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UNIVERSITY OF OXFORD

OxCarre Research Paper 35

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Is There Really A Green Paradox?

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IS THERE REALLY A GREEN PARADOX?[1]

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Abstract

In the absence of a CO2 tax, the anticipation of a cheaper renewable backstop increases current emissions of CO2. Since the date at which renewables are phased in is brought forward and more generally future emissions of CO2 will decrease, the effect on global warming is unclear. Green welfare falls if the backstop is relatively expensive and full exhaustion of fossil fuels is optimal, but may increase if the backstop is sufficiently cheap relative to the cost of extracting the last drop of fossil fuels plus marginal global warming damages as then it is attractive to leave more fossil fuels unexploited and thus limit CO2 emissions. We establish these results by analyzing depletion of non-renewable fossil fuels followed by a switch to a clean renewable backstop, paying attention to timing of the switch and the amount of fossil fuels remaining unexploited. We also discuss the potential for limit pricing when the non-renewable resource is owned by a monopolist. Finally, we show that if backstops are already used and more backstops become economically viable as the price of fossil fuels rises, a lower cost of the backstop will either postpone fossil fuel exhaustion or leave more fossil fuel in situ, thus boosting green welfare. However, if a market economy does not internalize global warming externalities and renewables have not kicked in yet, full exhaustion of fossil fuel will occur in finite time and a backstop subsidy always curbs green welfare.

Keywords: Green Paradox, Hotelling rule, non-renewable resource, renewable backstop, simultaneous use, global warming, carbon tax, monopoly, limit pricing

JEL codes: Q30, Q42, Q54

Revised 11 December 2011

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[1] We are grateful to Rob Aalbers, Alex Bowen, Christa Brunnschweiler, Reyer Gerlagh, Dieter Helm, Cameron Hepburn, Erik Verhoef and Amos Zemel for helpful suggestions on the costs and CO2 intensity of various energy sources, two anonymous referees, Lucas Bretschger, Reyer Gerlagh, Michael Hoel, Ngo Van Long, Stephen Salant, Hans-Werner Sinn, Tony Venables, participants in seminars at OxCarre, Groningen, CORE, Louvain, CPB, The Hague and CenDEF at the University of Amsterdam, and at the conferences SURED in Ascona and WCERE 2010 in Montreal for helpful comments on earlier versions of the paper, to Vincent van Goes for preparing figure 3.

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1. Introduction

The accumulation of $CO2$ due to the extraction and use of fossil fuels is the main cause of climate change. In a somewhat different context d'Arge and Kogiku (1973) cite Boulding (1971): ".. the 'pure' mining problem must be coupled with the 'pure' pollution problem and questions like these become relevant: which should we run out first, air to breathe or fossil fuels to pollute the air we breathe?". In the design of optimal climate policy one could neglect the exhaustibility of fossil fuels by arguing that they are abundant until the far future, as is the case for coal or oil from tar sands. However, this may lead to the failure of climate policy. In the absence of renewable resources such as solar or wind energy, some fossil fuels such as oil and gas are essentially available in limited amounts and their optimal intertemporal use needs to be determined in conjunction with any adverse effects this may have on global warming. The optimal policy of extracting such fossil fuels and combating climate change should take into account the order in which the fuels are to be extracted. In doing so, differences in extraction costs for the various sources of energy as well as differences in the contributions the resources make to climate change play a role. With the availability of renewable backstops these problems persist. In addition, the timing, order and speed of extraction in conjunction with the introduction of the backstop are crucial for future welfare.

Our aim is to first present a dynamic welfare analysis in a world where climate change poses a serious negative externality. We explicitly consider exhaustibility of some fossil fuels, but following the pioneering work of Tahvonen (1979) we also study renewable backstops[4]. Backstops are defined in our paper as renewable resources that are perfect substitutes for fossil fuel and not constrained by exhaustibility. By characterizing the first-best outcome and the time path of the optimal carbon tax, we obtain a natural benchmark for considering the second-best policy of subsidizing the carbon-free substitute in a market economy. This is motivated by Sinn's (2008ab) argument, building on the earlier contributions by Sinn (1981, 1982), that this, somewhat paradoxically, may have detrimental climate effects. Sinn's 'Green Paradox' has received a lot of attention both in the press and in academia. It is firmly linked to fighting climate change through fossil-fuel demand-reducing policies that are *intended* to flatten the time profile of carbon emissions. The impact of such policies is paradoxical if they steepen rather than flatten the extraction path of fossil fuel – *despite good intentions*. Sinn argues that the paradox occurs if and only if demand-reducing policies become more stringent over time. These arguments have recently also been scrutinized more rigorously by Hoel (2008) and Gerlagh (2011).

[4] Papers addressing externalities and exhaustibility, but abstracting from a backstop, include Krautkraemer (1985), who is mainly interested in preservation in view of amenity values, Withagen (1994), who shows that initial use of the exhaustible resource is smaller than without the externality, Ulph and Ulph (1994), who deal with optimal (dynamic) taxation of fossil fuels and their detrimental effect on the environment, and Sinclair (1994), who argues that with endogenous growth optimal fossil fuel taxes may fall rather than rise over time.

We first derive the socially optimal depletion paths for fossil fuels and use of renewable. To implement the optimum in a market economy in our model, this requires a rising carbon tax to internalize global warming externalities, but no subsidy on renewables. Our point of departure is then a market economy where the government has at its disposal a very special form of demand-reducing policy, i.e., a subsidy on renewable energy whose rate is constant over time, which it resorts to when it finds it impossible to levy a rising CO2 tax. Following Gerlagh (2011), one could distinguish a *weak* Green Paradox which occurs if fossil fuel is currently pumped up more quickly due to the anticipation of cheaper renewables from a *strong* Green Paradox which says that green welfare falls on account of the subsidy. In this specific, albeit realistic second-best setting we demonstrate that the anticipation of cheaper renewables leads to faster pumping of fossil fuel and raises current emissions of CO2 (weak Green Paradox). We provide examples where following an increase in the subsidy on the carbon-free backstop green welfare increases (no strong Green Paradox) if it is optimal to leave some of the fossil fuel reserves forever in the crust of the earth. But the subsidy leads to quicker exhaustion of fossil fuel and a fall in green welfare (strong Green Paradox) if fossil fuel is fully exhausted in finite time.

We emphasize the following features of our analysis.

In the first place, we study in detail the situation where marginal extraction costs of the non-renewable resource depend on the existing stock. It follows that lowering the cost of supplying the renewable backstop may lead to a positive remaining stock of fossil fuel reserves in case the backstop price is lower than the marginal extraction costs at low resource stocks.

Secondly, we focus on backstops which do not cause CO2 emissions such as solar or wind energy. [5] In first instance we consider the case where the unit production costs of the backstop are constant. Whether a backstop technology is cheap relative to fossil fuel depends on its own production cost and the total cost associated with fossil fuel, consisting of the stock dependent extraction costs and the damage caused by

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[5] We thus abstract from heavily polluting and expensive backstops (van der Ploeg and Withagen, 2011). An example of this is the tar sands, because their reserves are much larger than conventional oil and gas reserves. Although burning oil from tar sands yields same emissions as burning conventional oil, a lot of energy is used in producing oil from tar sands and therefore CO2 emissions are much higher. In a recent study for the European Commission, Brandt (2011) reports that emissions of greenhouse gases are on average 23% higher than from conventional oil, and in some cases much more. This has led the EU to consider a ban on oil from tar sands. Tar sands also adversely affect the livelihood of indigenous communities via large-scale leakage of toxic waste in groundwater and destruction of ancient forests larger than the size of England. We also abstract from coal which is heavily polluting (electricity from coal-fired plants are 30% higher than oil-fired plants), but cheap to exploit (depending on location and soil characteristics). Also, the process of making coal liquid so that it can be a substitute for oil in transportation takes a lot of energy.

the accumulation of CO2 emissions.[6] Therefore, the relative cheapness of backstops from a social perspective is changing over time with a decreasing stock of fossil fuel and an increasing stock of CO2.

We pay special attention to the case where these backstops are still relatively expensive, possibly not when it comes to the marginal production costs once capacity is installed but surely when it comes to the costs of increasing capacity, to do with intermittence and repair (especially of offshore wind mills). As far as the electricity industry is concerned, solar energy is currently 50% more expensive than conventional electricity, wind energy has the same cost and is (apart from the problem of intermittence) competitive, and biomass, CCS coal/gas and advanced natural gas combined cycle have mark-ups of 10%, 60% and 20%, respectively (Paltsev et al., 2009). These mark-ups for renewable energy sources are measured from a very low base and may be less when they account for a much larger market share, but higher in other applications. Another energy source that does not emit CO2 is nuclear energy, which is deemed to be rather competitive already, possibly due to the neglect of the cost to be incurred after the plants become obsolete including the cost of disposing of nuclear waste. We show that with this cost configuration the weak Green Paradox holds. In as far as advanced nuclear is much more expensive than conventional energy as suggested by a mark-up of 70% (Paltsev et al., 2009), our arguments suggest that the strong Green Paradox may not hold. In a sense carbon sequestration of electricity-generating industries may be viewed as an expensive backstop compared to conventional oil or gas but with lower CO2 emissions.

Thirdly, our policy recommendations follow straightforwardly from dynamic optimization. Optimal taxes correspond to the shadow prices that are generated in the social optimum. We will characterize them, also in a dynamic setting. However, on a worldwide scale these optimal taxes are difficult to implement. In the long run new technologies are indispensable and one could therefore advocate subsidizing the development of clean backstop technologies. In addition to earlier analyses we add a global social welfare perspective. A lower cost of supplying the backstop is beneficial, albeit not necessarily for green welfare. We give the conditions under which in a market economy a subsidy enhances social welfare when there is no carbon tax. However, unless the reduction in the cost of the backstop is realized in a costless way, the policy will in general not be first best.

Sinn (2008a) discusses a neoclassical optimal growth model with a non-renewable resource and a global warming externality, but abstracts from explicitly analyzing backstop alternatives. He argues that in as far they are imperfect substitutes[7], they are already incorporated in the demand function for oil/gas, and

[6] Useful studies on the costs of producing various renewable and non-renewable forms of energy are Shihab Eldin (2002, European Commission (2003), Neuhoff (2004) and Paltsev et al. (2009). Some care must be taken, since the costs vary much across studies. Part of the reason is that the costs depend on which application energy is used.

[7] In Section 5, introduced in point 5 below, we implicitly allow for backstops varying in production costs.

perfect substitutes such as bio-fuels will require too much of scarce resources – land – and will meet political opposition. This is why Sinn (2008a) concentrates on policies that limit the speed of extraction, not on policies that limit the total amount of extraction of fossil fuels, and focuses only on green rather than total welfare. Our main objective is to model backstops explicitly and to analyze the effects of subsidies and taxes on the use of the backstop on total as well as green welfare paying attention to situations where variations in backstop subsidies result in leaving more or less fossil fuel reserves in situ and in switching earlier or later from fossil fuel to the carbon-free backstop. Following Sinn (2008ab), we analyze what happens if a Hotelling ramp for taxes on C02 emissions is politically infeasible. If the government then resorts to subsidizing solar or wind energy, as is done on a large scale in Germany, depletion of fossil fuels may occur more rapidly and discounting then implies that climate change damages increase. If the atmosphere is already polluted with lots of CO2 emissions, we show that it is socially optimal to postpone depletion of fossil fuels to combat global warming.

Fourthly, some argue that the Green Paradox is the result of rational speculative behaviour of resource-owners under perfect competition (Sinn, 2008ab) or resource-owning monopolists (Gerlagh, 2011). We analyze the implications of imperfect competition more formally. When it comes to the theory of non-renewable resources, this is the case studied most extensively elsewhere in the literature. The reason is that neither the oil market dominated by OPEC nor the gas market dominated by Russia, Iran, Qatar and Venezuela can be characterized as competitive. It goes beyond the scope of the present paper to extend the cartel-fringe model (e.g., Groot et al., 2003) by allowing for a renewable backstop and the interactions with climate change. However, we do pay attention to the case of a resource-owning monopolist, in order to incorporate the phenomenon of limit pricing (cf., Salant, 1977; Hoel, 1978). We show that under monopoly the Green Paradox arises. Green welfare is negatively affected for high costs of the backstop, whereas the result for low backstop costs is ambiguous

Finally, we investigate what happens if there is a continuum of backstops coming on stream as the price of fossil fuels gradually rises over time. In this case, the analysis becomes more complicated as it will be optimal to have a phase where fossil fuels and the renewable backstop are used simultaneously if global warming externalities are properly internalized. We then show that even when it is optimal to fully exhaust fossil fuel reserves, green welfare might rise provided renewables are already being used alongside fossil fuels as lowering the backstop cost will either leave more fossil fuel in situ or will postpone exhaustion of fossil fuels. However, if there is an initial phase where only fossil fuel is used, subsidizing the backstop may lead to a fall in green welfare, especially if global warming externalities are not properly internalized and exhaustion of fossil fuel takes place in finite time.

The outline of the paper is as follows. Section 2 analyzes the socially optimal transition from conventional oil and gas to a clean renewable backstop, in the face of climate externalities and stock-dependent extraction costs.[8] CO2 emissions can harm health and productivity of workers and also damage productive capacity in other ways which can be modelled through a negative externality in production (cf., Heal, 1985; Sinn, 2008ab), but we follow the mainstream approach where damage adversely affects social welfare. We abstract from capital accumulation.[9] Section 3 studies the outcome in a market economy and shows how the social optimum can be sustained with a rising CO2 tax. It also shows that in the second-best situation where a rising CO2 tax is infeasible, subsidizing the backstop need not lead to a Green Paradox if this encourages private resource owners to leave more fossil fuels in the soil and to switch more quickly to the clean backstop. Section 4 offers some ideas on the Green Paradox and imperfect competition. Section 5 investigates the implications of a sequence of backstops becoming economically viable as the social price of fossil fuel rises due to an upwards sloping supply of renewables. It also contrasts the social optimum with the market outcome and investigates the effects of subsidizing the renewable backstop. Section 6 concludes and discusses policy implications.

2. Switching from dirty fossil fuels to a clean backstop: social optimum

To have a benchmark to evaluate climate policy in a market outcome in the next section, we first consider the optimal extraction of non-renewable resources (fossil fuels) with a renewable backstop kicking in once fossil fuels become too expensive. The backstop is, for the time being, a perfect substitute for the non-renewable resource and its supply is infinitely elastic, but in section 5 we consider the more general case where there is an upward-sloping supply schedule for renewables. To be able to assess the Sinn (2008a and 2008b) arguments about the Green Paradox properly, we include climate change externalities in social welfare. With quasi-linear preferences, the social planner's welfare function reads:

(1) $$\int_0^\infty e^{-\rho t}[U(q(t)+x(t))-G(S(t))q(t)-bx(t)-D(E(t))]dt$$

[8] Many papers address transitions from non-renewables to backstops (e.g., Heal, 1976; Tsur and Zemel, 2003, 2005), but they do not explicitly analyze environmental externalities.

[9] Golosov et al. (2010) also study a general equilibrium model of fossil taxes and a backstop fuel, but focus on capital accumulation and ignore exhaustibility of fossil reserves. They show that optimal ad-valorem taxes on oil consumption decline over time, which is a consequence of their assumption of natural decay of CO2 in the atmosphere. Van der Ploeg and Withagen (2010) also allow for capital accumulation, but do not assume natural decay of CO2. They focus, in contrast, on the optimal time of transition to a carbon-free economy and the optimal amount of fossil fuel left in situ, and find that the optimal carbon tax rises over time until the moment of transiton.

