MODELLING FOAM DRAINAGE

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Abstract

Foaming occurs in many distillation and absorption processes. In this paper, two basic building blocks that are needed to model foam drainage and hence foam stability are discussed. The first concerns the flow of liquid from the lamellae to the Plateau borders and the second describes the drainage flows that occur within the borders. The mathematical modelling involves a balance between gravity, diffusion, viscous forces, and varying surface tension effects with or without the presence of monolayers of surfactant. In some cases, mass transfer through the gas–liquid interface also causes foam stabilisation, and must be included. Our model allows us to clarify which mechanisms are most likely to dominate in both the lamellae and Plateau borders and hence to determine their evolution. The model provides a theoretical framework for the prediction of foam drainage and collapse rates. The analysis shows that significant foam stability can arise from small surface tension variations.

Keywords: Foam, thin films, surfactant, Marangoni, fluid flow, surface tension effects

INTRODUCTION

Unwanted foam, occurring in a distillation or absorption column, can cause substantial problems such as loss of throughput, or deterioration in product quality through loss of separation efficiency. The foam itself may be formed on trays or in downcomers, in inlet devices or reboilers. It is less common in packed columns, where the vapour is generally the continuous phase.

It is not easy to define what constitutes a foam. Many bubbly dispersions are fairly stable over considerable time intervals, but they would only be described by an operator as "foam" if their longevity was sufficient to cause practical difficulties in the processing equipment. It is regretably common that small increases in a foam's longevity can cause an increase in the net volume production of the foam that is catastrophic in practice. This sensitivity to the local operating conditions is the major cause of uncertainty in the prediction of "hold-up" (*i.e.* the gaseous volume fraction) and the consequent interfacial areas.

Foams are stabilised by surface effects which counteract drainage of liquid from the films, or lamellae, between adjacent bubbles, where the curvature of the interface is small, into the "Plateau borders" that bound these lamellae. In these borders, the curvature is large and this induces a large negative pressure (see figure 1 for a schematic of the geometry under consideration). The stabilisation mechanism may sometimes be caused by the formation of a solid surface layer, as in some ageing dry foams, or by surface tension gradients: it is only the latter phenomena with which we will be concerned in the following sections. Such surface tension gradients can arise in at least two ways.

1. Surfactant Stabilisation: If the liquid contains a soluble surfactant, whose molecules are present throughout the liquid and as monolayers at the gas/liquid interfaces,

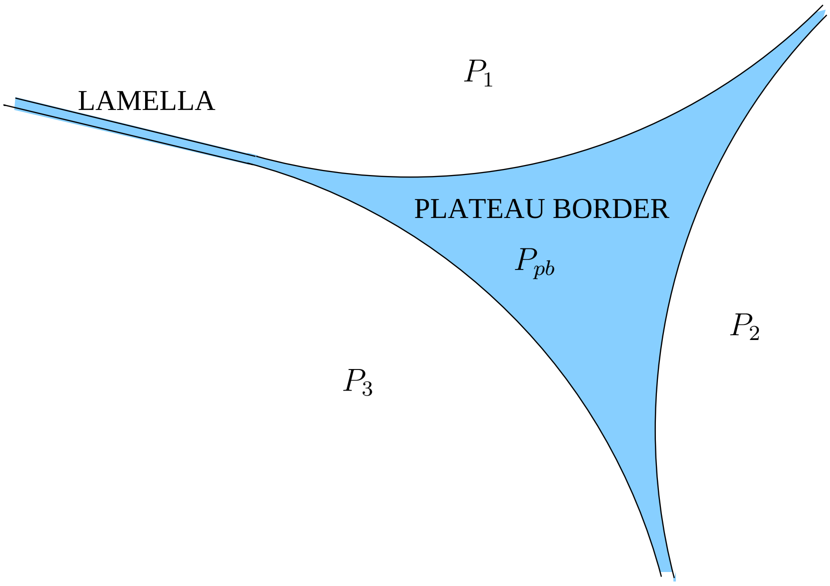

Figure 1: Schematic of a Plateau border, with P_i denoting the pressure in the i-th bubble, and P_{pb} denoting the pressure in the Plateau border

differences in surface concentration cause surface shear forces that may counteract drainage from the lamellae into the Plateau borders. The way in which this happens can be described by a qualitative model that incorporates the effects of curvature changes, bulk diffusion and the assumption of thermodynamic equilibrium of the surface concentration. At equilibrium there will be a thermodynamic relation between the bulk surfactant concentration near the surface and the surface concentration in the monolayer, generated by the surfactant's preference to accumulate at an interface.

