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BARGAINING AND SOCIAL STRUCTURE

Edoardo Gallo

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Manor Road Building, Oxford OX1 3UQ

Bargaining and social structure

Edoardo Gallo*

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Abstract

This paper presents a bargaining model between individuals belonging to different groups where the equilibrium outcome depends on the communication network within each group. Belonging to a group gives an informational advantage: connections help to gather information about past transactions and this information can be used to make more accurate demands in future bargaining rounds. In the long-term there is a unique stochastically stable equilibrium which depends on the peripheral or least connected individuals in each group. Comparative statics shows that a denser and more homogeneous network allows members of a group to obtain a better deal. An empirical analysis of the observed price differential between Asian and white buyers in New York's Fulton fish market is consistent with these predictions. An extension explores an alternative set-up where buyers and sellers belong to the same communication network: if the network is regular and the agents are homogeneous then the equilibrium division is 50-50.

Keywords: network, noncooperative bargaining, core-periphery networks, Fulton fish market, 50-50 division.
JEL: C73, C78, D83.

*Address: University of Oxford, Nuffield College, New Road, Oxford OX1 1NF, UK. Email: *edoardo.gallo@economics.ox.ac.uk*. I am very grateful to Peyton Young for his invaluable help and guidance throughout this project, and to Kathy Graddy for making available the Fulton fish market dataset. Thanks to Carlos Alós-Ferrer, Jean-Paul Carvalho, Aytek Erdil, Andrea Galeotti, Sanjeev Goyal and Margaret Meyer for helpful comments and suggestions. I also thank seminar and conference participants at the University of Oxford, Cambridge University, the 2008 Workshop of Internet and Network Economics, ACDD 2009, the Royal Economic Society 2009 Meeting and the Irish Economic Association 2009 Meeting. Any remaining errors are the sole responsibility of the author.

1 Introduction

Individuals who belong to close-knit groups often enjoy an advantage in many perfectly competitive markets. For instance, Greif [1994] describes how in the 11th century Maghribi traders joined into close-knit groups to facilitate trading across the Mediterranean in an environment characterized by a high degree of uncertainty and imperfect information. Rauch [2001] reviews empirical evidence that ethnic immigrant networks significantly increase international trade volumes, especially for commodities whose price is variable and/or uncertain. The goal of this paper is to explore one type of advantage that these groups provide and to relate the internal structure of interactions of a group to the observed market outcomes.

The core idea is that *belonging to a group gives an informational advantage*: individuals who belong to a group use their connections to gather information about the demands made in past transactions and they use this information to make optimal demands in future private bilateral negotiations. This set-up is relevant for perfectly competitive markets with a large number of individuals that are characterized by imperfect information, uncertainty on the price of the goods, and private bilateral negotiations. In these markets an individual is unable to collect information on the current price of a good due to the size of the market and the unobservability of private transactions, and therefore she turns to other members of her group to gather information about recent transactions before starting a trade.

Standard economic models assume away the presence and the role of the interactions among different agents in determining market outcomes. In classical models of perfectly competitive markets with complete information, prices seamlessly aggregate all the information present in the market: each individual is a price-taker who competes with all other individuals in the economy. At the other end of the spectrum, in classical bargaining models, the outcome of a transaction is fully determined by the attitude to risk, i.e. the functional form of the utility, of the two agents involved.[1] In these bargaining models individuals know the game they are playing, the utility or the distribution of utility of their opponent(s), and they are able to perform complex calculation on the strategies available to all the agents involved in the game. In a market with a large number of individuals these bargaining models would predict a staggering multiplicity of outcomes, corresponding to the thousands of possible combinations of utility pairs.

This paper develops a bargaining model between agents belonging to different groups based on the framework in Young [1993a]. It shows that the equilibrium outcome depends on the structure of interactions within each group: in the long-term every agent in a group receives the same share of the good, but the share varies across groups depending on their internal structure. As in classical models of perfect competition, there is a *large number of agents* trading a *homogeneous commodity* in a *perfectly competitive* environment. However, there is *incomplete information* on the price of the commodity: there are no posted prices and the agents *communicate* with each other to *learn* about the current price. As

[1]Rubinstein [1982] is the seminal paper, see Muthoo [1999] for an introduction to standard bargaining models.

in classical bargaining models, each transaction is a *private, bilateral negotiation* between two agents and the outcome in equilibrium will depend on the risk profile of the agents in each group. However, agents base their bid on *information on past transactions* they have collected *from other agents* in their group, and they are unaware of the game they are playing or of the utility profile of their opponent.

There are many markets that share these characteristics. A prominent example is markets in developing countries where there is a high variability in the price of goods due to exogenous factors affecting supply, and where the lack of strong institutions is an obstacle to the adoption of publicly displayed prices allowing the proliferation of decentralized bilateral transactions.[2] A second example is markets in illegal commodities: the need to perform secret transactions leads to imperfect information, private bilateral exchanges and fluctuations in price due to frequent disruptions of the supply chain.[3]

A third example is wholesale markets where transactions are private between one buyer and one seller and the prices are very sensitive to exogenous factors that affect the supply chain. For instance, wholesale fish markets are characterized by private, bilateral transactions and prices fluctuate widely due to exogenous factors such as wind and wave height that affect the volume of the daily catch of fish. Section 5 of this paper analyzes a dataset of prices from the Fulton wholesale fish market in New York and it argues that the predictions of this model shed light on the observed price differential between Asian and white buyers.[4]

The following two subsections give a more detailed overview of the model and survey the related literature respectively. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the bargaining solution and discusses its implications for the desirable network structure for the members of a group. Section 4 carries out the comparative statics analysis. Section 5 analyzes a dataset of prices in the Fulton wholesale fish market in New York and it shows corroborating evidence that the comparative statics predictions explain the observed pricing patterns. Section 6 investigates how results change if communication between buyers and sellers is allowed. Finally, section 7 concludes, and Appendix A contains the proofs omitted in the main text.

1.1 Overview of the model and results

The following is a more detailed description of the model. There is a population of agents formed by two disjoint groups of n_B buyers and n_S sellers. At each time t a buyer b and a seller s are randomly drawn to play the Nash demand game: b demands a fraction x_t and s demands a fraction y_t. If $x_t + y_t \leq 1$ then b and s get their demands, otherwise they get nothing. Note that the role of buyers and sellers is completely interchangeable:

[2] See, for example, Aker [2008] for evidence from grain markets in Niger.

[3] See, for example, Levitt and Venkatesh [2007] for evidence from the Chicago street-level prostitution market.

[4] Kirman [2001] provides a detailed description of wholesale fish markets. A different type of wholesale market that can be described by this model is, for example, the fruit and vegetable market investigated in Kirman et al. [2005].

for expositional purposes the following description of the model will focus on buyers, but the same applies to sellers as well.

Each buyer maximizes a well-behaved utility function. At each time t the buyer receives a sample of previous demands by sellers: she chooses an optimal reply to the cumulative distribution of demands with probability $1 - \epsilon$, and a non-optimal reply, i.e. a "mistake," with probability ϵ.[5] The amount of information that buyer b receives from another buyer b' is the realization of a Poisson process connecting b to b'. Thus, the total information sample that b receives before the bargaining round consists of all the information coming from the realization of the Poisson processes that connect b to the other buyers she communicates with. A network g^B is an abstract representation of the average communication flows over a long time in the group of buyers: the link $g_{bb'}$ is equal to the rate of the Poisson process connecting b to b'.

Theorem 1 proves that if the communication networks of buyers and sellers are connected and if they are not complete networks then the process without mistakes always converges to a convention. A convention means that each buyer always makes the same demand x and each seller always makes the same demand $1 - x$. The condition on the network structure guarantees that the information available to each player on the history of demands is sufficiently incomplete to avoid the whole process getting stuck in a cycle.

Theorem 2 proves that the process with mistakes converges to a unique stable division, which is the *asymmetric Nash bargaining solution (ANB)* with weights that depend on the network structure. Specifically, the weights are determined by the subset of peripheral agents in each group with the least number of and/or weakest communication links. A consequence of this result is that, given a budget of links to allocate, the desirable architectures for a group are *quasi-regular networks*, i.e. networks where all the agents are connected by strong links and have a very similar number of connections.

The solution in theorem 2 allows the exploration of how changes in the network structure affect the shares a group obtains in the stable division. The changes are modeled in terms of first and second order stochastic dominance shifts in the weighted degree distribution, i.e. variations in the relative frequencies of agents with different number of connections. Theorem 3 shows that individuals belonging to a group with a *denser and more homogeneous communication structure will fare better*.

Section 5 analyzes a dataset on transaction prices in the wholesale Fulton fish market in New York. A puzzling finding first highlighted by Graddy [1995] is that Asian buyers pay a significantly lower price than white buyers for the same product sold by a white seller. This model provides a rationale for the presence of the price difference. Buyers communicate within their ethnic group to learn the daily price of fish. Sociological evidence shows that the group of Asians is denser and therefore it is a better channel of information on the uncertain price of the product. Over time the Asians receive more information and they exploit this additional knowledge to make more accurate offers and obtain a lower price from the seller. Empirical analysis of the dataset shows that Asians learn the ongoing price more accurately and that the price differential emerges only after learning has taken

[5]Throughout the paper, the buyer is female and the seller is male.

place.

An extension explores how the theoretical predictions change if buyers and sellers belong to the same communication network, allowing in this way buyers to receive information from other buyers *and* sellers. The unique stable division is still the *ANB solution* in theorem 2, but the effect of varying the network structure is now different. A denser communication network leaves the ANB unchanged, but *a more homogeneous network narrows down the difference* between the shares of the two groups. If all agents are homogeneous and the network is a regular network, then the solution is the *50-50 division*. The desirable architectures for the buyers are *core-periphery networks*: the buyers form a core network where they are connected by strong links and they have a very similar number of connections, while the sellers are at the periphery where each one of them is connected by one link to a buyer.

1.2 Related literature

In previous contributions there are at least two complementary explanations of why belonging to a group leads to a competitive advantage in a market with a high degree of uncertainty. The first one was originally advanced by Greif [1994]: an individual trader in a group can rely on the other members of the group to inflict a costly punishment to a cheater by cutting all future trade between any member of the group and the cheater. He illustrates this with a simple model in a repeated game framework, and he draws on historical records to discuss its relevance for trading in the 11th century.

The second explanation is the core idea behind the model presented here: an individual in a group has access to information from other group members and this leads to a competitive advantage in a market where information is incomplete. Rauch and Casella [2003] proposed a model where information-sharing within ethnic groups influences resource allocation in international trade markets affected by incomplete information. Rauch and Trindade [2002] show that the information-sharing story fits observed international trade flows better than the collective punishment one. One of the key differences between this paper and these previous contributions is that it explicitly models the role of the network structure of interactions within a group.

Methodologically, this paper is based on the evolutionary bargaining framework first formulated by Young [1993a]. The bargaining procedure and the behavior of agents is the same as in Young's model: individuals from two groups of bargainers are randomly matched to play the Nash demand game and they make demands by choosing best replies based on an incomplete knowledge of precedents. The novel element introduced here is the modeling of the process by which agents receive information to play the game: information travels through a communication network that connects the agents in each group.

The introduction of the network requires the construction of a different Markov process to describe the evolution of the system. This demands a re-analysis of the Markov process in order to derive the equilibrium outcome in this set-up. The equilibrium outcome depends on the underlying network and therefore this allows the comparative statics analysis in section 4, which would not be possible in the model without the network. The primary advantage of introducing the network is that it opens the possibility of testing the comparative statics predictions of the model on real market data, as the empirical exercise carried out in section 5 illustrates.

Moreover, the extension in section 6 shows that making explicit the process by which agents receive information leads to further insights that are not accessible to the model without the network. Section 6 analyzes the case when buyers and sellers belong to the same communication network instead of being part of separate networks as in the basic model. The equilibrium outcome is unchanged making this modified version of the model indistinguishable from the basic version in the set-up without the network. However, the comparative statics predictions are different, and this type of analysis is only feasible in the model with the network.

In the economics of networks literature a number of papers investigate how a network that constrains agents' interactions affects the outcome of a bargaining process. Selected contributions include Calvó-Armengol [2001], Calvó-Armengol [2003], Corominas-Bosch [2004], Polanski [2007], Abreu and Manea [2008] and Manea [2008]. The framework adopted here is conceptually different. In all the references listed above, the network is a constraint on the *interactions* that agents are allowed to have. On the other hand, in this paper the network is a constraint on the *information* about past bargains that agents have as they enter a bargaining round. Moreover, the focus of this paper is also different. Previous work in the literature investigates how the position of *one* agent in a network affects her *individual* payoffs. Here the aim of the paper is to understand how the *overall* structural properties of the network determine the payoff that every individual in the *whole group* receives, independently on their position in the network.

The contribution of this paper is twofold. First, it constructs a model to investigate the role of communication structure in perfectly competitive markets with incomplete information. This provides a theoretical underpinning to previous empirical studies that emphasized the informational role of social structure in determining market outcomes. Secondly, it derives predictions on the effects of changes in the communication structure on the equilibrium outcome, and it tests these predictions with a basic empirical analysis of prices in a wholesale fish market. This is a preliminary step in a growing research agenda whose objective is to test the predictions of theoretical models of network structure on economic outcomes.

2 The Model

This section presents the main elements of the model: the network concepts and terminology used, the adaptive play bargaining process, and the Markov process which describes the evolution of the system.

Networks. A *weighted, undirected network* g is represented by a symmetric matrix $[g_{ij}]^{n \times n}$, where $g_{ij} \in \mathbb{R}_+$. The entry g_{ij} indicates the *strength* of the *communication link* between i and j. If $g_{ij} > 0$ then agents i and j are connected and they communicate directly with each other. If $g_{ij} = 0$ then i and j are not connected in the communication network. Throughout this paper let $g_{ii} \equiv \overline{g}$, i.e. an agent is connected with herself and the strength of this self-connection is the same for all agents.