Here, ρ is the nonnegative constant rate of time preference. The function U denotes instantaneous utility, depending on total energy use, consisting of the sum of q, the extraction rate of fossil fuel, properly scaled, and x, the rate of use of the backstop. It is assumed that $U' > 0$ and $U'' < 0$. Per unit extraction cost is denoted by G, which is a decreasing function of the remaining *in situ* stock S. The unit cost of supplying the backstop energy source is b. Finally, D indicates the global warming damages resulting from accumulated CO2 emissions E. The damage function is convex, so $D' > 0$ and $D'' > 0$. A widely used description of the accumulation of CO2 in the atmosphere reads $\dot{E}(t) = q(t) - \nu E(t)$. However, in order to get tractable solutions we set the rate of decay ν equal to zero. E_0 indicates the CO2 emissions that have taken place up to time zero. We also have $\dot{S}(t) = -q(t), q(t) \geq 0, S(t) \geq 0$ with an initial stock S_0. The resource constraint is written as

$$(2) \qquad \int_0^\infty q(t)\mathrm{d}t \leq S_0 .$$

Tahvonen (1997) is an important, but neglected contribution to the emerging literature on fossil fuel and backstops, in as far as he was the first to perform a deep analysis of the first-best problem posed here. The main difference between his and our model is that he allows for exponential decay of the stock of pollutants. Tahvonen (1997) finds that for high levels of initial pollution compared with the initial stock of the non-renewables, it is optimal to start with the backstop only. Else, it is best to start with only fossil fuel. If the backstop price is low then the backstop and fossil fuel are used simultaneously eventually, whereas with a high backstop price all fossil fuel is exhausted within finite time, after which the backstop takes over forever. In between the initial and the final phases several possibilities arise. Our outcomes of sections 2 and 3 regarding the social optimum differ from these outcomes because we abstract from decay of pollution. With exponential decay of pollution and a finite stock of fossil fuel, the stock of CO2 necessarily vanishes eventually, which, at low cost of the backstop, makes simultaneous use of fossil fuel and the backstop eventually optimal. In our model this does not occur, until we introduce strictly convex costs of the backstop in section 5. Furthermore, global warming damages do then not play a role in answering the question whether fossil fuel reserves get fully depleted or not. The drawback of working with exponential decay is that the model becomes more difficult to analyse from a technical point of view, because then two state variables play a role instead of only one, as is essentially the case in our model. This requires Tahvonen to use quadratic damage cost and linear marginal extraction costs, whereas we can obtain results for more general functional forms. Hoel and Kverndokk (1996) also allow for decay, but do not study sensitivity with respect to the cost of supplying the backstop.

Our model is also similar to that of Gerlagh (2011), which was one of the earliest analytical papers on the Green Paradox. His paper focuses on the market economy and assumes either stock-independent extraction costs or stock-dependent extraction costs but, in the latter case, in such a way that fossil fuel is never fully exhausted. Our approach is to first derive the social planner outcome in order to have a benchmark for discussing the "laissez-faire" outcome and the instruments that can be used to ensure that the market outcome replicates the first-best outcome. Moreover, we explicitly model damages from the accumulation of CO2, through the function D. The net present value of damages is equal to $\int_0^\infty e^{-\rho t} D(E(t))dt$ whereas Gerlagh (2011) assumes that it can be represented by $\int_0^\infty e^{-\rho t} \vartheta(t) q(t) dt$, where $\vartheta(t)$ is some (unspecified) function of time, satisfying the condition that $e^{-\rho t}\vartheta(t)$ is decreasing over time. We examine the case where in the market fossil fuel is fully exhausted, before the backstop kicks in, but also allow for the possibility that the market leaves fossil fuel in situ before switching to renewables. In contrast to Gerlagh (2011), we are not just interested in what happens to green welfare as the backstop gets cheaper, but we also derive the overall welfare effect of subsidizing the backstop. Hence, we consider the effects of improved technology (the backstop gets cheaper to produce) as well as of a subsidy, with its distorting effects elsewhere in the economy. Finally, we offer a detailed analysis of the implications of non-constant marginal cost of the backstop (section 5).

2.1. Characterizing the first-best optimum

Noting that $E(t) = E_0 + S_0 - S(t)$ we define the current-value Hamiltonian as

(3) $$H(q, x, S, \eta) \equiv U(q + x) - G(S)q - bx - D(E_0 + S_0 - S) - \eta q,$$

where η is the shadow price of the non-renewable resource. The necessary conditions for a social optimum[10] are:

(3a) $$U'(q(t) + x(t)) - b \le 0,\ x(t) \ge 0,\ \text{c.s.},$$

(3b) $$U'(q(t) + x(t)) - \eta(t) - G(S(t)) \le 0,\ q(t) \ge 0, \quad \text{c.s.},$$

(3c) $$\dot{\eta}(t) = \rho\eta(t) + G'(S(t))q(t) - D'(E(t)),$$

[10] Formally we should take into account that we have a pure state constraint on the stock of fossil fuel. If necessary in the sequel, this will be taken into account.

(3d) $\lim_{t\to\infty} e^{-\rho t}\eta(t)S(t) = 0,$

where c.s. refers to complementary slackness. The necessary conditions can be interpreted as follows. Equation (3a) says that the backstop is used unless the marginal utility of energy falls short of the supply cost b. Equation (3b) implies that no resource extraction takes place if the marginal utility of the resource is below marginal extraction cost plus the social cost of the resource. If fossil fuel extraction does take place, then the shadow price η corresponds to the rent, $U'(q+x)-G(S).$ The Hotelling rule, in its simple form, states that the return on keeping a marginal unit of fossil fuel in the ground, i.e., the expected increase in the scarcity rent of the non-renewable resource $\dot{\eta},$ must equal the social return on extracting a marginal unit of fossil fuel rate, $\rho\eta;$ equation (3c) indicates that in our more general model two further terms must be deducted from the return on extracting a marginal unit. The first term is the marginal increase in the cost of fossil fuel extraction as less accessible reserves have to be mined, $-G'(S)q.$ The second term represents the marginal global warming damages from burning an extra unit of fossil fuel, $D'(E).$ We will see that deducting these two extra terms from the right-hand side of the Hotelling rule makes it less attractive to keep fossil fuel in situ, so it will be socially optimal to deplete the stock of fossil fuels more slowly. The transversality condition (3d) states that the present value of the remaining stock of fossil fuel vanishes as time goes to infinity.

We assume that $U'(x)=b$ has a positive solution that will be denoted by $\bar{x}(b)>0$ with $\bar{x}'=1/U''<0.$ This means that it is profitable from a social welfare perspective to employ the backstop technology after extraction of fossil fuel has come to an end. Moreover, we assume that $U'(0)>G(0)+D'(E_0+S_0)/\rho.$ This means that in the absence of the backstop, extraction of fossil fuel will continue till full exhaustion. A weaker assumption would be $U'(0)>G(S_0)+D'(E_0)/\rho$, so that in the absence of the backstop it will initially be profitable to extract fossil fuel. But then we have to deal with the possibility of the use of fossil fuel coming to an end due to lack of contribution to social welfare rather than due to its cost becoming too high relative to the cost of the backstop. We now characterize the social optimum.

Proposition 1: The social optimum is characterized by an initial phase where only fossil fuel is used. After finite time *T* the backstop takes over indefinitely. The use of the backstop, the use and stock of fossil fuel, and the atmospheric CO2 concentration from time *T* on are given by:

(4) $x(t)=\bar{x}(b),\ q(t)=0,\ S(t)=S(T),$ and $E(t)=E_0+S_0-S(T),$ for all $t\geq T.$

Proof: See appendix.

A higher cost of the backstop thus implies less use of the backstop in the post-oil era. Furthermore, given our assumption of no natural decay in the atmospheric CO2 concentration, the stock of CO2 in the atmosphere remains what is at the time of the switch to the clean backstop and is higher if the amount of oil burnt in the oil era (i.e., $S_0 - S(T)$) has been more substantial. The reason why there is no simultaneous use of fossil fuel and the backstop is that then, on the one hand, the shadow price of fossil fuel decreases as fossil fuel reserves fall (from $\eta = b - G(S)$), and, on the other hand, the shadow price increases as the pollution level rises (as from the formal proof $\eta = D(E)$).

2.2. How much fossil fuel to leave in the soil?

With regard to what is left unexploited three possibilities arise: full exhaustion $(S(T)=0)$, partial exhaustion $(S_0 > S(T) > 0)$ and no extraction of fossil fuel at all $(S(T)=S_0)$. The following proposition provides the conditions under which each of these possibilities occurs.

Proposition 2: Full, partial and no exhaustion of fossil fuels occurs under the following conditions:

$$\text{if } b > G(0) + D'(E_0 + S_0)/\rho \text{ then } T > 0,\, S(T) = 0,$$

(5)
$$\text{if } G(S_0) + D'(E_0)/\rho < b < G(0) + D'(E_0 + S_0)/\rho \text{ then}$$
$$T > 0,\, S(T) = \bar{S} > 0, \text{where } D'\left(E_0 + S_0 - \bar{S}\right)/\rho = b - G\left(\bar{S}\right)$$

$$\text{if } G(S_0) + D'(E_0)/\rho > b \text{ then } T = 0,\; S(T) = S_0.$$

Proof: see appendix.

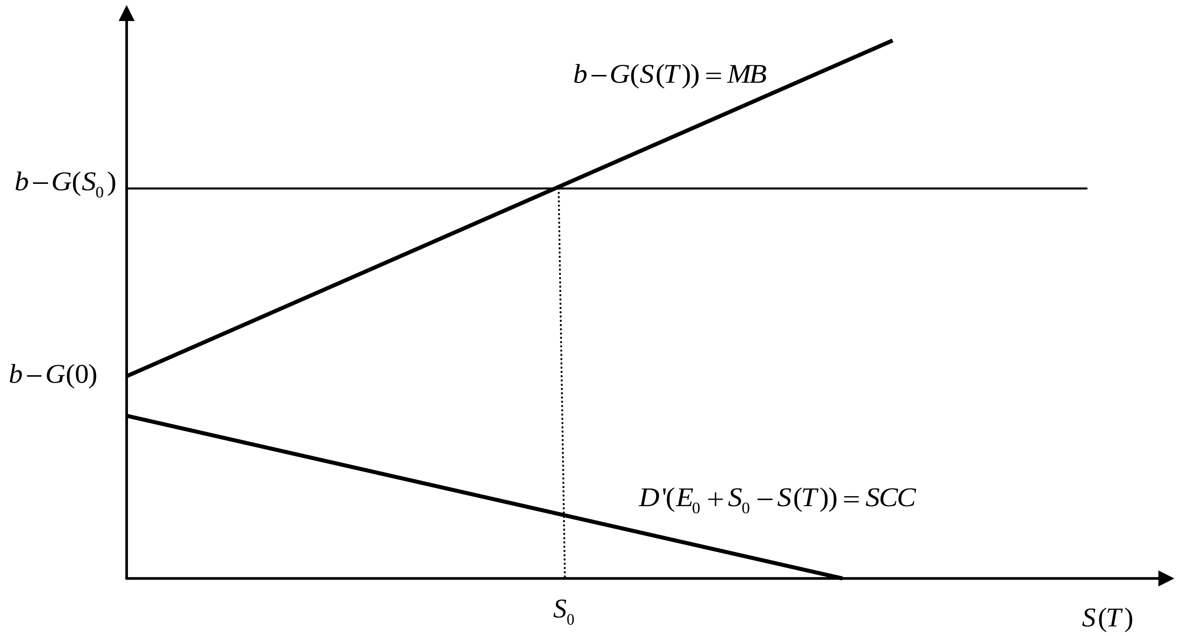
The interpretation is as follows. Suppose it is optimal to leave some, but not all fossil fuel unextracted. Then it must be the case that the present value of future marginal global warming damages of remaining fossil fuel reserves (the so-called ‘social cost of carbon’, *SCC*) equals the marginal benefit of extracting fossil fuel rather than using the backstop (*MB*) at the time of the switch to the backstop:

(5′)
$$SCC = D'\left(E_0 + S_0 - \bar{S}\right)/\rho = b - G\left(\bar{S}\right) \equiv MB.$$

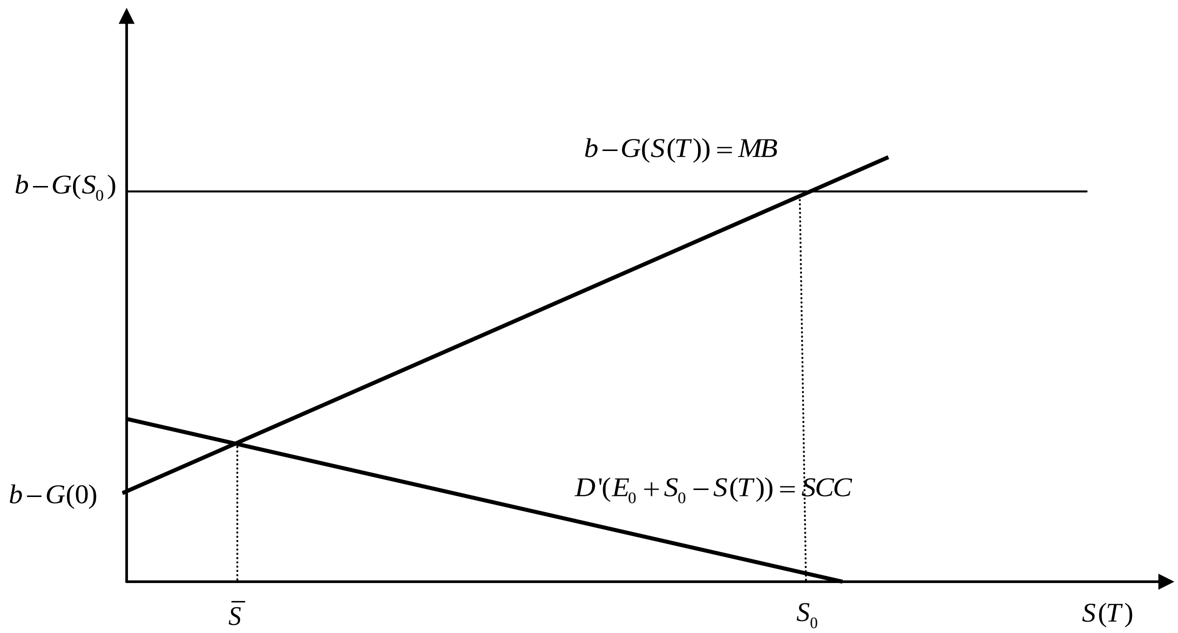
If this equation yields a positive solution, smaller than the initial resource stock, it is optimal to leave this amount in situ which is the second case portrayed in panel (b) of fig. 1. So the cost of supplying the backstop must be higher than the marginal cost of extracting the final drop of fossil fuel to cover the social cost of carbon. It could be that the solution $\bar{S}$ of equation (5′) is negative, e.g., if $b > G(0)$ and the aversion to global warming is small (i.e., preferences for a low CO2 stock are low or the rate used to discount marginal global warming is very high). The stock of fossil fuel will then be fully exhausted at the time the backstop takes over, so that $S(T) = 0$ which is the first case portrayed in panel (a) of fig.1.