2. Marangoni Stabilisation: Here there is no surfactant but there is mass transfer between the liquid and vapour phases of at least one volatile component. This surface evaporation causes the surface concentration of the relevant component to vary and, if the system is one in which this variation in surface concentration causes surface tension to be less at the Plateau border (the "Marangoni positive" case), then the surface tension gradient will oppose drainage.

Our objective in this paper is to construct mathematical models including the influence of both mechanisms on the drainage of the lamellae, from which we can make quantitative predictions. These models provide a consistent mathematical framework for future work where foam volumes and stability in actual systems will be determined.

First, we will derive the basic hydrodynamic equations that govern flow in lamellae and then we will discuss how to model the effects of either of the two stabilisation mechanisms. Next we describe how to close these drainage models by matching their solutions to flow in the high curvature Plateau borders and finally we will make some comments about flow in those borders. This work indicates those parameters to which the stability of the foam is most sensitive.

DERIVATION OF THIN FILM EQUATIONS

Consider the two-dimensional flow of an incompressible viscous liquid between two time-dependent free surfaces, given by $y = H(x,t) \pm \frac{1}{2}h(x,t)$. Thus, h denotes the thickness of the liquid film, and $y = H(x,t)$ is the equation of its centre-line. The following derivation is based on the long-wave approximation, whereby the lengthscale in the y-direction (*i.e.* across the film) is assumed to be much less than that in the x-direction (*i.e.* along the film). Physically, this assumption restricts our attention to a dry foam, that is, one with a very high gaseous volume fraction. This motivates the following nondimensionalisation of x, y, t, the liquid velocity field (u, v) and the liquid pressure p:

$$\begin{array}{lll} x = Lx', & y = \epsilon L y', & t = \frac{L}{U}t' \\ u = Uu', & v = \epsilon U v', & p = \mu \frac{U}{L} p', \end{array} \tag{1}$$

where μ is the liquid viscosity and $\epsilon \ll 1$ is the slenderness parameter, assumed given throughout this paper, and $O(10^{-3})$. L, μ and ρ, the liquid density are also assumed to be given as data, whilst U will be determined from the prevailing physical mechanism. In flows that are of practical interest in distillation columns, the velocities that occur are, at most, 10^{-3} ms^{-1} and we will restrict our attention to these. We will, of course, verify this restriction a posteriori.

After invoking (1), the two-dimensional Navier-Stokes equations take the form (on dropping primes, and where suffices denote partial differentiation)

$$u_x + v_y = 0, \tag{2}$$

$$\epsilon^2 \ \mathrm{Re}\,(u_t + uu_x + vu_y) + \epsilon^2 p_x = u_{yy} + \epsilon^2 u_{xx}, \tag{3}$$

$$\epsilon^2 \ \mathrm{Re}\,(v_t + uv_x + vv_y) + p_y = v_{yy} + \epsilon^2 v_{xx}, \tag{4}$$

where as usual $\mathrm{Re} = \rho U L/\mu$ is the Reynolds number.

On the two free surfaces we now apply continuity of normal and tangential stress. In so doing, we suppose the dimensionless ambient pressure on either side of the film is $P_\pm$ and that on each free surface there is a surface tension of the form

$$\sigma = \gamma + \Delta\gamma\sigma_\pm. \tag{5}$$

Thus the surface tension has a constant component (γ) and a variable ("Marangoni") component, $\Delta\gamma\sigma_\pm$, where $\sigma_\pm$ is the nondimensional surface tension, necessarily $O(1)$ or less. The dimensionless groups associated with these two effects are the capillary number and Marangoni number:

$$\mathrm{Ca} = \frac{\mu U}{\gamma}, \qquad \mathrm{Ma} = \frac{\Delta\gamma}{\mu U}, \tag{6}$$

and in terms of these the normal and tangential stress conditions on the free surfaces read

