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Pinning down the mass of Kepler-10c: the importance of sampling and model comparison

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ABSTRACT
Initial radial velocity (RV) characterization of the enigmatic planet Kepler-10c suggested a mass of $\sim 17\,\mathrm{M}_\oplus$, which was remarkably high for a planet with radius $2.32\,\mathrm{R}_\oplus$; further observations and subsequent analysis hinted at a (possibly much) lower mass, but masses derived using RVs from two different spectrographs (HARPS-N and HIRES) were incompatible at a 3σ level. We demonstrate here how such mass discrepancies may readily arise from suboptimal sampling and/or neglecting to model even a single coherent signal (stellar, planetary or otherwise) that may be present in RVs. We then present a plausible resolution of the mass discrepancy, and ultimately characterize Kepler-10c as having mass $7.37^{+1.32}_{-1.19}\,\mathrm{M}_\oplus$, and mean density $3.14^{+0.63}_{-0.55}\,\mathrm{g\,cm^{-3}}$.

Key words: methods: data analysis – techniques: radial velocities – stars: activity – stars: individual: Kepler-10 – planetary systems.

1 INTRODUCTION

Kepler-10 (KOI-72; hereafter K-10 for short) is a slowly rotating, Sun-like star that exhibits little stellar activity (Dumusque et al. 2014, hereafter D14). It is known to host at least two planets, viz. Kepler-10b and Kepler-10c.

Stony-iron world Kepler-10b (hereafter K-10b) – with orbital period 0.84 d, radius $1.48\,\mathrm{R}_\oplus$ and mass $\sim 4\,\mathrm{M}_\oplus$ – was the first unambiguously rocky exoplanet to be discovered, and also the first super-Earth discovered around a Sun-like star (Batalha et al. 2011).

Kepler-10c (hereafter K-10c) – with orbital period 45.29 d and radius $2.32\,\mathrm{R}_\oplus$ – has proven more enigmatic. Following its discovery and statistical validation as a planet (Batalha et al. 2011; Fressin et al. 2011), D14 used 148 HARPS-N radial velocities (RVs) spanning two observing seasons to infer a mass of $17.2 \pm 1.9\,\mathrm{M}_\oplus$. Given K-10c's radius, this was a striking result. Most planets with radii 2.0–2.5 $\mathrm{R}_\oplus$ have masses significantly lower than $17\,\mathrm{M}_\oplus$, with a weighted mean mass of $5.4\,\mathrm{M}_\oplus$ (Weiss & Marcy 2014); Weiss & Marcy's empirical mass-radius relation for planets between 1.5 and $4\,\mathrm{R}_\oplus$, viz. $M_\mathrm{p}/\mathrm{M}_\oplus = 2.69(R_\mathrm{p}/\mathrm{R}_\oplus)^{0.93}$, predicts a mass of $5.8\,\mathrm{M}_\oplus$ for K-10c. D14 interpreted the composition of K-10c as being mostly rock by mass, and regarded the planet as being the first evidence of a class of more massive solid planets with longer orbital periods.

Weiss et al. (2016, hereafter W16) built on the work of D14, adhering closely to the techniques employed by the latter authors, but adding 72 RVs from Keck-HIRES to the analysis, resulting in a combined RV baseline of 6 yr. Since it has been well established that both the HIRES and HARPS-N spectrometers are independently capable of accurate and precise measurement of low-amplitude planetary signals, it was a great surprise when W16 inferred a mass for K-10c of $5.69^{+3.19}_{-2.90}\,\mathrm{M}_\oplus$ (fitted RV semi-amplitude $K_\mathrm{c} = 1.09 \pm 0.58\,\mathrm{m\,s^{-1}}$) using the HIRES RVs alone, which was incompatible with D14's estimate of $17.2 \pm 1.9\,\mathrm{M}_\oplus$ ($K_\mathrm{c} = 3.26 \pm 0.36\,\mathrm{m\,s^{-1}}$) using the HARPS-N RVs alone.