The *neighborhood* of i in g is $L_i(g) = \{j \in N | g_{ij} > 0\}$.[6] $d_i(g) \equiv |L_i(g)|$ denotes the size of i's neighborhood, or the *degree* of i, in g. $z_i(g) \equiv \sum_{j \in L_i(g)} g_{ij}$ is the *weighted degree* of i in g. Let $Z(g) = \max_{i \in N} z_i$ be the maximum weighted degree of any agent in the network g. A *complete network* is a network that belongs to the class of networks $g^C = \{g | g_{ij} > 0, \forall i, j \in N\}$ where every pair of agents is connected. A *regular network* $g_{d,a}$ of degree d and link strength a is a network that belongs to the class of networks $\overline{g}_{d,a} = \{g | g_{ij} = \{0, a\}; d_i(g) \equiv d; \forall i, j \in N; a \in \mathbb{R}_+\}$.

The *weighted degree distribution* of a network is a description of the relative frequencies of agents that have different degrees. Let $p(z)$ denote the weighted degree distributions of network g, i.e. the fraction of nodes that have weighed degree z in network g. The comparative statics analysis in this paper will investigate changes in the communication structure that are captured by stochastic dominance shifts in this degree distribution. The following are more formal definitions of these notions.

Definition 1. A distribution p' first order stochastic dominates (FOSD) another distribution p if $\rho'(z) \leq \rho(z)$ for any $z \in [0, Z]$, where $\rho(z) = \sum_{z=0}^{Z} p(d)$ is the cumulative distribution of $p(z)$. The FOSD shift is variance-preserving if $Var[p(z)] = Var[p'(z)]$.

Definition 2. A distribution p' strictly second order stochastic dominates (SOSD) another distribution p if $\sum_{z=0}^{Z} \rho'(z) \leq \sum_{z=0}^{Z} \rho(z)$ for any $z \in [0, Z]$. The SOSD shift is mean-preserving if $\mu[p(z)] = \mu[p'(z)]$.

If $p(z)$ FOSD $p'(z)$ then a network g is *denser* than a network g'. Note that in the context of weighted networks denser means that agents in g have on average a higher number and/or stronger links than agents in g'. If $p(z)$ SOSD $p'(z)$ then a network g is *more homogeneous* than a network g'. Similarly, more homogeneous means that agents in g are more homogeneous in terms of the number and/or strength of their connections than agents in g'.

Adaptive play bargaining process. Consider two finite, non-empty and disjoint groups of individuals $B = \{1, ..., n_B\}$ and $S = \{1, ..., n_S\}$: the buyers and sellers. In each period t one buyer and one seller drawn at random meet to divide a pie of size normalized to one. They play the Nash demand game: b demands a fraction x_t and s demands a fraction y_t, if $x_t + y_t \leq 1$ then b and s get their demands, otherwise they get nothing. Assume that the set of possible divisions is discrete and finite, and let δ be the smallest possible division. The sequence $h = \{(x_1, y_1), ..., (x_t, y_t)\}$ is the complete *global history* up to and including period t. Each agent remembers the last m rounds of the bargaining game that she has played, where m stands for the memory of the player.

[6]Note that this definition is slightly different than the standard one adopted in the literature because it allows for i's neighborhood to include i as well. This is because in our framework agent i's own degree g_{ii} is allowed to be positive. This difference affects the ensuing definitions as well.

Agents receive information to play the game as follows. Suppose player $b \in B$ is picked to play the game at $t+1$: in the $\Delta t = 1$ time period she receives information from some of the other buyers in B about past bargaining rounds. Information arrival is modeled as a Poisson process. Specifically, in the $\Delta t = 1$ time interval, the probability $P(s_{bj}(\Delta t = 1) = k)$ that b receives a sample $s_{bj}(\Delta t = 1)$ of k past bargains from player j is equal to:

$$P(s_{bj}(\Delta t = 1) = k) = \frac{e^{-g_{bj}} g_{bj}^k}{k!}$$

where g_{bj} is the rate of arrival of information to b from j. By standard properties of Poisson processes, the expected amount of information b receives from j before each bargaining round is $E[P(s_{bj})] = g_{bj}$. Also, let $\sum_{j \in L_b(g)} E[P(s_{bj})] = E[P(s_b)] = \sum_{j \in L_b(g)} g_{bj} = z_b$ be the expected total amount of information b receives before each bargaining round. Clearly, at each point in time the realization of the Poisson process that determines how much information b receives from j may be higher or lower than g_{bj}, but over a long period of time the average amount of information per time period that b receives from j will be equal to g_{bj}. Thus, the network g captures the average information flows between each pair of agents in the group over a long period of time.

Agents are boundedly rational as they are not aware of the game they are embedded in and they base their decision exclusively on the information they receive. Specifically, agents do not have prior knowledge or beliefs about the utility function of the other side, and they do not know the distribution of utility functions in the general population. Agent b chooses an optimal reply to the cumulative probability distribution $G(y)$ of the demands y_j made by sellers in his sample, where $G(y) = \frac{h}{s_b(t)}$ if and only if there are exactly h demands y_l in the sample $s_b(t)$ such that $y_l \leq y$.

Agent b has a concave and strictly increasing von Neumann-Morgenstern utility function $u(x)$. Assume that $u(x)$ is defined for all $x \in [0, 1]$ and that it is normalized so that $u(0) = 0$. Buyer b's expected payoff from demanding x is then equal to $Eu(x) \equiv u(x)G(1-x)$. Thus, b chooses x_{t+1} so as to maximize $Eu(x)$, and if there are several values of x to choose from then each one of them is chosen with positive probability.

The set-up for seller s is analogous, and the utility function of the sellers will be denoted by $v(y)$.

Markov process. Let $\mathbf{S}$ be the state space, whose elements are sets of vectors $\mathbf{s} = \{v_1, ..., v_n\}$, where v_i stands for agent i's memory, which is a vector of size m, and $n \equiv n_B + n_S$. If $i \in B$ then $v_i = \{y^i_{k-m+1}, ..., y^i_k\}$, i.e. the entries of v_i are the m last demands made by sellers in bargaining rounds involving i. Similarly, if $i \in S$ then $v_i = \{x^i_{k-m+1}, ..., x^i_k\}$. Let $p_b(x \mid \mathbf{s})$ be agent b's best-reply distribution, i.e. $p_b(x \mid \mathbf{s}) > 0$ if and only if demanding x is b's best-reply to a sample received when the system is in state $\mathbf{s}$. Analogously, $p_s(y \mid \mathbf{s})$ is seller s's best-reply distribution.

Assume that the process starts at an arbitrary time $t_0 > n \cdot m$, and denote the initial state by $\mathbf{s}^0$. At each $t > t^0$, one pair of agents $(b, s) \in B \times S$ is drawn at random with probability $\pi(b, s)$, where $\pi(b, s) > 0$, $\forall (b, s) \in B \times S$. At time t, consider a state $\mathbf{s} = \{v_b, v_s, v_{-b}, v_{-s}\}$, where $v_b = \{y^b_{k-m+1}, ..., y^b_k\}$ and $v_s = \{x^s_{k-m+1}, ..., x^s_k\}$. Define $\mathbf{s}'$ to be a successor of $\mathbf{s}$ if it has the form $\mathbf{s}' = \{v'_b, v'_s, v_{-b}, v_{-s}\}$, where $v'_b = \{y^b_{k-m+2}, ..., y^b_{k+1}\}$ and $v'_s = \{x^s_{k-m+2}, ..., x^s_{k+1}\}$. The transition probability $P_{\mathbf{ss}'}$ of moving from state $\mathbf{s}$ to state $\mathbf{s}'$ is then equal to:

$$P_{\mathbf{ss}'} = \sum_{b \in B} \sum_{s \in S} \pi(b, s) p_b(x_{t+1} \mid \mathbf{s}) p_s(y_{t+1} \mid \mathbf{s}) \tag{1}$$

Mistakes. In the process described so far agents always give a best reply to the sample they happen to pick. In reality, people make mistakes for a variety of reasons: human beings are poor at computing probabilities and they might miscalculate the expected utility from an offer, they are prone to get distracted, they experiment, or sometimes they are outright irrational. The following is a more formal definition of a mistake.

Definition 3. Let $\mathbf{s} = \{v_b, v_s, v_{-b}, v_{-s}\}$ and let $\mathbf{s}' = \{v'_b, v'_s, v_{-b}, v_{-s}\}$ be a successor of $\mathbf{s}$, where $v_b = \{y^b_{k-m+1}, ..., y^b_k\}$, $v_s = \{x^s_{k-m+1}, ..., x^s_k\}$, $v'_b = \{y^b_{k-m+2}, ..., y^b_{k+1}\}$ and $v'_s = \{x^s_{k-m+2}, ..., x^s_{k+1}\}$. The demand x^s_{k+1} is a mistake by b if it is not a best response to any sample b could have received given that the system is in state $\mathbf{s}$. A mistake y^s_{k+1} by s is defined similarly.

Another concept that will be useful in the analysis of the perturbed process is the *resistance* in moving from one state $\mathbf{s}$ to another state $\mathbf{s}'$.

Definition 4. Let $\mathbf{s}$ and $\mathbf{s}'$ be two states of the system. The *resistance* $r(\mathbf{s}, \mathbf{s}')$ is the least number of mistakes required for the system to go from state $\mathbf{s}$ to $\mathbf{s}'$.

Note that if $\mathbf{s}'$ is a successor of $\mathbf{s}$ then $r(\mathbf{s}$,$\mathbf{s}$') $\in \{0, 1, 2\}$ as the maximum number of mistakes in any one-time transition is two, i.e. both the buyer and seller involved in that bargaining round make a mistake.

Now let ϵ be the absolute probability that agents in the model make mistakes, and let λ_b, λ_s be the relative probabilities that buyers and sellers do so respectively. Thus, $\epsilon\lambda_b$ and $\epsilon\lambda_s$ are the probabilities that buyers and sellers make a mistake. Denote by $q_b(x \mid \mathbf{s})$ the buyer's conditional probability of choosing x given that the current state is $\mathbf{s}$ and that she is not giving a best-response offer to the sample picked. Assume $\lambda_b, \lambda_s, \epsilon > 0$ and that $q_b(x \mid \mathbf{s})$, $q_s(y \mid \mathbf{s})$ have full support.

This process also yields a stationary Markov chain on $\mathbf{S}$ that can be described by the probability of moving from a state $\mathbf{s}$ to a successor state $\mathbf{s}$', similarly to equation (1) above. Assume that the process starts at an arbitrary time $t_0 > n \cdot m$, and denote the initial state by $\mathbf{s}^0$. At each $t > t^0$ one pair of agents $(b, s) \in B \times S$ is drawn at random with probability $\pi(b, s)$, where $\pi(b, s) > 0$, $\forall (b, s) \in B \times S$. Let $\mathbf{s}$ be the state at time t, and let $\mathbf{s}$' be a successor of $\mathbf{s}$, where $\mathbf{s}$ and $\mathbf{s}$' are defined above. The transition probability $P^{\epsilon}_{\mathbf{ss}'}$ of moving from state $\mathbf{s}$ to state $\mathbf{s}'$ is then equal to:

$$
\begin{aligned}
P^{\epsilon}_{\mathbf{ss}'} =& \sum_{b\in B}\sum_{s\in S}\pi(b,s)[(1-\epsilon\lambda_b)(1-\epsilon\lambda_s)p_b(x_{t+1}\mid \mathbf{s})p_s(y_{t+1}\mid \mathbf{s})+ \\
&+\epsilon\lambda_b(1-\epsilon\lambda_s)q_b(x_{t+1}\mid \mathbf{s})p_s(y_{t+1}\mid \mathbf{s})+\epsilon\lambda_s(1-\epsilon\lambda_b)q_s(x_{t+1}\mid \mathbf{s})p_b(y_{t+1}\mid \mathbf{s})+ \\
&+\epsilon^2\lambda_b\lambda_s q_b(x_{t+1}\mid \mathbf{s})q_s(y_{t+1}\mid \mathbf{s})] \qquad (2)
\end{aligned}
$$

Note that $\lim_{\epsilon\to 0} P^{\epsilon}_{\mathbf{ss}'} = P_{\mathbf{ss}'}$. It is also worthwhile to stress that this Markov process is more complex than it needs to be to generate the results presented in this paper because it allows for the relative probabilities that buyers and sellers make mistakes to vary due to the λ_b and λ_s factors. This heterogeneity does not affect the results because the analysis is asymptotic in the limit as mistakes go to zero.

3 Equilibrium analysis

This section presents the results of the equilibrium analysis. Section 3.1 shows that the process without mistakes converges to a convention as long as the network is not complete. Section 3.2 derives the stochastically stable division which crucially depends on the least connected agent(s) in each group. Section 3.3 characterizes the desirable communication network structure for the members of a group and discusses the relevance of this result to a long-standing debate in the sociology literature.

3.1 Convergence

First, consider the unperturbed process P. The first step in the analysis is to define an appropriate concept of stability for this system, and to show that in the long-term the process will reach it. Intuitively, the system will be in a stable state if, after a certain time t, any buyer will always make the same demand x because in any sample she receives of previous sellers' demands, the sellers have always demanded $1-x$, and vice versa for the sellers. The following definition states this more formally.

Definition 5. A state $\mathbf{s}$ is a convention if any $v_i \in \mathbf{s}$ with $i \in B$ is such that $v_i = (1-x, ..., 1-x)$, and any $v_j \in \mathbf{s}$ with $j \in S$ is such that $v_j = (x, ..., x)$, where $x \in D$, $0 < x < 1$. Hereafter, denote this convention by $\mathbf{x}$.

