nice solution of the η-problem.

First of all, a key observation is that the presence of open string loop corrections would definitely dominate over the tiny poly-instanton effects, and generate a potential for τ_1 which is also able to give rise to slow-roll inflation due to the 'extended no-scale structure' as studied in [106]. We shall however forbid the presence of these g_s corrections to K by not wrapping any D7-brane either on τ_1 or on any four-cycle intersecting the K3 or T^4 fibre (i.e. τ_2 in our case). In this way there is no open string localised on τ_1, and so we do not expect any τ_1-dependent open string loop correction to K.

However, closed string loops might still introduce a dependence on the fibre modulus since they correspond to loops of bulk Kaluza-Klein modes which cannot be avoided by construction. Hence it is crucial to study the behaviour of these g_s effects and their relative strength with respect to poly-instanton corrections. Following [27], the dependence of these corrections on the Kähler moduli can be estimated by using the one-loop Coleman-Weinberg potential [130]:

$$\delta V_{(g_s)} \simeq \Lambda^4 \, {\rm STr}(M^0) + \Lambda^2 \, {\rm STr}(M^2) \,, \tag{5.42}$$

where the first contribution vanishes since supersymmetry implies ${\rm STr}(M^0) = 0$. The cut-off Λ can be taken as the scale at which the 4D effective field theory description ceases to be valid. Given the hierarchy of scales described in [46] this should be M_{KK}^{6D}, the scale above which the theory becomes 6D:

$$\Lambda \equiv M_{KK}^{6D} \simeq \frac{M_s}{t_1^{1/2}} \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} M_p. \tag{5.43}$$

The supertrace can instead be approximated as $\mathrm{STr}(M^2) \simeq m_{3/2}^2 \simeq (W_0 M_p/\mathcal{V})^2$, and so the expression (5.42) takes the form:

$$\delta V_{(g_s)} \simeq (g_s C_{\rm loop})^2 \, W_0^2 \frac{\tau_1}{\mathcal{V}^4}\,, \tag{5.44}$$

where $C_{\rm loop}$ is an unknown function of the complex structure moduli which we had to introduce by hand since our argument does not allow us to constrain the dependence of the one-loop potential on the U-moduli [3]. However, this is not a problem since the complex structure moduli are flux-stabilised at tree-level, and so we can consider $C_{\rm loop}$ just as a constant parameter. This parameter is squared due to the extended no-scale structure [27] which implies the vanishing of the leading contribution proportional to $C_{\rm loop}$ (this corresponds to the vanishing of the $\mathcal{O}(\Lambda^4)$ term in the Coleman-Weinberg potential).

Therefore the loop-corrected potential for the canonically normalised inflaton becomes:

$$V = \frac{F_{\rm poly}}{\langle\mathcal{V}\rangle^{3+p}} \left[1 - e^{-c\left(e^{\frac{2}{\sqrt{3}}\hat\phi}-1\right)} \left(1 + c\left(e^{\frac{2\hat\phi}{\sqrt{3}}} - 1\right)\right)\right] + \frac{F_{\rm loop}}{\langle\mathcal{V}\rangle^4}\, e^{\frac{2}{\sqrt{3}}\hat\phi}\,, \tag{5.45}$$

where we recall that $c = 2\pi\langle\tau_1\rangle$, and we have defined:

$$F_{\rm loop} = \frac{c}{2\pi}\left(g_s C_{\rm loop} W_0\right)^2 \,. \tag{5.46}$$

As can be seen from Figure 5.1, the effect of the one-loop correction is to lift the inflationary region and to generate an inflection point at $\hat\phi = \hat\phi_{\rm ip}$ where $\partial^2 V/\partial\hat\phi^2 = 0$. The position of the inflection point as a function of the ratio

$$R \equiv F_{\rm loop}/F_{\rm poly} \tag{5.47}$$

can be approximated by [4]:

$$e^{\kappa\hat\psi_{\rm ip}} \simeq \frac{\kappa^3\langle\mathcal{V}\rangle^{1-p}}{R}\,, \tag{5.48}$$

[3]The factor of g_s can be derived by demanding that the one-loop corrected K scales as g_s^2 in string frame.

[4]We again use $\hat\psi$ instead of $\hat\phi$ for simplicity. Recall that at leading order the two quantities coincide (see Eq. (5.28)).

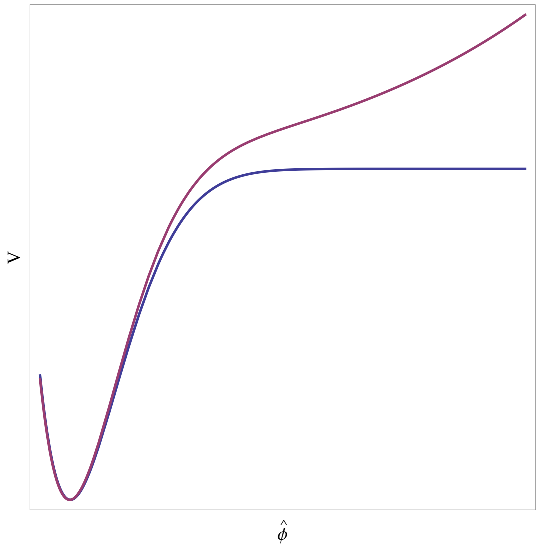

Figure 5.1: Sketch of the inflationary potential for an illustrative choice of the underlying parameters. In blue: the potential considering only the contribution from poly-instantons; in magenta: the potential including loop corrections.

showing correctly that $\hat{\phi}_{\rm ip}$ gets larger for smaller values of R, i.e. when the loops get weaker than the poly-instanton effects. The position of the inflection point is also dependent on the value of the overall volume via the parameter p, showing that the loop corrections get volume suppressed for $p < 1$. We shall focus on models where $p \lesssim 1$ since even though values of p much smaller than unity would render the loop effects unimportant, they would also make the inflaton heavier and bring $\langle \tau_1 \rangle$ in a regime where we no longer trust the effective field theory.

For $\hat{\phi} > \hat{\phi}_{\rm ip}$ the slow-roll parameter η becomes positive while ϵ stays negligibly small. Hence if inflation started at $\hat{\phi}_* > \hat{\phi}_{\rm ip}$, we would obtain a spectral index $n_s > 1$ which is incompatible with observations. We need therefore to focus on cases where the ratio R is small enough to have the inflection point lying outside the inflationary region, i.e. $\hat{\phi}_{\rm ip} > \hat{\phi}_*$. In this way, we can realise inflation with a potential generated by the poly-instantons in a region in which loop effects are negligible.

In Figure 5.2 we plot the slow-roll parameters and the spectral index for different indicative values of R. Given that the numerical study of the potential generated by poly-instanton effects gives $\hat{\phi}_* \simeq 0.9$ for $N_e \simeq 54$, Figure 5.2 reveals that the effect of loop corrections is negligible for $R \lesssim 10^{-3}$ (corresponding to $\hat{\phi}_{\rm ip} \gtrsim \hat{\phi}_*$).

From the definitions of $F_{\rm poly}$, Eq. (5.19), and $F_{\rm loop}$, Eq. (5.46), it follows that R

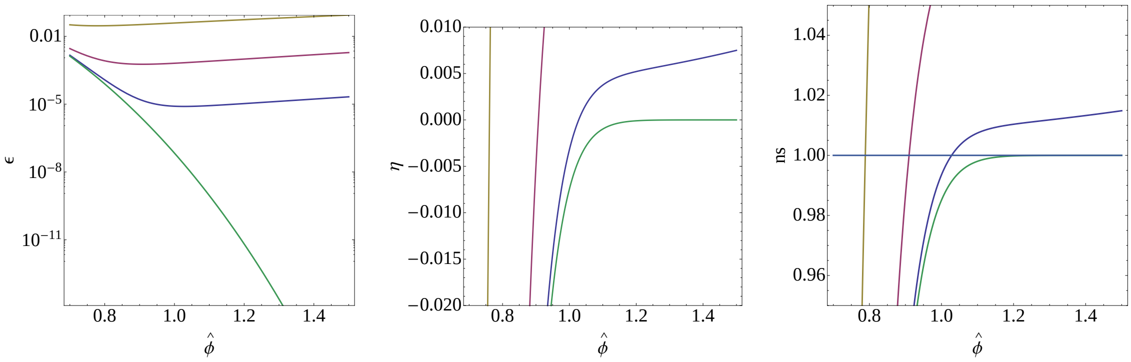

Figure 5.2: Slow-roll parameters and spectral index for different indicative values of R setting $p = 1$: blue $R = 10^{-3}$, magenta $R = 10^{-2}$, yellow $R = 10^{-1}$, green $R = 0$.

is naturally very small, being proportional to the small parameter $(g_s C_{\rm loop})^2 \ll 1$:

$$R \equiv \frac{F_{\rm loop}}{F_{\rm poly}} = (g_s C_{\rm loop})^2 \left(\frac{W_0\, c}{8\pi f_2^{p+1} r_1\, e^{-1}} \right) \ll 1\,, \tag{5.49}$$

since we expect all the parameters in the second parenthesis to be $\mathcal{O}(1-10)$. Thus this analysis shows that the loop corrections to the potential are a subleading effect that does not affect the inflationary dynamics.

We now present three illustrative parameter fits which we found numerically. All of them satisfy all our phenomenological and theoretical constraints and give rise to a viable inflationary model. Table 5.1 displays the underlying parameters, Table 5.2 the compactification properties, and Table 5.3 the inflationary footprints.

	W_0	A	B	a	b	C_1	n	ξ	g_s	α	β
$\mathcal{P}_1$	14	11	8	$2\pi/4$	$2\pi/5$	4	-2.308	1.5	0.15	$\sqrt{1.5}$	$\sqrt{1.5}$
$\mathcal{P}_2$	6	12	6	$2\pi/4$	$2\pi/5$	3	-1.198	1.5	0.15	$\sqrt{1.5}$	$\sqrt{1.5}$
$\mathcal{P}_3$	5	30	10	$2\pi/4$	$2\pi/5$	9	-3.090	1.5	0.2	0.717	1

Table 5.1: Sampling of points in parameter space.

	$\langle\tau_3\rangle$	$\langle\mathcal{V}\rangle$	$\langle\tau_1\rangle$	p	Z	r_1	R/C^2_{loop}
$\mathcal{P}_1$	4.57	1480	1.07	1	5.95	0.58	0.017
$\mathcal{P}_2$	4.68	1300	1.00	0.9	3.21	0.60	0.013
$\mathcal{P}_3$	5.70	1419	1.07	0.8	4.69	3.42	0.035

Table 5.2: Minima for the Kähler moduli generated for the microscopic parameters of Table 5.1.

	N_e	n_s	r	$A_{\rm COBE}$	$M_{\rm inf}$(GeV)
$\mathcal{P}_1$	54.2	0.959	9.56×10^{-6}	2.63×10^{-7}	1.81×10^{15}
$\mathcal{P}_2$	54.2	0.959	9.56×10^{-6}	2.26×10^{-7}	1.75×10^{15}
$\mathcal{P}_3$	54.2	0.959	9.56×10^{-6}	2.11×10^{-7}	1.72×10^{15}

Table 5.3: Inflationary observables for the fits of Table 5.1.

Comparing the results of Tables 5.1, 5.2 and 5.3 we see that it is possible to generate inflation at the right scale for natural values of the microscopic parameters, with the fibre modulus stabilised in the geometric regime. Furthermore we see in Table 5.2 that the values of the ratio R turn out to be always smaller than 10^{-3} for $C_{loop} \sim 0.1$, justifying the fact that we neglected string loop effects.

We stress that this was not possible in the model of [104, 105] even if the inflationary potential was still generated by non-perturbative effects, and so had a shape very similar to (5.27). The reason is that in [104,105] the parameter κ scales with the volume as $\kappa \sim \mathcal{V}^{1/2}$, so it is large for large values of the volume. Instead, in our case it results much smaller since it scales with the volume as $\kappa \sim \ln \mathcal{V}$. This is due to the different topological origin of the inflaton which in [104,105] was a blow-up mode whereas, in our case it is a fibre modulus. The fact that κ is smaller in our set-up, with respect to the model of [104, 105], has a two-fold implication:

- Our potential is steeper than the one of [104, 105] resulting in a larger value of the tensor-to-scalar ratio $r \sim 10^{-5}$ compared to the value $r \sim 10^{-10}$ of [104,105];

- Looking at the dependence of the slow-roll parameters (5.30) and (5.31) on κ, we realise that larger values of κ require larger values of $\hat{\phi}$ in order to satisfy the slow-roll conditions. Hence the inflationary region in our case is much closer to the minimum than in [104, 105]. This is the reason why, contrary to [104, 105], our model is not affected by the η-problem since we can avoid to experience regions in field space where string loop corrections dominate.

In this set-up, our numerical analysis confirms that the potential gives rise to negligible tensor contributions to the CMB and to a spectral index for scalar perturbations that is compatible with current observations. Notice that the fibre inflation model of [106] has also an inflationary potential with a shape very similar to (5.27). However in [106] the parameter κ is smaller and volume-independent: its value is $\kappa = 1/\sqrt{3}$. This gives a steeper potential with respect to our case, which is able to give rise to observable tensor modes.

5.4 Discussion

In this Chapter we derived a new model of inflation driven by a Kähler modulus within the context of type IIB Calabi-Yau flux compactifications. This class of inflationary models features an interesting solution to the η-problem since they are characterised by the presence of several directions which are naturally flat. In fact, due to the no-scale structure of the Kähler potential, all the $h^{(1,1)}$ directions in moduli space are flat at tree-level. Then the next-to-leading order effect, that breaks the no-scale structure, is an α' correction which lifts only the volume mode. Hence all the other $(h^{1,1}-1)$ directions in moduli space are flatter than the volume mode which sets the Hubble scale H. This implies that all these scalar fields are natural inflaton candidates since they turn out to be lighter than H. Moreover, the expansion of the pre-factor e^K of the F-term scalar potential generates higher-dimensional operators which depend only on the volume mode, and so do not introduce any dangerously large correction to the inflaton mass.

An inflationary potential for these remaining flat directions can be generated at subleading order either by non-perturbative corrections to the superpotential, as in Kähler moduli inflation [104, 105], or by perturbative string loop corrections to the Kähler potential, as in fibre inflation [106]. However, as noticed in [106], the model of [104, 105] is plagued by the η-problem since loop corrections dominate over the non-perturbative potential and spoil inflation.

In this Chapter we proposed a new way to generate an inflationary potential via non-perturbative effects which does not lead to an η-problem. The new ingredient is the inclusion of poly-instanton corrections to the superpotential [118] for Calabi-Yau manifolds with a K3 or T^4 fibre over a $\mathbb{P}^1$ base [46, 120]. The main difference with the model of [104, 105] is the topological nature of the inflaton field. In our case it is a fibre modulus, whereas in the case of [104, 105] it was a blow-up mode. Due to this difference, our potential is not affected by the η-problem. In fact in our case the inflationary region is much closer to the minimum where the string loop corrections are subdominant for natural values of the underlying parameters. Moreover, the different topology of the inflaton field is reflected also in a steeper potential with respect to Kähler moduli inflation. We therefore obtain a larger value for the tensor-to-scalar ratio $r \sim 10^{-5}$ compared to the value $r \sim 10^{-10}$ of [104, 105], even though it is still beyond the observational reach.