W16 concluded that some additional, time-correlated signal (possibly from stellar activity or additional planets) was present and led to the discrepant mass estimates for K-10c. This claim was supported by (i) the fact that masses inferred using RVs from *either* instrument were found to be time dependent, and (ii) $>5\sigma$ evidence for transit timing variations (TTVs) of K-10c (Kipping et al. 2015). W16 found that dynamical solutions including a third planet candidate, KOI-72.X, were very strongly favoured over a two-planet solution (based on Bayesian information criterion differentials); the TTVs and RVs were consistent with KOI-72.X having an orbital period of 24, 71 or 101 d, with 101 d being strongly favoured over the other periods. W16 inferred a likely mass of $\lesssim 7\,\mathrm{M}_\oplus$ for KOI-72.X, based on the best solutions from a partial exploration of the dynamical parameter space, with the parameters of K-10b fixed.

Even when including a third planet in their models, however, W16 were not able to reconcile the HIRES and HARPS-N masses for K-10c, so settled on a 'compromise' mass for K-10c of $13.98 \pm 1.79\,\mathrm{M}_\oplus$. We suggest the observed 3σ incompatibility between the HARPS-N and HIRES estimates for K-10c's mass points to an inadequate model under which at least one (if not both) of the inferred masses is incorrect, and that the true mass need not lie in the middle of the two incompatible mass posteriors.

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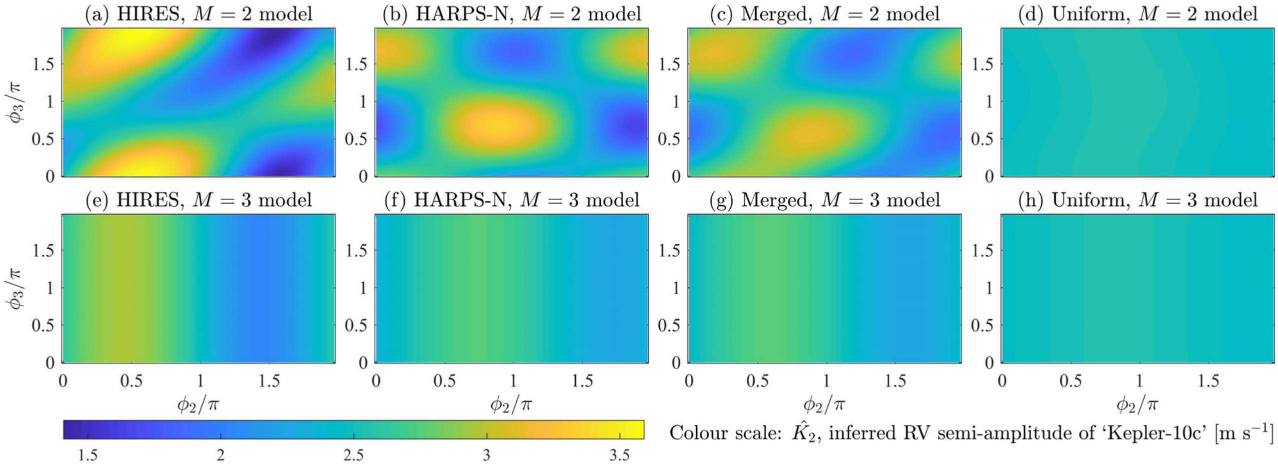

Figure 1. ML estimates for K_2 based on synthetic data comprising three sine waves, with four different sampling patterns [left to right, corresponding to sampling patterns (i)–(iv) listed in Section 2] and fitted with a two-sine model (upper panels) and a three-sine model (lower panels). ϕ_3 axis compressed to save space.

2 DOUBLE TROUBLE: IMPERFECT MODEL MEETS INADEQUATELY SAMPLED SIGNAL

To shed light directly on the effects of (i) suboptimal sampling and (ii) inference based on an imperfect physical model, consider synthetic RV data sets $\{(t_i, y_i)|i = 1, 2, \ldots, N\}$ generated as follows:

$$y_i = \sum_{j=1}^{M} K_j \sin\left(\frac{2\pi t_i}{P_j} + \phi_j\right); \tag{1}$$

y_i may be interpreted as the combined RV signal at time t_i due to M planets on zero-eccentricity orbits around a star. For planet j, the associated RV amplitude K_j would be determined by the planet's mass and inclination (assuming known stellar mass); P_j would correspond to the planet's orbital period; and ϕ_j would be determined by the planet's orbital phase in some coordinate system.