$$\begin{aligned} P_\pm - p + 2\frac{\epsilon^2(H_x \pm \frac{1}{2}h_x)^2 u_x - (H_x \pm \frac{1}{2}h_x)(u_y + \epsilon^2 v_x) + v_y}{1 + \epsilon^2(H_x \pm \frac{1}{2}h_x)^2} &= \\ \pm\left(\frac{\epsilon}{\mathrm{Ca}} + \epsilon Ma\sigma_\pm\right)\frac{H_{xx} \pm \frac{1}{2}h_{xx}}{\left[1 + \epsilon^2(H_x \pm \frac{1}{2}h_x)^2\right]^{3/2}}&, \end{aligned} \tag{7}$$

$$\left(1-\epsilon^2(H_x \pm \tfrac{1}{2}h_x)^2\right)(u_y+\epsilon^2 v_x)+2\epsilon^2(H_x \pm \tfrac{1}{2}h_x)(v_y-u_x) = \pm\epsilon\ \mathrm{Ma}\left(1+\epsilon^2(H_x \pm \tfrac{1}{2}h_x)^2\right)^{1/2}(\sigma_\pm)_x, \tag{8}$$

respectively.

We avoid concepts such as "surface viscosity"or "surface elasticity" (used by other authors, see Aris[1] , for example): indeed it is our contention that the important effects that result in foam stabilisation can be explained using *only* Marangoni stresses (generated by either of the mechanisms discussed in the introduction), whose experimental measurability are comparatively well-established compared to other "surface phenomena".

The fluid mechanics problem is closed by imposing a kinematic condition on the free surfaces. We include the possibility that liquid evaporates from these surfaces at a rate $E_0 e_\pm$, and hence introduce a dimensionless evaporation constant $E = E_0/\epsilon U$, in terms of which the kinematic conditions read

$$v = H_t \pm \tfrac{1}{2}h_t + u(H_x \pm \tfrac{1}{2}h_x) \pm E e_\pm \sqrt{1+\epsilon^2(H_x \pm \tfrac{1}{2}h_x)^2}. \tag{9}$$

Now a simplified one-dimensional model for the evolution of the film can be derived by expanding the dependent variables as asymptotic series in powers of the small parameter ϵ. With $U \sim O(10^{-3})$ ms^{-1}, Re and E are O(1), and the following equations (valid up to O(ϵ^2)) are obtained. First we have the physically obvious mass-conservation relation

$$h_t + (\bar{u}h)_x + E(e_+ + e_-) = 0, \tag{10}$$

where the axial velocity, u is related to its average $\bar{u}$ by

$$u = \bar{u} + \frac{\epsilon\ \mathrm{Ma}}{2}(\sigma_+ - \sigma_-)_x(y-H) + \frac{\epsilon^3}{4\ \mathrm{Ca}}h_{xxx}\left(-y^2+2Hy-H^2+\frac{1}{12}h^2\right). \tag{11}$$

The liquid pressure is

$$p = -\left[\frac{\epsilon}{2\ \mathrm{Ca}} + \frac{\epsilon\ \mathrm{Ma}}{4}(\sigma_+ + \sigma_-)\right]h_{xx} - \frac{\epsilon\ \mathrm{Ma}}{2}\left[(\sigma_+ - \sigma_-)_{xx}(y-H)\right] - 2u_x, \tag{12}$$

and the balance of axial momentum gives

$$(4h\bar{u}_x)_x + \frac{\mathrm{Ma}}{\epsilon}(\sigma_+ + \sigma_-)_x + \frac{1}{2}\frac{\epsilon}{\mathrm{Ca}}hh_{xxx} - \ \mathrm{Re}h(\bar{u}_t + \bar{u}\bar{u}_x) = 0. \tag{13}$$

The normal momentum balance gives a decoupled equation for H:

$$\frac{\Delta p}{\epsilon^2} + \ \mathrm{Re}hH_{tt} + 2\ \mathrm{Re}h\bar{u}H_{xt} + \left(\ \mathrm{Re}h\bar{u}^2 - 4h\bar{u}_x - \frac{2}{\epsilon\ \mathrm{Ca}} - \frac{\mathrm{Ma}}{\epsilon}(\sigma_+ + \sigma_-)\right)H_{xx} - \frac{\mathrm{Ma}}{2\epsilon}\left[(\sigma_+ - \sigma_-)\,h\right]_{xx} = 0. \tag{14}$$

where Δp is the pressure drop across the film. There are many dominant balances that occur within these equations. For example, setting $\Delta p \sim O(1)$, $\mathrm{Ca} \sim O(\epsilon^3)$ and $\mathrm{Ma} \sim O(\epsilon^{-1})$ leads to a lubrication flow, with velocities of the order 10^{-7} ms^{-1}, as in a classical bearing squeeze film.