This small value of r corresponds to a high inflationary scale, $M_{\rm inf} \simeq 10^{15}$ GeV, and a sub-Planckian motion of the inflaton in field space $\Delta\phi \simeq 0.5 M_p$. Horizon exit takes place around $N_e \simeq 54$ e-folding before the end of inflation where the spectral index assumes a value $n_s \simeq 0.96$ in agreement with current observations. At the end of inflation, the visible sector degrees of freedom get excited by the gravitational decay of the inflaton field which leads to a reheating temperature of the order $T_{\rm rh} \simeq 10^6$ GeV.

Finally it is worth emphasising that our analysis of the effect of string loop corrections on the inflationary potential relies on a low-energy estimate instead of a proper computation of string scattering amplitudes. Due to the relatively simple dependence of these loop corrections on the Kähler moduli, we believe the results we find here are likely to capture their leading behaviour. However, it would be very interesting to have more explicit calculations of g_s corrections to the Kähler potential for general Calabi-Yau manifolds.

Apart from inflation, we point out that our set-up possesses two crucial properties that can be applied to build models for quintessence. The first is that loop corrections to the potential can be controlled and the second is that the coupling of the inflaton field to visible sector degrees of freedom is weaker than Planck strength. In this way one could avoid the stringent bounds coming from fifth-force experiments. Hence it would be interesting to investigate if this framework could also give rise to a successful quintessence model. We tackle this issue in the next Chapter.

Chapter 6

A Model of Quintessence

This Chapter is based on the paper [15].

Present cosmological observations indicate that our Universe is currently accelerating. This accelerated expansion is driven by a mysterious form of energy density with negative pressure, dubbed 'dark energy', that accounts for about 70% of the total energy density of the Universe. The major part of the remaining 30% consists of cold non-baryonic dark matter, whose microscopic origin is not known, although it might be associated with new particles as supersymmetric partners, axions, string moduli or hidden sector degrees of freedom. Finally, ordinary baryonic matter accounts for only a few % of the total energy density.

The microscopic origin of dark energy is even less clear than that of dark matter. Many proposals have been put forward for explaining present day acceleration (see [131] for a review). We will focus here on quintessence models, in which the dynamics of scalar fields drives the present cosmological acceleration, as opposed to the simplest scenario in which the cosmological energy density is dominated today by a pure cosmological constant.

Both of these proposals, a rolling quintessence field and a non-zero cosmological constant, have their own virtues and shortcomings:

- *Cosmological Constant Problem*: If dark energy is given by a non-zero cosmological constant, in order to match the current cosmological observations, its value needs to be of the order $\langle V \rangle = \Lambda^4 \sim \text{meV}^4 \sim 10^{-120}\, M_P^4$. Whereas, if dark energy is provided by a quintessence model, in which a late-time acceleration is characterised by a scalar ϕ with kinetic energy smaller than its potential energy, the corresponding scalar mass has to be of the order $m \sim 10^{-33}\,\text{eV}$. The two values of Λ and m are related to each other since:

$$m^2 \sim V_{\phi\phi} \sim \frac{V}{\phi^2} \lesssim \frac{V}{M_P^2} \sim 10^{-120} M_P^2 \sim \left(10^{-33}\,\text{eV}\right)^2 . \tag{6.1}$$

The condition $\phi \gtrsim M_P$ follows from the Friedmann equation $3M_P^2 H^2 = (\dot{\phi}^2/2 + V) \simeq V$ and the equation of motion for ϕ, $0 = \ddot{\phi} + 3H\dot{\phi} + V_\phi \simeq 3H\dot{\phi} + V_\phi$. In fact, for $\dot{\phi}^2 \lesssim V$, we have:

$$\phi \sim \frac{V}{V_\phi} \sim \frac{V}{H\dot{\phi}} \gtrsim M_P . \tag{6.2}$$

The required smallness of these quantities (Λ or m, depending on the scenario under consideration) represents one of the biggest mysteries of modern physics since a naïve estimate of the vacuum energy would yield a result many orders of magnitude larger than the observed one: $\langle V \rangle \sim M_P^4$ for non-supersymmetric theories and $\langle V \rangle \sim \text{TeV}^4 \sim 10^{-60}\, M_P^4$ for theories with TeV-scale supersymmetry. In the quintessence case, one assumes that the vacuum energy is exactly zero and then explains how to obtain such a small mass for the rolling scalar. Indeed, scalar masses are notoriously hard to keep from getting large contributions when integrating out ultra-violet physics running in the loops. This is the issue of *technical naturalness*, which plays a crucial rôle in the determination of the Higgs mass and the solution of the famous η-problem in slow-roll inflation.

- *Coincidence Problem*: Observations indicate that the energy densities of dark matter and dark energy are of the same order of magnitude at present cosmological epochs. The reason why different forms of matter have similar properties today is a puzzle which goes under the name of 'coincidence problem'. Dark energy models based on a non-zero, pure cosmological constant have little to say about it, but quintessence models can in principle provide an answer to this problem. The idea is to consider 'tracking' solutions, in which the non-trivial dynamics of the scalar field leads its energy density to mimic radiation during the radiation-dominated era. The transition between radiation to matter domination then triggers quintessence towards assuming characteristics that match the observed behavior of dark energy.

- *Fifth-force constraints*: Despite providing a potential understanding of the present ratios of energy densities via tracking solutions, quintessence models are generically plagued by the problem of long-range unobserved fifth-forces which would be mediated by such a light scalar field [132]. The present mass bounds for scalars with Planck strength couplings to matter are $m \gtrsim \mathcal{O}(\text{meV})$ [11]. Furthermore, a rolling scalar would lead to a time variation of the constants of Nature, while these do not appear to drastically change between Big Bang Nucleosynthesis (BBN) and the present epoch. A possible solution to this problem involves protecting the quintessence field with a shift-symmetry [133]. This allows it to evade the constraints from fifth-force experiments and renders the quintessence potential stable against radiative corrections from UV physics. For models without this symmetry, in which the quintessence field is a scalar, it is harder to satisfy these constraints and explicit computation of couplings between the quintessence field and the visible sector matter is required.

Most of the attempts to realise quintessence within string theory rely on axions and shift symmetries [134–137]. In this Chapter we shall take a different approach and build the first string model where the quintessence field is a scalar parameterising the size of an internal cycle. We shall focus on LARGE Volume Scenarios (LVS) which emerge naturally in type IIB flux compactifications. The set-up of this model represents an interesting variation of the one derived in the previous Chapter [14] for inflationary purposes. In fact, as in [14], we focus on the string embedding [46] of the Supersymmetric Large Extra Dimension (SLED) proposal [138–140]. The field which was playing the rôle of the inflaton in [14] will now behave as quintessence.

In our model, quintessence is driven by a closed string modulus with a naturally small mass m protected against loop corrections. The mechanism for obtaining such a light scalar whose mass is radiatively stable is similar to the one of [141] and we shall outline it in Section 6.1. Contrary to the model of [141] where the quintessence field was the overall volume mode, in our case quintessence is driven by a fibre modulus that at leading order neither develops a potential nor couples to Standard Model (SM) matter. The potential for the quintessence field arises only at subleading order by means of non-perturbative and string loop effects, that also induce suppressed, weaker-than-Planckian couplings with ordinary matter. Thus, the model is able to avoid fifth-force constraints. An intuitive explanation for this non-standard behaviour for the quintessence scalar will be given in Section 6.1, while in Sections 6.2 and 6.3 we will develop our arguments at a more technical level.

The dynamics of the quintessence-dark matter system will then be analysed in Section 6.4 starting from the standard decoupled case and then moving to the more complicated coupled case where we shall also explain how our stringy set-up can allow for unsuppressed couplings of the quintessence field with dark matter degrees of freedom. Hence we will suggest a realisation of a coupled quintessence-dark matter model inspired by string theory. Finally we will conclude in Section 6.5.

6.1 Quintessence from anisotropic compactifications

This Section constitutes a qualitatively discussion of the ideas behind the construction of our quintessence model. The technical elements of the set-up will be analysed in the next sections.

6.1.1 SLED in a nutshell

Supersymmetric Large Extra Dimension (SLED) scenario (see [142] for a review) in its original form is a brane-world model with two large extra dimensions, in which supersymmetry is preserved in the bulk and non-linearly realised on the Standard Model brane. In these models the solutions to the hierarchy and cosmological constant problems are closely tied. The hierarchy problem is solved by having a low fundamental gravity scale (around the TeV), which can be achieved by having large extra dimensions. The observed value of the cosmological constant is then associated with the low supersymmetry breaking scale in the bulk, which is also a consequence of having large extra dimensions. This can be seen explicitly from considerations based on dimensional reduction which yield the following relation between the fundamental $(4+d)$-dimensional gravity scale M_* and the gravitino mass $m_{3/2}$:

$$m_{3/2} \sim \left(\frac{M_*}{M_P}\right)^2 M_P \,. \tag{6.3}$$

Setting $M_* \sim \mathcal{O}(\mathrm{TeV}) \sim 10^{-15} M_P$, the gravitino mass turns out to be very small, of the order of the dark energy scale $m_{3/2} \sim \Lambda^{1/4} \sim \mathcal{O}(\mathrm{meV}) \sim 10^{-30} M_P$. This implies that the bulk is nearly supersymmetric, given that supersymmetry is broken only at the dark energy scale. Quantum corrections to the vacuum energy from loops of bulk fields scale as:

$$V_{\rm loop} \sim \mathrm{Str}(M^2)\, \Lambda_{\rm UV}^2 \sim m_{3/2}^2\, M_{\rm KK}^2 \,, \tag{6.4}$$

where $\mathrm{Str}(M^2) \sim m_{3/2}^2$ and the cut-off is taken to be the scale at which a pure 4D description ceases to be valid: $\Lambda_{\rm UV} \sim M_{\rm KK}$. Given that the Kaluza-Klein scale is given by:

$$M_{\rm KK} \sim \left(\frac{m_{3/2}}{M_P}\right)^{1/2+1/d} M_P \,, \tag{6.5}$$

this is of the same order of $m_{3/2}$ for $d = 2$. This is also the case that for TeV-scale fundamental gravity leads to micron-sized extra dimensions which are at the edge of detectability in experiments that look for deviations from Newton's law. Hence in the case of two large extra dimensions, the loop generated potential (6.4) gives the correct observed order of magnitude of the cosmological constant since it scales as $V_{\rm loop} \sim m_{3/2}^4 \sim 10^{-120} M_P$. Notice that for energies above the cut-off $M_{\rm KK}$, the theory is effectively 6D and supersymmetric, and so the contributions to the vacuum energy from loops of bosons and fermions cancel among each other.

In order to have a viable solution of the cosmological constant problem, one has to make sure that (6.4) is the leading order contribution to the vacuum energy. In general, one would expect larger contributions coming directly from the tension of the Standard Model brane, of the order $T \sim \mathcal{O}(\text{TeV})^4$, and from the tree-level part of the bulk potential corresponding to the curvature of the extra dimensions induced by the presence of the TeV brane. This is a delicate issue (see e.g. [143]): recent progresses seem to suggest that consistent scenarios can be built along these lines by means of suitable couplings between bulk fields and the SM brane [144]. Since the rest of our discussion does not rely specifically on these details, we will not dwell on this issue any further.

The SLED framework opens the possibility to obtain a natural model of quintessence. In fact in 6D SLED models, the radion mode r develops a potential via loops of bulk fields which scale as (6.4), and so give it a mass of the order [141]:

$$m_r^2 \sim \frac{V_{\rm loop}}{M_P^2} \sim \frac{m_{3/2}^4}{M_P^2} \sim \left(10^{-33}\,\text{eV}\right)^2 . \tag{6.6}$$

This is the perfect scale for quintessence, since it is the typical mass scale expected for a quintessential scalar; for the arguments we outlined above, this small mass is *technically natural* since it is radiatively stable thanks to the unbroken supersymmetry in the bulk.

In the quintessence model based on the 6D SLED scenario the radion mode, being a scalar with Planck strength coupling to ordinary matter, would mediate long range fifth-forces. Its cosmological evolution would also lead to a time dependence of Newton's constant. The authors of [141] propose to cure this problem exploiting the fact that the couplings of the radion to matter are field dependent, approaching small values for appropriate cosmological evolutions of the radion field. The strongest constraint comes from the requirement that the Newton's constant does not change that much from BBN until today.

We argue that an automatic solution to the fifth-force problem emerges when embedding this quintessence scenario in the highly anisotropic string compactifications of [46].

6.1.2 Stringy SLED

A recent paper [46] discusses how to embed these 6D SLED scenarios within type IIB flux compactifications. The simplest Calabi-Yau three-fold that allows such a derivation has a fibered structure with a 4D K3 or T^4 fibre over a 2D $\mathbb{P}^1$ base. This structure allows to take a very anisotropic limit where the base is exponentially larger than the fibre, resulting in an intermediate 6D effective field theory when compactifying 10D type IIB string theory down to 4D. The authors of [46] show how to stabilise the geometric moduli in such a way to obtain the above mentioned anisotropic limit within the framework of the LARGE Volume Scenario (LVS) [29]. In this explicit string embedding the 6D effective field theory is far richer than the one considered in [141], due to the presence of additional closed string moduli on top of the radion (that in the IIB embedding corresponds to the volume mode).

The minimal scenario we consider in this work involves three Kähler moduli, which parameterise the size in string units of internal four-cycles, and a fibered Calabi-Yau volume of the form [120]:

$$\mathcal{V} = t_b \, \tau_f - \tau_s^{3/2} \, . \tag{6.7}$$

The fields defining the volume have the following roles and geometrical interpretation:

- τ_s is a del Pezzo divisor supporting non-perturbative effects of the form $W_{\rm np} = A \, e^{-aT_s}$, where $\tau_s = \mathrm{Re}(T_s)$ and A and a are τ_s-independent constants. These effects fix τ_s at 'small' values but still in a geometric regime above the string scale: $\langle \tau_s \rangle \sim \mathcal{O}(10)$.

- The overall volume mode $\mathcal{V} \simeq t_b \tau_f$ is a modulus fixed at exponentially large values, $\langle \mathcal{V} \rangle \sim e^{1/g_s} \gg 1$ for weak string coupling $g_s \lesssim 0.1$, by the interplay of non-perturbative effects and α' corrections to the Kähler potential [29]. The size of the 2D $\mathbb{P}^1$ base is given by t_b whereas the volume of the 4D K3 or T^4 fibre is given by τ_f. The value of $\mathcal{V}$ plays a crucial role in determining the scales and coupling strengths of our model, explaining in a natural way why the size of dark energy is so small.

- The fibre modulus τ_f plays the role of the quintessence field. It develops a potential only at subleading order due to poly-instanton corrections to the superpotential generated by an E3-instanton wrapping this non-rigid divisor: $W_{\rm np} = A\, e^{-a\left(T_s + C e^{-2\pi T_f}\right)}$, where again $\tau_f = {\rm Re}(T_f)$ and C is a τ_f-independent constant. These tiny effects fix the fibre modulus 'small', i.e. at the same order of magnitude of τ_s: $\langle\tau_f\rangle \sim \langle\tau_s\rangle$. This gives rise to a very anisotropic compactification with t_b exponentially larger than $\sqrt{\tau_f}$.