Suppose we set $M = 3$, and let the periods of the three mock planets be $P_1 = 0.84$ d, $P_2 = 45.29$ d and $P_3 = 101.36$ d; P_1 and P_2 correspond to the known orbital periods of K-10b and K-10c, respectively, while P_3 corresponds to the most likely orbital period (per W16) for planet candidate KOI-72.X. We further set $K_1 = K_2 = 2.5\,\mathrm{m\,s^{-1}}$, and $K_3 = 1.0\,\mathrm{m\,s^{-1}}$; the values for K_1 and K_2 are roughly the average of the RV semi-amplitudes of both K-10b and K-10c variously reported in the literature, while the value for K_3 is based on the most likely mass for KOI-72.X reported by W16. Lastly, instead of fixing the mutual phases of the planets, let us generate synthetic data by evaluating equation (1) over a grid of phases $(\phi_1, \phi_2, \phi_3) \in \Phi^3$, where $\Phi = [0, 0.1, 0.2, \ldots, 2\pi]$.

Now consider the problem of using such synthetic data to infer the value of K_2, i.e. the RV semi-amplitude of the second mock planet, using both a two-planet and a three-planet model, and with the synthetic signals sampled discretely using the following calendars:

(i) the real HIRES observing calendar ($N = 72$) for K-10;
(ii) the real HARPS-N observing calendar ($N = 148$) for K-10;
(iii) the combined HARPS-N/HIRES observing calendars and
(iv) 220 uniformly spaced observations with a $\sim$6 yr baseline (see Appendix A, online, for more details).

Thus we will produce an estimate for K_2, $\hat{K}_2$, using two different models (of orders $M = 2$ and $M = 3$), four different sampling patterns and many different input values for ϕ_j. In each case we assume that the periods and orbital phases of the planets are tightly constrained, as if performing RV follow-up of transiting planets, but that the planets' RV semi-amplitudes are not known a priori. For the two-planet model we assume only the presence of the $P_1 = 0.84$ d and $P_2 = 45.29$ d planets, while for the three-planet model we also assume the presence of a $P_3 = 101.36$ d planet.

We place flat priors on all free parameters, and to simplify computation, reduce the more general problem of finding posterior distributions to one of maximum likelihood (ML) estimation; our priors render ML estimates equivalent to maximum a posteriori (MAP) estimates. As a further convenience, we assume independent and identically distributed (i.i.d.) Gaussian measurement errors on our synthetic data (with arbitrarily small, constant standard deviation), then use a standard downhill simplex algorithm (with multiple starting points) to locate ML parameters through least-squares fitting.

In Fig. 1 we present the results of this fitting exercise, showing ML estimators of K_2, $\hat{K}_2$, for a range of values of ϕ_2 and ϕ_3; we fix ϕ_1 to permit visualization in two dimensions.[1] The following striking conclusions emerge.

(i) When using an inadequate model (i.e. $M = 2$) to fit the data, the inferred mass for the second mock planet is very sensitive to the mutual phases of that planet and the unobserved third planet, with $\hat{K}_2$ varying between $\sim$1.5 and 3.5 m s^{-1}.

(ii) For almost all possible mutual phases of the second and the unobserved third planet, $\hat{K}_{2,\mathrm{HIRES}}$ and $\hat{K}_{2,\mathrm{HARPS}}$ differ[2] by up to $\sim\pm 1\,\mathrm{m\,s^{-1}}$ (in the worst cases) and typically by $\sim\pm 60\,\mathrm{cm\,s^{-1}}$. For about 60 per cent of possible phase configurations, the HARPS-N and HIRES observing calendars either both result in overestimation or in underestimation of K_2 (i.e. the true value is *not* bracketed); for about 40 per cent of configurations, one calendar will lead to $\hat{K}_2 > K_2$ while the other leads to $\hat{K}_2 < K_2$.

[1] Given its short period $P_1 \ll P_2 < P_3$, the phase of the innermost mock planet does not have a significant effect on our inference about the properties of the other signals in our synthetic data. K_1 can be estimated more accurately and precisely than K_2, regardless of the phases of the other signals, and of the choice of observing calendar.