Figure 1: Marginal global warming damages and marginal benefits of oil extraction

(a) **Full exhaustion:** expensive backstop and modest climate challenge $b - G(0) > D'(E_0 + S_0)/\rho$



(b) **Partial exhaustion:** cheap backstop and acute climate challenge $b - G(0) < D'(E_0 + S_0)/\rho$



The third case arises if the backstop is very cheap and the climate challenge is acute. In that case, $b - G(S_0) < D'(E_0)/\rho,$ so it is optimal to never use fossil fuel and start using the backstop straight away.

Note that the *SCC* curve is strictly decreasing, whereas the *MB* curve is strictly increasing in the final stock of fossil fuel left in situ. Panel (a) depicts the case where marginal global warming damages after full exhaustion, $D'(E_0 + S_0)$, are not very important and/or we have a high value of the rate of discount ρ and where the cost of supplying fossil fuel, even at low levels of the stock, $G(0)$, is low relative to the cost of supplying the backstop. Then the *SCC* locus hits the vertical axis before it intersects the *MB* locus. In that case it is optimal to fully extract conventional fossil fuel reserves before switching to the backstop energy source. However, if marginal global warming damages are believed to be important and if prudent discounting is used (as advocated by the Stern Review (2007)), the *SCC* locus crosses the *MB* locus yielding a non-zero stock of fossil fuel left in situ ($S_0 > S(T) = \overline{S} > 0$) as portrayed in panel (b) of fig. 1. If the backstop is very cheap, the marginal cost of global warming is high and the rate used to discount global warming damages is very low, and the climate challenge is very acute, i.e., $b - G(S_0) < D'(E_0)/\rho,$ the intersection point lies to the right of S_0.

As technical progress reduces the cost of the backstop or a lower discount rate is used, one moves from a regime of full exhaustion to partial exhaustion and eventually to zero exhaustion of oil and gas reserves. Note that the final stock of fossil fuel does not depend on preferences regarding energy consumption.

2.3. When to switch from fossil fuel to the renewable backstop?

The question we address next is how the time at which extraction of fossil fuel stops, depends on several crucial parameters of the model. Most of our results are derived for general functional forms. However, we can solve the model explicitly by making use of the following functional forms for utility of fuel consumption, per-unit cost of extracting oil and global warming damages:

(6) $$U(y) = \alpha y - \beta y^2/2, \quad G(S) = \gamma - \delta S, \quad D(E) = \kappa E^2/2.$$

Therefore, $U'(\overline{x}) = b$ yields $\overline{x} = (\alpha - b)/\beta.$ Moreover, the assumption $U'(0) > G(0) + D'(E_0 + S_0)/\rho$ which says that the choke price of fuel exceeds the social cost of extracting the last drop of fossil fuel, now boils down to $\alpha > \gamma + \kappa(E_0 + S_0)/\rho.$ This ensures that the choke price is high enough, $\alpha > \gamma - \delta S_0 + \kappa E_0/\rho,$ to make it optimal to start using fossil fuel from the outset, such that $T > 0$. It also follows from proposition 2 that fossil fuel reserves will only be fully exhausted if the social cost of extracting the last drop of fossil fuel falls short of the cost of the backstop:

(7)
$$S(T) = 0 \text{ if } b > \gamma + \kappa(E_0 + S_0)/\rho$$
$$S(T) = \bar{S} \equiv \frac{\gamma + \kappa(E_0 + S_0)/\rho - b}{\delta + \kappa/\rho} \text{ if } b < \gamma + \kappa(E_0 + S_0)/\rho.$$

When it comes to the question which factors enhance fossil fuel depletion we obtain the following proposition.

Proposition 3: If $b > \gamma + \kappa(E_0 + S_0)/\rho,$ there is full exhaustion of fossil fuel reserves and the transition to a carbon-free economy occurs more quickly if α is marginally higher and γ, E_0, S_0 and b are marginally lower. If $b < \gamma + \kappa(E_0 + S_0)/\rho,$ there is partial exhaustion of fossil fuel reserves and the transition occurs quicker if α, γ or E_0 are marginally higher or S_0 and b are marginally lower.

Proof: see appendix.

As proposition 2 and our discussion of fig. 1 have established, if the backstop is more expensive than the last drop of fossil fuel including the social cost of carbon, there will be full exhaustion. In that case, the transition from fossil fuel to the backstop occurs more quickly with a smaller initial stock of fossil fuel reserves (lower S_0), a lower initial atmospheric concentration of CO2 (lower E_0), lower marginal cost of extracting the last drop of fossil fuel (lower γ) and a higher choke price of fossil fuels (higher α). Furthermore, a lower cost of the renewable backstop (lower b) induces a quicker switch from fossil fuel to the backstop as well.

However, if the backstop is cheaper than the social cost of extracting the last drop of fossil fuel, it will be optimal to leave some fossil fuel in situ forever. In that case, the effect of a higher choke price (higher α) on speeding up the transition to the carbon-free economy is unaffected. However, there are additional effects operating via the negative effect of the final stock of fossil fuels $\bar{S}$ on the transition date T. Most importantly, a lower cost of the backstop (lower b) implies that it is attractive from a social perspective to keep more fossil fuel in situ at the time of transition and this in itself reinforces the quickening of the transition from fossil fuels to the renewable backstop.[11] With partial exhaustion a lower marginal cost of extracting the last drop of fossil fuel (lower γ) makes it optimal to hold less fossil fuel reserves in situ at

[11] We show in proposition 4 below that this core result also holds for general functional forms.

the time of the switch (lower $\bar{S}$), and this postpones the date of transition to the backstop. In fact, proposition 3 establishes that with partial exhaustion this effect is strong enough to outweigh the speeding up of the transition to the backstop which would result in the absence of leaving fossil fuel in situ. Similarly, a lower initial stock of CO2 in the atmosphere (lower E_0) requires society to leave less fossil fuel in situ and thus now leads to a later rather than earlier transition to carbon-free renewables. Finally, a lower initial stock of fossil fuel reserves (lower S_0) lowers the stock of fossil fuel to be left in situ but in this case the slowing-down effect is not strong enough, and hence the transition towards the carbon-free society is more rapid (albeit not as quick as without leaving reserves in situ).

Note that, if society did not care about global warming damages (i.e., $\kappa = 0$), the initial stocks of CO2 in the air and in the ground, E_0 and S_0, do not affect the optimal amount of fossil fuel to be left in situ (which will be positive only if the backstop is cheaper than the last drop of fossil fuel). In that case, the transition to the carbon-free economy occurs more quickly if the initial stocks of CO2 in the ground or in the air and the extraction cost of the last drop of fossil fuel are lower.

2.4. Green welfare and the cost of supplying the backstop energy source

A lower cost of supplying the renewable backstop clearly increases *overall* social welfare in the socially optimal outcome. Here we consider the effects of a lower backstop cost on *green* welfare:

$$(8) \qquad \Lambda \equiv -\int_0^\infty e^{-\rho t} D(E(t))dt = -\int_0^T e^{-\rho t} D(E(t))dt - \frac{D(E(T))e^{-\rho T}}{\rho}.$$

The results carry over to the decentralized market economy discussed in section 3.

Consider first the case where the backstop is expensive and stays that way also after the marginal reduction in its cost, so that fossil fuel reserves will always be fully exhausted. Due to the lower cost of renewables bringing forward of the date of exhaustion of fossil fuel, fossil fuel is pumped up more quickly at each point of time until renewables are introduced. Due to discounting and convexity of global warming damages, we establish in proposition 4 that faster initial depletion of fossil fuel reserves must also lead to worsening of green welfare. However, this is not so clear if it is not optimal to fully exhaust fossil fuel reserves, i.e., if the backstop is cheap compared with the cost of extracting the final drop of oil and gas plus the social cost of carbon (the present value of marginal global warming damages). In that case, at all instants of time during the fossil-fuel phase, fossil fuel extraction is higher but more fossil fuel reserves are left in the earth.

Proposition 4: If $b > G(0) + D'(E_0 + S_0)/\rho,$ there is full exhaustion in which case a marginally lower cost of the backstop leads to more rapid fossil fuel depletion, earlier exhaustion of oil and gas reserves, quicker phasing in off renewables and an increase in global warming damages. If the backstop is cheap and the global warming challenge acute, $G(S_0) + D'(E_0)/\rho < b < G(0) + D'(E_0 + S_0)/\rho,$ there is partial exhaustion of fossil fuel reserves in which case a marginally lower cost of the backstop costs speeds up carbon extraction, but leads to a larger stock of fossil fuels left unextracted, and brings forward the transition towards a carbon-free economy. For our specific functional forms (6), global warming damages fall as *b* is reduced in the regime of partial exhaustion, but not necessarily monotonically.

Proof: see appendix.

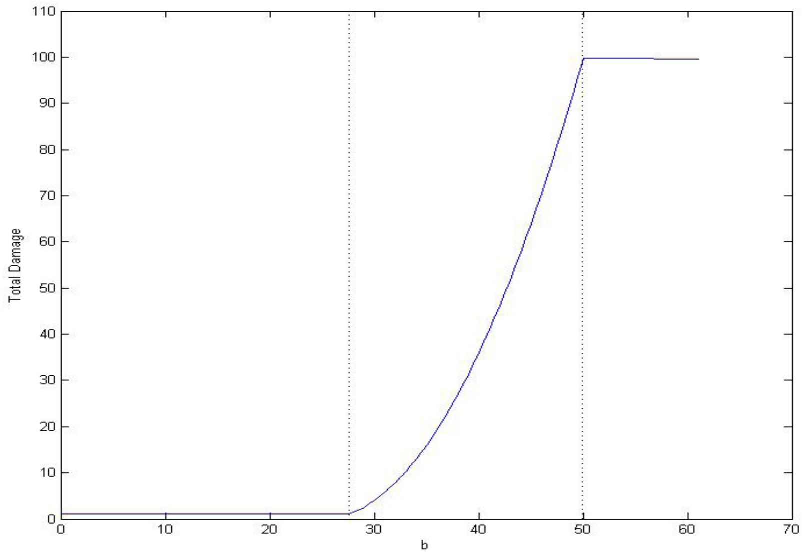
Writing global warming damages as $GWD \equiv \int_0^T D\big(E_0 + S_0 - S(t)\big)e^{-\rho t}dt + D\big(E_0 + S_0 - S(T)\big)e^{-\rho T}/\rho,$ we use the Leibniz integral rule to obtain:

$$(9) \qquad \frac{\partial GWD}{\partial b} = \underbrace{-\int_0^T D'\big(E_0 + S_0 - S(t)\big)\frac{\partial S(t)}{\partial b}e^{-\rho t}dt}_{(<0)} \underbrace{-D\big(E_0 + S_0 - S(T)\big)\frac{\partial S(T)}{\partial b}e^{-\rho T}}_{\geq 0}.$$

Consider first the case of full exhaustion of fossil fuel reserves which occurs if the resource cost of renewables exceeds the social cost of extracting the last drop of fossil fuel, $b > G(0) + D'(E_0 + S_0)/\rho.$ In that case, the second term in (9) drops out. We know from proposition 3 that with full exhaustion a marginally lower *b*, say *b′*, reduces the transition time from *T* to *T′* and still leads to exhaustion of the stock of reserves. We prove in the appendix that the extraction paths *q* and *q′* cannot cross and that at the time of transition more fossil fuel is taken out the earth on account of the lower b (from $U'(q+x) = b$), hence at each instant of time more fossil fuel is taken out of the earth and thus $S'(t) < S(t)$, $\forall 0 < t < T$. This establishes that the first term in (9) must be negative. Hence, a marginal reduction in the resource cost of renewables increases global warming damages. This results from faster pumping of fossil fuel.

Consider next the case of no exhaustion of fossil fuel reserves at all which occurs if renewables are cheaper than the social cost of the first drop of fossil fuel, $b < G(S_0) + D'(E_0)/\rho.$ Global warming damages are then not affected by a marginal change in the cost of renewables and simply equal $GWD = D(E_0)/\rho$ (see the left part of fig. 2). For our chosen parameter values, global warming damages rise only by a small amount as the cost of renewables is reduced if full exhaustion of fossil fuel reserves takes place (see the right part of fig. 2 where the downward slope is hardly noticeable), but fall very rapidly if partial exhaustion takes place (see the middle part of fig. 2).

Figure 2: Global warming damages and the cost of renewables



Key: $\alpha = 100, \beta = 1, \gamma = 30, \delta = 0.5, \kappa = 0.4, \rho = 0.2, S_0 = 9, E_0 = 1, b_1 = 27.5, b_2 = 50.$.

For the intermediate range where only partial exhaustion of fossil fuel reserves takes places, i.e., if $G(S_0) + D'(E_0)/\rho \equiv b_1 < b < G(0) + D'(E_0 + S_0)/\rho \equiv b_2$ holds, we know that global warming damages must increase in b at $b = b_1$ and that global warming damages at the beginning are less than at the end of this range, $GBW(b_1) < GBW(b_2)$. However, we cannot in general establish whether global warming damages rise *monotonically* over this range or not. We know from proposition 4 that the second term in (9) will be strictly positive as a marginally lower b will boost the stock of fossil fuel to be kept in situ. As far as the first term is concerned, proposition 4 states that a marginally lower b speeds up depletion q and thus leads initially to lower stocks of fossil fuel reserves. But extraction stops earlier and more fossil fuel is left in situ, $T' < T$ and $S'(T') > S(T)$. So, a lower cost of the backstop increases CO2 emissions faster initially but pollution will be lower eventually. If $S'(T') < S(T')$, we have $S'(t) < S(t), \forall 0 < t < T$ and the first term in (9) is strictly negative. Whether global warming damages increase monotonically then depends on whether the absolute value of the first term falls short of the second term in (9) throughout the range $b \in (b_1, b_2)$. In a wide variety of simulation experiments with a linear extraction technology we have been unable to find a non-monotonic rise in global warming damages over this range and in particular we have not been able to numerically find a range where global warming damages fall in a range of values of b only a bit smaller than b_2. Intuitively, a monotonic increase of global warming damages in the range $b \in (b_1, b_2)$ seems plausible, since an increase in b diminishes social welfare and given concavity of the utility function and convexity of the damage function this is achieved through both lower private welfare and more global warming damages.

One might wonder how the picture changes if the unit extraction cost of fossil fuel tends to infinity as the in-situ stock of fossil fuel gets fully depleted[12], e.g., $G(S) = \gamma + \delta / S$. In this case $b_2 = \infty$, but $b_1 > 0$ and finite. In the range $b < b_1$ oil is not used at all and no additional damages are caused beyond those generated by the pre-existing CO2 stock E_0. Therefore, in the range $b > b_1$ with b small, a higher production cost of the backstop increases total damages, as in fig. 2. To see what happens for large production cost, let us consider the case $b = U'(0)$ which implies that renewables are never used. If b is marginally lower, social welfare increases. Two effects can be distinguished: (i) higher initial extraction of fossil fuel which depresses green welfare (the first term in (9)); (ii) more fossil fuel left in situ which increases green welfare (the second term in (9)). Given concavity of the instantaneous utility function and convexity of the damage function, we conjecture that the second effect dominates as in fig. 2.

Proposition 4 establishes whether a cheaper backstop (e.g., due to technical progress) increases or reduces global warming damages; it always boosts total *social* welfare. It does not deal with backstop subsidies and does not relate to the Green Paradoxes in market economies, which are discussed in sections 3 and 4.