In summary, (10) and (13) comprise two equations for h and $\bar{u}$, and the velocity and pressure fields, u, p, and the centre–line H, are given by (11), (12) and (14) respectively. We seek solutions which exhibit symmetry about the centre–line, and so henceforth we set $H \equiv 0$, $\sigma_+ = \sigma_-$ and $e_+ = e_-$. In practical cases, where the bubbles are almost of equal size and the pressures within them are practically identical, Δp is small, and so may be set to zero in (10-14). Before the equations can be solved, the evaporation rates $e_\pm$ and surface tensions $\sigma_\pm$ must be specified. In the following two sections, we examine two physical mechanisms which determine $e_\pm$ and $\sigma_\pm$. We re-emphasise that throughout this section we have used values of U that are known externally from experiments. Only when we come to couple the thin film flow into the Plateau border will we be able to predict the value of U and hence check consistency.

THIN FILM WITH A SOLUBLE SURFACTANT

Here we give equations to be coupled to (10–14) if evaporation does not occur and the surface tension gradients are solely caused by the presence of a surfactant dissolved in an inert liquid. The bulk surfactant concentration C is governed by the convection–diffusion equation:

$$\epsilon^2 \ \mathrm{Pe}\,(C_t + uC_x + vC_y) = (DC_y)_y + \epsilon^2 (DC_x)_x. \tag{15}$$

Here, the Peclet number is defined by $Pe = LU/D_0$ and the diffusion coefficient is $D_0 D(C)$. A kinematic relation for C relates flux of bulk surfactant into the surface to the rate of change of surface concentration Γ and reads, on $y = \pm\frac{1}{2}h$,

$$\Gamma_t + \frac{1}{\sqrt{1 \pm \epsilon^2 \frac{1}{4} h_x^2}} \left\{ \frac{\Gamma}{\sqrt{1 \pm \epsilon^2 \frac{1}{4} h_x^2}} \left[u \pm \epsilon^2 \tfrac{1}{2} h_x v \right]_{y=\pm\frac{1}{2}h} \right\}_x$$

$$= \frac{S}{\sqrt{1 \pm \epsilon^2 \frac{1}{4} h_x^2}} \left[\mp C_y + \epsilon^2 \tfrac{1}{2} h_x C_x \right]. \tag{16}$$

where $S = D_0 C_0 / \epsilon U \eta_1$ and C is nondimensionalised with C_0 and η_1 is a surfactant-specific constant (used to nondimensionalise Γ).

Coupled with this is an empirical relation between Γ and C at the free surfaces, assuming they are at thermodynamic equilibrium. The Langmuir isotherm (see Adamson[2]) is usually employed:

$$\Gamma = \left[\frac{C}{\eta_3 + C} \right], \tag{17}$$

where $\eta_3 = \eta_2 / C_0$ and η_2 is another surfactant-specific constant (η_1 and η_2 can be deduced from an experimentally-generated Langmuir isotherm). We relate σ on the two free surfaces to Γ using the linearised Franklin equation (see Adamson[2]), $\sigma_\pm = 1 - \lambda\Gamma$. The model is now

$$h_t + (\bar{u}h)_x = 0, \tag{18}$$

$$u = \bar{u} + \frac{\epsilon^3}{4 \ \mathrm{Ca}} h_{xxx} \left(-y^2 + \frac{1}{12} h^2 \right), \tag{19}$$

$$(4h\bar{u}_x)_x - \frac{2 \ \mathrm{Ma}\lambda}{\epsilon} \Gamma_x + \frac{\epsilon}{2 \ \mathrm{Ca}} h h_{xxx} - \ \mathrm{Re} h(\bar{u}_t + \bar{u}\bar{u}_x) = 0, \tag{20}$$

and coupling these with (15), (16) and (17) now closes the problem for u, $\bar{u}$, h, C and Γ.