None of these three four-cycles can support the visible sector where supersymmetry is non-linearly realised via a proper configuration of branes wrapped around internal divisors.[1] In fact, the Standard Model cannot be localised neither on τ_s nor on τ_f due to the tension between chirality and the generation of non-perturbative effects [57]. Furthermore the four-cycle containing the base, $\tau_b \simeq t_b\sqrt{\tau_f}$, would give rise to an exponentially small gauge coupling $g^{-2} \simeq \tau_b \gg 1$. Thus we need to add a fourth Kähler modulus for the Standard Model, $\tau_{\rm SM}$. This divisor has to be fixed at small values in order to get a correct value of the gauge coupling. This can be done by exploiting D-terms and string loop corrections to the Kähler potential [145]. Therefore $\tau_{\rm SM}$ has the same features as τ_s apart from the fact that one cycle is fixed perturbatively whereas the other non-perturbatively. Hence we shall restrict ourselves to the simplest case with three moduli, having in mind that the modulus τ_s can represent a generic rigid and 'small' divisor which can be either $\tau_{\rm SM}$ or the one supporting the non-perturbative effects.

The structure of the scalar potential for the bulk fields (trading t_b for $\mathcal{V}$) is the following:

$$V = V_{\rm np}\left(\mathcal{V},\tau_s\right) + V_{\alpha'}\left(\mathcal{V},\tau_s\right) + V_{\rm poly}\left(\mathcal{V},\tau_s,\tau_f\right) + V_{\rm loop}\left(\mathcal{V},\tau_s,\tau_f\right)\,, \tag{6.8}$$

where the leading order effects are given by the non-perturbative and α' potentials, $V_{\rm np}$ and $V_{\alpha'}$, which depend only on $\mathcal{V}$ and τ_s and scale as:

$$V_{\rm lead} = V_{\rm np}\left(\mathcal{V},\tau_s\right) + V_{\alpha'}\left(\mathcal{V},\tau_s\right) \sim \mathcal{O}\left(m_{3/2}^3 M_P\right)\,. \tag{6.9}$$

$V_{\rm lead}$ is the familiar LVS potential and it determines the properties of the volume and the modulus τ_s at leading order in an inverse volume expansion. This implies that the basic features of SM physics, like gauge and Yukawa couplings, are determined at leading order by the VEV of τ_s. The fibre modulus τ_f develops a potential only at

[1]We need to focus on a non-linear realisation of supersymmetry where the low-energy effective field theory contains just the Standard Model without any superpartner since models with TeV-scale strings predict $m_{3/2} \sim$ meV, and so gravity mediation would give rise to soft terms of this unacceptably small order of magnitude.

subleading order due to poly-instanton effects and loops of closed strings propagating in the bulk. The subleading potential scales as (6.4) [2]:

$$V_{\rm sub} = V_{\rm poly}\left(\mathcal{V},\tau_s,\tau_f\right) + V_{\rm loop}\left(\mathcal{V},\tau_s,\tau_f\right) \sim \mathcal{O}\left(m_{3/2}^4\right)\,. \tag{6.10}$$

The potential (6.9) gives rise to a SUSY breaking AdS vacuum, which needs to be uplifted to a dS solution. This can be accomplished by means of one of the many mechanisms known in the literature (tension of anti branes at the tip of warped throats [28], D-terms from magnetised branes [146] or dilaton-dependent non-perturbative effects [147]). The leading order contribution to the vacuum energy would then be given by the subleading potential (6.10) which now naturally gives rise to the observed size of dark energy, since it scales as the bulk loop potential of 6D SLED models (see (6.4)).

A key difference between our scenarios and SLED constructions is the fact that the volume mode can no longer be used as the quintessence field, since it develops a potential at order $m_{3/2}^3 M_P$, and so the corresponding mode is too heavy to drive quintessence:

$$m_{\mathcal{V}}^2 \sim \frac{V_{\rm lead}}{M_P^2} \sim m_{3/2}^2\left(\frac{m_{3/2}}{M_P}\right) \sim \left(10^{-18}\,{\rm eV}\right)^2\,. \tag{6.11}$$

On the other hand the fibre modulus τ_f does *not* appear in the expression (6.9) for $V_{\rm lead}$, and so its mass has the perfect quintessential value which can be estimated from the subleading potential (6.10):

$$m_{\tau_f}^2 \sim \frac{V_{\rm sub}}{M_P^2} \sim m_{3/2}^2\left(\frac{m_{3/2}}{M_P}\right)^2 \sim \left(10^{-33}\,{\rm eV}\right)^2\,. \tag{6.12}$$

This expression is similar to (6.6) which gives the mass of the radion mode of standard 6D SLED models. Consequently the natural explanation of why the quintessence field is light and radiatively stable is, in practice, the same in both scenarios.

The main advantage of the string embedding of SLED is that the different topological origin for the quintessence field, and the structure of the scalar potential allow it to evade fifth-force constraints. The radion mode in the SLED proposal couples to ordinary matter with Planck strength, $1/M_P$. Instead, as discussed also in [46], the fibre modulus has a weaker-than-Planckian coupling of the order ϵ/M_P where $\epsilon \sim m_{3/2}/M_P \ll 1$. For this reason, this set-up is ideal for building a quintessence

[2] We do not include the contribution coming from loops of open string fields since by construction no D7-brane stack wraps either the fibre or the base divisor. Hence open strings are localised far away in the Calabi-Yau, and so do not develop any potential for τ_1. If that were the case, the internal manifold would have a isotropic shape since τ_1 would also be fixed at exponentially large values: $\langle\tau_1\rangle \simeq g_s^{4/3}\,\langle\mathcal{V}\rangle^{2/3} \gg 1$.

model since the rolling field can safely evade all the current bounds from fifth-force experiments.

The intuitive reason of the weakness of the fibre modulus coupling to ordinary matter is the fact that the leading order potential $V_{\rm lead}$ does not depend on τ_f (see (6.9)), and so there is no leading order mixing between τ_f and the volume mode $\mathcal{V}$. The mixing between τ_f and $\mathcal{V}$ arises only at subleading order at the level of $V_{\rm sub}$ (see (6.10)). Therefore the order of magnitude of this small mixing, $\epsilon \ll 1$, is weighted by the ratio between $V_{\rm sub}$ and $V_{\rm lead}$:

$$\epsilon \sim \frac{V_{\rm sub}}{V_{\rm lead}} \sim \mathcal{O}\left(\frac{m_{3/2}}{M_P}\right) . \tag{6.13}$$

Given that the volume mode couples to ordinary matter as $1/M_P$ and the coupling of the fibre modulus to SM particles is induced by the mixing between τ_f and $\mathcal{V}$, the coupling of the fibre modulus has to be suppressed with respect to the one of the volume mode by the small mixing parameter ϵ, explaining the weaker-than-Planckian nature of this interaction. In other words, the weakness of this coupling is reflecting the fact that τ_f is a flat direction at leading order and does not couple with ordinary matter at that level. On the other hand, there is the possibility that the quintessence field couples with Planckian strength to massive degrees of freedom like stable 6D Kaluza-Klein states. This enables the current setup to provide a stringy realisation of an interacting dark energy-dark matter system, if dark matter is made up of these Kaluza-Klein states.

We shall make this analysis more precise in Section 6.3 where we will compute the moduli couplings to Standard Model particles. These are extracted from the moduli dependence of the kinetic and mass terms of gauge bosons and fermions, using the canonical normalisation carried out in Appendix A.2.

After this outline of the general properties of this quintessence scenario, we next develop the technical details.

6.2 Explicit form of the quintessence potential

In this Section we develop our set-up by discussing the features of the potential that drives quintessence. As explained in Section 6.1, we work within the framework of the LARGE Volume Scenario of type IIB string theory [29] where the volume is fixed at exponentially large values by combining α' corrections to the Kähler potential K and non-perturbative contributions to the superpotential W.

We consider compactifications where the volume of the Calabi-Yau manifold has a fibered structure and is given by Eq. (6.7). We have seen in Chapter 5 that with this choice of internal manifold, the standard LVS framework stabilises only two of the three Kähler moduli. The remaining flat direction, which can be identified with the fibre modulus τ_f, is lifted either by string loop corrections to K [27, 45] or by poly-instanton effects [14, 46]. Here we focus our attention on the latter possibility, and investigate if the resulting potential for the fibre modulus τ_f allows us to provide a string theory realisation of quintessence. The analysis closely follows that of the previous Chapter, in which we studied the same scalar potential in the context of an inflationary model. For this reason, we will omit the full derivation of the scalar potential here, referring the reader to Chapter 5 for more details.

Let us briefly recall the qualitative aspects of the compactification that leads to the string realisation of the 6D SLED scenario [14]. We assume the following brane set-up: the cycle τ_s is wrapped by a stack of magnetised D7-branes where appropriate gauge fluxes give rise to two different gauge groups that separately undergo gaugino condensation. This gives rise to a race-track superpotential. Due to the existence of a Euclidean D3-instanton wrapping the fibre τ_f, this racetrack superpotential acquires further corrections. Following [118, 119], the gauge kinetic functions of the two condensing gauge groups receive non-perturbative corrections, resulting in a poly-instanton corrected superpotential of the form of Eq. (5.5). The Kähler potential is assumed to include the leading α' corrections that are crucial for LVS compactifications, Eq. (2.71).

The scalar potential can be show to factorize into:

$$V = V_{\mathcal{O}(\mathcal{V}^{-3})}\left(\tau_s, \mathcal{V}\right) + V_{\mathcal{O}(\mathcal{V}^{-3-p})}\left(\tau_f, \tau_s, \mathcal{V}\right) , \tag{6.14}$$

where the first term scales with the volume as $\mathcal{V}^{-3}$, while the other as $\mathcal{V}^{-3-p}$, with p a positive order one parameter. If $\mathcal{V}$ is large, the second piece is suppressed with respect to the first one: the previous equation reproduces the split between leading and subleading potentials we discussed in Section 6.1. The leading contribution to the scalar potential, $V_{\mathcal{O}(\mathcal{V}^{-3})}$, is the standard LVS potential that stabilises the blow-up modulus τ_s and the volume mode $\mathcal{V}$ at $\langle\tau_s\rangle \sim 1/g_s$ and $\langle\mathcal{V}\rangle \sim e^{1/g_s}$.

The subleading $V_{\mathcal{O}(\mathcal{V}^{-3-p})}$ term depends on all three directions of Kähler moduli space and takes the form of Eq. (5.8). Given that the mass spectrum in the Kähler moduli sector exhibits the following hierarchy $m_s^2 \sim \mathcal{V}^{-2} \gg m_{\mathcal{V}}^2 \sim \mathcal{V}^{-3} \gg m_f^2 \sim \mathcal{V}^{-3-p}$ with $p \sim \mathcal{O}(1)$ [14], one can set τ_s and $\mathcal{V}$ to their minima and so reduce the analysis of quintessence to the study of a single field dynamics for τ_f moving in the potential $V_{\mathcal{O}(\mathcal{V}^{-3-p})}(\tau_f, \langle\tau_s\rangle, \langle\mathcal{V}\rangle)$.

Following this procedure and performing the canonical normalisation of the fibre modulus (see Appendix A.2), the poly-instanton generated potential is written as:

$$V_{\rm poly} = \frac{F_{\rm poly}}{\langle\mathcal{V}\rangle^{3+p}}\left[1 - e^{-c\left(e^{\frac{2}{\sqrt{3}}\hat{\phi}}-1\right)}\left(1 + c\left(e^{\frac{2\hat{\phi}}{\sqrt{3}}} - 1\right)\right)\right], \tag{6.15}$$

where $\hat{\phi}$ is the field parametrising the shift of the canonically normalised fibre modulus from its minimum: $\hat{\phi} \equiv \phi - \langle\phi\rangle = \sqrt{3}/2\ln(1 + \hat{\tau}_f/\langle\tau_f\rangle)$. The constant c is given by $c = 2\pi\langle\tau_f\rangle = pb\langle\tau_s\rangle + 1$. $F_{\rm poly}$ is given in terms of the underlying parameters as:

$$F_{\rm poly} = \left(\frac{3\alpha W_0\sqrt{\langle\tau_s\rangle}f_1}{Z}\right)^{p+1} 4\,e^{-1}W_0(C_1 Z - nBb). \tag{6.16}$$

Equation (6.15) provides a first, extremely flat contribution to the quintessence potential we will be working with. Note that, as explained in Section 6.1, we included in Eq. (6.15) an uplifting term, $V_{\rm up} = F_{\rm poly}/\langle\mathcal{V}\rangle^{3+p}$ so to set the minimum of the quintessence potential to zero.

As pointed out in [14], there will be further contributions to the fibre modulus potential, in particular UV physics will add new ϕ dependent terms to the potential. These are the loop contributions that also featured in the study of the inflationary applications in this setup and that we have to take into account here too, since they can influence the dynamics of the quintessence field. These contributions can be estimated via the Coleman-Weinberg potential and are due to subleading g_s corrections to the Kähler potential. Supersymmetry reduces their size and following the estimate performed in [14] we obtain:

$$V_{\rm loop} \simeq \frac{F_{\rm loop}}{\langle\mathcal{V}\rangle^4}\, e^{\frac{2}{\sqrt{3}}\hat{\phi}}, \qquad \text{where} \qquad F_{\rm loop} = \frac{c}{2\pi}\left(g_s C_{\rm loop} W_0\right)^2. \tag{6.17}$$

The full potential for the fibre modulus that here corresponds to the quintessence field is then given by:

$$V_{\rm fibre} = V_{\rm poly} + V_{\rm loop}, \tag{6.18}$$

which we plot in Figure 6.1. Given that the gravitino mass is $m_{3/2} \sim M_P/\mathcal{V}$, this potential for $p \sim \mathcal{O}(1)$ scales as $V_{\rm fibre} \sim (M_P/\mathcal{V})^4 \sim m_{3/2}^4$, reproducing the behaviour described in Section 6.1.

For the study of the dynamics of the quintessence field it is convenient to divide the potential into three distinct regions:

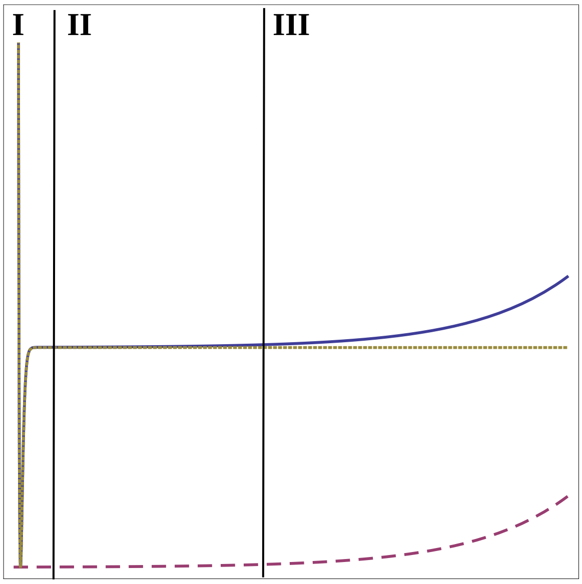

Figure 6.1: Quintessence potential for the fibre modulus. The dotted line represents the poly-instanton generated potential, the dashed line the loop potential from UV physics and the full line the total potential for the canonically normalised fibre modulus.

1. The region I corresponds to the vicinity of the minimum where the potential is very steep;
2. The region II is the poly-instanton dominated part of the potential;
3. In region III the loop corrections are the dominant contribution to V_{fibre}.

Notice that, as outlined in Section 6.1, we have obtained an extremely flat quintessential potential (above all in region II) in which supersymmetry in the extra-dimensions allows to tame the dangerous corrections that would spoil its flatness. The aim of Section 6.4 is to investigate whether the dynamics of the fibre modulus rolling down regions II and III gives a good description of the observed late time acceleration of Universe.