3. Climate policy in the competitive market outcome

To assess whether the social optimum can be sustained in a competitive market economy, we consider behaviour of households and resource owners. Households maximize $U(q+x)+C$ subject to the budget constraint, $C + p(q+x) \le A - T,$ where *C*, *p*, *A* and *T* denote consumer expenditures on all other commodities than oil, the market price of oil, endowment of households and lump-sum taxes, used to finance the subsidy, respectively. Households thus set $U'(q+x) = p,$ so the demand for fuel is a decreasing function of the market price of fuel (as $U'' < 0$). We assume that mining companies have access to the backstop. This is equivalent to having separate mining companies and other companies supplying the backstop in competition with each other. Taking the time paths of the price of oil *p*, the carbon tax τ and the backstop subsidy σ as given, the resource-owning firms maximize profits, $\int_0^\infty \left\{ p(t)(q(t)+x(t)) - (G(S(t)) + \tau(t))q(t) - (b - \sigma(t))x(t) \right\} e^{-\rho t} \mathrm{d}t,$ subject to the depletion equation (2). This yields the first-order conditions:

[12] We are grateful to one of the referees for suggesting this example. Note that in section 2.1 it has been assumed that unit extraction costs of fossil fuel are finite.

(3a′) $p(t) - b + \sigma(t) \le 0,\ x(t) \ge 0,$ c.s.,

(3b′) $p(t) - G(S(t)) - \tau(t) - \omega(t) \le 0,\ q(t) \ge 0,$ c.s.,

(3c′) $\dot{\omega}(t) = \rho\omega(t) + G'(S(t))q(t),$

(3d′) $\lim_{t\to\infty} e^{-\rho t}\omega(t)S(t) = 0,$

where ω is the private marginal value of the fossil fuel stock. Equation (3a′) says that the backstop is used unless the fuel price falls short of the supply price, net of the subsidy $b-\sigma$. Equation (3b′) says that fossil fuel extraction does not take place if the fuel price is below marginal extraction costs plus the CO2 tax. Equation (3c′) is the modified Hotelling rule, which states that the increase in the scarcity rent of fossil fuel equals the return $\rho\omega$ minus the increase in the marginal cost of extraction as less accessible reserves have to be mined, $-G'(S)q$. Comparing this with equation (3c), we see that the shadow price rises more quickly than in the social optimum so that depletion occurs too fast in a market economy unless the CO2 tax (or a market for CO2 permits) corrects for this externality.[13]

3.1. Sustaining the first-best outcome

We first characterize how the socially optimal outcome can be achieved in a market economy.

Proposition 5: The social optimum is sustained in a market economy by a CO2 tax ramp given by $0 < \dot{\tau}/\tau = \rho - D'(E)/\tau < \rho$ and $\sigma(t) = 0$.

Proof: See appendix.

The optimal rate of change in the carbon tax thus consists of a Hotelling term equal to the rate of time preference minus a term depending on marginal global warming damages.[14] It is thus socially optimal to have the CO2 tax rate growing at a slower rate than the discount rate. The optimal CO2 tax never falls.

3.2. Second-best outcome if a carbon tax is infeasible

Sinn (2008a and 2008b) argues that a (rapidly) rising CO2 tax may be tough to sell to the people. Instead, governments may resort to the second-best policy of a subsidy on the renewable backstop and financing

[13] We have assumed that owners of the fossil fuel take account of the rising costs of extraction as reserves diminish. This occurs with a large number of competitive mining firms each one of them owning a small mine with well defined property rights. If they do not do this, the price of fossil fuel rises even more quickly.
[14] Similar results have been obtained earlier (e.g., Hoel and Kverndokk, 1996).

this with lump-sum taxes whilst ruling out a CO2 tax. Sinn argues that such a subsidy leads to faster pumping of fossil fuel (weak Green Paradox) and therefore harms green welfare (strong Green Paradox). We will show that this argument only holds in the case that it is desirable to fully exhaust fossil fuel reserves. But if this is not the case, we will argue that there is not a strong but there is always a weak Green Paradox. Furthermore, we show that, contrary to what is suggested in the literature, a subsidy on the backstop can be beneficial from a social welfare perspective rather than from the narrower perspective of green welfare. We rule out a time-varying backstop subsidy or tax even though a constant backstop subsidy or tax may not be second-best optimal. Still, it is a fair first approximation. We now characterize the market outcome in which there is no carbon tax, but a backstop subsidy.

Proposition 6: Assume a CO2 tax is not feasible and a time-invariant subsidy σ is placed on the backstop. The market outcome then has an initial phase where only fossil fuel is used and after finite time T the economy switches to only using the backstop. Use of the backstop is a decreasing function of the cost of the backstop (net of subsidy), $x(t) = \overline{x}(b-\sigma).$ Full exhaustion of fossil fuel reserves occurs if $b-\sigma > G(0),$ partial exhaustion if $G(S_0) < b-\sigma < G(0)$ with $S_0 > S(T) > 0,$ and full preservation if $G(S_0) > b-\sigma.$ With the functional forms given in (6) full exhaustion takes longer (T larger) if the initial fossil fuel stock S_0 is high, the cost of the backstop net of the subsidy $b-\sigma$ is high, the maximal marginal extraction costs γ is high and the choke price for fuel α is low. In case of partial exhaustion, the transition occurs slower if α, γ or σ are marginally lower and S_0 and b marginally higher.

Proof: See appendix.

To illustrate this proposition, reconsider fig. 1. We note that the *SCC* is zero for the market outcome as the “laissez-faire” outcome does not internalize marginal global warming damages. The *SCC* locus thus corresponds to the horizontal axis. As fig. 1 is drawn for the case $b > \gamma$, reserves will be fully exhausted. For the case that $b < \gamma$, the *MB* line crosses the negative part of the vertical axis and the stock of fossil fuel reserves left in situ will equal $S(T) = (\gamma - b)/\delta$ provided this is less than S_0 (otherwise, fossil fuel reserves will never be extracted). Of course, less oil and gas will be left in situ in the competitive economy than in the social outcome which internalizes global warming damages. Introduction of a subsidy on the backstop in the market economy brings forward the date of the switch to the backstop, which is a manifestation of the (weak) Green Paradox. This makes it more likely that some of the fossil fuel reserves will be left in situ, and will lead to a bigger stock of unexploited fossil fuel reserves in case of partial exhaustion. Applying proposition 4 for the special case that global warming externalities are not internalized, we find that welfare decreases if fossil fuel reserves are fully exhausted. The next proposition characterizes the impact on social welfare of introducing a subsidy for the backstop.

Proposition 7: Assume a CO2 tax is infeasible. If the renewable backstop is always more expensive than carbon fuels (i.e., $b > G(0)$) and the atmospheric CO2 concentration is severely damaging in the margin, introducing a marginal tax on the backstop enhances welfare. If the backstop is cheaper than carbon fuels from the outset ($b < G(0)$), marginally subsidizing the backstop may decrease global warming damages, and if the stock of CO2 is severely damaging, marginally subsidizing the backstop enhances welfare**.**

Proof: See appendix.

Four remarks are in order. First, even for a market economy, we cannot be completely sure that a marginally lower cost of the backstop (for $b < G(0)$) decreases global warming damages, as explained right after proposition 4. Hence, the second part of the proposition on the effects of subsidizing renewables is stated with the appropriate condition, which is however is likely to hold over a large range of parameters. Second, once fossil fuel reserves are fully depleted, it becomes socially optimal to abolish the tax on the renewable backstop. This may lead to a credibility problem. This will also occur in case of a subsidy if $b < G(0)$ after depletion of the oil stock. Third, the results of proposition 7 do not depend on marginal extraction costs being stock-dependent. In case of our specification (6) with $\delta = \kappa = 0$ and $\alpha > b > \gamma$ there is always full exhaustion of oil reserves and from (9) we see that global warming damages increase if the backstop is subsidized in the market economy because $\frac{\partial S(t)}{\partial b} = \frac{1}{\rho\beta} e^{-\rho T}(e^{\rho t} - 1)\left[\frac{\alpha - b}{\alpha - \gamma + (\gamma - b)e^{-\rho T}}\right] > 0, \forall t < T.$ In that case, there is always a Green Paradox.

Fourth, it could be that the high global warming damage parameter needed in the proof to warrant a subsidy or a tax is such that a regime switch occurs.[15] Related to this is the observation that in case of an expensive backstop, an alternative policy is to subsidize the backstop to such an extent that it becomes cheaper than oil (i.e., $b > G(0) > b^*$). Then the policy is non-marginal, which might work for a very negative global warming externality, as can be illustrated in an example using the functional forms of section 2.2. We take the same parameters as in fig. 2 and set b = 30. In contrast to the *marginal* effects of a backstop subsidy or tax on welfare presented in proposition 7, fig. 3 plots the *non-marginal* effects of introducing a backstop subsidy and a backstop tax of varying orders of magnitude on welfare (net of the

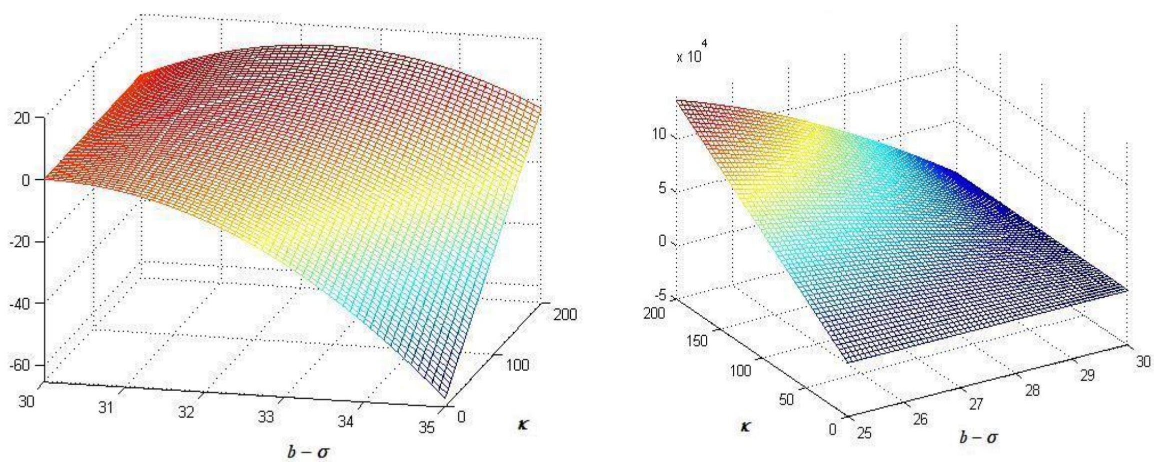
[15] For example, starting in the unlikely socially optimal regime with full exhaustion, the damage parameter needed to justify a tax might be such that we arrive in a regime where no exhaustion of fossil fuel takes place at all.

lump-sum taxes needed to finance the subsidy or the lump-sum transfers made possible by the tax on the backstop)[16] for different values of the damage parameter κ.

In panel (a) of fig. 3 we introduce a tax on the backstop, leading to the backstop cost being larger than γ . In the competitive economy fossil fuel reserves will then always be fully exhausted. The higher the tax and the cost of the backstop, the smaller is the initial rate of fossil fuel extraction and the later exhaustion of fossil fuel reserves takes place. This curbs emissions and implies an initial positive effect on green welfare. However, the private component of social welfare falls. With no or little concern about global warming (small κ), taxing the backstop always harms total welfare. Only if society cares a lot about global warming damages and the tax is not too high, is the welfare effect positive. Hence, if the increase in green welfare is large enough, it outweighs the fall in the private component of social welfare. However, too large backstop taxes lower social welfare even if society cares a lot about CO_2 damages (high κ). For $\kappa = 200$, welfare is maximized if the backstop tax equals 2.

Figure 3: Backstop subsidies can boost second-best social welfare

(a) Taxing the backstop (full exhaustion) **(b) Subsidizing the backstop (partial exhaustion)**



Panel (b) of fig. 3 deals with the case of a subsidy which leads to a lower backstop cost and therefore to partial exhaustion of reserves with a positive final stock of fossil fuels. With no concern about global warming ($\kappa = 0$) introducing a backstop subsidy affects social welfare negatively, since the competitive outcome is socially efficient. However, it turns out that for these parameter values, the net effect of introducing a backstop subsidy is rather small. We also see that even for relatively little concern about

[16] Since use of the backstop is given by $x(t) = (\alpha - b + \sigma) / \beta > 0$ for all $t \geq T$, we have to subtract $e^{-\rho T} \sigma(\alpha - b + \sigma) / \beta\rho$ from social welfare at time zero.

global warming, there is a substantial[17] welfare gain from introducing the backstop subsidy. This suggests that with $b > \gamma$ it is better from a welfare perspective to subsidize the backstop so that the effective cost is reduced below 30, rather than taxing the backstop.

Summing up, given the availability of a clean backstop, the appropriate way of realizing the first best is to implement a C02 tax ramp, not a backstop subsidy. If a carbon tax is infeasible and the government implements a constant subsidy on the backstop, this is welfare increasing if fossil fuel is relatively cheap and damages are high. Else, a tax on the backstop is in order to delay the moment of full exhaustion of fossil fuel. The only constellation under which the strong Green Paradox occurs is if the backstop is initially relative expensive so that all fossil fuel will be fully exhausted. Green welfare will fall as oil and gas reserves are more quickly exhausted, but overall welfare may increase. However, if the backstop is (made) cheap enough compared with current extraction costs of oil and gas, it is optimal to keep some fossil fuel reserves unexploited which benefits the environment.[18] Subsidizing the renewable backstop then means that the switch away from oil and gas to the clean backstop occurs more rapidly; and also that a bigger fraction of fossil fuel reserves remains in situ. CO2 emissions are less, so that global warming damages fall rather than rise (no strong Green Paradox).

4. Monopolistic supply of fossil fuels

So far, we have discussed socially optimal outcomes and outcomes that would prevail in a competitive market economy. Clearly on the markets for non-renewables imperfect competition prevails. Therefore, it is relevant to study the Green Paradox under the assumption of imperfect competition. As a first step we consider the case of a monopoly. It is well known that with monopolies in natural resource markets, limit pricing may occur (Salant, 1977; Hoel, 1978). This means that in the presence of a backstop technology with price b and constant marginal extraction costs of the non-renewable resource smaller than b, there is an initial phase until some T_1 where the monopolist keeps the market price of oil or gas below the cost of supplying the backstop price, and subsequently a final phase $(T_1, T_2]$ where the backstop price is undercut by an infinitely small margin. The instants of time (T_1, T_2) are determined endogenously by maximizing over the two parts of the trajectory. With stock-dependent extraction costs matters are more complicated. However, limit pricing may still occur. To see this, and to investigate its consequences, we consider a

[17] Note that in panel (b) the vertical axis is several orders of magnitude larger than in panel (a).

[18] An alternative to a subsidy on carbon-free renewables is to compensate the owner of non-renewable resources for keeping some of its reserves unexploited. Interestingly, Ecuador demanded at the 2009 United Nations Climate Change Conference in Copenhagen $4.5 billion as compensation to keep oil in the soil and thus preserve the Amazon rain forest and curb CO2 emissions by 410 million tons. In practice, mining companies also attempt to bribe indigenous people to accept their resources being plundered.

monopolist facing a linear inverse demand function $p(t) = \alpha - \beta q(t)$ and having unit extraction costs $G(S) = \gamma - \delta S.$ The cost of supplying the renewable backstop is b. The monopolist's problem is then

$$\max_{q,T} \int_0^T \exp(-\rho t)\left[\alpha - \beta q(t) - \gamma + \delta S(t)\right] q(t) \mathrm{d}t \tag{10}$$

subject to the depletion equation (2) and the inverse demand function $p(t) = \alpha - \beta q(t) \le b.$ The choke price is an upper limit on the fossil fuel price and the backstop price the lower limit. Note that the maximization also takes place with respect to the date T at which extraction definitely stops.