The following lemma shows that the convention $\mathbf{x}$ is an appropriate definition to work with because any $\mathbf{x}$ is an absorbing state of P.

Lemma 1. *Every convention $\mathbf{x}$ is an absorbing state of the Markov process P in (1).*

The following theorem shows that if information about the history of play is sufficiently incomplete then the process P converges to a convention. The incompleteness of information is delivered by the network structure: if the network is not complete then some agents do not receive information on past demands in rounds played by individuals that do not belong to their neighborhoods.

Theorem 1. *Assume both g^B and g^S are connected and they are not complete networks. The bargaining process converges almost surely to a convention.*

The example networks in figure 1 help understanding the intuition behind the proof. The goal is to show that from any initial state $\mathbf{s}$ there is a positive probability p independent of t of reaching a convention within a finite number of steps. The assumption that g^B is not a complete network implies that there are at least two agents b' and b'' such that $g_{b'b''} = 0$. Moreover, given that g^B is connected, there are at least two agents like b' and b'' such that the intersection of their neighborhoods includes at least one agent b. The same applies to the sellers' network, where agents s and s'' are the equivalent of agents b' and b'' respectively.

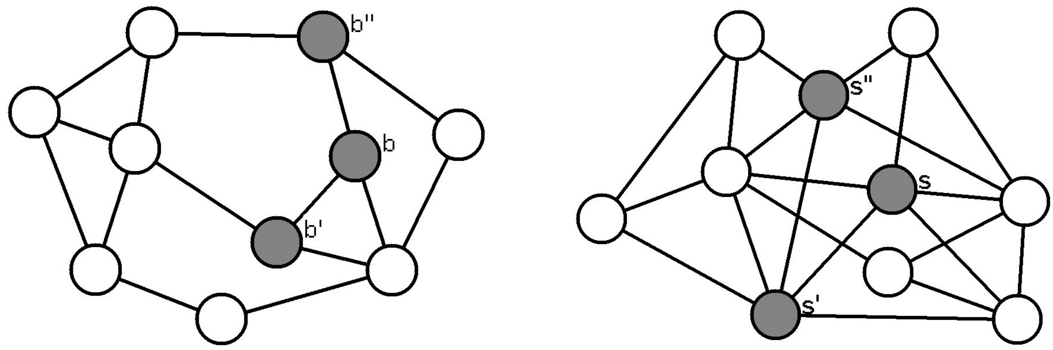

Figure 1: Example networks of buyers (left) and sellers (right). Gray-colored nodes are referred to in the text to give intuition about the proof.

Now, consider the following path which happens with positive probability from any state $\mathbf{s}$ at time t. First, b and s are picked to play the game for m consecutive periods, they draw samples σ and σ' respectively, they demand best-replies x and y respectively, and therefore they obtain a run $\xi = \{(x, y), ..., (x, y)\}$ such that $v_b = (y, ..., y)$ and $v_s = (x, ..., x)$. Second, b' and s' are picked to play the game for m consecutive periods, they draw samples from v_b and v_s each time, they demand best-replies $1 - y$ and $1 - x$ respectively, and therefore they obtain a run $\xi' = \{(1 - y, 1 - x), ..., (1 - y, 1 - x)\}$ such that $v_{b'} = (1 - x, ..., 1 - x)$ and $v_{s'} = (1 - y, ..., 1 - y)$. Third, b'' and s'' are picked to play the game for m consecutive periods, they draw sample v_b and $v_{s'}$ each time, they demand best-replies $1 - y$ and y respectively, and therefore they obtain a run $\xi'' = \{(1-y, y), ..., (1-y, y)\}$ such that $v_{b''} = (y, ..., y)$ and $v_{s''} = (1-y, ..., 1-y)$. Hereafter it is clear that there is a positive probability of reaching a convention $\mathbf{x} = (1 - y, y)$.

Theorem 1 in Young [1993b] proves adaptive play converges almost surely to a convention in any weakly acyclic game with n agents as long as information is sufficiently incomplete. In Young [1993b]'s the incompleteness of the information is given by bounds on the size of the sample the agents can draw to base their play on. Here the incompleteness of information is given by the network structure: if the network is not complete there will be agents who cannot sample some past rounds because they were played by agents in their group with whom they do not communicate.

Second, consider the perturbed process P^ϵ. Given that the distribution q_b and q_s have full support, P^ϵ is irreducible. Thus, P^ϵ has a unique stationary distribution. Moreover, P^ϵ is strongly ergodic, i.e. $\forall \mathbf{s} \in S$, μ_s^ϵ is with probability one the relative frequency with which state $\mathbf{s}$ will be observed in the first t periods as $t \to \infty$. The stability concept for this kind of perturbed process is a *stochastically stable convention*, which was introduced by Foster and Young [1990].

Definition 6. A convention $\mathbf{s}$ is stochastically stable if $\lim_{\epsilon\to 0} \mu_{\mathbf{s}}^\epsilon > 0$. A convention $\mathbf{s}$ is strongly stable if $\lim_{\epsilon\to 0} \mu_{\mathbf{s}}^\epsilon = 1$.

Intuitively, in the long-run stochastically stable conventions will be observed much more frequently than unstable conventions when the probability ϵ of mistakes is small. A strongly stable convention will be observed almost all the time. The technique to compute the stochastically stable conventions is standard and it will not be explained in detail below, see Young [1998] for an excellent introduction.

Construct a weighted, directed network $[r_{\mathbf{s}^i\mathbf{s}^j}]^{k\times k}$, where the nodes are the states $\mathbf{s} \in S$, the links are the resistances $r_{\mathbf{s}^i\mathbf{s}^j}$ connecting $\mathbf{s}^i$ to $\mathbf{s}^j$, and k is the total number of states in S. Define an ***x**-tree* $t_{\mathbf{x}} \in T_{\mathbf{x}}$ to be a collection of links in $[r_{\mathbf{s}^i\mathbf{s}^j}]^{k\times k}$ such that, from every node $\mathbf{x}' \neq \mathbf{x}$, there is a unique directed path to $\mathbf{x}$ and there are no cycles. This construction leads to the definition of the concept of *stochastic potential* of a convention $\mathbf{x}$.

Definition 7. The stochastic potential $\gamma(\mathbf{x})$ of a convention $\mathbf{x}$ is the least resistance among all $t_{\mathbf{x}} \in T_{\mathbf{x}}$. Mathematically:

$$\gamma(\mathbf{x}) = \min_{t_{\mathbf{x}}\in T_{\mathbf{x}}} \sum_{(\mathbf{x}',\mathbf{x}'')\in T_{\mathbf{x}}} r(\mathbf{x}',\mathbf{x}'') \tag{3}$$

Theorem 4 in Young [1993b] explains how to compute the stochastically stables states. The following is a special case of that result.

Theorem. ***[Young [1993b]]*** *Let μ^0 be a stationary distribution of the unperturbed process P. Then $\lim_{\epsilon\to 0} \mu_{\boldsymbol{s}}^\epsilon = \mu^0$. Moreover, $\mu^0 > 0$, i.e. $\boldsymbol{s}$ is stochastically stable, if and only if $\boldsymbol{s} = \boldsymbol{x}$ is a convention and $\gamma(\boldsymbol{x})$ has minimum stochastic potential among all conventions.*

3.2 Asymmetric Nash bargaining solution

Let us apply the methodology outlined above to find the division which the process will converge to. However, before proceeding with the analysis, let us impose a *mean-field assumption* to make the model analytically tractable. Given that information arrival to a buyer/seller about past bargains is a Poisson process, there are fluctuations in the total size of the sample received by the same buyer/seller in different bargaining rounds. The variability of an agents' information sample over time poses significant challenges to an analytical investigation of the model. The mean-field approach smoothes out these variations by assuming that the total amount of information an agent b receives is the same across bargaining rounds.

Technically, assume that the size of the information sample of the buyer b is constant and equal to the amount of information b receives in expectation given the Poisson processes involving b, i.e. $s_b(t) \equiv \sum_{j \in B} g_{bj} = z_b$. The same assumption holds for the seller s. It is important to point out that this assumption imposes no constraint on the variability of each individual Poisson process, but it fixes only the total amount of information that a player receives after the realization of all the Poisson processes. Thus, in some bargaining rounds player b may receive most of the information from her neighbor b', while in other rounds b' may not provide any information. However, the size of the information sample b receives before playing each bargaining round is always the same.

Define $B_{min} = \{j \in B \mid \lceil z_j \rceil \leq \lceil z_b \rceil, \forall b \in B\}$ to be the subset of buyers with the least weighted degree. Let $z_b^{min} = \lceil z_j \rceil$ for $j \in B_{min}$. Equivalent definitions apply to the sellers. Hereafter, also assume that the individual memory $m \geq \max\{z_b, z_s\}$, where $b \in B$ and $s \in S$. The first step is to compute the minimum resistance to moving from the convention $\mathbf{x}$ to the basin of a different convention $\mathbf{x}'$. This is done in the following lemma.

Lemma 2. *The minimum resistance to moving from x to a state in some other basin is $\lceil R(x) \rceil$, where:*

$$R(x) = \min\left\{ z_b^{min}\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}\frac{v(1-x)}{v(1-\delta)}, z_s^{min}\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} \quad (4)$$

The intuition is as follows. Some agents have to make mistakes in order for the system to move from one convention to a state in the basin of another convention. The agents who will switch with the least number of mistakes in their sample are the ones who receive the smallest samples. This explains the factors z_b^{min} and z_s^{min} in equation (4). Now, consider the case when some sellers make a mistake. The smallest mistake they can make is to demand a quantity δ more than the conventional demand $1-x$. If they do this, buyers will attempt to resist up to the point when the utility from getting the conventional amount x some of the time, i.e. when sellers do not make a mistake, is equal to the utility from getting the lower amount $x - \delta$ all the time. This gives the first term in equation (4). The third term is the equivalent of the first one, but this time the buyers make a mistake and demand δ more than the conventional amount x.

Another possibility is that some buyers make a mistake, but this time they demand less than the conventional amount x. The "worst" mistake, from the buyers' point of view, would be to demand the minimum amount δ. If they do this, sellers will only switch at the point when the utility from getting the higher amount $1 - \delta$ some of the time, i.e. when buyers make a mistake, is higher than the utility from getting the conventional amount x all the time. This gives the second term in equation (4). The careful reader will point out that there should also be a fourth term, i.e. the equivalent of the second one with the roles of buyers and sellers reversed. This is true, but it is not included in equation (4) because this term is never strictly smaller than the last term.

The expression for $R(x)$ in (4) is the minimum of three monotone functions: the first two are strictly decreasing in x, while the last one is strictly increasing in x. Thus, $R(x)$ is first strictly increasing and then strictly decreasing as x increases, so it achieves its maximum at a unique value on the subset D.[7] Using this fact, the following theorem shows that there is a unique stable division, which is the asymmetric Nash bargaining solution with weights depending on the agents in each group with the least weighted degrees.

Theorem 2. *There exists a unique stable division* $(x^*, 1-x^*)$. *It is the one that maximizes the following product:*

$$u^{z_b^{min}}(x)v^{z_s^{min}}(1-x) \tag{5}$$

In other words, it is the asymmetric Nash bargaining solution with weights z_b^{min} *and* z_s^{min}.

The intuition behind the solution is that if the precision δ is sufficiently small then over time the two groups will settle on a conventional division, which is the asymmetric Nash bargaining solution. This solution crucially depends on the communication networks that buyers and sellers use to learn about past bargaining rounds to determine what to demand once they are picked to play. More precisely, *ceteris paribus* (i.e. agents' risk-aversion in the two groups is the same), the share a group gets hinges on the agents in the group with the least number and/or weakest communication links. The reason is that these agents will be the least informed when it comes to play the game, and therefore they will be the most susceptible to respond to mistakes from the other side. Over time, this susceptibility weakens the bargaining position of the whole group.

The proof of the theorem follows from two lemmas from Young [1993a]. The first lemma shows that a division $(x, 1-x)$ is generically stable if and only if x maximizes the function $R(x)$ in equation (4). The second lemma shows that the maxima of $R(x)$ converge to the asymmetric Nash bargaining solution which maximizes the product in (5). This solution is clearly analogous to the one in theorem 3 in Young [1993a]. The key difference here is that the solution in theorem 2 above depends explicitly on the internal communication structure of the group of buyers/sellers. This allows the derivation of the comparative statics results in section 4 and the empirical analysis of the Fulton fish market.

It is worthwhile to point out that it is possible to derive an equilibrium division that depends on the network structure in a richer way than the number of direct connections of the agents. Define the *geodesic distance* $d_{ij}(g)$ between i and j in g as the minimum number of links that need to be used along some network path to connect i and j, and the *q-neighborhood* of an agent i as $L_i^q(g) = \{j \in N | d_{ij}(g) = q\}$. Let $\delta \in (0,1)$ and denote by $z_i^q(g,\delta) = z_i(g) + \sum_{k=2}^{q} \sum_{j \in L_i^k(g)} \delta^{k-1} z_j(g)$ the (discounted) *q-degree* of i, which is the discounted sum of the weighted degrees of agents at a geodesic distance q from i. It is straightforward to extend the model to a setting where a buyer hears information not just from her direct friends, but also from the friends of her friends, and so on up to a

[7] Technically, $R(x)$ can achieve its maximum at one value x^* or at two values x^* and $x^* + \delta$. As $\delta \to 0$ these two values clearly converge to a unique maximum x^*.

social distance q. In this extended model the statement of theorem 2 would be unchanged except for the weights that would be equal to z_b^{qmin} and z_s^{qmin}. This extended set-up makes the comparative statics analysis in section 4 more involved without adding any further insight, so this paper assumes throughout that $q = 1$.