Let us conclude this Section by determining the relevant scales in our set-up. As we explained, observations constrain the dark-energy scale to be of the order $\rho_{DE} \sim 10^{-120} M_P^4$. In order to use the fibre modulus as a dynamical model of dark-energy, this maps into the following condition on the amplitude of the potential:

$$\frac{F_{\text{poly}}}{\langle\mathcal{V}\rangle^{3+p}} \sim 10^{-121}. \tag{6.19}$$

This scale can be achieved by considering compactifications with very large volumes, as originally noted in [46], in particular for $p \sim \mathcal{O}(1)$ this yields:

$$\langle \mathcal{V} \rangle \sim 10^{30} \tag{6.20}$$

in string units. We stress that in our scenario the VEV of the volume depends exponentially on the microscopic parameters of the model (see eq. (5.10)), and such large values are easy to obtain (for example by lowering the string coupling to $g_s \sim 0.01$). So, once the minimum of the quintessence potential is set to zero, the tiny height of the potential, Eq. (6.19), is obtained straightforwardly! Notice that values of the volume of the order $\mathcal{V} \sim 10^{30}$ correspond to a string scale of the order the TeV-scale, $M_s \sim M_P/\sqrt{\mathcal{V}} \sim \mathcal{O}\,(\text{TeV})$, and a gravitino mass of the order the cosmological constant scale, $m_{3/2} \sim M_P/\mathcal{V} \sim \mathcal{O}\,(\text{meV})$, according to the discussion of Section 6.1. Such a low-value of the fundamental gravity scale poses a severe problem for obtaining the original period of inflation since both the amount of density perturbations and the resulting reheating temperature would be far too low. For example, the inflationary model developed in Chapter 5 requires values of the volume of the order $\mathcal{V} \sim 10^3$ corresponding to a GUT-scale M_s. The general idea that we have in mind to overcome this problem is to use the volume mode as the inflaton following the work of [112]. Thus the volume can take smaller values in the early Universe, and then roll to values of the order $\mathcal{V} \sim 10^{30}$ after the end of inflation, where the only field left over to roll down its minimum is the fibre modulus τ_f which drives quintessence.

A very large volume, apart from dynamically describing the smallness of the observed size of dark energy, has also the virtue of suppressing fifth-force couplings between the quintessence field and SM matter, and enlarging the very flat region II in the potential V_{fibre}. In the next Sections, we will discuss these features in detail.

6.3 Quintessence coupling to matter

The first step in the computation of the quintessence couplings to ordinary matter is the moduli canonical normalisation, which we perform in detail in Appendix A.2. The expression of the original moduli τ_f, $\mathcal{V}$, and τ_s in terms of the corresponding canonically normalised fields ϕ, $\chi_{\mathcal{V}}$ and χ_s can be obtained explicitly at leading order in an inverse volume expansion. Considering all the moduli in the vicinity of their minima, the canonical normalisation of the Standard Model modulus represented by τ_s, takes the form (see Eq. (A.20)):

$$\frac{\delta\tau_s}{\langle\tau_s\rangle} \sim \mathcal{O}\left(\mathcal{V}^{-p}\right)\delta\phi + \mathcal{O}(1)\,\delta\chi_{\mathcal{V}} + \mathcal{O}\left(\sqrt{\mathcal{V}}\right)\delta\chi_s\,, \tag{6.21}$$

where ϕ is the quintessence field, while $\chi_{\mathcal{V}}$ and χ_s are two heavy fields corresponding respectively to the volume mode and the small rigid divisor τ_s. The exact coefficients of (6.21) are be computed in Appendix A.2. What is crucial in the previous expression is that the canonically normalised quintessence field ϕ provides only a suppressed contribution to $\delta\tau_s$, essentially determined by the ratio of the leading and subleading scalar potentials of Eqs. (6.9) and (6.10). Since τ_s controls the properties of SM physics, this implies that the coupling of the quintessence field to Standard Model particles is very suppressed.

Before proving this explicitly, let us discuss a technical issue: during the quintessence dynamics the two heavy fields $\chi_{\mathcal{V}}$ and χ_s sit at their minima, but the rolling scalar ϕ can be far away from its minimum. Hence the fibre modulus has to be canonically normalised not just close to its minimum but for an arbitrary point in moduli space. This is a complicated computation whose result has already been qualitatively discussed in [148] where the authors used the poly-instanton effects for deriving a model of modulated reheating. The final upshot is that in regions far away from the minimum for large values of τ_f, the poly-instantons get more suppressed, and so the mixing with the other moduli also gets more suppressed. Thus the coupling of the fibre modulus to ordinary matter gets weaker in regions far from the minimum. In fact, if τ_f is far from its minimum then:

$$e^{-2\pi\tau_f} = e^{-2\pi\langle\tau_f\rangle} e^{-2\pi\hat{\tau}_f} \sim \mathcal{V}^{-p} e^{-2\pi\hat{\tau}_f} \,, \tag{6.22}$$

and so the mixing term in (6.21) would be even more suppressed than $\mathcal{V}^{-p}$.

6.3.1 Coupling to gauge bosons

We start by estimating the couplings to gauge bosons following [46]. Recalling that τ_s is the Standard Model cycle, the gauge kinetic terms are given by:

$$\mathcal{L}_{\text{gauge}} = -\frac{\tau_s}{M_P} F_{\mu\nu} F^{\mu\nu} \,. \tag{6.23}$$

After expanding τ_s around the minimum ($\tau_s = \langle\tau_s\rangle + \delta\tau_s$) and performing the canonical normalisation of the field strength tensor, we find that the interaction term is:

$$\mathcal{L} \supset \frac{\delta\tau_s}{4M_P\langle\tau_s\rangle} G_{\mu\nu} G^{\mu\nu}. \tag{6.24}$$

From Eq. (6.21) it follows that

$$\frac{\delta\tau_s}{\langle\tau_s\rangle} \sim \mathcal{O}\left(\mathcal{V}^{-p}\right) \delta\phi \,, \tag{6.25}$$

and so we find

$$\mathcal{L} \supset \mathcal{O}\left(\frac{1}{M_P\,\mathcal{V}^p}\right)\delta\phi\, G_{\mu\nu}G^{\mu\nu}. \tag{6.26}$$

We conclude that the coupling between the fibre modulus and the SM gauge bosons is weaker-than-Planckian for $p \sim \mathcal{O}(1)$, in accordance with [46], and therefore poses no issues with the constraints from fifth-force experiments.

6.3.2 Coupling to fermions

Let us now turn our attention to the coupling between the quintessence field and the Standard Model fermions, which we denote as C_i. The relevant part of the supergravity Lagrangian for the computation of this coupling is:

$$\mathcal{L} \supset \tilde{K}_{ij}\overline{C_i}i\gamma^\mu\partial_\mu C_j + \tilde{K}_{H\overline{H}}\partial^\mu H\partial_\mu\overline{H} + e^{K/2}\lambda_{ij}C_iC_jH, \tag{6.27}$$

where $\tilde{K}_{ij}$ and $\tilde{K}_{H\overline{H}}$ are generic functions of the moduli fields. Very little can be said about this theory without specifying $\tilde{K}_{ij}$ and $\tilde{K}_{H\overline{H}}$, and so we follow the work of [55], where the leading modular dependence of these two functions has been estimated through simple scaling arguments which we reviewed in Chapter 2. We simplify the set-up further by considering diagonal Kähler metrics and Yukawa couplings:

$$\mathcal{L} \supset \tilde{K}_{ii}\overline{C_i}i\gamma^\mu\partial_\mu C_i + \tilde{K}_{H\overline{H}}\partial^\mu H\partial_\mu\overline{H} + e^{K/2}\lambda\overline{C_i}C_iH. \tag{6.28}$$

Following [55] we take:

$$\tilde{K}_{ii} \sim \tilde{K}_{H\overline{H}} \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}}. \tag{6.29}$$

Expanding τ_s and $\mathcal{V}$ around their VEVs as $\tau_s = \langle\tau_s\rangle + \delta\tau_s$ and $\mathcal{V} = \langle\mathcal{V}\rangle + \delta\mathcal{V}$ we can write the Kähler metrics as:

$$\tilde{K}_{ii} \sim \tilde{K}_{H\overline{H}} \sim \tilde{K}_0\left(1 + \frac{\delta\tau_s}{3\langle\tau_s\rangle} - \frac{2\delta\mathcal{V}}{3\langle\mathcal{V}\rangle}\right) \qquad \text{where} \qquad \tilde{K}_0 \equiv \frac{\langle\tau_s\rangle^{1/3}}{\langle\mathcal{V}\rangle^{2/3}}. \tag{6.30}$$

This allows us to define the canonically normalised fields $c_i = \sqrt{\tilde{K}_0}C_i$ and $h = \sqrt{\tilde{K}_0}H$ and rewrite the Lagrangian as:

$$\mathcal{L} \supset \left(1 + \frac{\delta\tau_s}{3\langle\tau_s\rangle} - \frac{2\delta\mathcal{V}}{3\langle\mathcal{V}\rangle}\right)\overline{c_i}i\gamma^\mu\partial_\mu c_i + \partial^\mu h\partial_\mu\overline{h} + \frac{\lambda}{\mathcal{V}\tilde{K}_0^{3/2}}\overline{c_i}c_ih. \tag{6.31}$$

Setting $h = \langle h\rangle$, expanding $1/\mathcal{V} \sim 1/\langle\mathcal{V}\rangle(1-\delta\mathcal{V}/\langle\mathcal{V}\rangle)$ and defining the fermionic mass as $m_c \equiv \frac{\lambda\langle h\rangle}{\langle\tau_s\rangle^{1/2}\tilde{K}_0^{3/2}}$ this becomes:

$$\mathcal{L} \supset \overline{c_i}(i\gamma^\mu\partial_\mu + m_c)c_i + \left(\frac{\delta\tau_s}{3\langle\tau_s\rangle} - \frac{2\delta\mathcal{V}}{3\langle\mathcal{V}\rangle}\right)\overline{c_i}(i\gamma^\mu\partial_\mu + m_c)c_i - m_c\overline{c_i}c_i\left(\frac{\delta\tau_s}{3\langle\tau_s\rangle} + \frac{\delta\mathcal{V}}{3\langle\mathcal{V}\rangle}\right). \tag{6.32}$$

Note that the first term is the standard Lagrangian for a free fermion of mass m_c, the second term vanishes once we impose the equations of motion and the third term encodes the interaction between moduli and Standard Model fermions. Expressing the moduli in terms of the canonically normalised fields near the minimum, we find that the dominant term in the interaction with the quintessence field comes from the expression (6.21) for $\delta\tau_s$ term, since to leading order $\delta\mathcal{V}$ does not depend on the canonically normalised fibre modulus. This implies that the leading contribution to the dimensionless coupling between ordinary fermions and the quintessence field is:

$$\lambda_{\phi\bar{c}c} \sim \mathcal{O}\left(\frac{m_c}{M_P \mathcal{V}^p}\right), \tag{6.33}$$

which is weaker-than-Planckian for $p \sim \mathcal{O}(1)$ like the coupling to Standard Model gauge bosons.

The fact that the couplings to gauge bosons and fermions are suppressed by a scale higher than the Planck mass means that the current model is able to comfortably accommodate the constraints from fifth-force experiments, see e.g. [11], and provide a controlled stringy description of quintessence.

6.4 Dynamics of the quintessence-dark matter system

In this Section we study the dynamics of the quintessence-dark matter system by following the autonomous phase plane analysis originally developed in [149]. We start by analysing a model in which we neglect any direct coupling between the two dominant components of the energy density. Then we look at the case where there is a direct, Planck strength coupling between dark matter and dark energy, while at the same time dark energy couples extremely weakly to ordinary matter. In this way, we will outline a proposal of how to realise a coupled quintessence scenario within a string inspired framework.

We consider a flat FRW Universe filled with quintessence as described by the fibre modulus and a pressureless dark matter component. The time evolution of the system is determined by the Raychaudhuri equation, the continuity equation for the dark matter component, the Klein-Gordon equation for a homogeneous scalar field and the Friedmann constraint:

$$\dot{H} = -\frac{1}{2M_P^2}\left(\rho_{DM} + \dot{\phi}^2\right), \tag{6.34}$$

$$\dot{\rho}_{DM} = -3H\rho_{DM} + Q, \tag{6.35}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \frac{Q}{\dot{\phi}}, \tag{6.36}$$

$$\Omega_{DM} + \Omega_{DE} = 1, \tag{6.37}$$

where $\Omega \equiv \frac{\rho}{3H^2M_P^2}$ and Q parameterises the direct interaction between the dark matter and the quintessence components. Using $N = \ln a$ as the time variable and defining

$$x \equiv \frac{\phi'}{M_P\sqrt{6}}, \qquad y^2 \equiv \frac{V}{3M_P^2H^2}, \tag{6.38}$$

the equations of motion can be cast in the following form:

$$x'(N) = -3x - \frac{\partial V/\partial\phi}{V}\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x(2x^2 + (1 - x^2 - y^2)) + \frac{Q}{6H^3M_P^2x}, \tag{6.39}$$

$$y'(N) = \frac{\partial V/\partial\phi}{V}\sqrt{\frac{3}{2}}xy + \frac{3}{2}y(2x^2 + (1 - x^2 - y^2)), \tag{6.40}$$

$$H'(N) = -\frac{3H}{2}(2x^2 + (1 - x^2 - y^2)), \tag{6.41}$$

$$\phi'(N) = \sqrt{6}x. \tag{6.42}$$

In what follows we will both search analytically for instantaneous fixed points of the system and study the phase space trajectories numerically.

6.4.1 Decoupled quintessence and dark matter

We first analyse the case where a direct coupling between dark matter and dark energy is negligible, which corresponds to setting $Q = 0$.

The system possesses two non trivial instantaneous fixed points, $x'(N) = y'(N) = 0$, whose properties are listed in Table 6.1. These correspond to motion in the two distinct regions of the potential: point $\mathcal{A}$ corresponds to motion in the poly-instanton dominated part of the potential (labelled II in Figure 6.1) whereas point $\mathcal{B}$ describes motion in the region where the loop potential dominates (labelled region III in Figure 6.1). Note that the flatness of the poly-instanton potential implies that $\delta \equiv V'_{\text{poly}}/V_{\text{poly}} \sim 0$, and so the behavior of the fibre modulus approaches that of a pure cosmological constant.

From the definitions of Eq. (6.38), one sees that $x(N)$ ($y(N)$) is the ratio of the scalar field's kinetic (potential) energy to the total energy. We illustrate the behaviour

Point	(x_c, y_c)	Stability	Ω_ϕ	ω_ϕ
$\mathcal{A}$	$(\delta, \sqrt{1-\delta^2})$	stable node	1	-1
$\mathcal{B}$	$(2/\sqrt{18}, \sqrt{14/18})$	stable node	1	$-5/9$

Table 6.1: Fixed points for the quintessence dark-matter system.

of the system in Figure 6.2 for different sets of initial conditions. The region above the parabola corresponds to accelerating solutions and the dots represent the two instantaneous fixed points described in Table 6.1.