[2] In a few cases where we chose to explore full posterior distributions for K_2 (rather than just find ML estimators), estimates of K_2 obtained with the HARPS-N versus HIRES sampling often disagreed at a 2σ level.

(iii) When combining the HIRES and HARPS-N observations, $\hat{K}_2$ interpolates the values predicted by the separate data sets, yet may still differ from $K_2 = 2.5\,\mathrm{m\,s^{-1}}$ by up to $\sim\pm 65\,\mathrm{cm\,s^{-1}}$.

(iv) Notably, even when using the inadequate $M = 2$ model but the uniform observing cadence, $|\hat{K}_2 - K_2| < 5\,\mathrm{cm\,s^{-1}}, \forall\,\phi_j$.

(v) Equally notably, when using the correct $M = 3$ model, the inferred mass for the second planet is relatively *insensitive* to the observing calendar. Now we find $|\hat{K}_{2,\mathrm{HIRES}} - K_2| < 37\,\mathrm{cm\,s^{-1}}$ and $|\hat{K}_{2,\mathrm{HARPS}} - K_2| < 22\,\mathrm{cm\,s^{-1}}$; with the combined HARPS-N and HIRES observations, $|\hat{K}_2 - K_2| < 25\,\mathrm{cm\,s^{-1}}$ and with the calendar where $t_{i+1} - t_i = \mathrm{constant}$, $|\hat{K}_2 - K_2| < 4\,\mathrm{cm\,s^{-1}}, \forall\,\phi_j$.

Suppose we add another signal into the synthetic RV data, with amplitude $K_4 = 1.0\,\mathrm{m\,s^{-1}}$ and period $P_4 = 55$ d, as a simplistic representation of a non-evolving stellar activity signal [we could use a Gaussian process (GP) to synthesize a quasi-periodic signal instead, but such sophistication is not required for the present illustration], then repeat the exercise of trying to estimate K_2.

Now, neither the $M = 2$ nor the $M = 3$ model is adequate in that neither accounts for a fourth periodic signal present in the data. Accordingly, we find that inference about K_2 becomes even more sensitive to sampling. $\hat{K}_{2,\mathrm{HIRES}}$ and $\hat{K}_{2,\mathrm{HARPS}}$ now differ by up to $2\,\mathrm{m\,s^{-1}}$ under the $M = 2$ model, and by up to $1\,\mathrm{m\,s^{-1}}$ under the $M = 3$ model. As before, however, when using the uniform observing cadence, the resultant uniform phase coverage allows remarkably robust inference about K_2 to be made: e.g. $|\hat{K}_2 - K_2| < 7\,\mathrm{cm\,s^{-1}}\,\forall\,\phi_j$, even under the $M = 2$ model.

Thus far we have established the prevalence of sizeable differences in $\hat{K}_2$ when fitting *simplistic* synthetic signals with HARPS-N versus HIRES sampling, yet even larger differences will result when including in our synthetic data such details as photon noise, quasi-periodic stellar activity signals, instrumental noise, multiple undetected planets, planets with non-circular orbits, possible dynamical interactions between planets and more. For example, adding to our synthetic data white Gaussian noise at a level consistent with that estimated for the HARPS-N data set ($\sigma \sim 2\,\mathrm{m\,s^{-1}}$), then repeating the previous test, results in HARPS-N/HIRES discrepancies for $\hat{K}_2$ of up to $3\,\mathrm{m\,s^{-1}}$ under the $M = 2$ model, and up to $2.3\,\mathrm{m\,s^{-1}}$ under the $M = 3$ model; see Fig. 2, available online.

The upshot is that using an inadequate physical model, and/or suboptimal sampling, can lead to incorrect conclusions about the masses even of planets whose other properties are well constrained – and even when we have hundreds of RVs at our disposal. Moreover, through our choice of real sampling patterns, and realistic values for K_j and P_j, we have provided a plausible explanation for why W16 obtained discrepant masses for K-10c using HIRES versus HARPS-N RVs. Specifically, the real K-10 RVs likely contained not only K-10b and K-10c's signals, but one or more other coherent signals (KOI-72.X, a stellar signal, etc., as indeed adduced by W16) that interfered constructively or destructively with the signals of the known planets. The suboptimality of the phase coverage is easily checked by phasing the HIRES or HARPS-N observation times to the orbital period of K-10c; see Fig. 3. In principle, accounting for the other signals jointly with those of the known planets (i.e. using a more appropriate physical model), and/or obtaining more observations to provide more complete phase coverage of K-10c's signal, could have mitigated the discrepancy.