Proposition 8: Suppose that the owner of the non-renewable resource is a monopolist who is faced with a renewable backstop fuel over which it has no control. If the backstop price is high compared to the marginal cost of extracting fossil fuel ($b > \gamma$), lowering the cost of the backstop implies that it takes a shorter time to fully exhaust fossil fuels and therefore the Green Paradox prevails. If the backstop price is relatively low ($b < \gamma$), then initial extraction of fossil fuels is excessive, but it lasts shorter than before, the stock of remaining fossil fuel at the time of the switch is $S(T) = (\gamma - b)/\delta > 0,$ and the Green Paradox need not necessarily arise. In both cases there is a phase of limit pricing, where fossil fuels are priced marginally below the cost of the backstop.

Proof: See appendix.

It is interesting to see whether the Green Paradox is more prominent under monopoly than under perfect competition. Indeed, it is sometimes said that "the monopolist is the conservationist's best friend" (Solow, 1974; Dasgupta and Heal, 1979, p. 323)[19] and the question is whether this conjecture also holds when it comes to climate change and backstop technologies. With linear demand and zero marginal extraction costs, the initial market price will be higher under monopoly than under perfect competition. Moreover, it will take the monopolist longer to exhaust fossil fuel reserves which will be reinforced once account is taken of constant marginal extraction costs. It is easily established that also with a backstop and stock-dependent extraction costs, the price path under monopoly will initially be higher than under perfect competition.[20] So also with a backstop and stock-dependent extraction costs the monopolist will exhaust fossil fuel reserves at a later instant of time. However, for the total amount of fossil fuel left in situ, the

[19] In a general equilibrium model with capital accumulation but without exhaustibility of oil and in the presence of a backstop Hassler et al. (2010) also conclude that an oil monopoly is good for the environment.

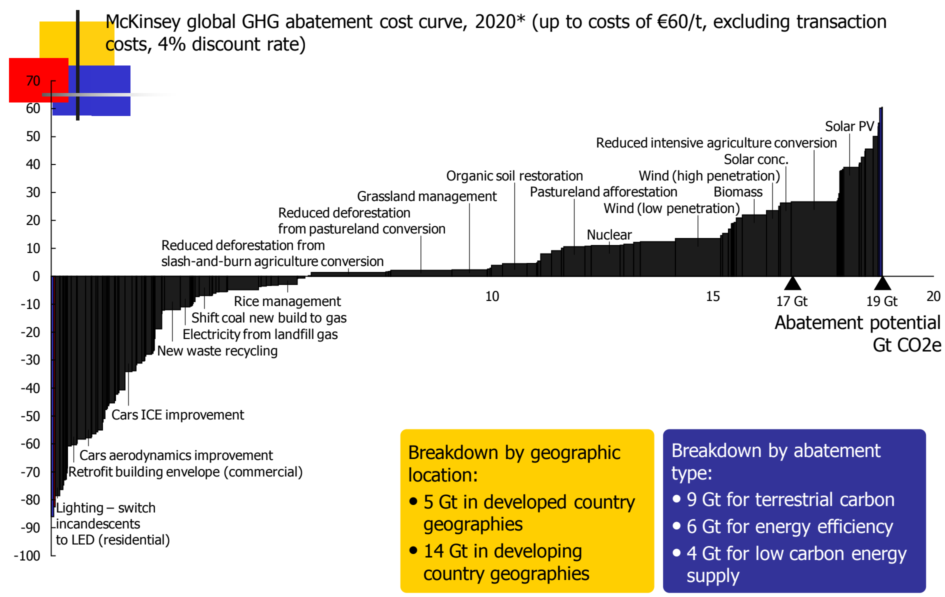
[20] As long as the backstop is not in use, the price paths under perfect competition and monopoly are: $\dot{p}^{pc} = \rho(p^{pc} - \gamma + \delta S^{pc})$ and $\dot{p}^{mon} = \rho(p^{mon} - \gamma + \delta S^{mon} - \beta q^{mon})$, respectively. If the price under perfect competition is initially larger than the monopoly price, this would remain so, implying that the price paths would never cross, which is at variance with the assumption of equal initial stocks.

market structure is irrelevant. Whether the exhaustion dates under monopoly and perfect competition come closer as the cost of the backstop is reduced, is left for further research.

5. Convex backstop production costs

Fig. 4 shows the McKinsey global greenhouse abatement cost curve. It indicates that lighting-switch, cars aerodynamics and rice management are already profitable to implement at the current cost of carbon, but that as the cost of carbon rises further grassland management, organic soil restoration and solar PV become profitable. To allow for such an abatement cost curve, we extend our model to have a convex production cost function for the backstop, $B(x)$ with $B' > 0$ and $B'' > 0$.

Figure 4: McKinsey global GHG abatement cost curve, 2020



Source: McKinsey Global GHG Abatement Cost Curve v2.0

The problem is then given by:

(11) $$\max \int_0^\infty e^{-\rho t}\left[U(q(t)+x(t)) - G(S(t))q(t) - B(x(t)) - D(E_0 + S_0 - S(t))\right]dt$$

subject to the resource constraint (2). The current-value Hamiltonian reads

(12) $$H(q,x,S,\lambda,\mu) \equiv U(q+x) - G(S)q - B(x) - D(E_0+S_0-S) - \eta q,$$

so that the necessary conditions for a social optimum become:

(3a″) $U'(q(t)+x(t))-B'(x)\le 0,\ x(t)\ge 0,$ c.s.,

(3b″) $U'(q(t)+x(t))-\eta(t)-G(S(t))\le 0,\ q(t)\ge 0,$ c.s.,

(3c″) $\dot{\eta}(t)=\rho\eta(t)+G'(S(t))q(t)-D'(E(t)),$

(3d″) $\lim_{t\to\infty} e^{-\rho t}\eta(t)S(t)=0.$

As before, we define $\bar{x}$ as the optimal use of the backstop when fossil fuel is not used. It follows from (3a″) that $U'(\bar{x})=B'(\bar{x})\equiv b>0.$ As in section 2, we suppose that $\bar{x}(b)>0.$

Proposition 9: With convex backstop production costs, the optimal sequence is to first have a phase $[0,T_1]$ where only fossil fuel is used, then a phase $[T_1,T_2]$ of simultaneous use of fossil fuel and the backstop, and finally a phase $[T_2,\infty)$ where only the backstop is used. The first phase or the third phase may be degenerate, so that $T_1=0$ or $T_2=\infty$, respectively. If the cost of using only the backstop b exceeds or equals the social cost of using the last drop of fossil fuel $G(0)+D'(E_0+S_0)/\rho$, fossil fuels will be fully exhausted: $S(T_2)=0$. Otherwise, there will be partial exhaustion of fossil fuels: $S(T_2)>0$.

Proof: See appendix.

Hence, the conditions for partial or full depletion are quite similar to those that we had before. However, we will show that the earlier derived consequences of the backstop becoming cheaper are not robust. To that end, we turn to the specific functional forms used in section 2.2 and furthermore suppose a quadratic cost function for the backstop, $B(x)=\psi x+\pi x^2/2$ with $\psi>0$ and $\pi>0.$ This implies the following use of the backstop and its cost in the carbon-free regime:

(17) $$\bar{x}=\frac{\alpha-\psi}{\beta+\pi}>0 \text{ and } b=\frac{\alpha\pi+\beta\psi}{\beta+\pi}>0.$$

The assumption $\bar{x}>0$ entails that the choke price of fuel α exceeds the cost of the first unit of the backstop ψ. We also define the following parameters:

(18) $$\bar{S}=\frac{\gamma+\kappa(E_0+S_0)/\rho-b}{\delta+\kappa/\rho},\quad \hat{s}_1\equiv\frac{1}{2}\rho+\frac{1}{2}\sqrt{\rho^2+4(\rho\delta+\kappa)/\hat{\beta}}<0>0,\quad \hat{s}_2\equiv\rho-\hat{s}_1<0,$$

with $\hat{\beta}\equiv\beta\pi/(\beta+\pi).$ We will first show that for a low initial resource stock it is optimal to initially have simultaneous use of fossil fuel and the backstop. It follows from (3a″) and (3b″) that along an interval of time with simultaneous use we have $\alpha - \beta(q(t) + x(t)) = \psi + \pi x(t).$ Upon using (3c″) we then get the second-order linear differential equation: $\hat{\beta}\ddot{S} - \rho\hat{\beta}\dot{S} - (\delta\rho + \kappa)S = \rho(b - \gamma) - \kappa(E_0 + S_0).$ The solution for the optimal resource stock is given by

(19) $\quad S(t) = \hat{K}_1 e^{\hat{s}_1 t} + \hat{K}_2 e^{\hat{s}_2 t} + \overline{S}$ and $q(t) = -\hat{s}_1 e^{\hat{s}_1 t}\hat{K}_1 - \hat{s}_2 e^{\hat{s}_2 t}\hat{K}_2.$

where $\hat{K}_1$ and $\hat{K}_2$ are constants to be determined. We now have the following proposition.

Proposition 10: Simultaneous use from the outset

I. *Asymptotic partial exhaustion:* Suppose $-(\alpha - \psi)/(\hat{s}_2\beta) + \overline{S} > S_0 > \overline{S} > 0.$ Then it is optimal to have simultaneous use from the outset forever ($T_1 = 0, T_2 = \infty$). The path for the fossil fuel resource stock is given by $S(t) = (S_0 - \overline{S})e^{\hat{s}_2 t} + \overline{S}$ and the stock approaches $\overline{S}$ asymptotically. Fossil fuel use is given by $q(t) = -(S_0 - \overline{S})\hat{s}_2 e^{\hat{s}_2 t}.$

II. *Full exhaustion in finite time:* Let $(\hat{K}_1, \hat{K}_2, T_2)$ solve $\overline{S} + \hat{K}_1 + \hat{K}_2 = S_0,$ $-\hat{s}_1 e^{\hat{s}_1 T_2}\hat{K}_1 - \hat{s}_2 e^{\hat{s}_2 T_2}\hat{K}_2 = 0,$ $e^{\hat{s}_1 T_2}\hat{K}_1 + e^{\hat{s}_2 T_2}\hat{K}_2 = 0.$ Suppose $-\hat{s}_1\hat{K}_1 - \hat{s}_2\hat{K}_2 \le (\alpha - \psi)/\beta.$ Then it is optimal to have simultaneous use from the outset with the backstop taking over and fossil fuels being fully exhausted in finite time T_2.

Moreover, backstop use follows in both cases from $x(t) = \dfrac{\alpha - \psi}{\beta + \pi} - \dfrac{\beta}{\beta + \pi}q(t) > 0.$ It is never optimal to start with simultaneous use and then the backstop taking over fully within finite time at a positive level of fossil fuel reserves in situ.

Proof: See appendix.

The intuition behind the proposition should be clear from (19). Suppose $\overline{S} > 0$. Take $\hat{K}_1 = 0$ and $\hat{K}_2 = S_0 - \overline{S}$. Then we have a solution to the necessary conditions with the stock approaching $\overline{S}$. Under the conditions mentioned under case I, we have a solution satisfying all the necessary conditions. Indeed, we can take $\hat{K}_1 = 0$ so that $S(t) \to \overline{S} > 0$ as $t \to \infty$. We must then have $\hat{K}_2 = S_0 - \overline{S}.$ Moreover, $q(0) = -\hat{s}_2\hat{K}_2 \le (\alpha - \psi)/\beta$ and therefore $x(0) > 0.$ This is an optimum, because the necessary conditions are also sufficient. The same holds for case II. But here $\overline{S} < 0,$ so that simultaneous use cannot go on forever. We will now first analyze the effects of a lower cost of the backstop technology using propositions 9 and 10. Second, proposition 10 suggests that for a high initial stock of fossil fuel it is not optimal to start with simultaneous use, but to use only fossil fuel initially. We will show below that this is indeed the case and also analyze the effect of a lower backstop cost for this case.

In our discussion we make use of the result that introducing a backstop subsidy, σ, in a market economy which does not internalize global warming externalities (i.e., acts as if $\kappa = 0$) is analytically equivalent to lowering the marginal cost of the backstop, ψ, in the socially optimal outcome in the economy without global warming externalities.

I. Backstop kicks in immediately and asymptotic partial exhaustion of fossil fuel

The conditions of case I of proposition 10 are relevant if the social cost of extracting the last drop of fossil fuel exceeds the cost of using only the backstop. A decrease in ψ, which is the marginal cost of the backstop at zero production, lowers b and has no impact on $\hat{s}_1$ or $\hat{s}_2$. The expression $(\alpha - \psi)/\beta$ gets larger, so that we remain in the regime with simultaneous use throughout. Hence, upon a decrease in the marginal cost of the backstop, the asymptotic stock of fossil fuel gets higher ($\overline{S}$ increases). More fossil fuel is left in the ground and there is no incentive to extract fossil fuel faster, so green welfare is boosted.

II. Backstop kicks in immediately and full exhaustion of fossil fuel in finite time

The conditions of case II of proposition 10 are relevant if the social cost of extracting the last drop of fossil fuel is less than the cost of using only the backstop. From equation (19) and the conditions $q(T_2) = S(T_2) = 0$ we can solve for $\hat{K}_1$ and $\hat{K}_2$ and obtain the paths of fossil fuel reserves and fossil fuel use:

$$(19') \qquad S(t) = \overline{S} - \overline{S}\left(\frac{\hat{s}_1 e^{\hat{s}_2(t-T_2)} - \hat{s}_2 e^{\hat{s}_1(t-T_2)}}{\hat{s}_1 - \hat{s}_2}\right) \text{ and } q(t) = \frac{\hat{s}_1\hat{s}_2}{\hat{s}_1 - \hat{s}_2}\overline{S}\left(e^{\hat{s}_2(t-T_2)} - e^{\hat{s}_1(t-T_2)}\right) > 0.$$

The time at which fossil fuel is fully exhausted T_2 follows from the condition $S(0) = S_0$ and making use of $\overline{S}$ from the first part of (18). Hence, T_2 can be found from:

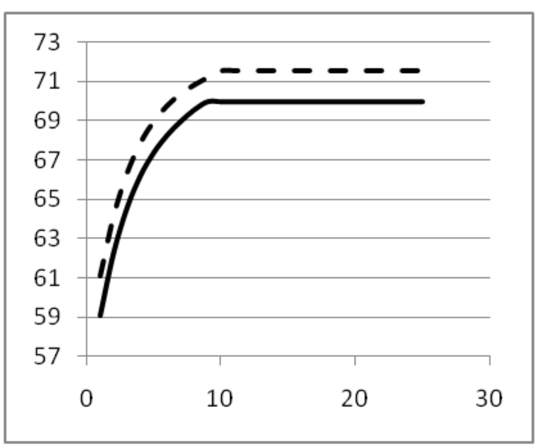
$$(20) \qquad \frac{\hat{s}_1 e^{-\hat{s}_2 T_2} - \hat{s}_2 e^{-\hat{s}_1 T_2}}{\hat{s}_1 - \hat{s}_2} = \frac{b - \gamma + \delta S_0 - \kappa E_0/\rho}{b - \gamma - \kappa(E_0 + S_0)/\rho}.$$

The left-hand side of (20) is increasing in T_2. The right-hand side of (20) is increasing in the initial oil stock, in the initial atmospheric CO2 concentration, in the marginal extraction cost of the last drop of fossil fuel, and decreasing in the cost of the backstop.[21] The latter implies that in the case at hand lower production costs of the backstop (due to lower ψ) give rise to *later* exhaustion, contrary to the case of constant marginal cost of the backstop. The reason is that now the cheaper backstop already substitutes for fossil fuel during the phase of simultaneous use, which boosts green welfare as on the one hand a non-polluting backstop is introduced more quickly and on the other hand the depletion of fossil fuel occurs more gradually. Hence, for high backstop costs, the results of proposition 4 are not robust with respect to more general cost functions. However, this case requires the backstop to be used from the outset and thus the initial resource stock should be small.