It is common practice (see Jensen and Grotberg[3]) to linearise the Langmuir isotherm (17), which is equivalent to considering large η_3, *i.e.* small bulk surfactant concentration C_0. Another interesting limit occurs if η_3 is taken to be small, which will occur at large bulk concentrations. In this limit, (17) gives

$$\Gamma \sim 1 + O(\eta_3), \tag{21}$$

and hence (16) implies that

$$\frac{\partial}{\partial x}\left\{u(x, \pm\tfrac{1}{2}h)\right\} \sim O(\eta_3, S). \tag{22}$$

This reduces to a no-slip condition at the free surfaces if both η_3 and S are small (that is, the surfactant is also relatively insoluble). Such a condition has been employed by previous authors (Schwarz & Princen[4]) in modelling soap films, but never previously justified from any systematic modelling of the free surface properties.

THIN FILM WITH A VOLATILE LIQUID COMPONENT

We consider the case in which there is no surfactant but the liquid in the film has two components with different evaporation rates and different surface tensions. We neglect variations in density and viscosity since it is the stabilising or destabilising effects of the surface tension gradients in which we are particularly interested. For simplicity we assume that only one liquid component is volatile, and denote its concentration by s, which therefore satisfies the convection-diffusion equation (15)

$$\epsilon^2 \ \mathrm{Pe}\,(s_t + us_x + vs_y) = (Ds_y)_y + \epsilon^2(Ds_x)_x. \tag{23}$$

At the free surfaces, $y = \pm\frac{1}{2}h$, the liquid evaporates at a rate $E_0 e(s)$ and hence the boundary condition for s is:

$$D\left(s_y \mp \epsilon^2 \frac{1}{2} h_x s_x\right) \pm \epsilon^2 \ \mathrm{Pe} E e(1-s)\sqrt{1 \pm \epsilon^2 \frac{1}{4} h_x^2} = 0. \tag{24}$$

The character of the leading-order problem for s is determined by the size of the relevant dimensionless parameters, here Pe and E. With $U \sim 10^{-3}\mathrm{ms}^{-1}$, both Pe and Pe$E$ are $O(10^3)$, which yields a particularly tractable problem. In this case, (23) and (24) dictate that s be independent of y to leading-order — this is the so-called well-mixed approximation. Then a one-dimensional evolution equation for $s(x,t)$ is obtained by considering (23) and (24) at $O(\epsilon)$:

$$\hat{\mathrm{P}}\mathrm{e} h\,(s_t + \bar{u}s_x) = -2\ \hat{\mathrm{P}}\mathrm{e} E(1-s)e \tag{25}$$

where $\hat{\mathrm{P}}\mathrm{e} = \epsilon\ \mathrm{Pe}$ and, as before, $\bar{u}$ is the y-averaged axial velocity.

We must specify a model for evaporation and a relationship between surface tension and concentration in order to close this system. As a first approximation, we assume that $e = s$, and that $\sigma = -\alpha s$; when $\alpha > 0$ ("positive Marangoni"), the surface tension gradients stabilise the film, and vice versa. The governing equations (10–13) give

$$h_t + (\bar{u}h)_x + 2Es = 0, \tag{26}$$

$$u = \bar{u} + \frac{\epsilon^3}{4\ \mathrm{Ca}} h_{xxx} \left(-y^2 + \frac{1}{12} h^2 \right), \tag{27}$$

$$\left(4h\bar{u}_x\right)_x - \frac{2\ \mathrm{Ma}\alpha}{\epsilon} s_x + \frac{\epsilon}{2\ \mathrm{Ca}} h h_{xxx} - \ \mathrm{Re} h(\bar{u}_t + \bar{u}\bar{u}_x) = 0, \tag{28}$$

with (25) closing the model.