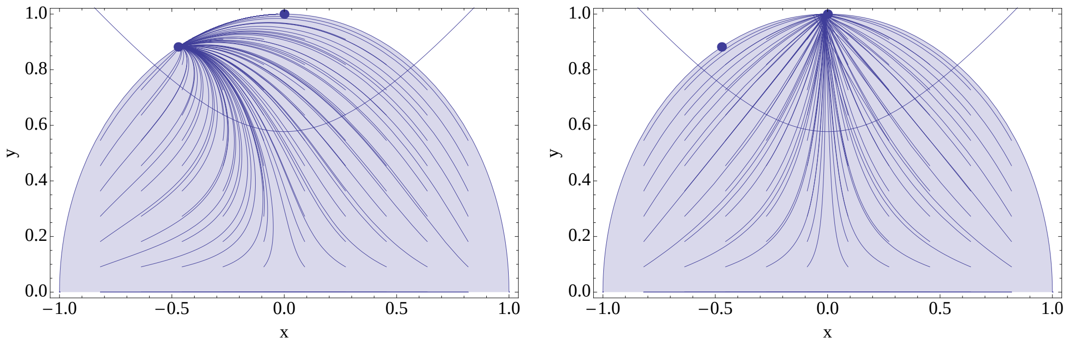

Figure 6.2: Phase plane evolution of the quintessence dark-matter system. Left: the fibre modulus starts in the loop dominated region of the potential; right: the fibre modulus starts in the poly-instanton dominated region of the potential.

We see that if the fibre modulus starts its evolution in the part of the potential that is dominated by the loop effects, the system will evolve towards the fixed point $\mathcal{B}$. This fixed point corresponds to an accelerated expansion solution, where the energy density is dominated by the scalar field, as preferred by current cosmological data. However the equation of state parameter, which is generically given by:

$$\omega_\phi = \frac{x-y}{x+y}, \tag{6.43}$$

is different from -1 for the fixed point $\mathcal{B}$. Thus, given the current observational constraints, this fixed point is not a viable candidate for dark energy. As the fibre modulus rolls down the loop generated potential it eventually enters a region in field space where the loop potential is suppressed relative to the poly-instanton generated potential. In this region the relevant fixed point to which the system will converge is $\mathcal{A}$, which corresponds to slow roll dynamics in the poly-instanton dominated part of the potential. The fixed point $\mathcal{A}$ describes a flat Universe filled with a negative pressure fluid with $\omega = -1$. Even though such trajectories display the same asymptotic behaviour in the future as the observed Universe, a numerical analysis of the system suggests they never go through a phase in which the conditions $\Omega_{DE} \sim 0.7$ and $\omega_\phi \sim -1$ are simultaneously verified. It then follows that trajectories that start with the quintessence field deep in the loop dominated region of potential do not provide a viable description of the late time Universe. On the other hand, if the initial conditions are such that the fibre modulus starts its evolution in a region of the potential where the loop effects are negligible, it will evolve towards the fixed point $\mathcal{A}$ without going through $\mathcal{B}$ first. As can be seen in Figure 6.2, these trajectories can go through the region of phase space where the energy density and the equation of state parameter of the quintessence field are in agreement with observational data. We then conclude that there are phase space trajectories for which the system evolves to a state where it provides a viable description of an accelerated expanding Universe, namely the ones corresponding to initial field values in the poly-instanton plateau. It is worth noting that from the discussion in Section 6.2 the width of the poly-instanton plateau is determined by the ratio $V_{\text{poly}}/V_{\text{loop}} \sim \mathcal{V}^{1-p}$ which can easily be $\gg 1$ given the large volumes required for quintessence phenomenology, $\mathcal{V} \sim 10^{30}$. There is therefore an appreciable range of initial conditions for the fibre modulus that yields a viable quintessential behaviour.

When looking at the late time composition of the Universe one is confronted with the coincidence problem: why is the dark energy density comparable to the dark matter density today? Dynamical models of dark energy often address this issue by allowing for scaling solutions: fixed points where the ratio of dark energy to dark matter density is constant. The existence of these scaling solutions is determined by the steepness of the dark-energy potential (V'/V) and by the equation of state for the matter component. In a Universe with pressureless cold dark matter and the fibre modulus playing the role of quintessence, there are no such fixed points as both regions of the quintessence potential are too shallow to support them. Therefore we are led to the conclusion that even though our model provides a realistic description of the current state of the Universe, it does not allow for a dynamical explanation of the coincidence problem: we just happen to be around at a time when Ω_{DE} and Ω_{DM} are of the same order and the Universe will eventually evolve to a state where it is filled with quintessence.

In these considerations about the distant future evolution of the system we have so far ignored the presence of the minimum of the potential, region I in Figure 6.1, to the left of the poly-instanton plateau. Its presence implies that in the very distant future the fibre modulus will eventually fall into its minimum, after which will follow a phase of oscillatory dynamics, in a process similar to post-inflationary reheating, albeit at a much lower energy scales.

6.4.2 Coupled quintessence and dark matter

We now analyse how a coupled quintessence-dark matter system affects the dynamics of the Universe [150]. Possible Planck suppressed couplings between dark energy and dark matter are allowed in our set-up if dark matter is constituted by bulk KK modes or moduli fields. In these highly anisotropic compactifications there are two kinds of KK modes: 6D KK modes and 10D KK modes. The 10D KK modes decay whereas the 6D KK modes are so long lived that can be considered as almost stable with respect to the age of the Universe [46]. Higher 6D KK states can also be produced from the thermal bath but they quickly decay to lower 6D KK states, which are then long-lived. There are also two light moduli which are stable with respect to the age of the Universe. One is the volume mode which is very light, with a mass of the order meV. Hence this modulus can be produced in the thermal bath or by simple scalar oscillation and so suffers from the cosmological moduli problem (CMP). The other light modulus is the fibre divisor which drives quintessence.

We then observe that in our scenario dark matter can be realised in terms of a mix of bulk 6D KK modes and the volume modulus, once the CMP associated with the latter is solved. The quintessence field would then indeed couple to dark matter, for example 6D KK modes, with Planck suppressed trilinear couplings of the form:

$$\frac{\psi}{M_P}\partial\phi\partial\phi\,, \tag{6.44}$$

where ϕ is the quintessence field and ψ the 6D KK mode.

After this qualitative discussion to motivate the coupling of dark energy to dark matter in our set-up, we can write the dark matter energy density as a function of the fibre modulus as:

$$\rho_{DM} = \rho_0 f(\phi), \tag{6.45}$$

where ρ_0 obeys the normal conservation equation:

$$\dot{\rho}_0 = -3H\rho_0, \tag{6.46}$$

and $f(\phi)$ parameterises the interaction between dark matter and the quintessence field. The conservation equation for dark matter is then given by:

$$\dot{\rho}_{DM} = -3H\rho_{DM} + \frac{\partial f}{\partial \phi}\frac{\dot{\phi}}{f(\phi)}\rho_{DM}. \tag{6.47}$$

By comparing Eqs. (6.35) and (6.47) we define $Q \equiv -\frac{\partial f}{\partial \phi}\frac{\dot{\phi}}{f(\phi)}\rho_{DM}$. In order to look for fixed points analytically, one must be able to write $\frac{\partial f}{f(\phi)\partial \phi}$ in terms of $x(N), y(N)$ and other dimensionless parameters. One simple case might be:

$$f(\phi) = e^{\alpha\phi/M_P} \,, \tag{6.48}$$

since this form for the coupling function implies that:

$$\frac{\partial f}{\partial \phi}\frac{\dot{\phi}}{f(\phi)}\rho_{DM} = \frac{\alpha}{M_P}\dot{\phi}\rho_{DM} \,, \tag{6.49}$$

allowing for an analytical study of the phase plane dynamics of the system. This is exactly what one expects to find if the dark matter mass is a function of the fibre modulus τ_f,

$$m_{DM} \sim \tau_f^m \sim e^{2m\phi/\sqrt{3}} \tag{6.50}$$

where in the last step we have written τ_f in terms of the canonically normalised field ϕ.

It then follows that $Q = -\alpha\sqrt{6}xH\rho_{DM}$ and Eq. (6.39) becomes:

$$x'(N) = -3x - \frac{\partial V/\partial\phi}{V}\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x(2x^2 + (1-x^2-y^2)) - \alpha\sqrt{3/2}(1-x^2-y^2). \tag{6.51}$$

The phase space for models with this form of the direct coupling Q has been studied in detail in [151], where a phenomenologically interesting fixed point located at $(x_c, y_c) = (\frac{\sqrt{6}}{2b}, \frac{\sqrt{9+6\alpha b}}{\sqrt{6}b})$ was found. Note that we have defined $\lambda \equiv -M_P\frac{\partial V}{V\partial\phi}$ and $b \equiv \lambda + \alpha$. Following [151] we find that this fixed point exists if:

$$b^2 \geq \frac{3}{2} \qquad \text{and} \qquad -\frac{3}{2} \leq \alpha b \leq b^2 - 3 \tag{6.52}$$

and is stable if:

$$b^2 \geq \frac{3}{2} \qquad \text{and} \qquad \left(-\frac{3}{2} < \alpha b < -B_{sgn(b)} \qquad \text{or} \qquad -B_{-sgn(b)} < \alpha b < b^2 - 3\right), \tag{6.53}$$

where

$$B_{\pm} = \frac{2b[\pm(b^2 - 3/2)^{3/2} - b(b^2 - 39/8)]}{4b^2 + 3/4}. \tag{6.54}$$

As in the decoupled case, we proceed by considering two different regimes, depending on which term in the potential dominates. For each case we shall then investigate whether the above conditions are met. In the regime in which the poly-instanton effects dominate over the loop potential (region II of Figure 6.1), $\lambda \sim 0$ to a very good accuracy. One sees that it is impossible to satisfy the existence conditions given by

Eq. (6.52). We then conclude that the poly-instanton potential is unable to support an interacting tracking solution. However if one looks at the regime where the loop generated potential dominates (region III of Figure 6.1), one has $\lambda = -2/\sqrt{3}$ and one sees that there are values of α for which a stable accelerating tracking solution exists. This region however is not very wide due to the requirement of stability. In fact we find that it is roughly $-\frac{5}{2\sqrt{3}} - 0.55 < \alpha < -\frac{5}{2\sqrt{3}}$. We must note that this is not so much a consequence of the shape of the potential as of the explicit form of Q we are assuming. Furthermore, we see that in this narrow stability region $0.93 < \Omega_{DE} < 1$, where the lower bound corresponds to $\alpha = -\frac{5}{2\sqrt{3}} - 0.55$ and the upper bound to $\alpha = -\frac{5}{2\sqrt{3}}$. It is worth noting that for $\alpha < 0$ the dark matter mass goes like $m_{DM} \propto 1/\tau_f^m$.

The behavior described above is illustrated in Figure 6.3, where it is clear that the presence of the interaction dramatically changes the trajectories in phase space. One also sees that the coupled system (magenta) converges to a fixed point which is different from the one of the decoupled case (blue lines).

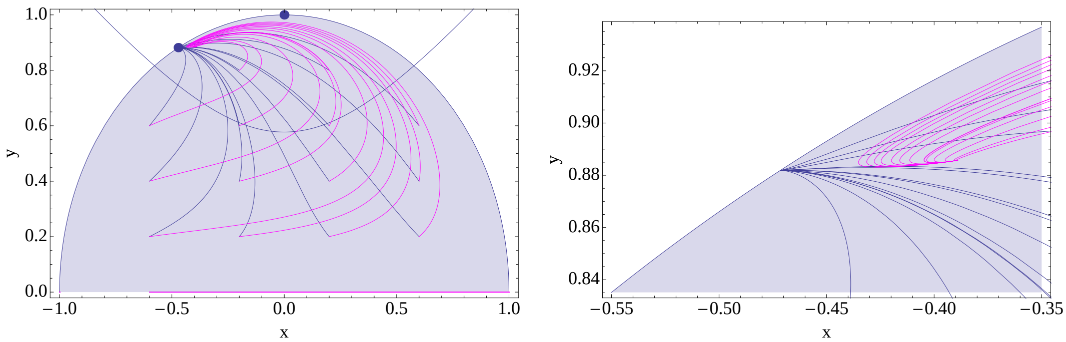

Figure 6.3: Phase plane evolution of the quintessence dark-matter system with interaction and $F_{\text{poly}} \gg F_{\text{loop}}$. Left: zoom into the region close to fixed point $\mathcal{B}$.

Once one considers the full potential, the fixed point described above is transient since as the field rolls down the loop generated potential it eventually enters the poly-instanton dominated region. So the phase-space trajectories start by converging to the fixed point $\mathcal{B}$ before evolving to the cosmological constant fixed point $\mathcal{A}$.

As in the non interacting case, the fibre modulus is able to provide suitable description of dark energy even though it does not address the coincidence problem. The introduction of the non minimal interaction term extends the existence of viable trajectories for cases when the field starts its evolution deep in the loop dominated part of the potential.

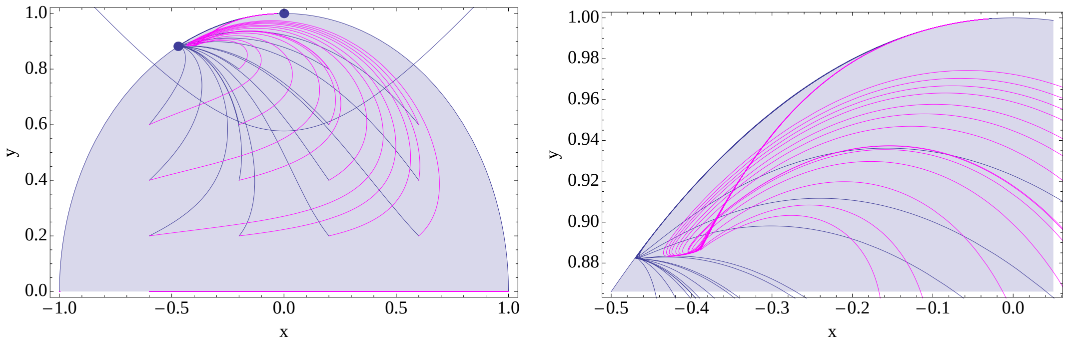

Figure 6.4: Phase plane evolution of the quintessence dark-matter system with interaction and $F_{\rm poly} \sim 10^3 F_{\rm loop}$. Left: zoom into the region close to fixed point $\mathcal{B}$.

6.5 Discussion

In this Chapter we presented a new quintessence model embedded in string theory where the mass of the rolling scalar field and its couplings to Standard Model degrees of freedom are functions of the volume of the internal space. Considering large volumes one can easily obtain a very light and weakly coupled scalar field suitable for quintessential physics.

We focused on a type IIB Large Volume Scenario with a very anisotropic shape of the extra dimensions since two of them are stabilised at values exponentially larger than the other four [46]. This leads to a string derivation of SLED scenarios [138–140] which have already turned out to be promising for obtaining models of quintessence [141]. In these scenarios, if TeV-scale gravity provides the solution to the hierarchy problem, the gravitino mass $m_{3/2}$ turns out to be of the observed order of magnitude of the cosmological constant: $m_{3/2} \sim \Lambda$. Therefore, the current value of dark energy Λ can be naturally reproduced if the quintessence field develops a potential at order $V \sim m_{3/2}^4$.

This mechanism, which ties the solution of the hierarchy problem to the one of the cosmological constant problem, relies on the existence of a compensation between the SM brane tension and its back-reaction. This issue is vital for the success of the current model and we have assumed it could be solved along the lines of [144].

In this Chapter, we showed that the fibre divisor τ_f indeed develops a potential at order $V \sim m_{3/2}^4$ via tiny poly-instanton corrections to the superpotential and closed string loop contributions to the Kähler potential. Furthermore, this field is radiatively stable since the bulk is nearly supersymmetric.