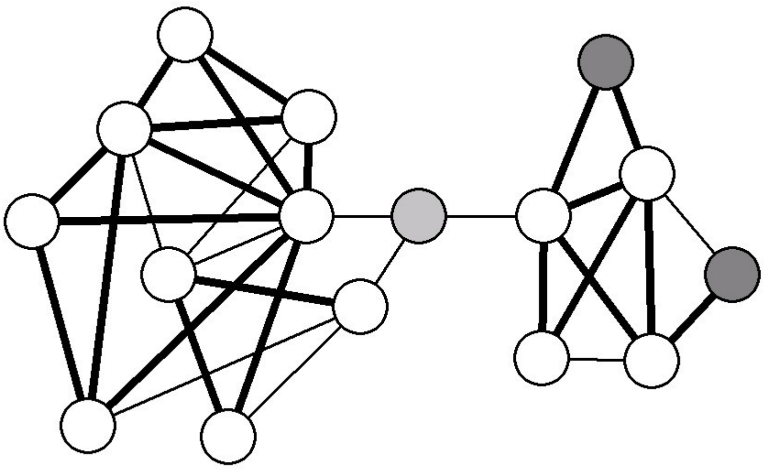

Figure 2: An example of a weighted network with 16 agents. There are two types of links: strong links (in bold) have weight 1 and weak links have weight 0.5. Color-coded nodes denote the agents belonging to the subset of least connected agents.

As an illustrative example, figure 2 is the graphical representation of a weighted network of, say, buyers. There are 16 agents, and there are two types of links: strong links with weight 1 and weak links with weight 0.5. The subset B_{min} of agents with the least information has three individuals, who are color-coded in the figure. Note that there are two typologies of agents who can belong to this subset. The first one is represented by the two agents color-coded in dark gray: they rely on just two sources for information on past bargaining rounds, and both of these sources belong to their own sub-community. They are strongly linked to these sources, but they are very susceptible to potential mistakes in the information coming from them. They are *peripheral* agents in the network, who rely excessively on information from their *own community*. On the other hand, the second typology is represented by the agent color-coded in light gray: she relies on a good number of sources from both sub-communities, but they are only weakly connected with her. She is connected to different parts of the network making her very exposed to any kind of information circulating in the network, including potential mistakes. She is an agent with *weak links* who is very susceptible to information flowing in the network because she connects *across communities*. Section 3.3 below will include further discussion on this.

Finally, as in standard bargaining models, the solution also depends on the utilities of the agents. *Ceteris paribus* (i.e. the least connected agents in each group have the same weighted degree), a group with less risk-averse agents will have a stronger bargaining position because agents who are less risk-averse are more likely to take chances, and therefore they are more demanding.

3.3 The weakness of weak ties

What is the desirable communication structure for the members of a group of individuals that engage in this bargaining process with another group? First, let us define a class of *quasi-regular networks*, which are "generated" by a given regular network.

Definition 8. Consider the set G of undirected networks with n nodes and at most L links. Let $g_{d,a}$ be a regular network with degree $d = \lfloor \frac{2L}{n} \rfloor$ and link strength a, i.e. it belong to $\overline{g}_{d,a}$ which is the class of largest regular networks in G. The network $g \in G$ is a quasi-regular network generated by $g_{d,a}$ if it can be obtained by randomly adding k links of any strength to $g_{d,a}$, where $k \in [0, L - \frac{n}{2})$.

A quasi-regular network is a network that is similar to a regular network in the sense that the links are distributed evenly among the nodes and there is minimal degree variation. Note that if $L/n \in \mathbb{N}$, i.e. the links can be exactly divided among the nodes, then the set of quasi-regular network coincides with the class of regular networks $\overline{g}_{d,a}$. If $L/n \notin \mathbb{N}$ then each node has at least as many links as in the generating regular network, and the remaining links are randomly assigned. The desirable communication structure for a group is a quasi-regular network, as the following corollary shows.

Corollary 1. *Fix a communication network g^S for the sellers. Consider the set G of all possible communication structures g^B among the n_B buyers such that the total number of links is $L < \frac{n_B}{2}(n_B - 1)$ and the strength of each link is in the range $[\underline{s}, \overline{s}]$, where $\underline{s}, \overline{s} \in \mathbb{N}$. The subset of networks $G_B \subset G$ that gives the highest share to buyers are the quasi-regular networks generated by regular networks in $\overline{g}_{d,\overline{s}}$, where $d = \left\lfloor \frac{2L}{n_B} \right\rfloor$. The same statement holds reversing the roles of buyers and sellers.*

For illustrative purposes it is easier to give the intuition for the case where $L/n_B \in \mathbb{N}$. First, the desirable network must have communication links of maximum strength because they carry more information about past rounds, decreasing in this way buyers' susceptibility to sellers' mistakes. Second, a regular network is desirable because it is the network where the buyers with the lowest degrees have the highest possible degree given the constraint L. Informally, (quasi)-regular networks are very "steady": they have no "weak points" that could be more susceptible to sellers' mistakes.

There is a long-standing debate in the sociological literature on what constitutes a desirable network to be embedded in for a group of individuals. A seminal paper by Granovetter [1973] introduced the idea that weak ties play an important role in networks because they connect individuals with few characteristics in common and that have non-overlapping neighborhoods, allowing them to access non-redundant information. For instance, Granovetter [1995] shows that individuals with many weak ties are better at finding employment through their social networks. A rough summary of this view is that networks with a significant fraction of *weak ties* and *high degree variability* are desirable because they facilitate the flow of information.

On the other hand, Coleman [1988] argues that close-knit, uniform networks formed by strong bonds are desirable. The rationale is that these strong connections and their even distribution make it easier to establish an informal, decentralized monitoring by all members of the flow of information. Moreover, there are no peripheral individuals who could be potential defectors. He gives the example of the network of wholesale diamond traders in New York: strong family, religious and community ties ensure that information about any 'cheating' will be quickly available to all the members leading to the exclusion of the cheater from the community. A rough summary of this view is that networks composed exclusively by *strong ties* and *minimal degree variability without peripheral individuals* are desirable because they facilitate monitoring of what is going on in the network.

In the context described by this model, corollary 1 shows that Coleman-type networks are the most desirable for the members of a group: quasi-regular networks exclusively formed by strong ties are desirable because they allow the effective sharing of information about past demands. However, it is important to understand that this is not an absolute statement about the two views, which are, in fact, complementary. There are two key aspects of this model which determine the desirability of a Coleman-type network. First, the new information that circulates in the network is negative: mistakes made by the other side that individuals in the group should not respond to. Second, the final outcome is the establishment of a norm for the whole group, so the important factor is how structural properties of the group as a whole, not the structural position of single agents, influence the outcome. A regular network with strong ties ensures that each player has a lot of information about the state of the system so that new negative information has a very low probability of affecting the group. Moreover, the regularity of the networks ensures that there are no weak points where negative information has a higher probability of "entering" the group. On the other hand, in a model where new information is positive and valuable (e.g. innovation, job opportunities) then the desirable network would probably be closer to the Granovetter's type because it would facilitate the effective circulation of positive information.

4 Comparative statics

In the network literature it has become popular to look at first order stochastic dominance (FOSD) and second order stochastic dominance (SOSD) shifts in the degree distribution, as defined in section 2, to analyze the effects of changes in the network structure.[8] If one considers a binary network, i.e. with 0-1 type of links, then a FOSD shift means, roughly speaking, looking at a network with "more" links, and a SOSD shift leads to a network with more "equally distributed" links. In this paper we consider weighted networks, which broaden the type of changes in network structure that can be analyzed: a FOSD shift means looking at a network with "more" and/or "stronger" links, and a SOSD shift leads to a network with a more equal distribution of the number and/or strength of links.

The following theorem shows how the asymmetric Nash bargaining solution (ANB)

[8] See, among others, Galeotti et al. [2006] for applications of this methodology.

in theorem 2 varies with changes in the degree distributions of the buyers and sellers' networks.

Theorem 3. *Let $(x^*, 1-x^*)$ be the ANB for sets of agents B and S that communicate through networks g^B and g^S with weighted degree distributions $p_b(z)$ and $p_s(z)$. Consider the weighted degree distributions $p'_b(z)$ and $p''_b(z)$ of networks g'^B and g''^B respectively, and let $p'_b(z)$ FOSD $p_b(z)$ and $p''_b(z)$ SOSD $p_b(z)$.*

(i) Let $(x'^*, 1-x'^*)$ be the ANB for sets of agents B and S with degree distributions $p'_b(z)$ and $p_s(z)$. Then $x'^* \geq x^*$.

(ii) Let $(x''^*, 1-x''^*)$ be the ANB for sets of agents B and S with degree distributions $p''_b(z)$ and $p_s(z)$. Then $x''^* \geq x^*$.

The same statement holds reversing the roles of buyers and sellers.

The theorem states that individuals who belong to a *denser* social group, i.e. with more numerous and/or stronger communication channels, will fare better. Similarly, individuals who belong to a *more homogeneous* social group, i.e. with more equally distributed connections in terms of number and/or strength of links, will also be better off. The intuition is that agents in these groups will have access to better information about the history of past deals experienced by other members in their group. Thanks to this informational advantage, they are less likely to respond to mistakes by the other side, and they are therefore able to maintain an advantageous bargaining position.

From the statement of theorem 2 it is also straightforward to show that a more general comparative statics result than theorem 3 is true: adding a link to the communication network g^B of a group B weakly increases the share individuals in that group obtain in equilibrium. There are two reasons for the choice to restrict the comparative statics analysis to shifts in the degree distribution. First, the same approach applies to the extension in section 6 where buyers and sellers belong to the same network, while the more general statement does not hold there. Second, the statement in theorem 3 is more suitable to empirical verification.

Stochastic dominance shift arguments are useful, but it is not straightforward to see how this type of result can be verified empirically. It is certainly not an option to artificially "engineer" a shift in the degree distribution of a network. Even in a controlled experimental setting, this would be very difficult due to the intrinsic complexity of a network with more than a few individuals. It is also very hard to imagine exogenous shocks on a social structure that would result in these stochastic dominance shifts.

However, from cross-sectional studies we know that homophily is a powerful determinant of social structure and that networks composed of different *types* of individuals often have internally different structures. Moreover, studies that compare cross-sections of different networks are much easier to undertake than tracking the evolution of a single network.

Only mild assumptions are required to extend the model to a context with several groups of buyers (or sellers). Assume that there is one group of sellers and that there are k

separate groups of buyers such that buyers communicate within their group but not across groups.[9] The group of sellers plays the Nash demand game with each group of buyers. The main assumption that is required is that each seller knows which group a buyer belongs to and he only receives information from other sellers on previous transactions with buyers from that group. Moreover, when a seller determines which offer to make to a buyer from a certain group, he does not use information from transactions with buyers in other groups. Mathematically, the whole system can be represented by k different processes that run "in parallel," and the dynamics/outcomes of one process are completely independent from the ones of the other processes. Clearly, the results in this paper apply to each one of these processes. The following corollary presents this set-up more formally and it states its implications.

Corollary 2. *Consider one group of sellers S who communicate through g^S, and k groups of buyers $B_1, ..., B_k$ who communicate through separate networks $g^1, \ldots, g^k$ with weighted degree distributions $p_1(z), \ldots, p_k(z)$ respectively. Assume $B_i \cap B_j = \emptyset$ and sellers know which group a buyer b belongs to. Then sellers will reach different conventions with different groups of buyers on the share x_i^* that buyers in B_i get. Moreover:*

(i) If $p_1(z)$ FOSD $p_2(z)$ FOSD $\ldots$ FOSD $p_k(z)$, then $x_1^* \geq x_2^* \geq \ldots \geq x_k^*$

(ii) If $p_1(z)$ SOSD $p_2(z)$ SOSD $\ldots$ SOSD $p_k(z)$, then $x_1^* \geq x_2^* \geq \ldots \geq x_k^*$

The same results hold for one group of buyers B playing the game with k groups of sellers.

Proof. Straightforward by applying theorems 2 and 3. □

This corollary states a clear and testable prediction of the model: in a market with different groups of buyers where communication only occurs within groups, buyers that belong to denser and/or more homogeneous groups will fare better. The next section explores how these predictions shed light on the observed pricing patterns in the Fulton wholesale fish market.

5 An Application: The Fulton wholesale fish market

Wholesale fish markets have historically attracted the attention of economists because they offer fertile ground for econometric tests of a competitive market.[10] They also have several characteristics that make them an ideal setting to test this model: *(i)* all transactions are private between one buyer and one seller; *(ii)* bargaining is minimal and usually consists of take-it-or-leave-it offers; *(iii)* they are perfectly competitive with a large number of buyers/sellers, low entry costs, and no search costs; *(iv)* products are very homogeneous; *(v)* the price of the product varies considerably from day to day and

[9] An equivalent set-up is to assume that there is one group of buyers B connected by a network g^B which is composed of k *components*.

[10] See Kirman [2001] for a review.

it depends on exogenous factors largely unknown to the market participants; *(vi)* there are no inventories so separate days can be considered independently.

Here we will analyze a dataset on transaction prices in the Fulton wholesale fish market in New York collected by Kathy Graddy in 1992.[11] The goal is to provide evidence that the predictions of the model in this paper provide an explanation for a puzzling finding found by Graddy [1995] in the pricing patterns observed in this market: Asian buyers pay a significantly lower price than white buyers for the same product sold by a white seller.