3 RECONCILING THE MASS ESTIMATES

We noted $>3\sigma$ evidence for linear correlations ($\rho \sim 30$ per cent) between the published HARPS-N RVs and (i) $\log R'_{\mathrm{HK}}$ index and (ii) bisector inverse slope (BIS) measurements; we did not find any similarly significant correlations in the HIRES RVs.

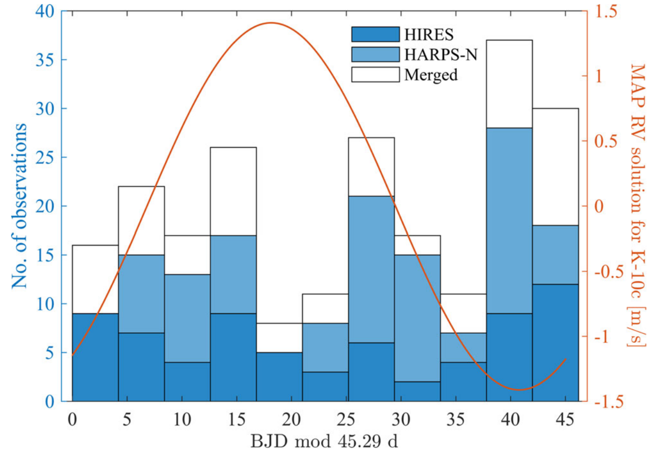

Figure 3. The uneven coverage of K-10c's orbital phase provided by existing HARPS-N and HIRES observations. (This representation does not by itself indicate whether the sampling would lead to mass under- or overestimation; this would require accounting for constructive or destructive interference between K-10c's signal and all other signals in the RVs.)

Whereas the models of D14 and W16 did not accommodate possible stellar activity signals in the RVs, we used the GP framework of Rajpaul et al. (2015, hereafter R15) to model jointly all available RV, $\log R'_{\mathrm{HK}}$ (in the case of HARPS-N) or S_{HK} (in the case of HIRES), and BIS time series, for a total of 660 data points. As in R15, we adopted a quasi-periodic covariance kernel, and non-informative priors were placed on all GP hyperparameters. GP amplitude parameters were also constrained to be smaller than the total variation seen in a given time series, and of the overall GP period we required $P > 20$ d (based on D14's lower limits on K-10's stellar rotation period). We additionally allowed at least two but up to five possible planetary signals in the RVs, modelled with Keplerian functions. We constrained the periods and periapsis passage times of two of the Keplerians to be consistent with the most precise values inferred from K-10b and K-10c's transits (Holczer et al. 2016; Morton et al. 2016), but left the other parameters free, with priors identical to those in W16's eccentric two-planet model. We adopted analogous uninformative priors for all parameters of the additional possible planets, ensuring only that planet periods did not overlap. Finally, we used the MULTINEST nested-sampling algorithm (Feroz & Hobson 2008; Feroz, Hobson & Bridges 2009; Feroz et al. 2013) to obtain a full joint posterior distribution for each model's parameters (and marginal posteriors for parameters of interest), and to compute a Bayesian evidence ($\mathcal{Z}$) for each model.

Of the numerous models we considered, we found only one in which estimates for *all* planet parameters were consistent within 1σ between the HARPS-N, HIRES and merged data sets: viz. a model including three planets, all with orbits consistent with circular, plus correlated noise. Significantly, this model was also favoured over others by Bayesian model comparison tests, and the period for the third planet in our model was 102 ± 1 d: in accord with the W16's favoured period for KOI-72.X (based on both analytical considerations and dynamical modelling), despite us not including this as prior information in our model. We summarize the marginal posteriors for this favoured model's planet parameters in Table 1; masses (for all three planets) and mean densities (for the transiting planets) were derived using the same stellar mass and planet radii as in W16.