We also showed that this quintessence model can evade the severe bounds coming from fifth-force experiments even if we are dealing with a scalar. The reason is the fact that the rigid four-cycle supporting the Standard Model does not intersect the fibre divisor, and so the quintessence scalar couples to ordinary matter only via its mixing with the volume mode which, in turn, induces its mixing with the SM cycle. However due to the fact that the fibre modulus is a flat direction at leading order which is lifted only at subleading order, the mixing between τ_f and $\mathcal{V}$ is suppressed, resulting in a weaker-than-Planckian coupling of the quintessence field to SM particles.

On the other hand, if dark matter is realised in terms of quasi-stable Kaluza-Klein states, direct couplings between dark energy and dark matter are allowed, leading to a scenario of coupled quintessence. We studied in detail the dynamics of the quintessence scalar in our set-up, investigating the nature of fixed points and the late time evolution of dark energy, showing that the main features of our scenario are compatible with observations.

To conclude, the model analysed here explicitly shows that concrete string theory constructions are able to lead to scalar theories of quintessence that, besides generating naturally flat scalar potentials, also provide couplings to Standard Model matter that are weak enough to satisfy present constraints on fifth forces.

Chapter 7

Conclusions and Outlook

In this Thesis we have studied some phenomenological aspects of Type IIB flux compactifications.

In Chapter 1 we have given a brief overview of the present day knowledge of physics both at high energies and on cosmological scales. We motivated the need to go beyond the standard model both in particle physics and cosmology in order to clarify some of the outstanding issues that remain unexplained both in the realm of the very small and of the very large.

We introduced in Chapter 2 the physics and geometry of flux compactifications which form the backbone of this Thesis. We sketched how the requirement of $\mathcal{N} = 1$ supersymmetry in four dimensions forces us to compactify the ten dimensional type IIB theory on Calabi-Yau orientifolds. Following the pioneering work of [25] we have illustrated how fluxes in the compact space can be used to stabilise some of the moduli. We have shown how the systematic inclusion of subleading corrections to the action of the moduli fields [26, 28, 29] allows for the stabilisation of all the moduli fields, which ultimately leads to reliable and quasi-realistic compactifications to four dimensions. We have also looked at the issue of supersymmetry breaking in particular the generation of soft terms in the visible sector due to the breaking of supersymmetry in the hidden/moduli sector.

The original work developed for this Thesis is presented in Chapters 3-6.

In Chapter 3 we have studied how the redefinition of the Kähler moduli affects the stabilisation mechanism and the breaking of supersymmetry. Redefinitions of the blow-up moduli were shown not to affect the existence and position of the minimum of the potential. The same cannot be said about redefinitions of the volume mode in Swiss cheese type geometries since the new terms, generated by the redefinition of the volume, dominate over the standard large volume scenario potential. We have also shown how the redefinition of the small modulus can lead to soft terms of the order of the gravitino mass.

Chapter 4 dealt with the issue of moduli destabilisation by very dense objects and in cosmological singularities. Even though this is a somewhat speculative topic, it could potentially be very rewarding since destabilised moduli would provide a unique window into the landscape. Through the study of the four dimensional field theory we have found that even the densest known astrophysical objects are unable to have an appreciable effect on the moduli potential and therefore do not destabilise the moduli fields. The requirement of higher energy densities leads us to the study of systems undergoing gravitational collapse such as the formation of black holes or positively curved Universes. We found that the gravitational collapse in a closed FRW universe would lead to super-inflationary decompactification of the internal space, and that during the collapse of a black hole there would be a region outside the horizon that would effectively be ten rather than four dimensional.

Chapter 5 and 6 deal with different phenomenological applications of the same fibered compactification. The novel ingredients are poly-instanton corrections to the superpotential. When combined with single instantonic contributions to the superpotential and higher derivative contributions to the Kähler potential, the poly-instantons allow to stabilise all the geometric moduli of the fibered compactification.

A model of inflation in the closed string sector was built in Chapter 5, with the fibre modulus playing the role of inflaton. In this context the inflationary potential is generated at subleading order by poly-instanton effects in the superpotential. We have shown that this model can accommodate the current observational constraints, provided the volume of the internal space is of the order of $10^3 l_s^6$. Furthermore we have seen that the corrections to the inflaton's potential coming from the UV physics can be tamed leading to a robust model of stringy inflation.

Given the similarities between the physics of inflation and that of quintessence, we have proposed in Chapter 6 a model of quintessence in a string theory. From the inflationary study of Chapter 5 we knew that the fibre modulus with a poly-instanton generated potential could provide a description of accelerated expansion and by allowing the volume of the internal space to be of the order of $10^{30} l_s^6$ we can have this expansion happening at the scale of dark energy. This however is not sufficient to have a controlled quintessence model, since light Planck coupled degrees of freedom give rise to problematic fifth forces. We have shown how this issue is addressed in our model, by having sub-Planckian couplings between the quintessence field and the standard model degrees of freedom. We have studied the dynamics of the resulting system and shown that it can broadly describe the current state of the Universe even though it is unable to address the "Why now?" question.

Fortunately there is a lot more work to be done.

Some ideas for the immediate future include an improved study of the back hole collapse dynamics, taking into consideration the dynamics of the metric. This will allow to test the robustness of the simple case treated in Chapter 4 and look for possible signals of decompactification.

In the context of the fibre inflation model discussed in Chapter 5 we have set the axion associated with the fibre modulus to its minimum. Given that the mass of this axion is comparable to that of the fibre modulus, an analysis of the fully dynamic system would be interesting. This would work as a test of the robustness of the results presented in Chapter 5 and would also allow us to look for different inflationary signatures.

In the model of Chapter 5, as in most models of inflation in supergravity/string theory, there is tension between the supersymmetry breaking scale and the inflationary scale: it is in general hard to get inflation around the Unification scale and supersymmetry around the TeV scale. Given that inflation in the Kähler moduli sector of type IIB string compactifications provides a very economical and elegant way of embedding inflation in a controlled string theory setup, it would be interesting to reconcile it with low energy supersymmetry. A possibility worth looking into would be having the dynamical dilaton varying between the inflationary epoch and today.

I plan to address these issues in the near future.

Appendix A

Canonical Normalisation

A.1 Swiss-cheese compactifications

The theory for the Kähler moduli sector is determined by the Lagrangian

$$\mathcal{L} = K_{i\bar{j}}\partial T_i \partial \bar{T}_j - V(T,\bar{T}) = \frac{1}{4}\frac{\partial^2 K}{\partial \tau_i \partial \tau_j}(\partial\tau_i\partial\tau_j + \partial b_i \partial b_j) - V(\tau, b) \tag{A.1}$$

where we have used the fact that $K_{i\bar{j}} \equiv \partial^2 K/\partial T_i \partial \bar{T}_j$ and $T_i = \tau_i + i b_i$. We are interested in finding the mass spectrum near the minimum of the large volume scenario potential, so we expand $\tau_i = \langle \tau_i \rangle + \delta\tau_i$ and $b_i = \langle b_i \rangle + \delta b_i$. The F-term potential can then be expanded to quadratic order in the δ's as

$$V(\tau, b) = \langle V \rangle + \frac{1}{2}\left\langle \frac{\partial^2 V}{\partial\tau_i\partial\tau_j}\right\rangle \delta\tau_i\delta\tau_j + \frac{1}{2}\left\langle \frac{\partial^2 V}{\partial b_i \partial b_j}\right\rangle \delta b_i \delta b_j + \left\langle \frac{\partial^2 V}{\partial b_i \partial \tau_j}\right\rangle \delta b_i \delta\tau_j \tag{A.2}$$

Defining the vector $\delta\psi \equiv (\delta\tau_i, \delta b_i)^T$ we can write the Lagrangian for the Kähler moduli sector as

$$\mathcal{L} = \frac{1}{2}\partial\delta\psi^T \mathbb{K} \partial\delta\psi - \langle V \rangle - \frac{1}{2}\delta\psi^T \mathbb{H} \delta\psi \tag{A.3}$$

where we defined

$$\mathbb{K} = \begin{pmatrix} \frac{1}{2}\frac{\partial^2 K}{\partial\tau_i\partial\tau_j} & 0 \\ 0 & \frac{1}{2}\frac{\partial^2 K}{\partial\tau_i\partial\tau_j} \end{pmatrix} \qquad \text{and} \qquad \mathbb{H} \equiv \begin{pmatrix} \langle\partial_{\tau\tau}V\rangle & \langle\partial_{\tau b}V\rangle \\ \langle\partial_{b\tau}V\rangle & \langle\partial_{bb}V\rangle \end{pmatrix} \tag{A.4}$$

Further introducing the transformation matrix U such that

$$(U^T)^{-1}\mathbb{K}U^{-1} = \mathbb{I} \qquad \text{and} \qquad m^2 = diag(m_i^2) = U\mathbb{K}^{-1}\mathbb{H}U^{-1}, \tag{A.5}$$

we see that U is the matrix that diagonalises the mass matrix $\mathbb{M}^2 \equiv \mathbb{K}^{-1}\mathbb{H}$ and that $\delta\phi$ defined by $U\delta\psi \equiv \delta\phi$ are the canonically normalised degrees of freedom in the vicinity of the minimum. The Lagrangian then takes the simple form

$$\mathcal{L} = \frac{1}{2}\partial\delta\phi^T\partial\delta\phi - \langle V \rangle - \frac{1}{2}\delta\phi^T m^2 \delta\phi. \tag{A.6}$$

In order to get the mass of the Kähler moduli and their respective axions all one needs to do is to specify the geometry of the compact space by writing the volume as a function of the moduli and to compute the eigenvalues of $\mathbb{M}^2$. For the simplest case of the two moduli Swiss cheese manifold presented above we find that

$$\mathbb{K}^{-1} = \begin{pmatrix} \frac{4\tau_b^2}{3} & 4\tau_b\tau_s & 0 \\ 4\tau_b\tau_s & \frac{8}{3}\sqrt{\tau_b^3\tau_s} & 0 \\ 0 & 0 & \frac{8}{3}\sqrt{\tau_b^3\tau_s} \end{pmatrix} \tag{A.7}$$

and that the Hessian matrix is given by

$$\mathbb{H} = \begin{pmatrix} \frac{9(1+2\epsilon)\nu W_0^2}{2\tau_b^{13/2}} & -\frac{3\left(1-5\epsilon+4\epsilon^2\right)\nu a_s W_0^2}{\tau_b^{11/2}} & 0 \\ -\frac{3\left(1-5\epsilon+4\epsilon^2\right)\nu a_s W_0^2}{\tau_b^{11/2}} & \frac{2\left(1-3\epsilon+6\epsilon^2\right)\nu a_s^2 W_0^2}{\tau_b^{9/2}} & 0 \\ 0 & 0 & -\frac{2(-1+\epsilon)\nu a_s^2 W_0^2}{\tau_b^{9/2}} \end{pmatrix}. \tag{A.8}$$

In computing Eq. (A.8) we have re-instated the axion dependence in the second term of the potential, Eq. (2.115) and set all the fields to their minima. We have defined $\epsilon \equiv \frac{1}{4a_s\tau_s} \ll 1$ and $\nu \equiv \frac{3\xi}{4g_s^{3/2}}$. We then find that the mass matrix can be written as an expansion in $\mathcal{V}^{-1}$ and ϵ as

$$\mathbb{M}^2 = \frac{2\nu a_s W_0^2 \tau_s}{3\tau_b^{9/2}} \begin{pmatrix} -9+63\epsilon & 6\left(1-5\epsilon+16\epsilon^2\right)a_s\tau_b & 0 \\ -6\left(1-5\epsilon+4\epsilon^2\right)\sqrt{\frac{\tau_b}{\tau_s}} & \frac{4\left(1-3\epsilon+6\epsilon^2\right)a_s\tau_b^{3/2}}{\sqrt{\tau_s}} & 0 \\ 0 & 0 & 8(1-\epsilon)\epsilon a_s^2\sqrt{\frac{\tau_b^3}{\epsilon a_s}} \end{pmatrix} \tag{A.9}$$

where we assume all fields are evaluated at the respective minima.

The eigenvalues are given by

$$m_{\chi_s}^2 = \frac{8W_0^2 a_s^2 \nu}{3}\frac{\sqrt{\langle\tau_s\rangle}}{\langle\mathcal{V}\rangle^2}, \qquad m_{\theta_s}^2 = \frac{8W_0^2 a_s^2 \nu}{3}\frac{\sqrt{\langle\tau_s\rangle}}{\langle\mathcal{V}\rangle^2}$$
$$m_{\chi_b}^2 = \frac{27}{4}\frac{W_0^2\nu}{a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}, \qquad m_{\theta_b}^2 = 0 \tag{A.10}$$

where ξ and θ denote the canonically normalised moduli and axions respectively. These can be related to τ and b by explicitly constructing the matrix U with the eigenvectors of $\mathbb{M}$ [129].

A.2 Fibered compactifications

In this section we compute the canonical normalisation for the fibered compactifications of Chapters 5 and 6.

The Lagrangian for the quadratic field fluctuations around their minima, $\tau_i = \langle\tau_i\rangle + \delta\tau_i$, takes the form:

$$\mathcal{L} = K_{ij}\partial_\mu\delta\tau_i\partial^\mu\delta\tau_i - \langle V\rangle - \frac{1}{2}V_{ij}\delta\tau_i\delta\tau_j\,, \tag{A.11}$$

where K_{ij} is the Kähler matrix, and V_{ij} the matrix of second derivatives of the potential. The leading terms in the Kähler matrix K_{ij} look like:

$$K_{ij} \simeq \begin{pmatrix} \frac{3}{8\tau_f^2} & -\frac{1}{4\mathcal{V}\tau_f} & -\frac{3\sqrt{\tau_s}}{8\mathcal{V}\tau_f} \\ -\frac{1}{4\mathcal{V}\tau_f} & \frac{1}{2\mathcal{V}^2} & 0 \\ -\frac{3\sqrt{\tau_s}}{8\mathcal{V}\tau_f} & 0 & \frac{3}{8\mathcal{V}\sqrt{\tau_s}} \end{pmatrix}, \tag{A.12}$$

where we used $(\tau_f, \mathcal{V}, \tau_s)$ as coordinates of the Kähler moduli space. We write the original moduli in terms of the canonically normalised fields around their minima as:

$$\begin{pmatrix} \delta\tau_f \\ \delta\mathcal{V} \\ \delta\tau_s \end{pmatrix} = \mathcal{C}\begin{pmatrix} \delta\phi \\ \delta\chi_{\mathcal{V}} \\ \delta\chi_s \end{pmatrix}, \tag{A.13}$$

where $\mathcal{C}$ is the matrix that diagonalises both the kinetic and the mass terms. It can be written as:

$$\mathcal{C} = \begin{pmatrix} \vec{v}_{(1)} & \vec{v}_{(2)} & \vec{v}_{(3)} \end{pmatrix}, \tag{A.14}$$

where $\vec{v}_{(i)}$ are the eigenvectors of the mass matrix $M_{ik} = \frac{1}{2}\,K_{ij}^{-1}V_{jk}$, normalised such that $\mathcal{C}^T_{ik}\,K_{km}\,\mathcal{C}_{mj} = \delta_{ij}$. The mass matrix reads explicitly:

$$M_{ij} = \frac{1}{2}\begin{pmatrix} -\frac{4\tau_f(f_8+2f_6\tau_f+2f_7\tau_s)\mathcal{V}}{\mathcal{V}^{3+p}\left(3\tau_s{}^{3/2}-2\mathcal{V}\right)} & -\frac{4(f_3+2f_4)\tau_f\sqrt{\tau_s}}{\left(3\tau_s{}^{3/2}-2\mathcal{V}\right)\mathcal{V}^3} & -\frac{4(f_4+2f_5)\tau_f}{3\tau_s{}^2\mathcal{V}^2-2\sqrt{\tau_s}\mathcal{V}^3} \\ -\frac{2\mathcal{V}\left(2(f_6\tau_f+f_7\tau_s)\mathcal{V}+3f_8\left(-\tau_s{}^{3/2}+\mathcal{V}\right)\right)}{\mathcal{V}^{3+p}\left(3\tau_s{}^{3/2}-2\mathcal{V}\right)} & \frac{6f_3\tau_s{}^2-2(3f_3+2f_4)\sqrt{\tau_s}\mathcal{V}}{\left(3\tau_s{}^{3/2}-2\mathcal{V}\right)\mathcal{V}^3} & \frac{6f_4\left(\tau_s{}^{3/2}-\mathcal{V}\right)-4f_5\mathcal{V}}{3\tau_s{}^2\mathcal{V}^2-2\sqrt{\tau_s}\mathcal{V}^3} \\ -\frac{4\left(3(f_8+2f_6\tau_f)\tau_s\mathcal{V}+4f_7\sqrt{\tau_s}\mathcal{V}^2\right)}{\mathcal{V}^{3+p}\left(9\tau_s{}^{3/2}-6\mathcal{V}\right)} & \frac{4\left(3f_3\tau_s{}^{3/2}+4f_4\mathcal{V}\right)}{3\mathcal{V}^3\left(-3\tau_s{}^{3/2}+2\mathcal{V}\right)} & -\frac{4\left(3f_4\tau_s{}^{3/2}+4f_5\mathcal{V}\right)}{3\mathcal{V}^2\left(3\tau_s{}^{5/2}-2\tau_s\mathcal{V}\right)} \end{pmatrix}. \tag{A.15}$$