The Fulton wholesale fish market in lower Manhattan is the largest in the United States with 100-200 million pounds of fish sold per year.[12] Transactions start at 3am and end at 9am; there are about 35 sellers and several hundred buyers. The Fulton fish market (FFM) dataset contains all 620 sales of whiting made by one white seller from April 13th till May 8th 1992. For each transaction the dataset contains the time of sale (month, day, hour and minutes); the price per pound; the quantity sold; customer information (unique identifier, ethnicity); the size (small, medium or large), type (king or normal) and quality (1-5 scale) of whiting; mode of transaction (cash or credit; in person or by phone); geographical location (Manhattan, Brooklyn, other) and type of establishment (store or fry shop) owned by buyer; total quantity that seller received and sold on that day.[13]

Only a subset of 132 observations has been used for the main analysis conducted below. Duplicate entries and sales with missing data were omitted. The few transactions carried out on the phone were also excluded. Following Graddy [1995], only sales involving medium sized normal whiting of medium quality were included in order to focus on a homogeneous product. A common characteristic of fish markets is the presence of large fluctuations in prices in the last 1-2 hours of the market depending on whether there is excess demand or supply on a given day. In order to avoid this effect the data was restricted to transactions carried out between 3am and 7am, which correspond to the busiest hours of the market.[14]

Graddy [1995] found a puzzling result from her analysis of the FFM: Asian buyers pay a significantly lower price than white buyers for the same product sold by a white seller. She investigates a number of potential determinants of this result, but in her concluding remarks she writes that "[...] price discrimination is present. The reason behind price discrimination is less clear" (p. 87). It is difficult to explain why the price difference is not arbitraged away in a market with a healthy competition, no obvious entry barriers,

[11] I am very grateful to Kathy Graddy for allowing me to use her dataset.

[12] In November 2005, the Fulton fish market moved from lower Manhattan to the Bronx, see Jacobs [2005] for a short account of the move.

[13] See Graddy [2006] for a more detailed description of the dataset and of the FFM.

[14] The goal of this data selection was to follow Graddy [1995] as closely as possible, but it is likely that there are very minor discrepancies between the two procedures. To be included in the subset of data analyzed here, transactions had to have the following characteristics: (i) time of trade and ethnicity of buyer are not missing; (ii) trade happened before 7am; (iii) trade was made in person; the fish was (iv) normal whiting of (v) medium quality (3 on a 1-5 scale) and of (vi) medium size. Moreover, one duplicate transaction was excluded. Steps (i)-(vi) reduced the dataset to $n = 132$ observations, while Graddy's selection procedure reduced it to $n = 131$.

low search costs and a homogeneous product.[15]

The model in this paper puts forward an alternative rationale for the observed price difference. There are *distinct groups of buyers* in the Fulton fish market depending on their ethnic group, and buyers communicate with other members of their group to learn the current price of fish. The *group of Asians is denser* and it is therefore a better channel of information on the uncertain price of the product. As the market unfolds the density of the communication network in their group gives the Asians an informational advantage: they *learn the ongoing price more accurately* and they exploit this additional knowledge to obtain a lower price from the sellers.

The argument will consist of four steps. First, we will document the price differential between Asians and whites by replicating the analysis in Graddy [1995]. Second, we will show that the price differential is not present in the first two hours of the market, and emerges only afterwards. We interpret this as evidence of learning: the price differential emerges only in the course of the market as the buyers learn the daily price of fish. Third, we will provide further evidence of learning by showing that the variability of price within the Asians decreases over time while the same does not hold for whites. Finally, we will draw on a number of studies to argue that the key competitive advantage of Asian buyers is that they belong to a denser social group than whites, in agreement with the predictions of the model.

The objective of the first step in this empirical analysis of the FFM is to reproduce the main finding in Graddy [1995]. This ensures consistency with the original study of the FFM and it confirms that the analysis carried out here correctly picks out the price differential between Asians and whites. The first column in table 1 reproduces the regression analysis in Graddy [1995]. The price of each of the $n = 132$ trades is regressed on the following independent variables: *TIM1, TIM2* and *TIM3* time dummies equal to 1 if the purchase was made before 5am, in the 5am-6am and in the 6am-7am time periods respectively; an $ASIAN$ dummy equal to 1 if the buyer is Asian; a $BLACK$ dummy equal to 1 if the buyer is black; a $CASH$ dummy equal to 1 if the purchase was paid in cash; a $MLOC$ dummy equal to 1 if the buyer is from Manhattan or Brooklyn; a $STORE$ dummy equal to 1 if the buyer's establishment is a store; and a dummy $DATE\ X$, not shown in the table to save space, if the purchase was made on day X.[16]

[15] In a recent contribution, Graddy and Hall [2009] construct a structural model to explain the pricing data in the FFM: their simulations match the observed prices very well. The key assumption they make to reproduce the price differential between Asians and whites is that Asians have a higher price elasticity of demand than whites. The approach they use is different from this paper, and the stories proposed in Graddy and Hall [2009] and in this paper to explain the price differential complement each other.

[16] Two variables present in Graddy's regression have not been included: $AVQUAN$ (i.e. average quantity purchased by the customer during the time period) and REG (i.e. the number of times the customer purchased during the time period). The results of the regressions presented here would not change if these variables were to be included.

TABLE 1 - Determinants of the Price of Whiting

Variables	(1) All times	(2) Before 5am	(3) From 5am till 7am
TIM1	-.0066	-	-
	(.0201)		
TIM2	.0244	-	-
	(.0201)		
TIM3	.0095	-	-
	(.0185)		
ASIAN	-.0488***	.0085	-.0455**
	(.0150)	(.0231)	(.0202)
BLACK	.0115	.0658	.0138
	(.0195)	(.0290)	(.0249)
CASH	.0249	-.0078	.0194
	(.0147)	(.0175)	(.0203)
MLOC	.0061	-.0214	.0069
	(.0147)	(.0204)	(.0175)
STORE	.0465***	.0514***	.0537**
	(.0128)	(.0187)	(.0165)
R^2	.990	.998	.986
Number of observations	132	38	86
Number of Asian buyers	70	22	48
Number of White buyers	49	20	29

Standard errors in brackets.
The coefficients on the date dummies are not reported.
*** Significant at the 0.01 level. ** Significant at the 0.05 level.
* Significant at the 0.1 level.

The coefficients in column 1 support Graddy [1995]'s conclusions. The Asian dummy is negatively correlated with price and statistically significant at the $p = 0.01$ level: Asian buyers get a price that is approximately 5% lower than white buyers. All the other controls are not significant, apart from the *STORE* and the date dummies that are strongly significant due to the strong dependence of the price of fish on daily conditions.[17] Note that the variables in the regression explain essentially all the variation in prices observed in the dataset.

However, the results in column 1 do not necessarily support the thesis that Asian buyers gain a competitive advantage through learning as opposed to some other mechanism. A much stronger piece of evidence for a learning story would show that *the price differential emerges over the course of the trading day as learning takes place.* The FFM dataset allows a quantitative test of this hypothesis. Column 2 in the table shows the coefficients for the same regression as column 1, but considering only trades that happened before 5am. Recall that the market starts around 3am and picks up around 4am, so transactions that happened before 5am can be considered as "early" trading.[18] The coefficient of the Asian dummy is now positive and insignificant: Asian buyers get a price that is slightly higher than white buyers, and *statistically there is no difference* between the two.

The price differential in favour of Asian buyers emerges later in the trading day. Column 3 in the table shows the same regression as column 1, but considering only trades that happened between 5am and 7am.[19] The coefficient of the Asian dummy is negative and statistically significant at the $p = 0.05$ level: after the first two hours of the market, the price differential emerges and the Asian buyers trade at significantly lower prices compared to white buyers. This is strong evidence in favour of the thesis that different rates of social learning within the two groups drive the emergence of the price differential: any alternative story based on individuals' and/or groups' characteristics would face the difficult task of explaining why these characteristics emerge only in the course of the trading day.

The FFM pricing data can offer a further piece of evidence in support of the social learning story: if buyers within the same group are learning the daily price of fish then *the variability of prices paid by members of the same group should decrease over time.* At the beginning of the market only a few transactions have taken place and therefore the scarce information on previous sellers' demands is of little guide to buyers: the prices that buyers pay should therefore vary a lot across buyers reflecting the lower accuracy of the small samples of information at their disposal. On the other hand, after some time buyers in the same group can provide abundant information about sellers' past demands: the prices that buyers pay should vary less reflecting the higher accuracy of buyers' demand

[17]A difference between these regressions and Graddy's is that here the coefficient on the $STORE$ dummy is positive and statistically significant in all regressions: store owners pay higher prices than non-store owners that buy fish for fry-shops. The likely explanation is that this is driven by an imperfect measure of quality: fry shops tend to purchase lower quality fish than store owners, and the classification by Graddy as medium quality (3 on a 1-5 scale) based on sight, feel and smell might cluster together fish of slightly different quality. In order to ensure that this has no effect on the Asian dummy coefficients, all regressions were repeated further restricting the sample to store owners only, who constitute the large majority of buyers. The results do not change.

[18]The coefficients of column 2 do not change substantially if "early" trading is defined as transactions that happened between 4am and 5am, excluding in this way the first hour of the market.

[19]As in Graddy [1995], trades after 7am are excluded from the analysis because of large fluctuations in prices in the last 1-2 hours of the market due to excess demand or supply.

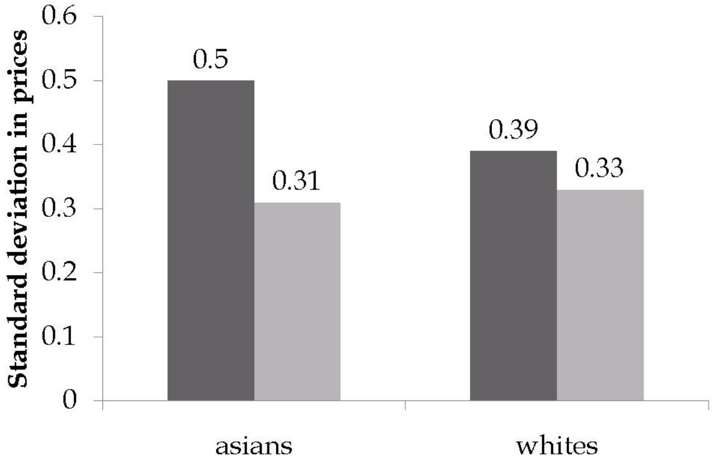

Figure 3: Standard deviation in prices paid by Asian and white buyers for the 4-5am (dark gray bar) and 6-7am (light gray bar) time periods.

which are now based on the large samples of information at their disposal. Given that social learning is faster within the Asian group, the variability of prices should decrease faster for Asians than for whites.

Figure 3 compares the standard deviation in prices for transactions between 4am and 5am (in dark gray) to the standard deviation for transactions between 6am and 7am (in light gray) for whites and Asians. They both decrease over time, but the decrease for Asian buyers is visibly larger. A two-sample variance comparison test rejects at the 99% confidence level the null hypothesis that for Asian buyers the standard deviation of prices at 4-5am is the same as the one at 6-7am. The same test for whites cannot reject the null hypothesis at the 90% confidence level. Moreover, the result does not hinge on the difference in initial standard deviations for whites and Asian buyers: the test cannot reject at the 90% confidence level the null hypothesis that the standard deviation of prices at 4-5am is the same for Asian and white buyers.[20]

The final step necessary to validate the social learning story advanced by the model is that social learning within the Asian buyers is faster than within the white buyers because the Asian buyers' group has a higher density of social connections. Unfortunately, there is no data available on the interactions among different buyers in the FFM data and therefore it is not possible to *test* this thesis directly. However, it is possible to use findings from other studies to provide *corroborating evidence* in support of this thesis.

First of all, Graddy's personal field observations of the FFM support the assumption that buyers are split in different groups along ethnic dimensions. She remarks that "[v]ery little social contact appears to take place between groups of Asian buyers and groups of white buyers" (p. 84) and "[b]uyers do not realize they are receiving better or worse prices than other buyers" (p. 83-84). It is not surprising that ethnicity is the driving force for

[20]The same results hold if we consider transactions *before 5am* instead of in the range 4-5am (this adds 6 additional observations). Taking the 4-5am, 5-6am and 6-7am time intervals separately, the test cannot reject at the 90% confidence level the null hypothesis that the standard deviation of prices is the same for Asian and white buyers. The results are the same if we use the robust equal variance test, which does not assume that the underlying distribution of prices is normal.

the formation of different groups among buyers. McPherson et al. [2001] review a large amount of empirical evidence that shows that "[h]omophily in race and ethnicity creates the strongest divides in our personal environments" (p. 415).

Social connections play an especially important role in business transactions in the overseas Asian community. Gordon Redding [1995]'s extensive study of overseas Chinese networks stresses that "co-operativeness [...] converts an otherwise disparate group of entrepreneurs into a significant economy" (p. 62) and "[p]ersonalism does in Asia what law does in the West [...] [w]ithout [what is termed *guanxi* or connections] nothing can be made to happen [...] the instinct of the Overseas Chinese to trust friends but no-one else is very deep-rooted" (p. 63). Redding [1995] (p. 65) explicitly states that one of the main purposes of social connections is to gather information in an uncertain environment:

> For the Overseas Chinese the uncertainties of the business environment mean that playing fields are not level. The markets are imperfect but they are at the same time full of opportunities and ideal for the exercise of entrepreneurial talent. Information here becomes crucial, and reliable information, ahead of the game, becomes an important trading currency in the process of building up networks for capital pooling, later information and risk sharing. [...] So the Chinese rules are: put your trust primarily in 'your own' people; seek the opportunities by trading rare information; share that information to build allegiances [...]