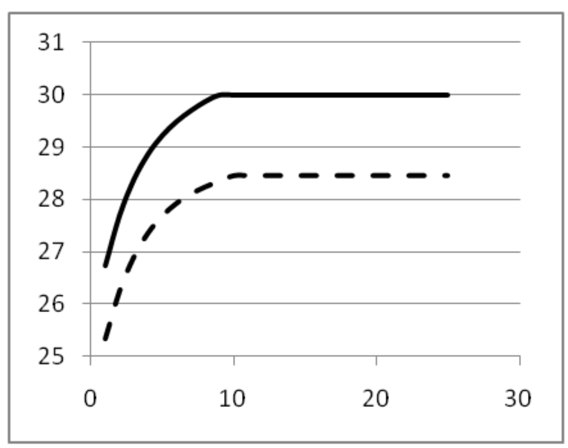
These insights are confirmed by the illustrative solution paths for use of the backstop and the social price of fossil fuel (i.e., the market price plus the optimal Pigouvian CO2 tax) plotted in fig. 5 with parameter values $\alpha = 100$, $\beta = 1, \gamma = 1$, $\delta = 0.5$, $\rho = 0.2$, $S_0 = 9$, $E_0 = 1$, $\kappa = 0.5$ and $\psi = 9$ or 7, so that $b = 30$ or 28.5 and $\overline{S} = -1.3$ or -0.8. The horizontal axes display time as 4*t*. Lowering the cost of the backstop (i.e., lowering ψ from 9 to 7; see dashed lines) thus postpones the date of exhaustion from 7.76 to 9. Backstop use is boosted while fuel prices are lower everywhere. So in contrast to the case where the marginal production cost of the backstop is constant (as in sections 2 and 3), lowering backstop costs in the regime of full exhaustion of fossil fuel reserves now does not yield lower green welfare, which is due to the fastened gradual phasing in of more and more expensive backstops from time zero.

Figure 5: Full exhaustion of fossil reserves in finite time and backstop kicking in at time zero

Backstop use



Price of fuel



[21] Both $\hat{s}_1$ and $\hat{s}_2$ depend on $\hat{\beta}$ and π, so we only consider variations in ψ.

III. Backstop kicks in later than fossil fuels and asymptotic partial exhaustion of fossil fuel

Now we consider the case where the initial stock is too large to warrant immediate simultaneous use, but where still a positive stock of fossil fuels is left in situ. So, suppose we have an initial interval of time with only fossil fuel use until T_1. From this point on, we have simultaneous use forever. The backstop thus kicks in at $t = T_1$ with $x(0) = 0$ after a phase of using only fossil fuel. It then rises asymptotically to $\bar{x}$. Fossil fuel use starts with $q(0) > (\alpha - \psi)/\beta$ then declines gradually reaching $q(T_1) = (\alpha - \psi)/\beta$ at $t = T_1$ and asymptotically approaching zero as $t \to \infty$. Fossil fuel stocks fall asymptotically from S_0 at time zero to $\bar{S}$. The differential equation for $t > T_1$ is $\hat{\beta}\ddot{S} - \rho\hat{\beta}\dot{S} - (\delta\rho + \kappa)S = \rho(b - \gamma) - \kappa(E_{T_1} + S_{T_1})$ $= \rho(b - \gamma) - \kappa(E_0 + S_0)$. Hence, with $\bar{S} > 0$, its solution is $S(t) = \hat{K}_2 e^{\hat{s}_2(t - T_1)} + \bar{S}$, $\forall t \geq T_1$ with $\hat{K}_2 = -(\alpha - \psi)/\hat{s}_2\beta.$ Hence $S(T_1) = \hat{K}_2 + \bar{S}.$ Now suppose we are in a regime with $T_1 > 0$ and that ψ falls. Hence, both $\bar{S}$ and $\hat{K}_2$ increase. Ultimately more fossil fuel is left in situ, and at the instant of time where simultaneous use starts, fossil fuel extraction gets higher. Hence, before the new T_1 extraction is increased. These two effects are compatible only if the new T_1 is smaller than the old one and the economy phases in renewables more quickly. Hence, initially we have more extraction, for a shorter period of time. Fig. 6 gives some illustrative solution trajectories with parameter values $\alpha = 200$, $\beta = 1$, $\gamma = 100$, $\delta = 1$, $\rho = 0.526$, $S_0 = 100$, $E_0 = 0$, $\kappa = 0.263$ and $\psi = 33$ or 30, and $\pi = 0.0118.$[22] The horizontal axes display time as 400 *t*.

These parameter values imply $\bar{x} = 165$ or 168, $b = 34.9$ or 32.0, $\bar{S}$ = 76.7 or 78.7, $s_2 = -0.66$ and $\hat{s}_2 = -9.56.$ For these parameter values $q(0)$ is 167 or 170, both of which exceed $(\alpha - \psi)/\beta$, so that there will be an initial phase where only fossil fuel is used.

Our theoretical insights are confirmed, so a lower cost of the backstop (lowering ψ from 33 to 30; see dashed lines) brings forward the date at which the backstop is phased in and also leaves more fossil fuel in situ in the long run. As a result of $-s_2$ being much smaller than $-\hat{s}_2$, fossil fuel use hardly diminishes with time during the first phase but it must fall much more rapidly as soon as the backstop is introduced

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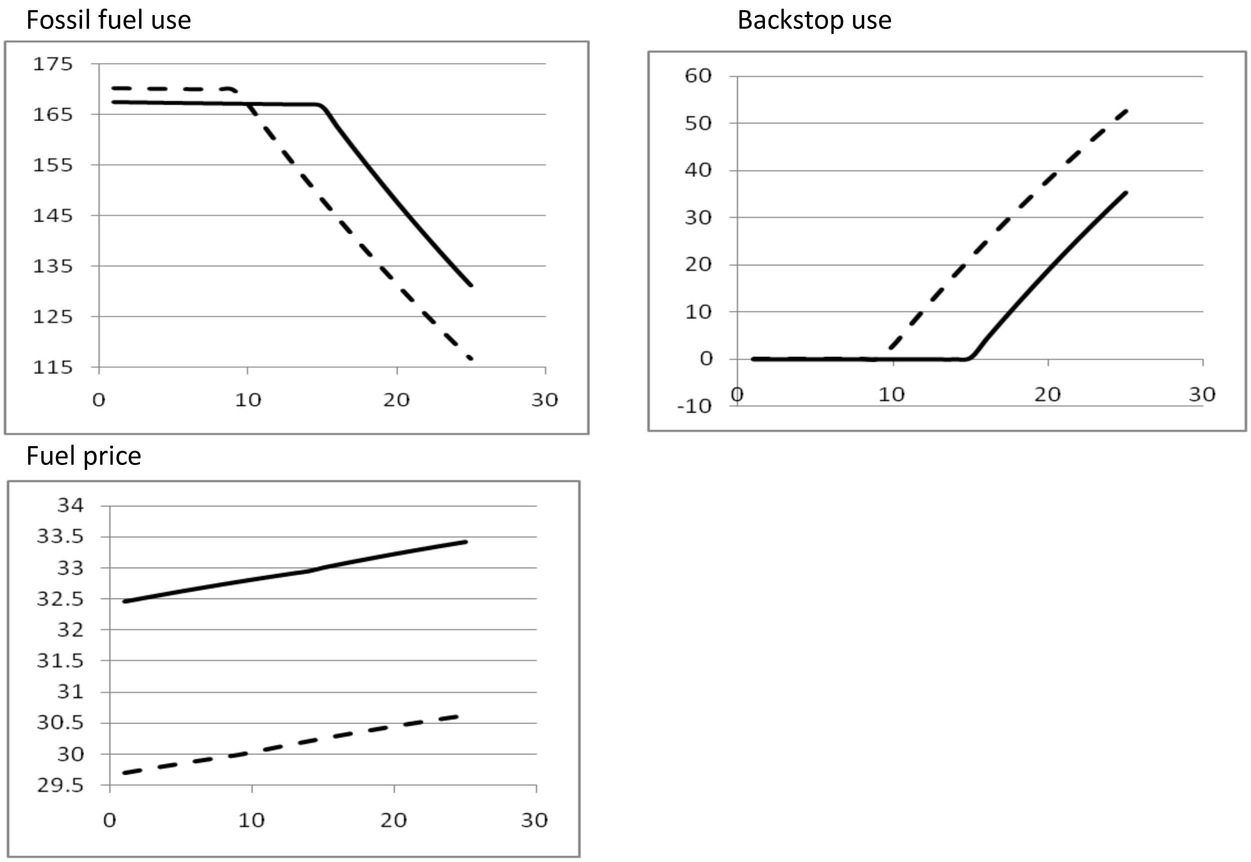
[22] We have chosen our parameters ρ, κ and S_0 in the following way. With the other parameters as above, we set $\kappa = \rho/2$ and $S_0 = 97$. We then have $S_0 > \bar{S} = 79 > 0$ and $\gamma - \delta S > 0.$ We choose ρ so that the economy is exactly on the boundary between an initial phase of using only fossil fuel and the backstop kicking in immediately, i.e., we pick ρ so that $q(0) = (\alpha - \psi)/\beta = 172 = -\hat{s}_2\hat{K}_2 = -18\hat{s}_2$ holds. Picking a slightly higher value of S_0, say, 100, then ensures that the former regime (case III) prevails. Picking a slightly lower value of S_0 ensures that case I prevails.

and phased in more and more. This ensures that the fuel price follows a smooth path during the two phases. Interestingly, the lower cost of the backstop pulls down the whole trajectory of fuel prices, including those that prevail in the first phase when the backstop has not been phased in. This is the Hotelling intertemporal arbitrage logic in action. Since the clean backstop is introduced more quickly and more aggressively and more fossil fuel is left in situ, green welfare increases. But also more fossil fuel is extracted in the initial phase, which reduces green welfare. Hence, the overall effect is not clear a priori. But our numerical exercises suggest that green welfare increases..

Figure 6: Asymptotic partial exhaustion of reserves with backstop kicking in at time $T_1 > 0$



IV. Backstop kicks in later than fossil fuels and full exhaustion of fossil fuels in finite time

The final case is where $\overline{S} < 0$ and the initial resource stock is too large to warrant the backstop kicking in immediately. Given the conditions $q(T_2) = S(T_2) = 0$ we can solve for $\hat{K}_1$ and $\hat{K}_2$ and obtain the paths of fossil fuel reserves and fossil fuel use, i.e., (19′) for $t \in [T_1, T_2]$. Making use of the expressions for b in (17) and $\overline{S}, \hat{s}_1$ and $\hat{s}_2$ in (18) and of the fact that $q(T_1)$ given below is a monotonically decreasing function of

$T_2 - T_1$, the boundary condition at the moment that renewables are phased in can be used to see how $T_2 - T_1$ depends on the various parameters: $q(T_1) = \frac{\hat{s}_1 \hat{s}_2}{\hat{s}_1 - \hat{s}_2} \overline{S} \left[e^{-\hat{s}_2 (T_2 - T_1)} - e^{-\hat{s}_1 (T_2 - T_1)} \right] = \frac{\alpha - \psi}{\beta} > 0.$ Hence,

$$\text{(21)} \qquad \frac{\hat{s}_1 \hat{s}_2}{\hat{s}_1 - \hat{s}_2} \left[e^{-\hat{s}_2 (T_2 - T_1)} - e^{-\hat{s}_1 (T_2 - T_1)} \right] = \frac{\alpha - \psi}{\beta} \left(\frac{\delta + \kappa / \rho}{\gamma - b + \kappa (E_0 + S_0) / \rho} \right)$$

The left-hand side is negative and decreasing in $T_2 - T_1$. The partial derivative of the right-hand side with respect to ψ taking account of $b = \frac{\alpha \pi + \beta \psi}{\beta + \pi}$ is equal to the sign of $\alpha - \gamma - \kappa(\mathrm{E}_0 + S_0) / \rho,$ which is positive as we assume that the choke price exceeds the social cost of extracting the first drop of oil. The right-hand side is decreasing in γ and E_0+S_0. Hence, the duration of the phase of simultaneous use of fossil fuel and the backstop is high if fossil fuel extraction is expensive (γ high), the initial stock of fossil fuel, S_0, and the initial atmospheric CO2 concentration, E_0, are high, and the marginal cost of renewables is low (ψ low). The boundary condition

$$q(T_1) = -\dot{S}(T_1) = (s_1 - s_2) \left[\frac{(S_0 + \Gamma) e^{s_2 T}}{1 - e^{(s_2 - s_1)T}} \right] - \left[\frac{\left(s_1 + s_2 e^{(s_2 - s_1)T} \right) \left(S_{T_1} + \Gamma \right)}{1 - e^{(s_2 - s_1)T}} \right] = \frac{\alpha - \psi}{\beta} \quad \text{where}$$

$\Gamma \equiv \frac{\alpha - \gamma - \kappa (E_0 + S_0) / \rho}{\delta + \kappa / \rho}$ (from (A1)) together with the final condition on $S(T_1)$ from (19′) allows one to calculate the values of $S(T_1)$ and T_1 and thus obtain the full solution for the three phases.

In order to get better intuition of the market outcome, we suppose that private agents do not internalize global warming externalities and therefore: set $\kappa = 0$. Setting the other parameters to α = 200, β = 1, γ = 100, δ = 1, ρ = 6, ψ = 50 or 45, π = 2, E_0 = 0 and S_0 = 100, we get *b* = 150 or 148.3 and $\overline{S}$ =–50 or –48.3.[23] Fig. 7 then gives the solution paths with the horizontal axes giving time as 0.66+*t*/400.