The quantities f_i are independent from the volume at leading order in an inverse volume expansion (see [15] for more details). In writing the previous mass matrix, we added to the leading part of the potential, Eq. (5.7), the subleading contribution due to polyinstanton corrections, Eq. (5.8).

Notice that, as expected, the first column of the mass matrix is suppressed by a factor $1/\mathcal{V}^{3+p}$, since it is due to the subleading poly-instanton potential. In other words, the first column of the matrix (A.15) would be zero in the absence of the subleading poly-instanton contribution.

It is straightforward, but long, to extract the eigenvalues and the eigenvectors of the mass matrix of Eq. (A.15), working at leading order in an expansion in inverse powers of the volume. The expressions of the eigenvectors turn out to be particularly complicated. It is convenient to present them at leading order in an expansion in the quantity τ_s, that we can assume to be small with respect to the other quantities. We find:

$$\mathcal{C} = \begin{pmatrix} \frac{2\tau_f}{\sqrt{3}} & \sqrt{\frac{2}{3}}\tau_f & 0 \\ \frac{c_0}{\mathcal{V}^{p-1}} & \sqrt{\frac{3}{2}}\mathcal{V} & -\tau_s{}^{3/4}\sqrt{3\mathcal{V}} \\ \frac{c_1}{\mathcal{V}^p} & \sqrt{6}\tau_s & \frac{2\tau_s{}^{1/4}\sqrt{\mathcal{V}}}{\sqrt{3}} \end{pmatrix}. \tag{A.16}$$

The two last entries in the first column are suppressed by powers of the volume, and are due to the effect of the poly-instanton corrections. The quantities c_0 and c_1 are independent from the volume (at leading order in a inverse volume expansion) and read:

$$c_0 = -\frac{\tau_s^{\frac{p}{2}-1}}{2e\pi W_0} 3^{-\frac{3}{2}+p}(-2aAC_1\pi + bB(c(-C_1+C_2)+2C_1\pi)) \left(\frac{W_0}{bB-aA}\right)^{1+p} \tag{A,17}$$

$$c_1 = \frac{4\,\tau_s^{\frac{p}{2}}\, 3^{-\frac{3}{2}+p}(2aAC_1\pi + bB(c(C_1-C_2)-2C_1\pi)) \left(\frac{W_0}{-aA+bB}\right)^p}{(aA-bB)e\pi}. \tag{A.18}$$

The lightest eigenvalue of the mass matrix scales as $m_\phi^2 \propto 1/\mathcal{V}^{3+p}$, and provides the effective mass for the quintessence field in the poly-instanton dominated region of the quintessence potential (region II of figure 6.1).

In this region, the volume and τ_s are stabilised at their minima since their mass is very large. From Eqs (A.13) and (A.16), one obtains that the displacement of the light field τ_f from its minimum can be expressed in terms of its canonical counterpart as:

$$\delta\tau_f \simeq \frac{2}{\sqrt{3}}\tau_f\,\delta\phi \qquad \Rightarrow \qquad \tau_f \simeq e^{\frac{2}{\sqrt{3}}\phi}, \tag{A.19}$$

justifying the canonical normalisation we used in Eq. (6.15). The diagonalising matrix (A.16), when compared with (A.13), also tells us that, within the approximations we are using, we can write the modulus τ_s in terms of canonically normalised fields as:

$$\frac{\delta\tau_s}{\langle\tau_s\rangle} \simeq \frac{c_1}{\mathcal{V}^p}\,\delta\phi + \sqrt{6}\,\delta\chi_{\mathcal{V}} + \frac{2\sqrt{\mathcal{V}}}{\sqrt{3}\langle\tau_s\rangle^{3/4}}\,\delta\chi_s\,, \tag{A.20}$$

reproducing the scalings of Eq. (6.21).

Bibliography

[1] Albert Einstein. The Foundation of the General Theory of Relativity. *Annalen Phys.*, 49:769–822, 1916.

[2] N. Jarosik, C.L. Bennett, J. Dunkley, B. Gold, M.R. Greason, et al. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Sky Maps, Systematic Errors, and Basic Results. *Astrophys.J.Suppl.*, 192:14, 2011.

[3] Claude Amsler et al. Review of Particle Physics. *Phys.Lett.*, B667:1–1340, 2008.

[4] Nima Arkani-Hamed, Savas Dimopoulos, and G.R. Dvali. The Hierarchy problem and new dimensions at a millimeter. *Phys.Lett.*, B429:263–272, 1998.

[5] Michael B. Green, J.H. Schwarz, and Edward Witten. Superstring theory. Vol. 1: Introduction. 1987.

[6] Michael B. Green, J.H. Schwarz, and Edward Witten. Superstring theory. Vol.2: Loop amplitudes, anomalies and phenomenology. 1987.

[7] K. Becker, M. Becker, and J.H. Schwarz. String theory and M-theory: A modern introduction. 2007.

[8] Luis E. Ibanez and Angel M. Uranga. String theory and particle physics: An introduction to string phenomenology. 2012.

[9] Theodor Kaluza. On the Problem of Unity in Physics. *Sitzungsber.Preuss.Akad.Wiss.Berlin (Math.Phys.)*, 1921:966–972, 1921.

[10] O. Klein. Quantum Theory and Five-Dimensional Theory of Relativity. (In German and English). *Z.Phys.*, 37:895–906, 1926.

[11] E. G. Adelberger, Blayne R. Heckel, and A. E. Nelson. Tests of the gravitational inverse-square law. *Ann. Rev. Nucl. Part. Sci.*, 53:77–121, 2003.

[12] Joseph P. Conlon and Francisco G. Pedro. Moduli Redefinitions and Moduli Stabilisation. *JHEP*, 1006:082, 2010.

[13] Joseph P. Conlon and Francisco G. Pedro. Moduli-Induced Vacuum Destabilisation. *JHEP*, 1105:079, 2011.

[14] Michele Cicoli, Francisco G. Pedro, and Gianmassimo Tasinato. Poly-instanton Inflation. *JCAP*, 1112:022, 2011.

[15] Michele Cicoli, Francisco G. Pedro, and Gianmassimo Tasinato. Natural Quintessence in String Theory. 2012.

[16] Peter G.O. Freund and Mark A. Rubin. Dynamics of Dimensional Reduction. *Phys.Lett.*, B97:233–235, 1980.

[17] Kasper Peeters. A Field-theory motivated approach to symbolic computer algebra. *Comput.Phys.Commun.*, 176:550–558, 2007.

[18] Kasper Peeters. Introducing Cadabra: A Symbolic computer algebra system for field theory problems. 2007.

[19] Philip Candelas and Xenia de la Ossa. MODULI SPACE OF CALABI-YAU MANIFOLDS. *Nucl.Phys.*, B355:455–481, 1991.

[20] Brian R. Greene. String theory on Calabi-Yau manifolds. pages 543–726, 1996.

[21] P. Candelas, Gary T. Horowitz, Andrew Strominger, and Edward Witten. Vacuum Configurations for Superstrings. *Nucl.Phys.*, B258:46–74, 1985.

[22] Robert Bohm, Holger Gunther, Carl Herrmann, and Jan Louis. Compactification of type IIB string theory on Calabi-Yau threefolds. *Nucl.Phys.*, B569:229–246, 2000.

[23] Thomas W. Grimm and Jan Louis. The effective action of N = 1 Calabi-Yau orientifolds. *Nucl. Phys.*, B699:387–426, 2004.

[24] Sergei Gukov, Cumrun Vafa, and Edward Witten. CFT's from Calabi-Yau four-folds. *Nucl. Phys.*, B584:69–108, 2000. [Erratum-ibid.B608:477-478,2001].

[25] Steven B. Giddings, Shamit Kachru, and Joseph Polchinski. Hierarchies from fluxes in string compactifications. *Phys. Rev.*, D66:106006, 2002.

[26] Katrin Becker, Melanie Becker, Michael Haack, and Jan Louis. Supersymmetry breaking and alpha'-corrections to flux induced potentials. *JHEP*, 06:060, 2002.

[27] Michele Cicoli, Joseph P. Conlon, and Fernando Quevedo. Systematics of String Loop Corrections in Type IIB Calabi- Yau Flux Compactifications. *JHEP*, 01:052, 2008.

[28] Shamit Kachru, Renata Kallosh, Andrei D. Linde, and Sandip P. Trivedi. De Sitter vacua in string theory. *Phys. Rev.*, D68:046005, 2003.

[29] Vijay Balasubramanian, Per Berglund, Joseph P. Conlon, and Fernando Quevedo. Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications. *JHEP*, 03:007, 2005.

[30] B. de Wit, D.J. Smit, and N.D. Hari Dass. Residual Supersymmetry of Compactified D=10 Supergravity. *Nucl.Phys.*, B283:165, 1987.

[31] Juan Martin Maldacena and Carlos Nunez. Supergravity description of field theories on curved manifolds and a no go theorem. *Int.J.Mod.Phys.*, A16:822–855, 2001.

[32] Ignatios Antoniadis, S. Ferrara, R. Minasian, and K.S. Narain. R**4 couplings in M and type II theories on Calabi-Yau spaces. *Nucl.Phys.*, B507:571–588, 1997.

[33] Joseph P. Conlon, Fernando Quevedo, and Kerim Suruliz. Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking. *JHEP*, 08:007, 2005.

[34] Marcus Berg, Michael Haack, and Boris Kors. On volume stabilization by quantum corrections. *Phys. Rev. Lett.*, 96:021601, 2006.

[35] Marcus Berg, Michael Haack, and Boris Kors. String loop corrections to Kaehler potentials in orientifolds. *JHEP*, 11:030, 2005.

[36] Marcus T. Grisaru, W. Siegel, and M. Rocek. Improved Methods for Supergraphs. *Nucl. Phys.*, B159:429, 1979.

[37] Nathan Seiberg. Naturalness Versus Supersymmetric Non-renormalization Theorems. *Phys. Lett.*, B318:469–475, 1993.

[38] Nathan Seiberg. The Power of holomorphy: Exact results in 4-D SUSY field theories. 1994.

[39] Edward Witten. Nonperturbative superpotentials in string theory. *Nucl.Phys.*, B474:343–360, 1996.

[40] Lars Gorlich, Shamit Kachru, Prasanta K. Tripathy, and Sandip P. Trivedi. Gaugino condensation and nonperturbative superpotentials in flux compactifications. *JHEP*, 0412:074, 2004.

[41] M. Cicoli and F. Quevedo. String moduli inflation: an overview. *Class. Quant. Grav.*, 28:204001, 2011.

[42] Joseph P. Conlon and Fernando Quevedo. On the explicit construction and statistics of Calabi-Yau flux vacua. *JHEP*, 0410:039, 2004.

[43] Konstantin Bobkov, Volker Braun, Piyush Kumar, and Stuart Raby. Stabilizing All Kahler Moduli in Type IIB Orientifolds. *JHEP*, 1012:056, 2010.

[44] Joseph P. Conlon. Quantum Gravity Constraints on Inflation. 2012.

[45] Michele Cicoli, Joseph P. Conlon, and Fernando Quevedo. General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation. *JHEP*, 10:105, 2008.

[46] M. Cicoli, C. P. Burgess, and F. Quevedo. Anisotropic Modulus Stabilisation: Strings at LHC Scales with Micron-sized Extra Dimensions. *JHEP*, 10:119, 2011.

[47] C. P. Burgess et al. Non-standard primordial fluctuations and nongaussianity in string inflation. *JHEP*, 08:045, 2010.

[48] Tom Banks, David B. Kaplan, and Ann E. Nelson. Cosmological implications of dynamical supersymmetry breaking. *Phys.Rev.*, D49:779–787, 1994.

[49] Lisa Randall and Scott D. Thomas. Solving the cosmological moduli problem with weak scale inflation. *Nucl.Phys.*, B449:229–247, 1995.

[50] Sanjeev K. Soni and H. Arthur Weldon. Analysis of the Supersymmetry Breaking Induced by N=1 Supergravity Theories. *Phys.Lett.*, B126:215, 1983. Revised version.

[51] G.F. Giudice and A. Masiero. A Natural Solution to the mu Problem in Supergravity Theories. *Phys.Lett.*, B206:480–484, 1988.

[52] A. Brignole, Luis E. Ibanez, and C. Munoz. Soft supersymmetry-breaking terms from supergravity and superstring models. 1997.

[53] Joseph P. Conlon, Shehu S. Abdussalam, Fernando Quevedo, and Kerim Suruliz. Soft SUSY Breaking Terms for Chiral Matter in IIB String Compactifications. *JHEP*, 0701:032, 2007.

[54] R. Blumenhagen, J. P. Conlon, S. Krippendorf, S. Moster, and F. Quevedo. SUSY Breaking in Local String/F-Theory Models. *JHEP*, 09:007, 2009.

[55] Joseph P. Conlon, Daniel Cremades, and Fernando Quevedo. Kaehler potentials of chiral matter fields for Calabi-Yau string compactifications. *JHEP*, 01:022, 2007.

[56] Ralph Blumenhagen. Gauge Coupling Unification in F-Theory Grand Unified Theories. *Phys.Rev.Lett.*, 102:071601, 2009.

[57] Ralph Blumenhagen, Sebastian Moster, and Erik Plauschinn. Moduli Stabilisation versus Chirality for MSSM like Type IIB Orientifolds. *JHEP*, 01:058, 2008.