The evidence on the importance of social connections for Asians in the US is not limited to the Chinese community. In their comprehensive "A Demographic Portrait of Asian Americans" Xie and Goyette [2004] stress that "[m]ost Asian Americans are recent immigrants and as such maintain a strong identity with their home culture [...] Ethnic communities offer many practical resources to immigrants, including [...] information in native languages, and entrepreneurial opportunities" (p. 66). Rauch [2001] reviews further evidence of the high density of ethnic Asian social networks.

The Asian buyers in the FFM seem to be no exception to the high density social groups observed among Asian immigrant communities. In her fieldwork Kathy Graddy observed that "Asian buyers certainly spoke to one another and congregated much more frequently than white buyers."[21] Even though all these studies do not provide conclusive quantitative evidence, they at least give broad support to the claim that the group of Asian buyers is part of a denser social network than the group of white buyers.

Summing up, this analysis provides corroborating evidence in support of a differential social learning story as a driver of the price differential between Asian and white buyers. In the first hours of the market, there is scarce information on the daily price of fish because very few transactions have taken place and therefore buyers cannot rely on their contacts to learn what the daily price is. After a good number of transactions have taken place, information on prices asked by sellers circulates among buyers who start learning the daily price of fish. The learning process occurs faster among Asian buyers, who are tightly connected with one another, and over time they cumulate an informational advantage that is reflected in the lower prices of their trades compared to white buyers.

[21] This quote is taken from a personal communication with Kathy Graddy.

It is not easy to find an alternative story that fits this analysis as well. For instance, an alternative rationale for the price differential could be that Asian buyers have better bargaining skills. Besides the fact that haggling is non-existant or minimal in this market, this story would have to explain why Asian buyers only employ their bargaining skills after 5am. Any other story based on individuals' and/or groups' characteristics that the analysis may not control for would have to explain why these characteristics become effective only after 5am.

Finally, it is worthwhile to point out and reiterate that there are several other contexts where the predictions of this model are relevant. A prominent example is international trade markets. James Rauch and other researchers have shown that the density of ethnic immigrants' social networks is an important determinant of international trade patterns.[22] Moreover, Rauch and Trindade [2002] explore two potential ways in which belonging to a social network may give a competitive advantage. The first one, proposed by Greif [1994], is a contract enforceability story: if a member of the social network has been cheated by a trader then every member of the network punishes the cheater by stopping future business with that trader. The second one is that the social network gives an informational advantage: being part of the social network gives access to key information to learn about the price of a product.

Rauch and Trindade [2002] are able to distinguish between the two mechanisms by comparing trade volumes in products with "reference prices" whose price is well-known to trade in products without "reference prices" whose price is uncertain. They find that ethnic Chinese social networks have a much larger effect on trade of products with uncertain price and they conclude that the main function of social networks is to provide an informational advantage. Similar research by Kumagai [2007] confirms that the same effect is present for Japanese ethnic networks. Furthermore, Kumagai [2007] shows that Japanese networks with higher density lead to a larger effect, in agreement with the comparative statics results presented in section 4.

6 A unique network of buyers and sellers

There are circumstances where buyers and sellers do not belong to different groups. For instance, in a small village everyone is included in the same community and therefore buyers would gather information from sellers as well as other buyers, and viceversa. This section investigates how the results in sections 3 and 4 change if buyers and sellers belong to the same communication network. Section 6.1 illustrates the changes to the model and derives the bargaining solution, section 6.2 discusses the implications for the desirable communication structure for the members of a group, and section 6.3 carries out the comparative statics analysis.

[22] See Rauch [2001] for a comprehensive review.

6.1 Set-up and bargaining solution

Only minimal changes to the model are required to describe a situation where buyers and sellers are part of the same communication network. First, consider the information arrival process. Assume player $b \in B$ is picked to play the game at time $t+1$: in the $\Delta t = 1$ time period she receives information from other buyers in B *and* other sellers in S about past bargaining rounds. As before, the expected total amount of information b receives before each bargaining round is equal to $\sum_{j \in L_b(g)} E[P(s_{bj})] = E[P(s_b)] = \sum_{j \in L_b(g)} g_{bj} = z_b$. The only difference is that here $z_b = \sum_{j \in B,S} g_{bj}$: b's sample comes from agents in both the buyers and sellers groups. The expected realizations of the Poisson processes define a *weighted, undirected network* of buyers *and* sellers, which is represented by a symmetric matrix $[g_{ij}]^{n \times n}$.

Second, consider the elements $\mathbf{s}$ of the state space S of the Markov process. Here, $\mathbf{s} = \{v_1, ..., v_{2n}\}$, i.e. for each agent i there are two vectors v_i and v_{2i} of size m. If $i = 1, ..., n$ then $v_i = \{y^i_{k-m+1}, ..., y^i_k\}$, i.e. if $i \in B$ then the entries of v_i are the m last demands made by sellers in bargaining rounds involving i, and if $i \in S$ then the entries of v_i are i's last m demands. Similarly, if $i = n+1, ..., 2n$ then $v_i = \{x^i_{k-m+1}, ..., x^i_k\}$, i.e. if $i \in S$ then the entries of v_i are the m last demands made by buyers in bargaining rounds involving i, and if $i \in B$ then the entries of v_i are i's last m demands. Assume that when a buyer $b \in B$ is picked to play the game, she receives a sample of information from her neighborhood about past demands made by *sellers*, i.e. the elements in the sample come from $v_1, ..., v_n$. Similarly, when a seller $s \in S$ is picked to play the game, he receives a sample of information from his neighborhood about past demands made by *buyers*, i.e. the elements in the sample come from $v_{n+1}, ..., v_{2n}$. Everything else works as before, the buyers (sellers) demand a best-reply to the cumulative distribution of sellers' (buyers') demands in the sample they received.

The first result is that the unique stable division is unchanged from the case of separate communication networks of buyers and sellers.

Theorem 4. *There exists a unique stable division $(x^*, 1-x^*)$. It is the one that maximizes the following product:*

$$u^{z_b^{min}}(x) v^{z_s^{min}}(1-x) \tag{6}$$

In other words, it is the asymmetric Nash bargaining solution with weights z_b^{min} and z_s^{min}.

Proof. The proof follows the same argument as the proof of theorem 2, and it is therefore omitted. □

Lemma 2 holds here as well with minor modifications, and the rest of the proof of theorem 2 follows. The solution is unchanged because what really matters to the deal a group obtains is the size of the smallest information sample among its members, i.e. the weighted degree of the agents with the least number and/or weakest communication links. Whether this information comes from members of the same group or of the other group is inconsequential for the split of the pie. As in theorem 2, the buyers with the minimum weighted degree will be the least informed and therefore they will be more susceptible to respond to mistakes from the sellers. Over time, this susceptibility weakens the bargaining position of the whole group of buyers, leading to the establishment of the conventional split that maximizes equation (6).

6.2 Core-periphery networks

The ANB solution is unchanged by the introduction of communication across groups. However, the desirable architecture for the group of buyers in this setting does not lead to the same outcome as in section 3.3 because now the sellers are part of this network. The corollary below shows that the desirable communication structures for the buyers are *core-periphery networks* where the buyers are at the core and the sellers at the periphery.

Before proceeding it is important to note that core-periphery networks appear in different works of the network literature, but there is unfortunately no agreed upon definition. Here we will adopt an informal definition that captures the essence of the concept: a network formed by a group of agents well connected with each other that form the core, and another group of poorly connected agents each of whom has at least one link with a core agent. The key characteristic of core-periphery networks is that they divide a society into two classes of individuals: on the one hand an elite of core individuals well-connected with each other, and on the other hand a group of peripheral individuals that are dependent on the elite and poorly connected with each other. It is intuitively clear why it would be desirable for the buyers to be at the core of a core-periphery network, the following corollary formalizes this intuition.

Corollary 3. *Consider the set G of all possible connected communication structures g such that the total number of links is $L \leq \frac{n_B}{2}(n_B - 1)$ and the strength of each link is in the range $[\underline{s}, \overline{s}]$, where $\underline{s}, \overline{s} \in \mathbb{N}$. The subset of networks $G_B \subset G$ that gives the highest share to buyers are core-periphery networks. The subset of n_B buyers forms the core which is a quasi-regular network generated by regular networks in $\overline{g}_{d,\overline{s}}$, where $d = \left\lfloor \frac{2L+n_s-1}{n_B} \right\rfloor$. The sellers form the periphery where (i) each $s \in S$ has one or very few links, (ii) each $s \in S$ has at least one link with a buyer and (iii) there is at least one $s \in S$ such that $d_s = 1$ with $g_{sb} = \underline{s}$. The same statement holds with the roles of buyers and sellers reversed.*

The intuition of the proof is the following. First, for the sellers to get the smallest possible share there must be at least a seller s_0 with only one weak link. Second, the sellers $s \in S \backslash s_0$ should have at least the lowest number of links needed for the network to be connected while at the same time take the least number of links away from the buyers. Thus each seller apart from s_0 is connected by one strong link to a buyer. Third, following the argument of corollary 1, the buyers should form a regular network with strong links to maximize the smallest weighted degree among all the buyers. The remaining links can be assigned at random (as long as none of them links to s_0) so the core is a quasi-regular network of buyers and each seller has only one or a few links.

The key for a group to obtain a high share is to create a close-knit "elite" of group members and leave the individuals of the other group at the periphery with minimal connections to the whole community. It helps to think of an example of a small village with landowners and tenants. If the former want to extract as much rent as possible from the latter then they should form a cliquish group amongst themselves, and prevent the formation of the same type of group among the tenants. Communication should happen almost exclusively within the landowners group with each landowner communicating only with one or a few tenants under her. The social structure within the group of landowners should be as uniform as possible so that there are no weak points for the tenants to exploit to improve on their share.

6.3 Comparative statics and the 50-50 split

The fact that the bargaining solution is unchanged when there is a unique network of buyers and sellers does not imply that the comparative statics results in section 4 apply here as well. In this setting changes in the social network structure affect both buyers *and* sellers, and therefore the comparative statics results will be different to the case of separate networks. The following is the equivalent statement of theorem 4 in the modified set-up where buyers and sellers belong to the same communication network.

Theorem 5. *Let $(x^*, 1-x^*)$ be the ANB for sets of agents B and S that communicate through network g with degree distribution $p(z)$.*

(i) If $p'(z)$ is a variance-preserving FOSD shift of $p(z)$ then $x'^* = x^*$.

(ii) If $p''(z)$ is a mean-preserving SOSD shift of $p(z)$ then:

- $x''^* \geq x^*$ *if* $x^* < \frac{1}{2}$, *and*
- $x''^* \leq x^*$ *if* $x^* > \frac{1}{2}$

A *denser* communication network without any changes in the variance of the distribution will leave the equilibrium *ANB* unchanged. This is because the weighted degrees of the least connected buyers and sellers will change in absolute value, but not in relative value to each other. On the other hand, a *more homogeneous* communication network with a degree distribution with the same mean will change the equilibrium because it will affect the relative values of the least connected buyers and sellers. Specifically, as the network becomes more homogeneous the difference between the shares of the two groups narrows down. Thus, societies with more homogeneous social groups have more equitable divisions. In the limit where the social network is regular, the split will be the division that maximizes the Nash bargaining solution. If the agents are homogeneous, this is the *50-50* division, as the following corollary shows.

Corollary 4. *Let g be a regular network with homogeneous agents, then 50-50 is the unique stable division.*

Proof. Let all agents have the same utility function $u(.)$. If g is regular then $\beta \equiv z_b^{min} = z_s^{min} \equiv \sigma$. Substituting this into equation (6) one obtains that the unique stable division $(x^*, 1-x^*)$ is the one that maximizes $u(x)u(1-x)$, which is clearly $x^* = 0.5$. □

Theorem 5 further highlights how the introduction of the communication network as a channel of information about past demands leads to new insights that are not accessible in the Young [1993a] framework. As the statements of theorems 2 and 4 make clear, the fact that buyers and sellers belong to separate or the same communication network has no impact on the long-term equilibrium division making these two cases indistinguishable in Young's framework. However, introducing the network allows to carry out the comparative statics that highlights how changes in the network affect the equilibrium division. The comparative statics clearly differs if buyers and sellers belong to the same network, and this leads to the insight of theorem 5 that a shift in the distribution of connections that makes the network more homogeneous narrows down the difference in the shares that buyers and sellers obtain.

In the extreme case of a regular communication network the equilibrium division is 50-50, which suggests that this well-observed phenomenon may be more prevalent in societies with a very flat and non-hierarchical social structure. Note that also here the mechanism that leads to the emergence of the 50-50 division differs from the mechanism leading to the equivalent statement in Young [1993a]. In Young's framework the 50-50 division emerges in societies where there are some individuals that exchange roles and, at different times, can be both buyers and sellers. On the other hand, in this model there is no exchange of roles and the mechanism leading to the emergence of the 50-50 division is the structure of the communication network that buyers and sellers are embedded in.

7 Conclusion

This paper has presented a model that investigates the informational advantage an individual derives from being part of a group in a large, perfectly competitive market characterized by incomplete information about the price of a homogeneous good. The communication patterns within the group determine the information the individual has before a private bilateral transaction, and the outcome of the bargaining hinges on the accuracy of this information. In the long-term equilibrium every member of the group obtains the same share of the good in each transaction, and therefore the group communication network critically determines the market outcome. More specifically, the equilibrium division depends on the number and the strength of the connections of the peripheral or least connected individuals in each group. An immediate consequence of this result is that individuals belonging to a group with a high density and a homogeneous distribution of connections fare better. Empirical evidence shows that this prediction explains the price differential between Asian and white buyers in the New York fish market. Finally, a modified setting analyzes the case where buyers and sellers are embedded in the same communication network: the peripheral individuals are again pivotal, and the more homogeneous the distribution of connections is the more similar are the shares of buyers and sellers.