We observe that a backstop subsidy (lower ψ) induces a higher value of fossil fuel use at the end of the first phase, but during the second phase fossil fuel is phased out more quickly. There is a quicker and more aggressive phasing in of renewables (at date 4.95 rather than 8.63). The second phase of simultaneous use of fossil fuel and renewables lasts longer (14.2 rather than 13.2 periods), which boosts green welfare. The date of exhaustion of fossil fuel reserves is brought forward (from date 21.88 to

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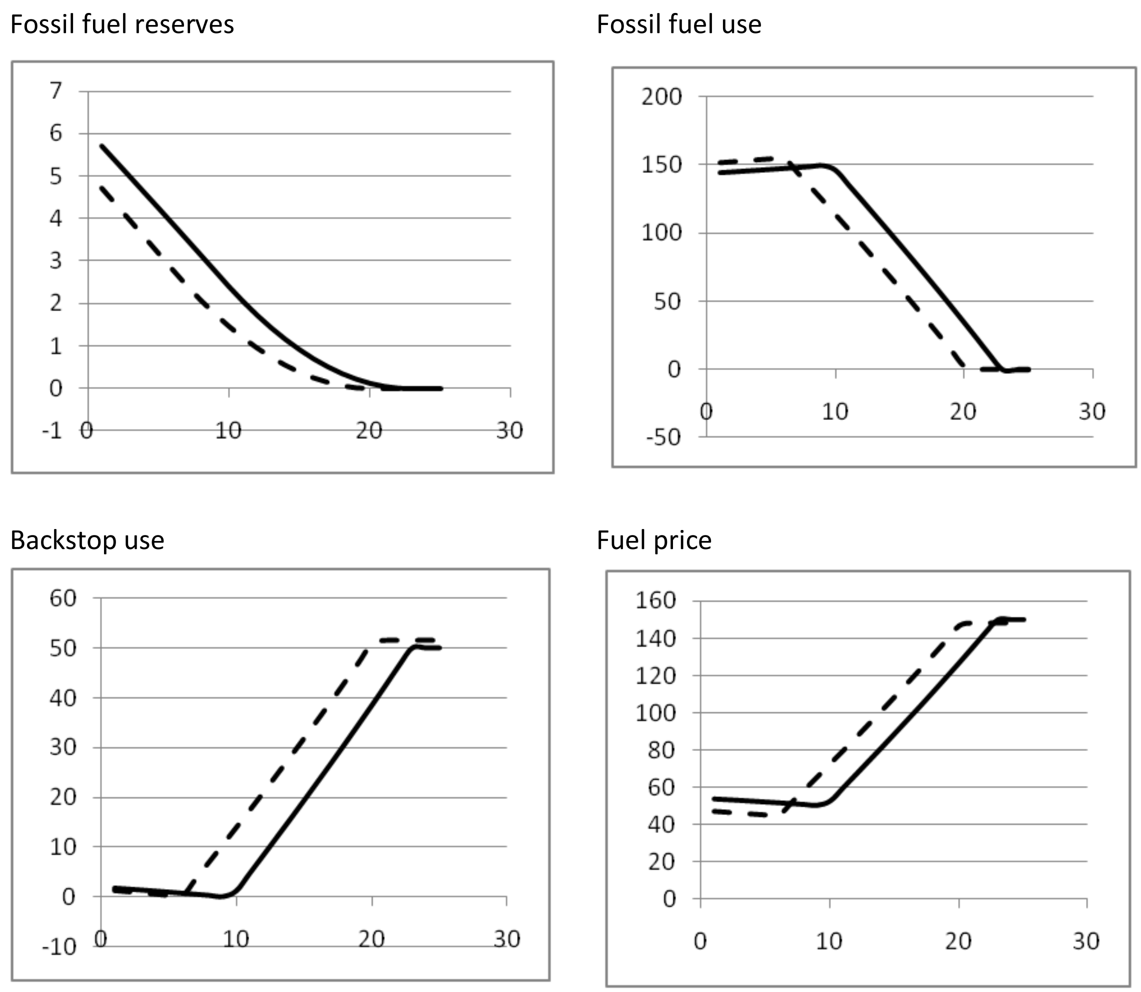
[23] Again, we chose ρ such that for S_0 = 97 the economy is exactly on the boundary where T_1 = 0. Raising S_0 a little puts the economy into regime IV whilst lowering it a little puts it into regime II.

19.18), which is bad for green welfare. However, as can be seen from the first panel of fig. 7, fossil fuel reserves are lower throughout the whole trajectory and therefore the second effect dominates the effect of a quicker and more aggressive phasing in of renewables. This result holds whenever the cost of using only the backstop, b, is bigger than the cost of extracting the drop of fossil fuel, γ. Fuel prices are lower during the initial phase where only fossil fuel is used, but higher during the phase of simultaneous use.

Figure 7: Full exhaustion with three phases of fuel, fuel/backstop and backstop use



The characterization of the four regimes and the effects of reducing the cost of the backstop in each of these regimes are summarized in Table 1. We conclude that a lower backstop cost is beneficial for green welfare in regimes I and II, but may be detrimental in regime III, whereas it is likely to be detrimental as well in regime IV where full exhaustion takes place in finite time and there is an initial phase where only fossil fuel is used it will definitely occur. Sadly, this fourth regime is the one which is most likely to prevail in market economies where the costs of global warming are not internalized by an appropriate carbon tax.

Table 1: Effects of lower marginal cost of renewable backstop

	Asymptotic partial exhaustion: $b < \gamma + \kappa(E_0 + S_0)/\rho, \bar{S} > 0$	Full exhaustion in finite time: $b > \gamma + \kappa(E_0 + S_0)/\rho, \bar{S} < 0$
Backstop kicks in immediately: low initial fossil fuel reserves	I. More fossil fuel is left in situ. Extraction speed unaffected.	II. Postpones exhaustion of fossil fuel. More aggressive phasing in of renewables.
First initial phase with only fossil fuel: high initial fossil fuel reserves	III. Initially more rapid fossil fuel extraction, but in the long run more fossil fuel is left in situ. Renewables are phased in more quickly (and more aggressively) at which point fossil fuel extraction speeds up.	IV. Initially more fossil fuel is left in the ground. Backstop is introduced more quickly and the simultaneous phase lasts longer. Fossil fuels are exhausted more quickly.

6. Conclusions

We show that a smaller initial stock of fossil fuel reserves, a positive shock to demand for energy fuels, and a lower cost of extracting fossil fuels, mean that fossil fuels are more rapidly exhausted in an economy with a clean backstop, having constant marginal cost. We also show that, if the atmosphere has already been polluted with a lot of CO2 emissions, it is socially optimal to postpone depletion of oil and gas in order to combat global warming. A market economy does not internalize the cost of global warming and thus depletes oil too fast. An anticipated reduction in the cost of renewables leads initially to fossil fuel being pumped up more quickly. The increased use of the backstop implies that just before they are phased in, fossil fuel use must rise as well. In this sense, the Green Paradox occurs always. However, global warming damages increase and green welfare falls, if there is full exhaustion of fossil fuel reserves, which happens if the cost of the backstop (e.g., solar or wind energy) is higher than the cost of extracting the last drop of fossil fuel including the social cost of carbon. Typically, this effect does not occur if it is optimal to only partially exhaust fossil fuel, which happens if the social cost of the renewables is low enough. We thus conclude that a worsening of green welfare is more likely to occur in the market economy, because then the social cost of carbon is not internalized and full exhaustion is more likely.

Hence, if, following Sinn, we suppose that a Hotelling ramp for taxes on C02 emissions is politically infeasible, the government might resort to subsidizing solar or wind energy, as is done on a large scale in

Germany, in an attempt to reduce greenhouse gas emissions. If, oil can be supplied at low cost, depletion of oil and gas will occur more rapidly and climate change damages increase. We have shown that in these circumstances also total welfare might decrease. If the concern for the environment is substantial, it is better to tax the clean backstop in order to postpone exhaustion. However, if a substantial subsidy renders the clean backstop cheaper than fossil fuel, total welfare will be enhanced if the concern for the environment is large enough. Therefore, to evaluate green policy measures a broad welfare perspective should be taken. However, Sheik Ahmed Zaki Yamani, the colourful former Saudi oil minister, has been quoted in the New York Times as saying "The Stone Age came to an end not for a lack of stones and the Oil Age will end, but not for lack of oil". It is this insight which lies at the heart of our critique of the Green Paradox. Fossil fuel will not and should perhaps not last forever if cheap and clean alternatives are available or become available in the future. We show that if the backstop is relatively cheap and low on C02 emissions compared to oil and gas, subsidizing the backstop leads to a bigger final in situ stock of oil and gas reserves and to a higher rate of extraction of oil and gas at the time that the economy switches to using the backstop. Subsidizing the backstop leads to less extraction so that not all oil and gas reserves will be extracted from the earth. Climate damages will now be less.

If the non-renewable resource is owned by a monopolist, limit pricing will occur. Moreover, due to our assumption of linear demand, initial monopolistic extraction is smaller than under perfect competition. The (weak) Green Paradox prevails but the welfare effects of a lower backstop cost depend on the backstop cost. Green welfare falls if the backstop cost is high compared with the initial marginal cost of extracting oil. Else, the result depends on other parameters of the model. This poses an interesting area for further research.

We offer also insights with increasing marginal cost of supplying the backstop. In that case, the optimal path is characterized by a first phase of only fossil fuels, a second phase with simultaneous use of fossil fuels and the backstop, and a third phase with only use of the backstop (where the first and third phases may be degenerate). Even if it is optimal to fully exhaust fossil fuel reserves, the Green Paradox need no longer hold provided renewable are already being used alongside fossil fuels as lowering the backstop cost will either postpone exhaustion of fossil fuels or lead to more fossil fuels to be left in situ. If renewables are not being used yet, a backstop subsidy will bring in the backstop more quickly alongside fossil fuels and for a longer period; but during the phase that only fossil fuels are being used, fossil fuel extraction will be higher. If some fossil fuel reserves are left in situ, we can therefore not say whether green welfare will fall or rise. But in the more likely market outcome where global warming externalities are not internalized and all fossil fuel reserves are fully exhausted in finite time, a backstop subsidy definitely reduces green welfare in line with the Green Paradox.

It may be worthwhile to extend our analysis in the following directions. First, it may be of interest to allow for imperfect substitution in the demand for the non-renewable and the backstop energy source. This may arise from concerns with security of energy supplies, diversification and/or intermittence of backstops such as wind and solar energy and will lead to the simultaneous use of both the non-renewable and the backstop. Second, it is important to investigate what happens if there are various types of backstop available at the same time. If it is possible to rank them, e.g., clean but competitive (nuclear), clean and expensive (wind, solar, advanced nuclear) and dirty and expensive (tar sands), it is best to go for the cleanest and cheapest backstop. However, with dirty and cheap backstops, matters are more complicated especially if we allow for upward-sloping supply schedules of the backstop. Third, given that once non-renewables are exhausted, it becomes attractive to abolish the tax on the backstop and therefore it is of interest to investigate credibility aspects of optimal climate change policies. Fourth, the analysis could be extended to an international context by analyzing issues of carbon leakage and ways to sustain international cooperation (see Hoel, 2008; Eichner and Pethig, 2009, 2010). Fifth, one could investigate the issues we addressed in this paper within the context of a Ramsey model with capital formation and pollution. Finally, one could use the analysis to empirically investigate the various policies that can be used to combat global warming.

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Appendix

Proof of proposition 1: Assume that there is simultaneous use of fossil fuel and the backstop. Equations (3a) and (3b) give $U'(q(t)+x(t)) = b = \eta(t) + G(S(t))$. Therefore $\dot{\eta}(t) = G'(S(t))q(t) < 0.$ But equation (3c) implies that $\dot{\eta}(t) = -G'(S(t))\dot{S}(t) = \rho\eta(t) + G'(S(t)q(t) - D'(E(t))$ or $\eta(t) = D'(E(t))/\rho.$ Hence, $\dot{\eta}(t) = D''(E(t))q(t)/\rho > 0.$ This is a contradiction. So at any instant of time t, $q(t)$ and $x(t)$ cannot both be positive . Furthermore, note that a transition from the backstop to the fossil fuel cannot take place. Once the backstop is in use, the state of the system no longer changes and there is no reason to fall back on fossil fuel. Also, note that as long as extraction takes place we have from equation (3a) $q(t) \geq \bar{x}$. Hence, as long as there is resource extraction, it is bounded from below by a positive constant, implying from the limited availability of the resource that extraction will come to an end within finite time, say at *T*. The above establishes equation (4) as well. Q.E.D.

Proof of proposition 2: To start with, let us assume that $S(T) > 0$. Then, we find from (3c) that $\eta(t) = D'(E(T))/\rho + (\eta(T) - D'(E(T))/\rho)e^{\rho(t-T)}$. Therefore, from the transversality condition (3d), $\lim_{t\to\infty} e^{-\rho t}\eta(t)S(t) = (\eta(T) - D'(E(T))/\rho)S(T)e^{-\rho T}$ implying $\eta(T) = D'(E(T))/\rho$. Using (3a) and (3b) we arrive at $G(S(T)) + D'(E_0 + S_0 - S(T))/\rho = b$. Hence, if this equation has a positive solution $\bar{S}$ smaller than S_0 it is optimal to leave this amount unexploited. The conditions of the proposition identify all three possible cases. Q.E.D.

Proof of proposition 3: For $t \in [0, T)$ we get from the first-order conditions (3a)-(3d) the following differential equation:

$$\rho\left[U'(-\dot{S}(t)) - G(S(t))\right] + U''(-\dot{S}(t))\ddot{S}(t) = D'(E_0 + S_0 - S(t)),$$

where the boundary conditions are $S(0) = S_0$ and $q(T) = -\dot{S}(T) = \bar{x}.$ With our specific functional forms the equation becomes $\beta\ddot{S} - \rho\beta\dot{S} - (\rho\delta+\kappa)S = \rho(\alpha-\gamma) - \kappa(E_0 + S_0).$ The solution for $t \leq T$ is given by $S(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} - \Gamma,$ where $\Gamma \equiv \dfrac{\alpha - \gamma - \kappa(E_0 + S_0)/\rho}{\delta + \kappa/\rho}.$ The characteristic equation $\beta s^2 - \rho\beta s - (\rho\delta+\kappa) = 0$ gives the roots $s_1 = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 + 4(\rho\delta+\kappa)/\beta} > \rho$ and $s_2 = \rho - s_1 < 0$. Using $K_1 + K_2 - \Gamma = S_0$ and $K_1 \exp(s_1 T) + K_2 \exp(s_2 T) - \Gamma = S(T)$, we solve for K_1 and K_2 and obtain for $t \in [0, T]$:

$$\text{(A1)} \qquad S(t) = \left[\frac{\left(S(T)+\Gamma\right)e^{-s_1 T} - (S_0+\Gamma)e^{(s_2-s_1)T}}{1-e^{(s_2-s_1)T}}\right]e^{s_1 t} + \left[\frac{S_0 + \Gamma - \left(S(T)+\Gamma\right)e^{-s_1 T}}{1-e^{(s_2-s_1)T}}\right]e^{s_2 t} - \Gamma.$$

Since $q(T) = -\dot{S}(T) = -s_1 K_1 e^{s_1 T} - s_2 K_2 e^{s_2 T} = \bar{x},$ we have:

$$\text{(A2)} \qquad \frac{\alpha - b}{\beta} = \frac{(S_0 + \Gamma)(s_1 - s_2)e^{s_2 T} + (S(T) + \Gamma)\left[s_2 e^{(s_2 - s_1)T} - s_1\right]}{1 - e^{(s_2 - s_1)T}}.$$

With $s_1 > \rho > 0$ and $s_2 < 0$ the denominator of the right-hand side is strictly positive and increasing in T. The derivative of the numerator with respect to T is $s_2(s_2 - s_1)e^{s_2 T}\left[S(T)e^{-s_1 T} - S_0 - \Gamma(1 - e^{-s_1 T})\right]$. This expression is negative since $\Gamma > 0$ by assumption and $S(T) < S_0$. Hence, the numerator is decreasing in T. So, the right-hand side of (A2) is decreasing in *T*. Equation (A2) can be solved to give the transition date as a function of the primitive parameters: $T = \mathrm{T}(S_0, E_0, b, \alpha, \gamma, \rho, \delta, \beta, \kappa)$. The signs of the partial derivatives with respect to ρ, δ, β and κ are difficult to determine, since they operate both via the eigenvalues s_1 and s_2 and via Γ, but the other partial derivatives can be signed as follows. We always have $S_0 + \Gamma \equiv \dfrac{\alpha - \gamma - \kappa E_0 / \rho + \delta S_0}{\delta + \kappa / \rho}$.