For films with larger aspect ratio, $\epsilon \sim 5 \times 10^{-2}$ or bigger, the well-mixed approximation breaks down and a two-dimensional problem for s must be solved. Putting $\epsilon^2\ \mathrm{Pe} = \ \mathrm{Pe}^*$, the leading-order problem for s reads

$$\begin{gathered} \mathrm{Pe}^* \left(s_t + u s_x + v s_y\right) = \left(D s_y\right)_y, \\ D s_y \pm \ \mathrm{Pe}^* E(1-s)e = 0 \quad \text{on } y = \pm\tfrac{1}{2}h. \end{gathered} \tag{29}$$

We note that if $\epsilon^3 /\ \mathrm{Ca} \ll 1$, so that u is uniform to leading order, (29) can be simplified somewhat via the transformation $y \mapsto h\zeta$.

MATCHING WITH PLATEAU BORDERS

We can only close our model and make predictions about U when we couple one of the above lamella models with the Plateau borders.

As the fluid approaches the Plateau border, the surfaces start to diverge rapidly and curvature effects become more important than in the lamella proper. The concept of a transition region between a lamella and a Plateau border has been employed by Barigou and Davidson[5]. In their work, an ad hoc "patching" is carried out between a capillary–static Plateau border, a transition region in which surface tension gradients are important (the free surfaces are effectively rigid), and a lamella described by an extensional flow with free boundaries. This leads to a drainage velocity from the lamella to the Plateau border given by

$$u = \frac{1}{32} \frac{\sigma h^2}{\mu x_0 a}, \tag{30}$$

where x_0 is the length of the lamella. Velocities given by (30) are $O(10^{-6})$ ms^{-1}, which is consistent with the earlier assumption that U be no greater than $O(10^{-3})$ ms^{-1}. Hence the typical time to rupture is $\sim 10^3 s$.

We have not found any literature in which a systematic mathematical matching is used to describe the transition region in a draining foam of practical interest. We note that Schwartz and Princen[4] have achieved such a matching in the case of a thin film/Plateau border flow that is driven by shear and in which implicit Marangoni effects are strong enough for the free surfaces to be perfectly rigid everywhere in the transition region. On the other hand, Howell[6] has considered the related problem of drainage from a bubble near the surface of a viscous liquid; there the transition region, capillary–static outer region and thin film all have stress free boundaries. The same analysis can be applied to a capillary foam if there are no surface tension gradient effects, with a lamella thinning in an extensional flow being matched to a capillary static Plateau border (see figure 2). The principal result is an expression for the rate at which liquid is sucked from the lamella into the plateau border:

$$u = \frac{3\sigma}{8\sqrt{2}\mu} \sqrt{\kappa h}, \tag{31}$$

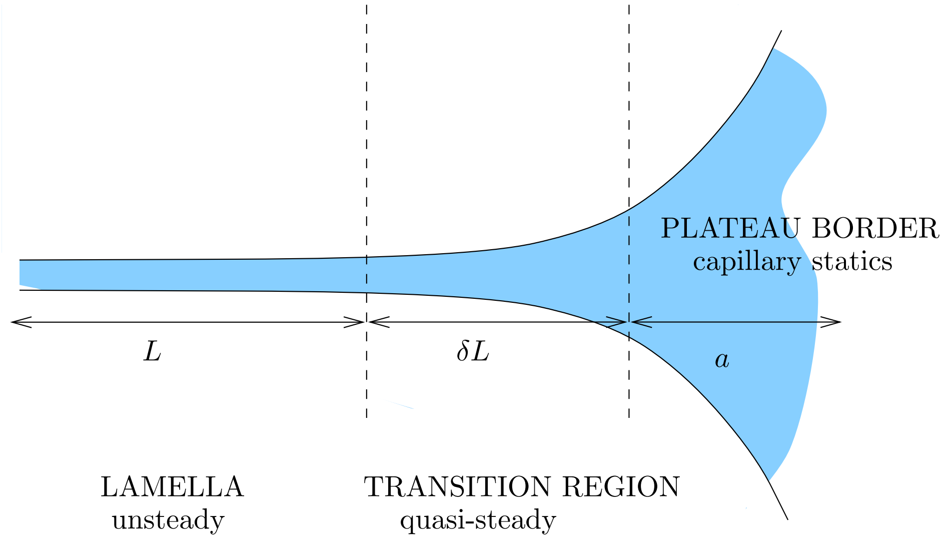

Figure 2: Schematic of the transition from a lamella to a Plateau border

where κ is the curvature of the Plateau border. This leads to velocities of the order 1 ms^{-1}, which violates our assumption about the size of U and demonstrates that surface tension gradient effects dominate the extensional flow in the transition region. The timescale for lamella rupture predicted by (31) is $\sim 10^{-2}s$.