[58] Frederik Denef, Michael R. Douglas, and Bogdan Florea. Building a better racetrack. *JHEP*, 06:034, 2004.

[59] Ralph Blumenhagen, Volker Braun, Thomas W. Grimm, and Timo Weigand. GUTs in Type IIB Orientifold Compactifications. *Nucl. Phys.*, B815:1–94, 2009.

[60] Andres Collinucci, Maximilian Kreuzer, Christoph Mayrhofer, and Nils-Ole Walliser. Four-modulus 'Swiss Cheese' chiral models. *JHEP*, 07:074, 2009.

[61] Michele Cicoli. String Loop Moduli Stabilisation and Cosmology in IIB Flux Compactifications. *Fortsch. Phys.*, 58:115–338, 2010.

[62] Marcus Berg, Michael Haack, and Enrico Pajer. Jumping Through Loops: On Soft Terms from Large Volume Compactifications. *JHEP*, 09:031, 2007.

[63] Aalok Misra and Pramod Shukla. Area Codes, Large Volume (Non-)Perturbative alpha'- and Instanton - Corrected Non-supersymmetric (A)dS minimum, the Inverse Problem and Fake Superpotentials for Multiple- Singular-Loci-Two-Parameter Calabi-Yau's. *Nucl. Phys.*, B799:165–198, 2008.

[64] Eran Palti, Gianmassimo Tasinato, and John Ward. WEAKLY-coupled IIA Flux Compactifications. *JHEP*, 06:084, 2008.

[65] Joseph P. Conlon. Gauge Threshold Corrections for Local String Models. *JHEP*, 04:059, 2009.

[66] Joseph P. Conlon and Eran Palti. Gauge Threshold Corrections for Local Orientifolds. *JHEP*, 09:019, 2009.

[67] Ignatios Antoniadis, C. Bachas, and E. Dudas. Gauge couplings in four-dimensional type I string orbifolds. *Nucl. Phys.*, B560:93–134, 1999.

[68] Nikolas Akerblom, Ralph Blumenhagen, Dieter Lust, and Maximilian Schmidt-Sommerfeld. Instantons and Holomorphic Couplings in Intersecting D- brane Models. *JHEP*, 08:044, 2007.

[69] L. Aparicio, D. G. Cerdeno, and L. E. Ibanez. Modulus-dominated SUSY-breaking soft terms in F-theory and their test at LHC. *JHEP*, 07:099, 2008.

[70] Justin Khoury and Amanda Weltman. Chameleon fields: Awaiting surprises for tests of gravity in space. *Phys.Rev.Lett.*, 93:171104, 2004.

[71] Ph. Brax, Carsten van de Bruck, and A.C. Davis. Is the radion a chameleon? *JCAP*, 0411:004, 2004.

[72] Daniel R. Green, Eva Silverstein, and David Starr. Attractor explosions and catalyzed vacuum decay. *Phys. Rev.*, D74:024004, 2006.

[73] Philippe Brax, Carsten van de Bruck, Anne-Christine Davis, and Douglas Shaw. The Dilaton and Modified Gravity. *Phys. Rev.*, D82:063519, 2010.

[74] Lilia Anguelova, Vincenzo Calo, and Michele Cicoli. LARGE Volume String Compactifications at Finite Temperature. *JCAP*, 0910:025, 2009.

[75] G. G. Raffelt. Stars as laboratories for fundamental physics: The astrophysics of neutrinos, axions, and other weakly interacting particles. Chicago, USA: Univ. Pr. (1996) 664 p.

[76] Renata Kallosh and Sergey Prokushkin. SuperCosmology. 2004.

[77] G. Veneziano. Scale factor duality for classical and quantum strings. *Phys.Lett.*, B265:287–294, 1991.

[78] M. Gasperini and G. Veneziano. Pre - big bang in string cosmology. *Astropart.Phys.*, 1:317–339, 1993.

[79] M. Gasperini and G. Veneziano. The Pre - big bang scenario in string cosmology. *Phys.Rept.*, 373:1–212, 2003.

[80] Alan H. Guth. The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Phys. Rev.*, D23:347–356, 1981.

[81] Andrei D. Linde. A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems. *Phys. Lett.*, B108:389–393, 1982.

[82] Andreas Albrecht and Paul J. Steinhardt. Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking. *Phys. Rev. Lett.*, 48:1220–1223, 1982.

[83] Edmund J. Copeland, Andrew R. Liddle, David H. Lyth, Ewan D. Stewart, and David Wands. False vacuum inflation with Einstein gravity. *Phys. Rev.*, D49:6410–6433, 1994.

[84] Renata Kallosh. On Inflation in String Theory. *Lect. Notes Phys.*, 738:119–156, 2008.

[85] Fernando Quevedo. Lectures on string/brane cosmology. *Class. Quant. Grav.*, 19:5721–5779, 2002.

[86] Daniel Baumann and Liam McAllister. Advances in Inflation in String Theory. *Ann. Rev. Nucl. Part. Sci.*, 59:67–94, 2009.

[87] Liam McAllister and Eva Silverstein. String Cosmology: A Review. *Gen. Rel. Grav.*, 40:565–605, 2008.

[88] P. Binetruy and M. K. Gaillard. Candidates for the Inflaton Field in Superstring Models. *Phys. Rev.*, D34:3069–3083, 1986.

[89] Tom Banks, M. Berkooz, S. H. Shenker, Gregory W. Moore, and P. J. Steinhardt. Modular cosmology. *Phys. Rev.*, D52:3548–3562, 1995.

[90] G. R. Dvali and S. H. Henry Tye. Brane inflation. *Phys. Lett.*, B450:72–82, 1999.

[91] C. P. Burgess et al. The Inflationary Brane-Antibrane Universe. *JHEP*, 07:047, 2001.

[92] G. R. Dvali, Q. Shafi, and S. Solganik. D-brane inflation. 2001.

[93] Carlos Herdeiro, Shinji Hirano, and Renata Kallosh. String theory and hybrid inflation / acceleration. *JHEP*, 12:027, 2001.

[94] C. P. Burgess, P. Martineau, F. Quevedo, G. Rajesh, and R. J. Zhang. Brane antibrane inflation in orbifold and orientifold models. *JHEP*, 03:052, 2002.

[95] Shamit Kachru et al. Towards inflation in string theory. *JCAP*, 0310:013, 2003.

[96] Hassan Firouzjahi and S. H. Henry Tye. Closer towards inflation in string theory. *Phys. Lett.*, B584:147–154, 2004.

[97] Norihiro Iizuka and Sandip P. Trivedi. An inflationary model in string theory. *Phys. Rev.*, D70:043519, 2004.

[98] C. P. Burgess, James M. Cline, H. Stoica, and F. Quevedo. Inflation in realistic D-brane models. *JHEP*, 09:033, 2004.

[99] Eva Silverstein and David Tong. Scalar Speed Limits and Cosmology: Acceleration from D- cceleration. *Phys. Rev.*, D70:103505, 2004.

[100] Mohsen Alishahiha, Eva Silverstein, and David Tong. DBI in the sky. *Phys. Rev.*, D70:123505, 2004.

[101] Katrin Becker, Melanie Becker, and Axel Krause. M-Theory Inflation from Multi M5-Brane Dynamics. *Nucl. Phys.*, B715:349–371, 2005.

[102] J. J. Blanco-Pillado et al. Racetrack inflation. *JHEP*, 11:063, 2004.

[103] J. J. Blanco-Pillado et al. Inflating in a better racetrack. *JHEP*, 09:002, 2006.

[104] Joseph P. Conlon and Fernando Quevedo. Kaehler moduli inflation. *JHEP*, 01:146, 2006.

[105] J. Richard Bond, Lev Kofman, Sergey Prokushkin, and Pascal M. Vaudrevange. Roulette inflation with Kaehler moduli and their axions. *Phys. Rev.*, D75:123511, 2007.

[106] M. Cicoli, C. P. Burgess, and F. Quevedo. Fibre Inflation: Observable Gravity Waves from IIB String Compactifications. *JCAP*, 0903:013, 2009.

[107] S. Dimopoulos, S. Kachru, J. McGreevy, and Jay G. Wacker. N-flation. *JCAP*, 0808:003, 2008.

[108] Thomas W. Grimm. Axion Inflation in Type II String Theory. *Phys. Rev.*, D77:126007, 2008.

[109] Eva Silverstein and Alexander Westphal. Monodromy in the CMB: Gravity Waves and String Inflation. *Phys. Rev.*, D78:106003, 2008.

[110] Liam McAllister, Eva Silverstein, and Alexander Westphal. Gravity Waves and Linear Inflation from Axion Monodromy. *Phys. Rev.*, D82:046003, 2010.

[111] Nemanja Kaloper, Albion Lawrence, and Lorenzo Sorbo. An Ignoble Approach to Large Field Inflation. *JCAP*, 1103:023, 2011.

[112] Joseph P. Conlon, Renata Kallosh, Andrei D. Linde, and Fernando Quevedo. Volume Modulus Inflation and the Gravitino Mass Problem. *JCAP*, 0809:011, 2008.

[113] A. Avgoustidis, D. Cremades, and F. Quevedo. Wilson line inflation. *Gen. Rel. Grav.*, 39:1203–1234, 2007.

[114] M. Badziak and M. Olechowski. Volume modulus inflation and a low scale of SUSY breaking. *JCAP*, 0807:021, 2008.

[115] Huan-Xiong Yang and Hong-Liang Ma. Two-field Káhler moduli inflation on large volume moduli stabilization. *JCAP*, 0808:024, 2008.

[116] Jonathan P. Hsu and Renata Kallosh. Volume stabilization and the origin of the inflaton shift symmetry in string theory. *JHEP*, 04:042, 2004.

[117] Renata Kallosh and Andrei D. Linde. Landscape, the scale of SUSY breaking, and inflation. *JHEP*, 12:004, 2004.

[118] Ralph Blumenhagen and Maximilian Schmidt-Sommerfeld. Power Towers of String Instantons for N=1 Vacua. *JHEP*, 07:027, 2008.

[119] Ralph Blumenhagen, Xin Gao, Thorsten Rahn, and Pramod Shukla. A Note on Poly-Instanton Effects in Type IIB Orientifolds on Calabi-Yau Threefolds. 2012.

[120] Michele Cicoli, Maximilian Kreuzer, and Christoph Mayrhofer. Toric K3-Fibred Calabi-Yau Manifolds with del Pezzo Divisors for String Compactifications. *JHEP*, 02:002, 2012.

[121] Michael B. Schulz. Calabi-Yau duals of torus orientifolds. *JHEP*, 05:023, 2006.

[122] Edward W. Kolb and Michael S. Turner. The Early universe. *Westview Press*, 1994.

[123] Temple He, Shamit Kachru, and Alexander Westphal. Gravity waves and the LHC: Towards high-scale inflation with low-energy SUSY. *JHEP*, 06:065, 2010.

[124] Stefan Antusch, Koushik Dutta, and Sebastian Halter. Combining High-scale Inflation with Low-energy SUSY. *JHEP*, 1203:105, 2012.

[125] David H. Lyth. What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy? *Phys. Rev. Lett.*, 78:1861–1863, 1997.

[126] Neil Barnaby, J. Richard Bond, Zhiqi Huang, and Lev Kofman. Preheating After Modular Inflation. *JCAP*, 0912:021, 2009.

[127] Michele Cicoli and Anupam Mazumdar. Reheating for Closed String Inflation. *JCAP*, 1009:025, 2010.

[128] Michele Cicoli and Anupam Mazumdar. Inflation in string theory: a graceful exit to the real world. *Phys. Rev.*, D83:063527, 2011.

[129] Joseph P. Conlon and Fernando Quevedo. Astrophysical and Cosmological Implications of Large Volume String Compactifications. *JCAP*, 0708:019, 2007.

[130] Sidney R. Coleman and Erick J. Weinberg. Radiative Corrections as the Origin of Spontaneous Symmetry Breaking. *Phys. Rev.*, D7:1888–1910, 1973.

[131] Edmund J. Copeland, M. Sami, and Shinji Tsujikawa. Dynamics of dark energy. *Int. J. Mod. Phys.*, D15:1753–1936, 2006.

[132] Christopher F. Kolda and David H. Lyth. Quintessential difficulties. *Phys. Lett.*, B458:197–201, 1999.

[133] Sean M. Carroll. Quintessence and the rest of the world. *Phys. Rev. Lett.*, 81:3067–3070, 1998.

[134] Kiwoon Choi. String or M theory axion as a quintessence. *Phys. Rev.*, D62:043509, 2000.

[135] Nemanja Kaloper and Lorenzo Sorbo. Where in the String Landscape is Quintessence. *Phys. Rev.*, D79:043528, 2009.

[136] Sudhakar Panda, Yoske Sumitomo, and Sandip P. Trivedi. Axions as Quintessence in String Theory. *Phys. Rev.*, D83:083506, 2011.

[137] Gaveshna Gupta, Sudhakar Panda, and Anjan A. Sen. Observational Constraints on Axions as Quintessence in String Theory. *Phys. Rev.*, D85:023501, 2012.

[138] Y. Aghababaie, C. P. Burgess, S. L. Parameswaran, and F. Quevedo. Towards a naturally small cosmological constant from branes in 6D supergravity. *Nucl. Phys.*, B680:389–414, 2004.

[139] Y. Aghababaie et al. Warped brane worlds in six dimensional supergravity. *JHEP*, 09:037, 2003.

[140] Jeremie Vinet and James M. Cline. Codimension-two branes in six-dimensional supergravity and the cosmological constant problem. *Phys. Rev.*, D71:064011, 2005.

[141] Andreas Albrecht, C. P. Burgess, F. Ravndal, and C. Skordis. Natural quintessence and large extra dimensions. *Phys. Rev.*, D65:123507, 2002.

[142] C. P. Burgess. Towards a natural theory of dark energy: Supersymmetric large extra dimensions. *AIP Conf. Proc.*, 743:417–449, 2005.

[143] Hans-Peter Nilles, Antonios Papazoglou, and Gianmassimo Tasinato. Selftuning and its footprints. *Nucl. Phys.*, B677:405–429, 2004.

[144] C. P. Burgess and L. van Nierop. Large Dimensions and Small Curvatures from Supersymmetric Brane Back-reaction. *JHEP*, 04:078, 2011.

[145] Michele Cicoli, Christoph Mayrhofer, and Roberto Valandro. Moduli Stabilisation for Chiral Global Models. *JHEP*, 02:062, 2012.

[146] C. P. Burgess, R. Kallosh, and F. Quevedo. de Sitter String Vacua from Supersymmetric D-terms. *JHEP*, 10:056, 2003.

[147] Michele Cicoli, Anshuman Maharana, F. Quevedo, and C. P. Burgess. De Sitter String Vacua from Dilaton-dependent Non- perturbative Effects. 2012.

[148] M. Cicoli, G. Tasinato, I. Zavala, C. P. Burgess, and F. Quevedo. Modulated Reheating and Large Non-Gaussianity in String Cosmology. 2012.

[149] Edmund J. Copeland, Andrew R Liddle, and David Wands. Exponential potentials and cosmological scaling solutions. *Phys. Rev.*, D57:4686–4690, 1998.

[150] Luca Amendola. Coupled quintessence. *Phys. Rev.*, D62:043511, 2000.

[151] Christian G. Boehmer, Gabriela Caldera-Cabral, Ruth Lazkoz, and Roy Maartens. Dynamics of dark energy with a coupling to dark matter. *Phys. Rev.*, D78:023505, 2008.