There are still several open questions for future work. First of all, the results presented here are valid in the limit where the probability of making a mistake tends to zero. This is a standard assumption in the evolutionary game literature and it allows the use of the powerful methodology in Young [1993b] to find the stochastically stable equilibrium.[23] In this model this assumption means that the least resistant path to equilibrium depends on the network structure only insofar as the structure determines the least number of mistakes needed to steer the system onto the equilibrium path. On the other hand, if the probability of a mistake did not tend to zero the least resistant path may depend on the network in a richer way. This is because the probability that agents switch to equilibrium play in a certain sequence would be of the same magnitude as the probability of a mistake, and the network structure determines the former as well. It is not clear whether this analysis would be analytically tractable, but it would be worthwhile to explore whether the results presented here extend to a regime where mistakes are more frequent.

Second, the analysis in this paper does not investigate the speed of convergence to equilibrium. A common critique of evolutionary models is that it may take a long time to converge to an equilibrium.[24] Simulations of the model indicate that convergence occurs after a few thousand runs for small groups of agents, but analytical bounds are difficult to obtain. The main hurdle is that the speed of convergence depends on the particular sequence of agents on the least resistant path: there are very many potential such sequences leading to different convergence speeds which depend on the network position of the agents in the sequence. More generally, the problem of computing bounds for the speed of convergence of Markov processes on networks is still largely unexplored.[25]

Third, the empirical analysis of the Fulton fish market is just a first step in the testing of the predictions of this model because it provides only corroborating evidence that the model explains the observed pricing pattern. The identification of networks effects in markets is a notoriously difficult task, and methodological advances in the econometric literature are necessary to tackle it properly. The option of conducting lab experiments to overcome the identification issue inherent in field data is usually not viable for network models, where it is difficult to reproduce social relations in a short time in the lab. However, the model in this paper may actually be quite suitable to an experimental investigation in the lab because the network here is simply a communication channel. It is relatively easy to construct protocols to constrain communication among subjects in the lab, and therefore it should be possible to create an artificial market where there are groups of traders with different internal communication structures. This is a promising potential direction for future work.

Network theorists have only recently started to examine models that investigate the role of network structure in determining market outcomes in markets with a large number of agents. In these models the mechanisms through which network structure affects market outcomes vary widely, reflecting the multiplicity of possible types of social interactions. This paper focused on the role of network structure as a carrier of market information. There are at least two future challenges that lie ahead for this literature. The first one is to

[23]However, see Young [1998] for a discussion of how the predictions of some benchmark evolutionary models may hold also in settings where the probability of mistakes does not tend to zero.

[24]See Ellison [1993] for a discussion.

[25]Golub and Jackson [2009] make some progress, see their discussion for more on this topic.

endogenize the network structure integrating these models with the literature on network formation. The second one is to identify empirically the predictions of these models using real market data or in laboratory settings. Hopefully, this model may serve as a starting point for future work in these directions.

A Appendix: Proofs

This appendix contains all the proofs omitted in the main body of the paper. Hereafter let $\delta = 10^{-p}$ ($p \in \mathbb{Z}_+$) be the precision of the demands, and assume $x_t, y_t \in D$, where D is the set of all p-place decimal fractions that are feasible demands.

Proof of Lemma 1. Suppose the process is in state $\mathbf{x}$ at time t, and pick any two agents $b \in B$ and $s \in S$ to play the Nash demand game at time $t+1$. For any sample b receives from her neighborhood, the cumulative distribution $G(y)$ of previous demands by sellers is a probability mass function with value 1 at $1-x$. Thus, b's best reply is always to demand x. Following a similar argument, the seller s' best reply is always $1-x$. It follows that the state of the system at $t+1$ is the same as it was at t, and therefore $\mathbf{x}$ is an absorbing state of P. □

Proof of Theorem 1. The goal is to show that from any initial state $\mathbf{s}$ there is a positive probability p independent of t of reaching a convention within a finite number of steps. Select individuals b, b', b_0 such that $b \in L_{b'} \cap L_{b_0}$ and $g_{b'b_0} = 0$. Similarly, select individuals s, s', s_0 such that $s' \in L_s \cap L_{s_0}$ and $g_{ss_0} = 0$. Note that such individuals must exist because by assumption the networks are connected and they are not complete networks. Figure 1 in section 3 illustrates two networks of buyers and sellers with individuals b, b', b_0 and s, s', s_0. Note that in figure 1 agents b_0 and s_0 are labeled b'' and s'' respectively. Consider the following steps from t onwards.

(i) $[t, t+m]$: There is a positive probability that b and s (or agents like them[26]) will play the game in every period $t \in [t, t+m]$. Also, there is a positive probability that b and s will draw samples σ and σ' respectively. Let x and y be the best replies of b and s to these samples respectively. Then there is a positive probability of obtaining a run of (x, y) for m periods in succession such that $v_b = (y, ..., y)$ and $v_s = (x, ..., x)$.[27]

(ii) $[t+m+1, t+2m]$: There is a positive probability that b' and s' (or agents like them[28]) will play the game in every period $t \in [t+m+1, t+2m]$. There is a positive probability that they will sample from $v_b = (y, ..., y)$ and $v_s = (x, ..., x)$

[26] A player $b_i \in B$ that is "like" b is such that $b_i \in L_{b'} \cap L_{b_0}$. This condition allows b_i to potentially collect the same sample of information σ as b. Similarly, a player s_i that is "like" s is such that $s_i \in L_{s' \setminus s_0}$, where $L_{s' \setminus s_0} = \{j \in N \mid j \in L_{s'}, j \notin L_{s_0}\}$.

[27] The argument here has been simplified on a number of dimensions for expositional purposes: 1) it is not necessary that the same pair of agents plays in each of the m rounds, it is sufficient that they are "like" b or s (see footnote above); 2) it is not necessary that the m rounds are consecutive, as long as there is a finite time between them and they are still in the state $\mathbf{s}$ at the end of the third step below; 3) if different agents are involved in these rounds, then the state $\mathbf{s}$ of the system at the end of this step will not be such that there are two vectors $v_b = (y, ..., y)$ and $v_s = (x, ..., x)$, but such that there are m entries of vectors $v_i \in \mathbf{s}$ equal to y and m entries of vectors $v_j \in \mathbf{s}$ equal to x, with $i \in B$ and $j \in S$. The same observations apply to the second step below.

[28] A player $b_i \in B$ that is "like" b' is such that $b_i \in L_{b \setminus b_0}$. This condition allows b_i to potentially collect the same sample of information ρ as b'. Similarly, a player s_i that is "like" s' is such that $s_i \in L_s \cap L_{s_0}$.

respectively. Thus, there is a positive probability of obtaining a run of $(1-y, 1-x)$ for m periods in succession such that $v_{b'} = (1-x, ..., 1-x)$ and $v_{s'} = (1-y, ..., 1-y)$.

(iii) $[t+2m+1, t+3m]$: There is a positive probability that b_0 and s_0 will play the game in every period $t \in [t+2m+1, t+3m]$. There is a positive probability that b_0 will sample from $v_b = (y, ..., y)$ and that s_0 will sample from $v_{s'} = (1-y, ..., 1-y)$. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $v_{b_0} = (y, ..., y)$ and $v_{s_0} = (1-y, ..., 1-y)$.

(iv) $[t+3m+1, t+4m]$: There is a positive probability that agents $b_1 \in L_{b_0}$ and $s_1 \in L_{s_0}$ play the game for the next m periods. There is a positive probability that their samples come from v_{b_0} and v_{s_0} respectively. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $v_{b_1} = (y, ..., y)$ and $v_{s_1} = (1-y, ..., 1-y)$.

(v) $[t+4m+1, t+5m]$: There is a positive probability that agents $b_2 \in \bigcup_{k=0}^{k=1} L_{b_k}$ and $s_2 \in \bigcup_{k=0}^{k=1} L_{s_k}$, with $b_2 \neq b_0, b_1$ and $s_2 \neq s_0, s_1$ play the game for the next m periods. There is a positive probability that their samples come from (v_{b_0}, v_{b_1}) and (v_{s_0}, v_{s_1}) respectively. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $v_{b_2} = (y, ..., y)$ and $v_{s_2} = (1-y, ..., 1-y)$.

(vi) Now iterate the following step for $p = 3, ..., n_{max} - 1$, where $n_{max} = \max\{n_B, n_S\}$. $[t+(p+2)m+1, t+(p+3)m]$: There is a positive probability that agents $b_p \in \bigcup_{k=0}^{k=p-1} L_{b_k}$ and $s_p \in \bigcup_{k=0}^{k=p-1} L_{s_k}$, with $b_p \neq b_0, ..., b_{p-1}$ and $s_p \neq s_0, ..., s_{p-1}$ play the game for the next m periods. There is a positive probability that their samples come from $(v_{b_0}, ..., v_{b_{p-1}})$ and $(v_{s_0}, ..., v_{s_{p-1}})$ respectively. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $v_{b_p} = (y, ..., y)$ and $v_{s_p} = (1-y, ..., 1-y)$.

At time $t + (n_{max}+2)m$ the state of the system is such that $v_i = (y, ..., y)$ $\forall i \in B$ and $v_j = (1-y, ..., 1-y)$ $\forall j \in S$, i.e. the system has reached a convention. Thus, from any initial state $\mathbf{s}$ there is a positive probability of reaching a convention within $[n_{max}+2]m$ periods. Given that the number of states is finite, there is a positive probability p of reaching a convention within $[n_{max}+2]m$ periods, which concludes the proof. □

Proof of Lemma 2. Suppose that the process is at the convention $\mathbf{x} = (x, 1-x)$, where $x \in D^0 = \{x \in D : \delta \leq x \leq 1-\delta\}$. Obviously, to move from $\mathbf{x}$ to another convention $\mathbf{x}' = (x', 1-x')$ the agents need to make mistakes. Without loss of generality, assume that the sellers make the mistakes. Let π be a path of least resistance from $\mathbf{x}$ to $\mathbf{x}'$, and let $\mathbf{s}$ be the first state on this path. In order to get to $\mathbf{s}$, a buyer b_0 must have received a sample σ where by mistake some sellers have demanded a quantity that differs from $1-x$, such that b_0's best reply to σ is to demand a quantity $x' \neq x$. The buyers who require the minimum number of mistakes to switch best reply are the ones receiving the smallest sample. Recall that $B_{min} = \{j \in B \mid \lceil z_j \rceil \leq \lceil z_b \rceil, \forall b \in B\}$ is the subset of buyers with the least weighted degree. Let $z_b^{min} \equiv z_j$ for $j \in B_{min}$ and let $b_0 \in B_{min}$. Denote by p the number of mistakes by sellers in σ.

Consider the sample σ and construct a different sample σ' such that every entry of σ that differs from $1-x$ is replaced by $1-x'$, and every entry of σ equal to $1-x$ stays the same. Note that if b_0's best reply to σ was x', then her best reply to σ' must also be x'. By the mean-field assumption, σ' is composed by a total of z_b^{min} demands: p demands are equal to $1-x'$ and $z_b^{min} - p$ are equal to $1-x$.

Now, let us construct an alternative path π' from $\mathbf{x}$ to $\mathbf{x}'$ such that π' is also a path of least resistance with p mistakes. Start with the system at the convention $\mathbf{x}$ at time t. Consider the time t_1 when the md_{b_0} bargaining rounds played by buyers $b \in L_{b_0}$ happened after t. Let p of these md_{b_0} bargaining rounds be such that the seller involved made a mistake and demanded $1-x'$. There is a positive probability that b_0 plays with seller $s_0 \in S$ at time t_1 and receives a sample σ', and therefore she plays the best-reply demand x. Moreover, there is a positive probability that in the next $m-1$ rounds that b_0 and s_0 are picked to play, they again play with each other. Moreover, there is a positive probability that in each of these rounds b_0 receives the sample σ', which could still be available, and plays the best-reply demand x'. Thus, at some time $t_2 > t_1$, $v_{s_0} = \{x', ..., x'\}$.

There is a positive probability that at time $t_3 > t_2$ agents b_0 and $s_1 \in L_{s_0}$ are picked to play, and that b_0 receives the sample σ' and s_1 receives his sample exclusively from v_{s_0}.[29] Thus, b_0 will play the best-reply demand x' and s_1 will play the best-reply demand $1-x'$. Moreover, there is a positive probability that in the next $m-1$ rounds that b_0 and s_1 are picked to play, they again play with each other. Moreover, there is a positive probability that in each of these rounds they receive the same samples they got at t_3, which could still be available, and they play the best-reply demands x' and $1-x'$ respectively. Thus, at some time $t_4 > t_3$, $v_{s_1} = \{x', ..., x'\}$ and $v_{b_0} = \{1-x', ..., 1-x'\}$.

Following the same argument as the proof of theorem 1 above, it is clear that the process can now converge to the new convention $\mathbf{x}$' without any further mistakes. Clearly, the same argument can be used to construct an alternative least-resistant path which starts with the buyers making q mistakes. In order to determine which least-resistant path requires the lowest number of mistakes, one has to compute these two numbers and choose the smallest. This leads us to consider four possible cases: two depending on whether the buyers or sellers make mistakes, and two depending on whether they ask a quantity higher or lower than what they get under the convention $\mathbf{x}$.