Experimental observations (see Barber and Hartland[7]) of foam of practical interest in distillation columns report rupture times of 1-10s, which lie between those predicted by (30) and (31). Since the two situations mentioned above represent an immobile and a completely free interface in the transition region respectively, it is reassuring that the corresponding velocities bound the observed fluid drainage velocities in distillation columns.

Instead of following Barigou and Davidson[5] or Howell[6], we proceed by basing the velocity scaling in the transition region on the two mechanisms described above, *i.e.* evaporation or surfactant replenishment (setting E or S equal to 1 respectively). We find that

$$U \sim \frac{E_0}{\sqrt{\kappa h_0}} \qquad \text{or} \qquad U \sim \frac{D_0 C_0}{\eta_1 \sqrt{\kappa h_0}} \tag{32}$$

respectively, where h_0 is a typical lamella thickness. In both these cases, $U \sim 10^{-3}$ ms^{-1}, which is within the above bounds and is consistent with the assumption about velocity size given at the start of the paper. With velocities of this size, the Marangoni number is typically $O(1)$ and the capillary number is $O(\frac{\epsilon L}{a})$, using the notation of figure 2. Considering (13) in this parameter regime leads us to the conclusion that *drainage in most practical foams is governed by a balance between capillary and surface forces within the transition region.* This transition region must then be matched with the capillary–dominated Plateau border and a lamella in which viscous and surface forces balance. However, the analysis is now more complicated than in Howell[6], since the hydrodynamic and concentration problems are coupled in the transition region via the dependence of the surface tension gradient on concentration.

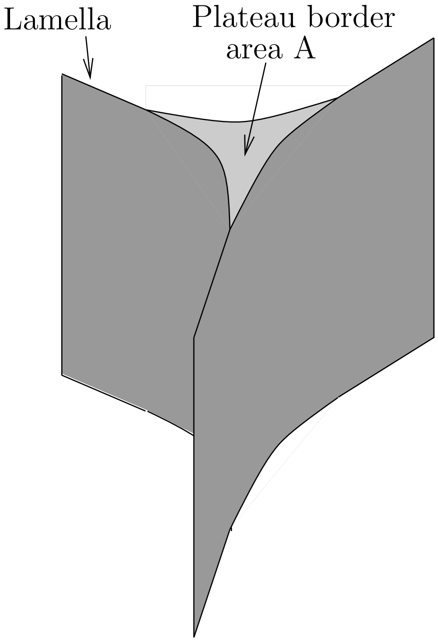

Figure 3: Schematic showing fluid in a 3-D Plateau border of cross–sectional area A

THREE–DIMENSIONAL EFFECTS: PLATEAU BORDER MODELLING

When looking at a two–dimensional foam, the Plateau borders are governed by capillary statics. When drainage is to be modelled, three–dimensional effects must be taken into account. We follow a similar approach to that described earlier to derive the leading order equations governing the evolution of the Plateau border area, A. The characteristic lengthscale, L, is the same as for the lamellae, but now the aspect ratio, ϵ', is different. We presume that the lengthscale of the transition region is small compared to the transverse lengthscale of a Plateau border (borne out by physical observations, see Barigou and Davidson[5] for a scaled drawing), so that the area described in this section is where the majority of the fluid within a Plateau border is situated, and corresponds to the area of the capillary static two–dimensional Plateau border. A schematic showing the fluid under consideration is shown in figure 3.