Table 1. Summaries of marginal posteriors for selected planet and GP parameters from our favoured model (three planets plus correlated noise). The planet parameters are the same as those from W16, while the GP parameters are as defined in R15; $K_{\rm GP} = \sqrt{V_r^2 + V_c^2}$ may be interpreted as the GP RV semi-amplitude; P is an overall period; $1/\lambda_p$ defines the harmonic complexity of the GP (behaviour is sinusoidal for $\lambda_p \gg 1$) and λ_e is the time-scale over which the GP signal evolves. Periapsis passage times are omitted: for K-10b and K-10c, these were effectively fixed in our models a priori via known transit times, while a periapsis passage time for non-transiting planet candidate KOI-72.X could not be well constrained, given its apparently circular orbit.

		HIRES		HARPS-N		Merged	
Parameter	Units	Median	$\pm\sigma$	Median	$\pm\sigma$	Median	$\pm\sigma$
K_b	m s^{-1}	2.39	$^{+0.30}_{-0.28}$	2.33	$\pm$0.16	2.32	$^{+0.21}_{-0.18}$
P_b	d	0.83748	$\pm$0.00003	0.837501	$\pm$0.000005	0.837501	$^{+0.000005}_{-0.000004}$
$\sqrt{e_b}\cos\omega_b$	–	0.000	$\pm$0.003	0.000	$\pm$0.003	0.000	$\pm$0.004
$\sqrt{e_b}\sin\omega_b$	–	0.000	$\pm$0.003	0.000	$\pm$0.003	0.000	$\pm$0.004
m_b	$M_\oplus$	3.33	$^{+0.40}_{-0.42}$	3.25	$^{+0.22}_{-0.23}$	3.24	$\pm$0.28
ρ_b	g cm^{-3}	5.65	$^{+0.94}_{-0.85}$	5.51	$^{+0.73}_{-0.64}$	5.48	$^{+0.78}_{-0.68}$
K_c	m s^{-1}	1.27	$^{+0.42}_{-0.35}$	1.64	$^{+0.42}_{-0.34}$	1.41	$^{+0.25}_{-0.23}$
P_c	d	45.2948	$\pm$0.0008	45.2940	$^{+0.0008}_{-0.0007}$	45.2946	$\pm$0.0008
$\sqrt{e_c}\cos\omega_c$	–	0.1	$\pm$0.2	0.0	$\pm$0.1	0.0	$\pm$0.1
$\sqrt{e_c}\sin\omega_c$	–	0.0	$\pm$0.2	0.1	$\pm$0.1	0.0	$\pm$0.1
m_c	$M_\oplus$	5.87	$^{+2.20}_{-1.82}$	8.59	$^{+2.19}_{-1.79}$	7.37	$^{+1.32}_{-1.19}$
ρ_c	g cm^{-3}	2.50	$^{+0.98}_{-0.78}$	3.66	$^{+0.98}_{-0.80}$	3.14	$^{+0.63}_{-0.55}$
K_X	m s^{-1}	1.30	$^{+0.51}_{-0.45}$	0.84	$^{+0.16}_{-0.14}$	0.85	$^{+0.24}_{-0.14}$
P_X	d	102	$^{+8}_{-7}$	101	$^{+6}_{-5}$	102	$\pm$1
$\sqrt{e_X}\cos\omega_X$	–	−0.1	$\pm$0.2	−0.1	$\pm$0.1	−0.1	$\pm$0.1
$\sqrt{e_X}\sin\omega_X$	–	−0.1	$\pm$0.2	0.0	$\pm$0.1	0.0	$\pm$0.1
m_X	$M_\oplus$	8.93	$^{+3.50}_{-3.15}$	5.80	$^{+1.20}_{-1.03}$	5.90	$^{+1.70}_{-1.01}$
$K_{\rm GP}$	m s^{-1}	0.09	$^{+0.22}_{-0.06}$	1.46	$\pm$0.17	1.68	$\pm$0.25
P	d	63	$\pm$10	55	$\pm$1	55.5	$\pm$0.8
λ_p	–	1.3	$^{+0.6}_{-0.3}$	0.33	$^{+0.04}_{-0.02}$	0.32	$^{+0.02}_{-0.01}$
λ_e	d	330	$\pm$100	86	$\pm$4	90	$\pm$6