For $b \geq \gamma + \kappa(E_0 + S_0)/\rho,$ we have $S(T) = 0$. Hence, $S(T) + \Gamma \equiv \dfrac{\alpha - \gamma - \kappa E_0 / \rho - \kappa S_0 / \rho}{\delta + \kappa / \rho}$. Using this expression and the one for $S_0 + \Gamma$ in (A2) yields:

$$\text{(A3)} \qquad \frac{\alpha - b}{\beta} = \frac{\alpha - \gamma - \kappa E_0/\rho}{\delta + \kappa/\rho}\left(\frac{(s_1 - s_2)e^{s_2 T} + s_2 e^{(s_2 - s_1)T} - s_1}{1 - e^{(s_2 - s_1)T}}\right) + \frac{\delta S_0}{\delta + \kappa/\rho}\left(\frac{(s_1 - s_2)e^{s_2 T}}{1 - e^{(s_2 - s_1)T}}\right) - \frac{\kappa S_0/\rho}{\delta + \kappa/\rho}\left(\frac{s_2 e^{(s_2 - s_1)T} - s_1}{1 - e^{(s_2 - s_1)T}}\right).$$

It is easily seen that the second factor of the first term on the right-hand side is negative, so that the right-hand side decreases in α and increases in γ, E_0 and S_0. Given that the right-hand side of (A2) and thus of (A3) is decreasing in T, we immediately see that an increase in α, a decrease in γ, a decrease in E_0, a decrease in S_0 and a decrease in b lead to a smaller T. Obviously, we deal with marginal changes so that the inequality $b \geq \gamma + \kappa(E_0 + S_0)/\rho$ is kept intact.

For $b < \gamma + \kappa(E_0 + S_0)/\rho,$ we have partial exhaustion with $S(T) = \bar{S}$ from (7). Hence, $S(T) + \Gamma = \dfrac{\alpha - b}{\delta + \kappa/\rho}$.

Using this expression and the one for $S_0 + \Gamma$ in (A2) gives:

$$\text{(A3}') \qquad \frac{\alpha - b}{\beta} = \frac{\alpha - \gamma - \kappa E_0/\rho + \delta S_0}{\delta + \kappa/\rho}\left(\frac{(s_1 - s_2)e^{s_2 T}}{1 - e^{(s_2 - s_1)T}}\right) + \frac{\alpha - b}{\delta + \kappa/\rho}\left(\frac{s_2 e^{(s_2 - s_1)T} - s_1}{1 - e^{(s_2 - s_1)T}}\right).$$

As in (A3), α appears with a negative sign on the right-hand side of (A3′). The signs for γ and E_0 are negative now, whereas those for b and S_0 are positive. Hence, *T* decreases with an increase in α, γ or E_0 and a decrease in b or S_0. Q.E.D.

Proof of proposition 4:

The marginal effect of the cost of the backstop on green welfare reads:

$$\text{(A4)} \qquad \frac{\partial \Lambda}{\partial b} = \int_0^T e^{-\rho t} D'(E(t)) \frac{\partial S(t)}{\partial b} dt + \frac{D'(E(T))e^{-\rho T}}{\rho} \frac{\partial S(T)}{\partial b}.$$

Note that the dependence of *T* on *b* does not appear, since the derivative of Λ with respect to *T* is zero. From here the proof proceeds in three steps. We first show that the extraction paths of fossil fuel corresponding with different backstop costs will not cross. Then we turn to the two cases mentioned in the proposition.

1. No crossing

Starting from backstop cost b and the corresponding optimum, consider a deviation from b given by b^*. Denote the corresponding optimum by asterisks. Suppose there exists $t^* > 0$ such that $q^*(t) < q(t)$ for all $t \in [0, t^*)$, $q^*(t^*) = q(t^*)$, and $q^*(t) \geq q(t)$ for all $t \in [t^*, \infty)$. Then $S^*(t^*) > S(t^*)$, $G(S^*(t^*)) < G(S(t^*))$ and $D'(E^*(t^*)) < D'(E(t^*))$. It follows from $U'(q^*(t^*)) = \eta^*(t^*) + G(S^*(t^*)) = U'(q(t^*)) = \eta(t^*) + G(S(t^*))$ that $\eta^*(t^*) > \eta(t^*)$. Then

(A5) $$\dot{q}^*(t^*) = \frac{\rho\eta^*(t^*) - D'(E^*(t^*))}{U''(q^*(t^*)} < \frac{\rho\eta(t^*) - D'(E(t^*))}{U''(q(t^*)} = \dot{q}(t^*) < 0.$$

But, since q^* crosses q from below, we have $\dot{q}^*(t^*) \geq \dot{q}(t^*)$. Hence, we obtain a contradiction. The paths of q and q^* thus do not cross.

2. Suppose $b > G(0) + D'(E_0 + S_0)/\rho$. Take some b^* satisfying $b > b^* > G(0) + D'(E_0 + S_0)/\rho$ and denote the corresponding socially optimal values by asterisks. Then fossil fuel reserves get fully exhausted, $S(T) = S^*(T^*) = 0$. This means that the second term in (A4) vanishes. Moreover, $q(T) = \bar{x} < \bar{x}^* = q^*(T^*)$. It must be the case that $q^*(t) > q(t)$ for all $t \leq T^*$. Indeed, if $q^*(t) < q(t)$ for some initial interval of time, then, in view of the requirement that all fossil fuel is exhausted, there must occur crossing, which has been ruled out above. Hence, $S^*(t) < S(t)$ for all $0 < t \leq T$ and $S^*(t) = S(t) = 0$ for all $t \geq T$ where $T > T^*$. The conclusion is that $\partial S(t)/\partial b > 0$ for all $t \leq T$, so $\partial \Lambda / \partial b > 0$.

3. Suppose $b < G(0) + D'(E_0 + S_0)/\rho$. Take some b^* satisfying $b > b^* > G(S_0) + D'(E_0)/\rho$ and denote the corresponding socially optimal values by asterisks. Reserves do not get fully exhausted, $S(T) > 0$. It follows from proposition 2 that a lower backstop price increases the final stock. Since, at the transition to the backstop, the rate of extraction is higher at b^* we have, due to the no crossing property, that initial extraction is higher. Q.E.D.

Proof of proposition 5: Comparing the optimality conditions for the market economy, (3a′)-(3d′), with those of the social optimum, (3a)-(3d), and using $U'(q + x) = p$, we see that to replicate the social optimum the CO2 tax at time t must equal $\tau(t) = \eta(t) - \omega(t)$ and $\sigma(t) = 0$ with revenue rebated in lump-sum fashion. Using this in equation (3a′), we get $\dot{\eta} - \dot{\tau} = \rho(\eta - \tau) + G'(S)q$. Substituting equation (3a), we obtain $\dot{\tau}/\tau = \rho - D'(E)/\tau < \rho$. Note that $\dot{\tau} > 0$, because otherwise τ would eventually become negative. Q.E.D.

Proof of proposition 6: Replace b by $b - \sigma$ and set $\kappa = 0$ in propositions 1, 2 and 3. Decentralization of the solutions is straightforward. Q.E.D.

Proof of proposition 7: We fix all parameters except b and write all variables in the competitive economy as functions of this b. We introduce a subsidy $\sigma > 0$ and define $b^* \equiv b - \sigma$. We also use κ, a pivotal parameter indicating the severity of damage. We decompose social welfare in the competitive economy into three parts: the private component of social welfare

(A6a) $$V(b) \equiv \int_0^\infty e^{-\rho t}\left[U(q(t;b) + x(t;b)) - G(S(t;b))q(t;b) - bx(t;b)\right]dt,$$

green welfare

(A6b) $$\Lambda(b) = -\int_0^\infty e^{-\rho t}\kappa D(E(t;b))dt,$$

and subsidies,

(A6c) $R(b-b^{*}) \equiv \int_{T(b^{*})}^{\infty} e^{-\rho t}(b-b^{*})\overline{x}^{*}dt = \frac{(b-b^{*})\overline{x}^{*}}{\rho} e^{-\rho T(b^{*})}$ with $U'(\overline{x}^{*}) = b^{*}$.

Suppose first that $G(0) > b$. Denote the total welfare difference by $\Delta(b, b^{*}, \kappa)$. Clearly, $\Delta(b, b^{*}, 0) < 0$ because without global warming externalities the market economy is socially optimal. We also observe that, for any given *b* and *b**, $\Delta(b, b^{*}, \kappa)$ is monotonic in *κ*. In the case at hand, $\Lambda(b^{*}) > \Lambda(b)$ by assumption. Hence, there exists a critical $\hat{\kappa}(b, b^{*})$ such that for $\kappa < \hat{\kappa}(b, b^{*})$ no subsidy should be given and for $\kappa > \hat{\kappa}(b, b^{*})$ a subsidy enhances social welfare.

Now assume $b > G(0)$, implying full exhaustion. If the backstop is subsidized in such a way that $b - \sigma > G(0)$ then, according to proposition 6, exhaustion will take place earlier, which is bad for social welfare if κ is large enough. Thus one should tax the backstop implying later exhaustion. Q.E.D.

Proof of proposition 8: The current value Hamiltonian reads $[\alpha - \beta q - \gamma + \delta S]q - \lambda q.$ A necessary condition for optimality is $\alpha - 2\beta q(t) - \gamma + \delta S(t) = \lambda(t)$ as long as $q(t) > 0$, $b > \gamma$ and $\alpha - \beta q(t) < b,$ where $\dot{\lambda}(t) = \rho\lambda(t) - \delta q(t).$ Moreover, at the time *T* when extraction comes to an end, the Hamiltonian vanishes:

(A7) $H(T) = [\alpha - \beta q(T) - \gamma + \delta S(T)]q(T) - \lambda(T)q(T) = 0.$

We consider first the case where $b > \gamma.$ Fossil fuels are then fully exhausted. Hence, at some instant of time *T* we have $S(T) = 0$ and $p(T) = \alpha - \beta q(T) = b$. Moreover, from $H(T) = 0$ we then have $\lambda(T) = b - \gamma$. If the solution would be interior ($q(t) > 0$ and $\alpha - \beta q(t) < b$) until exhaustion, meaning no limit pricing, we have $\lim_{t\to T} \lambda(t) = \lim_{t\to T} \alpha - 2\beta q(t) - \gamma + \delta S(t) = 2b - \alpha - \gamma.$ However, this contradicts $\lambda(T) = b - \gamma.$ Therefore, there must be a phase with limit pricing. Hence, there exist $0 < T_1 < T_2$ such that for $0 \le t \le T_1$ we have $p(t) < b,$ and for $T_1 \le t \le T_2$ we have $p(t) = b.$ A marginal decrease of the backstop price results in a smaller shadow price λ. Indeed a smaller backstop price makes the constraint on the monopolist more stringent and thereby lowers the shadow price of the non-renewable resource. Consequently, extraction increases during the first phase as well as in the second phase. Therefore it takes a shorter period of time to exhaust the resource. This result obtains a fortiori if limit pricing occurs from the outset.

Now consider the second case where $b < \gamma$. To have an interesting problem, suppose $\gamma - \delta S_0 < b < \gamma.$ Otherwise, extraction will never take place. At the time where the monopolist leaves the market (*T*), the price must equal the backstop price. Hence, $q(T) = (\alpha - b)/\beta.$ Moreover, it should not be profitable to extract anymore, $\alpha - \beta(\alpha - b)/\beta - \gamma + \delta S(T) = 0.$ Hence, $S(T) = (\gamma - b)/\delta.$ It follows, as before, that there is a phase with limit pricing. Regarding the green welfare, two countervailing effects are at work. On the one hand, a smaller backstop price increases the remaining stock of fossil fuels kept in situ, which constitutes a positive effect on green welfare. On the other hand, it increases the final extraction rate and thereby all extraction rates during the regime of limit pricing and of the extraction rates before limit pricing starts. This can only happen if non-renewables are taken out of exploitation earlier than before. Hence, initially extraction becomes larger, but lasts shorter than before. Q.E.D.

Proof of proposition 9: Clearly, once only the backstop is used, this will remain so. Moreover, then it follows from (3d″) that $x(t) = \overline{x}$. A transition from simultaneous use to use of only fossil fuel is ruled out by the following argument. Along an interval of simultaneous use we have from (3a″) and (3c″) that $(\dot{q} + \dot{x})U'' = \rho\eta - D'(E).$ The right-hand side of this expression is positive since otherwise η becomes negative eventually, which is not allowed.

Hence $q+x$ is decreasing. It then follows from (3d″) that q is decreasing and x increasing. A transition to only fossil fuel use then requires a downward jump in the use of the backstop and an upward jump in the use of oil. But (3c″) implies continuity of q+x whereas (3d″) requires an upward jump in q+x. Finally, there will never be a transition from only fossil fuel to only the backstop. To see this, suppose that a transition takes place at some instant of time t_1. Right before the transition we have $U'(q) \leq B'(0)$ and right after $U'(x) = B'(x)$. Again, continuity is violated. So, a generic sequence reads: $q > 0 \rightarrow (q > 0, x > 0) \rightarrow x > 0$ with transition dates T_1 and T_2 respectively. We could have $T_1 = 0$ and $T_2 = \infty$. But $x(T_1) = 0$ if $T_1 > 0$ and $q(T_2) = 0$ if $T_2 < \infty$, from continuity. If $b > G(0) + D'(E_0 + S_0)/\rho$ then $S(T_2) = 0$. Otherwise, it would pay to continue using fossil fuel and reduce the use of the backstop marginally. If $b < G(0) + D'(E_0 + S_0)/\rho$ then $S(T_2) > 0$. Q.E.D.

Proof of proposition 10: Along an interval of time with simultaneous use $\alpha - \beta(q(t) + x(t)) = \psi + \pi x(t)$. We thus get $\hat{\beta}\ddot{S} - \rho\hat{\beta}\dot{S} - (\delta\rho + \kappa)S = \rho(b-\gamma) - \kappa(E_0 + S_0)$. The solution is given by (19). Under the conditions mentioned under case I, we have a solution satisfying all the necessary conditions. Indeed, we can take $\hat{K}_1 = 0$. Then $S(t) \rightarrow \bar{S} > 0$ as $t \rightarrow \infty$. We must then have $\hat{K}_2 = S_0 - \bar{S}$. Moreover, $q(0) = -\hat{s}_2\hat{K}_2 \leq (\alpha - \psi)/\beta$ and therefore $x(0) > 0$. Hence, this is an optimum. The same holds for the conditions mentioned under case II. It cannot be optimal to start with simultaneous use and then the backstop taking over completely within finite time at a positive stock level. If this would occur, then at the transition we should have $b = \gamma - \delta\bar{S} + \kappa(E_0 + S_0 - \bar{S})/\rho$. Therefore, the solution of the differential equation gives $\bar{S} = \hat{K}_1 e^{\hat{s}_1 T_2} + \hat{K}_2 e^{\hat{s}_2 T_2} + \bar{S}$ but this is at variance with $q(T_2) = -\hat{s}_1\hat{K}_1 e^{\hat{s}_1 T_2} - \hat{s}_2\hat{K}_2 e^{\hat{s}_2 T_2}$ unless $T_2 \rightarrow \infty$. Q.E.D.