We assume again that $U \sim O(10^{-3})$ ms^{-1}, and that the flow is extensional. We include gravity effects, and impose the boundary conditions of zero shear, normal force balance and the kinematic condition on the free surface. We nondimensionalise A using $\epsilon'^2 L^2$ and, denoting length along the Plateau border by z, the resulting equations are

$$A_t + (Au)_z = 0 \tag{33}$$

which represents conservation of mass, and

$$3\left(Au_z\right)_z + A + \frac{\pi}{\mathrm{Ca}\epsilon' C_1^{\frac{1}{2}}}\left(A^{\frac{1}{2}}\right)_z - \mathrm{Re}A(u_t + uu_x) = 0 \tag{34}$$

which represents axial force balance. $C_1 = \sqrt{\sqrt{3} - \frac{\pi}{2}}$ and comes from the geometry of the system. Implicit in this model is the fact that the centre–line of the Plateau border is straight. A naive model for a foam may be obtained by assuming that a foam consists of connected layers of vertical Plateau borders, and that the drainage of fluid from each layer is given by the drainage from the model (33), (34) for one Plateau border, multiplied by the number of Plateau borders in the layer. This could be the building block for a better foam model, where the orientation of the Plateau borders is random. The fluid drainage from the lamellae could also be included, using a relationship similar to (31). We would simulate foam collapse by assuming that after the lamellae reach a critical thickness, they rupture, and the fluid contained within any Plateau borders held up by such a lamella would then be distributed into the layers below.

We note here that previous work modelling the Plateau borders (see Darton et al[8], Verbist et al[9,10]) assumed that there was no slip at the free surfaces, and from (22) we see that this may be justified in the limit of high surfactant concentration and small replenishment parameter S.

CONCLUSIONS

In this paper we have presented the ingredients of a systematic theoretical framework within which to model foam drainage. Our principal observations are that the basic hydrodynamics must be in accordance with (10–14), the basic mass transfer responsible for surface tension variations must be either (15–17) or (23,24) or a combination and that these equations must be coupled into the flow in Plateau borders via a model that generalises that described in this paper. It emerges that the key parameters are the capillary and Marangoni numbers ($(\mu U)/\gamma$ and $\Delta\gamma/(\mu U)$) and they need to be small and $O(1)$ respectively if drainage is to be so slow as to cause problems in typical industrial situations.

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LIST OF SYMBOLS

a	radius of curvature of Plateau border	m
A	Plateau border area	
C_0	bulk conc of surfactant	mol m^{-3}
C_1	Plateau border geometrical constant	
D_0	diffusivity	$\mathrm{m}^2\mathrm{s}^{-1}$
E	evaporation constant	
E_0	mass transfer coefficient	ms^{-1}
$e_\pm(x,t)$	evaporation rate on top and bottom surfaces	
g	gravity	ms^{-2}
$H(x,t)$	centreline of lamella	
$h(x,t)$	thickness of lamella	
h_0	typical lamella thickness	m
L	lengthscale	m
$p(x,y,t)$	pressure	
$s(x,y,t)$	concentration of volatile liquid component	
t	time	
U	velocity scale	ms^{-1}
$u(x,y,t)$	velocity along axis of lamella or Plateau border	
$\bar{u}(x,t)$	cross-sectionally averaged velocity	
$v(x,y,t)$	velocity perpendicular to axis of lamella or Plateau border	
x	coordinate along the axis of the lamella	
y	coordinate perpendicular to lamella axis	
z	coordinate along the axis of the Plateau border	
$\Gamma(x,t)$	surface concentration of surfactant	mol m^{-2}
γ	constant component of surface tension	Nm^{-1}
$\Delta\gamma$	surface tension change	Nm^{-1}
δ	transition region lengthscale	
ϵ	aspect ratio of lamella	
ϵ'	aspect ratio of Plateau border	
η_1	Langmuir constant	mol m^{-2}
η_2	Langmuir constant	mol m^{-3}
η_3	Langmuir constant	
κ	curvature of Plateau border	m^{-1}
λ	Franklin constant	
μ	viscosity	Pa s
ρ	density	kg m^{-3}
$\sigma_\pm(x,t)$	variable component of surface tension on top and bottom surfaces	

$$\mathrm{Ca} = \frac{\mu U}{\gamma} \quad E = \frac{E_0}{\epsilon U} \quad \mathrm{Ma} = \frac{\Delta\sigma}{\mu U} \quad \mathrm{Pe} = \frac{UL}{D_0} \quad \mathrm{Re} = \frac{\rho L U}{\mu} \quad S = \frac{D_0 C_0}{\eta_3 U \epsilon}$$