Additionally, we note the following. First, for the HIRES, HARPS-N and merged data sets, three-planet models were strongly favoured over two-planet models ($\Delta \ln \mathcal{Z} > 10$), which were in turn favoured over four- and five-planet models. Secondly, we obtained consistent parameters for all planets when splitting either the HIRES or HARPS data sets in two; presumably W16 found discrepant results because neither a third planet nor a nuisance signal model was included when performing the same test. Thirdly, a zero-amplitude GP component was favoured for the HIRES RVs ($\Delta \ln \mathcal{Z} \sim 3$), whereas a non-zero GP amplitude was favoured ($\Delta \ln \mathcal{Z} \gtrsim 10$) for the HARPS-N and merged RVs; the latter two cases suggested a GP period of $P = 55 \pm 1$ d.

We interpreted the third finding as evidence of the HARPS-N RVs being confounded by at least one semicoherent though not strictly periodic, $\gtrsim 1$ m s^{-1} nuisance signal that cannot be ascribed to a planet (for which a simpler Keplerian model would have sufficed, rather than a GP; a planetary signal should also have been simultaneously present in the HIRES RVs). Given the correlation observed between HARPS-N RVs and activity indicators, at least part of this signal could be due to stellar activity; it is unclear, though, whether $P = 55 \pm 1$ d corresponds to a stellar rotation period. An instrumental component to the signal also cannot be ruled out.[3] Either way, it appears that when *not* accounting for a correlated nuisance signal, and an apparent third planet (KOI-72.X), the amplitude of the signal ascribed to K-10c is forced to inflate artificially to absorb some of this appreciable variability in the discretely sampled RV signal.

Our mass and mean density estimates for K-10b are consistent with those of D14 and W16. Our mass estimate for K-10c ($7.37^{+1.32}_{-1.19}$ $M_\oplus$), however, is significantly lower than those from D14 and W16 (17.2 ± 1.9 and 13.98 ± 1.79 $M_\oplus$, respectively); accordingly, we also infer a significantly lower mean density of $\rho_c = 3.14^{+0.63}_{-0.55}$ g cm^{-3}. This implies a composition that is either consistent with a low-density solid planet with a significant fraction of volatiles in the form of e.g. water or methane, or a planet with a dense core and an extended gaseous envelope. K-10c would thus join a region of parameter space in the mass-radius diagram occupied by a number of other exoplanets with radii between 2.0 and 2.5 $R_\oplus$ that have similar mean densities to K-10c; see Fig. 4, available online.

Finally, our inferred mass of $5.90^{+1.70}_{-1.01}$ $M_\oplus$ for KOI-72.X is compatible with W16's point estimate of ~ 7 $M_\oplus$, though it remains to be established whether this is a genuine planet. We used the parameters from our Keplerian solutions as inputs to numerical *N*-body integrations using TTVFAST (Deck et al. 2014); we noted that the maximum difference between the RVs from a full dynamical simulation

[3] The small posterior uncertainty of ± 1 d may simply indicate that 55 d is the only GP period that does a reasonable job of modelling some (possibly complex) combination of nuisance signals. Regardless, given the similarity of the 55 d period to K-10c's orbital period, the nuisance and planet signals interfere strongly over time-scales of several months (an envelope with period 248 d would be predicted if the nuisance signal were sinusoidal).

and from our Keplerian solution was of the order $1\,\mathrm{cm\,s^{-1}}$ over 101 d. As this is two orders of magnitude below the RV noise floor, we concluded that full dynamical modelling would have yielded no constraints beyond those already derived by W16.

4 INSTRUMENTAL CONSIDERATIONS

W16 detrended the HIRES RVs by removing correlations between RVs and instrumental parameters, RV uncertainties and spectrum signal-to-noise ratio. The RVs published in W16 are these *detrended* RVs; the published RV uncertainties also already have jitter applied. We obtained both the pre-detrending RVs and the uncertainties without jitter from Weiss (personal communication), and re-ran the analyses described in Section 3. As before, we ended up favouring a three-planet plus correlated noise model strongly over all competing models, and the posterior distributions for the parameters of all three Keplerians were consistent ($<1\sigma$) with those obtained when using the detrended HIRES RVs.