- *Sellers make a mistaken demand* $1-x' < 1-x$
  Suppose sellers make p mistaken demands. Clearly, $p \leq z_b^{min}$, which is the sample size for the buyers with the smallest sample. As above, let $b_0 \in B_{min}$. Buyer b_0 therefore receives a sample of p mistaken demands $1-x'$ and $z_b^{min} - p$ conventional

[29] Note that s_1 can receive his sample exclusively from v_{s_0} only if the size m of this vector is larger than z_{s_0}. This is guaranteed by the assumption made in section 3.2 that the individual memory $m \geq \max\{z_b, z_s\}$, where $b \in B$ and $s \in S$. Note that a lower bound would also be sufficient, what is necessary is that m is large enough.

demands $1 - x$. If b_0 demands $x' > x$ then she expects to obtain utility $u(x')$ with probability (p/z_b^{min}). On the other hand, b_0 demands $x < x'$ then she expects to obtain utility $u(x)$ for sure (because if the seller makes a mistake and demands $1-x'$ then $1 - x' + x < 1$ and each player gets their demand). Thus, b_0 switches to x' if:

$$p \geq z_b^{min} \frac{u(x)}{u(x')}$$

The minimum p occurs with the largest possible $u(x')$, i.e. with $x' = 1 - \delta$, which is the largest possible mistake the sellers can make, so:

$$p = z_b^{min} \frac{u(x)}{u(1-\delta)} \tag{7}$$

- *Sellers make a mistaken demand* $1 - x' > 1 - x$
  Now suppose sellers make p mistaken demands, but they demand more than the conventional demand. Now, if b_0 demands $x' < x$ then she expects to obtain utility $u(x')$ for sure. On the other hand, if b_0 demands $x > x'$ then she expects to obtain utility $u(x)$ with probability $(z_b^{min} - p)/z_b^{min}$. Thus, b_0 switches to x' if:

$$p \geq z_b^{min} \left(1 - \frac{u(x')}{u(x)}\right)$$

  The minimum p occurs with the largest possible $u(x')$, i.e. with $x' = x - \delta$, which is the largest possible mistake $x' < x$ the sellers can make, so:

$$p = z_b^{min} \left(1 - \frac{u(x-\delta)}{u(x)}\right) \tag{8}$$

- *Buyers make a mistaken demand* $x' < x$
  Suppose buyers make q mistaken demands. Clearly, $q \leq z_s^{min}$, which is the sample size for the sellers with the smallest sample. Assume that $s_0 \in S_{min}$ where $S_{min} = \{j \in S \mid \lceil z_j \rceil \leq \lceil z_s \rceil, \forall s \in S\}$ is defined analogously to B_{min}. Seller s_0 therefore receives a sample of q mistaken demands x' and $z_s^{min} - q$ conventional demands x. If s_0 demands $1 - x' > 1 - x$ then she expects to obtain utility $v(1 - x')$ with probability (q/z_b^{min}). On the other hand, if s_0 demands $1 - x < 1 - x'$ then she expects to obtain utility $v(1 - x)$ for sure. Thus, s_0 switches to $1 - x'$ if:

$$q \geq z_s^{min} \frac{v(1-x)}{v(1-x')}$$

  The minimum q occurs with the largest possible $v(1 - x')$, i.e. with $x' = \delta$, which is the largest possible mistake the buyers can make, so:

$$q = z_s^{min} \frac{v(1-x)}{v(1-\delta)} \tag{9}$$

- *Buyers make a mistaken demand $x' > x$* Now suppose buyers make q mistaken demands, but they demand more than the conventional demand. Now, if s_0 demands $1 - x' < 1 - x$ then she expects to obtain utility $v(1 - x')$ for sure. On the other hand, if s_0 demands $1 - x > 1 - x'$ then she expects to obtain utility $v(1 - x')$ with probability $(z_s^{min} - q)/z_s^{min}$. Thus, s_0 switches to $1 - x'$ if:

$$q \geq z_s^{min}\left(1 - \frac{v(1-x')}{v(1-x)}\right)$$

  The minimum q occurs when the term in brackets is as small as possible, i.e. with $x' = x + \delta$, which is the smallest possible mistake the buyers can make, so:

$$q = z_s^{min}\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right) \tag{10}$$

Combining equations (7), (8), (9), and (10) it follows that the least number of mistakes necessary to move out of the convention $\mathbf{x}$ is $\lceil R(x) \rceil$, where $R(x)$ is equal to:

$$R(x) = \min\left\{ z_b^{min}\frac{u(x)}{u(1-\delta)}, z_b^{min}\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}\frac{v(1-x)}{v(1-\delta)}, z_s^{min}\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\}$$

It is straightforward to show that the first term is at least as large as the last one for all $x \in D^0$, so it can be ignored. Thus, the minimum resistance to move out of the $\mathbf{x}$ convention is $\lceil R(x) \rceil$, where $R(x)$ is given by:

$$R(x) = \min\left\{ z_b^{min}\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}\frac{v(1-x)}{v(1-\delta)}, z_s^{min}\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} \tag{11}$$

□

Proof of Theorem 2. Lemma 2 in Young [1993a] shows that a division $(x, 1 - x)$ is generically stable if and only if x maximizes the function $R(x)$ in (4). Lemma 3 in Young [1993a] shows that as $\delta \to 0$, the maxima of the function $R(x)$ converge to the asymmetric Nash bargaining solution in (5). The proofs of the equivalent statements to lemmas 2 and 3 for this model are essentially the same as in Young [1993a], and they are therefore omitted here. □

Proof of Corollary 1. Denote by G_Q the quasi-regular networks generated by regular networks in $\overline{g}_{d,a}$. The proof is by contradiction. Suppose there exists a network $g \in G$ such that $g \in G_B$ and $g \notin G_Q$. There are two possible cases:

(i) $g \in G_B$ and $G_Q \cap G_B = \emptyset$: If this is the case then $\min_{b \in B} z_b(g) > \min_{b \in B} z_b(\overline{g}_{d,a}) = \overline{s}d$, i.e. $\min_{b \in B} z_b(g) \geq \overline{s}d + \epsilon$. Given that the maximum link strength is $\overline{s}$, this implies that $\min_{b \in B} d_b(g) = \lfloor \frac{2L}{n_B} \rfloor + 1$ and the degree of all other buyers must be at least equal to this. But if this is the case then the total minimum number of links is $\frac{n_B}{2} \min_{b \in B} d_b(g) > L$, which is a contradiction.

(ii) $g \in G_B$ and $G_Q \subset G_B$: If this is the case then either $\min_{b\in B} z_b(g) > \min_{b\in B} z_b(\overline{g}_{d,a})$ or $\min_{b\in B} z_b(g) = \min_{b\in B} z_b(\overline{g}_{d,a})$. The argument above shows that the former leads to a contradiction, so suppose that $\min_{b\in B} z_b(g) = \min_{b\in B} z_b(\overline{g}_{d,a}) = \overline{s}d$. Thus, $\min_{b\in B} d_b(g) = d$ and the degree of all other buyers must be at least equal to this. The minimum total number of links for this to hold is $d \cdot n_B/2$, which leaves a maximum of $L - d \cdot n_B/2 = L - \lfloor L \rfloor$ links to assign. But this means that g is a quasi-regular network, no matter how the remaining links are assigned and we have a contradiction. □

Proof of Theorem 3. Let us look at *(i)* and *(ii)* separately.

(i) First, consider case $i = B$. The goal is to compare the $(x^*, 1-x^*)$ ANB solution for agents that communicate through networks g^B and g^S, and the $(x'^*, 1-x'^*)$ ANB solution for agents that communicate through networks g'^B and g^S, where $p'_b(z)$ FOSD $p_b(z)$. The claim is that $x'^* \geq x^*$. From equation (4) we have:

$$R(x) = \min\left\{ z_b^{min}(g^B)\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g^S)\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g^S)\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} \leq$$
$$\leq \min\left\{ z_b^{min}(g'^B)\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g^S)\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g^S)\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} =$$
$$= R'(x)$$

because, by definition of FOSD, $z_b^{min}(g^B) \leq z_b^{min}(g'^B)$. Thus, the unique division $(x'^*, 1-x'^*)$ that maximizes $R'(x)$ is such that $x'^* \geq x^*$, where $(x^*, 1-x^*)$ is the unique division that maximizes $R(x)$. The case $i = S$ is similar, and it is therefore omitted.

(ii) Again, consider the case $i = B$ first. The goal is to compare the $(x^*, 1-x^*)$ ANB solution for agents that communicate through networks g^B and g^S, and the $(x''^*, 1-x''^*)$ ANB solution for agents that communicate through networks g''^B and g^S, where $p''_b(z)$ SOSD $p(z)$. The claim is that $x''^* \geq x^*$. From equation (4) we have:

$$R(x) = \min\left\{ z_b^{min}(g^B)\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g^S)\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g^S)\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} \leq$$
$$\leq \min\left\{ z_b^{min}(g''^B)\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g^S)\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g^S)\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} =$$
$$= R''(x)$$

because, by definition of SOSD, $z_b^{min}(g^B) \leq z_b^{min}(g''^B)$. Thus, the unique division $(x''^*, 1-x''^*)$ that maximizes $R''(x)$ is such that $x''^* \geq x^*$, where $(x^*, 1-x^*)$ is the unique division that maximizes $R(x)$. The case $i = S$ is similar, and it is therefore omitted. □

Proof of Corollary 3. First, for the sellers to receive the least possible share there must be a seller s_0 such that $g_{s_0} = \underline{s}$, i.e. s_0 has only one weak link. Second, for the network to be connected each seller $s \in S \backslash s_0$ must have one link g_{si}, and, to maximize the number of links of buyers, let $i \in B$ and $g_{si} = \overline{s}$. Third, by corollary 1, the networks that maximize the buyers' share are quasi-regular networks generated by $\overline{g}_{d,\overline{s}}$, where $d = \left\lfloor \frac{2L+n_S-1}{2} \right\rfloor$.

Here, the addition of $n_S - 1$ takes into account the strong links buyers have with the sellers $s \in S \backslash s_0$. The same argument in the proof of corollary 1 shows that the existence of a network g which gives a weakly higher share to buyers and which is not a core-periphery network would lead to a contradiction. $\square$

Proof of Theorem 5. Let us look at *(i)* and *(ii)* separately.

(i) The goal is to compare the $(x^*, 1-x^*)$ ANB solution for agents that communicate through network g, and the $(x'^*, 1-x'^*)$ ANB solution for agents that communicate through network g', where $p'(z)$ FOSD $p(z)$ and $Var[p(z)] = Var[p'(z)]$. The claim is that $x'^* = x^*$. By definition of a variance-preserving FOSD shift, we have that $z_i(g) = \varsigma z_i(g')$ for each $i \in N$, where $\varsigma \in \mathbb{R}_+$. From equation (4) we have:

$$R'(x) = \min\left\{ z_b^{min}(g')\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g')\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g')\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} =$$
$$= \min\left\{ \varsigma z_b^{min}(g)\left(1 - \frac{u(x-\delta)}{u(x)}\right), \varsigma z_s^{min}(g)\frac{v(1-x)}{v(1-\delta)}, \varsigma z_s^{min}(g)\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} =$$
$$= \varsigma R(x)$$

Clearly, the variance-preserving FOSD shift is only a rescaling of $R(x)$ by a ς factor. Thus, the unique division $(x^*, 1-x^*)$ that maximizes $R(x)$ is also the division that maximizes $R'(x)$, i.e. $x^* = x'^*$.

(ii) First, assume that $x^* < \frac{1}{2}$. The goal is to compare the $(x^*, 1-x^*)$ ANB solution for agents that communicate through network g, and the $(x''^*, 1-x''^*)$ ANB solution for agents that communicate through network g'', where $p(g'')$ SOSD $p(g)$ and $\mu[p(g)] = \mu[p(g'')]$. The claim is that $x''^* \geq x^*$. By definition of a mean-preserving SOSD shift, we have that $z_i(g'') = \varsigma(z_i)z_i(g)$, where $i \in N$ and $\varsigma(z_i) \in \mathbb{R}_+$. Moreover, we have that: $\varsigma(z_i) > \varsigma(z_j) > 1$ if $z_i < z_j < \mu[p(g)]$, $\varsigma(z_j) < \varsigma(z_i) < 1$ if $z_j > z_i > \mu[p(g)]$, and $\varsigma(z_i) = 1$ if $z_i = \mu[p(g)]$.

From equation (4) we have:

$$R''(x) = \min\left\{ z_b^{min}(g'')\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g'')\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g'')\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)\right\} =$$
$$= \min\left\{ \varsigma(z_b^{min})z_b^{min}(g)\left(1 - \frac{u(x-\delta)}{u(x)}\right), \varsigma(z_s^{min})z_s^{min}(g)\frac{v(1-x)}{v(1-\delta)}, \varsigma(z_s^{min})z_s^{min}(g)\left(1 - \frac{v(.)}{v(.)}\right)\right\} =$$
$$= \varsigma(z_s^{min})\min\left\{ \frac{\varsigma(z_b^{min})}{\varsigma(z_s^{min})}z_b^{min}(g)\left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g)\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g)\left(1 - \frac{v(.)}{v(.)}\right)\right\} \tag{12}$$

where $\left(1 - \frac{v(.)}{v(.)}\right)$ stands for $\left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right)$. By simple inspection of the equation for $R(x)$ in (4) and the last line in (12) above, it is clear that the second and third term, which depend on $(1-x)$, are simply rescaled by a $\varsigma(z_s^{min})$ factor. However, the first term is rescaled by a factor of $\frac{\varsigma(z_b^{min})}{\varsigma(z_s^{min})} > 1$ since $\varsigma(z_b^{min}) > \varsigma(z_s^{min})$ because $z_b^{min} < z_s^{min} < \mu[p(g)]$. Thus, the only term dependent on x is rescaled by an additional factor greater than 1 in $R''(x)$, and therefore the unique division $(x''^*, 1 - x''^*)$ that maximizes $R''(x)$ must be such that $x''^* \geq x^*$. The case $x^* > \frac{1}{2}$ is similar and it is therefore omitted. $\square$

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