To explore the possibility of the HARPS-N data reduction pipeline contributing to the discrepancy, we applied a novel, template- and mask-free approach we are developing (paper in preparation) for extracting RVs from observed spectra. We model each observed spectrum non-parametrically, with shifts between all possible pairs of spectra included as parameters in the modelling (in addition to possible telluric, stellar activity and instrumental effects). Interestingly, we found that when modelling HARPS-N RVs extracted with our own pipeline versus the HARPS-N pipeline, our inferred RV semi-amplitude for K-10b was unchanged, but we reliably inferred $K_{\mathrm{c}} < 2\,\mathrm{m\,s^{-1}}$ even *without* a correlated noise (GP) component in our model. This suggests the possibility that at least part of the signal confounding K-10c's signal might be instrumental rather than stellar (and would explain why the same nuisance signal is absent from the HIRES RVs); given the preliminary nature of our pipeline, however, further investigation is required.

5 DISCUSSION AND CONCLUSIONS

Previous studies (e.g. Dawson & Fabrycky 2010) have explored the impact of irregular time sampling on planet *period* estimation; here we have demonstrated that a failure to account for one or more coherent signals (whether of stellar, planetary or instrumental origin) in RV data, and/or uneven phase coverage, can confound attempts to infer the *masses* of planets with known periods. We used synthetic data with sampling based on real observations to demonstrate how such difficulties could arise when characterizing planets in a system analogous to K-10; tests such as the ones we presented may readily be applied to other systems, to test the sensitivity of planet characterization to sampling and model selection.

By accounting for a time-correlated (stellar or instrumental) signal present in the HARPS-N K-10 RVs, as well as a likely third planet in the system, we were able to achieve full consistency between the Keplerian solutions for the HIRES, HARPS-N and combined RVs. The third planet included in our model has properties consistent with K-10c's TTVs; and although our model is more complex than the one used by W16 to model the RVs, it was nevertheless favoured over simpler models in Bayesian model comparison testing. While our proposed resolution of the K-10c mass discrepancy is a plausible one, it appears that (many) more RVs will be required for a definitive characterization of the K-10 system.

Whereas W16 suggested a strategy of employing a long observing baseline compared to time-correlated noise influences, we suggest it's also important to focus on obtaining more complete *phase coverage* of the relevant signals. As we demonstrated in Section 2, good phase coverage can permit robust inference about known planets, even when using a demonstrably inadequate physical model. While uniform cadence might not be feasible or desirable, e.g. to avoid aliasing, a long observing baseline and approximately uniform cadence would lead to good phase coverage even of planets with unknown orbital periods (see Appendix A, online, for more details). And while W16 suggested that a long baseline would help to average out spurious signals that may arise from stellar activity, we suggest it is strongly preferable to *model* these nuisance signals, as it is difficult to know a priori how these nuisance signals might interfere with signals of interest. Baselines and cadence aside, it seems all but essential to implement a variety of physical models (to account for varying numbers of possible planets, nuisance signals, etc.), and to compare systematically the evidence for the competing models.

Our findings may also have relevance to archival RV data sets, and indeed, this is not the first example of a system where inference has turned out to be extremely sensitive to both sampling and model choice, despite the availability of a large number of RVs (Rajpaul, Aigrain & Roberts 2016). Then again, K-10 might have been a relatively pathological case; as W16 noted, there were various hints (TTVs, K-10c mass discrepancy, etc.) that existing characterizations of the system were inadequate. Looking to the future, with a new generation of RV spectrographs with expected precisions of $10\,\mathrm{cm\,s^{-1}}$ soon to come online, optimized sampling strategies and careful model selection will clearly both be essential if these spectrographs are to be used for accurate characterization of small planets, especially those in potentially multiplanet systems. Moreover, it would be prudent to coordinate observations made by different teams with different telescopes, to minimize 'redundant' observations that do not contribute to improved coverage of a given planet's orbital phases.

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SUPPORTING INFORMATION

Supplementary data are available at *MNRASL* online.

Figure 2. As for Fig. 1, but now with *noisy* synthetic data comprising four rather than three sine waves.

Figure 4. Mass–radius relation for planets smaller than 3.2 $R_\oplus$, and mass determinations better than 20 per cent precision.
Appendix A. On uniform cadence and phase coverage.

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